

**ADDIS ABABA SCIENCE AND TECHNOLOGY UNIVERSITY
COLLEGE OF NATURAL AND SOCIAL SCIENCES**

APPLIED MATHEMATICS IB

WORKSHEET 4 (Practice Problems on Derivatives)

- Find the equations of the tangent (and normal) line(s) to the following curves at the indicated points.
 - $y = x^3$ at $x = 2$ c. $y = \log_2 x$ at $x = 4$ e. $y = \sin x$ at $x = \frac{\pi}{2}$
 - $y = \sqrt{x}$ at $x = \sqrt{2}$ d. $y = \cosh x$ at $x = 0$ f. $y = \sinh x$ at $x = 0$
- Using definition differentiate the following functions.
 - $y = \sqrt[3]{x}$ c. $y = e^x$ e. $y = \cosh x$
 - $y = \log_3 x$ d. $y = \sinh x$ f. $y = 2^x$
- If $f'(a) = 4$, then find
 - $\lim_{h \rightarrow 0} \frac{f(a+3h) - f(a)}{h}$ c. $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{\sqrt{x} - \sqrt{a}}$
 - $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a-h)}{h}$ d. $\lim_{h \rightarrow 0} \frac{f(a+6h) - f(a+3h)}{6h}$
- Find the positive integer n so that $\lim_{x \rightarrow 3} \frac{x^n - 3^n}{x - 3} = 108$.
- Find y' where
 - $y = \frac{1}{2-x}$ e. $y = e^{x^5}$ i. $y = \tan^{-1} \left(\frac{1-x}{1+x} \right)$
 - $y = \sin(2x^3)$ f. $y = 3^{\cos x}$ j. $y = \cosh^{-1}(\cos x^2)$
 - $y = \ln(\operatorname{sech}^2 x)$ g. $y = x^2 \tanh^{-1}(2x)$ k. $y = (\ln x)^{\ln x}$
 - $y = \sin(\ln(\cos(4x)))$ h. $y = \sin^{-1}(3x^2)$ l. $y = x^{x^x}$
- Find $\frac{d^2}{dx^2} f(x^3 - 2x^2 + 1)$ for differentiable function f satisfying $\frac{d}{dx} (f(x)) = 2x^2 + 3$.
- Given that $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 1$. Then find
 - $f(0)$ b. $f'(0)$
- Let f be a function for which $f(2) = -3$ and $f'(x) = \sqrt{x^2 + 5}$, if $g(x) = x^2 f\left(\frac{x}{x-1}\right)$. Find $g'(2)$.
- Determine a and b so that f is differentiable at $x = c$.
 - $f(x) = \begin{cases} \cos x, & x \geq 0 \\ ax + b, & x < 0 \end{cases}; c = 0$ b. $f(x) = \begin{cases} \frac{1}{|x|}, & x > 1 \\ ax^2 + b, & x \leq 1 \end{cases}; c = 1$
- Find $\frac{dy}{dt}$, if
 - $y = u^2 + 1, u = 2x, x = \sin t$ c. $y = \frac{1+u}{1-u}, u = s - \frac{1}{s}, s = \sqrt{t}$
 - $y = e^x + x^e, x = \cosh u, u = \sqrt{t}$ d. $y = \sinh^{-1} x, x = \sin x, x = 2t$
- Find $\frac{dy}{dx}$, if
 - $x^3 y + y^2 x = 2$ c. $\ln(\cos(2y)) = x + y$ e. $\ln(xy) + \sin(xy) = 0$
 - $\sin(x^2 + y) = y^2(3x + 1)$ d. $x^2 + 3x \cos y = \frac{x}{y}$ f. $\sinh(2y) + \cosh(2y) = 2x$
- Find all points on the graph of the given equation at which the tangent line is horizontal.
 - $x^4 + y^4 + 2 = 4xy^3$ b. $(x^2 + y^2)^2 = x^2 - y^2$
- Find all points on the graph of the given equation at which the tangent line is vertical.

- a. $x^2 + y^2 = 4$ b. $x^3y + 2y^4 - x^4 = 0$
14. Find the equations of the tangent (and normal) line(s) to the following curves at the indicated points.
- a. $xy + e^{x^2} \ln y = 5 \sin x$ at $(0,1)$ c. $x^2 + xy + y^2 - 3y = 6$ at $(-1, -1)$
 b. $(1+x)e^x + ye^{-x} = 1$ at $(0,0)$ d. $\sinh(xy) + \cosh(xy) = \cosh x$ at $(1,1)$
15. Find the indicated derivatives
- a. $\frac{d}{dx} \left(x^2 \frac{d}{dx} (3^x + 4x) \right)$ b. $\frac{d}{dx} \left((2 + 3x^2) \frac{d^3}{dx^3} (x^2 + 4x + 1) \right)$
16. Find a general formula for the higher order derivatives of:
- a. $f(x) = 2^x$ b. $f(x) = \cos 2x$ c. $f(x) = \ln |3x|$
17. Use L'Hopital's rule to evaluate the following limits.
- a. $\lim_{x \rightarrow 0} \frac{\sin x}{x}$ d. $\lim_{x \rightarrow 0} (1+x)^{1/x}$ g. $\lim_{x \rightarrow 0} \frac{\cos 2x - \cos 3}{x^2}$
 b. $\lim_{x \rightarrow \infty} \frac{e^x}{x^3}$ e. $\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt[3]{x}}$ h. $\lim_{x \rightarrow \infty} \frac{\ln(\ln 2x)}{x}$
 c. $\lim_{x \rightarrow 1^+} (\ln x)^{x-1}$ f. $\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$ i. $\lim_{x \rightarrow 0^+} (-\ln x)^x$
18. Find the critical number(s) and indicate the local extreme values of the functions:
- a. $f(x) = |3x + 4| \cdot x^3 - 6x^2 + 11x - 6$ e. $f(x) = \sin^2 x + 2 \cos x$
 b. $x^{4/5}(x-1)^2$ d. $f(x) = \sqrt[3]{x^2 - x}$ f. $f(x) = \ln x$
19. Find the absolute extreme values of f on the given interval.
- a. $f(x) = x^2 - 3x + 2$ on $\left[0, \frac{3}{2}\right]$ c. $f(x) = e^{-x} - e^{-2x}$, on $[0, 1]$
 b. $f(x) = \frac{x}{1+x^2}$ on $[0, 2]$ d. $f(x) = \tan^{-1} x + 1$, on $[0, 1]$
20. Let $f(x) = x^a(x-1)^b$, $a \geq 2$, $b \geq 2$. Determine the values of a and b for which their relative extreme values of f are $f(0)$, $f\left(\frac{1}{2}\right)$ and $f(1)$.
21. Show that $f(x) = 6x^7 + 5x^3 + 12x - 7$ has exactly one real root.
22. Sketch the following
- a. $y = x^3 - 6x^2 + 11x - 6$ b. $y = \frac{1+x^2}{1-x^2}$ c. $y = \tan^{-1} x$
23. Suppose a spherical balloon grows in such a way that after t seconds $v = 4\sqrt{t}$ (cubic inches). How fast is the radius changing after 64 seconds?
24. A window is made in the shape of a rectangle with a semi-circle attached at the top. If the perimeter of the window is 12m, find the dimensions that allow the maximum amount of light to enter.
25. A rectangle has two of its vertices on x-axis and the other two above the x-axis and on the parabola $y = 16 - x^2$. For all such possible rectangles, what are the dimensions of the rectangle to have maximum area?
26. A particle P moves in a straight line with position given by $s(t) = t^2 - 4t + 3$ cm from O, where $t \geq 0$, t is in seconds.
- a. Find expressions for the object's velocity and acceleration
 b. At what time(s) does the object reverse direction?
 c. Draw a motion diagram for the object.
 d. For what time intervals is the speed of the object decreasing?