

Chapter Four

Soil Permeability and seepage analysis

Many catastrophic failures in geotechnical engineering result from instability of soil masses due to groundwater flow. Lives are lost, infrastructures are damaged or destroyed, and major economic losses occur. In this chapter, you will study the basic principles of two-dimensional flow of water through soils. The topics covered here will help you to avoid pitfalls in the analyses and design of geotechnical systems where groundwater flow can lead to instability.

Sample Practical Situation A deep excavation is required for the construction of a building. The soil is silty sand with groundwater level just below ground level. The excavation cannot be made unless the sides are supported. You, a geotechnical engineer, are required to design the retaining structure for the excavation and to recommend a scheme to keep the inside of the excavation dry.

4.1 Permeability

Permeability is defined as the property of soil which permits flow of water through it. A soil is highly pervious when water can flow through it easily, e.g. **Gravels**. In an impervious soil, the permeability is very low and water cannot easily flow through it, e.g. **clays**. Permeability is a very important engineering property of soils. Knowledge of permeability is essential for,

- Settlement of buildings
- Yield of wells
- Seepage through and below the earth structures
- Earth pressures
- Uplift pressure under hydraulic structures

4.2 Some Definitions

1) Total Head

Darcy's law governs the flow of water through soils. But before we delve into Darcy's law, we will discuss an important principle in fluid mechanics – Bernoulli's principle – which is essential in understanding flow through soils.

If you cap one end of a tube, fill the tube with water, and then rest it on your table (Fig. 4.1), the height of water with reference to your table is called the pressure head (h_p).

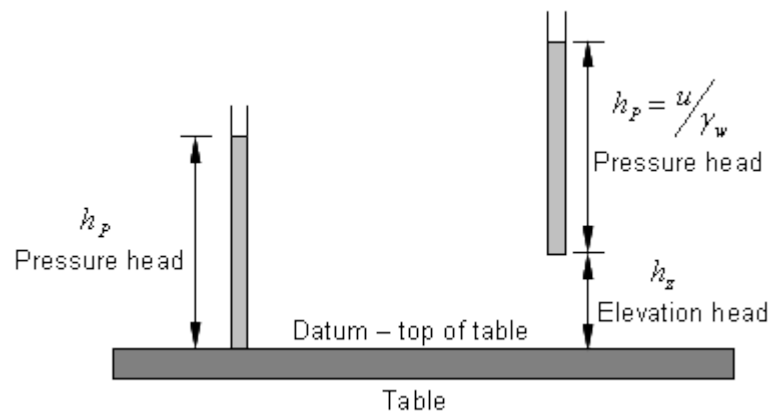


Figure 4.1: Illustration of elevation and pressure heads

Head refers to the mechanical energy per unit weight. If you raise the tube above the table, the mechanical energy or total head increases. You now have two components of total head - the pressure head (h_p) and the elevation head (h_z). If water were to flow through the tube with a velocity v , under steady state condition, then we have an additional head due to the velocity given as $v^2/2g$. The total head (sometimes called piezometric head), H , according to Bernoulli's principle is:

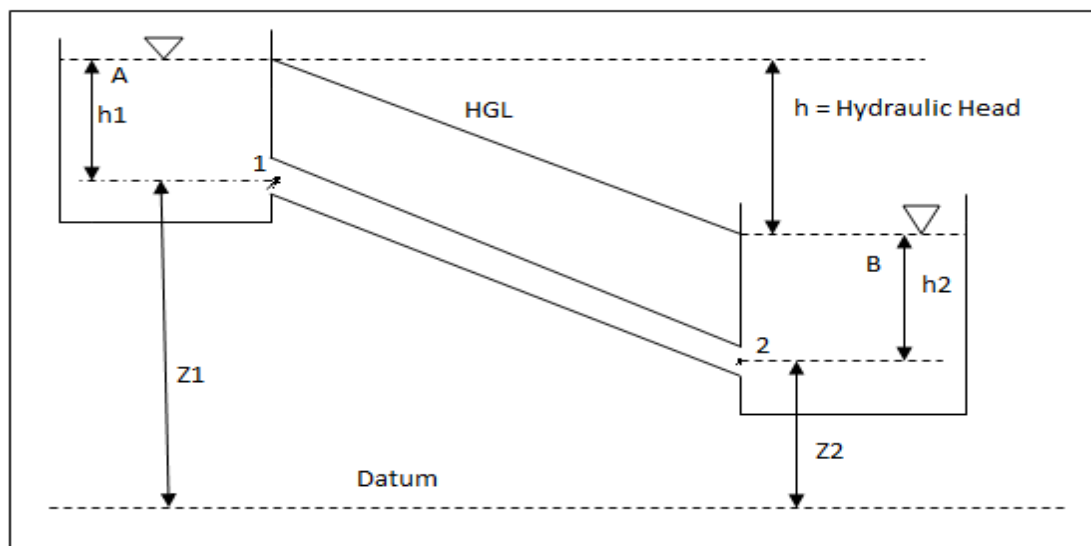


Figure 4.2 Various Heads

$$H = h_z + h_p + \frac{v^2}{2g}$$

The elevation or potential head is referenced to an arbitrary datum and the total head will change depending on the choice of the datum position. Therefore, it is essential that you identify your datum position in solutions to flow problems. Pressures are defined relative to atmospheric pressure. The velocity of flow through soils is generally small (< 1 cm/s) and we usually neglect the velocity head. The total head in soils is then

$$H = h_z + h_p = h_z + \frac{u}{\gamma_w}$$

Where $u = \gamma_w h_p$ is the pore water pressure.

Consider a cylinder containing a soil mass with water flowing through it at a constant rate as depicted in Fig. 4.3. If we connect two tubes, A and B, called piezometers, at a distance l apart, the water will rise to different heights in each of the tubes. The height of water in tube B near the exit is lower than A. Why? As the water flows through the soil, energy is dissipated through friction with the soil particles, resulting in a loss of head. The head loss between A and B, assuming decrease in head is positive and our datum is arbitrarily selected at the top of the cylinder, is $\Delta H = |(h_p)_B - (h_p)_A|$.

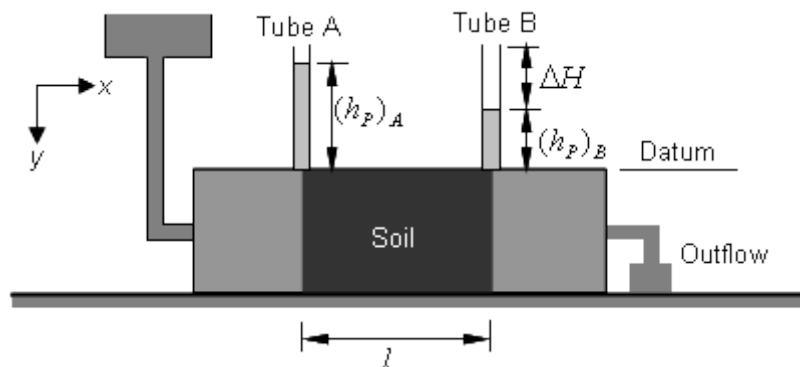


Figure 4.3: Head loss due to flow of water through soil.

2) Hydraulic Head (h)

Hydraulic head is equal to the difference in the elevations of water levels at the entry and exit points in a soil mass. Obviously, it is equal to the loss of head through the soil. The hydraulic head is also known as the effective head.

3) Hydraulic gradient (i)

The loss of head per unit length of flow through the soil is known as hydraulic gradient.

$$i = \frac{h}{L} \quad \text{where, } h = \text{Hydraulic Head, } L = \text{Length of soil specimen}$$

4) Percolation (Seepage)

The rain water after falling on the earth surface, some part of it seeps through the soil and meet the ground water. This process is called percolation. Seepage is the flow of water under gravitational forces in a permeable medium.

5) Discharge Velocity (v)

The discharge of water per unit of total cross-sectional area (A) of soil is known as discharge velocity.

$$V = \frac{q}{A}$$

Where, q = discharge per unit time, A = total cross-section of soil

6) Piping

Hydraulic structures, such as weirs and dams, built on pervious foundations sometimes fail by formation of a pipe-shaped channel in its foundation, known as piping failure.

Backward erosion of soil is caused by the percolating water, and the piping begins when the hydraulic gradient at exit (exit gradient) exceeds the critical gradient (i_c). The soil at the exit is removed by the percolating water. When the soil near the exit has been removed, the flow net gets modified. There is more concentration of the flow lines in the remaining soil mass, resulting in an increase of the exit gradient. This causes further removal of the soil. This process of backward erosion continues towards upstream reservoir and a sort of pipe is formed. A large amount of water rushes through the channel so formed and the hydraulic structure fails.

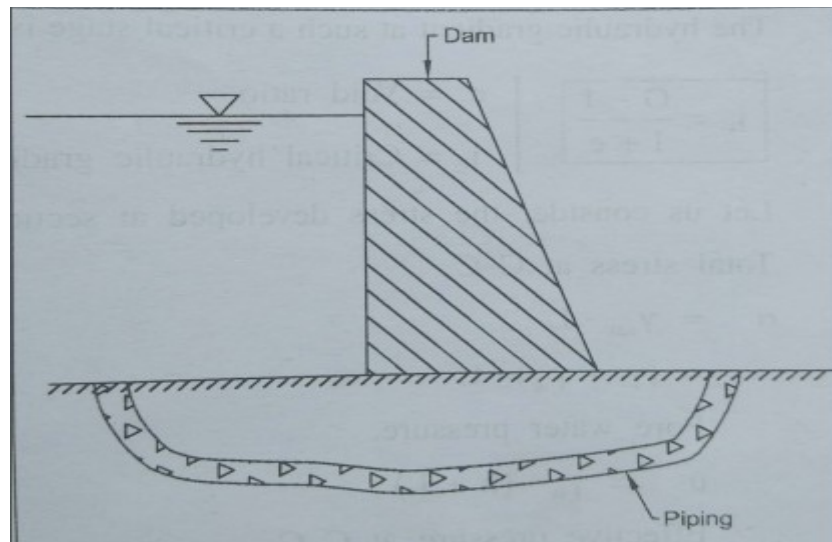


Figure 4.4 Piping

4.3 Darcy's Law

The law of flow of water through soil was first studied by Darcy in 1856. The Darcy's law is, **"For laminar flow through saturated soil mass, the discharge per unit time is proportional to the hydraulic gradient."**

$$q = k.i.A$$

$$\frac{q}{A} = k.i$$

$$V = k.i \dots \dots \dots \text{Darcy's law}$$

Where, q = discharge per unit time (rate of flow)

A = total cross-sectional area of soil mass.

i = Hydraulic gradient = h/L

k = Darcy's coefficient of Permeability

v = velocity of flow (discharge velocity)

4.3.1 coefficient of permeability

The average velocity of flow through total cross-sectional area of soil mass under unit hydraulic gradient is called coefficient of permeability (k).

$$v = k$$

The unit of k is cm/sec or m/day

Assumptions of Darcy's law

The following assumptions are made in Darcy's law

- 1) The soil is saturated

- 2) The flow through soil is laminar
- 3) The flow is continuous and steady
- 4) The total cross-sectional area of soil mass is considered

4.3.2 validity of Darcy's law

- 1) Darcy's law is valid if the flow through soils is laminar:

The flow of water through soils depends upon the dimension of particles. In fine-grained soils the dimensions of the interstices (voids) are very small and flow is necessarily laminar. In coarse-grained soils, the flow is also laminar. However, in very coarse grained soils, such as gravels, the flow may be turbulent. For flow through soils, the flow is laminar if the Reynolds number is less than unity.

$$\frac{v \cdot Da \cdot \rho_w}{\eta \cdot g} \leq 1$$

Where, v = velocity of flows in cm/sec

Da = diameter of average particle (cm)

ρ_w = density of water (gm/cc)

η = viscosity of water (gm.sec/cm²)

g = acceleration due to gravity (cm/cm²)

- 2) As per Allen Hazen, the maximum diameter of the particle for the flow to be laminar is about 0.5mm
- 3) It is valid for flow in clays, silts and fine sands. In coarse sands, gravels and boulders, the flow may be turbulent and Darcy's law may not be applicable.
- 4) For Darcy's law to be valid, the relationship between velocity (v) and hydraulic gradient (i) should be linear.

4.3.3 Factors affecting permeability of soils

The following factors affect the permeability of soils:

- 1) Particle size
- 2) Properties of pore fluids
- 3) Void ratio of soils
- 4) Shape of particles
- 5) Structure of soil mass
- 6) Degree of saturation

7) Impurities in water

4.3.4 Determination of Coefficient of Permeability

Typical values of k for various types of soil are shown in Table 4.1. This table illustrates the enormous range of values of permeability for soils.

Table 4.1 Typical values of coefficient of permeability

Soil Type	K values
Gravel	Greater than 10^{-2} m/sec
Sandy	10^{-6} m/sec to 10^{-2} m/sec
Silt	10^{-9} m/sec to 10^{-5} m/sec
Clay	10^{-11} m/sec to 10^{-8} m/sec

Homogeneous clays are practically impervious. Two popular uses of “impervious” clays are in dam construction to curtail the flow of water through the dam and as barriers in landfills to prevent migration of effluent to the surrounding area. Clean sands and gravels are pervious and can be used as drainage materials or soil filters.

Several empirical equations have been proposed for the evaluation of the coefficient of permeability. One of the earliest is that proposed by Hazen for uniform sands

$$K \text{ (cm/sec)} = C_1 D_{10}^2$$

Where, D_{10} is effective size in mm

C_1 is constant, varying from 1.0 to 1.5

The various methods to determine coefficient of permeability are as under.

1. Laboratory Methods

A) **Constant Head Permeability Test.** The value of k could be determined by means of the following equation and depicted in figure 4.5

$$Q = k i A$$

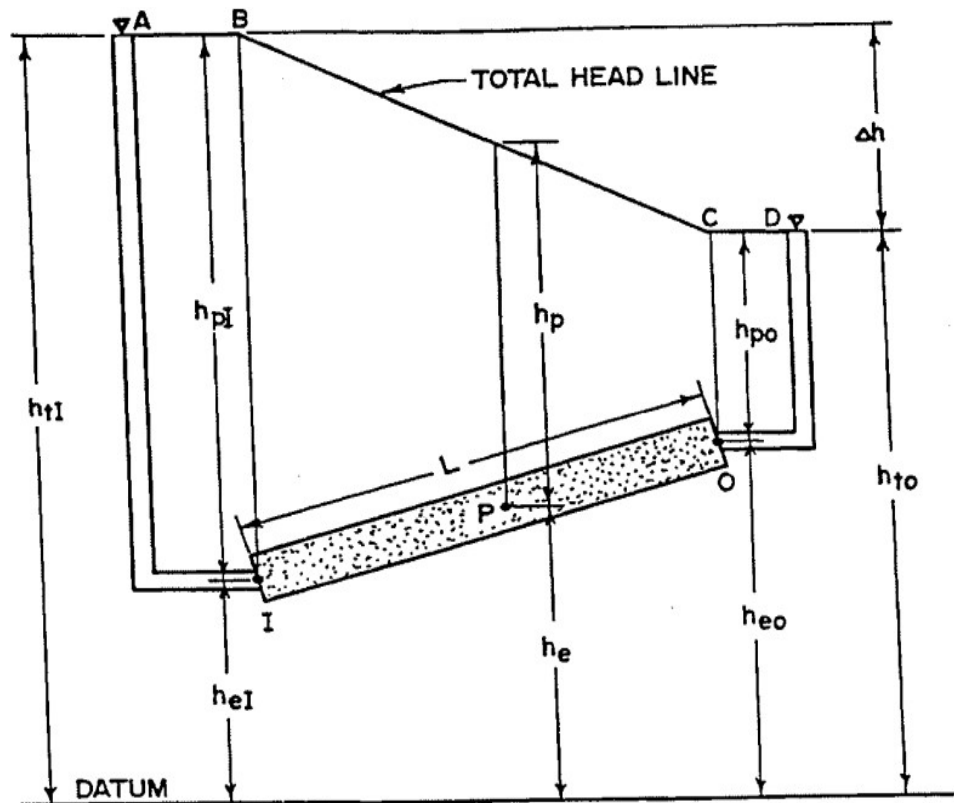


Figure 4. 5: One dimension seepage

The reservoir levels at A and D would be maintained constant allowing steady state seepage through the soil sample to be established. The rate of discharge Q would be observed over a convenient time interval.

The permeability could then be determined from

$$K = \frac{Q}{Ai} = \frac{QL}{A\Delta h}$$

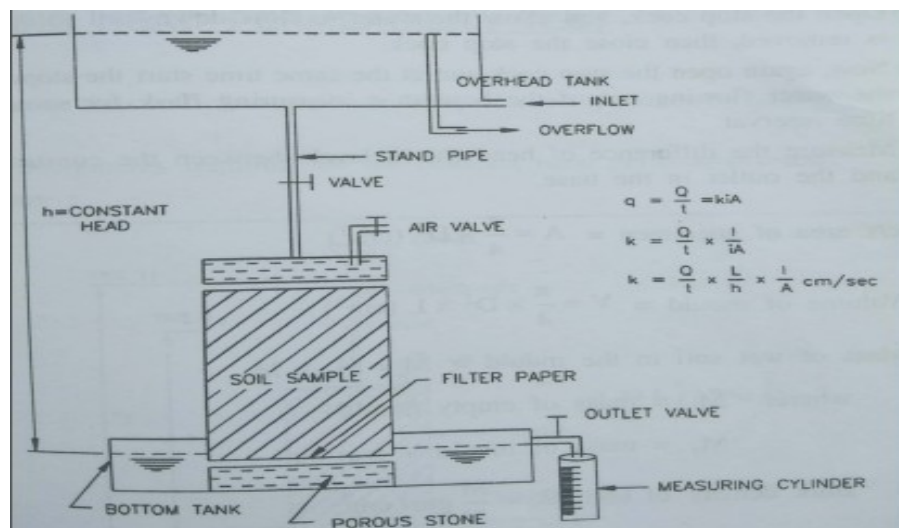


Figure 4.6 Constant head permeability test

$$\text{Coefficient of permeability } K = \frac{Q}{t} \frac{L}{h} \frac{1}{A} \text{ (cm/sec)}$$

Sr. No.	Quantity of water collected (Q) ml	Duration (t) sec	Constant head (h) cm	i=h/L	K = $\frac{Q}{t} \frac{1}{i.A}$
1.					
2.					
3.					

Results:

The coefficient of permeability of a given soil sample is -----cm/sec.

B) Falling Head Permeability Test:

The falling head permeability test differs from the constant head permeability test in that the upstream and/or downstream total heads do not remain constant throughout the test.

Figure 4.7 depicts a situation in which the upstream water level is allowed to fall in a standpipe having across-sectional area a.

The test commences with a head difference of h_i between the upstream and downstream water levels. At any time t after the commencement of the test the head difference will be symbolized by h . At the end of the test time t_1 , the head difference is h_f , throughout the test the downstream water level remains constant.

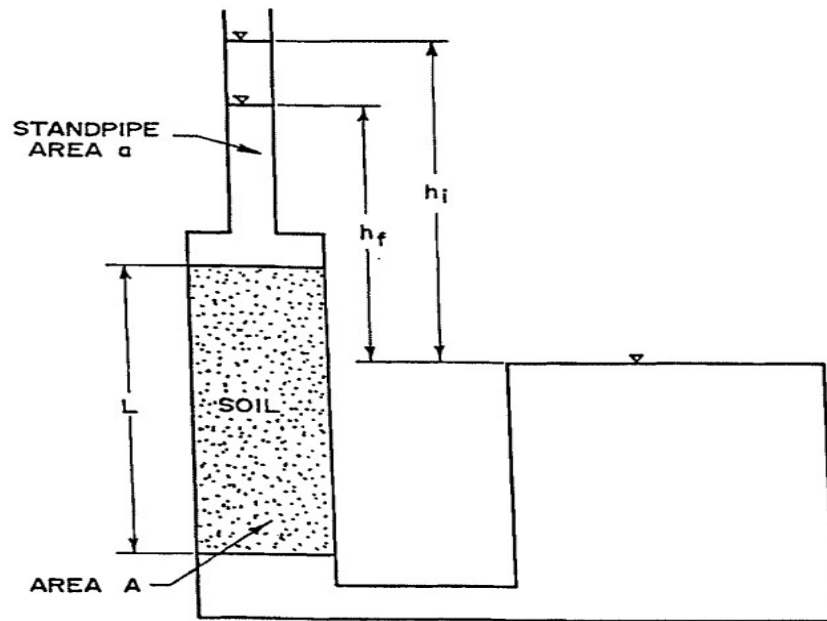


Fig. 4.7 Falling Head Permeability Test

At time t , the rate of discharge is

$$Q = k i A$$

$$= k \frac{h}{L} A$$

and this must be equal to the rate of discharge through the standpipe

$$Q = -a \frac{dh}{dt}$$

$$-a \frac{dh}{dt} = k \frac{h}{L} A$$

$$-a \int_{hi}^{hf} \frac{dh}{h} = \frac{KA}{L} \int_0^{t_1} dt$$

$$K = 2.3 \frac{aL}{At_1} \log_{10} \left(\frac{hi}{hf} \right)$$

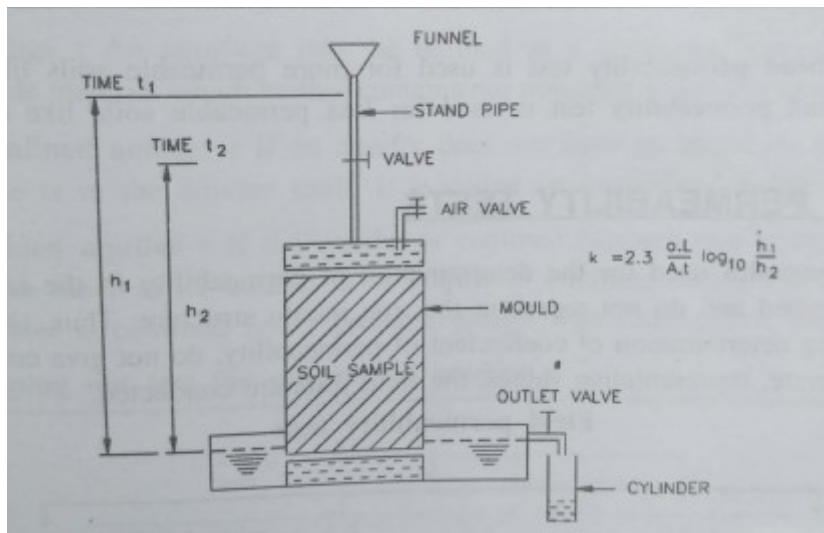


Figure 4.8: Falling head permeability test

$$\text{Coefficient of permeability (k)} = 2.3 \frac{a \cdot L}{A \cdot t} \log_{10} \frac{h_1}{h_2}$$

Where, a = c/s area of standpipe (cm^2)

$$A = \text{c/s area of soil sample} = \frac{\pi}{4} \cdot D^2 \text{ (cm}^2\text{)}$$

L = length of soil sample (cm)

t = time interval to fall head from h_1 to h_2

h_1 = initial head (cm)

h_2 = final head (cm)

Normally,

- Constant head permeability test is used for more permeable soils like sand
- Falling head permeability test is used for less permeable soils like clay.

2. Field Determination Methods

One of the many methods of determining k in the field is by means of a pump out test (USBR, 1974).

A horizontal stratum of pervious soil containing a water table overlies an impervious stratum.

A well is sunk to the bottom of the pervious material and water is pumped from the well at a constant rate Q until

steady state conditions are reached. Observation wells at radial distances r_1 and r_2 permit measurement of the heights h_1 and h_2 respectively.

Field determination method can be Pumping out tests and Pumping in tests

Pumping out tests

For large engineering projects, it is the usual practice to measure the permeability of soils by pumping out tests. This method is very useful for homogeneous coarse grained soils for which it is difficult to obtain undisturbed samples. In a pumping out test, the soil deposit over a large area is influenced, and therefore the results represent coefficient of permeability of a large mass of soil. However, the pumping out tests is very costly.

Aquifer: an aquifer may be defined as a geological formation that contains sufficient permeable materials which permits storage as well as movement of water through it under ordinary field conditions.

E.g. sands, gravels

Aquiclude: an aquiclude may be defined as a geological formation of relatively impermeable material which permits storage of water but it is not capable of transmitting water in sufficient quantity. For example, clay

Aquitard: a geological formation that has poor permeability, but through which seepage is possible, and hence, it does not yield water freely to wells. It may transmit vertically appreciable quantities of water to or from adjacent aquifers.

Sandy clay is an example of Aquitard.

Aquifuge: an Aquifuge may be defined as a geological formation of relatively impermeable material which neither contains nor transmits water. For example, solid rocks.

Unconfined aquifer: if an aquifer does not have an aquiclude at its top and the water table is in the aquifer itself, it is called an unconfined aquifer or free aquifer.

Confined aquifer: if the aquifer is confined between two aquicludes, one at its top and the other at its bottom, it is known as confined aquifer. In

confined aquifer, ground water is confined under pressure greater than atmospheric pressure.

(a) Pumping out test for unconfined aquifer

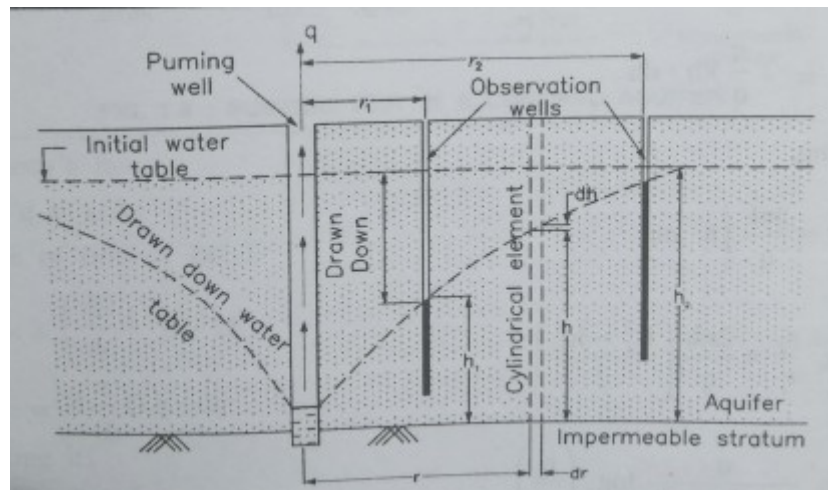


Figure 4.9: Unconfined flow pumping test

In an unconfined aquifer, a tube well is drilled up to the impervious stratum. The fuse used for the well is perforated so that water can enter the well. The tube is surrounded by a screed called strainer to check the flow of soil particles into the well. Water is pumped out of the tube well till a steady state is reached. At that stage, the discharge becomes constant and the water level in the well does not change. The water table which was originally horizontal before the pumping was started is depressed near the well. The water table near the well forms an inverted cone, known as the cone of depression. The maximum depression of the water table is known as the drawdown.

The expression for the coefficient of permeability can be derived by Dupuit's radial flow theory.

Consider the flow through an elementary cylinder of soil having radius r , thickness dr and height h .

Hydraulic gradient (i),

$$i = \frac{dh}{dr}$$

$$\text{Area of flow} = A = 2\pi rh$$

From Darcy's law,

$$q = KiA$$

$$= k \left(\frac{dh}{dr} \right) 2\pi r h$$

$$\square \frac{dr}{r} = 2 \frac{\pi}{q} k h \cdot dh$$

Integrating,

$$\int_1^2 \frac{dr}{r} = \frac{2\pi k}{q} \int_1^2 h \cdot dh$$

$$\text{Or } \log_e \frac{r_2}{r_1} = \frac{2\pi k}{q} \left(\frac{h_2^2 - h_1^2}{2} \right)$$

$$\square k = \frac{q}{\pi(h_2^2 - h_1^2)} \log_e \left(\frac{r_2}{r_1} \right)$$

$$\square k = \frac{2.3 q}{\pi(h_2^2 - h_1^2)} \log_{10} \left(\frac{r_2}{r_1} \right)$$

(b) Pumping out test for confined aquifer

A confined flow condition occurs when the aquifer is confined both above and below by permeable strata. Here, the drawdown surface is, for all values of r , above the upper surface of the aquifer.

Consider the flow after steady state is reached.

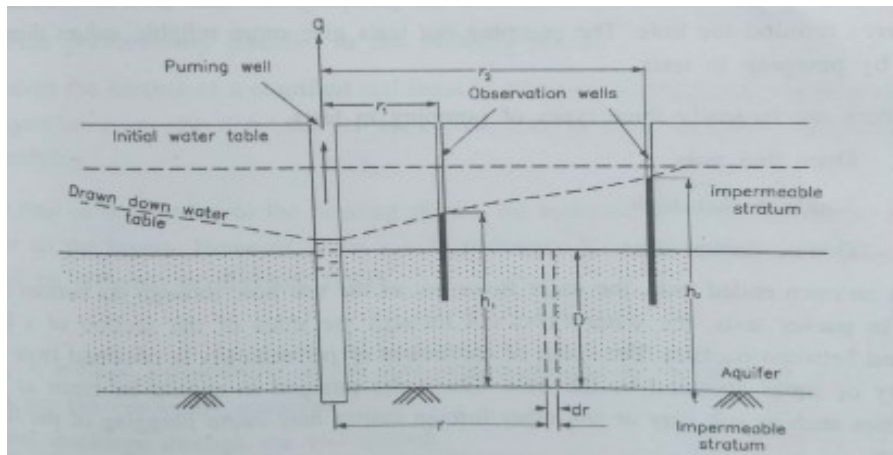


Figure 4.10: Pumping test in a confined aquifer

From Darcy's law,

$$q = k i A$$

$$\text{Area of flow} = 2\pi r D$$

$$\square q = k \left(\frac{dh}{dr} \right) 2\pi r D$$

$$\square \frac{dr}{r} = 2 \frac{\pi}{q} D k \cdot dh$$

Integrating it,

$$\int_1^2 \frac{dr}{r} = \frac{2\pi k}{q} \int_1^2 D \cdot dh$$

$$\log_e \frac{r_2}{r_1} = \frac{2\pi Dk}{q} (h_2 - h_1)$$

$$k = 2.3 \frac{q}{2\pi D(h_2 - h_1)} \log_{10} \left(\frac{r_2}{r_1} \right)$$

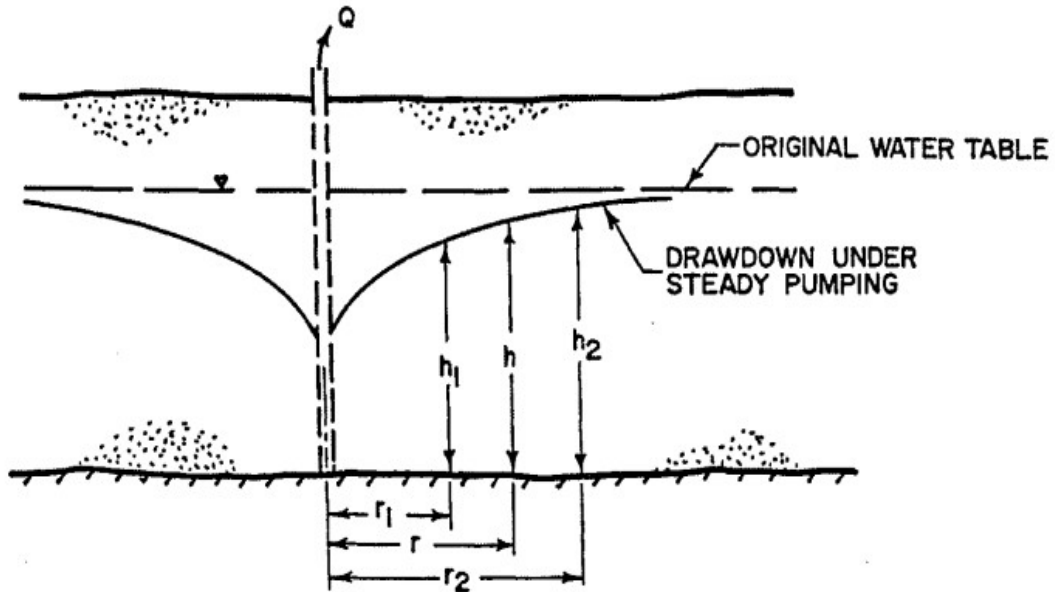


Fig. 4.11: Field Pump out Test

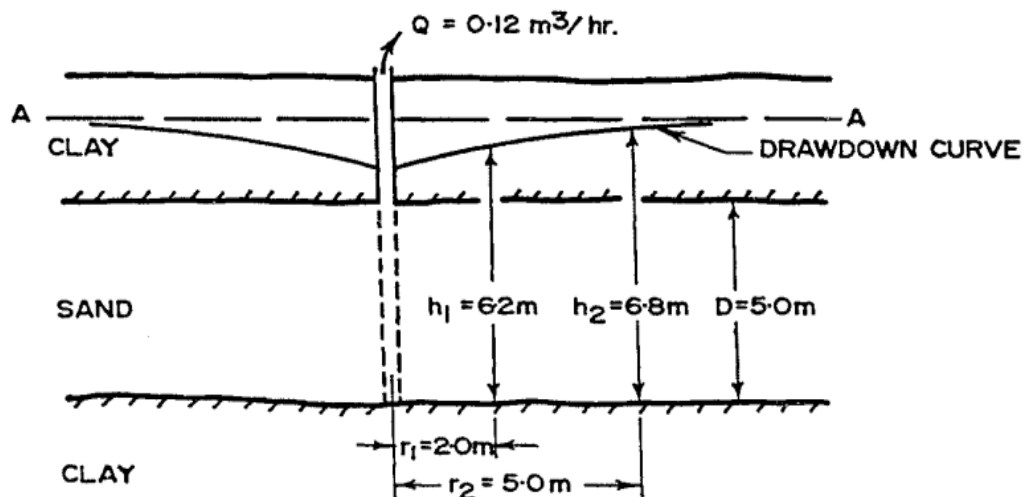


Fig. 4.12: Pump out Test in a Confined Aquifer

4.3.5 Permeability of Stratified Soil Deposits

In nature, soil mass may consists of several layers having different permeability. Their bedding planes may be horizontal, inclined or

vertical. The average permeability of the whole deposit parallel to the planes and perpendicular to the planes can be determined as follows.

(a) Average permeability parallel to the bedding planes

Consider the section of a stratified soil mass as shown in the following figure. Let the thickness of the layers be $z_1, z_2, z_3 \dots z_n$ and let $k_1, k_2, k_3 \dots k_n$ be their respective coefficients of permeability. For flow to be parallel to the bedding planes, the hydraulic gradient (i) will be the same for all the layers. However, since $v = ki$ and since k is different, the velocity of flow will be different in different layers.

Let,

K_h = average permeability of the soil deposit parallel to the bedding plane.

As per the continuity equation,

Total discharge through the soil deposit = sum of discharge through the individual layers.

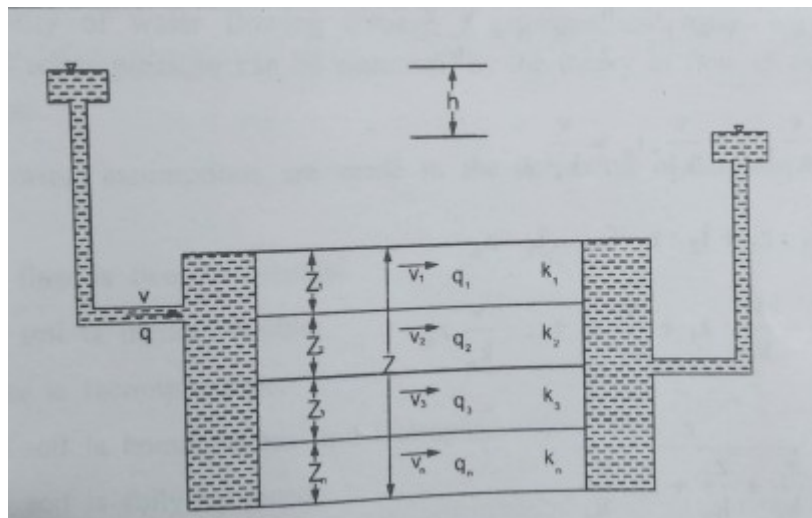


Figure 4.13: Flow parallel to bedding plane

$$q = q_1 + q_2 + \dots q_n$$

$$q = k_h \cdot i \cdot z, \quad z = z_1 + z_2 + \dots z_n$$

$$\square \quad k_h \cdot i \cdot z = k_1 \cdot i \cdot z_1 + k_2 \cdot i \cdot z_2 + \dots + k_n \cdot i \cdot z_n$$

$$K_h = \frac{k_1 \cdot z_1 + k_2 \cdot z_2 + \dots k_n \cdot z_n}{z}$$

(b) Average permeability perpendicular to the bedding planes

In this case, the velocity of flow, and hence the unit discharge will be the same through each layer. However, the hydraulic gradient and hence the head loss through each layer will be different.

Let, $h_1, h_2, \dots, h_n$ be the head loss through each layer.

$$h = h_1 + h_2 + h_3 + \dots + h_n$$

$$\text{but, } h_1 = i_1 \cdot Z_1, h_2 = i_2 \cdot Z_2 \text{ and } h_n = i_n \cdot Z_n$$

therefore,

$$h = i_1 \cdot Z_1 + i_2 \cdot Z_2 + \dots + i_n \cdot Z_n$$

let, k_v = average permeability perpendicular to the bedding plane

$$v = k_v \cdot i = k_v \cdot \frac{h}{Z} \text{ or } h = \frac{v \cdot Z}{k_v}$$

$$\text{also } i_1 = \frac{v}{k_1}, i_2 = \frac{v}{k_2}, i_n = \frac{v}{k_n}$$

$$\text{Therefore, } h = i_1 \cdot Z_1 + i_2 \cdot Z_2 + \dots + i_n \cdot Z_n$$

$$\frac{v \cdot Z}{k_v} = \frac{v}{k_1} \cdot Z_1 + \frac{v}{k_2} \cdot Z_2 + \dots + \frac{v}{k_n} \cdot Z_n$$

Therefore,

$$K_v = \frac{Z}{\frac{Z_1}{k_1} + \frac{Z_2}{k_2} + \dots + \frac{Z_n}{k_n}}$$

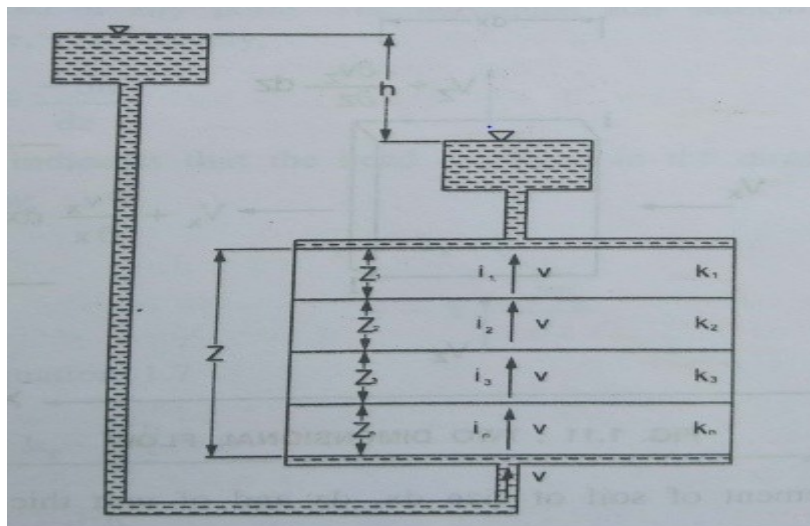


Figure 4.14 Flow perpendiculars to bedding plane

4.4 Laplace Equation (2-D flow)

The quantity of water flowing through a saturated soil mass, as well as the distribution of water pressure can be estimated by the theory of flow

of fluids through porous medium. The following assumptions are made in the derivation of Laplace equation for 2-D flow.

1. The flow is two-dimensional
2. The soil is incompressible
3. Water is incompressible
4. The soil is homogenous and isotropic
5. The soil is fully saturated
6. The flow is steady, i.e. the flow conditions do not change with time.
7. Darcy's law is valid

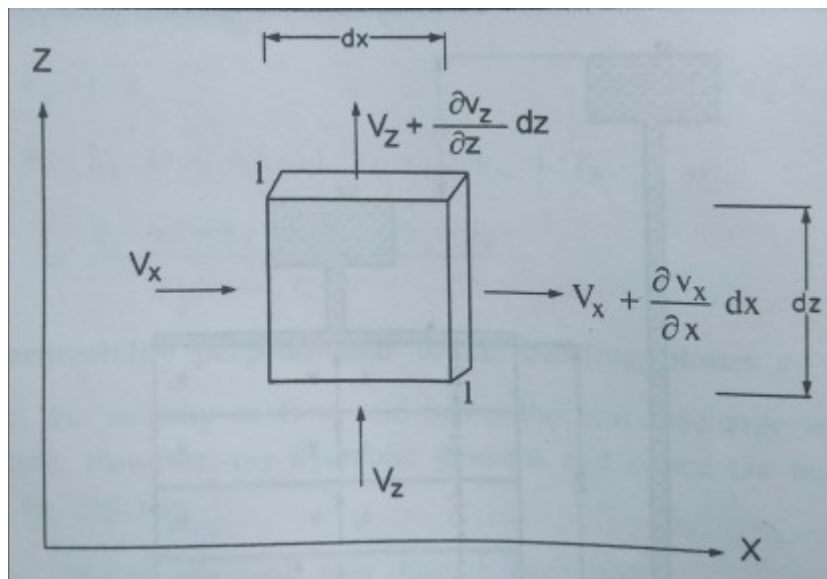


Figure 4.15 Two dimensional flows

Consider an element of soil of size dx , dz , and of unit thickness. Let the velocity of the inlet and outlet faces be V_x and $(V_x + \frac{\partial V_x}{\partial x} \cdot dx)$ in x -direction and v_z and $(v_z + \frac{\partial V_z}{\partial z} \cdot dz)$ in z -direction. As the flow is steady and the soil is incompressible, the discharge entering the element is equal to that leaving the element.

Therefore, $V_x \cdot (dz \cdot 1) + V_z \cdot (dx \cdot 1) = (V_x + \frac{\partial V_x}{\partial x} \cdot dx) (dz \cdot 1) + (V_z + \frac{\partial V_z}{\partial z} \cdot dz) (dx \cdot 1)$

$$(\frac{\partial V_x}{\partial x} + \frac{\partial V_z}{\partial z}) dx \cdot dz = 0$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_z}{\partial z} = 0 \dots\dots\dots (a)$$

This is the continuity equation for two-dimensional flow.

Let h be the head at any point. The horizontal and vertical components of the hydraulic gradient are, respectively,

$$i_x = \frac{-\partial h}{\partial x}, \quad i_z = \frac{-\partial h}{\partial z}$$

The minus sign indicates that the head decreases in the direction of flow.

From Darcy's law,

$$\begin{aligned} V_x &= K_x \cdot i_x & V_z &= K_z \cdot i_z \\ &= -K_x \cdot \frac{\partial h}{\partial x} & &= -K_z \cdot \frac{\partial h}{\partial z} \end{aligned}$$

Substituting in equation (a)

$$-K_x \cdot \frac{\partial^2 h}{\partial x^2} - K_z \cdot \frac{\partial^2 h}{\partial z^2} = 0$$

$$K_x \cdot \frac{\partial^2 h}{\partial x^2} + K_z \cdot \frac{\partial^2 h}{\partial z^2} = 0$$

As the soil is isotropic, $K_x = K_z$

$$\text{Therefore, } \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0 \dots\dots\dots (b)$$

This is the Laplace equation in terms of head h .

Sometimes, the Laplace equation is represented in terms of velocity potential ϕ , given by

$$\phi = -kh$$

$$\frac{\partial \phi}{\partial x} = V_x = -k \cdot \frac{\partial h}{\partial x}$$

$$\text{And } \frac{\partial \phi}{\partial z} = V_z = -k \cdot \frac{\partial h}{\partial z}$$

Substituting the values in equation (a)

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

This is the Laplace equation in terms of velocity potential. The graphical representation of the Laplace equation is a flow net.

4.5 FLOW NET

The general solution of the Laplace equation yields two set of curves orthogonal to each other. One set of curves (ψ - lines) represents the path

along which the individual particles of water seep through the soil are called stream lines or flow lines. Flow lines indicate the direction of flow. The other set of curves (ϕ - lines) represents lines of equal head and are termed **equipotential lines**. The head loss caused by water crossing two adjacent equipotential lines is termed as the potential drop.

The entire pattern of flowlines and equipotential lines is referred to as a flow net. Thus, a flow net is a graphical representation of direction of seepage and head at every point. The space between two adjacent flow lines is called the flow path or flow channel. The figure formed on the flow net between any two adjacent flow lines and adjacent equipotential lines is referred to as the field.

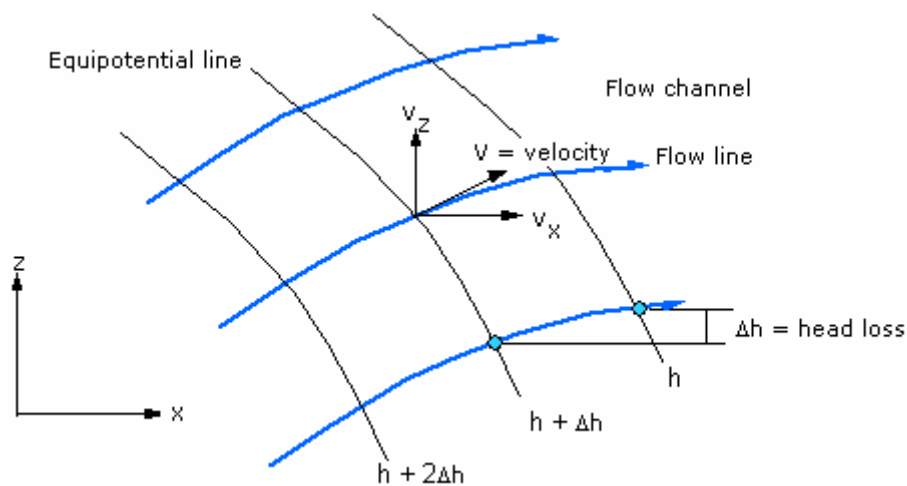


Figure 4.16: Illustration of flow terms.

Since flow lines are normal to equipotential lines, there can be no flow across flow lines. The rate of flow between any two flow lines is constant. The area between two flow lines is called a flow channel (fig. 4.17). Therefore, the rate of flow is constant in a flow channel.

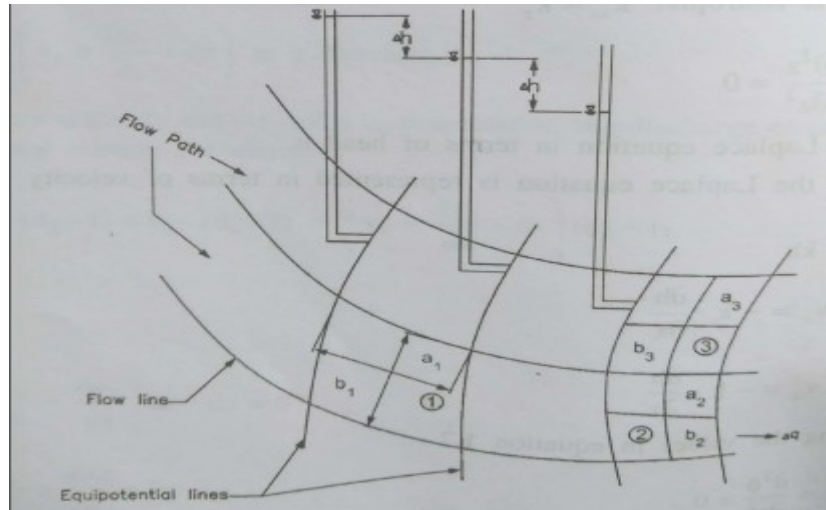


Figure 4.17: Part of flow net

Considering figure 4.3, each flow line starting at the upstream boundary with head h_1 , dissipates the head in viscous friction and attains head h_2 when terminating at the downstream boundary. All such lines are shown by continuous lines. Along each flow line there must be a point where the total head may have a specific value. A line connecting all such points of equal head represents the equipotential line, and such lines are shown by broken lines.

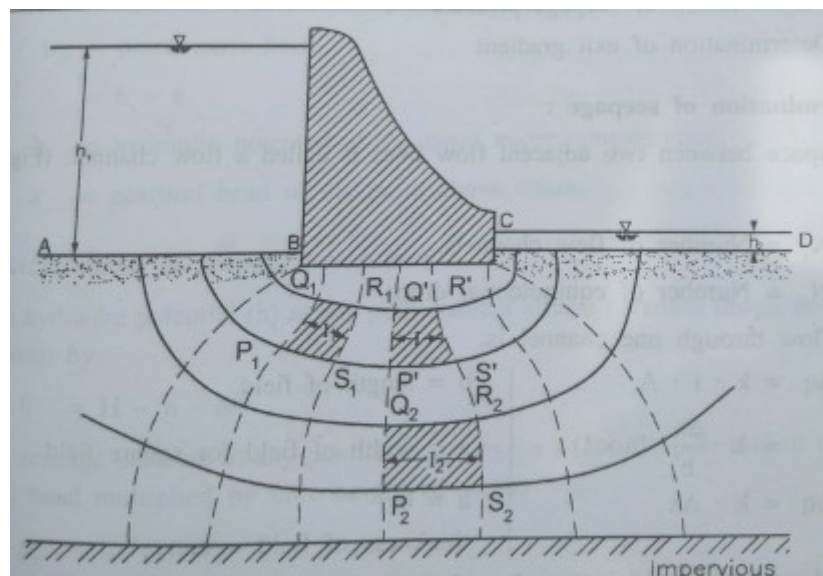


Figure 4.18: Flows under dam

4.5.1 Flow Net for Anisotropic Soil

The coefficient of permeability of stratified soil deposits parallel to the plane of stratification is generally greater than that normal to this plane. Such soils are anisotropic in permeability. Let us take the axes $x - x$ and z

- z parallel and perpendicular to the plane of stratification, respectively, therefore, $K_x > K_z$. From Darcy's law,

$$V_x = K_x \cdot i_x = -K_x \cdot \frac{\partial h}{\partial x} \text{ and}$$

$$V_z = K_z \cdot i_z = -K_z \cdot \frac{\partial h}{\partial z}$$

Substituting the values of V_x and V_z in the continuity equation $[(\frac{\partial v_x}{\partial x} +$

$$\frac{\partial v_z}{\partial z}) = 0]$$

$$\begin{aligned} \text{or} \quad & -k_x \frac{\partial^2 h}{\partial x^2} - k_z \frac{\partial^2 h}{\partial z^2} = 0 \\ & k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0 \end{aligned}$$

As the above equation is not Laplace's equation, the principles of flow net construction, as described in the preceding sections, are not applicable to anisotropic soils. The equation can however, be converted to Laplace's equation by transformation. Let the x coordinate be transformed to the new coordinate X_t by the transformation (figure 4.19)

$$X_t = x \sqrt{\frac{k_z}{k_x}}$$

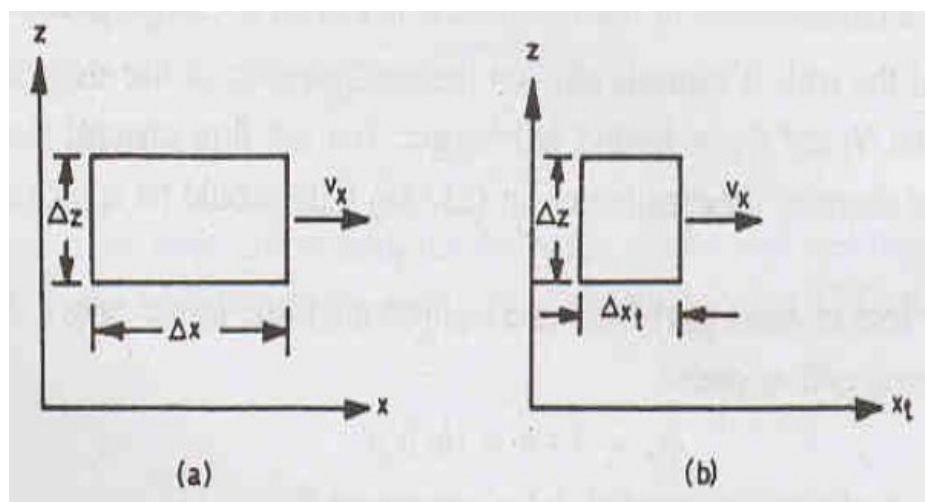


Figure 4.19: Transformations of Coordinates

The above equation can be written as

$$\left(\frac{k_x}{k_z}\right) \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

Or $\frac{\partial^2 h}{\partial x_t^2} + \frac{\partial^2 h}{\partial z^2} = 0$ this equation is the Laplace equation in x_t and z .

Therefore, the principles of flow net construction can be used for anisotropic soils after transformation.

Characteristics of flow net

1. The flow lines and equipotential lines meet at right angles to one another.
2. The fields are approximately squares, so that a circle can be drawn touching all the four sides of the square.
3. The quantity of water flowing through each flow channel (Δq) is the same.
4. The potential drop (head drop Δh) between two successive equipotential lines is also same.
5. Smaller the dimensions of the field, greater will be the hydraulic gradient and velocity of flow through it.
6. In a homogenous soil, every transition in the shape of the curves is smooth, being either elliptical or parabolic in shape.

Applications of flow net

The applications of flow net are:

1. Determination of seepage
2. Determination of uplift pressure
3. Determination of seepage pressure
4. Determination of exit gradient

1) Determination of seepage

The space between two adjacent flow lines is called a flow channel. (figure 4.17).

Let N_f = Number of flow channels

N_d = Number of equipotential drops

The flow through one channel is,

$$\Delta q = k \cdot i \cdot A$$

$$= k \cdot \frac{\Delta h}{b} \cdot (a \cdot 1)$$

$$\text{Therefore, } \Delta q = k \cdot \Delta h$$

Where, b = length of field

a = width of field for square field, $a = b$. thickness of field = unity

Let H = total head loss = $h_1 - h_2$

$$\Delta h = \frac{H}{N_d}, \text{ therefore, } \Delta q = k \cdot \frac{H}{N_d} \dots \dots \text{discharge through one channel}$$

The total discharge through the complete flow net is given as,

$$q = N_f \cdot \Delta q$$

$$= N_f \cdot k \cdot \frac{H}{N_d}$$

$$\text{Therefore, } q = k \cdot H \cdot \frac{N_f}{N_d}$$

$\frac{N_f}{N_d}$ is called the shape factor of the flow net

2) Determination of uplift pressure

The uplift pressure (hydrostatic pressure) at any point within the soil mass is given by

$$u = h_w \cdot \gamma_w$$

where, u = uplift pressure

h_w = piezometric head

$$= h - z$$

h = hydraulic potential at the point under consideration

z = position head of the point above datum

3) Determination of seepage pressure

The hydraulic potential (h) at any point located after n potential drops, each of value Δh is given by

$$h = H - n \cdot \Delta h$$

The seepage pressure at any point is equal to the hydraulic potential or the balance hydraulic head multiplied by unit weight of water.

$$P_s = h \cdot \gamma_w$$

$$= (H - n \cdot \Delta h) \cdot \gamma_w$$

The seepage pressure acts in the direction of flow

4) Determination of exit gradient

The maximum hydraulic gradient at downstream end of flow lines is termed the exit gradient. It is given by,

$$i_e = \frac{\Delta h}{l}$$

Where, Δh = potential drop

l = average length of last field in the flow net

Examples

1. Following observations are made during the falling head laboratory test.

- i) Diameter of soil specimen (D) = 100mm
- ii) Length of the soil specimen (L) = 125mm
- iii) Initial water head (h_1) = 120mm
- iv) Final water head (h_2) = 50mm
- v) Diameter of stand pipe (d) = 12mm
- vi) Time for water head to fall (t) = 250 seconds

Calculate coefficient of permeability of soil sample

Solution:

$$A = \frac{\pi}{4} \times 10^2 = 78.53 \text{ cm}^2$$

$$a = \frac{\pi}{4} \times 1.2^2 = 1.131 \text{ cm}^2$$

$$k = 2.3 \frac{a \cdot L}{A \cdot t} \log_{10} \frac{h_1}{h_2} = \underline{6.292 \cdot 10^{-4} \text{cm/sec}}$$

2. Calculate the value of the coefficient of permeability of a soil with effective diameter of 0.15mm.

Solution:

$$D_{10} = \text{effective diameter} = 0.15\text{mm} = 0.015\text{cm}$$

Taking $C = 125$

$$\begin{aligned} K &= C \cdot D_{10}^2 \\ &= 125 \cdot (0.015)^2 \\ &= \underline{0.0281 \text{cm/sec}} \end{aligned}$$

3. Calculate the value of k of a sample of 6cm height and 50cm^2 cross-sectional area, if a quantity of water of 430cm^3 flows down in 10 min under an effective constant head of 40cm. on oven drying the test specimen weighed 498g. Assume $G = 2.65$, calculate the discharge velocity and seepage velocity of water.

Solution:

$$Q = 430\text{cm}^3 (\text{ml})$$

$$L = 6\text{cm}$$

$$A = 50\text{cm}^2$$

$$t = 10\text{min.} = 600\text{sec.}$$

$$h = 40\text{cm}$$

$$\begin{aligned} k &= \frac{Q \cdot L \cdot 1}{t \cdot h \cdot A} \\ &= \underline{2.15 \cdot 10^{-3} \text{cm/sec}} \end{aligned}$$

Discharge velocity (v)

$$\begin{aligned} V &= k_i = k \cdot \frac{h}{L} \\ &= 2.15 \cdot 10^{-3} \cdot 40/6 \\ &= \underline{0.0143 \text{cm/sec}} \end{aligned}$$

$$\rho_d = \frac{Md}{V} = 498/(50 \cdot 6) = 1.66\text{gm/cm}^3$$

$$\rho_d = \frac{G \cdot \rho_w}{1+e}$$

$$e = 0.596$$

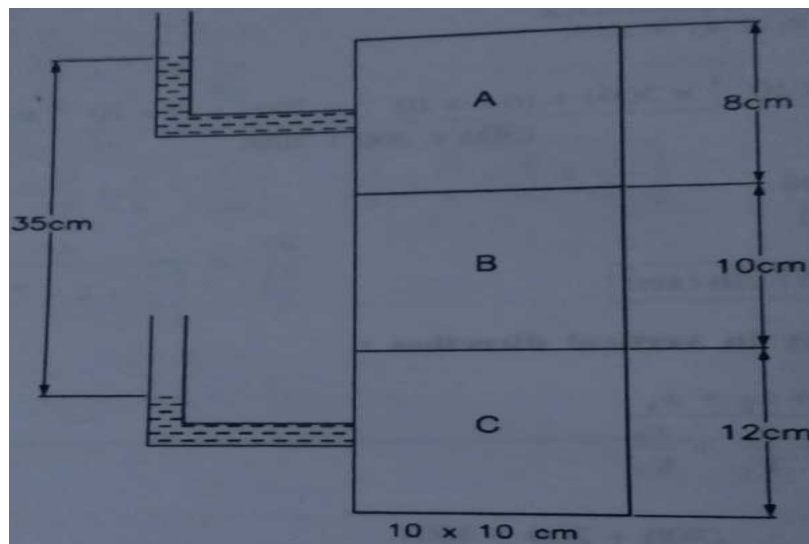
$$n = \frac{e}{1+e} = 0.373$$

Seepage velocity = V_s

$$V_s = \frac{v}{n} = \frac{0.143}{0.373} = \underline{0.0383 \text{ cm/sec}}$$

1. Three soil samples A, B and C with coefficient of permeability 2×10^{-2} , 3×10^{-3} , and $5 \times 10^{-4} \text{ cm/sec}$ are placed in a tube of cross-section $10 \times 10 \text{ cm}$ as shown in figure. Water is supplied through the apparatus such that the head difference is maintained at 35cm. find the rate of supply in lit/hour.

Solution:



$$h = 35 \text{ cm}$$

k = average coefficient of permeability perpendicular to plane

$$k = \frac{\frac{z_1}{k_1} + \frac{z_2}{k_2} + \frac{z_3}{k_3}}{\frac{z_1 + z_2 + z_3}{k_1 + k_2 + k_3}}$$

$$= \frac{8 + 10 + 12}{\frac{8}{2 \times 10^{-2}} + \frac{10}{3 \times 10^{-3}} + \frac{12}{5 \times 10^{-5}}}$$

$$K = \underline{1.23 \times 10^{-4} \text{ cm/sec}}$$

Rate of supply,

$$q = k_i A = k \cdot \frac{h}{L} \cdot A$$

$$= 1.23 \times 10^{-4} \times \frac{35}{30} \times (10 \times 10)$$

$$= 0.01435\text{cm}^3/\text{sec}$$

$$= \underline{\underline{0.0516}}\text{lit/hr}$$