

CHAPTER ONE

COST-VOLUME-PROFIT ANALYSIS

Cost-volume-profit (CVP) analysis studies the behavior and relationship among total revenues, total costs, and operating income as changes occur in the units sold, the selling price, the variable cost per unit, or the fixed costs of a product.

Assumptions of CVP analysis

The assumptions of CVP analysis for the purpose of simplification of the analysis are:

1. Changes in the levels of revenues and costs arise only because of changes in the number of product (or service) units sold. The number of units sold is the only revenue driver and the only cost driver. Just as a cost driver is any factor that affects costs, a revenue driver is a variable, such as volume, that causally affects revenues.
2. Total costs can be separated into two components: a fixed component that does not vary with units sold and a variable component that changes with respect to units sold.
3. When represented graphically, the behaviors of total revenues and total costs are linear (meaning they can be represented as a straight line) in relation to units sold within a relevant range (and time period).
4. Selling price, variable cost per unit, and total fixed costs (within a relevant range and time period) are known and constant.
5. The CVP analysis either covers a single product or assumes that the sales mix, when multiple products are sold, will remain constant as the level of total units sold changes. For the purpose of CVP analysis, we assume that the company produces and sales only one product or if there are more than one products, we assume a constant sales mix. Note that, **sales mix** refers to the relative proportion or combination of quantity of output produced and sold that constitute total sales.
6. Time value of money is not considered.
7. Inventory change is zero. This means that the quantity of output sold in the period is the same as the quantity of output produced in the same period.

Essentials of CVP Analysis

The major terms and key concepts that are used in CVP analysis are operating income, net income, and contribution margin.

✚ **Operating income:** is equal to total revenue from operation minus cost of goods sold and all operating expenses.

✚ **Net income:** is operating income plus non-operating revenues minus non-operating costs minus income tax. That is

$$\text{Net income} = \text{Operating income} + \text{non-operating revenue} - \text{non-operating cost} - \text{income tax}$$

From what is given above, for the purpose of CVP analysis, we assume non-operating revenue and non-operating cost as zero. Therefore,

$$\text{Net income} = \text{Operating income} - \text{income tax}$$

- ✚ **Contribution margin (CM):** The difference between total revenues and total variable costs is called contribution margin. That is,

$$\text{Contribution margin} = \text{Total revenues} - \text{Total variable costs}$$

Contribution margin indicates why operating income changes as the number of units sold changes.

- ✚ **Contribution margin per unit (UCM):** is a useful tool for calculating contribution margin and operating income. It is defined as,

$$\text{Contribution margin per unit} = \text{Selling price} - \text{Variable cost per unit}$$

Contribution margin per unit provides a second way to calculate contribution margin:

$$\text{Contribution margin} = \text{Contribution margin per unit} * \text{Number of units sold}$$

- ✚ **Contribution margin percentage (CM%):** is the contribution margin per unit divided by the unit selling price. That is

$$\text{Contribution margin percentage} = \frac{\text{Contribution margin per unit}}{\text{Selling price}}$$

- ✚ **Variable cost percentage (VC%):** is the unit variable cost divided by the unit selling price. That is

$$\text{VC\%} = \frac{\text{Unit variable cost}}{\text{Unit selling price}}$$

For the purpose of illustration, let's consider the following example.

Example: Emma Frost is considering selling GMAT Success, a test prep book and software package for the business school admission test, at a college fair in Chicago. Emma knows she can purchase this package from a wholesaler at \$120 per package, with the privilege of returning all unsold packages and receiving a full \$120 refund per package. She also knows that she must pay \$2,000 to the organizers for the booth rental at the fair. She will incur no other costs. Emma predicts that she can charge a price of \$200 per package for GMAT Success.

Required:

- A. Calculate the unit contribution margin
- B. Calculate the contribution margin percentage
- C. Using a contribution income statement, calculate the total contribution margin, and operating income (operating loss) assuming that
 - i. 0 units are sold
 - ii. 1 units are sold
 - iii. 5 units are sold
 - iv. 25 units are sold
 - v. 40 units are sold

Solutions: Notice that the booth-rental cost of \$2,000 is a fixed cost because it will not change no matter how many packages Emma sells. The cost of the package itself is a variable cost because it increases in proportion to the number of packages sold. Emma will incur a cost of \$120 for each package that she sells.

$$\begin{aligned}\text{A. Contribution margin per unit} &= \text{Selling price} - \text{Variable cost per unit} \\ &= \$200 - \$120 \\ &= \underline{\$80}\end{aligned}$$

Notice that, even before she gets to the fair, Emma incurs \$2,000 in fixed costs. Because the contribution margin per unit is \$80, Emma will recover \$80 for each package that she sells at the fair. Emma hopes to sell enough packages to fully recover the \$2,000 she spent for renting the booth and to then start making a profit.

$$\text{B. Contribution margin percentage} = \frac{\text{Contribution margin per unit}}{\text{Selling price}}$$

$$\text{CM\%} = \frac{\$80}{\$200} = 0.40 \text{ or } 40\%$$

Contribution margin percentage is the contribution margin per dollar of revenue. Emma earns 40% of each dollar of revenue (equal to 40 cents).

C.

	Number of Packages Sold				
	0	1	5	25	40
Revenues @ \$200 per package	\$ 0	\$ 200	\$ 1,000	\$ 5,000	\$ 8,000
Variable Costs @ \$120 per package	<u>0</u>	<u>120</u>	<u>600</u>	<u>3,000</u>	<u>4,800</u>
Contribution Margin @ \$80 per package	0	80	400	2,000	3,200
Fixed Costs \$2000	<u>2,000</u>	<u>2,000</u>	<u>2,000</u>	<u>2,000</u>	<u>2,000</u>
Operating income (loss)	<u>\$(2,000)</u>	<u>\$(1,920)</u>	<u>\$(1,600)</u>	<u>\$ 0</u>	<u>\$ 1,200</u>

Exhibit 1.1: Contribution Income Statement for Different Quantities of GMAT Success Packages Sold

Exhibit 1-1 presents contribution margins for different quantities of packages sold. The income statement in Exhibit 1-1 is called a contribution income statement because it groups costs into variable costs and fixed costs to highlight contribution margin. Each additional package sold from 0 to 1 to 5 increases contribution margin by \$80 per package, recovering more of the fixed costs and reducing the operating loss. If Emma sells 25 packages, contribution margin equals \$2,000 (\$80 per package*25 packages), exactly recovering fixed costs and resulting in \$0 operating income. If Emma sells 40 packages, contribution margin increases by another \$1,200 (\$3,200-\$2,000), all of which becomes operating income. As you look across Exhibit 1-1 from left to right, you see that the increase in contribution margin exactly equals the increase in operating income (or the decrease in operating loss).

Breakeven Point

The breakeven point (BEP) is that quantity of output sold at which total revenues equal total costs—that is, the quantity of output sold that results in \$0 of operating income. Why would managers be interested in

the breakeven point? Mainly because they want to avoid operating losses, and the breakeven point tells them what level of sales they must generate to avoid a loss. Breakeven point can be determined by using: the equation method, the contribution margin method and the graph method.

For the purpose of illustration of the three methods, using the data given for **Emma Frost** earlier, calculate the breakeven quantity and revenue of **Emma Frost** using:

1. Equation method
2. Contribution margin method
3. Graphic method

Solution:

Equation method

Each column in Exhibit 1-1 is expressed as an equation.

$$\text{Revenues} - \text{Variable costs} - \text{Fixed costs} = \text{Operating income}$$

How are revenues in each column calculated?

$$\text{Revenues} = \text{Unit Selling price (USP)} \times \text{Quantity of units sold (Q)}$$

How are variable costs in each column calculated?

$$\text{Variable costs} = \text{Variable cost per unit (VCU)} \times \text{Quantity of units sold (Q)}$$

So,

$$[(\text{USP} \times \text{Q}) - (\text{UVC} \times \text{Q})] - \text{Fixed costs} = \text{Operating income} \quad (\text{Equation 1})$$

Where, USP = Unit selling price

UVC = Unit variable cost

Q = Quantity of units sold

Setting operating income equal to \$0 and denoting quantity of output units that must be sold by Q,

$$(\$200 \times \text{Q}) - (\$120 \times \text{Q}) - \$2,000 = \$0$$

$$\$80 \times \text{Q} = \$2,000$$

$$\text{Q} = \$2,000 \div \$80 \text{ per unit} = \underline{\underline{25 \text{ units}}}$$

If Emma sells fewer than 25 units, she will incur a loss; if she sells 25 units, she will breakeven; and if she sells more than 25 units, she will make a profit. While this breakeven point is expressed in units, it can also be expressed in revenues as follows:

Breakeven revenue = Breakeven quantity X Unit selling price

$$= 25 \text{ units} \times \$200 \text{ selling price} = \underline{\underline{\$5,000}}$$

Contribution margin method

Rearranging equation 1,

$$[(USP - UVC) \times Q] - \text{Fixed costs} = \text{Operating income}$$

$$(\text{UCM} \times Q) - \text{Fixed costs} = \text{Operating income} \quad (\text{Equation 2})$$

At the breakeven point, operating income is by definition \$0 and so,

$$\text{UCM} \times \text{Breakeven number of units} = \text{Fixed cost} \quad (\text{Equation 3})$$

Rearranging equation 3 and entering the data,

$$\text{Breakeven number of units} = \frac{\text{Fixed costs}}{\text{UCM}} = \frac{\$2,000}{\$80} = \underline{25 \text{ units}}$$

Usually breakeven point in terms of revenues is calculated using contribution margin percentages. Recall that in the GMAT Success example,

$$\text{CM \%} = \frac{\text{UCM}}{\text{USP}} = \$80 \div \$200 = 0.40 \text{ or } 40\%$$

That is, 40% of each dollar of revenue, or 40 cents, is contribution margin. To breakeven, contribution margin must equal fixed costs of \$2,000. To earn \$2,000 of contribution margin, when \$1 of revenue earns \$0.40 of contribution margin, revenues must equal $\$2,000 \div 0.40 = \underline{\$5,000}$.

$$\text{Breakeven number of units} = \frac{\text{Fixed costs}}{\text{CM \%}} = \frac{\$2,000}{0.40} = \underline{\$5,000}$$

Graph Method

In the graph method, we represent total costs and total revenues graphically. Each is shown as a line on a graph. Exhibit 1-2 illustrates the graph method for GMAT Success. Because we have assumed that total costs and total revenues behave in a linear fashion, we need only two points to plot the line representing each of them.

1. **Total costs line:** The total costs line is the sum of fixed costs and variable costs. Fixed costs are \$2,000 for all quantities of units sold within the relevant range. To plot the total costs line, use as one point the \$2,000 fixed costs at zero units sold (point A) because variable costs are \$0 when no units are sold. Select a second point by choosing any other convenient output level (say, 40 units sold) and determine the corresponding total costs. Total variable costs at this output level are \$4,800 (40 units X \$120 per unit). Remember, fixed costs are \$2,000 at all quantities of units sold within the relevant range, so total costs at 40 units sold equal \$6,800 (\$2,000 + \$4,800), which is point B in Exhibit 1-2. The total costs line is the straight line from point A through point B.
2. **Total revenues line:** One convenient starting point is \$0 revenues at 0 units sold, which is point C in Exhibit 2-2. Select a second point by choosing any other convenient output level and

determining the corresponding total revenues. At 40 units sold, total revenues are \$8,000 (\$200 per unit X 40 units), which is point D in Exhibit 2-2. The total revenues line is the straight line from point C through point D.

Profit or loss at any sales level can be determined by the vertical distance between the two lines at that level in Exhibit 3-2. For quantities fewer than 25 units sold, total costs exceed total revenues, and the purple area indicates operating losses. For quantities greater than 25 units sold, total revenues exceed total costs, and the blue-green area indicates operating incomes. At 25 units sold, total revenues equal total costs. Emma will break even by selling 25 packages.

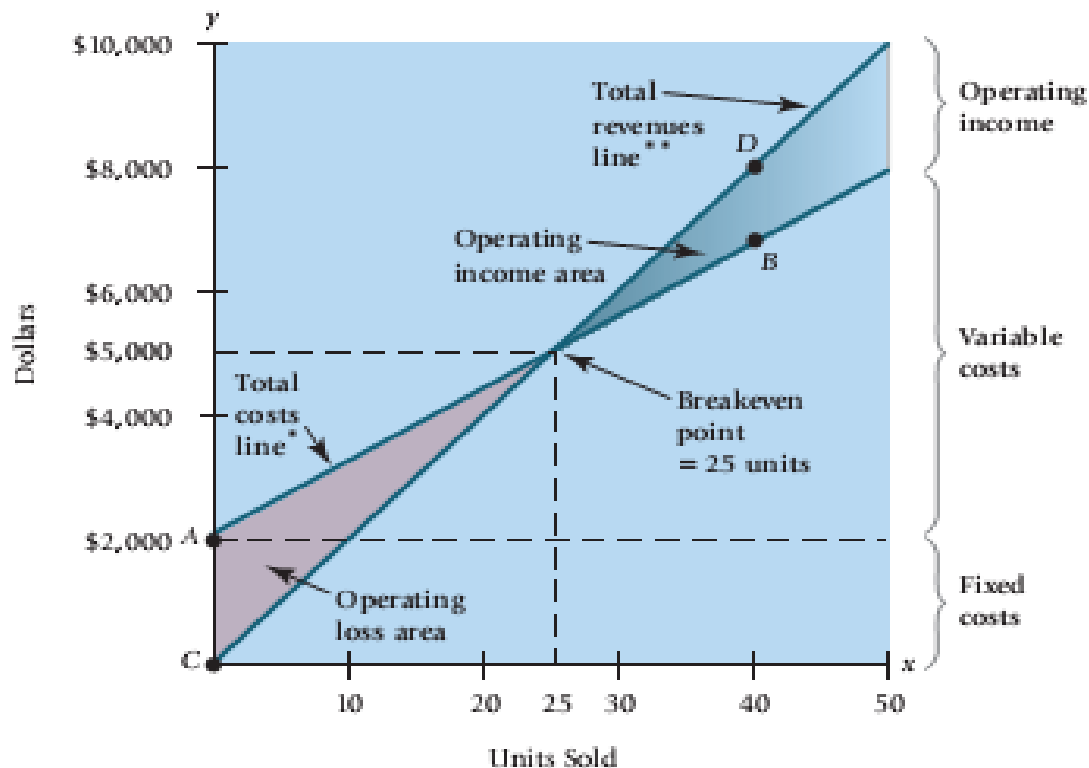


Exhibit 1.2: CVP graph

While the breakeven point tells managers how much they must sell to avoid a loss, managers are equally interested in how they will achieve the operating income targets underlying their strategies and plans. In our example, selling 25 units at a price of \$200 assures Emma that she will not lose money if she rents the booth. This news is comforting, but we next describe how Emma determines how much she needs to sell to achieve a targeted amount of operating income.

Target Operating Income

We illustrate target operating income calculations by asking the following question: How many units must Emma sell to earn an operating income of \$1,200? One approach is to keep plugging in different quantities into Exhibit 1-1 and check when operating income equals \$1,200. Exhibit 1-1 shows that

operating income is \$1,200 when 40 packages are sold. A more convenient approach is to use equation 1 provided earlier.

$$[(\text{USP} \times Q) - (\text{UVC} \times Q)] - \text{Fixed costs} = \text{Target operating income (TOI)} \quad (\text{Equation 1})$$

We denote by Q the unknown quantity of units Emma must sell to earn an operating income of \$1,200. Selling price is \$200, variable cost per package is \$120, fixed costs are \$2,000, and target operating income is \$1,200. Substituting these values into equation 1, we have

$$(\$200 \times Q) - (\$120 \times Q) - \$2,000 = \$1,200$$

$$\$80 \times Q = \$2,000 + \$1,200 = \$3,200$$

$$Q = \$3,200 \div \$80 \text{ per unit} = \underline{40 \text{ units}}$$

Alternatively, we could use equation 2,

$$(\text{UCM} \times Q) - \text{Fixed costs} = \text{Target operating income} \quad (\text{Equation 2})$$

Given a target operating income (\$1,200 in this case), we can rearrange terms to get equation 4.

$$\text{Quantity of units required to be sold} = \frac{\text{Fixed costs} + \text{TOI}}{\text{UCM}} \quad (\text{Equation 4})$$

$$\text{Quantity of units required to be sold} = \frac{\$2,000 + \$1,200}{\$80 \text{ per unit}} = \underline{40 \text{ units}}$$

Proof:

Revenues, \$200 per unit * 40 units	\$8,000
Variable costs, \$120 per unit * 40 units.....	<u>4,800</u>
Contribution margin, \$80 per unit *40 units	3,200
Fixed costs	<u>2,000</u>
Operating income	<u>\$1,200</u>

The revenues needed to earn an operating income of \$1,200 can also be calculated by using:

$$\begin{aligned} \text{Revenues needed to earn an OI of \$1,200} &= \text{Quantity of units required to be sold} \times \text{USP} \\ &= 40 \text{ units} \times \$200 = \underline{\$8,000} \end{aligned}$$

Or

$$\begin{aligned} \text{Revenues needed to earn an OI of \$1,200} &= \frac{\text{FC} + \text{TOI}}{\text{CM} \%} \\ &= \frac{\$2,000 + \$1,200}{0.4} = \underline{\$8,000} \end{aligned}$$

The graph in Exhibit 1-2 is very difficult to use to answer the question: How many units must Emma sell to earn an operating income of \$1,200? Why? Because it is not easy to determine from the graph the precise point at which the difference between the total revenues line and the total costs line equals \$1,200. However, recasting Exhibit 1-2 in the form of a profit-volume (PV) graph makes it easier to answer this question.

APV graph shows how changes in the quantity of units sold affect operating income. Exhibit 1-3 is the PV graph for GMAT Success (fixed costs, \$2,000; selling price, \$200; and variable cost per unit, \$120). The PV line can be drawn using two points. One convenient point (M) is the operating loss at 0 units sold, which is equal to the fixed costs of \$2,000, shown at $-\$2,000$ on the vertical axis. A second convenient point (N) is the breakeven point, which is 25 units in our example. The PV line is the straight line from point M through point N. To find the number of units Emma must sell to earn an operating income of \$1,200, draw a horizontal line parallel to the x-axis corresponding to \$1,200 on the vertical axis (that's the y-axis). At the point where this line intersects the PV line, draw a vertical line down to the horizontal axis (that's the x-axis). The vertical line intersects the x-axis at 40 units, indicating that by selling 40 units Emma will earn an operating income of \$1,200.

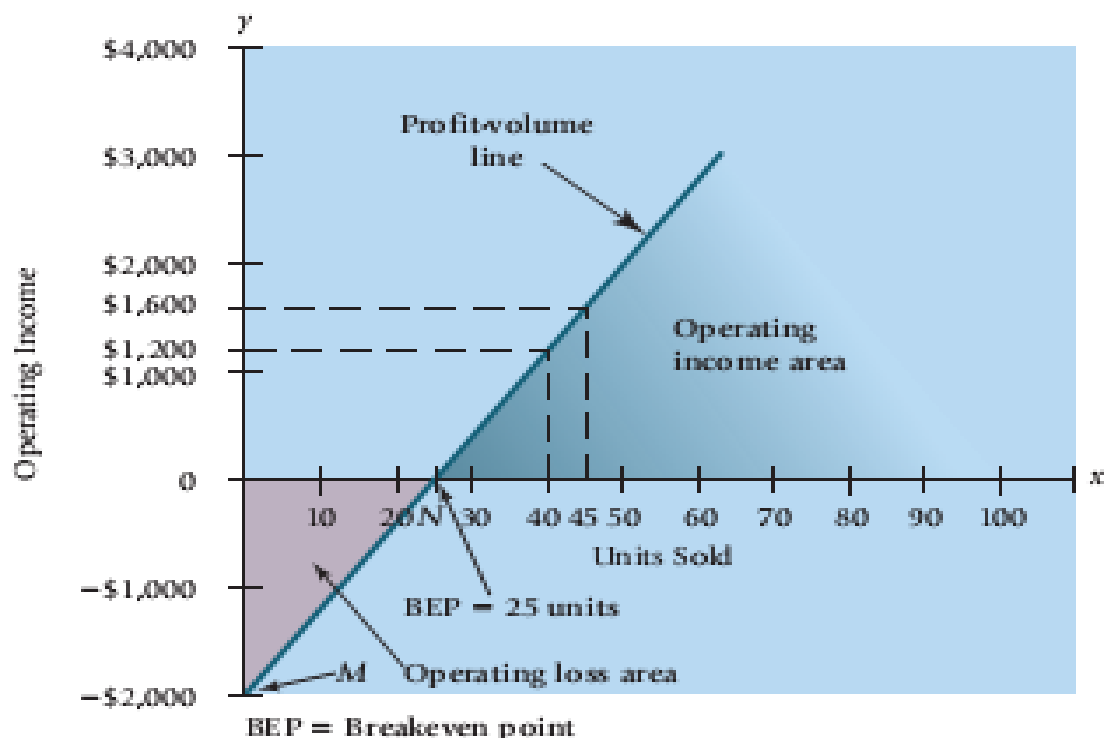


Exhibit 1.3: Profit-Volume Graph for GMAT Success

Target Net Income and Income Taxes

Net income is operating income plus non-operating revenues (such as interest revenue) minus non-operating costs (such as interest cost) minus income taxes. For simplicity, throughout this chapter we assume non-operating revenues and non-operating costs are zero. Thus,

$$\text{Net income} = \text{Operating income} - \text{Income taxes}$$

Until now, we have ignored the effect of income taxes in our CVP analysis. In many companies, the income targets for managers in their strategic plans are expressed in terms of net income. That's because top management wants subordinate managers to take into account the effects their decisions have on operating income after income taxes. Some decisions may not result in large operating incomes, but they may have favorable tax consequences, making them attractive on a net income basis—the measure that drives shareholders' dividends and returns.

To make net income evaluations, CVP calculations for target income must be stated in terms of target net income instead of target operating income. **For example**, Emma may be interested in knowing the quantity of units she must sell to earn a net income of \$960, assuming an income tax rate of 40%.

$$\text{Target net income} = \text{Target operating income} - \text{Income tax}$$

$$\text{Target net income} = (\text{Target operating income}) - (\text{Target OI} \times \text{Tax rate})$$

$$\text{Target net income} = (\text{Target operating income}) \times (1 - \text{Tax rate})$$

$$\text{Target operating income} = \frac{\text{Target net income}}{1 - \text{Tax rate}}$$

$$= \frac{\$960}{1 - 0.4} = \$1,600$$

In other words, to earn a target net income of \$960, Emma's target operating income is \$1,600.

Proof:	Target operating income.....	\$1,600
	Tax at 40% (0.40 X \$1,600).....	<u>640</u>
	Target net income.....	<u>\$960</u>

The key step is to take the target net income number and convert it into the corresponding target operating income number. We can then use equation 1 for target operating income and substitute numbers from our GMAT Success example.

$$[(\text{USP} \times \text{Q}) - (\text{UVC} \times \text{Q})] - \text{Fixed costs} = \text{Target Operating income} \quad (\text{Equation 1})$$

$$(\$200 \times \text{Q}) - (\$120 \times \text{Q}) - \$2,000 = \$1,600$$

$$\$80 \times \text{Q} = \$3,600$$

$$\text{Q} = \$3,600 \div \$80 \text{ per unit} = \underline{45 \text{ units}}$$

Alternatively we can calculate the number of units Emma must sell by using the contribution margin method and equation 4:

$$\text{Quantity of units required to be sold} = \frac{\text{Fixed costs} + \text{TOI}}{\text{UCM}} \quad (\text{Equation 4})$$

$$= \frac{\$2,000 + \$1,600}{\$80 \text{ per unit}} = \underline{45 \text{ units}}$$

Proof:

Revenues, \$200 per unit X 45 units.....	\$9,000
Variable costs, \$120 per unit X 45 units	<u>5,400</u>
Contribution margin.....	3,600
Fixed costs.....	<u>2,000</u>
Operating income.....	1,600
Income taxes, (\$1,600 * 0.40).....	<u>640</u>
Net income.....	<u>\$960</u>

The revenues needed to earn a net income of \$960 can also be calculated by using:

$$\begin{aligned}\text{Revenues needed to earn a NI of \$960} &= \text{Quantity of units required to be sold} \times \text{USP} \\ &= 45 \text{ units} \times \$200 = \underline{\$9,000}\end{aligned}$$

Or

$$\text{Revenues needed to earn a NI of \$960} = \frac{FC + TOI}{CM\%}$$

$$= \frac{FC + \frac{TNI}{1 - TR}}{CM\%}$$

$$= \frac{2000 + \frac{960}{1 - 0.4}}{0.4} = \underline{\$9,000}$$

Emma can also use the PV graph in Exhibit 1-3. To earn target operating income of \$1,600, Emma needs to sell 45 units.

Focusing the analysis on target net income instead of target operating income will not change the breakeven point. That's because, by definition, operating income at the breakeven point is \$0, and no income taxes are paid when there is no operating income.

Using CVP Analysis for Decision Making

We return to our GMAT Success example to illustrate how CVP analysis can be used for strategic decisions concerning advertising and selling price.

Decision to Advertise

Suppose Emma anticipates selling 40 units at the fair. Exhibit 1-3 indicates that Emma's operating income will be \$1,200. Emma is considering placing an advertisement describing the product and its features in the fair brochure. The advertisement will be a fixed cost of \$500. Emma thinks that advertising

will increase sales by 10% to 44 packages. **Should Emma advertise?** The following table presents the CVP analysis.

	40 Packages Sold with No Advertising (1)	44 Packages Sold with Advertising (2)	Difference (3) - (2) = (1)
Revenues (\$200 X 40; \$200 X 44)	\$8,000	\$8,800	\$800
Variable costs (\$120 X 40; \$120 X 44)	<u>4,800</u>	<u>5,280</u>	<u>480</u>
Contribution margin (\$80 * 40; \$80 * 44)	3,200	3,520	320
Fixed costs	<u>2,000</u>	<u>2,500</u>	<u>500</u>
Operating income	<u>\$1,200</u>	<u>\$1,020</u>	<u>\$(180)</u>

Operating income will decrease from \$1,200 to \$1,020, so **Emma should not advertise**. Note that Emma could focus only on the difference column and come to the same conclusion: If Emma advertises, contribution margin will increase by \$320 (revenues, \$800 - variable costs, \$480), and fixed costs will increase by \$500, resulting in a \$180 decrease in operating income.

As you become more familiar with CVP analysis, try evaluating decisions based on differences rather than mechanically working through the contribution income statement. Analyzing differences gets to the heart of CVP analysis and sharpens intuition by focusing only on the revenues and costs that will change as a result of a decision.

Decision to Reduce Selling Price

Having decided not to advertise, Emma is considering whether to reduce the selling price to \$175. At this price, she thinks she will sell 50 units. At this quantity, the test-prep package wholesaler who supplies GMAT Success will sell the packages to Emma for \$115 per unit instead of \$120. **Should Emma reduce the selling price?**

Contribution margin from lowering price to \$175: (\$175- \$115) per unit * 50 units	\$3,000
Contribution margin from maintaining price at \$200: (\$200 - \$120) per unit * 40 units	<u>3,200</u>
Change in contribution margin from lowering price	<u>\$ (200)</u>

Decreasing the price will reduce contribution margin by \$200 and, because the fixed costs of \$2,000 will not change, it will also reduce operating income by \$200. **Emma should not reduce the selling price.**

Sensitivity Analysis and Margin of Safety

Before choosing strategies and plans about how to implement strategies, managers frequently analyze the sensitivity of their decisions to changes in underlying assumptions. Sensitivity analysis is a “what-if” technique that managers use to examine how an outcome will change if the original predicted data are not achieved or if an underlying assumption changes. In the context of CVP analysis, sensitivity analysis answers questions such as, “What will operating income be if the quantity of units sold decreases by 5% from the original prediction?” and “What will operating income be if variable cost per unit increases by

10%?” Sensitivity analysis broadens managers’ perspectives to possible outcomes that might occur before costs are committed.

One aspect of sensitivity analysis is **margin of safety**. The margin of safety is the excess of an organization’s expected future sales (in either revenue or units) above the breakeven point.

$$\text{Margin of safety in revenues} = \text{Budgeted (or actual) revenues} - \text{Breakeven revenues}$$

$$\text{Margin of safety (in units)} = \text{Budgeted (or actual) sales quantity} - \text{Breakeven quantity}$$

The margin of safety answers the “what-if” question: If budgeted revenues are above breakeven and drop, how far can they fall below budget before the breakeven point is reached? Sales might decrease as a result of a competitor introducing a better product, or poorly executed marketing programs, and so on. **Assume that Emma** has fixed costs of \$2,000, a selling price of \$200, and variable cost per unit of \$120. From Exhibit 1-1, if Emma sells 40 units, budgeted revenues are \$8,000 and budgeted operating income is \$1,200. The breakeven point is 25 units or \$5,000 in total revenues.

$$\text{Margin of safety} = \text{Budgeted revenues} - \text{Breakeven revenues} = \$8,000 - \$5,000 = \underline{\$3,000}$$

$$\text{Margin of safety (in units)} = \text{Budgeted sales (units)} - \text{Breakeven sales (units)} = 40 - 25 = \underline{15 \text{ units}}$$

Sometimes margin of safety is expressed as a percentage:

$$\text{Margin of safety percentage} = \frac{\text{Margin of safety in dollars}}{\text{Budgeted (or actual) revenues}}$$

$$\text{In our example, margin of safety percentage} = \frac{\$3,000}{\$8,000} = \underline{37.5\%}$$

This result means that revenues would have to decrease substantially, by 37.5%, to reach breakeven revenues. The high margin of safety gives Emma confidence that she is unlikely to suffer a loss.

If, however, Emma expects to sell only 30 units, budgeted revenues would be \$6,000 (\$200 per unit X 30 units) and the margin of safety would equal:

$$\text{Budgeted revenues} - \text{Breakeven revenues} = \$6,000 - \$5,000 = \underline{\$1,000}$$

$$\text{Margin of safety percentage} = \frac{\text{Margin of safety in dollars}}{\text{Budgeted (or actual) revenues}} = \frac{\$1,000}{\$6,000} = \underline{16.67\%}$$

The analysis implies that if revenues decrease by more than 16.67%, Emma would suffer a loss. A low margin of safety increases the risk of a loss. If Emma does not have the tolerance for this level of risk, she will prefer not to rent a booth at the fair.

Sensitivity analysis is a simple approach to recognizing uncertainty, which is the possibility that an actual amount will deviate from an expected amount. Sensitivity analysis gives managers a good feel for the risks involved.

Cost Planning and CVP

Managers have the ability to choose the levels of fixed and variable costs in their cost structures. This is a strategic decision. In this section, we describe various factors that managers and management accountants consider as they make this decision.

Alternative Fixed-Cost/Variable-Cost Structures

CVP-based sensitivity analysis highlights the risks and returns as fixed costs are substituted for variable costs in a company's cost structure. **As an example in Exhibit 1-4 below, compare line 6 and line 11.**

	Fixed cost	Variable cost	Number of units required to be sold at \$200 selling price to breakeven point	Number of units required to be sold at \$200 selling price to earn a TOI of \$1,200
Line 6	\$2,000	\$120	25	50
Line 11	\$2,800	\$100	28	48

Compared to line 6, line 11, with higher fixed costs, has more risk of loss (has a higher breakeven point) but requires fewer units to be sold (48 versus 50) to earn operating income of \$2,000. CVP analysis can help managers evaluate various fixed-cost/variable-cost structures. We next consider the effects of these choices in more detail. ***Suppose the Chicago college fair organizers offer Emma three rental alternatives:***

Option 1: \$2,000 fixed fee

Option 2: \$800 fixed fee plus 15% of GMAT Success revenues

Option 3: 25% of GMAT Success revenues with no fixed fee

Emma's variable cost per unit is \$120. Emma is interested in how her choice of a rental agreement will affect the income she earns and the risks she faces. Exhibit 1-5 graphically depicts the profit-volume relationship for each option. The line representing the relationship between units sold and operating income for Option 1 is the same as the line in the PV graph shown in Exhibit 1-3 (fixed costs of \$2,000 and contribution margin per unit of \$80). The line representing Option 2 shows fixed costs of \$800 and a contribution margin per unit of \$50 [selling price, \$200, minus variable cost per unit, \$120, minus variable rental fees per unit, \$30, $(0.15 \times \$200)$]. The line representing Option 3 has fixed costs of \$0 and a contribution margin per unit of \$30 [$\$200 - \$120 - \$50$ ($0.25 \times \$200$)].

Option 3 has the lowest breakeven point (0 units), and Option 1 has the highest breakeven point (25 units). Option 1 has the highest risk of loss if sales are low, but it also has the highest contribution margin per unit (\$80) and hence the highest operating income when sales are high (greater than 40 units).

The choice among Options 1, 2, and 3 is a strategic decision that Emma faces. As in most strategic decisions, what she decides now will significantly affect her operating income (or loss), depending on the demand for GMAT Success. Faced with this uncertainty, Emma's choice will be influenced by her confidence in the level of demand for GMAT Success and her willingness to risk losses if demand is low. For example, if Emma's tolerance for risk is high, she will choose Option 1 with its high potential rewards. If, however, Emma is averse to taking risk, she will prefer Option 3, where the rewards are smaller if sales are high but where she never suffers a loss if sales are low.

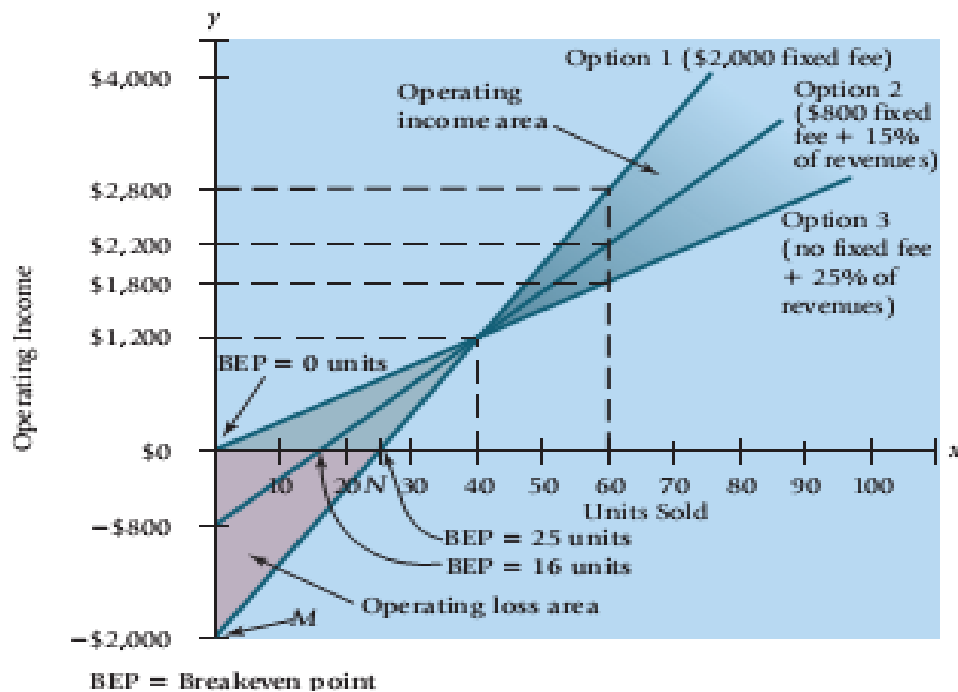


Exhibit 1-5: Profit-Volume Graph for Alternative Rental Options for GMAT Success

Operating Leverage

The risk-return trade-off across alternative cost structures can be measured as operating leverage. **Operating leverage** describes the effects that fixed costs have on changes in operating income as changes occur in units sold and contribution margin. Organizations with a high proportion of fixed costs in their cost structures, as is the case under Option 1, have high operating leverage. The line representing Option 1 in Exhibit 1-5 is the steepest of the three lines. Small increases in sales lead to large increases in operating income. Small decreases in sales result in relatively large decreases in operating income, leading to a greater risk of operating losses. At any given level of sales,

$$\text{Degree of operating leverage} = \frac{\text{Contribution margin}}{\text{Operating income}}$$

The following table shows the degree of operating leverage at sales of 40 units for the three rental options.

	Option 1	Option 2	Option 3
Contribution margin per unit	\$80	\$50	\$30
Contribution margin (row 1 X 40 units)	\$3,200	\$2,000	\$1,200
Operating income (from Exhibit 1-5)	\$1,200	\$1,200	\$1,200
Degree of operating leverage (row 2 ÷ row 3)	$\frac{\$3,200}{\$1,200} = 2.67$	$\frac{\$2,000}{\$1,200} = 1.67$	$\frac{\$1,200}{\$1,200} = 1$

These results indicate that, when sales are 40 units, a percentage change in sales and contribution margin will result in 2.67 times that percentage change in operating income for Option 1, but the same percentage change (1.00) in operating income for Option 3. Consider, for example, a sales increase of 50% from 40 to 60 units. Contribution margin will increase by 50% under each option. Operating income, however, will increase by $2.67 \times 50\% = 133\%$ from \$1,200 to \$2,800 in Option 1, but it will increase by only $1.00 \times 50\% = 50\%$ from \$1,200 to \$1,800 in Option 3 (see Exhibit 1-5). The degree of operating leverage at a given level of sales helps managers calculate the effect of sales fluctuations on operating income.

Sales Mix Analysis

Sales mix is the relative proportion of quantities of products (or services) that constitute total unit sales. If the proportions of the mix change, the cost volume profit relationships also change.

Unlike in the single product (or service) situation, there is no unique breakeven number of units for a multiple-product situations. The breakeven quantity depends on the sales mix. One possible assumption is that the budgeted sales mix will not change at different levels of total unit sales.

For the illustration purpose, consider the following information obtained from the accountant of Ramos Company. Suppose Ramos Co. is currently producing and selling two different products, known as product A and product B. The budgeted income statement of the company is shown below:

	Product A	Product B	Total
Sales in units	300,000	75,000	375,000
Sales @\$8, and \$5	2,400,000	375,000	2,775,000
Variable expenses @\$7 & \$3	<u>2,100,000</u>	<u>225,000</u>	<u>2,325,000</u>
Contribution margin @\$1 & \$2	300,000	150,000	450,000
Fixed costs			<u>180,000</u>
Operating income			<u>\$270,000</u>

Additional information: the company assumed that 4 units of product A are produced and sold for every 1 unit of product B, and this budgeted sales (relative proportion of 4:1) will not change at different levels of total sales.

Required: Calculate the breakeven quantity using:

- Equation method.
- Weighted-average contribution margin per unit method.
- Weighted-average contribution margin percentage method.

Notice that, as the sales mix will not be changed, there are a constant proportion of 4 units of product A, for every one unit of product B. Therefore, **Product A = 4 Product B.**

Solution:

A. Equation method

In equation method, we have

$$\text{Revenue} - \text{variable cost} - \text{fixed cost} = \text{operating income}$$

Notice that here we have two different products with two different selling price and variable cost. But only one common fixed cost for both product. Having this in mind, we can rewrite the above equation as:

$$(\text{Revenue of product A} + \text{Revenue of product B}) - (\text{Variable cost of product A} + \text{Variable cost of product B}) - \text{Fixed cost} = \text{Operating income}$$

Let Q be the number of units of product B to breakeven, then the number of units of product A to breakeven is equal to 4Q, substituting this,

$$= [8(4Q) + 5(Q)] - [7(4Q) + 3(Q)] - 180,000 = 0$$

$$= [32Q + 5Q] - [28Q + 3Q] - 180,000 = 0$$

$$= (37Q - 31Q) - 180,000 = 0$$

$$Q = \underline{30,000 \text{ units}}$$

Therefore,

The number of units of product A to breakeven = $4Q = 4 \times 30,000 = \underline{120,000 \text{ units}}$, and

The number of units of product B to breakeven = $Q = 1 \times 30,000 = \underline{30,000 \text{ units}}$

B. Weighted-average contribution margin per unit method

Weighted average contribution margin per unit (WACMPU) =

$$\frac{(\text{Product As UCM} \times \text{number of product A sold}) + (\text{Product BS UCM} \times \text{number of product B sold})}{\text{Number of product A sold} + \text{Number of product B sold}}$$

$$= \frac{(\$1 \times 300,000) + (\$2 \times 75,000)}{300,000 + 75,000}$$

$$= \frac{450,000}{375,000} = \underline{\$1.2} \text{ Therefore,}$$

The breakeven point = $FC \div \text{WACMPU}$

The breakeven point = $\$180,000 \div \$1.2 = \underline{150,000 \text{ units}}$

Because the ratio of the quantity of product A to product B is 4:1 (80%: 20%), the breakeven quantity of product A is $80\% \times 150,000 = 120,000 \text{ units}$. And the breakeven quantity of product B is $20\% \times 150,000 = 30,000 \text{ units}$.

C. Weighted-average contribution margin percentage method

Weighted-average contribution margin percentage (WACM %) = $\frac{\text{Total contribution margin}}{\text{Total Revenue}}$

$$= 450,000 \div 2,775,000 = 0.162162162$$

Total revenue required to breakeven = FC $\div$ WACM%, = $180,000 \div 0.162162162 = \underline{\$1,110,000}$

The \$1,110,000 revenue is based on the ratio of 86.5%:13.5% (2,400,000:373,000). Hence, the breakeven revenue should be apportioned in the ratio of 86.5%:13.5%. This amounts to the breakeven revenue of \$960,000 (86.5% X 1,110,000) of product A and \$150,000 (13.5 X 1,110,000) of product B.

At a selling price of \$8 for product A and \$5 for product B, the breakeven quantity equals to 30,000 units (\$150,000/\$5) for product B and 120,000 units (\$960,000/\$8) for product A.

Managers usually want to maximize the sales of all their products. Faced with limited resource and time, however, executives prefer to generate the most profitable sales mix achievable. Profitability of a given product helps guide executive who must decide to emphasize or de-emphasize particular products. For example given limited production facilities or limited time of sales personnel, should we emphasize on product A or B? These decisions may be affected by other factors beyond the contribution margin per unit of product.

In general, other things being equal, for any given total quantity of units sold, if the sales mix shifts toward unit with higher contribution margins operating income will be higher.

CVP analysis in service and nonprofit organizations

Thus far, our CVP analysis has focused on a merchandising company mainly. CVP can also be applied to decisions by manufacturing companies like Ramos Co. or Harar Brewery factory, service companies like Commercial Bank of Ethiopia, and nonprofit organizations like the United Way. To apply CVP analysis in service and nonprofit organizations, we need to focus on measuring their output, which is different from the tangible units sold by manufacturing and merchandising companies.

Examples of output measures in various service and nonprofit industries are as follows:

<u>Industry</u>	<u>Measure of Output</u>
Airlines	Passenger-miles
Hotels/motels	Room-nights occupied
Hospitals	Patient-days
Universities	Student credit-hours

Example: Consider an agency of the Massachusetts Department of Social Welfare with a \$900,000 budget appropriation (its revenues) for 2011. This nonprofit agency's purpose is to assist handicapped people seeking employment. On average, the agency supplements each person's income by \$5,000 annually. The agency's only other costs are fixed costs of rent and administrative salaries equal to \$270,000. The agency manager wants to know how many people could be assisted in 2011. We can use CVP analysis here by setting operating income to \$0. Let Q be the number of handicapped people to be assisted:

$$\text{Revenues} - \text{Variable costs} - \text{Fixed costs} = \text{Operating income}$$

$$\$900,000 - \$5,000Q - \$270,000 = 0$$

$$\$5,000Q = \$900,000 - \$270,000 = \$630,000$$

$$Q = \$630,000 \div \$5,000 \text{ per person} = \underline{126 \text{ people}}$$

Suppose the manager is concerned that the total budget appropriation for 2012 will be reduced by 15% to \$900,000 $\times (1 - 0.15) = \$765,000$. The manager wants to know how many handicapped people could be assisted with this reduced budget. Assume the same amount of monetary assistance per person:

$$\$765,000 - \$5,000Q - \$270,000 = 0$$

$$\$5,000Q = \$765,000 - \$270,000 = \$495,000$$

$$Q = \$495,000 \div \$5,000 \text{ per person} = \underline{99 \text{ people}}$$

Note the following two characteristics of the CVP relationships in this nonprofit situation:

1. The percentage drop in the number of people assisted, $(126 - 99) \div 126$, or 21.4%, is greater than the 15% reduction in the budget appropriation. It is greater because the \$270,000 in fixed costs still must be paid, leaving a proportionately lower budget to assist people. The percentage drop in service exceeds the percentage drop in budget appropriation.

2. Given the reduced budget appropriation (revenues) of \$765,000, the manager can adjust operations to stay within this appropriation in one or more of three basic ways: (a) reduce the number of people assisted from the current 126, (b) reduce the variable cost (the extent of assistance per person) from the current \$5,000 per person, or (c) reduce the total fixed costs from the current \$270,000.

Limitation of CVP Analysis

The following are the limitations of Cost Volume Profit Analysis:

1. Segregation of total costs into its fixed and variable components is difficult to do.
2. Fixed costs are unlikely to stay constant as output increases beyond a certain range of activity.
3. The analysis is restricted to the relevant range specified and beyond that the results can be unreliable
4. Besides volume, other elements like inflation, efficiency, capacity and technology can affect costs
5. Impractical to assume sales mix remain constant since this depend on the changing demand levels.
6. The assumption of linear property of total cost and total revenue relies on the assumption that unit variable cost and selling price are constant. However, this is likely to be valid within relevant range only.