

Haik General Secondary and Preparatory School

Grade 12 Physics

Unit 2- Oscillation and Wave

Suplimentary Note



Beats

Compiled by Wondye. Y.

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7.6.3 Beats

When two waves of nearly equal frequencies travelling in a medium along the same direction superimpose upon each other, beats are produced. The amplitude of the resultant sound at a point rises and falls regularly.

The intensity of the resultant sound at a point rises and falls regularly with time. When the intensity rises to maximum we call it as waxing of sound, when it falls to minimum we call it as waning of sound.

The phenomenon of waxing and waning of sound due to interference of two sound waves of nearly equal frequencies are called beats. The number of beats produced per second is called beat frequency, which is equal to the difference in frequencies of two waves.

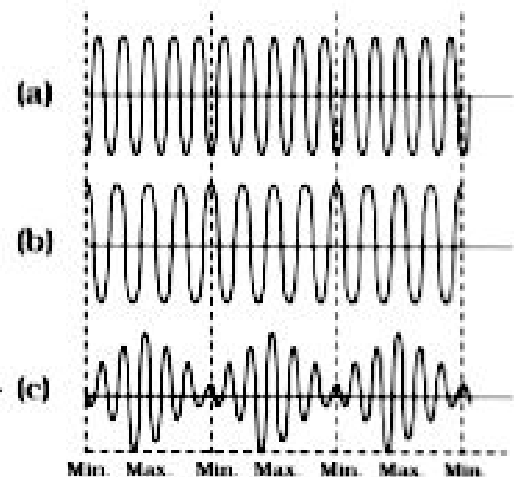


Fig. 7.12 Graphical representation of beats

Analytical method

Let us consider two waves of slightly different frequencies n_1 and n_2 ($n_1 - n_2 < 10$) having equal amplitude travelling in a medium in the same direction.

At time $t = 0$, both waves travel in same phase.

The equations of the two waves are

$$y_1 = a \sin \omega_1 t$$

$$y_1 = a \sin (2\pi n_1)t \quad \dots(1)$$

$$\begin{aligned} y_2 &= a \sin \omega_2 t \\ &= a \sin (2\pi n_2)t \end{aligned} \quad \dots(2)$$

When the two waves superimpose, the resultant displacement is given by

$$\begin{aligned} y &= y_1 + y_2 \\ y &= a \sin (2\pi n_1)t + a \sin (2\pi n_2)t \end{aligned} \quad \dots(3)$$

Therefore

$$y = 2a \sin 2\pi \left(\frac{n_1 + n_2}{2} \right)t \cos 2\pi \left(\frac{n_1 - n_2}{2} \right)t \quad \dots(4)$$

Substitute $A = 2a \cos 2\pi \left(\frac{n_1 - n_2}{2} \right)t$ and $n = \frac{n_1 + n_2}{2}$ in equation (4)

$$\therefore y = A \sin 2\pi nt$$

This represents a simple harmonic wave of frequency $n = \frac{n_1 + n_2}{2}$ and amplitude A which changes with time.

(i) The resultant amplitude is maximum (i.e) $\pm 2a$, if

$$\cos 2\pi \left[\frac{n_1 - n_2}{2} \right]t = \pm 1$$

$$\therefore 2\pi \left[\frac{n_1 - n_2}{2} \right]t = m\pi$$

(where $m = 0, 1, 2, \dots$) or $(n_1 - n_2)t = m$

The first maximum is obtained at $t_1 = 0$

The second maximum is obtained at

$$t_2 = \frac{1}{n_1 - n_2}$$

The third maximum at $t_3 = \frac{2}{n_1 - n_2}$ and so on.

The time interval between two successive maxima is

$$t_2 - t_1 = t_3 - t_2 = \frac{1}{n_1 - n_2}$$

Hence the number of beats produced per second is equal to the reciprocal of the time interval between two successive maxima.

(ii) The resultant amplitude is minimum (i.e) equal to zero, if

$$\cos 2\pi \left(\frac{n_1 - n_2}{2} \right) t = 0$$

$$(i.e) 2\pi \left(\frac{n_1 - n_2}{2} \right) t = \frac{\pi}{2} + m\pi = (2m + 1) \frac{\pi}{2} \text{ or } (n_1 - n_2)t = \frac{(2m + 1)}{2}$$

where $m = 0, 1, 2 \dots$

The first minimum is obtained at

$$t_1' = \frac{1}{2(n_1 - n_2)}$$

The second minimum is obtained at

$$t_2' = \frac{3}{2(n_1 - n_2)}$$

The third minimum is obtained at

$$t_3' = \frac{5}{2(n_1 - n_2)} \text{ and so on}$$

Time interval between two successive minima is

$$t_2' - t_1' = t_3' - t_2' = \frac{1}{n_1 - n_2}$$

Hence, the number of beats produced per second is equal to the reciprocal of time interval between two successive minima.

7.6.4 Uses of beats

(i) The phenomenon of beats is useful in tuning two vibrating bodies in unison. For example, a sonometer wire can be tuned in unison with a tuning fork by observing the beats. When an excited tuning fork is kept on the sonometer and if the sonometer wire is also excited, beats are heard, when the frequencies are nearly equal. If the length of the wire is adjusted carefully so that the number of beats gradually decreases to zero, then the two are said to be in unison. Most of the musical instruments are made to be in unison based on this method.

(ii) The frequency of a tuning fork can be found using beats. A standard tuning fork of frequency N is excited along with the experimental fork. If the number of beats per second is n , then the frequency of experimental tuning fork is $N \pm n$. The experimental tuning

fork is then loaded with a little bees' wax, thereby decreasing its frequency. Now the observations are repeated. If the number of beats increases, then the frequency of the experimental tuning fork is $N - n$, and if the number of beats decreases its frequency is $N + n$.