

Applied Mathematics IB

Practice Problems on Limits and Continuity

1) Use the formal definition of limits to show that:

(a) $\lim_{x \rightarrow 1} (2x + 3) = 5$ (b) $\lim_{x \rightarrow 4} (x^2 - x) = 12$ (c) $\lim_{x \rightarrow 1} (x^2 + 1) = 2$

2) Evaluate each of the following limits if it exists.

a) $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x^3 - x^2 - 6x}$ b) $\lim_{x \rightarrow 1} \frac{x^4 - 1}{x^3 - 1}$ c) $\lim_{x \rightarrow -3} \frac{x^4 - 6x^2 - 27}{x^3 + 3x^2 + x + 3}$

d) $\lim_{x \rightarrow 0} \frac{x}{\sqrt{x+2} - \sqrt{2}}$ e) $\lim_{x \rightarrow -\infty} x \sin\left(\frac{1}{x}\right)$ f) $\lim_{x \rightarrow 0} \frac{xe^x}{\tan x}$

g) $\lim_{x \rightarrow 0} \frac{2\sin x - \sin 2x}{3x^2}$ h) $\lim_{x \rightarrow \infty} \left(\frac{x}{x-4}\right)^{3x}$ i) $\lim_{x \rightarrow 0} \left(\frac{1 - \cos x}{x^2}\right)$

3) Find the limits (if it exists) of the following functions at the indicated point(s) a.

(a) $f(x) = \frac{|x+1|}{x+1}$; $a = -1$

(b) $g(x) = \begin{cases} 2 & \text{if } x < -5 \\ x + 2 & \text{if } -5 \leq x < 0 \\ \frac{\sin 2x}{x} & \text{if } x > 0 \end{cases}$; $a = -6, -5, 1, 0$.

4) Show that the function $f(x) = \sqrt{9 - x^2}$ is continuous on the interval $[-3, 3]$.

5) Determine the continuity of f at $x = a$ from the right, left and both sides.

(a) $f(x) = \frac{\sqrt{4-2x}}{4+x}$; $a = 2, -4$.

(b) $f(x) = \begin{cases} x + 3 & \text{if } x > 3 \\ 6 & \text{if } x = 3 \\ x - 3 & \text{if } x < 3 \end{cases}$; $a = 3$.

$$(c) f(x) = \begin{cases} \frac{x^2-x-6}{x-3} & ; x \neq 3 \\ 5 & ; x = 3 \end{cases} ; a=3.$$

6. Find the horizontal and vertical asymptotes (if any) of the graphs of each of the following functions.

$$a) f(x) = \frac{x^3-x}{x^2-6x+5}$$

$$b) f(x) = \frac{2x^2+x-1}{x^2+x-2}$$

7) Find the value(s) of k for which the function f is continuous at $x=a$.

$$(a) f(x) = \begin{cases} x^2 - kx - 1 & ; x \leq 3 \\ 2 + kx - x^2 & ; x \geq 2 \end{cases} ; a=2.$$

$$(b) f(x) = \begin{cases} \frac{\sin(k^2-5k)x}{3x} & ; x \neq 0 \\ kx - 2 & ; x = 0 \end{cases} ; a=0.$$

$$(c) f(x) = \begin{cases} x^2 - 2k + 1, & \text{if } x \geq 2 \\ 2 + 3kx - kx^2, & \text{if } x < 2 \end{cases} ; a=2.$$

8) Show that each of the following equations has at least one solution in the given interval.

$$(a) x^3 - 4x + 1 = 0 ; [1, 2].$$

$$(b) -x^3 - x^2 - 2x = 1 ; [-1, 1].$$

$$(c) \cos x = \sin x + \frac{1}{2} ; [0, \pi].$$