

ADDIS ABABA UNIVERSITY
Department of Mathematics
Applied Mathematics I (Math 1041)

Worksheet 2 2021/22 AY

1. Let $A = \begin{bmatrix} -1 & 2 \\ 3 & 0 \\ 1 & 5 \end{bmatrix}$ and $\begin{bmatrix} 3 & -5 & 7 \\ 1 & 1 & -3 \end{bmatrix}$.

Find a) $2A - B^t$, $A^t + 3B$

b) $AB, (AB)^t, A^t B^t, B^t A^t$

2. (a) Find x, y, z and w if $3\begin{pmatrix} x & y \\ z & -w \end{pmatrix} = \begin{pmatrix} x & 6 \\ -1 & 2w \end{pmatrix} + \begin{pmatrix} 4 & x+y \\ z+w & 3 \end{pmatrix}$

(b) Let $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 5 & 0 \\ 0 & 2 & 4 \end{bmatrix}$. If $(2A - 3B)^t = 6I_3$, then find matrix B.

3. Find a matrix D such that $A = DB$ where $A = \begin{bmatrix} 1 & -1 \\ -2 & 3 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$.

4. Let $A = \begin{pmatrix} 4 & 5 & 3 & 7 \\ -1 & 3 & 2 & -1 \\ 2 & 3 & 2 & 1 \end{pmatrix}$. Find the reduced row echelon form of A.

5. Let $A = \begin{pmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{pmatrix}$. Then, find A^n for any positive integer n .

6. Let $A = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ -2 & -4 & -2 \end{bmatrix}$. Write A as a sum of symmetric and skew-symmetric matrices.

7. Let A and B be square matrices of order n . Express the following in terms of $\det A$ and $\det B$.

a) $\det(4A^t B^5)$

b) $\det(\text{adj}A)$

8. Given that $\begin{bmatrix} 1 & -2 & -3 \\ 2 & 1 & -2 \\ 0 & 2 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 4 \\ -2 & 1 & -8 \\ 3 & 0 & 7 \end{bmatrix} = 2I_3$, find $\det(A)$.

9. Find the determinant of the following matrices

a) $\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} 5 & -2 \\ 4 & -1 \end{pmatrix}$ b) $\begin{bmatrix} abc & a^2 & ac \\ bc & ac & c^2 \\ b^2 & ab & bc \end{bmatrix}$ c) $\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$ d) $\begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix}$

10. Find the rank of the following matrices

a) $\begin{pmatrix} -1 & 0 \\ 2 & 5 \end{pmatrix}$ b) $\begin{pmatrix} 2 & -3 & 1 \\ -4 & 6 & -2 \end{pmatrix}$ c) $\begin{pmatrix} -1 & 0 & -1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$ d) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \\ 2 & 3 \\ 2 & -1 \end{pmatrix}$

11. If it exists, find the inverse of the following matrices.

a) $\begin{pmatrix} 2 & 4 \\ 6 & -1 \end{pmatrix}$ b) $\begin{pmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & 3 \end{pmatrix}$ c) $\begin{pmatrix} 4 & -2 & 5 \\ 1 & 1 & -2 \\ 6 & 1 & -1 \end{pmatrix}$

12. Find a 3×1 matrix X with entries not all zero such that

$$AX = 3X, \text{ where } A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{bmatrix}.$$

13. Let $A = \begin{bmatrix} 2 & 2 & t \\ 0 & -t & -3 \\ t & 0 & -1 \end{bmatrix}$. Find the set of values of t for which the homogeneous system of linear equation $AX = 0$ has non trivial solution.

14. Let $M = \begin{bmatrix} a & 0 & b & 2 \\ a & a & 4 & 4 \\ 0 & a & 2 & b \end{bmatrix}$ be the augmented matrix for a linear system. For what values of a and b the system will have:

a) no solution? b) unique solution? c) infinitely many solutions?

15. Solve the following system of equations using either Gauss's Method or Cramer's Rule, whenever possible.

a) $\begin{cases} 3x - 7y = 2 \\ 2x - 5y = -1 \end{cases}$ b) $\begin{cases} 4x + 6y + z = 2 \\ 2x + y - 4z = 3 \\ 3x - 2y + 5z = 8 \end{cases}$ c) $\begin{cases} 3x + 3y + 4z = 3 \\ 4x + 6y = 2 \\ 8x + z = 2 \end{cases}$

d) $\begin{cases} 3x - 9y = 1 \\ -2x + 6y = -3 \end{cases}$ e) $\begin{cases} 2x + 3y + z = 1 \\ x + 4y - z = 0 \end{cases}$ f) $\begin{cases} x + y + 2z - w = -2 \\ 2x + y - 2z - 2w = -2 \\ x - w = -1 \end{cases}$

16. Find the characteristic polynomial, eigenvalues and eigenvectors of

a) $A = \begin{pmatrix} -2 & 0 \\ 1 & -2 \end{pmatrix}$ b) $A = \begin{pmatrix} 3 & 4 \\ 1 & 3 \end{pmatrix}$ c) $B = \begin{pmatrix} 0 & -2 \\ 1 & 3 \end{pmatrix}$ d) $B = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ -2 & 0 & -1 \end{pmatrix}$

17. Find the determinant, adjoint and inverse (if exists) of each of the following matrices

a) $\begin{bmatrix} -1 & 3 \\ 8 & 5 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ c) $\begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$