



Trigonometry

• **Angle** : The angle covered by the revolving line OP is $\theta = \angle POX$

$$1^\circ = 60' \text{ (minute)} ; 1' = 60'' \text{ (second)}$$

$$\Rightarrow 1 \text{ rad} = \frac{180^\circ}{\pi} \approx 57.3^\circ$$

$$\Rightarrow \text{Angle } \theta^\circ \text{ to Radian multiply it by } \frac{\pi}{180^\circ}$$

$$\Rightarrow \text{Angle Radian to } \theta^\circ \text{ multiplying it by } \frac{180^\circ}{\pi}$$

• **Trigonometrical Ratios** :

$$\sin \theta = \frac{P}{H} = \frac{MP}{OP} \quad \cos \theta = \frac{B}{H} = \frac{OM}{OP}$$

$$\tan \theta = \frac{P}{B} = \frac{MP}{OM} \quad \cot \theta = \frac{B}{P} = \frac{OM}{MP}$$

$$\sec \theta = \frac{H}{B} = \frac{OP}{OM} \quad \operatorname{cosec} \theta = \frac{H}{P} = \frac{OP}{MP}$$

$$\Rightarrow \sin^2 \theta + \cos^2 \theta = 1 ; 1 + \tan^2 \theta = \sec^2 ; 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

• **Table : Trigonometry Standard angles from 0° to 180°**

• **Four Quadrants and ASTC Rule** :

In 1st quadrant, all trigonometric ratios are positive.

$\Rightarrow$ In 2nd quadrant, only $\sin \theta$ and $\operatorname{cosec} \theta$ are positive.

$\Rightarrow$ In 3rd quadrant, only $\tan \theta$ and $\cot \theta$ are positive.

$\Rightarrow$ In 4th quadrant, only $\cos \theta$ and $\sec \theta$ are positive.

	90°	
II nd quadrant	sin	I st quadrant All
180°	tan	0°
III rd quadrant	cos	IV th quadrant
	270°	

• **Important trigonometric formula** :

Angle (θ)	0	30°	45°	60°	90°	120°	135°	150°	180°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	(not defined)	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

• **Range of trigonometric functions** :

$$\Rightarrow \sin \theta = \frac{P}{H} \text{ and } P \leq H$$

$$\text{So, } -1 \leq \sin \theta \leq 1$$

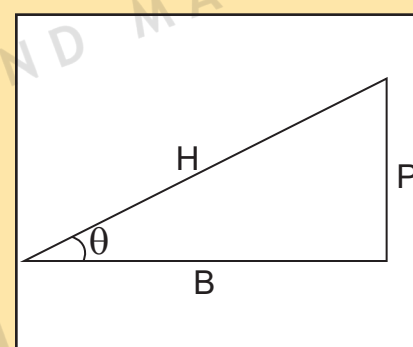
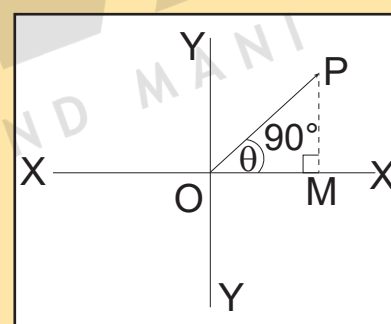
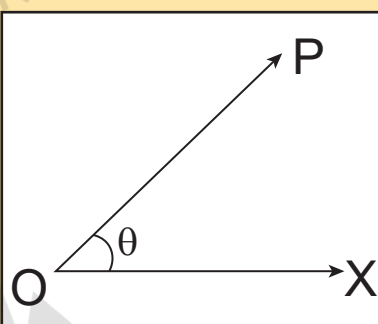
$$\Rightarrow \cos \theta = \frac{B}{H} \text{ and } B \leq H, \text{ So } -1 \leq \cos \theta \leq 1$$

$$\Rightarrow \tan \theta = \frac{P}{B} \text{ So, } -\infty < \tan \theta < \infty$$

• **Small Angle Approximation** :

If θ is small ($\theta < 45^\circ$)

$$\sin \theta \approx \theta ; \cos \approx 1 \text{ and } \tan \theta \approx \theta$$



BASIC MATHEMATICS

Coordinate Geometry

- **Origin** : ANY fixed point from which all measurements are taken from this.
- **Axis** : ANY fixed direction passing through origin.

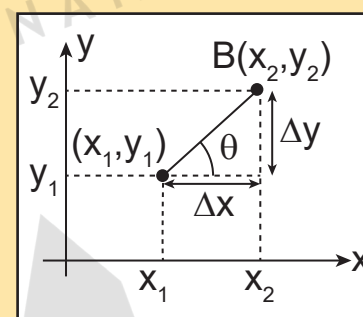
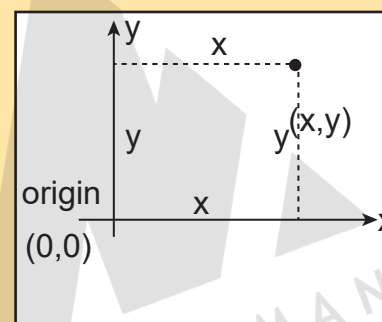
- **Distance Formula** :

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{In 3-d (space) } - d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- **Slope of a line** :

$$m = \tan \theta = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



• **Important Formulae of Differentiation** :

$$\Rightarrow \frac{d}{dx}(\sin x) = \cos x \Rightarrow \frac{d}{dx}(\cos x) = -\sin x \quad (\#) \frac{d}{dx}(\log_e x) = \frac{1}{x}$$

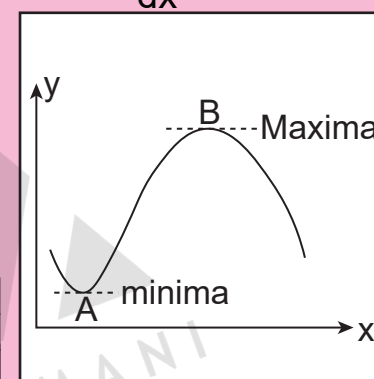
$$\Rightarrow \frac{d}{dx}(\tan x) = \sec^2 x \Rightarrow \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x \quad (\#) \frac{d}{dx}(e^x) = e^x$$

$$\Rightarrow \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x \quad (\#) \frac{d}{dx}(e^{ax}) = ae^{ax}$$

• **Concept of Maxima and Minima**

$$\Rightarrow \text{Condition for minima : } \left[\frac{dy}{dx} = 0 \text{ and } \frac{d^2y}{dx^2} > 0 \right]$$

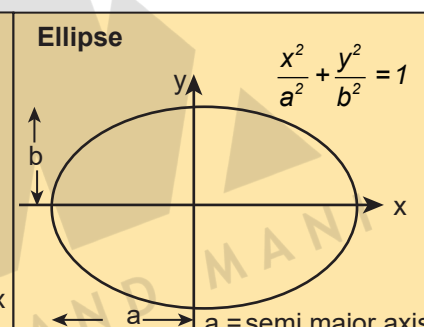
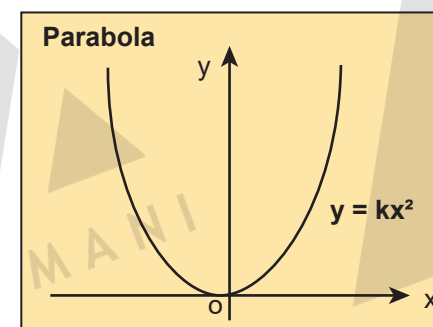
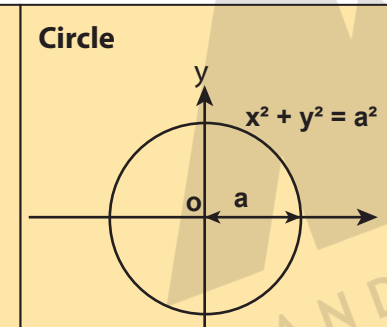
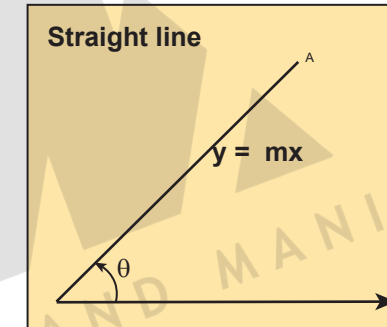
$$\Rightarrow \text{Condition for maxima : } \left[\frac{dy}{dx} = 0 \text{ and } \frac{d^2y}{dx^2} < 0 \right]$$



Algebra

• **Quadratic Equation and its Solutions** :
An algebraic equation of 2nd order is called a quadratic equation. Equation = $ax^2 + bx + c = 0$

$$\text{General Solution } \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



Differentiation

• **Physical Meaning of $\frac{dy}{dx}$**

(i) The ratio of small change in the function y and the variable x is called the average rate of change y w.r.t. x .

(ii) When $\Delta x \rightarrow 0$. The limiting value of $\frac{\Delta y}{\Delta x}$ is $\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$

• **Main Formulas of Differentiation**

$$1. \frac{d}{dx}(K) = 0 \quad K = \text{constant}$$

$$2. \frac{d}{dx}(KU) = K \frac{dU}{dx} \quad [U \text{ is a function of } x]$$

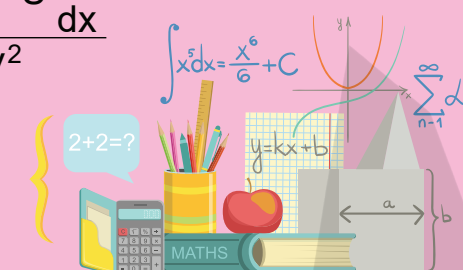
$$3. \frac{d}{dx}(U \pm V \pm W) = \frac{dU}{dx} \pm \frac{dV}{dx} \pm \frac{dW}{dx}$$

where U, V and W are functions of x .

$$4. \frac{d}{dx}(UV) = U \frac{dV}{dx} + V \frac{dU}{dx}$$

$$5. \frac{d}{dx}\left(\frac{U}{V}\right) = \frac{V \frac{dU}{dx} - U \frac{dV}{dx}}{V^2}$$

$$6. \frac{d}{dx}(x^n) = nx^{n-1}$$



$$\text{gives } x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow b^2 - 4ac \geq 0 \text{ For real roots.}$$

$$\Rightarrow b^2 - 4ac < 0 \text{ For Imaginary roots.}$$

• **Binomial Expression** :

An algebraic expression having two terms only. Example : $(a + b), (a + b)^4, (2x - 3y)^2$ etc.

• **Binomial Theorem** :

$$(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2 \times 1}a^{n-2}b^2 + \dots$$

Standard Graphs and their Equations

Integration

If I is the integration of $f(x)$ with respect to x then $I = \int f(x)dx$

• **Main Formulae of Integration** :

$$1. \int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$$

$$2. \int \sin x dx = -\cos x + c, c = \text{constant}$$

$$3. \int \cos x dx = \sin x + c$$

$$4. \int \frac{1}{x} dx = \log_e x + c$$

$$5. \int e^x dx = e^x + c$$

• **Define Integrals** :

$$\text{If } \frac{d}{dx}(f(x)) = f'(x) \text{ then } \int_a^b f'(x)dx = [f(x)]_a^b$$

is called definite integral.

• **Area Under Curve** :

$$\int_a^b f(x)dx = \text{Shaded area between curve and x-axis.}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2 \times 1} + \dots$$

• **Logarithm Main Formulae** :

$$\log mn = \log m + \log n \quad \log m^n = n \log m$$

$$\log \frac{m}{n} = \log m - \log n \quad \log_e m = 2.303 \log_{10} m$$

• **Arithmetic Progression (AP)** :

$$(AP) = a + a + d + a + 2d + \dots + a + (n-1)d$$

where a = first term d = common difference

$$\Rightarrow \text{Sum of } n \text{ term's } S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + n^{\text{th}} \text{ term}]$$

$$(i) \text{ Sum of first } n \text{ natural number's } - S_n = \frac{n(n+1)}{2}$$

$$(ii) \text{ Sum of squares of first } n \rightarrow S_{n^2} = \frac{n(n+1)(2n+1)}{6}$$

• **Geometric Progression (GP)**

$$(GP) = a, ar, ar^2, ar^3, \dots, ar^{n-1}; a = \text{first term}, r = \text{common ratio}$$

$$\text{Sum of } n \text{ terms } \rightarrow S_n = \frac{a(1-r^n)}{1-r}; \text{ For } 0 \leq |r| < 1$$

$$\text{Sum of } \infty \text{ terms } S_{\infty} = \frac{a}{1-r}$$