# GRAVITATION

## Gravitational Force

$F = \frac{GM_1M_2}{r^2}$ $\quad G = 6.67 \times 10^{-11}$

- central force, long range,
- form act$^n$ n$\times$n pair
- mediated by graviton
- Conservative in nature
- principal of superposition is valid

↳




Hollow sphere
$F_{net}$ on $m = 0$



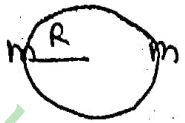

Solid sphere
$F_{net}$ on $m \neq 0$

$F \propto \frac{M^2}{r^2}$ $\quad F \propto \rho r^4$

↳

$m \xrightarrow{x} \underset{r}{\square} nm$

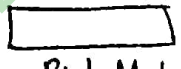
$f_0$ at $x = \frac{r}{\sqrt{n}+1}$




$F_{net}$ = centripetal.

$mr\omega^2 = \frac{G \cdot m^2}{(2r)^2}$

↳



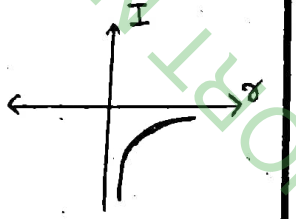

$F_m = \frac{GmM}{(d)(L+d)}$ $\quad$ ME'

## (I) GRAVITATIONAL FIED INTENSITY

- force on unit mass $I = F/m_0$
- vector
- directed along force $\quad M^0LT^{-2} \quad N/kg$
-

### 1. Point mass (M)

$\vec{I} = -\frac{GM}{r^2}\hat{r}$



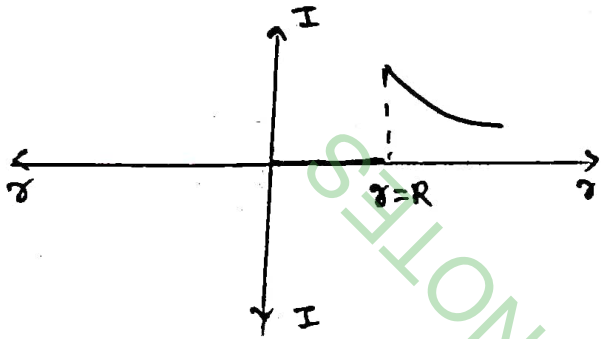

### 2. Hollow Sphere (M,R)

Outside $(r>R) \Rightarrow I = GM/r^2$

surface $(r=R) \Rightarrow I = GM/R^2$

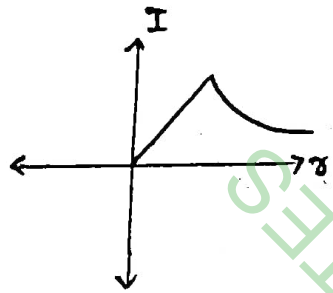
inside $(r<R)$

↳ $M=0 \rightarrow I=0$

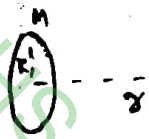



### 3. Solid sphere (M,R)

- Outside $(r>R) \Rightarrow I = GM/r^2$
- surface $(r=R) \Rightarrow I = GM/R^2$
- inside $(r<R) \Rightarrow I = \frac{GMr}{R^3}$




### 4. On the axis of Ring → (M, R)



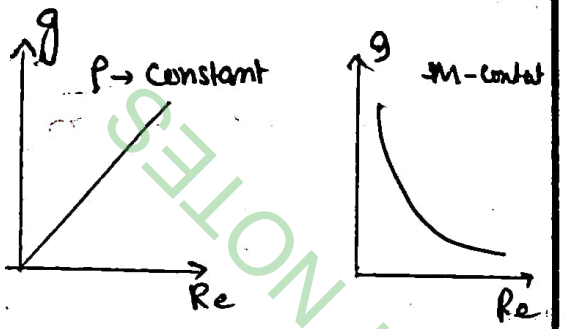

$I = \frac{GMx}{(R^2+x^2)^{3/2}}$

- $R \ggg x$ $\quad I = \frac{GMx}{R^3}$
- $x \ggg R$ $\quad I = \frac{GM}{x^2}$
- $I_{max}$ at $\Rightarrow x = R/\sqrt{2}$

## • ACCELARATION DUE TO GRAVITY • (g)

$$g = a = \frac{F}{m_0} = \frac{GM_e}{R_e^2} \Rightarrow GM_e = gR_e^2$$

$$g = \frac{4}{3}\pi\rho G R_e$$




## Variation of 'g' with height

$$g' = \frac{GM_e}{(R_e+h)^2} \Rightarrow g' = \frac{gR_e^2}{(R_e+h)^2}$$

∴ h → from surface of earth.

Case I → $h \ll\ll R$ (5 to 10 km)

$$g' = g$$

II → $h \ll R$ (upto 5% of $R_e$)

$$g_h = g\left(1 - \frac{2h}{R}\right)$$ MR Ratta

$$\frac{\Delta g}{g} = 2\frac{\Delta R}{R}$$

## Variation of 'g' with depth 'd'

$$g' = \frac{GM_e}{r^2} \qquad r = R - h$$

$$g' = g\left(1 - \frac{d}{R}\right) \qquad \frac{\Delta g}{g} = -\frac{d}{R} \times 100$$

## Variation due to shape of Earth

$R_p = R_e + 21$ $\qquad g_{pole} > g_{equator}$

## Variation due to Rotational effect

$$g' = g_0 - \omega^2 R\cos^2\theta$$

Pole: $g' = g$

Equator: $g' = g_0 - \omega^2 R$

Case I → $g'_e = 0$

$$\omega = \sqrt{\frac{g_0}{R}} \qquad T = 84 \text{ mm}, 1.41 \text{ hr}$$

$$\frac{\Delta g}{g_0} \times 100 = \frac{\omega^2 R}{g_0} \times 100$$

## GRAVITATIONAL POTENTIAL ENERGY (U)

- depend upon frame of reference
- −ve of W by cons force

### gravitation potential (V)

gravitational potential energy per unit mass. • Scalar Quantity

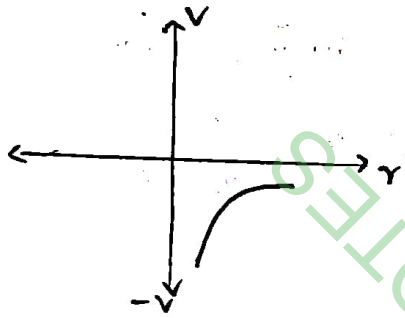
$$V = \frac{U}{m} \qquad I = \frac{F}{m}$$

$$\cdot I = -\frac{dV}{dr} \qquad \cdot dV = \int I \cdot dr$$

Potential due to a point Mass

M • ---- r ---- V = ?

$$V = -\frac{GM}{r}$$

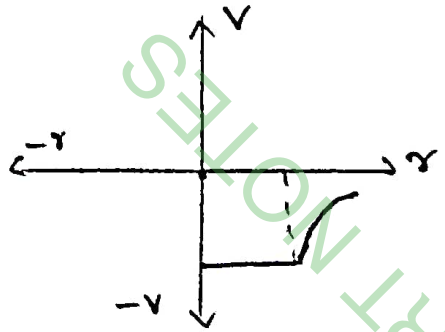


1. Hollow sphere (R, M)

Outside → $r > R$ $\quad V = -\frac{GM}{r}$

Surface → $r = R$ $\quad V = -\frac{GM}{R}$

Inside → $r < R$ $\quad V = -\frac{GM}{R}$



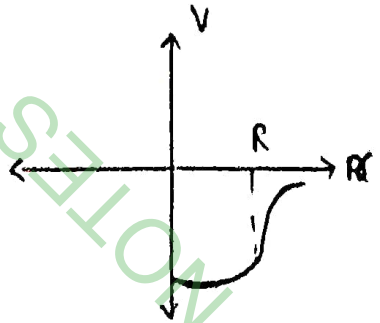

2. Solid sphere (M, R)

Outside → $r > R$ $\quad V = -\frac{GM}{r}$

Surface → $r = R$ $\quad V_s = -GM/R$

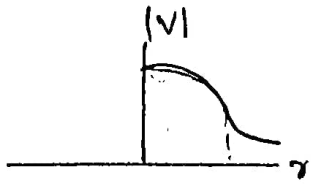
inside → $r < R$ $\quad V = -\frac{GM}{2R^3}(3R^2 - r^2)$

$V_0 = \frac{3}{2}V_s$




On the Axis of Ring (M, R) at distance x

$$V_x = \frac{-GM}{(R^2 + x^2)^{1/2}}$$




MR* →

- Change in potential dont depend upo ref
- If potential is constant I is zero
- potential of given configuraa.

No. of configuration = $\frac{n(n-1)}{2}$

n → no of mass

Δ Gravitational potential energy (U)

$$U = \frac{MgH}{\left(1 + \frac{H}{R}\right)}$$

∴ COME




$$V = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$



velocity at centre $\quad V = \sqrt{\frac{GM}{R}}$

Escape velocity..

- don't depend upon the mass of object and angle of projection
- use COME if initial point changes

$$V = \sqrt{\frac{2GM}{R}} = R\sqrt{\frac{8}{3}\rho\pi G}$$

- $V \propto \frac{1}{\sqrt{R}}$ (M-cont) • $V \propto R$ (ρ-cont)

for Blackhole

$V_e \geq c$

↳ $V > V_e$

$V_I = \sqrt{V^2 - V_e^2}$

↓ intestellar speed

↳ $V < V_e$

$H = \frac{R}{\left(\frac{V_e^2}{V^2} - 1\right)}$

## Energy of Sattelite

$V = \sqrt{\frac{GM}{R+h}}$ (dont depend upon mass of Sattelite)

$V_e = \sqrt{2}\, V_o$

41 % than orbital

$KE = \frac{GMm}{2r}$ $\quad U = -\frac{GMm}{r}$

$TE = -\frac{GMm}{2r}$

Sattelite on circular path

KE = constant — M.E → constant
PE = constant — $\bar{v}$ → variable
L = constant — W = 0
$\tau = 0$ — Speed = const.
$\bar{P}$ → variable — Power = 0

Time period

$m\omega^2 r = \frac{GMm}{r^2}$

$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$ $\quad T^2 \propto r^3$

$T = 2\pi\sqrt{\frac{r^3}{GM}}$

## KEPLER'S LAWS OF PLANETORY MOTION

1. Law of orbit – All planet move in elliptical orbits, with the sun at the foci of the ellipse

$a = \frac{r_{max} + r_{min}}{2}$ $\quad \frac{r_{max}}{r_{min}} = \frac{a(1+e)}{a(1-e)}$

Closest poin → perihelion farthest → aphelion

2. Law of Area → equal area in equal interval of time

$Vr$ = constant $\quad L$ = constant

$\frac{L}{2m} = \frac{dA}{dt}$




$\omega \propto \frac{1}{r^2}$ $\quad \frac{\omega_1}{\omega_2} = \left(\frac{r_2}{r_1}\right)^2$

$V \propto \frac{1}{r}$ $\quad \frac{V_1}{V_2} = \frac{r_2}{r_1}$

3. Law of period →

$T^2 = \frac{4\pi^2}{GM} a^3$

$T^2 \propto a^3$ ↳ semi major axis

↳ Work done = $\Delta K.E$

Geostationary / parking / Communicath
$h = 6R$ $\quad T = 24$ hr $\quad V = 3.1$ km/hr
Cover $\frac{1}{3}^{rd}$ area of earth.
Relay sattelite

Polar sattelite
About pole
T = 100 min.

Binding Energy
→ B.E = $|TE| = |KE| = \left|\frac{V}{2}\right|$