Assignment 2

1. Let A and B be square matrices of order n. Express the following in terms of $\det A$ and $\det B$

a) $\det(4A^tB^5)$ b) $\det(adjA)$

2. Given that $\begin{pmatrix} 1 & -2 & -3 \\ 2 & 1 & -2 \\ 0 & 2 & 0 \end{pmatrix}$ A $\begin{pmatrix} 1 & 0 & 3 \\ -2 & 1 & -8 \\ 3 & 0 & 7 \end{pmatrix} = 2I_3$, find det(A).

3. Find the rank of the following matrices

(a) $\begin{pmatrix} -1 & 0 \\ 2 & 5 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & -3 & 1 \\ -4 & 6 & -2 \end{pmatrix}$ (c) $\begin{pmatrix} -1 & 0 & -1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \\ 2 & 3 \\ 2 & 3 \end{pmatrix}$

4. Let $M = \begin{pmatrix} a & 0 & b & 2 \\ a & a & 4 & 4 \\ 0 & a & 2 & b \end{pmatrix}$ be the augmented matrix for a linear system. For what values of a and b the system will have: a) no solution? b) unique solution? c) infinitely many solutions?

5. Solve the following system of equations using either Gauss's Method or Cramer's Rule, whenever possible.

$$\begin{cases} x + y + 2z - w = -2 \\ 2x + y - 2z - 2w = -2 \\ 3x - 3w = -3 \end{cases}$$

8. Find the characteristic polynomial, eigenvalues and eigenvectors of

a) $A = \begin{pmatrix} -2 & 0 \\ 1 & -2 \end{pmatrix}$ b) $B = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ -2 & 0 & -1 \end{pmatrix}$