

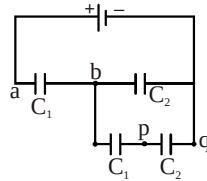
EXERCISE # (S-1)

CAPACITOR CIRCUIT

HCV Worked out Examples (Chapter No.-31 - 2,3,4,5,6,7,8,9,10,11,13,14)

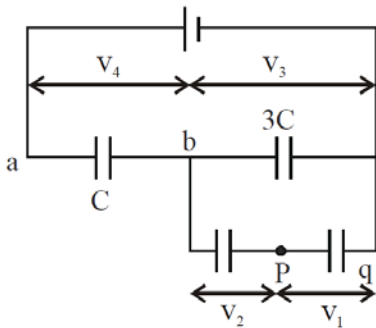
HCV Exercises (Chapter No.31 - 6, 8, 9 11, 13, 15, 16, 20, 22, 24, 25, 28, 33, 38, 40, 42, 44, 46)

1. In the given network if potential difference between p and q is 2V and $C_2 = 3C_1$. Then find the potential difference between a & b.



Ans. 30 V

Sol. $V_1 : V_2 = \frac{1}{3C} : \frac{1}{C}$



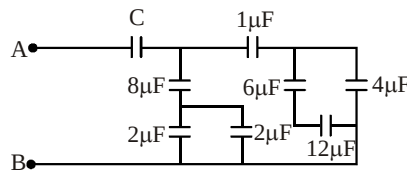
$$V_1 : V_2 = 1 : 3$$

$$V_1 = \frac{1}{4} V_3 \Rightarrow V_3 = 8V$$

$$V_3 : V_4 = \frac{4}{15C} : \frac{1}{C} \Rightarrow \frac{8}{V_4} = \frac{4}{15}$$

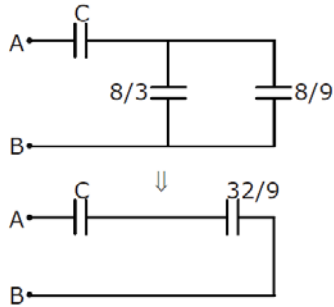
$$V_4 = 30V$$

2. In the following circuit, the resultant capacitance between A and B is $1 \mu F$. Find the value of C.



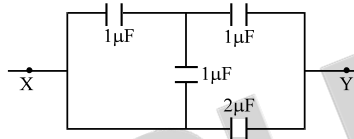
Ans. $\frac{32}{23} \mu F$

Sol.



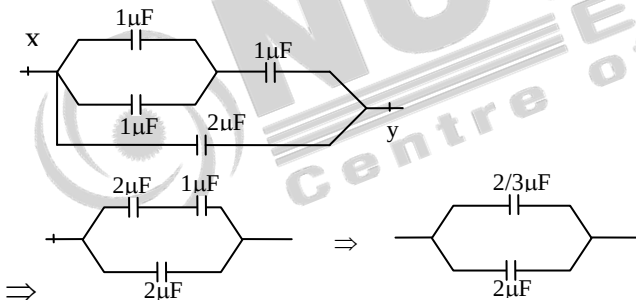
$$\Rightarrow \frac{\frac{32}{9}C}{\frac{32}{9} + C} = 1 \Rightarrow C = \frac{32}{23}$$

3. The figure shows a circuit consisting of four capacitors. Find the effective capacitance between X and Y.



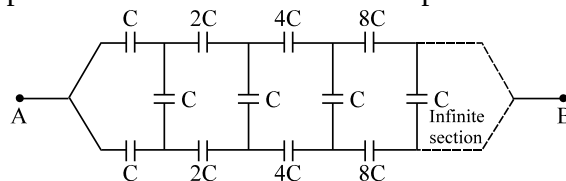
Ans. $\frac{8}{3} \mu\text{F}$

Sol. The equivalent circuit diagram is :



$$C_{eq} = \frac{2}{3} + 2 = \frac{8}{3} \mu\text{F}$$

4. Find the equivalent capacitance of the circuit between point A and B.



Ans. C

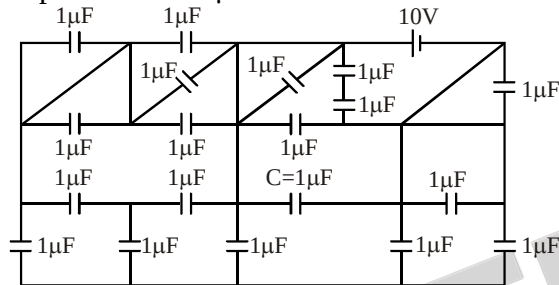
Sol. $\therefore \frac{1}{C_{eq}} = \frac{1}{C} + \frac{2}{2C} + \frac{1}{4C} + \dots$

$$= \frac{1}{C \left[1 + \frac{1}{2} \dots \dots \right]}$$

$$\frac{1}{C_{eq}} = \frac{1}{C \left(1 - \frac{1}{2} \right)} \Rightarrow C_{eq} = \frac{C}{2}$$

$$C_{eq} = C$$

5. Find the charge on the capacitor $C = 1 \mu\text{F}$ in the circuit shown in the figure.

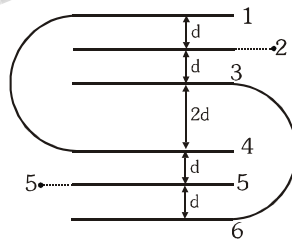


Ans. $10 \mu\text{C}$

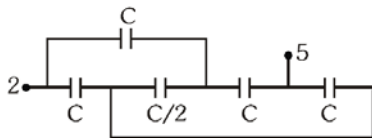
Sol. Starting with writing potential difference across the 10V battery, we get potential difference across the shown $C = 1 \mu\text{F}$ capacitor equal to 10 V.

$$\therefore Q = CV = 10 \mu\text{C}$$

6. There are six plates of equal area A and separation between the plates is d ($d \ll A$) are arranged as shown in figure. The equivalent capacitance between points 2 and 5, is $\propto \frac{\epsilon_0 A}{d}$. Then find the value of α .



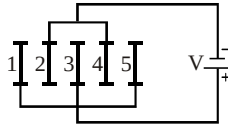
Ans. 1



Sol.

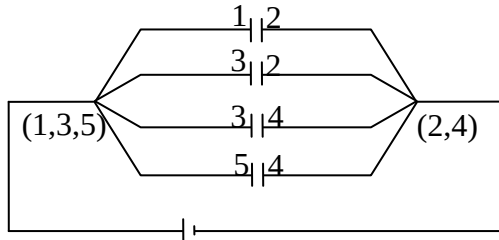
Redraw the circuit $C_{eq} = C = \frac{\epsilon_0 A}{d}$

7. Five identical capacitor plates, each of area A , are arranged such that adjacent plates are at a distance ' d ' apart, the plates are connected to a source of emf V as shown in figure. The charge on plate 1 is _____ and that on plate 4 is _____.



Ans. $\frac{A \epsilon_0 V}{d}$, $-\frac{2A \epsilon_0 V}{d}$

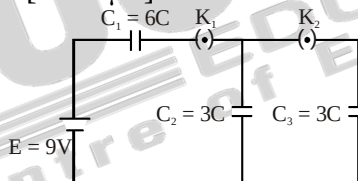
Sol. The equivalent circuit diagram is



$$C_{eq} = 4C = \frac{4\epsilon_0 A}{d}, \quad Q \text{ on plate 1} = \left(\frac{\epsilon_0 A}{d} \right) V$$

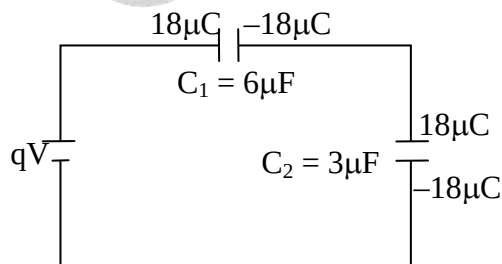
$$Q \text{ on plate 4} = 2 \left[\frac{-\epsilon_0 A}{d} \right] V$$

8. In the circuit shown in figure, initially K_1 is closed and K_2 is open. What are the charges on each capacitors. Then K_1 was opened and K_2 was closed (order is important), What will be the charge on each capacitor now? [$C = 1\mu F$]



Ans. $Q_1 = 18 \mu C$, $Q_2 = 18 \mu C$, $Q_3 = 0 \mu C$, $Q_2' = Q_3' = 9 \mu C$

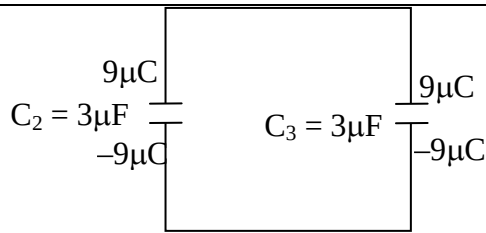
Sol. Initial circuit diagram when K_1 closed & K_2 open



$$C_{eq} = \frac{6 \times 3}{6 + 3} = 2 \mu F$$

$$Q = CV = 2 \times 9 = 18 \mu C$$

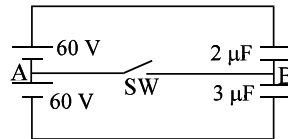
After opening K_1 & closing K_2 , the circuit becomes



C_2 & C_3 are in parallel

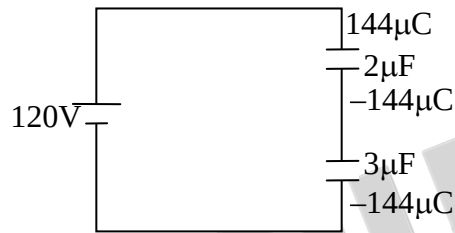
$\therefore$ charge on C_2 divides in C_2 & C_3

9. In the circuit shown in the figure, initially SW is open. When the switch is closed, the charge passing through the switch _____ in the direction _____ to _____.



Ans. $60 \mu\text{C}$, A to B

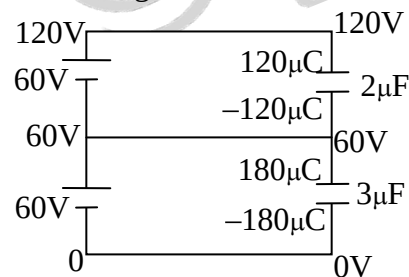
Sol. Initial charges on Capacitors with Switch open



$$C_{eq} = \frac{2 \times 3}{2 + 3} = \frac{6}{5} \mu\text{F}$$

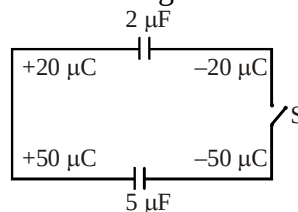
$$Q = C_{eq} V = \frac{6}{5} \times 120 = 144 \mu\text{C}$$

On closing Switch the final charges are :



On calculating, $\Rightarrow Q_{\text{flow}} = 60 \mu\text{C}$ from A to B.

10. Find heat produced in the circuit shown in figure on closing the switch S.



Ans. 0

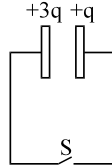
Sol. $70 = 7 \times V_f$

$V_f = 10V$

Heat

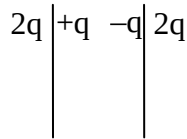
$$= \left\{ \frac{1}{2 \times 2} (20)^2 + \frac{1 \times (50)^2}{2 \times 5} - \frac{1}{2} \times 7 \times (10)^2 \right\} = 0$$

- 11.** The two identical parallel plates are given charges as shown in figure. If the plate area of either face of each plate is A and separation between plates is d , then find the amount of heat liberated after closing the switch.



Ans. $\frac{1}{2} \frac{q^2 d}{\epsilon_0 A}$

Sol. Distribution of charges on plates before closing the switch



Energy stored inside the capacitor

$$= \frac{1}{2} \epsilon_0 E^2 (Ad)$$

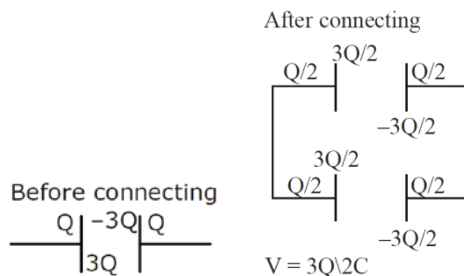
$$= \frac{1}{2} \epsilon_0 \left(\frac{q}{A \epsilon_0} \right)^2 Ad = \frac{q^2 d}{2A \epsilon_0}$$

On closing the switch the above energy dissipates in the form of heat.

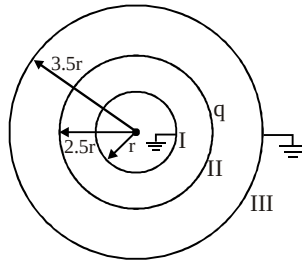
- 12.** The plates of a parallel plate capacitor are given charges $+4Q$ and $-2Q$. The capacitor is then connected across an uncharged capacitor of same capacitance as first one ($= C$). Find the final potential difference between the plates of the first capacitor.

Ans. $3Q / 2C$

Sol.



13. Figure shows three concentric conducting spherical shells with inner and outer shells earthed and the middle shell is given a charge q . Find the electrostatic energy of the system stored in the region I and II.



Ans. $U_I = \frac{3kq_1^2}{10r}$ where $q_1 = -\frac{4q}{25}$; $U_{II} = 2K(q + q_1)^2/35r$

Sol. Let charge on sphere I be q_1 and on sphere III be q_3

$$V_{S_1} = 0 = \frac{Kq_1}{r} + \frac{Kq}{2.5r} + \frac{Kq_3}{3.5r} = 0$$

$$\Rightarrow \boxed{q_1 + \frac{2}{7}q_3 = \frac{-2q}{5}} \quad \dots(1)$$

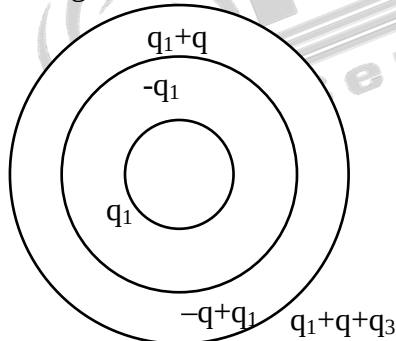
$$V_{S_3} = 0$$

$$\frac{Kq_1}{3.5r} + \frac{Kq}{3.5r} + \frac{Kq_3}{3.5r} = 0$$

$$\boxed{q_1 + q_3 = -q} \quad \dots(2)$$

Solving (1) & (2) $\Rightarrow q_1 = -4q/25$; $q_3 = -21q/25$

Charge Distribution



$$E_I = \frac{q^2}{2C_I}, \quad C_I = \frac{4\pi\epsilon_0 r_1 r_2}{r_2 - r_1}$$

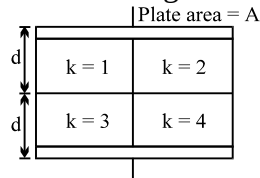
$$E_{II} = \frac{(q + q_1)^2}{2C_{II}}, \quad C_{II} = \frac{4\pi\epsilon_0 r_2 r_3}{r_3 - r_2}$$

DI-ELECTRIC

HCV Worked out Examples(Chapter No.31 - 17, 18, 19, 20, 21, 22, 23)

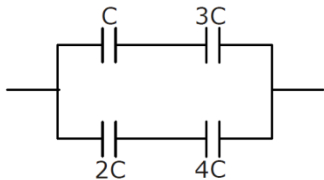
HCV Exercises (Chapter No. 31 - 51, 52, 54, 56, 57, 59, 60, 61, 64, 66, 67, 68)

14. Find the capacitance of the system shown in figure.



Ans. $\frac{25 \epsilon_0 A}{24 d}$

Sol. Let $\frac{\epsilon_0 A}{2d} = C$ {Area = $A/2$ }



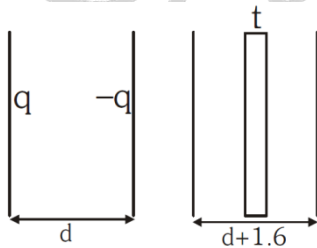
$$C_{eq} = \frac{3C^2}{C+3C} + \frac{(2C)(4C)}{2C+4C}$$

$$= \frac{3}{4}C + \frac{4}{3}C = \frac{25 \epsilon_0 A}{24 d}$$

15. The plates of a parallel plate capacitor are charged upto 100 volt. A 2 mm thick plate is inserted between the plates, then to maintain the same potential difference, the distance between the capacitor plates is increased by 1.6 mm. Find the dielectric constant of the plate.

Ans. 5

Sol. $C = \frac{\epsilon_0 A}{d}$; $q = C \times 100 = \frac{\epsilon_0 A}{d} \times 100$



New capacitance $C_1 = \frac{\epsilon_0 A}{(d+1.6) + t \left(\frac{1}{k} - 1 \right)}$ where $t = 2 \text{ mm}$

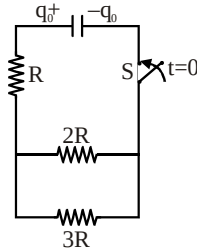
$$100 = \frac{q}{C_1} = \frac{\left[\frac{\epsilon_0 A}{d} \times 100 \right]}{C_1} \text{ Solving } K = 5$$

RC CIRCUIT

HCV Worked out Examples(Chapter No.32- 30, 31, 32, 33)

HCV Exercises (Chapter No.32 - 66,67,74,79,81,82,84)

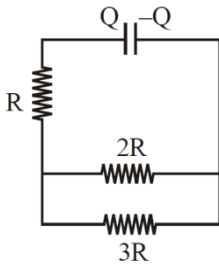
16. At $t = 0$ charge on capacitor is q_0 . Now switch S is closed. Heat loss in $3R$ is $x \times 10^{-6}$ J. Then find the value of x . [Given $q_0 = 15\mu\text{C}$, $C = 6/55 \mu\text{F}$]



Ans. 225

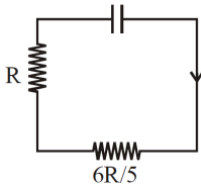
Sol. $U = \frac{1}{2} \frac{(15)^2 \times 10^{-12}}{6 \times 10^{-6}} \times 55 = 1031.25 \mu\text{J}$

Energy in $\frac{6R}{5}$ is $\frac{U \times \frac{6R}{5}}{\frac{11R}{5}} = \frac{U \times 6}{11}$



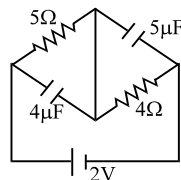
Energy emitted in $3R$

$E = \frac{U \times 6}{11} \times \frac{6R}{5 \times 3R}$



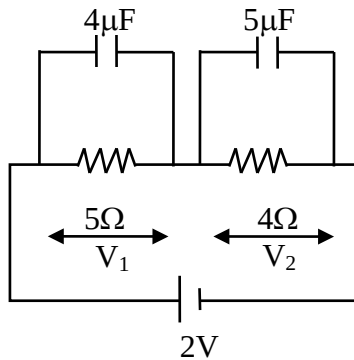
$= \frac{6U}{11} \times \frac{6}{15}$
 $= 225 \times 10^{-6} \text{ J}$

17. Find the ratio between the energy stored in $5 \mu\text{F}$ capacitor to the $4 \mu\text{F}$ capacitor in the given circuit in steady state.



Ans. 0.8

Sol. Redrawing the circuit diagram



In steady state, current in the circuit

$$i = \frac{2}{9} \text{ A}$$

potential difference across $4\mu\text{F}$ capacitor = $i(5)$

$$V_1 = \frac{10}{9} \text{ V}$$

Potential difference across $5\mu\text{F}$ capacitor = $i(4)$

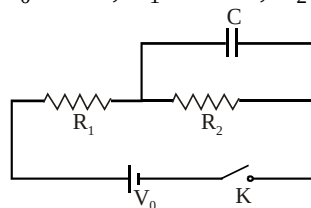
$$V_2 = \frac{8}{9} \text{ V}$$

$$\text{energy stored in } 4\mu\text{F} = \frac{1}{2}(4)(V_1)^2 = E_1$$

$$\text{energy stored in } 5\mu\text{F} = \frac{1}{2}(5)(V_2)^2 = E_2$$

$$\frac{E_2}{E_1} = \frac{5V_2^2}{4V_1^2} = \frac{5 \times 64}{4 \times 100} = 0.8$$

- 18.** In the connection shown in the *figure* the switch K is open and the capacitor is uncharged. Then we close the switch and let the capacitor charge up to the maximum and open the switch again. Then (Use the following data : $V_0 = 30 \text{ V}$, $R_1 = 10 \text{ k}\Omega$, $R_2 = 5 \text{ k}\Omega$.)



- (i) the current through R_1 be I_1 immediately after closing the switch
- (ii) the current through R_2 be I_2 a long time after the switch was closed
- (iii) the current through R_2 be I_3 immediately after reopening the switch

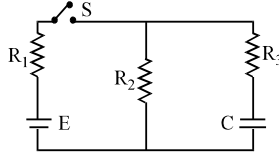
Find the value of $\frac{I_1}{I_2 I_3}$ (in ampere^{-1})

Ans. 750

Sol. Just after closing switch no current flows through R_2 so $I_1 = 3\text{mA}$
Long time after closing switch no current flows through C so $I_2 = 2\text{mA}$

Directly after re-opening the switch no current flows through R_1 and the capacitor will discharge through R_2 so $I_3 = 2\text{mA}$

19. In the circuit shown in figure $R_1 = R_2 = 6R_3 = 300\text{ M}\Omega$, $C = 0.01\text{ }\mu\text{F}$ and $E = 10\text{V}$. The switch is closed at $t = 0$, find



- (a) Charge on capacitor as a function of time.
(b) energy of the capacitor at $t = 20\text{s}$.

Ans. (a) $q = 0.05(1 - e^{-t/2})\text{ }\mu\text{C}$; (b) $0.125\text{ }\mu\text{J}$

Sol. $R_1 = 300\text{M}\Omega$, $R_2 = 300\text{M}\Omega$, $R_3 = 50\text{M}\Omega$

$$\tau = (R_{eq})C = 200\text{M}\Omega \times (0.01\text{ }\mu\text{F})$$

$$= 200 \times 10^6 \times 0.01 \times 10^{-6}$$

$$\tau = 2\text{ sec}$$

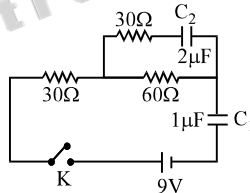
$$(a) q = Q_{max}(1 - e^{-t/RC})$$

$$q = 0.05(1 - e^{-t/2})$$

- (b) energy stored at $t = 20\text{ sec}$

$$E = \frac{q^2}{2C} = 0.125\text{ }\mu\text{J}$$

20. For the arrangement shown in the figure, the key is closed at $t = 0$. C_2 is initially uncharged while C_1 has a charge of $2\text{ }\mu\text{C}$.



- (a) Find the current coming out of the battery just after switch is closed.
(b) Find the charge on the capacitors in the steady state condition.

Ans. (a) $\frac{7}{50}\text{ A}$ or $\frac{11}{50}\text{ A}$, (b) $Q_1 = 9\text{ }\mu\text{C}$, $Q_2 = 0$

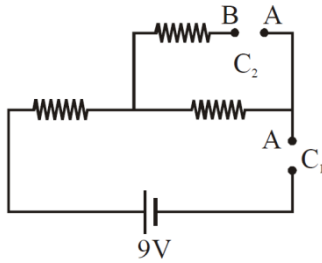
Sol. (b) In steady state there will be no current in the circuit

P.d across $C_2 = 0$

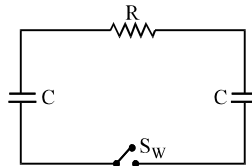
So charge = 0

P.d across $C_1 = 9\text{V}$

$Q = 9\text{ }\mu\text{C}$



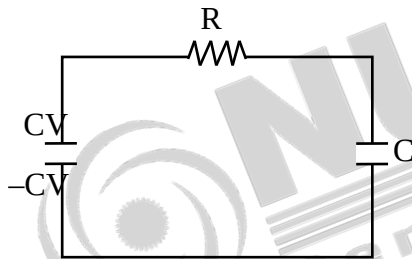
21. In the circuit shown in figure the capacitance of each capacitor is equal to C and resistance R . One of the capacitors was charge to a voltage V and then at the moment $t = 0$ was shorted by means of the switch S . Find:



- (a) the current in the circuit as a function of time t .
(b) the amount of generated heat.

Ans. (a) $I = \frac{V_0}{R} e^{-2t/RC}$; (b) $\frac{1}{4} CV_0^2$

Sol. $t = 0$

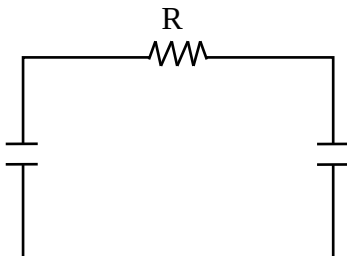


In discharging

$$i = i_0 e^{-t/(R_q C_{eq})}$$

$$i = \frac{V_0}{R} e^{-2t/RC}$$

At steady state



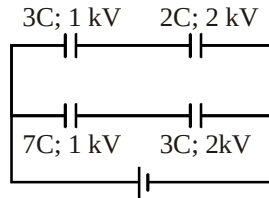
Initial energy stored

$$\epsilon_i = \frac{1}{2} CV^2$$

$$\epsilon_f = \frac{1}{4} CV^2$$

$$\text{Heat} = \frac{1}{4} CV^2$$

22. The diagram shows four capacitors with capacitances and break down voltages as mentioned. What should be the maximum value of the external emf source such that no capacitor breaks down?



Ans. 2.5 kV

Sol. $Q_{1\max} = 3C \times 10^3 \text{ C}$
 $Q_{2\max} = 4C \times 10^3 \text{ C}$
 $Q_{\max}$ for first branch $3C \times 10^3 \text{ C}$

$$V_{\max_1} = \frac{3C \times 10^3 \times 5C}{6C^2} = \frac{5}{2} \text{ KV}$$

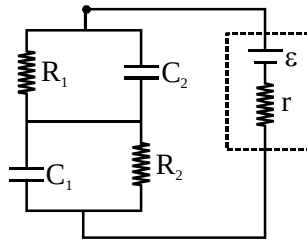
Similarly for second branch

$$Q_{3\max} = 7C \times 10^3 \text{ C} \quad Q_{4\max} = 6C \times 10^3 \text{ C}$$

$$V_{\max_2} = \frac{6C \times 10^3}{21C^2} \times 10C = \frac{20}{7} \text{ kV}$$

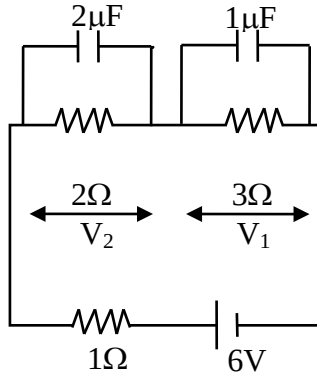
The two branches are in parallel. So in order to find max value of voltage for which no capacitor breaks down $V_{\max_1} < V_{\max_2}$.

23. In the given circuit, $R_1 = 2\Omega$, $R_2 = 3\Omega$, $r = 1\Omega$, $\varepsilon = 6\text{V}$, $C_1 = 1\mu\text{F}$ and $C_2 = 2\mu\text{F}$. In steady state the ratio of energy stored in the capacitors C_2 and C_1 is $\frac{x}{9}$. Then x will be.



Ans. 8

Sol.



At steady state $i = \frac{6}{6} = 1\text{A}$

$$V_2 = i(2) = 2 \text{ volt}$$

$$V_1 = i(3) = 3 \text{ volt}$$

$$E_{C_2} = \frac{1}{2}(2)(2)^2 = 4$$

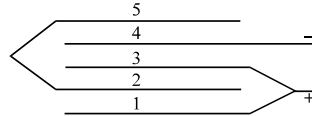
$$E_{C_1} = \frac{1}{2}(1)(3)^2 = \frac{9}{2}$$

$\frac{E_{C_2}}{E_{C_1}} = \frac{8}{9}$

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EXERCISE # (S-2)

1. Five identical conducting plates 1, 2, 3, 4 & 5 are fixed parallel to and equidistant from each other (see figure). Plates 2 & 5 are connected by a conductor while 1 & 3 are joined by another conductor. The junction of 1 & 3 and the plate 4 are connected to a source of constant e.m.f. V_0 . Find ;

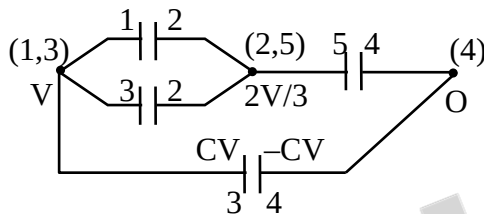


- (i) the effective capacity of the system between the terminals of the source.
(ii) the charges on plates 3 & 5.

Given d = distance between any 2 successive plates & A = area of either face of each plate.

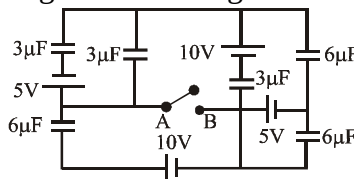
Ans. (i) $\frac{5}{3} \left(\frac{\epsilon_0 A}{d} \right)$; (ii) $Q_3 = \frac{4}{3} \left(\frac{\epsilon_0 A V_a}{d} \right)$, $Q_5 = \frac{2}{3} \left(\frac{\epsilon_0 A V_a}{d} \right)$

Sol. Equivalent circuit Diagram



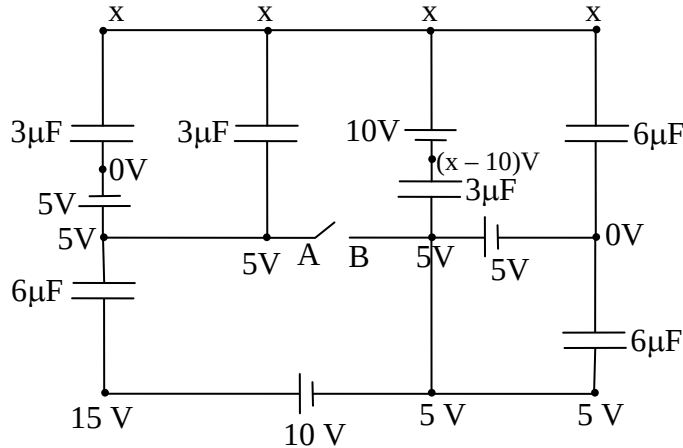
(i) $C_{eq} = \frac{2C \times C}{3C} + C = \frac{5C}{3}$
(ii) $\frac{CV}{3} + CV = \frac{4CV}{3} = \frac{4}{3} \left(\frac{\epsilon_0 A}{d} \right) V$
(iii) $C \left(\frac{2V}{3} \right) + CV = \frac{2CV}{3} = \frac{5}{3} \left(\frac{\epsilon_0 A}{d} \right) V$

2. In the given circuit, find the charge flown through the switch from A to B when it is closed.



Ans. 69 mC

Sol.



After closing the switch, let the potential of upper most point be x , hence for isolated capacitors ($3\mu\text{F}$, $3\mu\text{F}$, $3\mu\text{F}$, $6\mu\text{F}$) connected to this point, sum of charges is zero.

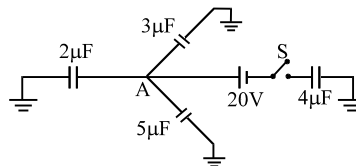
$$\Sigma Q = 0$$

$$\begin{aligned} \Rightarrow 3(x-0) + 3(x-5) + 3(x-10-5) + 6(x-0) &= 0 \\ \Rightarrow 2x-5 + x-15 + 2x &= 0 \\ \Rightarrow 5x &= 20 \\ \Rightarrow x &= 4\text{V} \end{aligned}$$

For the 3 capacitors ($3\mu\text{F}$, $3\mu\text{F}$, $6\mu\text{F}$) to the left of the switch, initially sum of charges was zero, hence the final total charge will come through the switch:

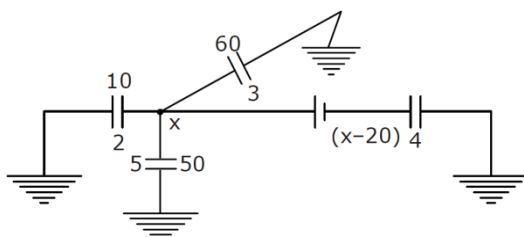
$$\begin{aligned} \text{So, Charge flown from A to B} &= -(\Sigma Q_f - \Sigma Q_i) \quad (\text{charges on the mentioned 3 capacitors}) \\ &= -(3(0-x) + 3(5-x) + 6(5-15) - 0) \\ &= -(-12 + 3 - 60) = +69 \mu\text{C} \end{aligned}$$

3. Three capacitors of $2\mu\text{F}$, $3\mu\text{F}$ and $5\mu\text{F}$ are independently charged with batteries of emf's 5V, 20V and 10V respectively. After disconnecting from the voltage sources. These capacitors are connected as shown in figure with their positive polarity plates are connected to A and negative polarity is earthed. Now a battery of 20V and an uncharged capacitor of $4\mu\text{F}$ capacitance are connected to the junction A as shown with a switch S. When switch is closed, find :



- (a) the potential of the junction A.
(b) final charges on all four capacitors.

Ans. (a) $\frac{100}{7}$ volts; (b) 28.56 mC, 42.84 mC, 71.4 mC, 22.88 mC



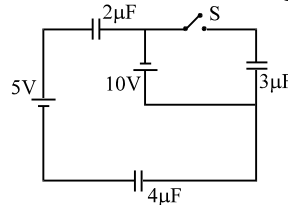
Sol.

$$2x + 3x + 5x + 4(x - 20) = 120$$

$$14x = 200$$

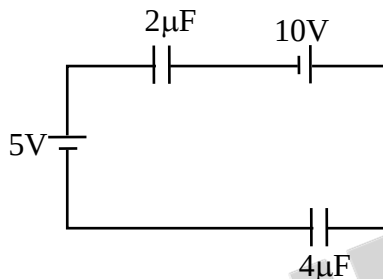
$$x = \frac{100}{7} \text{ Volt}$$

4. In the circuit shown in figure, find the amount of heat generated when switch S is closed.

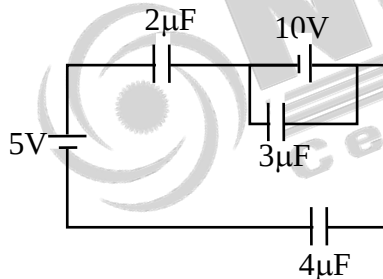


Ans. 150 μJ

Sol. Before



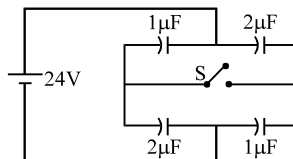
After



The only difference on closing the switch is that the charge flows through 3μF capacitor

$$\begin{aligned} \text{Heat} &= CV^2 - \frac{1}{2} CV^2 = \frac{1}{2} CV^2 \\ &= 150 \mu\text{J} \end{aligned}$$

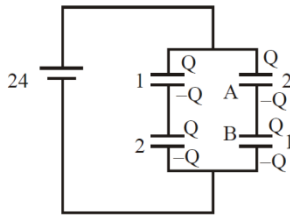
5. The connections shown in figure are established with the switch S open. How much charge will flow through the switch if it is closed?



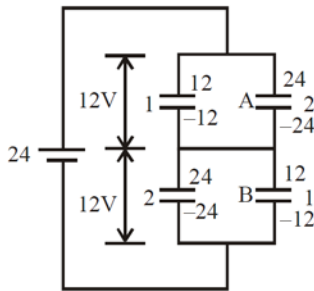
Ans. 12μC

Sol. Initial circuit

$$Q = \left(\frac{2 \times 1}{2 + 1} \right) \times 24 = 16C$$



Final circuit



Initially charge on A & B combined is 0 and finally at is -12

So charge flowing is $12 \mu C$

6. A potential difference of 300 V is applied between the plates of a plane capacitor spaced 1 cm apart. A plane parallel glass plate with a thickness of 0.5 cm and a plane parallel paraffin plate with a thickness of 0.5 cm are placed in the space between the capacitor plates find :

- Intensity of electric field in each layer.
- The drop of potential in each layer.
- The surface charge density of the charge on capacitor the plates. Given that : $k_{\text{glass}} = 6$, $k_{\text{paraffin}} = 2$

Ans. (i) $1.5 \times 10^4 \text{ V/m}$, $4.5 \times 10^4 \text{ V/m}$, (ii) 75 V, 225 V, (iii) $8 \times 10^{-7} \text{ C/m}^2$

Sol. $C_1 = \frac{A\epsilon_0 k_g}{d}$, $C_2 = \frac{A\epsilon_0 k_p}{d}$

$$C_{\text{eff}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\left(\frac{A\epsilon_0}{d} \right)^2 k_g k_p}{\frac{A\epsilon_0}{d} (k_g + k_p)} = \frac{A\epsilon_0 (k_g \times k_p)}{d(k_g + k_p)}$$

$$Q = C_{\text{eff}} \times V_0$$

$$V_1 = \frac{C_{\text{eff}} V_0}{C_1} = \frac{k_p}{k_g + k_p} V_0 = \frac{2}{8} \times 300 = 75V$$

$$V_2 = \frac{C_{\text{eff}} V_0}{C_2} = \frac{6}{8} \times 300 = 225V$$

$$E_1 = \frac{V_1}{d} = \frac{75}{0.5 \times 10^{-2}} = 15000 \text{ V/m}$$

$$E_2 = \frac{V_2}{d} = \frac{225}{0.5 \times 10^{-2}} = 45000 \text{ V/m}$$

$$\sigma_1 = \sigma_2 = \frac{Q}{A}$$

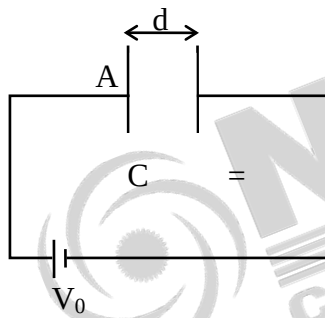
$$= \frac{\epsilon_0 (12)}{8 \times 0.5 \times 10^{-2}} \times 300 = 90000 \quad \epsilon_0 = 8 \times 10^{-7} \text{ C/m}^2$$

7. A parallel plate capacitor has plates with area A & separation d . A battery charges the plates to a potential difference of V_0 . The battery is then disconnected & a di-electric slab of constant K & thickness d is introduced. Calculate the positive work done by the system (capacitor + slab) on the man who introduces the slab.

Ans. $W = \frac{1}{2} C_0 V_0^2 \left(1 - \frac{1}{K} \right)$

Sol.

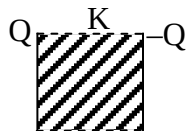
Initial



$$Q = CV_0$$

$$Q = \left(\frac{\epsilon_0 A}{d} \right) V_0$$

final



$$Q = \left(\frac{\epsilon_0 A}{d} \right) V_0 = C_f V_f = \left(\frac{K \epsilon_0 A}{d} \right) V_f$$

$$V_f = \frac{V_0}{K}$$

$$\epsilon_i = \frac{1}{2} CV^2, \quad \epsilon_f = \frac{1}{2} (K C) \left(\frac{V}{K} \right)^2$$

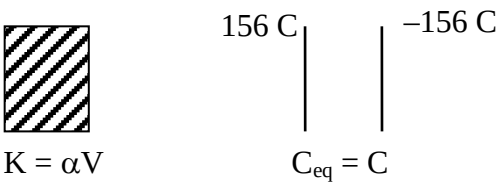
$$= \frac{1}{K} \left(\frac{1}{2} CV^2 \right)$$

$$\text{Work} = \Delta E = \frac{1}{2} CV^2 \left(1 - \frac{1}{K} \right)$$

$$= \frac{1}{2} C_0 V_0^2 \left(1 - \frac{1}{K} \right)$$

8. A parallel plate capacitor is filled by a di-electric whose relative permittivity varies with the applied voltage according to the law $= \alpha V$, where $\alpha = 1$ per volt. The same (but containing no di-electric) capacitor charged to a voltage $V = 156$ volt is connected in parallel to the first "non-linear" uncharged capacitor. Determine the final voltage V_f (in volts) across the capacitors.

Ans. 12 Volt

Sol. 

$K = \alpha V$
 $C_{eq} = C$

After Both are connected charge on then remains equal to 156 C

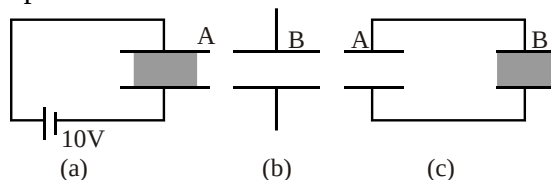
$$Q_1 + Q_2 = 156 \text{ C}$$

$$(\alpha CV)V + CV = 156C$$

$$V^2 + V = 156$$

$$V = 12 \text{ volt}$$

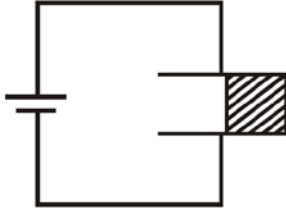
9. Two parallel plate capacitors A & B have the same separation $d = 8.85 \times 10^{-4}$ m between the plates. The plate areas of A & B are 0.04 m^2 & 0.02 m^2 respectively. A slab of di-electric constant (relative permittivity) $K=9$ has dimensions such that it can exactly fill the space between the plates of capacitor B.



- The di-electric slab is placed inside A as shown in the figure (a) A is then charged to a potential difference of 110 volt. Calculate the capacitance of A and the energy stored in it.
- The battery is disconnected & then the di-electric slab is removed from A. Find the work done by the external agency in removing the slab from A.
- The same di-electric slab is now placed inside B, filling it completely. The two capacitors A & B are then connected as shown in figure (c). Calculate the energy stored in the system.

Ans. (i) $0.2 \times 10^{-8} \text{ F}$, $1.2 \times 10^{-5} \text{ J}$; (ii) $4.84 \times 10^{-5} \text{ J}$; (iii) $1.1 \times 10^{-5} \text{ J}$

Sol. (i) $C_{eq} = C_1 + C_2$



$$= \frac{k \epsilon_0 \frac{A}{2}}{d} + \frac{\epsilon_0 A}{2d}$$

$$= \frac{\epsilon_0 A}{2d} (1+k)$$

$$= \frac{8.85 \times 10^{-12} \times 0.04}{2 \times 8.85 \times 10^{-4}} (1+9)$$

$$= 0.2 \times 10^{-8} \text{ F}$$

$$U = \frac{1}{2} C_{eq} \times 110^2 = 1.2 \times 10^{-5} \text{ J}$$

(ii) Charge on capacitor A

$$Q = 0.2 \times 10^{-8} \times 110$$

$$u_i = 1.2 \times 10^{-5} \text{ J}$$

When battery is removed charge remains same

$$u_f = \frac{1}{2} \frac{Q^2}{C_f}$$

$$C_f = \frac{\epsilon_0 \times 0.04}{8.85 \times 10^{-4}}$$

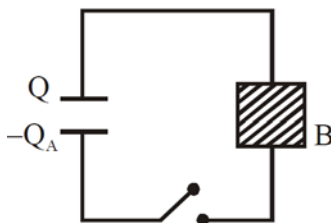
$$W = U_f - U_i$$

$$= 4.84 \times 10^{-5} \text{ J}$$

$$(iii) C_A = \frac{\epsilon_0 \times 0.04}{d}$$

$$C_B = \frac{k \epsilon_0 \times 0.02}{d}$$

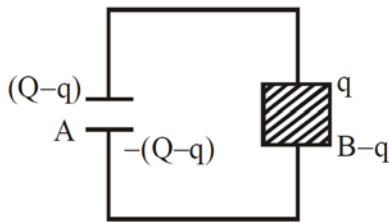
$$Q = 2.2 \times 10^{-7}$$



When the switch is closed P.d across both capacitor becomes same

$$\frac{(Q-q)}{C_A} = \frac{q}{C_B}$$

and



$$U_f = \frac{1}{2} \frac{q^2}{C_B} + \frac{1}{2} \left(\frac{Q-q}{C_A} \right)^2$$

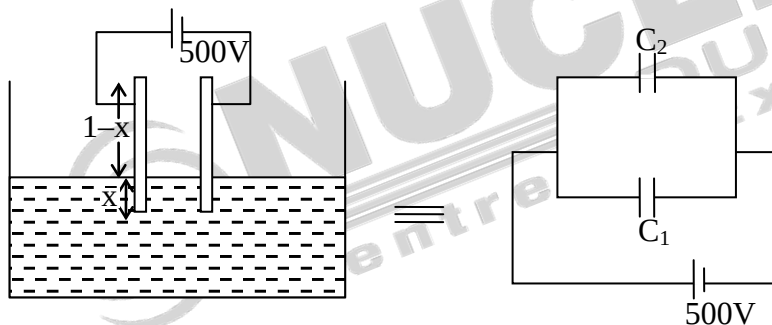
$$= 1.1 \times 10^{-5} \text{ J}$$

10. Two square metallic plates of 1 m side are kept 0.01 m apart, like a parallel plate capacitor, in air in such a way that one of their edges is perpendicular, to an oil surface in a tank filled with an insulating oil. The plates are connected to a battery of e.m.f. 500 volt. The plates are then lowered vertically into the oil at a speed of 0.001 m/s. Calculate the current drawn from the battery during the process.

[dielectric constant of oil = 11, $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N}^2 \text{ m}^2$]

Ans. 4.425×10^{-9} Ampere

Sol.



$$C_1 = K \frac{\epsilon_0 (x \times 1)}{0.01} = 1100 \epsilon_0 x$$

$$C_2 = \frac{\epsilon_0 (1-x) \times 1}{0.01} = 100 \epsilon_0 (1-x)$$

$$C_{eq} = C_1 + C_2 = 100 \epsilon_0 (10x + 1)$$

$$Q = C_{eq} E = 100 \epsilon_0 (10x + 1) \times 500$$

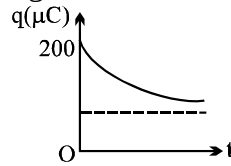
$$i = \frac{dQ}{dt} = 100 \epsilon_0 \times 10 \times 500 \times \frac{dx}{dt}$$

$$= 5 \times 10^5 \times 8.85 \times 10^{-12} \times (0.001)$$

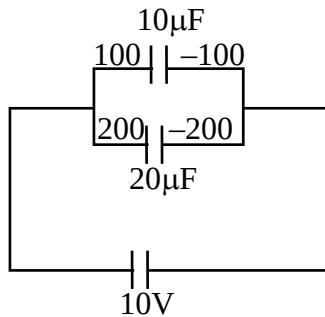
$$= 4.425 \times 10^{-9} \text{ A}$$

11. A $10 \mu\text{F}$ and $20 \mu\text{F}$ capacitor are connected to a 10 V cell in parallel for some time after which the capacitors are disconnected from the cell and reconnected at $t = 0$ with each other, in series, through wires of finite resistance. The +ve plate of the first capacitor is connected to the -ve

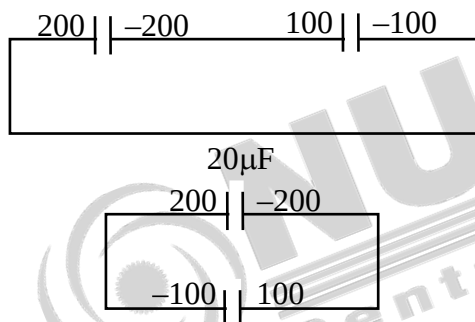
plate of the second capacitor. Draw the graph which best describes the charge on the +ve plate of the $20\ \mu\text{F}$ capacitor with increasing time.



Ans.
Sol.



final



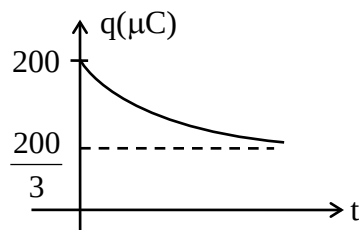
$\Rightarrow$

$$V_f = \frac{200 - 100}{(20 + 10)} = \frac{10}{3} \text{ Volt}$$

$$V_i = 10 \text{ Volt} \quad V_f = \frac{10}{3} \text{ Volt}$$

$\therefore$ charge on the capacitor will decrease

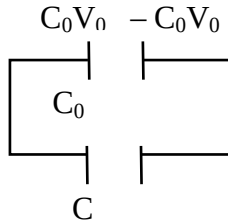
$$Q_i = 200\ \mu\text{C} \quad Q_f = \frac{200}{3}\ \mu\text{C}$$



12. A capacitor of capacitance C_0 is charged to a potential V_0 and then isolated. A small capacitor C is then charged from C_0 , discharged & charged again, the process being repeated n times.

The potential of the large capacitor has now fallen to V . Find the capacitance of the small capacitor. If $V_0 = 100$ volt, $V = 35$ volt, find the value of n for $C_0 = 0.2 \mu\text{F}$ & $C = 0.01075 \mu\text{F}$. Is it possible to remove charge on C_0 this way?

Ans. $C = C_0 \left[\left(\frac{V_0}{V} \right)^{1/n} - 1 \right] = 0.01078 \mu\text{F}$, $n = 20$, No



Sol.

$$C_0 V_0 = (C_0 + C)V$$

$$V = \frac{C_0 V_0}{C_0 + C}$$

$$\therefore Q_f \text{ on first capacitor} = \frac{C_0^2 V}{C_0 + C}$$

Again connecting with uncharge capacitor of capacitance C .

$$V' = \frac{C_0^2 V_0}{C_0 + C} = (C_0 + C)V'$$

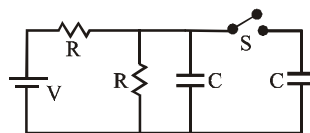
$$\Rightarrow V' = \frac{C_0^2 V_0}{(C_0 + C)^2}$$

After n Repeats

$$V_n = \frac{C_0^n V_0}{(C_0 + C)^n}$$

$$\therefore C = C_0 \left[\left(\frac{V}{V_0} \right)^{\frac{1}{n}} - 1 \right]$$

13. In the figure shown initially switch is open for a long time. Now the switch is closed at $t = 0$. Find the charge on the rightmost capacitor as a function of time given that it was initially unchanged.

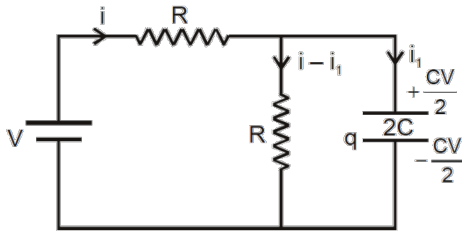


Ans. $q = \frac{CV}{2} \left(1 - \frac{1}{2} e^{-t/RC} \right)$

Sol. $iR + (i - i_1)R = V$

$$\Rightarrow 2iR - i_1 R = V \quad \dots(i)$$

$$\frac{q}{2C} = (i - i_1)R \quad \dots(ii)$$

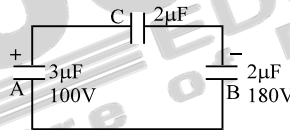


from (1) & (2)

$$2\left(\frac{q}{2C} + i_1 R\right) - i_1 R = V \Rightarrow \frac{q}{C} + i_1 R = V \Rightarrow \int_{\frac{CV}{2}}^q \frac{dq}{CV - q} = \int_0^t \frac{dt}{RC}$$

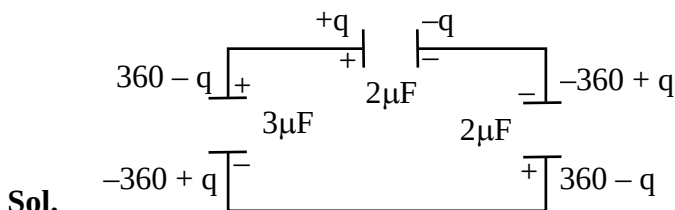
$$\Rightarrow q = CV \left[1 - \frac{1}{2} e^{-\frac{t}{RC}} \right] \quad \therefore q' = \frac{q}{2} = \frac{CV}{2} \left[1 - \frac{1}{2} e^{-\frac{t}{RC}} \right]$$

14. Two capacitors A and B with capacities $3 \mu\text{F}$ and $2 \mu\text{F}$ are charged to a potential difference of 100 V and 180 V respectively. The plates of the capacitors are connected as shown in figure with one wire from each capacitor free. The upper plate of A is positive and that of B is negative. An uncharged $2 \mu\text{F}$ capacitor C with lead wires falls on the free ends to complete the circuit. Calculate :



- The final charges on the three capacitors
- The amount of electrostatic energy stored in the system before and after the completion of the circuit.

Ans. $Q_A = 90 \mu\text{C}$, $Q_B = 150 \mu\text{C}$, $Q_C = 210 \mu\text{C}$, $U_i = 47.4 \text{ mJ}$, $U_f = 18 \text{ mJ}$



Sol.

$$\text{KVL} \quad \frac{300 - q}{3} - \frac{q}{2} + \frac{360 - q}{2} = 0$$

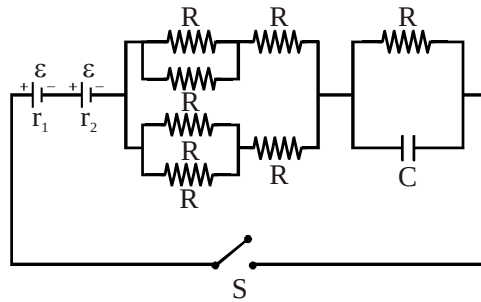
$$q = 210 \mu\text{F}$$

$$\therefore Q_A = 90 \mu\text{C}, Q_B = 150 \mu\text{C}, Q_C = 210 \mu\text{C}$$

$$U_L = \frac{1}{2} C_A V_A^2 + \frac{1}{2} C_B V_B^2 = 47.4 \text{ mJ}$$

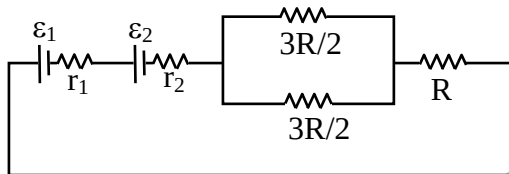
$$U_f = \frac{Q_A^2}{2C_A} + \frac{Q_B^2}{2C_B} + \frac{Q_C^2}{2C_C} = 18 \text{ mJ}$$

15. In the circuit shown in the figure initially switch S is open and capacitor is uncharged. Internal resistances of the cells are r_1 and r_2 , their emf's are equal to ε . The potential difference across the cell of internal resistance r_1 becomes zero long time after closing the switch. Find the value of R in terms of other known physical quantities. All symbols have their usual meaning.



Ans. $\frac{4}{7}(r_1 - r_2)$

Sol. After longtime the circuit becomes

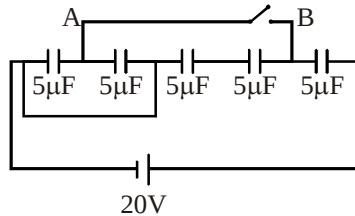


$$R_{eq} = 7R/4 + r_1 + r_2$$

$$i = \frac{\varepsilon_1 + \varepsilon_2}{R_{eq}}$$

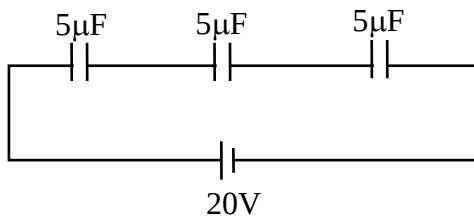
Given that $\varepsilon_1 - ir_1 = 0 \quad \therefore R = \frac{4}{7}(r_1 - r_2)$

16. Find the charge which flows from point A to B, when switch is closed.

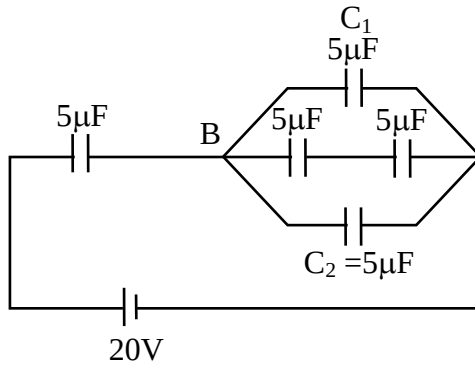


Ans. $\frac{-400}{7} \mu C$

Sol. Initially



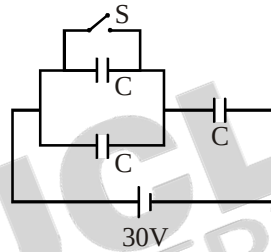
Finally



The charge flown through A to B, will be charge on C_1 and C_2

$$Q = Q_{C_1} + Q_{C_2} = -\frac{400}{7} \mu\text{C}$$

17. The capacitors each having capacitance $C=2\mu\text{F}$ are connected with a battery of emf 30V as shown in figure. When the switch S is closed. Find



- (i) the amount of charge flown through the battery
- (ii) the heat generated in the circuit
- (iii) the energy supplied by the battery
- (iv) the amount of charge flown through the switch S

Ans. (i) $20\mu\text{C}$, (ii) 0.3mJ , (iii) 0.6mJ , (iv) $40\mu\text{C}$

Sol. Before After

$$C_{\text{eq}} = \frac{2C}{3} \quad C_{\text{eq}} = C$$

$$(i) \theta = C_f V_f - C_i V_i = \frac{C}{3} \times 30 = 10C$$

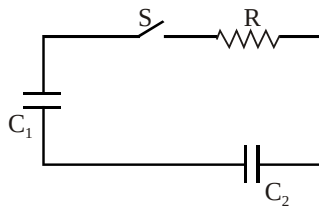
$$= 20 \mu\text{C}$$

$$(ii) \text{Heat} = (\text{WD})_{\text{Battery}} \\ = 0.3 \text{ mJ}$$

$$(iii) \text{Energy supplied} = (\text{WD})_{\text{Battery}} \\ = (Q_{\text{flown}}) (\Delta V) \\ = (20\mu\text{C}) (30) \\ = 0.6 \text{ mJ}$$

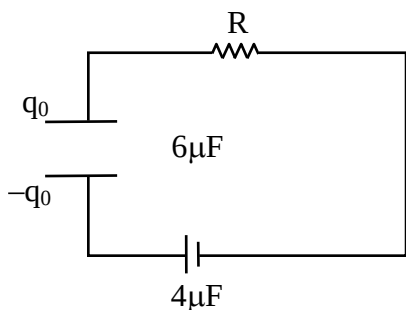
$$(iv) Q_{\text{flown}} = (60\mu\text{C} - 20\mu\text{C}) \\ = 40\mu\text{C}$$

18. Two capacitors C_1 ($6 \mu\text{F}$ & initial charge $q_0 = \left(\frac{30e}{e-1}\right) \mu\text{C}$) & C_2 ($4 \mu\text{F}$ & initial uncharged) are joined in series with resistance R (80Ω) as shown in figure. Switch S is closed at $t = 0$. Find charge on C_2 (in μC) at $t = 192 \mu\text{s}$.

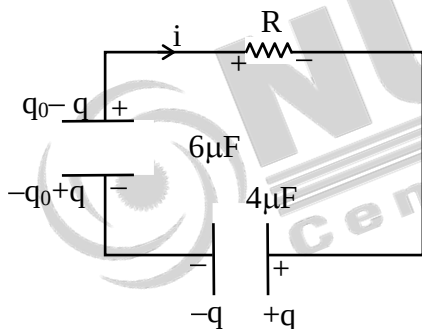


Ans. 012

Sol. $t = 0$



$t = t$



$$\frac{q_0 - q}{6} - iR - \frac{q}{4} = 0$$

$$i = \frac{dq}{dt}$$

$$\frac{-q}{4} + \frac{q_0 - q}{6} = R \left(\frac{dq}{dt} \right)$$

$$\frac{4q_0 - 10q}{24R} = \frac{dq}{dt}$$

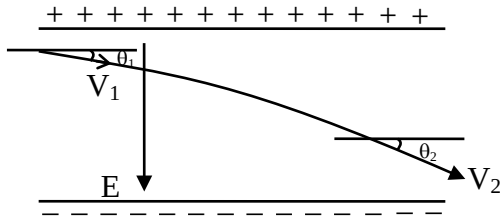
$$\int_0^q \frac{dq}{4q_0 - 10q} = \int_0^t \frac{dt}{24R}$$

$$q = 12 \mu\text{C}$$

at $t = 192 \mu\text{s}$

19. If an electron enters into a space between the plates of a parallel plate capacitor at an angle θ_1 with the plates and leaves at an angle θ_2 to the plates the ratio of its kinetic energy while entering the capacitor to that while leaving will be—

Ans. $\sec^2 \theta_1 / \sec^2 \theta_2 = \cos^2 \theta_2 / \cos^2 \theta_1$



Sol.

$$V_1 \cos \theta_1 = V_2 \cos \theta_2$$

$$V_2 = \frac{V_1 \cos \theta_1}{\cos \theta_2}$$

$$KE_i = \frac{1}{2} m V_1^2$$

$$KE_f = \frac{1}{2} m V_2^2$$

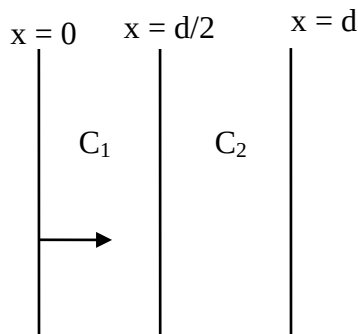
$$\frac{KE_i}{KE_f} = \frac{\cos^2 \theta_2}{\cos^2 \theta_1}$$

20. Calculate the capacitance of a parallel plate condenser, with plate area A and distance between plates d , when filled with a medium whose permittivity varies as ;

$$\epsilon(x) = \epsilon_0 + \beta x \quad \text{for } 0 < x < \frac{d}{2}$$

$$\epsilon(x) = \epsilon_0 + \beta(d - x) \quad \text{for } \frac{d}{2} < x < d$$

Ans. $\frac{\beta A}{2} \ln \left(\frac{2\epsilon_0 + \beta d}{2\epsilon_0} \right)$



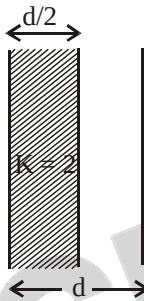
Sol.

$$dC = \left(\frac{\epsilon_0}{dx} A \right) (\epsilon_0 + \beta x)$$

$$\int \frac{1}{dC} = \int_0^{d/2} \frac{dx}{\epsilon_0 A(\epsilon_0 + Bx)} + \int_{d/2}^d \frac{dx}{\epsilon_0 A(\epsilon_0 + B(d-x))}$$

$$C_{eq} = \frac{BA}{2 \ln \left(\frac{2\epsilon_0 + Bd}{2\epsilon_0} \right)}$$

21. A certain series RC circuit is formed using a resistance R, a capacitor without dielectric having a capacitance $C = 2F$ and a battery of emf $E = 3V$. The circuit is completed and it is allowed to attain the steady state. After this, at $t = 0$, half the thickness of the capacitor is filled with a dielectric of constant $K = 2$ as shown in the figure. The system is again allowed to attain a steady state. What will be the heat generated (in joule) in the circuit between $t = 0$ and $t = \infty$?



Ans 3

Sol. Initial charge on capacitor = CE

$$\text{Initial potential energy of capacitor} = \frac{CE^2}{2}$$

$$\text{Now } C = \frac{\epsilon_0 A}{d}$$

$$\text{And new capacitance : } C' = \frac{\epsilon_0 A}{d/2}, \text{ in series with } \frac{\epsilon_0 kA}{d/2}$$

$$\Rightarrow C' = \frac{2\epsilon_0 A}{d} \text{ series } \frac{4\epsilon_0 A}{d}$$

$$= \frac{\epsilon_0 A}{d} \left(\frac{2 \times 4}{2 + 4} \right) = \frac{4\epsilon_0 A}{3d} = \frac{4}{3}C \Rightarrow \text{New charge} = \frac{4}{3}CE$$

$$\text{New energy} = \frac{1}{2} \times \frac{4}{3}CE^2 = \frac{2}{3}CE^2$$

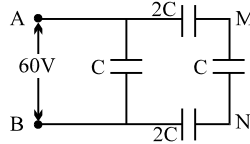
$$\text{Now } W_{\text{bat}} = \Delta H + \Delta U$$

$$\Rightarrow E \left(\frac{4}{3}CE - CE \right) = \Delta H + \frac{2}{3}CE^2 - \frac{1}{2}CE^2 \Rightarrow \Delta H = \frac{1}{6}CE^2$$

EXERCISE # (O-1)

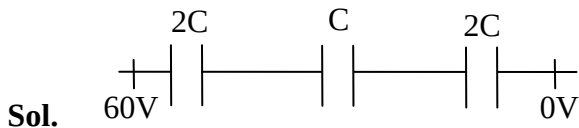
CAPACITOR CIRCUIT

1. In the circuit shown, a potential difference of 60V is applied across AB. The potential difference between the point M and N is



- (A) 10 V (B) 15 V (C) 20 V (D) 30 V

Ans. D



In series charge on all the 3 capacitors & will be same

$$\therefore V_1 + V_2 + V_3 = 60$$

$$\frac{Q}{2C} + \frac{Q}{C} + \frac{Q}{2C} = 60$$

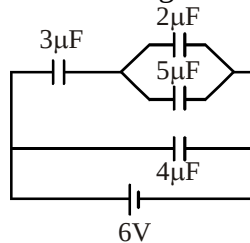
$$\frac{Q}{C}[2] = 60$$

$$\boxed{Q = 30C}$$

Potential difference across capacitance 'C'

$$= \frac{Q}{C} = \frac{30C}{C} = 30 \text{ V}$$

2. In the circuit shown in figure, the ratio of charges on $5\mu\text{F}$ and $4\mu\text{F}$ capacitor is:



- (A) 4/5 (B) 3/5 (C) 3/8 (D) 1/2

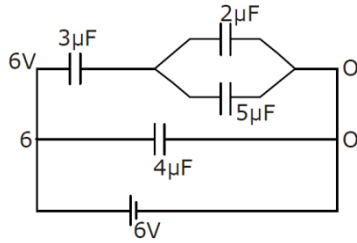
Ans C

Sol. For charge in $5\mu\text{F}$ capacitor

$$C_1 : C_2 = 2 : 5$$

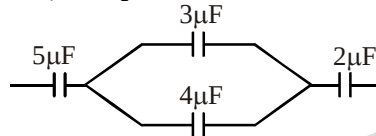
$$\frac{q_1}{q_2} = \frac{C_1}{C_2}$$

$$q_2 = \frac{5 \times 18}{10}$$



charge on $5\mu\text{F}$ capacitor is $9\mu\text{C}$
 charge on $4\mu\text{F}$ capacitor is $24\mu\text{C}$
 Ratio of charges = $9 : 24 = 3 : 8$

3. If charge on left plane of the $5\mu\text{F}$ capacitor in the circuit segment shown in the figure is $-20\mu\text{C}$, the charge on the right plate of $3\mu\text{F}$ capacitor is



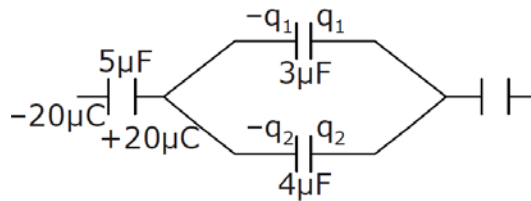
(A) $+8.57\mu\text{C}$

(B) $-8.57\mu\text{C}$

(C) $+11.42\mu\text{C}$

(D) $-11.42\mu\text{C}$

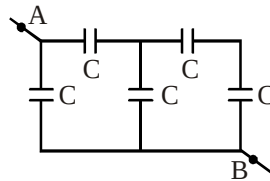
Ans. A
 Sol.



$$q_1 : q_2 = 3 : 4$$

$$q_1 = \frac{3}{7} \times 20\mu\text{C}$$

4. What is the equivalent capacitance of the system of capacitors between A & B as shown in the figure.



(A) $\frac{7}{6}C$

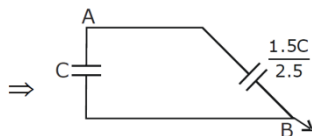
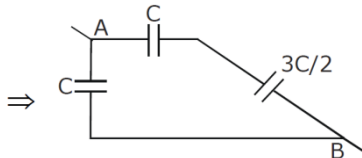
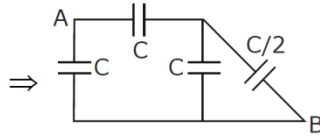
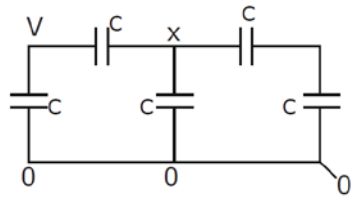
(B) $1.6C$

(C) C

(D) None

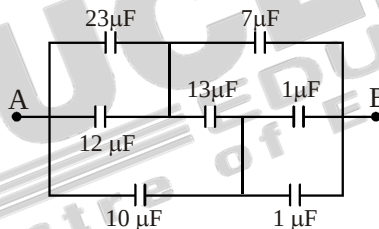
Ans. B

Sol. Solving the circuit using following steps



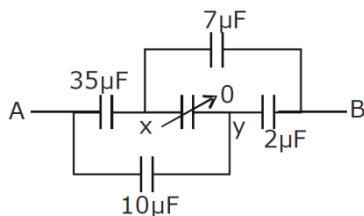
Resultant capacitance of the circuit = $1.6C$

5. Find the equivalent capacitance across A & B

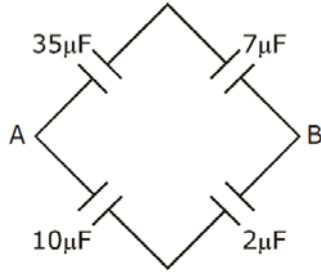


- (A) $\frac{28}{3} \mu\text{F}$ (B) $\frac{15}{2} \mu\text{F}$ (C) $15 \mu\text{F}$ (D) None

Ans. B
Sol.

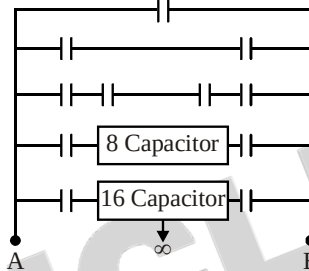


As the resulting circuit is a Wheat stone bridge hence current in 13mF capacitor is zero. Hence the circuit now reduces to



The resultant capacitance is $\frac{35}{6} + \frac{10}{6} = \frac{45}{6} = \frac{15}{2} \mu\text{F}$

6. An infinite number of identical capacitors each of capacitance $1 \mu\text{F}$ are connected as shown in figure. Then the equivalent capacitance between A and B is

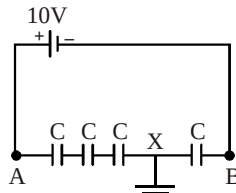


- (A) $1 \mu\text{F}$ (B) $2 \mu\text{F}$ (C) $1/2 \mu\text{F}$ (D) ∞

Ans. B

Sol. $C_{eq} = C + \frac{C}{2} + \frac{C}{4} + \frac{C}{8} + \frac{C}{16} + \dots (C = 1 \mu\text{F})$
 $= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$ (Sum of infinite C.P = $\frac{a}{1-r}$)
 $= \frac{1}{1 - \frac{1}{2}} = 2 \mu\text{F}$ [B]

7. Four identical capacitors are connected in series with a battery of emf 10V. The point X is earthed. Then the potential of point A is



- (A) 10 V (B) 7.5 V (C) -7.5 V (D) 0 V

Ans. B

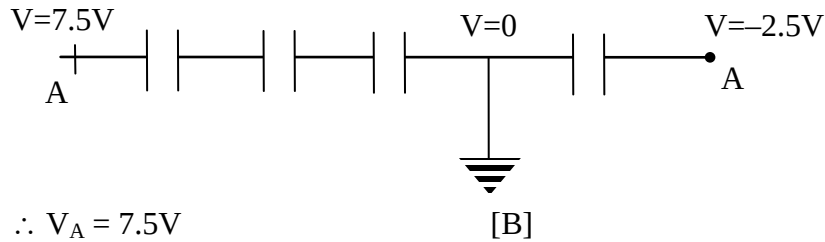
Sol. All the four capacitors are in series, $\therefore$ charge is same, and since Capacitance is also same $\therefore$

$\left(V = \frac{Q}{C} \right)$, Potential will also be same

$\therefore V_1 + V_2 + V_3 + V_4 = 10$

$$4V = 10$$

$$V = 2.5 \text{ volts}$$



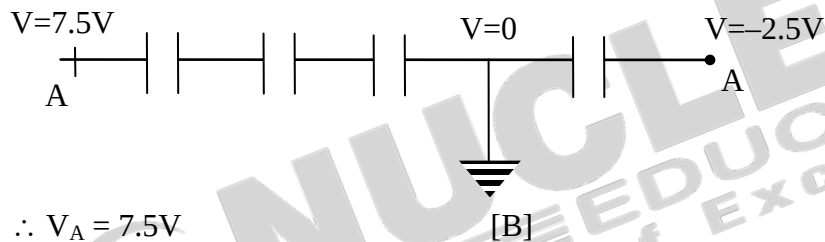
Sol. All the four capacitors are in series, $\therefore$ charge is same, and since Capacitance is also same $\therefore$

$$\left(V = \frac{Q}{C} \right), \text{ Potential will also be same}$$

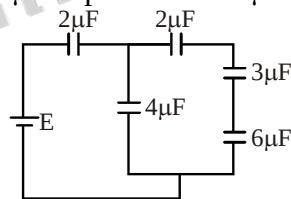
$$\therefore V_1 + V_2 + V_3 + V_4 = 10$$

$$4V = 10$$

$$V = 2.5 \text{ volts}$$



8. In the given circuit if charge on $6\mu F$ capacitor is $10\mu C$, then charge on $4\mu F$ capacitor will be



(A) $40\mu C$

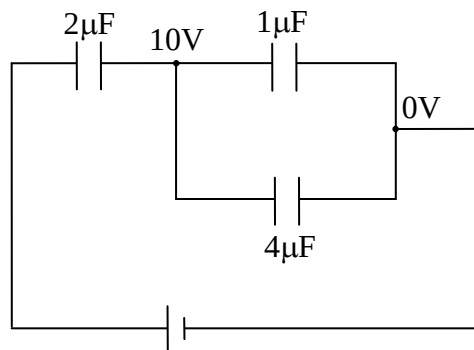
(B) $10\mu C$

(C) $20\mu C$

(D) $30\mu C$

Ans. A

Sol. The circuit can be redrawn as



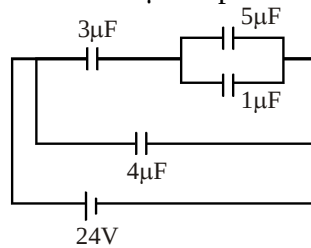
$$\Delta V_{1\mu F} = \Delta V_{4\mu F}$$

$$\Rightarrow \frac{Q_{1\mu F}}{C_1} = \frac{Q_{4\mu F}}{C_2}$$

$$\frac{10}{1} = \frac{Q_{4\mu F}}{4}$$

$$Q_{4\mu F} = 40\mu C$$

9. In the circuit shown, the energy stored in $1\mu F$ capacitor is



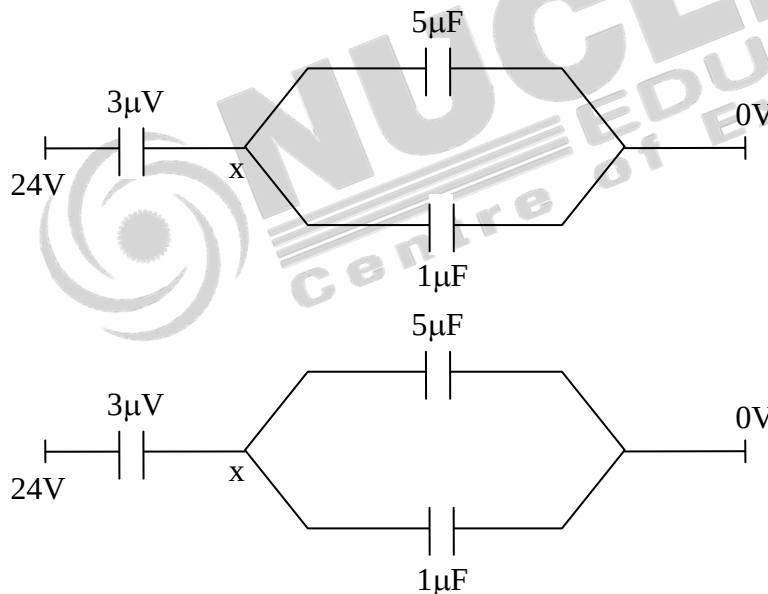
(A) $40\mu J$

(B) $64\mu J$

(C) $32\mu J$

(D) None

Ans
Sol.



On solving Potential $x = 8$ Volts

$$\text{Energy stored} = \frac{1}{2} CV^2$$

$$= \frac{1}{2} (1)(8)^2 = 32\mu J$$

10. A capacitor of capacitance C is initially charged to a potential difference of V volt. Now it is connected to a battery of $2V$ Volt with opposite polarity. The ratio of heat generated to the final energy stored in the capacitor will be

(A) 1.75

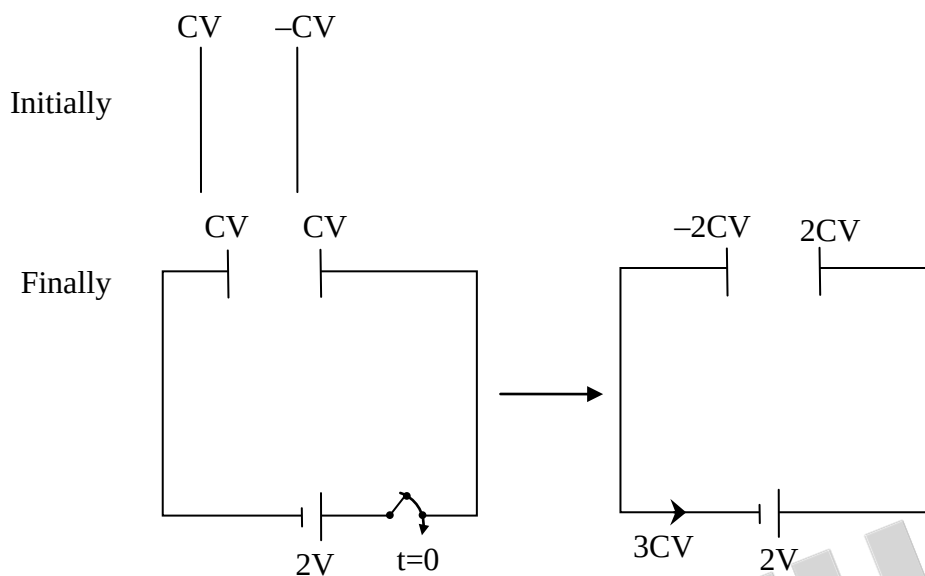
(B) 2.25

(C) 2.5

(D) 1/2

Ans. B

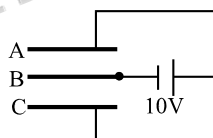
Sol.



$$\epsilon_i = \frac{1}{2} CV^2, \quad E_f = 2CV^2, \quad W_{\text{battery}} = (3CV)(2V) = 6CV^2$$

$$\text{Heat} = 4.5 CV^2 \quad \boxed{\frac{\text{Heat}}{\epsilon_f} = 2.25}$$

11. Three plates A, B and C each of area 0.1 m^2 are separated by 0.885 mm from each other as shown in the figure. A 10 V battery is used to charge the system. The energy stored in the system is



(A) $1 \mu\text{J}$

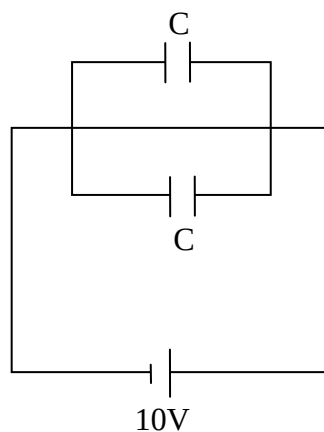
(B) $10^{-1} \mu\text{J}$

(C) $10^{-2} \mu\text{J}$

(D) $10^{-3} \mu\text{J}$

Ans B

Sol. The equivalent circuit diagram is



$$C = \frac{\epsilon_0 A}{d}$$

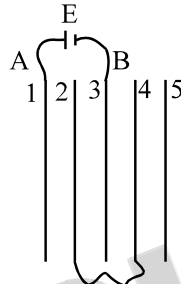
$$C = 10^{-9} \text{ faraday}$$

$$E = \frac{1}{2} CV^2 + \frac{1}{2} CV^2 = CV^2$$

$$= 10^{-9}(100) = 10^{-7} \text{ J}$$

$$= 0.1 \times 10^{-6} \text{ J} = 0.1 \mu\text{J} \quad [\text{B}]$$

12. Five conducting parallel plates having area A and separation between them d , are placed as shown in the figure. Plate number 2 and 4 are connected wire and between point A and B, a cell of emf E is connected. The charge flow through the cell is



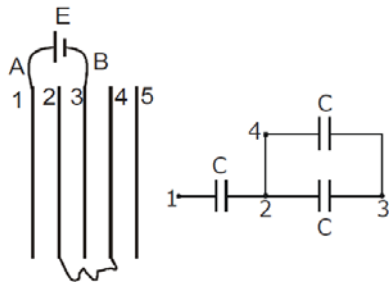
(A) $\frac{3 \epsilon_0 AE}{4 d}$

(B) $\frac{2 \epsilon_0 AE}{3 d}$

(C) $\frac{4 \epsilon_0 AE}{d}$

(D) $\frac{\epsilon_0 AE}{2d}$

Ans
Sol.



$$C_{eq} = \frac{2C \times C}{3C} = \frac{2 \epsilon_0 A}{3d}$$

$$Q = \frac{2}{3} \times \frac{\epsilon_0 A}{d} \times E$$

13. A parallel plate capacitor has an electric field of 10^5 V/m between the plates. If the charge on the capacitor plate is $1 \mu\text{C}$, then the force on each capacitor plate is
(A) 0.1 Nt (B) 0.05 Nt (C) 0.02 Nt (D) 0.01 Nt

Ans

B

Sol. Force between the plates is given by

$$\frac{\sigma^2 A}{2 \epsilon_0} \text{ or}$$

$$F = q \frac{E}{2} = \frac{1 \times 10^{-6} \times 10^5}{2}$$

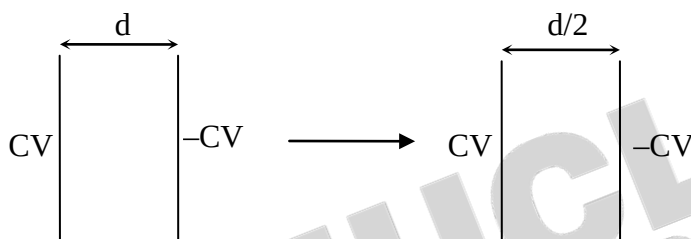
$$\left[\frac{E}{2} \text{ as electric field is due to charges on a single plate is to be written} \right] \frac{0.1}{2} N = 0.05 N$$

14. A parallel-plate capacitor is connected to a resistance less circuit with a battery until the capacitor is fully charged. The battery is then disconnected from the circuit and the plates of the capacitor are moved to half of their original separation using insulated gloves. Let V_{new} be the potential difference across the capacitor plates when the plates have moved. Let V_{old} be the potential difference across the capacitor plates when they were connected to the battery. $\frac{V_{\text{new}}}{V_{\text{old}}} =$

- (A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2

Ans. B

Sol.



$$C = \frac{\epsilon_0 A}{d}$$

$$C' = \frac{\epsilon_0 A}{d/2} = 2C$$

$$V_{\text{old}} = \frac{CV}{C} = V$$

$$V_{\text{new}} = \frac{CV}{2C} = \frac{V}{2}$$

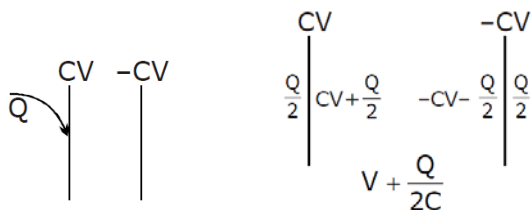
$$\boxed{\frac{V_{\text{new}}}{V_{\text{old}}} = \frac{1}{2}}$$

15. A capacitor of capacitance C is charged to a potential difference V from a cell and then disconnected from it. A charge $+Q$ is now given to its positive plate. The potential difference across the capacitor is now

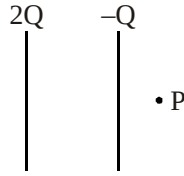
- (A) V (B) $V + \frac{Q}{C}$ (C) $V + \frac{Q}{2C}$ (D) $V - \frac{Q}{C}$, if $V < CV$

Ans C

Sol.



16. In the figure shown the plates of a parallel plate capacitor have unequal charges. Its capacitance is 'C'. P is a point outside the capacitor and close to the plate of charge -Q. The distance between the plates is 'd' then which statement is wrong



(A) A point charge at point 'P' will experience electric force due to capacitor

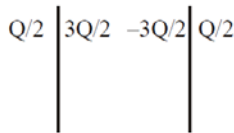
(B) The potential difference between the plates will be $\frac{3Q}{2C}$

(C) The energy stored in the electric field in the region between the plates is $\frac{9Q^2}{8C}$

(D) The force on one plate due to the other plate is $\frac{Q^2}{2\pi\epsilon_0 d^2}$

Ans D

Sol.



$$\Delta V = \frac{3Q}{2C}$$

$$U = \frac{\frac{1}{2} \left(\frac{3Q}{2} \right)^2}{C}$$

17. Consider a capacitor connected with a battery, capacitor is in steady state. Now plates of capacitor are drawn apart so as to double the separation in two cases.

Case :

(i) Battery remains connected

(ii) Battery is disconnected

Mark the **CORRECT** statement.

- (A) In case (i) energy of capacitor increases
(B) In case (i) work done by battery is positive
(C) In case (ii) energy of capacitor increases
(D) In case (ii) potential difference across capacitor decreases

Ans C

Sol. In case-i potential difference remains constant & $U = \frac{1}{2} CV^2$

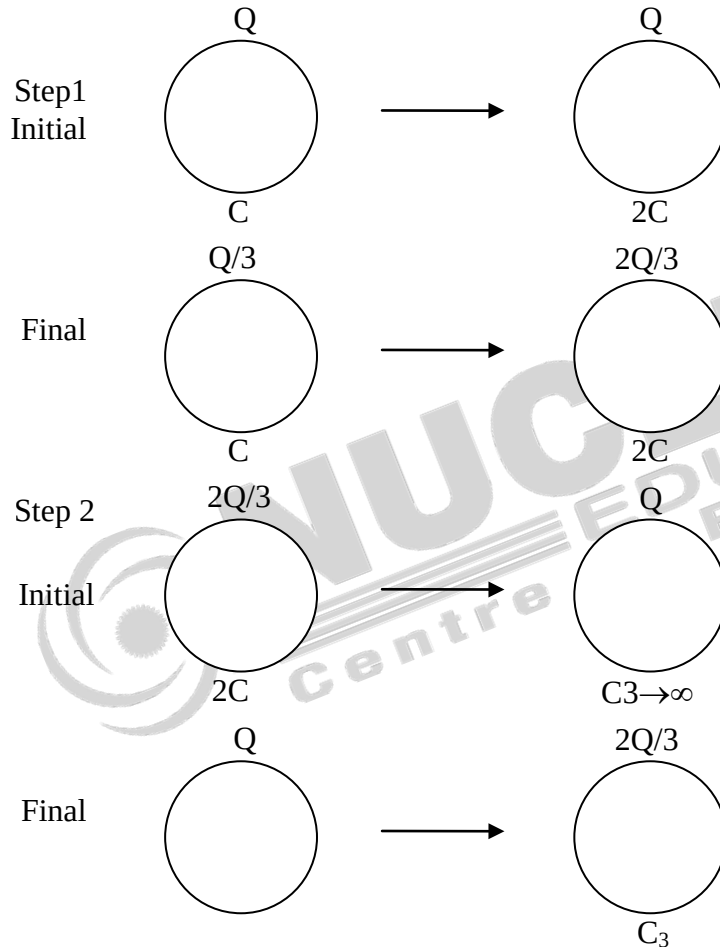
in case-ii charge remains constant & $U = \frac{Q^2}{2C}$

18. A conducting body 1 has some initial charge Q , and its capacitance is C . There are two other conducting bodies, 2 and 3, having capacitances : $C_2 = 2C$ and $C_3 \rightarrow \infty$. Bodies 2 and 3 are initially uncharged. "Body 2 is touched with body 1. Then, body 2 is removed from body 1 and touched with body 3, and then removed." This process is repeated N times. Then, the charge on body 1 at the end must be

(A) $Q/3^N$ (B) $Q/3^{N-1}$ (C) Q/N^3 (D) None

Ans A

Sol. On touching the Capacitors charge flows between them, until the potential on them becomes same



On doing the process for one time, charge on capacitor $C_1 = \frac{Q}{3}$

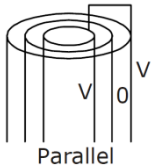
On repeating the process 'x' times, $Q_f = \frac{Q}{3^N}$

19. Three long concentric conducting cylindrical shells have radii R , $2R$ and R . Inner and outer shells are connected to each other. The capacitance across middle and inner shells per unit length is:

- (A) $\frac{1}{3} \frac{\epsilon_0}{\ln 2}$ (B) $\frac{6\pi \epsilon_0}{\ln 2}$ (C) $\frac{\pi \epsilon_0}{2 \ln 2}$ (D) None

Ans B

Sol. $C = \frac{2\pi \epsilon_0}{\ln b/a} = \frac{2\pi \epsilon_0}{\ln 2R/R} + \frac{2\pi \epsilon_0}{\ln \frac{2\sqrt{2}}{2R} R}$

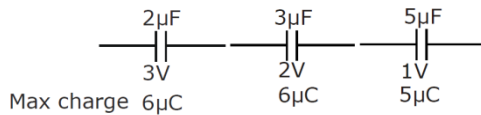


$= \frac{2\pi \epsilon_0}{\ln 2} [1 + 2] = \frac{6\pi \epsilon_0}{\ln 2}$

20. Three capacitors $2 \mu\text{F}$, $3 \mu\text{F}$ and $5 \mu\text{F}$ can withstand voltages to 3V, 2V and 1V respectively. Their series combination can withstand a maximum voltage equal to
(A) 5 Volts (B) $(31/6)$ Volts (C) $(26/5)$ Volts (D) None

Ans. B

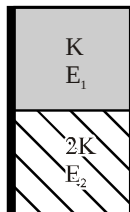
Sol.



Hence maximum charge that the series can with stand is $5 \mu\text{C}$. So break down voltage = $5 \times \frac{31}{30} = \frac{31}{6}$ volt

DI-ELECTRIC

21. A parallel plate capacitor is connected from a cell and then isolated from it. Two dielectric slabs of dielectric constant K and $2K$ are now introduce in the region between upper half and lower half of the plate (as shown in figure). The electric field intensity in upper half of dielectric is E_1 and lower half is E_2 then



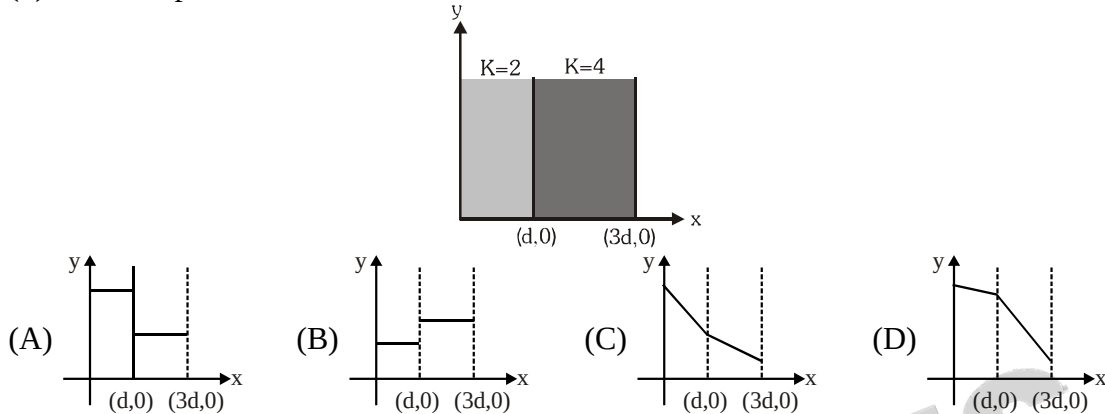
- (A) $E_1 = 2E_2$
(B) Electrostatic potential energy of upper half is less than that of lower half
(C) Induced charges on both slabs are same
(D) Charge distribution on the plates remains same after insertion of dielectric

Both capacitors are in parallel

Ans. B

Sol. Both capacitors are in parallel

22. A parallel plate capacitor has two layers of dielectric as shown in figure. This capacitor is connected across a battery. The graph which shows the variation of electric field (E) & distance (x) from left plate.



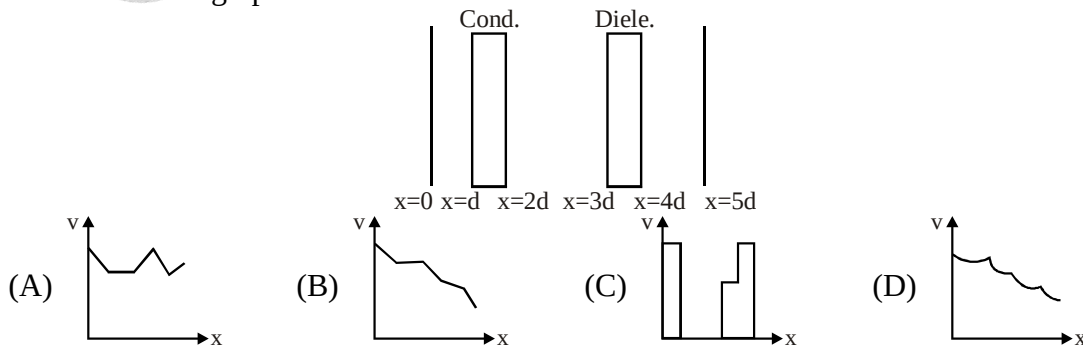
Ans A

Sol. In dielectric $E = \frac{\sigma}{K\epsilon_0}$

For region (1) $E_1 = \frac{\sigma}{2\epsilon_0}$

For region (2) $E_2 = \frac{\sigma}{4\epsilon_0}$ [A]

23. The distance between plates of a parallel plate capacitor is 5d. Let the positively charged plate is at $x=0$ and negatively charged plate is at $x=5d$. Two slabs one of conductor and other of a dielectric of equal thickness d are inserted between the plates as shown in figure. Potential versus distance graph will look like :



Ans B

Sol. $\Delta V = E(d)$

$$E_1 = \frac{\sigma}{\epsilon_0}, E_2 = 0, E_3 = \frac{\sigma}{\epsilon_0}, E_4 = \frac{\sigma}{\epsilon_0}, E_5 = \frac{\sigma}{\epsilon_0}$$

Region (1), (3) & (5) have same slope of straight-line.

$\therefore$ Graph (B) is correct.

24. A capacitor stores $60\mu\text{C}$ charge when connected across a battery. When the gap between the plates is filled with a dielectric, a charge of $120\mu\text{C}$ flows through the battery. The dielectric constant of the material inserted is :

(A) 1 (B) 2 (C) 3 (D) None

Ans C

Sol. As the potential difference is constant hence we can say that

$$Q_1 = 60 \mu\text{C} = V \times C \quad \dots(1)$$

Now there is already $60 \mu\text{C}$ on the capacitor.

More $120 \mu\text{C}$ charge flows from battery. Hence net charge on capacitor is

$$Q_2 = 180 \mu\text{C} = V \times KC \quad \dots(2)$$

$$(2) / (1) \Rightarrow 3 = K$$

25. Condenser A has a capacity of $15 \mu\text{F}$ when it is filled with a medium of dielectric constant 15. Another condenser B has a capacity $1 \mu\text{F}$ with air between the plates. Both are charged separately by a battery of 100V . After charging, both are connected in parallel without the battery and the dielectric material being removed. The common potential now is

(A) 400V (B) 800V (C) 1200V (D) 1600V

Ans B

Sol. Charge on $15 \mu\text{F}$ capacitor A = $1500 \mu\text{C}$.

Charge on capacitor B = $100 \mu\text{C}$.

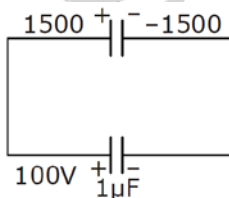
When they are connected with dielectric removed from A.

Capacitance of A now becomes $1 \mu\text{F}$.

$$C_{iA} = \frac{\epsilon_0 A \cdot 15}{d} = 15C = 15\mu\text{F}$$

$$C_{fA} = \frac{\epsilon_0 A}{d} C = 1\mu\text{F}$$

Q remains constant

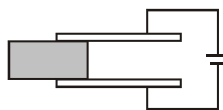


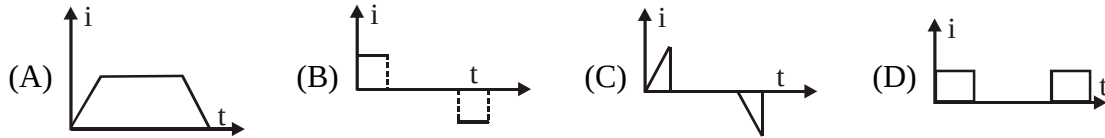
$$Q_{\text{net}} = C_{\text{eq}} \times V_{\text{common}}$$

$$1500 + 100 = (1 + 1)V$$

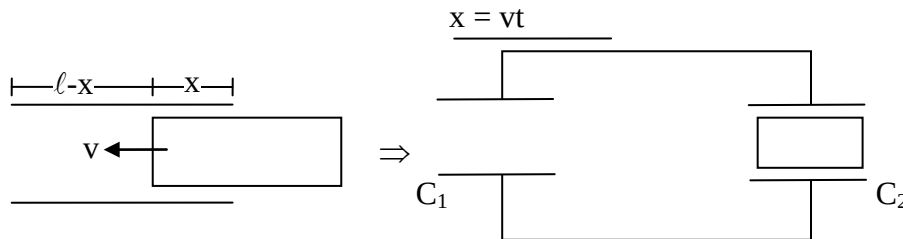
$$V = 800 \text{ Volt}$$

26. A dielectric slab of area A passes between the capacitor plates of area $2A$ with a constant speed v . The variation of current (i) through the circuit as function of time (t) can be qualitatively represented as :





Ans. B
Sol.



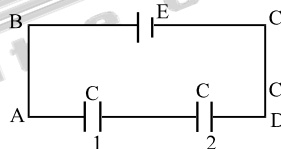
$$C_{eq} = C_1 + C_2$$

$$C_{eq} = \frac{K \epsilon_0 (\ell - vt) b}{d} + \frac{\epsilon_0 vtb}{d}$$

$$C_{eq} = \frac{\epsilon_0 b}{d} (K\ell + vt(1-K))$$

$$Q = v_{cq} \Rightarrow i = \frac{dq}{dt} = \text{const}$$

27. In the adjoining figure, capacitor (1) and (2) have a capacitance 'C' each. When the dielectric of dielectric constant K is inserted between the plates of one of the capacitor, the total charge flowing through battery is



(A) $\frac{KCE}{K+1}$ from B to C

(B) $\frac{KCE}{K+1}$ from C to B

(C) $\frac{(K-1)CE}{2(K+1)}$ from B to C

(D) $\frac{(K-1)CE}{2(K+1)}$ from C to B

Ans. D

Sol. $C_{eq} = \frac{C}{2}$

$$Q_1 = \frac{C}{2} E$$

Now

$$C_1 = KC, \quad C_2 = C$$

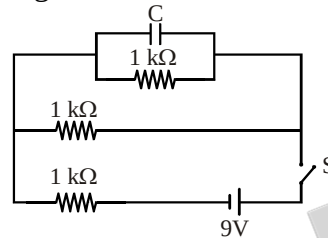
$$C_{eq} = \frac{KC \cdot C}{KC + C} = \frac{KC}{(K+1)}$$

$$Q_2 = \frac{KCE}{(K+1)} \rightarrow \text{clearly } Q_2 < Q_1$$

$$Q_1 - Q_2 = CE \left(\frac{1}{2} - \frac{K}{K+1} \right) \quad \text{from C to B}$$

$$= \frac{(K-1)CE}{2(K+1)}$$

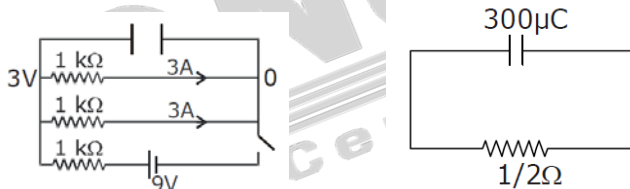
28. A capacitor $C = 100 \mu\text{F}$ is connected to three resistor each of resistance $1 \text{ k}\Omega$ and a battery of emf 9V . The switch S has been closed for long time so as to charge the capacitor. When switch S is opened, the capacitor discharges with time constant



- (A) 33 ms (B) 5 ms (C) 3.3 ms (D) 50 ms

Ans. D

Sol. To calculate charge on capacitor consider that capacitor acts as open circuit when completely charged and calculate drop across it which comes out to be 3V .
When s is opened i.e. discharging circuit

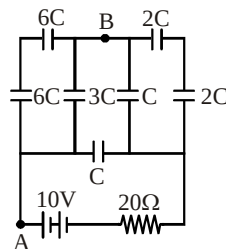


$$\tau = R_{eq}C$$

$$= \frac{1}{2} \times 100 \times 10^{-6} \times 10^3 = 50 \times 10^{-3} = 50\text{ms.}$$

RC CIRCUIT

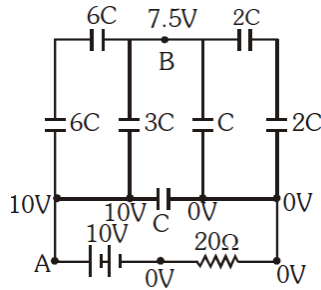
29. For the circuit shown here, the potential difference between points A and B is



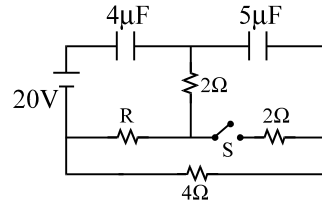
- (A) 2.5 V (B) 7.5 V (C) 10 V (D) Zero

Ans. A

Sol.



30. Find heat produced on closing the switch S



- (A) 0.0002 J (B) 0.0005 J (C) 0.00075 (D) zero

Ans

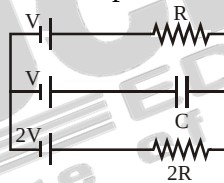
D

Sol.

In both case circuit behave as open circuit.

So, Net heat produced = 0

31. In the given circuit, with steady current the potential drop across the capacitor must be :

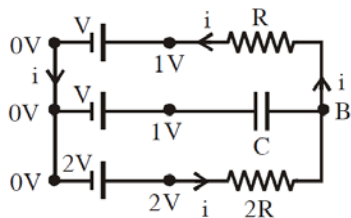


- (A) V (B) $\frac{V}{2}$ (C) $\frac{V}{3}$ (D) $\frac{2V}{3}$

Ans

C

Sol.

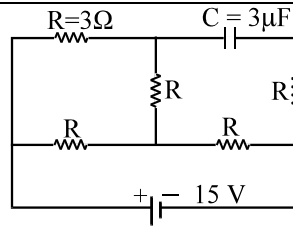


$$i = \frac{V}{3R}$$

$$V_B = 2V - \frac{V}{3R} \times 2R = \frac{4V}{3}$$

$$\text{Potential difference across capacitor } \frac{4V}{3} - V = \frac{V}{3}$$

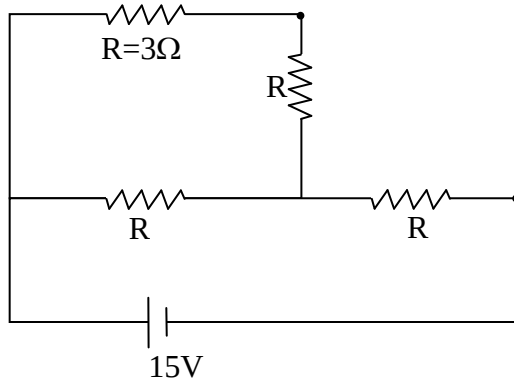
32. In the circuit shown, the cell is ideal, with emf = 15 V. Each resistance is of 3Ω . The potential difference across the capacitor is



- (A) zero (B) 9 V (C) 12 V (D) 15 V

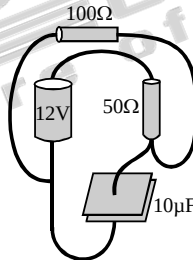
Ans. C

Sol. At Steady state.



Potential difference across point A & B at steady state is 9V, therefore potential difference across capacitor is also 9V.

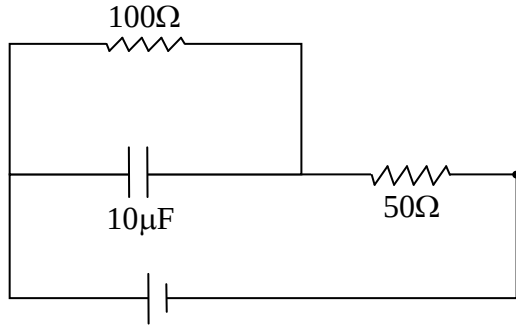
33. In the circuit shown in figure.



- (A) In steady state, there is no current in the 100 Ω resistor
(B) In steady state, the current in 100 Ω resistor is 0.08 A
(C) In steady state, there is no current in the 50 Ω resistor
(D) In steady state, the current in 50 Ω resistor is 0.04 A

Ans. B

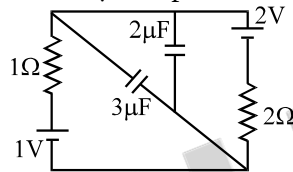
Sol. The equivalent circuit diagram is.



at steady state $R_{eq} = 150$

$$(i)_{100\Omega} = (i)_{50\Omega} = i = \frac{12}{150} = 0.08 \text{ Amp}$$

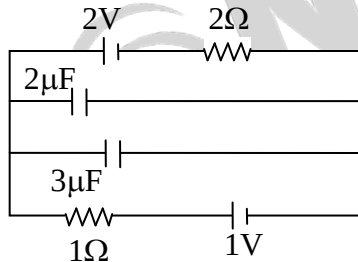
34. In the circuit shown, the charge on the $3\mu\text{F}$ capacitor at steady state will be



- (A) $6 \mu\text{C}$ (B) $4 \mu\text{C}$ (C) $\frac{2}{3} \mu\text{C}$ (D) $3 \mu\text{C}$

Ans. B

Sol. Equivalent Circuit diagram



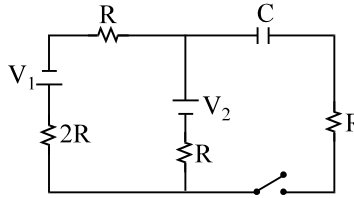
$$i = \frac{2-1}{3} = \frac{1}{3} \text{ A}$$

Potential difference across Capacitors is equal to Potential difference across terminals of 1 volt

$$\text{Battery} = \varepsilon = ir = 1 + 1\left(\frac{1}{3}\right) = \frac{4}{3} \text{ Volt}$$

$$Q_{3\mu\text{F}} = CV = 3\left(\frac{4}{3}\right) = 4\mu\text{C}$$

35. In the transient circuit shown the time constant of the circuit is :



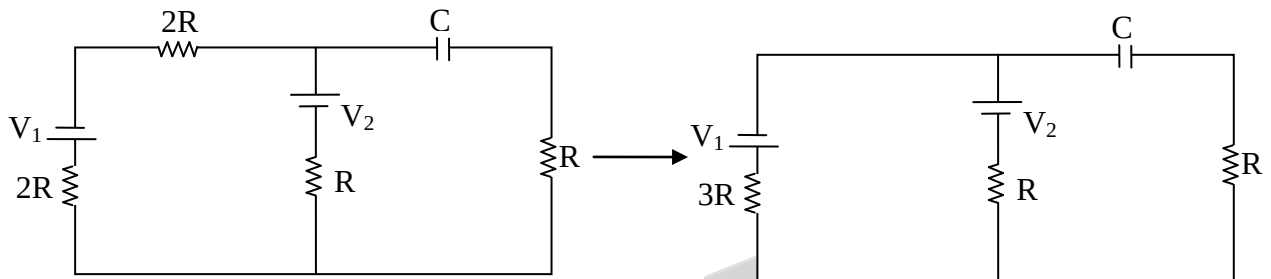
(A) $\frac{5}{3} RC$

(B) $\frac{5}{2} RC$

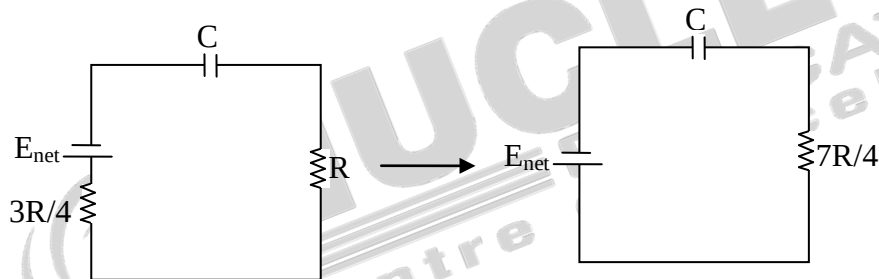
(C) $\frac{7}{4} RC$

(D) $\frac{7}{3} RC$

Ans. C
Sol.



V_1 and V_2 are in parallel, $\therefore$ Circuit diagram becomes

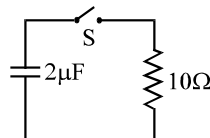


$$\tau = \left(\frac{7R}{4} \right) C$$

Solving, we get

$$t = RC \ln \left(\frac{1+n}{n} \right)$$

36. In the R-C circuit shown in the figure the total energy of $3.6 \times 10^{-3} \text{ J}$ is dissipated in the 10Ω resistor when the switch S is closed. The initial charge on the capacitor is



(A) $60 \mu\text{C}$

(B) $120 \mu\text{C}$

(C) $60\sqrt{2} \mu\text{C}$

(D) $\frac{60}{\sqrt{2}} \mu\text{C}$

Ans. B

Sol. $\frac{Q^2}{2C} = 3.6 \times 10^{-3} \text{J}$
 $Q_2 = (2 \times 2 \times 10^{-6}) \times 3.6 \times 10^{-3}$
 $\boxed{Q = 120 \mu\text{C}}$

37. A charged capacitor is allowed to discharge through a resistor by closing the key at the instant $t = 0$. At the instant $t = (\ln 4) \mu\text{s}$, the reading of the ammeter falls half the initial value. The resistance of the ammeter is equal to

(A) $1 \text{ M}\Omega$ (B) 1Ω (C) 2Ω (D) $2 \text{ M}\Omega$

Ans. C

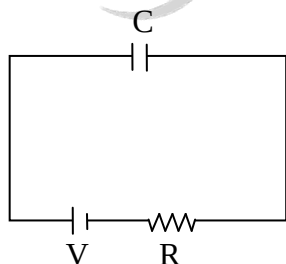
Sol. $i = i_0 e^{-\frac{t}{RC}}$
 $R = r_A + 2, C = 0.5 \times 10^{-6} \text{F}$
 at, $t = (\ln 4) \mu\text{s}$, $i = \frac{i_0}{2}$
 $\therefore \frac{i_0}{2} = i_0 e^{-\frac{(\ln 4) \times 10^{-6}}{(r_A + 2)(.5 \times 10^{-6})}}$
 Solving, we get $\boxed{r_A = 252}$

38. A capacitor of capacitance C is charged by a battery whose internal resistance is R . The time after which potential difference across resistor becomes n times to that across capacitor is

(A) $RC \ln \left(\frac{1+n}{n} \right)$ (B) $RC \ln \left(\frac{n}{1+n} \right)$ (C) $RC \ln \left(\frac{n}{n-1} \right)$ (D) $RC \ln(1+n)$

Ans. A

Sol.



$$V_R = iR = \left(\frac{V}{R} e^{-\frac{t}{RC}} \right) R$$

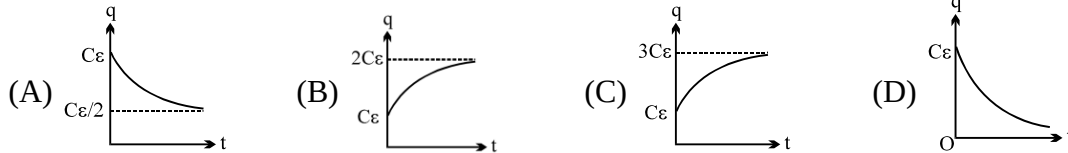
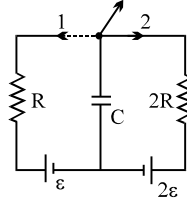
$$V_R = V e^{-\frac{t}{RC}}$$

$$V_R - iR = \left(\frac{V}{R} e^{-\frac{t}{RC}} \right)$$

$$V_C = \frac{Q}{C} V (1 - e^{-\frac{t}{RC}})$$

$$\text{Given } V_R = nV_C$$

39. In the circuit shown, the switch is shifted from position 1 $\rightarrow$ 2 at $t = 0$. The switch was initially in position 1 since a long time. The graph between charge on capacitor C and time ' t ' is

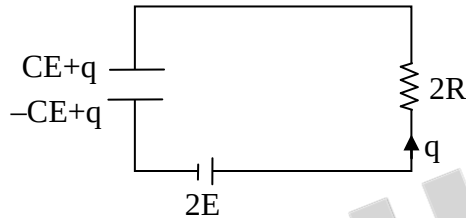


Ans. B

Sol. When switch is shifted to position - 2 charge present of capacitor is Q

$$Q = CE$$

Now after time t .



Applying KVL

$$2E - 2iR - \frac{CE + q}{C} = 0$$

$$2iR = E - \frac{q}{C}$$

$$2RC \cdot \frac{dq}{dt} = (CE - q)$$

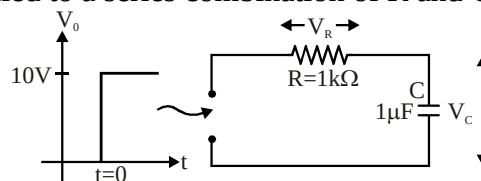
$$\int_0^q \frac{dq}{CE - q} = \frac{1}{2RC} \int_0^t dt$$

$$\ln\left(\frac{CE - q}{CE}\right) = -\frac{1}{2RC} \cdot t$$

$$CE - q = CE e^{-\frac{t}{\tau}}$$

$$q = CE \left(1 - e^{-\frac{t}{\tau}}\right)$$

40. A step voltage V_0 is applied to a series combination of R and C as shown in the figure. Then,



- (A) After sufficiently long time $V_R = 0$
 (B) As time passes V_R decreases as $\left(\frac{1}{t}\right)$
 (C) After 1 ms, $V_C = 6.3$ volt (approximately)
 (D) Initially current through R is 10 mA

Ans. ACD

Sol. (A) After long time current in the circuit is zero. Hence $= V_R = 0$
 (B) This is case of charging of capacitor.

$$q = q_0 \left(1 - e^{-t/\tau}\right)$$

$$V_R \propto e^{-t/\tau}$$

(C) After 1ms.

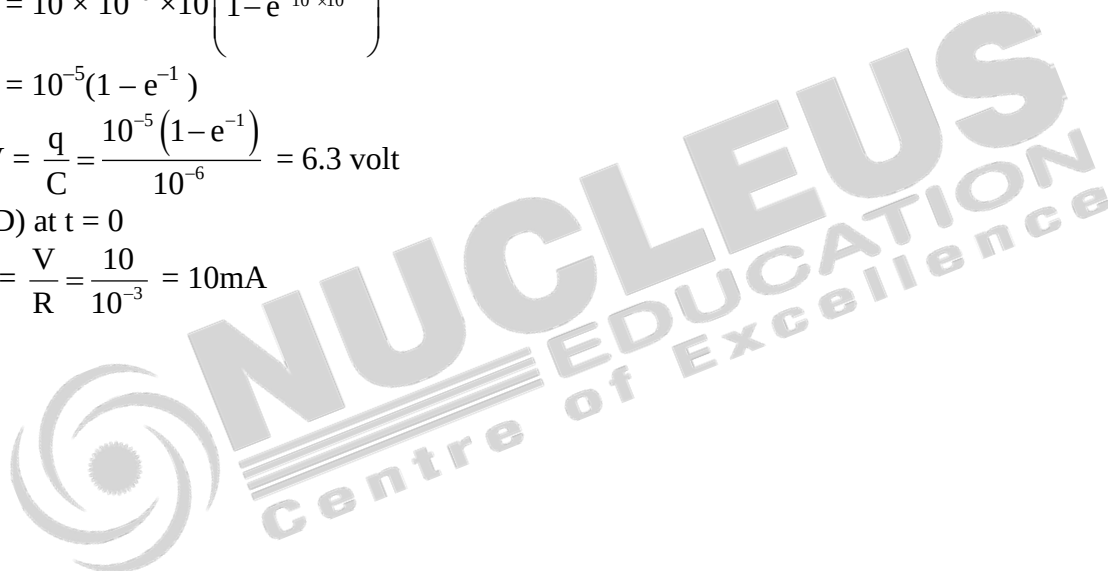
$$q = 10 \times 10^{-6} \times 10 \left(1 - e^{-\frac{10^{-3}}{10^3 \times 10^{-6}}}\right)$$

$$q = 10^{-5} (1 - e^{-1})$$

$$V = \frac{q}{C} = \frac{10^{-5} (1 - e^{-1})}{10^{-6}} = 6.3 \text{ volt}$$

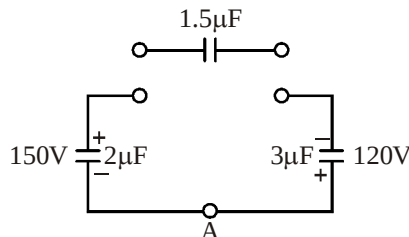
(D) at $t = 0$

$$i = \frac{V}{R} = \frac{10}{10^{-3}} = 10 \text{ mA}$$



EXERCISE # (O-2)

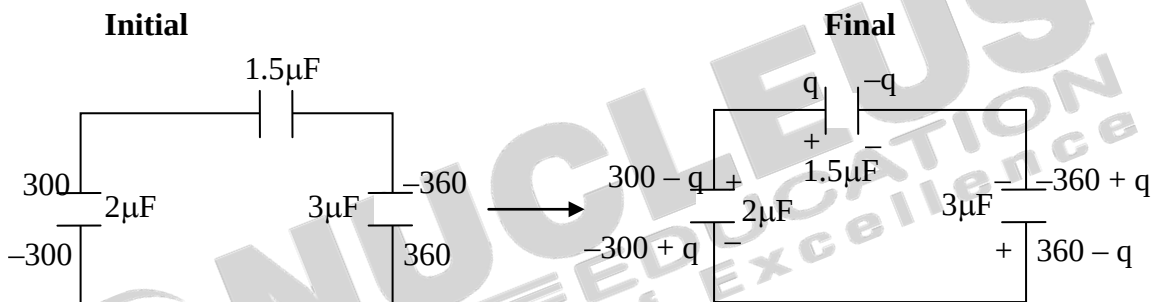
1. Two capacitors of $2\ \mu\text{F}$ and $3\ \mu\text{F}$ are charged to 150 volt and 120 volt respectively. The plates of capacitor are connected as shown in the figure. A discharged capacitor of capacity $1.5\ \mu\text{F}$ falls to the free ends of the wire. Then



- (A) charge on the $1.5\ \mu\text{F}$ capacitor is $180\ \mu\text{C}$
 (B) charge on the $2\ \mu\text{F}$ capacitor is $120\ \mu\text{C}$
 (C) positive charge flows through A from right to left.
 (D) positive charge flows through A from left to right.

Ans. ABC

Sol.



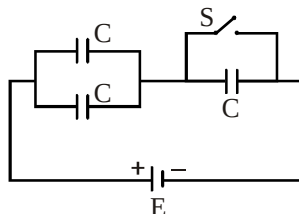
Applying KVL

$$\frac{300 - q}{2} - \frac{q}{1.5} + \frac{360 - q}{3} = 0$$

$$\Rightarrow q = 180\ \mu\text{C}$$

A, B, C

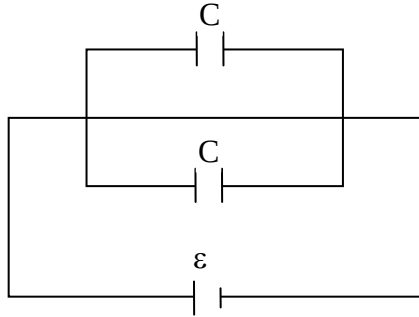
2. In the circuit shown, each capacitor has a capacitance C . The emf of the cell is E . If the switch S is closed



- (A) positive charge will flow out of the positive terminal of the cell
 (B) positive charge will enter the positive terminal of the cell
 (C) the amount of charge flowing through the cell will be CE .
 (D) the amount of charge flowing through the cell will be $\frac{4}{3}CE$.

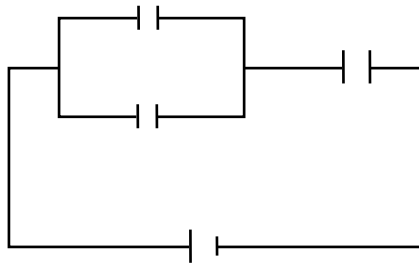
Ans. A, D

Sol. After closing the switch



$$C_{eq} = 2C$$

Before closing the switch

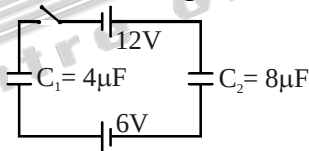


$$C_{eq} = \frac{2C}{3}$$

$$\text{Charge flown} = 2C\varepsilon - \left(\frac{2}{3}C\right)\varepsilon = \frac{4C\varepsilon}{3}$$

Ans. A, D

3. In the circuit shown initially C_1, C_2 are uncharged. After closing the switch



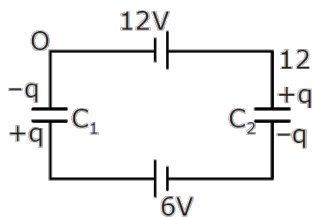
- (A) The charge on C_2 is greater than that on C_1
 (B) The charge on C_1 and C_2 are the same
 (C) The potential drops across C_1 and C_2 are the same
 (D) The potential drops across C_2 is greater than that across C_1

Ans. B

Sol. In series charge will be same

$$12 - \frac{q}{8} + 6 - \frac{q}{4} = 0$$

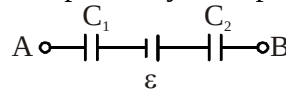
$$q = 48\mu\text{C}$$



$$V_{C_2} = \frac{48}{8} = 6V$$

$$V_{C_1} = \frac{48}{8} = 12V$$

4. A circuit shown in the figure consists of a battery of emf 10 V and two capacitance C_1 and C_2 of capacitances $1.0 \mu F$ and $2.0 \mu F$ respectively. The potential difference $V_A - V_B$ is 5V



- (A) charge on capacitor C_1 is equal to charge on capacitor C_2
 (B) Voltage across capacitor C_1 is 5V.
 (C) Voltage across capacitor C_2 is 10 V
 (D) Energy stored in capacitor C_1 is two times the energy stored in capacitor C_2 .

Ans. AD

Sol. As the capacitance are in series hence charge on both of them will be same.

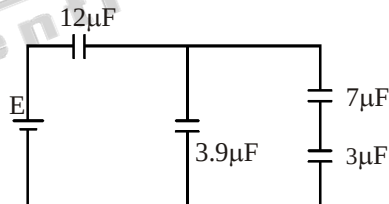
$$E = \frac{Q^2}{2C}$$

$$V_1 : V_2 = \frac{1}{1} : \frac{1}{2},$$

$$V_1 = \frac{2}{3} \times 15 = 10V$$

$$V_2 = 5V$$

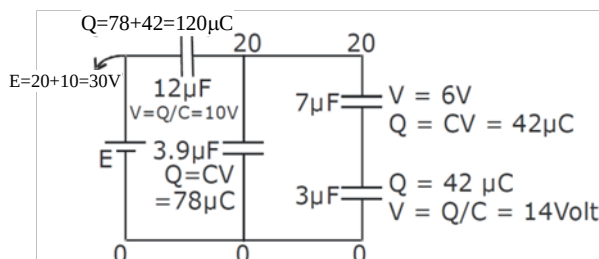
5. Four capacitors and a battery are connected as shown. The potential drop across the $7 \mu F$ capacitor is 6 V. Then the :



- (A) potential difference across the $3 \mu F$ capacitor is 10 V
 (B) charge on the $3 \mu F$ capacitor is $42 \mu C$
 (C) e.m.f. of the battery is 30 V
 (D) potential difference across the $12 \mu F$ capacitor is 10 V.

Ans. BCD

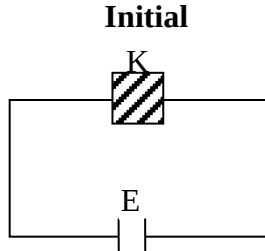
Sol. From the diagram



6. The capacitance of a parallel plate capacitor is C when the region between the plate has air. This region is now filled with a dielectric slab of dielectric constant k . The capacitor is connected to a cell of emf E , and the slab is taken out
- (A) charge $CE(k - 1)$ flows through the cell
 (B) energy $E^2C(k - 1)$ is absorbed by the cell.
 (C) the energy stored in the capacitor is reduced by $E^2C(k - 1)$
 (D) the external agent has to do $\frac{1}{2} E^2C(k - 1)$ amount of work to take the slab out.

Ans. ABD

Sol.



$$C_i = KC$$

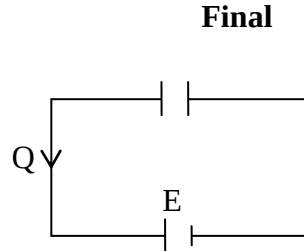
$$Q_i = KCE$$

$$(\text{Energy})_i = \frac{1}{2} (KC)E^2$$

$$(\text{Charge flown}) \Delta Q = KCE - CE = (K - 1) CE, \text{ into the cell.}$$

$$\begin{aligned} \text{Energy absorbed} &= (\Delta Q)(\Delta V) = ((K - 1) CE) E \\ &= E^2 C(K - 1) \end{aligned}$$

$$\text{Heat} = \frac{1}{2} E^2 C (K - 1)$$



$$C_f = C$$

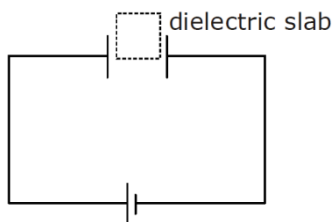
$$Q_f = CE$$

$$(\text{Energy})_f = \frac{1}{2} C(E)^2$$

7. A parallel plate air-core capacitor is connected across a source of constant potential difference. When a dielectric plate is introduced between the two plates then :
- (A) some charge from the capacitor will flow back into the source.
 (B) some extra charge from the source will flow back into the capacitor.
 (C) the electric field intensity between the two plate does not change.
 (D) the electric field intensity between the two plates will decrease.

Ans. BC

Sol.



$$V = \text{const.}$$

$$C = \frac{\epsilon_0 k A}{d}$$

$$E = \frac{V}{d} = \text{const.}$$

$$C \uparrow, \quad Q = CV \uparrow$$

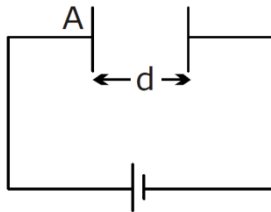
8. A parallel plate capacitor of plate area A and plate separation d is charged to potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. If Q , E and W denote respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted) and the work done on the system, in question, in the process of inserting the slab, then

(A) $Q = \frac{\epsilon_0 AV}{d}$ (B) $Q = \frac{\epsilon_0 KAV}{d}$ (C) $E = \frac{V}{Kd}$ (D) $W = -\frac{\epsilon_0 AV^2}{2d} \left(1 - \frac{1}{K}\right)$

Ans. ACD

Sol. $U = \frac{1}{2} CV^2$

$$U_i = \frac{1}{2} \frac{\epsilon_0 A}{d} V^2$$



After inserting slab $Q = \text{const.}$

$$U_f = \frac{Q^2}{2C} \left\{ Q = \frac{\epsilon_0 AV}{d} \right\}$$

$$U_f = \frac{\epsilon_0^2 A^2 V^2 d}{2d^2 k \epsilon_0 A} = \frac{\epsilon_0 AV^2}{2dk}$$

$$W.D. = U_f - U_i$$

$$= -\frac{\epsilon_0 AV^2}{2d} \left(1 - \frac{1}{k}\right)$$

9. A parallel plate capacitor has a parallel slab of copper inserted between and parallel to the two plates, without touching the plates. The capacity of the capacitor after the introduction of the copper sheet is :

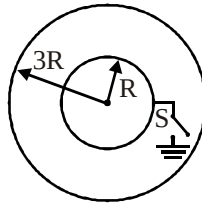
- (A) minimum when the copper slab touches one of the plates.
(B) maximum when the copper slab touches one of the plates.
(C) invariant for all positions of the slab between the plates.
(D) greater than that before introducing the slab.

Ans. CD

Sol. $C = \frac{\epsilon_0 A}{d - t + t/K}$

Independent of Position

10. Two thin conducting shells of radii R and $3R$ are shown in the figure. The outer shell carries a charge $+Q$ and the inner shell is neutral. The inner shell is earthed with the help of a switch S .



- (A) With the switch S open, the potential of the inner sphere is equal to that of the outer.
 (B) When the switch S is closed, the potential of the inner sphere becomes zero.
 (C) With the switch S closed, the charge attained by the inner sphere is $-Q/3$.
 (D) By closing the switch the capacitance of the system increases.

Ans. ABCD

Sol. (a) $V_i = \frac{kQ}{3R}$ $V_o = \frac{kQ}{3R}$

(b) Earthing means $V = 0$

(c) $\frac{kq'}{R} + \frac{kQ}{3R} = 0 \Rightarrow q' = -q/3$

(d) energy between the spheres increases.

- 11.** A parallel-plate capacitor is connected to a cell. Its positive plate A and its negative plate B have charges $+Q$ and $-Q$ respectively. A third plate C, identical to A and B, with charge $+Q$, is now introduced midway between A and B, parallel to them. Which of the following are correct?

(A) The charge on the inner face of B is now $-\frac{3Q}{2}$

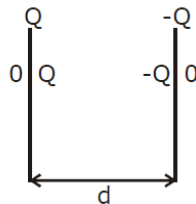
(B) There is no change in the potential difference between A and B.

(C) The potential difference between A and C is one-third of the potential difference between B and C.

(D) The charge on the inner face of A is now $Q/2$.

Ans ABCD

Sol. Initially

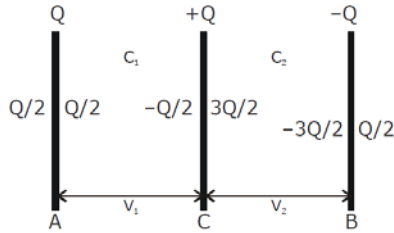


$$\Rightarrow C = \frac{\epsilon_0 A}{d}$$

$$V = \frac{Q}{C}$$

$$\frac{Qd}{\epsilon_0 A}$$

Finally



$$C_1 = C_2 = \frac{2\epsilon_0 A}{d}$$

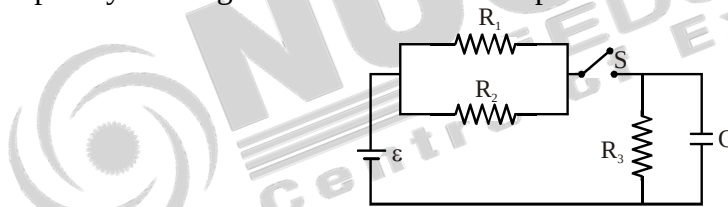
$$V_1 = \frac{Qd}{2\epsilon_0 A}$$

$$V_2 = \frac{3Qd}{2\epsilon_0 A}$$

$$V = V_1 + V_2 = \frac{d}{\epsilon_0 A}$$

$$V_f = V_i$$

12. The circuit shown in the figure consists of a battery of emf $\epsilon = 10$ V; a capacitor of capacitance $C = 1.0$ μF and three resistor of values $R_1 = 2\Omega$, $R_2 = 2\Omega$ and $R_3 = 1\Omega$. Initially the capacitor is completely uncharged and the switch S is open. The switch S is closed at $t = 0$.



- (A) The current through resistor R_3 at the moment the switch closed is zero.
(B) The current through resistor R_3 a long time after the switch closed is 5A.
(C) The ratio of current through R_1 and R_2 is always constant.
(D) The maximum charge on the capacitor during the operation is $5\mu\text{C}$.

Ans. ABCD

Sol. **An initial state**

$$Q_{\text{capacitor}} = 0, \Delta V_{\text{cap}} = 0 \Rightarrow i_{R_3} = 0$$

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2} = 1\Omega$$

An Steady state

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2} + R_3 = 2\Omega$$

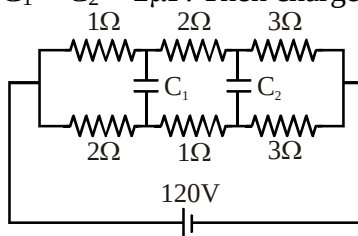
$$R_{\text{eq}} \uparrow \text{ Current } \downarrow$$

$$i_{\text{initial}} = 10 \text{ A}, i_{\text{final}} = \frac{10}{2} = 5 \text{ A}$$

$$\Delta V_{R_3} = \Delta V_{\text{cap}} = 5 \times 1 = 5 \text{ volt}$$

$$Q = CV = (1) (5) = I\mu\text{f}$$

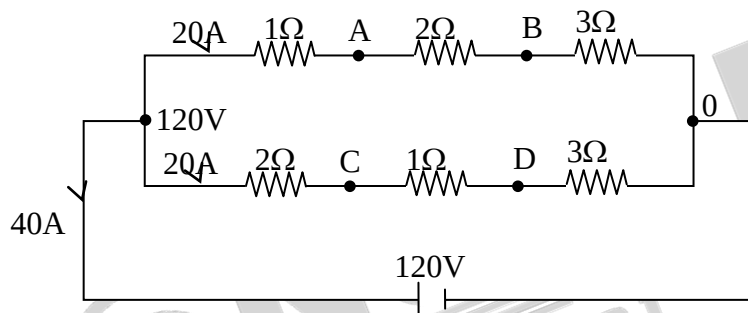
13. In the circuit shown in figure $C_1 = C_2 = 2\mu\text{F}$. Then charge stored in



- (A) capacitor C_1 is zero
(B) capacitor C_2 is zero
(C) both capacitor is zero
(D) capacitor C_1 is $40\mu\text{C}$

Ans. BD

Sol.



Circuit, at steady state

$$R_{\text{eq}} = 3\Omega$$

$$i = \frac{120}{3} = 40 \text{ A}$$

$$V_A = 100 \text{ V}, V_B = 60 \text{ V},$$

$$V_C = 80 \text{ V}, V_D = 60 \text{ V}$$

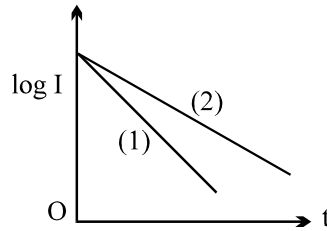
$$(\Delta V)_{C_1} = V_A - V_C = 20 \text{ V}$$

$$Q_{C_1} = 40\mu\text{C}$$

$$(\Delta V)_{C_2} = 0$$

$$Q_{C_2} = 0$$

14. A capacitor of capacity C is charged to a steady potential difference V and connected in series with an open key and a pure resistor ' R '. At time $t = 0$, the key is closed. If I = current at time t , a plot of $\log I$ against ' t ' is as shown in (1) in the graph. Later one of the parameters i.e. V , R or C is changed keeping the other two constant, and the graph (2) is recorded. Then



- (A) C is reduced (B) C is increased (C) R is reduced (D) R is increased

Ans. B

Sol. $i = i_0 e^{-t/RC}$

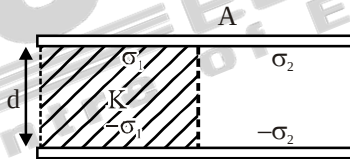
$$\ell n i = \ell n i_0 - \frac{t}{RC}$$

Intercept is same in graph

$$\Rightarrow i_0 \text{ is same} \Rightarrow R \text{ is same} \left(i = \frac{V}{R} \right)$$

And magnitude slope of graph $\left(\frac{1}{RC} \right) \downarrow \text{decreased} \Rightarrow C \uparrow \text{increased}$

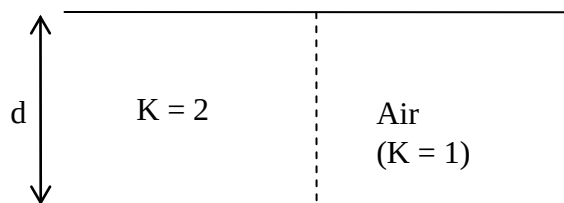
15. A parallel plate capacitor of area A and separation d is charged to potential difference V and removed from the charging source. A dielectric slab of constant $K = 2$, thickness d and area A/2 is inserted, as shown in the figure. Let σ_1 be free charge density at the conductor-dielectric surface and σ_2 be the charge density at the conductor-vacuum surface.



- (A) The electric field has the same value inside the dielectric as that in the free space between the plates.
 (B) The ratio $\frac{\sigma_1}{\sigma_2}$ is equal to $\frac{2}{1}$
 (C) The new capacitance is $\frac{3\epsilon_0 A}{2d}$
 (D) The new potential difference between the plates is $\frac{2}{3} V$.

Ans. ABCD

Sol.



$$C_{eq} = \frac{\epsilon_0 \left(\frac{A}{2} \right)}{d} + \left[\frac{\epsilon_0 \left(\frac{A}{1} \right)}{d} \right] K = \frac{3 \epsilon_0 A}{2 d}$$

$$E_K = \frac{\sigma_1}{K \epsilon_0} \quad E_{air} = \frac{\sigma_2}{\epsilon_0}$$

$\therefore \Delta V$ is same.

$$\frac{\sigma_1}{K \epsilon_0} \cdot d = \frac{\sigma_2}{\epsilon_0} \cdot d$$

$$\frac{\sigma_1}{\sigma_2} = K = 2$$

$\therefore$ Charge on capacitor remain same

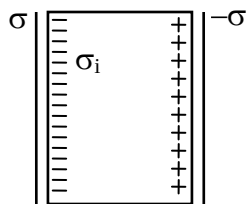
$$C_1 V_1 = C_2 V_2$$

$$C V = \left(\frac{3}{2} C \right) V_2 \Rightarrow V_2 = \frac{2V}{3}$$

16. A dielectric slab fills the space between the plates of a parallel-plate capacitor. The magnitude of the bound charge on the slab is 75% of the magnitude of the free charge on the plates. The capacitance of the capacitor is $480 \mu\text{F}$ with the slab inserted. The maximum charge that can be stored on the capacitor is $240 \epsilon_0 L^2 E_{\text{max}}$, where E_{max} is the breakdown field. Choose the **CORRECT** statement(s)
- (A) The dielectric constant for the dielectric slab is 4
- (B) Without the dielectric, the capacitance of the capacitor would be $360 \mu\text{F}$.
- (C) The plate area is $60 L^2$
- (D) If the dielectric slab is having the same area as the capacitor plate but the width half that of the capacitor, the capacitance would be $192 \mu\text{F}$.

Ans. ACD

Sol.



$$\sigma_i = \sigma \left(1 - \frac{1}{K} \right) = \frac{3}{4} (\sigma)$$

$$1 - \frac{1}{K} = \frac{3}{4}$$

$$\Rightarrow K = 4 \quad \dots\dots\dots (A)$$

$$KC = 480 \mu\text{F} = 4C$$

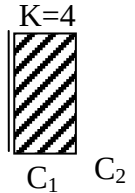
$$\Rightarrow C = 120 \mu\text{F}$$

$$Q_{\text{stored}} = K (CV)$$

$$240\epsilon_0 L^2 E = 4 \left(\frac{\epsilon_0 A}{d} \right) (Ed)$$

$$A = 60L^2 \quad \dots\dots\dots(C)$$

$$(D) \frac{\epsilon_0 A}{d} = 120\mu F$$

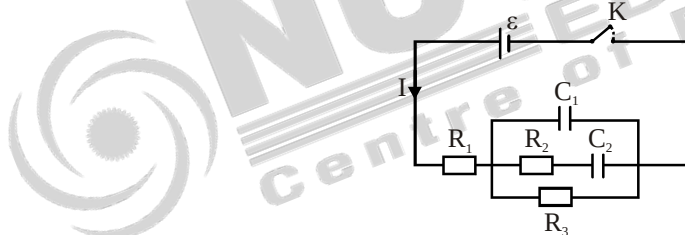


$$C_1 = K \left(\frac{\epsilon_0 A}{d/2} \right) = 960\mu F$$

$$C_2 = \left(\frac{\epsilon_0 A}{d/2} \right) = 240\mu F$$

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{960 \times 240}{960 + 240} = 192\mu F$$

17. Mark the CORRECT statement(s) regarding the current I through the battery in the circuit shown in figure.



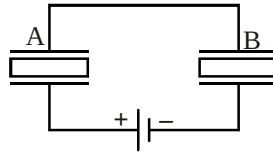
- (A) Immediately after the key K is closed, $I = \frac{\epsilon}{R_1}$
- (B) Immediately after the key K is closed, $I = \frac{\epsilon}{R_1 + R_3}$
- (C) Long time after key K is closed, $I = \frac{\epsilon}{R_1 + R_3}$
- (D) Long time after key K is closed, $I = \frac{\epsilon}{R_1 + R_2}$

Ans. AC

Sol. Immediately after closing the switch, C_1 behave like wire $i = \frac{E}{R_1}$

long time after R_3 become working $i = \frac{E}{R + R_3}$

18. Figure shows two identical capacitors A & B with identical dielectric inserted between them and are connected to a battery. Now the slab of capacitor B is pulled out with battery connected :



- (A) During the process current flows counter clockwise in the circuit.
(B) Finally charge on capacitor B will be less than that on capacitor A.
(C) Electric field in capacitor A reduces in magnitude.
(D) During the process, internal energy of the battery increases.

Ans. ACD

Sol. $C_A = KC$, $C_B = KC$

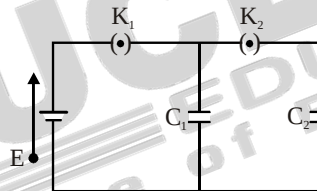
$$(C_A)_f = KC, \quad (C_B)_f = C$$

$$C_{eq} = \frac{KC}{2}$$

$$C_{eq} = \frac{(KC) \times (C)}{KC + C} = \frac{KC}{K+1}$$

$C_{eq} \downarrow$ se $\Rightarrow$ charge flows into the battery from positive terminal.

19. In the circuit shown in Fig. initially key K_1 is closed and key K_2 is open. Then K_1 is opened and K_2 is closed (order is important). [Take Q'_1 and Q'_2 as charges on C_1 and C_2 and V_1 and V_2 as voltage respectively. Then



- (A) charge on C_1 gets redistributed such that $V_1 = V_2$
(B) charge on C_1 gets redistributed such that $Q'_1 = Q'_2$
(C) charge on C_1 gets redistributed such that $C_1 V_1 + C_2 V_2 = C_1 E$
(D) charge on C_1 gets redistributed such that $Q'_1 + Q'_2 = Q$

Ans. AD

Sol. Initially, $Q = C_1 E$

When K_1 opened & K_2 closed, the charge Q is distributed between both capacitors, such that

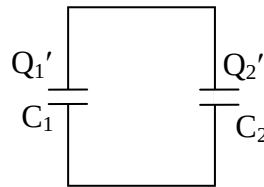
$$V_1 = V_2$$

$$\Rightarrow \frac{Q'_1}{C_1} = \frac{Q'_2}{C_2}$$

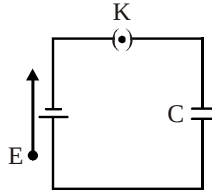
$$\Rightarrow Q'_1 \neq Q'_2$$

$$\text{Also } Q'_1 + Q'_2 = Q$$

$$\Rightarrow C_1 V_1 + C_2 V_2 = C_1 E$$



20. A parallel plate capacitor is connected to a battery as shown in Fig. Consider two situations:
A: Key K is kept closed and plates of capacitors are moved apart using insulating handle.
B: Key K is opened and plates of capacitors are moved apart using insulating handle.
Choose the correct option(s).

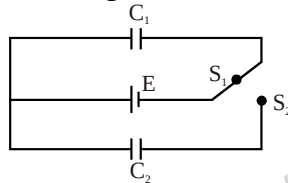


- (A) In A : Q remains same but C changes.
 (B) In B : V remains same but C changes.
 (C) In A : V remains same and hence Q changes.
 (D) In B : Q remains same and hence V changes.

Ans. CD

Sol. **Case (A)** : K_1 is closed and hence battery is connected. $\therefore$ V remains same.
Case (B) : Battery disconnected, hence charge on the capacitor plates remains same.

21. As shown in circuit C_2 is initially uncharged. Now switch is shifted from position S_1 to S_2 then



- (A) Charge distribution will not take place
 (B) Charge & energy distribution will take place but total energy & charge of system (circuit) will remain constant
 (C) Charge will decrease
 (D) Circuit will release some energy

Ans. D

Sol. Charge distribution will take place and due to this U_i and U_f will be different.

22. A parallel plate capacitor is charged from a cell and then isolated from it. The separation between the plate is now increased

- (A) the force of attraction between the plates will decrease
 (B) The field in the region between the plates will not change
 (C) The energy stored in the capacitor will increase
 (D) The potential difference between the plates will decreases.

Ans. BC

Sol. $q = CV$

Now when capacitor will isolated from cell charge on capacitor will remain constant.

$$C = \frac{\epsilon_0 A}{d} \Rightarrow U = \frac{q^2}{2C} = \frac{q^2}{2\epsilon_0 A} \cdot d \Rightarrow U \rightarrow \text{increase.}$$

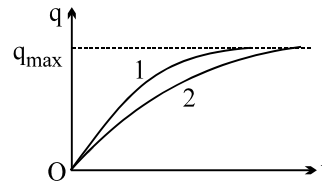
$$V = \frac{q}{C} = \frac{q}{\epsilon_0 A} \cdot d \Rightarrow V \rightarrow \text{increase.}$$

$$E = \frac{\bar{V}}{\epsilon_0} = \frac{q}{A \epsilon_0} \Rightarrow E \rightarrow \text{increase same}$$

$$F = q \cdot E \Rightarrow F \rightarrow \text{remains same}$$

Paragraph for Q. No. 23 to 24

The charge across the capacitor in two different RC circuits 1 and 2 are plotted as shown in figure.



23. Choose the correct statement(s) related to the two circuits.

- (A) Both the capacitors are charged to the same charge.
(B) The emf's of cells in both the circuit are equal.
(C) The emf's of the cells may be different.
(D) The emf E_1 is more than E_2

Ans. AC

Sol. For charging,

$$Q = Q_{\max} (1 - e^{-t/RC})$$

$$= CE (1 - e^{-t/RC})$$

As $Q_{\max 1} = Q_{\max 2}$

$$\Rightarrow C_1 E_1 = C_2 E_2$$

Hence E_1 & E_2 may be different

24. Identify the correct statement(s) related to the R_1 , R_2 , C_1 and C_2 of the two RC circuits.

- (A) $R_1 > R_2$ if $E_1 = E_2$ (B) $C_1 < C_2$ if $E_1 = E_2$ (C) $R_1 C_1 > R_2 C_2$ (D) $\frac{R_1}{R_2} < \frac{C_2}{C_1}$

Ans. D

Sol. As the maximum charge reaches earlier in circuit 1

So $\tau_1 < \tau_2$

$$\Rightarrow R_1 C_1 < R_2 C_2$$

$$\Rightarrow \frac{R_1}{R_2} < \frac{C_2}{C_1}$$

25. A parallel plate capacitor is connected to a battery. The quantities charge, voltage, electric field and energy associated with the capacitor are given by Q_0 , V_0 , E_0 and U_0 respectively. A dielectric slab is introduced between plates of capacitor but battery is still in connection. The corresponding quantities now given by Q , V , E and U related to previous ones are

- (A) $Q > Q_0$ (B) $V > V_0$ (C) $E > E_0$ (D) $U < U_0$

Ans. A

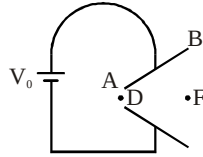
Sol. Charge increase as Capacitance increases

$$C_i = CV_0$$

$$C_f = (KL) (V_0)$$

$$\therefore Q > Q_0$$

26. In the given figure, a capacitor of non-parallel plates is shown. The plates of capacitor are connected by a cell of emf V_0 . If σ denotes surface charge density and E denotes electric field. Then



- (A) $\sigma_A > \sigma_B$ (B) $E_f > E_D$ (C) $E_F = E_D$ (D) $\sigma_A = \sigma_B$

Ans. A

Sol. $E_D = \frac{V}{d_A}$ $E_F = \frac{V}{d_B}$

$d_A \rightarrow$ distance between plates at A

$d_B \rightarrow$ distance between plates at B

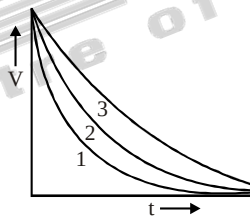
Since, $d_B > d_A$

$$E_D > E_F$$

$$\frac{\sigma_A}{\epsilon_0} > \frac{\sigma_B}{\epsilon_0}$$

$$\sigma_A > \sigma_B$$

27. Three identical capacitors A, B and C are charged to the same potential and then made to discharge through three resistances R_A , R_B and R_C , where $R_A > R_B > R_C$. Their potential differences (V) are plotted against time t , giving the curves 1, 2 and 3. The relations between A, B, C and 1, 2, 3 is/are -



- (A) $1 \rightarrow A$ (B) $2 \rightarrow B$ (C) $1 \rightarrow C$ (D) $3 \rightarrow A$

Ans. BCD

Sol. $\tau = RC$ As $R \uparrow$, $\tau \uparrow$

$$\therefore \tau_A > \tau_B > \tau_C$$

$\therefore$ A will discharge slowest and C will discharge fastest.

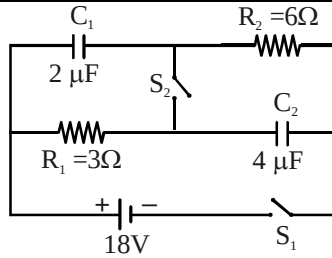
$$\therefore 1 \rightarrow C$$

$$2 \rightarrow B$$

$$3 \rightarrow A$$

Paragraph for Q. 28 to 30

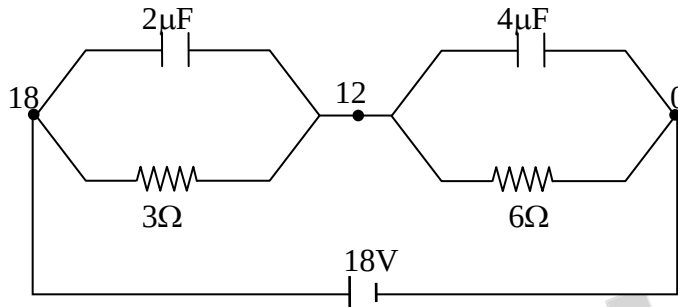
In the circuit shown initially the switches are open and capacitors are uncharged. Switches S_1 and S_2 are closed simultaneously at $t = 0$.



28. Charge on capacitor C_2 at steady state
(A) $12 \mu\text{C}$ (B) $24 \mu\text{C}$ (C) $48 \mu\text{C}$ (D) None of these

Ans. C

Sol. Equivalent circuit diagram



$$Q_{C_2} = (12 - 0)(4\mu\text{F}) = 48\mu\text{C}$$

$$Q_{C_1} = (18 - 12)(2) = 12\mu\text{C}$$

29. Now switch S_2 is opened after a long time interval then charge flow through the S_1 is
(A) $12 \mu\text{C}$ (B) $24 \mu\text{C}$ (C) $36 \mu\text{C}$ (D) $48 \mu\text{C}$

Ans. D

Sol. After opening the switch

$$Q_{C_1} = 2 \times 18 = 36\mu\text{C}$$

$$Q_{C_2} = 4 \times 18 = 72\mu\text{C}$$

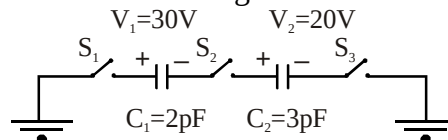
$$Q_f = (36 - 12) + (72 - 48) = 24 + 24 = 48 \mu\text{C}$$

30. In above question the amount of heat dissipated in resistors
(A) $216 \mu\text{J}$ (B) $272 \mu\text{J}$ (C) $68 \mu\text{J}$ (D) None of these

Ans. A

Sol. Heat = $W_B - (\epsilon_f - \epsilon_i) = 216 \mu\text{J}$

31. For the circuit shown, which of the following statements is true ?



- (A) with S_1 closed, $V_1 = 15 \text{ V}$, $V_2 = 20 \text{ V}$
(B) with S_3 closed, $V_1 = V_2 = 25 \text{ V}$
(C) with S_1 & S_2 closed, $V_1 = V_2 = 0$
(D) with S_1 & S_2 closed, $V_1 = 30 \text{ V}$, $V_2 = 20 \text{ V}$

Ans. D

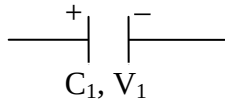
Sol. There will be no effect until all the 3 switches are closed.

32. Two identical capacitors, have the same capacitance C . One of them is charged to potential V_1 and the other to V_2 . The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is

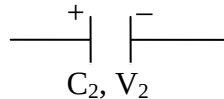
(A) $\frac{1}{4}C(V_1 - V_2)^2$ (B) $\frac{1}{4}C(V_1^2 + V_2^2)$ (C) $\frac{1}{4}C(V_1 - V_2)^2$ (D) $\frac{1}{4}C(V_1 + V_2)^2$

Ans. C

Sol. Initial



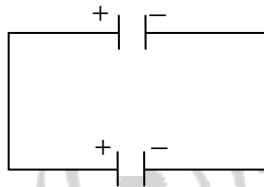
$$\epsilon = \frac{1}{2}C_1V_1^2,$$



$$\epsilon = \frac{1}{2}C_2V_2^2$$

$$(\text{Energy})_i = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

Finally



$$C_1V_1 + C_2V_2 = (C_1 + C_2)V$$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

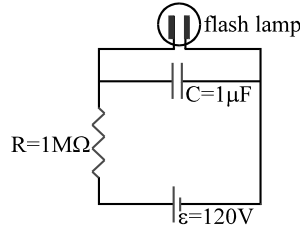
$$\epsilon_f = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2$$

$$\Delta E = \epsilon_i - \epsilon_f = \frac{1}{4}C(V_1 - V_2)^2$$

Paragraph for Q. 33 to 36

A highway emergency flasher uses a 120 volt battery, a $1 \text{ M}\Omega$ resistor, a $1 \mu\text{F}$ capacitor, a neon flash lamp in the circuit shown in the figure. The flash lamp has a resistance more than $10^{10} \Omega$ when the voltage across it is less than 110V. Above 110 V, the neon gas ionizes, the lamp's resistance drops to 10Ω , and the capacitor discharges completely. Until the capacitor voltage reaches the breakdown voltage $V_b = 110 \text{ V}$, the large resistance of the flash lamp ensures that it draws a negligible current. The capacitor charges as if the lamp were absent. At V_b , however, the lamp resistance quickly becomes negligible, and the capacitor discharges through the lamp as if the battery and the series resistor were absent. The time between the flashes is the time for

the capacitor to charge to V_b . The flash duration is roughly the time for the capacitor to discharge through the lamp, or about 3 time constant of the capacitor-lamp circuit. The flash energy is the stored energy in the capacitor at 110 volt.



33. The flash interval is found by solving for the time when the capacitor voltage is $V_b = 110$ V.

$V_b = \varepsilon(1 - e^{-t/CR})$, $\ln 12 = 2.5$). Flash interval is

- (A) 2 s (B) 2/5 s (C) 5/2 s (D) 1 s

Ans. C

Sol. $V_b = \varepsilon(1 - e^{-t/RC})$

$V_b = 110$ V, $C = 1 \mu\text{F}$, $\varepsilon = 120$ V, $R = 1 \text{ M}\Omega$

$$110 = 120 - 120 e^{-t/1}$$

$$\frac{1}{12} = e^{-t}$$

$$t = \ln 12$$

$$t = 2.5 \text{ sec}$$

34. Time constant (τ_c) of the capacitor-lamp circuit is

- (A) 20 μs (B) 15 μs (C) 30 μs (D) 10 μs

Ans. D

Sol. $\tau = R_{\text{lamp}} C$
 $= 10 \times (1 \mu\text{F})$
 $= 10 \mu\text{s}$

35. Flash duration is

- (A) 10 μs (B) 20 μs (C) 30 μs (D) 5 μs

Ans. C

Sol. Flash duration = 3 (τ)
 $= 3 \times 10 \mu\text{s}$
 $= 30 \mu\text{s}$

36. The energy in the flash is

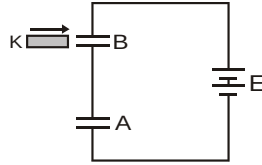
- (A) 6.1 mJ (B) 6.1 J (C) 3 mJ (D) 12.2 mJ

Ans. A

Sol. Energy = $\frac{1}{2}(C)(110)^2 = \frac{1}{2} \times (10^{-6})(110)^2 = 6.1 \text{ mJ}$

Paragraph for Q. 37 to 41

Two identical capacitor A and B, each of capacitance 'C' are connected in series. The combination connected to a battery of emf E. A dielectric slab of dielectric constant K is introduced between the plates of capacitor B to cover entire space between the plates.



37. After introduction of dielectric slab in B, the ratio of capacitance of A and B is
(A) 1:1 (B) 1:K (C) K:1 (D) 1:√K

Ans. B

Sol. $C_A = C$ $C_B = KC$
 $\frac{C_A}{C_B} = \frac{1}{K}$

38. After introduction of dielectric slab in B, the ratio of potential differences across A and B will be
(A) 1:1 (B) 1:K (C) K:1 (D) 1:√K

Ans. C

Sol. In series charge is same

$$Q_A = Q_B$$

$$C_A V_A = C_B V_B$$

$$\frac{V_A}{V_B} = \frac{C_B}{C_A} = K$$

39. The ratio of potential differences across A before and after the introduction of dielectric slab in B will be
(A) 1:1 (B) 1:K (C) (K+1):2 (D) K+1:2K

Ans. D

Sol. $(\Delta V_A)_i = \frac{V}{2}$
 $(\Delta V_A)_f = \left(\frac{K}{K+1} \right) V$
Ratio = $\frac{K+1}{2K}$

40. The ratio of potential difference across B before and after the introduction of dielectric slab in B will be
(A) 1:1 (B) K:1 (C) (K+1):2 (D) K+1:2K

Ans. C

Sol. $(\Delta V_B)_i = \frac{V}{2}$
 $(\Delta V_B)_f = V - \left(\frac{K}{K+1} \right) V = \frac{V}{K+1}$

$$\text{Ratio} = \frac{K+1}{2}$$

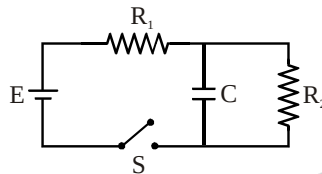
41. The ratio of energy stored in capacitors A and B after the introduction of dielectric slab in B is
(A) 1:1 (B) 1:K (C) K:1 (D) $(K+1)^2 : K^2$

Ans. C

Sol.
$$\frac{\epsilon_A}{\epsilon_B} = \frac{\frac{Q_A^2}{2C_A}}{\frac{Q_B^2}{2C_B}} = \frac{C_B}{C_A} = K$$

Paragraph for Q. 42 to 44

In the circuit as shown in figure the switch is closed at $t = 0$.



42. At the instant of closing the switch
(A) the battery delivers maximum current.
(B) no current flows through C
(C) Voltage drop across R_2 is zero.
(D) the current through the battery decreases with time finally becomes zero.

Ans. AC

Sol.
$$t = 0 \Rightarrow R_{eq} = R_1$$

$$t = \infty \Rightarrow R_{eq} = R_1 + R_2$$

$$i_{t=0} = \frac{\epsilon}{R_1} \quad i_{t=\infty} = \frac{\epsilon}{R_1 + R_2}$$

$\therefore$ current through battery is maximum at $t = 0$

$$V_{R_2} = V_C = \frac{q}{C} = (\text{at } t = 0)$$

(A,C)

43. A long time after closing the switch
(A) voltage drop across the capacitor is E.
(B) current through the battery is $\frac{E}{R_1 + R_2}$
(C) energy stored in the capacitor is $\frac{1}{2}C \left(\frac{R_2 E}{R_1 + R_2} \right)^2$
(D) current through the capacitor becomes zero.

Ans. BCD

Sol. (B) $t = \infty \Rightarrow i = \frac{E}{R_1 + R_2}$

$$(C) E = \frac{1}{2} C (V_{R_2})^2 = \frac{1}{2} C (iR_2)^2$$

$$= \frac{1}{2} C \left(\frac{R_2 E}{R_1 + R_2} \right)^2$$

(D) Capacitor becomes full charged

$$\therefore \frac{dq}{dt} = 0$$

44. A capacitor of capacity C_0 is connected to a battery of emf V_0 . When steady state is attained a dielectric slab of dielectric constant K is slowly introduced in the capacitor. Mark the Correct statement(s), in final steady state

(A) Magnitude of induced charge on the each surface of slab is $C_0 V_0 (K - 1)$

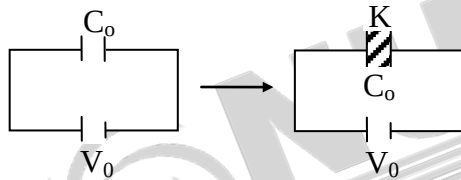
(B) Net electric force due to induced charges on one the plate is zero.

(C) Force of attraction between plates of capacitor is $\frac{K(C_0 V_0)^2}{2 \epsilon_0 A}$

(D) Net field due to induced charges in dielectric slab is $\frac{8V_0(k-1)^2}{K \epsilon_0 A}$

Ans. AB

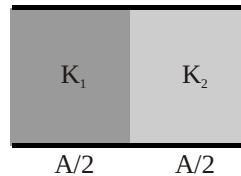
Sol.



$$(A) Q_1 = Q \left(1 - \frac{1}{K} \right) = KC_0 V_0 \left(1 - \frac{1}{K} \right) = C_0 V_0 (K - 1)$$

(B) $F_{\text{net}} = 0$ (E due to induced charges outside the dielectric will be zero)

45. Inside a parallel plate capacitor of capacitance C (without dielectrics) dielectrics are filled which have dielectric constant k_1 and k_2 such that $k_1 > k_2$. The capacitor is now connected to a battery of voltage V then



(A) New capacitance of the capacitor will be $\left(\frac{k_1 + k_2}{2} \right) C$

(B) Field in dielectric 1 will be more than the field in dielectric 2

(C) Surface charge density on the capacitor plates will be uniform

(D) Field in both the dielectric regions will be the same.

Ans. AD

Sol. $C_1 = K_1 \left(\frac{\epsilon_0 A / 2}{d} \right)$ $C_2 = K_2 \left(\frac{\epsilon_0 A / 2}{d} \right)$

$$C_{eq} = C_1 + C_2 = \left(\frac{K_1 + K_2}{2} \right) \left(\frac{\epsilon_0 A}{d} \right) = \left(\frac{K_1 + K_2}{2} \right) C \quad (A)$$

(B) E will be same $= \frac{\Delta V}{d}$

(C) $\left(\frac{\sigma_1}{\epsilon_0} \right) \frac{1}{K_1} = \left(\frac{\sigma_2}{\epsilon_0} \right) \frac{1}{K_2}$

$\therefore \sigma_1$ and σ_2 are not same

46.

Column-I

- (A) Plates of an isolated, charged, parallel plate, (P) air core capacitor are slowly pulled apart.
 (B) A dielectric is slowly inserted inside an (Q) isolated and charged parallel plate air cored capacitor to completely fill the space between plates.
 (C) Plates of a parallel plate capacitor connected (R) across a battery are slowly pulled apart.
 (D) A dielectric slab is slowly inserted inside a (S) parallel plate capacitor connected across a battery to completely fill the space between plates. (T) Electric field in the region decreases.

Column-II

- Electric energy stored inside capacitor increases in the process.
 Force between the two plates of the capacitor remain unchanged.
 Electric field in the region between plates remain unchanged.
 Total electric energy stored inside capacitor decreases in the process.

Ans.

(A) $\rightarrow$ PQR; (B) $\rightarrow$ QST; (C) $\rightarrow$ ST; (D) $\rightarrow$ PR

Sol.

For (A) : Field remains same as charge is same. Hence energy stored increases. (P) & (Q) as field is same and (R).

For (B) : Since distance between plate and charge remains same and field decreases, (S & T)

$$F = \frac{Q^2}{2\epsilon_0 A}, \text{ charge is same hence force remain same (Q).}$$

For (C) : Since connected to battery P.D. remains same and as distance increases the electric field decreases. (T) Also as capacitance decreases the charge also decreases decreasing the force of interaction and energy inside decrease (S).

For (D) : Capacitance increases and P.D. remaining same, thus charge and energy increases (P) and force of interaction increases. Field remains same as distance and P.D. remains same (R) and energy increases.

47.

Column-I shows some parameters after completion of event and **Column-II** shows the arrangements to which these parameters belong.

Column-I

- (A) Work done by external agent is negative (P)

Column-II

Dielectric is being removed slowly from a charged capacitor

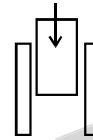
(B) Electric field between plates increases

(Q) Dielectric being introduced slowly while battery is connected



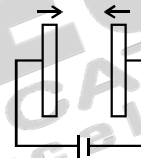
(C) Energy of capacitor decreases

(R) Dielectric is being introduced slowly in a charged capacitor



(D) Force between plates remains constant

(S) Plates of capacitor are being slowly moved closer



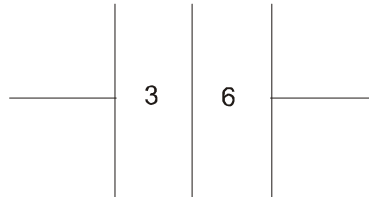
(T) Plates of capacitor are being moved slowly in a charged capacitor

Ans. (A) → QRST ; (B) → PS ; (C) → RT ; (D) → PRT

EXERCISE # (JM)

1. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is 'd'. The space between the plates is now filled with two dielectrics. One of the dielectrics has dielectric constant $k_1 = 3$ and thickness $d/3$ while the other one has dielectric constant $k_2 = 6$ and thickness $2d/3$. Capacitance of the capacitor is now : [AIEEE-2008, 4/144]

$C = 9 \text{ PF}$



- (1) 45 pF (2*) 40.5 pF (3) 20.25 pF (4) 1.8 pF

Sol. $C' = \frac{A \epsilon_0}{\frac{d_1}{3} + \frac{d_2}{6}} = \frac{A \epsilon_0}{\frac{d}{9} + \frac{2d}{18}} = \frac{18A \epsilon_0}{4d}$

$C' = 40.5 \text{ PF}$

Correct choice is (3).

2. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be

[AIEEE-2010, 4/144]

- (1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) 2

Ans. (3)

Sol. $U_0 = \frac{q_0^2}{2C}$ $U = \frac{q_0^2 e^{-2t_1/\tau}}{2C} = \frac{U_0}{2} = \frac{q_0^2}{4C} \Rightarrow e^{-2t_1/\tau} = \frac{1}{2}$

$t_1 = \frac{\tau}{2} \ln 2 \dots (1)$

and $q = q_0 e^{-t_2/\tau}$

$\frac{q_0}{4} = q_0 e^{-t_2/\tau},$

$e^{-t_2/\tau} = \frac{1}{4}$

$t_2 = 2\tau \ln 2 \dots (2)$

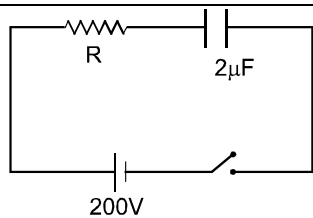
$\frac{t_1}{t_2} = \frac{1}{4}$

3. A resistor 'R' and $2\mu\text{F}$ capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5s after the switch has been closed. ($\log_{10} 2.5 = 0.4$) [AIEEE - 2011, 4/120, -1]

- (1) $1.3 \times 10^4 \Omega$ (2) $1.7 \times 10^5 \Omega$ (3) $2.7 \times 10^6 \Omega$ (4) $3.3 \times 10^7 \Omega$

Ans. (3)

Sol.



$$v = 200(1 - e^{-t/\tau})$$

$$120 = 200(1 - e^{-t/\tau})$$

$$e^{-t/\tau} = \frac{200 - 120}{200} = \frac{80}{200}$$

$$t/\tau = \log(2.5) = 0.4$$

$$5 = (0.4) \times R \times 2 \times 10^{-6}$$

$$\Rightarrow R = \frac{5}{(0.4) \times 2 \times 10^{-6}}$$

$$= R = 2.7 \times 10^6$$

4. Combination of two identical capacitors, a resistor R and a dc voltage source of voltage 6V is used in an experiment on a (C – R) circuit. It is found that for a parallel combination of the capacitor the time in which the voltage of the fully charged combination reduces to half its original voltage is 10 second. For series combination the time needed for reducing the voltage of the fully charged series combination by half is : **[AIEEE 2011, 11 May; 4, –1]**
- (1) 10 second (2) 5 second (3) 2.5 second (4) 20 second

Ans. (3)

Sol. Time constant for parallel combination = 2RC

$$\text{Time constant for series combination} = \frac{RC}{2}$$

In first case :

$$V = V_0 e^{-\frac{t_1}{2RC}} = \frac{V_0}{2} \quad \text{.....(i)}$$

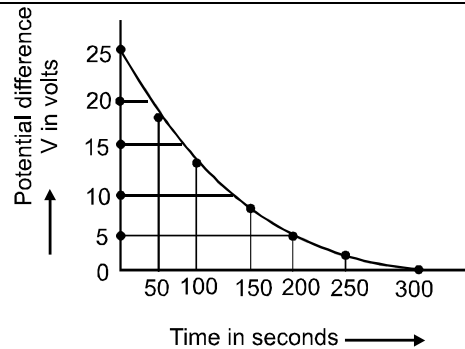
In second case :

$$V = V_0 e^{-\frac{t_2}{(RC/2)}} = \frac{V_0}{2} \quad \text{.....(ii)}$$

From (i) & (ii),

$$\frac{t_1}{2RC} = \frac{t_2}{(RC/2)} \quad \Rightarrow \quad t_2 = \frac{t_1}{4} = \frac{10}{4} = 2.5 \text{ sec.}$$

5. The figure shows an experimental plot discharging of a capacitor in an RC circuit. The time constant τ of this circuit lies between : **[AIEEE 2012 ; 4/120, –1]**



- (1) 150 sec and 200 sec
(3) 50 sec and 100 sec

- (2) 0 and 50 sec
(4) 100 sec and 150 sec

Ans.

Sol.

$$Q = c\epsilon_0 e^{-t/cR}$$

$$4\epsilon = 4\epsilon_0 e^{-t/\tau}$$

$$\epsilon = \epsilon_0 e^{-t/\tau}$$

When $t = 0 \Rightarrow \epsilon_0 = 25$

$$\epsilon = \epsilon_0 = 25 e^{-\frac{200}{\tau}}$$

when $t = 200 \Rightarrow \epsilon = 5$

$$5 = 25$$

$$\ln 5 = \frac{200}{\tau}$$

$$\tau = \frac{200}{\ln 5} = \frac{200}{\ln 10 - \ln 2}$$

$$= \frac{200}{\ln 10 - 0.693}$$

Alternative :

Time constant is the time in which 63% discharging is completed.

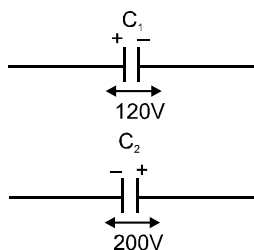
So remaining charge = $0.37 \times 25 = 9.25$ V

Which time in $100 < t < 150$ sec.

6. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then : **[Capacitance]**

- (1) $5C_1 = 3C_2$ (2) $3C_1 = 5C_2$ (3) $3C_1 + 5C_2 = 0$ (4) $9C_1 = 4C_2$

Sol.



For potential to be made zero, after connection

$$120C_1 = 200C_2$$

$$\Rightarrow 3C_1 = 5C_2$$

Ans. (2)

7. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to : **[JEE-Mains 2014]**

(1) $6 \times 10^{-7} \text{ C/m}^2$ (2) $3 \times 10^{-7} \text{ C/m}^2$ (3) $3 \times 10^4 \text{ C/m}^2$ (4) $6 \times 10^4 \text{ C/m}^2$

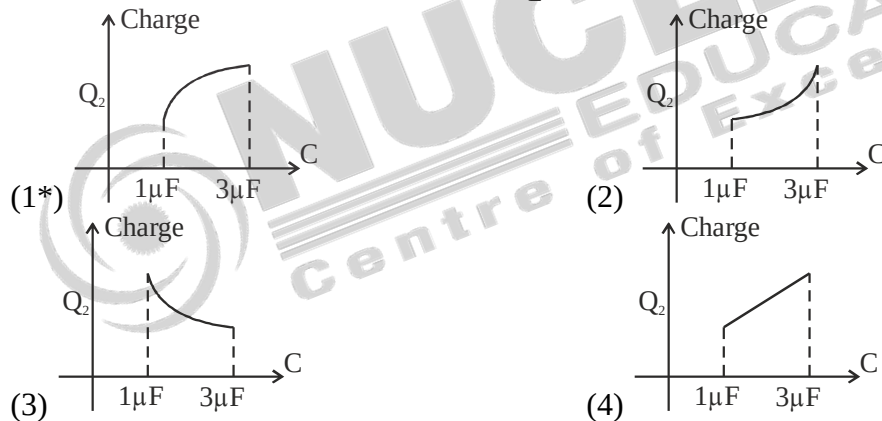
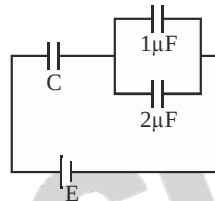
Ans. (1)

Sol. Electric field inside dielectric $\frac{\sigma}{K\epsilon_0} = 3 \times 10^4$

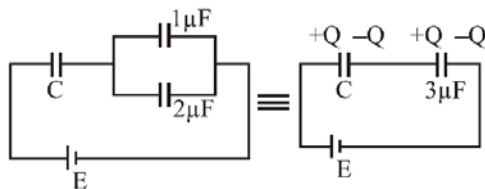
$$\Rightarrow \sigma = 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4$$

$$= 6 \times 10^{-7} \text{ C/m}^2$$

8. In the given circuit, charge Q_2 on the $2\mu\text{F}$ capacitor changes as C is varied from $1\mu\text{F}$ to $3\mu\text{F}$. Q_2 as a function 'C' is given properly by : (figures are drawn schematically and are not to scale) **[JEE MAIN 2015]**



Ans. 1



Sol.

$$C_{eq} = \frac{3C}{3+C} \quad Q = C_{eq} \cdot E$$

$$Q_2 = \frac{2CE}{(C+3)} = \frac{3CE}{(C+3)}$$

$$= 2E \left[1 - \frac{3}{C+3} \right]$$

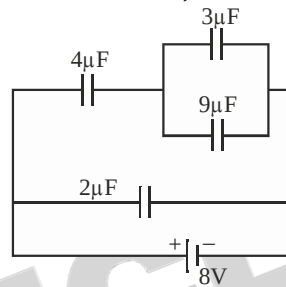
$$Q_{2/\text{when } C = 1\mu\text{F}} = 2E \left[\frac{1}{4} \right] = \frac{E}{2}$$

$$Q_{2/\text{when } C = 3\mu\text{F}} = 2E \left[\frac{1}{2} \right] = E$$

$$\frac{dQ}{dC} = -\frac{2E \cdot 3}{(C+3)^2} (-1) = \frac{6E}{(C+3)^2} > 0$$

$$\frac{d^2\theta}{dC^2} = \frac{6E(-2)}{(C+3)^3} = -\frac{12E}{(C+3)^2} < 0$$

9. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the $4\mu\text{F}$ and $9\mu\text{F}$ capacitors), at a point distant 30 m from it, would equal : **[JEE MAIN 2016]**



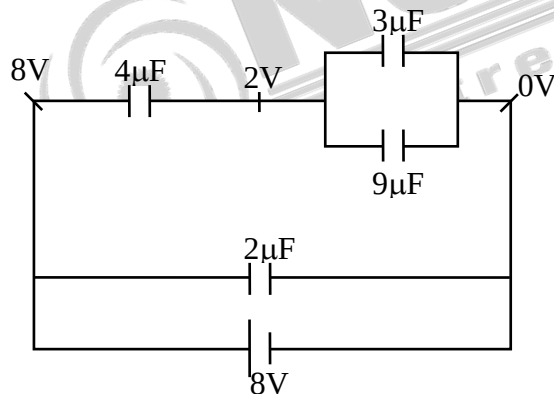
(1) 360 N/C

(2*) 420 N/C

(3) 480 N/C

(4) 240 N/C

Ans.
Sol.



$$Q_{4\mu\text{F}} = 24\mu\text{C}, \quad Q_{9\mu\text{F}} = 18\mu\text{C}$$

$$Q = 24 + 18 = 42\mu\text{C}$$

$$E = \frac{KQ}{r^2} = \frac{9 \times 10^9 \times 42 \times 10^{-6}}{900} = 420 \text{ N/C}$$

10. A capacitance of $2\mu\text{F}$ is required in an electrical circuit across a potential difference of 1.0 kV. A large number of $1\mu\text{F}$ capacitors are available which can withstand a potential difference of not more than 300 V. **[JEE MAIN 2017]**

The minimum number of capacitors required to achieve this is :

(1) 2

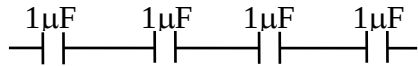
(2) 16

(3) 24

(4*) 32

Ans. 4

Sol. We need 4 capacitors in single branch to be connected across 1000 V



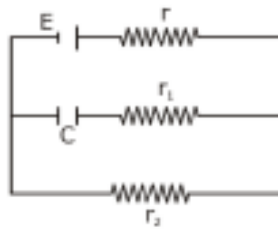
$$C_{eq} \text{ of single branch} = \frac{1}{4} \mu F$$

$$\text{we need } C_{eq} = 2 \mu F = \left(\frac{1}{4} \right) (m \text{ branches})$$

$m = 8$ branches in parallel

$$\text{Total No. of Capacitor} = 8 \times 4 = 32$$

11. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be : [JEE MAIN 2017]



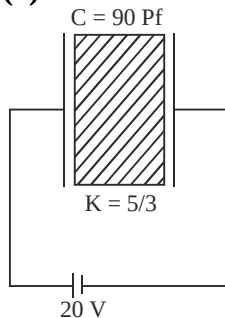
- (1) CE (2) $CE \frac{r_1}{(r_2 + r)}$ (3) $CE \frac{r_2}{(r + r_2)}$ (4) $CE \frac{r_1}{(r_1 + r)}$

Ans. 3

12. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be : [JEE Main - 2018]

- (1) 0.9 nC (2) 1.2 nC (3) 0.3 nC (4) 2.4 nC

Ans. (2)



Sol.

$$\begin{aligned} C_{eq} &= KC \\ &= \frac{5}{3} \times 90 \\ &= 150 \text{ pF} \\ Q &= C_{eq} \cdot V \\ &= 150 \times 20 \\ &= 3000 \times 10^{-12} \text{ C} \end{aligned}$$

$$= 3 \times 10^{-9} \text{ C}$$

$$Q = 3nC$$

$$\left[\sigma_i = \sigma \left(1 - \frac{1}{k} \right) \right] \times A$$

$$\sigma_i \times A = \sigma A \left(1 - \frac{1}{k} \right)$$

$$Q_i = Q \left(1 - \frac{1}{k} \right)$$

$$= 3nC \left(1 - \frac{1}{5/3} \right)$$

$$= 1.2 \text{ nC}$$

13. A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d ($d \ll a$). The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure. Capacitance of this capacitor is : [JEE Main-2019]

(1) $\frac{1}{2} \frac{k \epsilon_0 a^2}{d}$

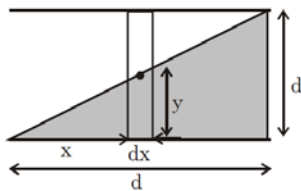
(2) $\frac{k \epsilon_0 a^2}{d} \ln K$

(3) $\frac{k \epsilon_0 a^2}{d(K-1)} \ln K$

(4) $\frac{k \epsilon_0 a^2}{2d(K+1)}$

Ans. (3)

Sol.



$$\frac{y}{x} = \frac{d}{a}$$

$$y = \frac{d}{a} x$$

$$dy = \frac{d}{a} (dx)$$

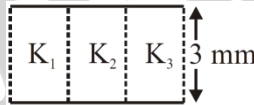
$$\frac{1}{dc} = \frac{y}{k \epsilon_0 adx} + \frac{(d-y)}{\epsilon_0 adx}$$

$$\frac{1}{dc} = \frac{1}{\epsilon_0 adx} \left(\frac{y}{k} + d - y \right)$$

$$\int dc = \int \frac{\epsilon_0 adx}{\frac{y}{k} + d - y}$$

$$\begin{aligned}
 C &= \epsilon_0 a \cdot \frac{a}{d} \int_0^d \frac{dy}{d + y \left(\frac{1}{k} - 1 \right)} \\
 &= \frac{\epsilon_0 a^2}{\left(\frac{1}{k} - 1 \right) d} \left[\ln \left(d + y \left(\frac{1}{k} - 1 \right) \right) \right]_0^d \\
 &= \frac{k \epsilon_0 a^2}{(1-k)d} \ln \left(\frac{d + d \left(\frac{1}{k} - 1 \right)}{d} \right) \\
 &= \frac{k \epsilon_0 a^2}{(1-k)d} \ln \left(\frac{1}{k} \right) = \frac{k \epsilon_0 a^2 \ln k}{(k-1)d}
 \end{aligned}$$

14. A parallel plate capacitor is of area 6 cm^2 and a separation 3 mm . The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants $K_1 = 10$, $K_2 = 12$ and $K_3 = 14$. The dielectric constant of a material which when fully inserted in above capacitor, gives same capacitance would be :



[JEE Main-2019]

- (1) 4 (2) 14 (3) 36 (4) 12

Ans. (4)

Sol. Let dielectric constant of material used be K .

$$\begin{aligned}
 \therefore \frac{10\epsilon_0 A/3}{d} + \frac{13\epsilon_0 A/3}{d} + \frac{14\epsilon_0 A/3}{d} &= \frac{K\epsilon_0 A}{d} \\
 \Rightarrow K &= 12
 \end{aligned}$$

15. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates the work done by the capacitor on the slab is :

[JEE Main-2019]

- (1) 692 pJ (2) 560 pJ (3) 508 pJ (4) 600 pJ

Ans. (3)

Sol. $q = 10 \times 12 \times 10^{-12}$

$$U_i = \frac{1}{2} \times 12 \times 10 \times 10 \times 10^{-12} = 600 \text{ pJ}$$

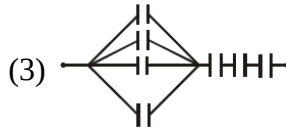
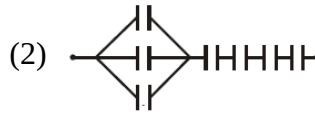
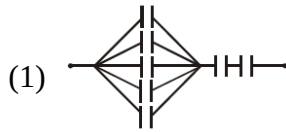
charge on capacitance will remain same

$$U_f = \frac{Q^2}{2C} = \frac{(10 \times 12 \times 10^{-12})^2}{2 \times 12 \times 10^{-12} \times 6.5} = 92.3 \text{ pJ}$$

$$w = (600 - 92.3) \text{ pJ} = 507.7 \text{ pJ}$$

16. Seven capacitors, each of capacitance $2 \mu\text{F}$, are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{6}{13}\right) \mu\text{F}$. Which of the combinations, shown in figures below, will achieve the desired value ?

[JEE Main-2019]



Ans. (2)

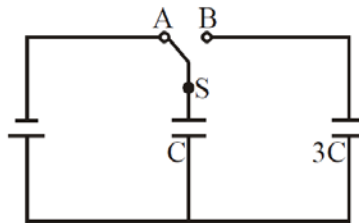
Sol. By Hit and trial method

Option-2 is correct :

$$\frac{1}{C_{eq}} = \frac{1}{3c} + \frac{1}{c} + \frac{1}{c} + \frac{1}{c} + \frac{1}{c}$$

$$\Rightarrow \frac{3c}{13} = \frac{6}{13} \mu\text{F}$$

17. In the figure shown, after the switch 'S' is turned from position 'A' to position 'B', the energy dissipated in the circuit in terms of capacitance 'C' and total charge 'Q' is:



[JEE Main-2019]

(1) $\frac{3 Q^2}{8 C}$

(2) $\frac{5 Q^2}{8 C}$

(3) $\frac{1 Q^2}{8 C}$

(4) $\frac{3 Q^2}{4 C}$

Ans. (1)

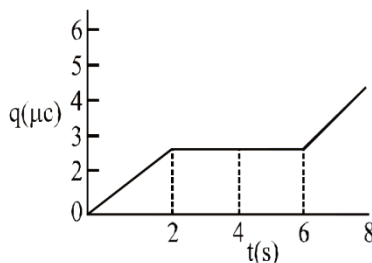
Sol. $U_i = \frac{Q^2}{2C}$

$$U_f = \frac{1}{2} \frac{\left(\frac{Q}{4}\right)^2}{C} + \frac{1}{2} \frac{\left(\frac{3Q}{4}\right)^2}{3C}$$

$$U_f = \frac{Q^2}{8C}$$

$$\Delta H = U_i - U_f = \frac{3Q^2}{8C}$$

18. The charge on a capacitor plate in a circuit, as a function of time, is shown in the figure:
What is the value of current at $t = 4$ s ?



[JEE Main-2019]

- (1) $1.5\mu\text{A}$ (2) $2\mu\text{A}$ (3) zero (4) $3\mu\text{A}$

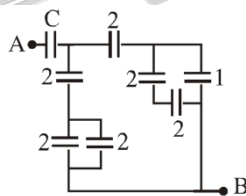
Ans. (3)

Sol. $i = \frac{dq}{dt}$ = slope of q - t curve

At $t = 4$ s, slope is zero, $\therefore$ current is Zero.

19. In the circuit shown, find C if the effective capacitance of the whole circuit is to be $0.5\mu\text{F}$. All values in the circuit are in μF .

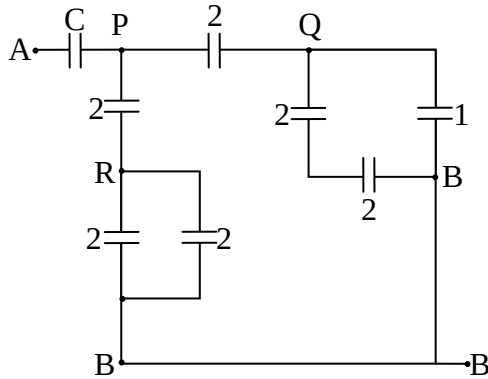
[JEE Main-2019]



- (1) $\frac{7}{11}\mu\text{F}$ (2) $4\mu\text{F}$ (3) $\frac{7}{10}\mu\text{F}$ (4) $\frac{6}{5}\mu\text{F}$

Ans. (1)

Sol.



$$C_{QB} = \frac{2 \times 2}{2 + 2} + 1 = 2 \mu\text{F}$$

$$\Rightarrow C_{PQB} = \frac{2 \times 2}{2 + 2} = 1 \mu\text{F}$$

$$C_{RB} = 2 + 2 = 4 \mu\text{F}$$

$$\Rightarrow C_{PRB} = \frac{2 \times 4}{2 + 4} = \frac{4}{3} \mu\text{F}$$

$$\text{So } C_{PB} = C_{PQB} + C_{PRB} = \frac{4}{3} + 1 = \frac{7}{3} \mu\text{F}$$

$$\text{Now, } C_{eq} = 0.5 \mu\text{F}$$

$$\Rightarrow 0.5 \mu\text{F} = \frac{C \times C_{PB}}{C + C_{PB}}$$

$$\Rightarrow C = \frac{7}{11} \mu\text{F}$$

20. A parallel plate capacitor with plates of area 1m^2 each, area t a separation of 0.1 m . If the electric field between the plates is 100 N/C , the magnitude of charge each plate is :-

$$(\text{Take } \epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} - \text{m}^2})$$

[JEE Main-2019]

- (1) $7.85 \times 10^{-10}\text{ C}$ (2) $9.85 \times 10^{-10}\text{ C}$ (3) $8.85 \times 10^{-10}\text{ C}$ (2) $6.85 \times 10^{-10}\text{ C}$

Ans. (3)

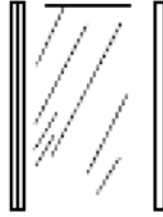
Sol. $E = \frac{Q}{A\epsilon_0}$

$$\Rightarrow 100 = \frac{Q}{1 \times 8.85 \times 10^{-12}}$$

$$\Rightarrow Q = 8.85 \times 10^{-10}\text{ C}$$

21. A parallel plate capacitor has plates of area A separated by distance ' d ' between them. It is filled with a dielectric which has a dielectric constant that varies as $k(x) = K(1 + \alpha x)$ where ' x ' is the distance measured from one of the plates. If $(\alpha d) \ll 1$, the total capacitance of the system is best given by the expression :

[JEE Main-2020]

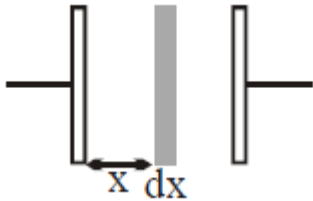


- (1) $\frac{AK\epsilon_0}{d} \left(1 + \frac{\alpha d}{2}\right)$ (2) $\frac{A\epsilon_0 K}{d} \left(1 + \frac{\alpha^2 d^2}{2}\right)$
(3) $\frac{A\epsilon_0 K}{d} \left(1 + \left(\frac{\alpha d}{2}\right)^2\right)$ (4) $\frac{AK\epsilon_0}{d} (1 + \alpha d)$

Ans. (1)

Sol. As K is variable we take plate element of Area A and thickness dx at distance x
Capacitance of element

$$dC = \frac{(A)K(1 + \alpha x)\epsilon_0}{dx}$$



Now all such elements are in series so equivalent capacitance

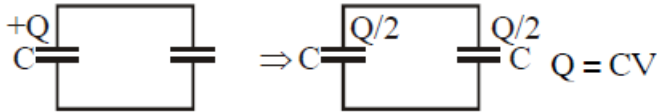
$$\begin{aligned} \frac{1}{C} &= \int \frac{1}{dC} = \int_0^d \frac{dx}{AK\epsilon_0(1 + \alpha x)} \\ \frac{1}{C} &= \frac{1}{\alpha AK\epsilon_0} \ln \left(\frac{1 + \alpha d}{1} \right) \\ &= \frac{1}{C} = \frac{1}{\alpha AK\epsilon_0} \left(\alpha d - \frac{(\alpha d)^2}{2} + \frac{(\alpha d)^3}{3} + \dots \right) \\ \Rightarrow \frac{1}{C} &= \frac{\alpha d}{\alpha AK\epsilon_0} \left(1 - \frac{\alpha d}{2} + \frac{(\alpha d)^2}{3} + \dots \right) \\ \frac{1}{C} &= \frac{d}{AK\epsilon_0} \left(1 - \frac{\alpha d}{2} \right) \\ C &= \frac{AK\epsilon_0}{d} \left(1 + \frac{\alpha d}{2} \right) \end{aligned}$$

22. A 60 pF capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged 60 pF capacitor in parallel. The electrostatic energy that is lost in this process by the time the charge is redistributed between them is (in nJ)

[JEE Main-2020]

Ans. (6)

Sol.



$$\begin{aligned}\Delta Q_L &= \frac{Q^2}{2C} - \left[\frac{(Q/2)^2}{2C} \times 2 \right] = \frac{Q^2}{4C} \\ &= \frac{1}{4} CV^2 \\ &= \frac{1}{4} \times 60 \times 10^{-12} \times 4 \times 10^2 \\ &= 6 \text{ nJ}\end{aligned}$$

23. Effective capacitance of parallel combination of two capacitors C_1 and C_2 is 10 μF . When these capacitors are individually connected to a voltage source of 1 V, the energy stored in the capacitor C_2 is 4 times that of C_1 . If these capacitors are connected in series, their effective capacitance will be:

[JEE Main-2020]

- (1) 1.6 μF (2) 3.2 μF (3) 8.4 μF (4) 4.2 μF

Ans. (1)

Sol. In parallel combination,

$$C_{eq} = C_1 + C_2 = 10 \quad \dots(1)$$

Given

$$(\text{Energy Stored})_2 = 4 \times (\text{Energy Stored})_1$$

$$\frac{1}{2} C_2 V^2 = 4 \times \frac{1}{2} C_1 V^2 \Rightarrow C_2 = 4C_1 \quad \dots(2)$$

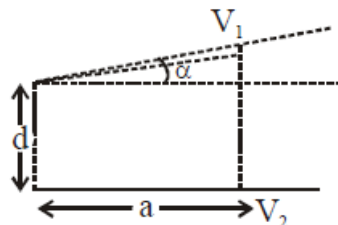
From (1) and (2)

$$C_2 = 8 \mu\text{f} \text{ and } C_1 = 2 \mu\text{f}$$

$$\text{For series combination, } C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = 1.6 \mu\text{f}$$

24. A capacitor L_s made of two square plates each of side 'a' making a very small angle α between them, as shown in figure. The capacitance will be close to :

[JEE Main-2020]



$$(1) \frac{\epsilon_0 a^2}{d} \left(1 - \frac{\alpha a}{4d}\right) \quad (2) \frac{\epsilon_0 a^2}{d} \left(1 + \frac{\alpha a}{d}\right) \quad (3) \frac{\epsilon_0 a^2}{d} \left(1 - \frac{\alpha a}{2d}\right) \quad (4) \frac{\epsilon_0 a^2}{d} \left(1 - \frac{3\alpha a}{2d}\right)$$

Ans. (3)

Sol. Take a small element of thickness dx at distance x .
Capacitance for small element $dx = dc$

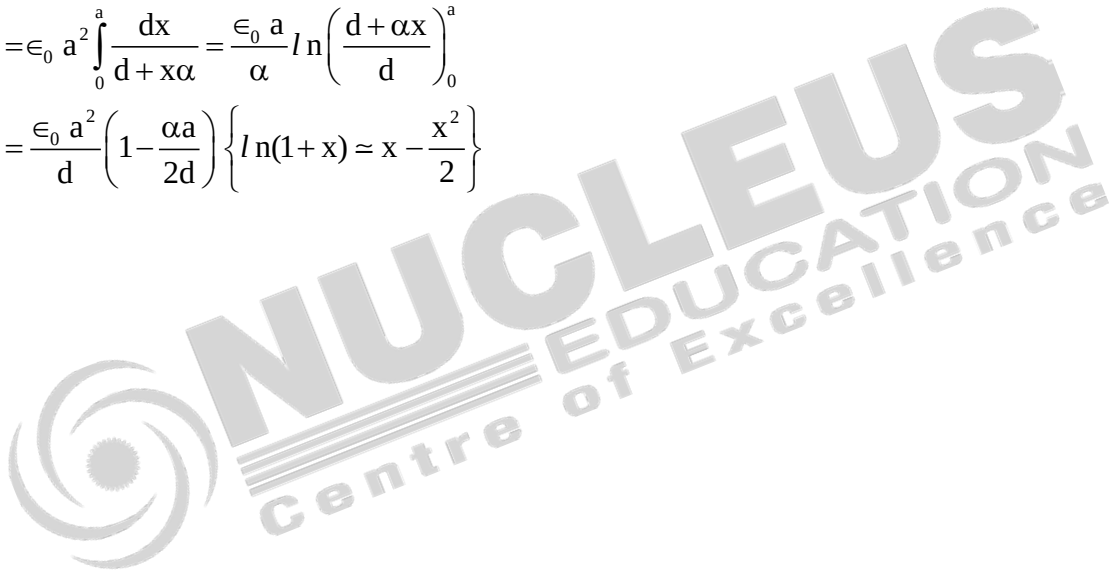
$$d_c = \frac{\epsilon_0 a d \alpha}{d + x \alpha}$$

$$c = \int_0^a \frac{\epsilon_0 a d \alpha}{d + x \alpha}$$

$$c = \int_0^a dc$$

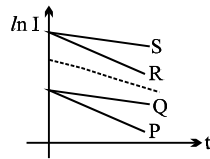
$$c = \epsilon_0 a^2 \int_0^a \frac{dx}{d + x \alpha} = \frac{\epsilon_0 a}{\alpha} \ln \left(\frac{d + \alpha x}{d} \right)_0^a$$

$$c = \frac{\epsilon_0 a^2}{d} \left(1 - \frac{\alpha a}{2d}\right) \left\{ \ln(1+x) \approx x - \frac{x^2}{2} \right\}$$



EXERCISE # (JA)

1. In an RC circuit while charging, the graph of $\ln I$ versus time is as shown by the dotted line in the diagram where I is the current. When the value of the resistance is doubled, which of the solid curves best represents the variation of $\ln I$ versus time? **[IIT JEE-2004 (Scr)]**



- (A) P (B) Q (C) R (D) S

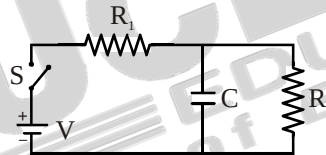
Ans. B

Sol. $\log i = \frac{-t}{RC} + \log \left(\frac{E}{R} \right)$; Compare with $y = mx + c$

$$\Rightarrow |m| = \frac{1}{RC}; c = \log \left(\frac{E}{R} \right)$$

as R increases, $|m|$ and C both decreases so (B) is the correct option

2. In the given circuit, the switch S is closed at time $t = 0$. The charge Q on the capacitor at any instant t is given by $Q(t) = Q_0 (1 - e^{-\alpha t})$. Find the value of Q_0 and α in terms of given parameters shown in the circuit. **[IIT JEE-2005]**



Ans. $Q_0 = \frac{CVR_2}{R_1 + R_2}, \alpha = \frac{R_1 + R_2}{CR_1R_2}$

Sol. $Q_0 = \frac{CV}{R_1 + R_2} \times R_2$

$$\alpha = \frac{1}{R_{eq}C} = \frac{R_1 + R_2}{R_1R_2C}$$

$$Q_{max} = \frac{CV}{R_1 + R_2} \times R_2, R_{eq} = \frac{R_1R_2}{R_1 + R_2}$$

3. An uncharged capacitor of capacitance $4\mu F$, a battery of emf 12 volt and a resistor of $2.5 M\Omega$ are connected in series. The time after which $v_C = 3v_R$ is (take $\ln 2 = 0.693$) **[IIT JEE-2005 (Scr)]**

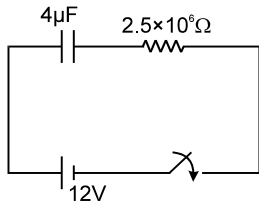
- (A) 6.93 sec. (B) 13.86 sec. (C) 20.52 sec. (D) none of these

Ans. B

Sol. $v_C + v_R = 12$

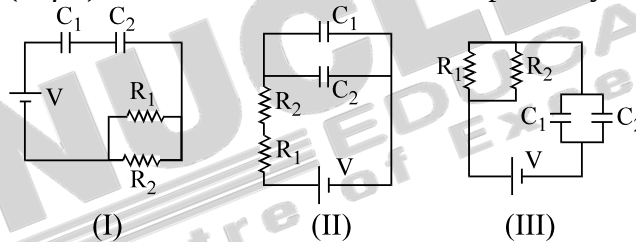
$$v_C + \frac{v_C}{3} = 12 \Rightarrow v_C = 9 \text{ volt}$$

$$V_C = \frac{q}{C} = \frac{CE(1 - e^{-t/RC})}{C}$$



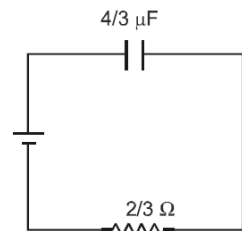
$$\begin{aligned} V_c &= 2V_r \\ &= E(1 - e^{-t/RC}) \\ 9 &= 12(1 - e^{-t/4 \times 2.5}) \\ 9 &= 12 - 12e^{-t/10} \\ 3 &= 12e^{-t/10} ; e^{-t/10} = \frac{1}{4} \\ \frac{t}{10} &= \ln 4 = 2 \ln 2 = 2 \times .693 \\ t &= 2 \times 6.93 = 13.86 \end{aligned}$$

4. Given : $R_1 = 1\Omega$, $R_2 = 2\Omega$, $C_1 = 2\mu F$, $C_2 = 4\mu F$ [IIT JEE-2006]
The time constants (in μS) for the circuits I, II, III are respectively



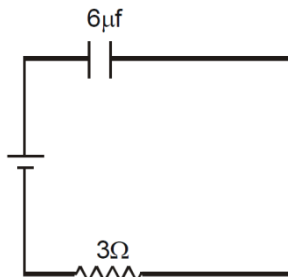
- (A) 18, 8/9, 4 (B) 18, 4, 8/9 (C) 4, 8/9, 18 (D) 8/9, 18, 4

Ans. D



Sol.

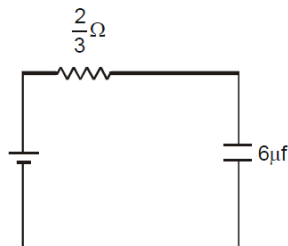
$$\text{For fig. 1 } R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{2}{3} \Rightarrow$$



$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{4}{3} \Rightarrow \tau = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$$

For fig. 2 $R_{eq} = R_1 + R_2 = 3$

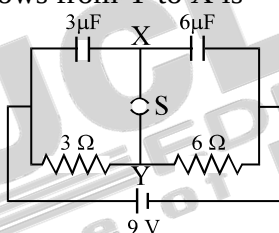
$$C_{eq} = C_1 + C_2 = 6 \Rightarrow \tau = 18$$



For fig. 3 $R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{2}{3}$

$$C_{eq} = C_1 + C_2 = 6 \Rightarrow \tau = 6 \times \frac{2}{3} = 4$$

5. A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is [IIT JEE-2007]



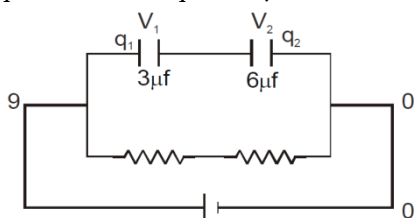
- (A) 0 (B) 54 μC (C) 27 μC (D) 81 μC

Ans. C

Sol.

When S open

$$q_1 = 18\text{mf} \Rightarrow q_2 = 18\mu\text{f}$$

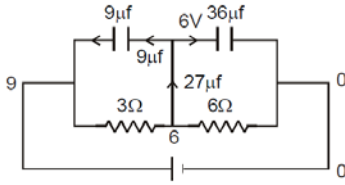


$$V_1 : V_2 = \frac{1}{3} : \frac{1}{6}$$

$$V_1 : V_2 = 2 : 1$$

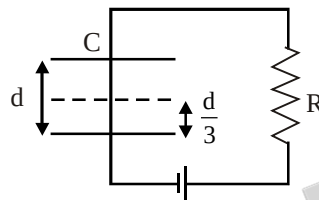
$$V_1 = \frac{2}{3} \times 9 = 6\text{volt}$$

$$V_2 = 3\text{volt}$$



When S closed

6. A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant $K = 2$. The level of liquid is $\frac{d}{3}$ initially. Suppose the liquid level decreases at a constant speed V , the time constant as a function of time t is [IIT JEE-2008]



- (A) $\frac{6\epsilon_0 R}{5d + 3Vt}$ (B) $\frac{(15d + 9Vt)\epsilon_0 R}{2d^2 - 3dVt - 9V^2t^2}$ (C) $\frac{6\epsilon_0 R}{5d - 3Vt}$ (D) $\frac{(15d - 9Vt)\epsilon_0 R}{2d^2 + 3dVt - 9V^2t^2}$

Ans. A

Sol.

$$\frac{1}{C_{eq^n}} = \frac{\left(\frac{d}{3} - Vt\right)}{2\epsilon_0 A} + \frac{\left(\frac{2d}{3} + Vt\right)}{\epsilon_0 A} = \frac{1}{2\epsilon_0 A}$$

$$\left(\frac{d}{3} - Vt + \frac{4d}{3} + 2Vt\right)$$

$$\frac{1}{C_{eq^n}} = \frac{1}{2\epsilon_0 A} \left(\frac{5d}{3} + vt\right)$$

$$\Rightarrow C_{eq^n} = \frac{6\epsilon_0 A}{(5d + 3Vt)} \Rightarrow \tau = \frac{6\epsilon_0 AR}{(5d + 3Vt)}$$

7. **Statement-1** : For practical, the earth is used as a reference at zero potential in electrical circuits.
and

Statement-2 : The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by $\frac{Q}{4\pi\epsilon_0 R}$ [IIT JEE-2008]

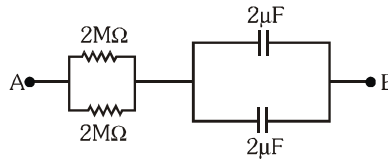
- (A) Statement-1 is True, Statement-2 is True ; statement-2 is a correct explanation for statement-1
(B) Statement-1 is True, Statement-2 is True ; statement-2 is NOT a correct explanation for statement-1

- (C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

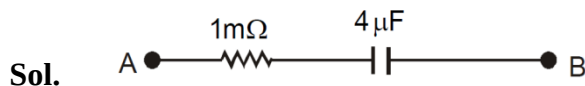
Ans. A

Sol. both statement are correct

8. At time $t = 0$, a battery of 10V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them become 4V? [Take : $\ln 5 = 1.6$, $\ln 3 = 1.1$]
[IIT JEE-2010]



Ans. 2

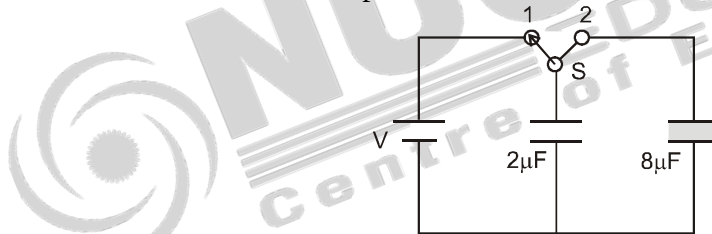


$$Q = Q_0(1 - e^{-t/RC})$$

$$16 \mu Q = 40 \mu Q (1 - e^{-t/4})$$

$$t = 2 \text{ sec}$$

9. A $2\mu\text{F}$ capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is
[IIT JEE-2010]



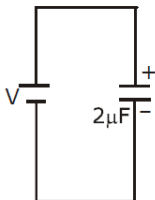
- (A) 0% (B) 20% (C) 75% (D) 80%

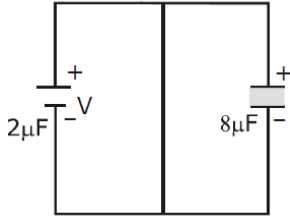
Ans. D

Sol. When connected to 1

$$\text{Energy stored} = \frac{1}{2} \times 2 \times 10^{-6} \times V^2 = V^2 \mu\text{J}$$

When connected to 2





If common potential is V' then from charge conservation $(8 + 2) V' = 2V$

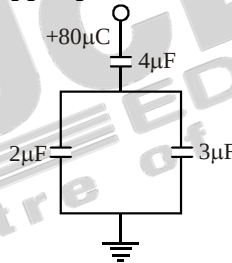
$$V' = \frac{V}{5}$$

$$\text{Total final energy} = \frac{1}{2} \times 2 \times \frac{V^2}{25} + \frac{1}{2} \times 8 \times \frac{V^2}{25} = \frac{V^2}{5}$$

$$\text{Energy dissipated} = \frac{V^2 - V'^2}{V^2} \times 100$$

$$= \frac{4V^2}{5V^2} \times 100 = 80\%$$

10. In the given circuit, a charge of $+80 \mu\text{C}$ is given to the upper plate of the $4 \mu\text{F}$ capacitor. Then in the steady state, the charge on the upper plate of the $3 \mu\text{F}$ capacitor is : [IIT JEE-2012]



- (A) $+32 \mu\text{C}$ (B) $+40 \mu\text{C}$ (C) $+48 \mu\text{C}$ (D) $+80 \mu\text{C}$

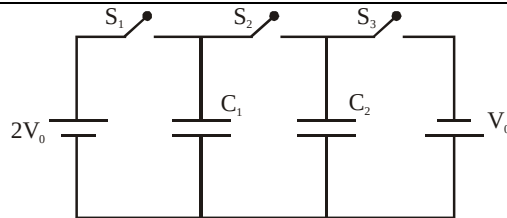
Ans. C

Sol. $q_3 = \frac{C_3}{C_2 + C_3} \cdot Q$

$$q_3 = \frac{3}{3+2} \times 80 = \frac{3}{5} \times 80$$

$$= 48 \mu\text{C}$$

11. In the circuit shown in the figure, there are two parallel plate capacitors each of the capacitance C . The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2 . After some time, S_2 is released and then S_3 is pressed. After some time, [IIT JEE-2013]

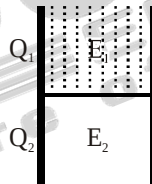


- (A) the charge on the upper plate of C_1 is $2CV_0$
 (B) the charge on the upper plate of C_1 is CV_0
 (C) the charge on the upper plate of C_2 is 0.
 (D) the charge on the upper plate of C_2 is $-CV_0$

Ans. BD

Sol. When switch S_1 is released charge on C_1 is $2CV_0$ (on upper plate)
 When switch S_2 is released charge on C_1 is CV_0 (on upper plate) and charge on C_2 is CV_0 (on upper plate)
 When switch S_3 is released charge on C_1 is CV_0 (on upper plate) and charge on C_2 is $-CV_0$ (on upper plate)

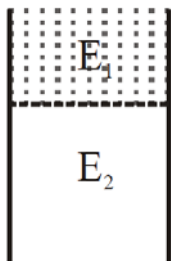
12. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers $1/3$ of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1 . When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2 . The electric field in the dielectric is E_1 and that in the other portion is E_2 . Choose the correct option/options, ignoring edge effects. [IIT JEE- 2014]



- (A) $\frac{E_1}{E_2} = 1$ (B) $\frac{E_1}{E_2} = \frac{1}{K}$ (C) $\frac{Q_1}{Q_2} = \frac{3}{K}$ (D) $\frac{C}{C_1} = \frac{2+K}{K}$

Ans. AD

Sol.



$$C_1 = \frac{K\epsilon_0(A/3)}{d} \text{ (With dielectric)}$$

$$\& \text{ let } C_2 = \frac{\epsilon_0(2A/3)}{d} \text{ (without dielectric)}$$

$$C = \frac{K\epsilon_0 A/3}{d} + \frac{\epsilon_0 2A/3}{d} = \frac{\epsilon_0 A/3}{d} [K+2]$$

$$\therefore \frac{C}{C_1} = \frac{K+2}{K}$$

As potential difference is same and gap is same.

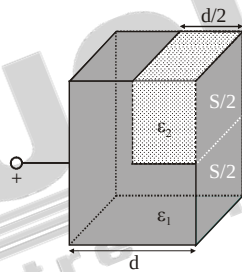
$$\therefore E_1 = E_2$$

$$\therefore \frac{E_1}{E_2} = 1$$

$$Q_1 = C_1 V, Q_2 = C_2 V$$

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{K}{2}$$

13. A parallel plate capacitor having plates of area S and plate separation d , has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\epsilon_1 = 2$ and $\epsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2 . The ratio $\frac{C_2}{C_1}$ is **[IIT JEE-2015]**



(A) $6/5$

(B) $5/3$

(C) $7/5$

(D) $7/3$

Ans. D

Sol. $\frac{5\epsilon_0}{d} = C_1$

in new setup we have 3 different setups

$$5/2, \frac{d}{2}, \epsilon_1$$

$$5/2, \frac{d}{2}, \epsilon_2$$

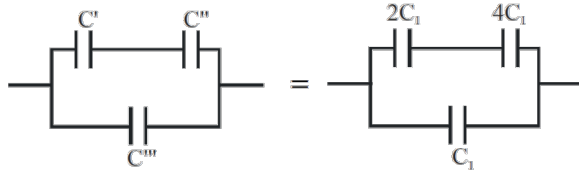
$$5/2, d, \epsilon_1$$



$$C' = \frac{\frac{5}{2} \epsilon_1 \epsilon_0}{d/2} = 2C_1$$

$$C'' = \frac{\frac{5}{2} \epsilon_2 \epsilon_0}{d/2} = 4C_1$$

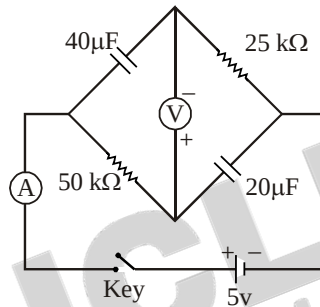
$$C''' = \frac{\frac{5}{2} \epsilon_1 \epsilon_0}{d} = C_1$$



$$C_2 = \left(\frac{1}{2C_1} + \frac{1}{4C_1} \right)^{-1} + C_1 = \frac{14}{6}C_1 = \frac{7}{3}C_1$$

$$\therefore \frac{C_2}{C_1} = \frac{7}{3}$$

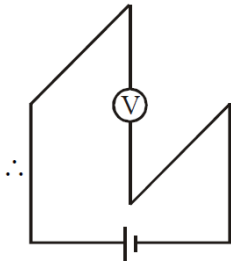
14. In the circuit shown below, the key is pressed at time $t = 0$. Which of the following statement(s) is (are) true? [IIT JEE-2016]



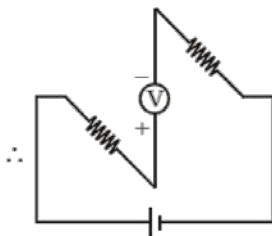
- (A) The voltmeter displays -5 V as soon as the key is pressed, and displays $+5$ V after a long time
(B) The voltmeter will display 0 V at time $t = \ln 2$ seconds
(C) The current in the ammeter becomes $1/e$ of the initial value after 1 second
(D) The current in the ammeter becomes zero after a long time

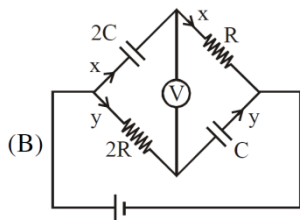
Ans. ABCD

Sol. (A) At $t = 0$, capacitor acts as short-circuit



At $t \rightarrow \infty$, capacitor acts as open circuit & no current flows through voltmeter.





$$q_x = 2CV(1 - e^{-t/2CR}) \quad x = \frac{V}{R} e^{-t/2CR}$$

$$q_y = CV(1 - e^{-t/2CR}) \quad y = \frac{V}{2R} e^{-t/2CR}$$

$$\Delta V = -y2R + \frac{q_x}{2C}$$

$$= V[1 - 2e^{-t/2CR}] = 0$$

(C) $\tau = 1 \text{ sec}$

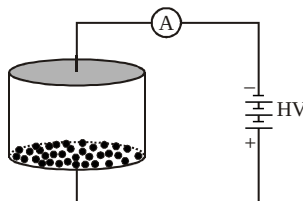
So by $i = i_0 e^{-t/\tau}$ current at $t = 1 \text{ sec}$ is $= i_0/e$

(D) After long time no current flows since both capacitor & voltmeter does not allow.

Paragraph for Question Nos. 15 to 16

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius $r \ll h$. Now a high voltage source (HV) is connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface, the balls will get charged, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. (Ignore gravity)

[IIT JEE-2016]



15. Which one of the following statements is correct?

- (A) The balls will stick to the top plate and remain there
- (B) The balls will bounce back to the bottom plate carrying the same charge they went up with
- (C) The balls will bounce back to the bottom plate carrying the opposite charge they went up with
- (D) The balls will execute simple harmonic motion between the two plates

Ans. C

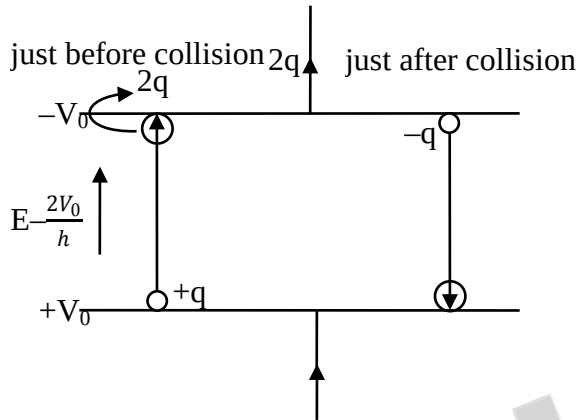
Sol. As the ball collides with upper plate, all its positive charge is neutralized with the negative plate, and to become at same potential as the upper plate, it acquires negative charge from the

plate, magnitude of which is same as the previous +ve charge it carried, as the magnitude of potential is same in both cases. Then plates will bounce back due to electrostatic repulsion.

16. The average current in the steady state registered by the ammeter in the circuit will be
 (A) zero (B) proportional to the potential V_0
 (C) proportional to $V_0^{1/2}$ (D) proportional to V_0^2

Ans. D

Sol.



$$\frac{kq}{r} = V_0 = q = \frac{V_0 r}{k}$$

$$\frac{1}{2} \left(\frac{qE}{m} \right) t^2 = h$$

$$\frac{1}{2} \frac{V_0 r}{K} \frac{2V_0}{hm} t^2 = h$$

$$t^2 = \frac{h^2 mk}{V_0^2 r}$$

$$t = \frac{h}{V_0} \sqrt{\frac{mk}{r}}$$

During every collision $2q$ charge will flow from circuit.

$$\text{Average current } I_{\text{avg}} = \frac{2q}{t} = \frac{2V_0^2}{h} \frac{e\sqrt{r}}{mk\sqrt{k}}$$

$$I_{\text{avg}} \propto V_0^2$$

Paragraph for Q.No. 17 to 18

Consider a simple RC circuit as shown in figure 1.

Process : In the circuit the switch S is closed at $t = 0$ and the capacitor is fully charged to voltage V_0 (i.e., charging continues for time $T \gg RC$). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C .

Process 2: In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time

$T \gg RC$. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time $T \gg RC$. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1. These two processes are depicted in figure-2. [IIT JEE-2017]

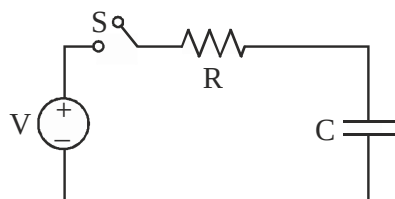


Figure 1

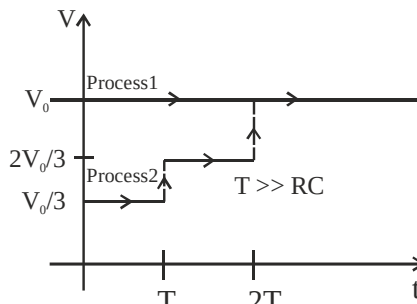
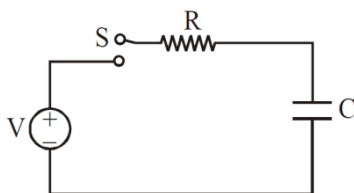


Figure 2

17. In Process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by:

(A) $E_C = E_D$ (B) $E_C = E_D \ln 2$ (C) $E_C = E_D$ (D) $E_C = 2E_D$

Ans. A



Sol.

When switch is closed for a very long time capacitor will get fully charged & charge on capacitor will be $q = CV$

$$\text{Energy stored in capacitor } \epsilon_C = \frac{1}{2} CV^2 \quad \dots(i)$$

$$\text{Work done by battery } (\omega) = Vq = VCV = CV^2$$

$$\text{dissipated across resistance } \epsilon_D = (\text{work done by battery}) - (\text{energy store})$$

$$\epsilon_D = CV^2 - \frac{1}{2} CV^2 = \frac{1}{2} CV^2 \quad \dots(ii)$$

from (i) & (ii)

$$\epsilon_D = \epsilon_C$$

18. In Process 2, total energy dissipated across the resistance E_D is:

(A) $E_D = \frac{1}{2} CV_0^2$ (B) $E_D = 3 \left(\frac{1}{2} CV_0^2 \right)$ (C) $E_D = \frac{1}{3} \left(\frac{1}{2} CV_0^2 \right)$ (D) $E_D = 3CV_0^2$

Ans. C

Sol. For process (1)

$$\text{Charge on capacitor} = \frac{CV_0}{3}$$

$$\text{energy stored in capacitor} = \frac{1}{2} C \frac{V_0^2}{9} = \frac{CV_0^2}{18}$$

$$\text{work done by battery} = \frac{CV_0}{3} \times \frac{V}{3} = \frac{CV_0^2}{9}$$

$$\text{Heat loss} = \frac{CV_0^2}{9} - \frac{CV_0^2}{18} = \frac{CV_0^2}{18}$$

For process (2)

$$\text{Charge on capacitor} = \frac{2CV_0}{3}$$

$$\text{Extra charge flow through battery} = \frac{CV_0}{3}$$

$$\text{Work done by battery} : \frac{CV_0}{3} \cdot \frac{2V_0}{3} = \frac{2CV_0^2}{9}$$

$$\text{Final energy store in capacitor} : \frac{1}{2} C \left(\frac{2V_0}{3} \right)^2 = \frac{4CV_0^2}{18}$$

$$\text{energy store in process 2} : \frac{4CV_0^2}{18} - \frac{CV_0^2}{18} = \frac{3CV_0^2}{18}$$

Heat loss in process (2) = work done by battery in process (2) – energy store in capacitor process (2)

$$= \frac{2CV_0^2}{9} - \frac{3CV_0^2}{18} = \frac{CV_0^2}{18}$$

For process (3)

$$\text{Charge on capacitor} = CV_0$$

$$\text{extra charge flow through battery} : CV_0 - \frac{2CV_0}{3} = \frac{CV_0}{3}$$

$$\text{work done by battery in this process} : \left(\frac{CV_0}{3} \right) (V_0) = \frac{CV_0^2}{3}$$

$$\text{find energy store in capacitor} : \frac{1}{2} CV_0^2$$

$$\text{energy stored in this process} : \frac{1}{2} CV_0^2 - \frac{4CV_0^2}{18} = \frac{5CV_0^2}{18}$$

$$\text{heat loss in process (3)} : \frac{CV_0^2}{3} - \frac{5CV_0^2}{18} = \frac{CV_0^2}{18}$$

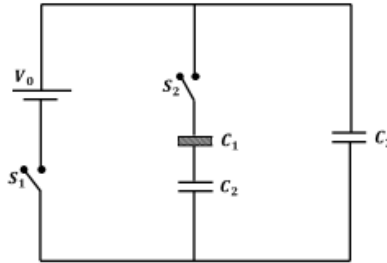
$$\text{Now total heat loss (E}_D\text{)} : \frac{CV_0^2}{18} + \frac{CV_0^2}{18} + \frac{CV_0^2}{18} = \frac{CV_0^2}{6}$$

$$\text{final energy store in capacitor} : \frac{1}{2} CV_0^2$$

$$\text{so we can say that } E_D = \frac{1}{3} \left(\frac{1}{2} CV_0^2 \right)$$

19. Three identical capacitors C_1, C_2 and C_3 have a capacitance of $1.0 \mu F$ each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity ϵ_r . The cell electromotive force (emf) $V_0 = 8V$. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be $5 \mu C$. The value of $\epsilon_r =$ _____.

[JEE-Advanced- 2018]



Ans. 1.50

Sol.

$$q_0 = C_3 V_0 = C(8) = 8 \times 1 \mu C$$

by using KVL, we can write

$$\frac{q_0 - q_1}{C_3} = \frac{q_1}{\epsilon_r C_1} + \frac{q_1}{C_2}$$

$$\frac{q_0 - q_1}{C} = \frac{q_1}{\epsilon_r C} + \frac{q_1}{C}$$

$$q_0 - q_1 = \frac{q_1}{\epsilon_r} + q_1 \quad \dots(1)$$

given $q_0 - q_1 = 5$

$$8 - q_1 = 5$$

$$q_1 = 3$$

$$8 - 3 = \frac{3}{\epsilon_r} + 3$$

$$2 = \frac{3}{\epsilon_r}$$

$$\epsilon_r = \frac{3}{2} = 1.50$$