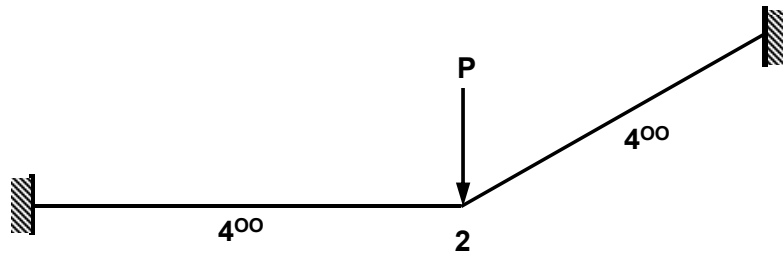
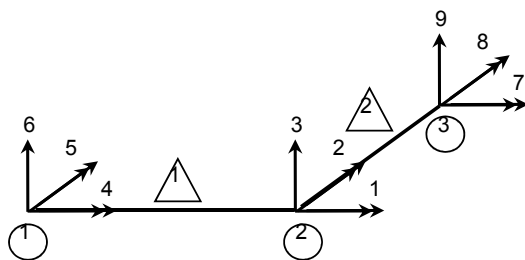


Contoh Soal

Diketahui struktur balok silang seperti gambar berikut, jika balok terbuat dari beton bertulang dan dianggap cukup kaku (rotasi pada titik 2 diabaikan) dan lendutan yang diijinkan pada titik tersebut adalah sebesar 4 cm, berapakah nilai P yang diperbolehkan bekerja pada struktur tersebut?

**Data Perhitungan :**

1. Ukuran Penampang Balok = 40/60 (cm)
2. Poisson Ratio (ν) = 0,3
3. Momen Inersia (I_y) = $\frac{1}{12}bh^3 = \frac{1}{12} \cdot 40 \cdot 60^3 = 720.000 \text{ cm}^4$
4. Faktor Konstanta Torsi (β) = $\frac{1}{3} - 0,21 \frac{b}{h} \left[1 - \frac{b^4}{12h^4} \right]$
 $= \frac{1}{3} - 0,21 \frac{40}{60} \left[1 - \frac{40^4}{12 \cdot 60^4} \right] = 0,19564$
5. Konstanta Torsi (I_x) = $\beta hb^3 = 0,19564 \cdot 60 \cdot 40^3 = 751.249 \text{ cm}^4$
6. Modulus Elastisitas (E) = 100.000 kg/cm²
7. Modulus Geser (G) = $\frac{E}{2(1+\nu)} = \frac{100.000}{2(1+0,3)} = 38.462 \text{ kg/cm}^2$

Penomoran (terhadap sistim koordinat Global)**Tabel Perhitungan**

i	j	k	α	C_x	C_y	$\frac{GI_x}{L}$	$\frac{12EI_y}{L^3}$	$\frac{6EI_y}{L^2}$	$\frac{4EI_y}{L}$	$\frac{2EI_y}{L}$
1	1	2	0o	1	0	96.315.130	32	4.800.000	960.000.000	480.000.000
2	2	3	90o	0	1	72.236.348	13.5	2.700.000	720.000.000	360.000.000

Matriks Kekakuan Batang dalam Koordinat Lokal [SM]_i

$$[SM]_i = \begin{bmatrix} +\frac{GI_x}{L} & 0 & 0 & -\frac{GI_x}{L} & 0 & 0 \\ 0 & +\frac{4EI_y}{L} & -\frac{6EI_y}{L^2} & 0 & +\frac{2EI_y}{L} & +\frac{6EI_y}{L^2} \\ 0 & -\frac{6EI_y}{L^2} & +\frac{12EI_y}{L^3} & 0 & -\frac{6EI_y}{L^2} & -\frac{12EI_y}{L^3} \\ -\frac{GI_x}{L} & 0 & 0 & +\frac{GI_x}{L} & 0 & 0 \\ 0 & +\frac{2EI_y}{L} & -\frac{6EI_y}{L^2} & 0 & +\frac{4EI_y}{L} & +\frac{6EI_y}{L^2} \\ 0 & -\frac{6EI_y}{L^2} & -\frac{12EI_y}{L^3} & 0 & +\frac{6EI_y}{L^2} & +\frac{12EI_y}{L^3} \end{bmatrix}$$

Batang 1

$$[SM]_1 = 10^6 \begin{bmatrix} 96,315 & 0 & 0 & -96,315 & 0 & 0 \\ 0 & 960 & -4,8 & 0 & 480 & 4,8 \\ 0 & -4,8 & 0,032 & 0 & -4,8 & -0,032 \\ -96,315 & 0 & 0 & 96,315 & 0 & 0 \\ 0 & 480 & -4,8 & 0 & 960 & 4,8 \\ 0 & 4,8 & -0,032 & 0 & 4,8 & 0,032 \end{bmatrix} \begin{matrix} 4 \\ 5 \\ 6 \\ 1 \\ 2 \\ 3 \end{matrix}$$

Batang 2

$$[SM]_4 = 10^6 \begin{bmatrix} 72,236 & 0 & 0 & -72,236 & 0 & 0 \\ 0 & 720 & -2,7 & 0 & 360 & 2,7 \\ 0 & -2,7 & 0,0135 & 0 & -2,7 & -0,0135 \\ -72,236 & 0 & 0 & 72,236 & 0 & 0 \\ 0 & 360 & -2,7 & 0 & 720 & 2,7 \\ 0 & 2,7 & -0,0135 & 0 & 2,7 & 0,0135 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 7 \\ 8 \\ 9 \end{matrix}$$

Matriks Rotasi Transformasi [R_T]

$$[R_T] = \begin{bmatrix} +c_x & +c_y & 0 & 0 & 0 & 0 \\ -c_y & +c_x & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & +c_x & +c_y & 0 \\ 0 & 0 & 0 & -c_y & +c_x & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 \end{bmatrix}$$

$$\alpha = 0^\circ$$

$$[R_T] = \begin{bmatrix} +1 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 \end{bmatrix}$$

$$\alpha = 90^\circ$$

$$[R_T] = \begin{bmatrix} 0 & +1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 \end{bmatrix}$$

Matriks Rotasi Transformasi Transpose $[R_T]^T$

$$[R_T]^T = \begin{bmatrix} +c_x & -c_y & 0 & 0 & 0 & 0 \\ +c_y & +c_x & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & +c_x & -c_y & 0 \\ 0 & 0 & 0 & +c_y & +c_x & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 \end{bmatrix}$$

$$\alpha = 0^\circ$$

$$[R_T]^T = \begin{bmatrix} +1 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 \end{bmatrix}$$

$$\alpha = 90^\circ$$

$$[R_T]^T = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ +1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & +1 \end{bmatrix}$$

Matriks Kekakuan Batang dalam Koordinat Global $[SM_S]_i$

$$[SM_S]_i = [R_T]^T [SM]_i [R_T]$$

Batang 1

$$[SM_S]_1 = 10^6 \begin{bmatrix} 96,315 & 0 & 0 & -96,315 & 0 & 0 \\ 0 & 960 & -4,8 & 0 & 480 & 4,8 \\ 0 & -4,8 & 0,032 & 0 & -4,8 & -0,032 \\ -96,315 & 0 & 0 & 96,315 & 0 & 0 \\ 0 & 480 & -4,8 & 0 & 960 & 4,8 \\ 0 & 4,8 & -0,032 & 0 & 4,8 & 0,032 \end{bmatrix} \begin{matrix} 4 \\ 5 \\ 6 \\ 1 \\ 2 \\ 3 \end{matrix}$$

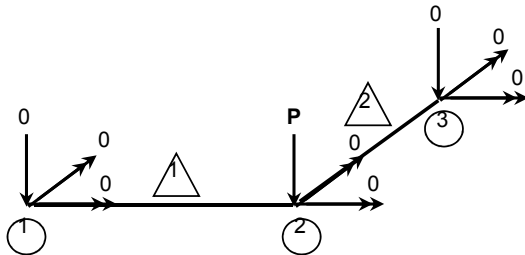
Batang 2

$$[SM_S]_2 = 10^6 \begin{bmatrix} 720 & 0 & 2,7 & 360 & 0 & -2,7 \\ 0 & 72,236 & 0 & 0 & -72,236 & 0 \\ 2,7 & 0 & 0,0135 & 2,7 & 0 & -0,0135 \\ 360 & 0 & 2,7 & 720 & 0 & -2,7 \\ 0 & -72,236 & 0 & 0 & 72,236 & 0 \\ -2,7 & 0 & -0,0135 & -2,7 & 0 & 0,0135 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 7 \\ 8 \\ 9 \end{matrix}$$

Perakitan Matriks Kekakuan Struktur $[S_j]$

Catatan : untuk penyusunan matriks kekakuan hanya dilakukan untuk sub matriks $[S_{FF}]$ dan $[S_{RF}]$ dan tidak diperlukan penataan ulang.

$$[S_j] = 10^6 \begin{bmatrix} 1632,63 & 0 & 0 & 1 \\ 0 & 2064,47 & 0 & 2 \\ 0 & 0 & 0,091 & 3 \\ -96,315 & 0 & 0 & 4 \\ 0 & 480 & 4,8 & 5 \\ 0 & -4,8 & -0,032 & 6 \\ 360 & 0 & 2,7 & 7 \\ 0 & -72,236 & 0 & 8 \\ -2,7 & 0 & -0,0135 & 9 \\ 1 & 2 & 3 & \end{bmatrix} \begin{matrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ [S_{RF}] \end{matrix}$$

Analisis Beban Join $\{A_c\}$ 

Karena pada struktur balok silang tersebut hanya diberikan gaya pada titik 2 berupa beban terpusat P, maka $\{A_c\} = \{A_j\}$. Selanjutnya :

$$\{A_c\} = \begin{bmatrix} 0 & 1 \\ 0 & 2 \\ -P & 3 \\ 0 & 4 \\ 0 & 5 \\ 0 & 6 \\ 0 & 7 \\ 0 & 8 \\ 0 & 9 \end{bmatrix} \begin{matrix} \\ \\ \{A_{FC}\} \\ \\ \\ \{A_{RC}\} \\ \\ \end{matrix}$$

Matriks Perpindahan $\{D_F\}$

$$\{D_F\} = [S_{FF}]^{-1} \{A_{FC}\}$$

$$\begin{Bmatrix} 0 \\ 0 \\ -4 \end{Bmatrix} = 10^6 \begin{bmatrix} 1632,63 & 0 & 0 \\ 0 & 2064,47 & 0 \\ 0 & 0 & 0,091 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ -P \end{bmatrix}$$

Dari persamaan di atas dapat disederhanakan menjadi persamaan linier biasa untuk mencari nilai **P** :

$$\mathbf{P} = 0.091 \times 4 \times 10^6$$

$$= 3,64 \times 10^5 \text{ kg}$$

Jadi besarnya beban **P** yang diperbolehkan bekerja pada struktur tersebut adalah **$3,64 \times 10^5 \text{ kg}$** .