

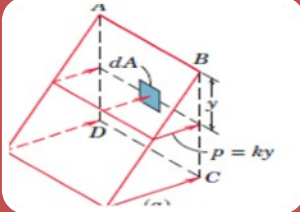
BAHIR DAR INSTITUTE OF TECHNOLOGY
FACULTY OF CIVIL AND WATER RESOURCE ENGINEERING

ENGINEERING MECHANICS I (STATICS)

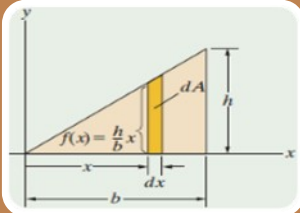
CHAPTER 6

AREA MOMENT OF INERTIA

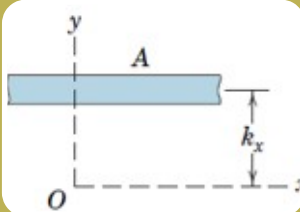
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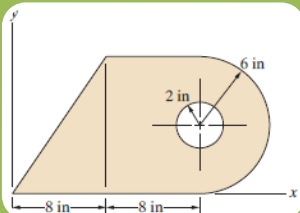
Introduction



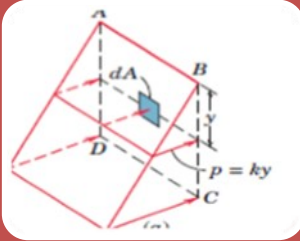
Rectangular and Polar Moments of Inertia



Radius of gyration

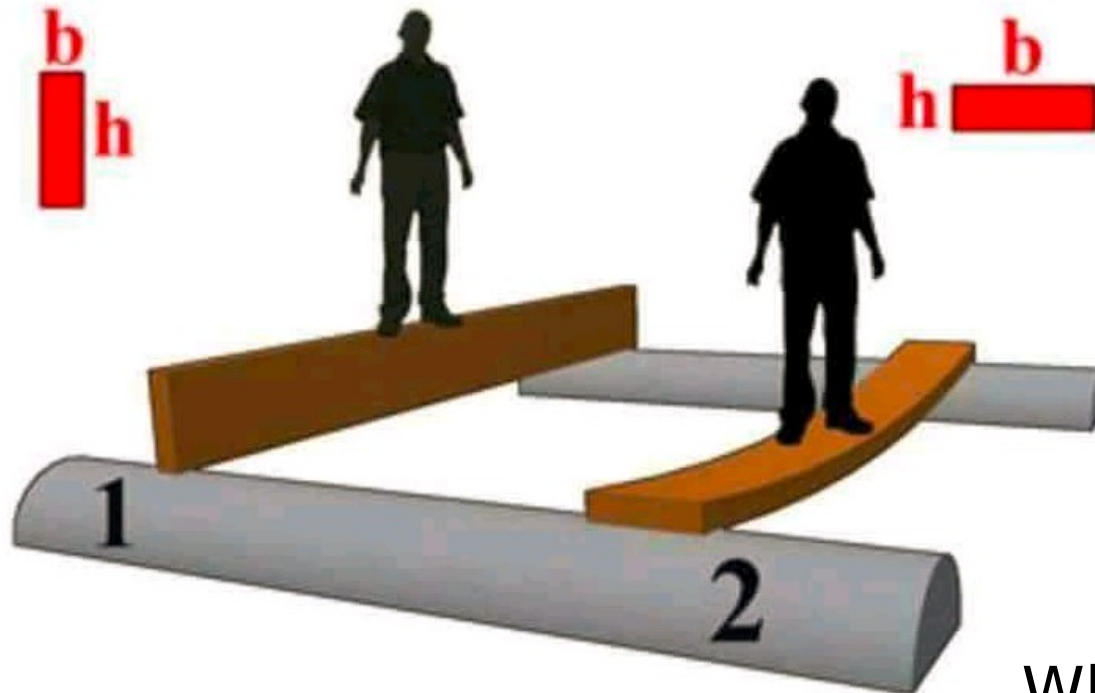


Composite Areas

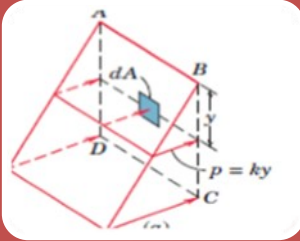


Introduction

What do you see here?

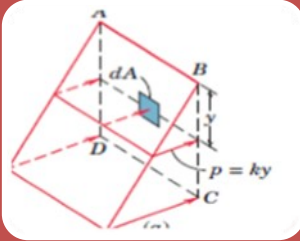


Which beam deflects more and why?



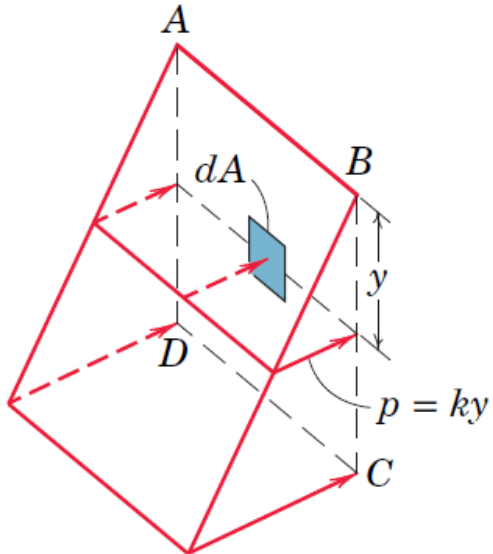
Introduction

- Area moments of inertia are a measure of the distribution of a two-dimensional area around a particular axis.
- Fundamentally, the portions of a shape which are located farther from the axis are more important than the parts which are closer.
- The primary application is in structural engineering and machine design where they are used to determine a structural member's stiffness. Another application is in Fluid Mechanics where they are used to determine the effect of pressure on a submerged surface.



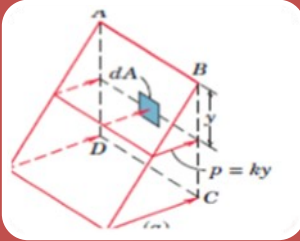
Introduction

- Moments of inertia of areas are used in the study of distributed forces and in calculating deflections of beams.
- It is often necessary to calculate the moment of a distributed force about some axis either in or perpendicular to the plane of the area.



Consider the action of liquid pressure on a plane surface:

- The intensity of the force is proportional to the distance of line of action of the force from the moment arm.

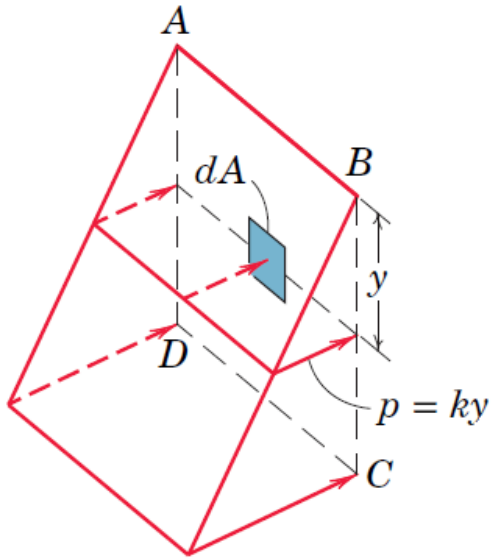


Introduction

- Pressure on the small surface area dA , p

$$F = p \cdot dA$$

$$F = ky \cdot dA$$

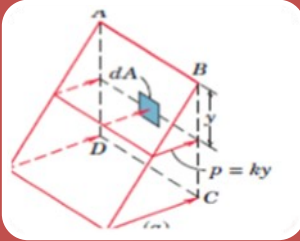


- Moment of differential area about AB

$$M_{AB} = F \cdot y = ky \cdot dA \cdot y = ky^2 dA$$

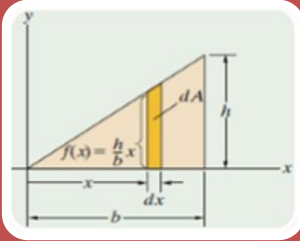
$M = k \int y^2 dA$ is called the area moment of inertia or second moment of area

$$M = k \int y^2 dA$$



Introduction

- **Area moment of inertia** is the integral of the part of a body whose squared distance from the reference point is multiplied by the cross sectional area.
- Also called as second moment of area
- It is a measure of how an area is distributed about a particular axis
$$M = k \int y^2 dA$$
- It describe the capacity of a cross-section to resist bending.
 - Moment of inertia about x axis, I_x
 - Moment of inertia about y axis, I_y

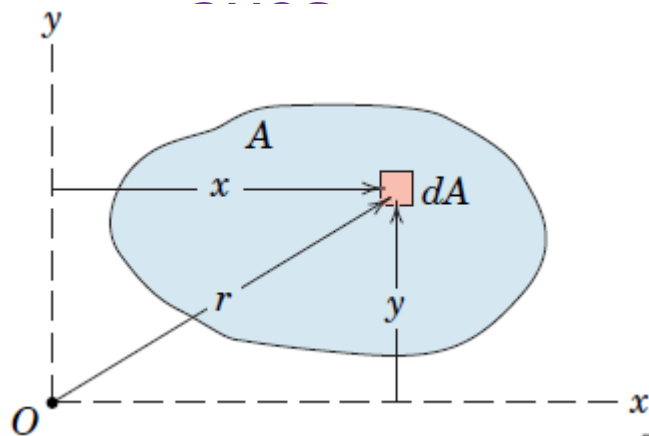


Rectangular and Polar Moments of Inertia

- The moments of inertia of the element dA about the x - and y -

$$dI_x = y^2 dA \text{ and } dI_y = x^2$$

dA



$$I_x = \int y^2 dA$$

$$I_y = \int x^2 dA$$

- I_x and I_y are called rectangular moments of inertia

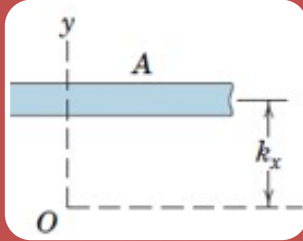
- I_z is called polar moments of inertia

$$I_z = \int r^2 dA$$

$$x^2 + y^2 = r^2$$

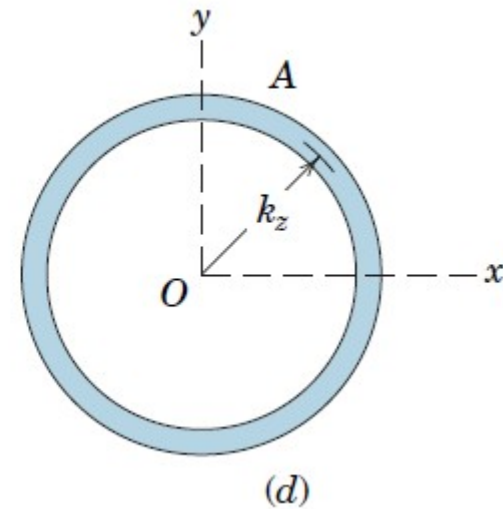
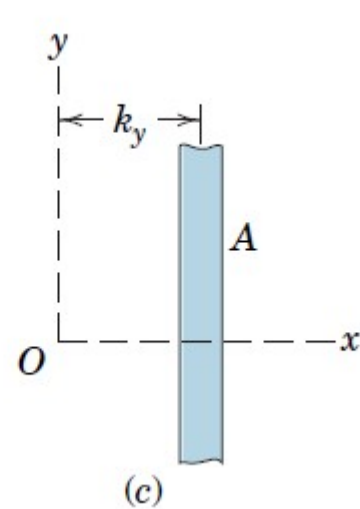
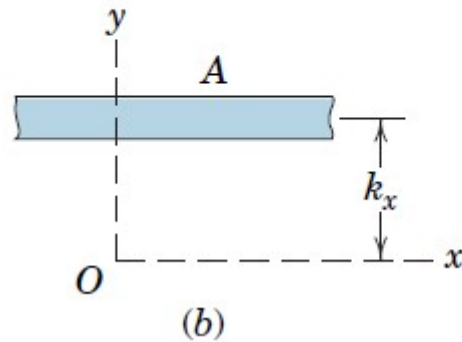
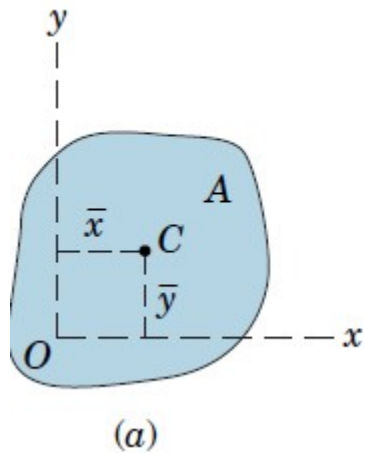
$$I_z = I_x + I_y$$

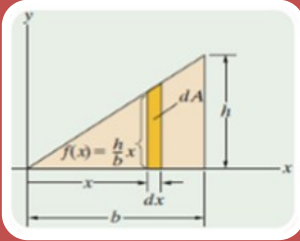
- Unit of moment of inertia is L^4



Radius of Gyration

- Radius of gyration represents the theoretical distance at which we can condense the entire area of cross-section into a narrow strip.





Radius of Gyration

- The radius of gyration of an area with respect to a particular axis is the square root of the quotient of the moment of inertia divided by the area as shown below.

$$I_x = k_x^2 A$$

$$I_y = k_y^2 A$$

$$I_z = k_z^2 A$$

or

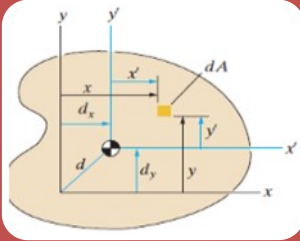
$$k_x = \sqrt{I_x/A}$$

$$k_y = \sqrt{I_y/A}$$

$$k_z = \sqrt{I_z/A}$$

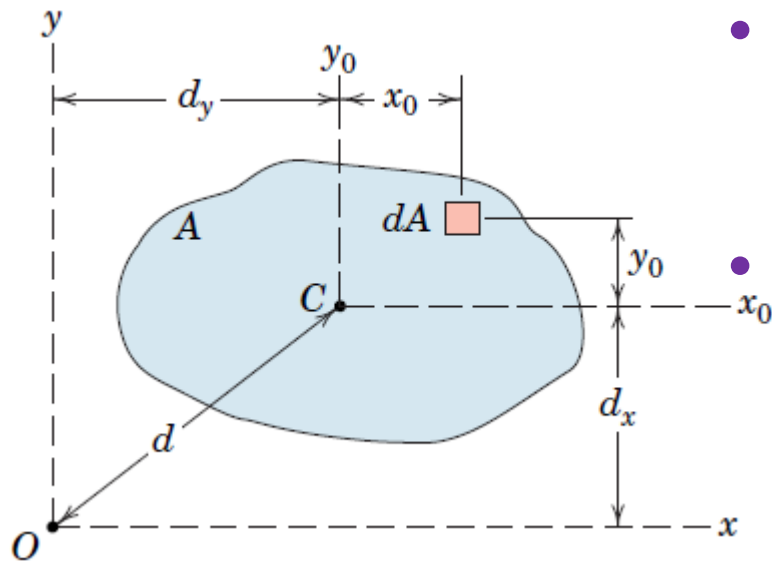
$$k_z^2 = k_x^2 + k_y^2$$

- K_x - radius of Gyration about the x axis
- K_y - radius of Gyration about the y axis



Transfer of Axes

- The moment of inertia of an area about a non centroidal axis may be expressed in terms of moment of inertia about a parallel centroidal

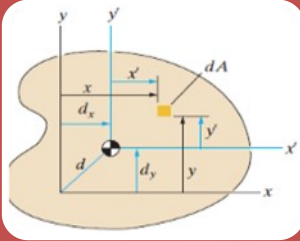


- x_0 - y_0 axes pass through the centroid C of the area.
- The moments of inertia of the area about the parallel axes are given by

$$dI_x = (y_0 + d_x)^2 dA$$

Expanding and integrating give us

$$I_x = \int y_0^2 dA + 2d_x \int y_0 dA + d_x^2 \int dA$$



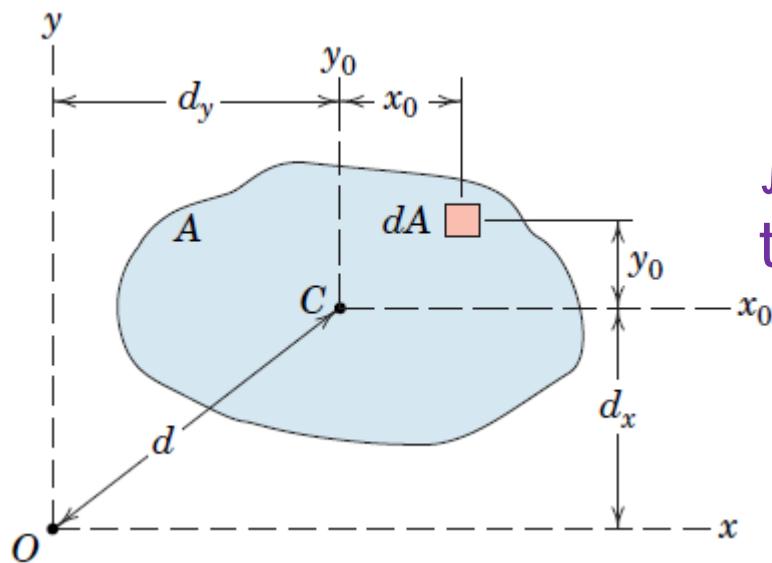
Transfer of Axes

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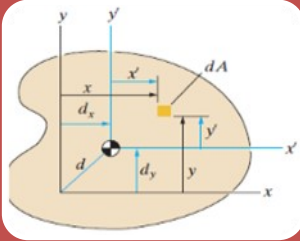
$\int y_0 dA = Ay_0$ and y_0 is automatically zero with the centroid on the x_0 -axis.



$$\begin{aligned} I_x &= \bar{I}_x + Ad_x^2 \\ I_y &= \bar{I}_y + Ad_y^2 \end{aligned}$$

→ parallel axis theorem

$$I_z = \bar{I}_z + Ad^2$$



Transfer of Axes

Axes between which the transfer is made:

- Must be parallel
- One of the axes must pass through the centroid.

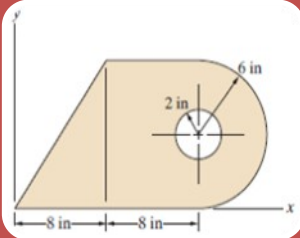
If transfer is required between two parallel axes, neither of which passes through the centroid, it is first necessary to transfer to the centroidal axis and then to the second axis.

For radii of gyration:

$$\bar{k} :$$

- radius of gyration

about a centroidal axis



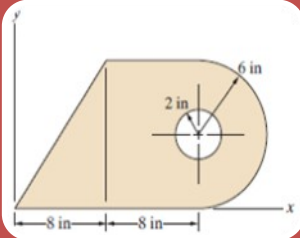
Composite Areas

The moment of inertia of a composite area about a particular axis is the sum of the moment of inertia of its components about the same axis.

Part	Area, A	d_x	d_y	Ad_x^2	Ad_y^2	$\bar{I}_x$	$\bar{I}_y$
Sums	ΣA			ΣAd_x^2	ΣAd_y^2	$\Sigma \bar{I}_x$	$\Sigma \bar{I}_y$

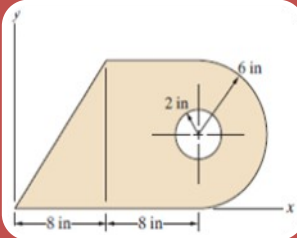
$$I_x = \Sigma \bar{I}_x + \Sigma Ad_x^2$$

$$I_y = \Sigma \bar{I}_y + \Sigma Ad_y^2$$

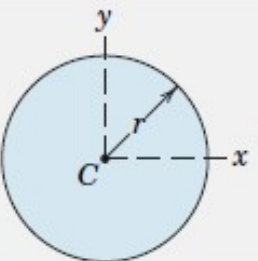
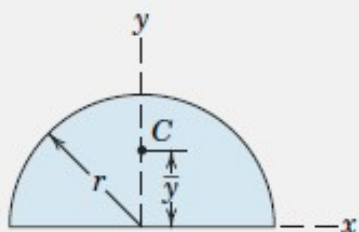
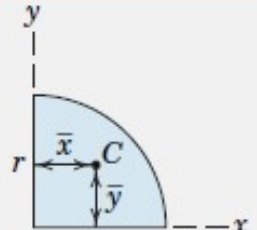


Properties of plane figures

Figure	Centroid	Area Moments of Inertia
Rectangular Area	—	$I_x = \frac{bh^3}{3}$ $\bar{I}_x = \frac{bh^3}{12}$ $\bar{I}_z = \frac{bh}{12}(b^2 + h^2)$
Triangular Area	$\bar{x} = \frac{a + b}{3}$ $\bar{y} = \frac{h}{3}$	$I_x = \frac{bh^3}{12}$ $\bar{I}_x = \frac{bh^3}{36}$ $I_{x_1} = \frac{bh^3}{4}$



Properties of plane figures

Figure	Centroid	Area Moments of Inertia
Circular Area 	—	$I_x = I_y = \frac{\pi r^4}{4}$ $I_z = \frac{\pi r^4}{2}$
Semicircular Area 	$\bar{y} = \frac{4r}{3\pi}$	$I_x = I_y = \frac{\pi r^4}{8}$ $\bar{I}_x = \left(\frac{\pi}{8} - \frac{8}{9\pi} \right) r^4$ $I_z = \frac{\pi r^4}{4}$
Quarter-Circular Area 	$\bar{x} = \bar{y} = \frac{4r}{3\pi}$	$I_x = I_y = \frac{\pi r^4}{16}$ $\bar{I}_x = \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi} \right) r^4$ $I_z = \frac{\pi r^4}{8}$