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Steel Beam Design

Six Easy Steps

Steel beam design is about selecting the lightest steel beam that will support the load without exceeding the bending strength or shear strength of the material, and without exceeding the maximum allowable deflection for the beam. We want the lightest beam because it is generally the cheapest. We can solve these problems with a 6-step process.

Step 1 Identify all loads and design constraints (yield strength, maximum allowable deflection Δ_{max}, beam length L, etc.).

Step 2 Draw the load diagram and calculate all reactions.

Step 3 Draw the shear and moment diagrams, and calculate V_{max} and M_{max}. If the loading conditions are right, use the Formula Method to find these values.

Step 4 Calculate the plastic section modulus Z_x required to support the applied moment. Select the lightest steel beam from the Appendix that supports M_{max} and has enough stiffness to limit Δ_{max} (if deflection is a constraint).

Step 5 Include the beam weight in new drawings of the load, shear, and moment diagrams. Check that the beam can support the applied loads and its own weight, and that it still meets the maximum deflection constraint.

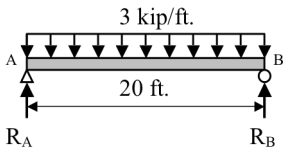
Step 6 Calculate the shear strength of the selected beam, and check that the beam will support more shear load than is applied.

Example #1

Select the lightest W-beam that will support a uniformly distributed load of 3 kip/ft. on a simply-supported span of 20 ft. The beam is rolled high-strength, low-alloy steel (HSLA).

Step 1 We know the loading and length; the steel has a yield strength $\sigma_{YS} = 50$ ksi . The maximum beam deflection Δ_{max} is not specified.

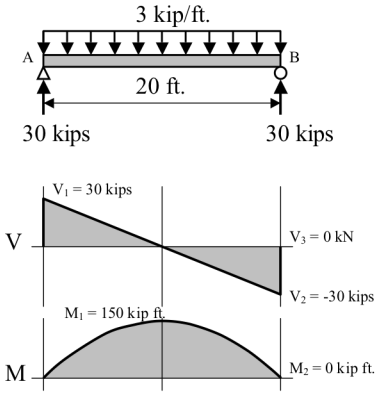
Step 2 The total load on the beam is $\frac{3 \text{ kip}}{\text{ft.}} \frac{20 \text{ ft.}}{} = 60$ kips . Since the loading is symmetrical, $R_A = R_B = 30$ kips .



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Step 3 The shear diagram for a uniform distributed load is two triangles. The moment diagram is a parabola, where M_{max} is the area of the shear diagram up to the midspan, or the area of the left-hand triangle. Since the area of a triangle is the base times the height divided by two, $M_{max} = \frac{30 \text{ kips} \times 10 \text{ ft.}}{2} = 150 \text{ kip ft.}$



Step 4 The moment strength of a steel beam is $M_R = 0.6\sigma_{YS}Z_x$. We can rewrite the equation to find the value of Z_x required to support the applied moment.

$$\text{Required } Z_x = \frac{M}{0.6\sigma_{YS}} = \frac{1.67M}{\sigma_{YS}}$$

$$= \frac{1.67 \times 150 \text{ kip ft.}}{50 \text{ kips}} \frac{\text{in.}^2}{\text{ft.}} \frac{12 \text{ in.}}{\text{ft.}}$$

$$= 60.1 \text{ in.}^3$$

Beam	Z_x (in.^{3})
W18×40	78.4
W12×50	72.4
W10×54	66.6
W16×36	64.0
W12×40	57.5

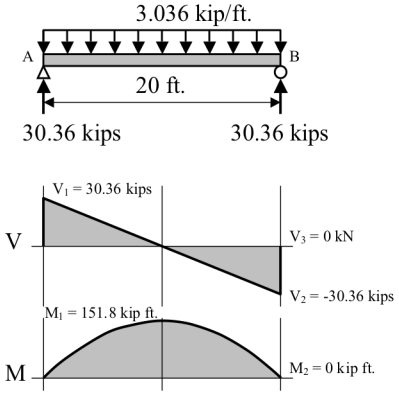
Appendix I lists W-beams in decreasing order of plastic section modulus Z_x. Look for a beam with a slightly larger Z_x than the required value. In this case, the lightest beam is W16×36, with a weight of 36 lb./ft., or 0.036 kips/ft.

Step 5 We can add the beam weight to the applied uniform distributed load, for a total of 3.036 kips/ft. The total load on the beam is $\frac{3.036 \text{ kip}}{\text{ft.}} \frac{20 \text{ ft.}}{} = 60.72 \text{ kips}$. Since the loading is symmetrical, $R_A = R_B = 30.36 \text{ kips}$. The maximum moment is

$$M_{max} = \frac{30.36 \text{ kips} \times 10 \text{ ft.}}{2} = 151.8 \text{ kip ft.}$$

$$\text{Required } Z_x = \frac{1.67 \times 151.8 \text{ kip ft.}}{50 \text{ kips}} \frac{\text{in.}^2}{\text{ft.}} \frac{12 \text{ in.}}{\text{ft.}} = 60.8 \text{ in.}^3$$

, which is less than Z_x of the selected beam. As long as we have more than we need, the beam will survive. If the new required Z_x had been 66 in.^{3}, then we would have to select a different beam.



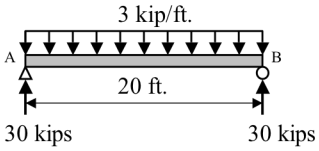
Step 6 We know the beam will support the load without exceeding its bending strength; now we need to check shear strength. For wide-flange steel W-beams, $V_{applied} \le 0.4\sigma_{YS}dt_w$ where d is the beam depth and t_w is the thickness of the web. Find these dimensions in Appendix A. A W16×36 beam can support a shear load of $0.4 \times 50 \frac{\text{kips}}{\text{in.}^2} \times 15.86 \text{ in.} \times 0.295 \text{ in.} = 93.6 \text{ kips}$. Since the actual shear load of 30.36 kips is less than 93.6 kips, the beam will not fail in shear.

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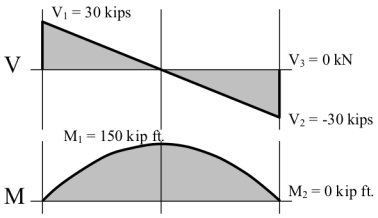
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Example #2

Select the lightest W-beam that will support a uniformly distributed load of 3 kip/ft. on a simply-supported span of 20 ft. *and* deflect no more than 0.6 inches. The beam is rolled high-strength, low-alloy steel (HSLA).



Steps 1-4 The first few steps are identical to Example #1 because the beam loading and length are the same. However, we have an additional constraint of $\Delta_{max} = 0.6$ in. From Appendix H, Case #1, the maximum deflection for a simply-supported beam with a uniform distributed load is $\Delta_{max} = \frac{5wL^4}{384EI}$. We can rewrite the equation to find the moment of inertia required to limit the maximum deflection.



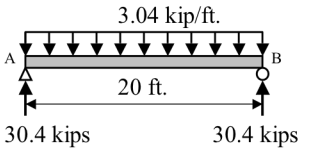
$$\text{Required } I = \frac{5wL^4}{384E\Delta_{max}}$$

$$= \frac{5}{384}\frac{3\text{ kip}}{\text{ft.}}\frac{(20\text{ ft.})^4\text{ in.}^2}{30\times10^3\text{ kip}}\frac{}{0.6\text{ in.}}\frac{(12\text{ in.})^3}{\text{ft.}^3} = 600\text{ in.}^4$$

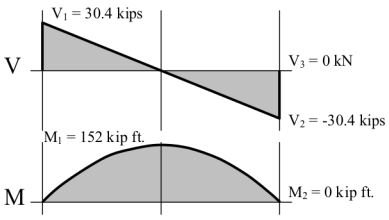
The moment of inertia of a W16×36 is 448 in.^{4}, which is not enough. Instead, we need to select a beam with a moment of inertia greater than 600 in.^{4}, such as W18×40, which has a weight of 40 lb./ft. or 0.040 kip/ft.

Beam	Z_x (in.^{3})	I_x (in.^{4})
W18×40	78.4	612
W12×50	72.4	394
W10×54	66.6	303
W16×36	64.0	448
W12×40	57.5	310

Step 5 Add the beam weight to the applied uniform distributed load, for a total of 3.040 kips/ft. The total load on the beam is $\frac{3.040\text{ kip}}{\text{ft.}}\frac{20\text{ ft.}}{} = 60.8$ kips. Since the loading is symmetrical, $R_A = R_B = 30.4$ kips. The maximum moment is



$$M_{max} = \frac{30.4\text{ kips}\times10\text{ ft.}}{2} = 152\text{ kip ft.}$$



$$\text{Required } Z_x = \frac{1.67\times152\text{ kip ft.}}{}\frac{\text{in.}^2}{50\text{ kips}}\frac{12\text{ in.}}{\text{ft.}} = 60.9\text{ in.}^3$$, which is less than Z_x of the selected beam, so the beam is strong enough in bending.

Checking for deflection,

$$\text{Required } I = \frac{5}{384}\frac{3.04\text{ kip}}{\text{ft.}}\frac{(20\text{ ft.})^4\text{ in.}^2}{30\times10^3\text{ kip}}\frac{}{0.6\text{ in.}}\frac{(12\text{ in.})^3}{\text{ft.}^3} = 608\text{ in.}^4$$

Since the beam's moment of inertia is more than the required value, the beam meets the deflection criterion.

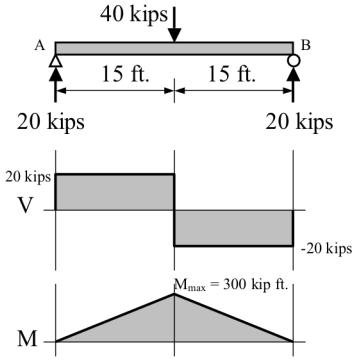
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Step 6 Use $V_{applied} \le 0.4\sigma_{YS}dt_w$ to find the shear strength. A W18×40 beam can support a shear load of $0.4 \times 50 \frac{\text{kips}}{\text{in.}^2} \times 17.9 \text{ in.} \times 0.315 \text{ in.} = 113 \text{ kips}$. Since the actual shear load of 30.4 kips is less than 113 kips, the beam will not fail in shear.

Examples #1 and #2 are the easiest types to solve because the weight of the beam is a uniform distributed load, therefore the load, shear, and moment diagrams have the same shape after the beam weight is included.

Example #3

Select the lightest W-beam that will support a point load of 40 kips at the midspan of a simply-supported 30 foot span.



Step 1 $P = 40$ kips, $L = 30$ ft., $\sigma_{YS} = 50$ ksi, Δ_{max} is not specified.

Step 2 The total load on the beam is 40 kips. Since the loading is symmetrical, $R_A = R_B = \frac{40 \text{ kips}}{2} = 20 \text{ kips}$.

Step 3 The shear diagram for a point load at the midspan is two rectangles. The moment diagram is a triangle, where M_{max} is the area of the shear diagram up to the midspan, or the area of the left-hand rectangle: $M_{max} = 20 \text{ kips} \times 15 \text{ ft.} = 300 \text{ kip ft.}$

Step 4 Calculate the needed plastic section modulus:

$$\text{Required } Z_x = \frac{1.67M}{\sigma_{YS}} = \frac{1.67 \times 300 \text{ kip ft.}}{50 \text{ kips}} \frac{\text{in.}^2}{\text{ft.}} \frac{12 \text{ in.}}{\text{ft.}}$$
$$= 120.2 \text{ in.}^3$$

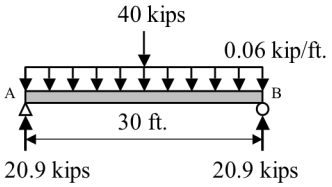
Select W18×60, with a weight of 60 lb./ft., or 0.06 kips/ft.

Beam	Z_x (in.^{3})
W21×62	144
W14×74	126
W18×60	123
W21×50	110

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Step 5 Redraw the load, shear, and moment diagrams to include the weight of the beam. Add the beam weight to the point load for a total load of $40 \text{ kips} + \frac{0.06 \text{ kip}}{\text{ft.}} \frac{20 \text{ ft.}}{} = 41.8 \text{ kips}$. Since the loading is symmetrical, $R_A = R_B = 20.9 \text{ kips}$.

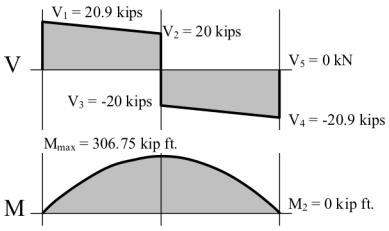


$$V_1 = R_A = R_B = 20.9 \text{ kips}$$

$$V_2 = V_1 - \frac{0.06 \text{ kip}}{\text{ft.}} \frac{15 \text{ ft.}}{} = 20 \text{ kips}$$

$$V_3 = V_2 - 40 \text{ kips} = -20 \text{ kips}$$

$$V_4 = V_3 + 20.9 \text{ kips} = 0 \text{ kips}$$



The maximum moment is the area of the left-hand trapezoid, which is the average height times the base:

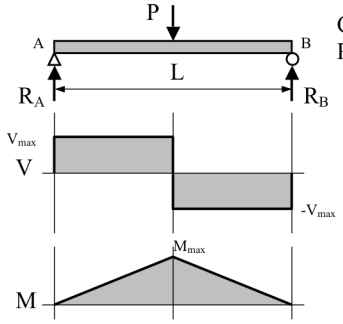
$$M_{max} = \frac{20.9 \text{ kips} + 20 \text{ kips}}{2} 15 \text{ ft.} = 306.75 \text{ kip ft.}$$

$$\text{Required } Z_x = \frac{1.67 \times 306.75 \text{ kip ft.}}{} \frac{\text{in.}^2}{50 \text{ kips}} \frac{12 \text{ in.}}{\text{ft.}} = 122.9 \text{ in.}^3,$$

which is slightly less than Z_x of the selected beam, so the beam is strong enough in bending.

Step 6 Use $V_{applied} \le 0.4 \sigma_{YS} d t_w$ to find the shear strength. A W18×60 beam can support a shear load of $0.4 \times 50 \frac{\text{kips}}{\text{in.}^2} \times 18.24 \text{ in.} \times 0.415 \text{ in.} = 151 \text{ kips}$. Since the actual shear load of 20.9 kips is less than 151 kips, the beam will not fail in shear.

Another way to solve this problem is to use the Formula Method and Superposition. For **Step 2** and **Step 3**, Case #5 in Appendix H gives us



Case #5
Point load at midspan

$$R_A = R_B = V_{max} = \frac{P}{2} = \frac{40 \text{ kips}}{2} = 20 \text{ kips}$$

$$M_{max} = \frac{PL}{4} = \frac{40 \text{ kips} \times 30 \text{ ft.}}{4} = 300 \text{ kip ft.}$$

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Solve **Step 4** as before. For **Step 5**, we can add the reaction forces for the two cases:

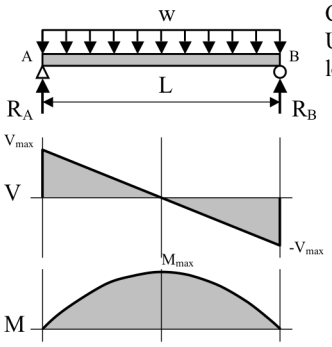
$$R_A = R_B = \frac{P}{2} + \frac{wL}{2} = \frac{40 \text{ kips}}{2} + \frac{0.06 \text{ kips}}{\text{ft.}} \frac{30 \text{ ft.}}{2} = 20.9 \text{ kips}$$

The maximum shear load occurs at the same place in both shear diagrams (the ends of the beams) and is equal to the reactions, so $V_{max} = R_A = R_B = 20.9 \text{ kips}$.

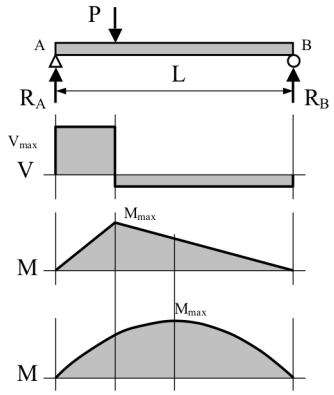
The maximum moment occurs at the same place in both moment diagrams (the midspan), so we can add the maximum moments for both cases:

$$M_{max} = \frac{PL}{4} + \frac{wL^2}{8} = \frac{40 \text{ kips} \times 30 \text{ ft.}}{4} + \frac{0.06 \text{ kips}}{\text{ft.}} \frac{(30 \text{ ft.})^2}{8}$$
$$= 306.75 \text{ kip ft.}$$

This use of the Formula Method and Superposition *only* works for shear loads when the maximum values of the two cases occur at the same location; likewise for maximum moment. For example, if the initial loading is a point load which is not at the midspan as in Case #6, then the maximum shear load $V_{max} = R_A$, but the maximum moment is not at the midspan. You can add the maximum shear loads of Cases #1 and #6 because they coincide, but you cannot add the maximum moments because they do not coincide.



Case #1
Uniform distributed load



Case #6
Point load at any point

Case #6
Max. moment is at the point load

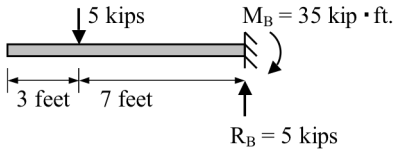
Case #1 (uniform distributed load)
Max. moment is at the midspan

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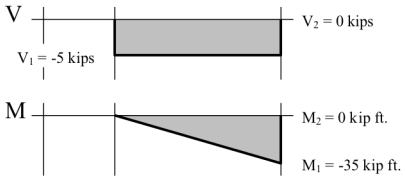
Example #4

Select the lightest W-beam that will support a point load of 5 kips 3 feet from the end of a 10-foot cantilever beam. The maximum deflection is 0.50 inches.

Step 1 $P = 5$ kips , $L = 10$ ft. , location of the point load $a = 3$ ft. , $\sigma_{YS} = 50$ ksi , $\Delta = 0.5$ in. .



Step 2 The total load on the beam is 5 kips, so the force reaction $R_B = P = 5$ kips . The point load is 7 feet from the wall, so the moment reaction $M_B = 5 \text{ kips} \times 7 \text{ ft.} = 35 \text{ kip ft.}$



Step 3 The shear diagram is a rectangle. The moment diagram is a triangle, where M_{max} is the area of the shear diagram: $M_{max} = -5 \text{ kips} \times 7 \text{ ft.} = -35 \text{ kip ft.}$

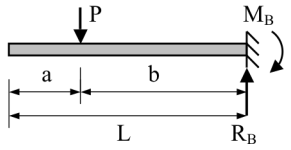
Step 4 Calculate the needed plastic section modulus:

$$\text{Required } Z_x = \frac{1.67M}{\sigma_{YS}} = \frac{1.67 \times 35 \text{ kip ft.}}{50 \text{ kips}} \frac{\text{in.}^2}{\text{ft.}} \frac{12 \text{ in.}}{\text{ft.}} = 14.0 \text{ in.}^3$$

W6×25 has a Z_x of 18.9 in.^{3}, which meets the requirements, but W12×16 is 36% lighter, with a weight of 16 lb./ft., or 0.016 kips/ft.

Beam	Z_x (in.^{3})	I_x (in.^{4})
W8×24	23.2	82.8
W12×16	20.1	103
W6×25	18.9	53.8
W10×12	12.6	53.8

We also need to calculate the required moment of inertia so that the beam does not deflect more than 0.50 inches. From Case #13 in Appendix H, $\Delta_{max} = \frac{Pb^2}{6EI}(3L - b)$. Recalculating, we get



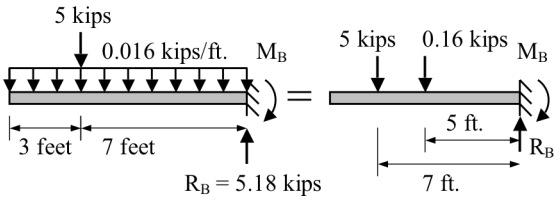
$$\text{Required } I = \frac{Pb^2}{6E\Delta_{max}}$$

$$= \frac{5 \text{ kips} (7 \text{ ft.})^2}{6} \frac{\text{in.}^2}{30 \times 10^3 \text{ kip}} \frac{}{0.5 \text{ in.}} \frac{(12 \text{ in.})^2}{\text{ft.}^2} = 0.39 \text{ in.}^4$$

The selected beam easily passes this requirement.

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Step 5 Redraw the load, shear, and moment diagrams to include the weight of the beam. Draw an Equivalent Load Diagram to find R_B and M_B. Add the beam weight to the point load for a total load of $R_B = 5 \text{ kips} + \frac{0.016 \text{ kip}}{\text{ft.}} \frac{10 \text{ ft.}}{} = 5.16 \text{ kips}$. The moment reaction $M_B = 5 \text{ kips} \times 7 \text{ ft.} + 0.16 \text{ kips} \times 5 \text{ ft.} = 35.8 \text{ kip ft.}$



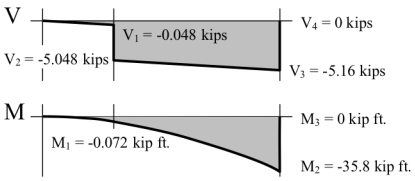
The shear diagram consists of a triangle and a trapezoid.

$$V_1 = \frac{-0.016 \text{ kip}}{\text{ft.}} \frac{3 \text{ ft.}}{} = -0.048 \text{ kips}$$

$$V_2 = V_1 - 5 \text{ kips} = -5.048 \text{ kips}$$

$$V_3 = V_2 - \frac{0.016 \text{ kip}}{\text{ft.}} \frac{7 \text{ ft.}}{} = -5.16 \text{ kips}$$

$$V_4 = V_3 + 5.16 \text{ kips} = 0 \text{ kips}$$



The moment diagram consists of two parabolas.

$$M_1 = \frac{-0.048 \text{ kip}}{2} \frac{3 \text{ ft.}}{} = -0.072 \text{ kip ft.}$$

$$M_2 = M_1 - \frac{5.048 \text{ kips} + 5.16 \text{ kips}}{2} \frac{7 \text{ ft.}}{} = -35.8 \text{ kip ft.}$$

$$M_3 = M_2 + 35.8 \text{ kip ft.} = 0 \text{ kip ft..}$$

The maximum moment is $M_B = 35.8 \text{ kip ft.}$.

$$\text{Required } Z_x = \frac{1.67 \times 35.8 \text{ kip ft.}}{} \frac{\text{in.}^2}{50 \text{ kips}} \frac{12 \text{ in.}}{\text{ft.}} = 14.3 \text{ in.}^3, \text{ which}$$

is less than Z_x of the selected beam, so the beam is strong enough in bending.

Step 6 Use $V_{applied} \le 0.4 \sigma_{YS} d t_w$ to find the shear strength. A W12×16 beam can support a shear load of $0.4 \times 50 \frac{\text{kips}}{\text{in.}^2} \times 11.99 \text{ in.} \times 0.220 \text{ in.} = 53 \text{ kips}$. Since the actual shear load of 5.16 kips is less than 53 kips, the beam will not fail in shear.

Symbols, Terminology, & Typical Units

Δ	Beam deflection	in.	mm
σ_{YS}	Yield strength	psi, ksi	MPa
a	Distance along a beam from the left end to the point load	ft., in.	m, mm
b	Distance along a beam from the point load to the right end	ft., in.	m, mm
d	Beam depth	in.	mm
E	Young's modulus (modulus of elasticity)	psi, ksi	GPa
I	Moment of inertia	ft.lb., ft.kips	kNm
L	Length	lb., kips	N, kN
M	Moment	lb./ft., kip/ft.	N/m, kN/m
P	Point load	lb., kips	N, kN
S	Section modulus	in.^3	mm^3
t_w	Web thickness (of a beam)	in.	mm
Z	Plastic section modulus	in.^3	mm^3