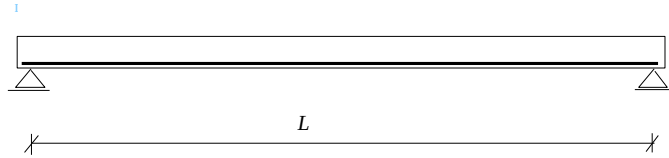


Beam Design Example 1

Simple span beam

Beam is loaded by equally distributed permanent load $g = 12,0$ kN/m and live load $q = 15,0$ kN/m. The span $L = 6,0$ m. Height of the beam is $h = 480$ mm and width $b = 240$ mm. The design strength for concrete is $f_{cd} = 22,0$ MPa and for reinforcement bars $f_{yd} = 455,0$ MPa. (CC2, combination factor for live loads $\psi_o = 0,7$).



Determine required amount of reinforcement steel (A_s) in most stressed cross-section.

Solution.

Only one live load (combination factor is not needed)

$$q_d = 1,35 \cdot K_{FI} \cdot g + 1,5 \cdot K_{FI} \cdot q \approx 38,7 \text{ kN/m} . \quad (1)$$

Max bending moment in the middle

$$M_d = \frac{q_d \cdot L^2}{8} \approx 174,2 \text{ kNm} . \quad (2)$$

The effective height d is approximated to be as $d = h - 50\text{mm} \approx 430$ mm .

Relative bending moment is then:

$$\mu = \frac{M_d}{bd^2 f_{cd}} \approx 0,178 \quad (\text{dimensionless quantity}) \quad (3)$$

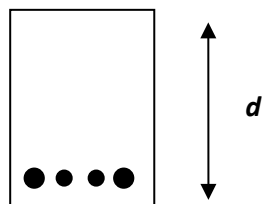
Mechanical reinforcement ratio is determined from formula:

$$\omega = 1 - \sqrt{1 - 2\mu} \approx 0,198 = \beta . \quad (4)$$

Required area of the steel

$$A_s = \frac{M_d}{f_{yd} d (1 - \frac{\omega}{2})} = 990 \text{ mm}^2 \quad (5)$$

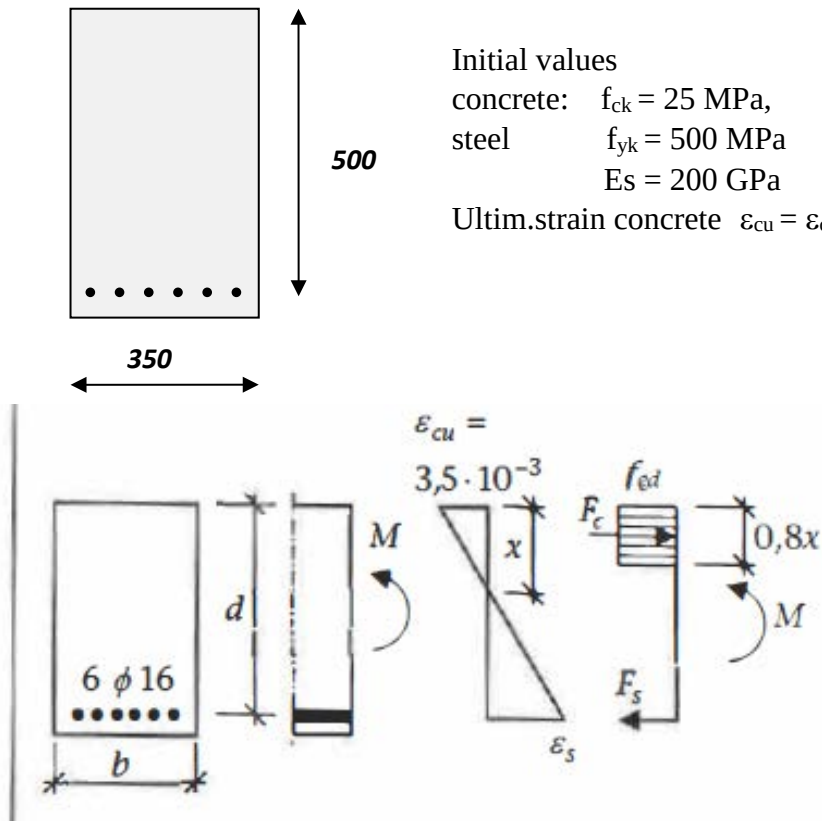
Select reinforcing bars: for example $2\phi 20 + 2\phi 16$, when the area of steel A_s is 1030 mm^2 .



Beam Design Example 2

Moment capacity of a given cross-section (byggkonstruktion)

Determine the load carrying capacity in ultimate limit state of the concrete beam with cross-section values are shown in figure. Concrete is C 25, Steelbars: 6f16, B500B. Consequence class CC2.



Initial values

concrete: $f_{ck} = 25 \text{ MPa}$, $\gamma_c = 1,50 \Rightarrow f_{cd} = 16,7 \text{ MPa}$

steel $f_{yk} = 500 \text{ MPa}$ $\gamma_c = 1,15 \Rightarrow f_{yd} = 435 \text{ MPa}$

$E_s = 200 \text{ GPa}$

Ultim.strain concrete $\epsilon_{cu} = \epsilon_{cu3} = 3,5 \text{ ‰}$

Amount of steel: $A_s = 6 \pi 8^2 = 1206 \text{ mm}^2$

is the beam normally reinforced? Calculate ρ ;

$$\rho = \frac{A_s}{bd} = 6,89 \cdot 10^{-3}$$

Balanced geometric reinforcement is

$$\rho_{bal} = 0,8 \frac{f_{cd}}{f_{yd}} \frac{\epsilon_{cu} E_s}{f_{yd} + \epsilon_{cu} E_s} = 18,9 \cdot 10^{-3}$$

consequently it is normal reinforced because

$$\rho < \rho_{bal} \quad \text{OK}$$

neutral axis can be determined from equilibrium equation:

$$x = \frac{A_s f_{yd}}{f_{cd} 0,8b} = 112 \text{ mm}$$

Moment capacity of the cross-section is

$$M_u = A_s f_{yd} (d - 0,4x) = 238,8 \text{ kNm}$$

Beam Design Example 3.1

Moment capacity of a given cross-section (byggkonstruktion)

The beam with cross-section in the figure is constructed of concrete C 30 and steelbars B500B is used. CC2.

- Determine the moment carrying capacity at ultimate limit state if the cross-section is balanced reinforced.
- Determine reinforcement required to resist external design bending moment $M_{ed} = 265 \text{ kNm}$

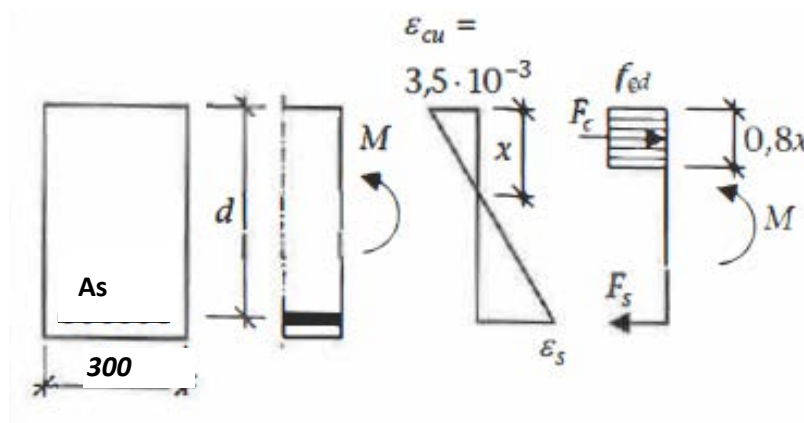
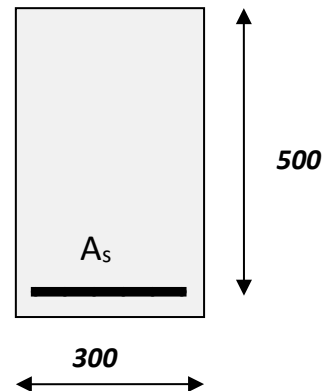
Initial values

concrete: $f_{ck} = 30 \text{ MPa}$, $\gamma_c = 1,50 \Rightarrow f_{cd} = 20 \text{ MPa}$

steel $f_{yk} = 500 \text{ MPa}$ $\gamma_s = 1,15 \Rightarrow f_{yd} = 435 \text{ MPa}$

$E_s = 200 \text{ GPa}$

Ultim.strain in concrete $\epsilon_{cu} = \epsilon_{cu3} = 3,5 \text{ ‰}$



Balance the internal forces by equilibrium equations

$$x_{bal} \frac{\epsilon_{cu} E_s d}{f_{yd} + \epsilon_{cu} E_s} = 308 \text{ mm}$$

Moment capacity of balanced reinforcement can be calculated either moment equilibrium equations:

$$M = F_s (d - 0,4x) \text{ or}$$

$$M = F_c (d - 0,4x) = f_{cd} 0,8x b (d - 0,4x)$$

by using latter moment balance

$$2400 x - 1920 x^2 = M = 265 \text{ kNm}$$

solving second order equation for x; the position of N.A. comes

$$x = 122 \text{ mm}$$

Checking the strain in reinforcement, if it has yielded:

$$\epsilon_s = \frac{d - x}{x} \epsilon_{cu} = 10,8 \cdot 10^{-3}$$

yield strain for steel is

$$\epsilon_{sy} = \frac{f_{yd}}{E_s} = 2,6 \cdot 10^{-3} \quad \text{steel has yielded and stress in steel is } \sigma = f_{yd}$$

Required steel area

$$f_{cd} 0,8xb = A_s f_{yd} \Rightarrow A_s = 1346 \text{ mm}^2$$

Beam Design Example 3.2

Moment capacity alternative solutions (byggkonstruktion)

a) Mechanic reinforcement ratio can be calculated from:

$$\omega_{bal} = 0,8 \frac{\epsilon_{cu} E_s}{f_{yd} + \epsilon_{cu} E_s} = 0,8 \frac{3,5 \cdot 10^{-3} \cdot 200 \cdot 10^3}{435 + 3,5 \cdot 10^{-3} \cdot 200 \cdot 10^3} = 0,493$$

Moment capacity of the balanced cross-section can be solved using equation:

$$M = b d^2 f_{cd} \omega_{bal} (1 - 0,5 \omega_{bal}) = 0,3 \cdot 0,5^2 \cdot 20 \cdot 10^3 \cdot 0,493 \cdot (1 - 0,5 \cdot 0,493) = 557 \text{ kNm}$$

b) Required reinforcement when $M_{Ed} = 265 \text{ kNm}$.

$$\mu = \frac{M}{b d^2 f_{cd}} = \frac{265}{0,3 \cdot 0,5^2 \cdot 20 \cdot 10^3} = 0,177$$

geometric reinforcement ratio

$$\omega = 1 - \sqrt{1 - 2\mu} = 0,196 < \omega_{bal} \quad \text{check OK}$$

Required steel amount:

$$A_s = \frac{M_d}{f_{yd} d (1 - \frac{\omega}{2})} = 1351 \text{ mm}^2$$