

EXERCISE # (S-1)

HCV Worked out Examples(Chapter No. 9 :- 1,2,3)

1. Four particles of mass 5, 3, 2, 4 kg are at the points (1, 6), (-1, 5), (2, -3), (-1, -4). Find the coordinates of their centre of mass.

Ans. $\left(\frac{1}{7}, \frac{23}{14}\right)$

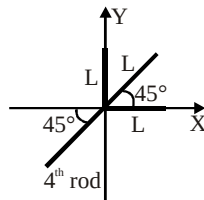
Sol.
$$x_{cm} = \frac{(5 \times 1) + (3 \times (-1)) + 2 \times 2 + 4 \times (-1)}{14} = \frac{1}{7}$$
$$y_{cm} = \frac{5 \times 6 + 3 \times 5 - 2 \times 3 - 4 \times 4}{14} = \frac{23}{14}$$

2. A rigid body consists of a 3 kg mass connected to a 2 kg mass by a massless rod. The 3kg mass is located at $\vec{r}_1 = (2\hat{i} + 5\hat{j})$ m and the 2 kg mass at $\vec{r}_2 = (4\hat{i} + 2\hat{j})$ m. Find the length of rod and the coordinates of the centre of mass.

Ans. $\sqrt{13}\text{m}, \left(\frac{14}{5}, \frac{19}{5}\right)$

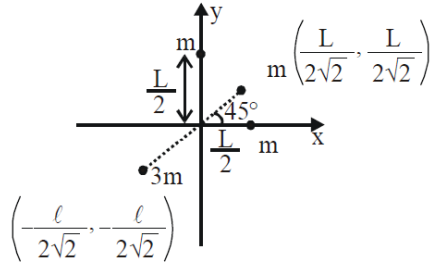
Sol.
$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1 = 2\hat{i} - 3\hat{j}$$
$$\therefore \ell = |\Delta \vec{r}| = \sqrt{4+9} = \sqrt{13}$$
$$x_{cm} = \frac{3 \times 2 + 2 \times 4}{5} = \frac{14}{5} = 2.8$$
$$y_{cm} = \frac{3 \times 5 + 2 \times 2}{5} = \frac{19}{5} = 3.8$$

3. Three identical uniform rods of the same mass M and length L are arranged in xy plane as shown in the figure. A fourth uniform rod of mass 3M has been placed as shown in the xy plane. What should be the value of the length of the fourth rod such that the center of mass of all the four rods lie at the origin?



Ans. $\frac{L(\sqrt{2} + 1)}{3}$

Sol.



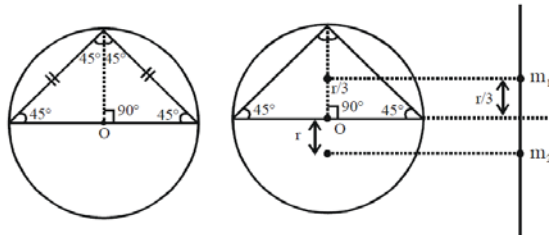
$$x_{cm} = 0 \Rightarrow m \frac{L}{2} + \frac{mL}{2\sqrt{2}} - 3m \frac{\ell}{2\sqrt{2}} = 0$$

$$\Rightarrow \frac{3\ell}{2\sqrt{2}} = \frac{L}{2} \left(1 + \frac{1}{\sqrt{2}} \right) \Rightarrow \ell = \frac{(\sqrt{2}+1)}{3} L$$

4. From a circle of radius a , an isosceles right angled triangle with the hypotenuse as the diameter of the circle is removed. The distance of the centre of gravity of the remaining position from the centre of the circle is

Ans. $\frac{a}{3(\pi-1)}$

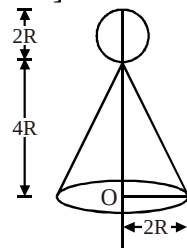
Sol.



$$m_1 \cdot \frac{R}{3} = m_2 r \Rightarrow r = \frac{m_1 R}{m_2 3}$$

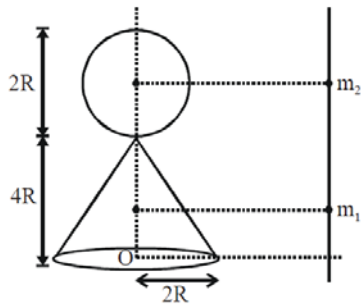
$$\Rightarrow r = \frac{A_1}{A_2} \cdot \frac{R}{3} = \left(\frac{\frac{1}{2} \times 2R \times R}{\pi R^2 - R^2} \right) \frac{R}{3} \Rightarrow r = \frac{R}{3(\pi-1)}$$

5. A man has constructed a toy as shown in figure. If density of the material of the sphere is 12 times of the cone compute the position of the centre of mass. [Centre of mass of a cone of height h is at height of $h/4$ from its base.]



Ans. $4R$ from O

Sol. $x_{cm} = 0$



$$y_{cm} = \frac{m_1 \cdot \left(\frac{4R}{4}\right) + m_2 \times 5R}{m_1 + m_2} = \left[\frac{V_1 \rho_1 + 5V_2 \rho_2}{V_1 \rho_1 + V_2 \rho_2} \right] R$$

$$m_1 = \frac{1}{3} \times \pi \times (2R)^2 \times 4R \times \rho = \frac{16}{3} \pi R^3 \rho$$

$$m_2 = \frac{4\pi}{3} \cdot R^3 \times 12\rho = 16\pi R^3 \rho$$

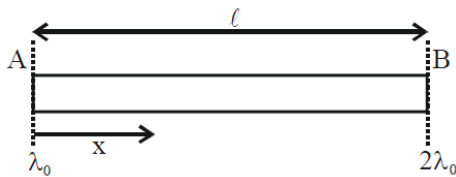
$$\therefore y_{cm} = \frac{\frac{16}{3} \pi R^3 \rho R + 16\pi R^3 5R}{\frac{16}{3} \pi R^3 \rho + 16\pi R^3} = \frac{R \left(\frac{1}{3} + 5 \right)}{\left(\frac{1}{3} + 1 \right)} = 4R$$

$$y_{cm} = 4R$$

6. The linear mass density of a ladder of length ℓ increases uniformly from one end A to the other end B,
 (i) Form an expression for linear mass density as a function of distance x from end A where linear mass density λ_0 . The density at one end being twice that of the other end.
 (ii) Find the position of the centre of mass from end A.

Ans. (i) $\lambda(x) = \lambda_0 + \frac{\lambda x}{L}$, (ii) $\frac{5}{9}L$

Sol.



$$\lambda(x) = \lambda_0 + \frac{(2\lambda_0 - \lambda_0)}{\ell} x = \lambda_0 + \frac{\lambda_0 x}{\ell}$$

$$\lambda = \lambda_0 \left(1 + \frac{x}{\ell} \right)$$

$$x_{cm} = \frac{\int x dm}{\int dm} = \frac{\int x(\lambda dx)}{\int \lambda dx}$$

$$= \frac{\int_0^\ell x \lambda_0 \left(1 + \frac{x}{\ell}\right) dx}{\int_0^\ell \lambda_0 \left(1 + \frac{x}{\ell}\right) dx} = \frac{\left(\frac{\ell^2}{2} + \frac{\ell^2}{3}\right)}{\ell + \frac{\ell}{2}}$$

$$= \frac{\frac{6\ell^2}{3}}{\frac{3\ell}{2}} = \frac{5}{9}\ell$$

7. Mass centers of a system of three particles of masses 1, 2, 3 kg is at the point (1 m, 2 m, 3 m) and mass center of another group of two particles of masses 2 kg and 3 kg is at point (-1 m, 3 m, -2 m). Where a 5 kg particle should be placed, so that mass center of the system of all these six particles shifts to mass center of the first system?

Ans. (3m, 1 m, 8 m)

Sol. $x_{cm} = 1$ $m_1 = 6 \text{ kg}$ $m_2 = 5 \text{ kg}$ $m_3 = 5 \text{ kg}$
 $y_{cm} = 2$ (1, 2, 3) (-1, 3, -2) (x, y, z)
 $z_{cm} = 3$

$$x_{cm} = \frac{6 \times 1 + 5 \times (-1) + 5x}{16} = 1$$

$$\Rightarrow x = 3$$

$$y_{cm} = \frac{6 \times 2 + 5 \times 3 + 5y}{16} = 2 \Rightarrow y = 1$$

$$z_{cm} = \frac{6 \times 3 + 5 \times 2 + 5.3}{16} = 3$$

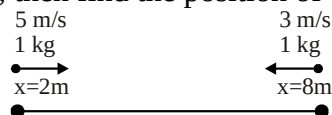
$$\Rightarrow Z_{cm} = 8$$

$$\therefore (x, y, z) = (3, 1, 8)$$

HCV Exercises (Chapter No. 9 :- 1 to 6, 11)

HCV Worked out Examples (Chapter No. 9 :- 4, 5, 6, 8, 9)

8. The figure shows the positions and velocities of two particles. If the particles move under the mutual attraction of each other, then find the position of centre of mass at $t = 1 \text{ s}$.



Ans. $x = 6 \text{ m}$

Sol. $(x_{cm})_i = \frac{1 \times 2 + 1 \times 8}{1 + 1} = 5 \text{ m}$

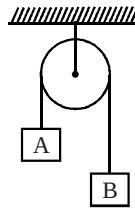
$$(u_{cm})_i = \frac{1 \times 5 - 1 \times 3}{1 + 1} = 1 \text{ m/s}$$

$$\therefore \vec{F}_{\text{ext}} = 0 \Rightarrow a_{cm} = 0$$

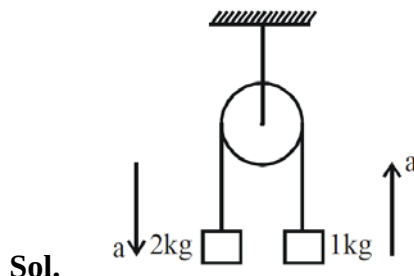
$$\therefore \text{At } t = 1 \text{ sec}$$

$$(x_{cm})_f = (x_{cm})_i + (u_{cm})_i t + 0 = 5 + 1 \times 1 = 6 \text{ m}$$

9. In the arrangement shown in the figure, $m_A = 2 \text{ kg}$ and $m_B = 1 \text{ kg}$. String is light and inextensible. Find the acceleration of centre of mass of both the blocks. Neglect friction everywhere.



Ans. $g/9$ downwards

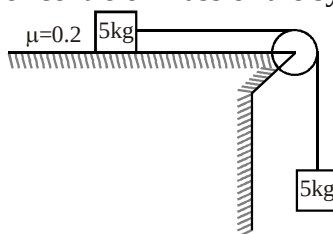


Sol.

$$a = \frac{2g - 1g}{3} = g/3$$

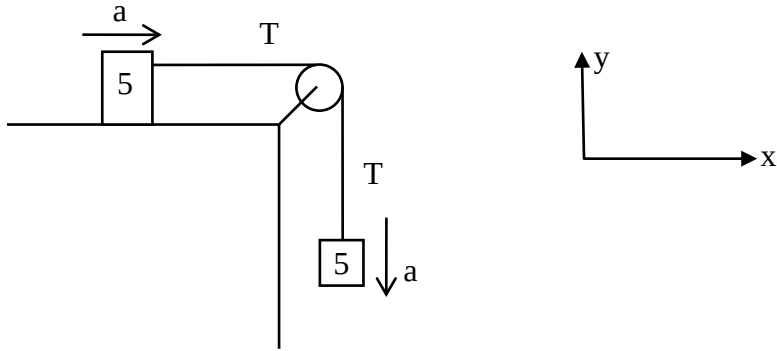
$$a_{cm} = \frac{2a - 1 \times a}{3} = \frac{a}{3} = \frac{g}{9}$$

10. The magnitude of acceleration of centre of mass of the system is ($g = 10 \text{ ms}^{-2}$)



Ans. $2\sqrt{2}$

Sol.



$$T - f = ma$$

$$T - f = 5a$$

$$T - f = 5a \quad \dots(1)$$

$$5g - T = 5a \quad \dots(2)$$

$$F = 0.2 \times 5g \quad \dots(3)$$

using equation (1), (2) & (3)

$$a = 4\text{m/s}^2$$

$$\vec{a}_{\text{cm}} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2}{m_1 + m_2}$$

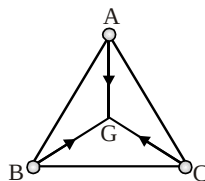
$$= \frac{5 \times 4\hat{i} + 5(-4\hat{j})}{5 + 5}$$

$$= \frac{20\hat{i} - 20\hat{j}}{10} = 2\hat{i} - 2\hat{j}$$

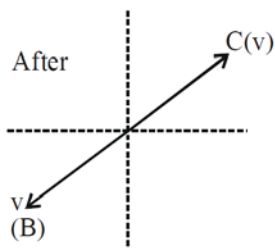
$$|\vec{a}_{\text{cm}}| = 2\sqrt{2} \text{ m/s}^2$$

11. Three particles A, B and C of equal mass move with equal speed v along the medians of an equilateral triangle as shown in fig. They collide at the centroid G of the triangle. After the collision, A comes to rest, B retraces its path with the speed v . What is the velocity of C ?

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Ans. $\vec{V}_C = -\vec{V}_B$



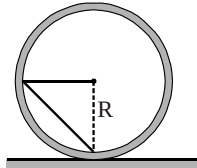
Sol.

In order to make $P_f = 0$

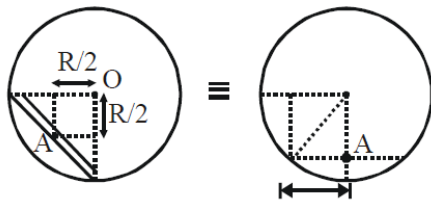
C, moves just opposite to B with v.

$$\Rightarrow \vec{v}_C = -\vec{v}_B$$

12. Inside a hollow uniform sphere of inner radius R a uniform rod of length $R\sqrt{2}$ is released from the state of rest as shown. The mass of the rod is same as that of the sphere. Assume friction to be absent everywhere. Horizontal displacement of sphere with respect to earth in the time in which the rod becomes horizontal, is



Ans. $\frac{R}{4}$



Sol.

Let displacement of sphere = x (right)

Displacement of rod = x + R/2

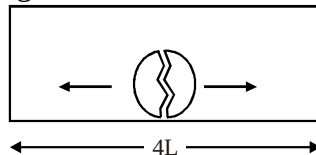
x + R/2 (right)

$$\Sigma mx = 0$$

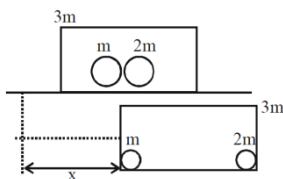
$$\Rightarrow mx + m(x + R/2) = 0$$

$$\Rightarrow x = -R/4 \Rightarrow R/4 \text{ left}$$

13. A bomb of mass 3m is kept inside a closed box of mass 3m and length 4L at its centre. It explodes in two parts of mass m & 2m. The two parts move in opposite direction and stick to the opposite side of the walls of box. Box is kept on a smooth horizontal surface. What is the distance moved by the box during this time interval.



Ans. $\frac{L}{3}$



Sol.

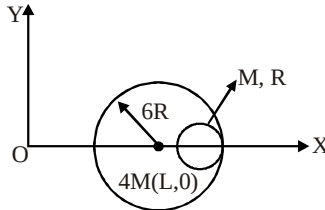
system [Box + two parts]

$$3mx + m(x - 2L) + 2m(x + 2L) = 0$$

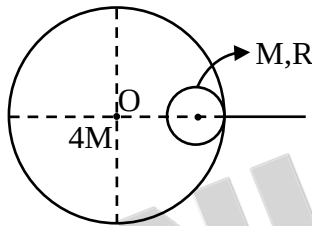
$$\Rightarrow 6x + 2L = 0$$

$$\Rightarrow x = -\frac{L}{3}$$

14. A small sphere of radius R is held against the inner surface of a larger sphere of radius $6R$. The masses of large and small spheres are $4M$ and M respectively. This arrangement is placed on a horizontal table. There is no friction between any surfaces of contact. The small sphere is now released. Find the co-ordinates of the centre of the larger sphere when the smaller sphere reaches the other extreme position. [JEE - 96]



Ans. $(L + 2R, 0)$
Sol.

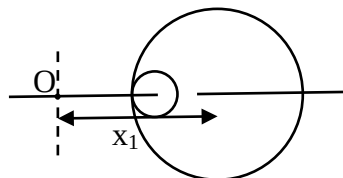


$$F_{\text{ext}} = 0 \text{ [In horizontal direction]}$$

$$\vec{U}_{\text{cm}} = 0 \text{ [In horizontal direction]}$$

$$\Rightarrow \vec{S}_{\text{cm}} = 0 \text{ [In horizontal direction]}$$

$$\Rightarrow \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = 0$$



$$\Rightarrow \frac{4mx_1 \times M(x_1 - 10R)}{4m_1 + m} = 0 \quad \Rightarrow \quad x_1 = \frac{10R}{5} = 2R.$$

$$\Rightarrow \text{Displacement of larger sphere} = 2R$$

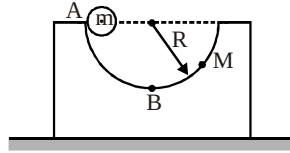
$$\Rightarrow \text{Coordinate of its center} = (L + 2R, 0)$$

HCV Exercises (Chapter No. 9 : -7,8,10,12,13)

HCV Worked out Examples(Chapter No. 9 :- 11,12,13,14,15)

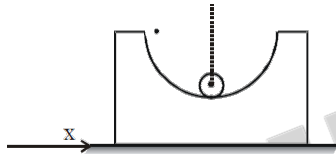
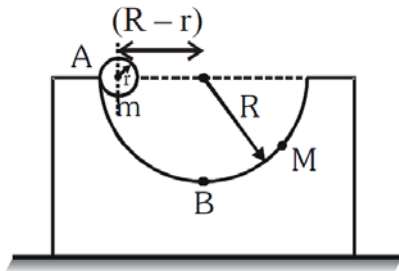
15. A block of mass M with a semicircular track of radius R , rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest at the top point A (see

Fig). The cylinder slips on the semicircular frictionless track. How far has the block moved when the cylinder reaches the bottom (point B) of the track ? How fast is the block moving when the cylinder reaches the bottom of the track?
[JEE - 83]



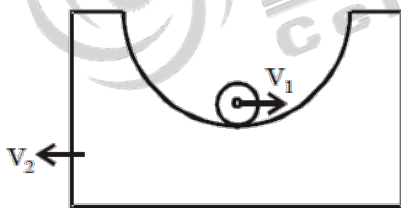
Ans. $\frac{m(R-r)}{M+m}, m\sqrt{\frac{2g(R-r)}{M(M+m)}}$

Sol.



$$\Rightarrow M \cdot x + m(x + R - r) = 0$$

$$\Rightarrow x = \frac{m(R-r)}{(M+m)}$$



v_1 & $v_2 \Rightarrow$ w.r.t. ground $\Rightarrow$

$(P)_H \Rightarrow$ Consumed $P_f = P_i$

$$mv_1 - Mv_2 = 0$$

$$mv_1 = Mv_2 \dots (i)$$

Energy conservation [for system]

$$mg(R-r) = \frac{1}{2}Mv_2^2 + \frac{1}{2}mv_1^2 \dots (ii)$$

$$\text{from (1)} \Rightarrow v_2 = \frac{m}{M} \cdot v_1$$

$$\text{from (2)} \quad mg(R-r) = \frac{1}{2}M\left(\frac{m}{M}v_1\right)^2 + mv_1^2 \dots (ii)$$

from (i)

$$v_2 = \frac{m}{M} \cdot v_1$$

from (2) $mg(R - r) =$

$$\frac{1}{2}M\left(\frac{m}{M}v_1\right)^2 + mv_1^2$$

$$mg(R - r) = \frac{1}{2}mv_1^2\left(1 + \frac{m}{M}\right)$$

$$\Rightarrow v_1 = \sqrt{\frac{2mg(R - r)M}{m(M + m)}}$$

$$\Rightarrow v_2 = \frac{m}{M}v_1 = \frac{m}{M}\sqrt{\frac{2Mg(R - r)}{M + m}}$$

$$\Rightarrow v_2 = m\sqrt{\frac{2g(R - r)}{M(m + M)}}$$

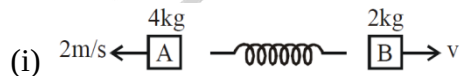
16. Two cars initially at rest are free to move in the x direction. Car A has mass 4 kg and car B has mass 2 kg. They are tied together, compressing a spring in between them. When the string holding them together is burned, car A moves off with a speed of 2 m/s.

(i) With what speed does car B leave.

(ii) How much energy was stored in the spring before it was burned.

Ans. (i) 4 m/s, (ii) 24 J

Sol.



system (A + B + Spring) $\Rightarrow$

$$F_{\text{ext}} = 0 \Rightarrow 2v - 2 \times 4 = 0$$

$$\Rightarrow v = 4 \text{ m/s} = v_B$$

(ii) Total energy of spring is obtained as the kinetic energy of A & B

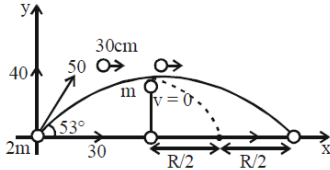
$$\therefore K_{\text{spring}} = \frac{1}{2}m_A v_A^2 + \frac{1}{2}mv_B^2 = \frac{1}{2} \times 4 \times 2^2 + \frac{1}{2} \times 2 \times 4^2 = 8 + 16 = 24 \text{ J}$$

17. A 24 kg projectile is fired at an angle of 53° above the horizontal with an initial speed of 50 m/s. At the highest point in its trajectory, the projectile explodes into two fragments of equal mass, the first of which falls vertically with zero initial speed.

(i) How far from the point of firing does the second fragment strike the ground? (Assume the ground is level.)

(ii) How much energy was released during the explosion?

Ans. (i) 360 m, (ii) 10800 J



Sol.

$$(i) \text{ Range } R = \frac{50^2 \times 2 \sin 53^\circ \cdot \cos 53^\circ}{10} = 250 \times 2 \times 0.6 \times 0.8$$

$$R = 240\text{m}$$

$$\therefore d = R + \frac{R}{2} = \frac{3R}{2} = 360\text{m}$$

(ii) From $P_f = P_i$ in horizontal

$$2m \cdot 30 = mv$$

$$\Rightarrow v = 60 \text{ m/s}$$

Just before explosion

$$K_i = \frac{1}{2} \cdot 2m \cdot v_x^2 \quad g m = 24 \Rightarrow m = 12$$

$$= \frac{1}{2} \times 240 \times (30)^2 = \frac{1}{2} \times 21600 \text{ J} = 10800 \text{ J}$$

Just after explosion

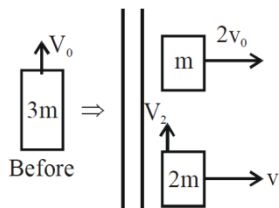
$$K_f = \frac{1}{2} mv^2 = \frac{1}{2} \times 12 \times 60^2 = 21600 \text{ J}$$

$$\therefore \text{Energy released} = K_f - K_i = 21600 - 10800 = 10800 \text{ J}$$

18. A spaceship is moving with constant speed v_0 in gravity free space along +Y-axis suddenly shoots out one third of its part with speed $2v_0$ along + X-axis. Find the speed of the remaining part.

Ans. $\frac{\sqrt{13}}{2} v_0$

Sol.



Gravity free

$$F_{\text{ext}} = 0$$

$$\Rightarrow P \Rightarrow \text{conserved}$$

$$\text{Along } x \Rightarrow 0 = m \cdot 2v_0 + 2mv_1$$

$$\Rightarrow v_1 = v_0$$

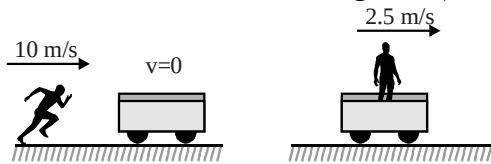
$$\text{along } y \Rightarrow 3mv_0 = 2mv_2$$

$$\Rightarrow v_2 = \frac{3v_0}{2}$$

$$\therefore \vec{v}_{2m} = -v_0 \hat{i} + \frac{3v_0}{g} \hat{j}$$

$$\therefore |\vec{v}_{2m}| = \sqrt{v_0^2 + \frac{9v_0^2}{4}} = \frac{\sqrt{13}}{2} v_0$$

19. A 50 kg boy runs at a speed of 10 m/s and jumps onto a cart as shown in the figure. The cart is initially at rest. If the speed of the cart with the boy on it is 2.50 m/s, what is the mass of the cart? (Assuming friction is absent between cart and ground)



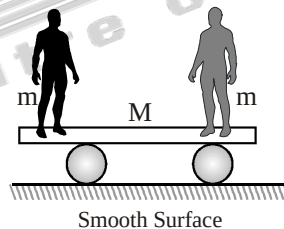
Ans. 150 kg

Sol. Conservation of momentum

$$50 \times 10 = (50 + m) 2.5$$

$$m = 150 \text{ kg.}$$

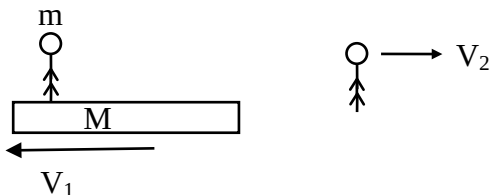
20. Two men of mass m are standing on the two corners of platform of length ' L ' and mass ' M '. They jump one by one from the platform in either direction with velocity u with respect to the final state of platform. Then find



- (a) Speed of platform after the first person jumps
(b) Speed of the platform after the 2nd person jumps to the other side
(c) Work done by 1st person on the platform after he jumps

Ans. (a) $\frac{mu}{M+2m}$; (b) $mu \left[\frac{1}{M+2m} - \frac{1}{M+m} \right]$; (c) $\frac{1}{2} \frac{m^2 u^2}{(M+2m)^2} (M+m)$

Sol. (a) Momentum cons.



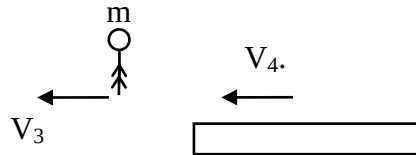
$$(m + M)V_1 = mV_2 \quad \dots(1)$$

$$\text{Also Relative velocity} = V_1 + V_2 = U \quad \dots(2)$$

using eq (1) & (2)

$$V_1 = V_1 = \frac{mu}{M+2m}$$

(b)



$$mV_3 + mV_4 = (m + M)V \quad \dots(3)$$

Also relative velocity

$$V_3 - V_4 = U \quad \dots(4)$$

use of (3) & (4)

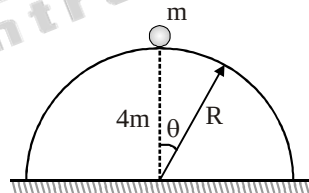
$$V_4 = mv \left[\frac{1}{M+2m} - \frac{1}{M+m} \right]$$

work energy theorem

(c) Work done = change in KE

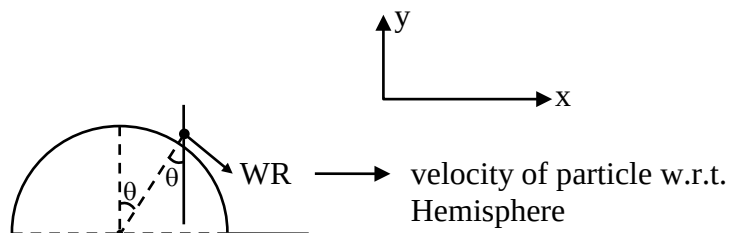
$$\text{Work done} = \frac{1}{2} (m + M)V_1^2 - 0$$

21. A hemisphere of radius R and of mass $4m$ is free to slide with its base on a smooth horizontal table. A particle of mass m is placed on the top of the hemisphere. Find the angular velocity of the particle relative to hemisphere as function of angular displacement θ , when velocity of hemisphere has become v .



Ans. $\frac{5v}{R \cos \theta}$

Sol.



velocity of particle in gnomist frame

$$= (\omega R \cos \theta - V) \hat{i} - \omega R \sin \theta \hat{j}$$

Momentum cons.

$$4m V = m(\omega R \cos\theta - V) \quad \dots(1)$$

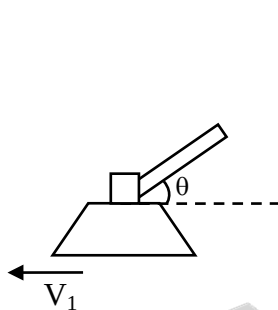
Using eq. (1)

$$\omega = \frac{5V}{R \cos\theta}$$

22. A cannon of mass $5m$ (including a shell of mass m) is at rest on a smooth horizontal ground, fires the shell with its barrel at an angle θ with the horizontal at a velocity u relative to cannon. Find the horizontal distance of the point where shell strikes the ground from the initial position of the cannon:

Ans. $\frac{4u^2 \sin 2\theta}{5g}$

Sol.



Velocity of shell w.r.t ground = $(V \cos\theta - V_1) \hat{i} + V \sin\theta \hat{j}$

Momentum Cons.

$$4mV_1 = m(u \cos\theta - V_1)$$

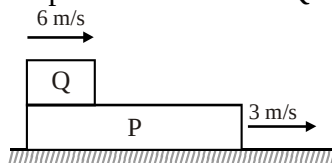
$$\Rightarrow V_1 = \frac{u \cos\theta}{5}$$

Velocity Projectile w.r.t. ground = $\frac{4V \cos\theta \hat{i} + V \sin\theta \hat{j}}{5}$

Horizontal distance = (Horizontal velocity) $\times$ time of flight

$$= \left(\frac{4}{5} V \cos\theta \right) \times \frac{2V \sin\theta}{g} = \frac{4}{5} \frac{V^2 \sin 2\theta}{g}$$

23. A plank P and block Q are arranged as shown on a smooth table top. They are given velocities 3 m/s and 6 m/s respectively. The length of plank is 1m and block is of negligible size. After some time when the block has reached the other end of plank it stops slipping on plank. Find the coefficient of friction between plank P and block Q if mass of plank is double of block).



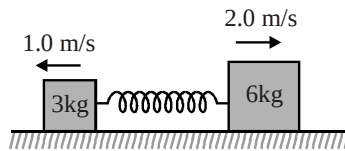
Ans. 0.3

Sol. $m \times 6 + 2m \times 3 = 3m \times V_C$
 $V_C = 4$
 with respect to P
 $0 = 3^2 - 2 \times (3\mu g/2) \times L$
 $\mu = 0.3$

HCV Exercises (Chapter No. 9 :- 14,15,18,19,20,21,22,23,26,28,29,38)

HCV Worked out Examples(Chapter No. 9 :- 7,18,19)

24. Two blocks of mass 3 kg and 6 kg respectively are placed on a smooth horizontal surface. They are connected by a light spring of force constant $k = 200 \text{ N/m}$. Initially the spring is unstretched. The indicated velocities are imparted to the blocks. The maximum extension of the spring will be



Ans. 30 cm

Sol. By using reduced mass concept

$$\frac{1}{2} \mu u_{\text{rel}}^2 = \frac{1}{2} K x_{\text{max}}^2 \Rightarrow x_{\text{max}} = u_{\text{rel}} \sqrt{\frac{\mu}{k}} = \sqrt{\frac{\left(\frac{3 \times 6}{3+6}\right)}{200}} = 3 \sqrt{\frac{2}{200}} = 0.3 \text{ m}$$

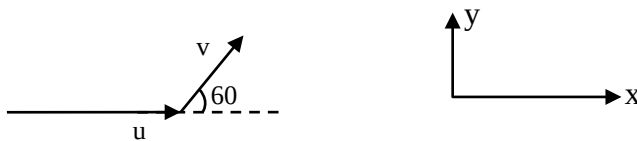
HCV Exercises (Chapter No. 9 :- 41,46,49,50,51,52)

HCV Worked out Examples(Chapter No. 9 :- 10,17,24)

25. A bullet of mass m strikes an obstruction and deviates off at 60° to its original direction. If its speed is also changed from u to v , find the magnitude of the impulse acting on the bullet.

Ans. $m \times \sqrt{u^2 - uv + v^2}$

Sol.



$$P_1 = mv \hat{i}$$

$$P_f = mv \cos \theta \hat{i} + mv \sin \theta \hat{j}$$

$$\text{Implies} = P_f - P_i = (mv \cos \theta - mv) \hat{i} + mv \sin \theta \hat{j}$$

Magnitude of Implies

$$= \sqrt{(mv \cos \theta - mv)^2 + (mv \sin \theta)^2}$$

$$= m \sqrt{u^2 - uv + v^2}$$

26. Bullets of mass 10 g each are fired from a machine gun at rate of 60 bullets/minute. The muzzle velocity of bullets is 100 m/s. The thrust force due to firing bullets experienced by the person holding the gun stationary is_____.

Ans. 1 N

Sol. Change in momentum of single bullet = $mv_f - m \times 0 = 10 \times 10^{-3} \times 100$

$\therefore$ Change in momentum of 60 bullets = 60 N₁ sec

$$\therefore (\Delta P)_{\text{per sec}} = \frac{60}{60} = 1 = \frac{dP}{dt}$$

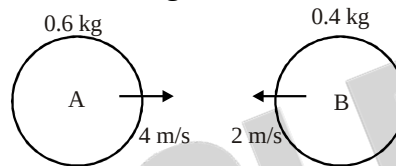
$\therefore$ Force exerted an bullet = 1N

$\therefore$ Force on person = 1N

HCV Exercises (Chapter No. 9 :-24,42,57)

HCV Worked out Examples(Chapter No. 9 - 16,20,21,22,23)

27. The velocities of two steel balls before impact are shown. If after head on impact the velocity of ball B is observed to be 3 m/s to the right, the coefficient of restitution is



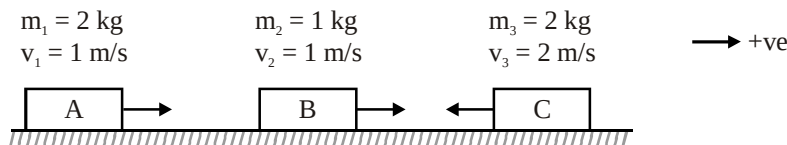
Ans. $\frac{7}{18}$

Sol. By conservation of linear momentum

$$(0.6)(4) + (0.4)(-2) = (0.6)(v_A) + (0.4)(3)$$

$$\Rightarrow v_A = \frac{2}{3} \text{ \& } e = \frac{v_{\text{sep}}}{v_{\text{app}}} = \frac{3 - v_A}{4 + 2} = \frac{7}{18}$$

28. Three carts move on a frictionless track with inertias and velocities as shown. The carts collide and stick together after successive collisions.



- (i) Find loss of mechanical energy when B & C stick together.
(ii) Find magnitude of impulse experienced by A when it sticks to combined mass (B & C).

Ans. (i) 3 J, (ii) $\frac{12}{5}$ N-s

Sol. $\frac{60}{60} = 1 = \frac{dP}{dt}$

(i) For B & C $\Rightarrow \bar{P}$ conser flux

$$-2 \times 2 = 3v$$

$$\Rightarrow v = 1\text{m/s}$$

$\therefore$ Loss of PE = Loss of KE ($\because \Delta PE = 0$)

$$\Delta K = \frac{1}{2} \times 3 \times 1^2 - \left(\frac{1}{2} \times 1 \times 1^2 + \frac{1}{2} \times 2 \times 2^2 \right) = \frac{3}{2} - \frac{9}{2}$$

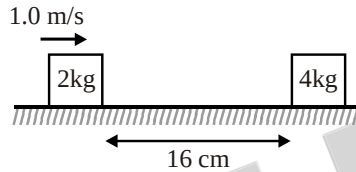
$$\Delta K = -3J$$

$$(ii) \quad \begin{array}{c} \xrightarrow{1\text{m/s}} \\ \boxed{A} \end{array} \quad \begin{array}{c} \xleftarrow{1\text{m/s}} \\ \boxed{B+C} \end{array} \Rightarrow \begin{array}{c} \xrightarrow{\quad} \\ \boxed{A+B+C} \end{array}$$

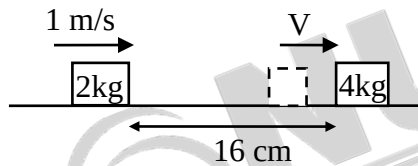
$$2 \times 1 - 3 \times 1 = 5v \Rightarrow v = -\frac{1}{5} \text{ m/s}$$

$$\therefore I_A = \Delta P_A = 2 \times \frac{1}{5} - (-2 \times 1) = \frac{12}{5} \text{ N.S}$$

29. The friction coefficient between the horizontal surface and each of the block shown in the figure is 0.2. The collision between the blocks is perfectly elastic. Find the separation (in cm) between them when they come to rest.



Ans. 005
Sol.



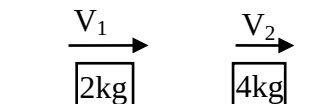
$$\begin{aligned} a &= \frac{f}{m} \\ &= \frac{\mu mg}{m} \\ &= \mu g \text{ m/s}^2 \\ &= 2 \text{ m/s}^2 \end{aligned}$$

$$V^2 = 1^2 + 2(-2)(0.16)$$

$$V^2 = 1 - 0.64$$

$$= 0.36 \Rightarrow V = 0.6 \text{ m/s}$$

Momentum conservation during the collision



$$2 \times 0.6 = 2V_1 + 4V_2$$

$$1.2 = 2V_1 + 4V_2$$

$$0.6 = V_1 + 2V_2 \quad \dots(1)$$

$$e = \frac{V_2 - V_1}{0.6} = 1$$

$$\Rightarrow V_2 - V_1 = 0.6 \quad \dots(2)$$

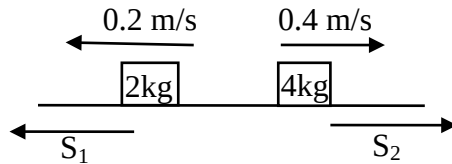
Eq, (1) + (2)

$$3V_2 = 1.2$$

$$V_2 = 0.4 \text{ m/s}$$

$$V_1 = -0.2 \text{ m/s.}$$

Collision



$$S_1 = \frac{(0.2)^2}{2 \times 2}$$

$$S_2 = \frac{(0.4)^2}{2 \times 2}$$

Separation between them = $S_1 + S_2$

30. A body is thrown vertically upwards from ground with a speed of 10 m/s. If coefficient of restitution of ground, $e = 1/2$. Find
- the total distance travelled by the time it almost stops.
 - time elapsed (after the ball has been thrown) when it is at its subsequent maximum height for the third time.

Ans. (i) $\frac{40}{3}$ m; (ii) $t_e = 3.25$ s

Sol.



Initial speed = $U = 10$ m/s.

Speed after first collision = eu

Speed after second collision = e^2u

$$\Rightarrow \text{distance travelled before first collision} = \frac{2V^2}{2g}$$

distance travelled between first & second

$$\text{collision} = \frac{2(eu)^2}{2g}$$

distance travelled between second & third

$$\text{Collision} = \frac{2(e^2u)^2}{2g}$$

and So on

total distance travelled

$$= \frac{2(u^2)}{2g} + \frac{2(ev)^2}{2g} + \frac{2(e^2v)^2}{2g} + \dots$$

$$= \frac{2u^2}{2g} [1 + e^2 + e^4 + e^6 + \dots]$$

$$= \frac{u^2}{g} \left[\frac{1}{1-e^2} \right] = \frac{4u^2}{3g}$$

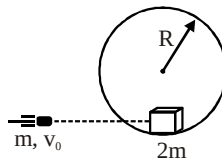
(ii) time = $\frac{2u}{g} + \frac{2eu}{g} + \frac{e^2v}{g}$

$$= 2 + 1 + \frac{1}{4}$$

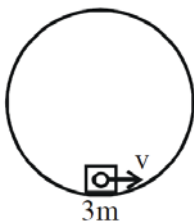
$$= \frac{13}{4}$$

31. A small block of mass $2m$ initially rests at the bottom of a fixed circular, vertical track, which has a radius of R . The contact surface between the mass and the loop is frictionless. A bullet of mass m strikes the block horizontally with initial speed v_0 and remain embedded in the block as the block and the bullet circle the loop. Determine each of the following in terms of m , v_0 , R and g .

- (i) The speed of the masses immediately after the impact.
(ii) The minimum initial speed of the bullet if the block and the bullet are to successfully execute a complete ride on the loop



Ans. (i) $\frac{v_0}{3}$, (ii) $3\sqrt{5gR}$

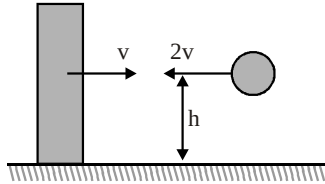


Sol.

$$mv_0 \Rightarrow 0 = 3mv \Rightarrow v = \frac{v_0}{3}$$

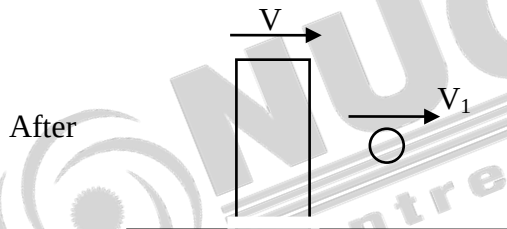
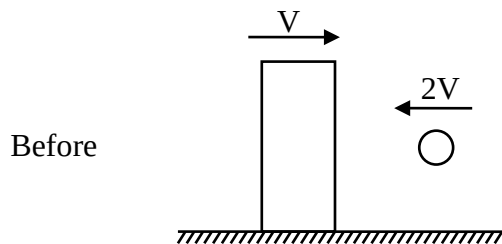
(ii) For successful motion $\Rightarrow v = \sqrt{5gR} \Rightarrow \frac{v_0}{3} = \sqrt{5gR} \Rightarrow v_0 = 3\sqrt{5gR}$

32. A ball collides elastically with a massive wall moving towards it with a velocity v . The collision occurs at a height of h above ground level and the velocity of the ball just before collision is $2v$ in horizontal direction as shown in figure. Then the distance from the foot of the wall and the point on the ground where the ball lands (at the instant the ball lands) is :



Ans. $3v\sqrt{\frac{2h}{g}}$

Sol.



$$e = \frac{V_1 - V}{3V} = 1$$

$$V_1 = 4V$$

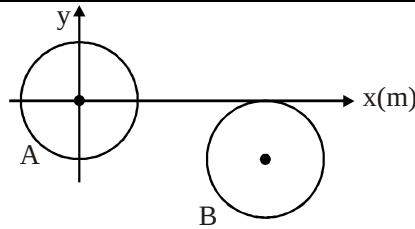
No Relation velocity of ball w.r.t. wall
after collision = $3v$

$$\text{time of flight} = \sqrt{\frac{2h}{g}}$$

Relation Horizontal separation between ball & wall

$$= 3v\sqrt{\frac{2h}{g}}$$

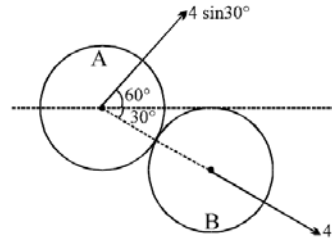
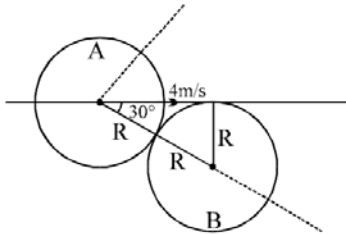
33. Two smooth balls A and B, each of mass m and radius R , have their centres at $(0,0,R)$ and at $(5R,-R,R)$ respectively, in a coordinate system as shown. Ball A, moving along positive x axis, collides with ball B. Just before the collision, speed of ball A is 4 m/s and ball B is stationary. The collision between the balls is elastic. Find Velocity of the ball A just after the collision and impulse of the force exerted by A on B during the collision.



Ans. $(\hat{i} + \sqrt{3}\hat{j}) \text{ m/s}, (3m\hat{i} - \sqrt{3}m\hat{j}) \text{ kg-m/s}$

Sol. Before collision

After collision



$$v_A = 4 \sin 30^\circ [\cos 60^\circ \hat{i} + \sin 60^\circ \hat{j}]; v_A = \hat{i} + \sqrt{3}\hat{j}$$

$$\vec{J}_{A \text{ on } B} = m\vec{V}_{B_f} - \vec{V}_{B_i} = m[4 \cos 30^\circ (\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j}) - 0] = (3m\hat{i} - \sqrt{3}m\hat{j}) \text{ kg-m/s}$$

(D) There is a net loss in the kinetic energy of the two particle system in the collision.

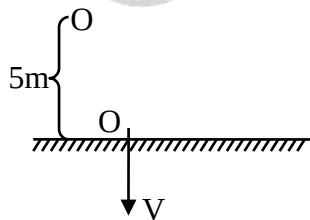
34. A ball of mass 100 g is dropped from a height $h = 5\text{m}$ on a inelastic floor. If the coefficient of restitution is $e = 0.5$ then. Find

(a) Time of entire motion in seconds

(b) Average speed of the entire motion in ms^{-1}

Ans. (a) 3; (b) $\frac{25}{9}$

Sol.



$$\begin{aligned} u &= \sqrt{2gh} \\ &= \sqrt{2 \times 10 \times 5} \\ &= 10 \text{ m/s.} \end{aligned}$$

$$\text{time of motion} = \sqrt{\frac{2h}{g}} + \frac{2ev}{g} + \frac{2e^2v}{g} + \frac{2e^3v}{g} + \dots \infty$$

$$= 1 + \frac{2v}{g} [e + e^2 + e^3 + e^4 + \dots]$$

$$= 1 + \frac{2 \times 10}{10} \left[\frac{e}{1-e} \right]$$

$$= 1 + 2 \left[\frac{1/2}{1 - \frac{1}{2}} \right]$$

$$= 3 \text{ sec.}$$

Total distance travelled

$$= 5 + 2 \frac{2(ev)^2}{2g} + \frac{2(e^2v)^2}{2g} + \frac{2(e^2v)^2}{2g} + \dots$$

$$= 5 + \frac{2e^2v^2}{2g} [1 + e^2 + e^4 + e^6 + \dots]$$

$$= 5 + 2 \times \frac{\frac{1}{4} \times 10^2}{2 \times 10} \left[\frac{1}{1 - e^2} \right]$$

$$= 5 + \frac{5}{2} \left[\frac{1}{1 - \frac{1}{4}} \right]$$

$$= 5 + \frac{5}{2} \left[\frac{4}{3} \right]$$

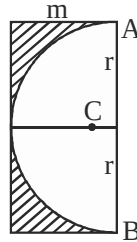
$$= 5 + \frac{10}{3} = \frac{25}{3} \text{ m.}$$

$$\text{Average speed} = \frac{\text{distance}}{\text{time}} = \frac{\left(\frac{25}{3} \right)}{3} = \frac{25}{9} \text{ m/s.}$$

HCV Exercises (Chapter No. 9 :- 25,31,32,33,34,36,37,39,43,44,45,47,48,53-56,58 to 64)

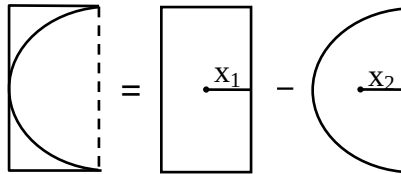
EXERCISE # (S-2)

1. Determine the centre of gravity of a thin homogeneous plate having the form of a rectangle with sides r and $2r$ from which a semicircle with a radius r is cut out of figure.



Ans. $C = \frac{2r}{3(4-\pi)}$

Sol.



$$A_1 = 2r^2 \quad A_2 = \frac{\pi r^2}{2}$$

$$x_1 = \left(\frac{r}{2}\right) \quad x_2 = \frac{4R}{3\pi}$$

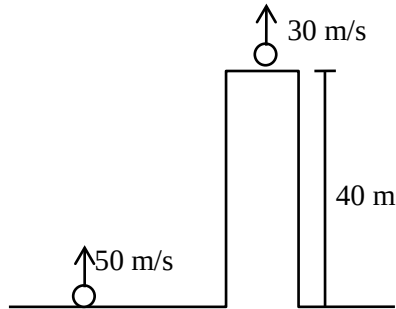
$$x_{cm} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} = \frac{(2r^2)\left(\frac{r}{2}\right) - \frac{\pi r^2}{2}\left(\frac{4R}{3\pi}\right)}{2r^2 - \frac{\pi r^2}{2}}$$

$$= \frac{r^3 - \frac{2r^3}{3}}{(4-\pi)\frac{r^2}{2}} = \frac{\frac{r^3}{3}}{(4-\pi)\frac{r^2}{2}} = \frac{2r}{3(4-\pi)}$$

2. Two balls of equal masses are projected upward simultaneously, one from the ground with speed 50 m/s and other from a 40 m high tower with initial speed 30 m/s. Find the maximum height attained by their centre of mass.

Ans. 100 m

Sol.



$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{m(50) + m(30)}{m + m}$$

$$= 40 \text{ m/s (upward)}$$

Initial position of com = 20 m from ground.

$$\text{Max. height achieved from initial position} = \frac{(40)^2}{2g} = 80\text{m}$$

Max. height reached from ground = 20 + 80 = 100 m.

3. A man whose mass is m kg jumps vertically into air from a sitting position in which his centre of mass is at a height h_1 from the ground. When his feet are just about to leave the ground his centre of mass is h_2 from the ground and finally rises to h_3 when he is at the top of the jump. What is the average upward force exerted by the ground on him?

Ans. $\frac{mg(h_3 - h_1)}{(h_2 - h_1)}$

Sol. Initially at Rest $\Rightarrow v_i = 0$

Let v = velocity of com when the contact is about to be lost.

Max. height of com after the jumps = $(h_3 - h_2)$

$$\frac{v^2}{2g} = (h_3 - h_2)$$

$$v = \sqrt{2g(h_3 - h_2)}$$

When he moves from sitting position to standing position, displacement of com = $h_2 - h_1$

a_{am} = acc. of com when it has contact with ground.

$$v_{cm}^2 = u_{cm}^2 + 2a_{cm}S$$

$$\Rightarrow (\sqrt{2g(h_3 - h_2)})^2 = 0^2 + 2a_{cm}(h_2 - h_1)$$

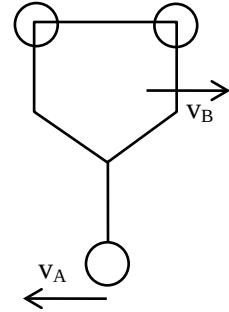
$$\Rightarrow a_{am} = \frac{g(h_3 - h_2)}{(h_3 - h_1)}$$

$$F_{net} = ma$$

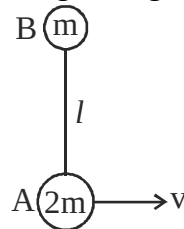
$$\Rightarrow F_{\text{net}} = m \times g \frac{(h_3 - h_2)}{h_2 - h_1}$$

$$\Rightarrow F - mg = mg \frac{(h_3 - h_2)}{h_2 - h_1}$$

$$\Rightarrow F = mg + mg \frac{(h_3 - h_2)}{h_2 - h_1}$$



4. Two masses A and B connected with an inextensible string of length ℓ lie on a smooth horizontal plane. A is given a velocity of v m/s along the ground perpendicular to line AB as shown in figure. Find the tension in string during their subsequent motion.



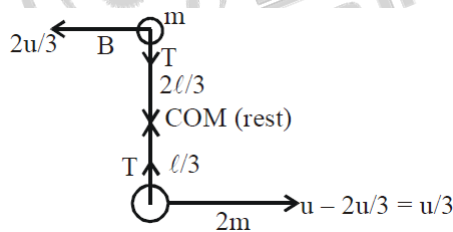
Ans. $\frac{2mv^2}{3\ell}$

Sol. In fram of COM $\Rightarrow$ (B moves in a circle of radii $\frac{2\ell}{3}$ and A moves in a circle of $\frac{\ell}{3}$

$$F_{\text{ext}} = 0$$

$$\Rightarrow a_{\text{cm}} = 0$$

n free of COM

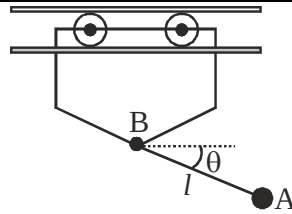


$$v_{\text{cm}} = \frac{2mu}{3m} = \frac{2u}{3}$$

$$\text{for A} \Rightarrow T = \frac{2m(u/3)^2}{\ell/3} = \frac{2mu^2}{3\ell}$$

$$\text{or For B} \Rightarrow T = \frac{m(2u/3)^2}{2\ell/3} = \frac{2mu^2}{3\ell}$$

5. The simple pendulum A of mass m_A and length ℓ is suspended from the trolley B of mass m_B . If the system is released from rest at $\theta = 0$, determine the velocity v_B of the trolley and tension in the string when $\theta = 90^\circ$. Friction is negligible.



Ans. $v_B = \frac{m_A}{m_B} \sqrt{\frac{2gl}{1 + m_A/m_B}}$, $T = 3m_A g + \frac{2m_A^2 g}{m_B}$

Sol. Momentum conservation

$$m_A v_A = m_B v_B \quad \dots(1)$$

Work Energy Theorem

$$\left(\frac{1}{2} m_B v_B^2 + \frac{1}{2} m_A v_A^2 \right) - 0 = m_A g \ell \quad \dots(2)$$

Using equation (1) and (2)

$$v_B = \frac{m_A}{m_B} \sqrt{\frac{2g\ell}{1 + \frac{m_A}{m_B}}}$$

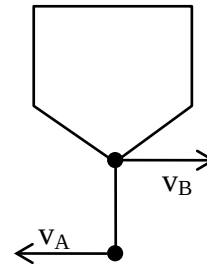
$$v_A = \sqrt{\frac{2g\ell}{1 + \frac{m_A}{m_B}}}$$

Point A moves in circular w.r.t. B.

$$T - m_A g = m_A \frac{(v_A + v_B)^2}{\ell}$$

$$T - m_A g + m_A \frac{(v_A + v_B)^2}{\ell}$$

$$T = 3m_A g + \frac{2m_A^2 g}{m_B}$$

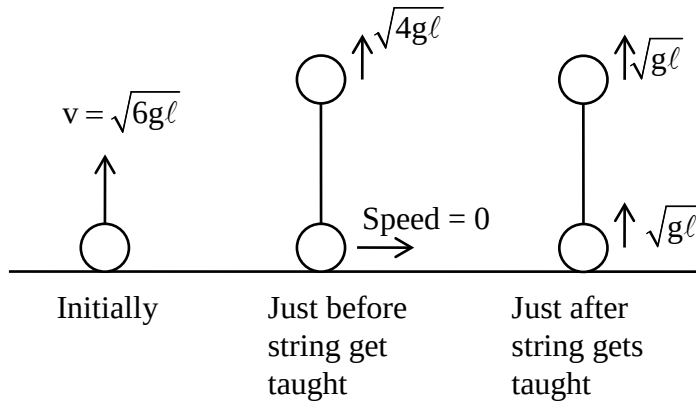


6. Two bodies of same mass tied with an inelastic string of length ℓ lie together. One of them is projected vertically upwards with velocity $\sqrt{6g\ell}$. Find the maximum height up to which the centre of mass of system of the two masses rises.

Ans. ℓ

Sol. V_{cm} = Velocity of con just after string gets taugt.

$$V_{cm} = \sqrt{g\ell}$$



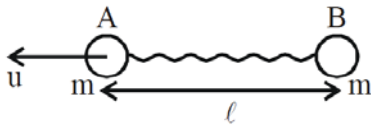
$$\text{Max. Height achieved by com from initial position} = \frac{(V_{\text{cm}})^2}{2g} = \frac{(\sqrt{g\ell})^2}{2g} = \left(\frac{\ell}{2}\right)$$

$$\text{Initial position of com after string gets taught} = \frac{\ell}{2}$$

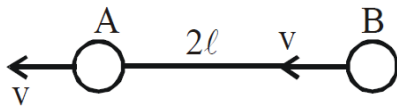
$$\Rightarrow \text{Max height of com from ground} = \frac{\ell}{2} + \frac{\ell}{2} = \ell$$

7. Two particles, each of mass m , are connected by a light inextensible string of length 2ℓ . Initially they lie on a smooth horizontal table at points A and B distant ℓ apart. The particle at A is projected across the table with velocity u . Find the speed with which the second particle begins to move if the direction of u is :-
- along BA.
 - at an angle of 120° with AB.
 - perpendicular to AB. In each case calculate (in terms of m & u) the impulsive tension in the string.

Ans. (i) $\frac{u}{2}, \frac{mu}{2}$ (ii) $\frac{\sqrt{13}}{8}u, \frac{\sqrt{13}}{8}mu$ (iii) $\frac{\sqrt{3}}{4}u, \frac{mu\sqrt{3}}{4}$



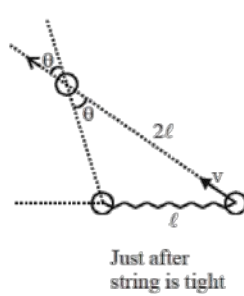
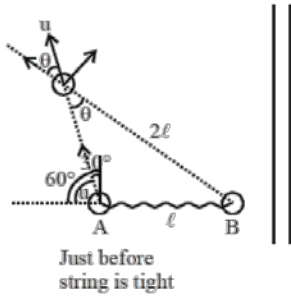
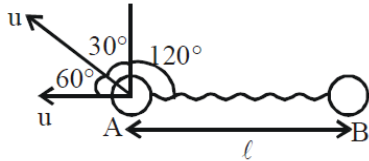
Sol. (i)



$$2mv = mu \Rightarrow v = \frac{u}{2}$$

$$\therefore \text{impulse of tension} = (\Delta P)_B = mv = mu/2$$

(ii)



$$\text{Sine rule} \Rightarrow \frac{\sin \theta}{\ell} = \frac{\sin 120^\circ}{2\ell} \Rightarrow \sin \theta = \frac{\sqrt{3}}{4}$$

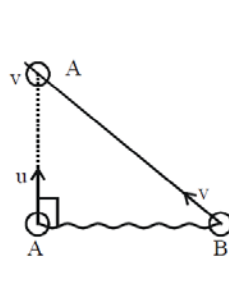
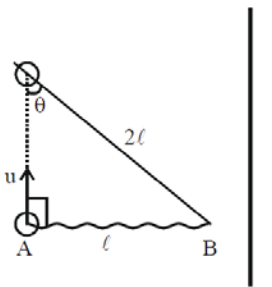
$$\Rightarrow \cos \theta = \frac{\sqrt{3}}{4}$$

$$2m \cdot v = mu \cos \theta$$

$$v = \frac{u \cos \theta}{2} = \frac{u}{2} \times \frac{\sqrt{13}}{4} = \frac{u\sqrt{13}}{8}$$

$$\therefore \text{Impulse of tension} = (\Delta P)_B = \frac{m \cdot u \sqrt{13}}{8}$$

(iii)



Just before

$$\sin \theta = \frac{\ell}{2\ell} \Rightarrow \theta = 30^\circ$$

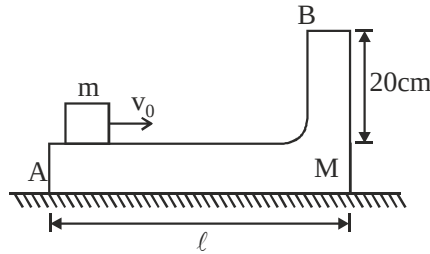
from $\vec{P}$ conservation along string just after tight

$$2mv = mu \cos \theta = mu \cos 30^\circ$$

$$v = \frac{u\sqrt{3}}{4}$$

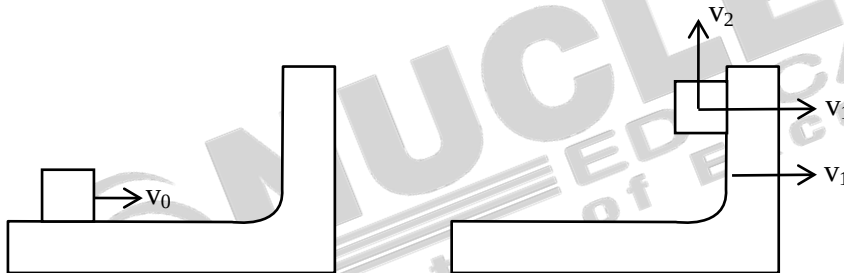
$$\therefore \text{Impulse of tension} = (\Delta P)_B = mv = \frac{mu\sqrt{3}}{4}$$

8. A wedge of mass $M = 2m$ rests on a smooth horizontal plane. A small block of mass m rests over it at left end A as shown in figure. A sharp impulse is applied on the block, due to which it starts moving to the right with velocity $v_0 = 6 \text{ m/s}$. At highest point of its trajectory, the block collides with a particle of same mass m moving vertically downwards with velocity $v = 2 \text{ m/s}$ and gets stuck with it. If the combined body lands at the end point A of body of mass M , calculate length ℓ . Neglect friction ($g = 10 \text{ m/s}^2$)



Ans. 40 cm

Sol.



Momentum Conservation

$$mv_0 = mv_1 + 2mv_1$$

$$v_1 = \frac{v_0}{3} \quad \dots(1)$$

$$v_1 = 2 \text{ m/s}$$

Work Energy Theorem

$$\frac{1}{2}m(v_1^2 + v_2^2) + \frac{1}{2}2m v_1^2 - \frac{1}{2}m v_0^2 = -mg(0.2)$$

$$\frac{v_1^2 + v_2^2}{2} + v_1^2 - \frac{v_0^2}{2} = -g(0.2)$$

$$\frac{3v_1^2}{2} + \frac{v_2^2}{2} - v_0^2 = -2$$

$$\frac{3}{2}(2)^2 + \frac{v_2^2}{2} - 36 = -2$$

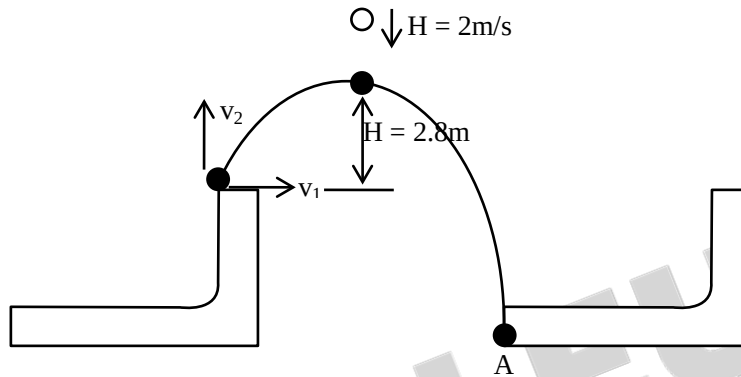
$$6 + \frac{v_2^2}{2} - 36 = -2$$

$$v_2^2 = 56$$

$$v_2 = \sqrt{56}$$

$$v_1 = 2 \text{ m/s}$$

$$H = \frac{v_2^2}{2g} = \frac{56}{20} = 2.8$$



At highest point, Momentum Conservation

$$m \times 2 = m \times v + mv$$

$$\Rightarrow v = 1 \text{ m/s downward.}$$

Time taken to come down

$$-3 = -1 \times t - \frac{1}{2} \times 10 \times t^2$$

$$\Rightarrow 5t^2 + t - 3 = 0$$

$$t = -1 + \frac{\sqrt{1^2 - (5)(-3)}}{2 \times 5}$$

$$= \frac{-1 + 4}{10} = \frac{3}{10} \text{ sec.}$$

At the topmost point, speed of block in horizontal direction.

Momentum Conservation

$$m(2) = (2m) v'$$

$$v' = 1 \text{ m/s}$$

After Reaching top most point, Relative velocity between block and wedge in horizontal direction = $2 - 1 = 1 \text{ m/s}$

Relative displacement in horizontal direction between block and wedge = ℓ .

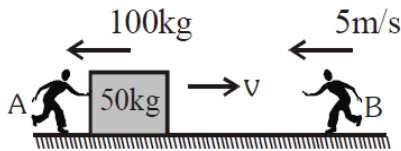
$$\ell = v_{\text{rel}} \times t$$

$$\ell = 1 \times \frac{3}{10} = \frac{3}{10} \text{ m}$$

9. Two persons A and B each of mass 100 kg are on a frictionless horizontal surface. Person A at rest is holding a block of mass 50 kg, suddenly pushes the block with some velocity (v) towards the person B approaching at a velocity of 5 m/s. B catches the block & slows down. Now the separation between A and B becomes constant. Find the speed v (in m/s).

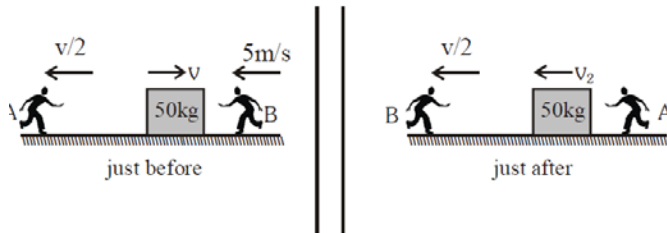


Ans. 4



Sol.

$$100v_1 = 50v \Rightarrow v_1 = \frac{v}{2}$$



$$\text{between block \& B} \Rightarrow 50v - 100 \times 5 = -150v_2$$

$$\Rightarrow v_2 = -\left(\frac{v}{3} - \frac{10}{3}\right) = \frac{-v+10}{3}$$

$$\because \text{separation between A \& B} \Rightarrow \text{const}$$

$$\Rightarrow v_{A/B} = 0 \Rightarrow v_{Ag} - v_{Bg} = 0$$

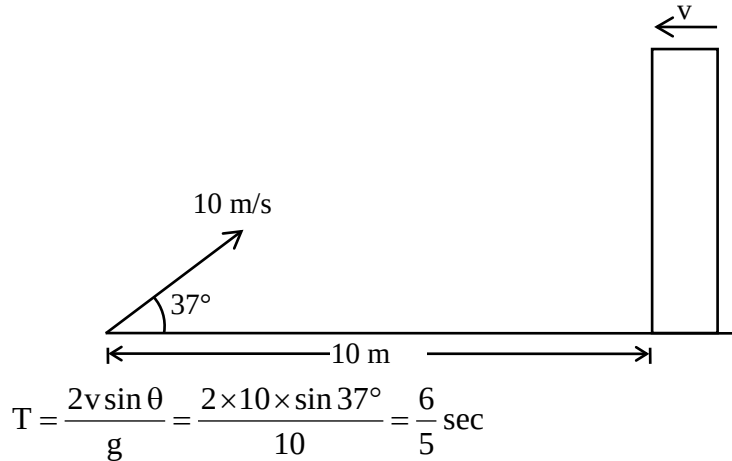
$$\Rightarrow \frac{v}{2} = \frac{-v+10}{3}$$

$$\Rightarrow 5v = 20 \Rightarrow v = 4 \text{ m/s}$$

10. A massive vertical wall is approaching a man at a speed u . When it is at a distance of 10 m, the man throws a ball with speed 10 m/s at an angle of 37° which after completely elastic rebound reaches back directly into his hands. Find the velocity u of the wall.

Ans. $13/3$ m/sec

Sol.



Let the time taken by ball to reach wall = x

$$\Rightarrow (10 \cos 37^\circ)x + U \times x = 10$$

$$\Rightarrow 8x + Ux = 10 \quad \dots(1)$$

Also at collision

$$\frac{v - u}{10 \cos 37^\circ + u} = e = 1 \quad [v = \text{speed of ball after collision}]$$

$$\frac{v - u}{8 + u} = 1 \Rightarrow v = 2u + 8$$

$$\Rightarrow (2u + 8) \left(\frac{6}{5} - x \right) = \text{distance travelled by ball in horizontal after collision}$$

$$= (10 \cos 37^\circ)x$$

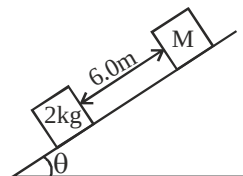
$$\Rightarrow (2u + 8) \left(\frac{6}{5} - x \right) = 8x \quad \dots(2)$$

using equation (1) and (2)

11. Two blocks of mass 2kg and M are at rest on an inclined plane and are separated by a distance of 6m as shown. The coefficient of friction between each of the blocks and the inclined plane is 0.25. The 2kg block is given a velocity of 10 m/s up the inclined plane. It collides with M, comes back and has a velocity of 1 m/s when it reaches its initial position. The other block M after the collision moves 0.5 m up and comes to rest. Calculate the coefficient of restitution between the blocks and the mass of the block M.

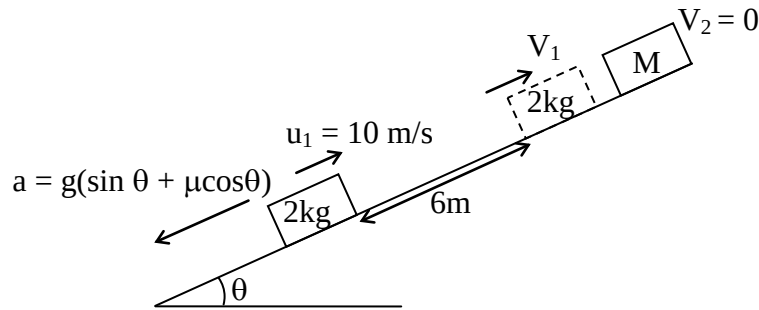
[Take $\sin \theta \approx \tan \theta = 0.05$ and $g = 10 \text{ m/s}^2$]

[JEE-99]



Ans. $e = \frac{5 + \sqrt{3}}{8}, M = \frac{26}{\sqrt{3}} \text{ kg}$

Sol. Before collision

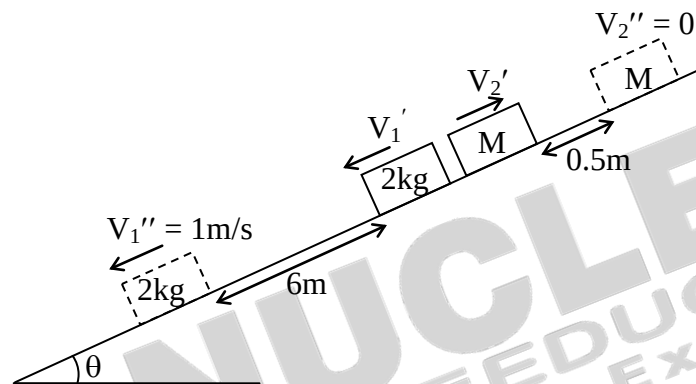


$$v_1^2 = 10^2 - 2g(\sin \theta + 0.25 \cos \theta) \times 6$$

$$v_1^2 = 100 - 2 \times 10(0.05 + 0.25 \times 1) \times 6$$

$$v_1 = 8 \text{ m/s}$$

After collision



For block M

$$0 = v_2'^2 - 2g(\sin \theta + 0.25 \cos \theta) \times \frac{1}{2}$$

$$0 = v_2'^2 - 2 \times 10(0.05 + 0.25 \times 1) \times \frac{1}{2}$$

$$v_2' = \sqrt{3}$$

For Block 2kg

$$1^2 = v_1'^2 - 2g(\mu \cos \theta - \sin \theta) \times 6$$

$$1^2 = v_1'^2 - 2 \times 10(0.25 \times 1 - 0.05) \times 6$$

$$1^2 = v_1'^2 - 24$$

$$v_1' = 5 \text{ m/s}$$

So

$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}} = \frac{5 + \sqrt{3}}{8 - 0}$$

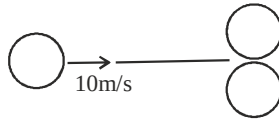
$$\Rightarrow e = \frac{5 + \sqrt{3}}{8}$$

And at the time just before and after the collision, by Linear momentum conservation along the incline.

$$(2 \times 8) + 0 = \sqrt{3}M - 5 \times 2$$

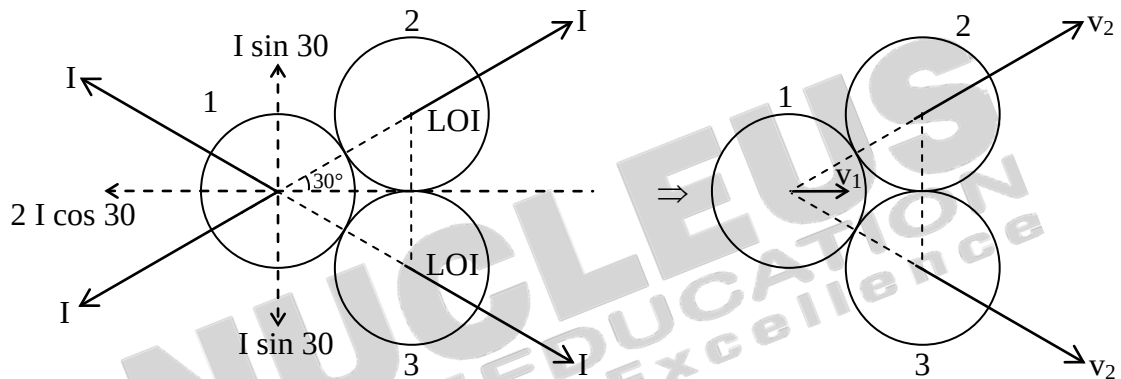
$$M = \frac{26}{\sqrt{3}} \text{ kg}$$

12. A ball with initial speed of 10 m/s collides elastically with two other identical ball whose centres are on a line perpendicular to the initial velocity and which are initially in contact with each other. All the three ball are lying on a smooth horizontal table. The first ball is aimed directly at the contact point of the other two balls. All the balls are smooth. Find the velocities of the three balls after the collision.



Ans. $V_1 = -2 \text{ m/s}$ $V_2 = 4\sqrt{3} \text{ m/s}$ $V_3 = 4\sqrt{3} \text{ m/s}$

Sol.



$$-2I \cos 30 = mv_1 - mu \quad \dots(1)$$

$$I = mv_2 - 0 \quad \dots(2)$$

from equation (1) & (2)

$$-2mv_2 \cos 30 = mv_1 - mu$$

$$-\sqrt{3}V_2 = v_1 - u \quad \dots(3)$$

And Newton's law of collision along the line of impact (LOI)

$$1 = \frac{v_2 - v_1 \cos 30}{u \cos 30}$$

$$\Rightarrow 2v_2 = \sqrt{3}u + \sqrt{3}v_1 \quad \dots(4)$$

from eq (3) & (4)

$$v_2 = \frac{2\sqrt{3}u}{5} = \frac{2\sqrt{3} \times 10}{5} = 4\sqrt{3} \text{ m/s}$$

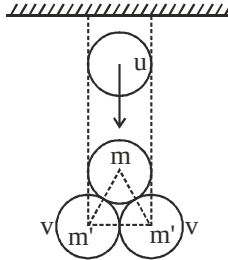
Now from eq (3)

$$-\sqrt{3}v_2 = v_1 - u$$

$$\Rightarrow -\sqrt{3} \times 4\sqrt{3} = v_1 - 10$$

$$\Rightarrow v_1 = -2 \text{ m/s}$$

13. Two equal spheres of mass m' are suspended by vertical strings so that they are in contact with their centers at same level. A third equal spheres of mass m falls vertically and strikes elastically the other two simultaneously so that their centres at the instant of impact form an equilateral triangle in a vertical plane. If u is the velocity of m just before impact, find the velocities just after impact and the impulse of tension of the strings.



Ans. $\frac{5u}{7}, \frac{2\sqrt{3}u}{7}, \frac{2\sqrt{3}u}{7}, \text{Impulse} = \frac{6mu}{7}$

Sol.

Just After Collision

Sphere A	For Sphere B
$Ndt \cos 60^\circ = mv$ $\Rightarrow Ndt = 2mv \quad \dots(1)$ $Ndt \sin 60^\circ - Tdt = 0$	$-2Ndt \sin 60^\circ = mv_1 - mv \quad \dots(2)$

Also

$$e = \frac{V_{sep}}{V_{app}}$$

$$1 = \frac{(V \cos 60^\circ) - v_1 \cos 30^\circ}{V \cos 30^\circ}$$

$$\frac{V\sqrt{3}}{2} = \frac{v}{2} - \frac{v_1\sqrt{3}}{2}$$

$$\Rightarrow U\sqrt{3} = v - v_1\sqrt{3} \quad \dots(3)$$

Using equation (1) and (2)

$$-2 \cdot (2mv) \sin 60^\circ = mv_1 - mv$$

$$-4mv \frac{\sqrt{3}}{2} = mv_1 - mU$$

$$-2\sqrt{3}v = v_1 - U$$

$$\Rightarrow v_1 = U - 2\sqrt{3}v$$

In equation (3)

$$\Rightarrow U\sqrt{3} - v - (U - 2\sqrt{3}v)\sqrt{3}$$

$$\Rightarrow U\sqrt{3} = v - U\sqrt{3} + 6v$$

$$\Rightarrow (2\sqrt{3})U = 7v$$

$$v = \frac{2\sqrt{3}}{7}U$$

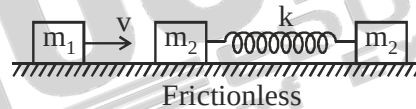
$$v_1 = \frac{5U}{7}$$

$$\text{Impulse of tension} = Tdt = Ndt \sin 60^\circ$$

$$= (2mv) \sin 60^\circ$$

$$= \frac{6mU}{7}$$

14. Mass m_1 hits & sticks with m_2 while sliding horizontally with velocity v along the common line of centres of the three equal masses ($m_1 = m_2 = m_3 = m$). Initially masses m_2 and m_3 are stationary and the spring is unstretched. Find the



- (i) velocities of m_1 , m_2 and m_3 immediately after impact.
- (ii) maximum kinetic energy of m_3 .
- (iii) minimum kinetic energy of m_2 .
- (iv) maximum compression of the spring.

Ans. (i) $\frac{v}{2}, \frac{v}{2}, 0$ (ii) $\frac{2mv^2}{9}$ (iii) $\frac{mv^2}{72}$, (iv) $x = \sqrt{\frac{m}{6k}} \cdot v$

Sol. (i)

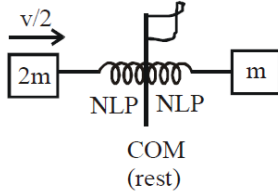


$$mv = 2mv_1 \quad v_1 = v/2$$

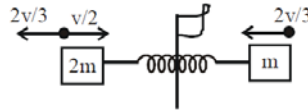
$$\text{just after collision} \Rightarrow v_{m_1} = v_{m_2} = v/2$$

$$v_{m_3} = 0$$

(ii)



$$v_{cm} = \frac{2m \cdot \frac{v}{2}}{3m} = \frac{2v}{3} \rightarrow (\text{right})$$



$\therefore$ In COM frame

$$\text{Let } +z \text{ Range (COM frame)} \left[-\frac{v}{6}, \frac{v}{6} \right] \quad \left[-\frac{2v}{3}, \frac{2v}{3} \right]$$

$$\text{Range in (ground)} \left[-\frac{v}{6} + \frac{2v}{3}, \frac{v}{6} + \frac{2v}{3} \right] \left[-\frac{2v}{3} + \frac{2v}{3}, \frac{2v}{3} + \frac{2v}{3} \right]$$

$$\left[\frac{v}{6}, \frac{v}{2} \right] \quad [0, 4v]$$

$\therefore$ Max. velocity of m_3 during its merfiro

$$\therefore (k_3)_{\max} = \frac{1}{2} \cdot m \cdot \left(\frac{2v}{3} \right)^2 = 2mv^2$$

$$\therefore (k_3)_{\max} = \frac{2}{9} mv^2$$

(iii) According to option (ii)

$$\left[\frac{v}{6}, \frac{v}{2} \right] \text{ main velocity of combined mass after collision} = \frac{v}{6}$$

$$\Rightarrow (v_2)_{\min} = \frac{v}{6}$$

(min of m_2)

$$\therefore (k_2)_{\min} = \frac{1}{2} \times m \left(\frac{v}{6} \right)^2 = \frac{mv^2}{72}$$

max. compression $\Rightarrow$ at max. compression $\Rightarrow v_1 = v_2 = v_3 = v_c$

$$\frac{1}{2} uv_{\text{rel}2}^2 = \frac{1}{2} mx^2$$

$$\Rightarrow \frac{1}{2} \times \frac{2m \cdot m}{3m} \left(\frac{v}{2} \right)^2 = \frac{1}{2} kx^2$$

$$\Rightarrow x = v \sqrt{\frac{m}{6k}}$$

OR

solve from basic

$$2m \frac{v}{2} = 3mv_c \Rightarrow v_c = \frac{v}{3}$$

$$\text{wet } \frac{1}{2}k(O^2 - x^2) = \frac{1}{2}m.v_c^2$$

$$\Rightarrow -\frac{1}{2}kx^2 = \frac{3}{2}m\left(\frac{v}{3}\right)^2 - \frac{mv^2}{4}$$

$$\Rightarrow -kx^2 = -\frac{mv^2}{6} \Rightarrow x = v\sqrt{\frac{m}{6k}}$$

(iv) Initial condition is just after collision of m_1 & m_2 max less of KE in collision of m_1 & m_2

15. A sphere of mass m is moving with a velocity $4\hat{i} + 3\hat{j} - 5\hat{k}$ when it hits a smooth wall and rebounds with velocity $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the impulse it receives. Find also the coefficient of restitution between the sphere and the wall.

Ans. Impulse = $m(-2\hat{i} - \hat{j} + 8\hat{k})$, $e = \frac{6}{17}$

Sol. Impulse = $\vec{p}_f - \vec{p}_i$

$$= m\vec{V}_f - m\vec{V}_i$$

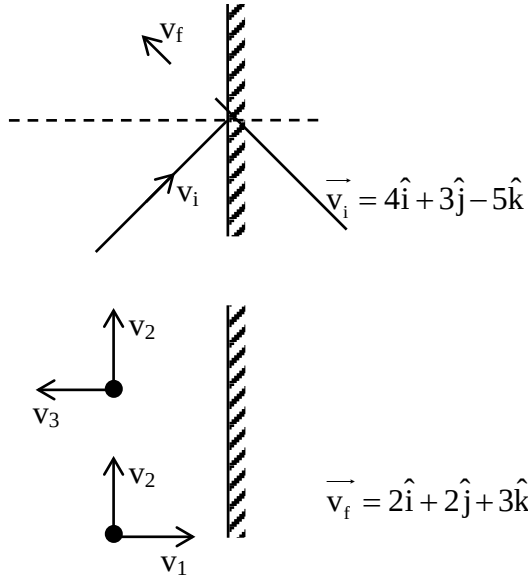
$$= m(2\hat{i} + 2\hat{j} + 3\hat{k}) - m(4\hat{i} + 3\hat{j} - 5\hat{k})$$

$$= m(-2\hat{i} - \hat{j} + 8\hat{k})$$

$$\vec{I} = m(-2\hat{i} - \hat{j} + 8\hat{k})$$

$$\hat{I} = \frac{m(-2\hat{i} - \hat{j} + 8\hat{k})}{m\sqrt{2^2 + 1^2 + 8^2}} = \frac{-2\hat{i} - \hat{j} + 8\hat{k}}{\sqrt{69}}$$

After



Before

$$e = \frac{V_3}{V_2}$$

$$V_3 = |\vec{V}_f \cdot \hat{I}|$$

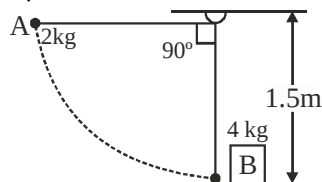
$$V_1 = |\vec{V}_i \cdot \hat{I}|$$

$$e = \frac{(2\hat{i} + 2\hat{j} + 3\hat{k}) \cdot \frac{(-2\hat{i} - \hat{j} + 8\hat{k})}{\sqrt{69}}}{4\hat{i} + 3\hat{j} - 5\hat{k} \cdot \frac{(-2\hat{i} - \hat{j} + 8\hat{k})}{\sqrt{69}}}$$

$$= \frac{-4 - 2 + 24}{-8 - 3 - 40}$$

$$= \frac{18}{51} = \frac{6}{17}$$

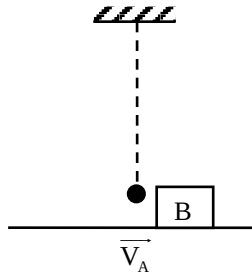
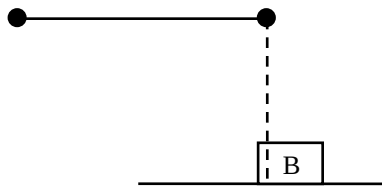
- 16.** A sphere A is released from rest in the position shown and strikes the block B which is at rest. If $e = 0.75$ between A and B and $\mu_k = 0.5$ between B and the support, determine



- the velocity of A just after the impact
- the maximum displacement of B after the impact.

Ans. $V_A = \sqrt{\frac{g}{12}} \text{ m/s}$ $S_{\max} = \frac{49}{48} \text{ m}$

Sol. Just before collision



Just after collision

Let velocities of A and B be V_A and V_B

Momentum Conservation

$$m_A V_A = m_A v_A + m_B v_B$$

$$2 \cdot \sqrt{2g(1.5)} = 2V_A + 4V_B \quad \dots(1)$$

Also

$$e = \frac{V_B - V_A}{\sqrt{2g(1.5)}} = 0.75 \quad \dots(2)$$

Using (1) and (2)

$$V_A = \sqrt{\frac{g}{12}} \text{ m/s}$$

$$\text{Retardation of B after collision} = \frac{\text{friction}}{m_B}$$

$$= \frac{\mu_k m_3 g}{m_B} = \mu_k g = 5 \text{ m/s}^2$$

Final speed of B = 0

$$\text{Using } V^2 = u^2 + 2as.$$

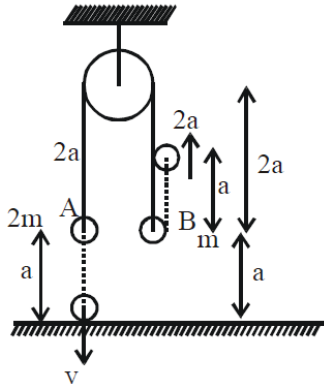
$$S = \frac{v_B^2}{2a} = \frac{49}{48} \text{ m}$$

17. Two particles A and B of mass $2m$ and m respectively are attached to the ends of a light inextensible string of length $4a$ which passes over a small smooth peg at a height $3a$ from an

inelastic table. The system is released from rest with each particle at a height a from the table. Find

- The speed of B when A strikes the table.
- The time that elapses before A first hits the table.
- The time for which A is resting on the table after the first collision & before it is first jerked off.

Ans. (i) $\sqrt{\frac{2ag}{3}}$ (ii) $\frac{3v}{g}$, (iii) $\frac{2v}{g}$



Sol.

- (i) when A strikes $\Rightarrow$ A move a down & B move a P

$$\text{WET} \Rightarrow 2mga - mga = \frac{1}{2} \cdot 2mv^2 + \frac{1}{2} mv^2$$

$$\Rightarrow mga = \frac{3}{2} mv^2 \Rightarrow v = \sqrt{\frac{2}{3} ga}$$

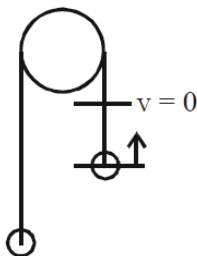
- (ii) for time between release of block A to just before A strikes.

$$a_A = a_B = \frac{2mg - mg}{3m} = \frac{g}{3}$$

$$\therefore \text{for A} \Rightarrow a = b, \quad \frac{g}{3} \cdot 2t = \sqrt{\frac{6a}{g}}$$

$$= \sqrt{\left(\frac{2}{3} ag\right) \cdot \frac{9}{g^2}} = \frac{3}{g} \cdot v$$

- (iii) Just after impact $\Rightarrow v_B = v = \sqrt{\frac{2}{3} ga}$ upwards and $a_B = g$ downwards



$$0 = v - gt$$

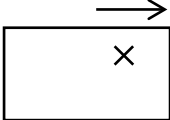
$$\Rightarrow t = \frac{v}{g}$$

$$\text{so } T = 2t \text{ (go+come)}$$

$$T = \frac{2v}{g}$$

- 18.** A cylindrical solid of mass 10^{-2} kg and cross-sectional area 10^{-4} m² is moving parallel to its axis (the x-axis) with a uniform speed of 10^3 m/s in the positive direction. At $t = 0$, its front face passes the plane $x = 0$. The region to the right of this plane is filled with the dust particle of uniform density 10^{-3} kg/m³. When a dust particles collides with the face of the cylinder, it sticks to its surface. Assuming that the dimensions of the cylinder remain practically unchanged and that the dust sticks only to the front face of the cylinder find the x-coordinate of the front of the cylinder at $t = 150$ s. [JEE-98]

Ans. 10^5 m

Sol. 

$$\text{Mass} = (M + \rho_{AX})$$

Momentum Conversation

$$MV_0 = (M + \rho_{AX})v$$

$$v = \left(\frac{M}{M + \rho_{AX}} \right) V_0$$

$$\frac{dx}{dt} = \frac{MV_0}{M + \rho_{AX}}$$

$$\int dx(M + \rho_{AX}) = \int MV_0 dt$$

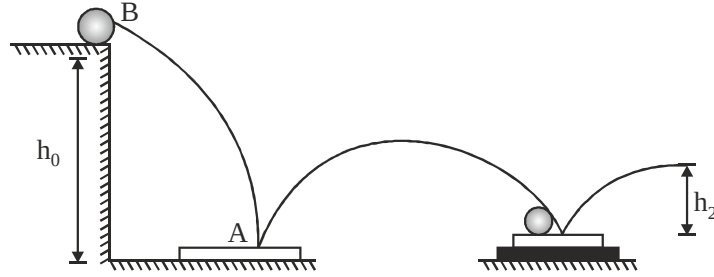
$$MX + \frac{\rho_{AX}^2}{2} = MV_0 t \quad \text{at } t = 150$$

$$MX + \frac{\rho_{AX}^2}{2} = MV_0 (150)$$

By solving

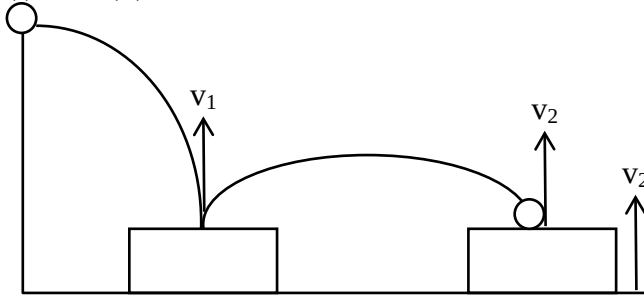
$$X = 10^5 \text{ m}$$

- 19.** A 70 g ball B dropped from a height $h_0 = 9$ m reaches a height $h_2 = 0.25$ m after bouncing twice from identical 210 g plates. Plate A rests directly on hard ground, while plate C rests on a foam-rubber mat. Determine.



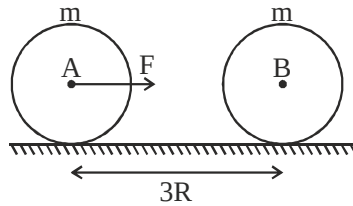
- (i) the coefficient of restitution between the ball and the plates.
(ii) the height h_1 of the ball's first bounce.

Ans.
Sol.



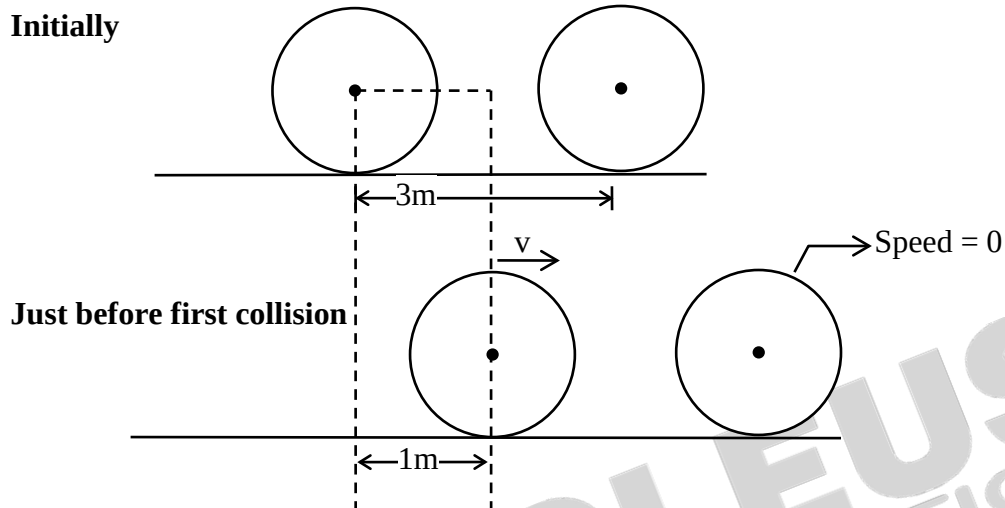
$$\begin{aligned} \text{(i)} \quad v_1 &= e\sqrt{2gh_0} \\ \frac{70}{1000} v_1 &= \left(\frac{70}{1000}\right) v_2 + \frac{210}{1000} v_2 \\ v_1 &= 4v_2 \\ \text{Also } \frac{v_2^2}{2g} &= h_2 \\ v_2^2 &= \sqrt{2gh_2} \\ \Rightarrow v_1 &= 4\sqrt{2gh_2} \\ \Rightarrow e\sqrt{2gh_0} &= 4\sqrt{2gh_2} \\ \Rightarrow e &= 4\sqrt{\frac{h_2}{h_0}} = 4 \times \frac{0.5}{3} = \frac{2}{3} \\ \text{(ii)} \quad h_1 &= \frac{v_1^2}{2g} = \frac{(4\sqrt{2gh_2})^2}{2g} \\ &= 16h_2 \\ &= 16 \times 0.25 = 4 \text{ m} \end{aligned}$$

20. Two identical spheres A & B of mass 50 kg and radius 1 m each lie on a smooth horizontal surface as shown in figure. The initial distance between their centres is 3 m. Starting from $t = 0$ a constant force $F = 100 \text{ N}$ acts on the sphere A towards right. The coefficient of restitution is $e = 0.5$. Find the time interval (in sec) between the first collision and the instant after which collision between both the spheres cease.



Ans. 2 sec

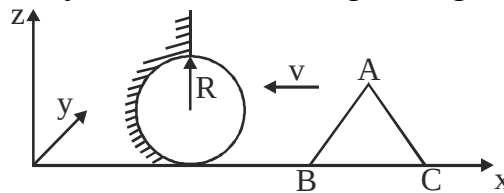
Sol. Initially



$$a = \frac{F}{m} = \frac{100}{50} = 2 \text{ m/s}^2$$

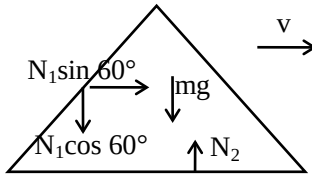
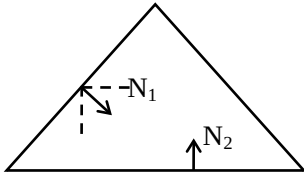
$$v = \sqrt{2as} = \sqrt{2(2) \times 1} = 2 \text{ m/s}$$

21. A uniform wedge of mass m and triangular cross-section ($AB = BC = CA = 2R$) is moving with a constant velocity $-v\hat{i}$ towards a sphere of radius R fixed on a smooth horizontal table as shown in the figure. The wedge makes an elastic collision with the fixed sphere and returns along the same path without any rotation. Neglect all friction and suppose that the wedge remains in contact with the sphere for a time Δt . Find the average force $\vec{F}$ and also the average normal reaction $\vec{N}$ exerted by the table on the wedge during the time Δt .



Ans. $\vec{F} = \frac{4mv}{\sqrt{3}\Delta t} \left(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{k} \right)$, $\vec{N} = \left(\frac{2mg}{\sqrt{3}\Delta t} + mg \right) \hat{k}$

Sol.



In Horizontal

$$(N_1 \sin 60^\circ) dt = mv_1 - (-mv)$$

$$= m(v_1 + v) \quad \dots(1)$$

In Vertical

$$N_1 \cos 60^\circ dt + mg dt = N_2 dt \quad \dots(2)$$

Also $e = 1 = \frac{V_{\text{sep}}}{V_{\text{app}}}$

$V_{\text{app}} = \text{Component of velocity along common normal}$
 $= v \sin 60^\circ$

$$\Rightarrow e = 1 = \frac{V_1 \cos 30^\circ}{V \sin 60^\circ} \Rightarrow V_1 = v$$

In equation (1)

$$N_1 \frac{\sqrt{3}}{2} dt = m(2v)$$

$$N_1 = \frac{4mv}{\sqrt{3}dt}$$

$$\vec{N}_1 = \frac{4mv}{\sqrt{3}dt} \left(\frac{\sqrt{3}}{2} \hat{i} - \frac{1}{2} \hat{k} \right)$$

In equation (2)

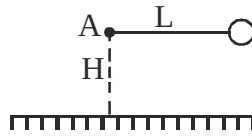
$$\left(\frac{4mv}{\sqrt{3}dt} \right) \cos 60^\circ dt + mg dt = N_2 dt$$

$$\frac{2mv}{\sqrt{3}} + mg dt = N_2 dt$$

$$\Rightarrow N_2 = \frac{2mv}{\sqrt{3}dt} + mg$$

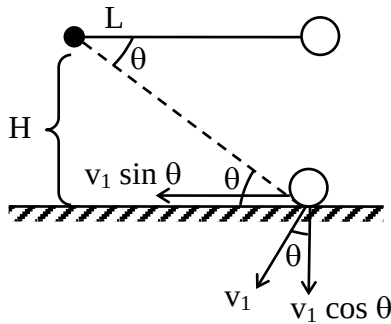
$$\Rightarrow \vec{N}_2 = \left(\frac{2mv}{\sqrt{3}dt} + mg \right) \hat{k}$$

22. A is a fixed point at a height H above a perfectly inelastic smooth horizontal plane. A light inextensible string of length L ($> H$) has one end attached to A and other is to a heavy particle. The particle is held at the level of A with string just taut and released from rest. Find the height of the particle above the plane when it is next instantaneously at rest.

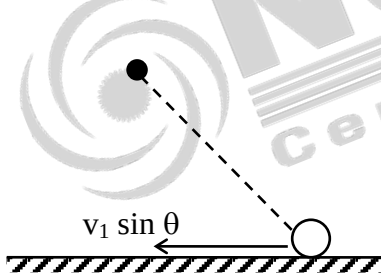


Ans. $\frac{H^5}{L^4}$

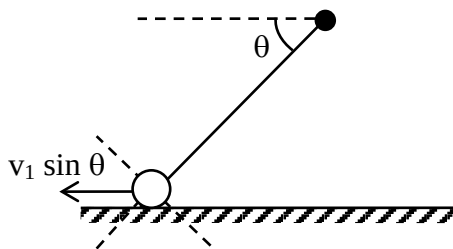
Sol. $v_1 = \sqrt{2gH}$



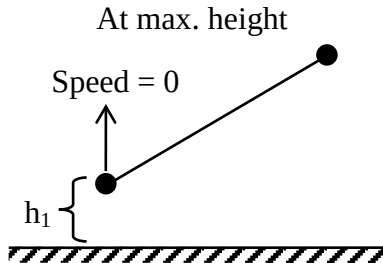
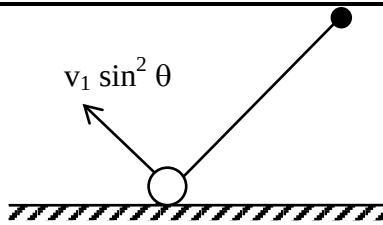
Just After collision $\sin \theta = \frac{H}{L}$



Just before the string gets taut



Just after the string gets taut



Work Energy theorem

$$\frac{1}{2}m(0)^2 - \frac{1}{2}m(v_1 \sin^2 \theta)^2 = -mgh_1$$

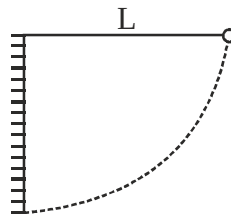
$$-\frac{1}{2}m(2gH) \sin^4 \theta = -mgh_1$$

$$h_1 = H \sin^4 \theta$$

$$= H \cdot \frac{H^4}{L^4}$$

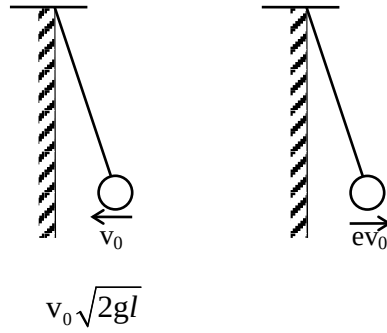
$$= \frac{H^5}{L^4}$$

23. A simple pendulum is suspended from a peg on a vertical wall. The pendulum is pulled away from the wall to a horizontal position (see figure) and released. The ball hits the wall, the coefficient of restitution being $\frac{2}{\sqrt{5}}$. What is the minimum number of collisions after which the amplitude of oscillations becomes less than 60° . [JEE-87]

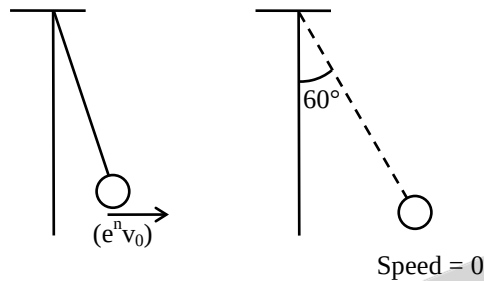


Ans. 4

Sol. Before



After n collisions



Work Energy Theorem

$KE_f - KE_i = \text{Work done by gravity}$

$$0 - \frac{1}{2} m (e^n v_0)^2 = -mg l (1 - \cos 60^\circ)$$

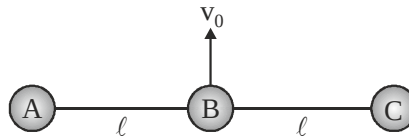
$$-\frac{1}{2} m (e^{2n}) v_0^2 = -mg l \left(1 - \frac{1}{2}\right)$$

$$-\frac{1}{2} m e^{2n} v_0^2 = -mg \frac{l}{2}$$

$$-\frac{1}{2} m (e^{2n}) (2gl) = -\frac{mg l}{2}$$

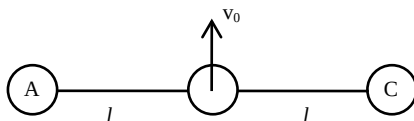
$$e^{2n} = \frac{1}{2}$$

24. Three identical balls each of mass $m = 0.5 \text{ kg}$ are connected with each other as shown in figure and rest over a smooth horizontal table. At moment $t = 0$, ball B is imparted a horizontal velocity $v_0 = 9 \text{ ms}^{-1}$. Calculate velocity of A just before it collides with ball C.

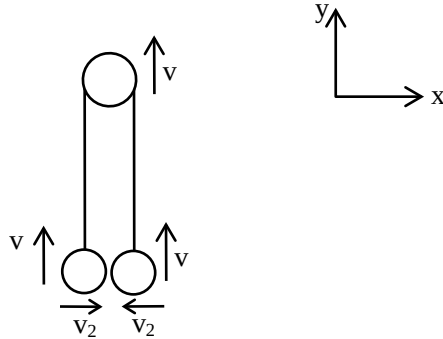


Ans. 6 m/s

Sol. Before



After



Momentum Conservation in y direction

$$\Rightarrow mv_0 = mv_1 + mv_1 + mv_1$$

$$\Rightarrow v_1 = \frac{v_0}{3}$$

$$v_1 = 3 \text{ m/s}$$

.....(1)

Energy conservation

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}m(\sqrt{v_1^2} + \sqrt{v_2^2})^2 + \frac{1}{2}m(\sqrt{v_1^2} + \sqrt{v_2^2})^2 \quad \text{.....(2)}$$

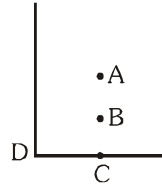
using equation (1) and (2)

$$v_2 = \sqrt{27} \text{ m/s}$$

$$\text{Speed} = \sqrt{v_1^2 + v_2^2} = \sqrt{3^2 + (27)^2} = 6 \text{ m/s}$$

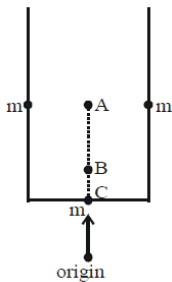
EXERCISE # (O-1)

1. A thick uniform wire is bent into the shape of the letter "U" as shown. Which point indicates the location of the center of mass of this wire? A is the midpoint of the line joining mid points of two parallel sides of 'U' shaped wire.



- (A) D (B) A (C) B (D) C

Ans.
Sol.



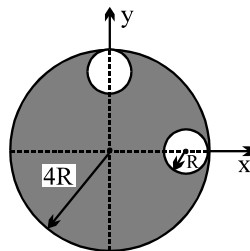
$$x_{cm} = \frac{m \times \frac{L}{9} - \frac{mL}{2}}{3m} = 0$$

$$y_{cm} = \frac{\frac{m\ell}{2} + \frac{m\ell}{2}}{3m}$$

$$y_{cm} = \frac{\ell}{3}$$

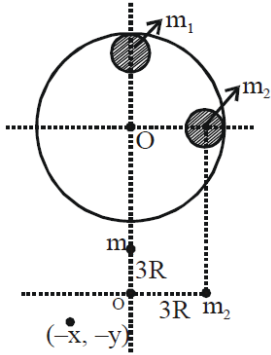
Point (B)

2. From the circular disc of radius $4R$ two small disc of radius R are cut off. The centre of mass of the new structure will be:



- (A) $i\frac{R}{5} + j\frac{R}{5}$ (B) $-i\frac{R}{5} + j\frac{R}{5}$ (C) $-\frac{3R}{14}(\hat{i} + \hat{j})$ (D) None of these

Ans. (C)



$$m_2 \cdot 3R = m_3 \cdot x$$

$$\Rightarrow x = \left(\frac{m_2}{m_3} \right) \cdot 3R = \frac{A_2}{A_3} (3R)$$

$$\Rightarrow x = \left(\frac{\pi R^2}{\pi(4R)^2 - \pi R^2 \times 2} \right) \cdot 3R$$

$$x = \frac{3R}{14}$$

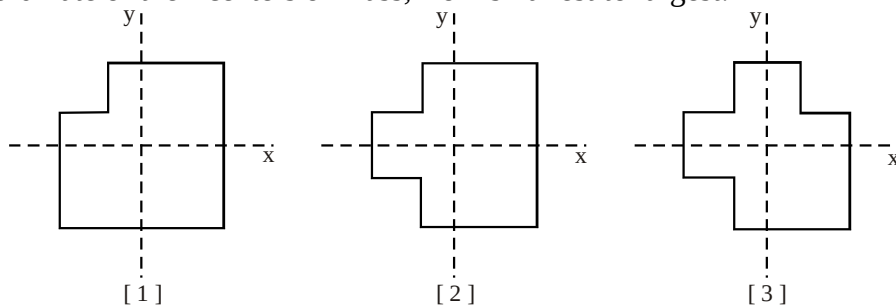
$$\text{and } m_1 \cdot 3R = m_3 \cdot y \Rightarrow y = \left(\frac{m_1}{m_3} \right) \cdot 3R$$

$$\Rightarrow y = \left(\frac{\pi R^2}{\pi(4R)^2 - \pi R^2 \times 2} \right) \cdot 3R$$

$$\Rightarrow y = \frac{3R}{14}$$

$$\therefore \text{Co-ordinate of point } m_3 = -\frac{3R}{14}(\hat{i} + \hat{j})$$

3. A machinist starts with three identical square plates but cuts one corner from one of them, two corners from the second, and three corners from the third. Rank the three plates according to the x-coordinate of their centers of mass, from smallest to largest.



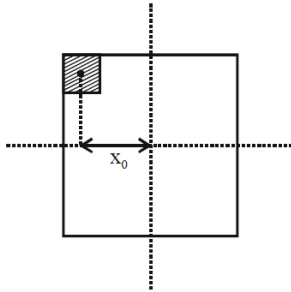
(A) 3, 1, 2

(B) 1, 3, 2

(C) 3, 2, 1

(D) 1 and 3 tie, then 2

Ans. (B)

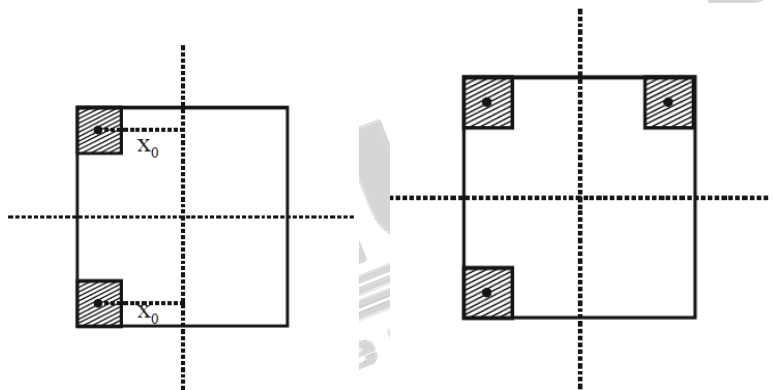


Sol.

$$x_{cm_1} = \frac{M \times 0 - m(-x_0)}{M - m} = \left(\frac{mx_0}{M - m} \right) = \frac{x_0}{\frac{M}{m} - 1}$$

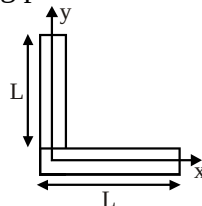
$$x_{cm_2} = \frac{M \times 0 - m(-x_0) - m_0(-x_0)}{M - 2m_0}$$

$$x_{cm_2} = \frac{2mx_0}{(M - 2m)} = \frac{x_0}{\frac{M}{2m} - 1}$$



$$x_{cm_3} = \frac{Mx_0 - m(-x_0) - m(-x_0) - mx_0}{M - 8m} = \left(\frac{mx_0}{M - 3m} \right) = \frac{x_0}{\frac{M}{m} - 3}$$

4. Centre of mass of two thin uniform rods of same length but made up of different materials & kept as shown, can be, if the meeting point is the origin of co-ordinates



(A) $(L/2, L/2)$

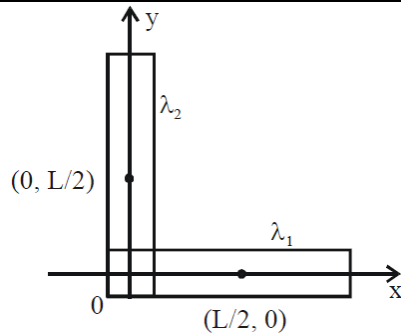
(B) $(2L/3, L/2)$

(C) $(L/3, L/3)$

(D) $(L/3, L/6)$

Ans. (D)

Sol.



$$x_{cm} = \frac{m_1 \cdot \frac{L}{2}}{m_1 + m_2} = \frac{(\lambda_1 \cdot L) \cdot L/2}{(\lambda_1 \cdot L + \lambda_2 \cdot L)} = \left(\frac{\lambda_1}{\lambda_1 + \lambda_2} \right) \cdot \frac{L}{2}$$

$$y_{cm} = \frac{m_2 \cdot \frac{L}{2}}{m_1 + m_2} = \left(\frac{\lambda_2 \cdot L}{\lambda_1 \cdot L + \lambda_2 \cdot L} \right) \frac{L}{2} = \left(\frac{\lambda_2}{\lambda_1 + \lambda_2} \right) \cdot \frac{L}{2}$$

$$x_{cm} + y_{cm} = \left(\frac{\lambda_1 + \lambda_2}{\lambda_1 + \lambda_2} \right) \cdot \frac{L}{2}$$

$$\Rightarrow x_{cm} + y_{cm} = \frac{L}{2}$$

5. Seven identical birds are flying south together at constant velocity. A hunter shoots one of them, which immediately dies and falls to the ground. The other six continue flying south at the original velocity. After the one bird has hit the ground, the centre of mass of all seven birds
- (A) continues south at the original speed, but is now located some distance behind the flying birds
 (B) continues south, but at 6/7 the original velocity
 (C) continues south, but at 1/7 the original velocity
 (D) stops with the dead bird

Ans. (B)

Sol. $(v_{cm})_f = \frac{m \cdot 0 + (m \cdot v) \times 6}{7m} = \frac{6}{7}v$

$$(v_{cm})_i = \frac{(mv) \times 7}{7m} = v$$

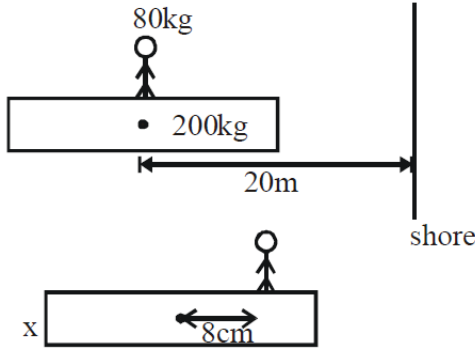
$$\therefore (v_{cm})_f = \frac{6}{7}(v_{cm})_i$$

6. A man weighing 80 kg is standing at the centre of a flat boat and he is 20 m from the shore. He walks 8 m on the boat towards the shore and then halts. The boat weight 200 kg. How far is he from the shore at the end of this time?

(A) 11.2 m (B) 13.8 m (C) 14.3 m (D) 15.4 m

Ans. (C)

Sol. Let the displacement of boat be x towards the shore



$$80(x + 8) + 200x = 0$$

$$\Rightarrow x = \frac{-640}{280} = -\frac{16}{7}$$

$$\therefore (\Delta x) = 8 - \frac{16}{7} = \frac{40}{7}$$

$\therefore$ New distance from shore

$$= 20 - \frac{40}{7} = \frac{100}{7} = 14.28\text{m} \approx 14.3\text{m}$$

7. There are some passengers inside a stationary railway compartment. The track is frictionless. The centre of mass of the compartment itself (without the passengers) is C_1 , while the centre of mass of the 'compartment plus passengers' system is C_2 . If the passengers move about inside the compartment along the track.

- (A) both C_1 and C_2 will move with respect to the ground.
 (B) neither C_1 nor C_2 will move with respect to the ground.
 (C) C_1 will move but C_2 will be stationary with respect to the ground.
 (D) C_2 will move but C_1 will be stationary with respect to the ground.

Ans. (C)

Sol. C_1 : may move, as passenger exert frictional force on floor of compartment.

C_2 : do not move, as there is no external force on compartment + passenger

8. A non-zero external force acts on a system of particles. The velocity and acceleration of the centre of mass are found to be v_0 and a_c respectively at any instant t . It is possible that

- (i) $v_0 = 0, a_c = 0$ (ii) $v_0 \neq 0, a_c = 0$ (iii) $v_0 = 0, a_c \neq 0$ (iv) $v_0 \neq 0, a_c \neq 0$

Then

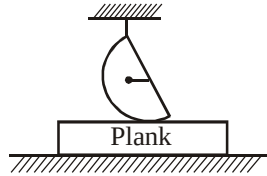
- (A) (iii) and (iv) are true. (B) (i) and (ii) are true.
 (C) (i) and (iii) are true. (D) (ii), (iii) and (iv) are true.

Ans. (A)

Sol. $\vec{F}_{\text{ext}} \neq 0 \Rightarrow \vec{a}_{\text{cm}} \neq 0$

$\vec{V}_{\text{cm}}(t)$ may or may not be zero at any instant.

9. Lower surface of a plank is rough and lying at rest on a rough horizontal surface. Upper surface of the plank is smooth and has a smooth hemisphere placed over it through a light string as shown in the figure. After the string is burnt, trajectory of centre of mass of the sphere is :-



- (A) a circle (B) an ellipse (C) a straight line (D) a parabola

Ans. (C)

Sol. On sphere there is no external force in horizontal direction

$$(a_{cm} = 0) \Rightarrow (v_{cm} = \text{constant} = 0)$$

& $(f_{ext})_{\text{vertical}} \neq 0 \Rightarrow \text{COM will fall vertically downward}$

$\therefore$ Path of COM $\Rightarrow$ Straight line & vertical downwards

10. Three interacting particles of masses 100 g, 200 g and 400 g each have a velocity of 20 m/s magnitude along the positive direction of x-axis, y-axis and z-axis. Due to force of interaction the third particle stops moving. The velocity of the second particle is $(10\hat{j} + 5\hat{k})$. What is the velocity of the first particle?

- (A) $20\hat{i} + 20\hat{j} + 70\hat{k}$ (B) $10\hat{i} + 20\hat{j} + 8\hat{k}$ (C) $30\hat{i} + 10\hat{j} + 7\hat{k}$ (D) $15\hat{i} + 5\hat{j} + 60\hat{k}$

Ans. (A)

Sol. Initial $\begin{cases} m_1 = 0.1, & \bar{v}_1 = 20\hat{i} \\ m_2 = 0.2\text{kg}, & \bar{v}_2 = 20\hat{j} \\ m_3 = 0.4\text{kg}, & \bar{v}_3 = 20\hat{k} \end{cases}$

final $\begin{cases} \bar{v}_3 = 0 \\ \bar{v}_2 = 10\hat{j} + 5\hat{k} \\ \bar{v}_1 = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} \end{cases}$

$$\therefore f_{\text{ext}} = 0 \Rightarrow (\bar{v}_{cm})_f = (\bar{v}_{cm})_i$$

$$\Rightarrow (v_x)_{cm\ i} = (v_x)_{cm\ f} \Rightarrow \frac{0.1 \times 20}{0.7} = \frac{0.1 \times v_x}{0.7}$$

$$\Rightarrow (v_y)_{cm\ i} = (v_y)_{cm\ f}$$

$$\Rightarrow \frac{0.2 \times 20}{0.7} = \frac{0.2 \times 10 + 0.1 \times v_y}{0.7}$$

$$\Rightarrow v_y = 20 \text{ m/s}$$

$$(v_z)_{cm\ i} = (v_z)_{cm\ f}$$

$$\Rightarrow \frac{0.4 \times 20}{0.7} = \frac{0.2 \times 5 + 0.1 \times v_z}{0.7}$$

$$\Rightarrow v_2 = 70 \hat{k}$$

$$= \vec{v}_1 = 20\hat{i} + 20\hat{j} + 70\hat{k}$$

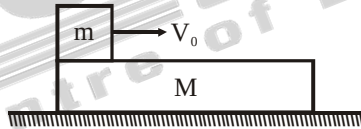
11. A system of N particles is free from any external forces.
- (i) Which of the following is true for the magnitude of the total momentum of the system?
- (A) It must be zero
(B) It could be non-zero, but it must be constant
(C) It could be non-zero, and it might not be constant
(D) The answer depends on the nature of the internal forces in the system
- (ii) Which of the following must be true for the sum of the magnitudes of the momenta of the individual particles in the system?
- (A) It must be zero
(B) It could be non-zero, but it must be constant
(C) It could be non-zero, and it might not be constant
(D) It could be zero, even if the magnitude of the total momentum is not zero

Ans. (i) B; (ii) C

Sol. (1) $\vec{f}_{\text{ext}} = 0$

$$\Rightarrow \vec{P}_{\text{system}} \Rightarrow \text{conserved}$$

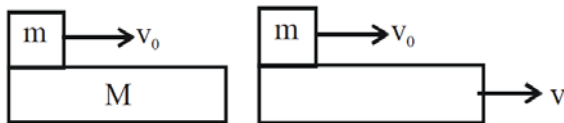
12. The coefficient of friction between the block and plank is μ and ground is smooth. The value of μ is such that block becomes stationary with respect to plank before it reaches the other end. Then which of the following statement is **incorrect**.



- (A) The work done by friction on the block is negative.
(B) The work done by friction on the plank is positive.
(C) The net work done by friction is negative.
(D) Net work done by the friction is zero.

Ans. (D)

Sol.



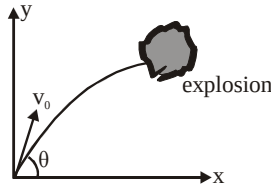
$$mv_0 = (M + m)v \Rightarrow v = \frac{mv_0}{(M + m)}$$

$$(\Delta K)_{\text{system}} = \frac{1}{2}(M + m)v^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}(M + m)\left[\frac{mv_0}{M + m}\right]^2 - \frac{1}{2}mv_0^2$$

$$= \frac{1}{2}\left(\frac{m^2}{M + m}\right)v_0^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}mv_0^2\left[\frac{m}{M + m} - 1\right]$$

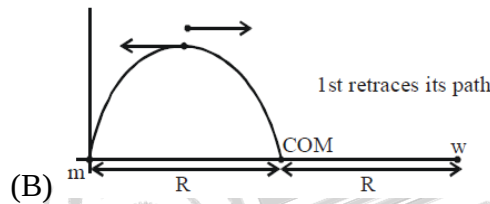
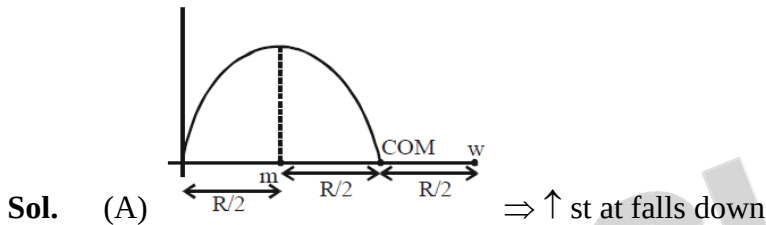
$\therefore$ Total work of friction is

13. A projectile is projected in x-y plane with velocity v_0 . At top most point of its trajectory projectile explodes into two identical fragments. Both the fragments land simultaneously on ground and stick there. Taking point of projection as origin and R as range of projectile if explosion had not taken place. Which of the following can not be position vectors of two pieces, when they land on ground.



- (A) $\frac{R}{2}\hat{i}, \frac{3R}{2}\hat{i}$ (B) $0\hat{i}, 2R\hat{i}$ (C) $R\hat{i} - R\hat{k}, R\hat{i} + R\hat{k}$ (D) $2R\hat{i} + \frac{R}{2}\hat{k}, R\hat{i} - \frac{R}{2}\hat{k}$

Ans. (D)



(C) $X_{cm} = \frac{mR + mR}{2m} = R$

$Y_{cm} = 0$

$Y_{cm} = \frac{-mR + mR}{2m}$

$\therefore$ This is the possible poster

(D) $X_{cm} = \frac{2R \cdot m + mB}{2m} = \frac{3R}{2}$

14. Ball A of mass 5.0 kilograms moving at 20 m/s collides with ball B of unknown mass moving at 10m/s in the same direction. After the collision, ball A moves at 10 m/s and ball B at 15 m/s, both still in the same direction. What is the mass of ball B?

- (A) 6.0 kg (B) 10 kg (C) 2.0 kg (D) 12 kg

Ans. (B)

Sol. $5 \times 20 + m \times 10 = 5 \times 10 + 15m$
 $5m = 50 \Rightarrow m = 10$

15. A boy hits a baseball with a bat and imparts an impulse J to the ball. The boy hits the ball again with the same force, except that the ball and the bat are in contact for twice the amount of time as in the first hit. The new impulse equals:

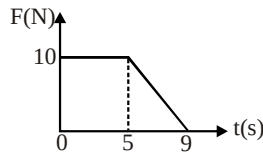
(A) half the original impulse (B) the original impulse
(C) twice the original impulse (D) four times the original impulse

Ans. (C)

Sol. $T = \int F \cdot dt = F \cdot t$

$T' = F \cdot t' = F \cdot 2t = 2Ft = 2J$

16. A body of mass 4 kg is acted on by a force which varies as shown in the graph below. The momentum acquired is



(A) 280 N-s (B) 140 N-s (C) 70 N-s (D) 210 N-s

Ans. (C)

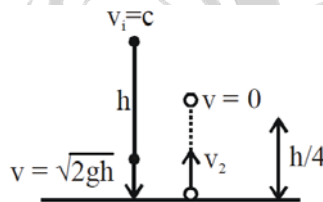
Sol. Momentum acquired = Area (Δ) = $5 \times 10 + \frac{1}{2} \times 4 \times 10 = 50 + 20 = 70 \text{ N-s}$

17. Two balls of same mass are dropped from the same height h , on to the floor. The first ball bounces to a height $h/4$, after the collision & the second ball to a height $h/16$. The impulse applied by the first & second ball on the floor are I_1 and I_2 respectively. Then

(A) $5I_1 = 6I_2$ (B) $6I_1 = 5I_2$ (C) $I_1 = 2I_2$ (D) $2I_1 = I_2$

Ans. (A)

Sol.

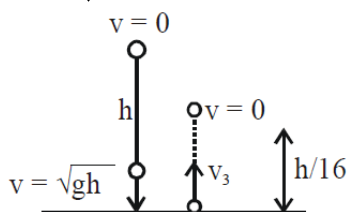


$$0^2 = v_2^2 - 2g \cdot h/4$$

$$\Rightarrow v_2 = \sqrt{gh/2}$$

$$I_1 = \bar{p}_f - \bar{p}_i = m \cdot \sqrt{\frac{gh}{2}} + m \cdot \sqrt{2gh}$$

$$I_1 = \frac{3m\sqrt{gh}}{\sqrt{2}}$$



$$0^2 = v_3^2 - 2g \frac{h}{16}$$

$$\Rightarrow v_3 = \frac{\sqrt{gh}}{2\sqrt{2}}$$

$$\therefore I_2 = m \cdot \frac{\sqrt{2gh}}{4} + m\sqrt{2gh}$$

$$I_2 = \frac{5m\sqrt{2gh}}{4}$$

$$\text{Now } \frac{I_1}{I_2} = \frac{\frac{3}{\sqrt{2}}}{\frac{5}{4}\sqrt{2}} = \frac{6}{5}$$

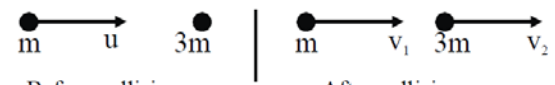
$$\Rightarrow 5I_1 = 6I_2$$

18. A smooth small spherical ball of mass m , moving with velocity u collides head on with another small spherical ball of mass m , which was initially at rest. One - third of the initial kinetic energy of the system is lost. The coefficient of restitution between the spheres is

- (A) $\frac{1}{3}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{2}$ (D) zero

Ans. (B)

Sol.



Before collision After collision

$$e = \frac{V_2 - V_1}{u} \quad \dots(i)$$

$$mu = mv_1 + 3mv_2 \quad \dots(ii)$$

$$\frac{1}{3} \times \left(\frac{1}{2} mu^2 \right) = \frac{1}{2} mv_1^2 + \frac{1}{2} 3mv_2^2 \quad \dots(iii)$$

Solving, $v_1 = \frac{(1-3e)}{4}u$, $v_2 = \frac{(1+e)}{4}u$

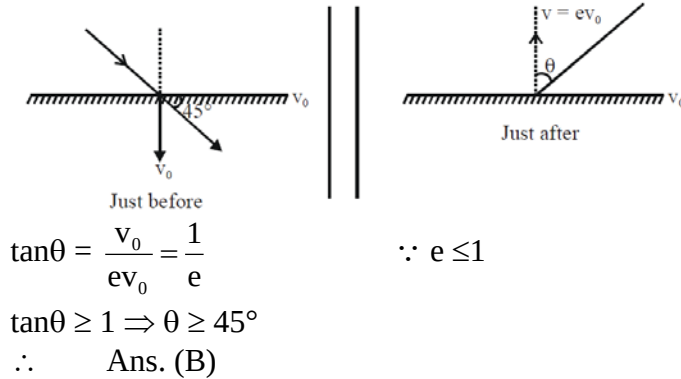
Alternate : $\Delta KE = \frac{1}{2} \mu v_{\text{rel}}^2 (1-e^2)$ [μ = reduced mass, v_{rel} = initial relative velocity]

19. A ball strikes a smooth horizontal ground at an angle of 45° with the vertical. What cannot be the possible angle of its velocity with the vertical after the collision. (Assume $e \leq 1$).

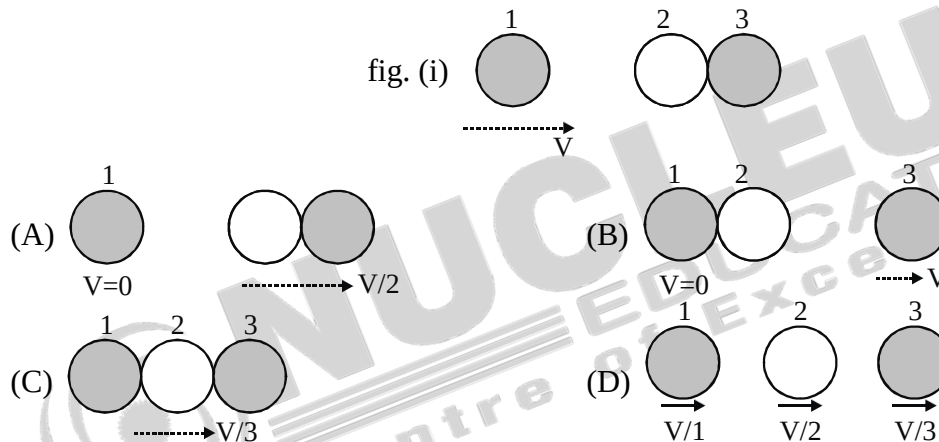
- (A) 45° (B) 30° (C) 53° (D) 60°

Ans. (B)

Sol.



20. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed V as shown in figure (i). If the collision is elastic, which of the following is a possible result after collision?



Ans. (B)

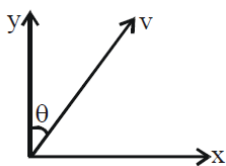
Sol. Momentum of first ball bearing is transfer to second & this momentum is finally transfer to 3rd. Ball bearing (see "Newton's cradle" for details)

21. A ball is projected from ground with a velocity V at an angle θ to the vertical. On its path it makes an elastic collision with a vertical wall and returns to ground. The total time of flight of the ball is

(A) $\frac{2v \sin \theta}{g}$ (B) $\frac{2v \cos \theta}{g}$ (C) $\frac{v \sin 2\theta}{g}$ (D) $\frac{v \cos \theta}{g}$

Ans. (B)

Sol.

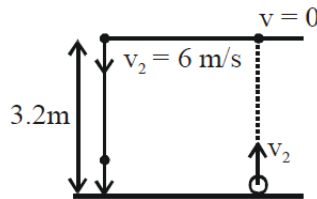


vertical direction of motion is not affected due to collision

$$\therefore t = T = \frac{2v \sin(90 - \theta)}{g} = \frac{2u \cos \theta}{g}$$

22. A ball is thrown downwards with initial speed = 6 m/s, from a point at height = 3.2 m above a horizontal floor. If the ball rebounds back to the same height then coefficient of restitution equals to
(A) 1/2 (B) 0.75 (C) 0.8 (D) None

Ans.
Sol.



$$0^2 = v_2^2 - 2 \times 10 \times 3.2$$

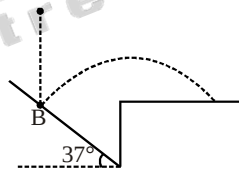
$$\Rightarrow v_2 = 8 \text{ m/s}$$

$$\therefore e = \frac{8}{10} = 0.8$$

$$v_1 = \sqrt{36 + 2 \times 10 \times 3.2} = \sqrt{100}$$

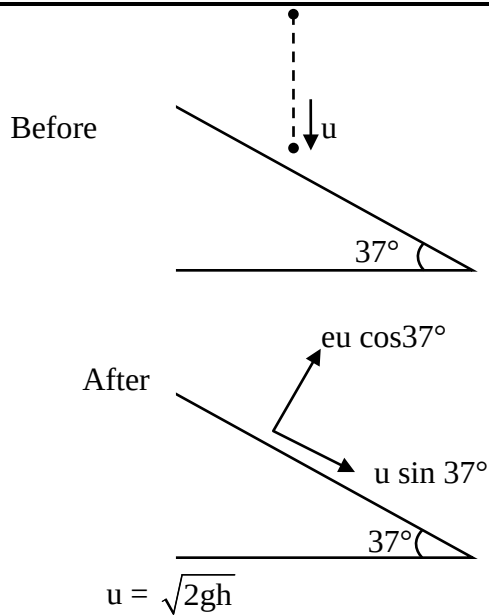
$$v_1 = 10 \text{ m/s}$$

23. A small ball is dropped from a height h onto a rigid frictionless plate at B and rebounds. Find the maximum height reached after rebound if the coefficient of restitution between the ball and the plate is $e = 0.75$:



- (A) 0 (B) $3h / 25$ (C) $3h / 125$ (D) $9h / 625$

Ans.
Sol.



Net vertical component velocity = $eV \cos^2 37^\circ - V \sin^2 37^\circ$

$$= \left(\frac{3}{25} \right) v$$

$$\begin{aligned} \text{max height} &= \frac{V_y^2}{2g} = \left(\frac{9}{625} \right) \frac{v^2}{2g} \\ &= \left(\frac{9}{625} \right) \frac{2gh}{2g} \\ &= \left(\frac{9}{625} \times h \right) \end{aligned}$$

24. A particle is projected from a smooth horizontal surface with velocity v at an angle θ from horizontal. Coefficient of restitution between the surface and ball is e . The distance of the point where ball strikes the surface second time from the point of projection is

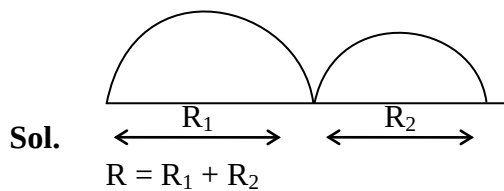
(A) $\frac{v^2 \sin 2\theta (1 + e^2)}{g}$

(B) $\frac{v^2 \sin 2\theta (1 + e^4)}{g}$

(C) $\frac{v^2 \sin 2\theta (1 + e^3)}{g}$

(D) $\frac{v^2 \sin 2\theta (1 + e)}{g}$

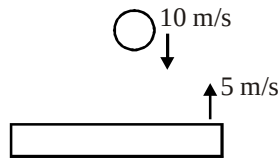
Ans. (D)



$$= V \cos \theta \left(\frac{2V \sin \theta}{g} \right) + V \cos \theta \left(\frac{2eV \sin \theta}{g} \right)$$

$$= \frac{V^2 \sin 2\theta (1+e)}{g}$$

25. A ball of mass 1kg strikes a heavy platform, elastically, moving upwards with a velocity of 5m/s. The speed of the ball just before the collision is 10m/s downwards. Then the impulse imparted by the platform on the ball is



- (A) 15 N – s (B) 10 N – s (C) 20 N – s (D) 30 N – s

Ans. (D)

Sol. Let velocity of ball just after the collision is $V \uparrow$

In elastic collision

Velocity of separation = Velocity of approach

$$V - 5 = 15$$

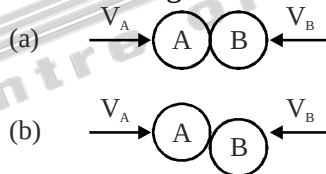
$$V = 20 \text{ m/s}$$

I_p = change in linear momentum of ball

$$= (1 \times 20) - (1 \times (-10))$$

$$= 30 \text{ NS}$$

26. Two bodies, A and B, collide as shown in figures a and b below :



Circle the true statement :

- (A) They exert equal and opposite forces on each other in (a) but not in (b)
 (B) They exert equal and opposite force on each other in (b) but not in (a)
 (C) They exert equal and opposite force on each other in both (a) and (b)
 (D) The forces are equal and opposite to each other in (a), but only the components of the forces parallel to the velocities are equal in (b).

Ans. (C)

Sol. During collision both ball apply impulsive normal on each other which is equal in magnitude and opposite in direction.

27. A mass 'm' moves with a velocity 'v' and collides elastically with another identical mass at rest. After collision the 1st mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision

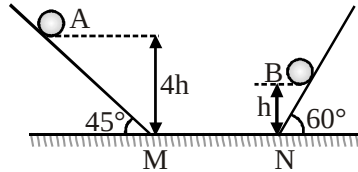
- (A) $\frac{2v}{\sqrt{3}}$ (B) $\frac{v}{\sqrt{3}}$ (C) $v\sqrt{\frac{2}{3}}$

(D) the situation of the problem is not possible without external impulse

Ans. (D)

Sol. Final energy becomes more than initial energy.

28. Two identical balls are released from positions as shown. They collide elastically on horizontal surface. Ratio of heights attained by A & B after collision is (All surfaces are smooth, neglect energy loss at M & N)



(A) 1 : 4

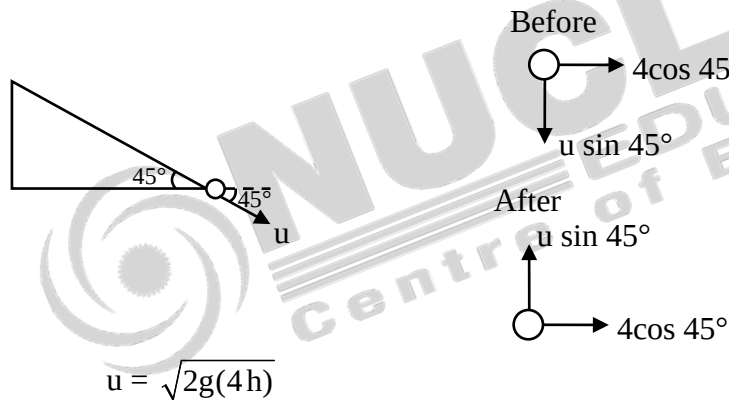
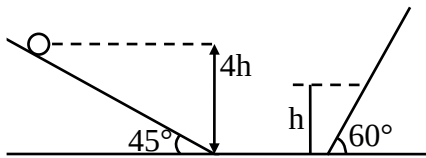
(B) 2 : 1

(C) 4 : 13

(D) 2 : 5

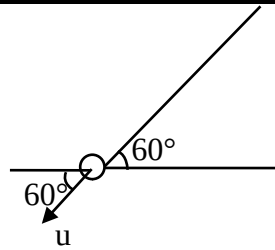
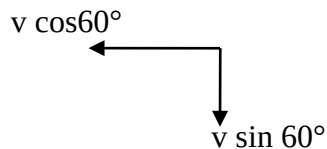
Ans. (A)

Sol.



$$\begin{aligned} \text{Height of A} &= \frac{(u \sin 45^\circ)^2}{2g} \\ &= \frac{V^2 \times \frac{1}{2}}{2g} = \frac{2g(4h) \frac{1}{2}}{2g} \\ &= 2h. \end{aligned}$$

Before



After

$$v = \sqrt{2gh}$$

$$\text{max height} = \frac{(V \sin 60^\circ)^2}{2g}$$

$$= \frac{V^2 \cdot \frac{3}{4}}{2g}$$

$$= \frac{(2gh) \cdot \frac{3}{4}}{2g}$$

$$= \frac{3h}{4}$$

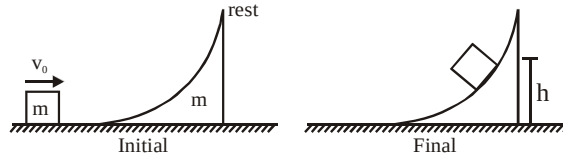
MULTIPLE CORRECT TYPE QUESTIONS

29. Two charges moving under their only own mutual attraction separated by large distance initially. Then choose the correct statement(s)
- (A) If both are free, mechanical energy is conserved.
 - (B) If one is fixed and other is free, mechanical energy is conserved.
 - (C) If one is fixed and other is free, momentum is conserved.
 - (D) If both are free momentum is conserved.

Ans. (ABD)

Sol. consider both charge as system. If both are free, net external force is zero so, mechanical energy and momentum is conserved.
If one is fixed, net external force is not zero so momentum is not conserved, but work done by external force is zero, so mechanical energy is conserved.

30. In the arrangement shown, horizontal surface is smooth, but friction is present between the block and the surface of the wedge. Block is given velocity v_0 at $t = 0$. After achieving height 'h' on the wedge, block comes to rest with respect to wedge at $t = t_0$. Then from $t = 0$ to $t = t_0$:

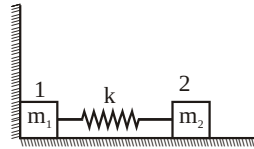


- (A) Work done by friction on the block is negative
(B) Work done by friction on the wedge is negative
(C) Work done by block on the wedge is positive
(D) Work done by wedge on the block is positive

Ans. (AC)

Sol. Since kinetic energy of block decreases.
Since kinetic energy of wedge increases.

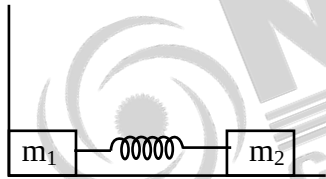
31. Two bars connected by a weightless spring of stiffness k rest on a smooth horizontal plane as shown in figure. Bar 2 is shifted a small distance x to the left and then released. The velocity of the centre of mass of the system after bar 1 breaks off the wall is -



- (A) $\frac{x\sqrt{m_2 k}}{m_1 + m_2}$ (B) $x\sqrt{\frac{k}{m_1 + m_2}}$ (C) zero (D) $\frac{\sqrt{m_1 k}}{m_1 + m_2}$

Ans. (A)

Sol.



$$\frac{1}{2} kx^2 = \frac{1}{2} m_2 v^2$$

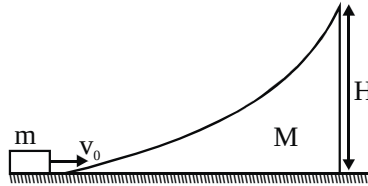
$$\Rightarrow v = x\sqrt{\frac{k}{m_2}}$$

$$V_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$= \frac{m_1(0) + m_2 x\sqrt{\frac{k}{m_2}}}{m_1 + m_2}$$

$$= \frac{x\sqrt{m_2 k}}{m_1 + m_2}$$

32. Figure shows a block of mass m projected with velocity v_0 towards a wedge. Consider all the surfaces to be smooth. Block does not have sufficient energy to negotiate (over come) wedge. Mark the correct option(s)



- (A) when block is at the maximum height on wedge, block and wedge have velocity equal to velocity of centre of mass of block wedge system
(B) wedge acquires maximum speed with respect to ground when block returns to lowest point on wedge.
(C) momentum of wedge and block is conserved at all times
(D) centre of mass of wedge and block remains stationary

Ans. (AB)

- Sol.** (i) in horizontal direction momentum is conserved.
(ii) when block is at maximum height it would be stationary w.r.t. plank. So $v_{\text{block}} = v_{\text{plank}} = v_{\text{cm}}$
(iii) $PE + KE = \text{constant}$
(iv) $v_{\text{cm}} \neq 0$

COMPREHENSION BASED QUESTIONS

Paragraph for Question 33 to 35

2 kg and 3 kg blocks are placed on a smooth horizontal surface and connected by spring which is unstretched initially. The blocks are imparted velocities as shown in the figure.



33. The maximum energy stored in the spring in the subsequent motion will be
(A) $5v_0^2$ (B) $15v_0^2$ (C) zero (D) $10v_0^2$

Ans. (B)

34. Maximum speed of 3 kg block in the subsequent motion will be
(A) v_0 (B) $2v_0$ (C) $3v_0$ (D) $4v_0$

Ans. (C)

35. Maximum speed of 2 kg block in the subsequent motion will be
(A) v_0 (B) $2v_0$ (C) $3v_0$ (D) $4v_0$

Ans. (D)

33. to 35

Sol. Maximum energy will be stored in the spring when blocks will be moving with same velocity.

$$v_c = \frac{8v_0 - 3v_0}{5} = v_0 \text{ towards left}$$

$$\text{So, } U_{\max} = \frac{1}{2} \times 2 \times 16v_0^2 + \frac{1}{2} \times 3 \times v_0^2 - \frac{1}{2} \times 5 \times v_0^2$$

Velocity of centre of mass is v_0 towards left and both the blocks 2kg & 3 kg will perform simple harmonic motion with speed at mean position $3v_0$ and $2v_0$ respectively. So, maximum speed of 2 kg block will be $3v_0 + v_0 = 4v_0$ and maximum speed of 3 kg block will be $2v_0 + v_0 = 3v_0$.

Paragraph for Question 36 to 37

A projectile of mass "m" is projected from ground with a speed of 50 m/s at an angle of 53° with the horizontal. It breaks up into two equal parts at the highest point of the trajectory. One particle coming to rest immediately after the explosion.

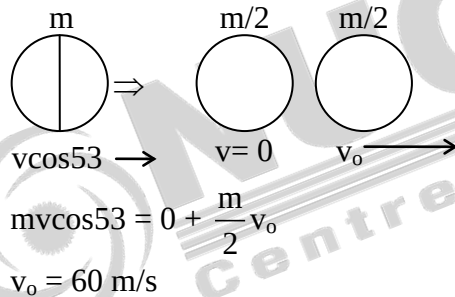
36. The ratio of the radii of curvatures of the moving particle just before and just after the explosion are:

(A) 1 : 4 (B) 1 : 3 (C) 2 : 3 (D) 4 : 9

Ans. (A)

Sol. Just before explosion $R_1 = \frac{(v \cos 53^\circ)^2}{g} = \frac{\left(50 \times \frac{3}{5}\right)^2}{g} = \frac{900}{g} \text{ m}$

By conservation of linear momentum in horizontal direction



Just after explosion $R_2 = \frac{v_0^2}{g} = \frac{3600}{g}$

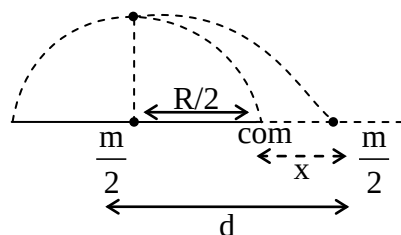
$$R_1 : R_2 = 1 : 4$$

37. The distance between the pieces of the projectile when they reach the ground are:

(A) 240 (B) 360 (C) 120 (D) none

Ans. (A)

Sol. Com divide the line of joining of two particle with inverse ratio of their mass.

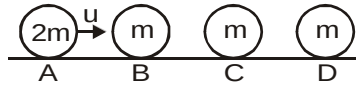


so $x = R/2$

$$\text{so } d = 2(R/2) = R = \frac{\left(2 \times 50 \times \frac{3}{5}\right) \left(50 \times \frac{4}{5}\right)}{g} = 240$$

MATRIX MATCH TYPE QUESTIONS

38. Four balls A, B, C and D are kept on a smooth horizontal surface as shown in figure. Ball A is given velocity u towards B



Column-I

- (A) Total impulse of all collisions on A
(B) Total impulse of all collisions on B
(C) Total impulse of all collision on C
(D) Total impulse of all collisions on D

Column-II

- (P) $\frac{4mu}{9}$
(Q) $\frac{4mu}{27}$
(R) $\frac{4mu}{3}$
(S) $\frac{52}{27}mu$

Ans. (A) $\rightarrow$ S; (B) $\rightarrow$ Q; (C) $\rightarrow$ P; (D) $\rightarrow$ R

Sol. In 1st collision between A & B

$$2mu = 2mv_A + 2mv_B \text{ \& } e = 1 = \frac{v_B - v_A}{u} \Rightarrow v_A = \frac{u}{3}, v_B = \frac{4u}{3}$$

Situation of all collisions is shown in figure.

Initial position				
1st collision				
2nd collision				
3rd Collision				
4th Collision				
5th collision				
6th collision				

$$\text{For (A) Total impulse on A} = 2m \left(u - \frac{u}{27} \right) = \frac{52}{27}mu$$

For (B) Total impulse on B = $m \left(\frac{4u}{27} - 0 \right) = \frac{4}{27} mu$

For (C) Total impulse on C = $m \left(\frac{4u}{9} - 0 \right) = \frac{4}{9} mu$

For (D) Total impulse on D = $m \left(\frac{4u}{3} - 0 \right) = \frac{4}{3} mu$

39. In Column-I, 4 situations are depicted and in column-II, 4 possible kinds of collision are listed. Match the situation with type of collision.

Column-I		Column-II	
	Before	After	
(A)			(P) Elastic
(B)			(Q) Perfectly Inelastic
(C)			(R) Partially elastic (S) Collision is not possible

Ans. (A)→Q; (B)→S; (C)→P;

Sol. $e = \frac{\text{velocity of separation}}{\text{velocity of approach}}$

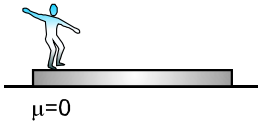
(A) $e = \frac{0}{1.5} = 0$ (Perfectly inelastic collision)

(B) $e = \frac{3}{-3} = -1$ (Collision no possible)

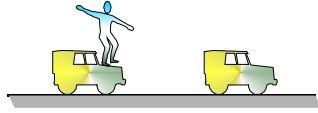
(C) $e = \frac{4}{4} = 1$ (Elastic collision)

40. In all cases in **column-I**, the blocks/plank/trolley are placed on the smooth horizontal surface.

Column-I	Column-II
(A) The initial velocities given to the blocks when spring is relaxed are as shown 	(P) Centre of mass of the complete system shown will not move horizontally
(B) A constant force is applied on 2 kg block. Springs are initially relaxed 	(Q) Centre of mass of the complete system shown will move horizontally
(C) Initially system is at rest. Man starts moving on a large plank with constant velocity.	(R) Mechanical energy of the system will be conserved



- (D) A man standing on one of the trolleys (initially at rest) jumps to the other with relative velocity of 4 m/s horizontally



- (S) Mechanical energy of the system will increase

- (T) Linear momentum of the complete system will always remain constant

Ans. (A)→PRT; (B)→QS; (C)→ PST; (D)→ PST

Sol.

(A) $V_{cm} = \frac{1(2) + 2(-1)}{1+2} = 0$

No external force

& No non conservation force

⇒ mechanical energy conservation

(B) $a_{cm} = \frac{\vec{F}}{m_{total}}$

⇒ Com moves in horizontal direction.

Also external force does work on system

⇒ Mechanical energy increases.

- (C) No external force on (man + Plank system) in horizontal direction

⇒ P momentum conservation & com does not move.

Mechanical energy of system increases.

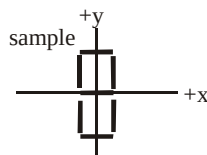
- (D) External force on entire system is zero.

⇒ Centre of mass remains at rest.

& momentum conservation

Mechanical Energy Increase.

41. On the left are statements about the location of the center of mass of the objects depicted on the right. The objects on the right are symbols constructed out of sticks of equal length and mass. The location of the center of mass is described using the coordinate system depicted in the sample.

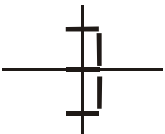
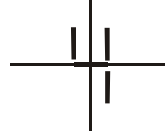
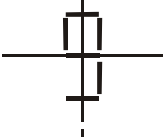
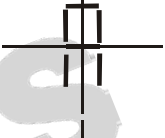


The centre of mass lies at $x = 0, y = 0$

Column I

- (A) The center of mass is at $x > 0$ and $y = 0$
- (B) The center of mass is at $x = 0$ and $y > 0$
- (C) The center of mass is at $x > 0$ and $y > 0$
- (D) The center of mass is at $x = 0$ and $y = 0$

Column II

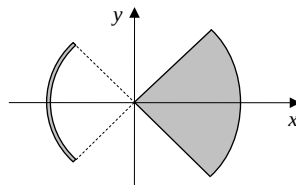
- (P) 
- (Q) 
- (R) 
- (S) 
- (T) 

Ans. (A)→P; (B)→S; (C)→ QR; (D)→ T

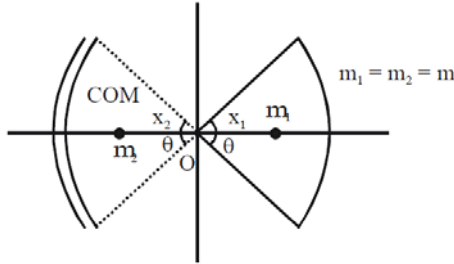
Sol. $X_{CM} = \frac{\sum_{i=1}^n m_i x_i}{\sum m_i}, Y_{CM} = \frac{\sum_{i=1}^n m_i y_i}{\sum m_i}$

EXERCISE # (O-2)

1. A sector cut from a uniform disk of radius 12 cm and a uniform rod of the same mass bent into shape of an arc are arranged facing each other as shown in the figure. If center of mass of the combination is at the origin, what is the radius of the arc?



- (A) 8 cm (B) 9 cm (C) 12 cm (D) 18 cm
- Ans.** (A)



Sol.

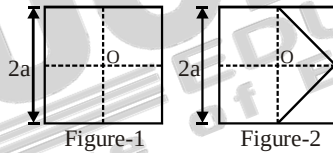
$$x_2 = \frac{2R_2}{\theta} \sin \frac{\theta}{2}, \quad x_1 = \frac{4R_1}{3\theta} \sin \frac{\theta}{2}$$

$$m_1 x_1 = m_2 x_2 \Rightarrow m \cdot \frac{4R_1}{3\theta} \sin \frac{\theta}{2} = m \cdot \frac{2R_2}{\theta} \sin \frac{\theta}{2}$$

$$\Rightarrow R_1 = \frac{3R_2}{2} \Rightarrow R_2 = \frac{2R_1}{3}$$

$$\Rightarrow R_2 = 2 \times \frac{12}{3} = 8 \text{ cm}$$

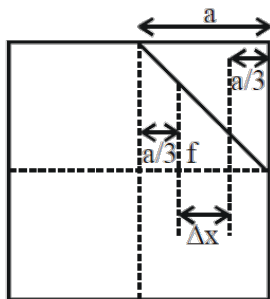
2. A piece of paper (shown in figure-1) is in form of a square. Two corners of this square are folded to make it appear like figure-2. Both corners are put together at centre of square 'O'. If O is taken to be (0, 0), the centre of mass of new system will be at



- (A) $\left(\frac{-a}{8}, 0\right)$ (B) $\left(\frac{-a}{6}, 0\right)$ (C) $\left(\frac{a}{12}, 0\right)$ (D) $\left(\frac{-a}{12}, 0\right)$

Ans.

Sol.



$$\Delta x = a - \left(\frac{a}{3} + \frac{a}{3}\right) = \frac{a}{3}$$

During the process shift in of triangular (heads)

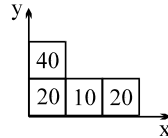
$$(\Delta x_1) = \Delta x_2 = a/3$$

$$\therefore (\Delta x)_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2 + m_3 \times 0}{m_1 + m_2 + m_3}$$

$$\Rightarrow (\Delta x)_{cm} = \frac{\frac{M}{8} \cdot \frac{a}{3} + \frac{M}{8} \cdot \frac{a}{3}}{M} = \frac{a}{12} \text{ towards left}$$

$$\therefore \text{New COM} = \left(-\frac{a}{12}, 0 \right)$$

3. Four cubes of side “a” each of mass 40gm, 20gm, 10gm and 20gm are arranged in XY plane as shown in figure. The coordinates of centre of mass of the combination with respect to O, are :



- (A) $19a/18, 17a/18$ (B) $17a/18, 11a/18$ (C) $17a/18, 13a/18$ (D) $13a/18, 17a/18$

Ans.

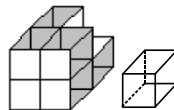
(A)

Sol.

Block	Its Com
40 gm	$\left(\frac{a}{2}, \frac{3a}{2} \right)$
20 gm	$\left(\frac{a}{2}, \frac{a}{2} \right)$
10 gm	$\left(\frac{3a}{2}, \frac{a}{2} \right)$
20 gm	$\left(\frac{5a}{2}, \frac{a}{2} \right)$

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i} = \frac{40 \left(\frac{a}{2}, \frac{3a}{2} \right) + 20 \left(\frac{a}{2}, \frac{a}{2} \right) + 10 \left(\frac{3a}{2}, \frac{a}{2} \right) + 20 \left(\frac{5a}{2}, \frac{a}{2} \right)}{40 + 20 + 10 + 20} = \left(\frac{19}{18}a, \frac{17}{18}a \right)$$

4. Eight solid uniform cubes of edge l are stacked together to form a single cube with center O. One cube is removed from this system. Distance of the centre of mass of remaining 7 cubes from O is



- (A) $\frac{7\sqrt{3}l}{16}$ (B) $\frac{\sqrt{3}l}{16}$ (C) $\frac{\sqrt{3}l}{14}$ (D) zero

Ans. (C)

Sol. Com of entire system, when single block is placed at its previous place is at O.

$$m_1 r_1 = m_2 r_2$$

$$7 m r_1 = m \left(\frac{\sqrt{3} \ell}{2} \right)$$

$$r_1 = \frac{\sqrt{3} \ell}{14}$$

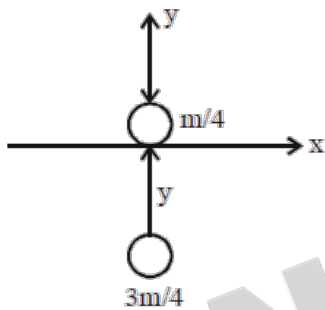
5. An isolated particle of mass m is moving in horizontal plane ($x - y$), along the x -axis, at a certain height above the ground. It suddenly explodes into two fragments of masses $\frac{m}{4}$ and $\frac{3m}{4}$.

An instant later, the smaller fragment is at $y = +15$ cm. The larger fragment at this instant is at :

- (A) $y = -5$ cm (B) $y = +20$ cm (C) $y = +5$ cm (D) $y = -20$ cm

Ans.

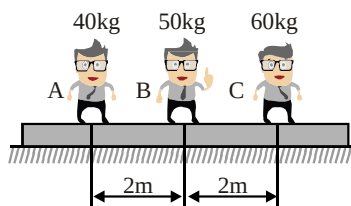
Sol.



$$\frac{m}{4} \times 15 = \frac{3m}{4} \times y \Rightarrow y = 5$$

$$\Rightarrow -5$$

6. Three men A, B & C of mass 40 kg, 50 kg & 60 kg are standing on a plank of mass 90 kg, which is kept on a smooth horizontal plane. If A & C exchange their positions then mass B will shift

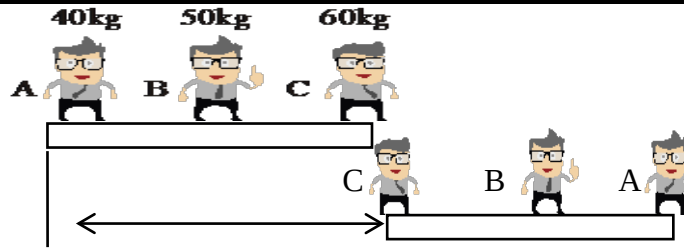


- (A) $\frac{1}{3}$ m towards left (B) $\frac{1}{3}$ m towards right
(C) will not move w.r.t. ground (D) $\frac{5}{3}$ m towards left

Ans.

Sol. $\because F_{\text{ext}} = 0$ [In horizontal]

$$\because U_{\text{cm}} = 0$$



$$\Rightarrow S_{cm} = 0$$

$$\Rightarrow m_1 s_1 + m_2 s_2 + m_3 s_3 + m_4 s_4 = 0$$

$$S_{\text{plank}} = x \quad S_B = x$$

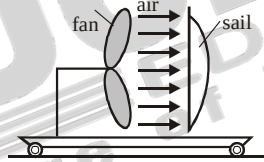
$$S_A = x + 4 \quad S_C = x - 4$$

$$90(x) + 40(x + 4) + 50(x) + 60(x - 4) = 0$$

$$24x + 16 - 24 = 0$$

$$x = \frac{8}{24} = \frac{1}{3}$$

7. A fan and a sail are mounted vertically on a cart that is initially at rest on a horizontal table as shown in the diagram. When the fan is turned on, an air stream is blown towards the right and is incident on the sail. The cart is free to move with negligible resistance forces. After the fan has been turned on the cart will

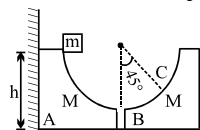


- (A) move to the right and then to the left (B) remain at rest
(C) move towards the right (D) move towards the left

Ans. (B)

Sol. Impulse given by fan to sail through air = Impulse given by air to fan.
 $\therefore$ Net impulse on system = zero
 $\Rightarrow$ system remains at rest

8. Two identical blocks having mass M each are smoothly conjugated and placed on a smooth horizontal floor as shown in figure. On the left of block A there is a wall. A small block of mass m is released from the position. Then velocity of block B is maximum



- (A) when m is at highest position on B
(B) when m is at lowest position and moving towards left w.r.t B
(C) when m is at point C

(D) is equal to $\frac{m\sqrt{2gh}}{m + M}$

Ans. (B)

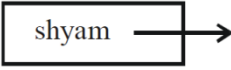
Sol. When block comes down wedge A, the wedge does not move due to wall.

When block climbs on wedge B, the wedge moves rightward due to normal force between block and wedge B.

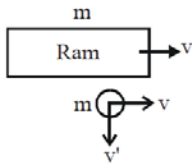
9. Two identical carts constrained to move on a straight line, on which sit two twins of same mass, are moving with same velocity. At some time snow begins to drop uniformly vertically downward. Ram, sitting on one of the trolleys, throws off the falling snow sideways and in the second cart shyam is a sleep. (Assume that friction is absent)

- (A) Cart carrying Ram will speed up while cart carrying shyam will slow down
(B) Cart carrying Ram will remain at the same speed while cart carrying shyam will slow down
(C) Cart carrying Ram will speed up while cart carrying shyam will remain at the same speed
(D) Cart carrying Ram as well as shyam will slow down

Ans. (D)

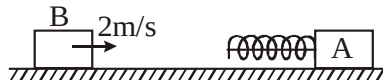
Sol. 

As snow falls mass of (trolley + snow) increases $\Rightarrow \frac{V}{P}$ decreases due to conservation.



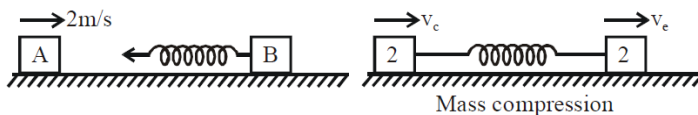
no extra force due to Ram on snow in direction of v no extra force due to snow on ram $\Rightarrow$ due throw of ice $\Rightarrow$ velocity of trolley in forward direction is not changed.

10. Figure shows a block A of mass 5 kg kept at rest on a horizontal smooth surface. A spring ($K = 200 \text{ N/m}$) which is compressed by 10 cm and tied with the help of a string to maintain the compression is attached to block A as shown in figure. Block B also of mass 5 kg moving with 2 m/s collides with A, as shown. During the collision the string breaks and after the collision the spring is in its natural state. Assume the bodies to be elastic and let the velocities of A and B be v_1 and v_2 respectively assuming positive direction towards right, after collision. Then

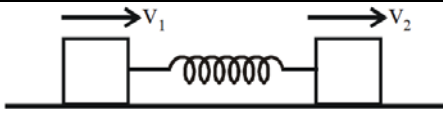


- (A) $v_1 + v_2 > 2$
(B) Initial kinetic energy of system = final kinetic energy of system
(C) $v_1^2 + v_2^2 = 4.4 \text{ (m/s)}^2$
(D) $v_1 - v_2 = 2$

Ans. (C)



Sol.



(A) System $m_1 + m_2 + \text{spring} \Rightarrow$ from momentum oxygen

$$5 \times 2 = 54 + 54 \Rightarrow v_1 + v_2 = 2$$

from energy $\Rightarrow (TE)_i = (TE)_f$

$$(C) \frac{1}{2} \times 5 \times 2^2 + \frac{1}{2} \times 200 \times (0.1)^2 = \frac{1}{2} \times 5 \times v_1^2 + \frac{1}{2} \times 5 \times v_2^2$$

$$\Rightarrow 20 + 2 = 5v_1^2 + 5v_2^2$$

(B) $(KE)_i + (PE)_i = (KE)_f$

11. An open water tight railway wagon of mass 5×10^3 kg coasts at an initial velocity 1.2 m/s without friction on a railway track. Rain drops fall vertically downwards into the wagon. The velocity of the wagon after it has collected 10^3 kg of water will be

(A) 0.5 m/s (B) 2 m/s (C) 1 m/s (D) 1.5 m/s

Ans. (C)

Sol. $(F_{\text{ext}})_H = 0 \quad \therefore \text{rain falls vertical}$

$$\therefore 5 \times 10^3 \times 1.2 = 6 \times 10^3 \times v$$

$$\Rightarrow v = 1.0 \text{ m/s}$$

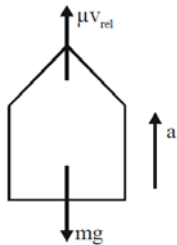
12. A rocket of mass 4000 kg is set for vertical firing. How much gas must be ejected per second so that the rocket may have initial upwards acceleration of magnitude 19.6 m/s^2 .

[Exhaust speed of fuel = 980 m/s .]

(A) 240 kg s^{-1} (B) 60 kg s^{-1} (C) 120 kg s^{-1} (D) None

Ans. (C)

Sol.



Initial $\Rightarrow$ mass = 4000

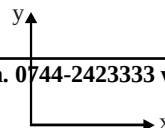
$$\mu v_{\text{rel}} - mg = ma$$

$$\mu \times 980 - 4000 \times 9.8$$

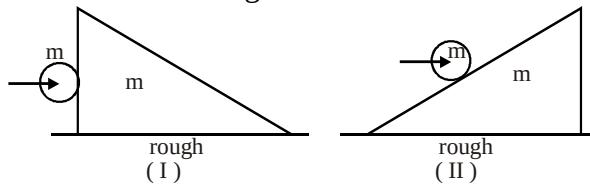
$$4000 \times 19.6$$

$$\Rightarrow \mu = 40 + 80 = 120$$

13. A ball of mass m collides horizontally with a stationary wedge on a rough horizontal surface, in the two orientations as shown. Neglect friction between ball and wedge. Two student comment



on system of ball and wedge in these situations



Saurav : Momentum of system in x-direction will change by significant amount in both cases.

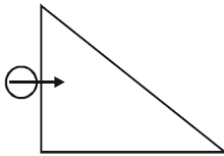
Rahul : There are no impulsive external forces in y-direction in both cases hence the total momentum of system in y-direction can be treated as conserved in both cases.

(A) **Saurav** is wrong and **Rahul** is correct (B) **Saurav** is correct and **Rahul** is wrong

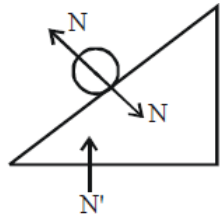
(C) Both are correct

(D) Both are wrong

Ans. (D)



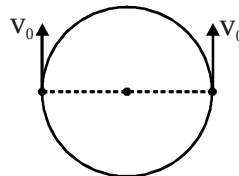
Sol.



No impulsive force (external) either in -ive x or in y-direction $P \Rightarrow$ constant

$N' \Rightarrow$ Impulsive $\Rightarrow f_r \Rightarrow$ Impulsive $\Rightarrow$ Momentum will change

14. Two particles of mass m , constrained to move along the circumference of a smooth circular hoop of equal mass m , are initially located at opposite ends of a diameter and given equal velocities v_0 shown in the figure. The entire arrangement is located in gravity free space. Their velocity just before collision is



(A) $\frac{1}{\sqrt{3}} v_0$

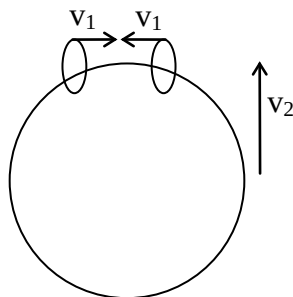
(B) $\frac{\sqrt{3}}{2} v_0$

(C) $\frac{2}{\sqrt{3}} v_0$

(D) $\frac{\sqrt{7}}{3} v_0$

Ans. (D)

Sol.



$3mv_2 = 2mv_0$

$$v_2 = \frac{2v_0}{3}$$

$$\frac{1}{2} \cdot m \cdot v_2^2 + 2 \times \frac{1}{2} m (v_1^2 + v_2^2) = \frac{1}{2} m v_0^2 \times 2$$

15. A uniform rope of linear mass density λ and length ℓ is coiled on a smooth horizontal surface. One end is pulled up with constant velocity v . Then the average power applied by the external agent in pulling the entire rope just off the ground is :



(A) $\frac{1}{2} \lambda \ell v^2 + \frac{\lambda \ell^2 g}{2}$ (B) $\lambda \ell g v$ (C) $\frac{1}{2} \lambda v^3 + \frac{\lambda \ell v g}{2}$ (D) $\lambda \ell g v + \frac{1}{2} \lambda v^3$

Ans. (C)

Sol. Time taken to pull up $= \frac{\ell}{v}$

W.E.T.

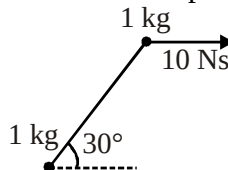
$$W_{\text{ext.}} + W_{\text{gravity}} = \frac{1}{2} (\lambda \ell) v^2 - 0$$

$$W_{\text{ext.}} - (\lambda \ell) g \frac{\ell}{2} = \frac{1}{2} \lambda \ell v^2$$

$$W_{\text{ext.}} = \lambda g \frac{\ell^2}{2} + \frac{1}{2} \lambda \ell v^2$$

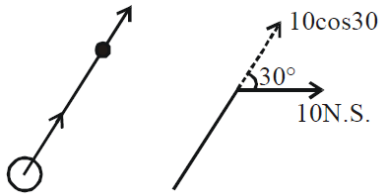
$$< \text{Power} > = \frac{W_{\text{ext.}}}{\text{Time}} = \frac{\lambda g \frac{\ell^2}{2} + \frac{1}{2} \lambda \ell v^2}{\left(\frac{\ell}{v} \right)} = \lambda g \frac{v \ell}{2} + \frac{1}{2} \lambda v^3$$

16. Two balls of masses 1 kg each are connected by an inextensible massless string. The system is resting on a smooth horizontal surface. An impulse of 10 Ns is applied to one of the balls at an angle 30° with the line joining two balls in horizontal direction as shown in the figure. Assuming that the string remains taut after the impulse, the magnitude of impulse of tension is :



(A) 6Ns (B) $\frac{5}{2} \sqrt{3}$ Ns (C) 5 Ns (D) $\frac{5}{\sqrt{3}}$ Ns

Ans. (B)



Sol.

$$2mv = 10 \cos 30^\circ$$

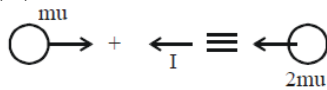
$$\Rightarrow mv = \frac{5\sqrt{3}}{2}$$

$$\therefore I_{\text{tension}} = (\Delta P)_1 = mv = \frac{5\sqrt{3}}{2}$$

17. A force exerts an impulse I on a particle changing its speed from u to $2u$. The applied force and the initial velocity are oppositely directed along the same line. The work done by the force is

- (A) $\frac{3}{2} I u$ (B) $\frac{1}{2} I u$ (C) $I u$ (D) $2I u$

Ans. (B)



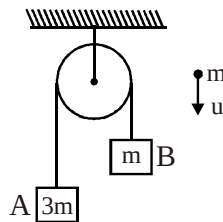
Sol.

$$\Rightarrow mu - I = -2mu \Rightarrow I = 3mu$$

$$W = \Delta KE = \frac{1}{2} m(2u)^2 - \frac{1}{2} mu^2 = \frac{3}{2} mu^2 = \left(\frac{3}{2} mu \right) \cdot u$$

$$W = \frac{I}{2} \cdot u$$

18. A system of two blocks A and B are connected by an inextensible massless strings as shown. The pulley is massless and frictionless. Initially the system is at rest when, a bullet of mass ' m ' moving with a velocity ' u ' as shown hits the block 'B' and gets embedded into it. The impulse imparted by tension force to the block of mass $3m$ is



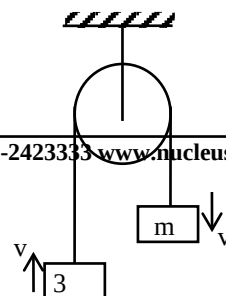
- (A) $\frac{5mu}{4}$ (B) $\frac{4mu}{5}$ (C) $\frac{2mu}{5}$ (D) $\frac{3mu}{5}$

Ans. (D)

Sol. Writing Impulse equation for tension

$$-Tdt = (m + m) v - mv \quad \dots(1)$$

$$Tdt = 3mv - 0 \quad \dots(2)$$

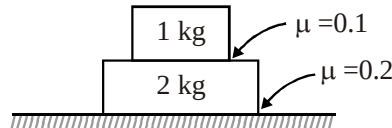


$$\Rightarrow 5mv = mu$$

$$\Rightarrow v = \frac{u}{5}$$

$$\Rightarrow \text{Impulse by tension on } 3m = 3m\left(\frac{u}{5}\right) = \frac{3mu}{5}$$

19. If both the blocks as shown in the given arrangement are given together a horizontal velocity towards right. If a_{cm} be the subsequent acceleration of the centre of mass of the system of blocks then a_{cm} equals



(A) 0 m/s^2

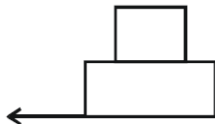
(B) m/s^2

(C) m/s^2

(D) 2 m/s^2

Ans.

(D)



Sol.

$$F_{\text{ext}} = f_k = \mu N = 0.2 \times 30 = 6 \text{ N}$$

$$\therefore a_{cm} = \frac{6}{3} = 2 \text{ m/s}^2$$

20. In adjacent figure a boy, on a horizontal platform A, kept on a smooth horizontal surface, holds a rope attached to a box B. Boy pulls the rope with a constant force of 50N. The coefficient of friction between boy and platform is 0.5. (Mass of boy = 80 kg, mass of platform = 120kg and mass of box = 100 kg)



(A) Velocity of platform relative to box after 4 sec. is 2m/s

(B) Velocity of boy relative to platform after 4sec is 2m/s

(C) Friction force between boy and platform is 30N

(D) Friction force between boy and platform is 50N

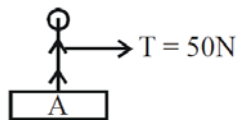
Ans.

(C)

Sol.

Both block & boy move together.

$$\Rightarrow (80 + 120)a = 50$$



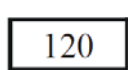
$$\Rightarrow a = \frac{1}{4}$$

$$f = 120 \times a = t$$

$$f = \frac{120}{4} = 30 \text{ N}$$

For no relative slipping

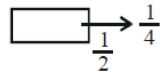
$$\text{common } a = \frac{50}{(80+120)} = \frac{1}{4} \text{ m/s}^2$$

 $\therefore fr = 120 \times a = 30 \text{ N}$

$$f_{\max} \mu N = 0.5 \times 800 = 40 \text{ N}$$

$$\Rightarrow \text{No relative slipping} \Rightarrow a_A = \frac{1}{4}$$

$$a_B = \frac{50}{100} = \frac{1}{2} \text{ m/s}^2$$



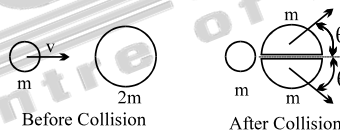
(A) w.r.t. B $\Rightarrow$

$$a_{AB} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \Rightarrow v_{AB} = 0 + \frac{1}{4} \times 4 = 1 \text{ m/s}$$

(B) $\bar{v}_{\text{boy/P}} = 0$ ($\because$ No relative slipping)

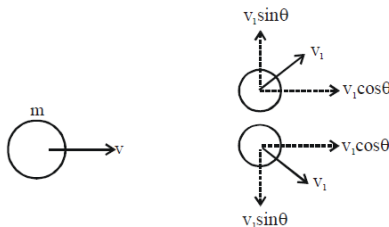
(C) $fr = 30 \text{ N}$

21. A particle of mass m is moving along the x -axis with speed v when it collides with a particle of mass $2m$ initially at rest. After the collisions, the first particle has come to rest, and the second particle has split into two equal-mass pieces that move at equal angles $\theta > 0$ with the x -axis, as shown in the figure. Which of the following statements correctly describes the speeds of the two pieces?



- (A) Each piece moves with speed v
(B) One of the pieces moves with speed v , the other moves with speed less than v
(C) Each piece moves with speed $v/2$
(D) Each piece moves with speed greater than $v/2$

Ans. (D)



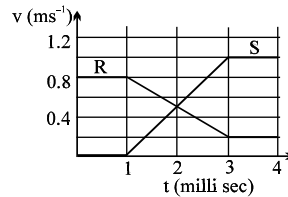
Sol.

$$m \cdot 2v_1 \cos \theta = mv$$

$$\Rightarrow v_1 = \frac{v}{2 \cos \theta}$$

$$\because \cos \theta < 1 \Rightarrow v_1 > \frac{v}{2}$$

22. The diagram shows the velocity-time graph for two masses R and S that collided elastically. Which of the following statements is true?



- (I) R and S moved in the same direction after the collision.
 (II) Kinetic energy of the system (R & S) is minimum at $t = 2$ milli sec.
 (III) The mass of R was greater than mass of S.
 (A) I only
 (B) II only
 (C) I and II only
 (D) I, II and III

Ans. (D)

Sol. Momentum conservation

$$M_S U_S + M_R U_R = M_S V_S + M_R V_R$$

$$M_S(0) + M_R(0.8) = M_S(1) + M_R(0.2)$$

$$M_R(0.6) = M_S$$

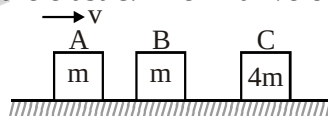
$$\Rightarrow M_S < M_R$$

Also velocity of both blocks are positive after collision $\Rightarrow$ they are moving in same direction.

Also during the collision.

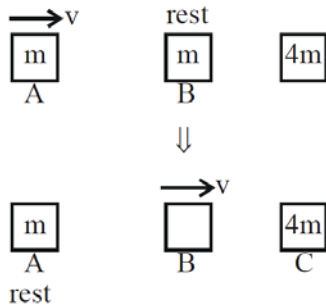
At min KE of system, velocity of both particles should be same.

23. Three blocks are initially placed as shown in the figure. Block A has mass m and initial velocity v to the right. Block B with mass m and block C with mass $4m$ are both initially at rest. Neglect friction. All collisions are elastic. The final velocity of block A is



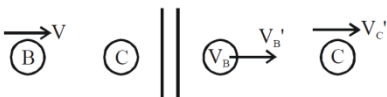
- (A) $0.6v$ to the left
 (B) $1.4v$ to the left
 (C) v to the left
 (D) $0.4v$ to the right

Ans. (A)



Sol.

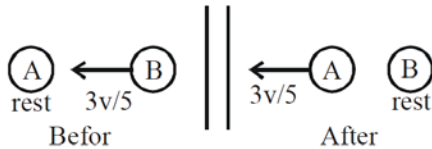
for B – C collision



$$v_{\text{com}} = \frac{mv}{5m} = \frac{v}{5}$$

$$v'_B = 2v_C - v_1 = 2 \times \frac{v}{5} - v = -\frac{3v}{5}$$

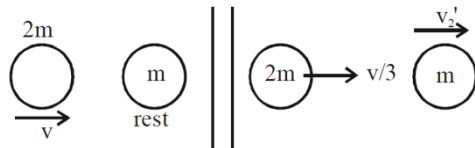
B returns with $\frac{3v}{5}$



24. Two billiard balls undergo a head-on collision. Ball 1 is twice as heavy as ball 2. Initially, ball 1 moves with a speed v towards ball 2 which is at rest. Immediately after the collision, ball 1 travels at a speed of $v/3$ in the same direction. What type of collision has occurred?
 (A) inelastic (B) elastic (C) completely inelastic
 (D) cannot be determined from the information given

Ans. (B)

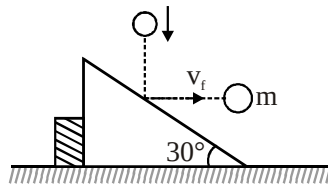
Sol.



$$v'_1 = (1+e)v_C - v_1$$

$$\frac{v}{3} = (1+e) \cdot \frac{2v}{3} - e v \Rightarrow e = 1$$

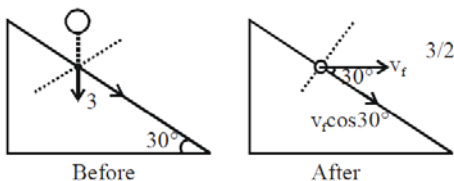
25. As shown in the figure a body of mass m moving vertically with speed 3 m/s hits a smooth fixed inclined plane and rebounds with a velocity v_f in the horizontal direction. If $\angle$ of inclined is 30° , the velocity v_f will be



- (A) 3 m/s
 (C) $\frac{1}{\sqrt{3}} \text{ m/s}$

- (B) $\sqrt{3} \text{ m/s}$
 (D) this is not possible

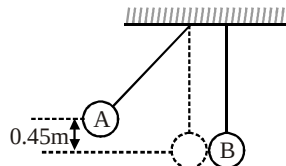
Ans. (B)



Sol.

$$v_f \cos 30^\circ = \frac{3}{2} \Rightarrow v_f = \sqrt{3} \text{ m/s}$$

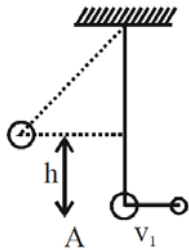
26. Two massless string of length 5 m hang from the ceiling very near to each other as shown in the figure. Two balls A and B of masses 0.25 kg and 0.5 kg are attached to the string. The ball A is released from rest at a height 0.45 m as shown in the figure. The collision between two balls is completely elastic. Immediately after the collision, the kinetic energy of ball B is 1 J. The velocity of ball A just after the collision is



- (A) 5 ms^{-1} to the right
(C) 1 ms^{-1} to the right
(D)

- (B) 5 ms^{-1} to the left
(D) 1 ms^{-1} to the left

Ans.



Sol.

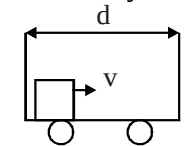
Just before collision

$$\text{WET} \Rightarrow v_A = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.45}$$

$$v_A = 3 \text{ m/s} = v_1$$

$$\therefore v_1' = 2 \cdot \frac{v}{3} - v = -\frac{v}{3} = -1 \text{ m/s}$$

27. In a smooth stationary cart of length d , a small block is projected along its length with velocity v towards front. Coefficient of restitution for each collision is e . The cart rests on a smooth ground and can move freely. The time taken by block to come to rest w.r.t. cart is



(A) $\frac{ed}{(1-e)v}$

(B) $\frac{ed}{(1+e)v}$

(C) $\frac{d}{e}$

(D) infinite

Ans. (D)

Sol. Total time = $\frac{d}{v} + \frac{d}{ev} + \frac{d}{e^2v} + \dots$

$$= \frac{d}{v} \left(1 + \frac{1}{e} + \frac{1}{e^2} + \dots \right)$$

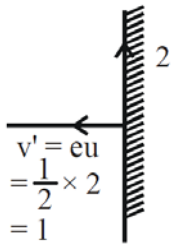


Since $\frac{1}{e} > 1 \Rightarrow$ total time = infinite.

28. A smooth sphere is moving on a horizontal surface with a velocity vector $(2\hat{i} + 2\hat{j})$ m/s immediately before it hit a vertical wall. The wall is parallel to vector $\hat{j}$ and coefficient of restitution between the sphere and the wall is $e = 1/2$. The velocity of the sphere after it hits the wall is

(A) $\hat{i} - \hat{j}$ (B) $-\hat{i} + 2\hat{j}$ (C) $-\hat{i} - \hat{j}$ (D) $2\hat{i} - \hat{j}$

Ans. (B)

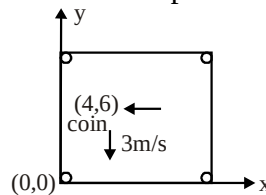


Sol.

$\therefore$ smooth sphere $\Rightarrow f = 0 \Rightarrow v_y = \text{uncylindrical}$

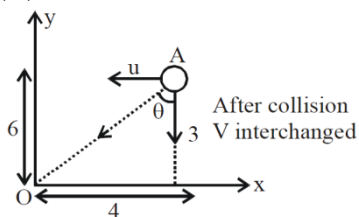
$\therefore \vec{v}_f = -\hat{i} + 2\hat{j}$

29. On a smooth carom board, a coin moving in negative y-direction with a speed of 3 m/s is being hit at the point (4, 6) by a striker moving along negative x-axis. The line joining centres of the coin and the striker just before the collision is parallel to x-axis. After collision the coin goes into the hole located at the origin. Masses of the striker and the coin are equal. Considering the collision to be elastic, the initial and final speeds of the striker in m/s will be



(A) (1.2, 0) (B) (2, 0) (C) (3, 0) (D) None of these

Ans. (B)



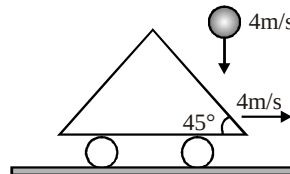
Sol.

Resultant of 4 & 3 is along AO.

$$\tan\theta = \frac{4}{6} = \frac{4}{3} \Rightarrow u = 2 \text{ m/s}$$

$$\text{striker } v = \begin{cases} u = 2 \Rightarrow \text{Before} \\ 0 \Rightarrow \text{After} \end{cases}$$

30. A small ball falling vertically downward with constant velocity 4m/s strikes elastically a massive inclined cart moving with velocity 4m/s horizontally as shown. The velocity of the rebound of the ball is



- (A) $4\sqrt{2}$ m/s. (B) $4\sqrt{3}$ m/s (C) 4 m/s (D) $4\sqrt{5}$ m/s

Ans. (D)

Sol. $e = \frac{v_{\text{sep}}}{v_{\text{app}}}$

$$v_{\text{app}} = 4 \cos 45^\circ + 4 \cos 45^\circ$$

$$= \frac{8}{\sqrt{2}}$$

$$v_{\text{sep}} = v_1 - 4 \cos 45^\circ$$

$$e = 1 = \frac{v_1 - \frac{4}{\sqrt{2}}}{\frac{8}{\sqrt{2}}} \Rightarrow v_1 = \frac{12}{\sqrt{2}}$$

v_1 = velocity of ball $\perp$ ar to the surface.

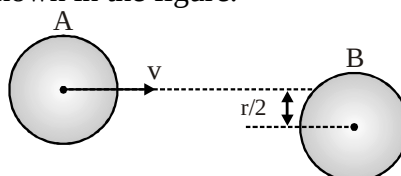
v_2 = velocity of ball along the surface.

$$= 4 \sin 45^\circ = \frac{4}{\sqrt{2}} \text{ m/s}$$

$$\text{Speed} = \sqrt{v_1^2 + v_2^2}$$

$$= \sqrt{\left(\frac{12}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2} = \sqrt{\frac{160}{2}} = \sqrt{80}$$

31. A disk A of radius r moving on perfectly smooth surface at a speed v undergoes an elastic collision with an identical stationary disk B. Find the velocity of the disk B after collision if the impact parameter is $r/2$ as shown in the figure.



(A) $\frac{\sqrt{15}}{4}v$

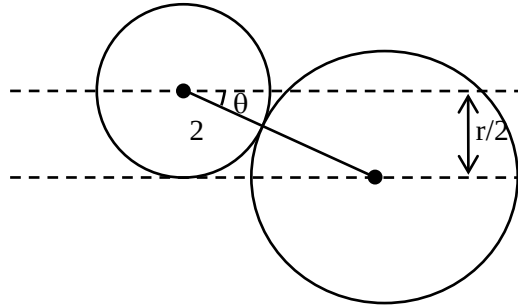
(B) $\frac{v}{4}$

(C) $\frac{v}{2}$

(D) $\frac{\sqrt{3}v}{2}$

Ans. (A)

Sol. Masses are equal and elastic collision.



$$\sin \theta = \frac{\left(\frac{r}{2}\right)}{2r} = \frac{1}{4}$$

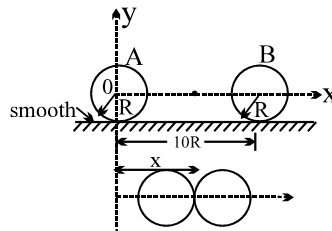
$$\Rightarrow \cos \theta = \frac{\sqrt{15}}{4}$$

$\Rightarrow$ velocity will be interchanged along line of collision.

$$\text{velocity of A along LOC} = v \cos \theta = v \frac{\sqrt{15}}{4}$$

= this velocity will be given to B.

32. Two uniform non conducting balls A & B have identical size having radius R but made of different density material (density of A = 2 density of B). The ball A is +vely charged & ball B is -vely charged. The balls are released on the horizontal smooth surface at the separation 10R as shown in figure. Because of mutual attraction the balls start moving towards each other. They will collide at a point.



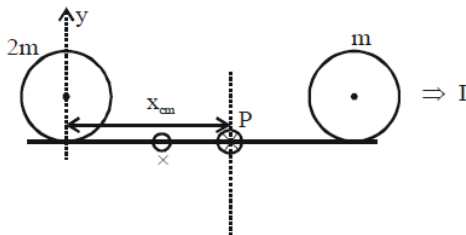
(A) $x = \frac{10R}{3}$

(B) $x = \frac{11R}{3}$

(C) $x = 5R$

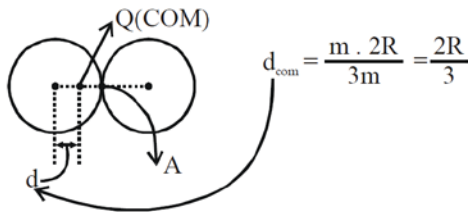
(D) $x = \frac{7R}{5}$

Ans. (B)



Sol.

$$X_{cm} = \frac{2m \cdot 0 + m \cdot 10R}{3m}$$

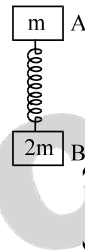


$$f_{\text{ext}} = 0 \Rightarrow a_{\text{cm}} = 0 \Rightarrow v_{\text{cm}} = \text{constant} = 0$$

$$\Rightarrow (\Delta x)_{\text{cm}} = 0$$

$$\therefore \text{Distance of A} = X_{\text{COM}} + (R - d) = \frac{10R}{3} + \left(R - \frac{2R}{3}\right) = \frac{11R}{3}$$

33. From what minimum height h must the system be released when spring is unstretched so that after perfectly inelastic collision ($e = 0$) with ground, B may be lifted off the ground (Spring constant = k).

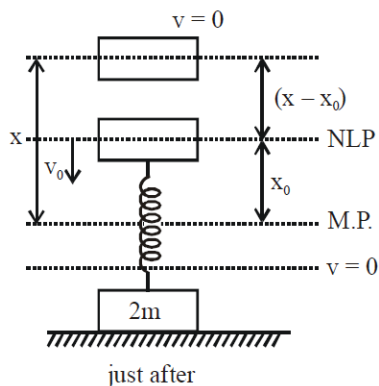


- (A) $\text{mg}/(4\text{k})$ (B) $4\text{mg}/\text{k}$ (C) $\text{mg}/(2\text{k})$ (D) none

Ans.

Sol.

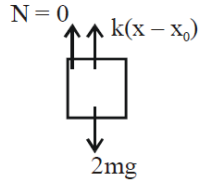
(A)



$$kx_0 = mg$$

$$x_0 = \frac{mg}{k}$$

compression from MP = Elongation from MP = x



$$\text{From W.E.T.} \Rightarrow mg(x + x_0) + \frac{1}{2}k[O^2 - (x + x_0)^2] = 0 - \frac{1}{2}mv_0^2 = -mgh$$

$$\Rightarrow \frac{1}{2}k(x + x_0)^2 - mg(x + x_0) - mgh = 0 \Rightarrow (x + x_0)$$

$$\text{From (1)} \quad kx - kx_0 = 2mg \Rightarrow kx - k \cdot \frac{mg}{k} = 2mg$$

$$\Rightarrow x = \frac{3mg}{k}$$

$$\therefore x + x_0 = \frac{3mg}{k} + \frac{mg}{k} = \frac{4mg}{k}$$

Now from W.E.T. $\Rightarrow$ from NLP to max compression

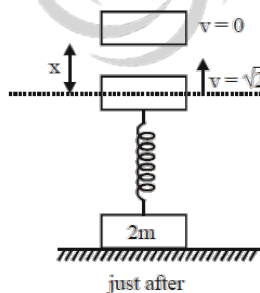
$$mg(x + x_0) + \frac{1}{2}k[O^2 - (x + x_0)^2] = 0 - \frac{1}{2}mv_0^2$$

$$\Rightarrow mg \cdot \frac{4mg}{k} - \frac{1}{2}k \cdot \frac{(4mg)^2}{k^2} = -\frac{1}{2}mv_0^2$$

$$\Rightarrow -\frac{4(mg)^2}{k} = -\frac{1}{2}mv_0^2$$

$$\Rightarrow \frac{8m^2g^2}{k} = m \cdot 2gh \Rightarrow h = \frac{4mg}{k}$$

Alternate Solution



NLP After collision with the ground and during upward journey of upper block, shown will be the condition.

$$\frac{1}{2}m(2gh) = mgh + \frac{1}{2}kx^2 \quad \dots(i)$$

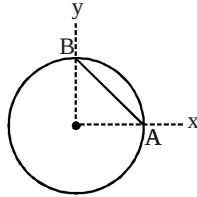
$$kx = 2mg \quad \dots(ii)$$

From (i) & (ii)

$$h = \frac{4mg}{k}$$

MULTIPLE CORRECT TYPE QUESTIONS

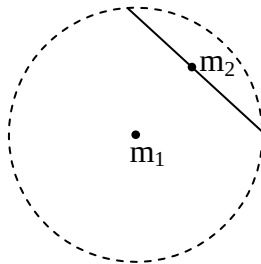
34. An object comprises of a uniform ring of radius R and its uniform chord AB (not necessarily made of the same material) as shown. Which of the following can not be the centre of mass of the object



- (A) $(R/3, R/3)$ (B) $(R/3, R/2)$ (C) $(R/4, R/4)$ (D) $(R/\sqrt{2}, R/\sqrt{2})$

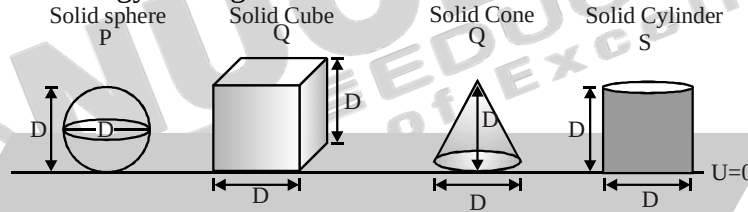
Ans. (BD)

Sol.



com always lie on $y = x$ and lie between mass m_1 & m_2 .

35. Assuming potential energy 'U' at ground level to be zero.



All objects are made up of same material.

U_P = Potential energy of solid sphere

U_Q = Potential energy of solid cube

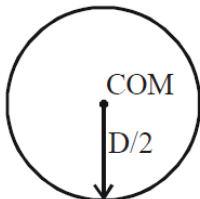
U_R = Potential energy of solid cone

U_S = Potential energy of solid cylinder

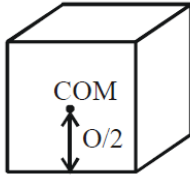
- (A) $U_S > U_P$ (B) $U_Q > U_S$ (C) $U_P > U_Q$ (D) $U_S > U_R$

Ans. (ABD)

Sol.

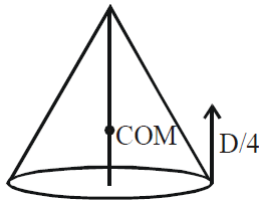


$$U_P = mgR = \frac{4\pi}{3} \left(\frac{D}{2} \right)^3 \rho g \frac{D}{2} = \frac{\pi}{12} \rho D^4 g$$

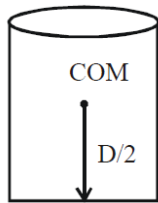


$$U_Q = mg \cdot \frac{D}{2} = D^3 \cdot \rho \cdot g \cdot \frac{D}{2}$$

$$= \frac{1}{2} \rho \cdot D^4 \cdot g$$



$$U_R = mg \cdot \frac{D}{4} = \frac{1}{3} \pi \left(\frac{D}{2} \right)^2 \times D \rho \times g \cdot \frac{D}{4} = \frac{\pi}{48} \rho \cdot D^4 \cdot g$$



$$U_S = m g \cdot \frac{D}{2} = \pi \left(\frac{D}{2} \right)^2 \cdot D \rho \cdot g \cdot \frac{D}{2} = \frac{\pi}{8} \rho \cdot D^4 \cdot g$$

(A) $U_S > U_P$

(B) $\frac{1}{2} = 0.5, \frac{\pi}{8} = \frac{3.5v}{8} = 0.4$

$\therefore U_Q > U_S$

(C) $U_P < U_Q$

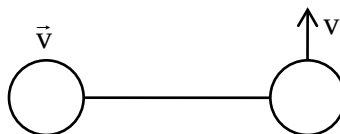
(D) $U_S > U_R$

36. Two identical balls are interconnected with a massless and inextensible thread of length L . The system is in gravity free space with the thread just taut. Each ball is imparted a velocity v , one towards the other ball and the other perpendicular to the first, at $t = 0$. Then,

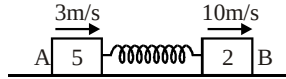
- (A) the thread will become taut at $t = (L/v)$
 (B) the thread will become taut at some time $t < (L/v)$.
 (C) the thread will always remain taut for $t > (L/v)$.
 (D) the kinetic energy of the system will always remain mv^2 .

Ans. (AC)

Sol. The thread will become taut at $t = \frac{L}{v}$ and it will remain taut after that.



37. Two blocks A (5kg) and B(2kg) attached to the ends of a spring constant 1120N/m are placed on a smooth horizontal plane with the spring undeformed. Simultaneously velocities of 3m/s and 10m/s along the line of the spring in the same direction are imparted to A and B then



- (A) when the extension of the spring is maximum the velocities of A and B are zero.
(B) the maximum extension of the spring is 25 cm.
(C) maximum extension and maximum compression occur alternately.
(D) the minimum speed of B is 0.

Ans. (BCD)

Sol. At max extension velocity of both block is same (v).

momentum conservation

$$5 \times 3 + 2 \times 10 = 5v + 2v$$

$$v = 5 \text{ m/s}$$

Gain in potential energy = loss in KE

$$\frac{1}{2}Kx^2 = \left(\frac{1}{2} \times 5 \times 3^2 + \frac{1}{2} \times 2 \times 10^2 \right) - \frac{1}{2}(5+2)v^2$$

$$\Rightarrow x = 25 \text{ cm}$$

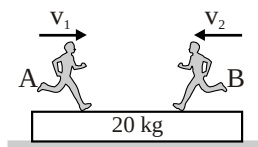
$$v_{\text{com}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{5 \times 3 + 2 \times 10}{5 + 2} = 5 \text{ m/s}$$

$v_{B/\text{com}}$ initially = 5 m/s

Range $v_{B/\text{com}} = 0$ to 10

$$\Rightarrow \min v_{B/\text{ground}} = 0$$

38. In the figure shown the system is at rest initially. Two persons 'A' and 'B' of masses 40 kg each move with speeds v_1 and v_2 respectively towards each other on a plank lying on a smooth horizontal surface as shown in figure. Plank travels a distance of 20 m towards right direction in 5 sec. (Here v_1 and v_2 are given with respect to the plank). Then the possible condition(s) can be



(A) $v_1 = 0 \text{ m/s}$, $v_2 = 10 \text{ m/s}$

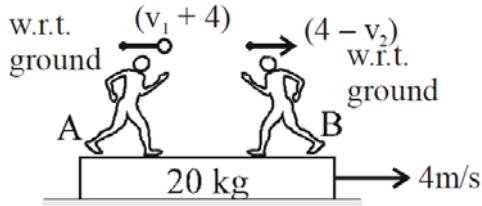
(B) $v_1 = 5 \text{ m/s}$, $v_2 = 15 \text{ m/s}$

(C) $v_1 = 10 \text{ m/s}$, $v_2 = 20 \text{ m/s}$

(D) $v_1 = 2 \text{ m/s}$, $v_2 = 12 \text{ m/s}$

Ans. (ABCD)

Sol. Velocity of plank w.r.t. ground = $\frac{20}{5} = 4 \text{ m/s}$ right.



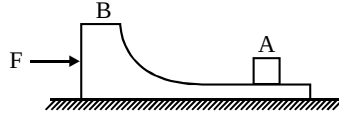
$$P_f = 0$$

$$\Rightarrow 40(v_1 + 4) + 40(4 - v_2) + 20 \times 4 = 0$$

$$\Rightarrow 40(v_1 - v_2) + 320 + 80 = 0$$

$$\Rightarrow v_1 - v_2 = -10$$

39. Two blocks A & B are placed on a smooth surface as shown, horizontal force F is applied on B when A & B are initially at rest. Friction between A & B is absent (in subsequent motion):



- (A) Kinetic energy of system of A & B increases
(B) Momentum of the system of A & B remains constant in frame of center of mass of A & B
(C) Kinetic energy of system of A & B remain constant in frame of center of mass of A & B
(D) Kinetic energy of A remains constant in B frame

Ans. (AB)

Sol. $\because$ External force does some positive work.

$\Rightarrow$ KE of system increases.

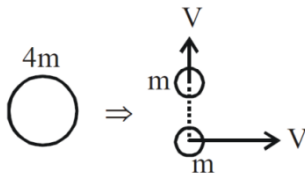
$\because$ In com frame, force on system is zero.

$\Rightarrow$ Momentum conservation in com frame.

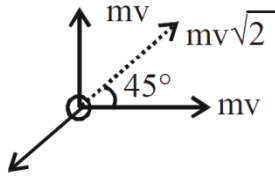
40. A blast breaks a body initially at rest of mass 0.5 kg into three pieces, two smaller pieces of equal mass and the third double the mass of either of small piece. After the blast the two smaller masses move at right angles to one another with equal speed. Find the statements that is/are true for this case assuming that the energy of blast is totally transferred to masses.

- (A) All the three pieces share the energy of blast equally
(B) The speed of bigger mass is $\sqrt{2}$ times the speed of either of the smaller mass
(C) The direction of motion of bigger mass makes an angle of 135° with the direction of smaller pieces
(D) The bigger piece carries double the energy of either piece.

Ans. (AC)



Sol.



In order to make

$$P_{\text{system}} = 0$$

Bigger particle

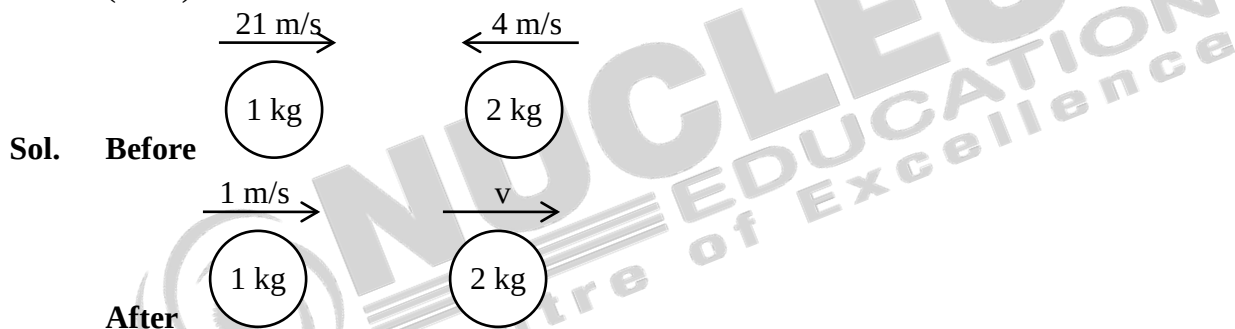
moves opposite to $mv\sqrt{2}$

$$\therefore 2mv_2 = mv\sqrt{2}$$

41. Two balls A and B having masses 1 kg and 2 kg, moving with speeds 21 m/s and 4 m/s respectively in opposite direction, collide head on. After collision A moves with a speed of 1 m/s in the same direction, then correct statements is :

- (A) The velocity of B after collision is 6 m/s opposite to its direction of motion before collision.
(B) The coefficient of restitution is 0.2.
(C) The loss of kinetic energy due to collision is 200 J.
(D) The impulse of the force between the two balls is 40 Ns.

Ans. (ABC)



Momentum conservation

$$1 \times 21 + 2(-4) = 1 \times 1 + 2 \times v$$

$$13 = 1 + 2v$$

$$v = 6 \text{ m/s}$$

$$e = \frac{v-1}{21+4} = \frac{6-1}{25} = \frac{1}{5}$$

$$\text{loss in KE} = KE_i - KE_{\text{final}}$$

$$\text{Impulse} = \text{Change in momentum}$$

$$= (1 \times 1) - 1 \times (21)$$

$$= -20 \text{ (m/s) kg}$$

42. In an inelastic collision,
(A) the velocity of both the particles may be same after the collision
(B) kinetic energy is not conserved
(C) linear momentum of the system is conserved.
(D) velocity of separation will be less than velocity of approach.

Ans. (ABCD)

Sol. If $e = 0 \Rightarrow$ velocity of both object will be same.

In inelastic collision, KE is lost.

$$e = \frac{v_{\text{sep}}}{v_{\text{app}}} < 1 \text{ (for inelastic)}$$

$$v_{\text{sep}} < v_{\text{app}}$$

43. In a one-dimensional collision between two particles, their relative velocity is $\vec{v}_1$ before the collision and $\vec{v}_2$ after the collision

(A) $\vec{v}_1 = \vec{v}_2$ if the collision is elastic

(B) $\vec{v}_1 = -\vec{v}_2$ if the collision is elastic

(C) $|\vec{v}_2| = |\vec{v}_1|$ in all cases

(D) $\vec{v}_1 = -k\vec{v}_2$ in all cases, where $k \geq 1$

Ans. (BD)

Sol.
$$e = -\frac{\vec{v}_2}{\vec{v}_1}$$

$$\vec{v}_2 = -e\vec{v}_1$$

$$\Rightarrow \vec{v}_1 = \left(\frac{-1}{e}\right)\vec{v}_2$$

$$\therefore e \leq 1$$

$$\Rightarrow \frac{1}{e} \geq 1$$

44. A particle moving with kinetic energy = 3 joule makes an elastic head on collision with a stationary particle which has twice its mass during the impact.

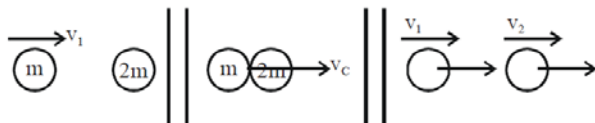
(A) The minimum kinetic energy of the system is 1 joule.

(B) The maximum elastic potential energy of the system is 2 joule.

(C) Momentum and total kinetic energy of the system are conserved at every instant.

(D) The ratio of kinetic energy to potential energy of the system first decreases and then increases.

Ans. (ABD)



Sol.

Maximum compression (RE $\Rightarrow$ Max $\Rightarrow$ KE $\Rightarrow$ Min)

$$3mv_C = mv_1$$

$$\Rightarrow v_C = \frac{v_1}{3}$$

$$\frac{1}{2}mv_1^2 = 3, (KE)_{\min} = \frac{1}{2} \cdot 3mv_C^2 = \frac{3}{2}m\left(\frac{v_1}{3}\right)^2 = \frac{1}{6}mv_1^2 = \frac{1}{3} \times \left(\frac{1}{2}mv_1^2\right) = \frac{1}{3} \times 3$$

$$\Rightarrow (KE)_{\min} = 1\text{J}$$

$$(PE)_{\max} = 3 - 1 = 2J$$

(C) $P \Rightarrow$ Always conserved

KE $\Rightarrow$ During collision changes

$$PE \uparrow \text{ \& } KE \downarrow \Rightarrow \left(\frac{PE}{KE} \right) \uparrow \Rightarrow \frac{KE}{PE} \downarrow$$

45. In a one dimensional collision between two identical particles A and B, B is stationary and A has momentum p before impact. During impact, B gives impulse J to A.

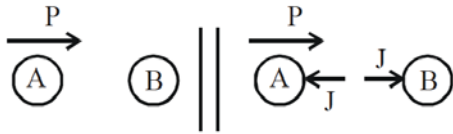
(A) The total momentum of the 'A plus B' system is p before and after the impact, and $(p-J)$ during the impact.

(B) During the impact A gives impulse of magnitude J to B

(C) The coefficient of restitution is $\frac{2J}{p} - 1$

(D) The coefficient of restitution is $\frac{J}{p} + 1$

Ans. (BC)



Sol.

(A) $P_{\text{system}} = P - J + J = P$

(B) Impulse given to B = J

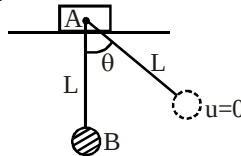
(C) $v_1 = P/m$, $v_2 = 0$, $v_1' = \frac{P-J}{m}$, $v_2' = \frac{J}{m}$

$$m_A = m_B = m$$

$$e = \frac{v_2' - v_1'}{v_1 - v_2} = \frac{J/m - (P-J)/m}{P/m} = \frac{2J - P}{P} = \frac{2J}{P} - 1$$

Paragraph for Question 46 to 48

A small ball B of mass m is suspended with light inelastic string of length L from a block A of same mass m which can move on smooth horizontal surface as shown in the figure. The ball is displaced by angle θ from lowest position & then released.

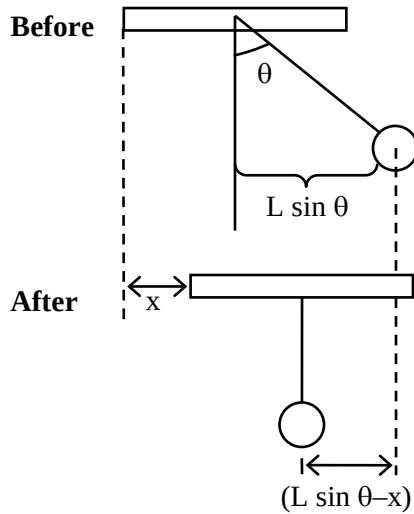


46. The displacement of block when ball reaches the lowest position is

- (A) $\frac{L \sin \theta}{2}$ (B) $L \sin \theta$ (C) $L \cos \theta$ (D) $\frac{L \cos \theta}{2}$

Ans. (A)

Sol. $S_{\text{cm}} = 0$ [Horizontal direction]



$$m_1 x_1 = m_2 x_2$$

$$mx = m(L \sin \theta - x)$$

$$x = \frac{L \sin \theta}{2}$$

47. Maximum velocity of block during subsequent motion of the system after release of ball is

(A) $[gL(1 - \cos \theta)]^{1/2}$

(B) $[2gL(1 - \cos \theta)]^{1/2}$

(C) $[gL \cos \theta]^{1/2}$

(D) informations are insufficient to decide

Ans. (A)

Sol. Max velocity of system when the ball reaches its lowest point of motion.

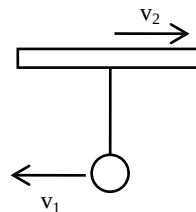
$$mv_1 = mv_2$$

$$\Rightarrow v_1 = v_2$$

$$\text{Also } \Delta KE = W_{\text{All}}$$

$$\left(\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \right) - 0 = mgL(1 - \cos \theta)$$

$$v_1 = \sqrt{gL(1 - \cos \theta)}$$



48. The displacement of centre of mass of A + B system till the string becomes vertical is

(A) zero

(B) $L(1 - \cos \theta)$

(C) $\frac{L}{2}(1 - \sin \theta)$

(D) $\frac{L}{2}(1 - \cos \theta)$

Ans. (D)

Sol. COM does not move in horizontal.

It moves in vertical.

$$(S_{\text{um}})_y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{m(L(1 - \cos \theta)) + m(0)}{2m} = \frac{L}{2}(1 - \cos \theta)$$

Paragraph for Question Nos. 49 to 50

A uniform chain of length $2L$ is hanging in equilibrium position, if end B is given a slightly downward displacement the imbalance causes an acceleration. Here pulley is small and smooth & string is inextensible

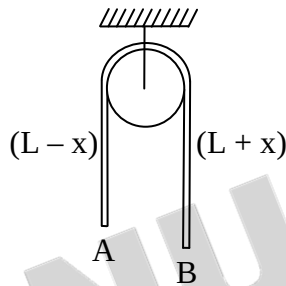


49. The acceleration of end B when it has been displaced by distance x , is

- (A) $\frac{x}{L}g$ (B) $\frac{2x}{L}g$ (C) $\frac{x}{2L}g$ (D) g

Ans. (A)

Sol. $a = \left(\frac{2x}{2L}\right)g = \frac{x}{L}g$



50. The velocity v of the chain when it slips out of the pulley (height of pulley from floor $> 2L$)

- (A) $\sqrt{\frac{gL}{2}}$ (B) $\sqrt{2gL}$ (C) $\sqrt{gL}$ (D) None of these

Ans. (C)

Sol. $\frac{v dv}{dx} = \frac{x}{L}g$

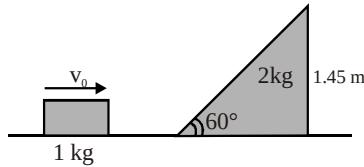
$$\int_0^v v dv = \frac{g}{L} \int_0^L x dx$$

$$\frac{v^2}{2} = \frac{g}{L} \left(\frac{L^2}{2} \right)$$

$$v = \sqrt{gL}$$

Paragraph for Question Nos. 51 to 53

A block of mass 1 kg is moving towards a movable wedge of mass 2 kg as shown in figure. All surfaces are smooth. When the block leaves the wedge from top, its velocity is making an angle $\theta = 30^\circ$ with horizontal.

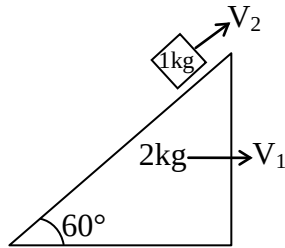


51. The value of v_0 in m/s is

- (A) 4 (B) 7 (C) 10 (D) 9

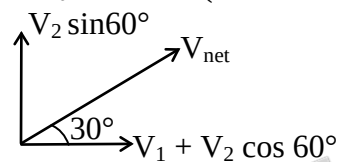
Ans. (B)

Sol.



using momentum conservation in horizontal direction.

$$1.V_0 = 2V_1 + 1(V_2 \cos 60^\circ + V_1) \quad \dots(i)$$



$$\tan 30^\circ = \frac{V_2 \sin 60^\circ}{V_1 + V_2 \cos 60^\circ}$$

$$\frac{1}{\sqrt{3}} = \frac{V_2 \times \frac{\sqrt{3}}{2}}{V_1 + \frac{V_2}{2}} \Rightarrow \frac{2V_1 + V_2}{2} = \frac{3V_2}{2}$$

$$2V_2 = 2V_1$$

$$V_1 = V_2 \quad \dots(ii)$$

equation (i) can be written as.

$$V_0 = 2V_1 + \left(\frac{V_1}{2} + V_1 \right)$$

$$V_0 = 2V_1 + \frac{3V_1}{2} = \frac{7V_1}{2} \Rightarrow V_1 = \frac{2V_0}{7} \quad \dots(3)$$

Using energy conservation.

$$\frac{1}{2} \times 1 \times V_0^2 = \frac{1}{2} \times 2 \times V_1^2 + \frac{1}{2} \times 1 \times (V_1^2 + V_2^2 + 2V_1V_2 \cos 60^\circ) + 1 \times g \times (1.45)$$

$$\frac{1}{2} \times 1 \times V_0^2 = V_1^2 + \frac{1}{2} (3V_1^2) + 14.5$$

$$\frac{1}{2} V_0^2 = \frac{5}{2} \left(\frac{2V_0}{7} \right)^2 + 14.5$$

$$\frac{1}{2} V_0^2 - \frac{5}{2} \times \frac{4}{49} V_0^2 = 14.5$$

$$\frac{1}{2} \left(\frac{29V_0^2}{49} \right) = 14.5$$

$$V_0^2 = \frac{14.5 \times 49 \times 2}{29} = 49$$

$$V_0 = 7 \text{ m/s.}$$

52. To what maximum height will the block rise:

- (A) 1.9 m (B) 2.7 m (C) 1.6 m (D) 1.45 m

Ans. (C)

Sol. for maximum height block and wedge move together.

$$V = \frac{V_0}{3} = \frac{7}{3} \text{ m/s}$$

$$\frac{1}{2} \times 1 \times V_0^2 = \frac{1}{2} \times 3 \times \left(\frac{7}{3} \right)^2 + 1 \times g \times h$$

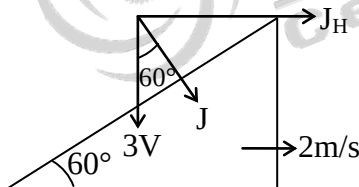
$$gh = \frac{49}{2} - \frac{49}{6} = \frac{49}{3} \Rightarrow h = \frac{49}{30} \text{ m} = 1.633 \text{ m.}$$

53. In the whole process let J be the magnitude of net impulse given to the block by the wedge, J_H its horizontal component and J_V its vertical component. Then :

- (A) $J = \frac{8}{\sqrt{3}} \text{ N-s}$ (B) $J_H = 4 \text{ N-s}$ (C) $J_V = \frac{4}{\sqrt{3}} \text{ N-s}$ (D) All of these

Ans. (D)

Sol.



$$J_H = 2 \times 2 = 4 \text{ N-s}$$

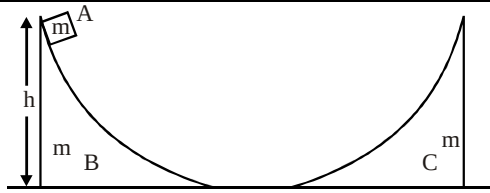
$$J \sin 60 = J_H = 4$$

$$J = \frac{8}{V_3} \text{ N-s}$$

$$J_V = J \cos 60 = \frac{8}{\sqrt{3}} \times \frac{1}{2} = \frac{4}{\sqrt{3}} \text{ N-s}$$

Paragraph for Question Nos. 54 to 56

Block A is placed on wedge B at a height h above ground. Block and the two wedges are all of same mass m . Neglect friction every where :



54. Find velocity of B when A has slid down from it :

(A) $\sqrt{gh}$

(B) $\sqrt{\frac{gh}{2}}$

(C) $\frac{\sqrt{gh}}{2}$

(D) None of these

Ans. (A)

Sol. By conservation of linear momentum in horizontal direction

$$0 = m\vec{V}_A + m\vec{V}_B = 0$$

$$\vec{V}_A = -\vec{V}_B$$

By conservation of mechanical energy

$$mgh = \frac{1}{2}mV_A^2 + \frac{1}{2}mV_B^2$$

$$2gh = 2V_B^2$$

$$V_B = \sqrt{gh}$$

55. Find maximum height upto which block A rises on wedge C :

(A) h

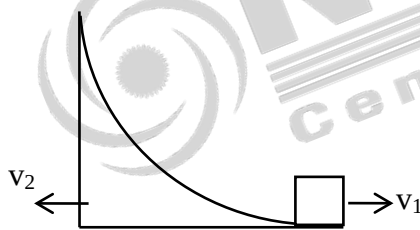
(B) $\frac{h}{2}$

(C) $\frac{h}{4}$

(D) None of these

Ans. (C)

Sol. Momentum conservation



$$mv_1 = mv_2$$

$$v_1 = v_2$$

Also Work Energy Theorem

$$mgh = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$mgh = \frac{1}{2}mv_2^2 + \frac{1}{2}mv_2^2$$

$$\Rightarrow v_2 = \sqrt{gh_1}$$

56. Find velocity of A when it has slid down to ground from wedge C :

(A) 0

(B) $\sqrt{\frac{gh}{2}}$

(C) $\sqrt{\frac{gh}{4}}$

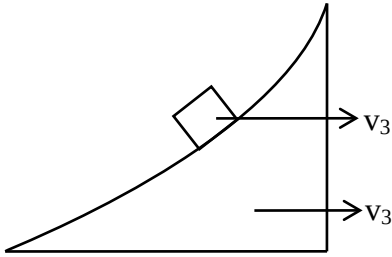
(D) None of these

Ans. (A)

Sol. At max height velocity of A w.r.t.

$$C = 0$$

$\Rightarrow$ A and C will have some velocity.



Momentum conservation

$$mv_1 = mv_3 + mv_3$$

$$\Rightarrow v_3 = \frac{v_1}{2} = \frac{\sqrt{gh}}{2} \quad \text{Also Work Energy Theorem}$$

$$mgh = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$mgh = \frac{1}{2}mv_2^2 + \frac{1}{2}mv_2^2$$

$$\Rightarrow v_2 = \sqrt{gh_1}$$

W.E.T.

$$\Delta KE = W_{All}$$

$$\left(\frac{1}{2}mv_3^2 + \frac{1}{2}mv_3^2 \right) - \frac{1}{2}mv_1^2 = -mgh_1$$

$$mv_3^2 - \frac{1}{2}mv_1^2 = -mgh_1$$

$$mv_3^2 - \frac{1}{2}mv_1^2 = -mgh_1$$

$$\frac{gh}{4} = -\frac{gh}{2} = -gh_1$$

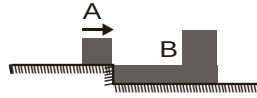
$$\frac{-gh}{4} = -gh_1$$

$$\frac{-gh}{4} = -gh_1$$

Paragraph for Question Nos. 57 to 59

As shown, a block (Object-A) of mass m_A moves on a frictionless plane at initial speed v_0 , and lands onto a cart (Object-B in the figure, mass m_B , length L , initially at rest) smoothly.

$$[m_A = m_B = m]$$

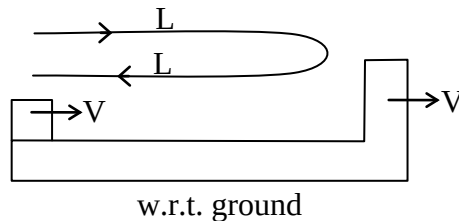


Ignore the size of the block. The friction coefficient between A and B is μ and the cart is on a frictionless plane. The block collides elastically on the fixed wall w.r.t. cart at the end of the cart and eventually falls off the cart.

57. Find the minimum value of v_0 such that the block can indeed fall off the cart.
(A) $\sqrt{2\mu g\ell}$ (B) $3\sqrt{2\mu g\ell}$ (C) $7\sqrt{2\mu g\ell}$ (D) $2\sqrt{2\mu g\ell}$

Ans. (D)

Sol. At the time of fall block A come to rest w.r.t. cart B. And Both moves with same velocity V wrt. ground.



Linear momentum conservation

$$m_A V_0 + 0 = (m_A + m_B) V$$

$$V = \frac{mV_0}{m+m} = \frac{V_0}{2}$$

Collision is elastic so no energy loss during collision

$$\mu mg (2L) = \frac{1}{2} m V_0^2 - \left[\frac{1}{2} (2m) \left(\frac{V_0}{2} \right)^2 \right]$$

$$V_0 = 2 \sqrt{2\mu g\ell}$$

58. Suppose the initial speed is larger than the minimum value in (Q. 57), then loss in the kinetic energy of the system is

- (A) $\mu mg\ell$ (B) $2\mu mg\ell$ (C) $4\mu mg\ell$ (D) $9\mu mg\ell$

Ans. (B)

Sol. Loss in kinetic energy = work done by kinetic friction.

$$\begin{aligned} KE_{\text{Loss}} &= \mu mg (2L) \\ &= 2\mu mgL \end{aligned}$$

59. If the initial speed is less than the minimum value in (Q. 57), find the time the block spends on the cart.

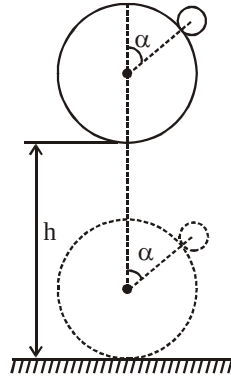
- (A) $2\ell/v_0$ (B) $8\ell/v_0$ (C) $3\ell/v_0$ (D) infinite

Ans. (D)

Sol. Block A will not fall and Both move with same velocity $V = \frac{V_0}{2}$ (wrt. ground) for infinite time.

Paragraph for Question Nos. 60 & 61

A large heavy sphere and a small light sphere are dropped together onto a flat surface from a height h . The radius of spheres is much smaller than height h . The large sphere collides with the surface with velocity v_0 and immediately thereafter with the small sphere. The spheres are dropped so that all motion is vertical before the second collision, and the small sphere hits the larger sphere at an angle α from its uppermost point, as shown in the diagram. All collisions are perfectly elastic and there is no surface friction between the spheres.

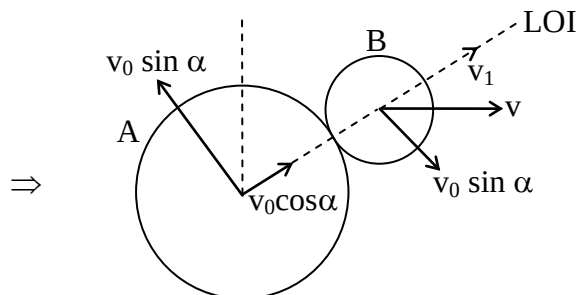
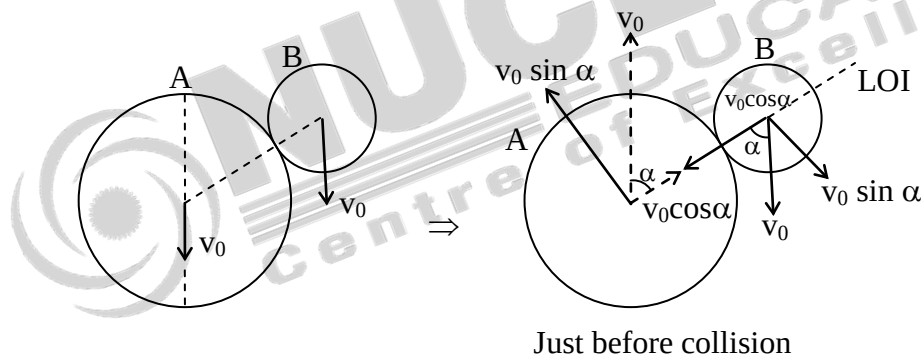


60. The angle made by velocity vector of small sphere with the vertical just after the second collision in the frame of large sphere is.

(A) α (B) 2α (C) $3\alpha/2$ (D) zero

Ans. (B)

Sol.

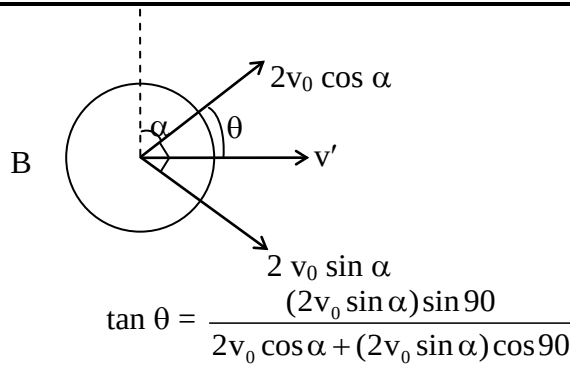


Along the line of impact

$$I = \frac{v_1 - v_0 \cos \alpha}{v_0 \cos \alpha + v_0 \cos \alpha}$$

$$v_1 = 3v_0 \cos \alpha \quad \dots(1)$$

So in the frame of large sphere



$$= \frac{2v_0 \sin \alpha}{2v_0 \cos \alpha}$$

$$\tan \theta = \tan \theta$$

$$\theta = \alpha$$

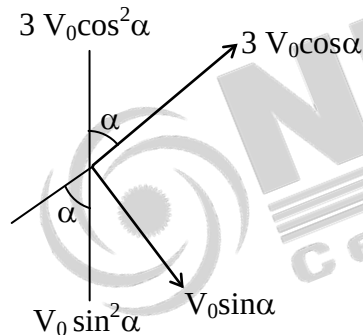
$$\text{from vertical, angle of } v' \Rightarrow \alpha + \theta = 2\alpha.$$

61. Find the vertical velocity of smaller sphere just after the collision with respect to ground.

- (A) $2v_0 \cos \alpha$ (B) $2v_0 \cos 2\alpha + v_0$ (C) $v_0 \cos \alpha + v_0$ (D) $v_0 \cos 2\alpha + v_0$

Ans. (B)

Sol.



Net vertical velocity

$$= 3V_0 \cos^2 \alpha - V_0 \sin^2 \alpha$$

$$= V_0 \cos^2 \alpha + 2V_0 \cos^2 \alpha - 2V_0 \sin^2 \alpha + V_0 \sin^2 \alpha$$

$$= V_0 + 2V_0 (\cos^2 \alpha - \sin^2 \alpha)$$

$$= 2V_0 \cos 2\alpha + V_0$$

MATRIX MATCH TYPE QUESTIONS

62. A particle of mass m , kinetic energy K and momentum p collides head on elastically with another particle of mass $2m$ at rest. After collision :

Column-I

- (A) Momentum of first particle
(B) Momentum of second particle
(C) Kinetic energy of first particle
(D) Kinetic energy of second particle

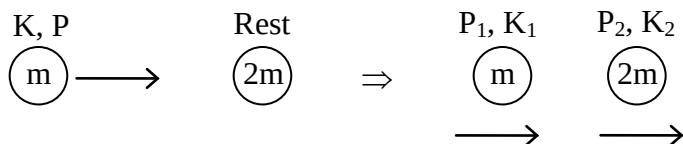
Column-II

- (P) $\frac{3}{4}p$
(Q) $-\frac{K}{9}$
(R) $-\frac{p}{3}$
(S) $\frac{8K}{9}$

(T) None

Ans. (A)→(R); (B)→(T); (C)→(T); (D)→(S)

Sol. $P = \sqrt{2mK} \Rightarrow K = \frac{P^2}{2m} \dots\dots(1)$



Linear momentum conservation

$$m \left(\frac{P}{m} \right) + 0 = m \left(\frac{P_1}{m} \right) + 2m \left(\frac{P_2}{2m} \right)$$

$$\Rightarrow P = P_1 + P_2 \dots\dots(2)$$

By Newton law of collision

$$\frac{P}{m} = \frac{P_2}{2m} - \frac{P_1}{m}$$

$$P = \frac{P_2}{2} - P_1 \dots\dots(3)$$

from (2) & (3)

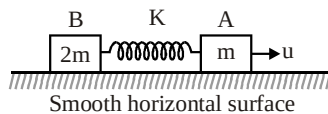
$$P_2 = \frac{4P}{3}$$

$$P_1 = -\frac{P}{3}$$

And $K_1 = \frac{P_1^2}{2m} = \frac{1}{9} \left(\frac{P^2}{2m} \right) = \frac{K}{9}$

$$K_2 = \frac{P_2^2}{2(2m)} = \frac{8}{9} \left(\frac{P^2}{2m} \right) = \frac{8K}{9}$$

- 63.** Two blocks A and B of mass m and $2m$ respectively are connected by a massless spring of spring constant K . This system lies over a smooth horizontal surface. At $t = 0$ the block A has velocity u towards right as shown while the speed of block B is zero, and the length of spring is equal to its natural length at that instant. In each situation of column-I, certain statements are given and corresponding results are given in column II. Match the statements in column I corresponding results in column II.



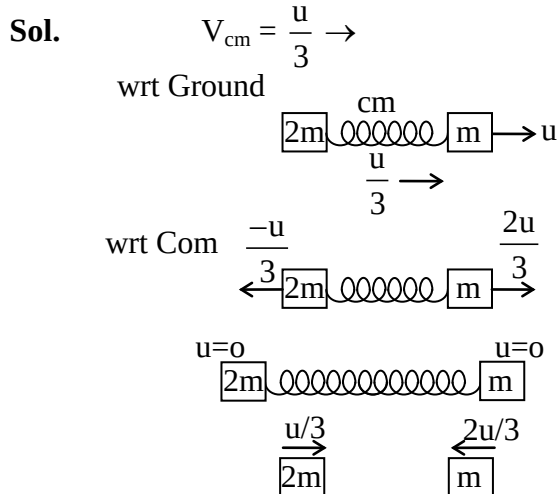
Column I

- (A) The velocity of block A
(B) The velocity of block B
(C) The kinetic energy of system of two blocks
(D) The potential energy of spring

Column II

- (P) Can never be zero
(Q) may be zero at certain instants of time
(R) is minimum at maximum compression of spring
(S) is maximum at maximum extension of Spring

Ans. (A)→(Q); (B)→(Q); (C)→(P,R); (D)→(Q,S)



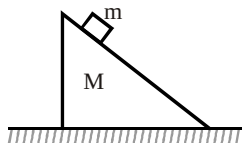
Block B	Block A
Velocity $V_{min} = 0$	Velocity $V_{min} = \frac{-u}{3}$
Velocity $V_{man} = \frac{2u}{3}$	Velocity $V_{man} = u$

Initial momentum = mu , momentum is conserved, so both block will never come to rest simultaneously, So KE can never be zero of system.

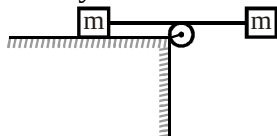
64. In each situation of column-I, a system involving two bodies given. All the strings and pulleys are light and friction is absent every where. Initially each body of every system is at rest. Consider the system in all situation of column I from rest till any collision occurs. Then match the statement in column-I with the corresponding results in column-II.

Column I

- (A) The block plus wedge system is placed over smooth horizontal surface. After the system is released from rest, the centre of mass of system



- (B) The string connecting both the blocks of mass m is horizontal. Left block is placed over smooth horizontal table as shown. After the two block system is released from rest, the centre of mass of system.



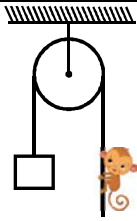
- (C) The block and monkey have same mass. The monkey starts climbing up the rope. After the monkey starts climbing up the centre of mass of monkey + block system.

Column II

- (P) Shift towards right

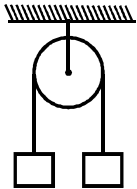
- (Q) Shift downwards

- (R) Shift upwards



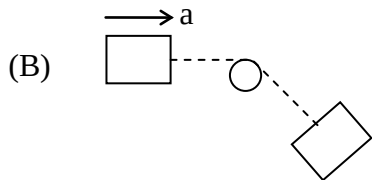
- (D) Both block of mass m are initially at rest. The left block is given initial velocity u downwards, then the centre of mass of two block system afterwards.

- (S) Does not shift

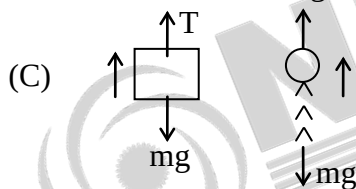


Ans. (A) $\rightarrow$ Q; (B) $\rightarrow$ PQ; (C) $\rightarrow$ R; (D) $\rightarrow$ S;

Sol. (A) F_{net} is in downward direction so, com of system will shift in downward direction.



com will shift rightward and downward.

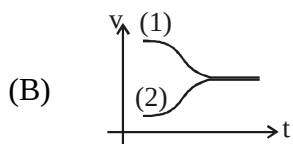
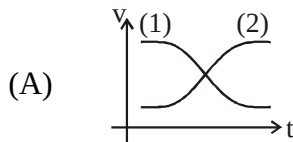


If monkey move in upward direction block will also move in upward direction.

- (D) Since initially both block is at rest, it will remain at rest ($a = 0$), Hence com will not move.

65. Assume that 2 bodies collide head on. The graph of their velocities with time are shown in column-I match them with appropriate situation in column-II

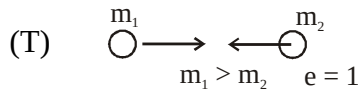
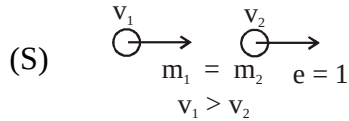
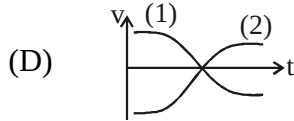
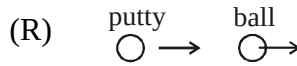
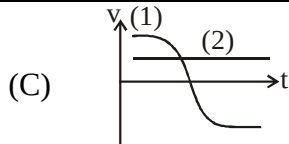
Column-I



Column-II

- (P) $m_1 < m_2 \quad 0 < e < 1$

- (Q) m_1 $\rightarrow$ wall $\leftarrow$ 2nd body is large



Ans. (A)→S; (B)→R; (C)→Q; (D)→P

Sol. (P) $0 < e < 1$

Initially, if we take velocity of m_1 is positive, velocity of m_2 will be negative.
final kinetic energy after collision will less than total initial kinetic energy.

(Q) Velocity of wall remain constant.

(R) Putty sticks to ball and finally they move together.

(S) After collision particle will exchange their velocities.

EXERCISE # (JM)

1. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass M and, a body C of mass M. The centre of mass of bodies B and C taken together shifts compared to that of body A towards- [AIEEE - 2005]
- (1) depends on height of breaking (2) does not shift
(3) body C (4) body B

Ans. (2)

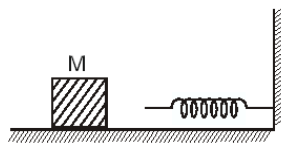
Sol. No horizontal external force is acting, Hence there will be no shift.

$$a_{cm} = 0$$

$$\text{since } v_{cm} = 0$$

$$\text{Hence } x_{cm} = 0$$

2. The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant k and compresses it by length L. The maximum momentum of the block after collision is- [AIEEE - 2005]



- (1) $\sqrt{Mk}L$ (2) $\frac{kL^2}{2M}$ (3) zero (4) $\frac{ML^2}{k}$

Ans. (1)

Sol. If only conservative forces act on a system, total mechanical energy remains constant –

$$K + U = E(\text{constant})$$

$$\Delta K + \Delta U = 0$$

$$\Delta K = -\Delta U$$

$$\text{Let initial velocity of block is } v \text{ then initial energy} = \frac{1}{2}mv^2$$

$$\text{Final potential energy} = \frac{1}{2}kx^2$$

From energy conservation

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$\therefore v = \sqrt{\frac{k}{m}} \cdot x \quad \because x = L$$

$$\Rightarrow \text{maximum } v = \left(\frac{\sqrt{k}}{m} \right) L$$

Therefore maximum momentum = mv

$$= \sqrt{mk}L$$

3. A bomb of mass 16 kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is 4 ms^{-1} . The kinetic energy of the other mass is- [AIEEE - 2006]
(1) 144 J (2) 288 J (3) 192 J (4) 96 J

Ans. (2)

Sol. $m_1 v_1 = m_2 v_2$

$$v_2 = \frac{12 \times 4}{4} \text{ m/s} = 12 \text{ m/s}$$

$$\text{K.E} = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \times 4 \times 12 \times 12 = 288 \text{ J}$$

4. Consider a two particle system with particles having masses m_1 and m_2 . If the first particle is pushed towards the centre of mass through a distance d , by what distance should the second particle be moved, so as to keep the centre of mass at the same position ? [AIEEE - 2006]

- (1) $\frac{m_2}{m_1} d$ (2) $\frac{m_1}{m_1 + m_2} d$ (3) $\frac{m_1}{m_2} d$ (4) d

Ans. (3)

Sol. $m_1 d_1 = m_2 d_2 \Rightarrow d_2 = \frac{m_1}{m_2} d$

5. A circular disc of radius R is removed from a bigger circular disc of radius $2R$ such that the circumferences of the discs coincide. The centre of mass of the new disc is αR from the centre of the bigger disc. The value of α is :- [AIEEE - 2007]

- (1) $1/3$ (2) $1/2$ (3) $1/6$ (4) $1/4$

Ans. (1)

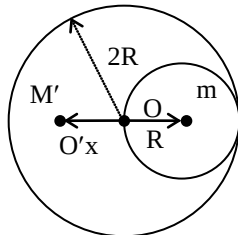
Sol. $(M + m) = M = \pi(2R)^2 \cdot \sigma$ (where σ = mass per unit area)

$$m = \sigma R^2 \cdot \sigma, M = 3\pi R^2 \sigma$$

$$\frac{3\pi R^2 \sigma \cdot x + \pi R^2 \sigma \cdot R}{M} = 0$$

Because for the full disc, the centre of mass is at the centre O.

$$\Rightarrow x = -\frac{R}{3} = \alpha R \therefore |\alpha| = \left| \frac{-1}{3} \right|$$



The centre of mass is at $R/3$ to the left on the diameter of the original disc.

6. A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is:- [AIEEE - 2008]

- (1) 0.16 J (2) 1.00 J (3) 0.67 J (4) 0.34 J

Ans. (3)

Sol. $m_1 u_1 + m_2 u_2 = (m_1 + m_2)v$
 $v = 2/3 \text{ m/s}$

$$\text{Energy loss} = \frac{1}{2}(0.5) \times (2)^2 - \frac{1}{2}(1.5) \times \left(\frac{2}{3}\right)^2$$

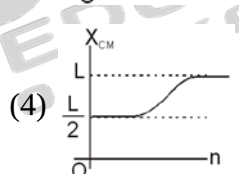
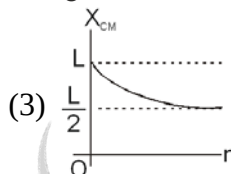
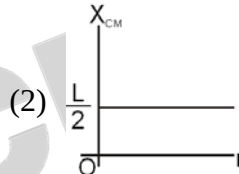
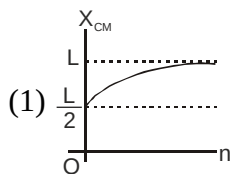
$$= 0.67 \text{ J}$$

$m_1 u_1 + m_2 u_2 = (m_1 + m_2)v$
 $v = 2/3 \text{ m/s}$

$$\text{Energy loss} = \frac{1}{2}(0.5) \times (2)^2 - \frac{1}{2}(1.5) \times \left(\frac{2}{3}\right)^2$$

$$= 0.67 \text{ J}$$

7. A thin rod of length 'L' is lying along the x-axis with its ends at $x = 0$ and $x = L$. Its linear density (mass/length) varies with x as $k\left(\frac{x}{L}\right)^n$ where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against 'n', which of the following graphs best approximates the dependence of x_{CM} on n ? [AIEEE - 2008]



Ans.

(3)

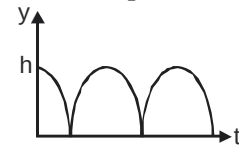
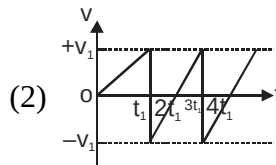
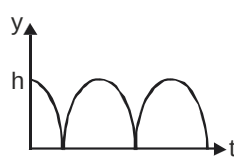
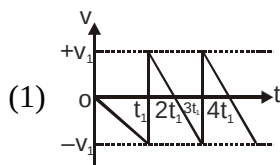
Sol.
$$x_{CM} = \frac{\int x dm}{\int dm} = \frac{\int_0^L x k \left(\frac{x}{L}\right)^n dx}{\int_0^L k \left(\frac{x}{L}\right)^n dx} = \frac{(n+1)L}{(n+2)}$$

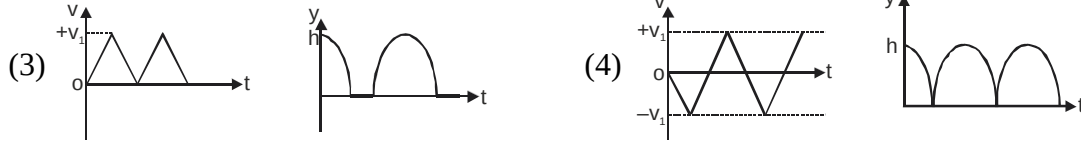
$n = 0, x_{cm} = \frac{L}{2}$ and if

$n \rightarrow \infty x_{cm} = L$

8. Consider a rubber ball freely falling from a height $h = 4.9 \text{ m}$ onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time and the height as a function of time will be :

[AIEEE - 2009]





Ans. (1)

Sol. Slope of velocity-time graph will remain constant except during collision where it will not be defined. Also variation of height with time will be parabolic.

9. **Directions :** Question number 9 contain Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1 : Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.

[AIEEE - 2010]

- (1) Statement-1 is true, Statement-2 is false
- (2) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1
- (3) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1
- (4) Statement-1 is false, Statement-2 is true

Ans. (2)

Sol. By conservation of momentum, magnitude of momentum can not become zero.
So kinetic energy can not be lost completely

10. This question has Statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes the two Statements. [JEE Mains-2013]

Statement - I : A point particle of mass m moving with speed v collides with stationary point particle of mass M . If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f = \left(\frac{m}{M+m}\right)$.

Statement - II : Maximum energy loss occurs when the particles get stuck together as a result of the collision.

- (1) Statement-I is true, Statement-II is true, Statement-II is a correct explanation of Statement-I.
- (2) Statement-I is true, Statement-II is true, Statement-II is a not correct explanation of Statement-I.
- (3) Statement-I is true, Statement-II is false.
- (4) Statement-I is false, Statement-II is true.

Ans. (4)

Sol. Maximum energy loss $= \frac{p^2}{2m} - \frac{p^2}{2(m+M)}$

$$= \frac{p^2}{2m} \left[\frac{M}{(m+M)} \right] = \frac{1}{2}mv^2 \left\{ \frac{M}{m+M} \right\} \quad \left(f = \frac{M}{m+M} \right)$$

Hence Statement -1 is wrong and statement 2 is correct Hence

Sol. Maximum energy loss = $\frac{p^2}{2m} - \frac{p^2}{2(m+M)}$

$$\frac{p^2}{2m} \left[\frac{M}{(m+M)} \right] = \frac{1}{2}mv^2 \left\{ \frac{M}{m+M} \right\} \quad \left(f = \frac{M}{m+M} \right)$$

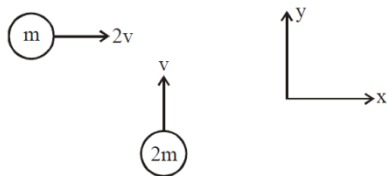
Hence statement-1 is wrong and statement-2 is correct.

- 11.** A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y direction with speed v . If the collisions perfectly inelastic, the percentage loss in the energy during the collision is close to : **[JEE Mains-2015]**

- (1) 56 % (2) 62% (3) 44% (4) 50%

Ans. (1)

Sol. Before collision



Kinetic energy = $\frac{1}{2}m(2v)^2 + \frac{1}{2}2m(v)^2 = 3mv^2$

After collision

Applying momentum conservation for inelastic collision

$$2m\hat{j} + m2v\hat{i} = 3m\vec{v}_f$$

$$|\vec{v}_f| = \sqrt{\frac{8}{9}}v$$

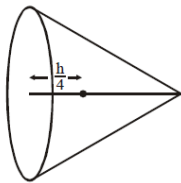
$$K_f = \frac{1}{2} \times 3m \times (v_f^2) = \frac{4mv^2}{3}$$

$$\% \Delta K = \frac{K_i - K_f}{K_i} = \frac{5mv^2/3}{3mv^2} = \frac{5}{9} = 56\%$$

- 12.** Distance of the centre of mass of a solid uniform cone from its vertex is z_0 . If the radius of its base is R and its height is h then z_0 is equal to :- **[JEE Mains-2015]**

- (1) $\frac{5h}{8}$ (2) $\frac{3h^2}{8R}$ (3) $\frac{h^2}{4R}$ (4) $\frac{3h}{4}$

Ans. (4)



Sol.

for solid cone c.m. is $\frac{h}{4}$ from base

$$\text{so } z_0 = h - \frac{h}{4} = \frac{3h}{4}$$

13. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision is :

[JEE Mains-2018]

- (1) $\frac{v_0}{\sqrt{2}}$ (2) $\frac{v_0}{4}$ (3) $\sqrt{2}v_0$ (4) $\frac{v_0}{2}$

Ans. (3)

Sol. By conservation of linear momentum

$$mv_0 + 0 = mv_1 + mv_2$$

$$v_0 = v_1 + v_2 \quad \dots(1)$$

$$\frac{3}{2} \left[\frac{1}{2} mv_0^2 \right] = \frac{1}{2} mv_1^2 + \frac{1}{2} mv_2^2$$

$$\frac{3}{2} v_0^2 = v_1^2 + v_2^2 \quad \dots(2)$$

Solving equation (1) and (2)

$$v_1 = \frac{v_0}{2} (1 + \sqrt{2})$$

$$v_2 = \frac{v_0}{2} (1 - \sqrt{2})$$

$$\vec{v}_{\text{rel}} = \vec{v}_1 - \vec{v}_2$$

$$\frac{v_0}{2} [1 + \sqrt{2} - 1 + \sqrt{2}] = \frac{v_0}{2} \times 2\sqrt{2} = \sqrt{2}v_0$$

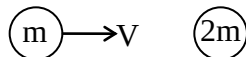
14. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d ; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c . The values of p_d and p_c are respectively : [JEE-Main - 2018]

- (1) (0, 1) (2) (.89, .28) (3) (.28, .89) (4) (0, 0)

Ans. (2)

Sol. Case-I

JBC



JAC



$$2V_2 - V_1 = V$$

$$V_2 + V_1 = V$$

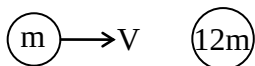
$$3V_2 = 2V$$

$$V_2 = \frac{2V}{3}$$

$$P_d = \frac{\frac{1}{2}mV^2 - \frac{1}{2}mV_1^2}{\frac{1}{2}mV^2} = \frac{1 - \frac{1}{9}}{1} = 0.89$$

Case-II

JBC



JAC



$$12V_2 - V_1 = V$$

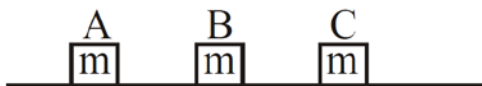
$$V_2 + V_1 = V$$

$$13V_2 = 2V$$

$$V_2 = \frac{2V}{13}$$

$$V_1 = V - \frac{2V}{13} = \frac{11V}{13} \Rightarrow P_c = \frac{\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \frac{1 - \frac{121}{169}}{1} = \frac{48}{169} = 0.28$$

15. Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M . Block A is given an initial speed v towards B due to which it collides with B perfectly in elastically. The combined mass collides with C, also perfectly in elastically $\frac{5}{6}$ th of the initial kinetic energy is lost in whole process. What is value of M/m ?



[JEE Main-2019]

(1) 4

(2) 5

(3) 3

(4) 2

Ans. (1)

Sol. $k_i = \frac{1}{2}mv_0^2$

From linear momentum conservation

$$mv_0 = (2m + M)v_f$$

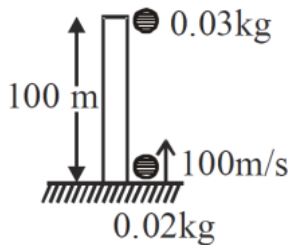
$$\Rightarrow v_f = \frac{mv_0}{2m + M}$$

$$\frac{k_i}{k_f} = 6 \Rightarrow \frac{\frac{1}{2}mv_0^2}{\frac{1}{2}(2m + M)\left(\frac{mv_0}{2m + M}\right)^2} = 6 \Rightarrow \frac{2m + M}{m} = 6 \Rightarrow \frac{M}{m} = 4$$

16. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity 100 ms^{-1} , from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is : ($g = 10 \text{ ms}^{-2}$)
- (1) 40 m (2) 20 m (3) 10 m (4) 30 m

[JEE Main-2019]

Ans. (1)
Sol.



Time taken for the particles to collide,

$$t = \frac{d}{v_{\text{rel}}} = \frac{100}{100} = 1 \text{ sec}$$

Speed of wood just before collision = $gt = 10 \text{ m/s}$

& speed of bullet just before collision $v = 100 \text{ m/s}$

$= 100 - 10 = 90 \text{ m/s}$

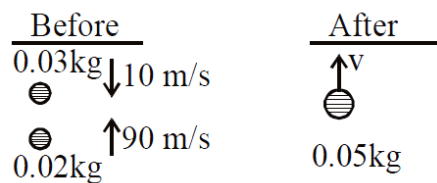
Now, conservation of linear momentum just before and after the collision -

$$-(0.03)(10) + (0.02)(90) = (0.05)v$$

$$\Rightarrow 150 = 5v$$

$$\Rightarrow v = 30 \text{ m/s}$$

$$\text{Max. height reached by body } h = \frac{v^2}{2g}$$



$$h = \frac{30 \times 30}{2 \times 10} = 45 \text{ m}$$

$\therefore$ Height above tower = 40 m

17. A particle of mass m is moving in a straight line with momentum p . Starting at time $t = 0$, a force $F = kt$ acts in the same direction on the moving particle during time interval T so that its momentum changes from p to $3p$. Here k is a constant. The value of T is :-

[JEE Main-2019]

- (1) $\sqrt{\frac{2k}{p}}$ (2) $2\sqrt{\frac{k}{p}}$ (3) $\sqrt{\frac{2p}{k}}$ (4) $2\sqrt{\frac{p}{k}}$

Ans. (4)

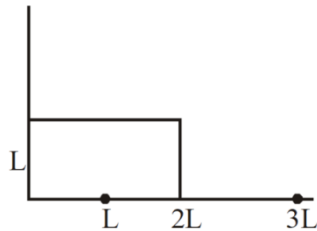
Sol. $\frac{dp}{dt} = F = kt$

$$\int_p^{3p} dp = \int_0^T F \cdot dt$$

$$2p = \left[\frac{Kt^2}{2} \right]_0^T$$

$$T = 2\sqrt{\frac{p}{k}}$$

- 18.** The position vector of the centre of mass $\vec{r}_{cm}$ of an symmetric uniform bar of negligible area of cross-section as shown in figure is : [JEE Main-2019]



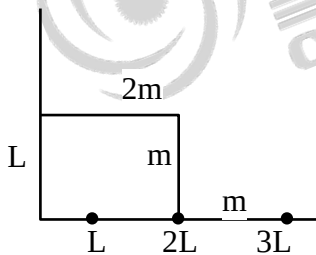
(1) $\vec{r}_{cm} = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$

(2) $\vec{r}_{cm} = \frac{11}{8}L\hat{x} + \frac{3}{8}L\hat{y}$

(3) $\vec{r}_{cm} = \frac{5}{8}L\hat{x} + \frac{13}{8}L\hat{y}$

(4) $\vec{r}_{cm} = \frac{3}{8}L\hat{x} + \frac{11}{8}L\hat{y}$

Ans.
Sol.



$$\vec{r}_{cm} = \frac{2m[L\hat{i} + L\hat{j}] + m\left[2L\hat{i} + \frac{L}{2}\hat{j}\right] + m\left[\frac{5L}{2}\hat{i}\right]}{4}$$

$$\vec{r}_{cm} = \frac{13}{8}L\hat{i} + \frac{5}{8}L\hat{j}$$

- 19.** An alpha-particle of mass m suffers 1-dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing, 64% of its initial kinetic energy. The mass of the nucleus is :- [JEE Main-2019]

(1) 3.5 m

(2) 2 m

(3) 1.5 m

(4) 4 m

Ans. (4)

Sol. Let α particle was coming with vel. v_0
 $K' = 0.36K_0$

$$\Rightarrow \frac{1}{2}m(v')^2 = 0.36 \times \frac{1}{2}mv_0^2$$

$$\Rightarrow v' = 0.6v_0 \text{ (In opposite direction)}$$

Now,

$$\text{KE of nucleus} = 0.64K_0$$

$$\Rightarrow \frac{1}{2}Mv^2 = 0.64 \times \frac{1}{2}mv_0^2$$

$$\Rightarrow \sqrt{M}v = 0.8\sqrt{m}v_0$$

By momentum conservation

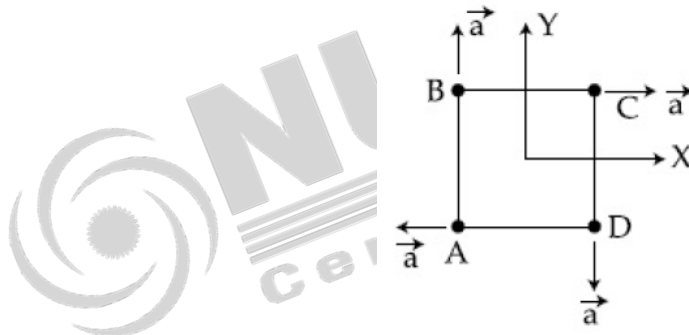
$$\Rightarrow P_i = P_f$$

$$\Rightarrow mv_0 = Mv - mv'$$

Solving above equations

$$\Rightarrow M = 4m$$

- 20.** Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with direction as shown. The acceleration of the centre of mass of the particles is [JEE Main-2019]



(1) $a(\hat{i} + \hat{j})$

(2) Zero

(3) $\frac{a}{5}(\hat{i} - \hat{j})$

(4) $\frac{a}{5}(\hat{i} + \hat{j})$

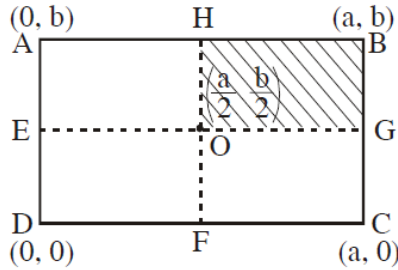
Ans. (3)

Sol. $\vec{a}_A = -a\hat{i}$, $\vec{a}_B = a\hat{j}$, $\vec{a}_C = a\hat{i}$, $\vec{a}_D = -a\hat{j}$

$$\vec{a}_{cm} = \frac{m_a \vec{a}_a + m_b \vec{a}_b + m_c \vec{a}_c + m_d \vec{a}_d}{m_a + m_b + m_c + m_d}$$

$$\vec{a}_{cm} = \frac{-ma\hat{i} + 2ma\hat{j} + 3ma\hat{i} - 4ma\hat{j}}{10m} = \frac{2ma\hat{i} - 2ma\hat{j}}{10m} = \frac{a}{5}(\hat{i} - \hat{j})$$

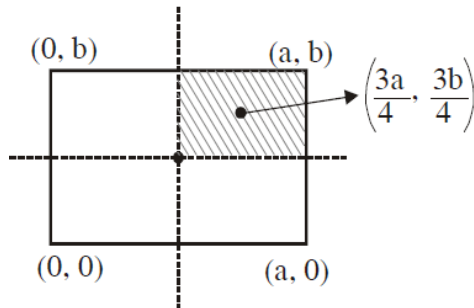
- 21.** A uniform rectangular thin sheet ABCD of mass M has length a and breadth b , as shown in the figure. If the shaded portion HBGO is cut-off, the coordinates of the centre of mass of the remaining portion will be : [JEE Main-2019]



- (1) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$ (2) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (3) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$ (4) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$

Ans. (3)

Sol.



$$x = \frac{M \frac{a}{2} - \frac{M}{4} \times \frac{3a}{4}}{M - \frac{M}{4}}$$

$$x = \frac{\frac{a}{2} - \frac{3a}{4}}{\frac{3}{4}} = \frac{\frac{2a - 3a}{4}}{\frac{3}{4}} = \frac{-\frac{a}{4}}{\frac{3}{4}} = -\frac{a}{3}$$

$$y = \frac{M \frac{b}{2} - \frac{M}{4} \times \frac{3b}{4}}{M - \frac{M}{4}} = \frac{\frac{b}{2} - \frac{3b}{4}}{\frac{3}{4}} = \frac{\frac{2b - 3b}{4}}{\frac{3}{4}} = \frac{-\frac{b}{4}}{\frac{3}{4}} = -\frac{b}{3}$$

22. A body of mass m_1 moving with an unknown velocity of $v_1 \hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2 \hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3 \hat{i}$ and $v_4 \hat{i}$, respectively. If $m_2 = 0.5 m_1$ and $v_3 = 0.5 v_1$, then v_1 is :

[JEE Main-2019]

- (1) $v_4 + v_2$ (2) $v_4 - v_2$ (3) $v_4 - \frac{v_2}{2}$ (4) $v_4 - \frac{v_2}{4}$

Ans. (3)

Sol. Applying linear momentum conservation

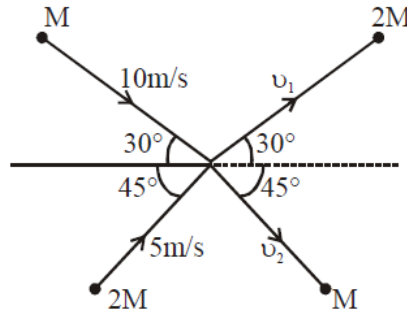
$$m_1 v_1 \hat{i} + m_2 v_2 \hat{i} = m_1 v_3 \hat{i} + m_2 v_4 \hat{i}$$

$$m_1 v_1 + 0.5 m_1 v_2 = m_1 (0.5 v_1) + 0.5 m_1 v_4$$

$$0.5 m_1 v_1 = 0.5 m_1 (v_4 - v_2)$$

$$v_1 = v_4 - v_2$$

23. Two particles, of masses M and $2M$, moving, as shown, with speeds of 10 m/s and 5 m/s , collide elastically at the origin. After the collision, they move along the indicated directions with speeds v_1 and v_2 are nearly : [JEE Main-2019]



(1) 3.2 m/s and 12.6 m/s

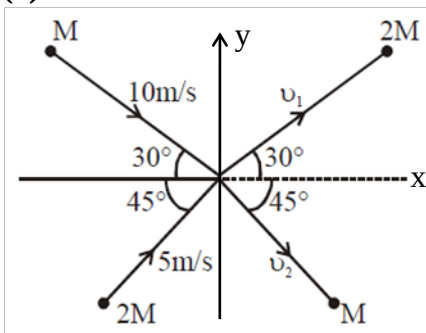
(3) 6.5 m/s and 6.3 m/s

(2) 3.2 m/s and 6.3 m/s

(4) 6.5 m/s and 3.2 m/s

Ans. (4)

Sol.



Applying momentum conservation along x-axis

$$m \times 10 \cdot \cos 30^\circ + 2m \times 5 \times \cos 45^\circ = 2m \times V_1 \times \cos 30^\circ + m \times V_2 \times \cos 45^\circ$$

$$10 \times \frac{\sqrt{3}}{2} + 2 \times 5 \times \frac{1}{\sqrt{2}} = 2 \times V_1 \times \frac{\sqrt{3}}{2} + V_2 \times \frac{1}{\sqrt{2}}$$

$$5\sqrt{3} + 5\sqrt{2} = \sqrt{3}V_1 + \frac{V_2}{\sqrt{2}} \quad \dots(i)$$

Applying momentum conservation along Y-axis

$$2m \times 5 \cdot \sin 45^\circ - m \times 10 \times \sin 30^\circ = 2m \times V_1 \times \sin 30^\circ - mV_2 \cdot \sin 45^\circ$$

$$10 \times \frac{1}{\sqrt{2}} - \frac{10}{2} = \frac{2V_1}{2} - \frac{V_2}{\sqrt{2}}$$

$$5\sqrt{2} - 5 = V_1 - \frac{V_2}{\sqrt{2}} \quad \dots(ii)$$

Add equation (i) and (ii)

$$5\sqrt{3} + 5\sqrt{2} + 5\sqrt{2} - 5 = (\sqrt{3} + 1)V_1$$

$$V_1 = \frac{5\sqrt{3} + 10\sqrt{2} - 5}{\sqrt{3} + 1} = 6.51 \text{ m/s}$$

Put value of V_1 in equation (ii)

$$5(\sqrt{2} - 1) = V_1 - \frac{V_2}{\sqrt{2}}$$

$$2.07 = 6.51 - \frac{V_2}{\sqrt{2}}$$

$$\frac{V_2}{\sqrt{2}} = 6.51 - 2.07 = 4.44$$

$$V_2 = 6.27 \approx 6.3 \text{ m/s}$$

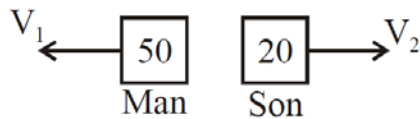
24. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is

[JEE Main-2019]

- (1) 0.20 ms^{-1} (2) 0.28 ms^{-1} (3) 0.47 ms^{-1} (4) 0.14 ms^{-1}

Ans. (1)

Sol.

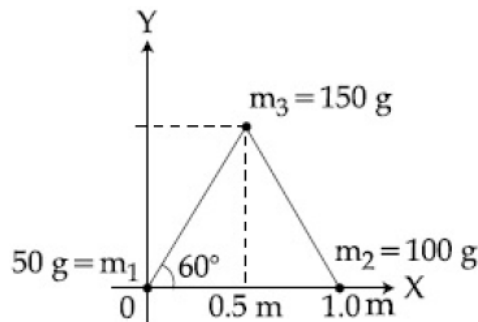


$$\Rightarrow 0 = 50V_1 - 20V_2 \text{ and } V_1 + V_2 = 0.7$$

$$\Rightarrow V_1 = 0.2$$

25. Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be :

[JEE Main-2019]



- (1) $\left(\frac{\sqrt{3}}{8} \text{ m}, \frac{7}{12} \text{ m}\right)$ (2) $\left(\frac{\sqrt{3}}{4} \text{ m}, \frac{5}{12} \text{ m}\right)$ (3) $\left(\frac{7}{12} \text{ m}, \frac{\sqrt{3}}{8} \text{ m}\right)$ (4) $\left(\frac{7}{12} \text{ m}, \frac{\sqrt{3}}{4} \text{ m}\right)$

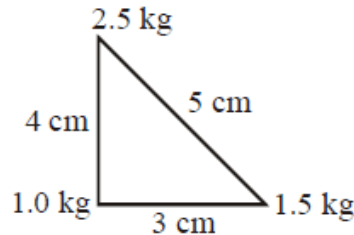
Ans. (4)

Sol.
$$X_{\text{CM}} = \frac{50(0) + 150(0.5) + 1(100)}{300}$$

$$Y_{\text{CM}} = \frac{50(0) + 150(0.5 \tan 60^\circ) + 100(0)}{300}$$

$$\Rightarrow X_{CM}, Y_{CM} = \left(\frac{7}{12}, \frac{\sqrt{3}}{4} \right)$$

26. Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point: [JEE Main-2020]



- (1) 0.6 cm right and 2.0 cm above 1 kg mass
(2) 2.0 cm right and 0.9 cm above 1 kg mass
(3) 0.9 cm right and 2.0 cm above 1 kg mass
(4) 1.5 cm right and 1.2 cm above 1 kg mas

Ans. (3)

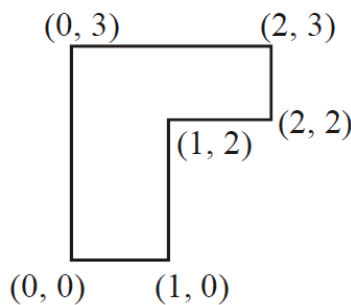
Sol. $X_{CM} = \frac{1 \times 0 + 1.5 \times 3 + 2.5 \times 0}{5}$

$$X_{CM} = 0.9 \text{ cm}$$

$$Y_{CM} = \frac{1 \times 0 + 1.5 \times 0 + 2.5 \times 4}{5}$$

$$Y_{CM} = 2 \text{ cm}$$

27. The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4 kg. (The coordinates of the same are shown in figure) are: [JEE Main-2020]

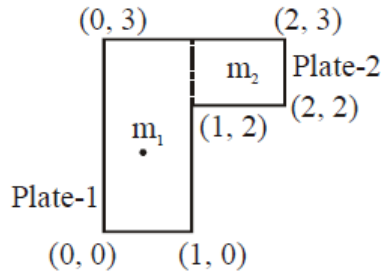


- (1) (1.25 m, 1.50 m) (2) (1m, 1.75 m) (3) (0.75 m, 1.75 m) (4) (0.75 m, 0.75 m)

Ans. (3)

Sol. $m_1 = 3\text{kg}$

$$m_2 = 1\text{kg}$$



Mass of plate-1 is assumed to be concentrated at (0.5, 1.5)

Mass of plate-2 is assumed to be concentrated at (1.5, 2.5).

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{3 \times 0.5 + 1 \times 1.5}{4} = 0.75$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{3 \times 1.5 + 1 \times 2.5}{4} = 1.75$$

28. A body A, of mass $m = 0.1$ kg has an initial velocity of $3\hat{i} \text{ ms}^{-1}$. It collides elastically with another body, B of the same mass which has an initial velocity of $5\hat{j} \text{ ms}^{-1}$. After collision, A moves with a velocity $\vec{v} = 4(\hat{i} + \hat{j})$. The energy of B after collision is written as $\frac{x}{10}$ J. The value of x is _____ . [JEE Main-2020]

Ans. (1.00)

Sol. Using Conservation of linear momentum,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

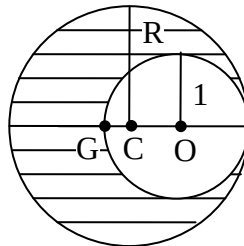
$$(0.1)(3\hat{i}) + (0.1)(5\hat{j}) = (0.1)(4(\hat{i} + \hat{j})) + (0.1)(\vec{v})$$

$$\therefore \vec{v} = -\hat{i} + \hat{j}$$

$$\therefore \text{Speed of B after collision, } |\vec{v}| = \sqrt{2} \text{ m/s}$$

$$\therefore \text{Kinetic Energy of B is } = \frac{1}{2}(0.1)(\sqrt{2})^2 = 0.1 \text{ J} = \frac{1}{10}$$

29. As shown in fig. when a spherical cavity (centred at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e on the surface of the cavity. R can be determined by the equation : [JEE Main-2020]



$$(1) (R^2 - R - 1)(2 - R) = 1$$

$$(3) (R^2 + R - 1)(2 - R) = 1$$

$$(2) (R^2 + R + 1)(2 - R) = 1$$

$$(4) (R^2 - R + 1)(2 - R) = 1$$

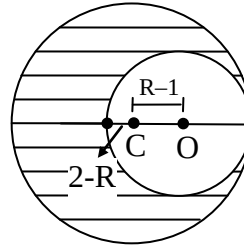
Ans. (3)

Sol. from COM

$$-(2-R) = \frac{\delta \cdot \frac{4}{3} \pi R \cdot 0 - \delta \cdot \frac{4}{3} \pi \cdot 1^3 \times (R-1)}{\delta \cdot \frac{4}{3} \pi R^3 - \delta \cdot \frac{4}{3} \pi \cdot 1^3}$$

$$\text{Or } (R^3 - 1)(2 - R) = R - 1$$

$$(R^2 + R + 1)(2 - R) = 1$$



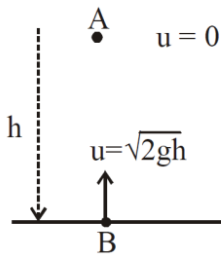
30. A particle of mass m is dropped from a height h above the ground. At the same time another article of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is :

[JEE Main-2020]

- (1) $\sqrt{\frac{3}{4}}$ (2) $\frac{1}{2}$ (3) $\sqrt{\frac{3}{2}}$ (4) $\sqrt{\frac{1}{2}}$

Ans. (3)

Sol.



Particles will collide after time $t_0 = \frac{h}{\sqrt{2gh}}$

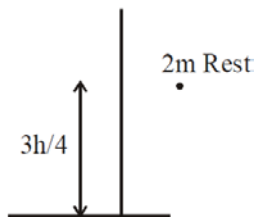
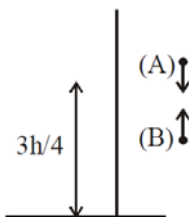
at collision, $v_A = gt_0$

$v_B = u_B - gt_0$

$\Rightarrow v_A = -v_B$

Before collision

After collision



Time taken by combined mass to reach the ground time = $\sqrt{\frac{2 \times 3h/4}{g}} = \sqrt{\frac{3h}{2g}}$

31. Two particles of equal mass m have respective initial velocities $u \hat{i}$ and $u \left(\frac{\hat{i} + \hat{j}}{2} \right)$. They collide completely inelastically. The energy lost in the process is : [JEE Main-2020]

(1) $\sqrt{\frac{2}{3}} mu^2$ (2) $\frac{1}{8} mu^2$ (3) $\frac{3}{4} mu^2$ (4) $\frac{1}{3} mu^2$

Ans. (2)

Sol. From momentum conservation

$$mu \hat{i} + mu \left(\frac{\hat{i} + \hat{j}}{2} \right) = (m + m) \bar{v}$$

$$\Rightarrow \bar{v} = \frac{3}{4} u \hat{i} + \frac{u}{4} \hat{j}$$

$$\Rightarrow |\bar{v}| = \frac{u}{4} \sqrt{10}$$

$$\text{Final kinetic energy} = \frac{1}{2} 2m \left(\frac{u}{4} \sqrt{10} \right)^2 = \frac{5}{8} mu^2$$

$$\text{Initial kinetic energy} = \frac{1}{2} mu^2 + \frac{1}{2} m \left(\frac{u}{\sqrt{2}} \right)^2 = \frac{6}{8} mu^2$$

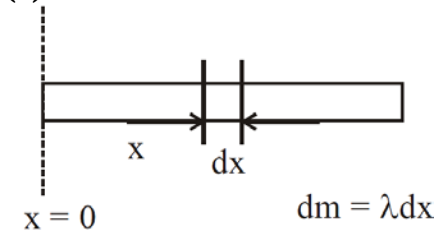
$$\text{Loss in K.E.} = k_i - k_f = \frac{1}{8} mu^2$$

32. A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b \left(\frac{x}{L} \right)^2$ where a and b are constants and $0 \leq x \leq L$. The value of x for the centre of mass of the rod is at: [JEE Main-2020]

(1) $\frac{4}{3} \left(\frac{a+b}{2a+3b} \right) L$ (2) $\frac{3}{4} \left(\frac{2a+b}{3a+b} \right) L$ (3) $\frac{3}{2} \left(\frac{2a+b}{3a+b} \right) L$ (4) $\frac{3}{2} \left(\frac{a+b}{2a+b} \right) L$

Ans. (2)

Sol.



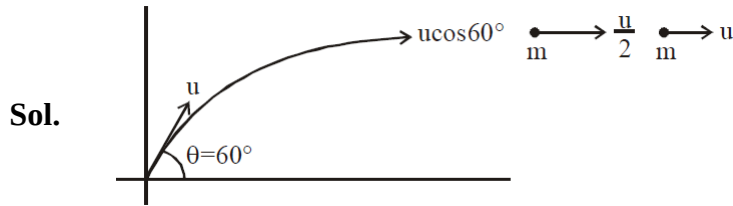
$$x_{cm} = \frac{\int x dm}{\int dm} = \frac{\int (\lambda dx) x}{\int dm}$$

$$= \frac{\int_0^L \left(a + \frac{bx^2}{L^2} \right) x dx}{\int_0^L \left(a + \frac{bx^2}{L^2} \right) dx} = \frac{\frac{aL^2}{2} + \frac{b}{L^2} \cdot \frac{L^4}{4}}{aL + \frac{b}{L^2} \cdot \frac{L^3}{3}} = \frac{\left(\frac{4a+2b}{8} \right) L}{\frac{(3a+b)}{3}} = \frac{3}{4} \frac{(2a+b)L}{(3a+b)}$$

33. A particle of mass m is projected with a speed u from the ground at an angle $\theta = \frac{\pi}{3}$ w.r.t. horizontal (x -axis). When it has reached its maximum height, it collides completely in elastically with another particle of the same mass and velocity $u\hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is: [JEE Main-2020]

- (1) $2\sqrt{2} \frac{u^2}{g}$ (2) $\frac{3\sqrt{3}}{8} \frac{u^2}{g}$ (3) $\frac{3\sqrt{2}}{4} \frac{u^2}{g}$ (4) $\frac{5}{8} \frac{u^2}{g}$

Ans. (2)



By momentum conservation,

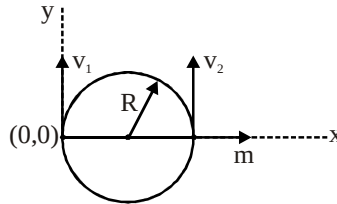
$$\frac{mu}{2} + mu = 2mv'$$

$$v' = \frac{3v}{4}$$

$$\text{Range after collision} = \frac{3v}{4} \sqrt{\frac{2H}{g}} = \frac{3v}{4} \sqrt{\frac{2 \cdot u^2 \sin^2 60^\circ}{g}} = \frac{3}{4} \cdot \frac{\sqrt{3}}{2} \cdot \frac{u^2}{g} = \frac{3\sqrt{3}u^2}{8g}$$

EXERCISE # (JA)

1. A particle of mass m , moving in a circular path of radius R with a constant speed v_2 is located at point $(2R, 0)$ at time $t = 0$ and a man starts moving with a velocity v_1 along the positive y -axis from origin at time $t = 0$. Calculate the linear momentum of the particle w.r.t. man as a function of time. [IIT JEE-2003]



Ans. $\vec{P}_{PM} = m\vec{v}_{PM} = -mv_2 \sin \omega t \hat{i} + m(v_2 \cos \omega t - v_1) \hat{j}$

Sol. Angular speed of particle about centre of the circle,

$$\omega = \frac{v_2}{R}, \theta = \omega t = \frac{v_2}{R} t$$

$$\vec{v}_p = (-v_2 \sin \theta \hat{i} + v_2 \cos \theta \hat{j})$$

or $\vec{v}_p = \left(-v_2 \sin \frac{v_2}{R} t \hat{i} + v_2 \cos \frac{v_2}{R} t \hat{j} \right)$

and $\vec{v}_m = v_1 \hat{j}$

$\therefore$ linear momentum of particle w.r.t. man as a function of time is

$$\vec{L}_{pm} = (m\vec{v}_p - m\vec{v}_m) = m \left[\left(-v_2 \sin \frac{v_2}{R} t \right) \hat{i} + \left(v_2 \cos \frac{v_2}{R} t - v_1 \right) \hat{j} \right]$$

2. **Statement-I :** In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision. [IIT JEE-2007]

Statement-II : In a elastic collision, the linear momentum of the system is conserved

- (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

Ans. (B or D)

Sol. According to Newton's Law

$$e = \frac{\vec{v}_2 - \vec{v}_1}{\vec{u}_1 - \vec{u}_2}$$

For elastic collision coefficient of restitution $e = 1$ so

$$\vec{v}_2 - \vec{v}_1 = \vec{u}_1 - \vec{u}_2 \quad \text{Statement - 1 is correct}$$

Linear momentum is conserved in both elastic & non elastic collision but it's not the explanation of statement -1 so it is not the correct explanation of the statement A.

3. **Statement I :** If there is no external torque on a body about its centre of mass, then the velocity of the centre of mass remains constant. [IIT JEE-2007]

Statement II : The linear momentum of an isolated system remains constant.

- (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is not a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

Ans. (D)

Sol. If there is no external torque on body than angular velocity of body remains constant.
and if there is no external force on body than linear velocity of body remains constant.

4. A particle moves in the X-Y plane under the influence of a force such that its linear momentum is $\vec{p}(t) = A[\hat{i} \cos(kt) - \hat{j} \sin(kt)]$, where A and k are constants. The angle between the force and the momentum is : [IIT JEE-2007]

- (A) 0° (B) 30° (C) 45° (D) 90°

Ans. (D)

Sol. As $\vec{p}_1 + \vec{p}_2 = \vec{0}$ so $\vec{p}_1' + \vec{p}_2' = \vec{0}$

For (A) $\vec{p}_1' + \vec{p}_2' = (a_1 + a_2)\hat{i} + (b_1 + b_2)\hat{j} + c_1\hat{k}$

For (B) $\vec{p}_1' + \vec{p}_2' = (a_1 + a_2)\hat{i} + (b_1 + b_2)\hat{j}$

For (C) $\vec{p}_1' + \vec{p}_2' = (c_1 + c_2)\hat{k}$

For (D) $\vec{p}_1' + \vec{p}_2' = (a_1 + a_2)\hat{i} + 2b_1\hat{j}$

But $a_1, b_1, c_1, a_2, b_2, c_2 \neq 0$

Therefore (A) & (B) is not possible to get $\vec{p}_1' + \vec{p}_2' = \vec{0}$

5. Two balls, having linear momenta $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$, undergo a collision in free space. There is no external force acting on the balls. Let $\vec{p}_1'$ and $\vec{p}_2'$ be their final momenta. The following option(s) is (are) **NOT ALLOWED** for any non-zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2 .

[IIT JEE-2008]

(A) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$
 $\vec{p}_2' = a_2\hat{i} + b_2\hat{j}$

(B) $\vec{p}_1' = c_1\hat{k}$
 $\vec{p}_2' = c_2\hat{k}$

(C) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$
 $\vec{p}_2' = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$

(D) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j}$
 $\vec{p}_2' = a_2\hat{i} + b_1\hat{j}$

Ans. (AD)

Sol. $\vec{p}_1 = p\hat{i}$

$\vec{p}_2 = -p\hat{i}$

as there is no external force so momentum will remain conserved

$$\vec{P}'_1 + \vec{P}'_2 = \vec{P}_1 + \vec{P}_2$$

$$\vec{P}'_1 + \vec{P}'_2 = 0$$

Now from option

$$(A) \vec{P}'_1 + \vec{P}'_2 = (a_1 + a_2)\hat{i} + (b_1 + b_2)\hat{j} + c_1\hat{k}$$

$$(B) \vec{P}'_1 + \vec{P}'_2 = (c_1 + c_2)\hat{k}$$

$$(C) \vec{P}'_1 + \vec{P}'_2 = (a_1 + a_2)\hat{i} + (b_1 + b_2)\hat{j}$$

$$(D) \vec{P}'_1 + \vec{P}'_2 = (a_1 + a_2)\hat{i} + 2b_1\hat{j}$$

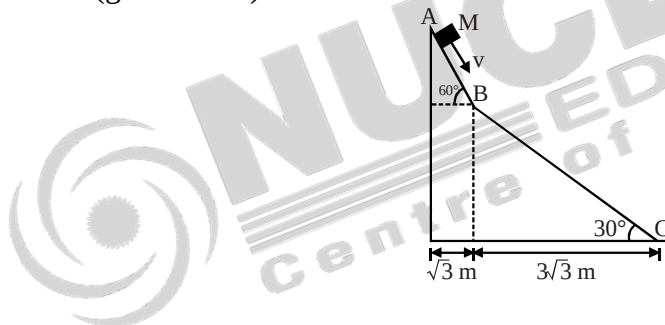
and it is given that $a_1, b_1, c_1, a_2, b_2, c_2 \neq 0$

in case of A and D it is not possible to get $\vec{P}'_1 + \vec{P}'_2 = 0$

Hence **Ans. (A) and (D)**

Comprehension based questions

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from 60° to 30° at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic ($g = 10 \text{ m/s}^2$) [IIT-JEE 2008]

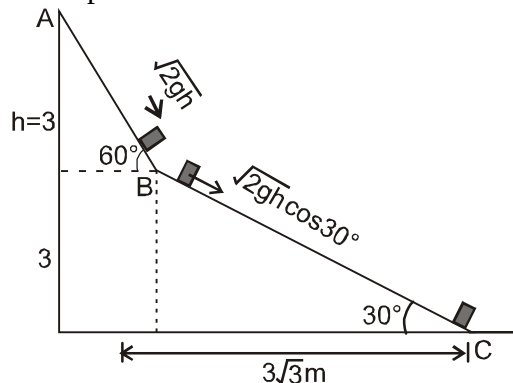


6. The speed of the block at point B immediately after it strikes the second incline is:

- (A) $\sqrt{60} \text{ m/s}$ (B) $\sqrt{45} \text{ m/s}$ (C) $\sqrt{30} \text{ m/s}$ (D) $\sqrt{15} \text{ m/s}$

Ans. (B)

Sol. At point B there is perfectly inelastic collision so component of velocity $\perp$ to incline plane becomes zero and component parallel to second surface is retained



velocity immediately after it strikes second incline

$$V = \sqrt{2gh} \cos 30^\circ = \sqrt{2 \times 10 \times 3} \times \frac{\sqrt{3}}{2} = \sqrt{\frac{2 \times 10 \times 9}{4}}$$

$$V = \sqrt{45} \text{ m/s}$$

7. The speed of the block at point C, immediately before it leaves the second incline is:

- (A) $\sqrt{120}$ m/s (B) $\sqrt{105}$ m/s (C) $\sqrt{30}$ m/s (D) $\sqrt{75}$ m/s

Ans. (B)

Sol. At point 'C'

$$V_C^2 = V_B^2 + 2gh$$

$$V_C^2 = 45 + 2 \times 10 \times 3$$

$$V_C = \sqrt{105} \text{ m/s}$$

8. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is:

- (A) $\sqrt{30}$ m/s (B) $\sqrt{15}$ m/s (C) 0 (D) $-\sqrt{15}$ m/s

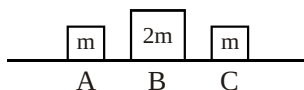
Ans. (C)

Sol. The block coming down from incline AB makes an angle 30° with incline BC. If the block collides with incline BC elastically, the angle of block after collision with the incline shall be 30° .

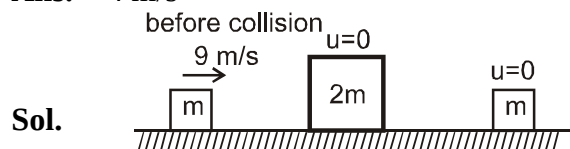
Hence just after collision with incline BC the velocity of block shall be horizontal. So immediately after the block strikes second inclined, its vertical component of velocity will be zero

9. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m , $2m$ and m , respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in m/s) of the object C.

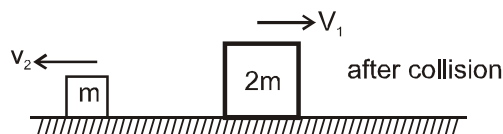
[IIT JEE-2009]



Ans. 4 m/s



Sol.



from momentum conservation :

$$9m = (2m) V_1 - (m)V_2$$

$$\Rightarrow 9 = 2V_1 - V_2 \quad \dots (1)$$

$$e = \frac{V_1 + V_2}{9} = 1 \quad \dots (2)$$

from eqn(1) and eqn(2)

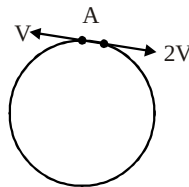
$$V_1 = 6 \text{ m/sec.}$$

for second collision between second block and third block :

$$(2m) 6 + m(0) = (2m + m) V_C \quad \Rightarrow \quad V_C = 4 \text{ m/sec.}$$

10. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than at A, these two particles will again reach the point A ?

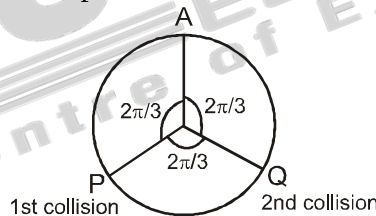
[IIT-JEE 2009]



- (A) 4 (B) 3 (C) 2 (D) 1

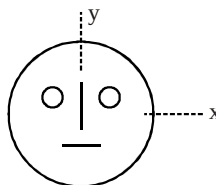
Ans. (C)

Sol. Since masses of particles are equal and collisions are elastic, so particles will exchange velocities after each collision. The first collision will be at a point P and second at point Q again and before third collision the particles will reach at A.



11. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments in m . The mass of the ink used to draw the outer circle is $6m$. The coordinates of the centres of the different parts are : outer circle $(0, 0)$, left inner circle $(-a, a)$, right inner circle (a, a) , vertical line $(0, 0)$ and horizontal line $(0, -a)$. The y -coordinates of the centre of mass of the ink in this drawing is:

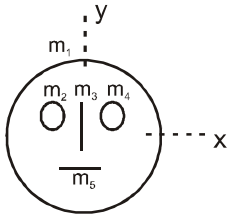
[IIT JEE- 2009]



- (A) $\frac{a}{10}$ (B) $\frac{a}{8}$ (C) $\frac{a}{12}$ (D) $\frac{a}{3}$

Ans. (A)

Sol.



$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4 + m_5 y_5}{m_1 + m_2 + m_3 + m_4 + m_5}$$

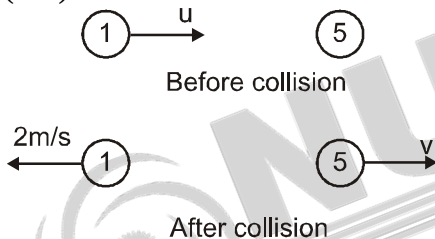
$$\Rightarrow y_{cm} = \frac{6m(0) + m(a) + m(0) + m(a) + m(-a)}{m + m + m + m + 6m} = \frac{a}{10}$$

12. A point mass of 1kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 m/s. Which of the following statement(s) is/are correct for the system of these two masses ? [IIT JEE-2010]

- (A) Total momentum of the system is 3 kg m/s
(B) Momentum of 5 kg mass after collision is 4 kg m/s
(C) Kinetic energy of the centre of mass is 0.75 J
(D) Total kinetic energy of the system is 4 J

Ans. (AC)

Sol.



Since collision is elastic, so $e = 1$

Velocity of approach = velocity of separation

So, $u = v + 2$ (i)

By momentum conservation :

$$1 \times u = 5v - 1 \times 2$$

$$u = 5v - 2$$

$$v + 2 = 5v - 2$$

So, $v = 1$ m/s

and $u = 3$ m/s

Momentum of system = $1 \times 3 = 3$ kgm/s

Momentum of 5kg after collision = $5 \times 1 = 5$ kgm/s

$$\text{So, kinetic energy of centre of mass} = \frac{1}{2} (m_1 + m_2) \left(\frac{m_1 u}{m_1 + m_2} \right)^2$$

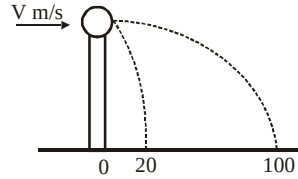
$$= \frac{1}{2} (1 + 5) \left(\frac{1 \times 3}{6} \right)^2$$

$$= 0.75 \text{ J}$$

$$\text{Total kinetic energy} = \frac{1}{2} \times 1 \times 3^2 = 4.5 \text{ J.}$$

13. A ball of mass 0.2 kg rests on a vertical post of height 5m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity V of the bullet is

[IIT JEE - 2011]



- (A) 250 m/s (B) $250\sqrt{2}$ m/s (C) 400 m/s (D) 500 m/s

Ans. (D)

Sol. $R = u\sqrt{\frac{2h}{g}}$

$$\Rightarrow 20 = V_1\sqrt{\frac{2 \times 5}{10}} \text{ and } 100 = V_2\sqrt{\frac{2 \times 5}{10}}$$

$$\Rightarrow V_1 = 20 \text{ m/s}, V_2 = 100 \text{ m/sec.}$$

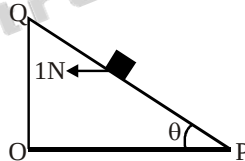
Applying momentum conservation just before and just after the collision

$$(0.01)(V) = (0.2)(20) + (0.01)(100)$$

$$V = 500 \text{ m/s}$$

14. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its center of mass as shown in the figure. The block remains stationary if (take $g = 10 \text{ m/s}^2$)

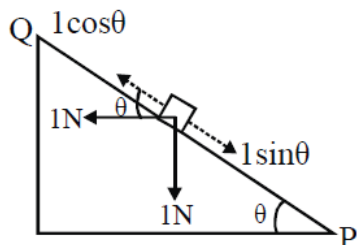
[IIT JEE-2012]



- (A) $\theta = 45^\circ$
 (B) $\theta > 45^\circ$ and a frictional force acts on the block towards P
 (C) $\theta > 45^\circ$ and a frictional force acts on the block towards Q
 (D) $\theta < 45^\circ$ and a frictional force acts on the block towards

Ans. (AC)

Sol.

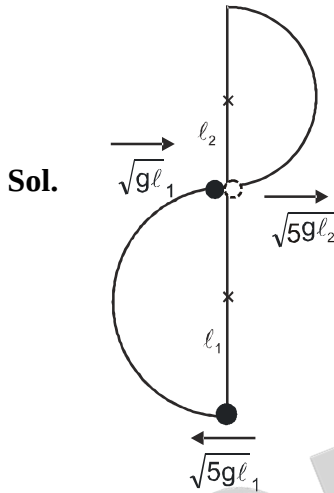


- (1) If $\sin\theta = \cos\theta \Rightarrow \theta = 45^\circ \Rightarrow$ no friction will act and the block will remain at rest.
 (2) If $\sin\theta > \cos\theta \Rightarrow \theta > 45^\circ \Rightarrow$ friction will act towards Q

(3) If $\sin\theta < \cos\theta \Rightarrow \theta < 45^\circ \Rightarrow$ friction will act towards P

15. A bob of mass m , suspended by a string of length ℓ_1 is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length ℓ_2 , which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio $\frac{\ell_1}{\ell_2}$ is [IIT JEE-2013]

Ans. 5

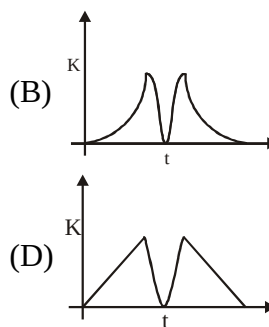
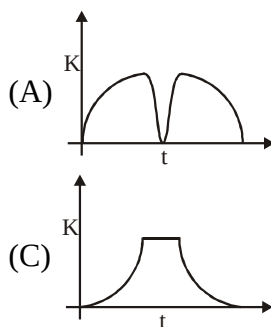


To complete the vertical circle

$$\sqrt{gl_1} = \sqrt{5gl_2}$$

$$\frac{\ell_1}{\ell_2} = 5$$

16. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figures are only illustrative and not to the scale. [JEE Adv. 2014]



Ans. (B)

$$t = 0$$

Sol. (Before collision) t $\downarrow$
 $v = gt$

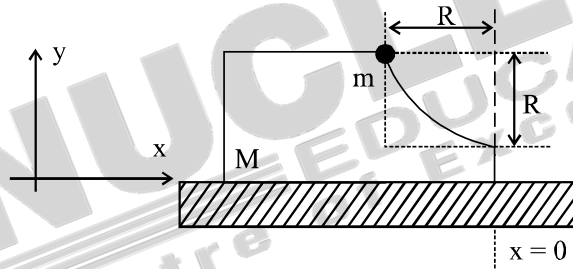
$$K = \frac{1}{2}mg^2t^2$$

$K \propto t^2$: parabolic graph

then during collision kinetic energy first decreases to elastic potential energy and then increases.

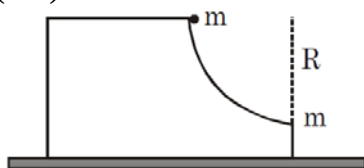
Most appropriate graph is B.

17. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at $x = 0$, in a co-ordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v . At that instant, which of the following options is/are correct ? [JEE Adv. 2017]



- (A) The x component of displacement of the center of mass of the block M is : $-\frac{mR}{M+m}$
- (B) The position of the point mass m is : $x = -\sqrt{2} \frac{mR}{M+m}$
- (C) The velocity of the point mass m is : $v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$
- (D) The velocity of the block M is : $V = -\frac{m}{M} \sqrt{2gR}$

Ans. (AC)

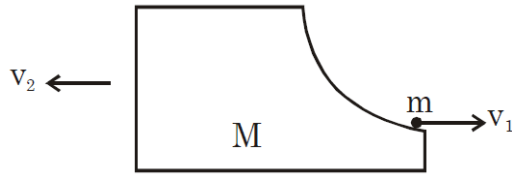


Sol.

$$M_S \Delta \bar{x}_{cm} = m_1 \Delta \bar{x}_1 + m_2 \Delta \bar{x}_2$$

$$0 = m(+R + \bar{x}) + m \bar{x}$$

$$\bar{x} = \frac{-mR}{M+m} \quad (\text{A) Ans.}$$



$$0 = m\bar{v}_1 + M\bar{v}_2$$

$$\bar{v}_2 = -\frac{m\bar{v}_1}{M}$$

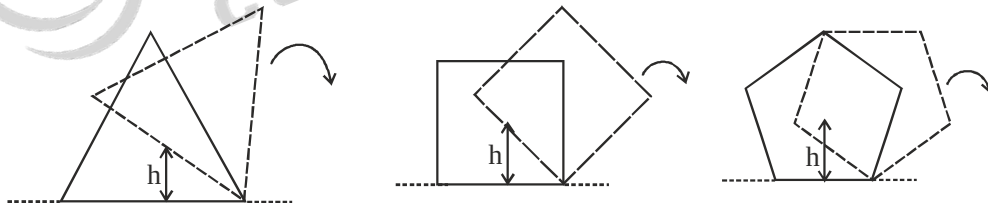
$$mgR = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2$$

$$mgR = \frac{1}{2}mv_1^2 + \frac{1}{2}M\left(\frac{mv_1}{M}\right)^2$$

$$mgR = \frac{1}{2}mv_1^2\left(1 + \frac{m}{M}\right)$$

$$\sqrt{\frac{2gR}{\left(1 + \frac{m}{M}\right)}} = v_1$$

18. Consider regular polygons with number of sides $n = 3, 4, 5 \dots$ as shown in the figure. The center of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ . Then Δ depends on n and h as **[JEE Adv. 2017]**



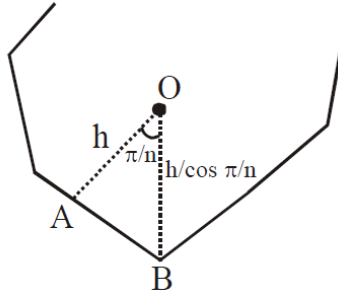
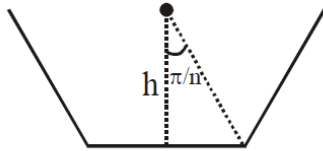
$$(A) \Delta = h \sin^2\left(\frac{\pi}{n}\right)$$

$$(B) \Delta = h \left(\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1 \right)$$

$$(C) \Delta = h \sin\left(\frac{2\pi}{n}\right)$$

$$(D) \Delta = h \tan^2\left(\frac{\pi}{2n}\right)$$

Ans. (B)
Sol.



$$OA = h$$

$$OB = \frac{h}{\cos \frac{\pi}{n}}$$

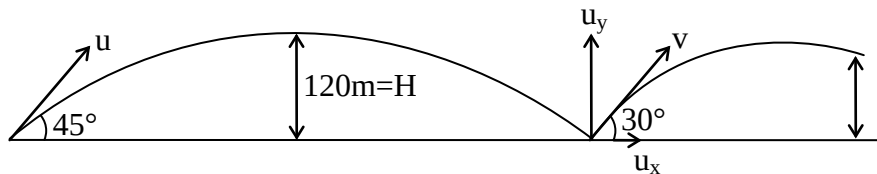
Initial height of COM = h

$$\text{Final height of COM} = \frac{h}{\cos \left(\frac{\pi}{n} \right)}$$

$$\therefore \Delta = \frac{h}{\cos \frac{\pi}{n}} - h \quad \Delta = h \left[\frac{1}{\cos \frac{\pi}{n}} - 1 \right]$$

19. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is _____.

[JEE Adv.(P-2) 2018]



Sol.

$$K_2 = \frac{1}{2} m u^2$$

$$\frac{U_y}{U_x} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$H = \frac{u^2 \sin^2 45^\circ}{2g}$$

$$\sqrt{3} u_y = u_x$$

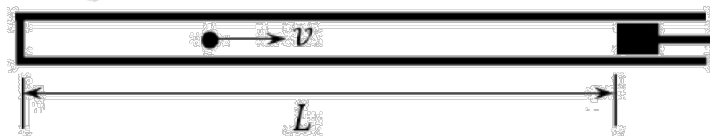
$$= \frac{u^2}{4g} = 120\text{m}$$

$$K_f = \frac{1}{2} m u^2 = \frac{1}{2} m (u_x^2 + u_y^2)$$

$$\begin{aligned}
 K_f &= \frac{1}{2} K_i \\
 \Rightarrow \frac{1}{2} m(u_x^2 + u_y^2) &= \frac{1}{2} \times \frac{1}{2} m u^2 \\
 \Rightarrow u_x^2 + u_y^2 &= \frac{u^2}{2} \\
 \Rightarrow \text{using } u_x &= \sqrt{3} u_y \\
 \Rightarrow 3u_y^2 + u_y^2 &= \frac{u^2}{2} \\
 \Rightarrow u_y^2 &= \frac{u^2}{8} \\
 h = \frac{u_y^2}{2g} = \frac{u^2}{16g} &= \frac{1}{4} \left(\frac{u^2}{4g} \right) = \frac{H}{4} = \frac{120}{4} = 30 \text{ m}
 \end{aligned}$$

20. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V \ll \frac{dL}{L} v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct?

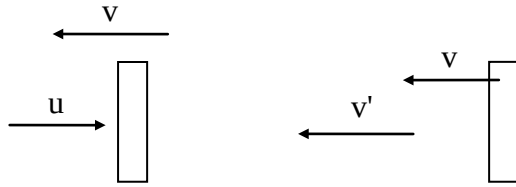
[JEE-Advanced-2019]



- (1) After each collision with the piston, the particle speed increases by $2V$
- (2) If the piston moves inward by dL , the particle speed increases by $2v \frac{dL}{L}$
- (3) The rate at which the particle strikes the piston is v/L
- (4) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2} L_0$

Ans. (1,4)

Sol. Before collision After collision



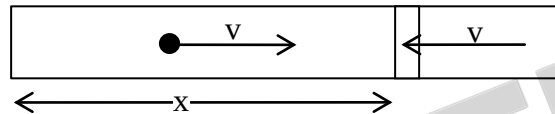
$$e = \frac{v' - v}{v + u} \Rightarrow 1 = \frac{v' - v}{v + u}$$

$$v' = 2v + u$$

Hence increment in velocity after every collision is $2v$.

Frequency of collision when piston is at distance x from the closed end is $f = \frac{v}{2x}$

Rate of collision (3 option is incorrect)



Time in moving the piston by distance dx further towards the closed end is $dt = \frac{dx}{v}$

$\therefore$ Total number of collision while piston moves dx is

$$\oint n = f \cdot dt$$

$$n = \left(\frac{v}{2x} \right) \frac{dx}{v}$$

$\Rightarrow$ Change in velocity in every collision is $2V$.

Hence change in velocity collide piston move dx is

$$dv = \left(\frac{v}{2x} \cdot \frac{dx}{v} \right) 2v = \frac{v}{n} dx$$

$$\int_{v_0}^v \frac{dv}{v} = - \int_L^{L/2} \frac{dx}{x}$$

(since x is decreasing, $\therefore dx$ will be negative)

$$\ln \left(\frac{v}{v_0} \right) = - \ln \left(\frac{1}{2} \right)$$

$$\ln \left(\frac{v}{v_0} \right) = \ln(2)$$

$$\Rightarrow v = 2v_0$$

Hence Kinetic energy will be 4 times.