

Water Hammer Analysis

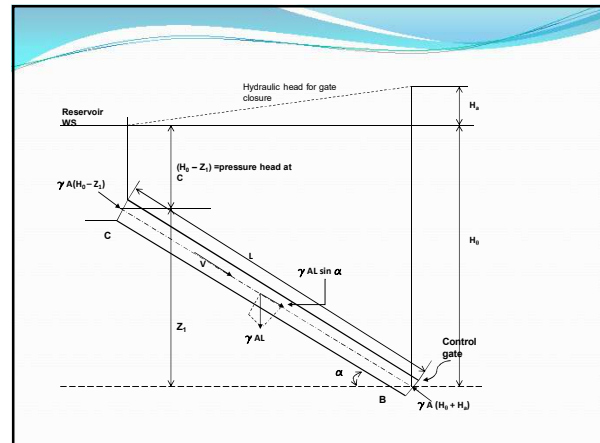
- Sudden shutdown of hydroelectric plants or changes in water flow through hydroelectric turbines may cause problem of rupture of penstocks due to water hammer.
- Water hammer is a phenomenon of pressure change in closed pipes caused when flowing water in a pipeline is decelerated by closing or opening a valve or changing the velocity of water rapidly in some manner.
- The phenomenon is accompanied by a series of positive and negative pressure waves which travel back and forth in the pipe system until they are damped out by friction.

- There are two approaches to the water hammer problem:
 1. The rigid water column theory, and
 2. The elastic water column theory.

The Rigid Water Column Theory

- Assumptions used in the theory:
 - i. The water in the pipe is incompressible, $\rightarrow$ density is constant
 - ii. The pipe walls do not stretch regardless of the pressure inside the pipe;
 - iii. The pipeline remains full of water at all times and the minimum pressure inside the pipe is in excess of the vapor pressure of water;

- iv. The hydraulic losses and velocity head are negligible when compared with the pressure changes;
 - v. The velocity of water in the direction of the axis of the pipe is uniform over any cross section of the pipe;
 - vi. The pressure is uniform over a transverse cross section of the pipe and is equal to the pressure at the center line of the pipe;
 - vii. The reservoir level remains constant during the gate movement time.
- Consider a pipeline of uniform area A and length L as shown in the figure below



- If the flow at the control gate is altered, an unbalanced external force will act at the gate on the mass of the water column.
- The deceleration, or the rate at which the velocity decreases per unit of time is, $-dv/dt$, where V is measured positive in the direction of flow.
- The mass of water decelerated in the entire length of the pipeline is $\gamma AL/g$.

- If H_a is the head rise at the gate, the unbalanced force acting on the water column is

$$\gamma A [(H_0 + H_a) - (H_0 - Z_1) - L \sin \alpha] \quad (1)$$

- But since $L \sin \alpha = Z_1$, the resulting unbalanced force on the column of water is $\gamma A H_a$.

- Then, from Newton's 2nd Law of motion,

$$H_a = - \frac{L}{g} \frac{dv}{dt} \quad (2)$$

- Prior to a closing movement of the control gate, the discharge into the atmosphere is

$$Q_0 = AV_0 = (C_d A_g)_0 \sqrt{2g H_0} \quad (3)$$

or $V_0 = B_0 \sqrt{H_0}$

Where

$$B_0 = \frac{(C_d A_g)_0 \sqrt{2g}}{A}$$

Then, at any instant during the gate movement,

$$V = B \sqrt{H_0 + H_a}$$

And

$$\frac{V}{V_0} = \frac{B}{B_0} \sqrt{1 + \frac{H_a}{H_0}} \quad (4)$$

- If the ratio B/B_0 is designated as τ

$$V = \tau V_0 \sqrt{1 + \frac{H_a}{H_0}} \quad (5)$$

where τ is a function of time which defines the ratio of the effective gate opening at any time during the gate closure to the effective gate opening at time zero.

- Equations (3) and (5) are the basic water hammer equations for gate closure as defined by the rigid water column theory.

- For partial or complete uniform gate closure,

$$\tau = 1 - \frac{V' t}{V_0 T} \quad (0 \leq t \leq T) \quad (6)$$

- Where T = the gate movement time

V' = the difference between the initial steady water velocity and the final steady velocity of water in the pipe

- Then, from equation (6);

$$V = \left(V_0 - \frac{V' t}{T} \right) \sqrt{1 + \frac{H_a}{H_0}} \quad (7)$$

- Now equations (2) and (7) are solved simultaneously for $(H_a)_{\max}$, the maximum head rise at the gate, by setting dH_a/dt equal to zero.

- Then,

$$\frac{(H_a)_{\max}}{H_0} = \frac{K_1}{2} + \sqrt{K_1 + \frac{K_1^2}{4}} \quad (8)$$

Where

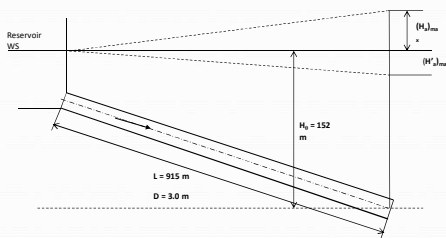
$$K_1 = \left(\frac{L V'}{g H_0 T} \right)^2$$

- A simple procedure for determining $(H'_a)_{\max}$, the maximum head drop at the gate due to a uniform gate opening, results in the equation

$$\frac{(H'_a)_{\max}}{H_0} = \frac{K_1}{2} - \sqrt{K_1 + \frac{K_1^2}{4}}$$

Example

Consider the pipeline shown in the figure below



- At this installation the initial flow of 43 m³/s is reduced to 14 m³/s by a uniform gate closure in 12 seconds.

- Find the maximum head rise, $(H_a)_{\max}$ and the maximum head drop, $(H'_a)_{\max}$.

Solution

$$V_0 = \frac{43}{\pi \times \left(\frac{3^2}{4}\right)} = 6.08 \text{ m/s}$$

$$V_1 = \frac{14}{\pi \times \left(\frac{3^2}{4}\right)} = 1.98 \text{ m/s}$$

$$K_1 = \left(\frac{L V'}{g H_0 T} \right)^2 = \left(\frac{915 \times (6.08 - 1.98)}{9.81 \times 152 \times 12} \right)^2 = 0.0441$$

- The maximum head rise, $(H_a)_{\max}$, is

$$\frac{(H_a)_{\max}}{H_0} = \frac{K_1}{2} + \sqrt{K_1 + \frac{K_1^2}{4}} = \frac{0.0441}{2} + \sqrt{0.0441 + \frac{(0.0441)^2}{4}} = 0.23$$

- Therefore, the maximum head rise, $(H_a)_{\max} = 0.23 \times 152 = 34.96 \approx 35 \text{ m}$

- If the control gate is opened uniformly in 12 seconds to increase the flow by $29 \text{ m}^3/\text{s}$ $(H'_a)_{\max}$ will be

$$\frac{(H'_a)_{\max}}{H_0} = \frac{K_1}{2} - \sqrt{K_1 + \frac{K_1^2}{4}} = \frac{0.0441}{2} - \sqrt{0.0441 + \frac{(0.0441)^2}{4}} = -0.1891$$

- $(H'_a)_{\max} = H_0 \times (-0.1891) = 152 \times (-0.1891) = -28.7 \approx 29 \text{ m}$
and the minimum head at the gate is $152 - 29 = 123 \text{ m}$

- NOTE:** For uniform gate operation, this theory gives satisfactory results only when T is greater than $L/1000$. For rapid gate movements, the elastic water column theory must be used.

Limitations

- The theory ignores the elastic effects due to sudden pressure changes. Thus any change in pressure is assumed to be transmitted instantaneously over the whole pipe length. In reality, the pressure transmission through the water mass is in the form of a compression wave, the velocity, V_c , of which is given by,

$$V_c = \sqrt{\frac{K}{\rho}}$$

- Where K = bulk modulus of water and ρ = density of water
- A pressure wave, thus needs a time L/V_c to traverse the whole length of the pipe and is not instantaneous as presumed.
- The critical time of closure, T_c , of the valve, i.e., the time taken by the pressure wave for a round trip from the valve to the reservoir and back is
$$T_c = \frac{2L}{V_c}$$
- If actual valve closure time T is greater than T_c , rigid water column theory gives fairly correct estimation of excess head built up due to valve closure
- Based on this,
 - If $T > T_c$, the valve closure is gradual,
 - If $T < T_c$, the valve closure is rapid.
- For rapid closure, elastic effect has to be considered.

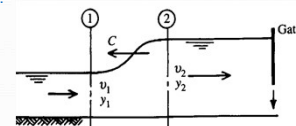
Canal Surges (Unsteady Flow in Open Channels)

- Flow in power canal will take time to respond to a change in turbine flow rate.
- Thus, if the canal is operating at a steady flow rate and the flow to the turbines is stopped suddenly, a **positive surge** will move upstream in the canal.
- These surges are hydraulic transients wherein the flow and the pressures change in the canal on account of the load variations in the hydropower station units.
- The direct result of surges is sudden **variation of water level** in the channel.
- Sufficient free board should be provided to prevent overtopping of the canal banks.**

- A positive surge is defined as one which leaves an increased water depth in its wake as the wave front passes, while a negative surge is one which leaves a shallower depth in its wake as the wave front passes.
- In the following simplified analysis of surge front movement, the effect of frictional resistance is neglected.

Upstream Positive Surge

- An upstream positive surge may be created in channel flow, for example, by the rapid closure of a gate, resulting in a step reduction in flow rate.
- The effect of this on the upstream side of the gate is the development of a wave front which travels upstream, as illustrated in Fig. below.



- Referring to Fig. above, the surge front is seen to leave in its wake an increased depth y_2 , hence the description 'positive'.
- By superimposing a downstream velocity c on the flow system, the flow regime is converted to an equivalent steady state, in which the wave front is now stationary.
- Applying the continuity and momentum principles to the control volume between sections 1 and 2, under the transformed steady state conditions:

$$\text{Continuity: } A_1(v_1 + c) = A_2(v_2 + c) \quad (i)$$

$$\text{Hence, } v_2 = \frac{A_1 v_1 - c(A_2 - A_1)}{A_2}$$

and

$$c = \frac{Q_1 - Q_2}{A_2 - A_1}$$

- Neglecting the flow friction force;

$$\text{Momentum: } \rho g \bar{y}_1 A_1 - \rho g \bar{y}_2 A_2 = \rho A_1(v_1 + c)(v_2 - v_1) \quad (ii)$$

- Solving equations (i) and (ii) for c and V_2 (eliminating v_2 in eq. (ii) & by substituting from eq. (i):

$$c = \left[g A_2 \frac{A_2 \bar{y}_2 - A_1 \bar{y}_1}{A_1(A_2 - A_1)} \right]^{1/2} - v_1 \quad (iii)$$

$$v_2 = v_1 - \left[\frac{g(A_2 - A_1)(A_2 \bar{y}_2 - A_1 \bar{y}_1)}{A_1 A_2} \right]^{1/2} \quad (iv)$$

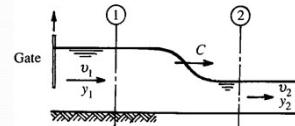
- For rectangular channel:

$$c = \left[\frac{g y_2 (y_2 + y_1)}{2 y_1} \right]^{1/2} - v_1 \quad (V)$$

- NOTE:** The continuity and momentum equations are not sufficient on their own to define the flow regime since there are three unknowns, c , y_2 , and v_2 (or Q_2). One of these must therefore be known to enable computation of the remaining two parameters.

Downstream Positive Surge

- A downstream positive surge is caused, for example, by sudden opening of a gate, which results in an instantaneous increase in discharge and flow depth downstream of the gate, as illustrated in Fig. below.



- Applying the same analytical approach as that used for the analysis of the upstream positive surge, the flow regime is transformed to an equivalent steady state by superimposing a backward velocity of magnitude c on the system.

- Thus, referring to the above Fig., the continuity and momentum principles can be applied to the control volume defined by sections 1 and 2.

$$\text{Continuity: } A_1(v_1 - c) = A_2(v_2 - c) \quad (vi)$$

$$\text{Hence, } c = \frac{Q_1 - Q_2}{A_1 - A_2} \quad (vii)$$

- Neglecting the boundary friction force:

$$\text{Momentum: } \rho g(A_1 \bar{y}_1 - A_2 \bar{y}_2) = \rho A_2(v_2 - c)(v_2 - v_1) \quad (viii)$$

- Solving equations (vi) and (viii) for c and v_1 :

$$c = \left[\frac{g A_1(A_1 \bar{y}_1 - A_2 \bar{y}_2)}{A_2(A_1 - A_2)} \right]^{1/2} + v_2 \quad (ix)$$

$$v_1 = \left[\frac{g(A_1 - A_2)(A_1 \bar{y}_1 - A_2 \bar{y}_2)}{A_1 A_2} \right]^{1/2} + v_2 \quad (x)$$

- For a rectangular section:

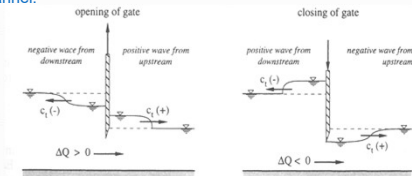
$$c = \left[\frac{g y_1}{2 y_2} (y_1 + y_2) \right]^{1/2} + v_2 \quad (xi)$$

$$v_1 = \left[\frac{g(y_1 - y_2)^2 (y_1 + y_2)}{2 y_1 y_2} \right]^{1/2} + v_2 \quad (xii)$$

Summary

- During the control of discharge by a gate, one encounters (Figure below):

 - By suddenly opening the gate, an increase in discharge; a negative wave from downstream forms itself in the upstream channel.
 - By suddenly closing the gate, a decrease in discharge; a positive wave from downstream forms itself in the upstream channel and a negative wave from upstream forms itself in the downstream channel.



Examples

Example 1:

A rectangular channel 4 m wide conveys a discharge of 25 m³/s at a depth of 3 m. The downstream discharge is suddenly reduced to 12 m³/s by partial closure of a gate. Determine the initial depth and celerity of the positive surge wave.

Solution

Given: $y_1 = 3.0$ m, $b = 4.0$ m, $Q_1 = 25$ m³/s, $Q_2 = 12$ m³/s

Required: y_2 and C

$$V_1 = \frac{Q_1}{A_1} = \frac{25}{4 \times 3} = 2.083 \text{ m/s}$$

$$V_2 = \frac{Q_2}{A_2} = \frac{12}{4 \times y_2} = \frac{3}{y_2}$$

$$(V_1 + C)y_1 = (v_2 + C)y_2 \rightarrow (2.083 + c) \times 3 = \left(\frac{3}{y_2} + c\right)y_2$$

$$\rightarrow c = \frac{3.249}{y_2 - 3}$$

- For rectangular channel, the expression for c is given by

$$c = \left[\frac{gy_2(y_2 + y_1)}{2y_1} \right]^{1/2} - v_1$$

$$\frac{3.249}{y_2 - 3} = \left[\frac{g y_2}{2} \left(\frac{y_2 + 3}{3} \right) \right]^{1/2} - 2.083$$

By trial and error $y_2 = 3.75$ m and $c = 4.35$ m/s

Therefore, the change in depth due to the surge, $\Delta y = 3.75 - 3.0 = 0.75$ m

Example 2:

A rectangular tailrace channel, 15 m wide, bed slope 0.0002 and Manning roughness coefficient 0.017 conveys a steady discharge of 45 m³/s from a hydropower station. A power increase results in a sudden increase in flow to the turbines to 100 m³/s. Determine the depth and celerity of the resulting surge wave in the channel.

Solution

Given: $b = 15$ m, slope, $s = 0.0002$, $n = 0.017$,

$Q_2 = 45$ m³/s, $Q_1 = 100$ m³/s

Required: y_1 and c

- Using Manning's equation, the depth of uniform flow under initial conditions at a discharge of 45 m³/s will be, $y_2 = 2.42$ m

- Then, from continuity equation for downstream positive surge,

$$(c - v_1)y_1 = (c - v_2)y_2$$

$$\rightarrow c = \frac{v_1 y_1 - v_2 y_2}{(y_1 - y_2)} = \frac{\frac{Q_1 y_1}{b y_1} - v_2 y_2}{(y_1 - y_2)}$$

$$v_2 = \frac{Q_2}{b y_2} = \frac{45}{15 \times 2.42} = 1.24 \text{ m/s}$$

- Now substituting this and other values in the equation above (equation of c), with $Q_1 = 100$ m³/s, we get,

$$c = \frac{\frac{100}{15} - 1.24 \times 2.42}{(y_1 - 2.42)} = \frac{6.67 - 3}{y_1 - 2.42} = \frac{3.67}{y_1 - 2.42}$$

- Another expression for c is given by

$$c = \left[\frac{g y_1}{2 y_2} (y_1 + y_2) \right]^{1/2} + v_2$$

- Substituting for c in the above equation, we have

$$\frac{3.67}{y_1 - 2.42} = \left[\frac{9.81 y_1}{2 \times 2.42} (y_1 + 2.42) \right]^{1/2} + 1.24$$

- By trial and error $y_1 = 2.95$ m, and $c = 6.92$ m/s
- Therefore, the increase in depth in the tailrace channel due to the surge, Δy , is

$$\Delta y = 2.95 - 2.42 = 0.53 \text{ m.}$$

Exercises

Ex. 1. A level, frictionless rectangular channel 3.0 m wide conveys a flow of 18 m³/s at a depth of 2.0 m. If the flow rate is suddenly reduced to 12 m³/s by partial gate closure, estimate the height and speed of the resulting surge wave.

Ex. 2. A rectangular channel 10 m wide, bed slope 0.0001, Manning's roughness coefficient 0.015, receives inflow from a reservoir with a gated inlet. When a steady discharge of 30 m³/s is being conveyed the gate is suddenly opened to release a discharge of 70 m³/s. Calculate the initial celerity and depth of the surge wave.