

Discrete Mathematics

Assignment 1

Given: July 12, 2022

Due: July 22, 2022

1. Let $a \in \mathbb{Z}$ and let $b \in \mathbb{Z}$. If $n \nmid ab$ (n does not divide ab) then $n \nmid a$ and $n \nmid b$.
2. Prove that the following propositions are true.
 - (a) The sum of any rational number and any irrational number is irrational.
 - (b) $\sqrt{13}$ is irrational.

3. Find the mistake in the following proof.
Statement. All natural numbers are divisible by 3.

Proof. Suppose, for the sake of contradiction, the statement were false. Let X be the set of counterexamples, i.e., $X = \{x \in \mathbb{N} \mid x \text{ is not divisible by } 3\}$. The supposition that the statement is false means that $X \neq \emptyset$. Since X is a non-empty set of natural numbers, it contains a least element x .

Note that $0 \notin X$ because 0 is divisible by 3. So $x \neq 0$. Now consider $x - 3$. Since $x - 3 < x$, it is not a counterexample to the statement. Therefore $x - 3$ is divisible by 3; that is, there is an integer a such that $x - 3 = 3a$. So $x = 3a + 3 = 3(a + 1)$ and x is divisible by 3, contradicting $x \in X$.

4. Prove or disprove the following.

Among five integers there are always three with sum divisible by 3.

5. Let a, b, c be integers satisfying $a^2 + b^2 = c^2$. Prove that abc must be even.
6. Let $x_1, x_2, \dots, x_n$ be n real numbers. Let $\bar{x} = (x_1 + x_2 + \dots + x_n)/n$ be their average. Use a proof by contradiction to prove that at least one of $x_1, x_2, \dots, x_n$ is greater than or equal to $\bar{x}$.