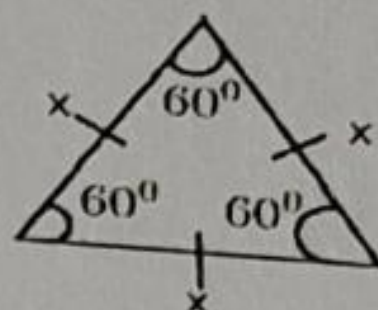
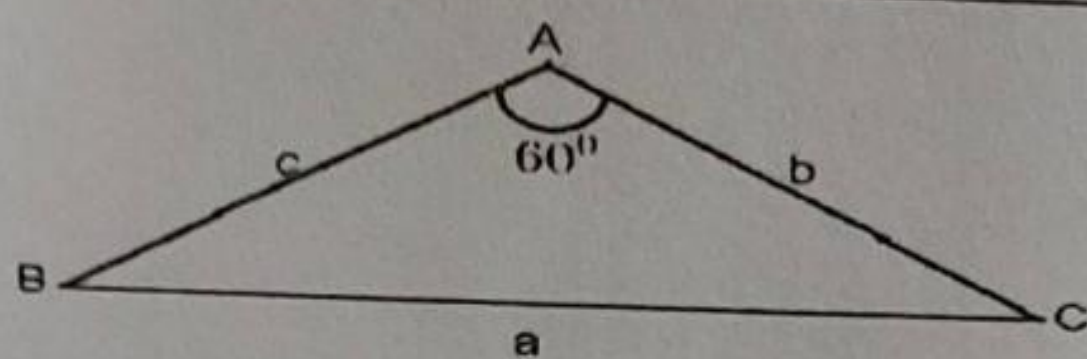


MARKING GUIDE FOR E.O.T-2 S.5 PURE MATHS 2023

1. $2x^2 + 5x - 8 = 0$ $a = 2, b = 5, c = -8$ Sum of roots; $\alpha + \beta = \frac{-b}{a} = \frac{-5}{2}$ Product of roots; $\alpha \beta = \frac{c}{a} = \frac{-8}{2}$ For the new roots: $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ Sum of roots = $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\left(\frac{-5}{2}\right)^2 - 2\left(\frac{-8}{2}\right)}{\frac{-8}{2}} = \frac{-57}{16}$ Product of roots = $\left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right) = 1$ $x^2 - (\text{sum of roots})x + \text{prd. of roots} = 0$ $x^2 - \left(\frac{-57}{16}\right)x + 1 = 0$; or $16x^2 + 57x + 16 = 0$	
2. From the L.H.S $\tan^2 A - \sin^2 A = (\tan A + \sin A)(\tan A - \sin A)$ $= \left(\frac{\sin A}{\cos A} + \sin A\right)\left(\frac{\sin A}{\cos A} - \sin A\right)$ $= \left(\frac{\sin A + \sin A \cos A}{\cos A}\right)\left(\frac{\sin A - \sin A \cos A}{\cos A}\right)$ $= \frac{\sin A(1 + \cos A) \sin A(1 - \cos A)}{\cos^2 A}$ $= \frac{\sin^2 A(1 + \cos A)(1 - \cos A)}{\cos^2 A}$ $= \frac{\sin^2 A}{\cos^2 A} (1^2 - \cos^2 A)$ $= \tan^2 A \sin^2 A$. As required	
3. $\log_3 x = p, \rightarrow 3^p = x$.....(i) $\log_{18} x = q \rightarrow 18^q = x$.....(ii) Eqn(i) = eqn(ii) $3^p = 18^q$ $3^p = (3 \times 6)^q$ Introducing logs to base 6 $\log_6 3^p = \log_6 (3 \times 6)^q$ $p \log_6 3 = q(\log_6 3 + \log_6 6)$ $p \log_6 3 = q \log_6 3 + q$ $p \log_6 3 - q \log_6 3 = q$ $(p - q) \log_6 3 = q$ $\log_6 3 = \frac{q}{p - q}$ as required	
4. $f(x) = x^3 + 4x - 5$ (i) $f(2) = (2)^3 + 4(2) - 5$ $= 11$ (ii) $f'(x) = 3x^2 + 4$ $f'(2) = 3(2)^2 + 4$ $= 16$	
5. $3p - q + 2r = 16$.....(i) $p + 2q - r = -1$.....(ii) $2p + 3q + r = 3$.....(iii) Accept any method Gaussian elimination $p = 4, q = -2, r = 1$	



6.

$$\text{Area of } ABC = \frac{1}{2}bc \sin 60^\circ \dots\dots\dots(i)$$

$$\text{Area of the equilateral triangle} = \frac{1}{2}x^2 \sin 60^\circ \dots\dots\dots(ii)$$

Since the two triangles have the same area, then $eqn(i) = eqn(ii)$

$$\frac{1}{2}bc \sin 60^\circ = \frac{1}{2}x^2 \sin 60^\circ$$

$$bc = x^2 \dots\dots\dots(iii)$$

$$\text{Also, } a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = b^2 + c^2 - 2x^2 \cos 60^\circ$$

$$a^2 = b^2 + c^2 - 2x^2 \left(\frac{1}{2}\right)$$

$$x^2 = b^2 + c^2 - a^2$$

$$b^2 + c^2 - a^2 = x^2. \text{ As required}$$

$$7. -4x^2 + 6x + 4 = 0$$

$$x^2 - \frac{3}{2}x - 1 = 0$$

$$x^2 - \frac{3}{2}x = 1$$

$$x^2 - \frac{3}{2}x + \left(-\frac{3}{4}\right)^2 = 1 + \left(-\frac{3}{4}\right)^2$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{25}{16}$$

$$\sqrt{\left(x - \frac{3}{4}\right)^2} = \pm \sqrt{\frac{25}{16}}$$

$$x - \frac{3}{4} = \pm \frac{5}{4}$$

$$x = \frac{3}{4} \pm \frac{5}{4}$$

$$\text{Either } x = 2 \text{ or } x = -\frac{1}{2}$$

$$8. y = \frac{2}{x^2}$$

$$y + \Delta y = \frac{2}{(x + \Delta x)^2}$$

$$\Delta y = \frac{2}{(x + \Delta x)^2} - y$$

$$\Delta y = \frac{2}{(x + \Delta x)^2} - \frac{2}{x^2}$$

$$\Delta y = \frac{2x^2 - 2(x^2 + 2x\Delta x + (\Delta x)^2)}{x^2(x + \Delta x)^2}$$

$$\Delta y = \frac{-4x\Delta x - 2(\Delta x)^2}{x^2(x + \Delta x)^2}$$

$$\Delta y = \frac{-\Delta x(4x + 2\Delta x)}{x^2(x + \Delta x)^2}$$

$$\frac{\Delta y}{\Delta x} = \frac{-(4x + 2\Delta x)}{x^2(x + \Delta x)^2}$$

$$\frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \left(\frac{\Delta y}{\Delta x}\right) = \frac{-(4x)}{x^2(x)^2} = \frac{-4}{x^3}$$

9. from the L.H.S

Let $\log_a x = y$

$$a^y = x \dots\dots\dots(i)$$

Introducing logs to base x

$$\log_x a^y = \log_x x$$

$$y \log_x a = 1$$

$$y = \frac{1}{\log_x a}$$

$$\text{Thus, } \log_a x = \frac{1}{\log_x a}$$

or

$$\log_a x = \frac{\log_x x}{\log_x a} = \frac{1}{\log_x a}$$

$$\text{For; } \log_{10} x + \log_x 100 = 3$$

$$\frac{1}{\log_x 10} + 2 \log_x 10 = 3$$

Let $\log_x 10$ be y

$$\frac{1}{y} + 2y = 3$$

$$1 + 2y^2 = 3y$$

$$2y^2 - 3y + 1 = 0$$

$$y = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 2 \times 1}}{2 \times 2} = \frac{3 \pm 1}{4}$$

$$\text{Either } y = \frac{3+1}{4} = 1$$

$$\text{Or } y = \frac{-1}{2}$$

$$\text{When } \log_x 10 = 1; x^1 = 10$$

$$x = 10$$

$$\text{When } \log_x 10 = \frac{-1}{2}$$

$$x^{\frac{-1}{2}} = 10$$

$$x^{\frac{-1}{2} \times \frac{-2}{1}} = 10^{-2} = \frac{1}{100}$$

$$\therefore x = 10 \text{ and } x = \frac{1}{100}$$

$$(b) 2^{(2+2x)} + 3(2^x) - 1 = 0$$

$$2^2(2^x)^2 + 3(2^x) - 1 = 0$$

$$\text{Let } 2^x = y$$

$$4y^2 + 3y - 1 = 0$$

$$y = \frac{-3 \pm \sqrt{(3)^2 - 4 \times 4 \times -1}}{2 \times 4} = \frac{-3 \pm 5}{8}$$

$$\text{Either; } y = \frac{-3+5}{8} = 1$$

$$\text{Or } y = \frac{-3-5}{8} = \frac{-1}{4}$$

$$\text{When } y = 1 = 2^x$$

$$2^x = 2^0$$

$$x = 0$$

$$\text{When } 2^x = \frac{-1}{4}$$

$$x \log 2 = \log \frac{-1}{4}; x \text{ has no real values}$$

$$\therefore x = 0$$

$$10. (a) \text{ let } P(x) = 2x^4 + 7x^3 + 6x^2 + ax + b$$

NB: $(x+1)^2$ has repeated roots hence $(x+1)$ is a factor of $P(x)$ and $P'(x)$

$$2x^4 + 7x^3 + 6x^2 + ax + b = (x+1)^2 Q(x)$$

$$\text{When } x+1 = 0$$

$$x = -1$$

$$P(-1) = 2(-1)^4 + 7(-1)^3 + 6(-1)^2 + a(-1) + b = 0$$

$$2 - 7 + 6 - a + b = 0$$

$$-a + b = -1 \dots \dots \dots (i)$$

$$P'(x) = 8x^3 + 21x^2 + 12x + a$$

$$P'(-1) = 8(-1)^3 + 21(-1)^2 + 12(-1) + a = 0$$

$$-8 + 21 - 12 + a = 0$$

$$1 + a = 0 \dots \dots \dots (ii)$$

Discard = y = -1
for y = 1/4
x = -2 *2^x = 2^-2*
x = -2

$$a = -1, b = -2$$

$$(b) f(x) = (x+5)q_1(x) - 16$$

$$f(-5) = -16 \dots\dots\dots (i)$$

$$f(x) = (x-3)q_2(x) + 8$$

$$f(3) = 8 \dots\dots\dots (ii)$$

$$D(x) = x^2 + 2x - 15 \text{ (D(x) is quadratic hence the remainder should be linear)}$$

$$= (x-3)(x+5)$$

$$\text{Let the remainder, } R(x) = ax + b$$

$$f(x) = (x-3)(x+5)Q(x) + ax + b \dots\dots\dots (iii)$$

$$f(3) = 0 + 3a + b = 8$$

$$3a + b = 8 \dots\dots\dots (*)$$

$$f(-5) = 0 - 5a + b = -16$$

$$-5a + b = -16 \dots\dots\dots (**)$$

Solving eqn(*) and eqn(**)

$$a = 3, \quad b = -1$$

$$\text{The remainder, } R(x) = 3x - 1$$

$$11.(a) \text{ let } 16 \sin \theta - 30 \cos \theta = R \sin(\theta - \alpha)$$

$$16 \sin \theta - 30 \cos \theta = R \sin \theta \cos \alpha - R \cos \theta \sin \alpha$$

Comparing coefficients;

$$\sin \theta; \quad 16 = R \cos \alpha \dots\dots\dots (i)$$

$$\cos \theta; \quad 30 = R \sin \alpha \dots\dots\dots (ii)$$

$$\text{Eqn(ii)} \div \text{Eqn(i)} \quad \tan \alpha = \frac{30}{16}$$

$$\alpha = \tan^{-1}(30 \div 16) = 61.93^\circ \text{ (2dp)}$$

$$(\text{Eqn(i)})^2 + (\text{Eqn(ii)})^2; R^2(\cos^2 \alpha + \sin^2 \alpha) = 16^2 + 30^2$$

$$R = +\sqrt{256 + 900} = 34$$

$$16 \sin \theta - 30 \cos \theta = 34 \sin(\theta - 61.93^\circ)$$

$$f(\theta) = \frac{12}{16 \sin \theta - 30 \cos \theta + 40} = \frac{12}{34 \sin(\theta - 61.93^\circ) + 40}$$

Maximum value of $f(\theta)$ occurs when $\sin(\theta - 61.93^\circ) = -1$

$$f_{\max}(\theta) = \frac{12}{34(-1) + 40} = \frac{12}{6} = 2$$

$$\text{When } \sin(\theta - 61.93^\circ) = -1$$

$$K.A = \sin^{-1}(1) = 90^\circ$$

$$\theta - 61.93^\circ = 270^\circ$$

$$\theta = 331.93^\circ \text{ or } 151.93^\circ$$

$$(ii) 16 \sin \theta - 30 \cos \theta - 17 = 0$$

$$34 \sin(\theta - 61.93^\circ) - 17 = 0$$

$$\sin(\theta - 61.93^\circ) = \frac{17}{34}$$

$$\theta - 61.93^\circ = \sin^{-1}(0.5)$$

$$\theta - 61.93^\circ = 30^\circ, 150^\circ$$

$$\theta = 91.93^\circ, 211.93^\circ$$

$$12.(i) \frac{6+4\sqrt{3}}{a+b\sqrt{3}} = 1 - \sqrt{3}$$

$$6 + 4\sqrt{3} = (1 - \sqrt{3})(a + b\sqrt{3})$$

$$6 + 4\sqrt{3} = a + b\sqrt{3} - a\sqrt{3} - 3b$$

$$6 + 4\sqrt{3} = (a - 3b) + (b - a)\sqrt{3}$$

Comparing terms on L.H.S and R.H.S

$$6 = a - 3b \dots\dots\dots (i)$$

$$4 = b - a \dots\dots\dots (ii)$$

Solving eqn(i) and eqn(ii)

$$a = -9, b = -5$$

min value when $\sin(\theta - 61.93^\circ) = -1$
 $f_{\min}(\theta) = \frac{12}{34(-1) + 40} = \frac{12}{6} = 2$
 $\sin(\theta - 61.93^\circ) = 1$
 $\theta = 90^\circ + 61.93^\circ$
 $\theta = 151.93^\circ$

$$(ii) \quad \sqrt{x+2} - \sqrt{3-x} = 1$$

$$\sqrt{x+2} = 1 + \sqrt{3-x}$$

$$(\sqrt{x+2})^2 = (1 + \sqrt{3-x})^2$$

$$x+2 = 1 + 2\sqrt{3-x} + 3-x$$

$$2x-2 = 2\sqrt{3-x}$$

$$(x-1)^2 = (\sqrt{3-x})^2$$

$$x^2 - 2x + 1 = 3 - x$$

$$x^2 - x - 2 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times -2}}{2 \times 1} = \frac{1 \pm 3}{2}$$

$$\text{Either; } x = \frac{1+3}{2} = 2 \quad \text{Or } x = \frac{1-3}{2} = -1$$

Testing

When $x = 2$

$$L.H.S = \sqrt{2+2} - \sqrt{3-2} = 2 - 1 = 1$$

L.H.S = R.H.S Thus, $x = 2$ is a solution to the equation

When $x = -1$

$$L.H.S = \sqrt{-1+2} - \sqrt{3-(-1)} = 1 - 2 = -1$$

L.H.S $\neq$ R.H.S thus $x = -1$ is not a solution to the equation

$\therefore x = 2$

(iii) Let $\pm(\sqrt{x_1} + \sqrt{x_2})$ be the square root of $5 + 2\sqrt{6}$

$$\pm(\sqrt{x_1} + \sqrt{x_2}) = \sqrt{(5 + 2\sqrt{6})}$$

$$\text{Squaring both side } (\pm(\sqrt{x_1} + \sqrt{x_2}))^2 = (\sqrt{(5 + 2\sqrt{6})})^2$$

$$x_1 + x_2 + 2\sqrt{x_1x_2} = 5 + 2\sqrt{6}$$

By comparing the terms on the two sides;

$$x_1 + x_2 = 5$$

$$x_1 = 5 - x_2 \dots \dots \dots (i)$$

$$x_1x_2 = 6 \dots \dots \dots (ii)$$

$$x_2(5 - x_2) = 6$$

$$5x_2 - x_2^2 = 6$$

$$x_2^2 - 5x_2 + 6 = 0$$

$$x_2 = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 1 \times 6}}{2 \times 1} = \frac{5 \pm 1}{2}$$

$$\text{Either; } x_2 = \frac{5+1}{2} = 3 \text{ or } x_2 = \frac{5-1}{2} = 2$$

$$\text{When } x_2 = 3, x_1 = 5 - 3 = 2$$

$$\text{When } x_2 = 2, x_1 = 5 - 2 = 3$$

Hence the square root of $5 + 2\sqrt{6}$ is $\pm(\sqrt{2} + \sqrt{3})$

$$13. y = (x+2)(x-1)^3$$

$$\text{Let } u = (x+2), \quad u^1 = 1$$

$$\text{Let } v = (x-1)^3, \quad v^1 = 3(x-1)^2(1) = 3(x-1)^2$$

$$\frac{dy}{dx} = uv^1 + vu^1$$

$$\frac{dy}{dx} = (x+2)3(x-1)^2 + (x-1)^3(1)$$

$$\frac{dy}{dx} = 3(x+2)(x-1)^2 + (x-1)^3$$

$$\text{when } x = 3, \quad \frac{dy}{dx} = 3(3+2)(3-1)^2 + (3-1)^3 = 3(5)(4) + 8$$

$$\frac{dy}{dx} = 68$$

$$(b) y = 2(x-x^2), \quad P(x,y), Q(x+\Delta x, y+\Delta y)$$

$$\text{Gradient of } PQ = \frac{(y+\Delta y)-y}{(x+\Delta x)-x}$$

$$\frac{\Delta y}{\Delta x} = \frac{2(x+\Delta x) - (x+\Delta x)^2 - 2(x-x^2)}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{2(x+\Delta x) - 2(x^2+2x\Delta x+(\Delta x)^2) - 2x+2x^2}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{2x+2\Delta x - 2x^2 - 4x\Delta x - 2(\Delta x)^2 - 2x+2x^2}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{2\Delta x - 4x\Delta x - 2(\Delta x)^2}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{(2 - 4x - 2\Delta x)\Delta x}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = 2 - 4x - 2\Delta x$$

The gradient of the curve at the point $P = \frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \left(\frac{\Delta y}{\Delta x} \right) = 2 - 4x$

14. (a)

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$a = (k^2 + k + 1), \quad b = (2k + 1), \quad c = (k^2 - 1)$$

$$\cos A = \frac{(2k+1)^2 + (k^2-1)^2 - (k^2+k+1)^2}{2(2k+1)(k^2-1)}$$

$$\cos A = \frac{(4k^2 + 4k + 1 + k^4 - 2k^2 + 1) - (k^4 + 2k^2(k+1) + (k+1)^2)}{2(2k^3 - 2k + k^2 - 1)}$$

$$\cos A = \frac{(2k^2 + 4k + k^4 - 2k^2 + 1) - (k^4 + 2k^3 + 2k^2 + k^2 + 2k + 1)}{2(2k^3 + k^2 - 2k - 1)}$$

$$\cos A = \frac{2k^2 + 4k + k^4 - 2k^2 + 1 - k^4 - 2k^3 - 3k^2 - 2k - 1}{2(2k^3 + k^2 - 2k - 1)}$$

$$\cos A = \frac{-2k^3 - k^2 + 2k + 1}{2(2k^3 + k^2 - 2k - 1)}$$

$$\cos A = \frac{-1(2k^3 + k^2 - 2k - 1)}{2(2k^3 + k^2 - 2k - 1)}$$

$$\cos A = \frac{-1}{2}$$

$$A = \cos^{-1} \left(\frac{-1}{2} \right) = 120^\circ \text{ as required.}$$

$$(b) \text{ From } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

$$a = 2R \sin A, \quad b = 2R \sin B, \quad c = 2R \sin C$$

From the L.H.S

$$a \cos A + b \cos B + c \cos C = R(2 \sin A \cos A + 2 \sin B \cos B + 2 \sin C \cos C)$$

$$= R(\sin 2A + \sin 2B + \sin 2C)$$

$$= R \left(2 \sin \left(\frac{2A+2B}{2} \right) \cos \left(\frac{2A-2B}{2} \right) + \sin 2C \right)$$

$$= R [2 \sin(A+B) \cos(A-B) + \sin 2C]$$

$$\text{But } A + B + C = 180^\circ, \quad A + B = (180^\circ - C)$$

$$a \cos A + b \cos B + c \cos C = R [2 \sin(180^\circ - C) \cos(A-B) + 2 \sin C \cos C]$$

$$= R [2 \sin C \cos(A-B) + 2 \sin C \cos C]$$

$$= 2R \sin C [\cos(A-B) + \cos(180^\circ - (A+B))]$$

$$= 2R \sin C [\cos(A-B) - \cos(A+B)]$$

$$= 2R \sin C [-2 \sin A \sin(-B)]$$

$$= 4R \sin A \sin B \sin C. \quad \text{As required}$$