Chapter 5

The Bohr Model of the Atom



Niels Henrik David Bohr (1885-1962) was the Danish physicist whose investigations of atomic structure earned him the 1922 Nobel Prize for physics. Bohr attended the University of Copenhagen (1903-11), then studied for a time under Ernest Rutherford. In 1920 he became the first director of the Institute of Theoretical Physics. Bohr's Institute became a gathering place for the world's top physicists, and he is considered one of the foremost scientists of modern physics, along with Albert Einstein, Erwin Schrödinger and Enrico Fermi. During World War II he left Denmark in 1943 and ended up in the United States and was sent to Los Alamos, New Mexico to join Robert Oppenheimer and others working to develop the atomic bomb. After the war he returned to Denmark and spent the rest of his career advocating the peaceful uses of atomic energy.One of Bohr's most famous students was Werner Heisenberg, the physicist who went on to lead Nazi Germany's attempt to develop the atomic bomb.

5.1 Line Spectra

When solids are heated, they emit radiation in which all wavelengths are present though with different intensities. This is because we are witnessing the collective behavior of a great many interacting atoms rather than the characteristic behavior of the individual atoms of a particular element. But in rarefied gas, since the atoms are far apart, the emitted radiation reflects the characteristic of the individual atoms or molecules present. When a low-pressure gas is excited, say by an electric discharge, the emitted radiation has a spectrum which contains certain discrete wavelengths only. The resulting pattern of light bands and dark areas is called a *line spectrum*. It has been known that all elements produce unique line spectrum of their own when burned in a flame or when excited in an electric discharge. This fact made optical spectroscopy an important area for analyzing the composition of an unknown substance. An empirical formula was found that specifies the wavelength in the hydrogen spectral lines:

$$\frac{1}{\lambda} = R\left(\frac{1}{k^2} - \frac{1}{n^2}\right) \tag{5.1}$$

where $R = 1.096776 \times 10^7 \mathrm{m}^{-1}$ is the *Rydberg Constant* and $n = k+1, k+2, \ldots$

Table: Hydrogen Series of Spectral Lines

Discoverer (year)	**Wavelength**	k	n
Lyman (1916)	ultraviolet	1	$2, 3, 4, \ldots$
Balmer (1885)	visible, ultraviolet	2	$3, 4, 5, \ldots$
Paschen (1908)	infrared	3	$4, 5, 6, \ldots$
Brackett(1922)	infrared	4	$5, 6, 7, \ldots$
Pfund (1924)	infrared	5	$6, 7, 8, \ldots$

Example 5.1: Show that the lines in the Lyman and Paschen series are in ultraviolet and infrared regions respectively. Also, find the wavelengths of the four

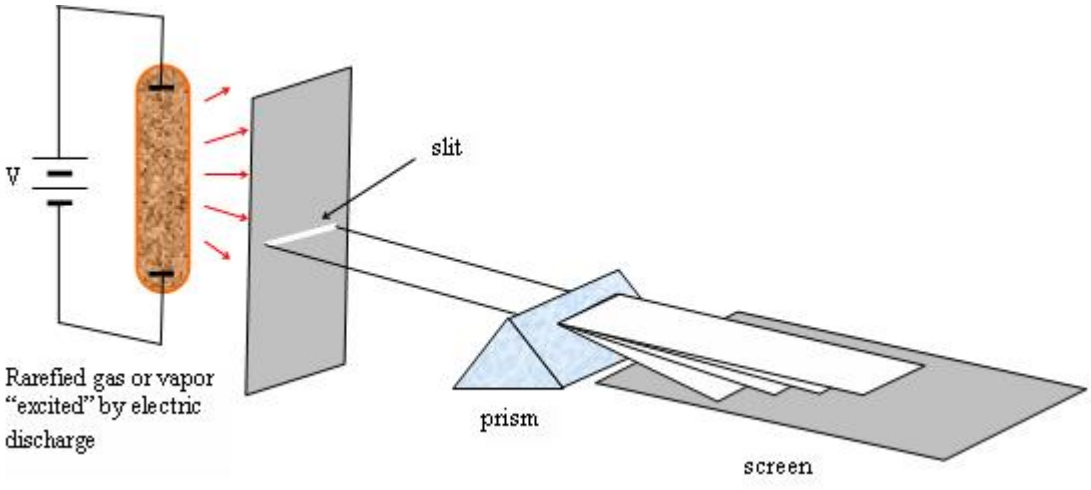


Figure 5.1: Schematic of an Optical Spectrometry

visible atomic hydrogen lines of the Balmer series. (visible light: 400 nm $\leq \lambda \leq$ 700 nm, $ultraviolet : \lambda < 400$ nm, $infrared : \lambda > 700$ nm)

5.2 Bohr Model of the Atom

The basic postulates of Bohr's theory are a combination of the ideas of classical physics and Planck's quantum theory of radiation.
The three postulates of Bohr's theory are:

1. Electrons revolve around the nucleus of an atom in a *non-radiating stationary orbits*. The dynamical equilibrium of the system is governed by the classical laws of physics, i.e.

$$\frac{mv^2}{r} = mr\omega^2 = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2} \tag{5.2}$$

2. Only those orbits are possible for which the angular momentum of the electron is an integral multiple of $\hbar(=\frac{h}{2\pi})$. i.e.

$$mvr_n = m\omega r_n^2 = n\hbar \tag{5.3}$$

   where n is an integer. In other words, angular momentum is quantized in those orbits.

3. Emission or absorption of energy occurs only in conjunction with a transition between two states. The frequency of the emitted or absorbed radiation is proportional to the difference in energy of the two stationary states.

$$E = E_i - E_f = hf \tag{5.4}$$

5.3 Energy Levels and Spectra

From Bohr's assumption, angular momentum is quantized;

$$\begin{aligned} L &= |\mathbf{r} \times \mathbf{p}| \\ \therefore mvr_n &= n\hbar \end{aligned}$$

n is called the *principal quantum number.*

$$v = \frac{n\hbar}{mr_n} \tag{5.5}$$

From §5.2 and §5.5,

$$v^2 = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{mr^2} = \frac{n^2\hbar^2}{m^2r_n^2}$$

$$\therefore r_n = \frac{4\pi\varepsilon_o\hbar^2}{me^2}n^2 \equiv a_o n^2 \tag{5.6}$$

a_o is called the *Bohr radius* for hydrogen atom. $r_1 = a_o = 0.529\times10^{-10}$ m = $0.529\mathring{A}$. This is the known size of the hydrogen atom!

Energies of the stationary states (§5.4) are:

$$E_n = -\frac{e^2}{8\pi\varepsilon_o r_n} = -\frac{e^2}{8\pi\varepsilon_o a_o n^2} \equiv -\frac{E_o}{n^2} \tag{5.7}$$

The lowest energy state ($n = 1$) is $E_1 = -E_o$ where

$$E_o = \frac{e^2}{8\pi\varepsilon_o a_o} = 13.6 \text{ eV}$$

This is the experimentally measured ionization (binding) energy of the hydrogen atom!

To obtain the spectral line wavelengths,

$$\begin{aligned} E_i - E_f &= hf = \frac{hc}{\lambda} \\ \Rightarrow \frac{1}{\lambda} &= \frac{1}{hc}(E_i - E_f) = \frac{1}{hc}\left(-\frac{E_o}{n_i^2} + \frac{E_o}{n_f^2}\right) \\ \therefore \frac{1}{\lambda} &= \frac{E_o}{hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \\ \text{but } \frac{E_o}{hc} &= \frac{13.6 \times 1.6022 \times 110^{-19}}{6.6261 \times 3 \times 10^8} \\ &= 1.097 \times 10^7 \text{m}^{-1} = R \text{ - the Rydberg Constant!} \end{aligned}$$

Therefore

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \tag{5.8}$$

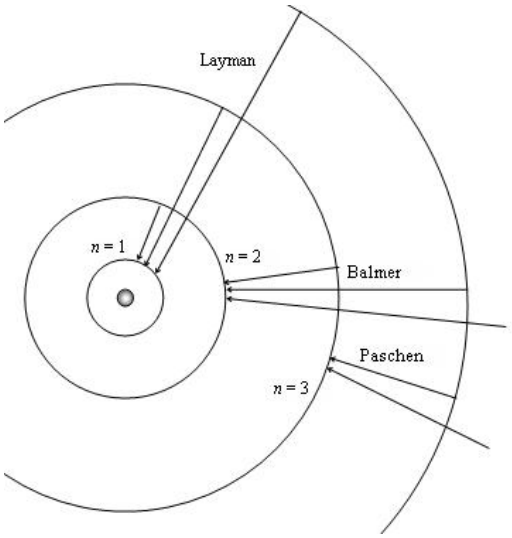


Figure 5.2: Electron orbits in Bohr atomic model

where

$$\begin{aligned} n_f &= 1,\ n_i = 2,3,4,\ldots \text{ Layman series} \\ n_f &= 2,\ n_i = 3,4,5,\ldots \text{ Balmer series} \\ n_f &= 3,\ n_i = 4,5,6,\ldots \text{ Paschen series} \\ n_f &= 4,\ n_i = 5,6,7,\ldots \text{ Brackett series} \\ n_f &= 5,\ n_i = 6,7,8,\ldots \text{ Pfund series} \end{aligned}$$

Instead of drawing orbits to the scale of their radii, it is customary to draw horizontal lines on an energy scale and call it *energy level diagram.*

5.4 The Correspondence Principle

In 1923, Bohr gave a new principle in an effort to make use of the classical theory as a limiting case to infer some properties of atomic systems. This principle, called the *correspondence principle*, asserts that the quantum and classical results agree in the region of high quantum numbers ($n \to \infty$). As an example, classically the frequency of the radiation emitted by an orbiting electron is equal to the orbital frequency f_{orb} of the electron around the nucleus;

$$f_{\text{classical}} = f_{\text{orb}} = \frac{\omega}{2\pi} = \frac{v}{2\pi r} \tag{5.9}$$

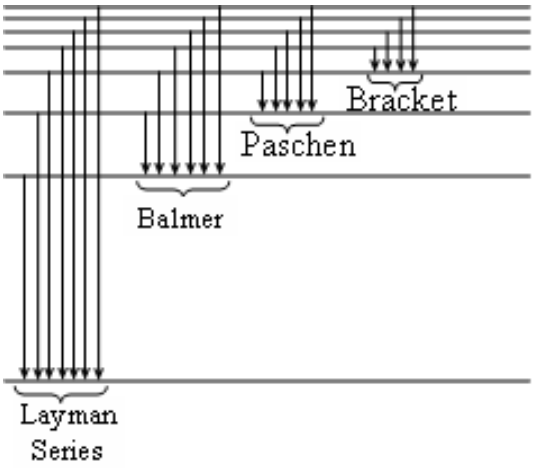


Figure 5.3: Energy Level Diagram

Substituting $v = \left(\frac{e^2}{4\pi\varepsilon_o mr}\right)^{\frac{1}{2}}$,

$$f_{\text{classical}} = \frac{1}{2\pi}\left(\frac{e^2}{4\pi\varepsilon_o mr^3}\right)^{\frac{1}{2}}$$

Inserting the orbital radius (§5.6),

$$f_{\text{classical}} = \frac{me^4}{4\pi\varepsilon_o^2 h^3}\frac{1}{n^3} \tag{5.10}$$

The frequency of the transition from $n_i = n$ to $n_f = n - p$ $(p = 1, 2, 3,)$ or $n \to n-1 \to n-2 \to n-3, \ldots$ is

$$\begin{aligned} f_{\text{Bohr}} &= \frac{E_o}{h}\left(\frac{1}{(n-p)^2} - \frac{1}{n^2}\right) \\ &= \frac{E_o}{h}\left(\frac{2np - p^2}{n^2(n-p)}\right) \end{aligned}$$

For large n, $(n >> p)$

$$f_{\text{Bohr}} \approx \frac{2npE_o}{hn^4} = \frac{2pE_o}{hn^3}$$

From §5.7 and §5.6

$$f_{\text{Bohr}} = \frac{me^4}{4\pi\varepsilon_o^2 h^3}\frac{p}{n^3} = f_{\text{classical}} \quad \text{for } p = 1\ ! \tag{5.11}$$

Harmonics of f_{Bohr} are radiated when $p = 2, 3, 4, \ldots$ So the frequencies of the radiated energy agree between classical theory and the Bohr model for large values of the quantum number n.

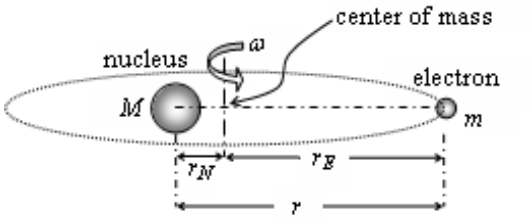


Figure 5.4: Finite nuclear mass motion

5.5 Nuclear Motion and Reduced Mass

Wavelength measurement for the atomic spectrum of hydrogen and hydrogenic atoms exhibit a small disagreement if an infinite nuclear mass is assumed. Since the nucleus has a finite mass, the electron and the nucleus rotate about a common center of mass that is located near the nucleus. The total angular momentum L of the system is

$$L = m\omega r_E^2 + M\omega r_N^2 \tag{5.12}$$

According to Bohr's second postulate (Eq. 5.3)

$$L = n\hbar$$

$$\Rightarrow m\omega r_E^2 + M\omega r_N^2 = n\hbar \tag{5.13}$$

but

$$mr_E = Mr_N \text{ (torque conserved)} \tag{5.14}$$

$$r = r_E + r_N \tag{5.15}$$

$$\Rightarrow r_E = \left(\frac{M}{M+m}\right) r \tag{5.16}$$

$$r_N = \left(\frac{m}{M+m}\right) r \tag{5.17}$$

So §5.13 becomes

$$\left(\frac{mM}{M+m}\right) \omega r^2 = n\hbar \tag{5.18}$$

The centripetal force depends on r_E while the electrostatic force depends on r;

$$\frac{mv_E^2}{r_E} = m\omega^2 r_E = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2}$$

$$\Rightarrow \left(\frac{mM}{M+m}\right) \omega r = \frac{1}{4\pi\varepsilon_o}\frac{e^2}{r^2} \tag{5.19}$$

If we let m', called the *reduced mass*, as

$$m' = \frac{mM}{M+m} \tag{5.20}$$

Then

$$E_n = -\frac{E_o}{n^2} = -\frac{m'e^4}{8\varepsilon_o^2 h^2}\frac{1}{n^2} \tag{5.21}$$

$$R = \frac{M}{M+m}R_\infty \tag{5.22}$$

where R_∞ is the Rydberg constant for infinite nuclear mass (§5.8) and should be replaced by R for finite nuclear mass.

Example 5.2: Calculate the wavelength for $n_i = 3 \rightarrow n_f = 2$ transition (called H_α line) for atoms of hydrogen, deuterium and tritium. Note that:

	mass
hydrogen	1.007825 u
deuterium	2.014102 u
tritium	3.016049 u
electron	0.0005485799 u

where $1\text{u} = 1.66054 \times 10^{-27}$ kg

5.6 Hydrogenic Atoms

The Bohr model may be applied to any single-electron atom (*hydrogenic*). The only change needed is in Coulomb force where e^2 is replaced by Ze^2 to account for the nuclear charge $+Ze$. Therefore,

$$F = \frac{1}{4\pi\varepsilon_o}\frac{Ze^2}{r^2} \tag{5.23}$$

$$E_n = -\frac{m'Z^2e^4}{4\pi\varepsilon_o}\frac{1}{n^2} \tag{5.24}$$

$$\frac{1}{\lambda} = Z^2R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \tag{5.25}$$

5.7 Failures of the Bohr Atomic Model

In spite of the fact that the Bohr atomic model gives satisfactory results for hydrogen atom, it suffers from some serious defects:

1. It can be applied to only hydrogen and hydrogenic atoms. It fails for most other elements.

2. It fail to explain the *fine structure* of even hydrogen atoms. (Latter explained by Sommerfield.)

3. It fails to explain the relative intensities of the spectral lines.

4. It could not explain the appearance of a large number of spectral lines when the source is placed under the effect of an electric and magnetic fields (*Stark* and *Zeeman effects*, respectively.)

5. It could not explain the binding of atoms into molecules.