



CHAPTER FOUR ANALYSIS OF SIMPLE STRUCTURES

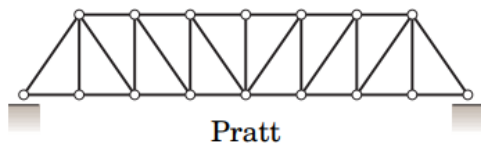
4.1. Introduction

- An engineering structure is any connected system of members built to support or transfer forces and to safely withstand the loads applied to it.
- To determine the forces internal to an engineering structure, we must dismember the structure and analyze separate free-body diagrams of individual members or combinations of members. This analysis requires careful application of Newton's third law, which states that each action is accompanied by an equal and opposite reaction.
- In this chapter we shall analyze the internal forces acting in several types of structures, namely, trusses, frames and simple machines.
- The structures considered here are only statically determinate structures, which do not have more supporting constraints than necessary to maintain an equilibrium configuration.

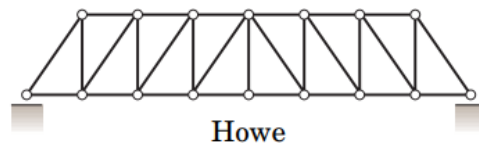
4.2. Trusses

4.2.1. Plane Trusses

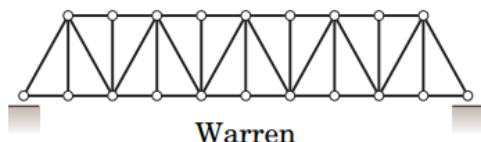
A **truss** is a framework composed of members joined at their ends to form a rigid structure. When the members of the truss lie essentially in a single plane, the truss is known as a **plane truss**.



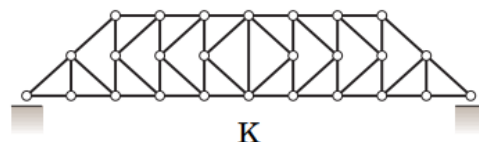
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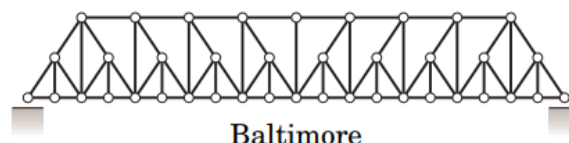
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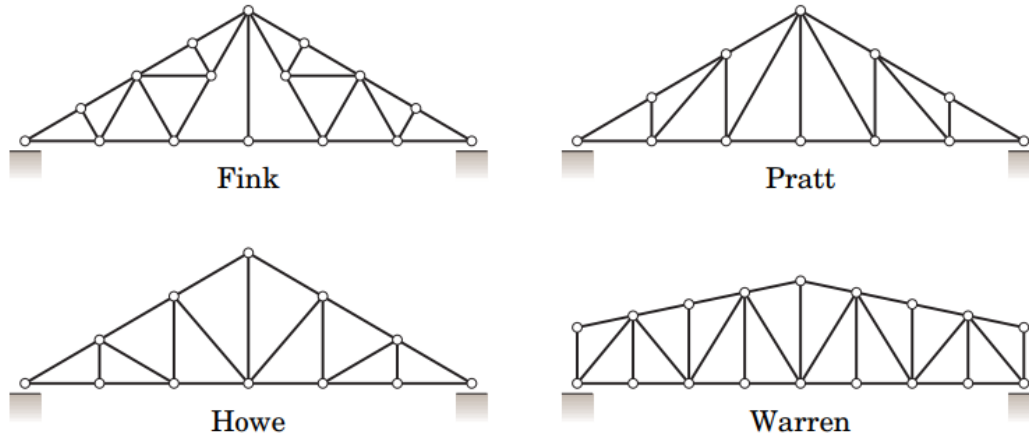
Commonly Used Bridge Trusses



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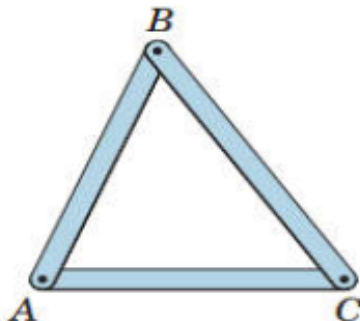
Bridges, roof supports, derricks and other such structures are common examples of trusses. Examples of commonly used trusses that can be analyzed as plane as plane trusses are.



Commonly Used Roof Trusses

The basic element of a plane truss is the triangle. Three bars joined by pins at their ends constituent a **rigid** frame. The term rigid is used to mean non-collapsible and also to mean that deformation of the members due to induced internal strains is negligible.

Thus, structures that are built from a basic triangle in a manner described are known as **simple trusses**. When more members are present than are needed to prevent collapse, the truss become statically indeterminate. A statically indeterminate truss cannot be analyzed by the equations of equilibrium alone. Additional members or supports which are not necessary for maintaining equilibrium configuration are called **redundant**.

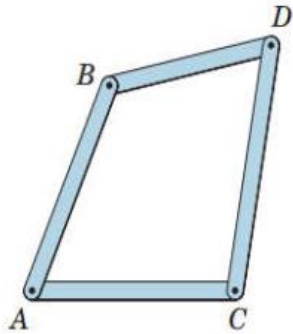


- Three bars joined by pins at their ends constitute a rigid frame.

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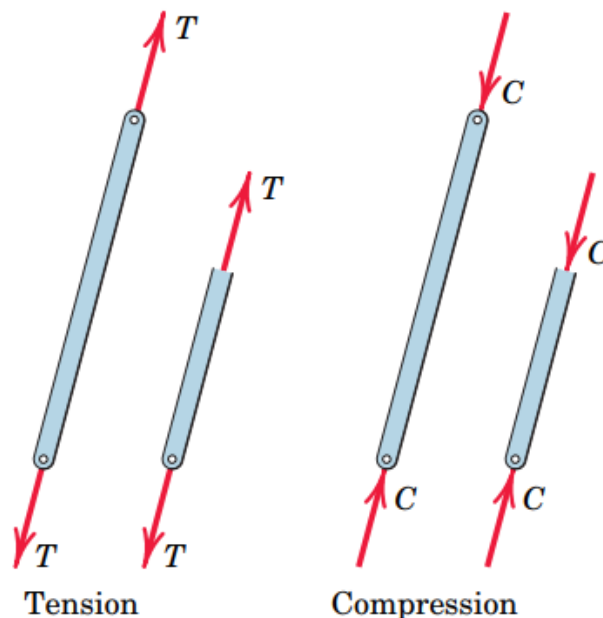


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- Four or more bars pin-jointed to form a polygon of many sides constitute a non-rigid frame.

Several assumptions are made in the force analysis of simple trusses. First, we assume all members to be two-force members. A two-force member is one in equilibrium under the action of two forces only – either tension or compression. Second, all externally forces are assumed to be applied at the pin connections not over the member. The weight of the truss members is assumed to be small as compared with the force it supports. But if the weight must be considered, the weight W of the member will be replaced with two forces, each $W/2$, if the member is uniform, with one force acting at each end of the member. These forces, in effect, are treated as loads externally applied to the pin connections. When welded or riveted connections are used to join structural members, the assumptions of a pin-jointed connection is usually satisfactory if the centerlines of the members are concurrent at the joint.



Two-Force Members



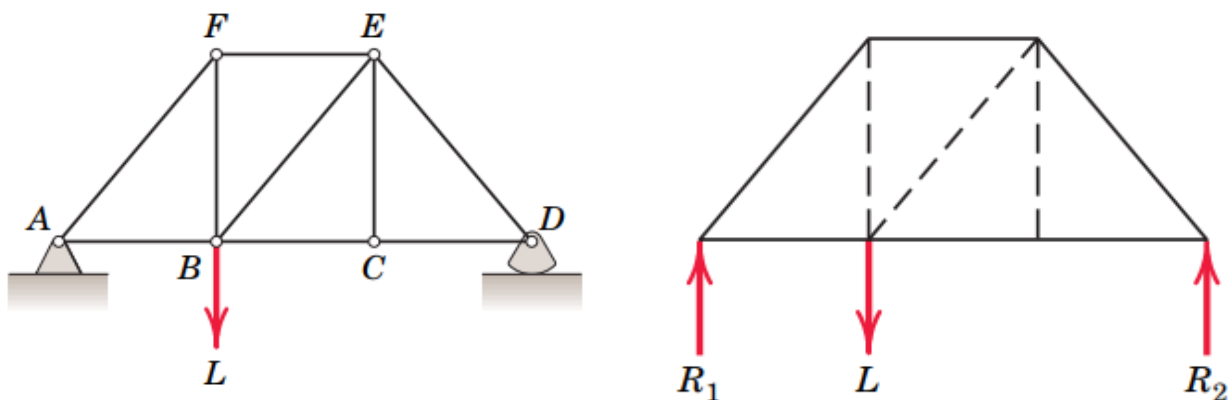
The external reactions are usually determined by computation from the equilibrium equations applied to the truss as a whole. Then the force analysis of the remainder of the truss is performed. Two methods for the force analysis of simple trusses are used – **Method of Joints** and **Method of Section**.

4.2.1.1 Method of Joints

This method for finding the forces in the members of a simple truss consists of satisfying the conditions of equilibrium for the forces acting on the connecting pin of each joint.

The method therefore deals with the equilibrium of concurrent forces and only two independent equilibrium equations are involved. ($\sum F_x = 0$ and $\sum F_y = 0$ for each joint). We begin the analysis with any joint where at least one known load exists and where not more than two unknown forces are present. Taking free body diagram of a joint, tension will always be indicated by an arrow away from the pin, and compression will always be indicated by an arrow toward the pin.

Consider a simple truss having 9 members and the free body diagram of the truss will be as shown below. The external reactions are usually determined first, by applying the equilibrium equations to the truss as a whole. Then the force analysis of the remainder of the truss is performed.

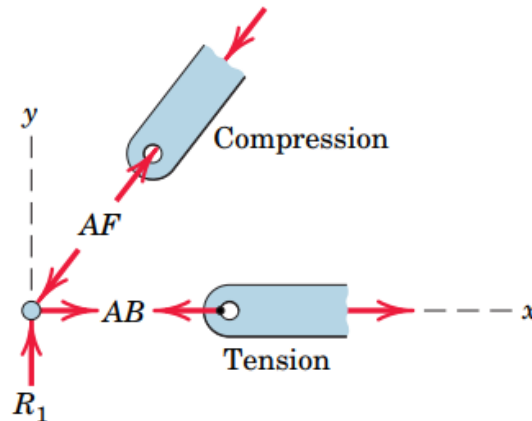




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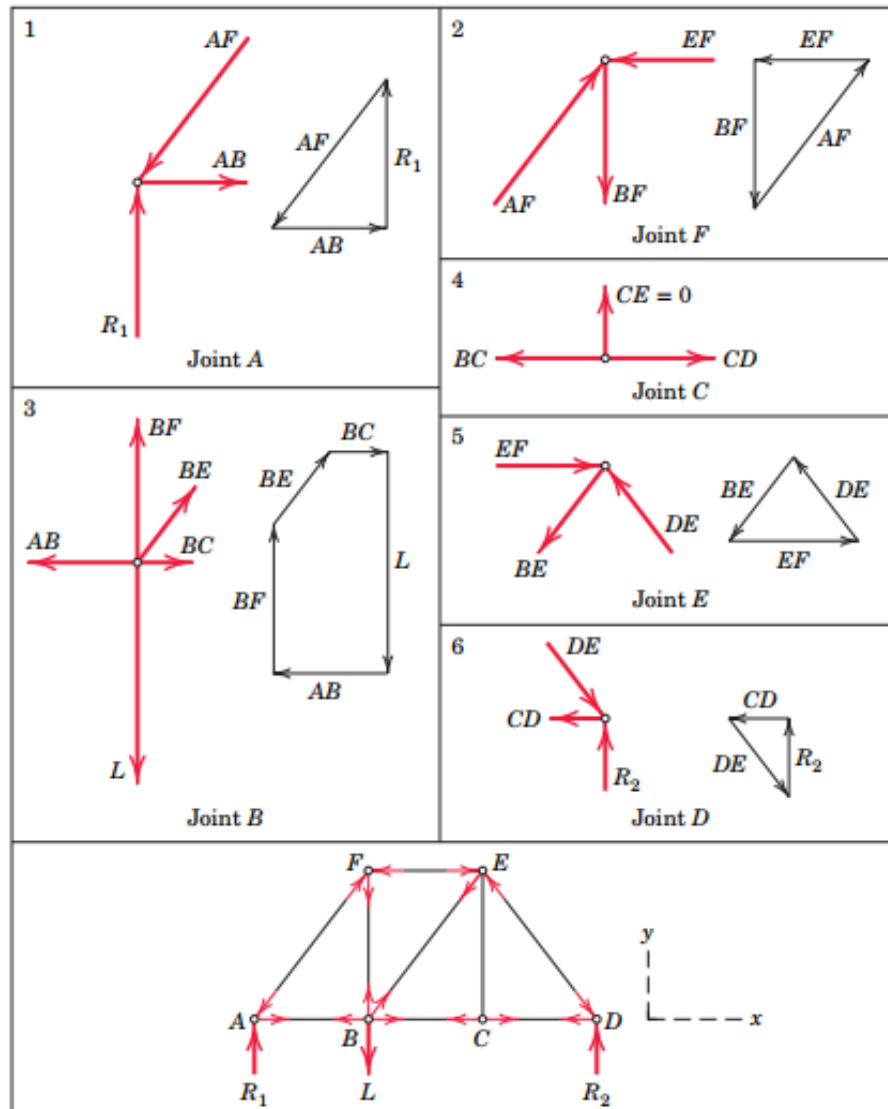
Considering joint A, there are two members AB and AF. The free –body diagrams of portions of members AF and AB are shown below.



The member AB actually makes contact on the left side of the pin, although the force AB is drawn from the right side and is shown acting away from the pin. Thus, if we consistently draw the force arrows on the same side of the pin as the member, then tension (such as AB) will be indicated by an arrow away from the pin, and compression (such as AF) will always be indicated by an arrow towards the pin. The magnitude of AF is obtained from the equation $\sum F_y = 0$ and AB is then found from $\sum F_x = 0$. After determining the force on member AF and AB, it is possible to proceed to the next joint (having only two unknowns) to determine the rest member forces. For this case the sequence will be analyzing joint F, B, C, E and D in the order.



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In some instances, it is not possible to initially assign the correct direction of one or both of the unknown forces acting on a given pin. In this event we may make an arbitrary assignment so that a negative value from the computation indicates that the assumed direction is incorrect.

If a plane truss has more external supports than are necessary to ensure a stable equilibrium configuration, the truss as a whole is statically indeterminate, and the extra supports constitute **external redundancy**. If a truss has more internal members than are necessary



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to prevent collapse when the truss is removed from its supports, then the extra members constitute **internal redundancy** and the truss is again statically indeterminate.

For a truss which is statically determinate externally, there is a definite relation between the number of its members and the number of its joints necessary for internal stability without redundancy. We can specify the equilibrium of each joint by two scalar force equations, there are $2j$ such equations for all trusses with j joints. For the entire truss composed of m two-force members and having the maximum number of three unknown support reactions, there are a total of $m+3$ unknowns (m tension or compression forces and three reactions). Thus, for any plane truss, the equation $m + 3 = 2j$ will be satisfied if the truss is statically determinate internally.

If $m + 3 > 2j$

- There are more members than independent equations
- The truss is statically indeterminate internally
- There are redundant members

If $m + 3 < 2j$

- There is deficiency of internal members
- The truss is unstable and collapse under load

4.2.1.2. Method of Sections

On the analysis of plane trusses by the method of joints, we took advantage of only two of the three equilibrium equations, since the procedure involves concurrent forces at each joint. We may take advantage of the third equation of equilibrium ($\sum M = 0$) by selecting an entire section of the truss for the free body in equilibrium under the action of a non-concurrent system of forces.

This method of sections has the basic advantage that the force in almost any desired member may be found directly from analysis if a section, which has cut that member. Thus,



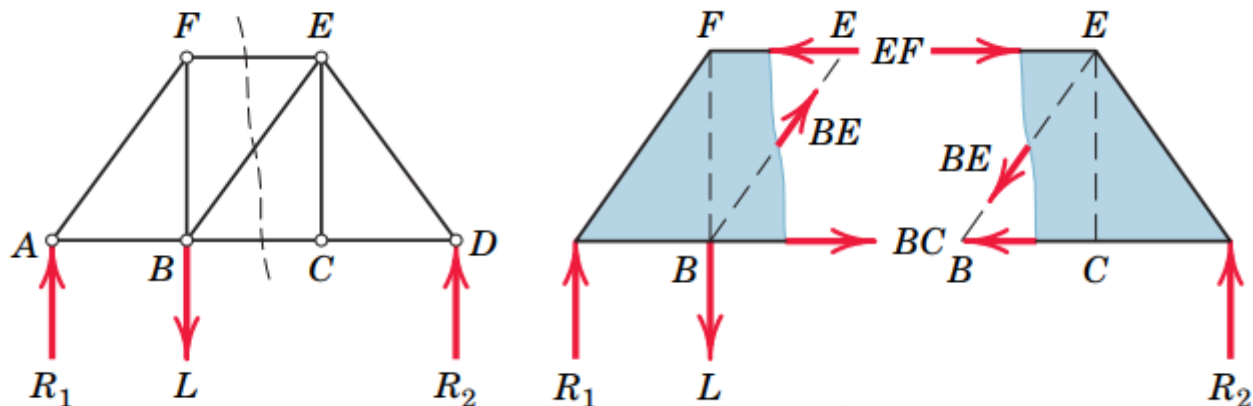
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it is not necessary to proceed with the calculation from joint to joint until the member in question has been reached.

In general, while choosing a section of a truss, not more than three members whose forces are unknown may be cut, since there are only three available independent equilibrium relations. It is essential to understand that in the method of sections an entire portion of the truss is considered as a single body in equilibrium. Thus, the members internal to the section are not involved in the analysis of the section as a whole.

Let us have a look at the analysis of a simple truss using the method of sections. For instance in order to determine the force in member BE, an imaginary section, indicated by the dashed line, is passed through the truss, cutting it in to two parts as shown below.



This section has cut three members whose forces are initially unknown. In order for the portions of the truss on each side of the section to remain in equilibrium, it is necessary to apply to each cut member the force which was exerted on it by the member cut away.

The left-hand section is in equilibrium under the action of the applied load L , the end reaction R_1 and the three forces exerted on the cut members by the right-hand section which has been removed. Thus the equation of moments about joint B eliminates three forces from the relation, and EF can be determined directly. The force BE is calculated



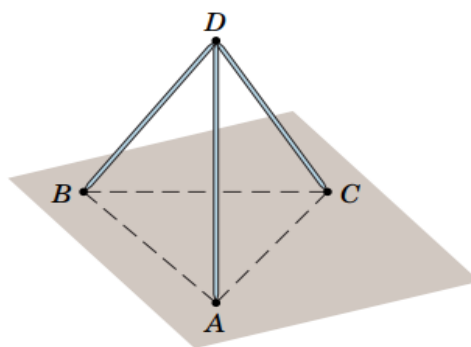
from the equilibrium equation for the y-direction. Finally, BC is determined by balancing moments about point E.

The right-hand section of the truss is also in equilibrium under the action of R_2 and the same three forces in the cut members applied in the directions opposite to those for the left-hand section. The proper sense for the horizontal forces can easily be seen from the balances of moments about point B and E.

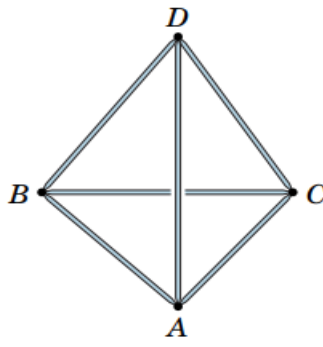
To classify the free body and the forces acting externally on it, the section is preferably passed through the members and not the joints. In some cases the methods of sections and joints can be combined for an efficient solution. It is not always possible to assign the proper sense of an unknown force when the free-body diagram of a section is initially drawn. With an arbitrary assignment made, a positive answer will verify the assumed sense and a negative result will indicate that the force is in the sense opposite to that assumed.

4.2.2. Space Trusses

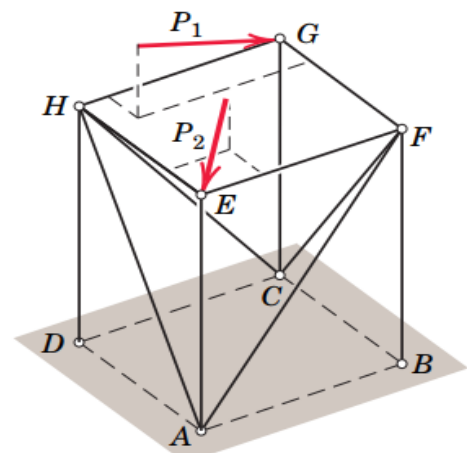
A space truss is the three-dimensional counterpart of the plane truss. The idealized space truss consists of rigid links connected at their ends by ball-and-socket joints. We saw that a triangle of pin-connected bars forms the basic non-collapsible unit for the plane truss. A space truss, on the other hand, requires six bars joined at their ends to form the edges of a tetrahedron for the basic non-collapsible unit.



(a)



(b)



(c)



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In fig. (a) the two bars AD and BD joined at D require a third support CD to keep the triangle ADB from rotating about AB. In fig. (b) the supporting base is replaced by three more bars AB, BC and AC to form a tetrahedron not dependent on the foundation for its own rigidity. We may form a new rigid unit to extend the structure with three additional concurrent bars whose ends are attached to three fixed joints on the existing structure. Thus in fig. (c) the bars AF, BF and CF are attached to the foundation and therefore fix point F in space. Likewise point H is fixed in space by the bars AH, DH and CH. The three additional bars CG, FG and HG are attached to the three fixed points C, F and H and therefore fix G in space. Point E is similarly established. We see now that the structure is entirely rigid. The two applied loads shown will result in all of the members.

Ideally there must be point support, such as that given by a ball-and-socket joint, for the connections of a space truss so that there will be no bending in the members. Again, as in riveted and welded connections for plane trusses, if the centerlines of joined members intersect at a point, we may justify the assumption of two force members under simple tension and compression.

The method of joints developed for plane trusses may be extended directly to space trusses by satisfying the complete vector equation; $\Sigma F = 0$ for each joint. We normally begin the analysis at a joint where at least one known force acts and not more than three unknown forces are present. Adjacent joints on which not more than three unknown forces act may be analyzed in turn. The above step-by-step joint technique tends to minimize the number of simultaneous equations which must be solved when the forces in all members of the space truss are to be determined. For this reason, such a judicious approach is recommended.

The method of sections developed in the previous article may also be applied to space trusses. The two vector equations $\Sigma F = 0$ and $\Sigma M = 0$ must be satisfied for any section of the truss, where the zero moment sum will hold for all moment axes. Since the two vector



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equations are equivalent to six scalar equations, we conclude that a section should in general not be passed through more than six members whose forces are unknown. The method of sections for space trusses is not widely used, however, because a moment axis can seldom be found which eliminates all but one unknown, as in the case of plane trusses.

4.3 Pin-ended Multi – force structures (Frames and simple machines)

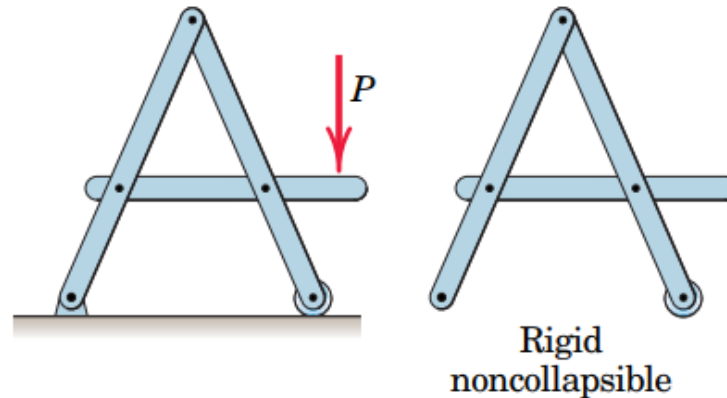
A structure is called a frame or machine if at least one of its individual members is a multi-force member. A multi-force member is defined as one with three or more forces acting on it or one with two or more forces and one or more couples acting on it. Frames are structures which are designed to support applied loads and are usually fixed in position. Machines are structures which contain moving parts and are designed to transmit forces or couples from input values to output values.

In this article attention is focused on the equilibrium of interconnected rigid bodies which contain multi force members. The forces acting on each member of a connected system are found by isolating the member with a free-body diagram and applying the established equations of equilibrium. The principle of action and reaction must be carefully observed when we represent the forces of interaction on the separate free-body diagrams.

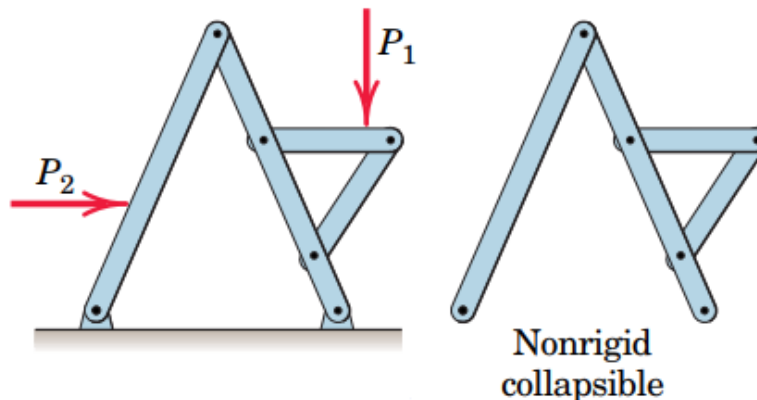
If the frame or machine constitutes a rigid unit by itself when removed from its supports, the analysis is best begun by establishing all the forces external to the structure considered as a single rigid body. We then dismember the structure and consider the equilibrium of each part separately. The equilibrium equations for the several parts will be related through the terms involving the forces of interaction.



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If the structure is not a rigid unit by itself but depends on its external supports for rigidity, as in the figure below, then the calculation of the external support reactions cannot be completed until the structure is dismembered and the individual parts are analyzed.



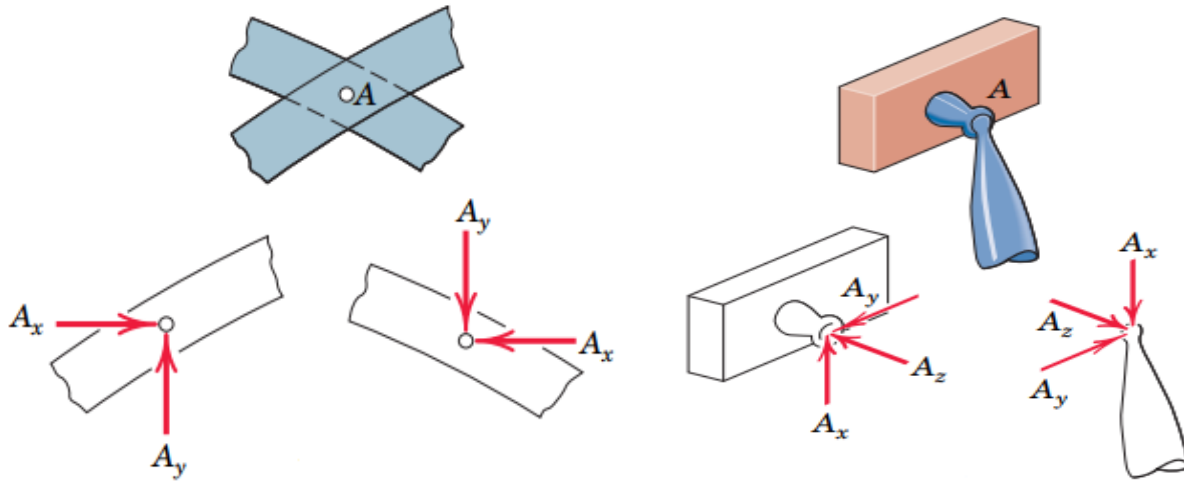
In most cases we find that the analysis of frames and machines is facilitated by representing the forces in terms of their rectangular components. It is not always possible to assign every force or its components in the proper sense when drawing the free body diagrams and it becomes necessary for us to make an arbitrary assignment. In any event it is absolutely necessary that a force be consistently represented on the diagrams for interacting bodies, which involve the force in question. For e.g., for two bodies connected by the pin, in the figure below, the force components must be consistently represented in opposite directions on the separate free-body diagrams.



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If we choose to use vector notation in labeling the forces, then we must be careful to use a plus sign for an action and a minus sign for the corresponding reaction.



Situations occasionally arise where it is necessary to solve two or more equations simultaneously in order to separate the unknowns. In most instances, however, we may avoid simultaneous solutions by careful choice of the member or group of members for the free-body diagram and by a careful choice of moment axes which will eliminate undesired terms from the equations.