

ACHIEVER'S EDUCATION CONSULTANT
08141710097
SUMMARY OF CIT342 WITH PAST QUESTIONS

In addition, it advocates some general guidelines for the amount of time you are likely to spend on each unit of the course in order to complete it successfully. It gives you guidance in respect of your Tutor-Marked Assignment which will be made available in the assignment file. There will be regular tutorial classes that are related to the course. It is advisable for you to attend these tutorial sessions.

The course will prepare you for the challenges you will meet in Mathematical Methods IV.

Course Aim

The aim of the course is to provide you with an understanding of Mathematical Methods IV.

Course Objectives

Each unit has specific objectives which are included at the beginning of the unit. You should read these objectives before you study the unit. You may wish to refer to them during your study to check on your progress. By doing so, you would have followed the instructions in the unit.

Working through This Course

Each unit contains Self-Assessment Exercises and at certain points in the course, you would be required to submit assignments for assessment purposes. The course should take you about a total of 17 weeks to complete. This course entails that you spend a lot of time to read. I would advice that you avail yourself of the opportunities of the tutorial classes provided by the University.

Presentation Schedule

Your course materials have important dates for the early and timely completion and submission of your TMAs and attending tutorials. You should remember that you are required to submit all your assignments by the stipulated time and date. You should guard against falling behind in your work.

Assessment

The first is made up of Self-Assessment Exercises, the second consists of the TutorMarked Assignments and third is the written examination/end of course You are advised to do the exercises. In tackling the assignments, you are expected to apply information, knowledge and

technique you gathered during the course. The assignments must be submitted to your facilitator for formal assessment in accordance with deadlines stated in the presentation schedule and the assignment file. The work you submit to your tutor for assessment will count for 30% of your total course work.

At the end of the course you will need to sit for a final or end of course examination of about three hours duration. This examination will count for 70% of your total course mark. You will be given four TMAs to answer. Three of these must be answered before you are allowed to sit for the end of course examination.

The TMA would be given to you by your facilitator and returned after you have done the assignment. Assignment questions for the units in this course are contained in your reading references and study units. Make sure that each assignment reaches your facilitator on or before the deadline given in the presentation schedule and assignment file. If for any reason you can not complete your work on time, contact your facilitator before the assignment is due to discuss the possibility of an extension.

Extension will not be granted after the due date.

Final Examination and Grading

All areas of the course will be assessed. Use the time between finishing the last unit and sitting for the examination, to revise the whole course.

100% Facilitators/Tutors and Tutorials

There are 16 hours of tutorials provided in support of this course. You will be notified of the dates, times and location of these tutorials as well as the name and phone number of your facilitator, as soon as you are allocated a tutorial group. Your facilitator will mark and comment on your assignments, keep a close watch on your progress and any difficulties you might face and provide assistance to you during the course. You are expected to mail your Tutor-Marked Assignment to your facilitator before the schedule date .

They will be marked by your tutor and returned to you as soon as possible. You should endeavour to attend the tutorials. You can raise any problem encountered in the course of your study. You will learn a lot from participating actively in discussions.

[...]

A differential equation is a functional equation where the unknown function or functions are present as derivatives with respect to a single variable in the case of an ordinary differential equation. The order of the highest derivative is called the order of the equation. Derivatives in a differential equation can occur in various ways and we do not admit equations where the unknown is subjected to other operations than algebraic and differential equations. A differential equation is a functional equation where the unknown function or functions are present as derivatives with respect to single variables in the case of an ordinary differential equation.

Consider the following six examples of functional equations involving derivatives.

[...]

Example 5 is not a differential equation as it stands but on differentiating will yield $f'' = \{1 + [f]^2\}^{1/2}$ which is a second order differential equation equivalent to example 4. Finally, example 6 is not a differential equation and is not reducible to such an equation by elementary means. The normal form of a first order differential equation is given as $y' = F$. Conversely, every such system can be writing as a first order vector differential equation.

The generalisation has the advantage of covering nth order equations.

We can consider differential equations in more general spaces than the

The intent of an existence theorem is to show that there exists a function which satisfies the equation in some neighborhood of point . We have examined differential equations in a general setting in this unit. This unit is important to the understanding of other units that would follow subsequently. This unit must be read carefully before proceeding to the other units.

If f satisfies the integral equation $f = y_0 + \int F[s, f]ds$, Find a differential satisfied by f .

THE FIXED POINT METHOD

In this unit, we shall use a topological method based on the contraction fixed point theorem. Apply the contraction fixed point theorem determine the existence of solutions for a given differential Equation solve correctly the tutor-marked assignment that follows.

We now define the transformation

You are supposed to master the concept developed in this unit before proceeding to the next unit. It can be extended to spaces of infinitely in many dimensions.

APPROXIMATIONS

What has been added is control of the limiting process. We know how often the process must be repeated to bring the result with the desired limit of tolerance. The method of trial and error can be traced back to Isaac Newton who was the first to be concerned with approximate solution of algebraic equation. The method of successive approximation was given by Emile Picard for differential equation in 1891.

This method soon became the standard method for proving existence and uniqueness theorems for all sorts of functional equations.

F is defined and continuous in

$F[s, f_{m-1}] \leq M$, $F[s, f_{m-1}]ds$, exist as a continuous function of x and its norm does not exceed $M(x - x_0) < M(b - a)$ by the choice of r .

This implies that f_m is also continuous satisfies $f_m = y_0$, $f_m - y_0 < b$

It follows that the approximation are well defined for all m .

$\lim_{m \rightarrow \infty} f_m(x) = y(x)$

From it follows that f satisfies and consequently, the differential equation and the initial condition. So to prove uniqueness we may suppose that g is a solution defined in some interval.

Therefore, f is the only solution of with $f = y_0$

Various questions arise when we want to use theorem above. You are required to read carefully before proceeding to the next unit. We have proved the existence of functional differential equations by successive approximation methods. Successive approximation method is essentially an iterative method that needs to be carefully designed to give a solution to the differential equation under consideration.

The Thomas-Fermi equation defined by $x^{1/2} y'' = y^{3/2}$ arises in nuclear physics.

Gamma function Beta function Factorial Notation

These functions are of very useful mathematical importance in solving differential equations and other applied mathematics problems. Define beta function, gamma function, and factorial notations apply these functions to solve mathematical problems.

[...]

Converges, so $\int_0^1 e^{-t} t^{z-1} dt$ is bounded.

$2 \leq z \leq 2 +$

The last quantity is independent of z and must be finite since the left side exists. Next, turn to polar co-ordinate for the iterated integration over the first quadrant in xy -plan.

[...]

In this unit, we have studied Gamma function, Beta function and Factorial notations. You are required to study these functions because you would be required to apply them in future. Study this area properly before moving to the next unit. The Beta function of p , and q is defined by the integral $B = \int_0^1 t^{p-1} (1-t)^{q-1} dt$, .

[...]

In this unit, we shall consider a class of function usually referred to as hyper-geometric functions. The series solution of the associated differential equation usually takes the form of a geometric series. Determine the differential equations that can give rise to hypergeometric functions explain the properties of this functions apply this function where necessary.

The solutions the $x^2 + [c -$

$y=0$ $dx - ab$ differential are generally called Hyper-geometric functions. Note that a , b , and c are fixed parameters.

$2 F1 = \sum$

For the moment, let c be not an integer for , the indicial equation has root zero and $i-c$.

[...]

You have learnt in this unit some properties of hyper-geometric functions. You are requested to study this unit properly before going to the next unit. Recall that you learnt about the class of differential equation, which usually give rise to hyper-geometric functions. You also learnt about the relations of this function to Gamma and Beta functions.

Study this unit properly before going to the next unit.

[...]

In solving differential equation, we often come across some problems which exhibit some characteristic which needed to be studied further. This is because of the wide applicability of this function in physics and applied mathematics. Identify Bessel functions correctly solve problems related to Bessel functions. $x^2 + y = 0$ is called Bessel's equation of index v .

[...]

Now we obtain the solution corresponding to the value $r = v$.

[...]

$+v$ is called the Bessel Function of the first kind of order v .

[...]

Let's associate with each term of the second series the term of the first series corresponding to $n = p + m$.

[...]

We have considered Bessel function in its general setting in this unit . You are required to read

this unit carefully before going to the next unit. Recall that Bessel functions are usually associated with a class of equations called Bessel equations. We gave some examples to enable you understand the contents of this unit.

We also examined another type of Bessel function usually referred to as Bessel Function of the First Kind. However, you are to master this unit properly before going into the next unit.

Deduce that $J_{n+1} =$

Obtain the general solution of each of the following equations in terms of Bessel functions, or if possible in terms of elementary functions.

[...]

The power of z in P can only occur in the term going from the p th term p down.

[...]

$-2x + ny = 0$ is called Legendry equation. We solve the equation with the singular point $x = 1$, we put $x - 1 = u$ and obtain the transformed equation.

We assume the solution point $y = \sum a_k u^k + c$

We are only interested here in the non-logarithmic solution.

[...]

We may now extend the range of this sum by taking k range from 0 to n . This extension will not affect the result, since the added terms are a polynomial of degree less than n and the n th derivative will vanish.

[...]

You have learnt about legendry polynomial and legendry functions in this unit. Read this unit properly before going to the next unit.

Some Examples of Partial Differential Equations

We shall only discuss certain partial differential equations which are used frequently in applied mathematics. In fact, we are going to discuss a kind of boundary value problems which enters modern applied mathematics at every turn. Recognise partial differential equations by type and character explain the methods of solving partial differential equations apply the knowledge in some other related field.

Applied Mathematics

Many linear problems in applied mathematics involve the solution of an equation obtained by specialising the form.

Poisson's Equation

'sheds function' involved in the elastic torsion of prismatic bars, with a suitably prescribed function f .

The Equation of Heat Conduction

$\alpha^2 \frac{\partial}{\partial t}$ This is satisfied, for example, by the temperature at a point of a homogeneous body and by the concentration of a diffused substance in the theory of diffusion, with a suitably prescribed constant θ .

The Telegraphic Equation

This is one dimensional specialisation of , and is satisfied by the potential in a telegraph cable, where $\lambda = Lc$ and $\mu = Rc$, if the Leakage is neglected . Differential equation of higher order, involving the operator Δ^2 , are rather frequently encountered, in particular, the bi-Laplacian equation in two dimensions. ∂_y is involved in many two dimensional problem of the theory of elasticity. The solution of a given problem must satisfy the proper differential equation, together with similarly prescribed boundary condition or initial conditions .

In spherical co-ordinates P, θ, ϕ related to x, y, z by the equations $x = P \sin \theta \cos \phi$, $y = P \sin \theta$, $z = P \cos \theta$.

[...]

$0 \leq x \leq p$ This is a straight bar of uniform cross section and homogenous material. The temperature v can be considered constant on any given cross section. α^2 is a constant known as $v = U$.

In which b is arbitrary

The partial differential equation has now been replaced by two ordinary differential equations. Boundary Conditions $xv = f$ $g = 0$ by , if $g = 0$ then u_x, t will be identically zero. It is not acceptable because it does not satisfy the equation . Thus it must satisfy the $f = 0$.

Applying the boundary condition and , we have $b_1 + b_2 = 0$ $f' b' 1e^{i\lambda x} + b_2 e^{i\lambda x} = 0$

The system has a non-trivial solution $k_1 = 0$ and $k_2 = 0$ always, but it is not acceptable u is identically zero.

Where n is an integer Hence the function $u_n = c_n \exp[-$

The functions u_n are sometimes called fundamental solution of the heat conduction problem

and . Each solution given by satisfies the differential equation and the boundary condition. From the known solutions, $u_1, u_2, \dots, u_n$, we may thus construct others with sufficiently strong convergence condition. It is true that even the infinite series $u = \sum u_n$ or $u = \sum c_n \exp(-\dots)$ is a solution of the differential equation.

We know how to compute b_n . We can satisfy the equation by choosing $c_n = b_n$ for each n .

[...]

We shall express U as the sum of the steady state temperature and another distribution w .

[...]

[when the ends of the bar are insulated so that there is no passage of heat through them].

So $U_n = C_n \exp(-\dots)$

These solution functions satisfy the differential equation and bounding conditions for any value of the constant C_n . We will assume that this is also line for convergent infinite sums of fundamental solution as well. Thus, the unknown coefficients in equation must be coefficients in the Fourier Cosine series of period $2e$ for f .

$X^2 U_{xx} = U_{tt}$

00 , provided h is twice continuously differentiable on.

Where the coefficients K_n are given in the Fourier Sine Series of period

Differentiate with respect to t and putting substitution. Where h is function obtained by extending the initial data f into as an odd function, and other values of x as a periodic function on period $2l$.

Then $h = \sum K_n \sin$

$-\cos U = \sum U_n = \sum \sin C_n \sin + K_n \cos$ Equation h is the function follows on adding the two equations. If U is continuous for $0 < x < l$ and $t > 0$ provided that h is requires that f is continuous on line interval. This requires that f is continuous on the interval and, since h is odd periodic extension of f , that f be zero at $x=0$ and $x=p$.

U is twice continuously differentiable $t=0$, we get $U_t = \sum$

The problem of finding a solution of Laplace equation which takes on given boundary values is known as Dirichlet problem.

If $K=0$ then $X'' = 0$ and $Y'' = 0$, and $U =$

The homogeneous boundary conditions $y=0$ and $y=b$ can be satisfied by $C_1 = C_2 = 0 \Rightarrow U$, is identically zero.

[...]

Again, the boundary condition at $y=0$ and $y=b$ lead to $C_1 = C_2 = 0$, so again U is zero, everywhere. Hence $K = \lambda^2$ is not acceptable.

[...]

Again, we assume that the separation constant must be real.

Suppose $K=0$ $r^2 R'' + rR'\theta = 0$

Further $r \rightarrow 0$ the term $\log r$ is unbounded.

If $K = \lambda^2$ then

This is not acceptable.

The boundary condition then requires that $f = U =$

The function f may be extended outside the interval.

[...]

You have been introduced to partial differential equation in this unit. You are required to study this unit properly because you will refer to it in your subsequent courses in mathematics. In this unit, various forms and types of partial differential equations were studied. You are required to study this unit properly and attempt all the exercises at the end of the unit.



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FACULTY OF SCIENCES

November, 2018

Course Code: CIT342

Course Title: Formal Languages and Automata Theory

Credit Units: 3

Time Allowed: 2½ Hours

Instruction: Answer Question 1 and any other four (4) questions

Question 1

- a) With the aid of a diagram, briefly describe the automata theory (8 marks)
- b) Clearly state the Godel incompleteness theorem (5 marks)
- c) Briefly explain the following terms: (9 marks)

Question 2

- a) List the three ways of defining a language (3marks)
- b) Define the following terms (9 marks)
Finite states ii. Infinite states iii. Stack memory

Question 3

- a) In a formal way, describe an automaton (6 marks)
- b) Explain the Regular Expressions (3 marks)
- c) Define context-sensitive grammars (3 marks)

Question 4

- a) List 6 types of Automata (6 marks)
- b) As touching regular expressions, state the precedence of the following operations relative to one another (3 marks)
- c) When is a grammar recursively enumerable? (3 marks)

Question 5

- a) List and define the two types of Push Down Automata (4 marks)
- b) A formal system can be regarded (i) complete; or (ii) incomplete. Discuss (5 marks)
- c) Considering that an automaton as a computer, state the way(s) it can handle non-determinism (3 marks)

Question 6

- a) Distinguish between regular grammar and context-free grammar (4 marks)
- b) State **two** of the ways of implementing a DFA. (4 marks)
- c) Explain the two types of Push Down Automata (4 marks)