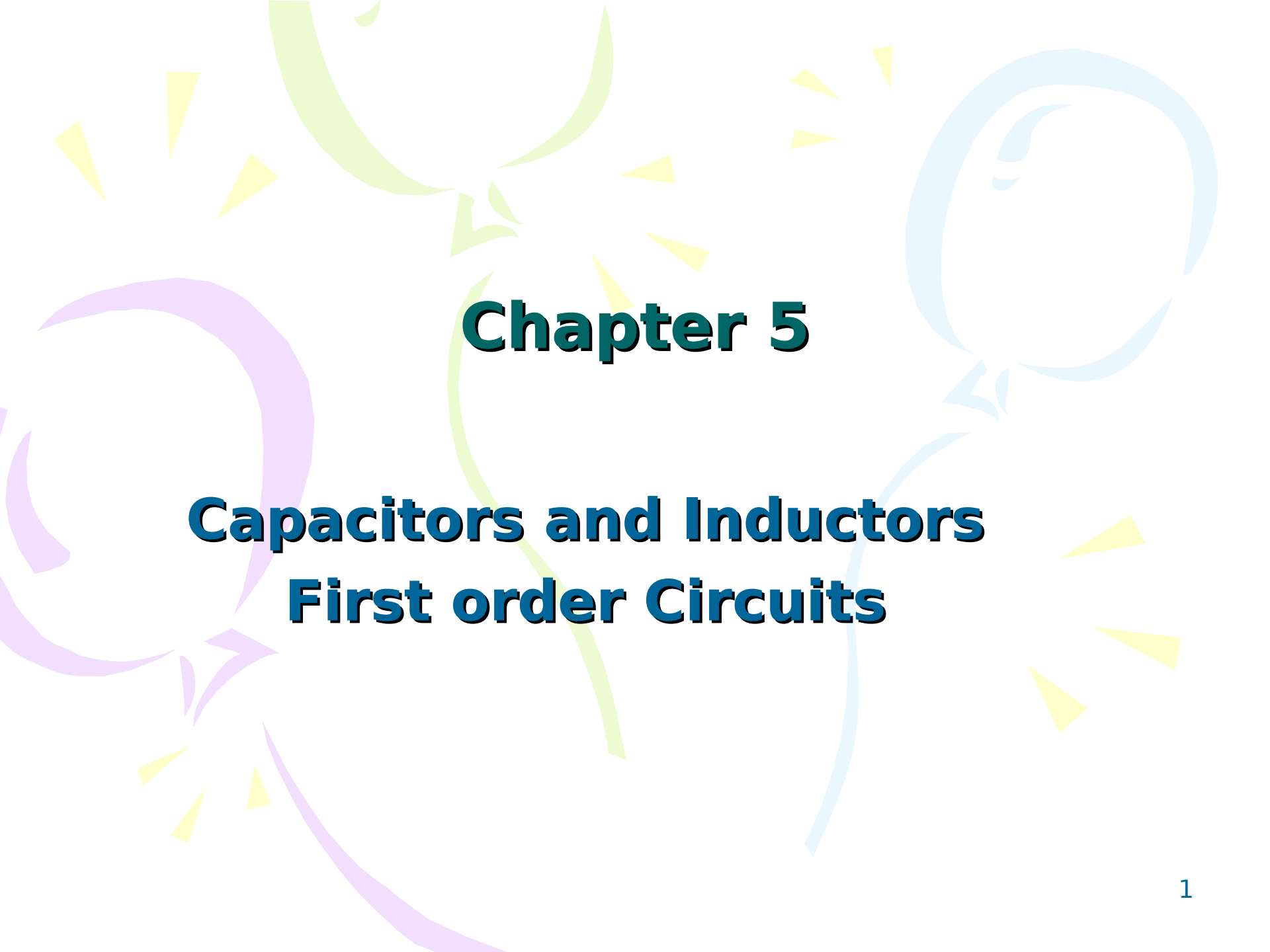


The background features several large, thick, curved lines in light green, light blue, and light purple. Scattered throughout are numerous small, yellow, triangular shapes, some pointing towards the center and others away from it, creating a dynamic, celebratory feel.

Chapter 5

Capacitors and Inductors

First order Circuits



Capacitors and Inductors

Chapter 5

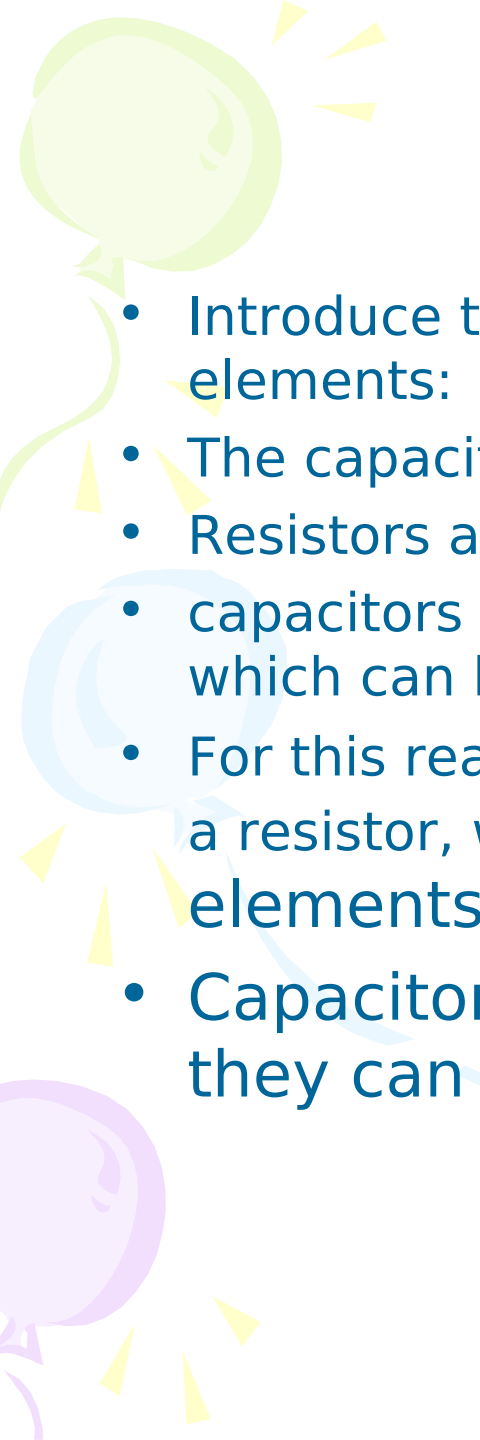
5.1 Capacitors

5.2 Series and Parallel Capacitors

5.3 Inductors

5.4 Series and Parallel Inductors

5.5 First order Circuits (Part-B)

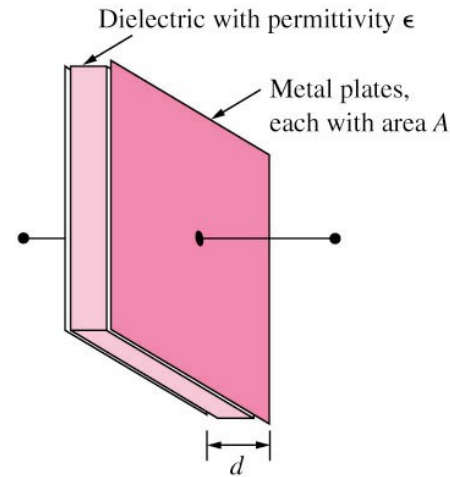
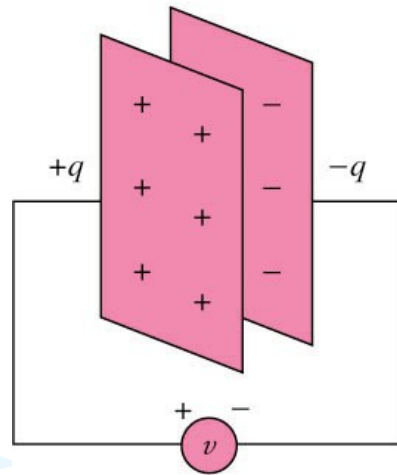


Introduction

- Introduce two new and important passive linear circuit elements:
- The capacitor and the inductor.
- Resistors are dissipate (disperse/Waste) energy.
- capacitors and inductors do not dissipate but store energy, which can be retrieved at a later time.
- For this reason, capacitors and inductors are In contrast to a resistor, which spends or dissipate- called *storage* elements.
- Capacitor and inductor can **store energy** only and they can **neither generate nor dissipate energy**.

5.1 Capacitors (1)

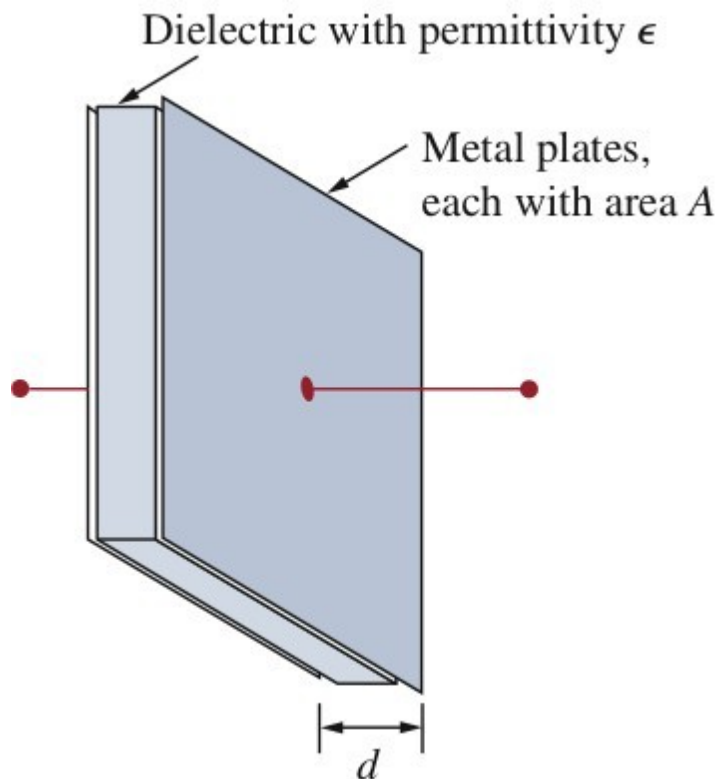
- A capacitor is a passive element designed to **store energy** in its **electric field**.



- A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).

5.2 Capacitors

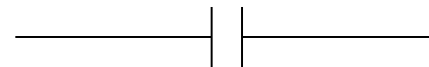
★ A capacitor consists of two **conducting plates** separated by **an insulator** (or dielectric).



$$C = \frac{\epsilon A}{d}$$

$$\epsilon = \epsilon_r \epsilon_0$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ (F/m)}$$



- ★ Common types of fixed-value capacitors.
- ★ Polyester capacitors are light in weight, stable, and their change with temperature is predictable.
- ★ Instead of polyester, other dielectric materials such as mica and polystyrene may be used.
- ★ Film capacitors are rolled and housed in metal or plastic films.
- ★ Electrolytic capacitors produce very high capacitance.



Figure 6.4 Fixed capacitors: (a) polyester capacitor, (b) ceramic capacitor, (c) electrolytic capacitor.
(Courtesy of Tech America.)

- ★ The most common types of variable capacitors.
- ★ The capacitance of a trimmer (or padder) capacitor or a glass piston capacitor is varied by turning the screw.
- ★ The trimmer capacitor is often placed in parallel with another capacitor so that the equivalent capacitance can be varied slightly.
- ★ The capacitance of the variable air capacitor (meshed plates) is varied by turning the shaft.
- ★ Variable capacitors are used in radio receivers allowing one to tune to various stations.
- ★ In addition, capacitors are used to block dc, pass ac, shift phase, store energy, start motors, and suppress noise.

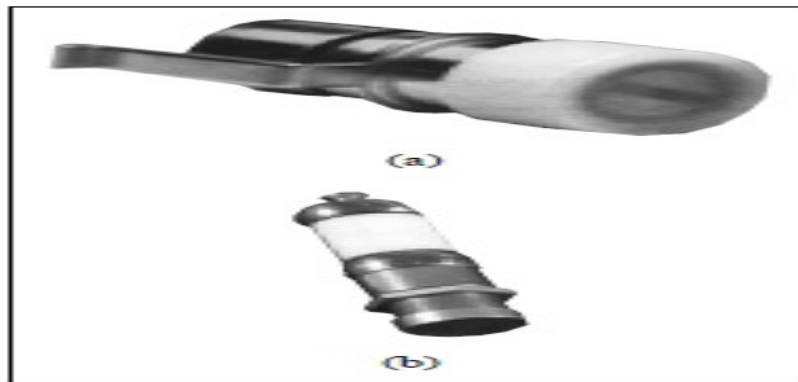


Figure 6.5 Variable capacitors:
(a) trimmer capacitor, (b) filmtrim capacitor.
(Courtesy of Johanson.)

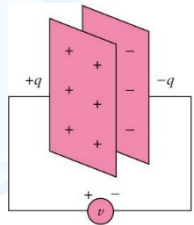
Three factors determine the value of the capacitance:

Capacitor voltage rating and capacitance are typically inversely related due to the relationships. Arcing occurs if d is small and V is high.

1. The surface area of the plates—the larger the area, the greater the capacitance.
2. The spacing between the plates—the smaller the spacing, the greater the capacitance.
3. The permittivity of the material—the higher the Permittivity, the greater the capacitance.

5.1 Capacitors (2)

- **Capacitance** C is the ratio of the charge q on one plate of a capacitor to the voltage difference v between the two plates, measured in farads (F).



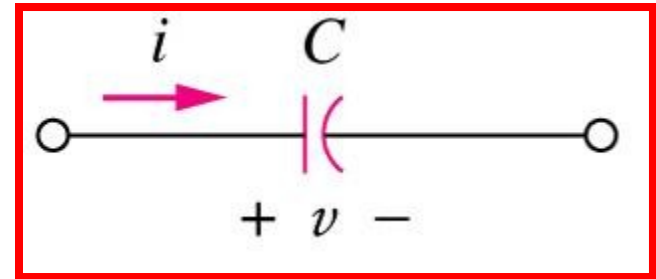
$$q = C v \quad \text{and}$$

$$C = \frac{\epsilon A}{d}$$

- Where ϵ is the permittivity of the dielectric material between the plates, A is the surface area of each plate, d is the distance between the plates.
- Unit: F, pF (10^{-12}), nF (10^{-9}), and μF (10^{-6})

5.1 Capacitors (3)

- If i is flowing into the +ve terminal of C
 - Charging $\Rightarrow i$ is +ve
 - Discharging $\Rightarrow i$ is -ve



- The current-voltage relationship of capacitor according to above convention is

$$i = C \frac{d v}{d t}$$

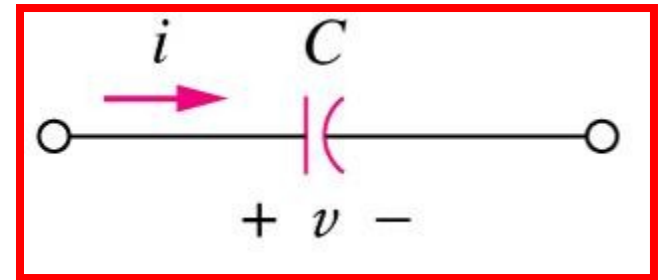
and

$$v = \frac{1}{C} \int_0^t i d t + v(t_0)$$

5.1 Capacitors (4)

- The energy, w , stored in the capacitor is

$$w = \frac{1}{2} C v^2$$



- A capacitor is
 - an **open circuit** to dc ($dv/dt = 0$).
 - its voltage **cannot change abruptly(shortly)**.

- (a) Calculate the charge stored on a 3-pF capacitor with 20 V across it.
(b) Find the energy stored in the capacitor.

Solution:

- (a) Since $q = Cv$,

$$q = 3 \times 10^{-12} \times 20 = 60 \text{ pC}$$

- (b) The energy stored is

$$w = \frac{1}{2}Cv^2 = \frac{1}{2} \times 3 \times 10^{-12} \times 400 = 600 \text{ pJ}$$

The voltage across a 5- μ F capacitor is

$$v(t) = 10 \cos 6000t \text{ V}$$

Calculate the current through it.

Solution:

By definition, the current is

$$\begin{aligned} i(t) &= C \frac{dv}{dt} = 5 \times 10^{-6} \frac{d}{dt}(10 \cos 6000t) \\ &= -5 \times 10^{-6} \times 6000 \times 10 \sin 6000t = -0.3 \sin 6000t \text{ A} \end{aligned}$$

Obtain the energy stored in each capacitor in Fig. 6.12(a) under dc conditions.

Solution:

Under dc conditions, we replace each capacitor with an open circuit, as shown in Fig. 6.12(b). The current through the series combination of the 2-k Ω and 4-k Ω resistors is obtained by current division as

$$i = \frac{3}{3 + 2 + 4}(6 \text{ mA}) = 2 \text{ mA}$$

Hence, the voltages v_1 and v_2 across the capacitors are

$$v_1 = 2000i = 4 \text{ V} \quad v_2 = 4000i = 8 \text{ V}$$

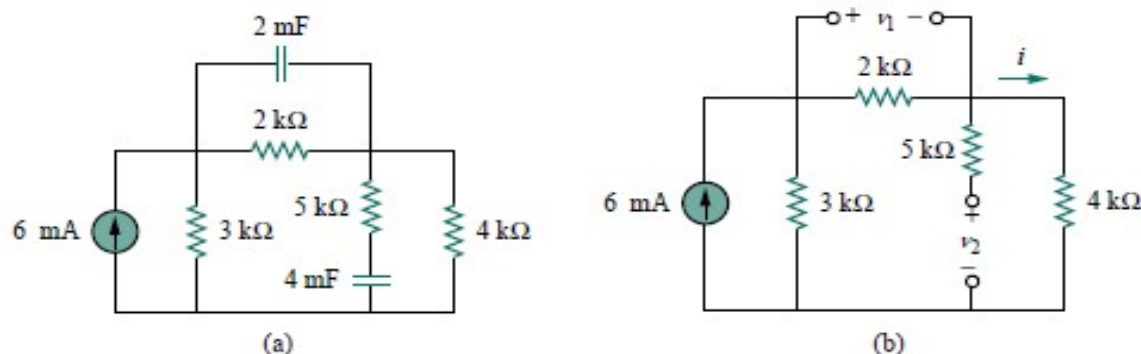


Figure 6.12 For Example 6.5.

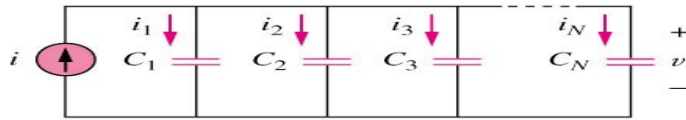
and the energies stored in them are

$$w_1 = \frac{1}{2}C_1v_1^2 = \frac{1}{2}(2 \times 10^{-3})(4)^2 = 16 \text{ mJ}$$

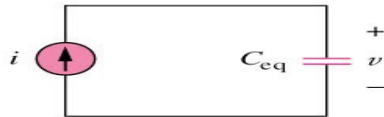
$$w_2 = \frac{1}{2}C_2v_2^2 = \frac{1}{2}(4 \times 10^{-3})(8)^2 = 128 \text{ mJ}$$

5.2 Series and Parallel Capacitors (1)

- The equivalent capacitance of N **parallel-connected** capacitors is the sum of the individual capacitances.



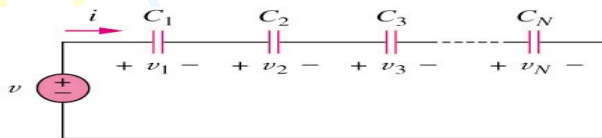
(a)



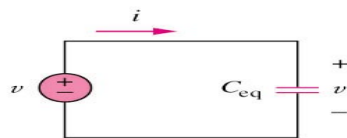
(b)

$$C_{eq} = C_1 + C_2 + \dots + C_N$$

The equivalent capacitance of N **series-connected** capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.



(a)



(b)

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$$

Find the equivalent capacitance seen between terminals a and b of the circuit in Fig. 6.16.

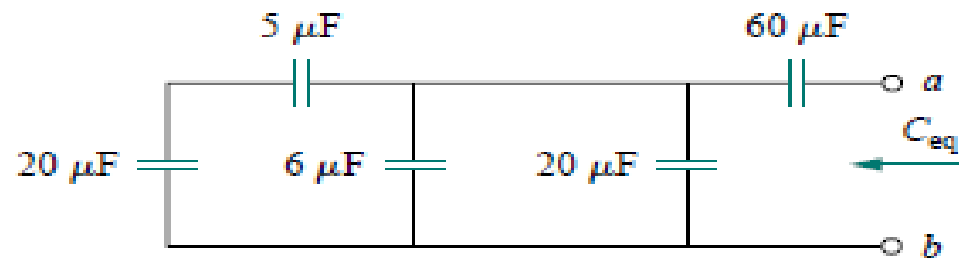


Figure 6.16 For Example 6.6.

Solution:

The $20\text{-}\mu\text{F}$ and $5\text{-}\mu\text{F}$ capacitors are in series; their equivalent capacitance is

$$\frac{20 \times 5}{20 + 5} = 4\ \mu\text{F}$$

This $4\text{-}\mu\text{F}$ capacitor is in parallel with the $6\text{-}\mu\text{F}$ and $20\text{-}\mu\text{F}$ capacitors; their combined capacitance is

$$4 + 6 + 20 = 30\ \mu\text{F}$$

This $30\text{-}\mu\text{F}$ capacitor is in series with the $60\text{-}\mu\text{F}$ capacitor. Hence, the equivalent capacitance for the entire circuit is

$$C_{eq} = \frac{30 \times 60}{30 + 60} = 20\ \mu\text{F}$$

find the voltage across each capacitor for the below circuit

Solution:

We first find the equivalent capacitance C_{eq} , shown in Fig. 6.19. The two parallel capacitors in Fig. 6.18 can be combined to get $40 + 20 = 60$ mF. This 60-mF capacitor is in series with the 20-mF and 30-mF capacitors. Thus,

$$C_{eq} = \frac{1}{\frac{1}{60} + \frac{1}{30} + \frac{1}{20}} \text{ mF} = 10 \text{ mF}$$

The total charge is

$$q = C_{eq}v = 10 \times 10^{-3} \times 30 = 0.3 \text{ C}$$

This is the charge on the 20-mF and 30-mF capacitors, because they are in series with the 30-V source. (A crude way to see this is to imagine that charge acts like current, since $i = dq/dt$.) Therefore,

$$v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15 \text{ V} \quad v_2 = \frac{q}{C_2} = \frac{0.3}{30 \times 10^{-3}} = 10 \text{ V}$$

Having determined v_1 and v_2 , we now use KVL to determine v_3 by

$$v_3 = 30 - v_1 - v_2 = 5 \text{ V}$$

Alternatively, since the 40-mF and 20-mF capacitors are in parallel, they have the same voltage v_3 and their combined capacitance is $40 + 20 = 60$ mF. This combined capacitance is in series with the 20-mF and 30-mF capacitors and consequently has the same charge on it. Hence,

$$v_3 = \frac{q}{60 \text{ mF}} = \frac{0.3}{60 \times 10^{-3}} = 5 \text{ V}$$

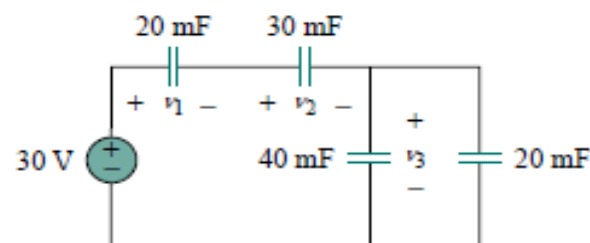


Figure 6.18 For Example 6.7.

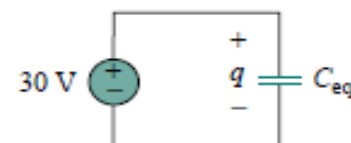


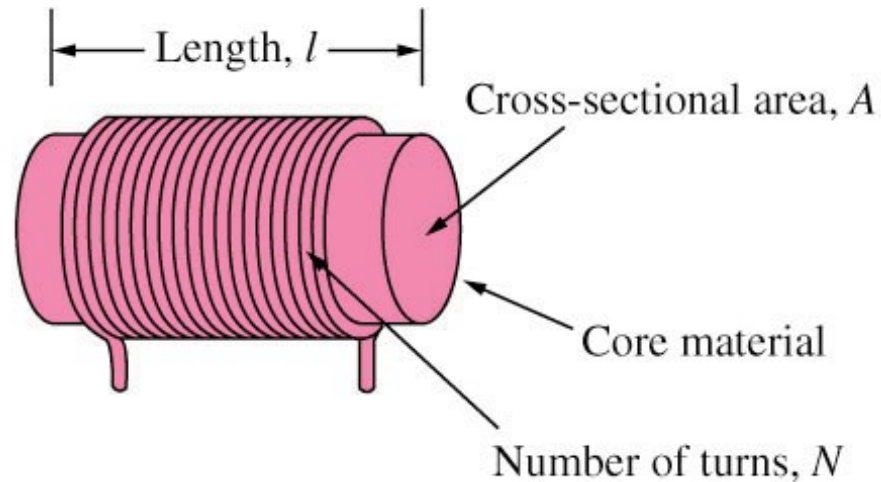
Figure 6.19 Equivalent circuit for Fig. 6.18.

Inductors

- An inductor is a passive element designed to store energy in its magnetic field.
- Inductors find numerous applications in electronic and power systems.
- They are used in power supplies, transformers, radios, TVs, radars, and electric motors.
- Any conductor of electric current has inductive properties and may be regarded as an inductor. But in order to enhance the inductive effect,
- a practical inductor is usually formed into a cylindrical coil with many turns of conducting wire

5.3 Inductors (1)

- An inductor is a passive element designed to store energy in its magnetic field.



- An inductor consists of a coil of conducting wire.

5.4 Inductors

- An inductor is made of a coil of conducting wire

$$L = \frac{N^2 \mu A}{l}$$

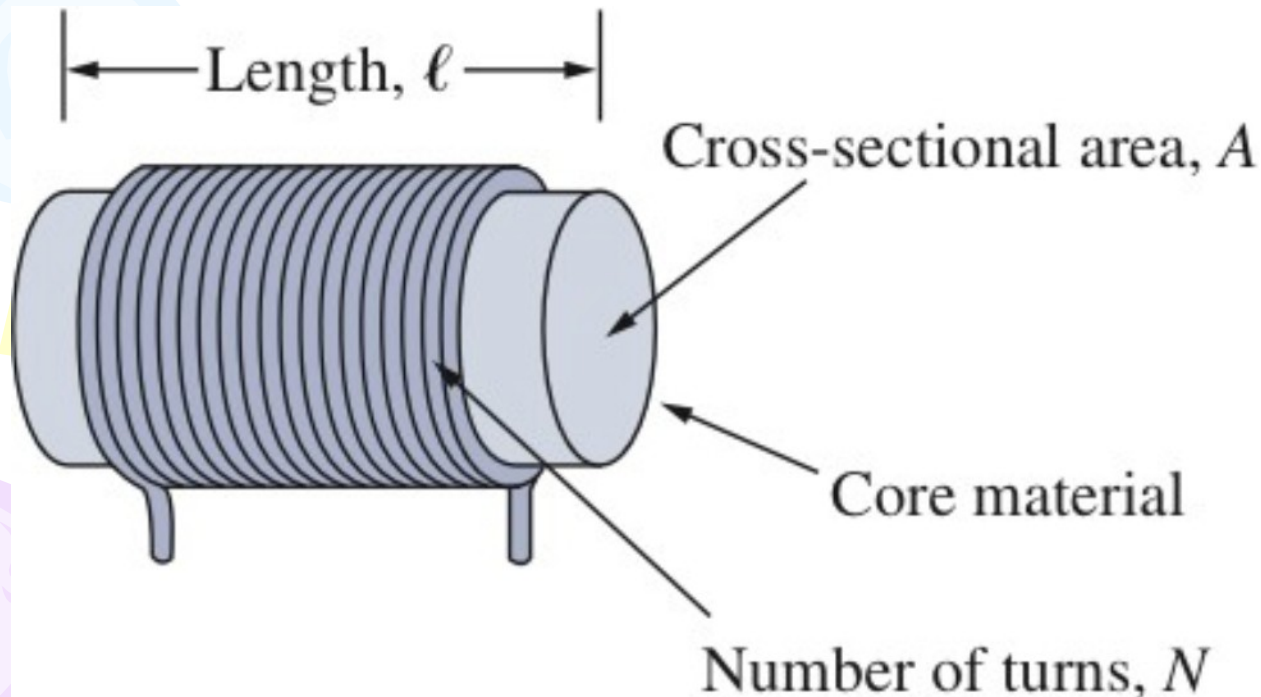
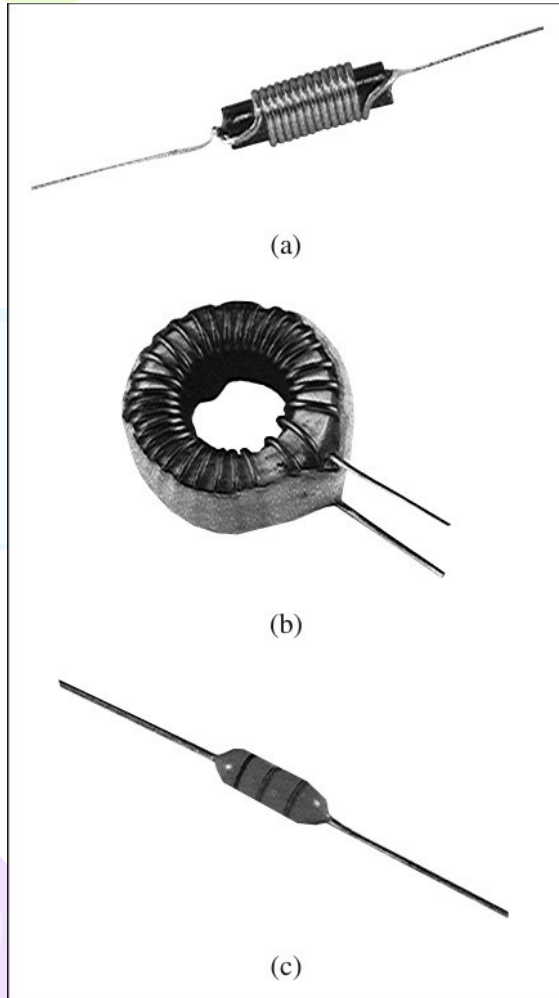


Fig 5.22



$$L = \frac{N^2 \mu A}{l}$$

$$\mu = \mu_r \mu_0$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ (H/m)}$$

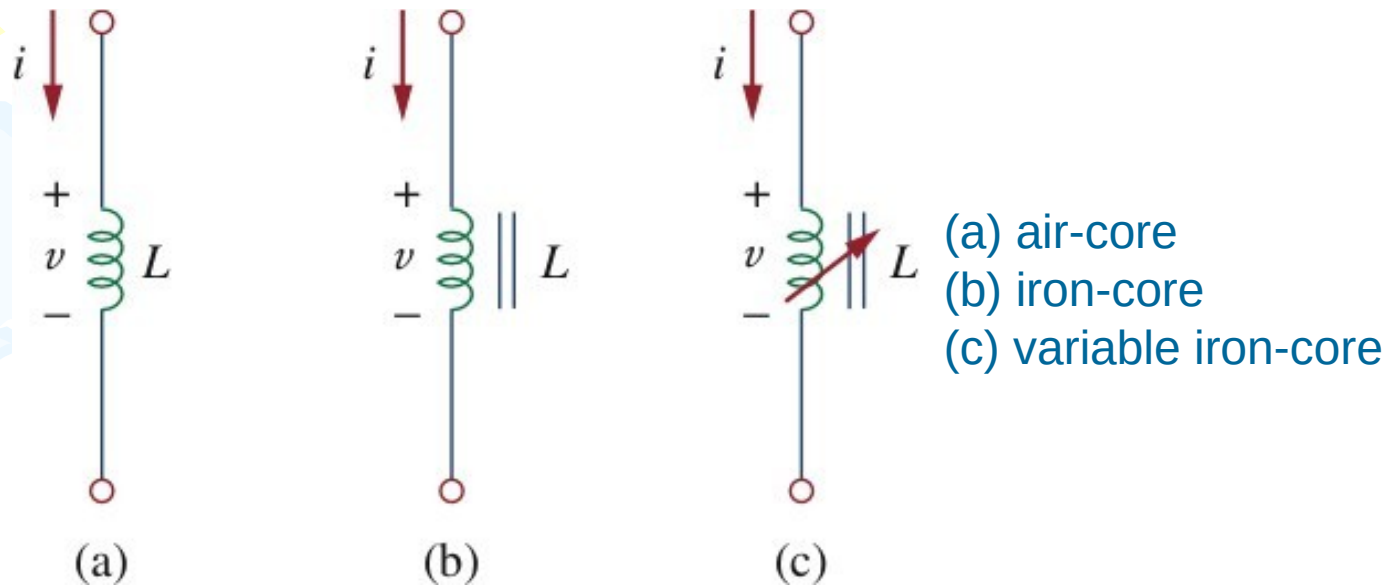
N : number of turns.

l : length.

A : cross - sectional area.

μ : permeability of the core

Fig 5.23

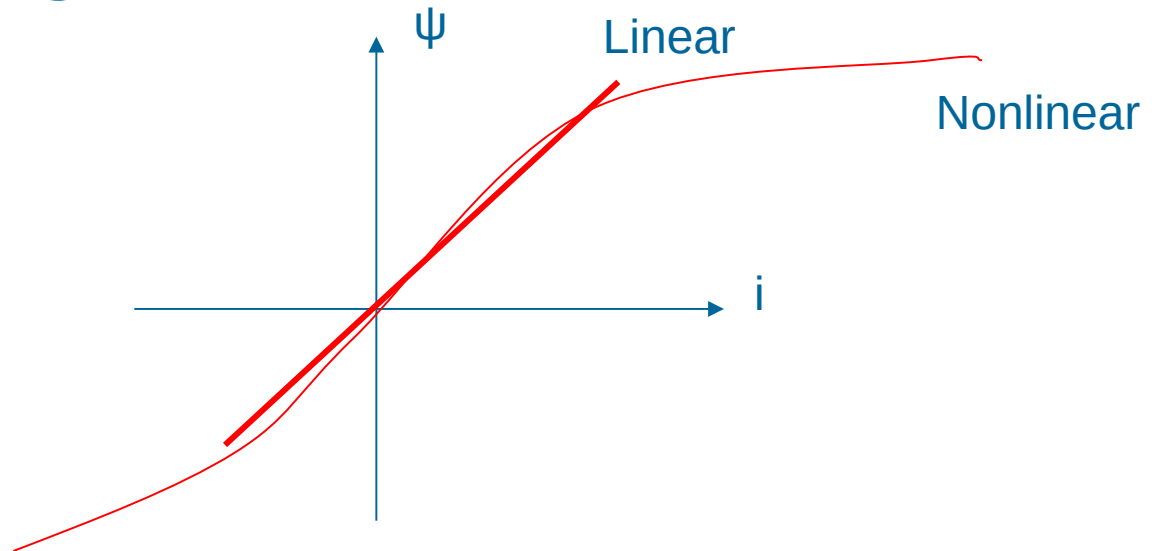


Flux in Inductors

- The relation between the flux in inductor and the current through the inductor is given below.

$$\phi = Li$$

1H = 1 Weber/A



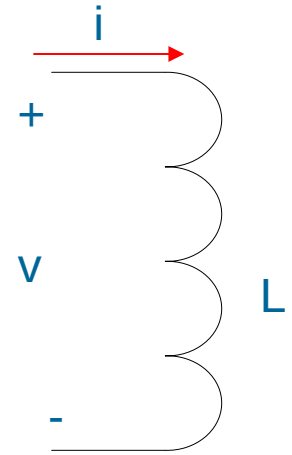


Energy Storage Form

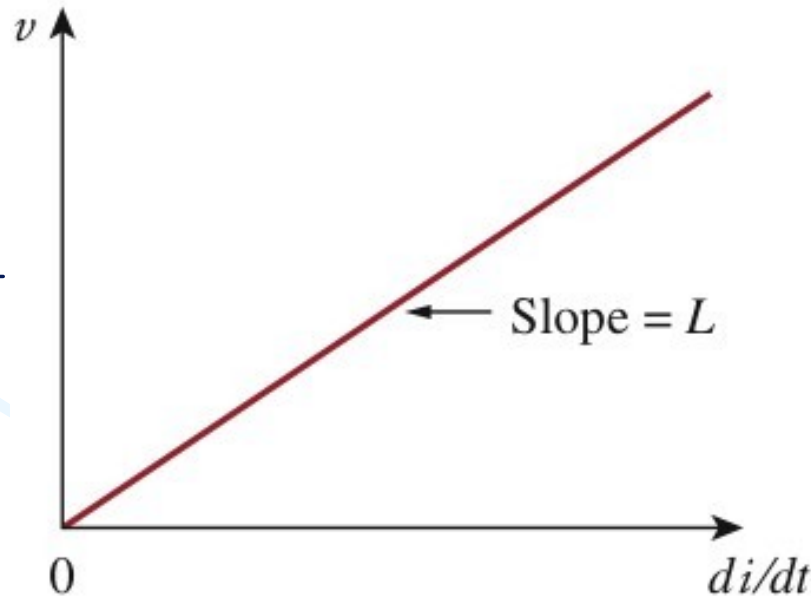
- An inductor is a passive element designed to store energy in the magnetic field while a capacitor stores energy in the electric field.

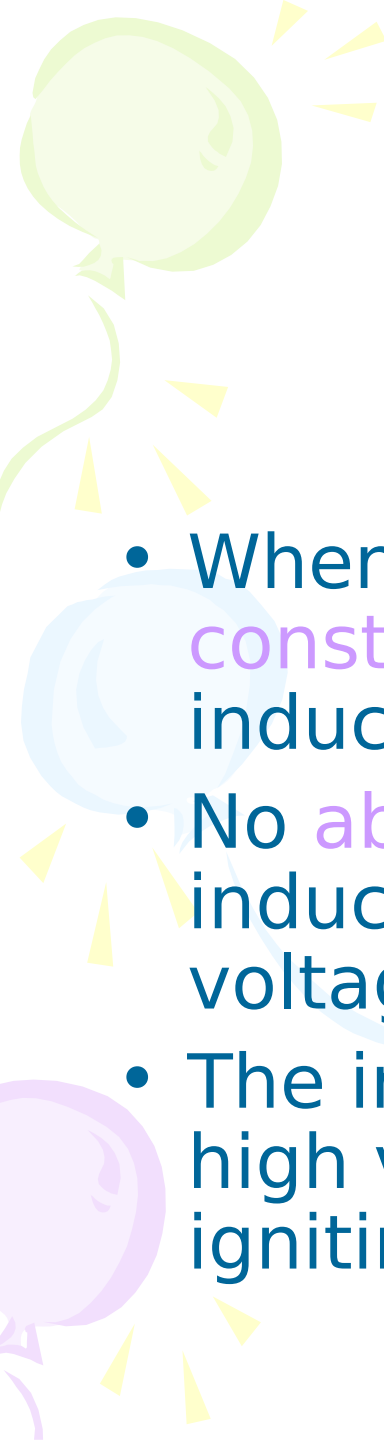
I-V Relation of Inductors

- An inductor consists of a coil of conducting wire.



$$v = \frac{d\phi}{dt} = L \frac{di}{dt}$$





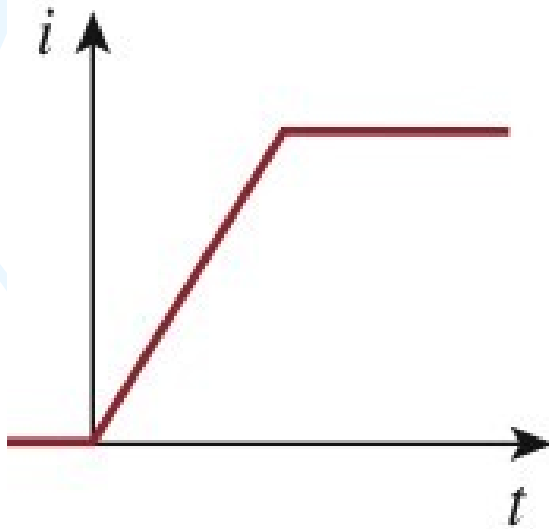
Physical Meaning

$$v = \frac{d\phi}{dt} = L \frac{di}{dt}$$

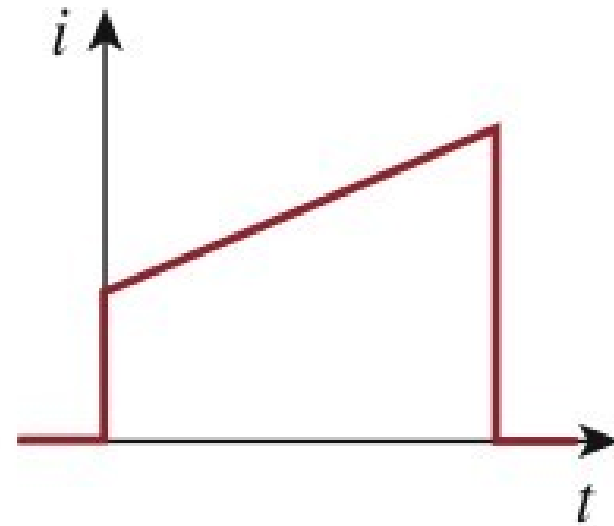
- When the current through an inductor is a constant, then the voltage across the inductor is zero, same as a short circuit.
- No abrupt change of the current through an inductor is possible except an infinite voltage across the inductor is applied.
- The inductor can be used to generate a high voltage, for example, used as an igniting element.

Fig 5.25

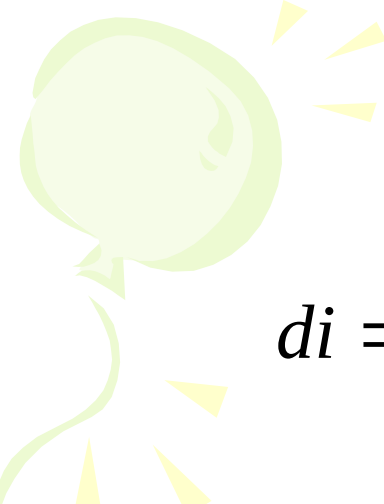
- An inductor are like a short circuit to dc.
- The current through an inductor cannot change instantaneously.

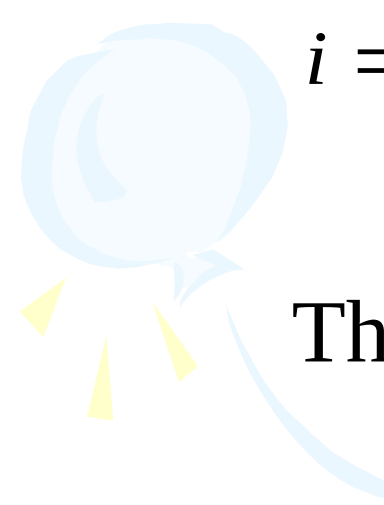


(a)

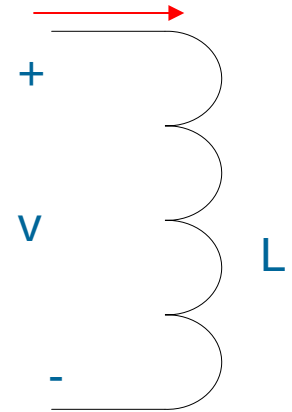


(b)

A green balloon with yellow streamers and confetti is located in the top-left corner of the slide.
$$di = \frac{1}{L} v dt \quad i = \frac{1}{L} \int_{-\infty}^t v(t) dt$$

A light blue balloon with yellow streamers and confetti is located in the middle-left corner of the slide.
$$i = \frac{1}{L} \int_{t_0}^t v(t) dt + i(t_0)$$

The inductor has memory.



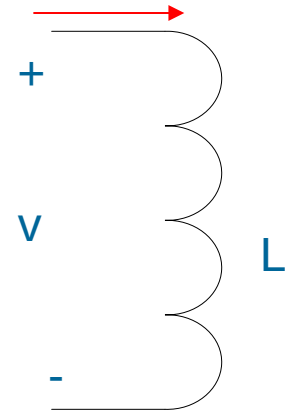
Energy Stored in an Inductor

$$P = vi = \left(L \frac{di}{dt} \right) i$$

$$w = \int_{-\infty}^t p dt = \int_{-\infty}^t \left(L \frac{di}{dt} \right) i dt$$

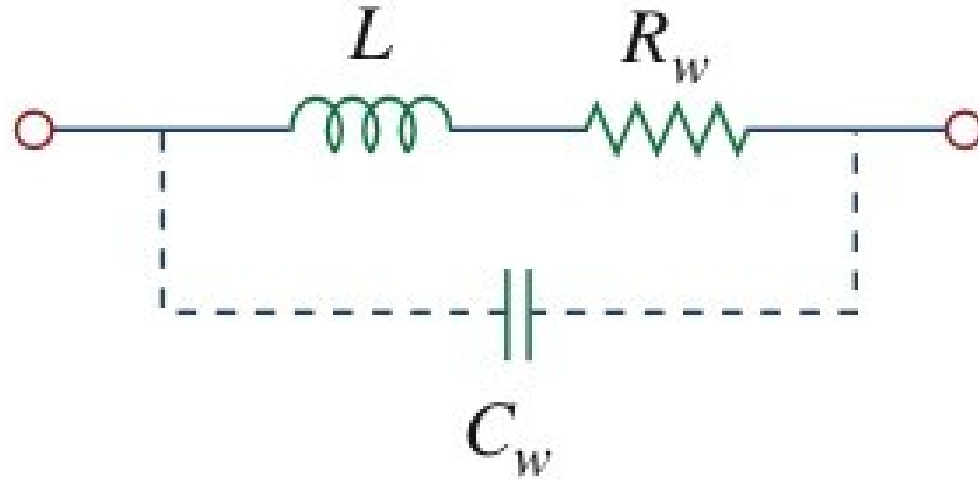
$$= L \int_{(-\infty)}^{i(t)} i di = \frac{1}{2} Li^2(t) - \frac{1}{2} Li^2(-\infty) \quad i(-\infty) = 0,$$

- The energy stored in an inductor



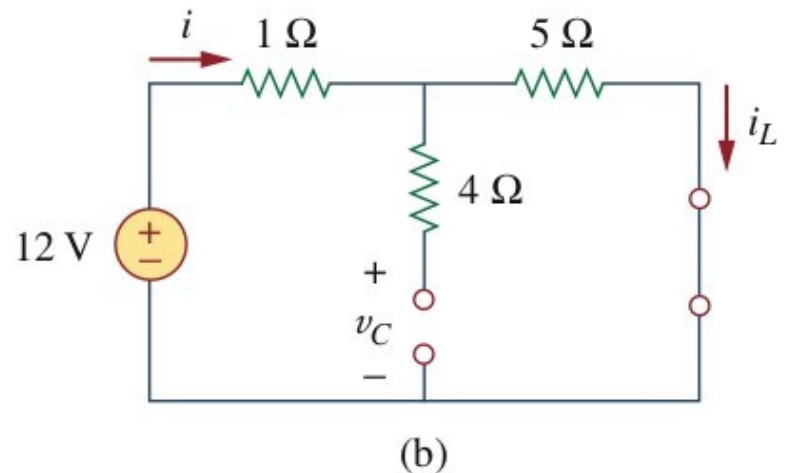
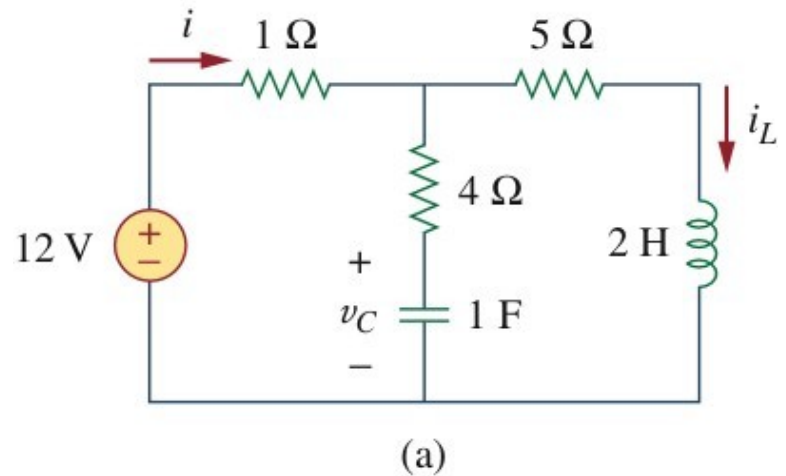
$$w(t) = \frac{1}{2} Li^2(t)$$

Model of a Practical Inductor



Example 5.10

- Consider the circuit in Fig 6.27(a). Under dc conditions, find:
(a) i , v_C , and i_L .
(b) the energy stored in the capacitor and inductor.



Example 5.10

Solution:

capacitor $\rightarrow$ open circuit

(a) Under dc condition : inductor $\rightarrow$ short circuit

$$i = i_L = \frac{12}{1+5} = 2A, v_c = 5i = 10V$$

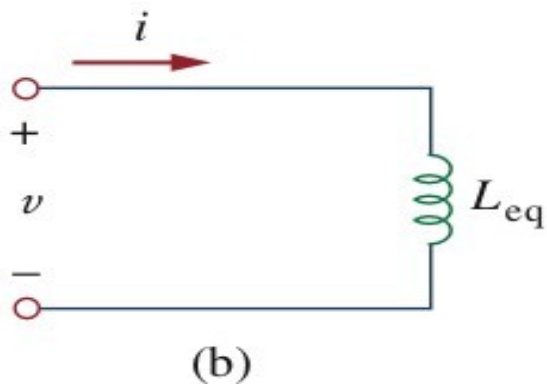
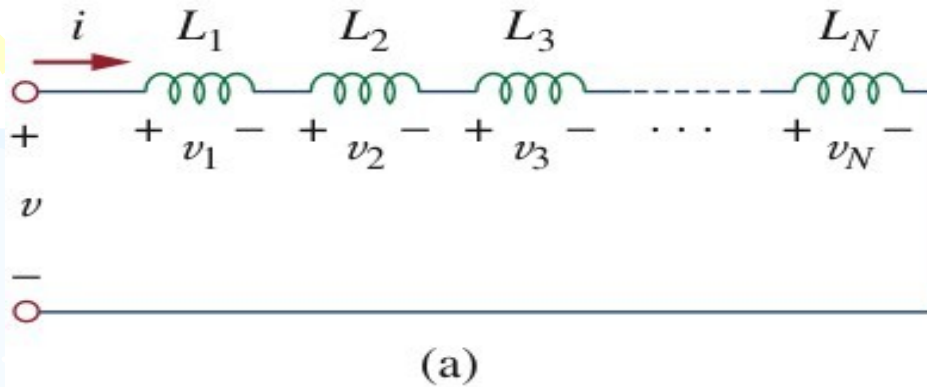
(b)

$$w_c = \frac{1}{2} C v_c^2 = \frac{1}{2} (1)(10^2) = 50J,$$

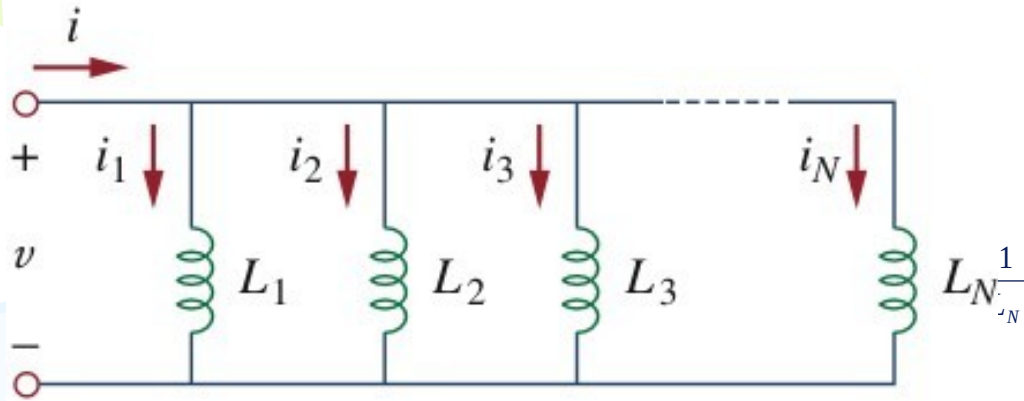
$$w_L = \frac{1}{2} L i^2 = \frac{1}{2} (2)(2^2) = 4J$$

Inductors in Series

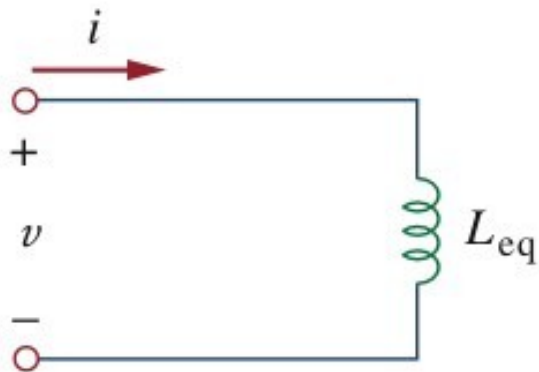
$$L_{eq} = L_1 + L_2 + L_3 + \dots + L_N$$



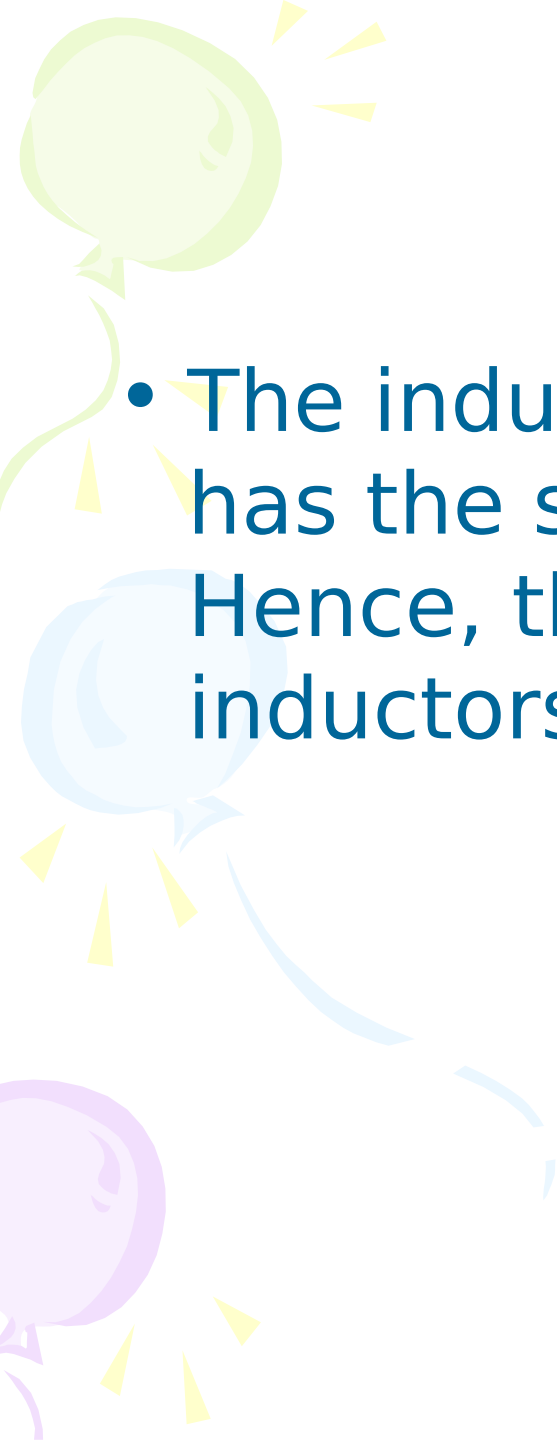
Inductors in Parallel



(a)

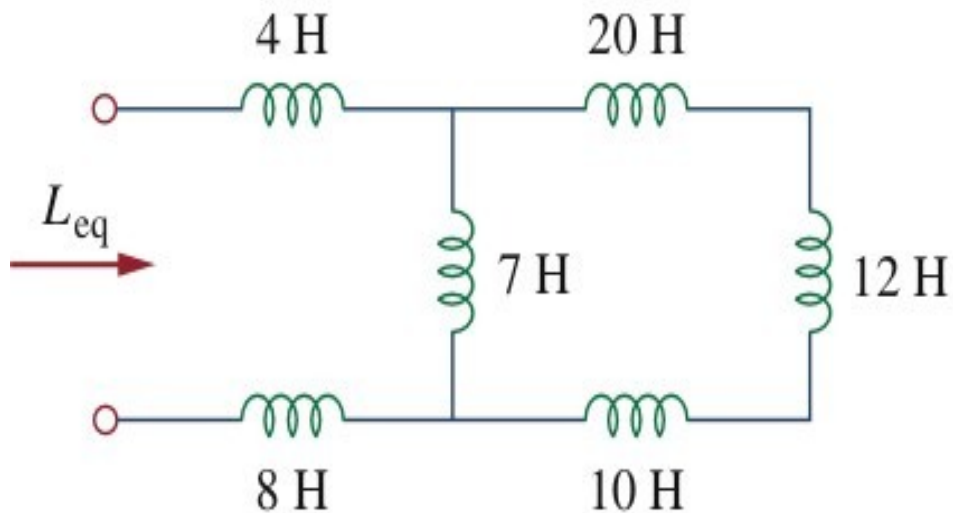


(b)

- 
- The inductor in various connection has the same effect as the resistor. Hence, the Y- Δ transformation of inductors can be similarly derived.

Example 6.11

★ Find the equivalent inductance of the circuit shown in Fig. 6.31.



Example 6.11

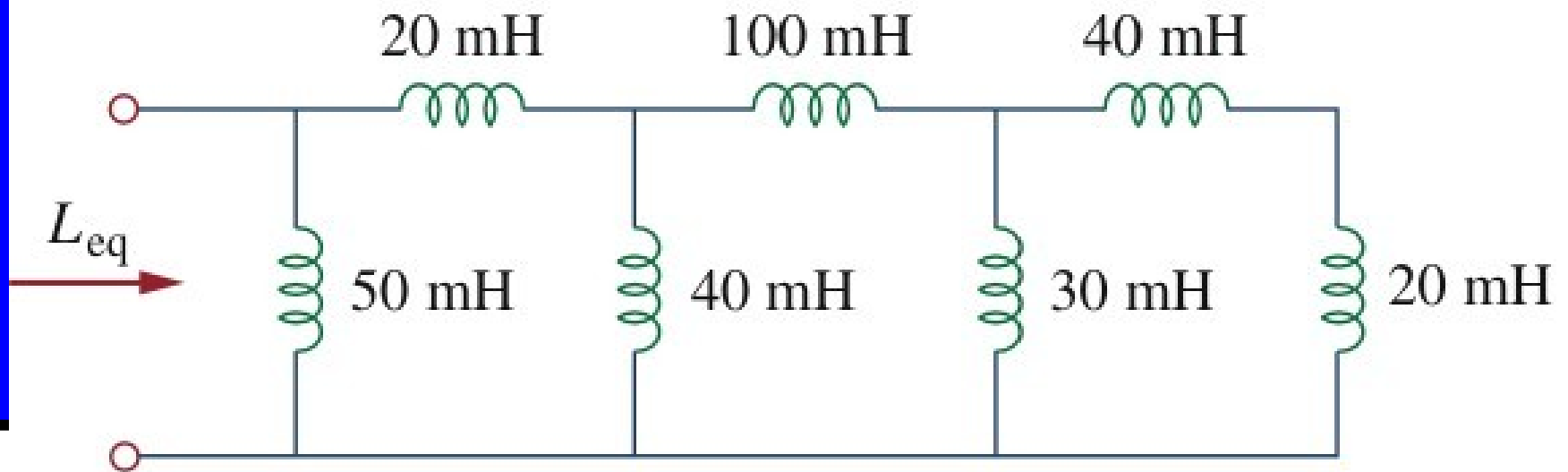
★ Solution:

Series : 20H, 12H, 10H
→ 42H

Parallel : $\frac{7 \times 42}{7 + 42} = 6H$

$\therefore L_{eq} = 4 + 6 + 8 = 18H$

Practice Problem 6.11

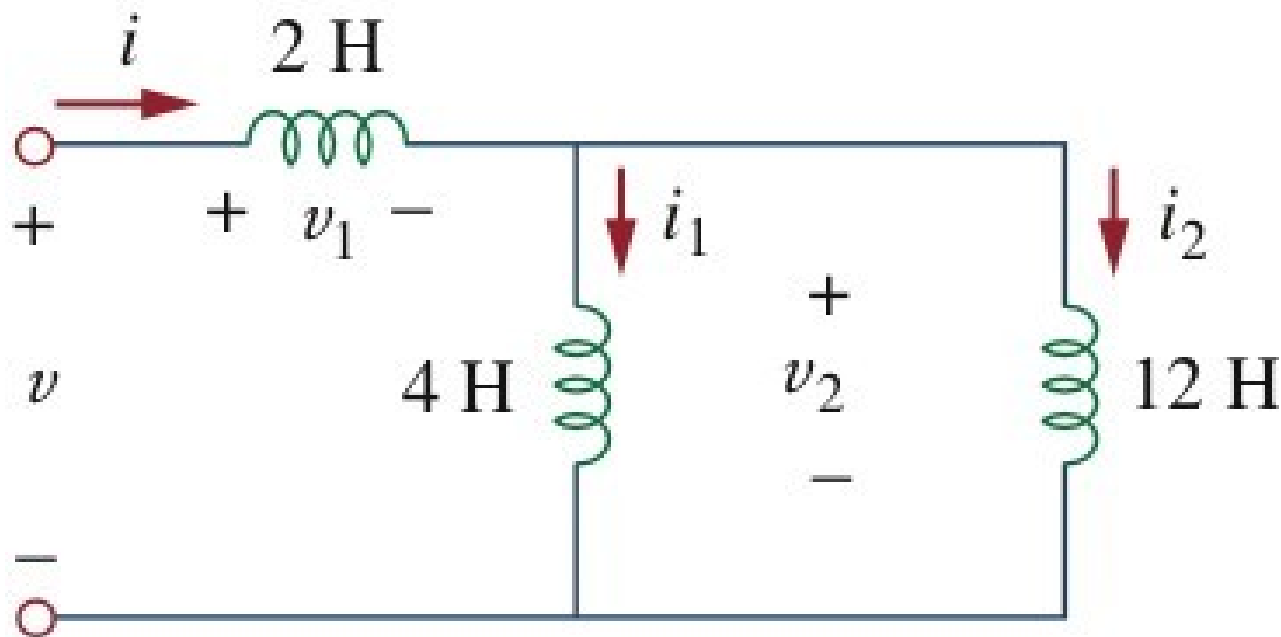


Example 5.12

★ Find the circuit in Fig. 5.33, $i(t) = 4(2 - e^{-10t})\text{mA}$.

If $i_2(0) = -1\text{mA}$, find : (a) $i_1(0)$

(b) $v(t)$, $v_1(t)$, and $v_2(t)$; (c) $i_1(t)$ and $i_2(t)$



Example 6.12

Solution:

$$(a) i(t) = 4(2 - e^{-10t}) \text{mA} \rightarrow i(0) = 4(2 - 1) = 4 \text{mA}.$$

$$\therefore i_1(0) = i(0) - i_2(0) = 4 - (-1) = 5 \text{mA}$$

(b) The equivalent inductance is

$$L_{eq} = 2 + 4 \parallel 12 = 2 + 3 = 5 \text{H}$$

$$\therefore v(t) = L_{eq} \frac{di}{dt} = 5(4)(-1)(-10)e^{-10t} \text{mV} = 200e^{-10t} \text{mV}$$

$$v_1(t) = 2 \frac{di}{dt} = 2(-4)(-10)e^{-10t} \text{mV} = 80e^{-10t} \text{mV}$$

$$\therefore v_2(t) = v(t) - v_1(t) = 120e^{-10t} \text{mV}$$

Example 6.12

$$(c) \ i = \frac{1}{L} \int v(t) dt + i(0) \Rightarrow$$

$$i_1(t) = \frac{1}{4} \int v_2 dt + i_1(0) = \frac{120}{4} \int e^{-10t} dt + 5 \text{ mA}$$

$$= -3e^{-10t} \Big|_0^t + 5 \text{ mA} = -3e^{-10t} + 3 + 5 = 8 - 3e^{-10t} \text{ mA}$$

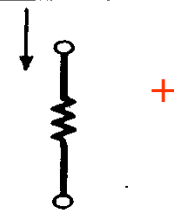
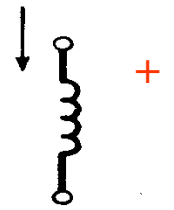
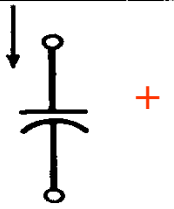
$$i_2(t) = \frac{1}{12} \int v_2 dt + i_2(0) = \frac{120}{12} \int e^{-10t} dt - 1 \text{ mA}$$

$$= -e^{-10t} \Big|_0^t - 1 \text{ mA} = -e^{-10t} + 1 - 1 = -e^{-10t} \text{ mA}$$

Note that $i_1(t) + i_2(t) = i(t)$

Current and voltage relationship for R, L, C

Current & Voltage Relationship

Circuit element	Units	Voltage	Current	Power
 Resistance	ohms (Ω)	$v = Ri$ (Ohm's law)	$i = \frac{v}{R}$	$p = vi = i^2 R$
 Inductance	henries (H)	$v = L \frac{di}{dt}$	$i = \frac{1}{L} \int v dt + k_1$	$p = vi = Li \frac{di}{dt}$
 Capacitance	farads (F)	$v = \frac{1}{C} \int i dt + k_2$	$i = C \frac{dv}{dt}$	$p = vi = Cv \frac{dv}{dt}$