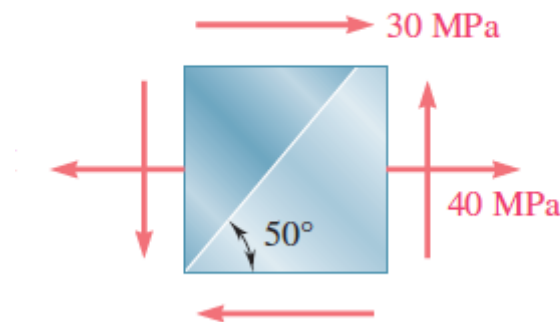


MECH 2040 Solid Mechanics I – Assignment 6 **Due on March 29, 2023**

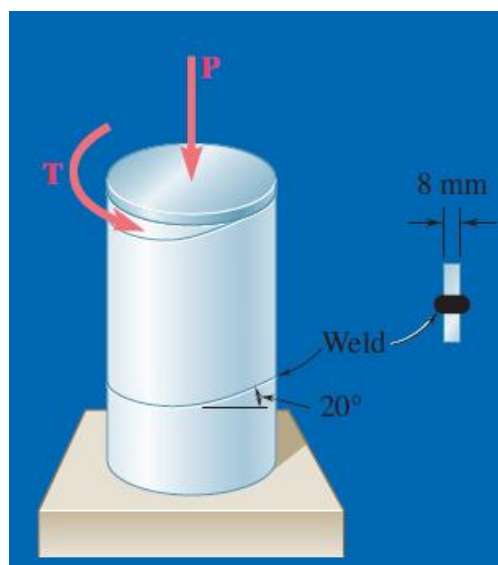
Question 1 (25 pts)

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element.

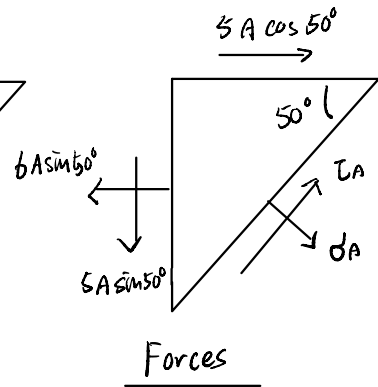
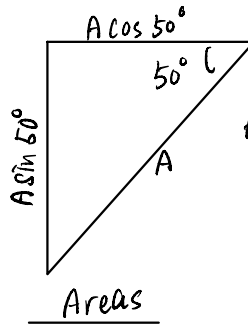
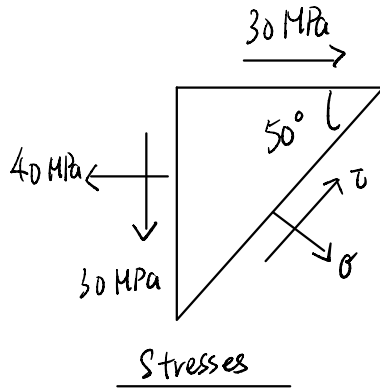


Question 2 (25 pts)

A steel pipe of 300-mm outer diameter is fabricated from 8-mm thick plate by welding along a helix that forms an angle of 20° with a plane perpendicular to the axis of the pipe. Knowing that a 250-kN axial force $\mathbf{P}$ and a 12-kN·m torque $\mathbf{T}$, each directed as shown, are applied to the pipe, determine the normal and in-plane shearing stresses in directions, respectively, normal and tangential to the weld.



Question 1



$$\uparrow \sum F = 0:$$

$$6A + 30A \cos 50^\circ \sin 50^\circ - 40A \sin 50^\circ \sin 50^\circ + 30A \sin 50^\circ \cos 50^\circ = 0$$

$$\begin{aligned} \sigma &= 40 \sin 50^\circ \sin 50^\circ - 30 \cos 50^\circ \sin 50^\circ - 30 \sin 50^\circ \cos 50^\circ \\ &= 2.753623 - (-7.595485) - (-7.595485) \\ &= 17.945 \text{ MPa}, \end{aligned}$$

$$\uparrow \sum F = 0$$

$$\tau A + 30A \cos 50^\circ \cos 50^\circ - 40A \sin 50^\circ \cos 50^\circ - 30A \sin 50^\circ \sin 50^\circ = 0$$

$$\begin{aligned} \tau &= 40 \sin 50^\circ \cos 50^\circ + 30 \sin 50^\circ \sin 50^\circ - 30 \cos 50^\circ \cos 50^\circ \\ &= -10.127313 + 2.065217 - 27.934783 \\ &= -35.997 \text{ MPa}, \end{aligned}$$

Question 2

$$d_2 = 0.3 \text{ m}$$

$$C_2 = \frac{1}{2} d_2 = 0.15 \text{ m}$$

$$t = 0.008 \text{ m}$$

$$C_1 = C_2 - t = 0.142$$

$$A = \pi (C_2^2 - C_1^2)$$

$$= \pi (0.15^2 - 0.142^2)$$

$$= 7.33876 \times 10^{-3} \text{ m}^2$$

$$J = \frac{\pi}{2} (C_2^4 - C_1^4)$$

$$= \frac{\pi}{2} (0.15^4 - 0.142^4)$$

$$= 1.5655 \times 10^{-4} \text{ m}^4$$

$$\sigma_y = -\frac{P}{A} = -\frac{250 \times 10^3}{7.33876 \times 10^{-3}}$$

$$= -34.066 \text{ MPa}$$

$$\tau_{xy} = \frac{\tau_{C_2}}{J} = \frac{12 \times 10^3 (0.15)}{1.5655 \times 10^{-4}}$$

$$= 11.498 \text{ MPa}$$

$$\sigma_x = 0, \sigma_y = -34.066 \text{ MPa}, \tau_{xy} = 11.498 \text{ MPa}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$= \frac{(-34.066)}{2} - \frac{(-(-34.066))}{2} \cos 2(20^\circ) - 11.498 \sin 2(20^\circ)$$

$$= -17.033 - 17.033 \cdot \cos(40^\circ) - 11.498 \sin(40^\circ)$$

$$= -17.033 - (-11.359956) - 8.567311$$

$$= -14.24 \text{ MPa}$$

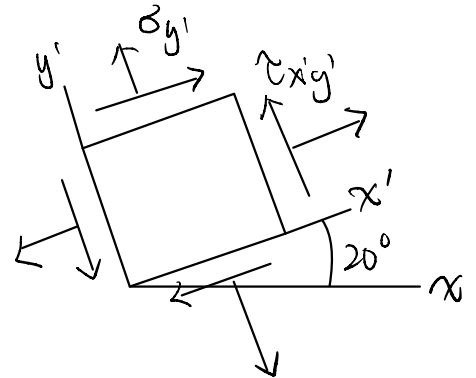
$$\tau_{xy'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= -\frac{(-(-34.066))}{2} \sin 40^\circ + 11.498 \cos 40^\circ$$

$$= -(17.033) \sin 40^\circ + (11.498) \cos 40^\circ$$

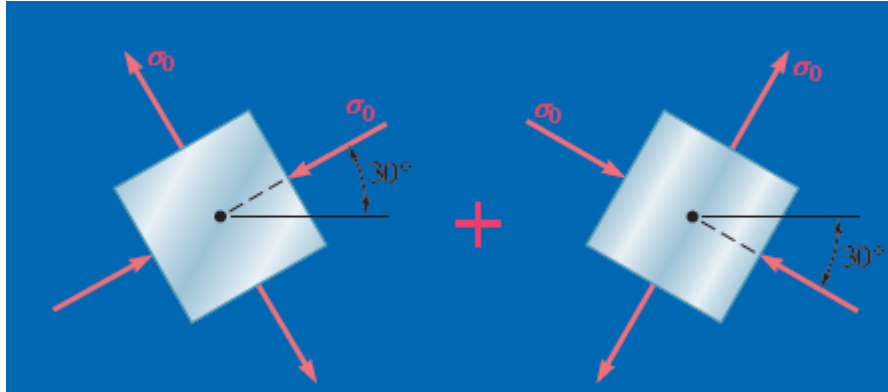
$$= -12.691512 + (-7.668454)$$

$$= -20.36 \text{ MPa}$$

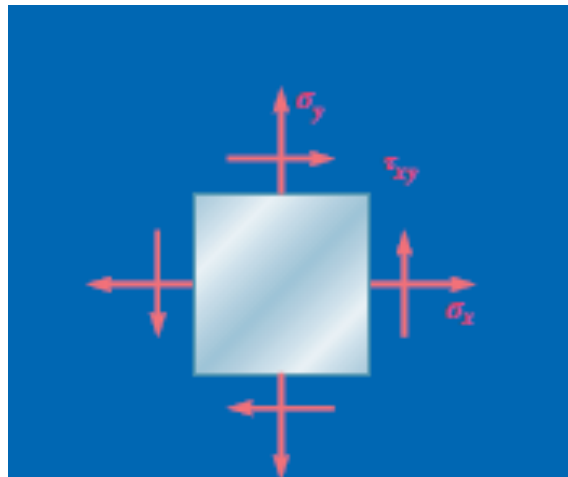


Question 3 (25 pts)

Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

**Question 4 (25 pts)**

For the state of stress shown, it is known that the normal and shearing stresses are directed as shown and that $\sigma_x = 14$ MPa, $\sigma_y = 9$ MPa, and $\sigma_{\min} = 5$ MPa. Determine (a) the orientation of the principal planes, (b) the principal stress $\sigma_{\max}$, (c) the maximum in-plane shearing stress.



Question 3

Let

$$\sigma_x = -\sigma_0$$

$$\sigma_y = \sigma_0$$

$$\tau_{xy} = 0$$

$$\theta = -30^\circ$$

$$\begin{aligned}\sigma_x &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{-\sigma_0 + \sigma_0}{2} + \frac{-\sigma_0 - \sigma_0}{2} \cos 2(-30) + 0 \cdot \sin 2(-30) \\ &= -\frac{2\sigma_0}{2} \cos 2(-30) \\ &= -0.5 \sigma_0 //\end{aligned}$$

$$\begin{aligned}\sigma_y &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{-\sigma_0 + \sigma_0}{2} - \frac{-\sigma_0 - \sigma_0}{2} \cos 2(-30) + 0 \cdot \sin 2(-30) \\ &= 0.5 \sigma_0 //\end{aligned}$$

$$\begin{aligned}\tau_{xy} &= -\left(\frac{\sigma_x - \sigma_y}{2} \sin 2\theta\right) + \tau_{xy} \cos 2\theta \\ &= -\frac{-\sigma_0 - \sigma_0}{2} \sin 2(-30) \\ &= -0.867 \sigma_0\end{aligned}$$

Right

$$\sigma_x = -\sigma_0$$

$$\sigma_y = \sigma_0$$

$$\tau_{xy} = 0$$

$$\theta = 30$$

$$\begin{aligned}\sigma_x &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{-\sigma_0 + \sigma_0}{2} + \frac{-\sigma_0 - \sigma_0}{2} \cos 2(30) + 0 \cdot \sin 2(30) \\ &= -\frac{2\sigma_0}{2} \cos 2(-30)\end{aligned}$$

$$= -0.5 \sigma_0 //$$

$$\begin{aligned}\sigma_y &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{-\sigma_0 + \sigma_0}{2} - \frac{-\sigma_0 - \sigma_0}{2} \cos 2(30) + 0 \cdot \sin 2(30) \\ &= 0.5 \sigma_0 //$$

$$\begin{aligned}\tau_{xy} &= -\left(\frac{\sigma_x - \sigma_y}{2} \sin 2\theta\right) + \tau_{xy} \cos 2\theta \\ &= -\frac{-\sigma_0 - \sigma_0}{2} \sin 2(30) \\ &= -0.867 \sigma_0 //\end{aligned}$$

Resulting stress :

$$\sigma_x = \sigma_x(L) + \sigma_x(R) = -0.5 \sigma_0 - 0.5 \sigma_0 = -\sigma_0$$

$$\sigma_y = \sigma_y(L) + \sigma_y(R) = 0.5 \sigma_0 + 0.5 \sigma_0 = \sigma_0$$

$$\tau_{xy} = \tau_{xy}(L) + \tau_{xy}(R) = -0.867 \sigma_0 + 0.867 \sigma_0 =$$

Superposition of principal stress:

$$\begin{aligned}\sigma_{1,2} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\frac{(\sigma_x - \sigma_y)^2}{4} + (\tau_{xy})^2} \\ &= \frac{-\sigma_0 + \sigma_0}{2} \pm \sqrt{\frac{(-\sigma_0 - \sigma_0)^2}{4} + 0^2} \\ &= \pm \sigma_0 //\end{aligned}$$

$$\sigma_1 = \sigma_0 \text{ (Max principal stress)}$$

$$\sigma_2 = -\sigma_0 \text{ (Min principal stress)}$$

$$\tau_{xy} = 0 + 0 = 0$$

Principal plane

$$\begin{aligned}\tan 2\theta &= \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \\ &= \frac{2(0)}{-\sigma_0 - \sigma_0} \\ &= 0\end{aligned}$$

$$\tan 2\theta = 0$$

$$\theta = 0^\circ \text{ and } 90^\circ \text{ (Min \& Max principal stress)}$$

Question 4

$$\sigma_{min} = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$5 = \frac{14+9}{2} - \sqrt{\left(\frac{14-9}{2}\right)^2 + (\tau_{xy})^2}$$

$$\sqrt{\left(\frac{14-9}{2}\right)^2 + \tau_{xy}^2} = \frac{14+9}{2} - 5$$

$$\begin{aligned}\tau_{xy} &= \sqrt{6.5^2 - \left(\frac{14-9}{2}\right)^2} \\ &= 6 \text{ MPa,}\end{aligned}$$

$$a) \tan 2\theta_{p1} = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}}$$

$$\tan 2\theta_{p1} = \frac{6}{\frac{14-9}{2}}$$

$$\tan 2\theta_{p1} = 2.4$$

$$2\theta_{p1} = 67.38^\circ$$

$$\theta_{p1} = 33.69^\circ,$$

$$\begin{aligned}\theta_{p2} &= \theta_{p1} + 90^\circ \\ &= 33.69^\circ + 90^\circ \\ &= 123.69^\circ,\end{aligned}$$

$$b) \sigma_{max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$= \frac{14+9}{2} + \sqrt{\left(\frac{14-9}{2}\right)^2 + 6^2}$$

$$= 18 \text{ MPa,}$$

$$c) \sigma_{in-plane} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$= \frac{18-5}{2}$$

$$= 6.5 \text{ MPa,}$$