CHAPTER TWO

STRESS CALCULATION: DESIGN FOR STATIC LOAD; DESIGN FOR FATIGUE LOAD

1. Principal Stresses for a Member Subjected to Bi-axial Stress

When a member is subjected to bi-axial stress (i.e. direct stress in two mutually perpendicular planes accompanied by a simple shear stress), then the normal and shear stresses are obtained as below

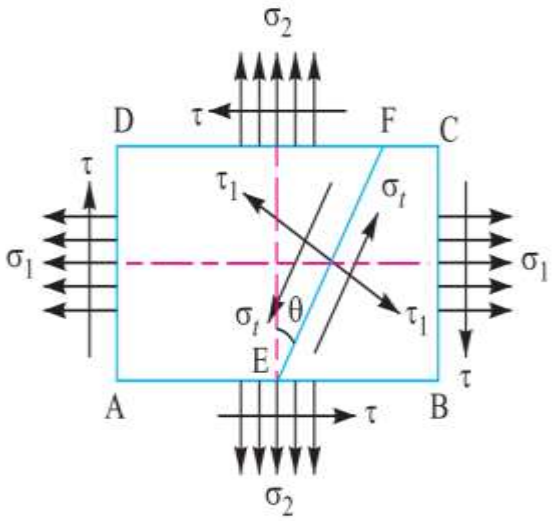


Fig A) Direct stress in two mutually perpendicular planes accompanied by a simple shear stress.

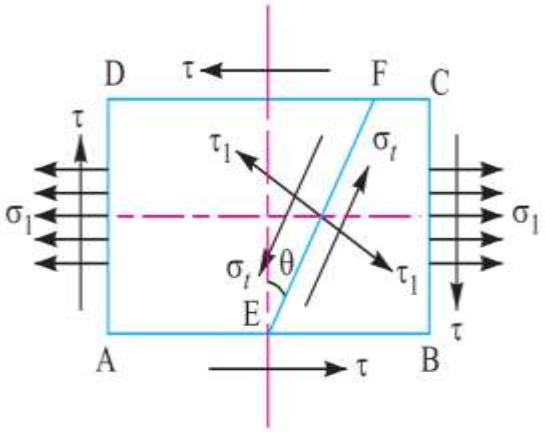


Fig B) Direct stress in one plane accompanied by a simple shear stress.

The Maximum principal (or normal) stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}$$

The Minimum principal (or normal) stress,

$$\sigma_{t2} = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}$$

The planes of maximum shear stress are at right angles to each other and are inclined at 45° to the principal planes. The maximum shear stress is given by one-half the algebraic difference between the principal stresses, i.e.

$$\tau_{max} = \frac{\sigma_{t1} - \sigma_{t2}}{2} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}$$

Note that: 1. when a member is subjected to direct stress in one plane accompanied by a simple shear stress as shown in Fig b, then the principal stresses are obtained by substituting $\sigma_2 = 0$ it reduced to.

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

2. Theories of Failure under Static Load

The principal theories of failure for a member subjected to bi-axial stress are as follows:

1. Maximum principal (or normal) stress theory (also known as Rankine's theory).
2. Maximum shear stress theory (also known as Guest's or Tresca's theory).
3. Maximum principal (or normal) strain theory (also known as Saint Venant theory).
4. Maximum strain energy theory (also known as Haigh's theory).
5. Maximum distortion energy theory (also known as Hencky and Von Mises theory).

1. Maximum Principal or Normal Stress Theory (Rankine's Theory).

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal or normal stress in a bi-axial stress system reaches the limiting strength of the material in a simple tension test. Since the limiting strength for ductile materials is yield point stress and for brittle materials (which do not have well defined yield point) the limiting strength is ultimate stress, therefore according to the above theory, taking factor of safety (F.S.) into consideration, the maximum principal or normal stress (σ_{t1}) in a bi-axial stress system is given by

$$\sigma_{t1} = \frac{\sigma_{yt}}{F.S.}, \text{ for ductile materials}$$

$$= \frac{\sigma_u}{F.S.}, \text{ for brittle materials}$$

Where

- σ_{t1} (maximum principal or normal stress)

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}$$

- σ_{yt} = Yield point stress in tension as determined from simple tension test, and
- σ_u= Ultimate stress.
- F.S= factor of safety

Since the maximum principal or normal stress theory is based on failure in tension or compression and ignores the possibility of failure due to shearing stress, therefore it is not used for ductile materials. However, for brittle materials which are relatively strong in shear but weak in tension or compression, this theory is generally used.

2. Maximum Shear Stress Theory (Guest's or Tresca's Theory)