



EEE 223 ELECTRICAL MACHINES I







COURSE OUTLINE

1. **Electromagnetism**: Magnetic circuits, flux, m.m.f., reluctance, B.H. relationship of magnetic materials, hysteresis loop. Kirchoffs laws applied to magnetic circuits, self and mutual inductance, induced e.m.f.

2. **Stored Energy, Force and Torque Calculations:** Flux - m.m.f. relationship in magnetic circuits and stored energy. Energy balance equation of an electromechanical system. Force and torque as rates of change of stored energy with respect to displacement illustrated by considering an electromagnetic plunger as an example. Analysis of magnetic circuit considering m.m.f. drops in iron parts and air gaps. Approximations of force calculations. Voltage-current relationship and conditions to be satisfied to avoid partial operation.

3. **DC Machines**: Construction, e.m.f. and torque equations. Performance characteristics of separately excited, shunt, series and compound machines. DC motors, necessity of a starter, construction of a starter, current variation during starting, speed control by variation of armature voltage and field current. Armature reaction, effect of brush shift. Calculation of magnetizing and cross-magnetizing ampere-turns. DC Machines heating, rating, losses, power flow, efficiency, braking. Applications of DC machines. Speed-power; load requirements. DC machine windings.





Assessment & Grading

Final Exam	70%
CAT'S AND ASSIGNMENTS	15%
Lab work	15%
TOTAL	100%

Textbooks (Reading list):

1. B.L. Theraja, A.K. Theraja: Electrical Technology, S. Chand & Co. Ltd.
2. Edward Hughes: Electrical Technology, ELBS
3. H. Cotton: Electrical Technology, CBS





1. ELECTROMAGNETISM: INTRODUCTION TO MACHINE THEORY







1.1 Introduction

- Electromagnetism is the energy conversion process whereby electrical energy is converted to mechanical energy and vice-versa by means of an electromagnetic system.
- In this topic we'll explain how mechanical forces can be developed by magnetic attraction and alignment and how rotary motion therefore be realized.

1.2 Conversion process in a machine

- An electromagnetic machine is one that links an electrical energy system to another energy system by providing a reversible means of energy flow in its magnetic field.
- The magnetic field is therefore the coupling between the two systems and is the mutual link.
- The energy transferred from the one system to the other is temporarily stored in the field and then released to the other system.
- An electromagnetic system can develop a mechanical force in two ways:

  (a) By alignment (b) By attraction

(a) Forces Of Alignment.

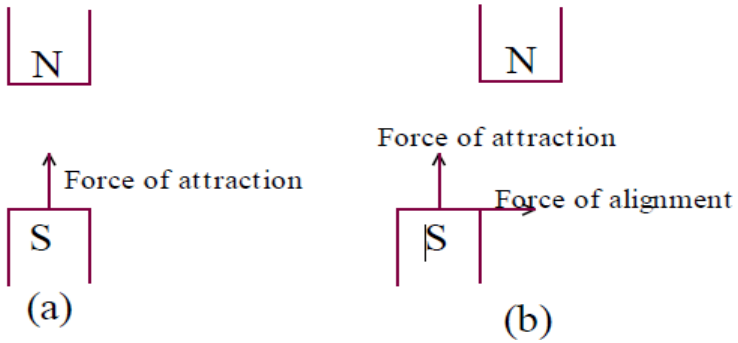


Fig 2.1 Force of attraction and alignment

- In fig 2.1(a) the two poles are situated opposite one another and a flux passes from one to the other and hence they are attracted towards one another.
- The force of alignment acts in any direction that will increase the magnetic energy stored in the arrangement





- In fig 2.1(a) it will try to bring the poles together since these decreases the reluctance of the air gap in the magnetic circuit and hence will increase the flux and consequently stored energy.
- In fig 2.1(b), poles are not situated opposite one another. The resultant forces tries to achieve greater stored energy by two component:

i) By attraction of poles towards one another

ii) By aligning of the poles laterally

- If the poles move laterally, the cross-sectional area of the air-gap is increased and the reluctance is reduced with consequent increase in the stored energy.

(b) Force of alignment in a reluctance motor

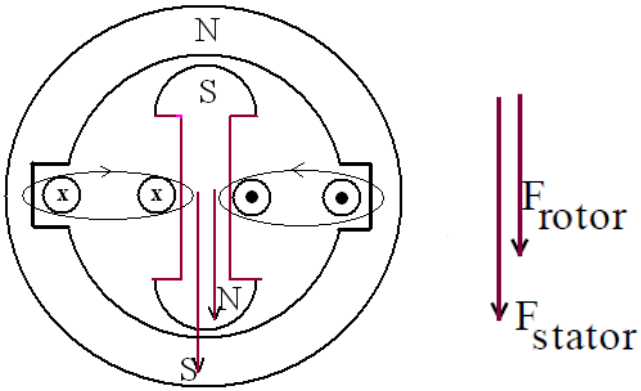


fig 2.2 **(a)**

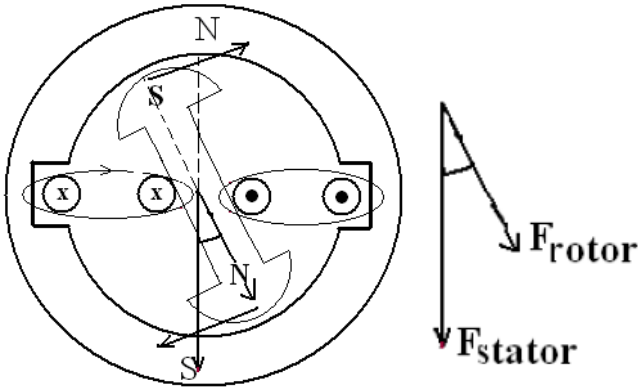


fig 2.2 (**b**)





- The centrally pivoted rotor magnet position as in fig 2.2(a) will not have any tendency to turn. Suppose that there is an angular displacement δ, between the two magnetic axes, fig 2.2(b).
- As indicated by the arrows, and also by the field lines round the conductors, forces will be exerted both attractive and repulsive tending to bring the magnets into alignment.
- The total force and the corresponding turning moment (torque) will vary with the angular position being a maximum when δ = **90°**

(c) Force of attraction

- Force of attraction has the advantage of simplicity in its application

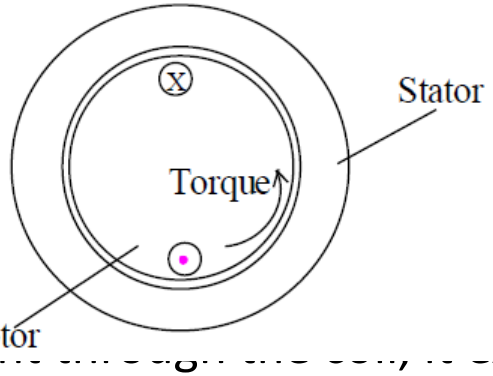


Fig 2.3

- Bypassing a c[illegible]periences a force on each of the coil sides resulting in a torque about the axis of rotation.
- A practical machine requires many conductors in order to develop sufficient torque.





1.3 The Magnetic Aspects

- The relationship between the flux density B in teslas (Wb/m2) and magnetizing field intensity H in amperes per meter (A/m) is given by

$$B = \mu\mu_0 H \text{.....2.1}$$

Where

$$\mu_0 = 4\pi.10^{-7}$$

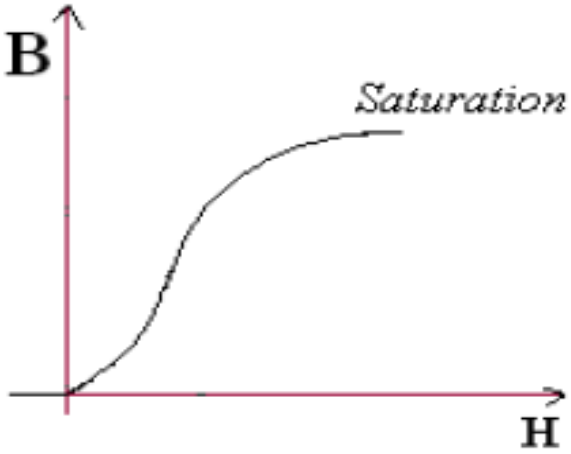
μ- Relative permeability

H-field intensity

- In an electrical machine the flux paths are either in air, hydrogen, water insulation or other non-magnetic material and also in iron for which μ varies with H and may have a value of 1000 or more.

Hysteresis loop

- The typical Magnetization curve, the relationship between B and H is highly none linear.



- If H is decreased and reversed, the curve does not retrace the same path.
- Energy expended in domain changes is not all recoverable and the domains retain their new magnetic axes. Consequently, when H=O, there is a residual value of flux density the magnitude of which, B_{res}, depends on the material, its crystal structure and the value of B.
- To bring B=O requires a coercive field intensity, coersivity, HC, which is negative.

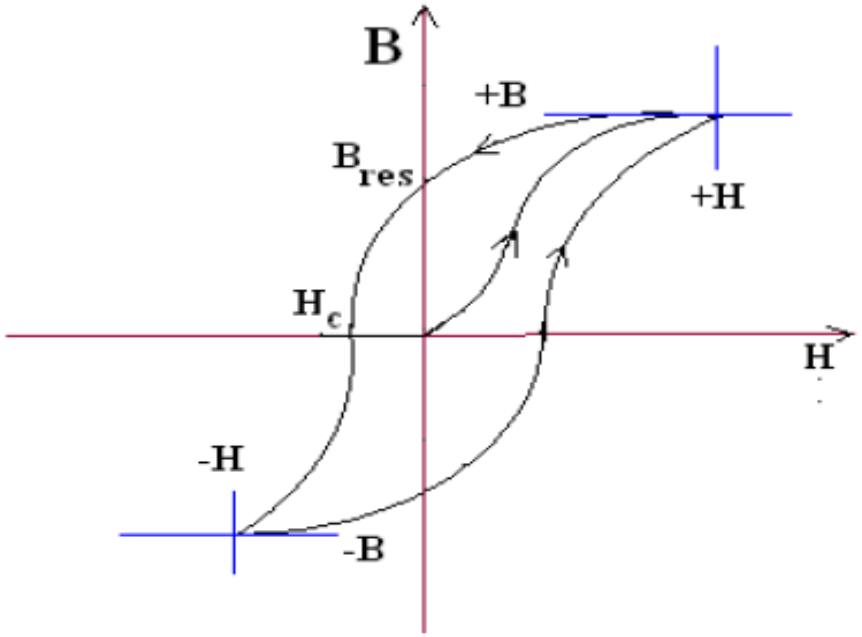


Fig 2.5 hysteresis loop

1.3.1 IRON LOSSES

a) Hysteresis loss

- If a ring of iron with cross-section area A, mean circumferential length is magnetized by a coil carrying *i* amperes through N turns, the supply must provide a component of voltage $e = N\frac{d\phi}{dt}$ to sustain a changing current and flux and to account for *i*R* work done against emf in time dt:

$$e.i.dt = N\frac{(B.A)}{dt}.\frac{Hl}{N}dt$$.....2.2

Where B- flux density

$$H = \frac{N.i}{l}\left[\frac{A}{m}\right]$$.....2.3.

(b) Eddy Current Loss

- Since iron is a conductor, a changing flux induces emfs and currents within the iron mass. These eddy current produce losses, heating and demagnetisations.

1.4 Magnetic Circuit Calculation

- Basically, there are two types of magnetic circuits, called series and parallel.
- Analogous to electric circuits a magnetic series circuit is one in which the flux set up by the current-carrying coil, the excitation, is common throughout the entire circuit. Parallel circuits are defined as those having more than one path for the flux to close.





- Air gaps may be present which have a significant influence on the excitation requirements.
- Air gaps may be introduced on purpose such as the air gap in rotating machine to separate the rotating member from the stationary one.
- Typical, in magnetic circuit calculations, it is desired to determine the required MMF or flux density for a specific condition.
- Since magnetic circuit can be represented by analogous electric circuit, methods of solution applicable to electric circuit can be extended to those for magnetic circuits.
- The major different is the nonlinear characteristic of the ferrous magnetic material of which the magnetic circuit is composed.
- It then becomes necessary to use B-H curve for the specific material used. Below is a typical B-H curve for sheet steel, cast steel and cast iron.

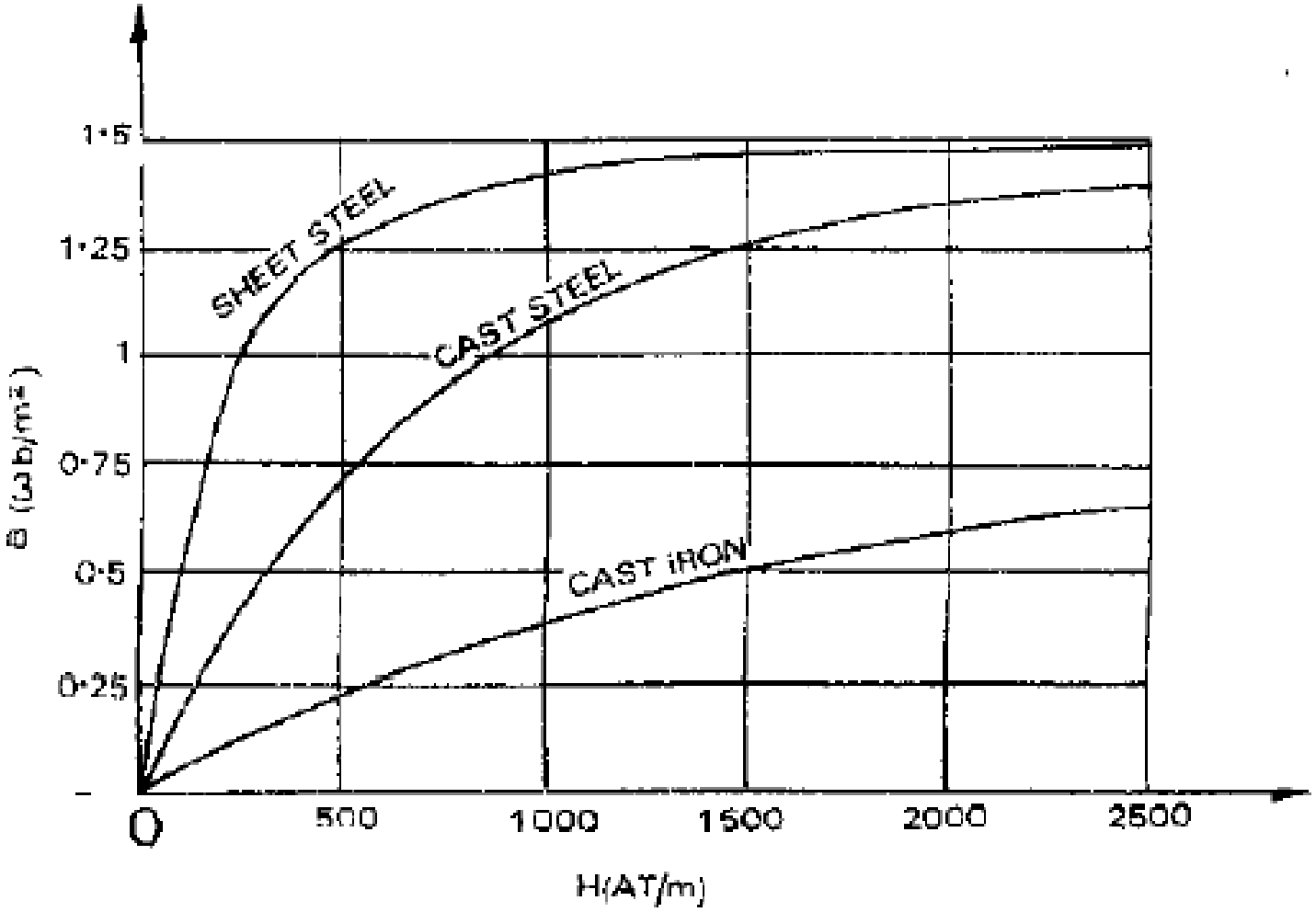
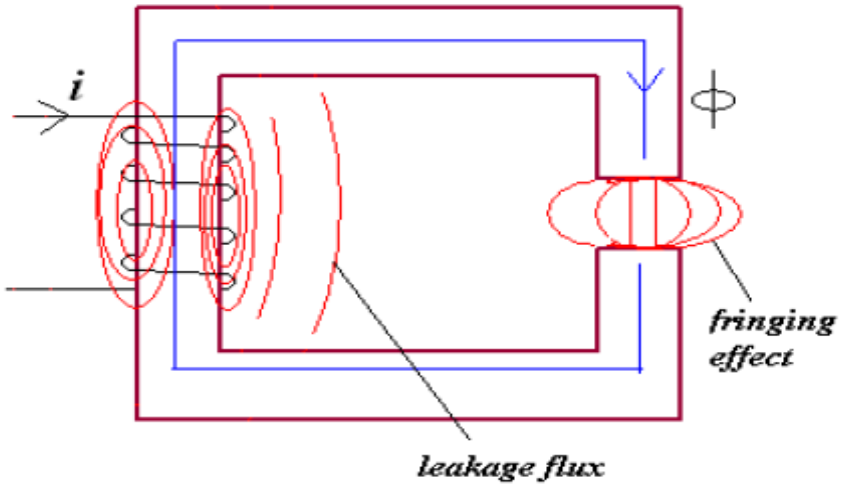


Fig 2.6 typical B-H curve for sheet steel, cast steel and cast iron.

1.4.1 Fringing and leakage fluxes

- In magnetic circuits there is a tendency for the flux to “leak out” of the magnetic path or “spread out” in the air gap.
- Leakage flux is not very effective and therefore requires a greater MMF.
- The spreading out of the flux in the air gap is called the fringing effect.







i. Hysteresis And Eddy Current Losses In Magnetic Circuits.

- For the purpose of calculation and basic understanding the following formulas are used.

1. For a volume V of magnetic material going through *f* cycles of magnetization per second the power dissipated in Hysteresis loss, Ph is

$$\boldsymbol{P_h = K_h . V . f . B_{max}^n} \text{..... 2.5}$$

where

K_h = Hysteresis constant, relating loop area to B_{max}^n

n = experimentally determined constant (1.5 – 2.5)

f = frequency of supply,

Bmax = maximum flux density

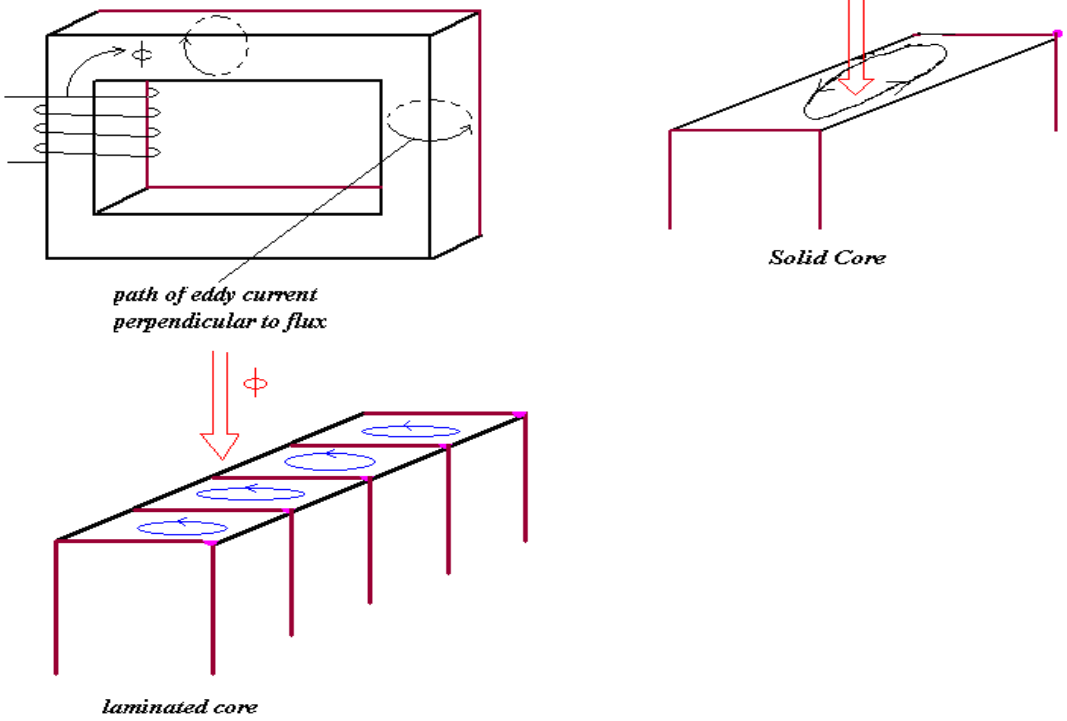
- Generally Hysteresis loop area is proportional to the power lost in that material.

2. For calculation of eddy current losses, Pe, an empirical equation is used:

$$P_e = K_e.(B_{max}.t.f)^2 \text{..... 2.6}$$

where

- K_e is a constant determined by the material.
- t – thickness of laminations.
- Generally the thinner the lamination the lower the eddy current losses, since eddy current losses are proportional to the square of the thickness. Also eddy current losses vary as the square of frequency whereas the Hysteresis losses vary directly with frequency.

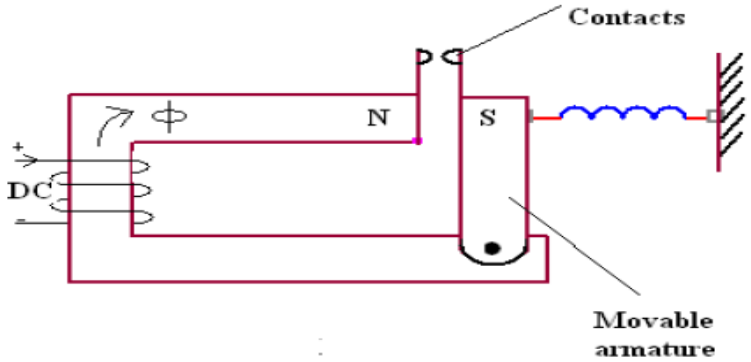


Eddy current tend to flow perpendicular to the flux and in such as to oppose any changes in the magnetic field.(Lenz's law)

- Both Hysteresis and eddy current losses (core losses) greatly affect operating temperatures, efficiencies and rating of magnetic devices.

1.5 Magnetic Forces Between Iron Surfaces

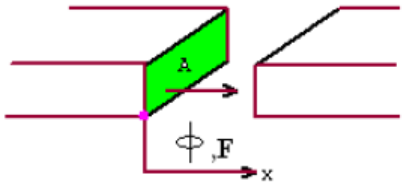
- Consider a fixed core with an excitation coil and a movable iron core called an armature.
- Assume that the two cores have the same cross-sectional area of square meters and are a small distance of *l* meters apart



- If the coil carries a current it will set up a flux density of B in the air gap. Mathematically the attractive force ***Fm*** between the two-iron surfaces is equal to:

$$\boldsymbol{F_m = \frac{B^2 A}{2\mu_0}[N]}\ldots\ldots$$

- To prove the above consider the force of alignment between two poles of a magnetic circuit as shown:



1) Assuming no fringing of flux the uniform flux density of the air-gap is given by: $\boldsymbol{B = \frac{\Phi}{A}}$

2) Suppose that the poles can be separated by a small distance dx, with B, Φ remaining constant. The work done due to the forces between the poles will then be:

$$dW_m = F.dx$$

3) Assume now that the magnetic core is ideal, that is, it is of infinite permeability and therefore requires no MMF to create a magnetic field in it. Thus the stored magnetic energy will be contained entirely in the air-gap.

- When the contacts separate the air gap is increased by a volume A*dx but since the flux density ,B, is constant the energy density is also constant. The increase in stored energy will be given by:

- $$dW_f = \frac{B^2}{2\mu_0}.A.dx$$

And since the system is ideal,

- $$dW_m = dW_f$$

- $$F.dx = \frac{B^2}{2\mu_0}.A.dx$$





Comparing this with $I = \frac{E}{R}$

We get $\mathcal{R} = \frac{1}{\mu_r \mu_0} \frac{l}{A}$

Analogy between Electric and Magnetic Circuits

Electric Circuits		Magnetic Circuits	
Quantity	*Units*	*Quantity*	*Units*
1. EMF, E	volts (V)	MMF, $\mathcal{F}$	ampere turn (At)
2. Current, I	ampere (A)	Flux, Φ	weber (Wb)
3. Resistance, R $R = \frac{1}{\sigma} \cdot \frac{l}{A}$	ohm (Ω)	Reluctance, $\mathcal{R}$ $\mathcal{R} = \frac{1}{\mu_r \mu_0} \cdot \frac{l}{A}$	ampere/weber (A/Wb)
4. Conductivity, σ	siemen/metre (S/m)	Permeabilty, μ	tesla metre per ampere (Tm/A)





Example 1

- Calculate the magnetomotive force (mmf) required to produce a flux of 0.015 Wb across an air gap of 2.5 mm long, having an effective area of 200 cm^2.

Solution :

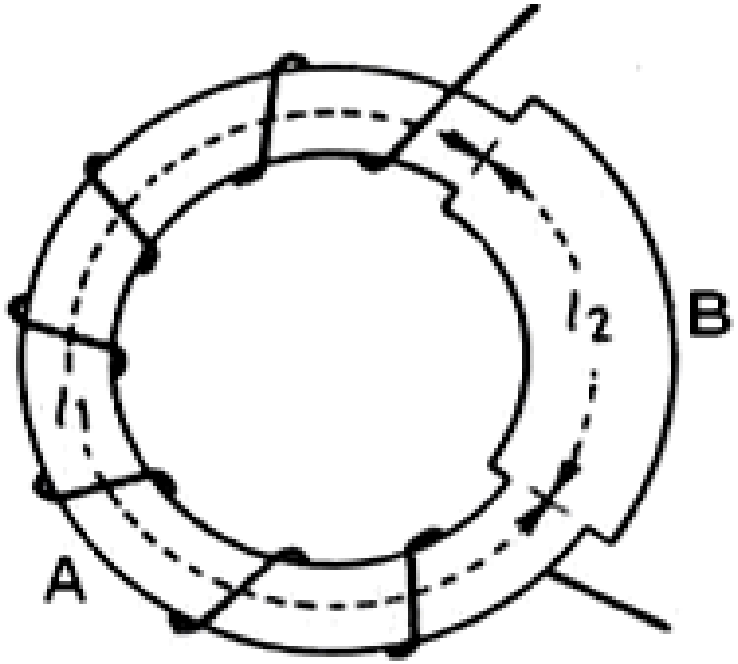
$$B = \frac{\Phi}{A} = \frac{0.015}{200 \times 10^{-4}} = 0.75 \text{ T}$$

$$H = \frac{B}{\mu_0} = \frac{0.75}{4\pi \times 10^{-7}} = 597\,000 \text{ A/m}$$

$$\mathcal{F} = Hl = 597\,000 \times 2.5 \times 10^{-3} = \mathbf{1492\ At}$$

Composite Magnetic Circuit

Case 1 :



$$\mathcal{R}_1 = \frac{l_1}{\mu_1 A_1}$$

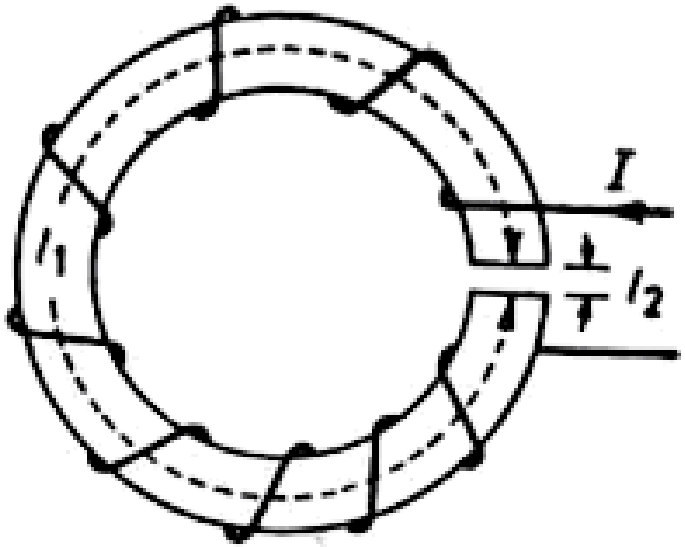
$$\mathcal{R}_2 = \frac{l_2}{\mu_2 A_2}$$

$$\therefore \text{Total Reluctance}, \mathcal{R} = \mathcal{R}_1 + \mathcal{R}_2 = \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2}$$

$$\therefore \text{ Total flux, } \Phi = \frac{\text{mmf of coil}}{\text{total reluctance}}$$

$$= \frac{\mathcal{F}}{\mathcal{R}} = \frac{NI}{\dfrac{l_1}{\mu_1 A_1} + \dfrac{l_2}{\mu_2 A_2}}$$

Case 2 : (with air gap)



Total reluctance,

$$\mathcal{R} = \frac{l_1}{\mu_1 A} + \frac{l_2}{\mu_0 A}$$

$$= \frac{1}{\mu_0 A}\left(\frac{l_1}{(\mu_1/\mu_0)} + l_2\right)$$

$$= \frac{1}{\mu_0 A}\left(\frac{l_1}{\mu_r} + l_2\right)$$





- Since the relative permeability μ_r (= μ_1/ μ_0) of steel is very large (of the order of thousand), the major contribution in the total reluctance $\mathcal{R}$ is by the air-gap, though its length l_2 may be quite small (say, a few millimetres).

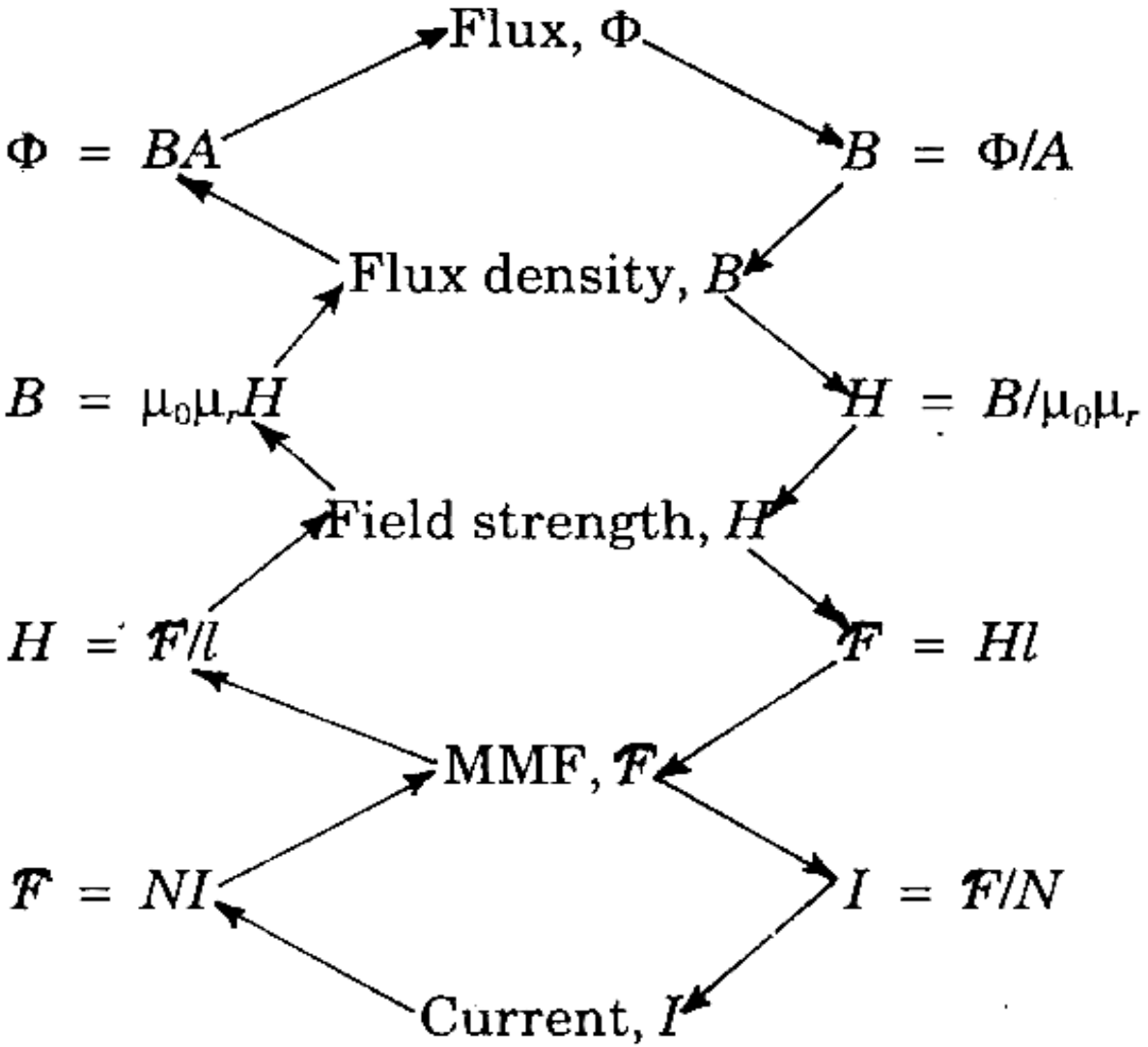




Kirchhoff's Laws

- ***Kirchhoff's Flux Law*** **(KFL) :** *The total magnetic flux towards a junction is equal to the total magnetic flux away from that junction.*
- ***Kirchhoff's Magnetomotive Force Law*** **(KML) :** *In a closed magnetic circuit, the algebraic sum of the product of the magnetic field strength and the length of each part of the circuit is equal to the resultant magnetomotive force.*

Steps to solve a problem on magnetic circuit



Example 1

Using the magnetic circuit concepts, analyze the toroid coil shown in Fig. 1. The magnetic circuit is analogous to the electrical circuit shown in Fig. 2, with a resistance connected across a voltage source.

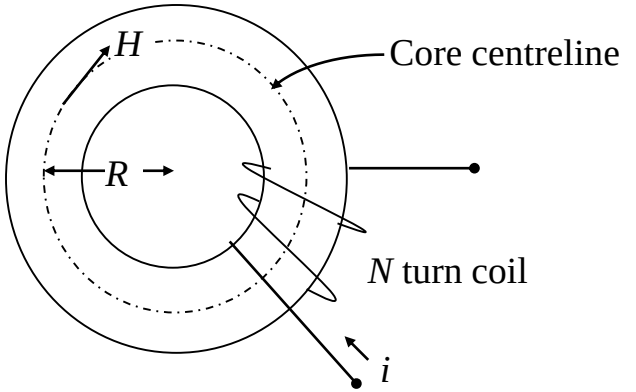


Fig. 1

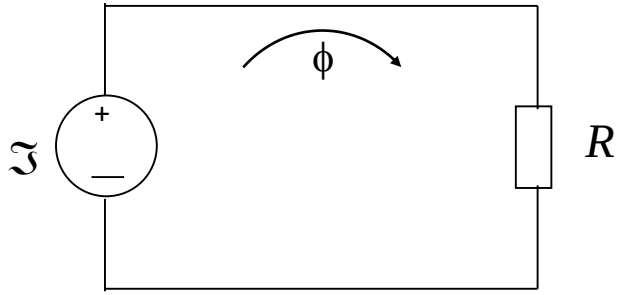


Fig. 2

Fig. 1. Coil on a toroid iron core

Fig. 2. The magnetic circuit for the toroid coil

Find the expressions for the reluctance and magnetic flux.

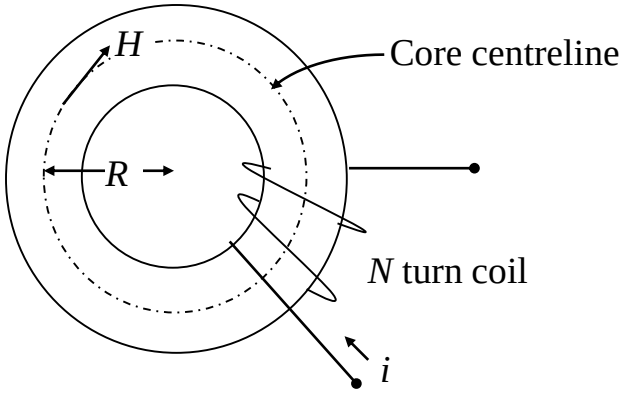
Solution to Example 1

The mean length of the magnetic path is: $l=2\pi R$

The cross-section of the core is circular with a radius, r.
Thus, the area of the cross section is: $A=\pi r^2$.

The expression for the reluctance (from Eq. 1-2):

$$\mathfrak{R} = \frac{l}{\mu A} = \frac{2\pi R}{\mu \pi r^2} = \frac{2R}{\mu r^2}$$

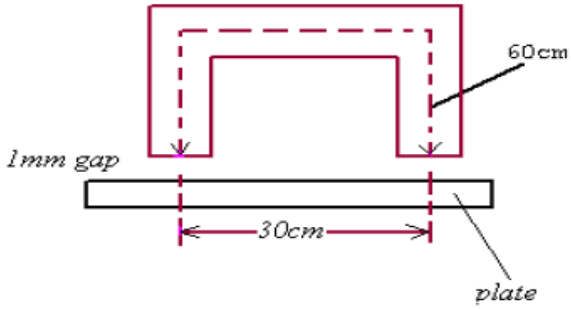


The mmf is: $\mathfrak{I}=Ni$

And the magnetic flux: $\phi = \frac{\mathfrak{I}}{\mathfrak{R}} = \frac{\mu N r^2 I}{2R}$

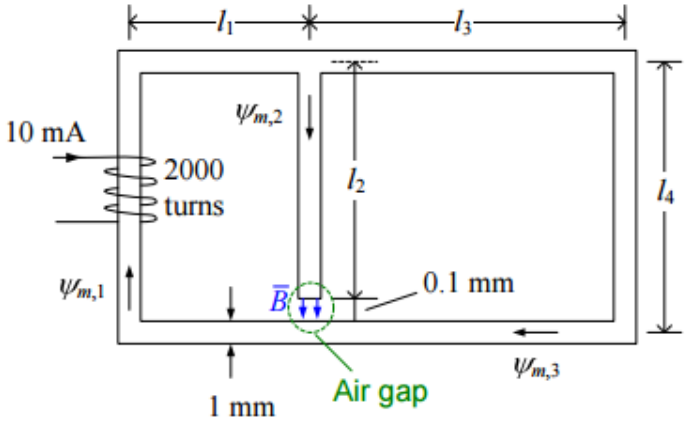
ASSIGNMENT

$$F = \frac{B^2}{2\mu_0}.A.$$



The poles faces of a lifting magnet shown have an area of *200cm2* which is also the cross sectional area of the *60 cm* long flux path in the cast steel magnet. The poles faces are *30 cm* apart. Determine the coil current required on the magnet to lift *267kg* of cast iron plate separated by *1mm* from the poles faces.

Example N1.1: Determine the magnetic flux through the air gap in the geometry shown below. The structure is assumed to have a square cross section of area 10^{-6} m^2, a core with $\mu_r = 1{,}000$, and dimensions $l_1 = 1$ cm, $l_3 = 3$ cm, and $l_4 = 2$ cm.



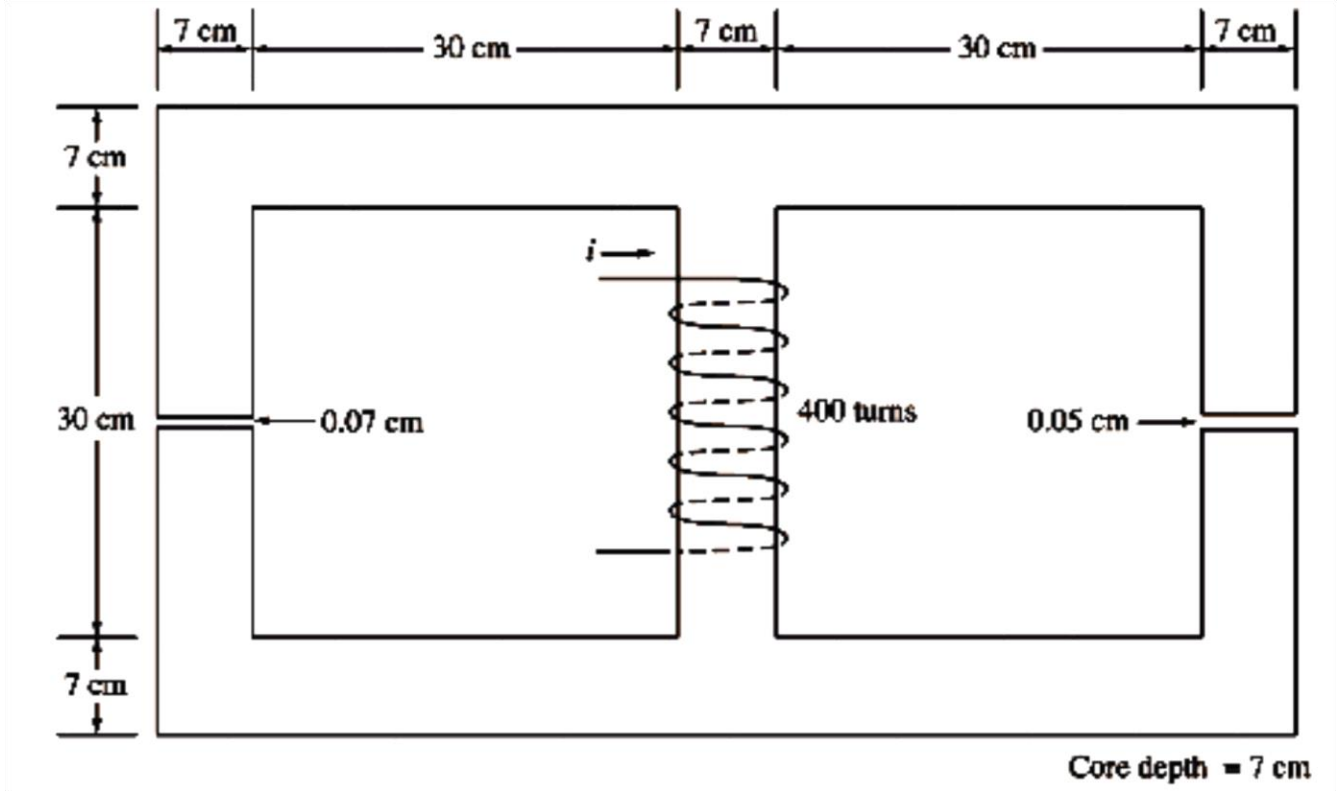








- A ferromagnetic core with a relative permeability of 2000 is shown in Figure 1. The dimensions are as shown in the diagram, and the depth of the core is 7 cm. The air gaps on the left and right sides of the core are 0.070 and 0.050 cm, respectively. Because of fringing effects, the effective area of the air gaps is 5 percent larger than their physical size. If there are **400** turns in the coil wrapped around the center leg of the core and if the current in the coil is 1.0 A, what is the flux in each of the left, center, and right legs of the core? What is the flux density in each air gap?


7 cm
30 cm
7 cm
30 cm
7 cm
7 cm
30 cm
7 cm
i
0.07 cm
400 turns
0.05 cm
Core depth = 7 cm









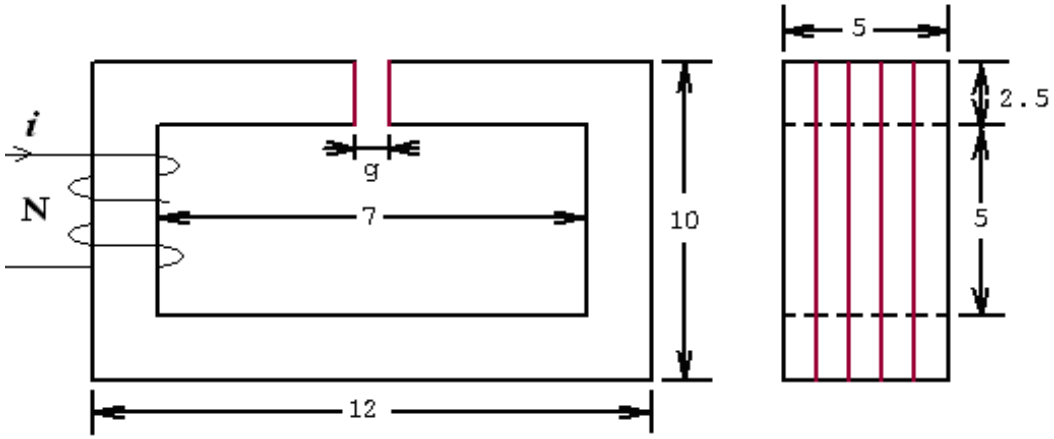


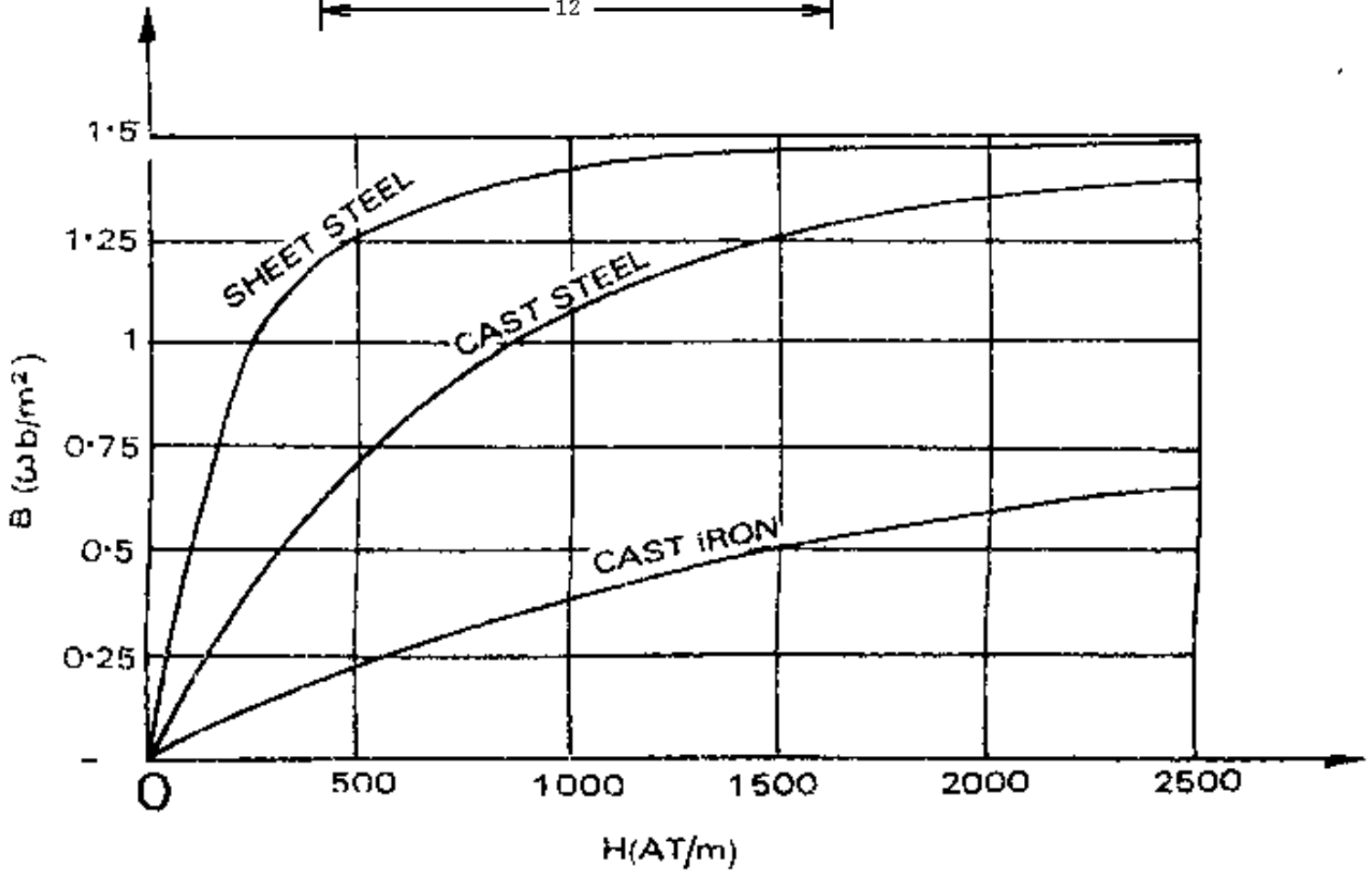


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- The magnetic circuit of the fig 1 below is comprised of a laminated core of cast iron with an air gap g=0.25 cm and has an exciting winding of 400 turns. The magnetization curve of cast iron is shown in fig 2. The presence of non magnetic material between the laminations is taken into account by 0.95 stacking factor. The measurements are in centimetres. Neglecting leakage but accounting for fringing, calculate the current in the exciting winding to produce a flux of $0.5*10^{-3}$ Wb in the core. What energy will be stored in the air gap?






i
N
g
7
10
12
5
2.5
5


1·5
1·25
1
0·75
0·5
0·25
-
B (ωb/m²)
SHEET STEEL
CAST STEEL
CAST IRON
O
500
1000
1500
2000
2500
H(AT/m)

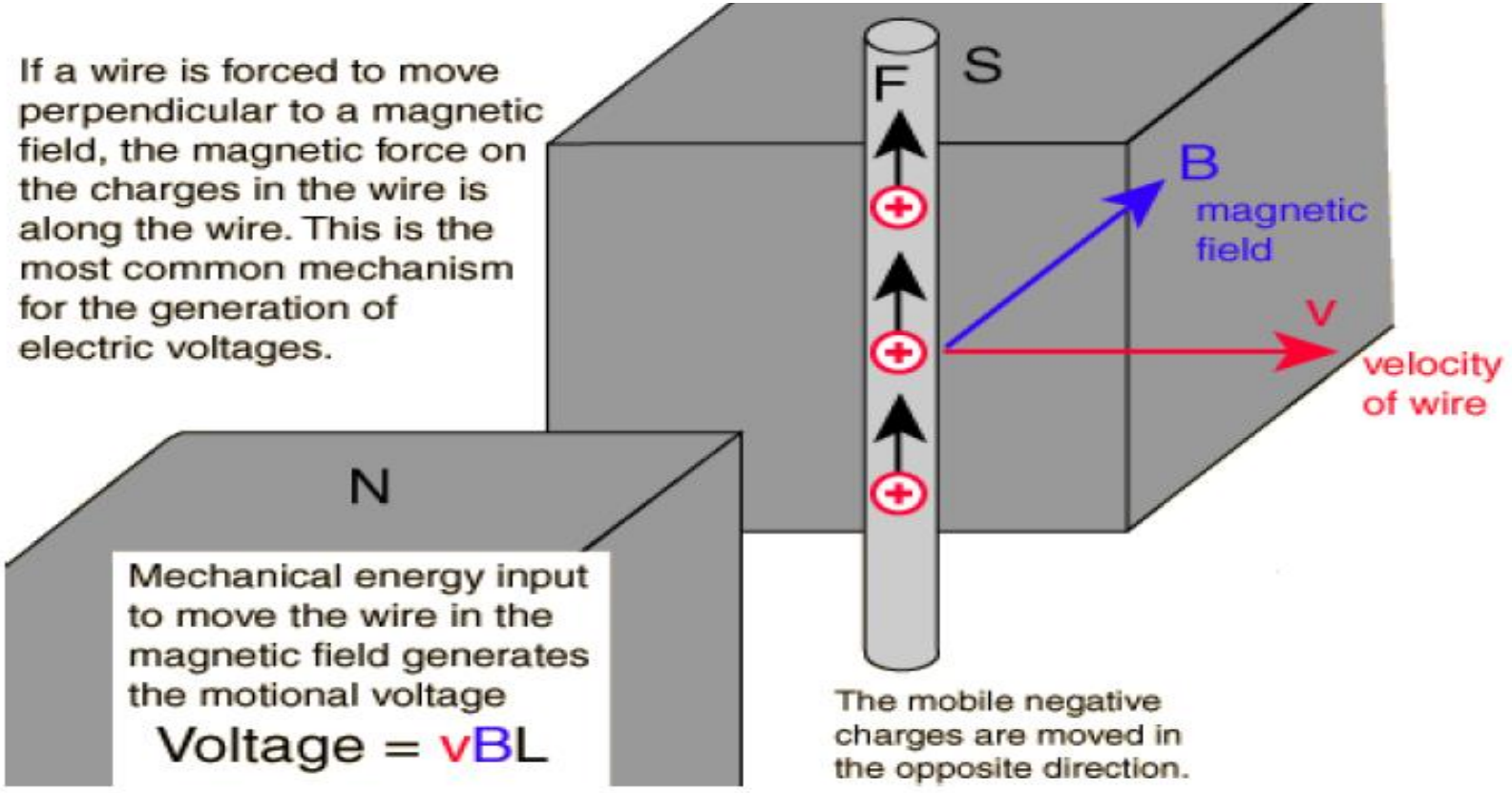




$$L_{iron} = 12 + 10 + 10 + (12 - 0.25) = 43.25cm$$

3. DC MACHINES

3.1 Principles of D.C. Machines:







- Consider a conductor of length **L** at right angles to a uniform magnetic field of density **B**. If the conductor moves as shown, the flux linking the conductor will change. According to Faradays' law an EMF is induced in a circuit placed in a magnetic field if either:

1. the flux linking the circuit changes

2. There is a relative motion between the circuit and the magnetic field such that the conductors forming the circuit cut across the field lines.

- In fig. 3.1, a voltage, **e** volts will be induced in the conductor and hence circuit because the conductor cuts the flux. If the conductor moves with a relative velocity **V** with respect to the field, then

$$\boldsymbol{e = B.l.v} \qquad \boldsymbol{\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.1}$$





where **B – M**agnetic flux density, **T**

$\boldsymbol{l}$**–** Conductor length in the field, **m**

$\boldsymbol{v}$**–** Relative velocity between the field and conductor**, m/s** and voltage polarity as indicated**.**

- The polarity is determined using the right hand rule (RHR). According to this rule if the right hand is held such that the thumb, forefinger and middle finger are mutually perpendicular, the forefinger points in the direction of the field, B, the thumb in the direction of the conductor motion with respect to field, the middle finger represents the direction of the generated EMF, i.e. toward the positive terminal. RHR is also called the generator rule.

3.1.1 Simple Generators:

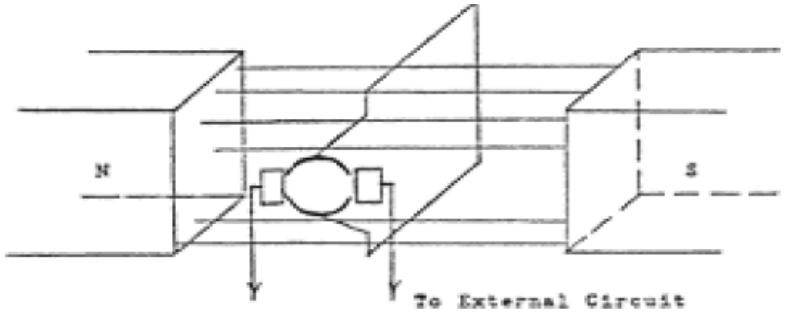


Figure 3.2 Brushes and Commutator provides DC voltage

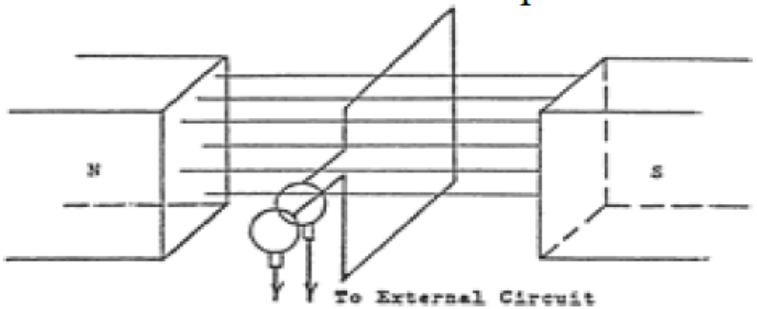


Figure 3.3 Brushes and slip rings provide AC

- To keep the torque on a DC motor from reversing every time the coil moves through the plane perpendicular to the magnetic field, a split-ring device called a **commutator** is used to reverse the current at that point. The electrical contacts to the rotating ring are called "**brushes**" since copper brush contacts were used in early motors. Modern motors normally use spring-loaded carbon contacts, but the historical name for the contacts has persisted.

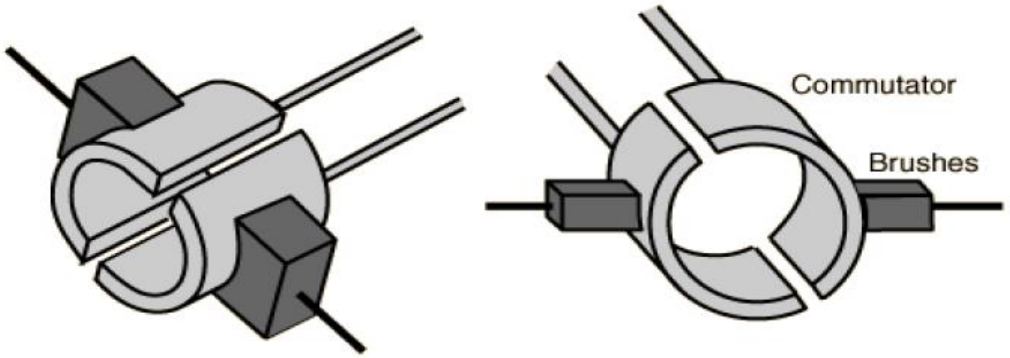


Fig 3.4 Commutator





- A simple AC Generator is illustrated below. The turning of a coil in a magnetic field produces motional e.m.f's in both sides of the coil which add. Since the component of the velocity perpendicular to the magnetic field changes sinusoidally with the rotation, the generated voltage is sinusoidal or AC.
- According to RHR for the clockwise rotation indicated, the direction of the generated voltage in the coil side under the North Pole will be directed from **a** to **b.** Similarly the direction of voltage induced under the South Pole will be directed from **c** to **d**
- When the conductor **a b** moves from under the North Pole to the South Pole, the direction of the induced **EMF** will reverse so that the current will flow from **b** to **a**.
- The current of externally connected load resistor via stationery brushes in contact with a pair of slip rings A and B will be alternating. The alternating current is not sinusoidal since the coil rotates in a non uniform field.

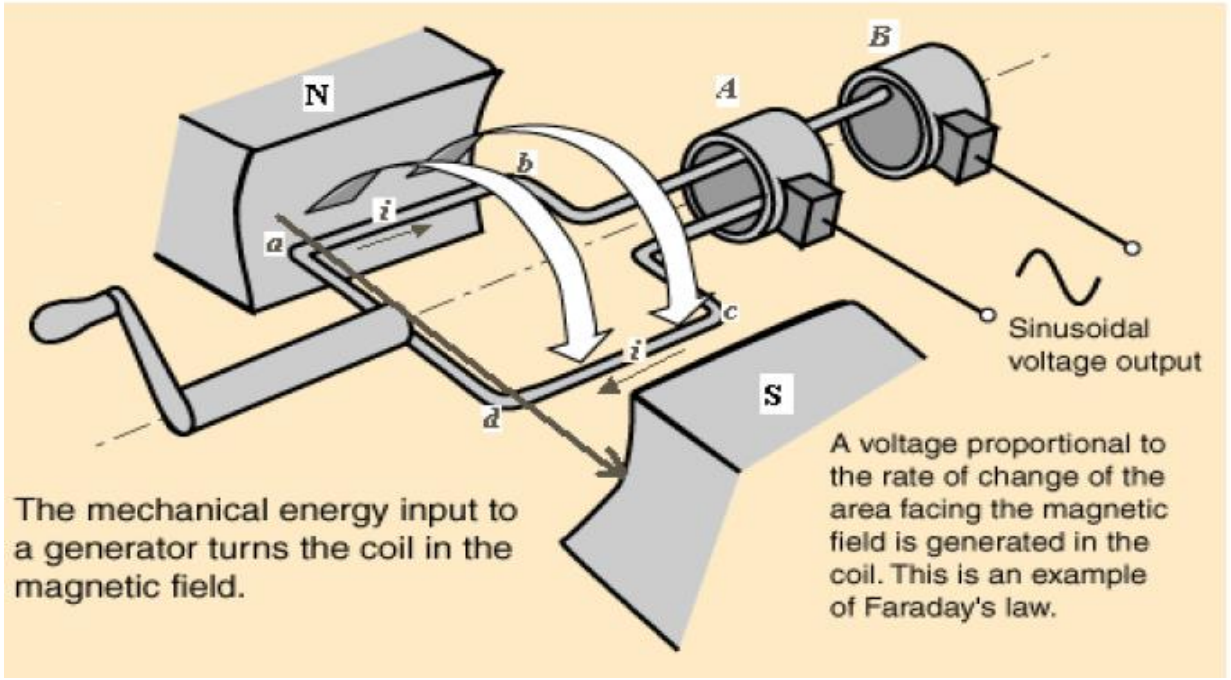
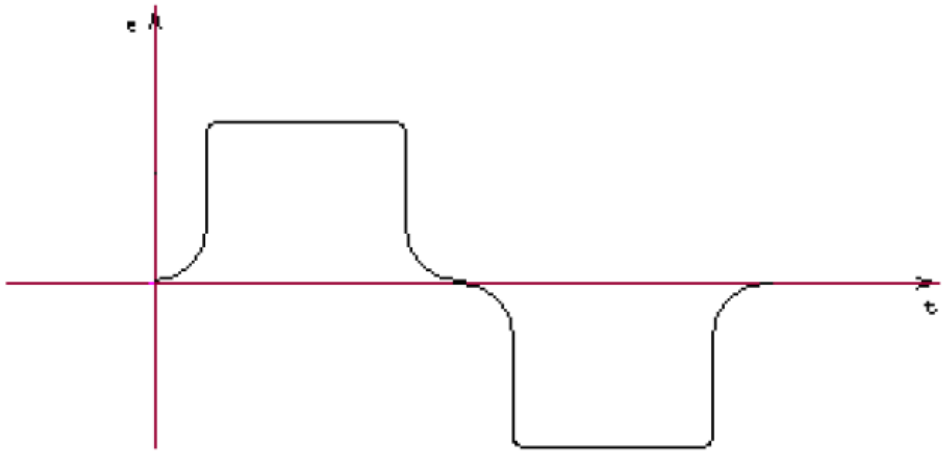


Fig 3.5 A simple AC Generator

- The field is more uniform at the conductor side location and the flux density more even, except for the inter-pole regions, that is, the regions between the poles



- **Fig 3.6**

Commutator action

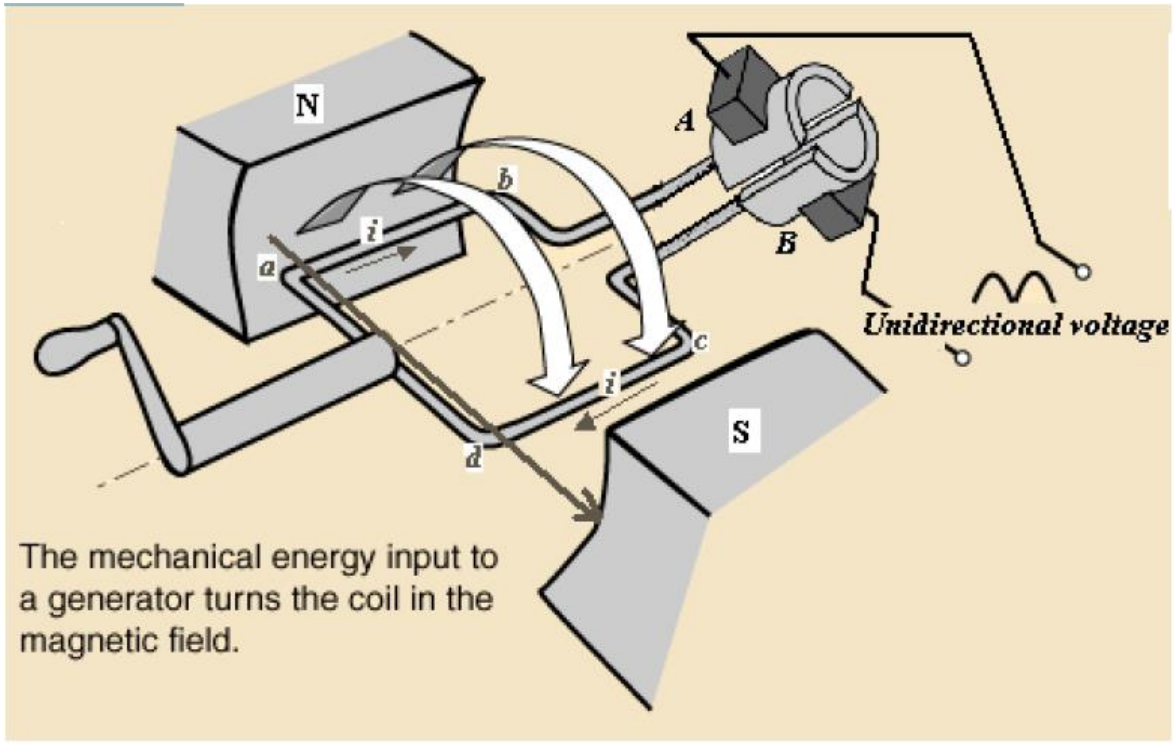


Fig 3.7 Commutator Action





- All practical rotating electrical generators generate alternating E.M.F's. To rectify the internal ac generated, to direct current, a mechanical rectifier, commutator, is used.
- The coil side **ab** is permanently connected to segment **A** of the commutator, and the coil side **cd** to **B**. The brushes connect the top and bottom when coil is in the horizontal position.
- Upon clockwise rotation, the split ring rotates with the armature and brushes and the poles are stationery. At position shown, current will flow through the coil from abcd to segment **B**, via the positive brush through the load to the negative brush to segment **A**. The current in the external load circuit flows from the lower brush to the upper brush.
- When the armature rotates so that the coil assumes a vertical position, the brushes will short circuit both segments, but the induced e.m.f is zero.

- When the armature rotates the vertical position, induced EMF in side **ab** and **cd** are reversed respectively, but the commutator segments to which they are connected simultaneously change positions under the stationery brushes. Hence both polarity of the brushes and the direction of the current in the load remain fixed. This is called the commutation process.

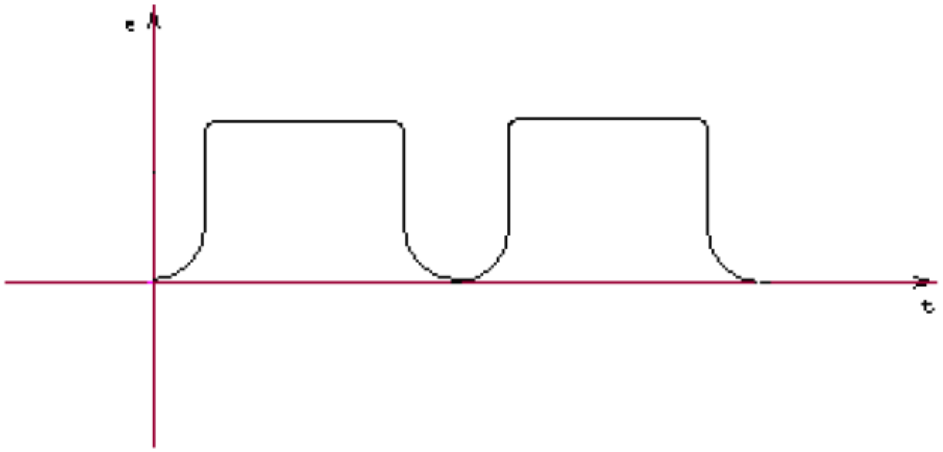


Fig 3.8 Unidirectional pulse voltage




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Armature Windings

- The resulting terminal voltage above is not a pure but is pulsating dc. To reduce this ripple, full armature winding distributed over the entire armature surface.

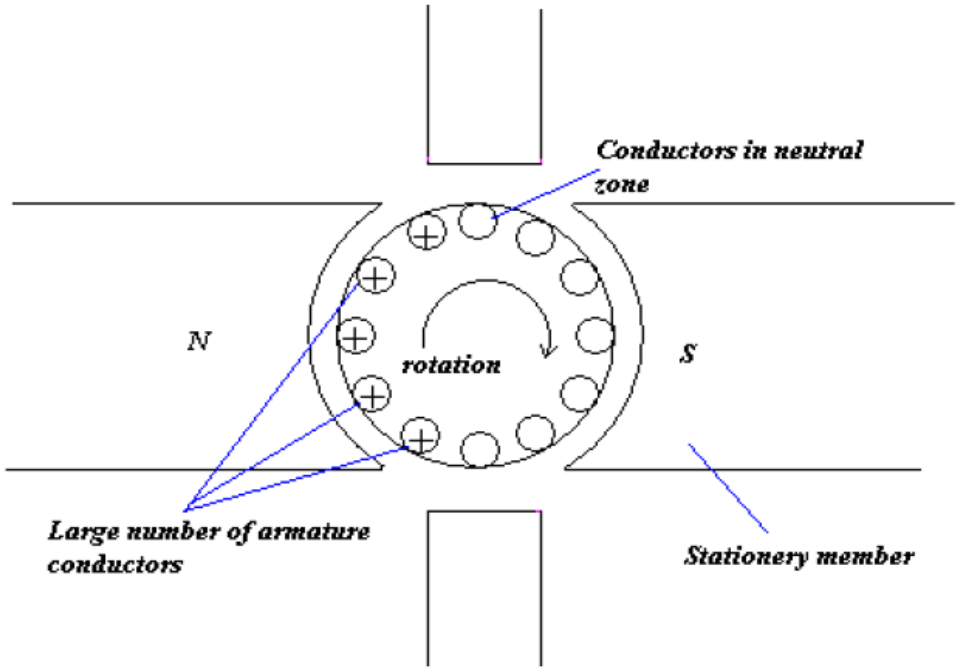


Fig 3.9 Armature windings

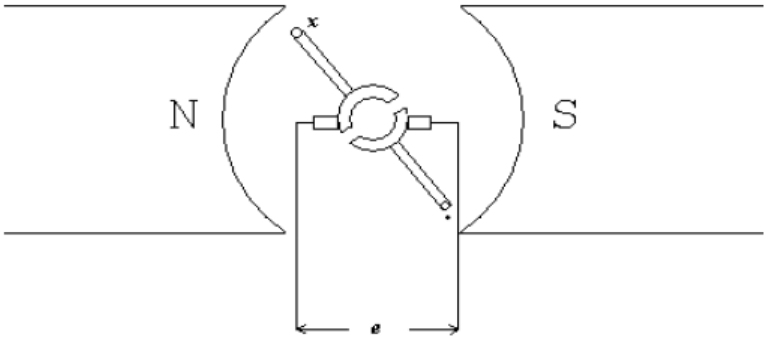




- With the armature rotating clockwise, all conductors under the north pole would have induced voltages directed into the page and all those facing the south pole have induced voltages directed out of the page.
- When all armature conductors are connected to form a continuous winding, the induced voltages will aid one another in the manner shown. The conductors generating EMF's are those that are moving opposite a pole. In each path the number of conductors simultaneously generating EMF's remain constant from instant to instant and are unaffected by the rotation of the armature, hence the voltage generated in the terminals is constant.
- At one instant in time there are some contacts in the inter-pole region which do not face either one of the poles mounted on the stationery member, the stator. These conductors are in neutral zone.
- The ends of each armature coil are connected to a multiple segmented commutator. The commutator is a distinctive element in the construction of the machines. It has major setbacks of limiting the output power of the dc generators and adding to its costs. When multiple coils are joined in series aiding, the terminal voltage will increase.

- Consider the relationship between the number of commutator segments and generated voltage wave.

a) Two segment commutator



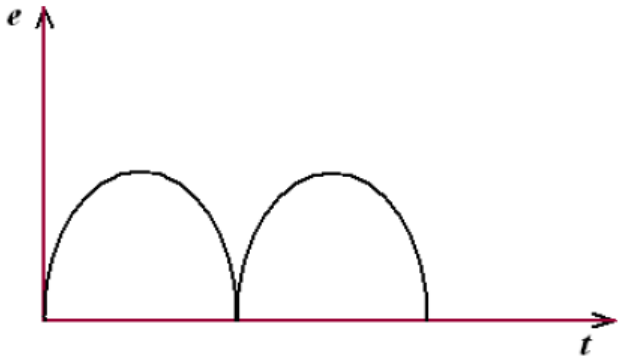


Fig 3.10

b) Four segment commutator

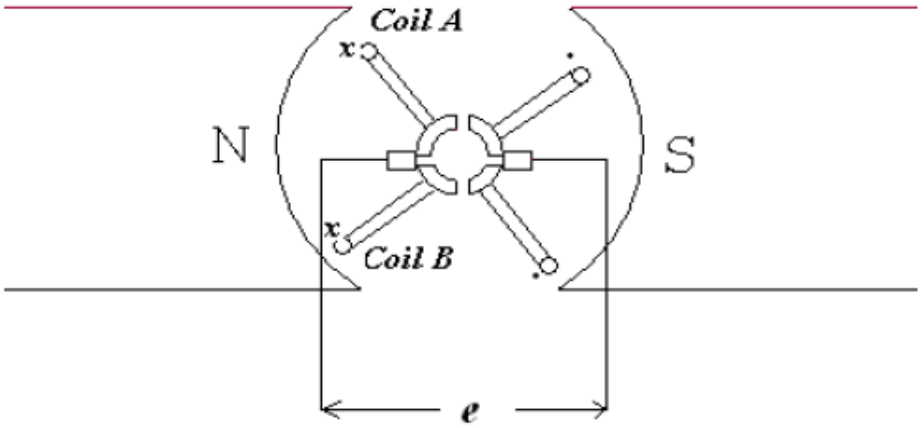
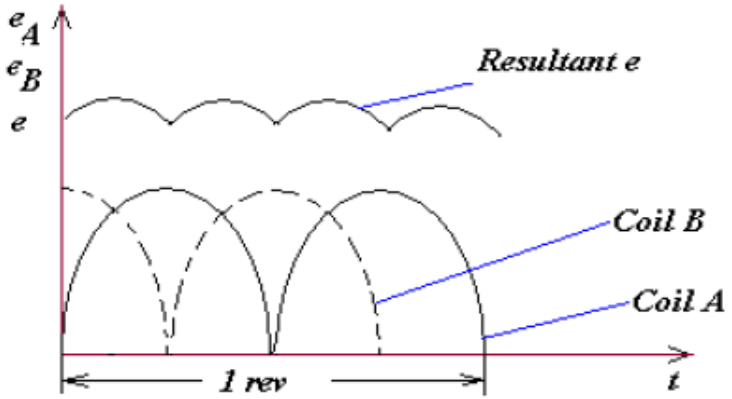
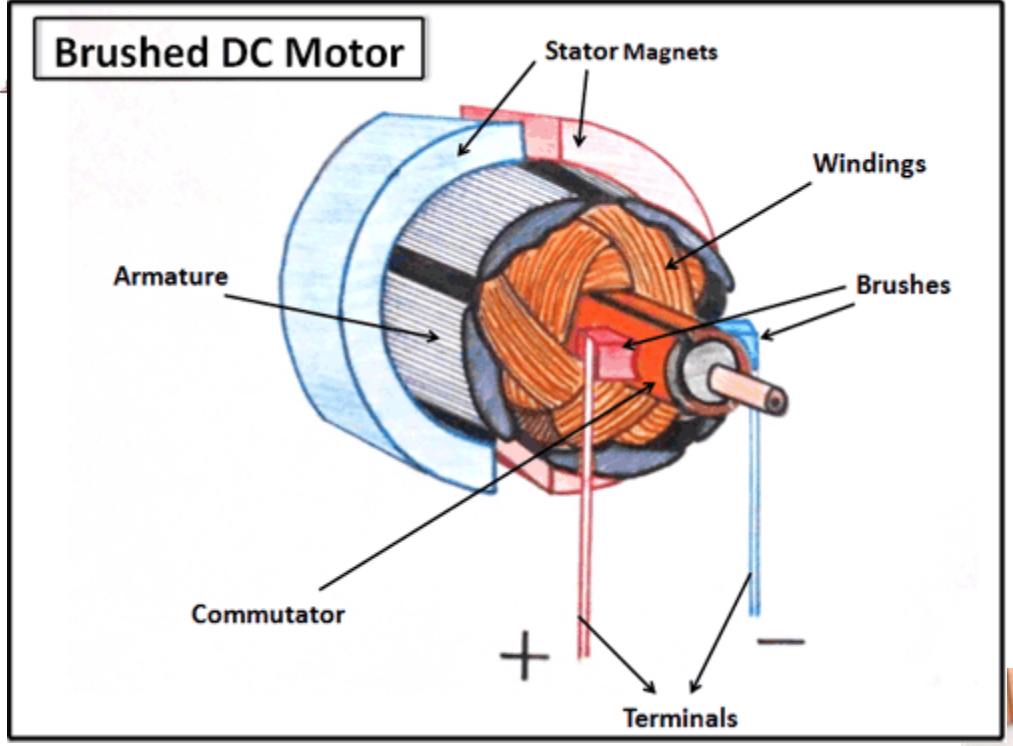


Fig 3.11 a, b



- Notice also as the number of segments increases the pulsations will be less pronounced. Generally when there are a great number of armature coils and commutator segments, the terminal voltage appearing at the brushes will approach a smooth unvarying dc voltage.


Brushed DC Motor
Stator Magnets
Windings
Armature
Brushes
Commutator
+
−
Terminals



- **Gramme Ring Winding** The old Gramme Ring type winding, now obsolete, is shown in Figure 3.12 and its equivalent circuit in Figure 3.13. It can be seen that there are an equal number of voltage-generating conductors on each side of the armature and the conductor voltages are additive from bottom to top on each side. There are two paths between the positive and negative brushes and the voltage per path is the generated voltage of the machine. Each path provides half of the current output.

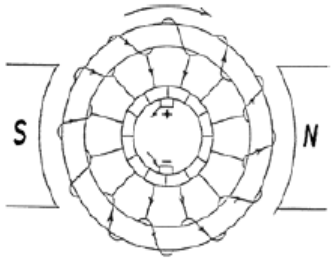


Fig 3.12 Two Pole Gramme Ring Winding

- a

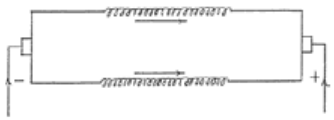


Fig 3.13 Equivalent Circuit, Two Pole Gramme Ring Winding

Drum Winding

- The Drum type winding is made of coils, one of which is illustrated in Figure 3.14. The straight portions of the coil are the parts rotating through the magnetic field in which the voltage is induced. Therefore, each single coil has two conductors. This has the advantage over the Gramme Ring winding where only one side of each coil is used as an active conductor. There are two classes of drum windings depending upon how the coils are connected to the commutator.

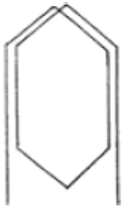


Fig 3.14 Drum Type Winding Coil

- When the end connections of the coils are brought to adjacent bars as shown in Figure 3.15, a lap or parallel winding is formed. In this type winding, there are as many paths through the armature as there are poles on the machine. Therefore, to obtain full use of this type winding, there must be as many brushes as there are poles, alternate brushes being positive and negative.





- Any winding can be illustrated in one of two forms, the circular form or the development form. A simplex lap winding is shown in Figure 3.16 (circular form) and Figure 3.17 (development form.)
- In this particular circular form, the flux cutting portions of the conductors are shown as straight lines radiating from the center and are numbered for convenience in connecting them to the commutator which is in the center of the diagram. The outermost connecting lines represent the end connections on the back of the armature and the inner connecting lines represent the connections on the front or commutator end of the armature. The development form of winding represents the armature as if it were split open and rolled out flat. It is somewhat simpler to understand but the continuity of the winding is broken. The lap winding is best suited for low voltage, high current ratings because of the number of parallel paths.

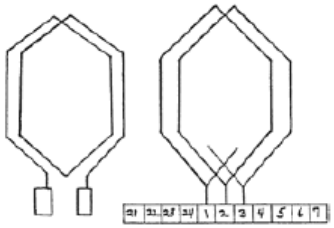


Fig 3.15 Lap Winding connected to commutator bars

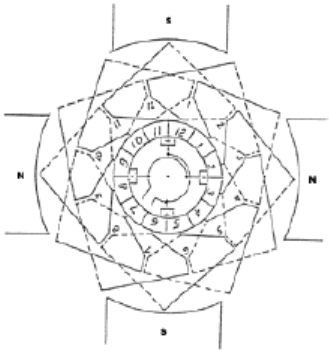


Figure 3.16 Simplex Lap Winding, Circular Form

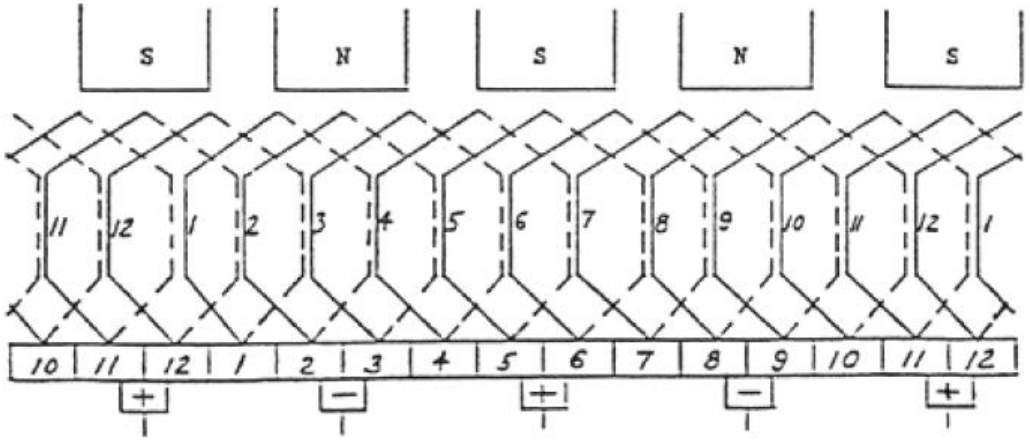


Figure 3.17. Simplex Lap Winding, Development Form

Wave Winding

- When the end connections of the coils are spread apart as shown in Figure 3.18 a wave or series winding is formed. In a wave winding there are only two paths regardless of the number of poles. Therefore, this type winding requires only two brushes but can use as many brushes as poles

- The simplex wave winding in Figure 16 (circular) and Figure 17 (development) shows that the connections to the armature do not lap back toward the coil but progress forward. The coil voltages are cumulative but it is necessary to travel several times around the armature and to traverse half the total winding in order to trace the path between the positive and negative brush. The wave winding is best suited for high voltage low current ratings since it has o

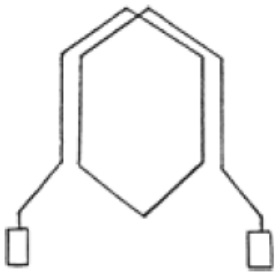


Figure 3.18. Wave Winding connected to commutator bars

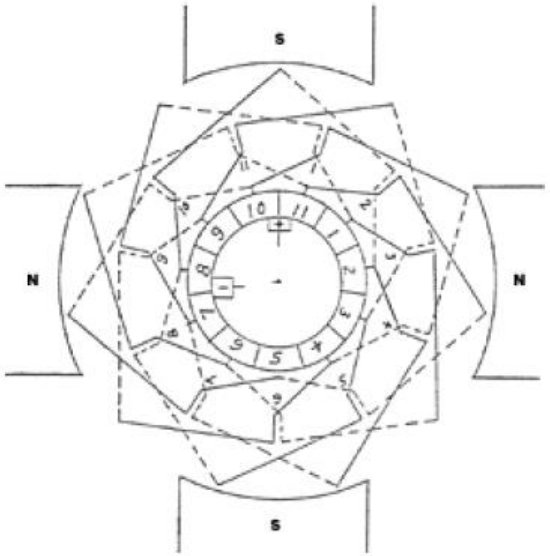


Figure 3.19. Simplex Wave Winding, Circular Form

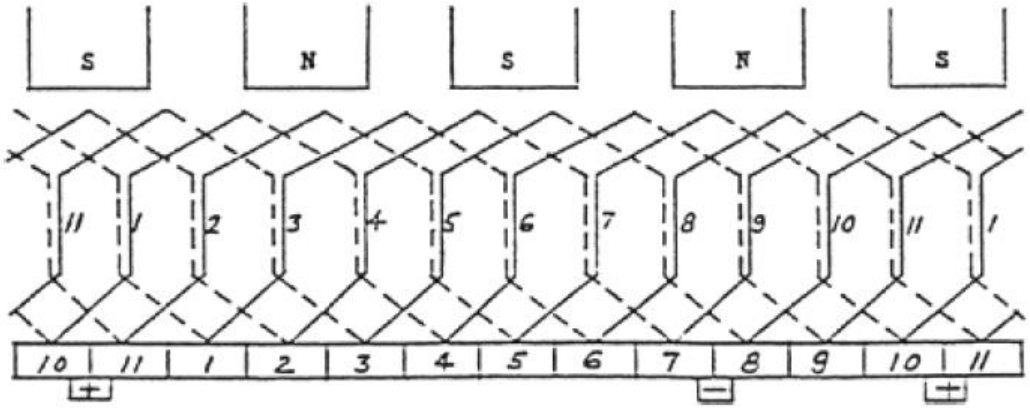


Figure 3.20 Simplex Wave Winding, Development Form

Slots and Coils

- The number and size of slots depend upon the generator or motor requirements. The slot has to be large enough to hold the correct number of conductors but at the same time, the tooth has to be large enough to pass the necessary magnetic flux

- Normally, in a simple winding, there are as many coils as there are slots. This means that each slot contains two coil sides, one side of each coil being at the top of a slot and the other at the bottom of a slot. Each coil may consist of one or more turns depending on the applied or generated voltage of the unit. A typical arrangement of coil sides and slots is shown in Figure 3.21. Solid lines represent the front end connections to the commutator and dotted lines represent the back end connections.

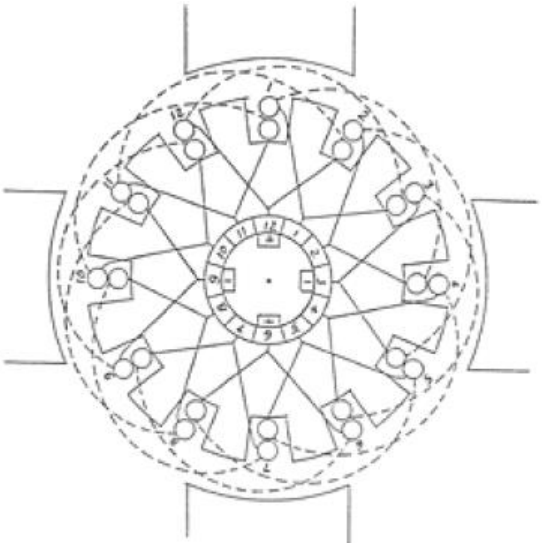


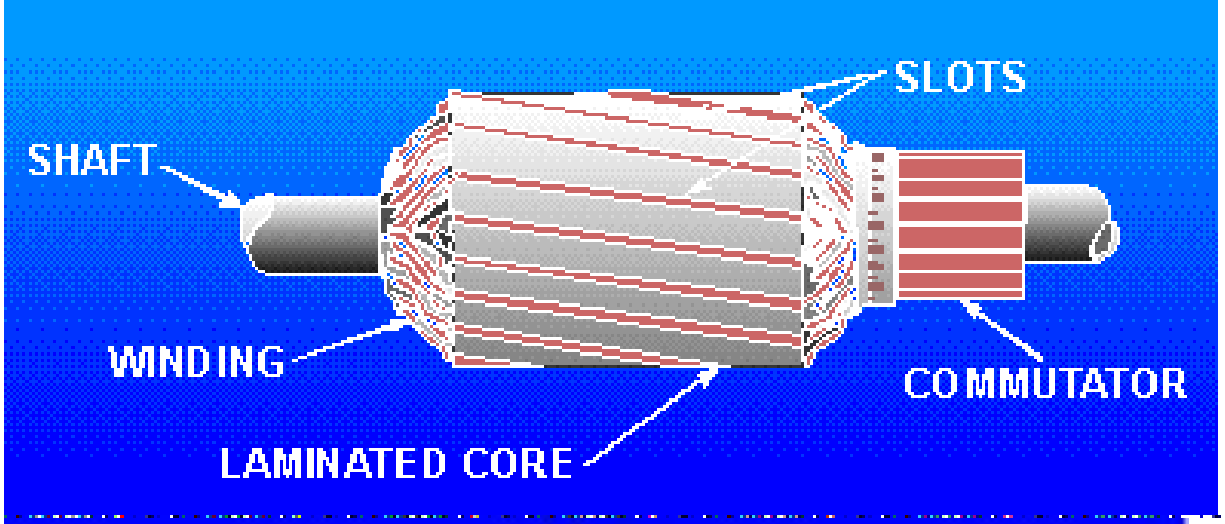
Figure 3.21. Coil Sides in Armature Slots

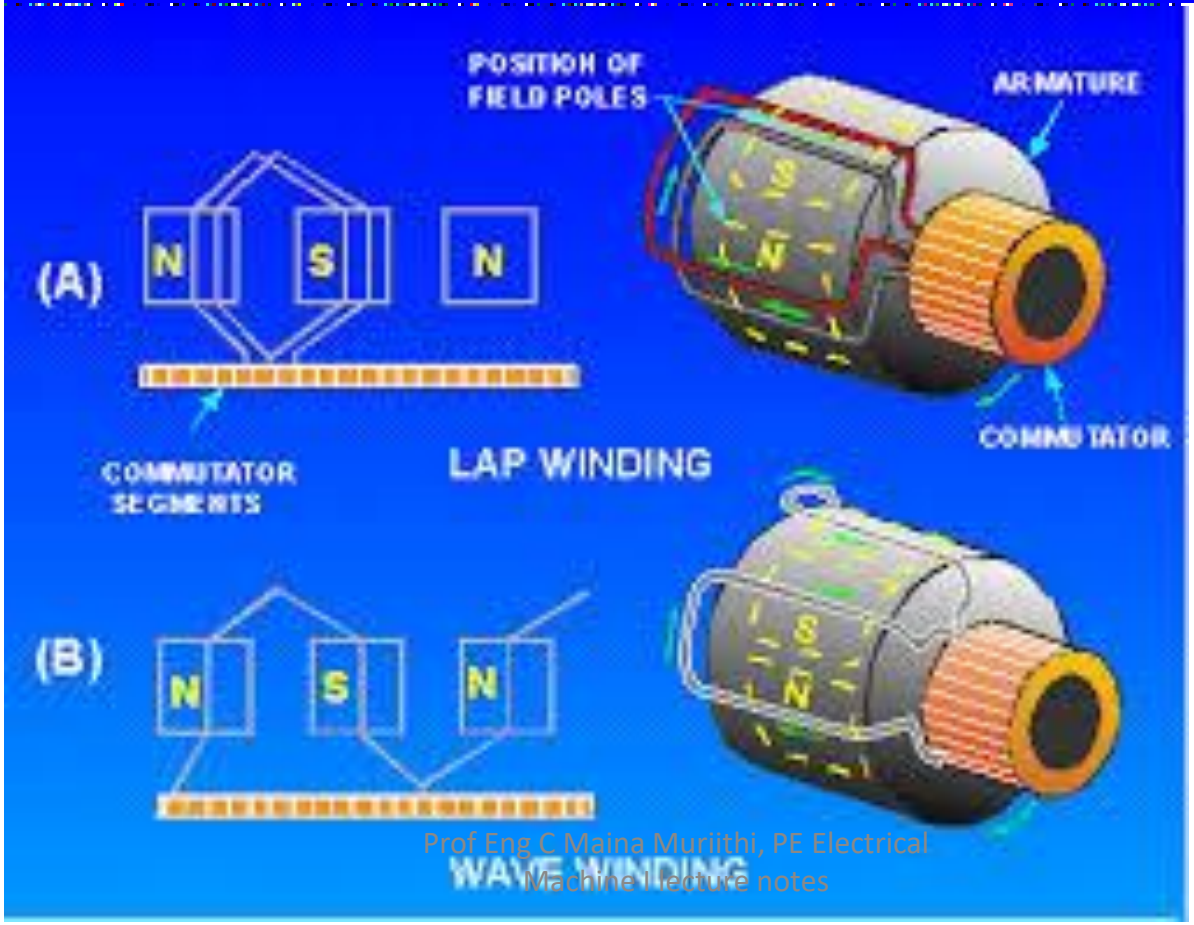


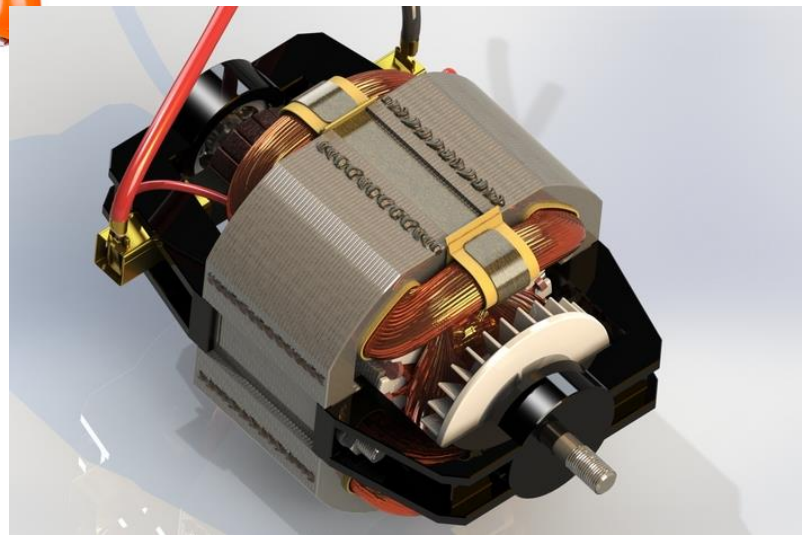


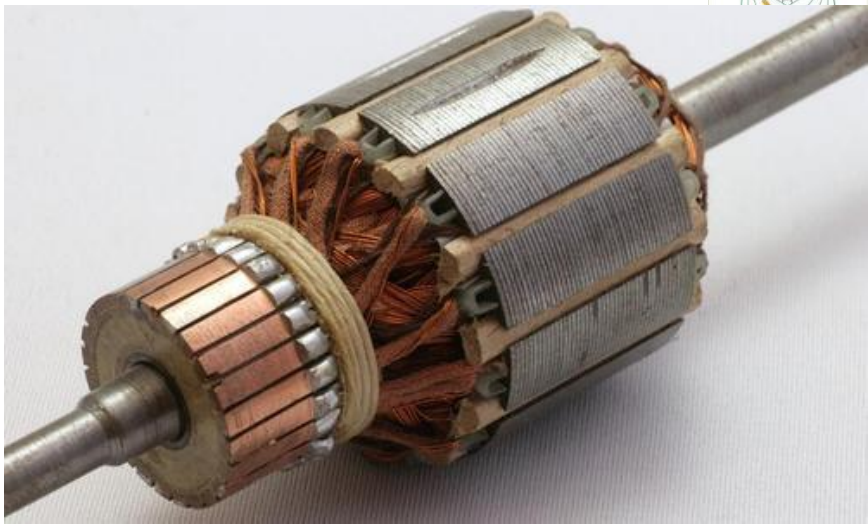
Slot Pitch

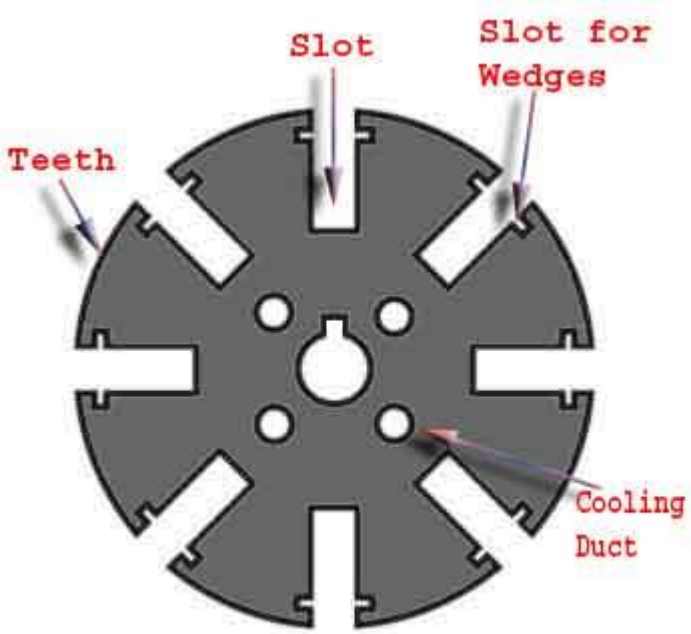
- Slot pitch refers to the number of slots spanned by each coil. For example, in Figure 3.21, the top of coil in slot 1 has its bottom in slot 4, therefore, the slot pitch is 1-4 or 3. Since the top of the coil is directly under the north pole and the bottom is directly under the south pole, the winding is known as a full pitch winding. In many cases, for various reasons, the pitch is reduced to less than full pitch.
-


SLOTS
SHAFT
WINDING
COMMUTATOR
LAMINATED CORE


POSITION OF FIELD POLES
ARMATURE
(A)
N
S
N
COMMUTATOR SEGMENTS
COMMUTATOR
LAP WINDING
(B)
N
S
N
WAVE WINDING






Slot
Slot for
Wedges
Teeth
Cooling
Duct



3.1.2. Generated voltages:

Faraday's law ($e = N\frac{\Delta\Phi}{\Delta t}$) tells us that the induced voltage depends on the rate at which the flux is cut , that is $\frac{\Delta\Phi}{\Delta t}$.

If there are Z conductors on the armature of a P-pole machine the total flux cut by each of the conductors is $\Phi \times P$.

When the rotational speed is n [rev/min] and the Z conductors are connected in **a** parallel paths, the voltage generated in each, conductor is $\frac{n\Phi P}{60}$.

If there are $\frac{Z}{a}$ such conductors in series per parallel path, the total generated EMF becomes:

$$E_a = \frac{ZP}{60a}\Phi n = K_G \Phi n \quad \text{.....3.2}$$

Where: Ea – Generated EMF

a – Number of parallel paths on armature (determined by type of winding, lap winding a= P, wave winding a = 2)

n – Speed of rotations, revolutions per minute

$K_G = \frac{ZP}{60a}$ – constant for a specific machine.





- **3.1.2. Generated voltages:**
- Faraday's law $(\boldsymbol{e = N\frac{\Delta\Phi}{\Delta t}})$ tells us that the induced voltage depends on the rate at which the flux is cut , that is $\frac{\Delta\Phi}{\Delta t}$
- If there are Z conductors on the armature of a P-pole machine the total flux cut by each of the conductors is $\boldsymbol{\Phi x P}$
- When the rotational speed is n [rev/min] and the Z conductors are connected in **a** parallel paths, the voltage generated in each, conductor is $\boldsymbol{\frac{n\Phi P}{60}}$.
- If there are such $\boldsymbol{\frac{Z}{a}}$conductors in series per parallel path, the total generated EMF becomes:

$$E_a = \frac{ZP}{60a}\Phi n = K_G \Phi n$$

Where:

Ea – Generated EMF

a – Number of parallel paths on armature (determined by type of winding, lap winding a= P, wave winding a = 2)

n – Speed of rotations, revolutions per minute

$\boldsymbol{K_G = \frac{ZP}{60a}}$ **is constant for a specific machine.**

- Notice that the generated voltage for a specific machine can be controlled by changing the prime mover speed n by variation of the field strength Φ, by means of the field excitation current.







3.1.3 Motor Action

- The function of a motor is to develop torque. In a dc motor the dc source is supplied to the brushes.

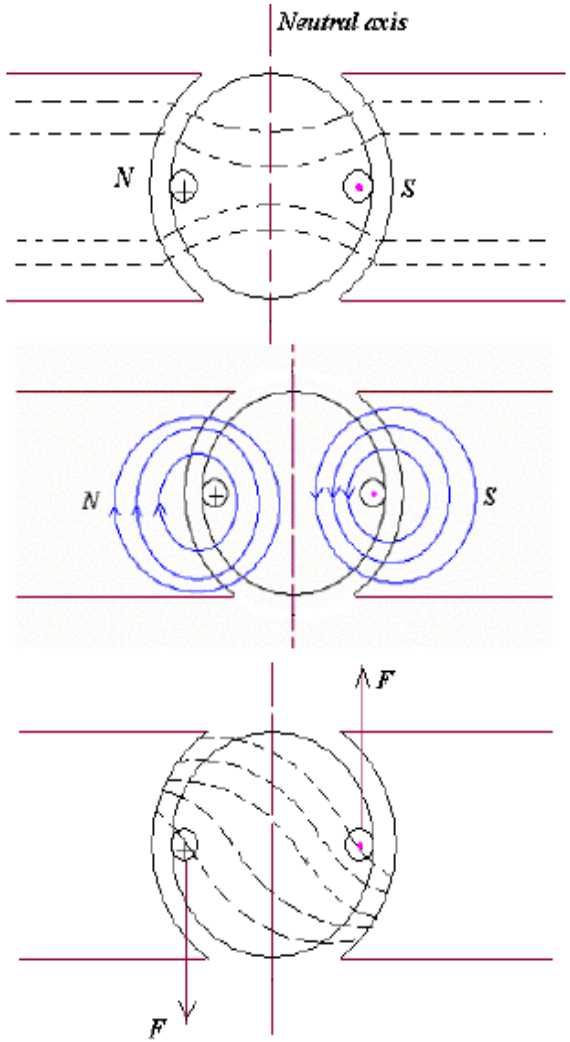


Fig 3.22 Production of Torque





- When placed in a uniform field, the current carrying conductor tends to distort that field. The conductor will experience a force tending to move it from the higher to the lower flux density.
- The armature being free to rotate on a shaft will therefore experience a torque that will produce anticlockwise rotation of the armature.
- The force depends on the field B and current I in the armature conductors. Since every conductor adds to the production of the total force, F must also depend on the conductor length L. Therefore:

$$F = BxLxI \ \ N \qquad 3.3$$

- Notice that the relationship between current direction, field direction and the developed force on the coils may be determined by means of the left hand rule (motor rule).
- LHR: The index finger points in the direction of the field, the second (middle) finger in the direction of the current and thumb points to the direction of Torque.





Counter EMF

- The developed force on the armature conductor causes the armature to rotate, resulting in a change of flux linking of those current carrying conductors. Thus, the motor also acts as a generator.
- The motor action is stronger than the generator action because the direction of the current in the armature winding is forced by the voltage supply.
- The generated EMF is opposed to the impressed voltage to such an extent as to limit the armature current to the value required to drive the shaft load. The generated EMF is called counter (back) EMF.

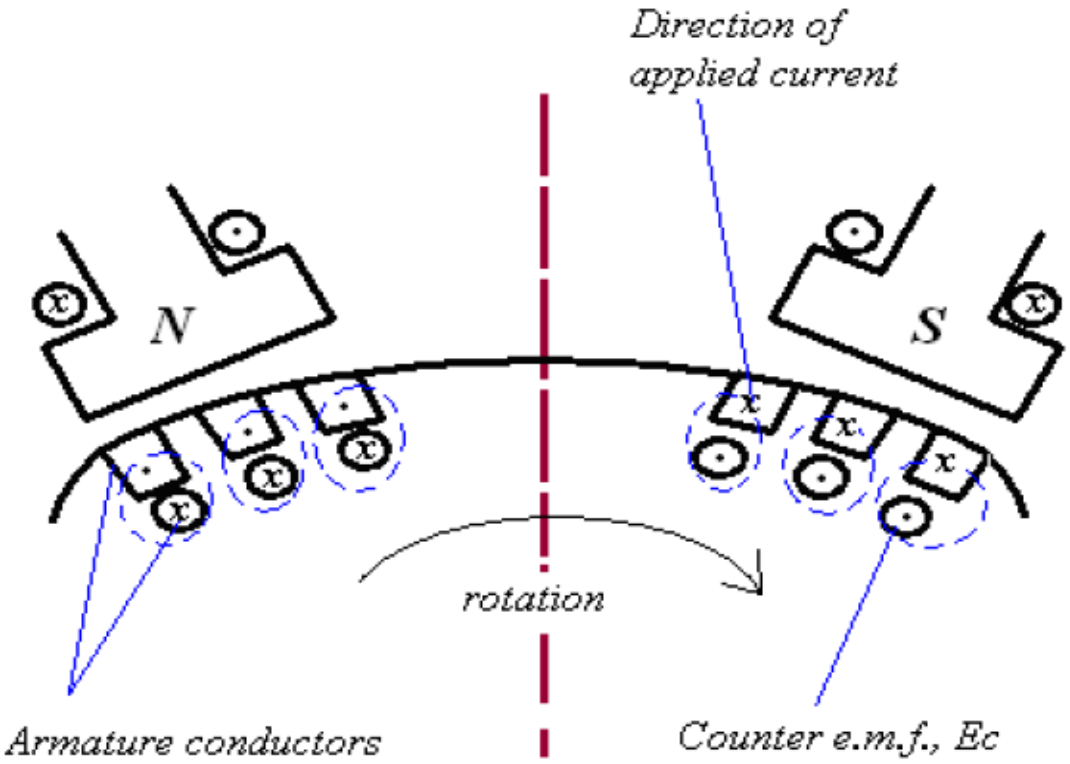


Fig 3.23 Counter e.m.f

- As the armature rotates, counter EMF's are generated in the very same conductors, since they cut the main pole magnetic fluxes which are responsible for motor action. The direction of the counter EMF's is determined by RHR.





- The alternate current $\boldsymbol{I_A}$ is then a function of the difference of the applied line voltage $\boldsymbol{V_L}$ and the counter EMF $\boldsymbol{E_C}$

  Thus, $\boldsymbol{I_A} = \frac{\boldsymbol{V_L - E_C}}{\boldsymbol{R_A}}$ 3.4

  where $\boldsymbol{R_A}$- Resistance of armature winding.

- In a dc motor the counter EMF limits the current to the extent dictated by the load power. In practical machines, the counter EMF will be usually 80-95% of the terminal voltage. The difference between $\boldsymbol{V_L}$ and $\boldsymbol{E_C}$ is a measure of the copper loss.

  $$\boldsymbol{I_A R_A = V_L - E_C} \quad 3.5$$

- By multiplying the equation above by IA and rearranging we get:1

  $$\boldsymbol{V_L I_A - {I_A}^2 R_A = E_C I_A} \quad 3.6$$

- or the developed power in volts, $\boldsymbol{E_C I_A}$ is given by the difference of the impressed armature power $\boldsymbol{V_L I_A}$ and the armature copper loss $\boldsymbol{{I_A}^2 R_A}$. Hence motors with a high percentage of $\boldsymbol{E_C}$ in terms of $\boldsymbol{V_L}$ operate at a higher efficiency.





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Torque Relations

- Power developed internally by the motor is

$$P_d = E_C I_A \qquad 3.7$$

- This power must be sufficient to drive the mechanical load on the motor shaft and overcome all mechanical losses which include core losses, windage (the resistance to overcome air friction when rotating) and friction losses in the bearing supporting the armature.
- Since generally, torque $T = Fr$ *then*;

$$T = BxL\ xI\ x\ r \quad (Nm) \qquad 3.8$$

Where $F = BxL\ xI$

- From generator equation, the generated emf is given by

$$E = BxL\ x\ v \qquad 3.9$$

- Dividing the torque equation by the generated emf equation gives

$$\text{or } \frac{E}{T} = \frac{BxLxv}{BxL\ xIx\ r} = \frac{\omega}{I} \quad or\ EI = T\omega \qquad 3.10$$

- That is, the developed electrical power is equal to the developed mechanical power.
- Energy conservation law is thus reinforced: electromechanical energy (electro mechanics) involves the interchange of energy between and electrical and mechanical system.
- Since,

$$E_C = \frac{ZP}{60a}\Phi n \qquad 3.11$$

Then

$$T = \frac{EI_A}{\omega} = \frac{ZP}{60ax2\pi}\Phi\, I_A = K_m \Phi\, I_A$$

Where,

$K_m = \frac{ZP}{60ax2\pi}$ is the motor constant.

I_A– armature current

- The torque, T, developed by the armature depends on the main pole field strength and the current in the armature for a specific machine.

Armature Reaction in DC Generators

- When a generator supplies an electrical load, the current carrying armature winding creates a magnetic field which interacts with the main pole field existing in the machine. This in turn sets up a resulting distorted field.
- Consider a two pole generator:

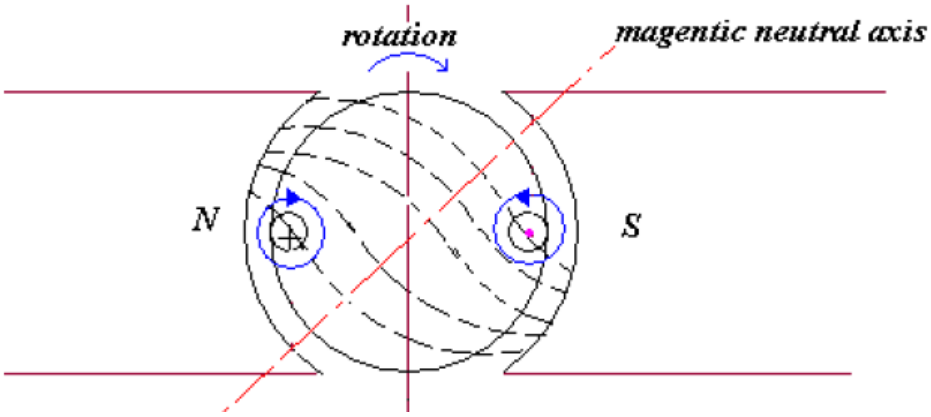
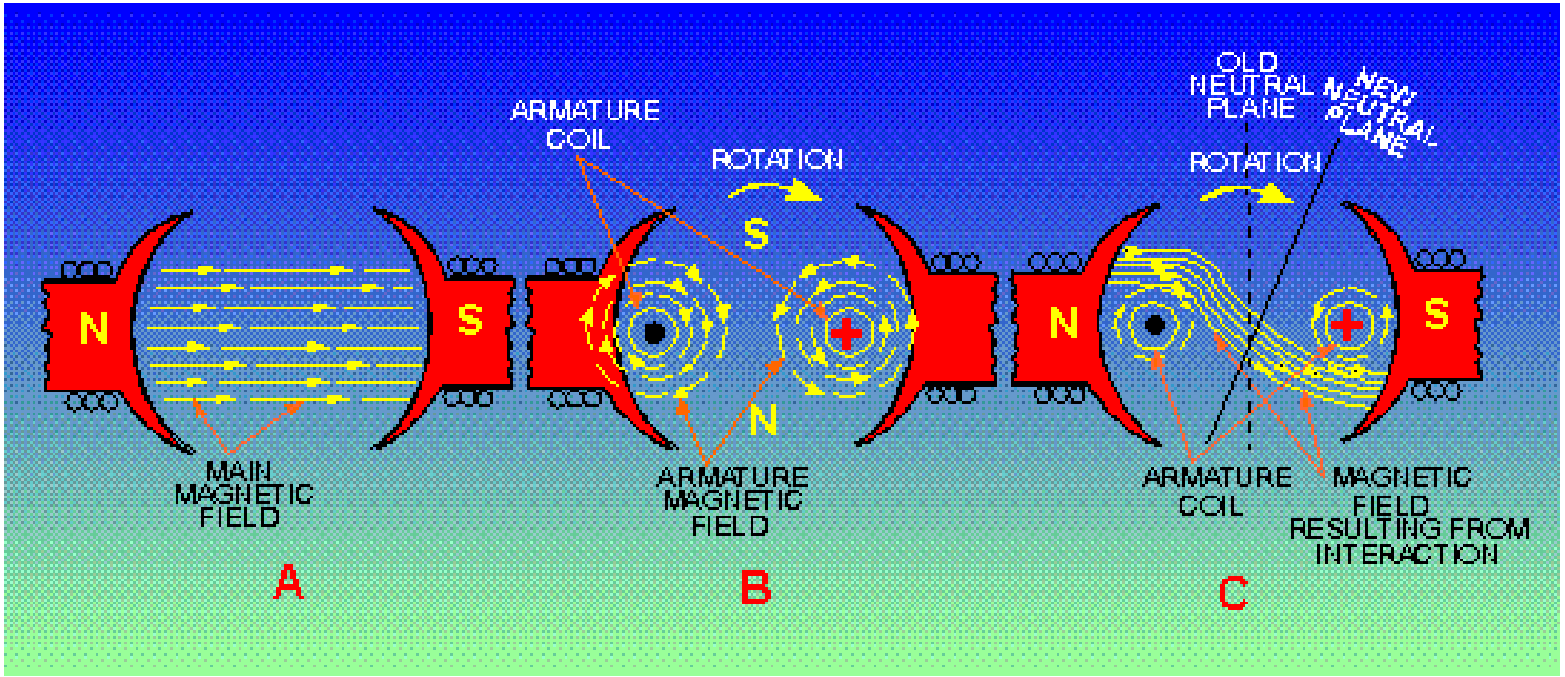


Fig 3.24 Armature reaction in a DC generator


ARMATURE
COIL
ROTATION
S
N
OLD
NEUTRAL
PLANE
NEW
NEUTRAL
PLANE
ROTATION
N
S
N
S
MAIN
MAGNETIC
FIELD
ARMATURE
MAGNETIC
FIELD
ARMATURE
COIL
MAGNETIC
FIELD
RESULTING FROM
INTERACTION
A
B
C





- The resultant distribution of flux is a combination of main pole flux and flux set up by armature current. Over the loading halves of the pole faces the armature flux in opposition to the main flux, thereby reducing the flux density. Over the trailing halves the two fluxes are in the same direction, so the flux density is increased.
- In a generator, the effect of armature reaction is to distort the field flux in the direction of rotation. For the motor, the magnetic neutral axis shifts against the direction of rotation.

Interpoles

- During the short time that the commutator segments, to which the armature coils are connected, are passing under the brush the current must be completely reversed in the coil.
- If this reversal is not complete an arc will occur at the commutator segment that moves under the brush. Sparking at the brushes causes the commutator to pit, increasing brush wear and commutator wear.

Illustration

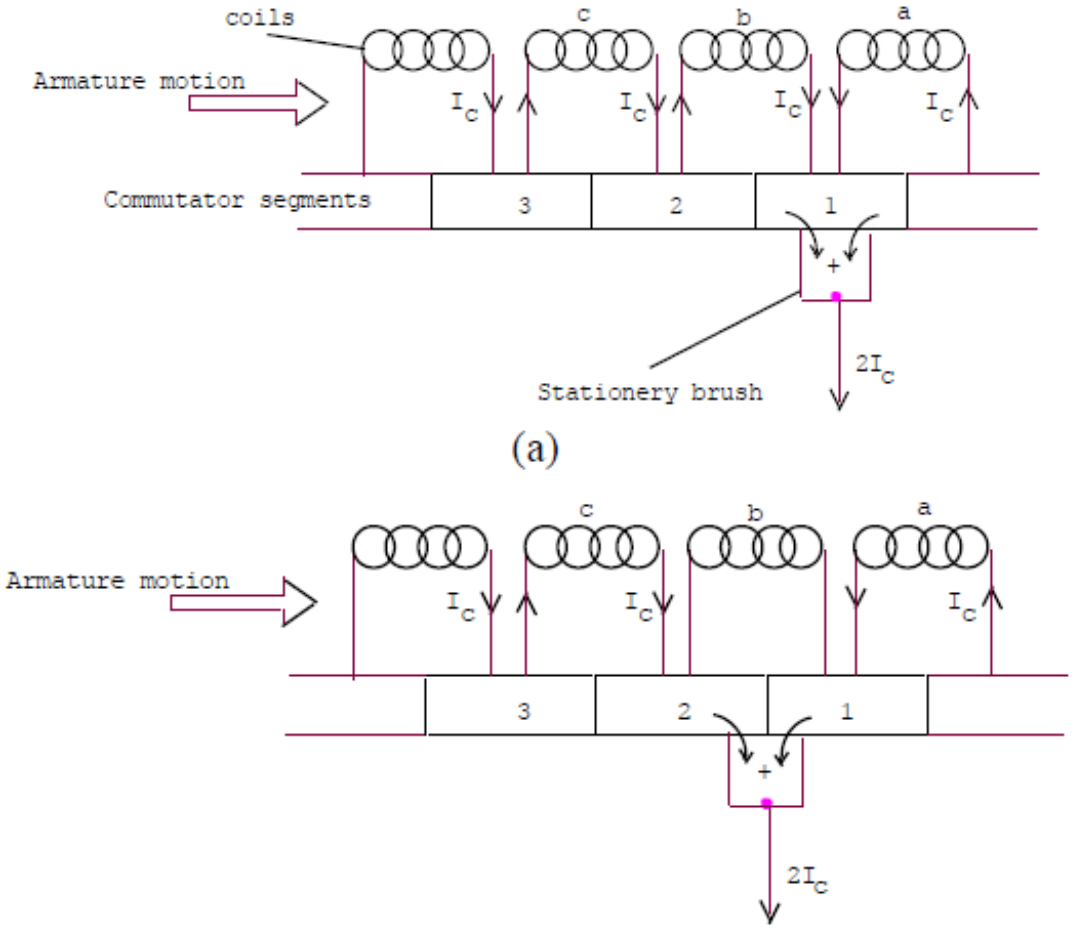


Fig 3.25 a, b

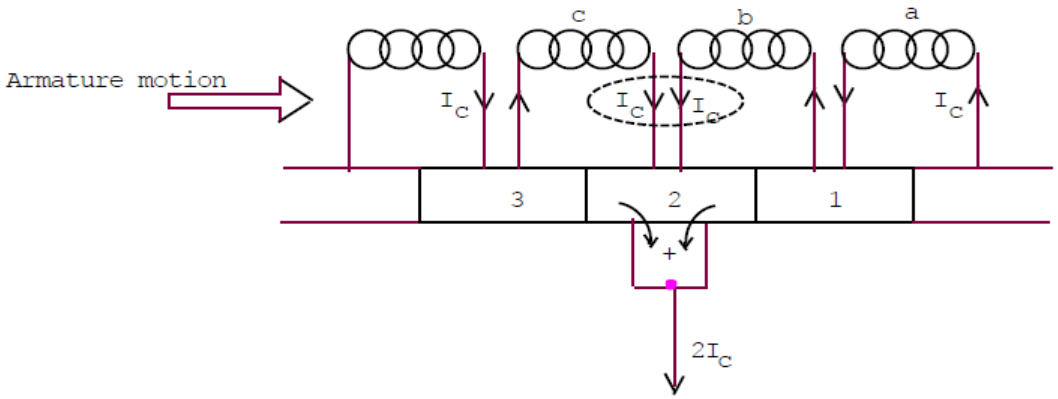


Fig 3.25 c

- The two ends of any coil are connected to adjacent commutator segments.
- As the armature rotates, the segments will move over the brush. The time it takes for the brush to move across the insulation between segments 1 and 2, the current in coil B must change from Ic in one direction to Ic in the opposite direction.





- The process begins as soon as the brush makes contact with segment 2 i.e. when the brushes are placed on the magnetic neutral because the coil is then not cutting any flux and no EMF is induced in it.
- But the insulated coils are placed in iron, hence their inductance is increased. As a result with large armature current and short commutation times a large EMF, reactance voltage, will be induced in the coil undergoing commutation.
- To overcome this reactance voltage, the brushes may be moved ahead of the neutral axis in the direction of rotation for a generator and backwards in a motor.





- Generally inter poles are used. They are connected such that they alternately have the same polarity as the following main pole in the direction of rotation for a generator, or the same polarity as the trailing pole in a motor. Thus an armature coil undergoing commutation [e.g. position b Fig 3.25] in a generator is cutting flux which is in the same direction as that of the next main pole. The result is an independent EMF in the coil that opposes the reactance voltage.
- Note also that since the reactance voltage depends on the armature current, the amount of commutation flux to counteract it is made to vary. In very large machines, there are also compensating winding which consist of a few turns placed in the main pole faces. The resulting field produced by this winding is opposite to that produced by the current in the armature conductors, hence helping to counteract the effect of armature reaction.

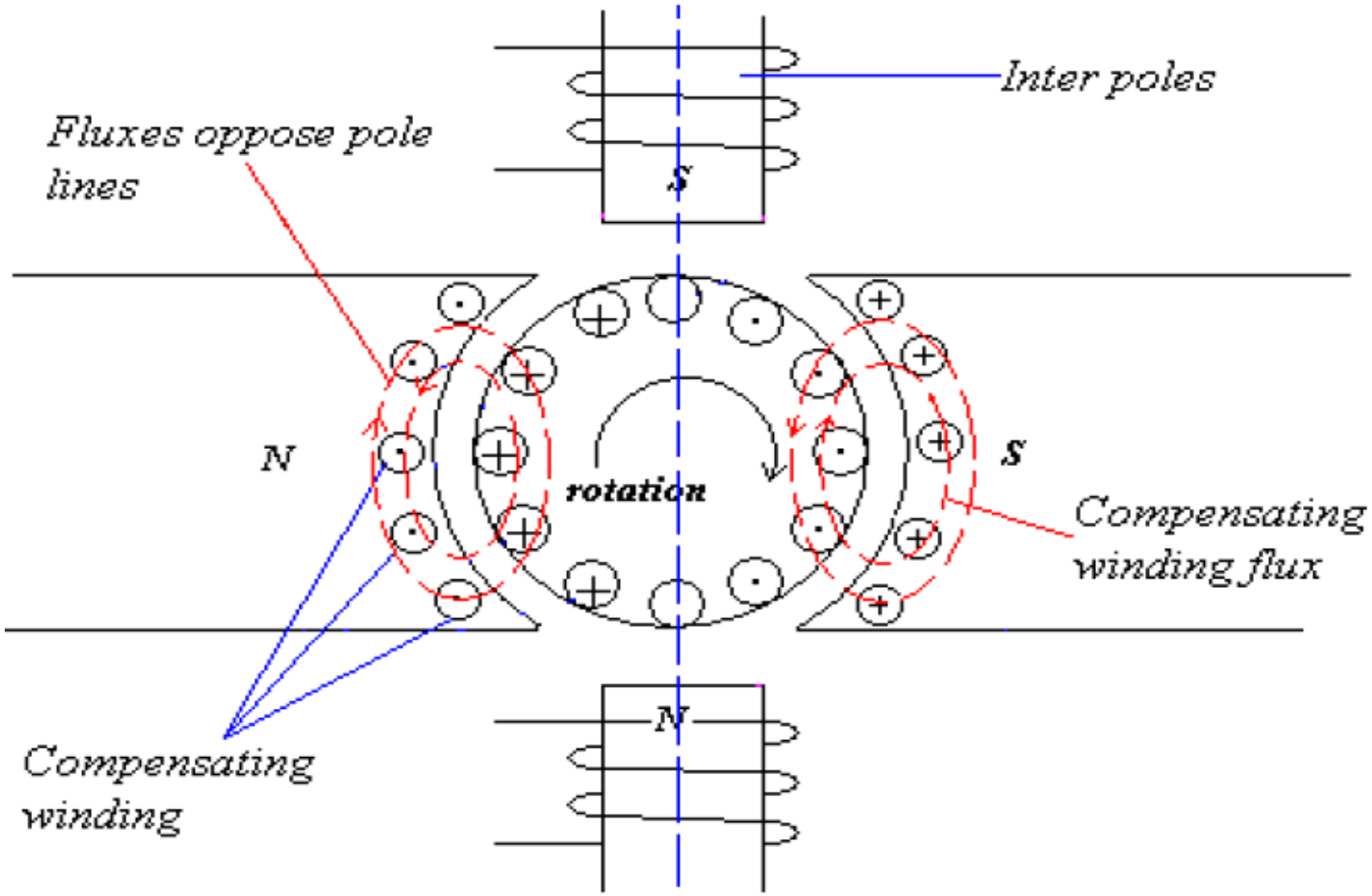


Fig 3.26 Inter poles in a Two-pole DC generator





3.2 D.C GENERATORS

- D.C generators are used for specified application and locals do power generation.
- This limitation is due to the commutator required to rectify the internal generated AC voltage, thereby making large-scale do power generators not feasible.

3.2.1 Voltage Regulation

- The generated EMF is proportional to both the flux per pole and the speed at which the generator is driven, namely, $E_G = K_G \Phi n$
- When n=const, EG depends directly on the flux, Φ.
- The flux in turn depends on the amount of field current flowing since the number of turns on the field winding is constant. This relation is non linear due to magnetic saturation after the field current reaches a certain value. The plot EG verses If is known as open-current characteristic of a generator.

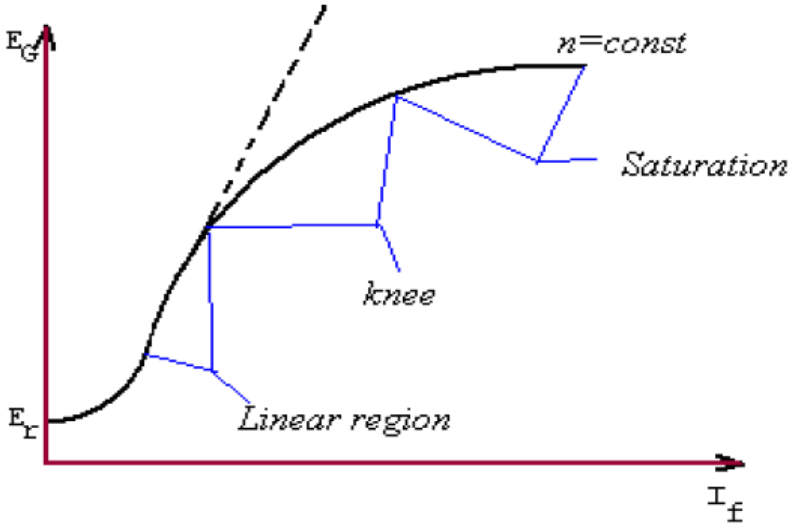


Fig 3.27 Open-circuit characteristic of a separately excited D.C. Machine.

- When If=0, that is, with the field open circuited a small voltage Er is measured, due to residual magnetism. As the field current increases the generated EMF increases linear up to the knee of the magnetization curve. Beyond this point, increasing the field current still further causes saturation of the magnetic structure to set in.

- If measurements of EG are taken during progressively decreasing value of If, a similar curve will be obtained but slightly above the existing one, due to Hysteresis. Upon loading a generator the terminal voltage will decrease due to the resistance of the armature winding and due to the armature reaction. The curve that shows the value of terminal voltage for various load currents is called the load or external characteristic of the generator.

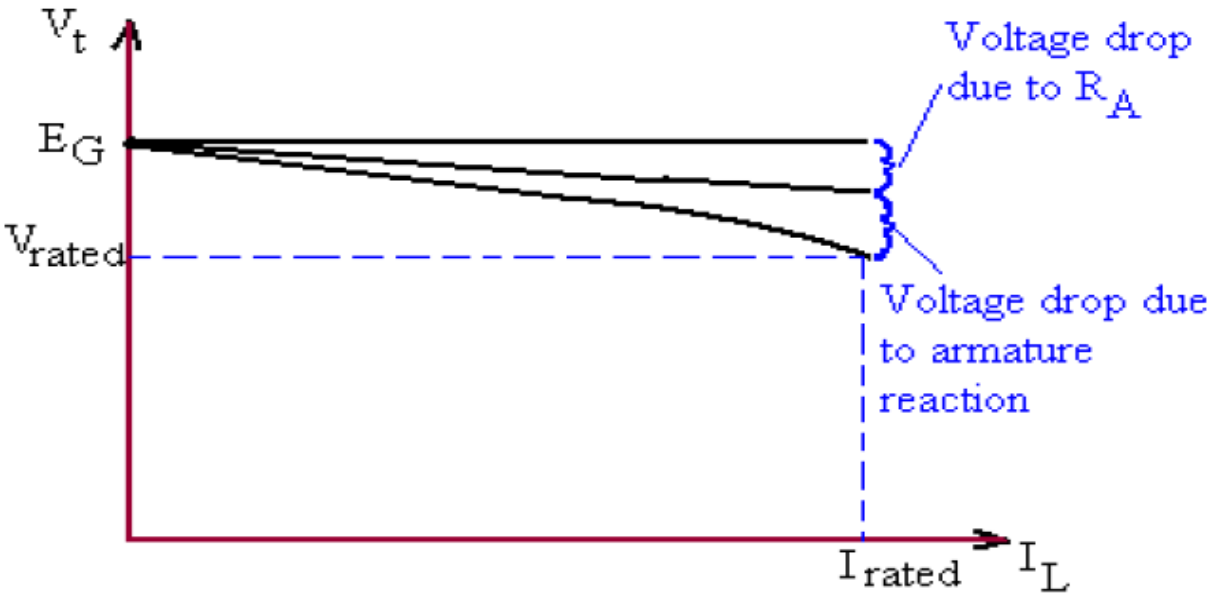


Fig 3.28 Load or external characteristic of a separately excited generator

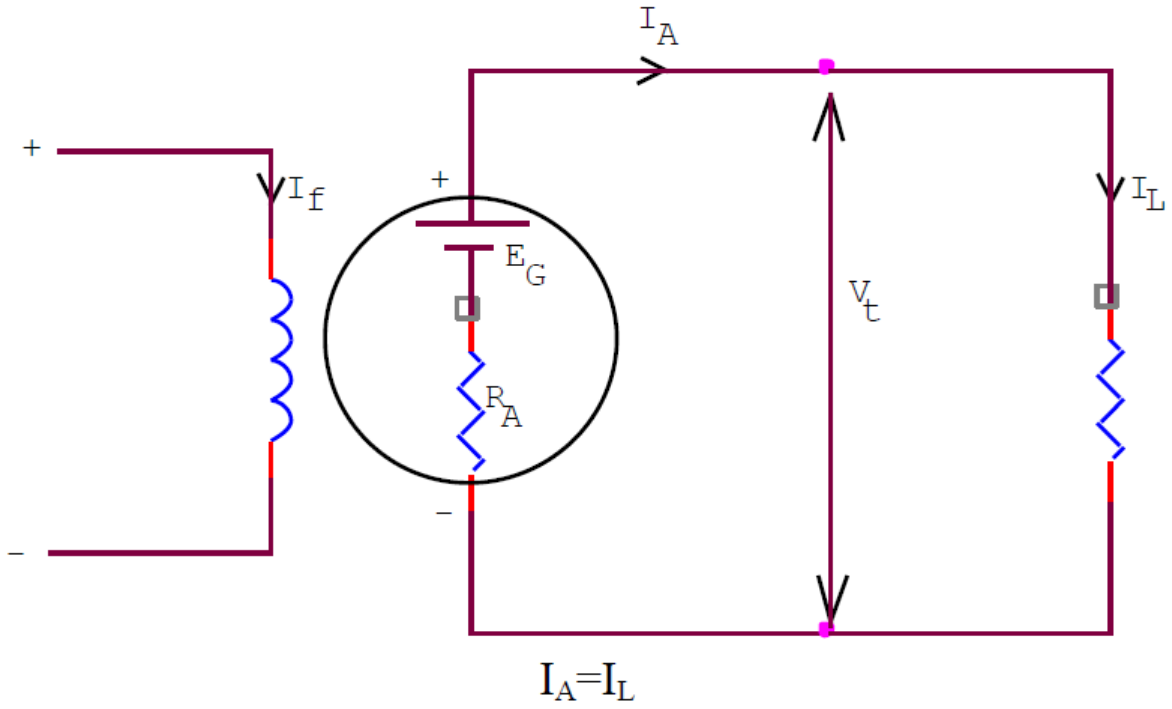


Fig 3.29 Circuit diagram of a separately excited DC generator





- Generally,

$$\boldsymbol{V_t = E_G - I_A R_A} \qquad \textbf{3.11}$$

- Where

$\boldsymbol{V_t}$– terminal voltage

$\boldsymbol{I_A}$ – armature current (or load current)

- Voltage regulation is the measure of how much the terminal voltage and changes from no-load condition and is given by

$$\boldsymbol{V_R = \frac{V_{NL} - V_{FL}}{V_{FL}} x\ 100\%} \qquad 3.12$$

- Where V_{NL}– No-load terminal voltage

  V_{FL}– Full load terminal voltage

- It is important to note that the generated voltage, EG, is not a constant voltage for with increasing loads on the generator armature reaction sets in and the drop in the armature winding resistance.



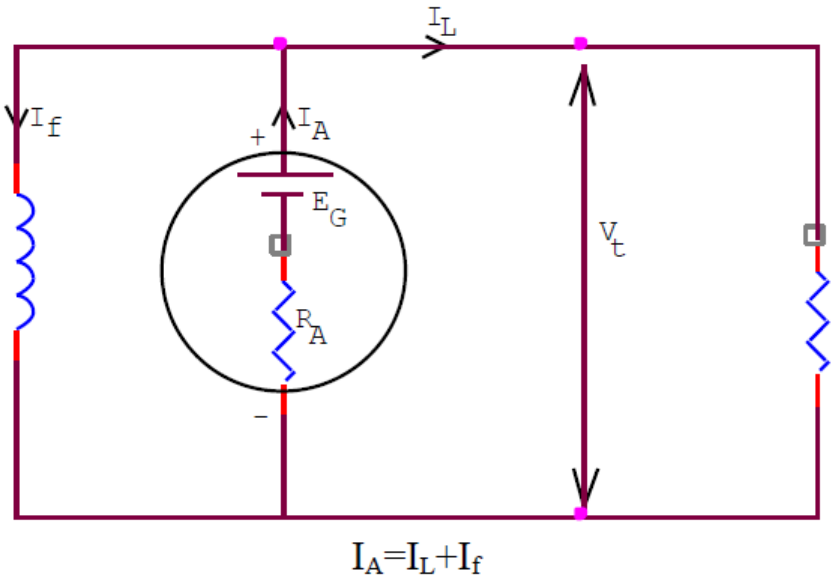


3.2.2 Types Of D.C Generators

- By the method employed to connect the field and armature winding generators can be divided into

1. Separately
2. Self- excited generators.

- In the separately excited generators the field winding is connected to a separate D.C. supply. It has the disadvantage of requiring an external do source and hence is used where a wide range of terminal voltages is required.
- Under the self-excited generators we have shunt generators. These have a shunt field winding connected in parallel with armature so that the machine provides its own excitation.



$I_A = I_L + I_f$

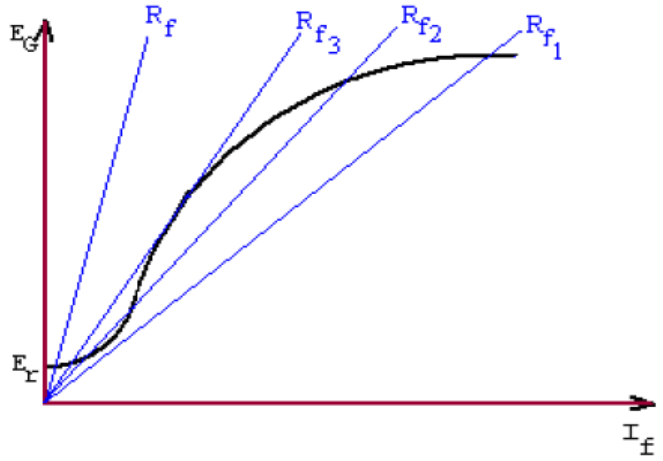
Fig 3.30 Circuit diagram of Shunt generator





- Connecting the field for self exultation the small voltage in the generators due to remnant magnetism will be applied to the shunt field and drive a small current through the field circuit.
- If this resulting small current in the shunt field is of a direction that it weakens the residual flux, the voltage remains near zero and there is not Vt build up, that is, the weak main pole opposes the residual flux. But if the weak main pole aids the residual flux the induced voltage will become larger. This in turn means more voltage applied to the main field and the terminal voltage increases rapidly to a large constant value.
- In the steady state the generated voltage causes a field current to flow that is just sufficient to develop a flux required for the generated EMF that causes the field current to flow.

- The circuit carries only DC current, hence the field current depends only on the field circuit resistance. This may consist of a field winding resistance Rf plus a field rheostat resistance R in series with it.
- For a given value of field circuit resistance RF, the field depends on the generated voltage in accordance to Ohm's law.



$R_f > R_{f3} > R_{f2} > R_{f1}$
$R_{f3} = R_{critical}$

Fig 3.32





- When the field circuit resistance is increased to Rf2 it changes the slope of the field resistance line and thereby the terminal voltage. The terminal voltage can thus be controlled by adjusting the field current.

The following can be noted as causes preventing the building up of the generated voltage:

1. In case of a new machine or an old machine where the residual flux is lost, magnetism is created by connecting a field winding to a separate DC source for a few seconds a process called flashing the field.

2. The value of EMF is extremely sensitive to variation of field resistance. The resistance Rf3 is called critical resistance and exceeding this Rf3 = Rcritical results in failure of the generated voltage to build up.

3. Low armature speed will prevent the generator voltage to build up. At a reduced speed the magnetization curve is adjusted according to the speed.

4. If the resulting small current in the shunt field is of a direction that it weakens the residual flux, the voltage remains near zero and there is not Vt build up, that is, the weak main pole opposes the residual flux.

- The variation of terminal voltage with load current for a shunt generators is greater than that of the separately excited generators.
- This is because apart from the voltage drop due to the armature resistance and armature reaction effect is there is an additional drop due to the flux decreasing with increased loading which in turn decreases the terminal voltage.

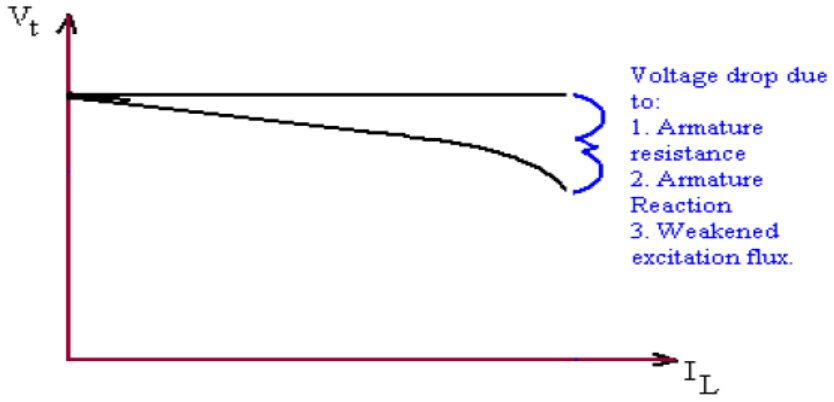


Fig 3.31 Load characteristic of a shunt generator

3.3 DC MOTORS

- **3.3.1 General review and introduction**

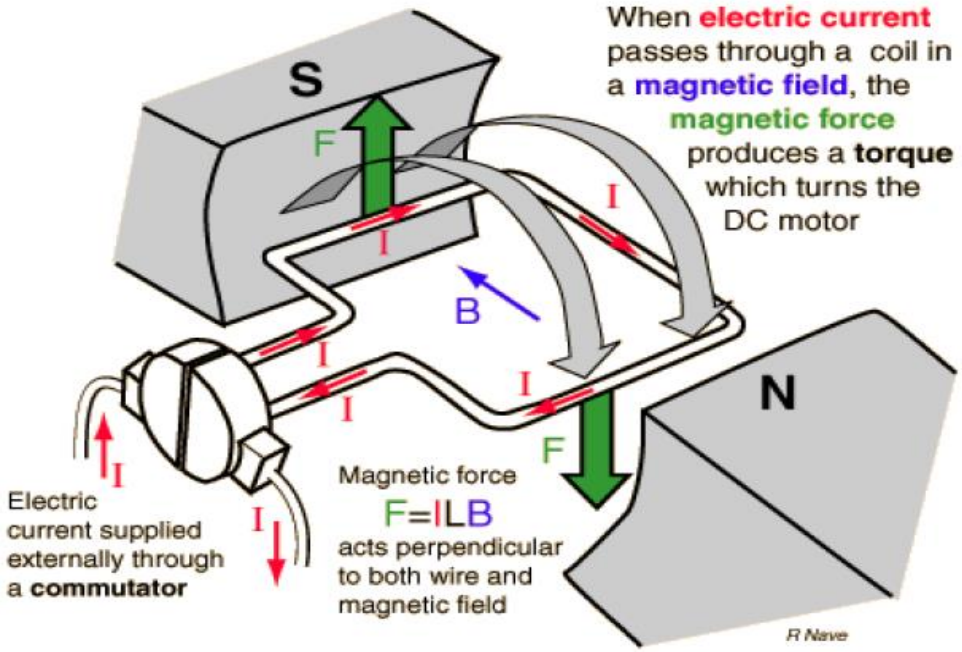


Fig 3.32 Motor Op

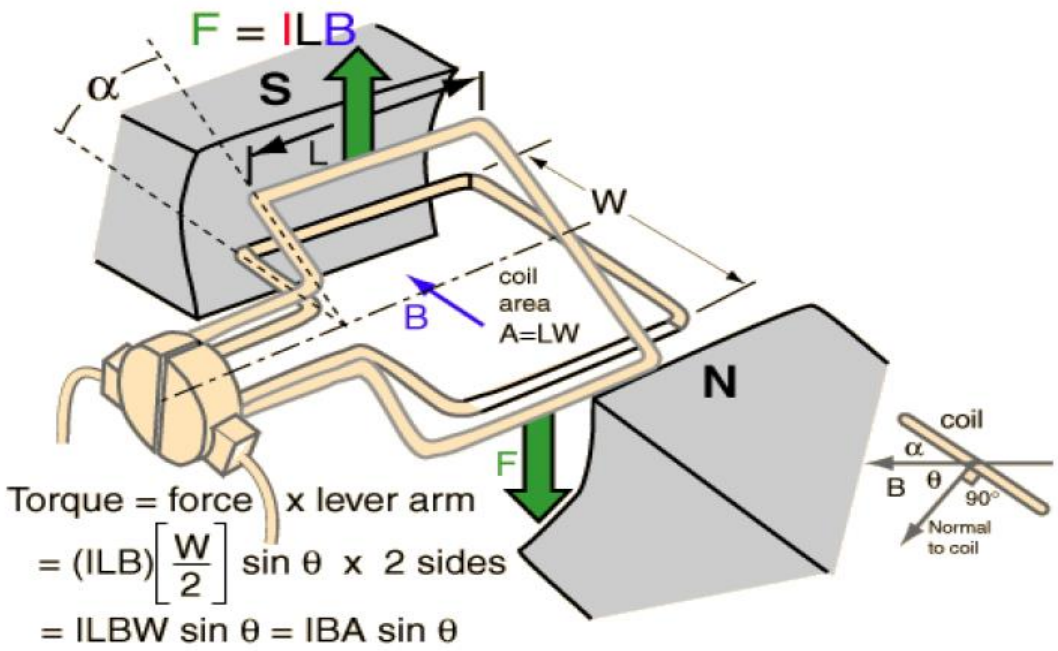


Fig 3.33 Production of torque in a DC motor





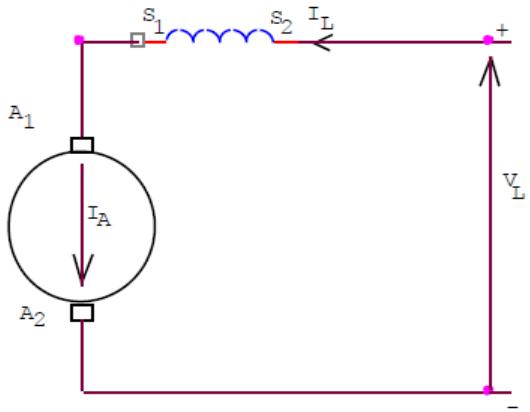
- D.C motors is similar to a DC generator. The only difference is that generators the EMF is greater then the terminal voltage, whereas in a motor the generated EMF counter EMF is less then the terminal voltage. The power flow is thus reversed, that is, the motor converts electrical energy into mechanical energy.
- DC motors are better suited for many industrial process that demand a high degree of flexibility in the control of speed and torque.
- DC motor can provide high starting torque as well as high decelerating torque for application requiring quick stoppage or reversals.

3.3.2 Classification Of DC Motors.

- In general there are three types of DC motors:

1. Series motors,
2. Shunt motors,
3. Compound motors.

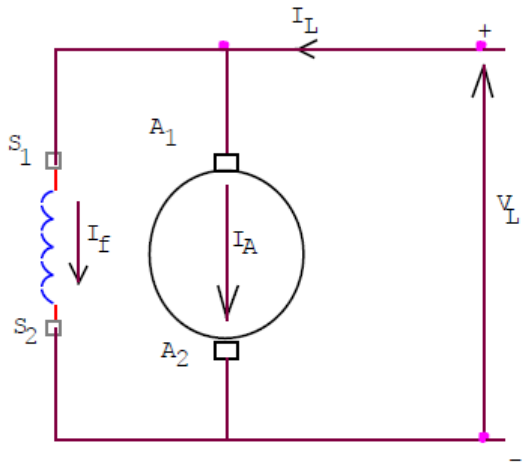
- The series motor is widely used because of its excellent starting – torque characteristic. The series machine has a field winding in series with the armature winding.



$I_L = I_A$

Fig 3.34

- If the field is connected in parallel with the armature winding, we obtain a shunt ma



$$I_L = I_A + I_f$$

Fig 3.35

- The compound motor has both field windings. These field windings may be connected so that the fields aid one another giving rise to a cumulative compound motor.

- Alternatively they may be connected so that the resulting fluxes oppose, as in the differentially compound motor.
- A compound motor may have a long shunt or a small shunt connection depending on whether the shunt field is connected before or after the series field winding.

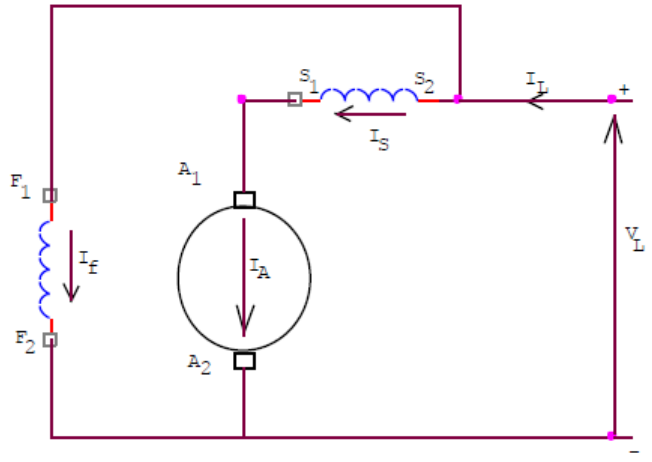


Fig 3.36

$I_S = I_A$

$I_L = I_A + I_f$





Starting Dc Motors

- At the instant of start up the counter EMF Ec is zero because the armature is not rotating. The counter EMF serves to limit the current in the armature winding to a value just sufficient to take care of the developed power needed to drive the load. Hence if we start motor directly across a supply line, it draws a large current that would cause mechanical shock to the armature and blow the fuse in the mains. For instance connecting directly to a 230 v mains line, a motor with an armature resistance equal to 0.25Ω, it would draw an armature current equivalent to;

$$I_A = \frac{230}{0.25} = 920A$$

- To avoid this from occurring, a resistance is connected in series with the armature to off set the absence of the counter EMF. As the motor comes up to speed, this resistance is taken out in steps because the counter EMF rises as the motor comes up to full speed.

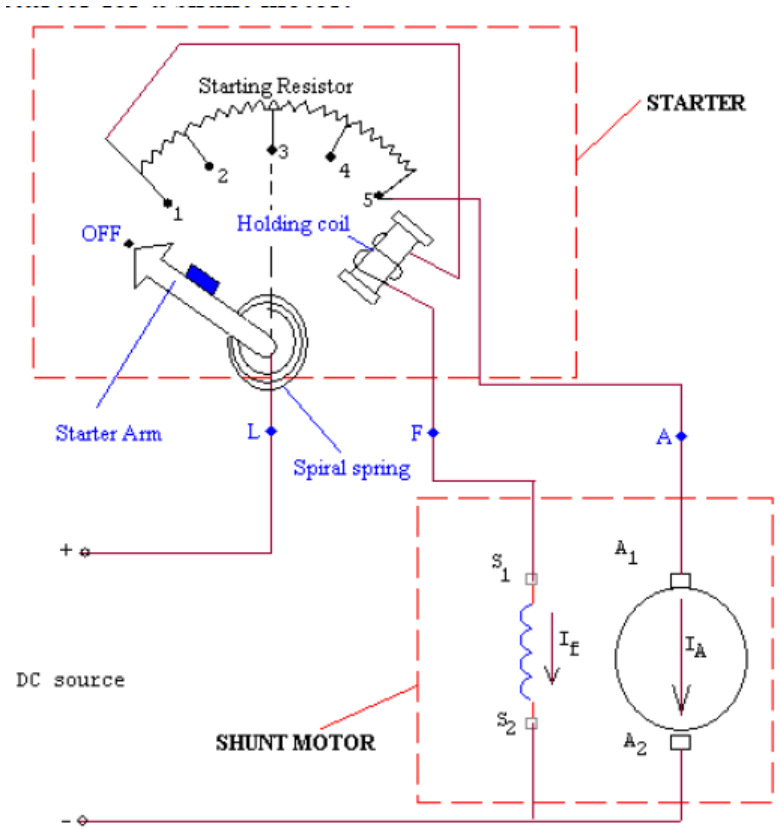


Fig 3.37 Manual starter for a shunt motor.

- To start the motor, the main switch is closed and the starter arm is moved to contact 1. The motor starts. After the armature has accelerated sufficiently on the first contact the starter arm is slowly moved to the following contacts until the iron keeper on the starter arm is held by the holding coil electromagnet. This occurs in a few seconds.

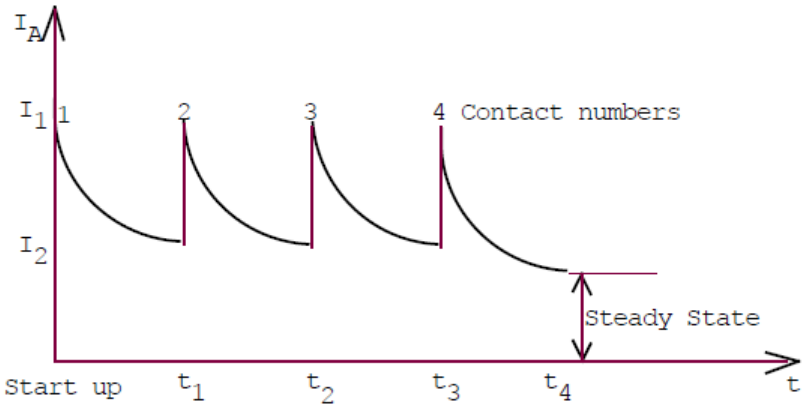


Fig 3.38

- In the final p circuit.

- When there is power failure or if the field circuit should be opened accidentally, the spring will return the starter arm to the off position, thereby shutting down the motor
- When the starter arm is on contact 1, the armature current;

$$I_1 = \frac{V_{supply}}{R_A + R_{starter}} \qquad 3.13$$

- Since the torque is proportional to the current and flux, maximum torque is available at start up to accelerate the armature.
- During acceleration period the counter EMF, Ec, and the armature current, IA decreases. When the current I1 has fallen to some pre-selected value I2 the starter arm is moved to contact 2, cutting down the resistance hence allowing rise of current to I1 again. The process is continued until the last contact is reached and the motor assumes steady state speed and current.

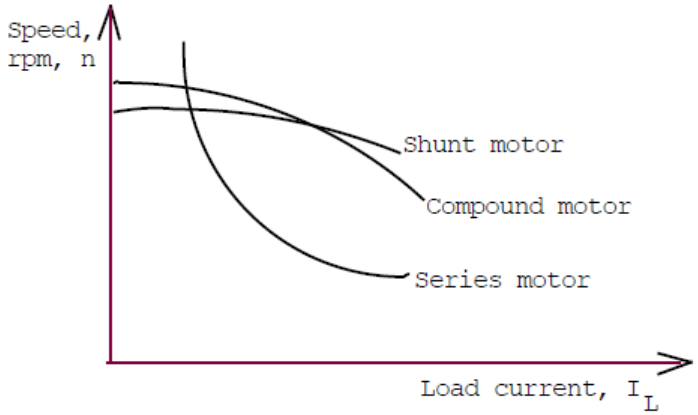
3.3.4 Speed Characteristics Of Dc Motors.

- When the mechanical load is removed from the motor, the motor speed, n, will increase. The amount by which it increases depends on the type of motor.
- Generally; $I_A = \frac{V_L - K\Phi n}{R_A}$ 3.14
- From the above equation we can derive the formula for motor speed as;

$$n = \frac{V_L - I_A R_A}{K\Phi} \quad 3.15$$

- In shunt motors, the flux Φ is only slightly affected by the armature current, while the IA RA drop rarely exceeds 5% of the line voltage. Therefore the maximum change in speed must be of the same order as IA and is about 8%.
- In a compound motor, the armature current IA and the flux Φ affect the speed. The effect of the series field is removed under no load conditions, thereby weakening the overall field flux. The result is then a larger increase in speed, since the speed is inversely proportional to the flux. The change is about 15- 20%.

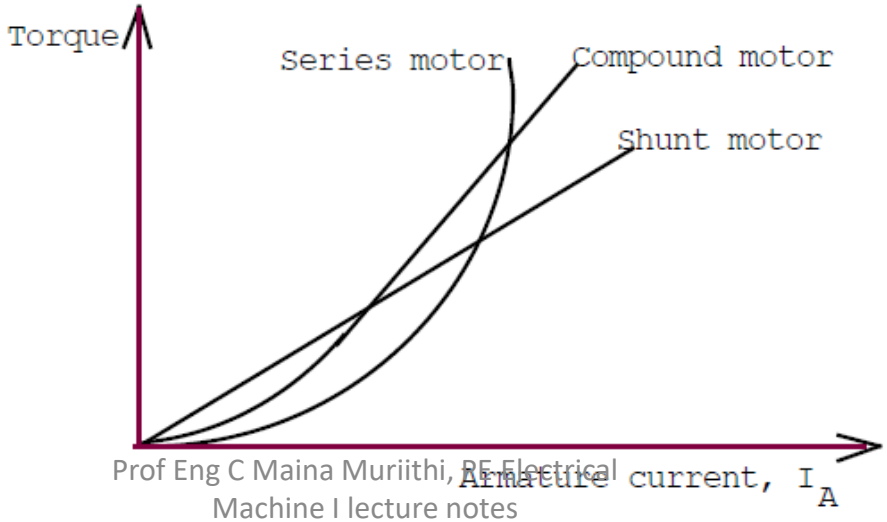
- In the case of series motor, speed rises rapidly because at no–load condition only residual magnetism flux is left. Hence series motors are not used in instance where the load can be disconnected accidentally.
- The general relationship between the speed and the load current for various motor types is shown below.



- ***Fig 3.39***

Torque – Load characteristic.

- *T= KΦIA*.....3.16
- The torque of a shunt motor depends only on the armature current. The graph is hence a straight line. In the compound motor (Cumulative type) the series field increases with loading, aiding the shunt field.
- In a series motor, neglecting saturation, the developed torque depends on the load current squared, because the flux in turn depends on the current.
- Hence *T= KΦIA=k(k1IA)IA=K'IA^2* 3.17



- ***Fig 3.40***





- For current exceeding rated current a series motor develops the largest torque.
- The speed regulation is given by;

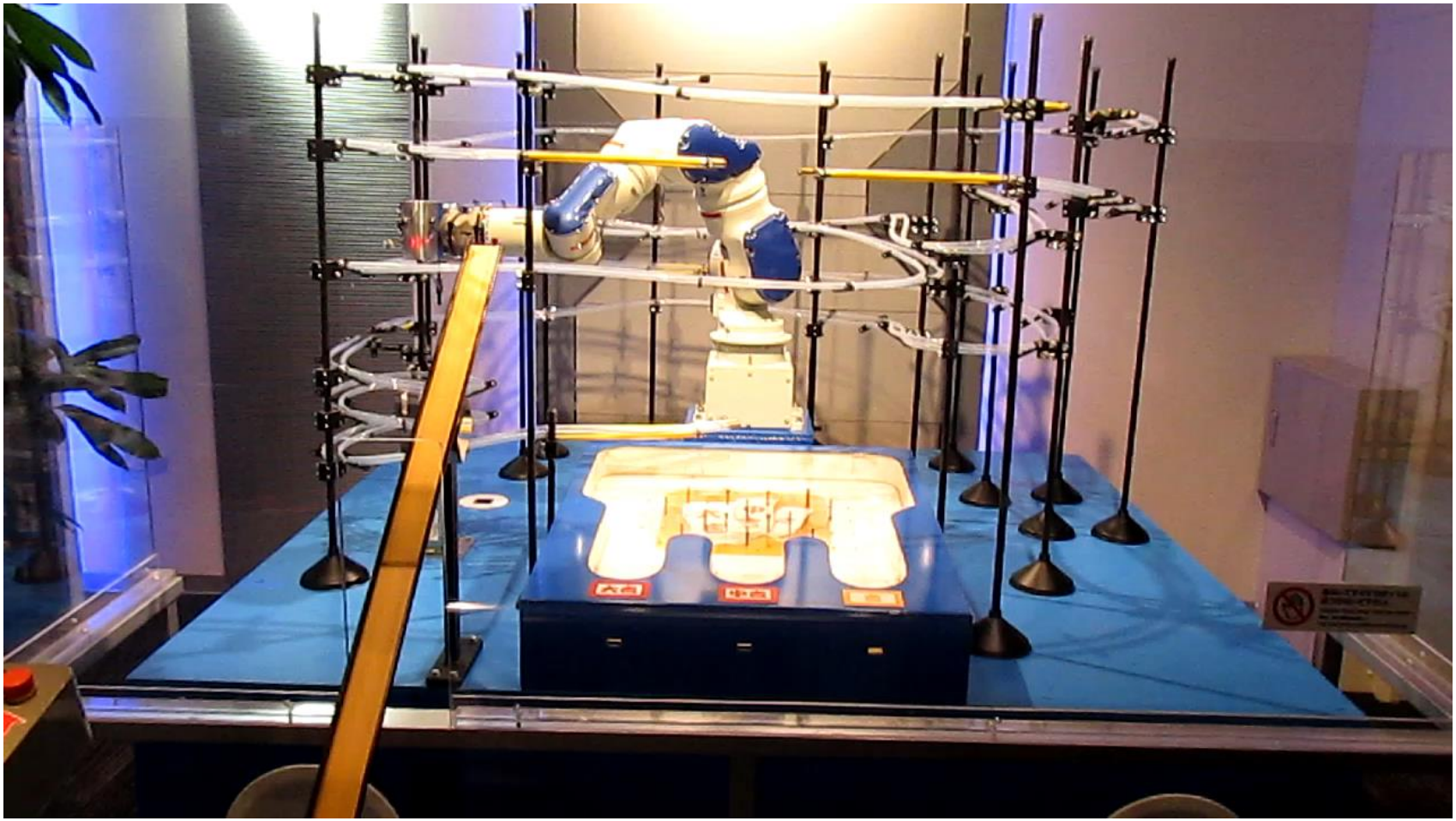
$$S_R = \frac{n_{NL} - n_{FL}}{n_{FL}} x \ 100\% \qquad 3.18$$

- Where n_{NL} - speed at no load in revolutions per minute

  n_{FL}- speed at full load in revolutions per minute.

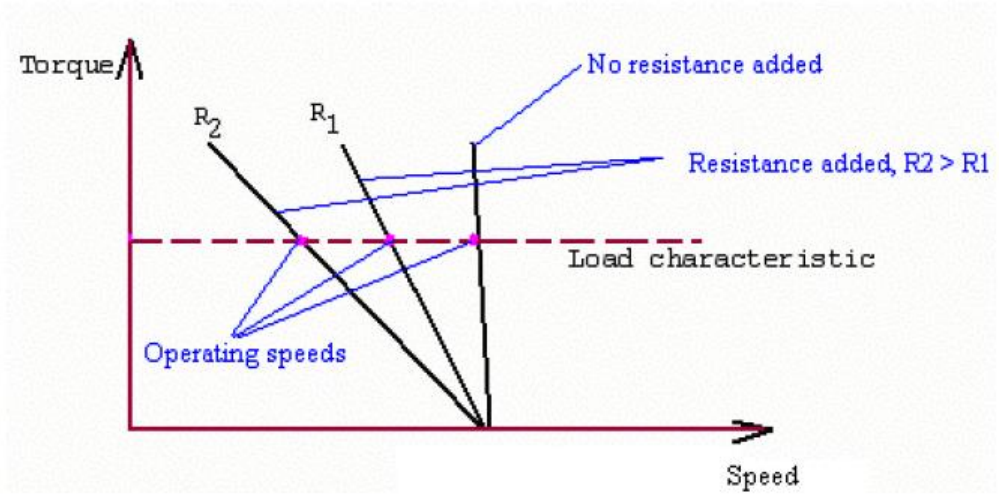
3.3.4 Speed Control Of Dc Motors.

- From the equation $n = \frac{V_L - I_A R_A}{K\Phi}$ there are 3 parameters V_L, R_A and Φ which can be used to control and / or change the speed.



1. **Increasing armatures circuit resistance.**

- In this method, increasing the armature resistance RA, results in a reduction of the steady–state speed, except for no-load condition. The field current is kept constant in this method.
- It is a relatively simple and cheap method of speed control.



- ***Fig 3.41***





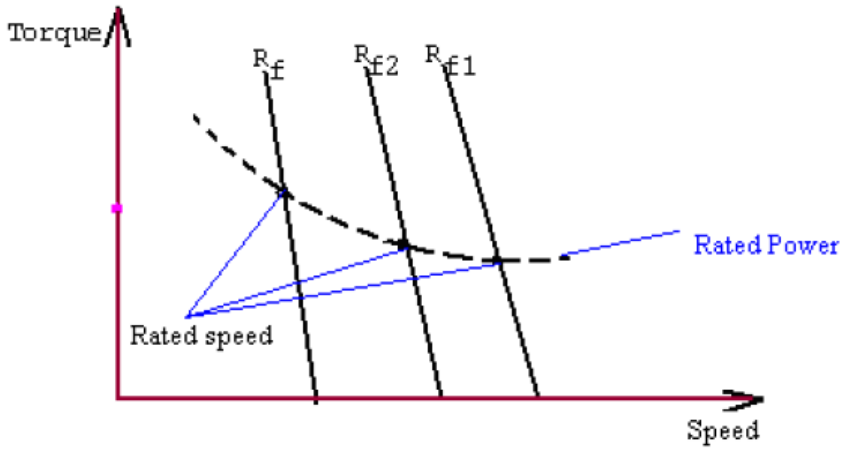
Disadvantages

1. By adding R, the speed of the motor, compared to that without connecting any resistance, is always lower.
2. Method is ineffective at no load
3. Adding a resistance means increased power losses, I2R.
4. Constant- speed characteristic of the motor is lost.

Speed control is limited to 50% of rated speed.

2. Changing The Flux Φ

- This is done by adding a field rheostat in the excitation windings. This decreases the field current and thus the flux **Φ**. Reduction of the flux, **Φ** increases the speed.



- ***Fig 3.42***

$R_{f2} > R_{f1} > R_f$





Disadvantages

1. We can only raise the speed at which the motor normally runs at a particular load.

2. The speed is increased without a corresponding reduction in shaft load leading to overloading of the motor,

At light loads or under no-load conditions speed can be varied about 300% above normal operating speeds

3. Changing the terminal voltage VL of the motor.

- Voltage control method lowers the speed in a similar fashion as the armature circuit resistance speed control method. The no-load speed and full load speed can be reduced to zero.

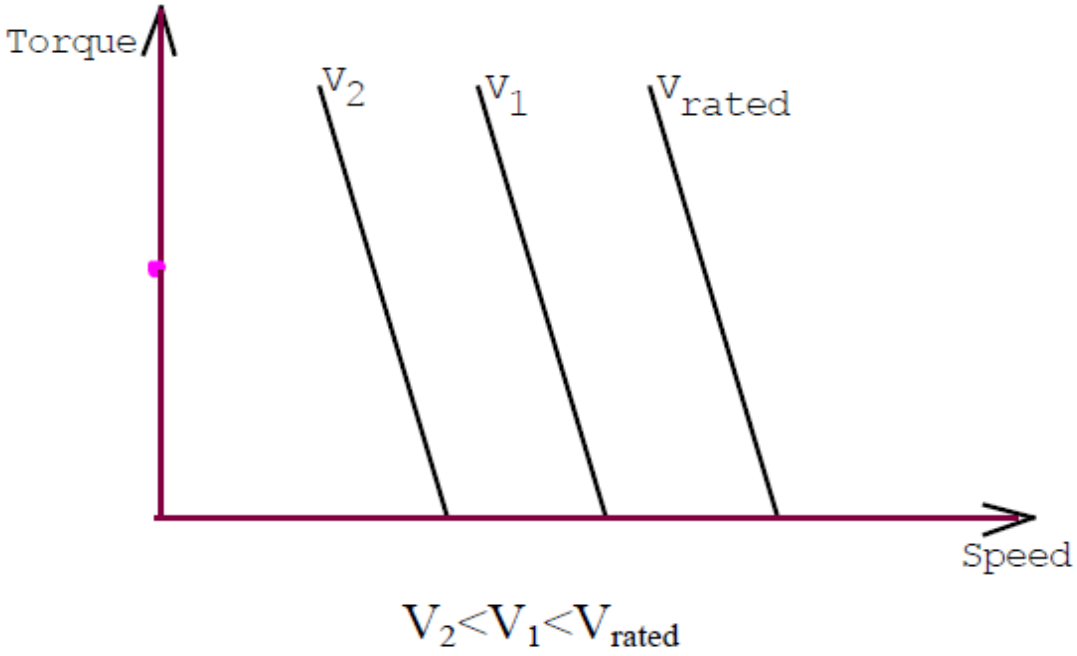
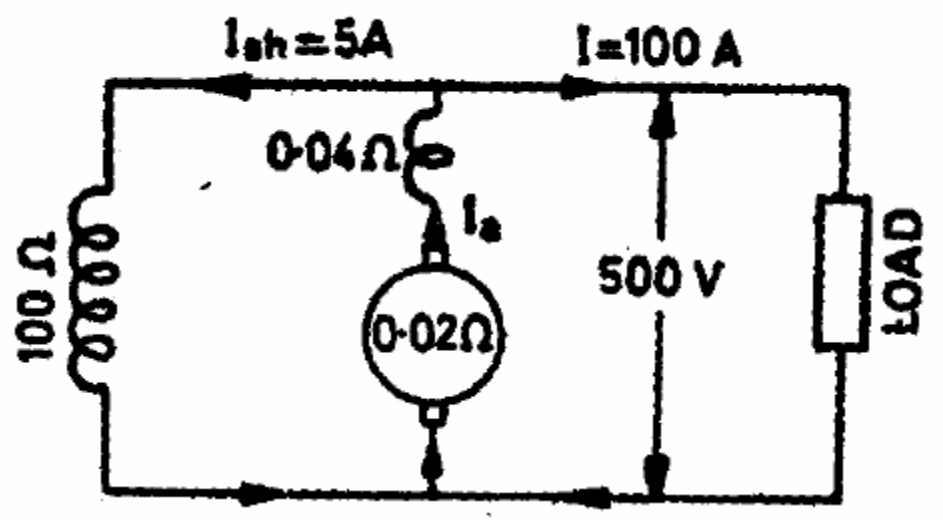


Fig 3.43

- The most effective way of speed control as seen from above will be a combination of at least two methods.

Example 3.1 *A 4 pole, long-shunt, compound gaterator supplys 100 A at a terminal voltage of 500 V. If armature resistance is 0.02Ω, series field resistance it 0.04 Ω and shunt field resistance 100Ω, find the generated EMF. Take drop per brush as 1 V, Neglect armature reaction.*



$$I_{sh} = \frac{500}{100} = 5\ A$$

Current through armature and series field winding is:

=100+5=105 A

Voltage drop on series field windings=105*0.04=4.2V

Armature voltage drop= 105 * 0.02=3.1 volt

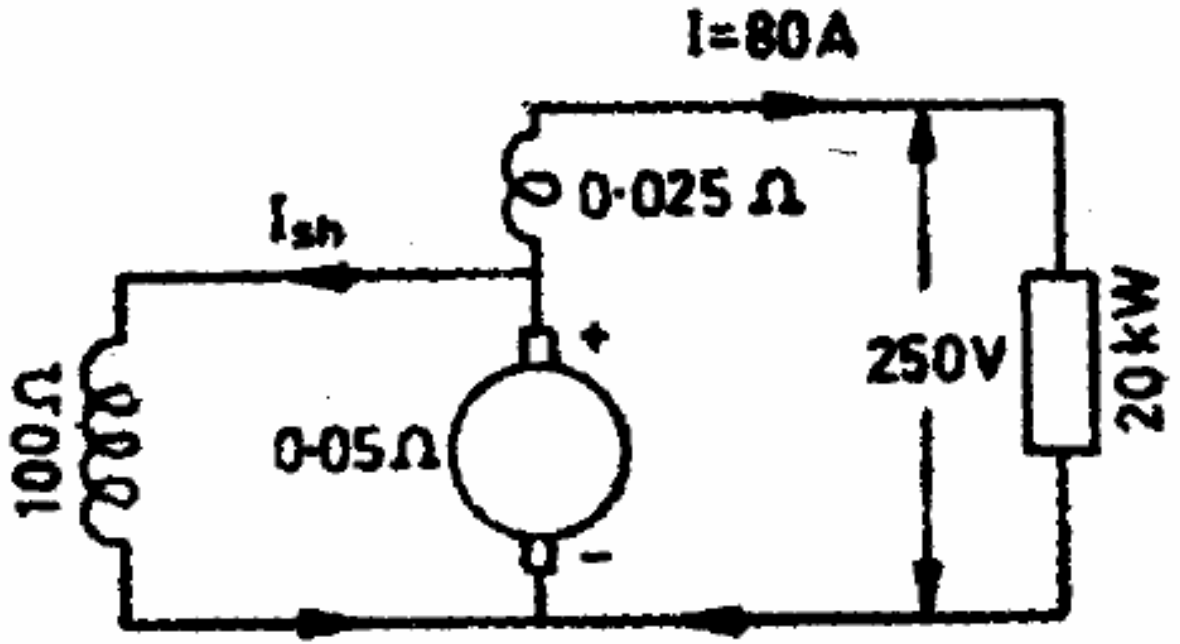
Drop at brushes= 2 * 1= 2 V

Now,

$$EMF = V + I_a R_a + series\ drop + brush\ drop$$

$$= 500 + 4.2 + 2.1 \qquad + 2 \qquad = 508.3\ Volt$$

Example 3.2 *A 20 kW compound generator works on full load with a terminal voltage of 250 V. The armature, series and shunt windings have resistances of 0.05Ω, 0.025Ω and 100 Ω respectively. Calculate the total EMF generated in the armature when the machine is connected as short shunt.*







Example 3.4 *A 4 pole, lap;wound, DC shunt generator has a useful flux per pole of 0.07 Wb. The armature winding consists of 220 turns each of 0.004 Ω resistance. Calculate the terminal voltage when running at 900 rpm if the armature current is 50 A.*

Solution:





Example 3.14 *A DC series motor takes 40 A at 220 V and runs at 800 rpm If the armature and field resistances are 0.2 Ω and 0.1Ω respectively and the iron and friction losses are 0.5 k W, find the torque developed in the armature. What will be the output of the motor?*



Solution:



Example 3.16 *A 460 V series motor runs at 500 rpm taking a current of 40 A. Calculate the speed and percentage change in torque if the load is reduced so that the motor is taking 30 A. Total resistance of the armature and field circuits is 0.8 Ω. Assume flux is proportional to the field current.*









Example 3.17 *A shunt generator delivers 50 kW at 250 V and 400 rpm The armature and field resistances are 0.02 Ω and 50 Ω respectively. Calculate the speed of the machine running as a shunt motor and taking 50 kW input at 250 V. Allow 1 volt per brush for contact drop.*

S

$$I_a = I + I_{sh} = 205\text{A},$$

$$I_a R_a = 205 * 0.02 = 4.1 \text{ V};$$

Brush drop=2 * 1 =2V

Induced EMF in armature =250 + 4.1 +2=256.1 V

Obviously, if this machine were to run as a motor at 400 rpm, it would have a back EMF of 256.1 V induced in its armature.

$$\therefore E_{a_1} = 256.1V,\ N_1 = 400\ \ rpm$$





Input line current I=50,000/250=200 A

$$I_{sh} = 250/50 = 5 \text{ A};$$

$$I_a = 200 - 5 = 195 \text{ A}$$

$$I_a R_a = 195 * 0.02 = 3.9 \ V$$

Brush drop=2 * 1 =2V

$$E_{a_2} = 25 - (3.9 + 2) = 244.1V$$

$$\frac{N_2}{N_1} = \frac{E_{a_2}}{E_{a_1}} * \frac{\phi_1}{\phi_2}, \ Since \ \phi_2 = \phi_1$$

$$Then, \ \frac{N_2}{N_1} = \frac{E_{a_2}}{E_{a_1}} \ Then, \ \frac{N_2}{400} = \frac{244.1}{256.1}$$

Then, $N_2 = 381.4 \ rpm$





EXAMPLE

A fan is driven by a 220V, 7hp series DC motor which draws 25A and runs at 300rpm when connected to a 220V supply with negligible internal resistance. The armature resistance of the DC motor is 0.6 Ohms and the resistance of the field is 0.4 Ohms. Neglecting rotational losses, determine:

- The armature voltage
- The power delivered to the fan
- The developed torque
- The copper losses in the motor
- The motor efficiency





(a.) The armature voltage of the series DC motor is given by

$$V_A = V_T - I_A(R_A + R_{FW} + R_{SC})$$
$$= 220 - 25(0.6 + 0.4 + 0) = 195 \text{ V}$$

(b.) The developed power is equal to the output power when rotational losses are neglected. Thus, the power delivered to the fan is

$$P_{out} = P_{dev} = V_A I_A = (195)(25) = 4.875 \text{ kW}$$
$$= \frac{4875}{746} = 6.53 \text{ hp}$$

Note that the output power for this motor at 300 rpm is slightly less than the rated power (the output power of this motor at rated voltage and speed is 7 hp).

(c.) The developed torque is equal to the developed power divided by the motor speed in rad/s.

$$\omega_m = 300 \frac{2\pi}{60} = 31.42 \text{ rad/s}$$

$$T_{dev} = \frac{P_{dev}}{\omega_m} = \frac{4875}{31.42} = 155.2 \text{ N-m}$$

(d.) The copper losses within the motor are those associated with the armature and field winding resistances.

$$P_{Cu} = I_A^2(R_A + R_{FW})$$
$$= 25^2(0.6 + 0.4) = 625 \text{ W}$$

(e.) The input power to the motor is

$$P_{in} = V_T I_T = V_T I_A = 220(25) = 5.5 \text{ kW}$$

The motor efficiency (neglecting rotational losses) is

$$\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{4.875}{5.5} \times 100 = 88.6\%$$





EXAMPLE

- A shunt DC motor rated at 240V runs at 1000 rev/min at no load. It is required to increase the speed of the drill to 1250rpm while drawing an armature current of 40A. The armature resistance and field resistance are 0.25 and 150 Ω respectively. Determine the resistance to be placed in series with the field so that the motor runs at this new speed. Assume the pole flux to be proportional to the field current and neglect rotational losses.

$$Ec = 240V, \ldots Ia = 0A$$

$$Ec = k\phi n = k\phi \times 1000 = 240$$

$$k\varphi = \frac{240}{1000} = 0.24$$

$$If = \frac{V_t}{Rf} = \frac{240}{150} = 1.8\text{A}$$

$$\varphi \alpha If$$

$$\frac{\varphi'}{\varphi} = \frac{If'}{If} \Rightarrow \varphi' = \frac{If'}{If} \times \varphi$$

$$Ec = V_t - IaRa = 240 - 40 \times 0.25 = 230V$$

$$Ec' = k\varphi\varphi n' = k \times \frac{If'}{If} \times \varphi \times n' = 0.24 \times \frac{If'}{1.8} \times 1500$$

$$If' = 1.15A$$

$$Rf' = V_t If' = 2401.15 = 208.7\Omega$$

$$\Delta Rf = Rf' - Rf = 208.7 - 150 = 58.7\Omega$$