

CHAPTER FOUR

Fundamental Principles of Traffic Flow

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4.1 Introduction

Traffic flow theory involves the development of mathematical relationships among the primary elements of a traffic stream: flow, density, and speed. These relationships help the traffic engineer in planning, designing, and evaluating the effectiveness of implementing traffic engineering measures on a highway system. Traffic flow theory is used in design to determine adequate lane lengths for storing left-turn vehicles on separate left-turn lanes, the average delay at intersections and freeway ramp merging areas, and changes in the level of freeway performance due to the installation of improved vehicular control devices on ramps. Another important application of traffic flow theory is simulation, where mathematical algorithms are used to study the complex interrelationships that exist among the elements of a traffic stream or network and to estimate the effect of changes in traffic flow on factors such as accidents, travel time, air pollution, and gasoline consumption.

4.2 Traffic Flow Elements

Let us first define the elements of traffic flow before discussing the relationships among them. Before we do that, though, we will describe the time-space diagram, which serves as a useful device for defining the elements of traffic flow.

The time-space diagram is a graph that describes the relationship between the location of vehicles in a traffic stream and the time as the vehicles progress along the highway.

Figure 6.1 ■ Time-Space Diagram

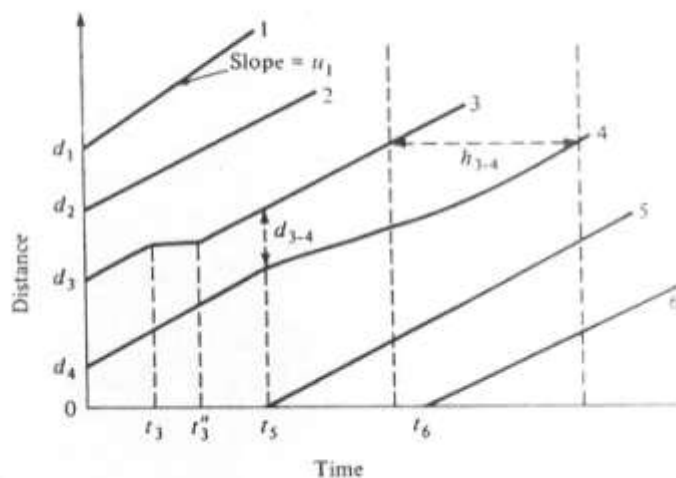


Figure 6.1 shows a time-space diagram for six vehicles, with distance plotted on the vertical axis and time on the horizontal axis. At time zero, vehicles 1, 2, 3, and 4 are at respective distances d_1 , d_2 , d_3 , and d_4 from a reference point, whereas vehicles 5 and 6, cross the reference point later at times t_5 and t_6 respectively.

The primary elements of traffic flow are **flow**, **density**, and **speed**. Another element associated with density, is the **gap** or **headway** between two vehicles in a traffic

The definitions of these elements follow.

Flow (q) is the equivalent hourly rate at which vehicles pass a point on a highway during a time period less than 1 hr. It can be determined by

$$q = \frac{n * 3600}{T} \text{ vph}$$

Where: n = the number of vehicles passing a point in the roadway in T secs; q = the equivalent hourly flow.

Density (k), sometimes referred to as **concentration**, is the number of vehicles traveling over a unit length of highway at an instant in time. The unit length is usually 1 mile thereby making vehicles per mile (vpm) the unit of density.

Speed (u) is the distance traveled by a vehicle during a unit of time. It can be expressed in miles per hour (mph), kilometers per hour (km/h), or feet per second (ft/sec). The speed of a vehicle at any time t is the slope of the time-space diagram for that vehicle at time t . Vehicles 1 and 2 in Figure 6.1, for example, are moving at a constant speed because the slopes of the associated graphs are constant. Vehicle 3 moves at a constant speed between time zero and time t_3 , then stops for the period t_3 to t'_3 (the slope of the graph equals zero), and then accelerates and eventually moves at a constant speed.

There are two types of mean speeds: **time mean speed** and **space mean speed**.

Time mean speed ($\bar{u}_t$) is the arithmetic mean of the speeds of vehicles passing a point on a highway during an interval of time. The time mean speed is found by

$$\bar{u}_t = \frac{1}{n} \sum_{i=1}^n u_i$$

Where: n = number of vehicles passing a point on the highway; u_i = speed of the i th vehicle (ft/sec)

Space mean speed ($\bar{u}_s$) is the harmonic mean of the speeds of vehicles passing a point in a highway during an interval of time. It is obtained by dividing the total distance traveled by two or more vehicles on a section of highway by the total time required by these vehicles to travel that distance. This is the speed that is involved in flow-density relationships.

The space mean speed is found by

$$\begin{aligned}\bar{u}_t &= \frac{n}{\sum_{i=1}^n (1/u_i)} \\ &= \frac{nL}{\sum_{i=1}^n t_i}\end{aligned}$$

Where: $\bar{u}_s$ = space mean speed (ft/sec); n = number of vehicles; t_i = the time it takes the i th vehicle to travel across a section of highway (sec); U_i = speed of the i th vehicle (ft/sec); L = length of section of highway (ft)

Time headway (h) is the difference between the time the front of a vehicle arrives at a point on the highway and the time the front of the next vehicle arrives at that same point. Time headway is usually expressed in seconds. For example, in the time-space diagram (Figure 6.1), the time headway between vehicles 3 and 4 at d_1 is h_{3-4} .

Space headway (d) is the distance between the front of a vehicle and the front of the following vehicle. It is usually expressed in feet. The space headway between vehicles 3 and 4 at time t_5 is d_{3-4} (see Figure 6.1).

4.3 Flow-Density Relationships

The general equation relating flow, density, and space mean speed is given as

- Flow = (density) x (space mean speed)

$$q = k * \bar{u}_s$$

Each of the variables in Eq. 6.5 also depends on several other factors, including the characteristics of the roadway, the characteristics of the vehicle, the characteristics of the driver, and environmental factors such as the weather.

Other relationships that exist among the traffic flow variables are given below.

- Space mean speed = (flow) x (space headway)

$$\bar{u}_s = q * \bar{d}$$

Where: $\bar{d} = (1/k) =$ average space headway

- Density = (flow) x (travel time for unit distance)

$$k = q * \bar{t}$$

Where:

$\bar{t}$ is the average time for unit distance

Average space headway = (space mean speed) x (average time headway)

$$\bar{d} = \bar{u}_s * \bar{h}$$

Average time headway = (average travel time for unit distance) x (average space headway)

$$\bar{h} = \bar{t} * \bar{d}$$

Fundamental Diagram of Traffic Flow

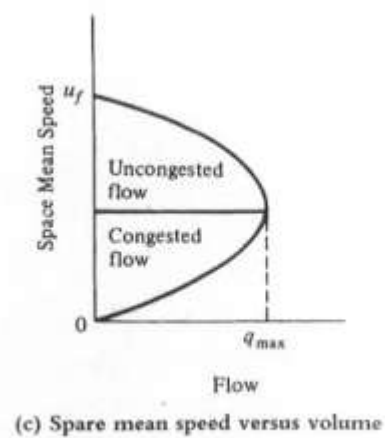
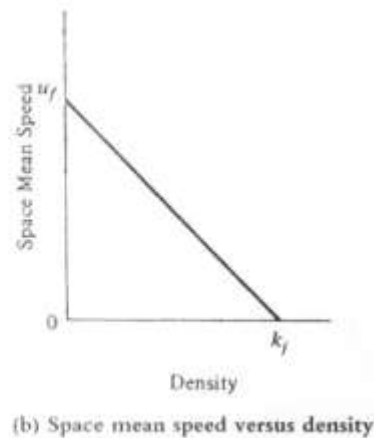
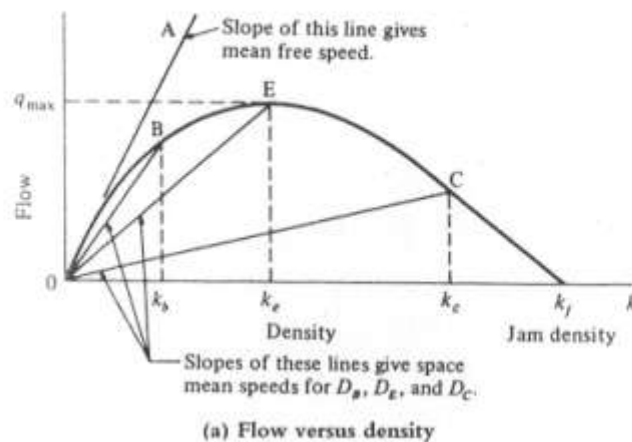
The relationship between the density (vpm) and the corresponding flow of traffic on a highway generally is referred to as the fundamental diagram of traffic flow. The following theory has been postulated with respect to the shape of the curve depicting this relationship.

1. When the density on the highway is zero, the flow is also zero because there are no vehicles on the highway.
2. As the density increases, the flow also increases.
3. However, when the density reaches its maximum, generally referred to as the jam density (k_j), the flow must be zero because vehicles will tend to line up end to end.
4. It follows that as density increases from zero, the flow will also initially increase from zero to a maximum value. Further continuous increase in density will result in continuous reduction of the flow, which will eventually be zero when the density is equal to the jam density. The shape of the curve therefore takes the form in Figure 6.3a.

A similar argument can be postulated for the general relationship between the space mean speed and the flow. When the flow is very low, there is little interaction between individual vehicles. Drivers are therefore free to travel at the maximum possible speed. The absolute maximum speed is obtained as the flow tends to zero, and it is known as the mean free speed (U_f). Continuous increase in flow will result in a continuous decrease in speed. A point will be reached, however, when further addition of vehicles will result in the reduction of the actual number of vehicles that pass a point on the highway (that is, reduction of flow). This results in congestion, and eventually both the speed and the flow become zero. Figure 6.3c shows this general relationship. Figure 6.3b shows the direct relationship between speed and density.

The slope of line OA is the speed as the density tends to zero and little interaction exists between vehicles. The slope of this line is therefore the mean free speed (U_f); it is the maximum speed that can be attained on the highway. The slope of line OE is the space mean speed for maximum flow. This maximum flow is the capacity of the highway. Thus it can be seen that it is desirable for highways to operate at densities not greater than that required for maximum flow.

Figure 6.3 ■ Fundamental Diagrams of Traffic Flow



Macroscopic Approach

The macroscopic approach considers traffic streams and develops algorithms that relate the flow to the density and space mean speeds. The two most commonly used macroscopic models are the Green shields and Greenberg models.

Green shields Model. Green shields carried out one of the earliest recorded works, in which he studied the relationship between speed and density. He hypothesized that a linear relationship existed between speed and density, which he expressed as

$$\bar{u}_s = u_f - \frac{u_f}{k_j} * k$$

Corresponding relationships for flow and density and for flow and speed can be developed. Since $q = \bar{u}_s k$ substituting $q / \bar{u}_s$, for k in equation gives

$$\bar{u}_s^2 = u_f \cdot \bar{u}_s - \frac{u_f}{k_j} * q$$

Also substituting q / k for $\bar{u}_s$ gives

$$q = u_f \cdot k - \frac{u_f}{k_j} * k^2$$

The above equations indicate that a linear relationship and is assumed for speed and density, then parabolic relationships are obtained between flow and density and between flow and speed. The shape of the curve shown will therefore be a parabola. The following equations can be used to determine the corresponding speed and the corresponding density for maximum flow.

$$\bar{u}_s^2 = u_f \cdot \bar{u}_s - \frac{u_f}{k_j} * q$$

Differentiating q with respect to $\bar{u}_s$ we obtain

$$2\bar{u}_s = u_f - \frac{u_f}{k_j} \frac{dq}{d\bar{u}_s}$$

That is,

$$\frac{dq}{d\bar{u}_s} = u_f \frac{k_j}{u_f} - 2\bar{u}_s \frac{k_j}{u_f} = k_j - 2\bar{u}_s \frac{k_j}{u_f}$$

For maximum flow,

$$\frac{dq}{d\bar{u}_s} = 0 \Rightarrow k_j = 2\bar{u}_s \frac{k_j}{u_f} \Rightarrow u_o = \frac{u_f}{2}$$

Thus, the space mean speed u_o at which the volume is maximum is equal to half the free mean speed.

$$q = u_f \cdot k - \frac{u_f}{k_j} * k^2$$

Differentiating q with respect to k , we obtain

$$\frac{dq}{dk} = u_f - 2k \frac{u_f}{k_j}$$

For maximum flow,

$$\frac{dq}{dk} = 0 \Rightarrow u_f = 2k \frac{u_f}{k_j} \Rightarrow k_o = \frac{k_j}{2}$$

Thus, at the maximum flow, the density k is half the jam density. The maximum flow for the Greenshields relationship can therefore be obtained from following equation.

$$q_{\max} = \frac{k_j u_f}{4}$$

Greenberg Model. Several researchers have used the analogy of fluid flow to develop macroscopic relationships for traffic flow. One of the major contributions using the fluid-flow analogy was developed by Greenberg in the form of:

$$\bar{u}_s = c \ln \frac{k_j}{k}$$

$$q = ck \ln \frac{k_j}{k}$$

Differentiating q with respect to k , we obtain

$$\frac{dq}{dk} = c \ln \frac{k_j}{k} - c$$

For maximum flow,

$$\frac{dq}{dk} = 0, \ln \frac{k_j}{k} = 1$$

Giving

$$\ln k_j = 1 + \ln k_o$$

4.4 Gap and Gap Acceptance

Thus far we have been considering the theory of traffic flow as it relates to the flow of vehicles in a single stream. Another important aspect of traffic flow is the interaction of vehicles as they join, leave, or cross a traffic stream. Examples of these include ramp vehicles merging onto an expressway stream, freeway vehicles leaving the freeway onto frontage roads, and the changing of lanes by vehicles on a multilane highway. The most important factor a driver considers in making any one of these maneuvers is the availability of a gap between two vehicles that, in the driver's judgment, is adequate for him or her to complete the maneuver. The evaluation of available gaps and the decision to carry out a specific maneuver within a particular gap are inherent in the concept of gap acceptance.

Following are the important measures that involve the concept of gap acceptance.

Merging is the process by which a vehicle in one traffic stream joins another traffic stream moving in the same direction, such as a ramp vehicle joining a freeway stream.

Diverging is the process by which a vehicle in a traffic stream leaves that traffic stream, such as a vehicle leaving the outside lane of an expressway.

Weaving is the process by which a vehicle first merges into a stream of traffic, obliquely crosses that stream, and then merges into a second stream moving in the same direction; for example, the maneuver required for a ramp vehicle to join the far side stream of flow on an expressway.

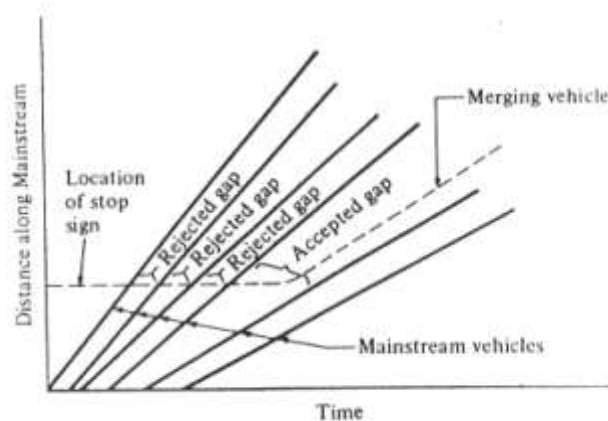
Gap is the headway in a major stream, which is evaluated by a vehicle driver in a minor stream who wishes to merge into the major stream. It is expressed either in units of time (time gap) or in units of distance (space gap).

Time lag is the difference between the time a vehicle that merges into a main traffic stream reaches a point on the highway in the area of merge and the time a vehicle in the main stream reaches the same point.

Space lag is the difference, at an instant of time, between the distance a merging vehicle is away from a reference point in the area of merge and the distance a vehicle in the main stream is away from the same point.

Figure 6.9 depicts the time-distance relationships for a vehicle at a stop sign waiting to merge and for vehicles on the near lane of the main traffic stream.

Figure 6.9 ■ Time-Space Diagrams for Vehicles in the Vicinity of a Stop Sign



A driver who intends to merge must first evaluate the gaps that become available to determine which gap (if any) is large enough to accept the vehicle, in his or her opinion. In accepting that gap, the driver feels that he or she will be able to complete the merging maneuver and safely join the main stream within the length of the gap. This phenomenon generally is referred to as **gap acceptance**. It is of importance when engineers are considering the delay of vehicles on minor roads wishing to join a major-road traffic stream at un-signalized intersections, and also the delay of ramp vehicles wishing to join expressways. It can also be used in timing the release of vehicles at an on-ramp of an expressway, such that the probability of the released vehicle finding an acceptable gap in arriving at the freeway shoulder lane is maximum.

4.5 Queuing Analysis

One of the major issues in the analysis of any traffic system is the analysis of delay. Delay is a more subtle concept. It may be defined as the difference between the actual travel time on a given segment and some ideal travel time of that segment. This raises the question as to what is the ideal travel time. In practice, the ideal travel time chosen will depend on the situation; in general, however, there are two particular travel times that seem best suited as benchmarks for comparison with the actual performance of the system. These are the travel time under free flow conditions and travel time at capacity. Most recent research has found that for highway systems, there is comparatively little difference between these two speeds. That being the case, the analysis of delay normally focuses on delay that results when demand exceeds its capacity; such delay is known as queuing delay, and may be studied by means of queuing theory. This theory involves the analysis of what is known as a queuing system, which is composed of a server; a stream of customers, who demand service; and a queue, or line of customers waiting to be served. The Fig. 1 shows a schematic diagram illustrating the concept of a queuing system.

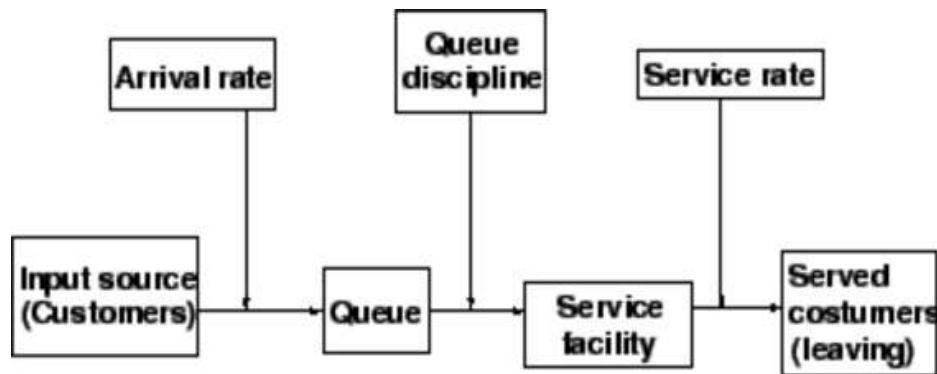


Figure: Components of a basic queuing system

For the analysis of a queuing system the input parameters are discussed in the following section.

Input parameters

- Mean arrival rate (λ)
- Mean service rate (μ)
- The number of servers (N)
- Queue discipline

Mean arrival rate is rate at which customers arrive at a service facility. It is expressed in flow (customers/hr or vehicles/hour) or time headway (seconds/customer or seconds/vehicle). If inter arrival time that is time headway (h) is known, the arrival rate can be found out from the equation: $\lambda = \frac{3600}{h}$ where λ is in veh/hr

Mean arrival rate can be specified as a deterministic distribution or probabilistic distribution and sometimes demand or input are substituted for arrival.

Mean service rate is the rate at which customers/vehicles depart from a transportation facility. It is expressed in flow (customers/hr or vehicles/hour) or time headway (seconds/customer or seconds/veh). If inter service time that is time headway (h) is known, the service rate can be found out from the equation: $\mu = \frac{3600}{h}$ where μ is in veh/hr

The number of servers that are being utilized should be specified and in the manner they work that is they work as parallel servers or series servers has to be specified.

Queue discipline is a parameter that explains how the customers arrive at a service facility.

The various types of queue disciplines are

- **First in first out (FIFO)** If the customers are served in the order of their arrival, then this is known as the first-come, first-served (FCFS) service discipline. Prepaid taxi queue at airports where a taxi is engaged on a first-come, first-served basis is an example of this discipline.
- **First in last out (FILO)** Sometimes, the customers are serviced in the reverse order of their entry so that the ones who join the last are served first. For example, the people who join an elevator first are the last ones to leave it.
- **Served in random order (SIRO)** under this rule customers are selected for service at random, irrespective of their arrivals in the service system. In this every customer in the queue is equally likely to be selected. The time of arrival of the customers is, therefore, of no relevance in such a case.
- **Priority scheduling** under this rule customers are grouped in priority classes on the basis of some attributes such as service time or urgency or according to some identifiable characteristic, and FIFO rule is used within each class to provide service. Treatment of VIPs in preference to other patients in a hospital is an example of priority service.