ETHIO NATIONAL SCHOOL

MATHEMATICS
NATIONAL EXAM (EUEE) QUESTIONS
FROM 2004 – 2011 E.C

COMPILED BY MR. SHISHAY

From grade 11

2004

Unit 1 and 2

1. If $x < 0$, then the simplest form of $f(x) = \frac{x-|x|}{x}$ is equal to:
   A. 2x B. 2 C. -2 D. 0
2. If $f(x) = \frac{\sqrt{x+2}}{x+2}$ and $g(x) = \frac{1}{x} - 2$, then $f(g(x))$ is equal to?
   A. $\sqrt{x} - 2$ B. $\sqrt{x} + 2$ C. $\sqrt{x}$ D. $\frac{\sqrt{x}}{x}$
3. If $f(x) = \ln\left(\frac{x}{x-1} + 2\right)$, for x> 1, then which of the following is the inverse of f?
   A. $g(x) = \frac{e^x-2}{e^x-3}$ B. $g(x) = \frac{e^x-2}{e^x+1}$ C. $g(x) = \frac{e^x}{e^x+1} - 2$ D. $g(x) = e^{\frac{x}{x-1}-2}$
4. Which of the following is a simplified form of $\frac{4-(a-2)^2}{a^4+2a^2+1} \div \frac{1-\frac{1}{a^2+1}}{a}$?
   A. $\frac{4-a}{a}$ B. $-\frac{a}{a^2+1}$ C. $-\frac{1}{a^2(a^2+1)}$ D. $\frac{4-a}{a^2+1}$
5. If $(x) = \frac{x-1}{(x-2)^2(x+1)}$, which of the following is true about f?
   A. Its graph has an oblique asymptote. C. The graph of f does not meet its asymptote.
   B. As x$\longrightarrow -1^+, f(x) \longrightarrow -\infty$. D. As x$\longrightarrow 2^-, f(x) \longrightarrow -\infty$

2005

6. What is the value of $|x| + 2x$ if x< 0?
   A. $-3x$ B. 3x C. $-x$ D. x
7. Which of the following functions has **No** vertical asymptote?
   A. $f(x) = \ln(x+1)$ C. $f(x) = \frac{x^2-9}{x-3}$
   B. $f(x) = \frac{x^2+1}{x^3+8}$ D. $f(x) = \frac{x-1}{x^2-x}$
8. If $f(x) = \frac{1}{e^x+1}$, then which of the following is equal to $f^{-1}(x)$?
   A. $\ln(1-x) - \ln(x)$ C. $\ln\left(\frac{1}{x+1}\right)$
   B. $e^{-x} + 1$ D. $\frac{1}{e^{-x}+1}$
9. What is the solution set of $\frac{2}{x} - \frac{x-2}{x^2-2x} = 1 - \frac{2x-2}{3x-2}$?
   A. $\{1, -2\}$ B. $\{1, 2\}$ C. $\{-1\}$ D. $\{1\}$
10. Given $f(x) = \ln(x-1)$ and $g(x) = \sqrt{1-2x}$, which one of the following is the domain of fog?
    A. $\{x \in \Re: x > 1\}$ C. $\{x \in \Re: x < 0\}$
    B. $\left\{x \in \Re: x \leq \frac{1}{2}\right\}$ D. $\left\{x \in \Re: x > \frac{1}{2}\right\}$

2006

11. Which of the following is expression is a polynomial expression?
    A. $x^2 - 3x + \sin x$ C. $\frac{2+\pi}{1+\pi^2}$
    B. $\frac{4x^3+12x^2-x}{\pi x^2}$ D. $2 - 3x^{\frac{2}{3}} + 7x^{\frac{5}{2}} + 3x^{-1}$
12. If $f(x) = \frac{x+1}{x-1}$ and $f(a) = 5$, then $f(2a)$ is equal to:
    A. 2 B. 4 C. 6 D. 8
13. If $f(x) = \sqrt[3]{1+e^{-x}}$, which of the following is equal to $f^{-1}(x)$?
    A. $\ln\left(\frac{1}{x^3-1}\right)$ C. $\ln(1-x^3)$
    B. $\frac{1}{\ln(x^3-1)}$ D. $(1+e^{-x})^3$
14. Which one of the following is true?
    A. A polynomial can have infinitely many vertical asymptotes.
    B. The graph of a rational function can never cross its horizontal asymptote?

C. The graph of $f(x)=\frac{3x-1}{x-1}$ has no horizontal asymptote.

D. The graph of $f(x)=\frac{x^3-x}{x^2-x}$ has no vertical asymptote.

15. Which of the following is true about the graph of $f(x)=\frac{x^2-1}{x-x^2}$?

A. $x=0$ and $x=1$ are its vertical asymptotes
B. y= 1 is its horizontal asymptote.
C. y= $x-1$ is its oblique asymptote.
D. It is almost the same as the horizontal line y= -1 as x⟶ $\pm\infty$.

2007

16. Which one of the following functions is one-to-one correspondence?

A. $f:\Re'\to\Re, f(x)=\tan x$, where $\Re'$ is the domain of f.
B. $g:\Re\to\Re, g(x)=2^x$
C. $h:[0.\infty)\to[0,\infty), h(x)=x^2$
D. $r:[0,\infty)\to[0,\infty), r(x)=x+5$

17. The inverse of the function defined by $g(x)=\frac{2x}{x+3}$ is equal to:

A. $g^{-1}(x)=-\frac{2x}{x-3}$
B. $g^{-1}(x)=-\frac{3x}{x-2}$
C. $g^{-1}(x)=-\frac{x+3}{2x}$
D. $g^{-1}(x)=\frac{x+2}{3x}$

18. If $p(x)=3x^2$ and $q(x)=x^2+x$, then what is the solution set of $\frac{p(x)}{3q(x)}-\frac{1}{x}=\frac{1}{q(x)}$?

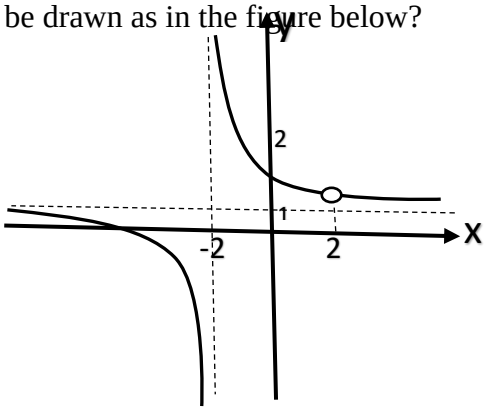
A. $\{-1,2\}$ B. $\{2\}$ C. $\{-3,2\}$ D. $\{-3\}$

19. The value(s) of x where the graph of the function $y=\frac{x^2-1}{x^3}$ crosses its horizontal asymptote is(are):

A. $x=-2$
B. $x=-1$ and $x=1$
C. $x=0$
D. $x=-\sqrt{2}$ and $x=1+\sqrt{2}$

20. Which of the following functions could most likely be drawn as in the figure below?

A. $f(x)=\frac{x+3}{x+2}$
B. $f(x)=\frac{x^2-2x}{x^2-4}$
C. $f(x)=\frac{-x^2-x+6}{x^2-4}$
D. $f(x)=\frac{x^2+x-6}{x^2-4}$



2008

21. Which of the following is one-to-one correspondence function from A= $[0,1]$ to B= $[1,2]$?

A. $f(x)=x$
B. $f(x)=\frac{1}{3}x^3+1$
C. $f(x)=2x+1$
D. $f(x)=x^2+1$

22. What is the solution set of $\frac{1-\frac{1}{x}}{1-\frac{1}{x^2}}=3x^2-\frac{x}{1+\frac{1}{x}}$?

A. $\left\{-1,\frac{1}{3}\right\}$ B. $\left\{\frac{1}{3}\right\}$ C. $\left\{3,\frac{-1}{3}\right\}$ D. $\left\{\frac{-1}{3}\right\}$

23. Suppose $f(x)=\frac{Q(x)}{x(x^2-1)}$ where Q(x) is a quadratic function. Which of the following is necessarily true about the graph of f?

A. x= $0, x=1$ and $x=-1$ are the vertical asymptotes of the graph of f.
B. The graph of f does not intersect with its horizontal asymptote.
C. The vertical asymptote of the graph of f is only $x=-1$ if Q(x)=x^2-x.
D. The vertical asymptote of the graph of f is only $x=1$ if Q(x)= $2x^2$.

24. Which one of the following is true about the horizontal asymptote(s) of the graphs of $y=\frac{|x|+2}{x}$?

A. $y=2$ is the only horizontal asymptote of the graph.

B. $y = 1$ and $y = -1$ are horizontal asymptotes of the graph.
C. $y = 2$ and $y = -2$ are horizontal asymptotes of the graph.
D. $y = 1$ is the only horizontal asymptote of the graph.

25. If $f: A \to B$ and $g: B \to C$ are functions, then which one of the following is true about the composition function?
A. Domain of $(gof) \subseteq$ Domain of f
B. Range of $(gof) \nsubseteq$ Range of g
C. Domain of $(gof) \nsubseteq$ Domain of f
D. Range of $(gof) \subseteq$ Range of f

26. If the point $(3, -2)$ is on the graph of $y = f(x)$, which point is on the graph of $y = f^{-1}(x)$?
A. $\left(\frac{1}{3}, -2\right)$ B. $(3, -1)$ C. $(-2, 3)$ D. $\left(3, -\frac{1}{2}\right)$

2009

27. Which one of the following is true about the graph of $f(x) = \frac{x^3 - x}{x^3(x-1)}$?
A. The vertical asymptotes of the graph are $x = 0$ and $x = 1$.
B. The horizontal asymptote of the graph is $y = 1$.
C. The graph intersects its horizontal asymptote at the point $(-1, 0)$.
D. The graph intersects the vertical line $x = 1$ at the point $(1, 2)$

28. What is the solution set of $\frac{1}{1+\frac{1}{x}} - \frac{1}{1-\frac{1}{x}} = \frac{x+\frac{1}{x}}{x-\frac{1}{x}}$?
A. $\{\}$ B. $\{-1\}$ C. $\{1\}$ D. $\{-1, 1\}$

29. Which of the following is the inverse of $f(x) = 8x^3 + 2$?
A. $f^{-1}(x) = \frac{1}{8x^3+2}$
B. $f^{-1}(x) = \frac{1}{2}\sqrt[3]{x-2}$
C. $f^{-1}(x) = 8x^{-3} - 2$
D. $f^{-1}(x) = \frac{1}{8}\sqrt[3]{x-2}$

30. If $f(x) = \sqrt{x^3}$ and $(fog)(x) = \sqrt[4]{x}$, then what is the value of $g(8)$?
A. $\sqrt[3]{2}$ B. 2 C. $\sqrt{2}$ D. $2\sqrt{2}$

31. Which of the following functions is one-to-one correspondence?
A. $f: [0, \infty) \to \Re$ defined by $f(x) = |x|$.
B. $f: \Re \to [0, \infty)$ defined by $f(x) = x^2$
C. $f: \Re \to [0, \infty)$ defined by $f(x) = 3^x$
D. $f: (0, \infty) \to \Re$ defined by $f(x) = \log_2 x$

2010

32. Which of the following is true about signum, absolute value and greatest integer functions?
A. $sgn(x) = \pm|x|$, for all x$\in \mathbb{R}$.
B. $sgn(x) \leq \lfloor x \rfloor$, for all x$\leq 0$
C. $|x| = xsgn(x)$, for all x$\in \mathbb{R}$.
D. $sgn(x) \leq \lfloor x \rfloor$, for all x$\geq 0$

33. What is the partial fraction decomposition of $\frac{x^2+x+1}{(x+2)(x^2+1)}$?
A. $\frac{3}{5(x+2)} + \frac{2x+1}{5(x^2+1)}$
B. $\frac{5}{3(x+2)} + \frac{2x+1}{3(x^2+1)}$
C. $\frac{2}{5(x+2)} + \frac{3x+1}{5(x^2+1)}$
D. $\frac{2}{3(x+2)} + \frac{2x+1}{3(x^2+1)}$

34. Which of the following is true about the graph of $f(x) = \frac{2x^3+2x^2+3x}{x^2+x}$?
A. The vertical asymptote of the graph is only $x = -1$ and its oblique asymptote is $y = 2x$.
B. The graph has y-intercept at (0, 3).
C. The graph has at least one x-intercept.
D. The vertical asymptotes of the graphs are at $x = 0$ and $x = -1$ but it has no vertical asymptote.

35. Let $f(x) = x - x^2$ and $g(x) = \frac{1}{x}$. Then $g(f(\frac{1}{x}))$ is equal to:
A. $x - x^2$ B. $\frac{x^2}{x-1}$ C. $\frac{1}{x^2-1}$ D. $\frac{x-1}{x^2}$

36. Let $f(x) = \frac{3x+1}{x-2}$. Then what is the range of $f(x)$?
A. $\mathbb{R}\backslash\{2\}$ B. $\mathbb{R}$ C. $\mathbb{R}\backslash\{3\}$ D. . $\mathbb{R}\backslash\left\{\frac{-1}{3}\right\}$

2011

37. Which of the following is equal to $f(x) = \sqrt{(x+4)^2}$, for every x∈ ℝ?
A. $g(x) = x + 4$ C. $g(x) = |x + 4|$
B. $g(x) = x + 2$ D. $g(x) = |x| + 4$

38. What is the simplified form of $\frac{a^{-1}b^{-1}}{a^{-3}-b^{-3}}$?
A. $\frac{a^2b^2}{b^2-a^2}$ C. $\frac{a^3-b^3}{a\,b}$
B. $\frac{a^2b^2}{b^3-a^3}$ D. $\frac{a^3-b^3}{a-b}$

39. If $f(x) = ax - b$ and $f^{-1}(x+1) = \frac{1}{2}x + 2$, for each x∈ ℜ, then what must be the value of a and b?
A. $a = \frac{1}{2}, b = -2$ C. $a = 1, b = 1$
B. $a = 2, b = 2$ D. $a = 2, b = 3$

40. Which of the following is true about the graph of $f(x) = \frac{x^2+5x+6}{x^2-4} + 3$?
A. The graph has a hole at $x = 2$.
B. The vertical asymptotes of the graph are $x = 2$ and $x = -2$.
C. The horizontal asymptote of the graph is $y = 4$.
D. The graph has y-intercept at $\left(0, -\frac{3}{2}\right)$.

41. If f is greatest integer function and g is absolute value function, then what is the value of $((fog)\left(\frac{3}{2}\right) + (gof)\left(-\frac{4}{3}\right)$?
A. 1 B. 3 C. -1 D. 2

Unit 3

2004

42. A line ℓ passes through (0, 5) and $(-5, 0)$. What is the acute angle between the y-axis and the line ℓ in radian measure?
A. $\frac{\pi}{4}$ B. $\frac{\pi}{3}$ C. $\frac{\pi}{2}$ D. $\frac{3\pi}{4}$

43. Consider a circle whose center is on the x –axis. If a line given by $y = x$ is tangent to the circle at the point (2, 2), what the equation of the circle?
A. $x^2 + y^2 = 8$ C. $(x-4)^2 + y^2 = 8$
B. $(x-2)^2 + y^2 = 4$ D. $(x-1)^2 + y^2 = 5$

44. What is the vertex and the equation of the directricx, respectively, of the parabola $x + y^2$+2y+1=0?
A. $(0, -1)$, x=$\frac{-1}{4}$ C. $(0, -1)$, x=$\frac{1}{4}$
B. $(-1, 0)$, y= $\frac{-1}{4}$ D. $(-1, 0)$, y= $\frac{1}{4}$

45. The orbit of Mercury around the sun forms an ellipse with eccentricity 0.206, length of major axis 1.16×10^8km and the sun at one focus. Which of the following is the best approximation of the maximum distance from Mercury to the sun?
A. 7.596×10^7km C. 8.695×10^7km
B. 5.695×10^7 km D. 6.995×10^7km

2005

46. If $x^2 - 6x + y^2 + k = 0$ is equation of a circle with radius 2, then what is the value of k?
A. 13 B. 5 C.4 D. -4

47. If a line with angle of inclination of $\frac{3\pi}{4}$ passes through (0. 1), which one of the following is the equation of the line?
A. $y = -x + 1$ B. $y = x + 1$ C.$y = -x - 1$ D. $y = x - 1$

48. A parabola with focus at (3, -1) has directrix $y = 3$. which one of the following is the equation of the parabola?
A. $(X-3)^2 = -4(Y+1)$ C. $(X-3)^2 = 4(Y+1)$
B. $(X-3)^2 = -8(Y-1)$ D. $(X-3)^2 = 8(Y-1)$

49. A satellite moves along a hyperbolic curve whose horizontal transverse axis is 24 km and an asymptote $y = \frac{5}{12}x + 2$. what is the eccentricity of the hyperbola?

A. $\frac{5}{13}$ B. $\frac{12}{13}$ C. $\frac{13}{12}$ D. $\frac{15}{3}$

50. For what value of b does the parabola $p(x) = ax^2 + x + b$ passes through the points $(-1, 5)$ and $(2, -1)$?

A. 9 B. 3 C. -3 D. -15

2006

51. What is the equation of the direcrix for the parabola whose equation is $y^2 + 8x + 6y + 25 = 0$?

A. $y = 3$ B. $x = 0$ C. $x = 2$ D. $x = 4$

52. If two lines $y = x$ *and* $y = x - 4$ are tangent to a circle at (2, 2) and (4, 0), respectively, then what is the equation of the circle?

A. $(x-2)^2 + y^2 = 4$ C. $(x-3)^2 + (y-1)^2 = 2$

B. $(x-4)^2 + (y-2)^2 = 4$ D. $(x-1)^2 + (y+1)^2 = 10$

53. A semi elliptical arc over a tunnel for a road through a mountain has a major axis of length 80 m and height of 30 m at the center. What is the equation of the semi-elliptical arc over the tunnel, if the center is considered as the origin?

A. $\frac{X^2}{6400} + \frac{Y^2}{900} = 1$ C. $\frac{X^2}{900} + \frac{Y^2}{6400} = 1$

B. $\frac{X^2}{1600} + \frac{Y^2}{900} = 1$ D. $\frac{X^2}{6400} + \frac{Y^2}{8100} = 1$

54. Let the equation $x^2 + 2x + y^2 = 8$ represents a circle. Then which one of the following lines cut the circle at exactly two points?

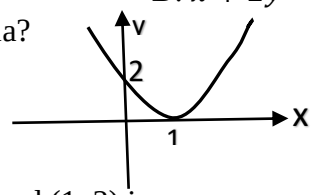
A. $4x + 3y + 19 = 0$ C. $2y = 5x + 43$

B. $3x + 4y + 14 = 0$ D. $2x = y - 50$

55. If the equation $(x-2)^2 - (y-2)^2 = 1$ represents a hyperbola, which one of the following represents the equation of an asymptote of the hyperbola?

A. $y = 4 - x$ B. $x + y = 1$ C. $x = 2 - y$ D. $x + 2y = 3$

56. Which of the equations below is represented by the following parabola?



A. $y = x^2 + 2$ C. $y = 2(x-1)^2$

B. $y = (2x-1)^2$ D. $y = (2x+1)^2$

2007

57. The equation of an ellipse with center at (1, 4) and vertices at (10, 4) and (1, 2) is :

A. $4(x-1)^2 + 81(y-4)^2 = 324$ C. $9(x-1)^2 + 4(y-4)^2 = 1$

B. $(x-1)^2 + 9(y-4)^2 = 4$ D. $2(x-1)^2 + 9(y-4)^2 = 4$

58. What is the focus of the parabola $y^2 + 4y + 8x = 4$?

A. $(1, -2)$ B. $(-1, -2)$ C. $(3, -2)$ D. $(-3, -2)$

59. Two perpendicular lines l_1 and l_2 are intersecting at (-1, 2). If the angle of inclination of l_1 is 45^0 then what is the equation of l_2?

A. $y = -x + 3$ B. $y = x + 3$ C. $y = -x + 1$ D. $y = x + 1$

60. Which of the following is true about a conic section represented by the equation $\frac{x^2}{k} + \frac{y^2}{k-9} = 1$?

A. it is a circle whose center is at the origin for some k∈R.

B. It is an ellipse whose major axis is vertical when k> 9.

C. it is a hyperbola whose foci are at (-3, 0) and (3, 0) when $0 < k < 9$.

D. It is a hyperbola whose foci are (-3k, 0) and (3k, 0) when $0 < k < 9$.

2008

61. The planet Mercury's orbit around the sun is an ellipse with eccentricity 0.206, length of major axis 11.6×10^8 km and the sun at one focus. What is the maximum distance from Mercury to the sun?

A. 6.99×10^8 . B. 6.99×10^7 C. 9.66×10^7 D. 9.66×10^8

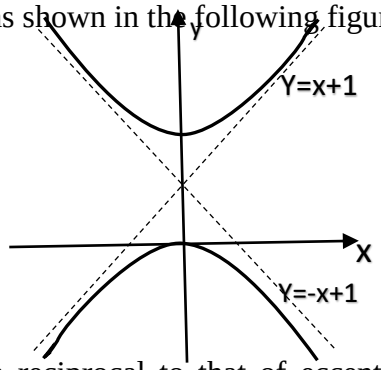
62. The equation of the line that passes through $(2, -1)$ and is perpendicular to $3x + 4y = 6$ is:

A. $-4x + 3y = 5$ C. $4x + 3y = 11$

B. $4x - 3y = 5$ D. $-4x + 3y = -11$

63. Which one of the following is the equation of a circle whose center is on the y-axis and radius 3?
A. $x^2 + y^2 + 6y = 0$ C. $x^2 + (y-2)^2 = 3$
B. $(x-2)^2 + y^2 = 9$ D. $x^2 - 2x + y^2 = 8$

64. The graph of a hyperbola and the lines of its asymptotes are as shown in the following figure. Which one of the following is an equation of the hyperbola?



A. $y^2 - 2y - x^2 = 0$
B. $y^2 - 3y - x^2 = 0$
C. $x^2 - (y-1)^2 = 1$
D. $(x-1)^2 - y^2 = 1$

2009

65. Suppose the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is a reciprocal to that of eccentricity of the ellipse $x^2 + 4y^2 = 4$. If the hyperbola passes through a focus of the ellipse, then what is the equation of the hyperbola?
A. $x^2 - 2y^2 = 2$ B. $x^2 - 3y^2 = 3$ C. $\frac{x^2}{3} - \frac{y^2}{2} = 1$ D. $\frac{x^2}{2} - \frac{y^2}{3} = 1$

66. What is the radius of the largest possible circle that can be inscribed in the ellipse given by $5(x-1)^2 + 3y^2 = 15$?
A. $\sqrt{3}$ B. $\sqrt{5}$ C. 3 D. 5

67. . What are the values of the center (C) and radius (r) of the circle $x^2 + y^2 - 4x + 6y = 5$?
A. $C = (-2, 3), r = 3\sqrt{2}$ C. $C = (2, -3), r = 2\sqrt{3}$
B. $C = (2, -3), r = 3\sqrt{2}$ D. $C = (-2, 3), r = 2\sqrt{3}$

68. . What is the equation of a line that passes through (a, a) in the xy-plane if it is parallel to a line that passes points through (a, b) and (b, a), where a≠b?
A. $y = x$ B. $y = -x$ C. $y = -x + 2a$ D. $y = 2x - a$

2010

69. What is the equation of the line that passes through (1, 1) and is parallel to $3y - x = 1$?
A. $x - 3y + 2 = 0$ B. $x + 3y = 4$ C. $3y - x + 2 = 0$ D. $3x - y = 2$

70. Which one of the following is the equation of the circle whose end points of a diameter are (0, -2) and (2, 2)?
A. $x^2 + y^2 = 4$ C. $(x-1)^2 + y^2 = 4$
B. $x^2 + y^2 - 2y - 4 = 0$ D. $x^2 + y^2 - 2x - 4 = 0$

71. . What is the area of the triangle in (sq. units) formed by the lines joining the vertex of the parabola $x^2 = -36y$ to the end points of the latus rectum?
A. 126 B. 261 C. 216 D. 162

72. A man running a race-course noted that the sum of the distances from the two flag posts from him is always 10 m and the distance between the flag posts is 8 m. what is the equation of the path traced by the man?
(Take the flag posts to be on the x-axis with the origin at their mid-point)
A. $\frac{X^2}{9} + \frac{Y^2}{25} = 1$ C. $\frac{X^2}{100} + \frac{Y^2}{64} = 1$
B. $\frac{X^2}{25} + \frac{Y^2}{9} = 1$ D. $\frac{X^2}{64} + \frac{Y^2}{100} = 1$

2011

73. Which one of the following is true about the pair of lines $3x + 9y - 24 = 0$ and $4x + 12y + 32 = 0$?
A. Perpendicular lines C. parallel and distinct lines
B. Intersecting lines D. representing the same lines

74. The center of a circle is on the line $y = 2x$ and the line $x = 1$ is tangent to the circle at the point (1, 6). How long is the radius of the circle?
A. 5 B. 4 C. 3 D. 2

75. If the circle passing through the point $(-1, 0)$ touches the y-axis at (0, 2), then what is the equation of the circle?

A. $x^2 + y^2 + 5x + 4y + 4 = 0$ C. $x^2 + y^2 - 5x - 4y + 4 = 0$

B. $x^2 + y^2 - 5x + 4y + 4 = 0$ D. $x^2 + y^2 + 5x - 4y + 4 = 0$

76. The earth's orbit has a semi-major axis $a \approx 149.6\ Gm$(gigameters) and an eccentricity of $e \approx 0.017$. What is the approximate value of the semi-minor axis?

A. 152.114Gm B. 145.32Gm C. 149.06Gm D. 149.58Gm

Unit 4

2004

77. Suppose the proposition $p \Rightarrow \neg q$ is false (F), which of the following is true?

A. $\neg q \wedge (p \Rightarrow q)$ C. $(\neg q \vee p) \Leftrightarrow q$

B. $\neg p \vee (q \Rightarrow \neg p)$ D. $(p \wedge q) \Leftrightarrow \neg q$

78. . Suppose e(x) ≡x is even; p(x) ≡x is prime; d(x)≡x is divisible by 2. Which one of the following has the truth value F on the set of natural numbers?

A. $(\exists x)[e(x) \wedge p(x)]$ C. $(\exists x)[e(x) \wedge \neg d(x)]$

B. $(\forall x)[e(x) \Rightarrow d(x)]$ D. $(\forall x)[e(x) \vee \neg d(x)]$

79. If x and y are non-negative integers, which of the following is **NOT** true?

A. $(\forall x)(\exists y)(y > x^2 - 1)$ C. $(\exists y)(\forall x)(y \leq x^2 - 1)$

B. $(\exists x)(\forall y)(y > x^2 - 1)$ D. $(\exists y)(\exists x)(y \leq x^2 - 1)$

80. . Consider the following argument form. Production is high if rain continues. Rain does not continue. Therefore, either production low or rain continues.

Let p: production is low

q: rain continues

The following table is also given about p and q.

row	P	Q	¬p	¬q	¬p⇒q	q⇒ ¬p	p∨q
1	T	T	F	F	T	F	T
2	T	F	F	T	T	T	T
3	F	T	T	F	T	T	T
4	F	F	T	T	F	T	F

Which of the following is necessarily true?

A. The argument form is valid due to row 2.

B. The argument form is valid due to row 2 and 3.

C. The argument form is invalid due to row 4.

D. The argument form is invalid due to row 1 and 3.

2005

81. For real numbers x and y, which one of the following statements is true?

A.$(\forall x)(\exists y)(x^2 + y + 1 = 0)$ C. $(\exists y)(\forall x)(x^2 + y + 1 = 0)$

B.$(\exists x)(\forall y)(x^2 + y + 1 = 0)$ D. $(\forall y)(\exists x)(x^2 + y + 1 = 0)$

82. Let p, q and r be propositions such that p ⇒$(r \vee \neg q)$ is false. Then which one of the following proposition is true?

A. P⇒r B. ¬r⇒q C. ¬p⇔q D. q⇔r

83. Consider the following argument:

"If he does not love her, she will not marry him. He loves her. Therefore, she will marry him."

If 'p≡ He loves her and 'q≡ she will marry him', which one of the following is correct representation of the argument form?

A. $\neg p \Rightarrow \neg q, p \vdash q$; valid argument
C. $p \Rightarrow q, p \vdash q$; valid argument
B. $\neg p \Rightarrow \neg q, p \vdash q$; invalid argument
D. $p \Rightarrow q, p \vdash q$; invalid argument

84. Suppose the following statements are the premises of an argument.
"He was lazy or he did not like the class room. If he was lazy, he could not pass the exam. He passed the exam."
Which one of the following could be a conclusion that makes the argument valid?
A. He did like the class room
C. If he was not lazy, he did like the class room
B. He did not like the class room.
D. He was not lazy and he did like the class room

2006

85. If $(p \vee q) \Rightarrow (\neg r \wedge r)$ is true, then which one of the following is necessarily true?
A. $(p \vee q) \Rightarrow q$ B. $\neg q \wedge r$ C. $\neg p \Leftrightarrow r$ D. $\neg p \vee r$

86. Which one of the following is a valid logical argument?
A. $p \Rightarrow q, q \vdash p$
C. $\neg p \wedge q, q \Rightarrow r \vdash r$
B. $p \Leftrightarrow q, p \Rightarrow q \vdash q$
D. $\neg p, p \vee q, r \Rightarrow q \vdash r$

87. Which one of the following is equivalent to $\neg[(\forall x)(p \Rightarrow q)]$?
A. $(\forall x)(\neg p(x) \Rightarrow \neg q(x))$
C. $(\exists x)(\neg p(x) \wedge q(x))$
B. $(\exists x)(\neg p(x) \Rightarrow \neg q(x))$
D. $(\exists x)(p(x) \wedge \neg q(x))$

2007

88. Suppose that p represents the statement "He missed the tournament." q represents the statement "He got the gold medal." And r represents the statement "He took a trip abroad." Then which of the following symbolic expression represents the statement: "If he takes a trip abroad and he does not miss the tournament, then he will get the gold medal."
A. $(r \Rightarrow q) \wedge \neg p$
C. $(r \wedge \neg p) \Rightarrow q$
B. $r \wedge (P \Rightarrow q)$
D. $\neg(r \vee p) \vee q$

89. For arbitrary propositions p and q, which one of the following is valid equivalence?
A. $\neg (p \Rightarrow q) \equiv (q \Rightarrow p)$
C. $(p \vee \neg q) \equiv (p \Rightarrow q)$
B. $[\neg (p \Rightarrow q) \wedge p)] \equiv (p \wedge \neg q)$
D. $[(p \vee q) \Rightarrow q] \equiv (p \Rightarrow \neg q)$

90. If each of the compound propositions $p \vee q, p \Rightarrow r$ and $\neg r$ is true, then which one of the following is True?
A. P B. q C. $q \Rightarrow p$ D. $p \wedge \neg r$

91. Which one of the following is not a tautology?
A. $[p \vee (q \Rightarrow r)] \Leftrightarrow [\neg p \Rightarrow (q \Rightarrow r)]$
C. $P \Rightarrow (P \Rightarrow q) \vee q$
B. $p \vee (q \Rightarrow \neg p)$
D. $[p \Leftrightarrow (q \wedge \neg r)] \Leftrightarrow [\neg p \Leftrightarrow (\neg q \vee r)]$

2008

92. Which one of the following compound proposition is tautology?
A. $(q \vee \neg q) \Rightarrow p$
C. $p \vee (q \wedge \neg q)$
B. $p \Rightarrow (q \vee \neg q)$
D. $p \Rightarrow (q \wedge \neg q)$

93. If the truth value of a proposition p is false, then which one of the following compound proposition has a truth value true?
A. $\neg p \wedge p$ B. $\neg p \Rightarrow P$ C. $\neg(\neg p \vee p)$ D. $p \Rightarrow \neg p$

94. What is the contra positive of "If $x \in \mathbb{N}$, then x is integer and x>0."?
A. If x is not integer or x<0, then $x \notin \mathbb{N}$.
C. If x is not integer or $x \leq 0$, then $x \notin \mathbb{N}$.
B. If x is integer and x>0, then $x \in \mathbb{N}$.
D If $x \notin \mathbb{N}$, then x is not integer and $x \leq 0$

95. The valid conclusion from the premises: $p \vee q,\ q \Rightarrow r,\ p \Rightarrow m, \neg m$ is……

A. $p \wedge (r \vee r)$ B. P$\wedge (p \wedge r)$ C. $q \wedge (p \wedge r)$ D. r$\wedge (p \vee q)$

2009

96. Let p and q stands for the statements "Nejat is intelligent" and "Almaz is hard working", respectively. Which of the following represent the statement "Almaz is hard working if Nejat is intelegent"?
A. $\neg p \wedge q$ B. $\neg p \vee q$ C. $p \wedge$q D. $\neg q \vee p$

97. Which of the following is a valid argument?
A. $\neg p \Rightarrow \neg q, q \vdash \neg$ p C.$\neg p \vee q, r \Rightarrow p, r \vdash \neg q$
B. $p \Rightarrow \neg q, p,\ r \Rightarrow q \vdash \neg r$ D.$\neg p, p \vee q, r \Rightarrow q \vdash \neg r$

98. Suppose "if $x \in A$, then $y \in B$" is a true statement. Then which one of the following is necessarily true?
A. $y \in B$. C. if $y \in B$, then $x \in A$.
B. If $x \notin A$, then $y \notin B$. D. If $y \notin B$, then $x \notin A$.

99. Consider the following compound open proposition: **P**(x)≡X is a prime number, **C**(x)≡X is a composite number, and **E**(x)≡X is an even number. Which one of the following has a truth value of True in the set of positive integers?
A. $(\forall x)\ [\ P(x) \Rightarrow \neg E(x)\ ]$ C.$(\exists x)\ [\ \neg P(x) \wedge \neg C(x)\ ]$
B. $\neg(\forall x)\ [\ C(x) \Rightarrow \neg P(x)\ ]$ D.$\neg(\exists x)\ [\ E(x) \wedge \neg C(x)\ ]$

2010

100. If the truth value of (p$\wedge \neg p) \Leftrightarrow\ [\ (q \vee \neg q) \Rightarrow r\ ]$ is True, then which one of the following must be True?
A. q B. P C. $\neg$q D. $\neg$ r

101. Suppose the following are premises of an argument:
He is healthy and he is not angry.
He is angry or his plan fails.
His plan does not fail if he does not travel abroad.
Given that the premises are true, which one of the following can be a conclusion that makes the argument valid?
A. His plan fails and he is not angry. C. He travels abroad.
B. His plan does not fail. D. His plan fails and he does not travel abroad.

102. Let $U = \mathbb{N}$ (the set of natural numbers) be the universe. Which one of the following propulsion is True?
A. $(\exists$x$)\ (x + x = x)$ C. $(\forall x)(\exists x)$(x$\div y = y \div x)$
B. $(\forall x)\ (y < x)$ D. $(\forall x)(\exists x)$(x$\div y = y \div x)$

2011

103. If $\neg p \Rightarrow r$ is false and $p \Leftrightarrow q$ is True, then which if the following is True?
A. $P \vee (\neg q \wedge r)$ B. $\neg p \wedge \left(q \Rightarrow r\right)$ C. $\neg p \Rightarrow (q \vee r)$ D. $p \Leftrightarrow (\neg q \vee r)$

104. Which one of the following is a valid argument?
A. If I don't change my oil regularly, my engine will die. My engine died. Thus, I didn't change my oil regularly.
B. If I am literate, then I can read and write. I can read but I can't write. Thus, I am not literate.
C. If you do every problem in the book, then you will learn the subject. You learned the subject. Thus, you did every problem in the book.
D. If it rains or snows, then my roof leaks. My roof is leaking. Thus, it is raining and snowing.

105. For real numbers x and y, which one of the following is NOT true?
A. $(\forall x)(\forall y)(y^2 + x^2 \geq -1)$ C. $(\exists x)(\forall y)(y \geq x^2 + 1)$
B. $(\forall x)(\exists y)\ (y \geq x^2 + 1)$ D. $(\exists x)(\exists y)(y \geq x^2 + 1)$

Unit 5

2004

106. A three-digit library identification card is to be printed from the numbers 0,1, 2,3,4,5 in a way that the first is non-zero and no number is to be repeated. How many such cards can be printed?
A. 100 B. 120 C. 150 D. 180

107. A student needs to select 3 books from 3 mathematics, 3 physics and 1 history book. What is the probability that one of them is mathematics and the other two are either physics or history books?

A. $\frac{3}{35}$ B. $\frac{9}{35}$ C. $\frac{15}{35}$ D. $\frac{18}{35}$

108. Items produced by a certain company are subjected to two kinds of defects D_1 and D_2. Out the total production, if 5% have defect D_1, 10% have defect D_2 and 2% have both defects, then what is the probability for an item that have defect D_2, given that it has defect D_1?

A. 0.2 B. 0.05 C. 0.1 D. 0.4

109. You are given a data on the age of students, in a primary school.

Age	8	10	11	12	13
Number of students	5	15	8	10	2

Which of the following is not true about the data?

A. The median is 10.5 C. The mean is 10.5

B. The mode is 10 D. The range is 5

110. The following is a table of simple frequency distribution of a data with variable x.

X	1	3	4	5	7
Frequency	2	5	6	5	2

The standard deviation of the data is equal to

A. $\sqrt{3}$ B. $\sqrt{2.3}$ C. 3 D. $\sqrt{3.6}$

111. A school has three class rooms for grade 11, namely, 11A, 11B, and 11C. The number of students in these classrooms is 28, 20 and 22, respectively. All the students took an examination and the average score of the students of 11A, 11B and 11C is 60, 70 and 70, respectively. What is the average score in this examination for all grade 11 students?

A. 66 B. 66.67 C. 65 D. 65.67

2005

112. Among students who took a quiz, 15 students scored 6, 20 students scored 7, 10 students scored 8 and 5 students scored 10. What is the average score of the students?

A. 7.8 B.7.5 C. 7.2 D. 7.0

113. How many four-digit even numbers can be formed from 1, 2, 3, 4, and 5 if the numbers start with 3?

A. 40 B. 50 C. 100 D. 120

114. A committee consisting of 3 students is to be selected from 10 candidates among which 4 are girls. What is the probability that at least one girl is selected.

A. $\frac{5}{6}$ B. $\frac{2}{3}$ C. $\frac{1}{3}$ D. $\frac{1}{6}$

115. A group of six students take their seats at random in a round table for a discussion. What is the probability that two specific students do not sit together?

A. $\frac{3}{5}$ B. $\frac{2}{3}$ C. $\frac{2}{5}$ D. $\frac{1}{3}$

116. The mark of students scored in an examination is grouped in class intervals as shown below.

Class interval(mark)	55-64	65-74	75-84	85-94	95-100
Number of students	8	12	20	6	4

What is the median of the mark?

A. 25.0 B. 75.5 C. 77.0 D. 79.5

117. A box contains 5 white 6 red and 4 black balls of identical size. If 3 balls are randomly taken out from the box one after the other, what is the probability that the first ball is white and both the second and the third are red?

A. $\frac{2}{15}$ B. $\frac{3}{15}$ C. $\frac{4}{75}$ D. $\frac{5}{91}$

2006

118. If the list of a measurement is 10, α, 5, α, 5, 10, 20, 15, 20, 5, with mean $\overline{x}$, then what is the value of α in terms of $\overline{x}$?

A. $10\overline{x} - 90$ B. $9\overline{x} - 90$ C. $5\overline{x} - 90$ D. $5\overline{x} - 45$

119. The following is the frequency distribution of a grouped data.

Class intervals	3-7	8-12	13-17	18-22
Frequency(f)	2	2	10	6

What is the mean and the standard deviation of the distribution, respectively?

A. 15, $2\sqrt{5}$ B. 15, $\sqrt{7.5}$ C. 12.5, $5\sqrt{2}$ D. 12.5, $\sqrt{15}$

120. If distinct codes (words) of eight letters are formed by rearranging the letters in the word 'ABBEBAYE', how many of the codes begin with B or Y?

A. 840 B. 630 C. 1680 D. 4220

121. If $\mathbf{Q_i}$, $\mathbf{D_i}$, and $\mathbf{P_i}$ are respectively the i^{th}-quartile, decile and percentile of a data arranged in increasing order, then which one of the following is necessarily true?

A. $Q2 = \frac{Q1+Q3}{2}$ C. $P_{25}>Q_1$

B. $D_3>P_{25}$ D. $Q = mean\ of\ the\ data$

122. A company produced 25,000 bulbs and randomly tested 2% of the product. Among the tested bulbs, if 40 have defect of D_1, 60 have defect of D_2 and 25 have both types of defects, what is the probability that a bulb produced by the company has **none** of the defects?

A. 0.95 B. 0.80 C. 0.85 D. 0.20

123. If **S** is a set with **10** elements and **A⊆S**, what is the probability that **A** has **3** or more elements?

A. $\frac{7}{10}$ B. $\frac{8}{11}$ C. $\frac{121}{128}$ D. $\frac{7}{128}$

2007

124. Different codes, each of which consisting of five characters, are to be generated in such a way that the first two characters are any of the English letters (A TO Z) and the remaining three are any of the digits (0, 1, . . . ,9). How many distinct codes can be generated so?

A. 468,000 B. 260 C. 676,000 D. 26! × 10!

125. The following is a set of data representing the average mark of 13 students: 91, 89, 93, 91, 87, 94, 92, 85, 91, 90, 96, 93, and 89. Then which of the following statements is true about the data?

A. The median is 90.5 C. The range of the mark is 11.

B. The upper quartile is 92 D. The mean is 91.5

126. A city has two daily newspapers, X and Y. the following information was obtained from a survey of 100 residents of the city: 35 people subscribe to X, 60 people subscribe to Y and 20 subscribe to both newspapers. Then how many of the people in the survey do not subscribe to either of the newspapers?

A. 5 B. 25 C. 40 D. 55

127. Suppose that the first 3 letters (A, B and C) and number digits are to be used to form car plates in a small town. How many different plates can be formed in a total that contain 1, 2 or 3 letters and then followed by 3 digits?

A. 3,000 B. 27,000 C. 39,000 D. 100,000

128. A measurement is grouped into five class intervals with the following frequency distribution.

Class interval	5-15	15-25	25-35	35-45	45-55
frequency	22	40	68	50	20

What are the **first** quartile $\mathbf{Q_1}$ and the **75th** percentile $\mathbf{P_{75}}$ of the measurement?

A. Q_1=20, P_{75}=40 C. Q_1=20, P_{75}=39

B. Q_1=22, P_{75}=40 D. Q_1=22, P_{75}=39

129. Three persons $\mathbf{P_1}$, $\mathbf{P_2}$ and $\mathbf{P_3}$ are firing at a target independently and have a probability **0.7**, **0.5** and **0.4**, respectively, of hitting the target. What is the probability that at least one of them hits the target?

A. 0.95 B. 0.85 C. 0.91 D. 0.99

2008

130. The following is a simple frequency distribution of a data with variable X.

X	3	5	6	7
Frequency	2	5	2	1

What are the mean ($\bar{x}$) and variance δ^2 of the data?

A. $\overline{X} = 5, \delta^2 = 0.7$ B. $\overline{X} = 6, \delta^2 = 1.4$ C. $\overline{X} = 6, \delta^2 = 0.7$ D. $\overline{X} = 5, \delta^2 = 1.4$

131. A box contains 10 items of which 3 are defective. If two items are randomly taken out of the box, what is the probability that both items are not defective?

A. $\frac{7}{10}$ B. $\frac{4}{7}$ C. $\frac{7}{15}$ D. $\frac{49}{100}$

132. Items produced by a certain company are subjected to two kinds of defects $\mathbf{D_1}$ and $\mathbf{D_2}$. Out of the total product **5%** have the defect $\mathbf{D_1}$, **10%** have the defect $\mathbf{D_2}$, and **2**% have both defects. What is the probability that a randomly selected item has neither defect $\mathbf{D_1}$ or $\mathbf{D_2}$?

A. 0.13 B. 0.5 C. 0.98 D. 0.87

133. There are three children in a room, ages three, four, and five, if a four- year-old child enters the room then which one of the following is true?

A. Mean age will stay the same but the standard deviation will increase.
B. Mean age will stay the same but the standard deviation will decrease.
C. Mean age and the standard deviation will increase.
D. Mean age and the standard deviation will stay the same.

134. In how many more ways can 4 people be arranged in a row than if they were arranged in a circle?

A. 1 B. 6 C. 18 D. 12

135. Two machines **A** and **B** work independently. The probability that both machines **A** and **B** work is **0.4.** if the conditional probability that machine **B** works given that machine **A** works is **0.5**, then the conditional probability that machine **A** works given that machine **B** works is……..

A. 0.8 B. 0 C. 0.5 D. 0.7

2009

136. A team of 10 researchers consists of 4 biologists and 6 chemists. If 3 persons are chosen randomly from the team, what is the probability that at least one is a biologist?

A. $\frac{2}{3}$ $\frac{2}{5}$ C. $\frac{5}{6}$ D. $\frac{7}{10}$

137. The probability that an electronic device produced by a company does not function properly is equal to 0.1. If two devices are bought, then what is the probability that at least one device function properly?

A. 0.81 B. 0.09 C. 0.18 D. 0.99

138. Two machines A and B produce respectively 60% and 40% of the total number of items of a factory. The percentages of defective of these machines are 2% and 5%, respectively. If an item is selected at random, then what is the probability that the item is defective?

A. 0.032 B. 0.07 C. 0.32 D. 0.426

139. In how many ways can a committee of 3 members be formed from 7 candidates?

A. 7 B. 21 C. 28 D. 35

140. The following is a frequency distribution table of a grouped data with variable X.

X	3-7	8-12	13-17	18-22
Frequency	4	6	8	2

What is the mean ($\overline{x}$) and the variance (δ^2) of the data respectively?

A. $\bar{x}, = 12, \delta^2 = 21$ C. $\bar{x} = 13, \delta^2 = 9$

B. $\bar{x} = 12, \delta^2 = 25$ D. $\bar{x} = 13, \delta^2 = 16$

141. The expenditure of 100 families is given below.

Expenditure	0-9	10-19	20-29	30-39	40-49
No. Of families	14	23	F_1	21	F_2

If the mode of the data is 23.5, what are the values of F_1 and F_2?

A. $F_1 = 27, F_2 = 15$ C. $F_1 = 25, F_2 = 17$

B. $F_1 = 15, F_2 = 27$ D. $F_1 = 17, F_2 = 25$

142. The first group of 10 children has a mean weight of 15.6 kg, and the second group of another 10 children has a mean weight of 16 kg, and the third group of children has a mean weight 20kg. If the mean weight of all children is 17 kg, what is the total number of children in all of the three groups?
A. 28 B. 29 C. 30 D. 32

2010

143. Let A and B be two events. Suppose that the probability that neither event occurs is $\frac{3}{8}$. What the probability that atleast one of the events occur?
A. $\frac{1}{8}$ B. $\frac{5}{8}$ C. $\frac{1}{4}$ D. $\frac{3}{4}$

144. The time needed to type a sample of 8 business letters in an office is 7, 8, 6, 8, 9, 7, 5, 6 minutes. What is the mean ($\bar{x}$) and the standard deviation (sd) of the data in minute?
A. $\bar{x} = 7,\ sd = \sqrt{1.5}$ C. $\bar{x} = 8,\ sd = \sqrt{2}$
B. $\bar{x} = 7,\ sd = \sqrt{2}$ D. $\bar{x} = 8,\ sd = \sqrt{1.5}$

145. A private college has 1000 students. 60% of these students are males, 45% of these students pay their payment by credit card including 175 females. What is the probability that the student is a male or a credit card user?
A. 0.675 B. 0.225 C. 0.325 D. 0.775

146. Let A $= \{1, 2, 3, 4, 5, 6, 7\}$, B$= \{7, 8, 9\}$ and C$= \{8, 9, 10\}$. If one of the numbers is deleted randomly from each of these sets, what is the probability that all of the three deleted numbers are even or all are multiple of 3?
A. $\frac{4}{5}$ B. $\frac{2}{21}$ C. $\frac{1}{9}$ D. $\frac{8}{63}$

147. The age distribution of students in a certain class is given below.

Age	10-14	15-19	20-24	25-29
No. Of students	2	10	6	7

What is the modal value of the distribution?
A. 17.83 B. 17.38 C. 18.37 D. 18.73

148. The variance of 20 observations is 5. If each observation is multiplied by 2, then what is the variance of the resulting observations?
A. 5 B. 10 C. 20 D. 40

2011

149. If there are two children in a family, what is the probability that there is at least one girl in the family?
A. $\frac{1}{2}$ B. $\frac{1}{4}$ C. $\frac{3}{4}$ D. $\frac{2}{3}$

150. Among 2000 students who took a regional exam, the percentile of certain student's score is 90. Which of the following is correct about the student's score?
A. The students have answered 90% of the questions correctly.
B. The student's score is greater than or equal to that of 1800 students.
C. The student's score is the same as the top 10% of the scores.
D. The score of the students is the as good as that of 90% the students.

151. The mark of 50 students is given below:

Marks	0-10	10-20	20-30	30-40	40-50
NO. Of students	5	8	f_1	10	f_2

If the median of the data is 26, what are the values of f_1 and f_2?
A. $f_1 = 7, f_2 = 20$ C. $f_1 = 15, f_2 = 12$
B. $f_1 = 12, f_2 = 15$ D. $f_1 = 20, f_2 = 7$

152. Suppose 2500 items are produced by a machine and 2% of the product are randomly selected and tested. If 5 of the tested items have a defect, then what is the probability that an item produced by a machine has No defect?
A. 0.80 B. 0.85 C. 0.90 D. 0.95

153. Fatuma can solve 90% of the problems given in a book and Mesfin can solve 70%. What is the probability that at least one of them will solve the problem?
A. 0.77 B. 0.87 C. 0.97 D. 0.67

154. There are three children in a room with, ages four, five, and six. If a five year old child enters the room, then which of the following statement is correct?
A. Mean age will stay the same the standard deviation will decrease.
B. Mean age will stay the same the standard deviation will increase.
C. Mean age will and standard deviation will increase.
D. Mean age and standard deviation will stay the same.

Unit 6

2004

155. Let A $= \begin{pmatrix} -2 & 0 & x \\ 2y & x+y & -4 \end{pmatrix}$ and B$= \begin{pmatrix} 1 & -y \\ 0 & 3 \\ 1-x & 2 \end{pmatrix}$ such that A+2B^T $= 0$. Then which of the following is the value of y?
A. 0 B. $-\frac{13}{2}$ C. -8 D. any real number

156. Let A and B be 3×3 matrices such that A$= \begin{pmatrix} 2 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & -1 & \frac{1}{2} \end{pmatrix}$ and$|B| = \frac{1}{10}$. Which of the following is equal to $|2AB^T|$
A. 1 B. 4 C. 100 D. 400

157. If $\begin{pmatrix} \alpha & 2 & \beta \\ 2 & 1 & 3 \\ -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -5 \\ 5 \\ 0 \end{pmatrix}$ and the determinant of the coefficient matrix is -5, then the value of x is equal to:
A. 3 B. $\alpha + \beta$ C. -5α D. 5

158. What is the solution set of the following system of equation $\begin{cases} x+y+2z=1 \\ x+2y+z=2 \\ -2x-2y-4z=-2 \end{cases}$?
A. $\{(0,1,0)\}$ C. $\{(-3k, k+1, k) \backslash k \in (-\infty, \infty)\}$
B. $(-\infty, \infty)$ D. $\{(3k, k-1, k) \backslash k \in (-\infty, \infty)\}$

2005

159. Suppose A$= \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$. If X is 2×2 matrix such that $AX - A^T = 2A$, then what is the value of X?
A. $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ B. $\begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}$ C. $\begin{pmatrix} 3 & 6 \\ 6 & 9 \end{pmatrix}$ D. $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

160. Suppose that A and B are 3×3 matrices, I is identity matrix of order 3 such that $AB = 2I$. If $|B| = 6$, what is det (A^T)?
A. $\frac{1}{3}$ B. $\frac{4}{3}$ C. 12 D. 48

161. Consider the system $\begin{cases} \alpha x + y + z = 1 \\ x + 2y + 4z = 0 \\ 5x - y + z = 0 \end{cases}$. If the determinant of the coefficient matrix is 2, then what is the solution of the system of the equations?
A. $\left(3\alpha, \frac{19\alpha}{2}, \frac{-11\alpha}{2}\right)$ B. $\left(3, \frac{19}{2}, \frac{-11}{2}\right)$ C. $\left(\frac{3}{\alpha}, \frac{-19}{2}, \frac{11}{2}\right)$ D. $\left(\frac{3}{2}, \frac{19}{2}, \frac{-9}{2}\right)$

2006

162. Let $A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ and $(2A+B)^T = A^T A$, then which of the following is equal to the value of B?
A. $\begin{pmatrix} 1 & 0 & -2 \\ 2 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}$ C. $\begin{pmatrix} 8 & 0 & -4 \\ 4 & 8 & 0 \\ 0 & 0 & -4 \end{pmatrix}$

$B.\ \begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}$ D. $\begin{pmatrix}1&2&0\\0&0&0\\-2&0&4\end{pmatrix}$

163. If $M=\begin{pmatrix}0&1&2\\3&-1&0\\5&2&4\end{pmatrix}$ and $A^TM=2I$, where A is 3×3 matrix and I is identity matrix of order 3, then what is the value of det(A)?

A. 0.2 B. $\frac{4}{17}$ C. 0.8 D. $\frac{1}{17}$

164. What should be the value of k so that the system of equation $\begin{cases}x-y+z=1\\-x+5y\pm 4z=1\\2x+2y-z=k\end{cases}$ has a solution?

A. 0 B. 1 C. -4 D. 4

165. Suppose $AX=b$, where A is a 3×3 matrix, $b=(\mathrm{b1},\mathrm{b2},\mathrm{b3})^T$ and $X=(x,y,z)^T$. Which of the following is necessarily true?

A. The system has a solution only when det(A)≠ 0.
B. The Cramer's is suitable to solve the system if two rows of A are identical.
C. If det(A)≠ 0 and the second column of A is multiple of b, then x= 0.
D. If b= 0, then X= $(0,0,0)^T$ is the only solution of the system.

2007

166. Consider the following system of equations: $\begin{cases}ax+by=2\\x+3y+2z=0\\2x+y+z=0\end{cases}$ if the determinant of the coefficient matrix is 2, then what is the solution set of the system?

A. $\{(1,3,-5)\}$ B. $\left\{\left(\frac{1}{a},\frac{1}{b},0\right)\right\}$ C. $\{(-2,-6,10)\}$ D. Ø

167. If $A=\begin{pmatrix}0&x&0\\1&-1&1\\0&y&-1\end{pmatrix}$ and $A^{-1}=\begin{pmatrix}1&1&1\\3&0&0\\2&0&-1\end{pmatrix}$, then what are the values of x and y?

A. $x=3,y=-2$ C. $x=-3,y=2$
B. $x=\frac{2}{3},y=\frac{1}{3}$ D. $x=\frac{1}{3},y=\frac{2}{3}$

168. If $=\begin{pmatrix}3&-2&8\\0&6&7\\0&4&5\end{pmatrix}$, then det($A^TA$) is equal to……

A. 12 B. 36 C. 30 D. 15

169. What is the solution set of the system $\begin{cases}x+y-z=1\\x+2y-3z=1\\2x+3y-4z=2\end{cases}$?

A. $\{(0,2,1)\}$ C. $\{(2k+1,-k,k)|k\in\Re\}$
B. $\{(1-k,2k,k)|k\in\mathcal{R}\}$ D. Ø

2008

170. If $A=\begin{pmatrix}2&7\\1&3\end{pmatrix}$ and $B^{-1}=\begin{pmatrix}1&1\\0&2\end{pmatrix}$, then $(AB)^{-1}$ is equal to:

A. $\begin{pmatrix}4&-3\\4&-5\end{pmatrix}$ B. $\begin{pmatrix}-2&5\\2&-4\end{pmatrix}$ C. $\begin{pmatrix}-3&11\\1&-3\end{pmatrix}$ D. $\begin{pmatrix}4&0\\0&4\end{pmatrix}$

171. Let A = $\begin{pmatrix}0&\alpha&\beta\\2&2&1\\3&1&2\end{pmatrix}$, $b=\begin{pmatrix}6\\0\\0\end{pmatrix}$, and X= $\begin{pmatrix}x\\y\\z\end{pmatrix}$. If det(A)=3, then what is the solution set of the system $\boldsymbol{AX}=b$?

A. $\{(6,-2,-8)^T\}$ B. $\left\{\left(0,\frac{1}{\alpha},\frac{1}{\beta}\right)^T\right\}$ C. $\{(-3,1,4)^T\}$ D. Ø

172. For any n×n square matrix A, which one of the following is true?

A. det(A)= $-\det(A^T)$, where A^T is the transpose of A.
B. If k is scalar, then det(kA)= $k^n\det(A)$.
C. If B is a matrix obtained from A by interchanging of two rows of A, det(B)=det(A).

D. If A is invertible, then det(A)=det(A^{-1}).

173. The solution of the system of linear equation of $\begin{cases} x-3y-2z=6 \\ 2x-4y-3z=8 \\ -3x+6y+8z=-5 \end{cases}$ is:

A. $\{(-1,-3,-2)\}$ C. $\{(1,-3,\ 2)\}$
B. $\{(-1,-3,\ 2)\}$ D. $\{(1,\ 3,-2)\}$

2009

174. If $A=(a_{ij})_{3\times3}$ is a square matrix with $A^{-1}=\begin{pmatrix}1&3&2\\1&1&3\\0&4&5\end{pmatrix}$, then what is the cofactor of a_{23}?

A. $-\frac{3}{14}$ B. $-\frac{2}{7}$ C. $\frac{2}{7}$ D. $-\frac{3}{7}$

175. When $\begin{vmatrix}a&b&c\\a&-a&a\\a&a&-a\end{vmatrix}=a^3$,and $a\neq0$, what is the solution of $\begin{pmatrix}a&b&c\\a&-a&a\\a&a&-a\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\0\\0\end{pmatrix}$

A. $\{(0,2a,2a)\}$ C. $\left\{\left(\frac{1}{a},\frac{-2}{a},\frac{2}{a}\right)\right\}$
B. $\left\{\left(\frac{1}{a},-2a,2a\right)\right\}$ D. $\left\{\left(0,\frac{2}{a},\frac{2}{a}\right)\right\}$

176. What are the values of γ and μ so that the system $\begin{cases} x+y+z=6 \\ x+2y+3z=10 \\ x+2y+\gamma z=\mu \end{cases}$, has infinitely many solutions?

A. $\gamma\neq3$ and $\mu\in\Re$ C. $\gamma=3$ and $\mu=10$
B. $\gamma=3$ and $\mu\neq10$ D. $\gamma\in\Re$ and $\mu=10$

2010

177. A salesman sold items x_1, x and x_3 , with different rates of commissions as shown in the table below.

Month	Sales of units			Total commission(in Birr)
	x_1	x_2	x_3	
February	90	100	20	800
March	130	50	40	900
April	60	100	30	850

What the rates of commission on items x_1, x_2 and x_3, respectively.

A. 4, 2 and 11 C. 4, 11 and 2
B. 2, 4 and 11 D. 11, 2 and 11

178. If A is a square matrix of order 3 and det(A)= 5, then what is the value of det(A.adj(A))?

A. 3 B. 5 C. 125 D. 25

179. Let A be a 3×3 invertible of matrix and B be any 3×3 matrix. If $|A|=a$ and, $|B|=b$, then which of the following is not true?

A. $|A^TA|=a^2$ C. $|A^{-1}B|=ab$
B. $|kA|=k^3|A|$, for any k∈ $\mathbb{R}$ D. if b =0, then B is not invertible

180. Let A be a 3×3 matrix and $|A|=-2$. Then what is the value of $|adj(A)|$?

A. -8 B.-2 C. $-\frac{1}{2}$ D. 4

2011

181. If $\begin{vmatrix}-1&1&2\\3&2&x\\2&4&1\end{vmatrix}=\begin{vmatrix}-x&3&2\\2&2&3\\1&-1&-2\end{vmatrix}$, then what is the value of x?

A. $\frac{1}{5}$ B. $-\frac{2}{5}$ C. $\frac{2}{3}$ D. $-\frac{2}{3}$

182. Consider the following system of equation: $\begin{cases} x-2y+z=1 \\ -x+y+z=3 \\ 3x-5y+z=k \end{cases}$. How much should be the value of k so that the system has a solution?

A. 7 B. 1 C. 0 D. -1

183. If A is a 3×3 matrix and det(A)= 5, then det($2A^TA$) is equal to:

A. 200 B. 100 C. 50 D. 20

184. If $2\begin{pmatrix}2x & x\\ -5 & -3\end{pmatrix}^{-1} = \begin{pmatrix}3 & 2\\ -5 & -4\end{pmatrix}$, then what is the value of x?

A. -2 B. 2 C. $\frac{1}{2}$ D. $-\frac{1}{2}$

Unit 7

2004

185. Which of the following is the simplest form of $\frac{4-3i}{3+4i} + \overline{1-2i}$?

A. $1+i$ B. $1+3i$ C. $2-i$ D. $1-3i$

186. Let z be a complex number. Which of the following is the solution set of $z^3 - iz = 0$?

A. $\left\{\pm\frac{1}{\sqrt{2}}(1+i)\right\}$ C. $\left\{\pm\sqrt{2}(1+i)\right\}$

B. $\left\{0, \pm\frac{1}{\sqrt{2}}(1-i)\right\}$ D. $\left\{0, \pm\frac{1}{\sqrt{2}}(1+i)\right\}$

187. In the set of complex numbers, the solution set of $x^2 - 2x + 5 = 0$?

A. Ø C. $\{1+2i,\ 1-2i\}$

B. $\{2+i,\ 2-i\}$ D. $\{2+4i,\ 2-4i\}$

2005

188. Which one of the following is the simplest form of $|3+4i| - \frac{25i}{3+4i}$?

A. $5-5I$ B. $5+5i$ C. $1+3i$ D. $1-3i$

189. If $z = \cos\left(\frac{\pi}{10}\right) + i\sin\left(\frac{\pi}{10}\right)$, then what is the value of z^5?

A. $\frac{\pi}{2} + \frac{\pi}{2}i$ B. $\frac{1}{2} + \frac{1}{2}i$ C. i D. $1+i$

190. In the set of complex numbers, which one of the following is the solution set of $z^3 - iz^2 + 2z = 0$?

A. $\{0\}$ B. $\{0, -i\}$ C. $\{0, -i, 2i\}$ D. $\{0, i, -2i\}$

2006

191. If $z = x + yi$, is a complex number, then $|z|^2 + \frac{1}{2}(z-\bar{z})^2 = 1$ is equivalent to which one of the following equation?

A. $x^2 - y^2 = 1$ C. $x^2 - y^2 = 2$

B. $x^2 - 3iy^2 = 1$ D. $2x^2 - y^2 = 2$

192. If $w = \frac{16i}{1+i} + (1-3i)^2$ and z= $|w| + \bar{w}$, which one of the following is the simplest form of z?

A. $\sqrt{2} + 2i$ B. $2 + 2i$ C. $4 - 2i$ D. $2 - 2i$

2007

193. If $z = \frac{3+i}{i-2}$ is a given complex number, then what is the conjugate, $\bar{z}$, of z?

A. $\bar{z} = \frac{-3+i}{2+i}$ C. $\bar{z} = -6 - 2i$

B. $\bar{z} = \frac{i-3}{2-i}$ D. $\bar{z} = -1 - i$

194. What is the principal argument of $(5+5i)^{11}$?

A. $\frac{\pi}{2}$ B $\frac{2\pi}{3}$ C. $\frac{\pi}{4}$ D. $\frac{3\pi}{4}$

195. What are the values of u and v that satisfy the equation: $\frac{u+3i}{4-2i} = \frac{2+vi}{20}$?

A. $u = 2,\ v = 3$ C. $u = 2,\ v = 16$

B. $u = -6,\ v = 10$ D. $u = -4,\ v = 6$

2008

196. In the set of complex numbers, what is the solution set of $x^2 + 4x + 5 = 0$?

A. Ø C. $\{2-i, 2+i\}$

B. $\{1-2i, 1+2i\}$ D. $\{-2-i, -2+i\}$

197. If $z = (1+i)^{10}$, then which of the following is equal to z?

A. $1+32i$ B. $32i$ C. $10i$ D. $1+10i$

198. If $z = \frac{2-4i}{1+i}$, then the modulus of the conjugate of z, $|\bar{z}|$ is:

A. $\sqrt{10}$ B. $3\sqrt{2}$ C. $2\sqrt{3}$ D. $2\sqrt{2}$

2009

199. If $z = \sqrt{2}\cos\left(\frac{\pi}{12}\right) + i\sqrt{2}\sin\left(\frac{\pi}{12}\right)$, then what is the value of z^3?

A. $2+2i$ C. $2\sqrt{2} + 2i\sqrt{2}$

B. $\sqrt{2}+i\sqrt{2}$ D. $3\sqrt{2}+3i\sqrt{2}$

200. Which one of the following is the conjugate of $|3+4i|-\frac{25i}{3+4i}$?
A. $5+3i$ B. $1+3i$ C. $3-5i$ D. $1-3i$

201. If $z_1=\frac{2-i}{1+i}$, $z_2=\frac{1+i}{1-i}$ then what is the value of z_1+2z_2?
A. $1+i$ B. $\frac{1+i}{2}$ C. $\frac{1-i}{2}$ D. $1-i$

2010

202. If $z=-3+4i$ and w= $1+2i$, then what is the value of $\frac{2z}{w}+\overline{w}$?
A. $2+3i$ B. $3+5i$ C. $3+2i$ D. $3-2i$

203. Let $z=\left(\frac{1-i}{1+i}\right)^{18}$, then what is the value of z?
A. -1 B. i C. $-i$ D. $1-i$

204. If $z=(1+i\sqrt{3})(1+i)$, then which one of the following is the polar representation of z?
A. $z=4(\cos(105°)+i\sin(105°))$ C. $z=2\sqrt{2}(\cos(15°)+i\sin(15°))$
B. $z=2\sqrt{2}(\cos(105°)+i\sin(105°))$ D. $z=4(\cos(75°)+i\sin(75°))$

2011

205. Which of the following is the multiplicative inverse of $z=\frac{3+4i}{4-5i}$?
A. $\frac{8}{25}-\frac{31}{25}i$ B. $-\frac{8}{25}+\frac{31}{25}i$ C. $-\frac{8}{25}-\frac{31}{25}i$ D. $\frac{8}{25}+\frac{31}{25}i$

206. Let z be a complex number and w= $3+4i$. If $\frac{z^2+1}{z+i}=|w|z-\frac{1}{i}\overline{w}$, then what is the value of z?
A. $-4-2i$ B. $4-2i$ C. $-1+i$ D. $-1-i$

207. What is the polar form of $\frac{7-i}{3-4i}$?
A. $\sqrt{2}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$ C. $2\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$
B. $\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ D. $2\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

Unit 8

2004

208. Which of the following is a vector that lies on the line through (0, 0) and (2, 4)?
A. $\vec{u}=(2,1)$ C. $\vec{u}=(\frac{1}{2},2)$
B. $\vec{u}=(-1,-2)$ D. $\vec{u}=(-2,-6)$

209. Let i and j be the standard unit vectors in the direction of positive X-axis and positive Y-axis, respectively, and $\overrightarrow{AB}$ be a vector from the point B(2,2). If $\vec{v}=3\overrightarrow{AB}+2j$, the unit vector in the direction of $\vec{v}$ is equal to:
A. $\left(\frac{3}{5},\frac{4}{5}\right)$ B. $\left(\frac{-3}{5},\frac{-4}{5}\right)$ C. $\left(\frac{-3}{5},\frac{4}{5}\right)$ D. $\left(\frac{3}{5},\frac{-4}{5}\right)$

210. Which of the following is a vector equation of the line tangent to the circle $x^2+y^2+2x-7=0$ at (1, 2)
A. $(x,y)=(0,3)+\lambda(-1,2)$ C. $(x,y)=(0,3)+\lambda(1,-1)$
B. $(x,y)=(1,2)+\lambda(2,-1)$ D. $(x,y)=(1,2)+\lambda(-1,2)$

211. Let ℓ be the line whose equation is $2x-y=10$. Which one of the following is the equation of the image of ℓ after a reflection in the line $y=2x-5$ followed by a rotation through the angle of 90° about the origin?
A. $x+2y=0$ B. $2x+y=0$ C. $x+2y=5$ D. $x-2y=5$

2005

212. *If* $\vec{u}=(-3,x)$ and $\vec{v}=(x,y-2)$ are vectors, what is the value of y so that $\vec{u}+\vec{v}=3\vec{u}-\frac{1}{2}\vec{v}$?
A. $\frac{2}{3}$ B. $-\frac{10}{3}$ C. -4 D. $-\frac{22}{3}$

213. A line given by a vector equation $r(x) = (0,3) + t(1,1)$ is tangent to a circle at point $(0,3)$. If the radius of the circle is $\sqrt{2}$, which one of the following is the of the circle?
A. $(1,4)$ B. $(1,-4)$ C. $(-1,2)$ D. $(1,2)$

214. What is the image of the ellipse whose equation is $2(x+2)^2 + (y-1)^2 = 2$ under a translation that takes (2, 1) to (4, 0) followed by a rotation of 90°?
A. $x^2 + 2y^2 = 2$ C. $2(x-4)^2 + y^2 = 2$
B. $2x^2 + y^2 = 2$ D. $(x-4)^2 + 2y^2 = 2$

215. If $\vec{A}$ is perpendicular to $\vec{B}$, what is the cosine of the angle between $\vec{A}$ and $\vec{A} - \vec{B}$?
A. $\frac{|\vec{A}-\vec{B}|}{|\vec{A}|}$ B. $\frac{|\vec{A}|}{|\vec{A}-\vec{B}|}$ C. $\frac{|\vec{A}-\vec{B}|}{|\vec{B}|}$ D. $\frac{|\vec{B}|}{|\vec{A}-\vec{B}|}$

216. Which of the following is necessarily true?
A. If $|\vec{A}| = |\vec{B}|$, then $\vec{A} = \vec{B}$. C. If $\vec{u}$ is a unit vector in the direction of $\vec{A}$, then $\vec{A}.\vec{u} = |\vec{A}|$
B. $|k\vec{A}| = k|\vec{A}|$, for any real number. D. If $\vec{A}$ is parallel to $\vec{B}$, then $\vec{A}.\vec{B} = 0$

2006

217. If a point (2, 5) is reflected under a line to the point $(-3,1)$, what is the line of reflection?
A. $2x + 3y = 7$ C. $8y + 10x = 19$
B. $x + 3y = 7$ D. $2x + 3y + 5 = 0$

218. If A= $(-2,3)$, B= $(3,1)$ and C is any other point on the plane, then which one of the following is the coordinate form of $\overrightarrow{AC} - \overrightarrow{BC}$?
A. $(-5,2)$ B. $(5,-2)$ C. $(1,4)$ D. $(-1,-4)$

219. What is the equation of a line that passes through the point $(-1,2)$, and parallel to the vector $(1,-1)$?
A. $2x - y = 1$ C. $x - 2y = 3$
B. $x + y - 1 = 0$ D. $y - 2x + 1 = 0$

220. What is the image of the line given by $(x,y) = (-1,0) + t(3,6), t \in \Re$, under the translation that takes (1, 0) to (0, 1) followed by the reflection about the line $y = 2x$?
A. $y = 2x + 3$ C. $y = 2x + 6$
B. $y = 2x - 3$ D. $y = 2x - 3$

221. If a translation T takes the circle $x^2 + y^2 - 2x + 6y + 3 = 0$ into the circle whose equation is $(x+2)^2 + (y-4)^2 = 7$, then what is the image of the origin under T?
A. $(-3,7)$ B. $(1,2)$ C. $(1.-3)$ D. $(-2,4)$

222. If ℓ is the line that passes through (0, 2) and parallel to $\vec{v} = i + 3j$ which of the following is true about ℓ and the circle $(x-2)^2 + (y-1)^2 = 5$?
A. ℓ is tangent to circle at (0, 2).
B. ℓ is tangent to the circle at some point p, where p$\neq (0,2)$.
C. ℓ Intersects the circle at exactly two points.
D. The distance between ℓ and the center of the circle is greater than $\sqrt{5}$.

223. Suppose $\vec{A} = 3i - 4j$ and $\vec{B}$ is a vector in the xy-plane such that the angle between $\vec{A}$ and $\vec{B}$ is $\frac{\pi}{3}$. $\vec{u}$ is a unit vector in the direction of $\vec{B}$, then $\vec{A}.(\vec{A} - 2\vec{u})$ is equal to:
A. 20 B. 5 C. 15 D. 30

2007

224. If $A = (1,-2), B = (-3,2)$ and $\vec{V}$ is a position vector such that $2\vec{V} + \overrightarrow{AB} = 0$, then $\vec{V}$ is equal to:
A. $(2,0)$ B.$(-1,0)$ C. $(-2,2)$ D. $(2,-2)$

225. If $\vec{A} = 4i - 3j$ and $\vec{u}$ is a unit vector such that $|\vec{A} + u|^2 = 27$, then the cosine of the angle between $\vec{A}$ and $\vec{u}$ is equal to……
A. 0.1 B. 0.2 C. 0.3 D. 0.4

226. What is the image of the ellipse $(x-1)^2 + 4y^2 = 1$ under the translation that takes (1, 1) and (0, 2) followed by the reflection through the x-axis?
A. $x^2 + 4(y-1)^2 = 1$ C. $x^2 + 4(y+1)^2 = 1$
B. $4x^2 + (y-1)^2 = 1$ D. $4x^2 + (y+1)^2 = 1$

227. What is the work done (in Joules) when a force of 50N is used to pull a crate 20m along a level path if the force is at an angle of 60°?

A. 360 B. 500 C. 760 D. $1500\sqrt{2}$

2008

228. The image of a figure with vertices A$(1,2)$, B$(3,6)$ C$(-1,2)$ and D$(-2,-2)$ after reflection across the x-axis is:

A. A'$(1,-2)$, B'$(-3,-6)$, C'$(1,-2)$ and D'$(2,2)$
B. A'$(1,-2)$, B'$(3,-6)$, C'$(-1,-2)$ and D'$(-2,2)$
C. A'$(-1,2)$, B'$(-3,6)$, C'$(1,-2)$ and D'$(2,-2)$
D. A'$(1,-2)$, B'$(3,6)$, C'$(-1,2)$ and D'$(-2,-2)$

229. If $\vec{V} = \overrightarrow{AB} + 3\overrightarrow{BA}$ where A and B are distinct points in the coordinate plane, then which one of the following is equal to $3\vec{V}$?

A. $6\overrightarrow{AB}$ B.$-6\overrightarrow{AB}$ C. $12\overrightarrow{AB}$ D. $-12\overrightarrow{AB}$

230. If $\vec{u}$ is a unit vector in the direction of $\vec{A}$ and $|\vec{A}| = 4$, $\vec{A}.\vec{u}$ is equal to……

A. $\frac{1}{4}$ B. 4 C. $\frac{1}{2}$ D. 2

231. If $\vec{A}$ and $\vec{B}$ are parallel vectors with opposite direction and $|\vec{B}| = |2\vec{A}|$, then $\vec{B} - \vec{A}$ is equal to:

A. $\vec{A}$ B. $-\vec{A}$ C. $-3\vec{A}$ D. $3\vec{A}$

2009

232. What is the translation vector u=(h, k) so that the equation $x^2 + 2y^2 + 6x - 8y + 15 = 0$ is transformed to an equation of the form $x^2 + 2y^2 + d = 0$ where d is constant?

A. u= $(-3,2)$ C. u= $(-2,3)$
B. u= $(3,2)$ D. u= $(2,-3)$

233. A line given by the vector the equation $(x,y) = (-t, 6+2t), t \in \Re$, is tangent to a circle at point (1, 4). What is the radius of the circle if its center is on the y-axis?

A. $\sqrt{5}$ B. $\frac{1}{2}\sqrt{5}$ C. $2\sqrt{5}$ D. $\sqrt{10}$

234. For two non-zero vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a} + \vec{b}| = |\vec{a}|$, then which of the following is true?

A. $2\vec{a}.\vec{b} = \vec{b}.\vec{b}$ C. $2\vec{a} + \vec{b}$ *and* $2\vec{a} + \vec{b}$ are parallel
B. $2\vec{a} + \vec{b}$ *and* $\vec{b}$ are perpendicular D. $\vec{a}.\vec{b} - \vec{b}.\vec{b} = 0$

235. Let $\overrightarrow{PQ}$ be a vector with initial point $P = (1,5)$ and terminal point $Q = (4,0)$. If $\vec{V} = xi + 2j$ is parallel to$\overrightarrow{PQ}$, then what is the value of x?

A. $-\frac{6}{5}$ B. -3 C. $-\frac{2}{5}$ D. 3

2010

236. Let ℓ be the line given by the vector equation $(x,y) = (-2,1) + \lambda(1,1), \lambda \in \mathbb{R}$. Which one of the following is the image of ℓ after being translated by the vector $u = (2,-1)$ followed by a rotation through 45° about the origin?

A. $y = \sqrt{2}\,x$ B. $y = \sqrt{2}$ C. $x = 0$ D. $y = -2\sqrt{2}$x

237. What is the image of the circle $x^2 + y^2 - 4x - 6y + 12 = 0$ when it is reflected with respect the line $y = -x$?

A. $(x+3)^2 + (y+2)^2 = 1$ C. $(x+2)^2 + (y+3)^2 = 1$
B. $(x-2)^2 + (y-3)^2 = 1$ D. $(x-3)^2 + (y-2)^2 = 1$

238. If the image of the line $2x - 3y = 7$ under a translation is $2x - 3y = 0$, which one of the following is a translation vector of the translation line?

A. $u = (2,-1)$ C. $u = (-1,2)$
B. $u = (-2,1)$ D. $u = (1,-2)$

239. If in, ΔABC, $AB = 3$, $BC = 4$ and m$(< B) = 60°$, then what are the length of AC and the cosine of $< A$, respectively?

A. $\sqrt{13}$ and $\frac{1}{\sqrt{13}}$ C. $\sqrt{13}$ and $\frac{6}{5\sqrt{13}}$
B. $\sqrt{13}$ and $\frac{-1}{\sqrt{13}}$ D. $\sqrt{13}$ and $\frac{-6}{5\sqrt{13}}$

240. What is the standard equation of the line passing through the point (2, 3) and parallel to the line given by $\begin{cases} x = 1 + 2\lambda \\ y = -2 - \lambda \end{cases}, \lambda \in \mathbb{R}$?

A. $\frac{x-2}{-1} = \frac{y-3}{2}$ C. $\frac{x-2}{2} = \frac{y-3}{-1}$

B. $\frac{x-1}{2} = \frac{y-3}{-2}$ D. $\frac{x-1}{1} = \frac{y-3}{-2}$

2011

241. Consider a rectangle ABCD with base vertices A=(0, 3) and B= $(4, 0)$ and the other vertices, C and D, in the first quadrant of the coordinate plane. If its height BC is half of the length of its base, then which of the following indicates the coordinate of the vertex C?

A. $\left(4, \frac{5}{2}\right)$ B. $\left(6, \frac{3}{2}\right)$ C. $\left(\frac{5}{2}, -2\right)$ D. $\left(\frac{11}{2}, 2\right)$

242. What is the image of the circle $x^2 + y^2 - 4x - 6x + 11 = 0$ when the origin is shifted to the point (1, 1) after translation of axes?

A. $x^2 + y^2 - 6x - 8y + 23 = 0$ C. $x^2 + y^2 + 6x + 8y - 23 = 0$

B. $x^2 + y^2 - 4x - 6y + 3 = 0$ D. $x^2 + y^2 - 4x - 2y + 3 = 0$

243. When the plane is rotated 45° about the point $(1,-2)$, then what would be the image of the point (2, 4)?

A. $\left(1 - \frac{5\sqrt{2}}{2}, -2 + \frac{7\sqrt{2}}{2}\right)$ C. $\left(\frac{5\sqrt{2}}{2}, \frac{7\sqrt{2}}{2}\right)$

B. $\left(1 - \frac{\sqrt{2}}{2}, -2 + \frac{\sqrt{2}}{2}\right)$ D. $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

244. Let ℓ be a line given by the equation $(x, y) = (1, 1) + t(\sqrt{3}, 1), t \in \mathbb{R}$, what is the equation of the image of ℓ after being rotated 15° about (1, 1) and then translated by the vector u= $(-1, 1)$?

A. $-x + y = 2$ C. $\sqrt{3}x - y = 2$

B. $x - y = 2$ D. $-x + \sqrt{3}y = 1$

Unit 9

2004

245. Which one of the following is a simplified form of $\csc\left(\frac{\pi}{2} - x\right)\cos x - sin^2 x$?

A. $2\cos x$ B.cos^2 C. $2\sin x$ D. $\sin 2x$

246. $\cos(\tan^{-1}(0.5))$ is equal to:

A. $\frac{1}{\sqrt{5}}$ B. $\frac{1}{\sqrt{3}}$ C. $\frac{2}{\sqrt{5}}$ D. $\frac{2}{\sqrt{3}}$

247. A patrol boat on a sea sailed from its station 10km to the south and then changed its course and sailed $5 + \sqrt{6}$ km in the direction of N60°E. What is the boat should travel in order to return to its station by the shortest route?

A. $4\sqrt{6}$ km B. 9 km C. $3\sqrt{6}$km D. 7 km

2005

248. If $f(x) = 2 - \frac{1}{2}\sin\left(\frac{\pi}{2}x\right)$, then which of the following is the amplitude and period of f, respectively?

A. $\frac{1}{2}$ and 4 B. $\frac{-1}{2}$ and 4 C. 2 and π D. $\frac{1}{2}$ and π

249. Which of the following is equal to $\sec\left(\frac{\pi}{2} - x\right) sin^3 x + \cos 2x$?

A. $2\cos x$ B. $2\sin x$ C. $cos^2 x$ D. $sin^2 x$

250. What is the solution set of $cos^2 x + \frac{1}{2} sin2x = 1$ in the interval $[0, 2\pi]$?

A. $\left\{0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}\right\}$ C. $\{0, \pi\}$

B. $\left\{0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi\right\}$ D. $\left\{0, \frac{\pi}{4}, \pi\right\}$

251. What is $\cot(\arcsin(x))$ if $0 < x < 1$?

A. $\frac{x}{\sqrt{1-x^2}}$ B. $\frac{\sqrt{1-x^2}}{x}$ C. $\sqrt{1 - x^2}$ D. $\frac{1}{\sqrt{1-x^2}}$

2006

252. Suppose an airplane is descending at a speed of 50 miles per hour at an angle of 30° below the horizontal line. What is the x- and y- components, respectively, of the velocity of the plane?

A. $50\sqrt{3}, 25$ B. $-25, 50\sqrt{3}$ C. $25, -25\sqrt{3}$ D. $-25\sqrt{3}, -25$

253. An observer on level ground is at a distance $10\sqrt{3}$ m from a building. The angle of elevation to the bottom of the windows on the second and third floors are 30° and 60°, respectively. What is the distance h between the bottoms of the windows?

A. 15m B. 20m C. $15\sqrt{3}$m D. 32m

254. If $\theta = 2\arctan\left(\frac{1}{2}\right)$, then which of the following is equal to $\sec(\theta)$?

A. $\frac{25}{3}$ B. $\frac{4}{5}$ C. $\frac{5}{3}$ D. $\frac{5}{4}$

2007

255. If angle θ is an acute angle of a right triangle, what is the length of the side adjacent to θ, given that the hypotenuse has 6 unit length and $\sec\theta = \frac{10}{3}$?

A. 1.8 units B. 2 units C. 18 units D. 20 units

256. What is the possible value of x that solves the equation: $\sin^{-1}x + \cos^{-1}\left(\frac{5}{3}\right) = \pi$?

A. $\frac{\pi}{3}$ B. $\frac{3}{5}$ C. $\frac{5\pi}{2}$ D. $\frac{4}{5}$

257. Two ships, one with angle of depression 60° due to east and the other with 30° due to west are observed from a plane 1000m above a sea. If the two ships are on the same line, what is the distance between the two ships?

A. $\frac{1}{\sqrt{3}}600$m B. 2000m C. $500\sqrt{3}$m D. $\frac{1}{\sqrt{3}}4000$m

258. What is the amplitude and period, respectively, of the graph of $f(x) = -6\sin x.\cos x$?

A. 3, π B. 6, π C. 3, $\frac{\pi}{2}$ D. 6, 2π

2008

259. If $\cot(\theta) = 2$, then which of the following is equal to $\csc(\theta)$?

$\sqrt{5}$ B. $\frac{2}{\sqrt{5}}$ C. $\frac{1}{\sqrt{5}}$ D. $\frac{1}{2}$

260. What is the amplitude and period, respectively, of the graph of $f(x) = 4\sin\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)$?

A. 4, $\frac{\pi}{3}$ B. 2, 3π C. 2, $\frac{2\pi}{3}$ D. 4, 3π

261. A boat on a sea sailed from its station toward North with constant speed of 80 km/hr. Another boat from the same station sailed 60° NE(North East) with constant speed of 100 km/hr. if the two boats start sailing at the same time, what is the straight distance between them after they have sailed for just 30 minutes?

A. $10\sqrt{42}$ km B. 90 km C. $10\sqrt{41}$ km D. $10\sqrt{21}$ km

262. What is the value of $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$?

A. $\frac{\pi}{4}$ B. $\frac{\pi}{2}$ C. $-\frac{\pi}{4}$ D. $-\frac{\pi}{2}$

2009

263. Which one of the following is true?

A. The amplitude of $f(x) = \sin 3x$ is 3. C. The period of $f(x) = \cos\left(\frac{1}{2}x - \frac{\pi}{3}\right)$ is 4π

B. The period of $f(x) = 2\sin 4x$ is π D. The amplitude of $f(x) = -5\cos(3x+2) - 2$ is 7

264. If $\theta = \arctan(2)$, then what is the value of $\sin(2\theta)$?

A. $\frac{2}{5}$ B. $\frac{4}{5}$ C. $\frac{4}{\sqrt{5}}$ D. $\frac{2}{\sqrt{5}}$

265. If $\cot\theta = \sqrt{8}$, and θ is in the first quadrant angle, then what is the value of $\csc\theta$?

A. $\frac{1}{3}$ B. 3 C. $\frac{\sqrt{8}}{3}$ D. $\frac{1}{\sqrt{8}}$

266. A patrol on a sea sailed from its station 7 km to the North; and changed its course and sailed $4\sqrt{2}$ km in the direction of 45° South-East. What is the shortest (straight) distance the boat should travel in order to return to its station?

A. 5 km B. 7 KM C. $5\sqrt{2}$ km D. $5+\sqrt{2}$ km

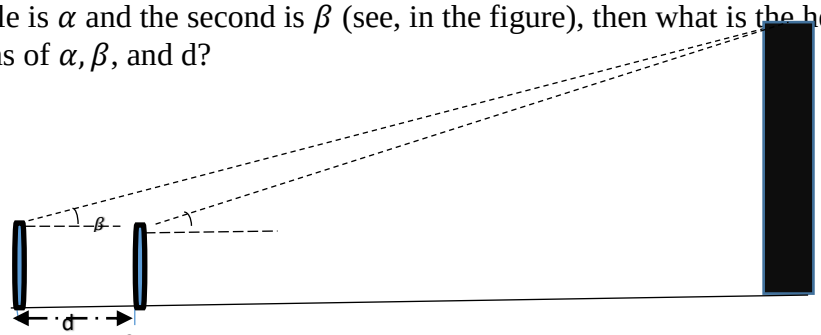
2010

267. What is the period (p) and the range(R) of $f(x) = 5\sin\left(\frac{1}{3}x + 2\right) + 3$?

A. $p = 6\pi,\ R = [-2, 8]$ C. $p = 6\pi,\ R = [-5, 5]$

B. $p = \frac{2\pi}{3},\ R = [-5, 5]$ D. $p = \frac{2\pi}{3},\ R = [-2, 8]$

268. In order to measure the height of a tower, suppose a surveyor takes two sightings from a transit 1m high which are positioned d meters apart on the same ground level as in the figure below. If the first measured angle is α and the second is β (see, in the figure), then what is the height of the tower (in meter) in terms of α, β, and d?



A. $\frac{\tan\alpha\tan\beta}{(\tan\alpha-\tan\beta)d}+1$ C. $\left(\frac{\tan\alpha-\tan\beta}{\tan\alpha\tan\beta}\right)d+1$

B. $\left(\frac{\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}\right)d+1$ D. $\left(\frac{\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}\right)d+1$

269. If θ is a fourth quadrant angle and $\sec\theta=\sqrt{2}$ then $\csc\theta$ is equal to?

A. $\frac{-1}{\sqrt{2}}$ B. $-\sqrt{2}$ C. $\frac{1}{\sqrt{2}}$ D. $\sqrt{2}$

2011

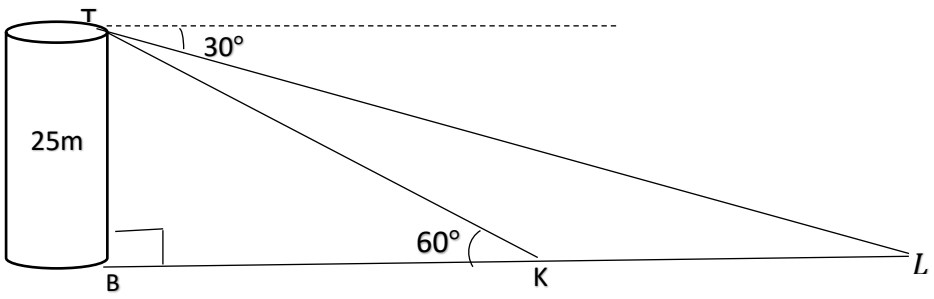
270. Ship A and B depart from the same point at the same time on the course N60°E and N40°E, respectively. If the speed of ship A is 20 km/hr and the speed of ship b is 30 km/hr, what is the distance between the two ships just after 30 minutes of their departure? **(you may take $\cos(40°)=0.77, \cos(20°)=0.94, \sin(20°)=0.34$)**

A. $\sqrt{40}$ km B.$\sqrt{43}$ C. $\sqrt{50}$ km D. $\sqrt{53}$ k

271. What is the value of $\cot 270° + 2\cos 90° + 4sec^2 180°$?

A. -2 B. 8 C. 4 D. 7

272. The diagram below is a representation of 25m vertical observation of tower TB and two cars K and L on a road. The angle of depression from T to car L is 30°. The angle of elevation from car K to the top of the tower is 60°. B, K and L lie in a straight line and lie on the same horizontal plane as the base of the tower.



What is the distance between the two cars?

A. $\frac{50\sqrt{3}}{3}$ M B. $50\sqrt{3}$ C. $\frac{50\sqrt{3}}{2}$ m D. $50+\sqrt{3}$ m

273. What is the solution set of $sin^2x-\sin x\cos x=0$?

A. $\left\{0,\pi,\frac{5\pi}{4},2\pi\right\}$ C. $\left\{0,\frac{\pi}{4},\pi,\frac{5\pi}{4},2\pi\right\}$

B. $\left\{0,\frac{\pi}{4},\pi,2\pi\right\}$ D. $\left\{0,\frac{\pi}{4},\pi,\frac{5\pi}{4}\right\}$

From grade 12

Unit 1

2004

274. Let $a_{n=}n + \cos n\pi, n = 1, 2, 3, \ldots$ be the n^{th} terms of a sequence. Then which of the following is true?
   A. The fifth term is 6
   B. The 10^{th} term is 9
   C. $a_7 + a_8 = 15$
   D. $a_7 - a_6 = 1$

275. What is the sum of the series $\sum_{n=0}^{\infty}(2^2 3^{2-n})$
   A. 6 B. 18 C. 27 D. ∞

276. If $\{A_n\}$ is an arithmetic sequence with first term A_1 =5 and the fifth term A_5 =21, then the partial sum$\sum_{n=1}^{30} A_n$ is equal to:
   A. 760 B. 780 C. 860 D. 870

277. A business association gets a net profit of Birr 3,000 at the end of each month. Just after the fifth year, its amount was Birr 230,000. What was the starting amount of the business (in Birr)?
   A. 47,000 B. 50,000 C. 53,000 D. 56,000

2005

278. If $\{a_n\}$ is a sequence such that $a_1 = 2$, and $a_{n+1=}a_n + 4$ for all $n \geq 1$, then $\sum_{n=1}^{35} a_n$ is equal to:
   A. 2460 B. 2458 C.2450 D. 2442

279. What is the 50^{th} term of the sequence 3, 10, 17, 24, 31 …?
   A. 310 B. 346 C. 510 D. 531

280. What is the sum of the series $\sum_{n=1}^{\infty}\left(\frac{2^{2n+1}}{5^{n-1}}\right)$?
   A. 40 B. 20 C. 10 D. 8

281. The population of certain country is 80 million with growth rate of 2% per year.

Given $(0.02)^9 = 5.12 \times 10^{-16}$,	$(1.02)^9 = 1.19$
$(0.02)^{10} = 1.024 \times 10^{-17}$,	$(1.02)^{10} = 1.22$

   Which of the following is the best approximation of the population (in million) after 10 years?
   A. 81.9 B. 86.8 C. 95.2 D. 97.6

2006

282. Which one of the following represents a geometric sequence?
   A. 3, 1,$\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$
   B. $\frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}, \frac{1}{6}, \ldots$
   C. 1, 3, 6, 10, 15, …
   D. $-3, 6, -9, 12, -15, \ldots$

283. What is the value of the sum $\sum_{n=1}^{\infty}\left(\frac{2^n+5^n}{10^n}\right)$
   A. 0.325 B. 1 C. $\frac{5}{4}$ D. $\frac{37}{9}$

284. What is the sum of the series $\sum_{n=1}^{\infty}(-1)^n (3)^{-2n}$?
   A. $-\frac{1}{8}$ B. -0.13 C. -0.1 D. $\frac{1}{8}$

2007

285. Which one of the following is an arithmetic sequence?
   A. 3, 5, 7, 11, …
   B. 3, 6, 12, 24, 48, …
   C. $-3, 6, -9, 12, -15, \ldots$
   D. 1, 3, 6, 10, 15, 21, …

286. A certain meeting hall has 20 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row, and so on. How many seats are there on the last (20^{th}) row of the hall?
   A. 46 B. 58 C. 760 D. 5240

287. What is the sum of all multiples of three between 20 and 200?
   A. 7,227 B. 6,570 C. 6,150 D. 5, 166

288. A ball is thrown vertically from ground up to a height of 16m. Each time it drops h meters, it rebounds 0.80h m. Noting that the ball travels every height of h twice, what is the total vertical distance travelled by the ball before it comes to rest?
   A. 40m B. 80m C. 160m D. 320m

2008

289. If $\{A_n\}_{n=1}^{\infty}$ is an arithmetic sequence such that its 1^{st} term $A_1 = -5$ and its 5^{th} term $A_5 = 15$, then its 11^{th} term A_{11} is equal to:
A. 40 B. 50 C. 45 D. 55

290. What is the sum of all multiples of 4 that are between 30 and 301?
A. 12,882 B. 11,288 C. 6,288 D. 6, 882

291. The n^{th} term of the sequence:1, -4, 9, -16, … is:
A. $a_n = (-2)^n$ C. $a_n = (-1)^{2n}n^2$
B. $a_n = (-1)^n n^2$ D. $a_n = (-1)^{n-1}n^2$

292. The sum of $\sum_{n=0}^{\infty} 5\left(\frac{2}{3}\right)^n$ is……..
A. 0 B. 15 C. $\frac{10}{3}$ D. 5

2009

293. Suppose a radioactive material loses one-third of its mass per year. If its current mass is 81 gram, then how much will its mass (in gram) just after 7 years?
A. 27 B. $\frac{1}{27}$ C. $\frac{128}{27}$ D. $\frac{128}{81}$

294. Which of the following is the sum of the series $5-\frac{10}{3}+\frac{20}{9}-\frac{40}{27}+-\cdots$?
A. -5 B.-3 C. 3 D. 5

295. What is the sum of $\sum_{n=1}^{30}(-1)^n\left(\frac{1}{n}+\frac{1}{n+1}\right)$
A. $-\frac{29}{30}$ B. $\frac{29}{30}$ C. $\frac{30}{31}$ D. $-\frac{30}{31}$

296. Which of the following relation holds for the sequence: $-10, -3, 4, 11, \ldots$?
A. $a_n = a_{n-1} - 8$ C. $a_n = a_{n-1} - 7$
B. $a_n = a_{n-1} + 7$ D. $a_n = a_{n-1} + 8$

2010

297. What is the value of $\sum_{n=2}^{20}\left(\frac{1}{n-1}-\frac{1}{n}\right)$?
A. $\frac{17}{20}$ B. $\frac{23}{20}$ C. $\frac{21}{20}$ D. $\frac{19}{20}$

298. The population of a certain city is increasing at a rate of 3% per year. If the population was 100,000 in 2010 E.C, then what will be the population in 2020 E.C?
(Given: $(1.03)^9 = 1.30$, $(1.30)^{10} = 134$, $(1.3)^9 = 10.60$, $(1.3)^{10} = 13.78$)
A. 134,000 B. 130,000 C. 1,060,000 D. 1,378.000

299. The sum of the first three consecutive terms of an arithmetic sequence is$\{A_n\}$, with $A_n > 0$ for all n, is 9 and the sum of their squares is 35, then what is the sum s_n of the first n terms?
A. $n^2 + 1$ B. $n^2 - 1$ C. n^2 D. $2n^2 + 1$

300. What is the sum of the series $\sum_{n=1}^{\infty}(3^n 4^{-n})$
A. ∞ B. $\frac{3}{16}$ C. 3 D. 4

2011

301. If $a_1 = 2, a_2 = 6, a_3 = 10, a_4 = 14,\ldots$, then $\sum_{n=1}^{100}(a_n) =$……..
A. 20,020 B. 20,200 C. 20.000 D. 22,000

302. Every day a person saves 5 cents more than the amount he saved on the previous day. His target is to save the total amount of 3225 cents by the end of 30 days. How much cents must to be the starting saving to meet the target?
A. 35 B. 25 C. 50 D. 60

303. If the second and fifth terms of a geometric progression are $-\frac{1}{2}$ and $\frac{1}{16}$, respectively, what is the sum of the first eight terms of the sequence?
A. $\frac{85}{128}$ B. $\frac{255}{256}$ C. $\frac{128}{85}$ D. $\frac{256}{255}$

304. What is the sum of the series $\sum_{k=0}^{\infty} 5\left(\frac{2^k+5^k}{10^k}\right)$?
A. 15 B. $\frac{25}{4}$ C. 5 D. $\frac{65}{4}$

Unit 2

2004

305. Which of the following is equal to $\lim_{n\to\infty}\frac{3\sqrt{n}-6n+5}{4n+1}$?

A. $-\frac{3}{2}$ B. 0 C. $\frac{3}{4}$ D. $\frac{5}{4}$

306. $\lim_{x\to 0}\frac{x\csc 3x}{x+1}$ is equal to:

A. $\frac{1}{3}$ B. 0 C. 1 D. 3

307. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x+4}$ is equal to:

A. e B. e^2 C. e^4 D. ∞

308. $\lim_{x\to 1^-}\frac{x^4-1}{|x-1|}$ is equal to:

A. 4 B. 0 C. -1 D. -4

309. Let $f(x)=\begin{cases}\frac{a}{x^2+1}, if\ x\le 1\\ \frac{x-1}{\sqrt{x}-1}, if\ x>1\end{cases}$ what is the value of a if f is continuous at x= 1?

A. 0 B. 0 C. 4 D. 8

2005

310. What is the value of $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{-\frac{x}{2}}$?

A. $\frac{1}{\sqrt{e}}$ B. $\sqrt{e}$ C. e^{-2} D. ∞

311. $\lim_{\to\infty}\frac{1-n-3n^2}{6n^2+1}$ is equal to:

A. $\frac{1}{6}$ B. $-\frac{1}{2}$ C. $-\frac{1}{6}$ D. $-\infty$

312. Let $(x)=\begin{cases}3^x+k,\ x\le 0\\ 3\frac{\sin(2x)}{x},\ x>0\end{cases}$, if f is continuous at $x=0$, then what is the value of k?

A. 6 B. 5 C. 2 D. 0

313. Which of the following is equal to $\lim_{x\to 1}\frac{\sqrt{x}-1}{x^2-1}$?

A. ∞ B. $\frac{1}{2}$ C. $\frac{1}{4}$ D. 0

314. Which of the following is equal to $\lim_{x\to 1}\frac{1-x}{1-\frac{1}{x^2}}$?

A. 1 B.0 C. $-\frac{1}{2}$ D. Doesn't exist

2006

315. The sequence $\left\{\frac{(n-1)(2n+1)}{1-n^2}\right\}_{n=1}^{\infty}$ converges to:

A. $-\infty$ B. -2 C. 0 D. 1

316. Given that $\lim_{x\to 3}f(x)=5$ and $\lim_{x\to 3}g(x)=11$, what is the value of $\lim_{x\to 3}\left(\frac{(f(x)-g(x))(g(x)-2f(x))}{g(x)^2-f(x)^2}\right)$?

A. $-\frac{66}{96}$ B.$-\frac{1}{16}$ C. 0 D. Doesn't exist

317. Let $f(x)=\begin{cases}a\frac{\sin x}{x-|x|}, & x<0\\ e^{-x}+\cos x, & x\ge 0\end{cases}$ if f is continuous at $x=0$, then what is the value of a?

A. 4 B.2 C. $-\frac{1}{2}$ D. -4

318. If $a_n=\left(\frac{n+3}{n+1}\right)^n$, then the limit of the sequence $\{a_n\}_{n=1}^{\infty}$ is equal to:

A. 1 B. $\frac{1}{2}e$ C. e^2 D. $+\infty$

319. Which of the following is equal to $\lim_{x\to\infty}\left(\frac{x}{x+2}\right)^{-3x}$?

A. e^6 B. e^{-3} C. $e^{\frac{-3}{2}}$ D. e^{-6}

2007

320. Which of the following sequences is a convergent sequence?

A. $1,\frac{1}{2},1,\frac{1}{3},1,\frac{1}{4},\dots$ C. $\left\{100^{109}-\frac{1}{100}n\right\}_{n=1}^{\infty}$

B. $\{(-1)^n\}_{n=1}^{\infty}$ D. $\left\{\sin\left(\frac{1}{n}\right)\right\}_{n=1}^{\infty}$

321. Let $f(x)=\begin{cases} a\frac{\sin 2x}{x}, & if\ x<0 \\ e^{2x}-2, & if\ x\geq 0 \end{cases}$ if f is continuous at $x=0$, then what is the value of a?

A. $\frac{1}{2}$ B. 2 C. $-\frac{1}{2}$ D. -2

322. Which one of the following is equal to $\lim_{x\to 9}\frac{x-9}{3-\sqrt{x}}$?

A. 6 B. -3 C. -6 D. ∞

323. Let $f(x)=\begin{cases} 3-e^{2x}, if & x<0.5 \\ \frac{2^x-5}{x+1}, & if\ \ x\geq 0.5 \end{cases}$. if c is a zero of f, that is $f(c)=0$, then which one of the following intervals must contain c?

A. $(-\infty,0]$ B.$[0,1]$ C. $[1,2]$ D. $[2,3]$

324. Which one of the following is equal to $\lim_{x\to\infty}\left(\frac{3x}{3x+2}\right)^{-3x}$

A. e^2 B. e^{-3} C. e^{-2} D. e^3

2008

325. The left hand side limit, $\lim_{x\to 0^-}\frac{xe^x-|x|}{x}$ is equal to:

A. 0 B. 2 C. 1 D. Doesn't exist

326. Which one of the following is equal to $\lim_{x\to 0}\frac{2x+\tan x}{x\sec x}$?

A. 2 B. 0 C. 1 D. 3

327. In which interval the sequence $\left\{\frac{(-1)^n}{3n}\right\}_{n=1}^{\infty}$ is bounded?

A. $\left[\frac{-1}{9},\frac{1}{12}\right]$ B. $\left[\frac{-1}{3},\frac{1}{6}\right]$ C. $\left[\frac{-1}{6},\frac{1}{3}\right]$ D. $\left[\frac{-1}{12},\frac{1}{9}\right]$

328. Which one of the following is true about the function $(x)=\begin{cases}\frac{x^2}{x}, & x\neq 0 \\ 0, & x=0\end{cases}$?

A. f is continuous except at $x=0$ C. f is continuous everywhere
B. f has an infinite discontinuity at $x=0$ D. f has a vertical asymptote at $x=0$

329. $\lim_{x\to\infty} x\sin\frac{1}{x}$ is equal to:

A. 0 B. 1 C. ∞ D. -1

2009

330. Which one of the following is a convergent sequence?

A. $\left\{\left(\frac{5}{3}\right)^n\right\}$ B.$\left\{\frac{n^2}{n+1}\right\}$ C. $\left\{\frac{2n}{n+1}\right\}$ D. $\left\{\frac{(-1)^n}{3}\right\}$

331. What is the value of k so that $f(x)=\begin{cases}\frac{\tan(2x)}{x}, & if\ x>0 \\ k-e^{2x}, & if\ x\leq 0\end{cases}$ is continuous at $x=0$?

A. 2 B. 3 C. 1 D. 0

332. If f is continuous at x= 0 and $g(x)=\sqrt{x}\left(2f(x)+\frac{3}{\sqrt{x}}\right)$ for all $x>0$, then what is the value of $\lim_{x\to 0^+} g(x)$?

A. 0 B. 2 C. 3 D. 5

333. What is the value of $\lim_{x\to 0}\frac{\sin x\cos 2x}{x^2+3x}$?

A. $\frac{1}{3}$ B. $\frac{2}{3}$ C. 1 D. 2

334. If $a\neq 0$, then what is the value of $\lim_{x\to a}\frac{x^2-a^2}{x^4-a^4}$?

A. $\frac{1}{a^2}$ B. $\frac{1}{2a^2}$ C. $\frac{1}{6a^2}$ D. 0

2010

335. Which one of the following is a convergent sequence?

A. $\left\{\frac{1+2^n}{3^n}\right\}$ B. $\left\{\frac{1}{n}+\sin(n)\right\}$ C. $\left\{\frac{1-3^n}{2^n}\right\}$ D. $\left\{\frac{(-1)^n}{2}\right\}$

336. What are the greatest lower bound and the least upper bound of the sequence $\left\{(-1)^n\left(1+\frac{1}{n}\right)\right\}$, respectively?

A. -2 and 2 B. $-\frac{3}{2}$ and 2 C. -2 and $\frac{3}{2}$ D. -2 and $-\frac{3}{2}$

337. What is the value of $\lim_{x\to 0} \frac{1}{x^2} sin^2\left(\frac{x}{2}\right)$?

A. $\frac{1}{2}$ B. $\frac{1}{4}$ C. 2 D. 4

338. What are the values of a and b so that the function $\begin{cases} x+1, & x<1 \\ ax+b, & 1\le x<2 \\ 3x, & x\ge 2 \end{cases}$ is continuous everywhere?

A. $a=4, b=-2$ C. $a=4, b=2$

B. $a=-4, b=-2$ D. $a=-4, b=2$

339. If $f(x)=\frac{|x|}{x}$ and $g(x)=\frac{x+2}{x^3-4x}$,then what is the value of $\lim_{x\to -2}(f(x)+g(x))$?

A. $-\frac{9}{8}$ B. ∞ C. $\frac{9}{8}$ D. $-\frac{7}{8}$

340. Let $\{a_n\}$ be a sequence with $a_1=a, a_2=f(a_1)=f(a), a_3=f(a_2)=f\big(f(a)\big), \dots, a_{n+1}=f(a_n)$, where f is continuous function. If $\lim\ a_n=5$, what is the value of $f(5)$?

A. 5 B. 5^n C. 5^{n+1} D. 1

341. What is the value of $\lim_{n\to\infty} \sum_{k=1}^{n} \left[\frac{2}{n}\left(\left(\frac{2k}{n}\right)^3+5\left(\frac{2k}{n}\right)\right)\right]$

A. 4 B. 14 C. 10 D. 18

2011

342. What is the greatest lower bound of the sequence $\left\{(-1)^n \frac{1}{n+1}\right\}_{n=1}^{\infty}$?

A. -1 B. 0 C. $-\frac{1}{2}$ D. $\frac{1}{2}$

343. What is the value of $\lim_{n\to\infty}\left(\frac{3^n+2^n}{6^n}\right)$?

A. 1 B. 0 C. ∞ D. $\frac{5}{6}$

344. What is the value of $\lim_{h\to 0} \frac{8\left(\frac{1}{2}+h\right)^2-8\left(\frac{1}{2}\right)^2}{h}$?

A. 0 B. 4 C. 8 D. The limit does not exist

345. What is the value of the left-hand side limit, $\lim_{x\to 0^-} \frac{\sin x}{x^2}\sqrt{4x^3+9x^2}$?

A. -3 B. 3 C. 2 D. Does not exist

346. What is the value of $\lim_{x\to 2} \frac{x^3-8}{x^2-4}$?

A. 2 B. 3 C. -3 D. The limit does not exist

347. What is the limit of the sequence $1, \frac{2}{2^2-1^2}, \frac{3}{3^2-2^2}, \frac{4}{4^2-3^2}, \dots$?

A. 0 B. 1 C. $\frac{1}{2}$ D. $\frac{3}{2}$

348. If $f(x)=x^3-2x+1$, then which one of the following is **Not** true?

A. $f(c)=0$ for some c$\in[-2,0]$ C. $f(c)=\frac{1}{4}$ for some c$\in[-1,0]$

B. $f(c)=\frac{1}{2}$ for some c$\in$ [0 1] D. $f(c)=3$ for some c$\in[1,2]$

Unit 3

2004

349. If $f(x)=\pi^2$, then $f'(x)$ is equal to:

A. 2π B. π^2 C. 1 D. 0

350. Let $(x)=\frac{6x}{x+a}$. For value of a is $f'(a)=1$?

A. $\frac{1}{3}$ B. $\frac{2}{3}$ C. $\frac{3}{2}$ D. 3

351. If $g(x)=\frac{f(x)}{x+1}+(f(x))^2$, $f\ (1)=8$ and $f'(1)=2$, then $g'(1)$ is equal to:

A. 36 B. 31 C. 25 D. 16

352. If $f(x)=xe^{3x}-\cos(2x)$, then $f''(0)$ is equal to:

A. 0 B. 2 C. 6 **D**. 10

2005

353. If $f(x) = x^2 + 2\ln x$, then what is $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$?

A. 5 B. 1 C. 2 D. 0

354. If $f(x) = e^{2x} + x - 3\cos(x)$, then $f''(x)$ is equal to:

A. $e^{2x} + 1 - 3\sin(x)$ C. $4e^{2x} - 3\cos(x)$

B. $e^{2x} + 1 + 3\sin(x)$ **D**. $4e^{2x} + 3\cos(x)$

355. If $g(x) = xf(x) - \sqrt{f(x)}$ and $f(x) = f'(2) = 4$, then which of the following is equal to $g'(x)$?

A. 11 B. 8 C. 2 D. 0

356. Which one of the following is the equation of the tangent line to the graph of $f(x) = \frac{1}{x+1} + \cos x$ at $(0, f(0))$?

A. $x + y = 1$ C. $x + y = 2$

B. $x - y = -2$ D. $x + 4y = 2$

357. Which one of the following is true about the derivative of $f(x) = x|x|$?

A. f is not differentiable at $x = 0$ C. $f'(x) = 2x$, for every $x \in (-\infty, \infty)$.

B. $f'(x) = 2|x|$, for every $x \in (-\infty, \infty)$. D. $f'(x) = |x| + x$, for every $x \in (-\infty, \infty)$.

2006

358. If $f(x) = \frac{x^2}{1+xg(x)}$, $g(2) = 1$ and $g'(2) = 10$, then which of the following is equal to $f'(2)$?

A. -8 B. $-\frac{8}{9}$ C. $\frac{4}{3}$ D. $\frac{8}{9}$

359. The simplified form of the derivative of $f(x) = \frac{1+\sin x}{\cos x}$ is:

A. $\sec x + \tan x$ **B**. $\frac{1+\sin x}{cos^2 x}$ C. $\frac{1}{1+\tan x}$ D. $\frac{\cos x}{sin^2 x}$

360. If $(x) = x^2\sqrt{2x+12}$, what is the slope of the tangent line to the graph of f at $x = 2$?

A. -4 B. 2 C. 18 **D**. 17

361. If $f(x) = e^{2x}\sin x$, then $f''(x)$ is equal to:

A. $3e^{2x}\sin x - 4e^{2x}\cos x$ **C**. $e^{2x}(3\sin x + 4\cos x)$

B. $4e^{2x}\sin x + 2e^{2x}\cos x$ D. $e^{2x}(4\sin x - 3\sin x)$

362. If $y = \sin(3x^2)$, then the simplified form of $\frac{d^2y}{dx^2}$ is:

A. $-6\sin(3x^2)$ **C**. $6\cos(3x^2) - 36x^2 sin(3x^2)$

B. $6\cos(6x) - 6sin(3x^2)$ D. $x^2\cos(3x^2) + 6sin(3x^2)$

363. If $F(x) = f(2x+2).g(1-x^2)$, with $f(2) = -3, f'(2) = 4, g(1) = -5$, and $g'(1) = 1$, then what is the value of $F'(0)$?

A. -40 B. -20 C. 0 D. 19

2007

364. If $f(x) = \ln(\sqrt{x^2+1})$, which of the following is equal to $f'(x)$?

A. $\frac{x}{\sqrt{x^2+1}}$ **B**. $\frac{x}{x^2+1}$ C. $\frac{2x}{\sqrt{x^2+1}}$ D. $\frac{2x}{x^2+1}$

365. If $F(x) = f(2x+2).g(1-x^2)$, with $f(2) = -3, f'(2) = 4, g(1) = -5$, and $g'(1) = 1$, then what is the value of $F'(0)$?

A. -40 B. -20 C. 0 D. 19

366. Suppose the function f has the property that $f(x+y) = f(x)f(y)$ for all values of x and y and that $f(0) = 2, f'(0) = 1$. Then which one of the following represents the formula for the derivative $f'(x)$?

A. $f'(x) = 2f'(x) + 1$ C. $f'(x) = f(x) + 2$

B. $f'(x) = f(x) + 2f'(x)$ D. $f'(x) = 2f(x) - 1$

367. For what value of a and b is the function $f(x) = \begin{cases} 1 - 3x^2, for & x \le 1 \\ ax + b, for & x > 1 \end{cases}$ differentiable at $x = 1$?

A. $a = 6, b = 0$ C. $a = 0, b = -2$

B. $a = -3, b = 1$ **D**. $a = -6, b = 4$

2008

368. If $f(x) = 2x^5 - 3x$, then $\lim_{x\to 1} \frac{f(x)-f(1)}{x-1}$ is equal to:
A. 1 B. -1 C. 7 D. ∞

369. If $f(x) = e^{3x}\cos x - \frac{x+\pi}{x^2+2}$, then $f'(0)$ is equal to……..
A. $3 - \frac{\pi}{2}$ B. $\frac{3}{2}$ C. $\frac{7}{4}$ D. $\frac{5}{2}$

370. If $f(x) = \ln(\sqrt{x^2-5})$, which one of the following is equal to $f'(x)$?
A. $\frac{x}{x^2-5}$ B. $\frac{-x}{\sqrt{x^2-5}}$ C. $\frac{2x}{\sqrt{x^2-5}}$ D. $\frac{-x}{x^2-5}$

371. $\frac{d}{dx}(\ln e^{2x})$ is equal to:
A. $\frac{1}{e^{2x}}$ B. $\frac{2}{e^{2x}}$ C. 2x D. 2

372. If $f(x) = 2 + |x-3|$ for all x, then what is the value of the derivative $f'(x)$ at $x = 3$?
A. -1 B. 1 C. 2 D. does not exist

2009

373. Let f be a differentiable function with $f(1) = -1$ and $f'(1) = 1$. If $g(x) = [f(2x+1)+2]^2$, then what is the value of $g'(x)$?
A. 4 B. 2 C. -2 D. -4

374. If $(x) = \ln(x^2+2)$, then what is the value of $f''(1)$?
A. $\frac{3}{2}$ B. $\frac{5}{9}$ C. $\frac{2}{3}$ D. $\frac{2}{9}$

375. If $x^2 + xy = 10$, then what is the value of $\frac{dy}{dx}$ when $x = 2$?
A. $-\frac{7}{2}$ B. $\frac{2}{7}$ C. $\frac{3}{2}$ D. $\frac{7}{2}$

376. What is the equation of the tangent line to the graph of $f(x) = 3x^2 + 4x - 5$ at (1, 2)?
A. $10x - y - 8 = 0$ C. $-10x - y - 8 = 0$
B. $-10x + y - 8 = 0$ D. $10x + y - 8 = 0$

377. If $f(x) = \pi^2 + 1$, then what is the value of $f'(x)$?
A. $2\pi + 1$ B. 2π C. 2 D. 0

2010

378. If a function f is differentiable at a, then what is the value of $\lim_{x\to a} f(x)$?
A. $f(a)$ B.$f'(a)$ C. $f'(a) - f(a)$ D. 0

379. What is the slope of the tangent line to the graph of $f(x) = 3e^x + \sin x + 2$ at (0, 5)?
A. 2 B. 3 C. 5 D. 4

380. If $f(x) = k\ln x + e^{\sin x}$ and $f''(x) = -1$, then what is the value of k?
A. $2\pi^2$ B. π^2 C. π D. 2π

381. Let $f(x) = \ln(x\sqrt{x})$. Then $f'(x)$ is equal to:
A. $\frac{2x}{3}$ B. $\frac{\sqrt{x}}{2}$ C. $\frac{2}{x\sqrt{x}}$ D. $\frac{3}{2x}$

382. Which one of the following is equal to $\frac{d}{dx}\log_2\sqrt{6x}$?
A. $\frac{1}{2x\ln(2)}$ B. $\frac{3}{2x\ln(2)}$ C. $\frac{3x}{2\ln(2)}$ D. $\frac{1}{6x\ln(2)}$

2011

383. Let $f(x) = 2e^x - k\sin x + 1$. If the equation of the tangent line to the graph of f at (0, 3) is $y = 5x + 3$, then what is the value of k?
A. 3 B.-3 C. -5 D. 2

384. If $f(x) = \ln(2^{\tan x})$, then what is the value of $f'(x)$?
A. $\ln 2$ B. $-2\ln 2$ C. $\frac{\ln 2}{2}$ D. 1

385. If $h(x) = \sqrt{1+\sqrt{x}}$, then which of the following is equal to $h'(x)$?
A. $\frac{1}{2\sqrt{1+\sqrt{x}}}$ B. $\frac{1}{4\sqrt{x+x\sqrt{x}}}$ C. $\frac{x}{2\sqrt{1+\sqrt{x}}}$ D. $\frac{x}{4\sqrt{x+x\sqrt{x}}}$

386. If $(x) = \frac{1}{x}$, then what is the value of $f^{(n)}(x)$?
A. $f^{(n)}(x) = \frac{(-1)^n n!}{x^n}$ C. $f^{(n)}(x) = \frac{(-1)^n n!}{x^{n-1}}$

B. $f^{(n)}(x) = \frac{(-1)^{n+1}(n+1)!}{x^{n+1}}$ D. $f^{(n)}(x) = \frac{(-1)^n n!}{x^{n+1}}$

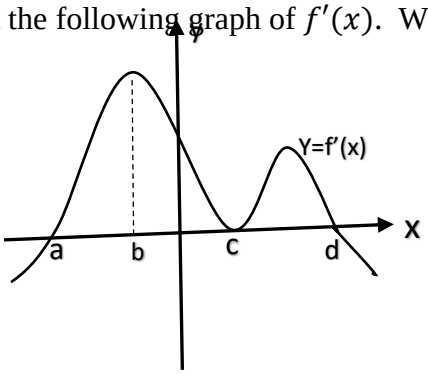
Unit 4

2004

387. Which of the following is necessarily true about a function $f(x)$?
A. If f is continuous at $x = a$, then it is differentiable at $x = a$.
B. If f is not differentiable at $x = a$, then $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$.
C. If f is differentiable at $x = a$, then $\lim_{x\to a^-} f(x) = f(a) = \lim_{x\to a^+} f(x)$.
D. If $f'(a) = 0$, then f attains its maximum value at $x = a$.

388. At what value(s) of x does $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ have a local maximum?
A. $x = 2$ B. $x = -1$ C. $x = 5$ D. $x = 2, x = -1$

389. Look at the following graph of $f'(x)$. Which of the following is true about f ?



A. f is increasing on $[c, d]$.
B. f is decreasing on $[b, c]$.
C. f has local minimum at d.
D. f has local extreme value at c.

390. A rectangular field of length l and width w meters for $w < l$. Has perimeter 400 meters. If a circular region of area w^2 is to be reserved for office purpose, what should be the length of the field (in meters) so that the area of the remaining region is maximum?
A. 50 B. 100 C. 120 D. 150

391. On which of the following intervals is the graph of $f(x) = \ln(x^2 + 1)$ concave upward?
A. $(-\infty,-1] \cup [1,\infty)$ B. $[0,\infty)$ C. $(-\infty,0]$ D. $[-1,1]$

392. Water is poured into a cylindrical tanker of radius 5m at a rate of $10m^3/min$ .what is the rate of change of the height of the level of water (in m/min) when it rises to 3m?
A. $\frac{3}{25\pi}$ B. $\frac{2}{5\pi}$ C. $\frac{1}{2\pi}$ D. $\frac{3}{5\pi}$

393. The volume of the solid which is generated when the region bounded by $y = \sqrt{x+1}$ and the x-axis from $x = 0$ to $x = 2$ is rotated bout the x-axis is equal to:
A. 4π B. 3π C. $\frac{4}{3}\pi$ D. $\frac{3}{4}\pi$

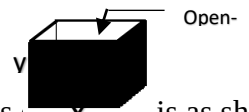
2005

394. On which of the following intervals does $f(x) = x^4 + 4x$ increase?
A. $(-\infty,-1]$ B. $(-\infty,0]$ C. $[-1,\infty)$ D. $(-\infty,\infty)$

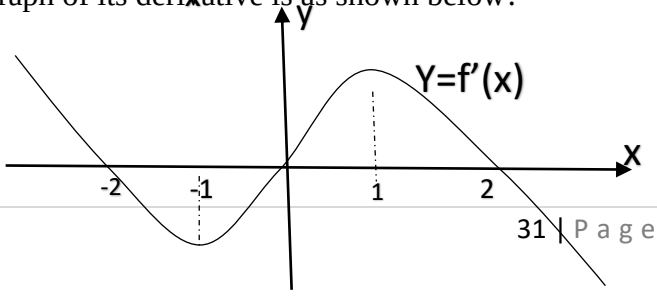
395. At which value(s) of x does $f(x) = \frac{1}{4}x^4 - 2x^2$ have a local maximum?
A. $x = 4$ B. $x = 0$ C. $x = -2, x = 2$ D. $x = 0,\ x = 2$

396. The volume V of a melting ice cube after t seconds is $V = 2000 - 4t + 0.2t^2$ (in cm^3). How fast is the volume changing (in cm^3/sec) when $t = 40\ seconds$?
A. 24 B. 15 C. -15 D. -24

397. A box seen below is to have a square base, an open top and volume of 32 cubic unit. If x is length of each side of its base and y is its height, how many units should x and y be in order to make the box with the smallest amount of material?
A. $x = 4,\ y = 2$ C. $x = \sqrt{8},\ y = 4$
B. $x = 2,\ y = 8$ D. $x = \sqrt{2},\ y = 16$



398. Suppose f is differentiable on $(-\infty,\infty)$ and the graph of its derivative is as shown below? Which of the following is true about f?
A. f *is decreasing on* $(-\infty,-1] \cup [1,\infty)$
B. f has a local minimum at $x = -2$
C. f is concave downward on $[0,\infty)$
D. f is concave up on $(-1, 1)$



2006

399. The total cost (in Birr) of producing x iron sheets per day is $C(x) = 1{,}000 + 10x - 0.5x^2$, $0 \leq x \leq 100$. What is the marginal (rate of change of) cost at a production level of 80 iron sheets?
A. 8.5 B. 20 C. 1,800 D. 5,800

400. Which one of the following is the set of all critical numbers of $f(x) = \frac{1}{3}x^3 - |4x - 1|$?
A. $\left\{\frac{1}{4}, 2\right\}$ B. $\left\{-2, \frac{1}{4}, 2\right\}$ C. $\{-2, 2\}$ D. $\left\{\frac{1}{4}\right\}$

401. If a box with a square base and open top is made from 1,200 cm^2 material, what is the largest volume of the box in cm^3?
A. 4,000 B. 8,000 C. 15,000 D. 3,000

402. Suppose equal squares are cut from each of the four corners of a square cardboard whose sides are 72 cm long. The resulting flaps are then folded up to form a box without a top. How long should be each of the four squares that has to be cut off to maximize the volume of the box?
A. 6 cm B. 12 cm C. 15 cm D. 24 cm

2007

403. What is the absolute maximum value of $f(x) = 2x^2 - x^4 - 4$ on $[0, 2]$?
A. -3 B. 3 C. -4 D. 12

404. The ozone level (in ppb-parts per billion) on a sunny day in a metropolitan area is given by the formula: $(t) = 80 + 12t - t^2$, t is time in hour and $t = 0$ corresponds to 9 A.M. What is the rate of increase of the ozone level after 3 hours (i.e at 12 A.M)?
A. 6ppb B. 12ppb C. 107ppb D. 113ppb

405. An object is moving along the parabola $y = \sqrt{2x}$ in the xy-plane. At what point on its path does the object become closest to the point (2, 0)?
A. $(3, \sqrt{6})$ B.$(1, 1)$ C. $(1, \sqrt{2})$ D. $(2, 2)$

406. Which one of the following is necessarily true?
A. If $f'(x) = 0$ for all x in the interval I, then $f(x) = 0$ for all x in I.
B. If $f(x) = x^2 \sin x + 5$, then there is $c \in (0, \pi)$ such that $f'(c) = 0$.
C. If $f(x) = e^x + x^2$, then f is increasing on $(-\infty, \infty)$.
D. If $f'(c) = 0$, then f attains its extreme value at $x = c$.

407. A ladder 6 m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at the rate (speed) of $\frac{1}{2}$ m/sec , how fast is the angle between the top of the ladder and the wall changing when the angle is $\frac{\pi}{4}$ rad?
A. $\frac{\sqrt{2}}{12}$ rad/sec B. $\frac{\sqrt{2}}{2}$ rad/sec C. $\frac{\sqrt{2}}{6}$ rad/sec D. $\frac{\sqrt{2}}{3}$ rad/sec

408. The number of shoes s that a factory can produce per day is a function of the number of hours t it operates:

$$s(t) = 40t \text{ for } 0 \leq t \leq 12.$$

The daily cost c, in Birr, to manufacture s shoes is given by the function

$$c(s) = 0.1s^2 + 90s + 800$$

If the factory operates for 10 hours, what is the cost it incurs in producing as much shoes it can within this time?
A. Birr 400 B. Birr 1,600 C. Birr 52,800 D. Birr 124,600

409. The total cost (in birr) of producing x radio sets per day is given by the expression $\frac{1}{4}x^2 + 35x + 25$ and the price per set at which they may be sold is given by $50 - \frac{1}{2}x$. What should be the daily output per day to a maximum total profit?
A. 50 B. 23 C. 10 D. 7

410. A company manufactures x computer sets per month. The monthly marginal profit (in Birr) is given by:

$$p'(x) = 165 - 0.1x, \text{for } 0 \leq x \leq 400.$$

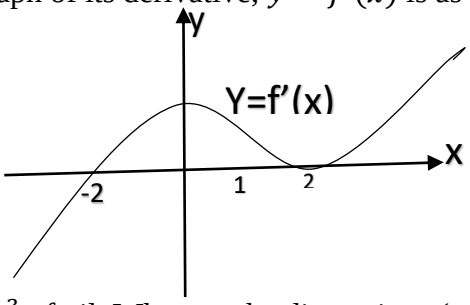
The company is currently manufacturing 10 sets of computers per month, but it is planning to increase production. What is the total change in the monthly profit if the monthly production is increased to 60 sets?

A. Birr 500 B. Birr 1, 865 C. Birr 8,075 D. Birr 18,635

2008

411. Let f be twice differentiable function on $\Re$. Which one of the following is necessarily true?
   A. If $f'(c) = 0$, at some c$\in \Re$, then f has relative extreme value at $x = c$.
   B. If $f'(x)$ is increasing, then the graph of $y = f(x)$ is concave upward.
   C. If $f'(x) = 0$, for all x$\in \Re$, then $f(x) = 0$ for all $x \in \Re$.
   D. If $f(x)$ is increasing, then $f''(x) \geq 0$ for all $\in \Re$.

412. Suppose $f(x)$ is differentiable on $(-\infty, \infty)$ and the graph of its derivative, $y = f'(x)$ is as shown in the figure below.



Which one of the following is true about $f(x)$?
   A. $f(x)$ is increasing on $(-\infty, 0) \cup (2, \infty)$.
   B. $f(x)$ has local extreme value at $x = 2$.
   C. $f(x)$ has a local minimum value at $x = -2$
   D. $f(x)$ has a local maximum value at $x = 0$

413. A closed cylindrical can is to be made to hold $1000cm^3$ of oil. What are the dimensions (r radius and h height) that will minimize the total surface area of the can?
   A. $r = \frac{\sqrt[3]{50}}{\pi}, h = 2\frac{\sqrt[3]{50}}{\pi}$
   B. $r = \frac{\sqrt[3]{500}}{\pi}, h = 2\frac{\sqrt[3]{500}}{\pi}$
   C. $r = \sqrt[3]{\frac{50}{\pi}}, h = 2\left(\sqrt[3]{\frac{50}{\pi}}\right)$
   D. $r = \sqrt[3]{\frac{500}{\pi}}, h = 2\left(\sqrt[3]{\frac{500}{\pi}}\right)$

414. The graph of $y = 5x^4 - x^5$ has a point of inflection at:
   A. $(3, 162)only$ B. $(4, 256)only$ C. $(0, 0)\ only$ D. $(0, 0)$ and $(3, 162)$

415. Which of the following is true about the function f defined by $f(x) = x^2 + e^{2x}$?
   A. f is decreasing for $x \geq 0$
   B. f is increasing for $x \leq 0$
   C. f has a relative minimum at $x = 0$
   D. f has a relative maximum at $x = 0$

2009

416. Which of the following is the set of critical numbers of $f(x) = \frac{4}{3}x^3 + |x|$?
   A. $\left\{\frac{1}{2}\right\}$ B. $\left\{0, \frac{1}{2}\right\}$ C. $\left\{0, -\frac{1}{2}\right\}$ D. $\left\{-\frac{1}{2}, 0, \frac{1}{2}\right\}$

417. Suppose f is continuous on $[2, 6]$ and the only solutions of the equation $f(x) = 7$ are $x = 2$ and $x = 5$. If $f(3) = 9$, then one of the following **CANNOT** be the value of $f(4)$:
   A. 5 B. 7.5 C. 8 D. 9

418. Suppose $f: (-\infty, \infty) \to \Re$ is differentiable and the graph of its derivative, $y = f'(x)$, is as shown in the figure below.

Which one of the following is true about f?
   A. f is increasing on $(1, \infty)$.
   B. f is concave upward on $(0, \infty)$.
   C. f has no relative maximum value.
   D. f has a relative minimum value at $x = 2$.

419. A tin can of volume $54\pi cm^3$ is to be made in the form of a circular cylinder that has both flat top and bottom. What is the base radius of the tin if it is to be made of the least amount of metal?
   A. 2 cm B. 3 cm C. 4 cm D. 6 cm

420. Air is being pumped into a spherical balloon so that its volume increases at a rate of 50 cm^3/se. How fast is the radius of the balloon increasing when the diameter is 5 cm?
   A. $\frac{1}{50\pi} cm/sec$ B. $\frac{1}{25\pi} cm/sec$ C. $\frac{5}{\pi} cm/sec$ D. $\frac{2}{\pi} cm/sec$

2010

421. What is the maximum value of the function $f(x) = x^4 - 2x^2$ on $[-2, 1]$?
   A. 8 B. 12 C. 24 D. 40

422. Which one of the following is NOT true about the function $f(x) = 3x^4 - 4x^3$?
   A. $(0, 0)$ is point of inflection of f.

B. 0 and 1 are critical numbers of f.

C. f is concave upward on $\left(0,\frac{2}{3}\right)$ and concave downward on $(-\infty,0)$ and $\left(\frac{2}{3},\infty\right)$

D. f is decreasing on $(-\infty,1)$ and increasing on $(1,\infty)$.

423. if $f(x)=\frac{1}{3}x^3+cx^2+ax+5$ has a local minimum value at $x=1$, then which one of the following is true about the possible values of a and c?

A. $a=3,\quad c=-2$ C. $a=-2c-1,\quad c<-1$

B. $a=-2c-1$, c is any real number. D. $a=-2c-1,\quad c>-1$

424. What is the maximum possible area of a rectangle in square units with diagonal of length 16 units?

A. 48 B. 128 C. 64 D. 256

2011

425. Which one of the following is the set of critical numbers of $(x)=\frac{3}{8}x^{\frac{8}{3}}-6x^{\frac{2}{3}}$?

A. $\{-2,2\}$ B. $\{-1,1\}$ C. $\{-1,0,1\}$ D. $\{-2,0,2\}$

426. If $f(x)=ax^3+\frac{b}{x}+5$ has a local minimum value at $(2,-3)$, what are the values of a and b?

A. $a=-\frac{1}{4},\ b=12$ C. $a=\frac{1}{4},\ b=12$

B. $a=\frac{-1}{4},\ b=-12$ D. $a=\frac{1}{4},\ b=-12$

427. At value of x does the function $f(x)=\frac{4x^3}{3}-x^4$ attains its maximum value?

A. -1 B. 0 C. 1 D. $\frac{4}{3}$

428. A man wants to fence two identical rectangular enclosures in a field alongside a straight river as shown in the following figure.



What is the maximum area of each enclosure that he can make with 192 m fencing material if the side along the river do not need a fence?

A. $1530m^2$ B. $1564m^2$ C. $1664m^2$ D. $1536m^2$

429. A water tank is a rectangular parallelepiped with base length 3m, width 2m and height 2.5m. if water is flowing into the tank at the rate of $0.12m^3$/sec , then how fast does the level of water rises up in the tank?

A. 0.02 m/sec B. 0.03 m/sec C. 0.04 m/sec D. 0.06 m/sec

Unit 5

2004

430. Which of the following is equal to $\int\frac{x}{x^2+2x+1}dx$?

A. $\ln(x^2+2x+1)+c$ C. $\ln(x^2+2x+1)-\frac{1}{x+1}-c$

B. $\ln|x+1|-\frac{1}{x+1}+c$ D. $\ln(x^2+2x+1)+\frac{1}{x+1}+c$

431. $\int\frac{e^{2x}-4x}{xe^{2x}}dx$ is equal to:

A. $\ln|x|+2e^{-2x}+c$ C. $\ln|x|+4e^{-2x}+c$

B. $\ln|x|-4e^{2x}+c$ D. $-\frac{1}{x^2}+2e^{-2x}+c$

432. Which of the following is equal to $\int_0^1\frac{(x-1)^2}{x+1}dx$?

A. $-\frac{3}{2}+\ln 16$ C. $-\frac{5}{2}+4\ln 2$

B. $-\frac{5}{2}+\ln 15$ D. $\frac{5}{2}+\ln 16$

433. If $f(x)=\begin{cases}\sin x, & x\le 0\\ 3x\sqrt{x^2+1}, & x>0\end{cases}$, which of the following is equal to $\int_{\frac{-\pi}{2}}^{\sqrt{8}}f(x)dx$?

A. 17 B. 25 C. 27 D. $3\sqrt{8}$

434. Which of the following is an anti-derivative of $f(x)=\tan x$?

A. $\frac{1}{2}tan^2x$ B. sec^2x C. $\ln(\cos x)$ D. $\ln(\sec x)$

435. The volume of the solid which is generated when the region bounded by $y=\sqrt{x+1}$ about the x-axis from $x=0$ to $x=2$ is rotated about the x-axis is equal to?

A. 4π B. 3π C. $\frac{4}{3}\pi$ D. $\frac{3}{4}\pi$

2005

436. What is the ant-derivative of $f(x)=\frac{2}{4x^2+4x+1}$?

A. $\frac{1}{2x+1}$ B. $\frac{-2}{2x+1}$ C. $-\frac{1}{2x+1}$ D. $\ln(4x^2+4x+1)$

437. Which one of the following is equal to $\int(1+x)3^x dx$?

A. $(1+x)3^x-3^x+c$ C. $(1+x)(3^x\log_3 e)-3^x(\log_3 e)^2+c$

B. $(1+x)3^x+3^x(\log_3 e)+c$ D. $(1+x)(3^x\log_3 e)-3^x(\log_3 e)+c$

438. Which one of the following is equal to $\int\frac{1}{x^2+x}dx$?

A. $\ln|x^2+x|+c$ C. $\ln|x|-\ln|x+1|+c$

B. $2\ln|x+1|+\ln|x|+c$ D. $\ln|x|+\ln|x+1|+c$

439. What is the value of $\int_0^{\frac{\pi}{2}}2x\cos x\,dx$?

A. $\pi-2$ B. $\frac{\pi}{2}+1$ C. $\pi+2$ D. $\frac{\pi}{2}-1$

440. Which one of the following is equal to $\int\frac{\ln(xe^x)}{x}dx$?

A. $\ln|x|+\frac{1}{2}e^x+c$ C. $\ln|x|+e^{2x}+c$

B. $\frac{1}{2}(\ln x)^2+x+c$ D. $-\frac{1}{x^2}+(\ln x)^2+c$

441. What is the area of the region between the graphs of $y=x^2$ and $y=-x+2$, where $0\le x\le 2$?

A. 3 B. 2 C. $\frac{3}{2}$ D. $\frac{2}{3}$

2006

442. A water tank is a circular cylinder with base radius 2m and height 3m. if the tank is empty and water is pumped into it at rate of 2 m^3/min, how long does it take for the tank to be full?

A. 1.5 min B. $\frac{3}{2}\pi$ min C. 6π min D. 12min

443. If $F(x)$ is anti-derivative of $f(x)=1-\frac{2}{x^2}$ and $F(1)=0$, then $F(2)$ is equal to:

A. 0 B. $\frac{1}{2}$ C. $-\frac{1}{2}$ D. 3

444. What is the area of the region between the graphs of $y=-x^2+2$ and $y=|x|$, where $-1\le x\le 2$?

A. $\frac{11}{6}$ B. $\frac{25}{6}$ C. $\frac{7}{3}$ D. $\frac{11}{3}$

445. What is the derivative of $(x)=\int_0^{(x^2+\pi)}\frac{dt}{\sin t+1}$?

A. $\frac{\cos x}{\sin(x^2+\pi)+1}$ C. $\frac{2x\cos x}{\sin(x^2+\pi)+1}$

B. $\frac{2x}{\sin(x^2+\pi)+1}$ D. $\int_0^{2x}\frac{dt}{\sin t+1}$

446. Which one of the following is equal to $\int_0^{\frac{\pi}{2}}(\frac{x-\sin x}{\sec x})dx$?

A. $\frac{\pi-3}{2}$ B. $\frac{\pi-1}{2}$ C. $\frac{3-\pi}{2}$ D. $\frac{\pi+3}{2}$

447. What is the area of the region between the graph of $f(x)=-x^2+4x-3$ and the x-axis from $x=0$ to $x=3$?

A. $-\frac{2}{3}$ B. $\frac{2}{3}$ C. $\frac{4}{3}$ D. $\frac{8}{3}$

448. Which one of the following is equal to $\int\frac{x+\ln(x+1)}{(x+1)^2}dx$?

A. $\ln(x+1)+\frac{x}{x+1}+c$ C. $(x+1)^2-\frac{x}{x+1}+c$

B. $(x+1)^2+\frac{x}{x+1}+c$ D. $\frac{x\ln(x+1)}{x+1}+c$

2007

449. If $f(x) = 2x(x^2+1)^4$ which one of the following is an anti-derivative of $f(x)$?
A. $\frac{2x}{5}(x^2+1)^5 + c$ C. $\frac{x}{5}(x^2+1)^5 + c$
B. $\frac{2}{5}(x^2+1)^5 + c$ D. $\frac{1}{5}(x^2+1)^5 + c$

450. What is the area of the region between the graphs of $y = \sin x$ and the x-axis where $0 \le x \le 2\pi$?
A. 4 B. 4π C. 2 D. 2π

451. If f and g are continuous on $\Re$ and $a, b \in \Re$, which one of the following is necessarily true?
A. If $\int_a^b f(x)dx = \int_a^b g(x)dx$, then $f(x) = g(x)$ for all x$\in [a,b]$.
B. If $f'(x) = g'(x)$ for all x$\in [a,b]$, then $\int_a^b f(x)dx = \int_a^b g(x)dx$.
C. If $f(x) \ge 3$ for all x$[-2,2]$, then $\int_{-2}^{2} f(x) \ge 12$.
D. $\int_a^b f(x)dx = \int_b^a f(x)dx$.

452. Which one of the following is equal to $\int \frac{\ln x + x^2 e^x}{x}$
A. $\frac{1}{2}ln^2x + e^x(x^2-1) + c$ C. $\frac{1}{x^2}\ln x + e^x(x-1) + c$
B. $\frac{1}{2}ln^2x + e^x(x-1) + c$ D $-\frac{1}{x^2}\ln x + e^x(x^2-1) + c$

453. The derivative of the function $F(x) = \int_{-x}^{x} \frac{1}{1+t}dt$ is equal to:
A. $\frac{2}{1-x^2}$ B. $\ln|1+x|$ C. $\frac{1}{1+x}$ D. $\ln\left|\frac{1+x}{1-x}\right|$

2008
454. Which of the following is equal to $\int \frac{(\ln x)^2 + x^2\cos x}{x}dx$?
A. $\frac{1}{x^2}\ln x + x\sin x - \cos x + c$ C. $\frac{1}{3}(\ln x)^3 + x\sin x + \cos x + c$
B. $\frac{1}{3}(\ln x)^3 + x\sin x - \cos x + c$ D. $\frac{1}{x^2}\ln x + x\sin x + \cos x + c$

455. The volume of the solid generated when the region bounded between the graph of $y = \begin{cases} x^2, & 0 \le x \le 2 \\ 4, & 2 \le x \le 3 \end{cases}$ and the x-axis is:
A. $\frac{32\pi}{5}$ B. $\frac{112\pi}{5}$ C. $\frac{112\pi}{3}$ D. $\frac{64\pi}{5}$

456. If $(x) = 3x^2\sqrt{x^3-1}$, then which one of the following is an anti-derivative of $f(x)$?
A. $\frac{3x}{2}(x^3-1)^{\frac{3}{2}} + c$ C. $\frac{3}{2}(x^3-1)^{\frac{3}{2}} + c$
B. $\frac{2}{3}(x^3-1)^{\frac{3}{2}}$ D. $\frac{3}{2}(x^3-1)^{\frac{2}{3}} + c$

457. $\int_0^3 (x+1)^{\frac{1}{2}}dx$ is equal to:
A. $\frac{21}{2}$ B. $\frac{14}{3}$ C. 7 D. $\frac{16}{3}$

458. Given $(x) = \begin{cases} -x+1, & x < 0 \\ \cos \pi x, & x \ge 0 \end{cases}$, then $\int_{-1}^{1} f(x)dx =$.....
A. $\frac{1}{2} + \frac{1}{\pi}$ B. $\frac{1}{2} - \frac{1}{\pi}$ C. $\frac{1}{2}$ D. $-\frac{1}{2}$

459. The value of $\int_0^1 (x+1)\, e^{(x^2+2x)}dx$ is:
A. $\frac{e^3}{2}$ B. $\frac{e^4-e}{2}$ C. $\frac{e^3-1}{2}$ D. $e^3 - 1$

2009
460. If $f(0) = -1, f(1) = 2$ and $f'(x)$ is continuous on $[0,1]$, then which of the following is equal to $\int_0^1 f'(x)\sqrt{2+f(x)}$ dx ?
A. $\frac{16}{3}$ B. $\frac{14}{3}$ C. $\frac{8}{3}$ D. $\frac{4}{3}$

461. What is the value of $\int_0^{\ln\sqrt{3}} \frac{e^x}{e^{-x}+e^x}dx$?
A. $\frac{1}{2}\ln 2$ B. $\ln 4$ C. $\ln 2$ D. 1

462. What is the value of the area of the region enclosed by the graph of $f(x) = e^x$ and $g(x) = x$ between the lines $x = -1$ and $x = 1$?

A. $\frac{e^2-1}{e}$ B.$e^2 - \frac{1}{e}$ C. $e^2 - \frac{1}{e} + 2$ D. $e - \frac{1}{e} + 2$

463. Which of the following is equal to the volume of the solid generated when the region bounded by the graph of $y = 2\sqrt{x+1}$ and the x-axis, when $0 \le x \le 2$, rotates about the x-axis?

A. 8π B. $8\pi^2$ C. 16π D. 16π

464. A particle moves along the x-axis with velocity given by $v(x) = 3t^2 + 6t$ for time $t \ge 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at $t = 1$?

A. 6 B. 9 C. 11 D. 12

465. Which of the following is equal to $\int x(e^x + \sin(x^2))dx$?

A. $e^x(x+1) + \frac{1}{2}\sin(x^2) + c$ C. $e^x(x-1) - \frac{1}{2}\cos(x^2) + c$

B. $e^x(x-1) - \frac{1}{2}\sin(x^2) + c$ D. $e^x(x-1) + \frac{1}{2}\cos(x^2) + c$

466. If $2 \le f'(x) \le 4$ for all values of x, then the value of $f(8) - f(2)$ is between which of the following numbers?

A. 14 and 24 B. 12 and 24 C. 12 and 18 D. 8 and 10

2010

467. What is the area of the region enclosed by the graph of $y^2 = x + 1$ and $y^2 = -x + 1$?

A. $\frac{3}{8} sq.units$ B. $\frac{4}{3} sq.units$ C. $\frac{8}{3} sq.units$ D. $\frac{3}{4} sq.units$

468. What is the value of $\int_1^2 \frac{x+4}{x(x+2)} dx$?

A. 2 B. $\ln 3$ C. $\ln 2$ D. $\ln 4 - \ln 2$

469. If $f'(x) = e^{x-1} + 3x^2 - \frac{1}{x}$ and $f(1) = 5$, what is $f(x)$?

A. $f(x) = e^{x-1} + 3x^2 + \frac{1}{x^2} + 2$ C. $f(x) = e^{x-1} + 3x^2 - \frac{1}{x} + 5$

B. $f(x) = e^{x-1} - x^3 + \ln x + 5$ D. $f(x) = e^{x-1} + x^3 - \ln x + 3$

470. What is the value of $\int_1^9 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$?

A. $e^3 - e$ B. $\frac{e^3}{3} - e$ C. $e\left(e^2 - \frac{1}{3}\right)$ D. $2(e^3 - e)$

471. What is the value of $\int x\sqrt{1-x^2}$dx?

A. $-\frac{1}{3}(1-x^2)^{\frac{3}{2}} + c$ C. $(1-x^2)^{\frac{3}{2}} + c$

B. $-2x\sqrt{1-x^2} + c$ D. $\frac{1}{2}\sqrt{1-x^2} + c$

472. What is the value of $\int \frac{1}{x}(\ln x + x^2 e^{-x})dx$?

A. $\frac{1}{2}x^2 \ln x - (x+1)e^{-x} + c$ C. $\frac{1}{2}x^2 \ln x + (2-x)e^{-x} + c$

B. $\frac{1}{2}ln^2 x - (x+1)e^{-x} + c$ D. $\frac{1}{2}ln^2 x + (2-x)e^{-x} + c$

473. A cylindrical tank whose inner diameter is 2 m and contains $4\pi m^3$ oil. If the oil is discharged from the tank at the rate of $\frac{2\pi}{3} m^3/min$, then how long (in min) does it take for the tank to be empty?

A. $\frac{4}{3}$ B. 4 C. 12 D. 6

2011

474. If $F(X) = \int_0^x e^{-t}\, dt$, then what is the value of $F'(x)$?

A. e^{-x} B. $-e^{-x} - 1$ C. $\frac{e^{-x+1}}{-x+1}$ D. $-e^{-x}$

475. What is the value of $\int_1^2 \frac{\ln x}{x^2}$dx?

A. $-\frac{\ln 2}{2} - \frac{1}{2}$ B. $\frac{\ln 2}{2} - \frac{1}{2}$ C. $-\frac{\ln 2}{2} + \frac{1}{2}$ D. $\frac{\ln 2}{2} + \frac{1}{2}$

476. What is the value of $\int \frac{x^{e^{-1}} + e^{x-1}}{x^e + e^x} dx$?

A. $\frac{1}{2}\ln(x^e + e^{x+1}) + c$ C. $\frac{1}{2}\ln((x+1)^e + e^{x+1}) + c$

B. $\frac{1}{2}\ln(x^e + e^x) + c$ D. $\frac{1}{2}\ln(x^{e+1} + e^{x+1}) + c$

477. What is the area of the region bounded by the lines $x = 0, x = 2, y = 1$ and the curve $y = e^{2x}$?

A. $\frac{e^4}{2} - \frac{5}{2}$ B. $\frac{e^2}{2} - \frac{1}{2}$ C. $\frac{e^3}{3} - \frac{1}{3}$ D. $\frac{e^4}{2} + \frac{5}{2}$

478. If the region enclosed by the graphs of $f(x) = x^2$ and $f(x) = x^3$ from $x = 0$ to $x = 1$ rotates about the x-axis, what is the volume of the solid revolution (in cubic units)?

A. $\frac{2\pi}{27}$ B. $\frac{2\pi}{25}$ C. $\frac{2\pi}{5}$ D. $\frac{2\pi}{35}$

479. What is the value of $\int 4x\left(\ln x + \frac{1}{x^2}\right)dx$?

A. $4x^2(\ln x + 1) - 2x^2 + c$ C. $x^2(2\ln x + 1) + 4\ln x + c$

B. $x^2(4\ln x - 1) + 2\ln x + c$ D. $x^2(2\ln x - 1) + 4\ln x + c$

Unit 6

2004

480. If a sphere with center C(0, 1, 1) intersects the z-axis at p(0, 0, 3), then the radius of the sphere is equal to:

A. 5 B. 3 C. $\sqrt{3}$ D. $\sqrt{5}$

481. Let ℓ_1 and ℓ_1 be two lines in space intersecting at the origin, (0, 0, 0). If ℓ_1 and ℓ_1 passes through A(1, 1, 0) and B(0, 1, 1), respectively, then the angle between ℓ_1 and ℓ_1 is equal to:

A. 30° B. 45° C. 60° D. 90°

482. Let $\vec{V} = 3i - 4k$, and $\overrightarrow{AB}$ is a vector from point A(0, 1, 2) to a point B in space. If $\overrightarrow{AB}$ is parallel to $\vec{V}$ and $|\overrightarrow{AB} = 10|$, then point B is at:

A. $(-6, -1, 10)$ C. $(6, 1, -6)$

B. $(6, -1, -10)$ D. $(-6, -1, 6)$

483. Let $\vec{a} = -i + 3k$ and $\vec{b} = -i + j$ be vectors in space. Which one of the following is the cosine of the angle between $\vec{a}$ and $\vec{a} - \vec{b}$?

A. $\frac{9}{10}$ B. $\frac{3}{5}$ C. $\frac{3}{\sqrt{10}}$ D. $-\frac{9}{10}$

484. Suppose A and B are the end points of a diameter of the sphere whose equation is $x^2 + y^2 + (z + 2)^2 = 1$. If A=(1, 0, -2), then B is equal to:

A. $(0, 1, -2)$ C. $(-1, 0, -2)$

B. $(0, 0, -1)$ D. $(0, -1, -2)$

2005

485. Suppose P(1, 2, 1) and Q(1, 0, 2) are points in space and $\vec{A} = \overrightarrow{PQ}$ .if $\vec{B}$ is parallel to $\overrightarrow{PQ}$ and $\vec{A}.\vec{B} = -10$, then which one of the following is true?

A. $\vec{A}$ and $\vec{B}$ have the same direction. C. $|\vec{B}| = \frac{1}{10}|\vec{A}|$

B. $|\vec{B}| = 10|\vec{A}|$ D. $|\vec{B}| = 2|\vec{A}|$

486. Which one of the following points is closer to the sphere $x^2 + y^2 + z^2 - 2x + 6z + 9 = 0$?

A. (1, 0, 0) B. (0, 0, 0) C. $(0, -1, 0)$ D. (0, 0, −1)

487. Suppose $\vec{A} = 2j - k$ and $\vec{B} = 5i + 15k$ where i, j and k are the standard unit vectors in the direction of positive x, y and z-axis, respectively. Which one of the following is the unit vector in the direction of $\vec{A} + \frac{1}{5}\vec{B}$?

A. $\frac{3}{5}i + \frac{4}{5}k$ C. $\frac{4}{5}j - \frac{3}{5}k$

B. $\frac{1}{3}i + \frac{2}{3}j + \frac{2}{3}k$ D. $\frac{2}{3}i - \frac{1}{3}j + \frac{2}{3}k$

488. Let $\vec{a} = 2i + (x - 1)j + k$ and $\vec{c} = i - j + yk$ be vectors. If $\vec{a}.\vec{c} = 0$ and $|\vec{a}| = 3$, which one of the following is possible value of y?

A. -4 B. -1 C. 3 D. 4

2006

489. Suppose $\vec{A} = 2i - j + 2k$ and $\vec{B}$ is a vector in space such that $|\vec{B}| = \vec{A}.\vec{B}$. if $\vec{u}$ is a unit vector in the direction of $|\vec{B}|$, then $|\vec{A} + \vec{u}|^2$ is equal to:

A. 16 B. 12 C. 10 D. 14

490. Suppose P and Q are points in space such that the midpoint of $\overrightarrow{PQ}$ is on the negative z-axis and the distance between P and Q is 6.if P= $(2,-1,0)$, then what is the coordinate of Q?

A. $(-2,1,4)$ C. $(2,-1,-6)$
B. $(2,-1,6)$ D. $(-2,1,-4)$

2007

491. If $p=(3,\alpha-1,\alpha+2)$ and $Q=(2\alpha+1,3,3\alpha)$ are points in space, what should be the value(s) of α so that the distance between the two points is 6?

A. $\alpha=-2$ or $\alpha=5$ C. $\alpha=-1$ or $\alpha=3$
B. $\alpha=0$ or $\alpha=5$ D. α = -3 or $\alpha=2$

492. If $(-1,2,2)$ and $(1,0,-2)$ are end points of a diameter of a sphere, then which one of the following is true about the sphere?

A. (0, 1, 0) is a point on the sphere.
B. The equation of the sphere is $x^2+(y-1)^2+z^2=6$.
C. The equation of the sphere is $x^2+(y-2)^2+z^2=6$.
D. The radius of the sphere is 6.

493. Suppose that the equation $x^2+y^2+z^2+2x+8z=6(y+1)$ represents a sphere. Where is the point $(1,-1,4)$ located relative to the sphere?

A. Inside the sphere. C. At the center of the sphere.
B. On the sphere. D. outside the sphere.

494. Suppose ℓ is the line through the center of the sphere $x^2+y^2+(z-2)^2=9$ and intersects the sphere at (1, 2, 4). What is the cosine of the angle between ℓ and positive z-axis?

A. $\frac{2}{3}$ B. $\frac{1}{3}$ C. $\frac{3}{5}$ D. $\frac{4}{5}$

2008

495. Let the angle between $\vec{V}=-2i-j+2k$ and $\overrightarrow{PQ}$ be 60°, where P and Q are points in space. If $\vec{V}.\overrightarrow{PQ}=2$, the what is the distance between P and Q?

A. $\frac{3}{4}$ B. $\frac{4}{5}$ C. $\frac{4}{3}$ D. $\frac{5}{4}$

496. What is the value of k, for which the two vectors $\vec{u}=\begin{pmatrix}1\\k\\-3\end{pmatrix}$ and $\vec{V}=\begin{pmatrix}2k\\-5\\4\end{pmatrix}$ are perpendicular?

A. 4 B. -4 C. 3 D. -3

497. If one of the end point of the line segment is $(3,2,-4)$ and the mid-point is $(4,1,-2)$, then the coordinate of the other end point is:

A. (5, 0, 0) B. (2, 0, 5) C. (5, 1, 2) D. (3, 1, 0)

2009

498. Let $\vec{A}$ and $\vec{B}$ be vectors in space such that $\vec{A}$ and $\vec{B}$ be vectors in space such that $\vec{A}\,.\,\vec{B}=-2$ and $\vec{B}=6i-7j+\sqrt{15}k$. If θ is an angle between $\vec{A}$ and $\vec{B}$, then what is the value of $|\vec{A}|$?

A. $\frac{1}{5}\cos\theta$ B. $\frac{1}{5\cos\theta}$ C. $-\frac{1}{5}\cos\theta$ D. $-\frac{1}{5\cos\theta}$

499. If $P\left(2,\sqrt{5},1\right)$ and $Q(3,0,9)$ are points on a sphere whose center is on z-axis, then which one of the following points is outside of the sphere?

A. $(-4,3,5)$ B. $(2,-2,1)$ C. $(3,1,1)$ D. $(0,0,0)$

500. If $A(x,0,2)$, $B(3,0,2)$ and $C\left(2,\sqrt{3},2\right)$ are vertices of equilateral triangle in space, then what is the value of x?

A. 5 B. 3 C. 2 D. 1

2010

501. Let A$(a,2,5)$, for $a>0$, be a point on the sphere $x^2+y^2+z^2-6z=0$ and C be center of the sphere. If $P(k,2,4)$

Is a point in space such that $\overrightarrow{PA}$ is perpendicular to $\overrightarrow{CA}$, what is the cosine of the angle between $\overrightarrow{PA}$ and $\overrightarrow{PC}$?

A. $\frac{5}{\sqrt{35}}$ B. $\frac{7}{\sqrt{35}}$ C. $\frac{7}{\sqrt{70}}$ D. $\frac{5}{\sqrt{70}}$

502. If the dot product of a vector $\vec{A}$ with the vectors $i-j+k$, $2i+j-3k$ and $i+j+k$ are 4, 0 and 2, respectively, what is $\vec{A}$?

A. $\vec{A} = (2,1,1)$ C. $\vec{A} = (-2,-1,1)$

B. $\vec{A} = (-2,1,-1)$ D. $\vec{A} = (2,-1,1)$

503. Let $P = (1,\alpha,\alpha)$ and $Q = (\alpha-1,1,1)$ be two points in space and the distance between P and Q is 3. Then what is the value(s) of α?

A. $\alpha = -1,\ \alpha = 9$ C. $\alpha = 3,\ \alpha = \frac{-1}{3}$

B. $\alpha = 1,\ \alpha = -9$ D. $\alpha = -3,\ \alpha = \frac{1}{3}$

2011

504. If the angle between the vectors $\vec{A} = (2,-1,1)$ and $\vec{B} = (1,1,\alpha)$ is $\frac{\pi}{3}$, then what is the value of α?

A. 1 B. -1 C. 2 D. -2

505. If the point $(\alpha,0,3)$ is on the sphere whose center is $(1,2,3)$ and radius 2, what is the value of α?

A. 0 B. 1 C. 2 D. -3

506. If $\vec{u} = 2j - k$ and $\vec{v} = i - 8j + 3k$, then what is the unit vector in the direction of $\mathbf{5u + v}$?

A. $i + 2j - 2k$ C. $\frac{1}{3}i + \frac{2}{3}j - \frac{2}{3}k$

B. $\frac{2}{3}i + \frac{1}{3}j - \frac{2}{3}k$ D. $\frac{1}{3}i + \frac{2}{3}j + \frac{2}{3}k$

Unit 7

2004

507. Let $a_n = n^2 - n$, when $n \in \mathbb{N}$ (the set of natural number). Which one of the following is true, when k is arbitrary chosen natural number and m is an integer?

A. a_n is not a multiple of 2 for some $n \in \mathbb{N}$ because $a_1 = 0$.

B. a_n is a multiple of 2 for all $n \in \mathbb{N}$ because $a_1 = 0$ and if $a_k = 2m$, then $a_{k+1} = 2(m+1)$.

C. a_n is a multiple of 2 for all $n \in \mathbb{N}$ because $a_1 = 0$ and if $a_k = 2m$, then $a_{k+1} = 2(m+k)$.

D. a_n is a multiple of 3 for all $n \in \mathbb{N}$ because $a_1 = 0$ and if $a_k = 3m$, then $a_{k+1} = 3(m+k-1)$.

508. Consider the following statement:

$\frac{x^2+2}{x} \neq 0$ for every real number x.

To show this, a person constructed the following proof.

"**Proof**: Take $x = 1$. Then $\frac{x^2+2}{x} = \frac{1^2+2}{1} \neq 0$.

In the same way, if we take $x = n$ for any real number n we get

$\frac{x^2+2}{x} = \frac{n^2+2}{n} \neq 0$.

It follows that $\frac{x^2+2}{x} \neq 0$, for real number x."

Which one of the following is true about the proof?

A. The proof is correct by the principle of induction.

B. The proof is correct by the method of exhaustion.

C. The proof is correct and it uses the method of direct proof.

D. It is not a valid proof since its argument cannot lead to the conclusion.

2005

509. Which one of the following describes the principle of Mathematical induction on the set of natural numbers?

A. If an assertion is true for a natural number n, then it is true for $n+1$.

B. If the assertion is true for 1 and it is true for $n+1$, then it is true for some n.

C. If the assertion holds for $n = 20$ and for any $n \geq 20$, then it is true for n implies true for $n+1$.

D. If an assertion is true for $n = 1$, and is true for $n = k$, whenever is true for $n = k+1$

2006

510. The following is an assertion of a person and his proof.

"for any natural number n, $n! < 10^n$.

Proof:

Step1. Let $n = 1$. Since $1! = 1$ and $10^1 = 10$, it is true that $1! < 10$.

Step2. Let $n = 2$. Since $2! = 2$ and $10^2 = 100$, it is true that $2! < 10^2$.

Step3. Let $n = 3$. Since $3! = 6$ and $10^3 = 1000$, it is true that $3! < 10^3$.

Step4. Continuing in this manner, we can see that whenever

$k! < 10^k$ is true, then $(k+1)! < 10^{k+1}$ is also true.

Therefore, by induction, n! $< 10^n$ for all natural numbers."

A. The proof is correct by the principle of mathematical induction, though step2 and step3 can be omitted.
B. The proof is correct by the principle of mathematical induction; and step2 and step3 are necessary since they provide additional information.
C. The proof is invalid because step4 did not justify the desired induction step.
D. The proof follows the technique of a proof by exhaustion.

2007

511. Consider the formula for a natural number n∈ ℕ:

$$2 + 4 + 8 + \cdots + 2^n = 2^{n+1} + 1$$

To proof this formula a person has used the following argument.
"Assume the formula is true for $n = k$, for some $k \in \mathbb{N}$. Then the person has shown that the formula is also true for $n = k + 1$. And then, the person has conclude that, by the principle of Mathematical induction, the formula is true for all natural number n∈ ℕ." Which one of the following is true about the above arguments?

A. The formula holds true through it does not work for $n = 1$.
B. Since the left-hand side is an even number and the right hand-hand side an odd number, the principle of Mathematical induction is false.
C. This is one example where the principle of mathematical induction fails to work.
D. The above formula does not work for all natural numbers for n∈ ℕ.

2008

512. Which one of the following is a valid assertion that can be proved by the principle of mathematical induction?

A. $2^n > 10n$ for every integer n such that $n \geq 6$.
B. $r^2 > 0$ for every real number r such that $r \geq 1$.
C. $n^2 + 10n > 2n^2$ for every natural number $n \geq 1$.
D. $2^n > 8n$ for every integer n such that $n \geq 3$.

513. Consider the following assertion of a person and his proof.
"If x and y are equal positive integers, then $x + y = y$."
Proof: The following steps and reasons are used to proof the assertion.

Step	Reason
1. $x = y$	Given hypothesis
2. $x^2 = xy$	Multiply both sides of (1) by x
3. $x^2 - y^2 = xy - y^2$	Subtract y^2 from both sides of (2)
4. $(x - y)(x + y) = (x - y)y$	Factor both sides of (3)
5. $x + y = y$	Divide both sides of (4) by $x - y$

Step 5 completes the proof.
Which one of the following is true about this proof?

A. It is a correct direct proof of the assertion.
B. It follows the technique of a proof by contradiction because the steps lead to a contradiction.
C. The proof is invalid because step 4 does not lead to step 5.
D. The proof is invalid because step 4 does not followed from step 3.

2009

514. Which one of the following is a valid assertion that can be proved by the principle of mathematical induction?

A. The sum of any two rational numbers is positive.
B. $r^2 \geq 1$, for every real number $r \geq 1$.
C. $n^2 \geq 4n$, for every integer $n \geq 4$.
D. $2^n \leq 4^n$, for every integer $n \leq 100$.

515. Consider the assertion: "The sum of positive irrational numbers is positive irrational number". Which one of the following is correct about the assertion?

A. Taking the irrational numbers such as $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}$ and so on, if we add any two of them, the sum is irrational. Therefore the assertion is true.
B. The sum of $1 + \sqrt{2}$ and $1 - \sqrt{2}$ is 2, which is rational. This is a counter example that disproves the assertion.

C. The sum of $\sqrt{7}$ and $\sqrt{2}$ is a counter example that shows the assertion is false.

D. The assertion can be disproved by taking the sum of $1+\sqrt{2}$ and $2-\sqrt{2}$.

2010

516. Let P(n) be an open proposition on the set of natural number (ℕ). Which one of the following is correct application of the principle of mathematical induction?

A. If P(1) is true for $n=1$; and if both P(n) and P(n+1) are true for a certain $n \in \mathbb{N}$, then P(n) is true for all $n \in \mathbb{N}$.

B. If P(10) is true; and if P(n) is true implies that P(n+1) is true, then P(n) is true for all $n \in \mathbb{N}$.

C. If P(1) is true; and P(n)⇒ P(n+1) is true for any $n \in \mathbb{N}$, then P(n) is true for all $n \in \mathbb{N}$.

D. If P(10) is true; and assuming P(n) is true for any $n > 10$ if it follows that P(n+1) is true, then P(n) is true for all $n > 10$.

517. Consider the following assertion:

$p+2^n$ is an odd number for any prime p and any $n \in \mathbb{N}$.

Which one of the following is correct about a prove or disprove of the assertion?

A. The assertion can be proved by direct method; because p is odd and $2^n = 2(2^{n-1})$ is even imply that $p+2^n$ is odd since the sum of even and odd is odd.

B. There is a counter example that disproves the assertion.

C. The assertion can be proved by the direct method; because if $n \notin \mathbb{N}$, then $2^n \notin \mathbb{N}$ and hence $p+2^n$ is not odd.

D. The assertion can be proved by the method of assertion.

2011

518. Which one of the following is a valid assertion that can be proved by the principle of mathematical induction?

A. $3n+25 < 3^n$, for every integer $n \geq 3$. C. $n^2 \leq 2^n$, for every integer $n \geq 1$.

B. $2^n > n+20$, for every integer $n \geq 4$. D. $n^3 - n$ is divisible by 6, for every integer $n \geq 1$.

519. Which of the following is a correct assertion that can be proved by the principle of mathematical induction?

A. $\frac{1}{n+1} \leq 1$, for each real number $n \geq 1$. C. $2^p - 1$ is prime for each prime integer p.

B. $m! \leq 4^m$, for each positive integer m. D. $k! \geq 2^k$, for each integer $k \geq 4$.

520. $\forall_n \in \mathbb{N}$, $3^n - 2$ is prime number that can be proved or disproved by which of the following mathematical proof?

A. Direct proof C. Disprove by counter example

B. Proof by exhaustion D. proof by contradiction.