

CHAPTER ONE

SHEAR STRENGTH OF SOILS

1.0 Introduction

The safety of any geotechnical structure is dependent on the strength of the soil. If the soil fails, a structure founded on it can **collapse**, endangering lives and causing economic damages. Soils fail either in **tension** or in shear. However, In the majority of soil mechanics problems (such as **bearing capacity**, **lateral pressure** against retaining walls, slope stability, etc.), only failure in shear requires consideration. The shear strength of soils is, therefore, of paramount importance to geotechnical engineers. The shear strength along any plane is mobilized by cohesion and frictional resistance to sliding between soil particles. The **cohesion c** and **angle of friction ϕ** of a soil are collectively known as **shear strength parameters**.

In this chapter we will define, describe, and determine the shear strength of soils. When you complete this chapter, you should be able to:

- Determine the shear strength of soils.
- Understand the difference between drained and undrained shear strength.
- Determine the type of shear test that best simulates field conditions.
- Interpret laboratory and field test results to obtain shear strength parameters.

Sample Practical Situation: You are the geotechnical engineer in charge of a soil exploration program for a dam and a housing project. You are expected to specify laboratory and field tests to determine the shear strength of the soil and to recommend soil strength parameters for the design of the dam.

1.1 Definitions of Key Terms

Shear strength of a soil (τ) is the maximum internal resistance to applied shearing stresses.

Angle of internal friction (ϕ) is the friction angle between soil particles.

Cohesion (c) is a measure of the forces that cement soil particles.

Undrained shear strength of a soil (S_u) is the shear strength of a soil when sheared at constant volume.

1.2 Coulomb's Frictional Law

You may recall Coulomb's frictional law from your courses in statics and physics. If a block of weight W is pushed horizontally on a plane (Fig. 1.1a), the horizontal force (H) required to initiating movement is:

$$H = \mu W \quad (1.1)$$

where μ is the coefficient of static friction between the block and the horizontal plane. The coefficient of friction μ is independent of the area of contact. It is, however, strongly dependent on **the nature of the surface in contact – the type of material, the condition of the surface**, and so on. Furthermore, in most materials the coefficient of static friction is larger than the kinetic coefficient. The angle between the resultant force R and the normal force N (Fig. 1.1) is called the friction angle,

$$\phi = \tan^{-1} \mu.$$

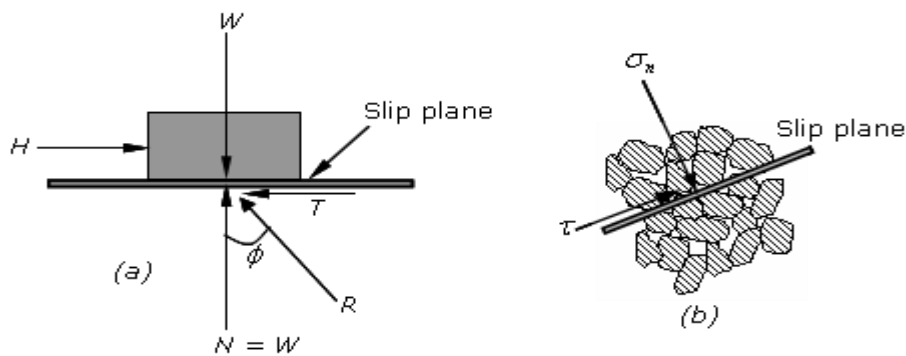


Figure 1.1: (a) Slip plane of a block. (b) A slip plane in a soil mass.

In terms of stresses, Coulomb's law is expressed as:

$$\tau_f = \sigma_n \tan \phi \quad (1.2)$$

Where $\tau_f (= T/A$, where T is the shear force at impending slip and A is the area of the plane parallel to T) is the shear stress when slip is initiated, and $\sigma_n (= N/A)$ is the normal stress on the plane on which slip is initiated. Coulomb's law requires the existence or the development of a critical sliding plane, also called **slip plane or failure plane**. In the case of the block the slip plane is at the interface between the block and the horizontal plane.

1.3 Mohr's Circle for Stress

The stress states at a point within a soil mass can be represented graphically by a very useful and widely used device known as **Mohr's circle for stress**. The stress state at a point is the set of stress vectors corresponding to all planes passing through that point. For simplicity, we will consider a two-dimensional element with stresses as shown in Fig. 1.2a. Let's draw Mohr's circle. First, we have to choose a sign convention. In soil mechanics, **compressive stresses** and **clockwise** shear are generally assumed to be **positive**. We will also assume that $\sigma_z > \sigma_x$.

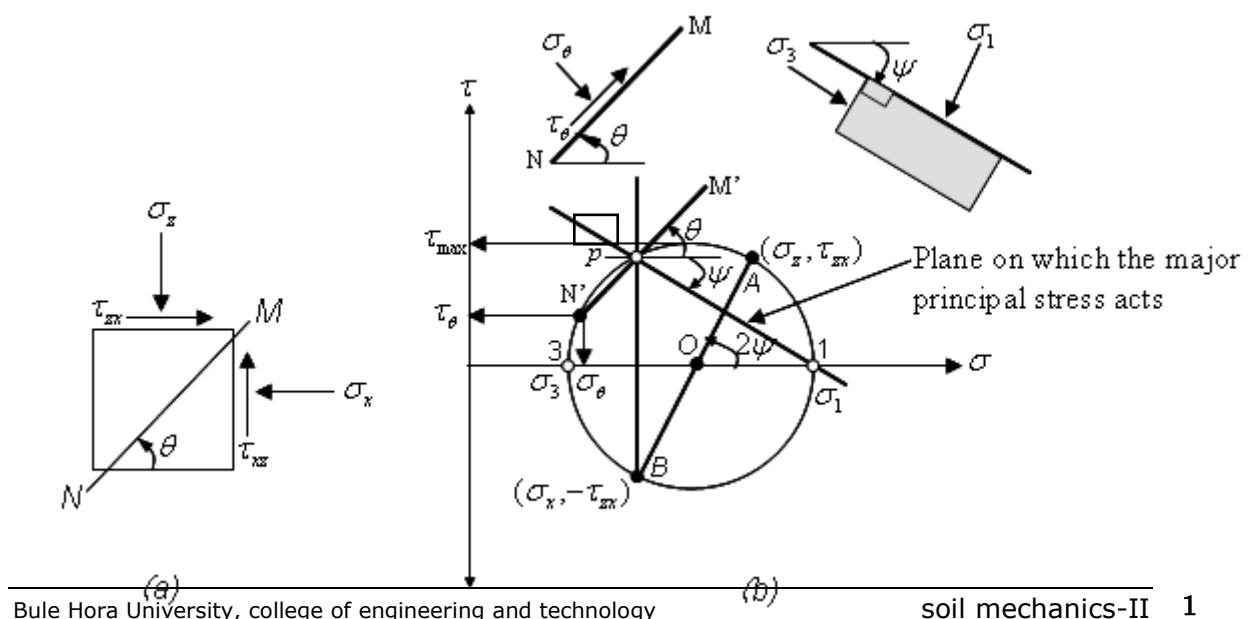


Figure 1.2: Stresses on two dimensional element and Mohr's circle.

The two coordinates of the circle are (σ_z, τ_{zx}) and $(\sigma_x, -\tau_{zx})$. Recall from your strength of materials course that, for equilibrium $\tau_{xz} = -\tau_{zx}$. Plot these two coordinates on a graph of shear stress (ordinate) and normal stress (abscissa) as shown by A and B in Fig. 1.2b. Draw a circle with AB as the diameter. The circle crosses the normal stress axis at 1 and 3, where shear stresses are equal to zero. The stresses at these points are the *major principal stress*, σ_1 , and the *minor principal stress*, σ_3 .

The principal stresses are related to the stresses σ_x, σ_z and τ_{zx} by the following relations:

$$\sigma_1 = \frac{\sigma_z + \sigma_x}{2} + \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{zx}^2} \quad (1.3)$$

$$\sigma_3 = \frac{\sigma_z + \sigma_x}{2} - \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{zx}^2} \quad (1.4)$$

The angle between the major principal stress plane and the horizontal plane (ψ) is:

$$\tan \psi = \frac{\tau_{zx}}{\sigma_1 - \sigma_x} \quad (1.5)$$

The stresses on a plane oriented at an angle θ to the *major principal stress plane* are:

$$\sigma_\theta = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta \quad (1.6)$$

$$\tau_\theta = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta \quad (1.7)$$

In the above equations θ is positive for clockwise orientation. The maximum shear stress is at the top of the circle with magnitude:

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} \quad (1.8)$$

The stress σ_z acts on the horizontal plane and the stress σ_x acts on the vertical plane. If we draw these planes in Mohr's circle, they intersect at a point, P. Point P is called the **pole of the Mohr circle**. It is a special point because any line passing through the pole will intersect Mohr's circle at a point that represents the stresses on a plane parallel to the line. Let us see how this works. Suppose we want to find the stresses on a plane inclined at angle θ to the horizontal plane as depicted by MN in Fig. 1.2a. Once we locate the pole, P, we can draw a line parallel to MN through P as shown by M'N' in Fig. 1.2b. The line M'N' intersects the circle at N' and the coordinates of N', $(\sigma_\theta, \tau_\theta)$ represent the normal and shear stresses on MN.

EXAMPLE 1.1

A sample of soil 100 mm×100 mm is subjected to the forces shown in Fig. E1.1a. Determine (a) σ_1, σ_3 and ψ ; (b) the maximum shear stress, and (c) the stresses on a plane oriented at 30° clockwise to the major principal stress plane.

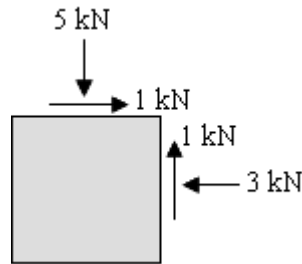


Figure E1.1a

Strategy. There are two approaches to solve this problem. You can either use Mohr's circle or the appropriate equations. Both approaches will be used here.

1.4 Mohr-Coulomb Failure Criteria

Coulomb (1776) suggested that the shear strength of a soil along a failure plane could be described by:

$$\tau_f = c + \sigma_n \tan \phi \quad (1.9)$$

where τ_f is the shear strength on the failure plane, σ_n is the stress normal to the plane, c is the cohesion and ϕ the angle of internal friction of the soil. The two parameters c and ϕ are called *shear strength parameters*.

To understand the concept behind Eq. (1.9), consider two blocks A and B (Fig. 1.3a) of unit area that are in contact with each other and are subjected to the normal and shear stresses shown. The interface between the blocks is not smooth and contains friction. Under a constant normal stress, the shear stress is increased from zero to the maximum τ_f , forcing the two blocks to slide along their contact area. When $\sigma_n = 0$, the shear stress has to be mobilized to a maximum value of c to make the sliding possible. If the friction angle between blocks A and B is ϕ then for the values of $\sigma > 0$, τ has to be increased to overcome the resistance to sliding $\sigma \times \tan \phi$ caused by friction (Coulomb's frictional law). Consequently, the summation of c and $\sigma \times \tan \phi$ represents the maximum shear stress needed to slide the two blocks on the plane of contact (slip or failure plane). In a real soil, if a predetermined sliding plane is forced to occur, the soil below and the soil above the failure plane will not act as rigid bodies but will deform, causing a volume change around the sliding and forming a *shear band* (Fig. 1.3b).

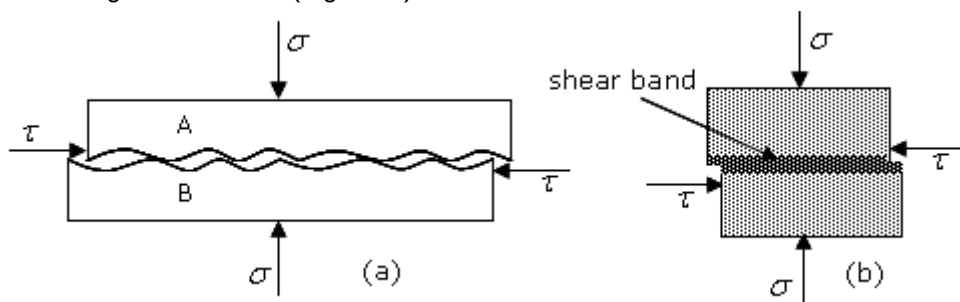


Figure 1.3: (a) Mechanical concept of sliding. (b) Soil deformation and a shear band.

In a coordinate system with σ_n plotted as abscissa and τ as ordinate, Eq. (1.9) is represented by the line shown in Fig. (1.4a). This equation was originally written in terms of total stress and was only partially successful in predicting the shear strength of real soils. Coulomb's failure criterion was subsequently redefined as:

$$\tau_f = c' + \sigma_n' \tan \phi' \quad (1.10)$$

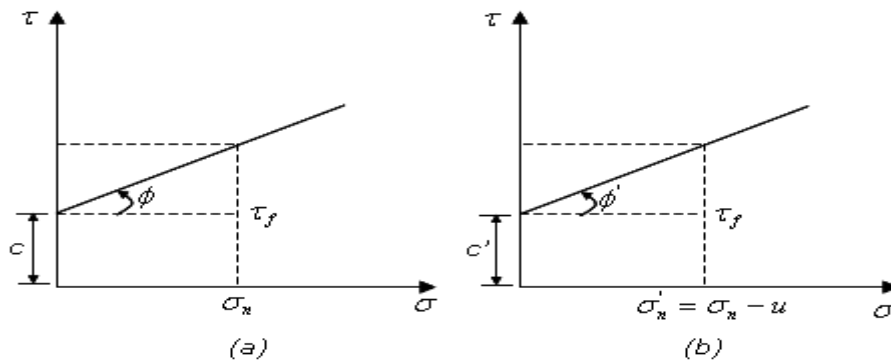


Figure 1.4: Coulomb's failure criteria: (a) total stress (b) effective stress.

Where: τ_f is the shear strength, σ_n' is the effective normal stress, c' is the effective cohesion, and ϕ' the effective angle of internal friction of the soil. In both the total and effective stress conditions, the shear stress is solely taken by the soil particles, since the liquid in the voids – which is normally water – has no resistance to shear. The tensile strength of soils is commonly ignored and therefore cohesion is the minimum shear strength at zero normal stresses.

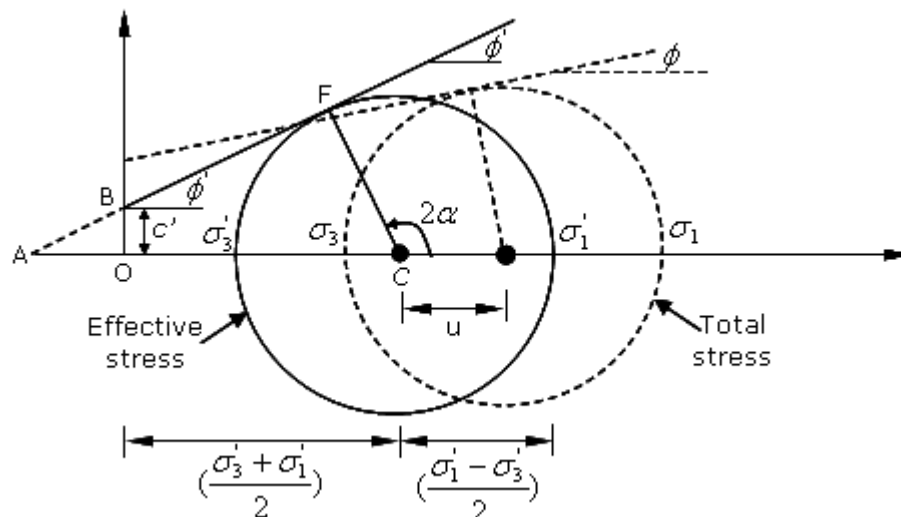


Figure 1.5: Mohr-Coulomb failure criterion.

Figure 1.5 shows the total and effective stress states at failure point represented by Mohr's circles. It is apparent that the shear stress at every plane in the total stress Mohr's circle is the same as in the effective stress Mohr's circle. The difference between normal stresses in two perpendicular directions in the total and effective stress is equal to:

$$\sigma_z - \sigma_x = (\sigma'_z + u) - (\sigma'_x + u) = \sigma'_z - \sigma'_x \quad (1.11)$$

Thus, the radii of both the total and effective stresses are identical. The horizontal distance of the two circles is equal to the pore water pressure u .

Any point F at the failure plane represents the **normal** and **shear stresses** on a failure plane at a specified point in a soil. These stresses must also satisfy the equilibrium conditions at the point, which is represented by Mohr's circle of stress. This implies that, at failure, Mohr's circle of stress must be tangent to the line expressed by Eq. 4.9 (or 4.10). This condition known as the **Mohr-Coulomb failure criterion** is shown in Fig. 1.5.

From geometry of Fig. 1.5, the theoretical angle between the failure plane and the major principal plane is given by the following equation:

$$\alpha = \left(\frac{90 + \phi'}{2} \right) = 45^\circ + \frac{\phi'}{2} \quad (1.12)$$

From figure 1.5, a relationship between the state of stress (σ'_1 and σ'_3) or (σ'_x, σ'_z , or τ_{zx}) and the shear strength parameters c' and ϕ' may be formulated by equating the radius of Mohr's circle R to the distance of the center of the circle from the failure envelope, CF , in which,

$$R = CF \Rightarrow \sqrt{\left(\frac{\sigma'_z - \sigma'_x}{2} \right)^2 + \tau_{zx}^2} = OB \cos \phi' + OC \sin \phi' \quad (1.13)$$

This equation can be written in terms of the principal stresses ($\tau_{zx} = 0$) as follows:

$$\sqrt{\left(\frac{\sigma'_1 - \sigma'_3}{2} \right)^2} = OB \cos \phi' + OC \sin \phi' \quad (1.14)$$

Considering $OB = c'$ and $OC = (\sigma'_1 + \sigma'_3)/2$, we have

$$\sigma'_1 - \sigma'_3 = 2c' \cos \phi' + (\sigma'_1 + \sigma'_3) \sin \phi' \quad (1.15)$$

Or,

$$\sigma'_1 = \sigma'_3 \frac{1 + \sin \phi'}{1 - \sin \phi'} + 2c' \frac{\cos \phi'}{1 - \sin \phi'} \Rightarrow \sigma'_1 = \sigma'_3 \frac{1 + \sin \phi'}{1 - \sin \phi'} + 2c' \sqrt{\frac{1 + \sin \phi'}{1 - \sin \phi'}} \quad (1.16)$$

$$\Rightarrow \sigma'_3 = \sigma'_1 \frac{1 - \sin \phi'}{1 + \sin \phi'} - 2c' \sqrt{\frac{1 - \sin \phi'}{1 + \sin \phi'}} \quad (1.17)$$

Or using some trigonometry manipulations,

$$\sigma'_1 = \sigma'_3 \tan^2 \left(45 + \frac{\phi'}{2} \right) + 2c' \tan \left(45 + \frac{\phi'}{2} \right) \quad (1.18)$$

$$\sigma_3' = \sigma_1' \tan^2(45 - \frac{\phi'}{2}) - 2c' \tan(45 - \frac{\phi'}{2}) \quad (1.19)$$

If the cohesion c' , is small or zero, then Eqs. (1.15 to 1.19) can be rearranged as follows:

$$\sin \phi' = \left(\frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} \right) \quad (1.20)$$

$$\frac{\sigma_3'}{\sigma_1'} = \frac{1 - \sin \phi'}{1 + \sin \phi'} \quad \text{or} \quad \frac{\sigma_1'}{\sigma_3'} = \frac{1 + \sin \phi'}{1 - \sin \phi'} \quad (1.21)$$

$$\frac{\sigma_3'}{\sigma_1'} = \tan^2(45 - \frac{\phi'}{2}) \quad \text{or} \quad \frac{\sigma_1'}{\sigma_3'} = \tan^2(45 + \frac{\phi'}{2}) \quad (1.22)$$

EXAMPLE 1.2

At a point in a soil mass, the total vertical and horizontal stresses are 240 kPa and 145 kPa respectively whilst the pore water pressure is 40 kPa. Shear stresses on the vertical and horizontal planes passing through this point are zero. Calculate the maximum excess pore water pressure to cause the failure of this point. What is the magnitude of the shear strength on the plane of failure? The effective shear strength parameters are $c' = 10$ kPa and $\phi' = 30^\circ$.

Strategy. You are given the initial stress state. You should first check whether the initial stress state is below the failure envelope, and then use the appropriate equations to calculate the excess pore water pressure and the shear strength at failure.

1.5 Drained and Undrained Shear strength

Drained condition occurs when the **excess pore water pressure** developed during loading of a soil dissipates, i.e. $\Delta u = 0$, resulting in volume changes in the soil. **Loose sands**, normally consolidated clays and lightly over consolidated clays tend to **compress** or **contract**, whilst dense sands and heavily over consolidated ($\text{OCR} > 2$) clays tend to expand during drained condition.

Undrained condition occurs when the excess pore water pressure cannot drain, at least quickly from the soil, i.e. $\Delta u \neq 0$. During undrained shearing, the volume of the soil remains constant. Consequently, the tendency towards volume change induces a pressure in the pore water. If the specimen tends to **compress** or **contract** during shear, then the induced pore water pressure is **positive**. It wants to contract and squeeze water out of the pores, but it can not. Positive pore water pressures occur in loose sands, normally consolidated clays and lightly over consolidated clays. If the specimen tends to **expand** and **swell** during shear, the induced pore water pressure is **negative**. It wants to expand and draw water into the pores, but it can not. Negative pore water pressures occur in dense sands and heavily over consolidated ($\text{OCR} > 2$) clays.

During the life of the geotechnical structure, called the **long-term condition**, the excess pore water pressure developed by a loading dissipates and drained condition applies. Clays usually take many years to dissipate the excess pore water pressure. During construction, and shortly after, called the **short-term condition**, soils with low permeability (fine-grained soils) do not have sufficient time for the excess pore water pressure to dissipate and undrained condition applies. The permeability of coarse-grained soils is sufficiently large that under static loading conditions the excess pore water

pressure dissipates quickly. Consequently undrained condition does not apply to clean coarse-grained soils under static loading. Dynamic loading, such as during an earthquake, is imposed so quickly that even coarse-grained soils do not have sufficient time to dissipate the excess pore water pressure and undrained condition applies.

The shear strength of a fine-grained soil under undrained condition is called the **undrained shear strength**, S_u . The undrained shear strength S_u is the radius of Mohr's total stress circle; that is:

$$S_u = \frac{\sigma_1 - \sigma_3}{2} = \frac{(\sigma'_1 + u) - (\sigma'_3 + u)}{2} = \frac{\sigma'_1 - \sigma'_3}{2} \quad (1.23)$$

1.6 Laboratory Shear Strength Tests

Different methods are available for testing shear strength of soils in a laboratory.

The following are the more commonly used testing methods:

1. Direct shear test.
2. Triaxial compression test.
3. Unconfined compression test.

1.6.1 Direct Shear Test

The direct shear test is the oldest and the simplest type of shear test. It was first devised by Coulomb (1776) for the study of shear strength. The test is performed in a **shear box**, illustrated in Figure 1.6. The box consists of two parts, one part **fixed** and the other **movable**. Usually the box is a square of sides equal to **5 cm**. The soil sample is placed in the box. A vertical normal force N is applied to the top of the sample through a metal platen resting on the top part of the box. Porous stones may be placed on the **top** and **bottom** part of the sample to facilitate drainage.

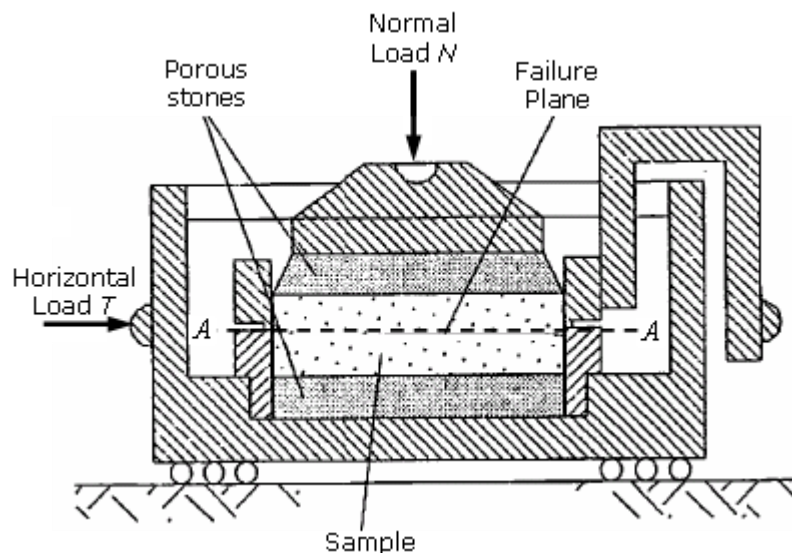


Figure 1.6: Schematic of direct shear apparatus.

The sample is subjected to shearing stress at the plane of separation AA (Fig. 1.6) by applying horizontal forces T . The horizontal force can be applied either at a **constant speed (strain controlled test)** or constant load (**stress controlled test**) until failure occurs in the soil. In most routine soil tests the strain controlled test is used. **Failure** is determined when the soil can not resist any further increment of horizontal force.

The above procedure is repeated for several values (**three or more**) of normal forces. By plotting the **normal stresses** and corresponding **shear stresses** from the results of such tests, a failure envelope is obtained as shown in Fig. 1.7. Note that the normal stress $\sigma_n = N/A$, and the shear stress $\tau = T/A$, where A is the cross-sectional area of the soil specimen. For example, if the normal and horizontal forces for the second test are represented by N_2 and T_2 , respectively, the coordinate point for test 2 is given as $(\sigma_2 = N_2/A, \tau_2 = T_2/A)$. The corresponding Mohr's circle for this case is shown in Fig. 1.7 and the shear strength parameters ϕ and c could be measured directly from this figure. It is not possible to obtain other deformation parameters such Young's modulus and Poisson's ratio from direct shear test.

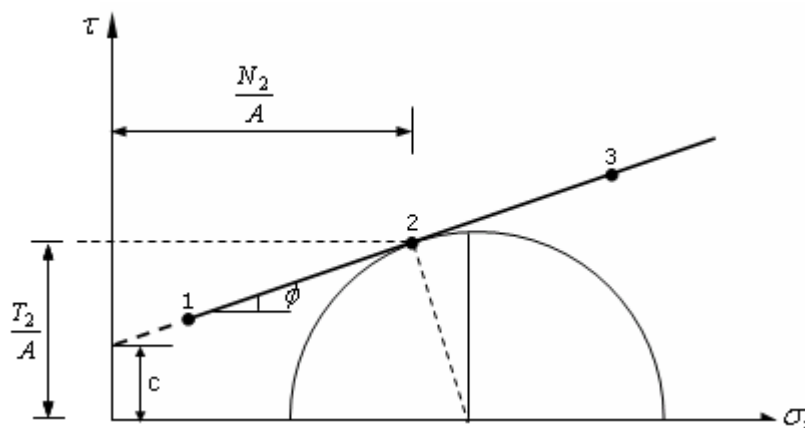


Figure 1.7: Plotted direct shear test results and a Mohr circle.

In direct shear test, drainage should be allowed through out the test because there is no way of sealing the specimen. Once the shear phase starts and one part of the specimen moves in relation to the other part, a gap opens. Water can flow through this gap, and drainage control becomes impossible. The only solution is, therefore, **to allow full drainage** throughout the test, and keep excess pore water pressure equal to zero. In sands, due to their **high permeability**, dissipation of excess pore water pressure is immediate, and the test can be conducted quickly. In clayey soils full drainage may require long testing time to allow for dissipation of excess pore water pressure. Some practical engineers still attempt to perform undrained direct shear test in clayey soils by shearing the soil very quickly. However, this may lead to totally erroneous results.

1.6.2 Triaxial Compression Test

A widely used apparatus to determine the shear strength parameters and the stress-strain behavior of soils is the **triaxial** apparatus. The essential features of a triaxial test apparatus together with a soil sample are shown in Fig. 1.8. The soil sample is protected by a **thin rubber membrane** and is subjected to pressure from water that occupies the volume of the chamber. This confining **water pressure** (also called **radial pressure**) enforces a condition of equality on two of the total principal stresses, i.e. $\sigma_2 = \sigma_3$. Vertical or axial stresses are applied by a loading ram (plunger), and therefore, the total major principal stress, σ_1 is the sum of the **confining pressures** and the **deviatoric stress** applied through the ram. In a traditional triaxial compression test, the confining pressure σ_3 is kept constant whilst the major principal stress σ_1 is increased incrementally by the loading ram until the sample fails.

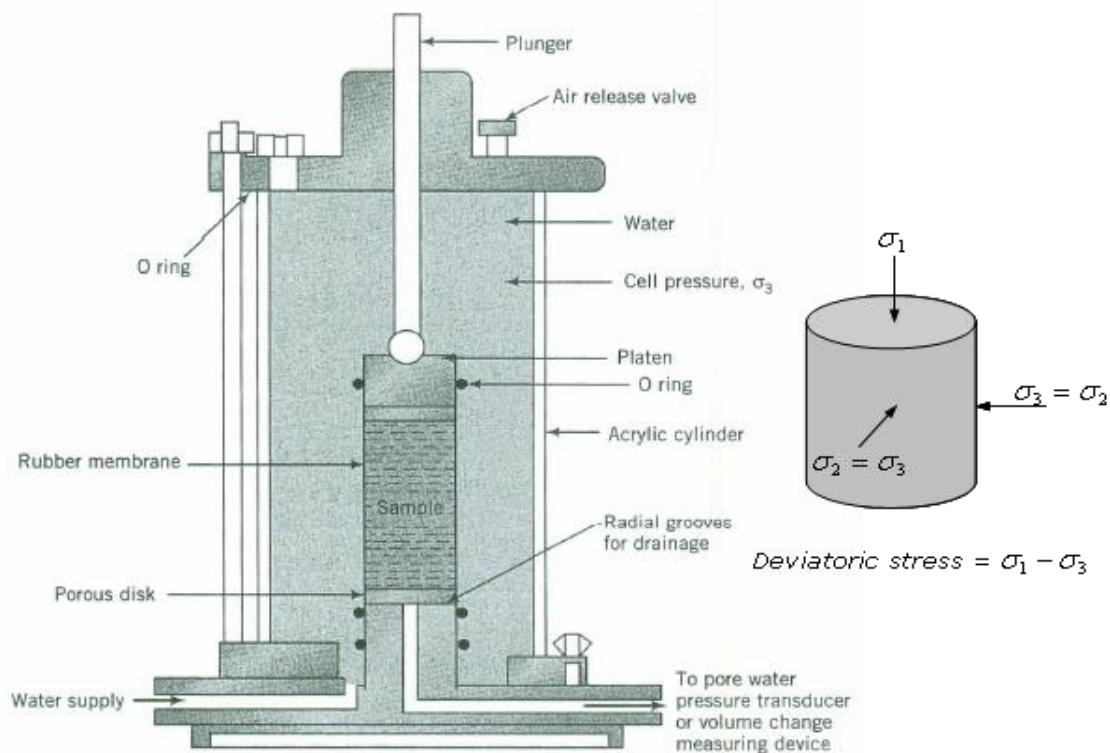


Figure 1.8: Schematic diagram of a triaxial compression apparatus (Budhu, pp. 225).

Facilities to measure the pore water pressure and the volume change at any stage of the test are available. To eliminate any end constraint effects on the test results, The height of the specimen is made to be twice the diameter. Specimen diameter are normally either **38 mm** or **100 mm**, however some cells have been manufactured to accommodate larger diameter. Specimens are either **undisturbed** or **remolded** depending on the type of material. To construct a failure envelope for a soil, a test has to be performed several times with different confining pressures using ideally identical samples.

The triaxial apparatus is versatile because we can

1. Independently control the applied axial and radial loads,
2. conduct tests under drained and undrained conditions, and
3. control the applied displacements or stresses.

Recorded measurements include deviatoric stress at different stages of the test, vertical displacement of the ram, volume change and pore water pressure. The average stresses and strains on a soil sample in a triaxial compression apparatus are as follows:

$$\text{Axial stress : } \sigma_1 = \frac{P_z}{A} + \sigma_3 \quad (1.24)$$

$$\text{Deviatoric stress : } \sigma_1 - \sigma_3 = \frac{P_z}{A} \quad (1.25)$$

$$\text{Axial strain : } \varepsilon_1 = \frac{\Delta z}{H_0} \quad (1.26)$$

$$\text{Radial strain : } \varepsilon_3 = \frac{\Delta r}{r_0} \quad (1.27)$$

$$\text{Volumetric strain : } \varepsilon_p = \frac{\Delta V}{V_0} = \varepsilon_1 + 2\varepsilon_3 \quad (1.28)$$

$$\text{Deviatoric strain : } \varepsilon_q = \frac{2}{3}(\varepsilon_1 - \varepsilon_3) \quad (1.29)$$

where P_z is the axial load on the ram, A is the cross-sectional **area of the soil sample**, r_0 is the initial radius of the soil sample, Δr is the change in radius, V_0 is the initial volume, ΔV is the change in volume, H_0 is the initial height, and Δz is the change in height. The area of the sample changes during loading, and at any given instance the area is:

$$A = \frac{V}{H} = \frac{V_0 - \Delta V}{H_0 - \Delta z} = \frac{V_0 \left(1 - \frac{\Delta V}{V_0}\right)}{H_0 \left(1 - \frac{\Delta z}{H_0}\right)} = \frac{A_0(1 - \varepsilon_p)}{1 - \varepsilon_1} \quad (1.30)$$

Where $A_0 (= \pi \cdot r_0^2)$ is the initial cross-sectional area and H is the current height of the sample.

A variety of stress paths can be applied to soil samples in the triaxial apparatus. However, only a few stress paths are used in practice to mimic typical geotechnical problems. We will discuss the tests most often used, why they are used, and typical results obtained.

(A) Consolidated Drained (CD) Test

The purpose of a CD test is to determine the **drained shear strength parameters** ϕ' and c' . The effective elastic moduli for drained condition E' is also obtained from this test. A CD test is performed in two stages. The first stage is consolidating the soil to a desired effective stress level appropriate to field conditions by pressurizing the water in the cell and allowing the soil sample to drain until the excess pore water pressure dissipates. In the second stage, the pressure in the cell (cell pressure or confining pressure) is kept constant and additional axial loads or displacements are

added very slowly until the soil sample fails. The displacement rate (or stress rate) used must be slow enough to allow the excess pore water pressure to dissipate. Because the permeability of **fine-grained** soils is much lower than coarse-grained soils, the displacement rate for testing fine-grained soils is much lower than for coarse-grained soils. Drainage of the excess pore water pressure is permitted throughout the test and the amount of **water expelled** is measured. Since the CD test is a drained test, a single test can take several days if the permeability of the soil is low (e.g. clays). The results of CD tests are used to determine the long-term stability of **slopes, foundations, retaining walls, excavations, and other earthworks**. For **remolded** and **normally consolidated** clays, the cohesion c' parameter from a CD test is essentially very small and can be assumed to be zero for all practical purposes.

(B) Consolidated Undrained (CU) Test

The purpose of a CU test is to determine both the undrained (c_u, ϕ_u) and drained (c', ϕ') shear strength parameters. The undrained elastic moduli E_u and effective elastic moduli E' are also obtained from this test. The CU test is conducted in a similar manner to the CD test except that after **isotropic consolidation**, the axial load is increased under undrained condition and the excess pore water pressure is measured. As explained in section 1.5, the excess pore pressure developed during shear can either be positive or negative. This happens because the sample tries to either contract or expand during shear. Positive pore pressures occur in loose sands and normally consolidated clays. Negative pore pressures occur in dense sands and heavily over consolidated clays.

The CU test is the most popular triaxial test because you can obtain both **drained** and **undrained shear strength parameters**, and most tests can be completed within a few minutes after consolidation compared with more than a day for a CD test. Fine-grained soils with low permeability must be sheared slowly to allow the excess pore water pressure to equilibrate throughout the test sample. The results from CU tests are used to analyze the stability of slopes, foundations, retaining walls, excavations and other earthworks. For remolded and normally consolidated clays, the cohesion c' parameter from a CU test is also essentially very small and can be assumed to be zero.

(C) Unconsolidated Undrained (UU) Test

The purpose of a UU test is to determine the **undrained shear strength (S_u)** of a saturated soil. The UU test consists of applying a cell pressure to the soil sample without drainage of pore water followed by increments of axial stress. The cell pressure is kept constant and the test is completed very quickly because in neither of the two stages – **consolidation** and **shearing** – is the excess pore pressure allowed to drain. In the UU test, pore water pressures are usually not measured.

1.6.3 Backpressure

Backpressure is a technique used for **saturating soil specimens**. It is accomplished by applying water pressure u_0 within the specimen, and at the same time changing the cell pressure σ_{cell} of an equal amount. Therefore, the net confining pressure $\sigma_c = \sigma_{cell} - u_0$ remains unchanged. Back pressuring has no influence on the calculations. In most cases, a backpressure of 300 kPa is sufficient to ensure specimen saturation and it should be applied in steps as shown in the

table below.

Stage	Backpressure u_0 (kPa)	Cell pressure σ_{cel} (kPa)	Confining pressure $\sigma_c = \sigma_{cel} - u_0$ (kPa)
0	0	100	100
1	20	120	100
2	40	140	100
3	80	180	100
*	*	*	*
*	*	*	*
10	300	400	100

1.6.4 Unconfined Compression (UC) Test

The UC test is the simplest and quickest test used to determine the shear strength of a **cohesive soil**. An undisturbed or remolded sample of cylindrical shape, about 38 mm in diameter and 76 mm in height is subjected to uniaxial compression until the soil fails. Since the sample is laterally unconfined, only cohesive soils can be tested. The sample is tested quickly and there is no drainage. Therefore, it is a special case of the UU test in which $\sigma_3=0$. However, rather than in a triaxial cell, the test is performed in a mechanical apparatus specially manufactured for this purpose. Figure 1.9 shows an unconfined compression test apparatus.

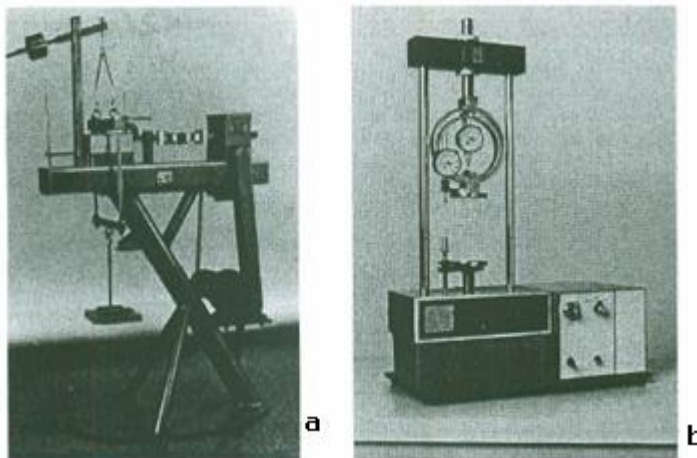


Figure 1.9: (a) Direct shear test apparatus, (b) UC test apparatus.

EXAMPLE 1.3

A CU test gave the following data. Sample diameter = 38 mm, and height = 76 mm. The pore pressures at failure (peak points) are 10, 61.6, 113.2 kPa for $\sigma_3=100, 200, 300$ kPa, respectively. Determine: (a) the deviatoric stress – axial strain curve and modulus of elasticity of the soil, and (b)

the shear strength parameters (effective and total).

Axial de- formation (mm)	Applied axial force (N)		
	$\sigma_3=100$ (kPa)	$\sigma_3=200$ (kPa)	$\sigma_3=300$ (kPa)
0.00	0.0	0.0	0.0
0.95	126.3	155.7	183.7
1.90	218.9	304.1	353.2
2.85	277.7	415.4	532.1
3.80	308.8	475.8	622.4
4.75	328.9	499.0	661.2
5.70	332.3	512.9	689.3
6.65	335.6	518.6	701.9
7.60	338.1	523.0	708.7
8.55	341.3	528.9	715.7
9.50	344.2	534.6	724.0

Example 1.4

The following result was obtained from CU tests on specimens of a saturated clay. Determine the shear strength parameters (effective and total).

σ_3 (kPa)	$\sigma'_1 - \sigma'_3$ (kPa) at peak	u (kPa) at peak
100	137	28
200	210	86
300	283	147

1.7 Field Tests

Sampling disturbances and sample preparation for laboratory tests significantly affect the shear strength parameters. Consequently, a variety of field tests have been developed to obtain more reliable soil shear strength parameters by testing soils in-situ. In the following sections some of the most popular field tests are described.

1.7.1 Shear Vane

In **soft and saturated clays**, where undisturbed specimen is difficult to obtain, the undrained shear strength is measured using a shear vane test. A diagrammatic view of the shear vane apparatus is shown in Fig. 1.20. It consists of four thin metal blades welded orthogonally (90°) to a rod where the height H is twice the diameter D (Fig. 1.20). Commonly used diameters are 38, 50 and 75 mm.

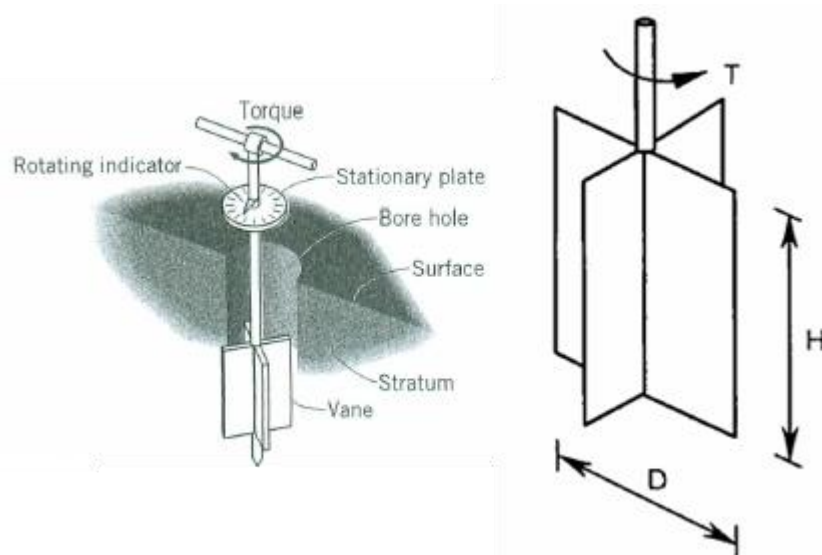


Figure 1.20: Shear vane apparatus.

The vane is pushed into the soil either at the ground surface or at the bottom of a borehole until totally embedded in the soil (at least 0.5 m). A torque T is applied by a torque head device (located above the soil surface and attached to the shear vane rod) and the vane is rotated at a slow rate of 6° per minute. As a result, shear stresses are mobilized on all surfaces of a cylindrical volume of the soil generated by the rotation. The maximum torque is measured by a suitable instrument and equals to the moment of the mobilized shear stress about the central axis of the apparatus. The undrained shear strength is calculated from:

$$S_u = \frac{T}{\pi D^2 (H/2 + D/6)} \quad (1.31)$$

1.7.2 Standard Penetration Test (SPT)

The Standard Penetration Test (SPT) was developed around 1927 and it is perhaps the most popular field test performed **mostly in coarse grained (or cohesionless) soils**. The SPT is performed by driving a standard split spoon sampler into the ground by blows from a drop hammer of mass 64 kg falling 760 mm (Fig. 1.21). The sampler is driven 150 mm into the soil at the bottom of a borehole, and the number of blows (N) required to drive it an additional 300 mm is counted. The number of blows N is called the standard penetration number.

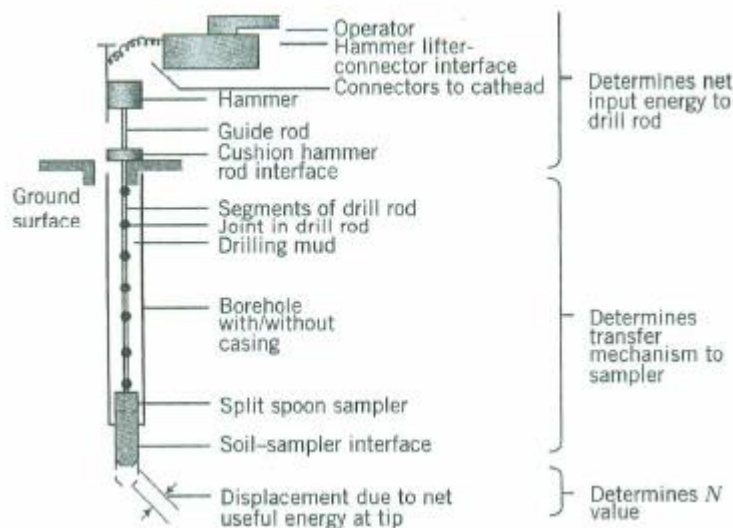


Figure 1.21: Standard Penetration Test (Budhu, 248)

Various corrections are applied to the N values to account for energy losses, overburden pressure, rod length, and so on. It is customary to correct the N values to a rod energy ratio of 60%. The rod energy ratio is – the ratio of the energy delivered to the split spoon sampler to the free falling energy of the hammer. The corrected N values are denoted as N_{60} . The N value is used to estimate the relative density, friction angle, and settlement in coarse grained soils. The test is very simple, but the results are difficult to interpret.

Typical correlation among N values, relative density, and ϕ' , for coarse grained soils are given in Table 1.2 below (Budhu, 248).

Table 1.2: Correlation of N , N_{60} , D_r , $\square$, and $\square'$ for coarse grained soils.

N	N_{60}	Description	$\square$ (kN/m ³)	D_r (%)	$\square'$ (°)
0 – 5	0 – 3	Very loose	11 – 13	0 – 15	26 – 28
5 – 10	3 – 9	Loose	14 – 16	16 – 35	29 – 34
10 – 30	9 – 25	Medium	17 – 19	36 – 65	35 – 40
30 – 50	25 – 45	Dense	20 – 21	66 – 85	38 – 45
> 50	> 45	Very dense	> 21	> 86	> 45

The SPT is mostly used in coarse grained soils. However, in some countries, for example, Japan and the United States, it is also used in fine-grained soils. Table 1.3 below shows, correlation of N_{60} and S_u for saturated fine grained soils.

Table 1.3: Correlation of N_{60} , and S_u for fine grained soils.

N_{60}	Description	S_u (kPa)
0 – 2	Very soft	< 10
3 – 5	Soft	10 – 25
6 – 9	Medium	25 – 50
10 – 15	Stiff	50 – 100
15 – 30	Very stiff	100 – 200
> 30	Extremely stiff	> 200

1.7.3 Cone Penetration Test (CPT)

The Cone Penetrometer Test (CPT) is an in situ test used for subsurface exploration in fine and **medium sands, soft silts and clays**. The apparatus consists of a cone with a 35.7 mm end diameter, projected area of 1000 mm² and 60° point angle (Fig. 1.22) that is attached to a rod. An outer sleeve encloses the rod.

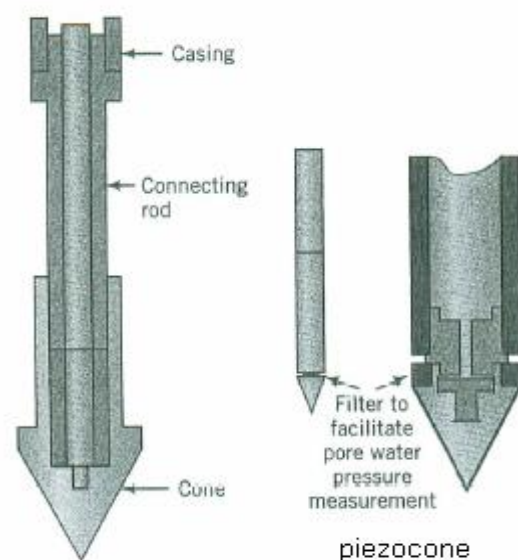


Figure 1.22: CPT apparatus (Budhu, 250)

The thrusts required to drive the cone and the sleeve 80 mm into the ground at a constant rate of **10 mm/s to 20 mm/s** are measured independently so that the end resistance or cone resistance and side friction or sleeve resistance may be estimated separately. A special type of the cone penetrometer, known as **piezocone** has porous elements inserted into the cone or sleeve to allow for pore water pressure measurements.

The cone resistance q_c is normally correlated with the undrained shear strength. One correlation equation is:

$$S_u = \frac{q_c - \sigma_z}{N_k} \quad (1.32)$$

where σ_z represents the total overburden pressure above the cone tip, and N_k is a cone factor that depends on the geometry of the cone and the rate of penetration. Average values of N_k as a function of plasticity index I_p can be estimated from

$$N_k = 19 - \frac{I_p - 10}{5}; \quad I_p > 10 \quad (1.33)$$

Results of cone penetrometer tests have also been correlated with the friction angle. A number of correlations exist. Based on published data for sand (Roberston and Campanella, 1983), you can estimate ϕ' using:

$$\phi' = 35^\circ + 11.5 \log \left(\frac{q_c}{30\sigma_{z0}} \right); \quad 25^\circ < \phi' < 50^\circ \quad (1.34)$$

Appendix: **Eccentric Loads**

Meyerhof (1963) proposed an approximate method for loads that are located off-centered (or eccentric loads).

