

CIRCLE THEOREM

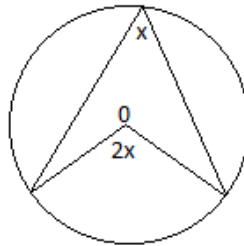
hint:

- Discuss circle properties such as Radius, Diameter, Circumference, Sector, Chords, Segment, and Tangent properties. Then move on to angles in a circle.

Theorems of angles in a Circle.

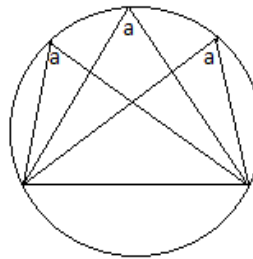
1. Angle at the centre theorem:

- Angle subtended by an arc at the centre is twice the angle subtended at the circumference.



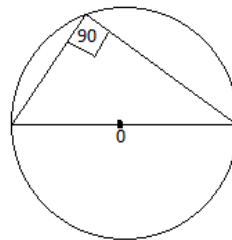
2. Angle in the same segment:

- Angles subtended by the same segment of a circle are equal.



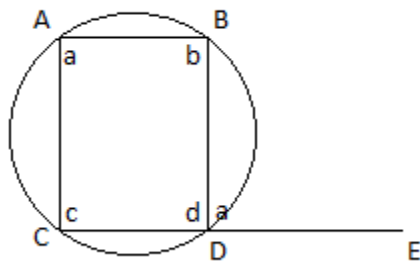
3. Angle in a semi-circle:

- Angle in a semi-circle is 90° .



4. Angles associated with a cyclic-quadrilateral:

- The opposite angles of a cyclic-quadrilateral are supplementary.
- The exterior angle of a cyclic-quadrilateral is equal to the opposite interior angle.



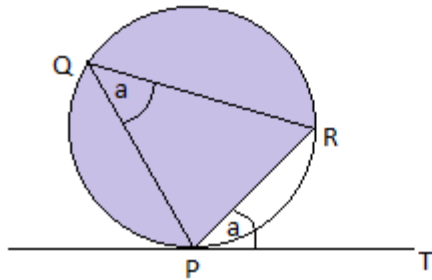
$$a + d = 180$$

$$b + c = 180$$

$$\angle CAB = \angle BDE$$

5. Alternate Segment theorem:

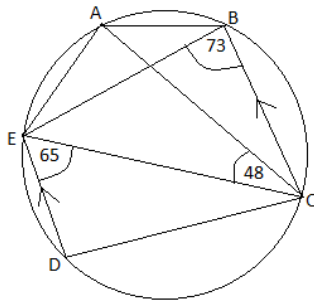
- The angle between a chord and a tangent at the point of contact is equal to any angle in the Alternate Segment.



$$\angle PQR = \angle TPR = a$$

Example

In the diagram, A, B, C, D and E lie on the circumference of a circle.



DE is parallel to CB. Angle ACE = 48° , angle CED = 65° and angle CBE = 73° . Calculate;

- a) $\angle ABE$ b) $\angle ACB$,
c) $\angle BEC$, d) $\angle CDE$

EXPECTED ANSWERS:

1) $DE \parallel CB$ Given

$\angle ACE = 48^\circ$ Given

$$\angle CED = 65^\circ \text{ Given}$$

$$\angle CBE = 73^\circ \quad \text{Given}$$

$$(a) \angle ABE = \angle ACE = 48^\circ \text{ (angles in the same segment)}$$

$$\therefore \angle ABE = 48^\circ$$

$$(b) \angle ACB = \angle BCE - \angle ACE$$

$$\text{Hint: But } \angle BCE = \angle DCE = 65^\circ$$

(alternate angles are equal)

$$\therefore \angle ACB = 65^\circ - 48^\circ$$

$$\therefore \angle ACB = 17^\circ$$

$$(c) \angle BEC + \angle BCE + \angle CBE = 180^\circ \text{ (sum of angles in a triangle)}$$

$$\therefore \angle BEC = 180^\circ - (\angle BCE + \angle CBE)$$

$$= 180^\circ -$$

$$180^\circ - 138^\circ$$

$$\therefore \angle BEC = 42^\circ$$

$$(d) \angle CDE + \angle CBE = 180^\circ \text{ (opposite angles of a cyclic quadrilateral)}$$

$$\therefore \angle CDE = 180^\circ -$$

$$\angle CDE = 180^\circ - 73^\circ$$

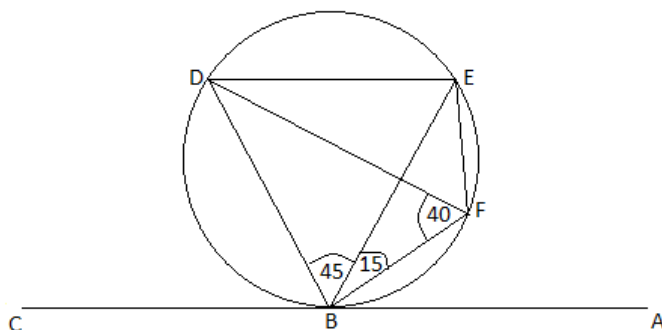
$$\therefore \angle CDE = 107^\circ$$

ACTIVITY 2

In the diagram below, ABC is a tangent to the circle BDEF at B, angle DFB = 40° ,

angle EBF = 15°

and angle DBE = 45° .



Find (a) angle DEB,

(b) angle DEF,

(c) angle ABF.

EXPECTED ANSWERS:

(a) angle DEB = angle DFB = **40°** (angles in the same segment)

(b) angle DEF + angle DBF = 180°

$\therefore \text{angle DEF} + 60^\circ = 180^\circ$ (opposite angles of a cyclic quadrilateral)

$\therefore \text{angle DEF} = 180^\circ - 60^\circ$

$\therefore \text{angle DEF} = 120^\circ$

(c) angle ABF = angle BDF (alternate segment theory)

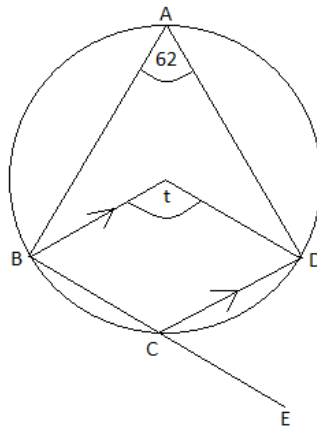
$= 180^\circ - (45^\circ + 15^\circ + 40^\circ)$

$\therefore \text{angle ABF} = 80^\circ$.

ACTIVITY 3

In the diagram below, A, B, C and D lie on the circumference of the circle, center O.

BO is parallel to CD, angle BAD = 62° and BCE is a straight line.



Calculate:

- (a) angle t,
- (b) angle BCD,
- (c) angle OBC,
- (d) angle DCE.

EXPECTED ANSWERS:

(a) angle t = $2 \times \text{angle BAD}$ (angle at the centre = $2 \times$ angle on the circumference)
 $\therefore \text{angle t} = 2 \times 62^\circ$

$\therefore \text{angle t} = 124^\circ$.

(b) angle BCD + angle BAD = 180° (opp. angles of a cyclic quad.)

$\therefore \text{angle BCD} = 180^\circ - 62^\circ$

$$\therefore \text{angle BCD} = 118^\circ.$$

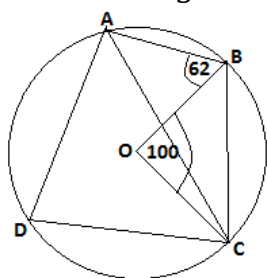
(c) angle $\text{OBC} = 180^\circ - 118^\circ$ (angles associated with parallel lines)

$$\therefore \text{angle OBC} = 62^\circ$$

(d) angle $\text{DCE} = \text{angle BAD} = 62^\circ$ (exterior angle of cyclic quad = opposite interior angle)

PRACTICE QUESTIONS:

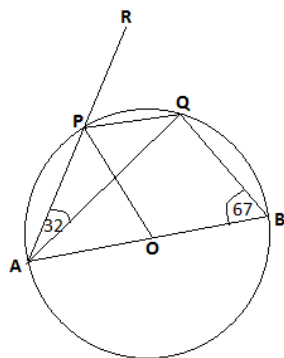
(1) O is the centre of the circle through A, B, C and D. Angle $\text{BOC} = 100^\circ$ and angle $\text{OBA} = 62^\circ$.



Calculate

(i) $\angle \text{BAC}$, (ii) $\angle \text{OCB}$, (iii) $\angle \text{ADC}$.

(2) In the diagram, AB is a diameter of the circle, centre O. P and Q are two points on the circle and APR is a straight line. Given that angle $\text{QBA} = 67^\circ$ and angle $\text{PAQ} = 32^\circ$,

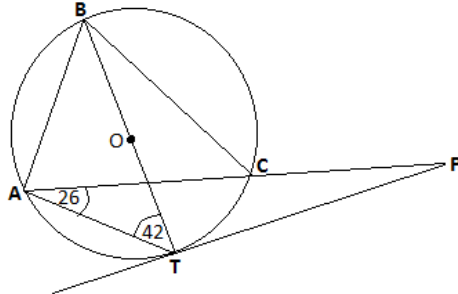


calculate

- (a) angle QAB ,
- (b) angle RPQ ,
- (c) angle POB .

(3) BT is a diameter of a circle and A and C are points on the circumference. The tangent to the circle at the point T meets AC produced at P.

Given that angle $ATB = 42^\circ$ and angle $CAT = 26^\circ$,



calculate

- (i) angle CBT,
- (ii) angle ABT,
- (iii) angle APT.

ANSWERS:

- (1) (i) 50° , (ii) 40° , (iii) 78° .
- (2) (a) 23° , (b) 67° , (b) 110° .
- (3) (i) 26° , (ii) 48° , (iii) 22°

Type equation here.