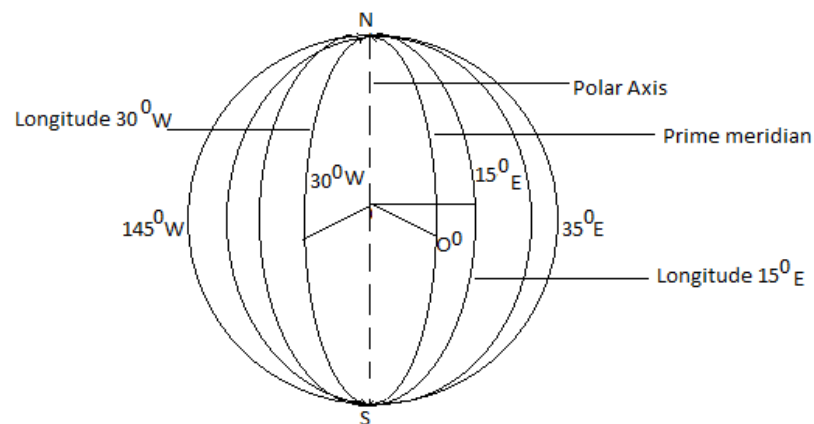


EARTH GEOMETRY

(i) THE EARTH

hints:

- The Earth is very nearly spherical.
- Mean Radius of the Earth is 6 370 km or 3 437nm.
- **Longitude:** The meridian that passes through Greenwich is called the **prime meridian**. All longitudes run through the north pole and the south pole. All longitudes are measured west or east of the Prime (Greenwich) Meridian.

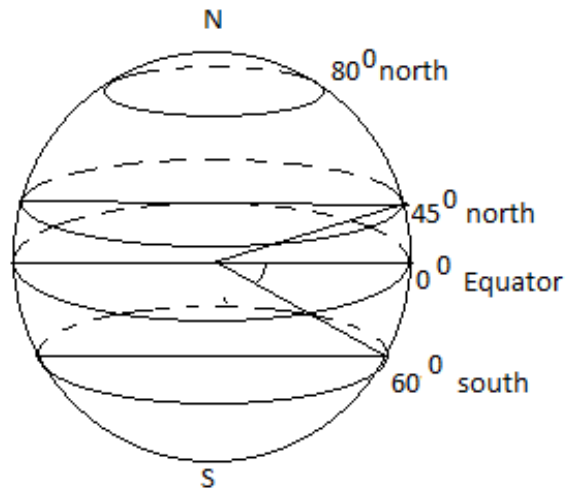


NOTE:

145°W and 35°E are DIAMETRICALLY OPPOSITE.

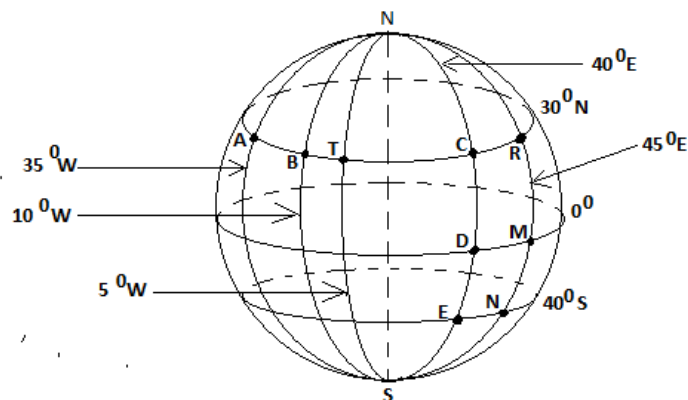
Together, they form a GREAT CIRCLE.

- **Latitude:** Latitude marks the distance (in degrees) given to a place north or south of the equator
- All latitudes parallel to the equator.
- Among the latitudes, the equator is the only great circle.



LEARNER ACTIVITY:

In the diagram below,



- (i) state the longitude of the points A, B, C, T and R.
- (ii) find the difference in longitude between A and T; A and R; E and N; B and C.
- (iii) find the difference in longitude between A and B; B and T; T and C; C and R.
- (iv) state the latitude of the points C, D, E, M, N and R.
- (v) find the difference in latitude between C and D; D and E; C and E; M and N; R and N.

DISTANCE BETWEEN POINTS ALONG THE SAME CIRCLE OF LONGITUDE.

NOTE:

- If the two given points are in the same hemisphere,
 $\theta = \text{difference in latitude}.$
- If the two given points are in different hemispheres,
 $\theta = \sum \text{of their angles of latitudes}.$

i) Distance in nautical miles

- along a Great circle, **1° of arc = 60 nm**
 $\therefore \text{distance between two points} = \theta \times 60 \text{ nm}.$

where θ is the difference in latitude between the given points.

or **distance between two points** $= \frac{\theta}{360} \times 2\pi R \text{ nm}.$

where θ is the difference in latitude and **R = 3 437 nm** is the radius of the Earth.

ii) Distance in kilometres (km)

- Radius of the Earth in kilometres = **6 370 km**
 $\therefore \text{distance between two points} = \frac{\theta}{360} \times 2\pi R \text{ km}$

with R = 6370 km.

iii) distance between any two points on the equator:

NOTE: Distance is calculated as in i) and ii), but

θ is the difference in Longitude between the two given points for points on the same side of the Prime Meridian, **OR** the sum of their Longitudes, if they are on opposite sides of the Prime Meridian.

iv) distance between points on the same circle of Latitude other than the Equator.

• **Fomulae:**

i) **distance** = $\theta \times 60 \text{ nm} \times \cos \alpha \text{ nm}$, where θ is the difference OR sum in longitude and α is the **angle** of Latitude.

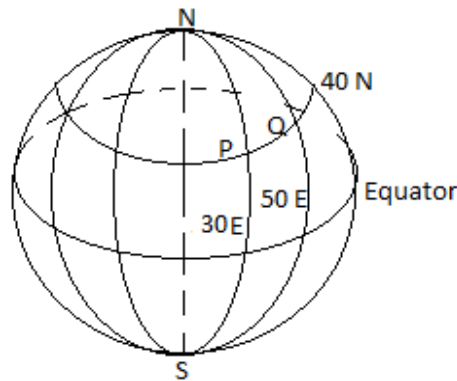
ii) **distance** = $\frac{\theta}{360} \times 2 \pi r$, but **r** = **R cos** α

$$\therefore \text{distance} = \frac{\theta}{360} \times 2 \pi R \cos \alpha$$

NOTE: In **nm** use R = 3437nm; in **km** use R = 6370km. and let $\pi = \frac{22}{7}$ or 3.142.

ACTIVITY 1.

Find the distance along a circle of latitude between P(40°N, 30°E) and Q(40°N, 50°E).



EXPECTED ANSWERS:

- difference in longitude $50^\circ - 30^\circ = 20^\circ$

- latitude 40°

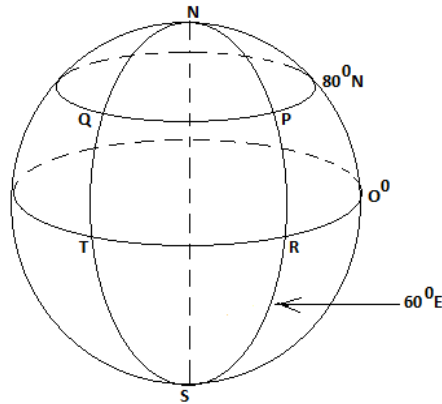
distance $\theta \times 60 \times \cos \alpha$

$\therefore$ distance $20^\circ \times 60 \times \cos 40^\circ$

$\therefore$ distance **919 nm.**

ACTIVITY 2.

The diagram below shows a wire model of the earth, The circle of latitude in the north is $80^{\circ}N$ and the circle of latitude 0° . The meridian $NPRS$ is $60^{\circ}E$ and meridian $NQTS$ is directly opposite $NPRS$.



- i) State the position of the point Q .
- ii) Find the distance along the circle of latitude $80^{\circ}N$ between Q and P in km .
- iii) Calculate the shortest distance between P and Q
- iv) If the time at R is $20:00hrs$, what is the time at T

EXPECTED ANSWERS:

i.

$$\therefore Q(80^{\circ}N, 120^{\circ}W)$$

ii.

$$= \frac{180}{360} \times 2 \times 3.142 \times 6370 \times \cos 80$$

$$= 3475.5 km$$

$$= 3480 km (3 s.f)$$

iii. SHORTEST DISTANCE BETWEEN P AND Q.

$$D = \frac{\theta}{360} \times 2\pi R$$

$$D = \frac{20}{360} \times 2 \times 3.142 \times 6370$$

$$D = 2224 km$$

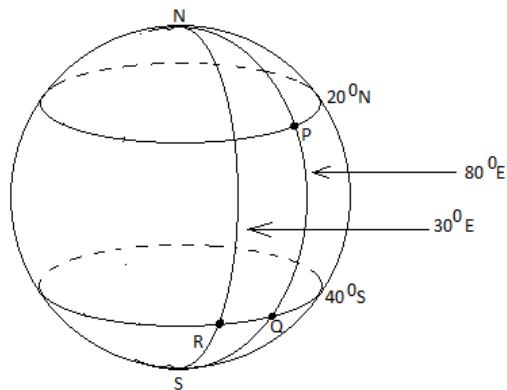
$$D = 2220 km (3 s.f)$$

$$\text{iv. } \frac{180}{15} = 12 \text{ hours}$$

$$\therefore 20:00 - 12:00 = 08:00 \text{ hours.}$$

ACTIVITY 3.

The diagram below is a sketch of the earth and on it are the points **P**(20°N, 80°E), **Q**(40°S, 80°E) and **R**(40° S, 30° E). [Use $\pi = 3.142$ & $R = 6370 \text{ km}$]



- (i) Calculate the distance QR in kilometres.
- (ii) An aeroplane starts from **P** and flies due west on the same latitude covering a distance of 1 232km to point **T**.
 - (a) Calculate the difference in angles between **P** and **T**,
 - (b) Find the position of **T**.

EXPECTED ANSWERS:

- (i) DISTANCE OF **PQ**:

-difference in longitude $80^\circ - 30^\circ$

$$50^\circ = \theta,$$

PQ is on latitude $40^\circ \text{ S} = \alpha,$

$$\text{Dist. PQ} = \frac{\theta}{360^\circ} \times 2\pi R \cos \alpha$$

$$\therefore \text{Dist. PQ} = \frac{50^\circ}{360^\circ} \times 2 \times 3.142 \times 6370 \cos 40^\circ$$

$$\therefore \text{Distance PQ} = 4258.8964 \text{ km}$$

$$\approx 4258.9 \text{ km (1 d.p.)}$$

- (ii) (a) Distance $\frac{\theta}{360^\circ} \times 2\pi R \cos \alpha$, but Dist = 1232km,

$$R =$$

$$6370 \text{ km,}$$

$$\alpha = 20^\circ.$$

$$\therefore 1232 = \frac{\theta}{360^\circ} \times 2 \times 3.142 \times 6370 \cos 20^\circ$$

$$\therefore \theta = \frac{104.4861975}{1232}$$

$$\therefore \theta = 11.79103106$$

$$\approx 11.8^\circ$$

$\therefore$ difference in angles between **P** and **T** $\approx 11.8^\circ$

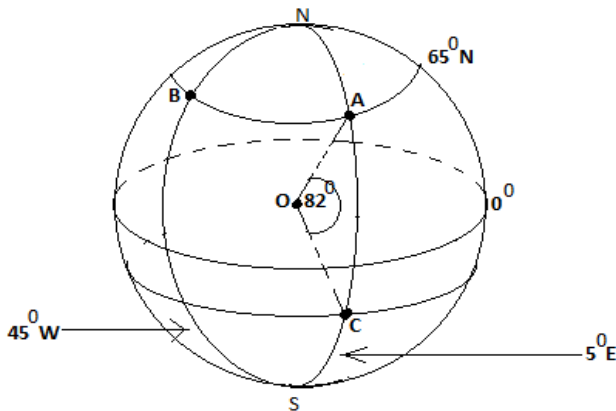
(b) **Position of T is: 80**

$$^\circ - 11.7910^\circ = 68.209^\circ \therefore T(20^\circ N, 68.2^\circ S)$$

ACTIVITY 4.

In the diagram below, A($65^\circ N, 5^\circ E$), B($65^\circ N, 45^\circ W$) and C are three points on the surface of the model of the earth and O is the centre of the model. The point C due south of A, is such that $\angle AOC = 82^\circ$.

$$[\pi = 3.142, R = 6370 \text{ km}]$$



- (i) State the longitude of A,
- (ii) Calculate the latitude of C,
- (iii) Calculate, in nautical miles, the shortest distance
 - (a) between A and C and measured along the common longitude,
 - (b) between A and B measured along the circle of latitude.

EXPECTED ANSWERS:

- (i) **Longitude of A $5^\circ E$**
- (ii) Latitude of C $82^\circ - 65^\circ = 17^\circ S$
- (iii) (a) 1° of arc = 60 nm,
difference in latitude = $82^\circ = \theta$,
 $\therefore$ Shortest distance between A and C:

$$\theta \times 60 \text{ nm} = 82^\circ \times 60 \text{ nm} = 4920 \text{ nm}.$$

(b) difference in longitude between A and B

$$= 45^\circ \text{ W} + 5^\circ \text{ E} = 50^\circ.$$

$\therefore$ distance along circle of latitude 65° N , between A and B

$$\theta \times 60 \text{ nm} \times \cos \alpha$$

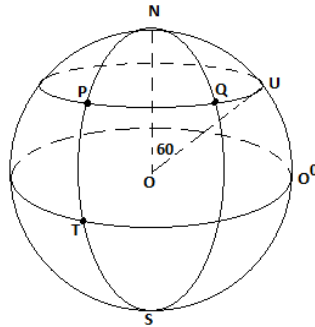
$$50^\circ \times 60 \text{ nm} \times \cos 65^\circ$$

$$1267.854785 \approx 1267.9 \text{ nm (1 d.p.)}$$

ACTIVITY 5

In the diagram below, the points P and Q lie on the same latitude, O is the Centre of

the earth and angle NOQ = 60° . (Take $\pi = 3.142$ and $R = 6370 \text{ km}$)



- (i) State the latitude where the points P and Q are lying.
- (ii) Find the distance between P and T.
- (iii) Given that the point P is on longitude 18° W and the time difference between P and Q is 5 hours, calculate the longitude on which Q lies.

EXPECTED ANSWERS:

- (i) Latitude where P and Q lie $90^\circ - 60^\circ$
 $30^\circ \text{ N}.$

- (ii) difference in latitude $30^\circ - 0^\circ$
 $30^\circ = \theta.$

$$\therefore \text{Distance between P and Q} = \frac{\theta}{360^\circ} \times 2\pi R$$

$$= \frac{30^\circ}{360^\circ} \times 2 \times 3.142 \times 6370 \text{ km}$$

$$3335.756667 \text{ km}$$

$$= 3335.76 \text{ km (2 d.p.)}$$

(iii) **Note:** $1 \text{ hr} = 15^\circ$.

$$5 \text{ hr} = y^\circ$$

$$\therefore y^\circ = 5 \text{ hr time difference} = 75^\circ$$

$$\therefore \text{longitude of Q} = 75^\circ - 18^\circ$$

$$\therefore \text{longitude of Q} = 57^\circ.$$

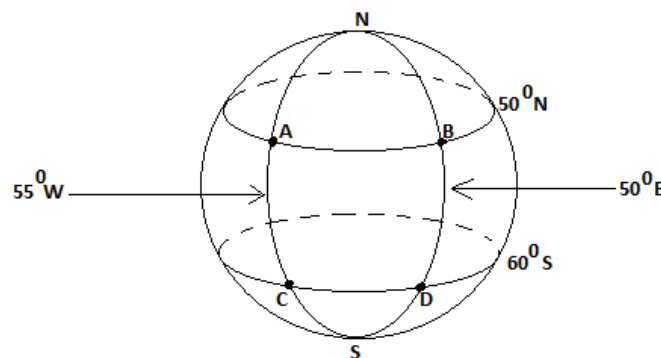
ACTIVITY 6

The diagram below shows a wire model of the earth. The circle of latitude in the north

is 50°N and the circle of latitude in the south is 60°S . A and C are on longitude 55°W

while B and D are on longitude 50°E .

(Take $\pi = 3.142$ and $R = 3437 \text{ nm}$)



- Write the positions, using longitudes and latitudes, of the points A and D.
- Calculate the difference in longitudes between A and B.
- Given that the time at town D is 09 20 hours, what would be the time at town C?
- Calculate the distance BD along the longitude 50°E in **nautical miles**.

EXPECTED ANSWER:

- A(50°N , 55°W) and D(60°S , 50°E)**
- Difference in longitudes between A and B $= 55^\circ + 50^\circ = 105^\circ$
- TIME AT C =
note: C(**60°S , 55°W**), D(**60°S , 50°E**)

Difference in longitudes between C and D = $55^\circ + 50^\circ = 105^\circ$

note: $1\text{ hr} = 15^\circ$

$$x\text{hrs} = 105^\circ$$

$$\therefore x\text{hrs} = \frac{105^\circ}{15^\circ} = 7\text{hrs} \text{ time difference,}$$

$\therefore$ TIME AT C = 09 20hrs $- 7\text{hrs}$ (*C is West of D, so subtract*)

$\therefore$ TIME AT C = **02 20hrs**.

(iv) Difference in latitudes = $50^\circ + 60^\circ = 110^\circ$,

$\therefore$ Distance between B & D = $110^\circ \times 60\text{nm} = 6600\text{nm}$.