ADDIS ABABA SCINCE AND TECHNOLOGY UNIVERSITY
APPLIED MATHEMATICS III
LINE AND SURFACE INTEGRALWORKSHEET

1) Evaluate $\int_C (2 + x^2 y)ds$ where C is the upper half of the unit circle $x^2 + y^2 = 1$.
2) Evaluate $\int_C (2x)ds$ where C = $C_1 \cup C_2$, C_1 being the arc of the parabola $y = x^2$ between (0, 0) and (1, 1) and C_2 being the vertical line from (1, 1) to (1, 2).
3) Find the work done by F (x, y, z) = (3y, 2x, 4z) over each path below.
   a) C_1 given by $\vec{r}(t) = (t, t, t), 0 \le t \le 1$
   b) C_2 given by $\vec{r}(t) = (t, t^2, t^4), 0 \le t \le 1$
4) Evaluate $\int_C \vec{F}.ds$
   a) $\vec{F} = yi + xj$, C is the graph $y = x^2$ from (1, 1) to (−2, 4).
   b) $\vec{F} = (y + x)i + (x - y)j$, C is given by $x = t^2, y = t^3, -1 \le t \le 1$.
5) Use the Fundamental Theorem of the Line Integral to evaluate:
   a) $\int_C y\sin(xy)dx + x\sin(xy)dy$; if C runs form (1, 2) to (3, 4)
   b) $\int_C \frac{x}{1+x^2+y^2+z^2}dx + \frac{y}{1+x^2+y^2+z^2}dy + \frac{z}{1+x^2+y^2+z^2}dz$; if C is parameterized by $\vec{r}(t) = ti + t^2 j + t^4 k$ for $0 \le t \le 1$.
6) Verify Green's Theorem for $\int_C [xy + y^2)dx + x^2 dy]$, where C is the boundary of the region bounded by y = x & $y = x^2$.
7) Use Green's Theorem to evaluate $\int_C [y - \sin x dx + \cos x dy]$, where C is the plane triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}$ and $y = \frac{2}{\pi}x$.
8) Suppose Σ is part of the paraboloid $z = 1 - x^2 - y^2$ that lies above the xy-plane and is oriented by the normal directed upward. The velocity of a fluid with constant density δ is given by $\vec{V}(x, y, z) = xi + yi + 2zk$. Determine the rate of mass flow through the surface Σ in the direction of $\vec{n}$.
9) Use Stokes's Theorem to evaluate $\int_C (x + y)dx + (2x - z)dy + (y + z)dz$, where C is the boundary of ΔABC with vertices A = (2, 0, 0), B = (0, 3, 0) and C = (0, 0, 6).
10) Use the Divergence theorem to evaluate $\iint_\Sigma \vec{F}.\vec{n}ds$, if $\vec{F} = 4xi - 2y^2 j + z^2 k$ and Σ is the surface bounding the region $x^2 + y^2 = 4, z = 0$ and $z = 3$.