CHAPTER-3
Image Measurement

Introduction

1. Introduction

- The solution of most photogrammetric problems generally requires some type of photographic measurement. For certain problems the measurements may simply be the lengths of lines between imaged points.
- However, rectangular coordinates of imaged points are the most common type of photographic measurement, and they are used directly in many photogrammetric equations.
- Photographic measurements are usually made on positives printed on paper, film, or glass, or in digital images manipulated on a computer.
- Measurement is done on positive paper most commonly, rarely on negative.

2. Coordinate System for Image Measurement

- For metric cameras with side fiducial marks, the commonly adopted reference system for photographic coordinates, is the rectangular axis system formed by joining opposite fiducial marks with straight lines,
- Rectangular coordinates are a very basic and useful type of photographic measurement. They are used in many different types of computations. As an example, they can be used to calculate the photo distances between points by using simple analytic geometry. Photographic distance ab on fig.3.1.can be determined (see on next page)

2. Coordinate System for Image Measurement

- The x axis is usually arbitrarily designated as the fiducial line
- most nearly parallel with the direction of flight, positive in the direction of flight.
- The positive y axis is 90° counterclockwise from positive x.
- The origin of the coordinate is the intersection of fiducial lines. This point is often called the indicated principal point.
- Image distance ab:

$$ab = \sqrt{(x_b - x_a)^2 + (y_b - y_a)^2}$$

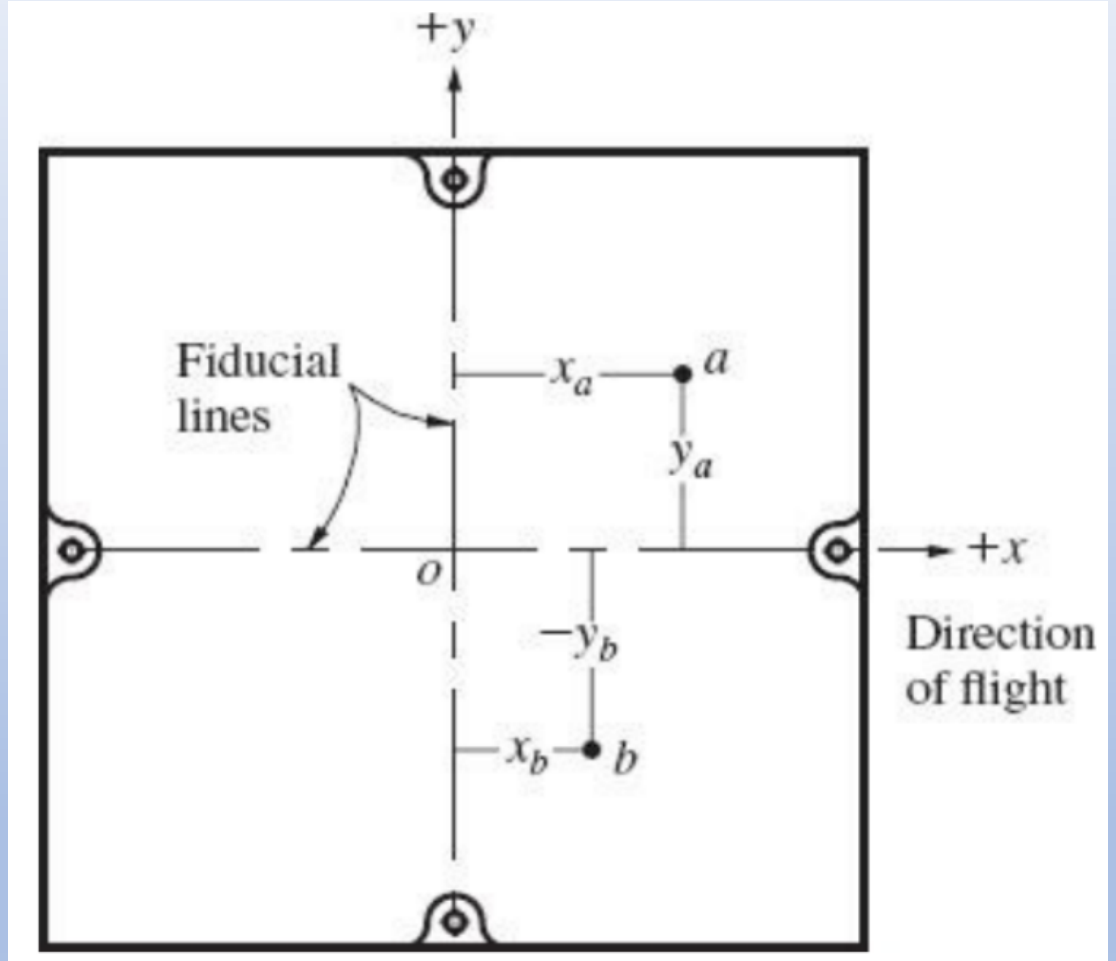


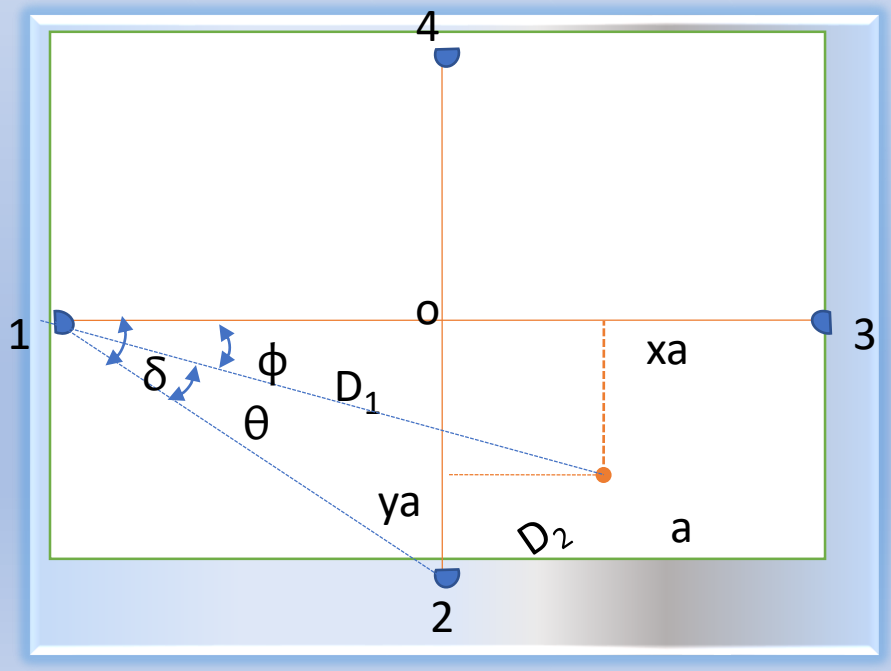
Fig.3.1:Photographic Coordinate system based on side fiducials.

3. Simple Scale for Photographic Measurement

- Variety of simple scales
- Depend on accuracy required
  - Low order accuracy
    - Engineer’s scale(metric +English Units)
    - Precision & accuracy can be enhanced by magnifying glasses
  - High order accuracy
    - Glass scale(fine graduation + magnifying eyepiece)
    - 0.1mm graduation

4. Trilaterative Method of Photo Coordinate Measurement

- Distances such as D1, D2 etc can be measured from fiducial marks to image point
- Fiducial marks coordinate can be obtained from camera calibration
- Image point coordinate can be calculated



$$D_{12} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\cos\theta = \frac{D_1^{\,2} + D_{12}^{\,2} - D2^2}{2D_1D_2}$$

$$\theta = \cos^{-1}\left(\frac{D_1^{\,2} + D_{12}^{\,2} - D2^2}{2D_1D_2}\right)$$

$$\delta = \tan^{-1}\left(\frac{\overline{o2}}{\overline{o1}}\right)$$

$$\phi = \delta - \theta$$

$$x_a = x_1 + D_1\cos\phi$$

$$y_a = y_1 + D_1\sin\phi$$

Refinement of Measured Image Coordinates

- Because of several effects, there will be systematic errors associated with practically all photographic measurements.
- Corrections may be applied to eliminate the effects of these systematic errors.

➢ Shrinkage Correction

➢ Reduction to Principal Point

➢ Lens Distortion Correction

Shrinkage Correction

Determined by Comparing

- Measured Photographic distance between Opposite fiducial marks
- Corresponding values determined in camera Calibration.

- The ratio $\left(\frac{x_c}{x_m}\right)$ is scale factor in x-direction.

- The ratio $\left(\frac{y_c}{y_m}\right)$ is scale factor in y-direction

$$x'_a = \left(\frac{x_c}{x_m}\right) x_a$$

$$y'_a = \left(\frac{y_c}{y_m}\right) y_a$$

Where

$x'_a, y'_a = Corrected\ image\ coordinate$

$x_a, y_a = measured\ image\ coordinate$

$x_c, y_c = Calibrated\ fiducial\ distance$

$x_m, y_m = Measured\ fiducial\ distance$

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Shrinkage Corr. Example

- On a Paper-print positive, the measured distance between fiducials (1 & 3) was 226.4mm & y between fiducials (2&4) was 225.693mm. These x and y distances determined in camera calibration were 225.433 & 225.693mm, respectively. Calculate shrinkage corrected coordinates of points a,b, and cwhose coordinates were measured on the paper print as follows:

Point	x,mm	y,mm	Corrt'd x,mm	Corrt'd y,mm
a	20.3	-92.1	*20.21329*	*-92.3837*
b	48.6	85.8	*48.39242*	*86.06426*
c	-111.1	-102.5	*-110.625*	*-102.816*

Solution:

$$x'_a = \left(\frac{x_c}{x_m}\right) x_a$$

$$y'_a = \left(\frac{y_c}{y_m}\right) y_a$$

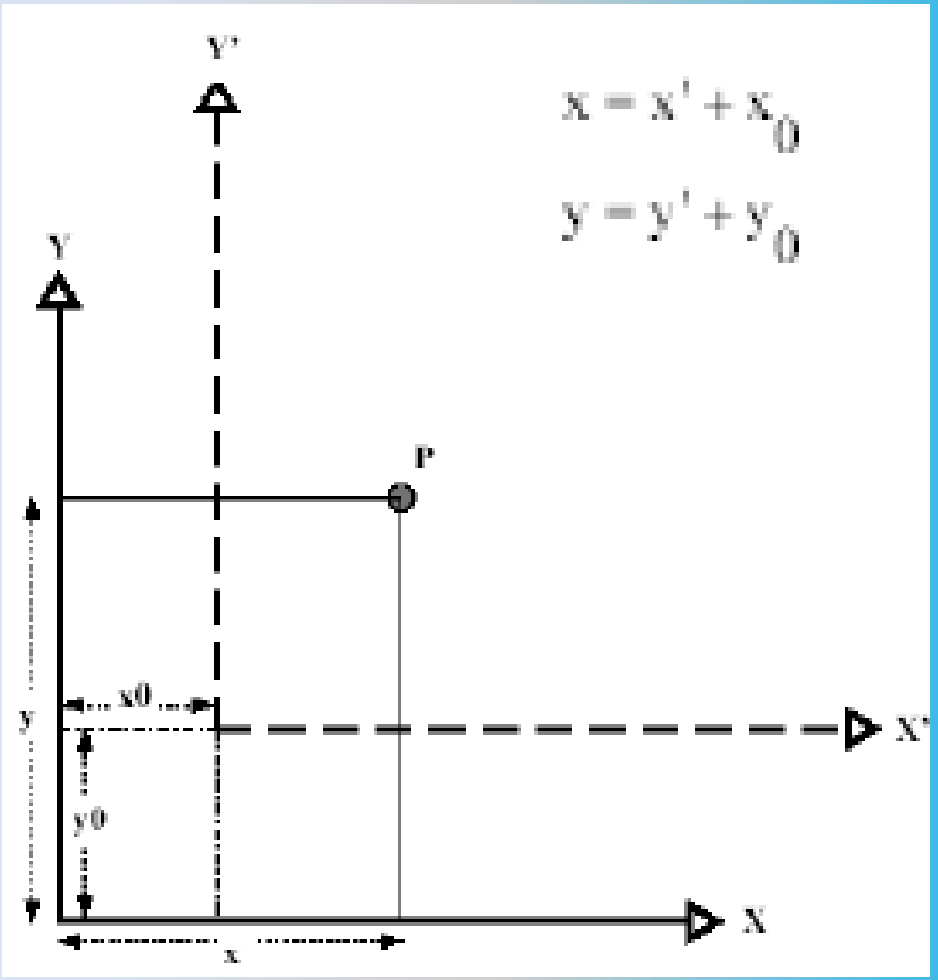
$$x'_i = \left(\frac{225.433}{226.4}\right) x_i = \mathbf{0.995729}x_i$$

$$y'_i = \left(\frac{225.693}{225.0}\right) y_i = \mathbf{1.00308}y_i$$

Reduction of Coordinates to an Origin at the Principal Point

- Photogrammetric equation assume an origin of photo coordinates at the **Principal Point(PP)**.

- Measured coordinates(from fiducial axis system) is reduced to axis whose origin is PP



Correction for Lens Distortion

- Causes imaged positions to be displaced from their ideal position
- In Modern Precision aerial camera distortion is **<5µm.**
- Metric camera has symmetrical radial lens distortion.
- Approach: lens distortion fit polynomial curve function.

$$\Delta r = k_1 r^1 + k_2 r^3 + k_3 r^5 + k_4 r^7$$

where:

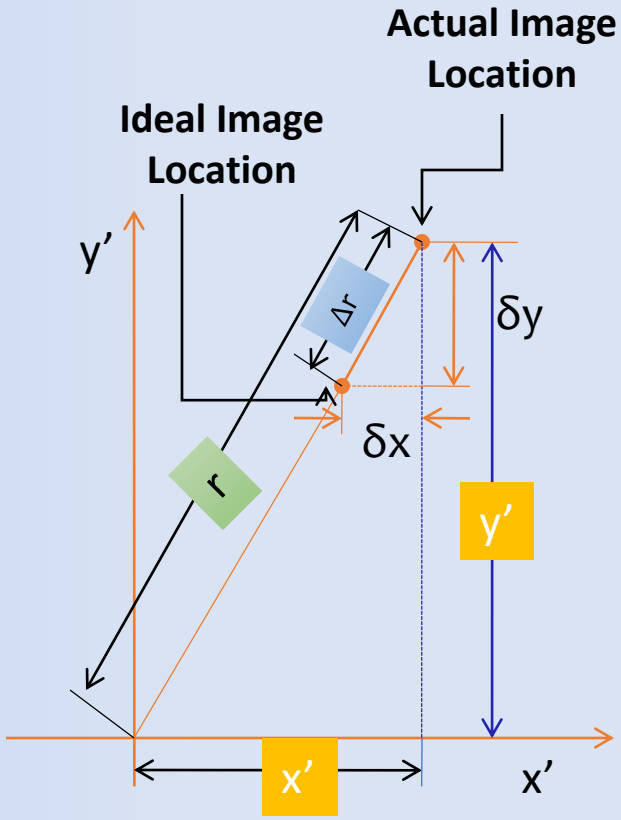
$\Delta r - amount\ of\ radial\ distortion$

$r - radial\ dis\tan ce\ from\ PP$

$k_1, k_2, k_3, \& k_4 - coefficients\ [calculated\ by\ least\ squares]$

Lens Distortion

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- Radial lens distortion

$$\Delta r = k_1 r^1 + k_2 r^3 + k_3 r^5 + k_4 r^7$$

The Components of Δr are δx & δy.

$$\frac{\Delta r}{r} = \frac{\delta_x}{x'} = \frac{\delta_y}{y'}$$

$$\delta_x = x' \frac{\Delta r}{r}$$

$$\delta_y = y' \frac{\Delta r}{r}$$

The Coordinate x_c and y_c

$$x_c = x' - \delta_x$$

$$y_c = y' - \delta_y$$

Example

A camera calibration report specifies the calibrated focal length f=153.206mm and coordinates of the calibrated principal point as x_o =0.008mm & y_o =-0.001mm. The report also lists mean radial lens distortion values given in column (a) and (b) of table below. Using these calibration values, compute the corrected coordinates for an image point having coordinates x=62.579mm, y=-80.916 relative to fiducial axes.

(a) Field angle(θ),°	(b) Δr, mm	(c) r,m
7.5	0.004	0.0202
15	0.007	0.0411
22.5	0.007	0.0635
30	0.001	0.0885
35	-0.003	0.1075
40	-0.004	0.1286

- Solution
- Compute r & write in column (c)
  - $r = f \tan\theta$
  - $r_{7.5} = 153.206 \tan 7.5$
    $= 0.0202m$
  - $r_{15} = 153.206 \tan 15$
    $= 0.0411m$

$$0.004 = 0.0202k_1 + 0.0202^3 k_2 + 0.0202^5 k_3 + 0.0202^7 k_4$$
$$0.004 = 0.0202k_1 + 0.0202^3 k_2 + 0.0202^5 k_3 + 0.0202^7 k_4$$
$$0.007 = 0.0411k_1 + 0.0411^3 k_2 + 0.0411^5 k_3 + 0.0411^7 k_4$$
$$0.007 = 0.0635k_1 + 0.0635^3 k_2 + 0.0635^5 k_3 + 0.0635^7 k_4$$
$$0.001 = 0.0885k_1 + 0.0885^3 k_2 + 0.0885^5 k_3 + 0.0885^7 k_4$$
$$-0.003 = 0.1073k_1 + 0.1073^3 k_2 + 0.1073^5 k_3 + 0.1073^7 k_4$$
$$-0.004 = 0.1286k_1 + 0.1286^3 k_2 + 0.1286^5 k_3 + 0.1286^7 k_4$$

- **In Matrix form:**

$$\underbrace{\begin{pmatrix} 0.0202 & 0.0202^3 & 0.0202^5 & 0.0202^7 \\ 0.0411 & 0.0411^3 & 0.0411^5 & 0.0411^7 \\ 0.0635 & 0.0635^3 & 0.0635^5 & 0.0635^7 \\ 0.0885 & 0.0885^3 & 0.0885^5 & 0.0885^7 \\ 0.1073 & 0.1073^3 & 0.1073^5 & 0.1073^7 \\ 0.1286 & 0.1286^3 & 0.1286^5 & 0.1286^7 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} 0.004 \\ 0.007 \\ 0.007 \\ 0.001 \\ -0.003 \\ -0.004 \end{pmatrix}}_{L}$$

$$\boldsymbol{X} = \left(\boldsymbol{A}^{\boldsymbol{T}} \boldsymbol{A}^{-1}\right)\left(\boldsymbol{A}^{\boldsymbol{T}} \boldsymbol{L}\right)$$

$$\begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{pmatrix} = \begin{pmatrix} 0.2296 \\ -0.3589 \\ 1018 \\ 12100 \end{pmatrix}$$

- Compute r from PP
  - Determine coordinates in refernce to PP
  - x’=x-xo; y’=y-yo
  - R=0.1023m

Solution con’t

- Compute Δr
  - Δr=-0.0021mm
- Compute δx & δy
  - δx= -0.0013mm
  - δy= 0.0017mm
- Compute Corrected Coordinate xc & yc
  - Xc=62.572mm
  - yc=-80.917mm