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THIRD EDITON

APPLIED THERMODYNAMICS



Onkar Singh



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APPLIED THERMODYNAMICS

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APPLIED THERMODYNAMICS

(THIRD EDITON)

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Preface to the Third Edition

I feel extremely encouraged at the good response to this textbook. Looking upon the feed back received from its readers third edition of the book is being presented here.

In this edition number of solved and unsolved problems have been added in some of the chapters and a few new topics have also been added.

I wish to express my sincere thanks to Professors and students for their valuable suggestions and recommending the book to their students and friends.

I strongly feel that the book would prove to be further useful to students. I would be obliged for the errors, omissions and suggestions brought to my notice for improvement of the book in its next edition.

Onkar Singh

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Preface to the First Edition

During teaching of the course of engineering thermodynamics and applied thermodynamics I have felt that the students at the undergraduate level of engineering and technology face difficulty in understanding the concepts of engineering thermodynamics and their applications in the course of applied thermodynamics. Also, the students face great difficulty in referring to the number of text-books for different topics. The present book is an effort in the direction of presenting the concepts of engineering thermodynamics and their applications in clear, concise and systematic manner at one place. Presentation is made in very simple and easily understandable language and well supported with wide ranging illustrations and numerical problems.

The subject matter in this book covers the syllabus of the basic and advanced course on engineering thermodynamics/thermal engineering being taught in different institutions and universities across the country. There are total 18 chapters in this book. The initial seven chapters cover the basic course on engineering thermodynamics and remaining chapters cover the advanced course in thermal engineering. These deal with "Fundamental concepts and definitions", "Zeroth law and thermodynamics", "First law of thermodynamics", "Second law of thermodynamics", "Entropy", "Availability and general thermodynamic relations", "Thermodynamic properties of pure substances", "Fuels and combustion", "Boilers and boiler calculations", "Vapour power cycles", "Gas power cycles", "Steam engines", "Nozzles", "Steam turbines, Steam condenser", "Reciprocating and rotatory compressors", "Introduction to internal combustion engines" and "Introduction to refrigeration and air conditioning". Each chapter has been provided with sufficient number of typical numerical problems of solved and unsolved type. The book is written in SI system of units and the various tables such as steam tables, refrigeration tables, Mollier chart, psychrometry chart etc. are also provided at the end of the book for quick reference. I hope that the students and teachers referring to this book will find it useful.

I am highly indebted to my family members for their continuous encouragement and cooperation during the preparation of manuscript. I would like to place on record my gratitude and apologies to my wife Parvin and kids Sneha and Prateek who patiently endured certain neglect and hardships due to my preoccupation with the preparation of this manuscript.

I am thankful to AICTE, New Delhi for the financial support provided to me in the Young Teacher Career Award.

I am also thankful to Mr. L.N. Mishra and other staff members of New Age International for their cooperation throughout the preparation of the textbook. At the end I thank to all those who supported directly or indirectly in the preparation of this book.

I shall be extremely grateful to all the readers of text book for their constructive criticism, indicating any errors and omissions etc. for improving its quality and form.

Onkar Singh

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1

Fundamental Concepts and Definitions

1.1 INTRODUCTION AND DEFINITION OF THERMODYNAMICS

Thermodynamics is a branch of science which deals with energy. Engineering thermodynamics is modified name of this science when applied to design and analysis of various energy conversion systems. Thermodynamics has basically a few fundamental laws and principles applied to a wide range of problems. Thermodynamics is core to engineering and allows understanding of the mechanism of energy conversion. It is really very difficult to identify any area where there is no interaction in terms of energy and matter. It is a science having its relevance in every walk of life. Thermodynamics can be classified as 'Classical thermodynamics' and 'Statistical thermodynamics'. Here in engineering systems analysis the classical thermodynamics is employed.

"Thermodynamics is the branch of physical science that deals with the various phenomena of energy and related properties of matter, especially of the laws of transformations of heat into other forms of energy and vice-versa."

Internal combustion engines employed in automobiles are a good example of the energy conversion equipments where fuel is being burnt inside the piston cylinder arrangement and chemical energy liberated by the fuel is used for getting the shaft work from crankshaft. Thermodynamics lets one know the answer for the questions as, what shall be the amount of work available from engine?, what shall be the efficiency of engine?, etc.

For analysing any system there are basically two approaches available in engineering thermodynamics. Approach of thermodynamic analysis means how the analyser considers the system. Macroscopic approach is the one in which complete system as a whole is considered and studied without caring for what is there constituting the system at microscopic level.

Contrary to this the microscopic approach is one having fragmented the system under consideration upto microscopic level and analysing the different constituent subsystems/microsystems. In this approach study is made at the microscopic level. For studying the system the microlevel studies are put together to see the influences on overall system. Thus, the statistical techniques are used for integrating the studies made at microscopic level. This is how the studies are taken up in statistical thermodynamics. In general it can be said that, Macroscopic approach analysis = $\sum$ (Microscopic approach analysis).

1.2 DIMENSIONS AND UNITS

"Dimension" refers to certain fundamental physical concepts that are involved in the process of nature and are more or less directly evident to our physical senses, thus dimension is used for characterizing

any physical quantity. Dimensions can be broadly classified as “primary dimensions” and “secondary or derived dimensions”. “Basic dimensions such as mass ‘M’, length ‘L’, time ‘t’ and temperature ‘T’ are called primary dimensions, while quantities which are described using primary dimensions are called secondary dimensions such as for energy, velocity, force, volume, etc”.

“Units” are the magnitudes assigned to the dimensions. Units assigned to “primary dimensions” are called “basic units” whereas units assigned to “secondary dimensions” are called “derived units”. Various systems of units have prevailed in the past such as FPS (Foot-Pound-Second), CGS (Centimetre-Gram-Second), MKS (Metre-Kilogram-Second) etc. but at present SI system (System-International) of units has been accepted world wide. Here in the present text also SI system of units has been used. Following table gives the basic and derived units in SI system of units.

Table 1.1 *SI system of units*

Quantity	*Unit*	*Symbol*
Basic Units		
Length (L)	Metre	m
Mass (M)	Kilogram	kg
Time (t)	Second	s
Temperature (T)	Kelvin	K
Plane angle	Radian	rad
Solid angle	Steradian	sr
Luminous intensity	Candela	cd
Molecular substance	Mole	mol.
Electric Current	Ampere	A
Derived Units		
Force (F)	Newton	N $\{kg.m/s^2\}$
Energy (E)	Joule	J $\{N.m = kg.\ m^2/s^2\}$
Power	Watt	W $\{J/s = kg.\ m^2/s^3\}$
Pressure	Pascal	Pa $\{N/m^2 = kg/(ms^2)\}$

Equivalence amongst the various systems of unit for basic units is shown in table 1.2.

Table 1. 2 *Various systems of units*

Unit - (Symbol)

Quantity	*SI*	*MKS*	*CGS*	*FPS*
Length	Metre (m)	Metre (m)	Centimetre (cm)	Foot (ft)
Mass	Kilogram (kg)	Kilogram (kg)	Gram (gm)	Pound (lb)
Time	Second (s)	Second (s)	Second (s)	Second (s)
Temperature	Kelvin (K)	Centigrade (^oC)	Centigrade (^oC)	Fahrenheit (^oF)

The various prefixes used with SI units are given as under :

Prefix	*Factor*	*Symbol*	*Prefix*	*Factor*	*Symbol*
deca	10	da	deci	10^{-1}	d
hecto	10^2	h	centi	10^{-2}	c
kilo	10^3	k	milli	10^{-3}	m
mega	10^6	M	micro	10^{-6}	μ
giga	10^9	G	nano	10^{-9}	n
tera	10^{12}	T	pico	10^{-12}	p
peta	10^{15}	P	femto	10^{-15}	f
exa	10^{18}	E	atto	10^{-18}	a

The conversion table for one unit into the other is given in table 1.3.

Table 1.3 *Unit conversion table*

1 ft = 0.3048 m	1 ft^2 = 0.09290 m^2
1 in = 0.0254 m	1 in^2 = 6.45 cm^2
1 lb = 453.6 gm	1 lb = 0.4536 kg
1 lbf = 4.45 N	1 kgf = 9.81 N
1 lbf/in^2 = 6.989 kN/m^2 = 0.0689 bar =	703 kgf/m^2
1 bar = 10^5 N/m^2 = 14.5038 $1bf/in^2$ =	0.9869 atm
= 1.0197 kgf/cm^2	
1 ft. lbf = 1.356 Joules	
1 Btu = 778.16 ft. lbf = 1.055 kJ	
1Btu/lb = 2.326 kJ/kg	
1 ft^3/lb = 0.0624 m^3/kg,	1 Cal = 4.18 J

1.3 CONCEPT OF CONTINUUM

In Macroscopic approach of thermodynamics the substance is considered to be continuous whereas every matter actually comprises of myriads of molecules with intermolecular spacing amongst them. For analyzing a substance in aggregate it shall be desired to use laws of motion for individual molecules and study at molecular level be put together statistically to get the influence upon aggregate. In statistical thermodynamics this microscopic approach is followed, although it is often too cumbersome for practical calculations.

In engineering thermodynamics where focus lies upon the gross behaviour of the system and substance in it, the statistical approach is to be kept aside and classical thermodynamics approach be followed. In classical thermodynamics, for analysis the atomic structure of substance is considered to be continuous. For facilitating the analysis this concept of continuum is used in which the substance is treated free from any kind of discontinuity. As this is an assumed state of continuum in substance so the order of analysis or scale of analysis becomes very important. Thus, in case the scale of analysis is large enough and the discontinuities are of the order of intermolecular spacing or mean free path then due to relative order of discontinuity being negligible it may be treated continuous.

In the situations when scale of analysis is too small such that even the intermolecular spacing or mean free path are not negligible i.e. the mean free path is of comparable size with smallest significant dimension in analysis then it can not be considered continuous and the microscopic approach for analysis should be followed. For example, whenever one deals with highly rarefied gases such as in rocket flight at very high altitudes or electron tubes, the concept of continuum of classical thermodynamics

should be dropped and statistical thermodynamics using microscopic approach should be followed. Thus, in general it can be said that the assumption of continuum is well suited for macroscopic approach where discontinuity at molecular level can be easily ignored as the scale of analysis is quite large. The concept of continuum is thus a convenient fiction which remains valid for most of engineering problems where only macroscopic or phenomenological informations are desired.

For example, let us see density at a point as a property of continuum. Let us take some mass of fluid Δm in some volume ΔV enveloping a point '*P*' in the continuous fluid. Average mass density of fluid within volume ΔV shall be the ratio $(\Delta m/\Delta V)$. Now let us shrink the volume ΔV enveloping the point to volume $\Delta V'$. It could be seen that upon reducing the volume, $\Delta V'$ may be so small as to contain relatively few molecules which may also keep on moving in and out of the considered very small volume, thus average density keeps on fluctuating with time. For such a situation the definite value of density can not be given.

Therefore, we may consider some limiting volume ΔV_{limit} such that the fluid around the point may be treated continuous and the average density at the point may be given by the ratio $\left(\frac{\Delta m}{\Delta V_{\text{limit}}}\right)$. Thus, it shows how the concept of continuum although fictitious is used for defining density at a point as given below,

$$\text{Average density at the point} = \lim_{\Delta V \to \Delta V_{\text{limit}}} \left(\frac{\Delta m}{\Delta V}\right)$$

1.4 SYSTEMS, SURROUNDINGS AND UNIVERSE

In thermodynamics the 'system' is defined as the quantity of matter or region in space upon which the attention is concentrated for the sake of analysis. These systems are also referred to as thermodynamic systems. For the study these systems are to be clearly defined using a real or hypothetical boundary. Every thing outside this real/hypothetical boundary is termed as the 'surroundings'. Thus, the surroundings may be defined as every thing surrounding the system. System and surroundings when put together result in universe.

$$\text{Universe} = \text{System} + \text{Surroundings}$$

The system is also some times defined as the control system and the boundary defined for separating it from surroundings is called control boundary, the volume enclosed within the boundary is control volume and the space enclosed within the boundary is called control space.

Based on the energy and mass interactions of the systems with surroundings/other systems across the boundary the system can be further classified as the open, close, and isolated system. The 'open system' is one in which the energy and mass interactions take place at the system boundary, for example automobile engine etc.

'Closed system' is the system having only energy interactions at its boundary, for example, boiling water in a closed pan etc. The mass interactions in such system are absent. 'Isolated system' refers to the system which neither has mass interaction nor energy interaction across system boundary, for example Thermos Flask etc. Thus, the isolated system does not interact with the surroundings/ systems in any way.

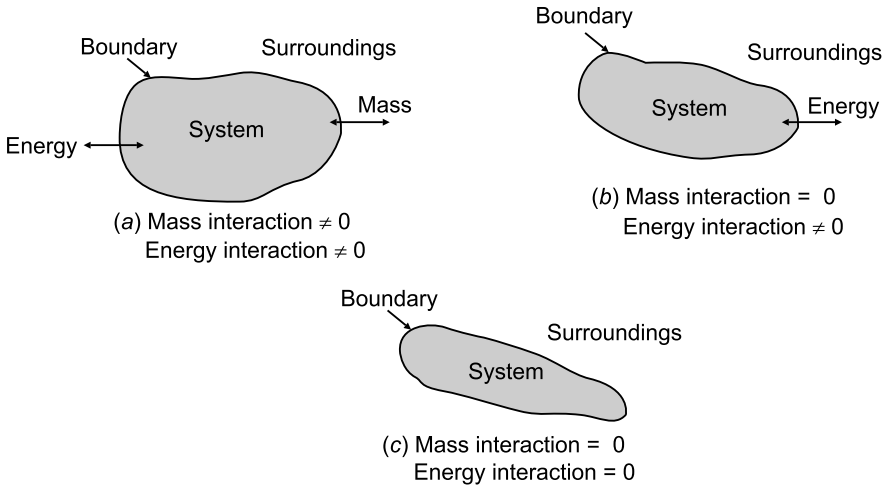


Fig. 1.1 *(a) Open system (b) Closed system (c) Isolated system*

1.5 PROPERTIES AND STATE

For defining any system certain parameters are needed. 'Properties' are those observable characteristics of the system which can be used for defining it. Thermodynamic properties are observable characteristics of the thermodynamic system. Pressure, temperature, volume, viscosity, modulus of elasticity etc. are the examples of property. These properties are some times observable directly and some times indirectly. Properties can be further classified as the 'intensive property' and 'extensive property'. The *intensive properties* are those properties which have same value for any part of the system or the properties that are independent of the mass of system are called intensive properties, e.g. pressure, temperature etc. *Extensive properties* on the other hand are those which depend upon the mass of system and do not maintain the same value for any path of the system. e.g. mass, volume, energy, enthalpy etc. These extensive properties when estimated on the unit mass basis result in intensive property which is also known as *specific property*, e.g. specific heat, specific volume, specific enthalpy etc.

'State' of a system indicates the specific condition of the system. To know the characteristics of the system quantitatively refers to knowing the state of system. Thus, when the properties of system are quantitatively defined then it refers to the 'state'. For completely specifying the state of a system number of properties may be required which depends upon the complexity of the system. Thermodynamic state in the same way refers to the quantitative definition of the thermodynamic properties of a thermodynamic system e.g. for defining a gas inside the cylinder one may have to define the state using pressure and temperature as 12 bar, 298 K. When the thermodynamic properties defining a state undergo a change in their values it is said to be the 'change of state'.

1.6 THERMODYNAMIC PATH, PROCESS AND CYCLE

Thermodynamic system undergoes changes due to the energy and mass interactions. Thermodynamic state of the system changes due to these interactions. The mode in which the change of state of a system takes place is termed as the *process* such as constant pressure process, constant volume process etc. Let us take gas contained in a cylinder and being heated up. The heating of gas in the cylinder shall result in change in state of gas as its pressure, temperature etc. shall increase. However, the mode in

which this change of state in gas takes place during heating shall be constant volume mode and hence the process shall be called constant volume heating process.

The *path* refers to the series of state changes through which the system passes during a process. Thus, path refers to the locii of various intermediate states passed through by a system during a process.

Cycle refers to a typical sequence of processes in such a fashion that the initial and final states are identical. Thus, a cycle is the one in which the processes occur one after the other so as to finally bring the system at the same state. Thermodynamic path in a cycle is in closed loop form. After the occurrence of a cyclic process system shall show no sign of the processes having occurred.

Mathematically, it can be said that the cyclic integral of any property in a cycle is zero, i.e.,

$\oint dp = 0$, where p is any thermodynamic property.

Thermodynamic processes, path and cycle are shown on *p-v* diagram in Fig. 1.2

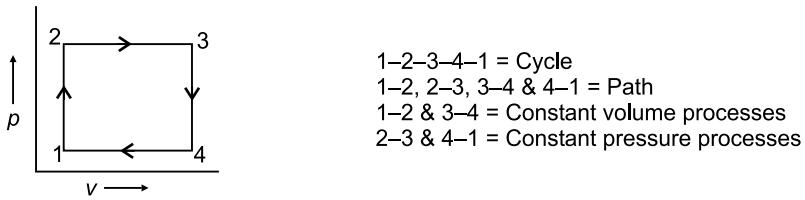


Fig. 1.2 *Thermodynamic process, path and cycle.*

1.7 THERMODYNAMIC EQUILIBRIUM

Equilibrium of a system refers to the situation in which it's "state" does not undergo any change in itself with passage of time without the aid of any external agent. Equilibrium state of a system can be examined by observing whether the change in state of the system occurs or not. If no change in state of system occurs then the system can be said in equilibrium. Thermodynamic equilibrium is a situation in which thermodynamic system does not undergo any change in its state. Let us consider a steel glass full of hot milk kept in open atmosphere. It is quite obvious that the heat from milk shall be continuously transferred to atmosphere till the temperature of milk, glass and atmosphere are not alike. During the transfer of heat from milk the temperature of milk could be seen to decrease continually. Temperature attains some final value and does not change any more. This is the equilibrium state at which the properties stop showing any change in themselves.

Generally, thermodynamic equilibrium of a system may be ensured by ensuring the mechanical, thermal, chemical and electrical equilibriums of the system. 'Mechanical equilibrium' of the system can be well understood from the principles of applied mechanics which say that the net force and moment shall be zero in case of such equilibrium. Thus, in the state of mechanical equilibrium the system does not have any tendency to change mechanical state as it is the state at which the applied forces and developed stresses are fully balanced.

'Thermal equilibrium' is that equilibrium which can be stated to be achieved if there is absence of any heat interactions. Thus, if the temperature states of the system do not change then thermal equilibrium is said to be attained. Equality of temperature of the two systems interacting with each other shall ensure thermal equilibrium.

'Chemical equilibrium' is the one which can be realized if the chemical potential of the systems interacting are same. The equality of forward rate of chemical reaction and backward rate of chemical reaction can be taken as criterion for ensuring the chemical equilibrium. Similar to this, in case the electrical potential of the systems interacting are same, the 'electrical equilibrium' is said be attained.

Thus, a system can be said to be in thermodynamic equilibrium if it is in mechanical, thermal, chemical and electrical equilibrium.

1.8 REVERSIBILITY AND IRREVERSIBILITY

Thermodynamic processes may have the change of state occuring in two ways. One is the change of state occuring so that if the system is to restore its original state, it can be had by reversing the factors responsible for occurrence of the process. Other change of state may occur such that the above restoration of original state is not possible. Thermodynamic system that is capable of restoring its original state by reversing the factors responsible for occurrence of the process is called reversible system and the thermodynamic process involved is called *reversible process.* Thus, upon reversal of a process there shall be no trace of the process being ocurred, i.e. state changes during the forward direction of occurrence of a process are exactly similar to the states passed through by the system during the reversed direction of the process. It is quite obvious that the such reversibility can be realised only if the system maintains its thermodynamic equilibrium throughout the occurrence of process.

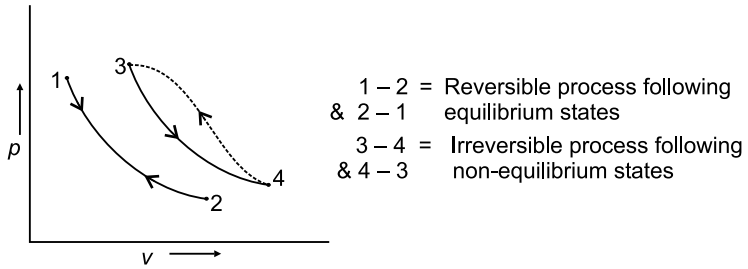


Fig. 1.3 *Reversible and Irreversible processes*

The *irreversibility* is the characteristics of the system which forbids system from retracing the same path upon reversal of the factors causing the state change. Thus, irreversible systems are those which do not maintain equilibrium during the occurrence of a process. Various factors responsible for the nonattainment of equilibrium are generally the reasons responsible for irreversibility. Presence of friction, dissipative effects etc. have been identified as a few of the prominent reasons for irreversibility. The reversible and irreversible processes are shown on *p-v* diagram in Fig. 1.3 by '1–2 and 2–1' and '3–4 and 4–3' respectively.

1.9 QUASI-STATIC PROCESS

Thermodynamic equilibrium of a system is very difficult to be realised during the occurrence of a thermodynamic process. It may be understood that this kind of equilibrium is rather practically impossible. In case such equilibrium could not be attained then the thermodynamic analysis cannot be done, as the exact analysis of a system not in equilibrium is impossible. 'Quasi-static' consideration is one of the ways to consider the real system as if it is behaving in thermodynamic equilibrium and thus permitting the thermodynamic study. Actually, system does not attain thermodynamic equilibrium only certain assumptions make it akin to a system in equilibrium, for the sake of study and analysis.

Quasi-static literally refers to "almost static" and the infinite slowness of the occurrence of a process is considered as the basic premise for attaining near equilibrium in the system. Here it is considered that the change in state of a system occurs at infinitely slow pace, thus consuming very large time for completion of the process. During the dead slow rate of state change the magnitude of change in a state

shall also be infinitely small. This infinitely small change in state when repeatedly undertaken one after the other consecutively, results in overall state change. Quasi-static process is considered to remain in thermodynamic equilibrium just because of infinitesimal state changes taking place during the occurrence of the process. Quasi static process can be understood from the following example.

Let us consider the heating of gas in a container with certain mass '*W*' kept on the top of lid (lid is such that it does not permit leakage across its interface with vessel wall) of the vessel as shown in Fig. 1.4. After certain amount of heat being added to the gas it is found that the lid gets raised up. Thermodynamic state change is shown in figure. The "change in state" is significant. During the change of state since the states could not be considered to be in equilibrium, hence for unsteady state of system, thermodynamic analysis could not be extended. Let us now assume that the total mass comprises of infinitesimal small masses of '*w*' such that all '*w*' masses put together become equal to *W.* Now let us start heat addition to vessel and as soon as the lifting of lid is observed put first fraction mass '*w*' over the lid so as to counter the lifting and estimate the state change. During this process it is found that the state change is negligible. Let us further add heat to the vessel and again put the second fraction mass '*w*' as soon as the lift is felt so as to counter it. Again the state change is seen to be negligible. Continue with the above process and at the end it shall be seen that all fraction masses '*w*' have been put over the lid, thus

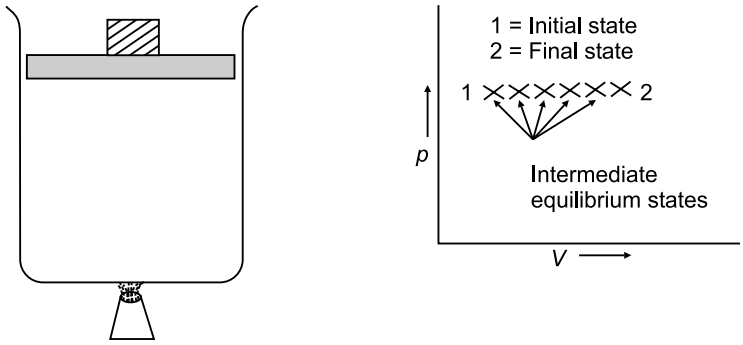


Fig. 1.4 *Quasi static process*

amounting to mass '*W*' kept over the lid of vessel and the state change occurred is exactly similar to the one which occurred when the mass kept over the lid was '*W*'. In this way the equilibrium nature of system can be maintained and the thermodynamic analysis can be carried out. *p-v* representation for the series of infinitesimal state changes occuring between states 1 and 2 is shown in Fig. 1.4.

1.10 SOME THERMODYNAMIC PROPERTIES

Pressure, temperature, density, volume etc. are some of the thermodynamic properties frequently used. Pressure is defined as the force per unit area. Mathematically, it can be given by the ratio of force applied on a area (say *F*) divided by the area (say *A*) as ;

$$p = F/A, \ (\text{N/m}^2).$$

In general during analysis one comes across the following four types of pressure,

(*i*) Atmospheric pressure (*ii*) Absolute pressure

(*iii*) Gauge pressure (*iv*) Vacuum pressure

Atmospheric pressure is defined as the pressure exerted by the atmosphere. It is said to be equal to 760 mm of mercury column at 0ºC for mercury density of 0.0135951 kg/cm^3, gravitational acceleration of 9.80665 m/s^2 and has magnitude of 1.013 bar (= 1.013×10^5 N/m^2). The instrument used for

measuring this pressure is called *barometer.* Italian scientist Torricelli was first to construct the barometer to measure the pressure. In his honour the pressure exerted by one millimeter column of mercury under atmospheric conditions is known as 'Torr' (1 atm = 760 Torr).

Absolute pressure of gas refers to the actual pressure of the gas. Let us consider a U-tube manometer as shown in Fig. 1.5. It shows the manometer with its one limb connected to bulb containing the gas while other limb is open to atmosphere. Fig. 1.5*a* describes a special case in which the pressure of the gas is more than the atmospheric pressure and it is the reason for the rise in level of mercury in the open limb. The difference in the pressure of fluid and atmosphere which is measurable by the rise of mercury column (= *h.d.g.* where *h* is the rise in mercury column, *d* is the density of mercury, *g* is the gravitational acceleration) is known as the *Gauge pressure.* Mathematically, it can be shown that,

Absolute pressure = Atmospheric pressure + Gauge pressure

Figure 1.5*b* shows another typical case in which the pressure of gas is less than the atmospheric pressure and that is why the mercury column is depressed in the open limb. The amount by which the pressure of gas is less than the atmospheric pressure is called *Vacuum pressure.* Thus, the vacuum pressure is the negative gauge pressure. Mathematically it can be shown by,

Absolute pressure = Atmospheric pressure – Vacuum pressure

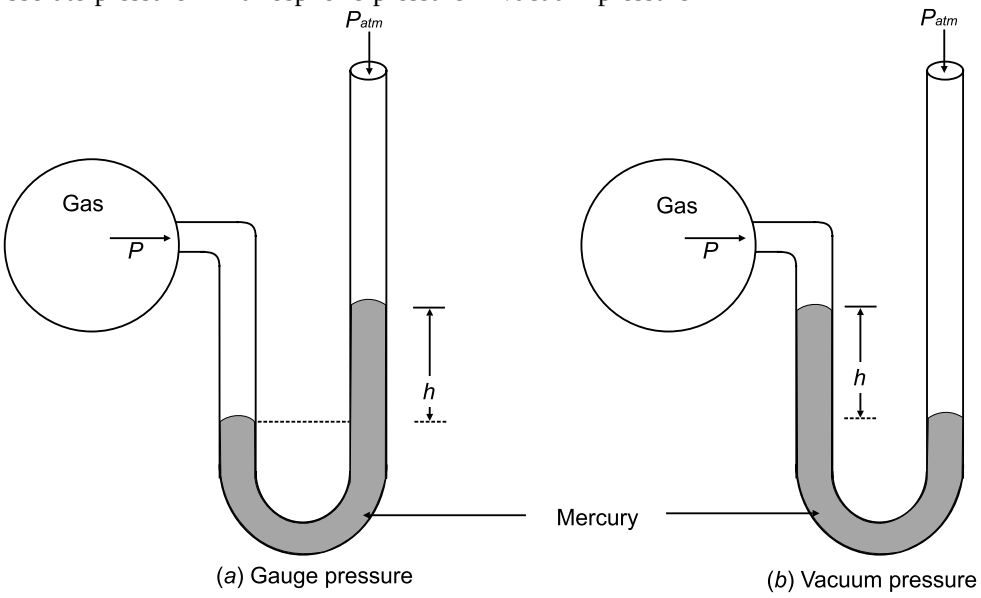


Fig. 1.5 *U-tube manometer*

The bar chart shown in Fig. 1.6 further clarifies the interrelationship amongst the different pressures.

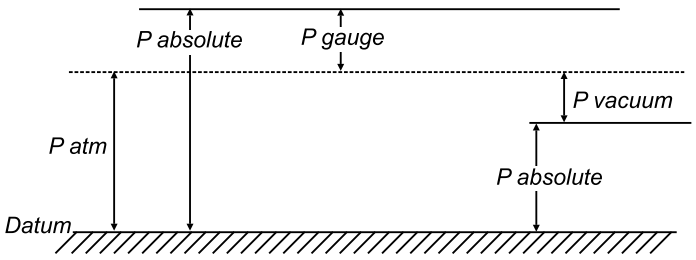


Fig. 1.6 *Different pressures*

Pressure could also be measured by a Bourdan tube. Bourdan tube has a flattened cross section (oval) closed at one end. Other end of tube connects to the region whose pressure is to be measured. Gas whose pressure is being measured acts on inside of tube surface, thus causing it to change its section from oval section to circular section. Pressure exerted by gas works against tube stresses and air pressure. This change in cross-section from elliptical to circular causes straightening of tube and thus deflecting free end of tube through some distance '*d*' as shown in figure 1.7. This deflection in free end of tube measures the pressure difference between gas pressure and atmospheric pressure. Generally this free end of tube is connected to an indicating hand sweeping over a graduated dial showing the gauge pressure directly.

Temperature is another thermodynamic property which is normally used in Kelvin scale in engineering thermodynamic analysis. It is dealt in detail in subsequent chapter.

Density which refers to the mass per unit volume is the ratio of mass and volume occupied. Its units are kg/m^3.

Density = (Mass/Volume)

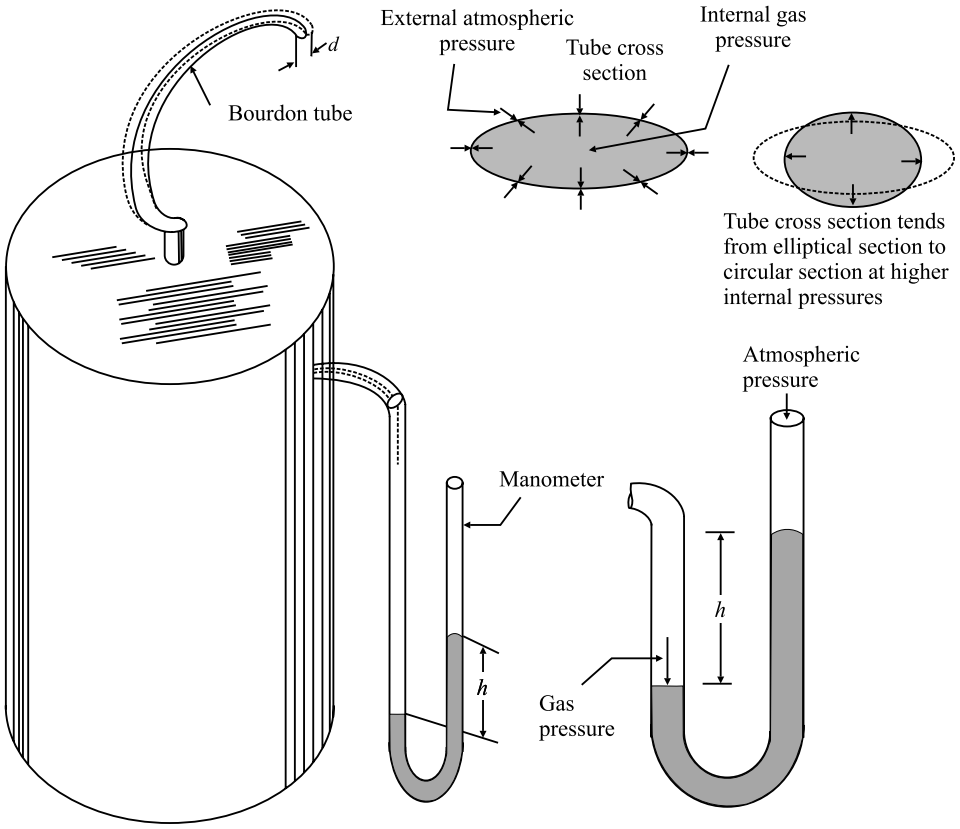


Fig. 1.7 *Bourdan tube for pressure measurement*

The *specific volume* is the volume per unit mass of the substance. It is defined by ratio of the volume occupied and the mass of substance. Its units are m^3/kg.

Specific volume = (Volume/Mass)

Density or specific volume conform to the definitive specification of a thermodynamic property and are capable of getting associated with other properties such as temperature, pressure and internal

energy. Also, the volume occupied by a material is a measure of distance between molecules and thus indicates their molecular energy.

Weight is actually the force due to gravity acting on any substance. Thus, it is the product of mass and gravitational acceleration. Its units are Newtons.

$$\text{Weight} = (\text{mass} \times \text{gravitational acceleration})$$

Specific weight of a substance is the ratio of weight of substance and volume of substance.

$$\text{Specific weight} = (\text{Weight/Volume})$$
$$= (\text{density} \times \text{gravitational acceleration})$$

Specific gravity is defined as the ratio of the density of any substance and standard density of some reference substance. For solids and liquids the water density at some specified temperature say 0ºC or 4ºC is taken as standard density.

1.11 ENERGY AND ITS FORMS

"*Energy* is usually defined as the ability to do mechanical work". It is indeed quite difficult to precisely define the "energy". We feel energy at every moment and can sense it very oftenly.

Another broader definition of energy says that "energy refers to the capacity for producing effects."

Total energy at any moment may be the algebraic summation of the different forms of energy. Conversion of energy from one to other is also possible. In thermodynamics we are primarily interested in studying the change in total energy of a system. Thus, for analysis relative value of energy is considered instead of absolute value.

Energy can be classified in the following general categories;

(*a*) *Energy in transition*: It refers to the energy that is in process of transition between substances or regions because of some driving potential, such as difference or gradient of force, or of temperature, or of electrical potential etc. For example heat, work etc.

(*b*) *Energy stored in particular mass*: It refers to the potential and kinetic energy associated with masses that are elevated or moving with respect to the earth.

Apart from above broad classification the energy can also be categorised into various forms.

(*i*) *Macroscopic energy*: It refers to the energy possessed by a system considered at macroscopic level such as kinetic energy, potential energy etc.

(*ii*) *Microscopic energy*: It refers to the energy defined at molecular level. Summation of energy at molecular level or microscopic energy results in internal energy.

Some of the popular forms of energy are described below :

Potential energy: This type of energy is based on relative position of bodies in a system, i.e. elevation in a gravitational field.

Potential energy for a mass m at elevation z is given as :

$$\text{P.E.} = m.g.z$$

Here g is the gravitational acceleration and elevation is measured from some reference point.

Kinetic energy: It is based on the relative movement of bodies. For a mass m moving with certain velocity c it could be mathematically expressed as;

$$\text{K.E.} = (1/2)\ m.c^2$$

Internal energy: Internal energy of a system is the energy associated with the molecular structure at molecular level.

Let us study fall of a 'weight' from certain height on the floor. Upon hitting the floor, 'weight' comes to dead stop and its potential energy and kinetic energy both reduce to zero. Question arises, where does the vanishing potential energy and kinetic energy go upon 'weight' touching the floor. If we touch the point of contact between 'weight' and floor, it is observed that both these points are slightly hotter than before impact. Thus, it is obvious that the energy has changed its form from potential and kinetic to internal energy and causes rise in temperature of 'weight' and floor at the points of contact.

Internal energy is sum of energy associated with molecules which may have translational, vibrational and rotational motions etc. and respective energies causing these motions.

Internal energy may be thus comprising of sensible energy, latent energy, chemical energy, nuclear energy etc. '*Sensible energy*' refers to the kinetic energy associated with molecules. '*Latent energy*' refers to the energy associated with phase of a substance.

'*Chemical energy*' refers to the energy associated with molecular bonds. '*Nuclear energy*' refers to the energy associated with the bonds within nucleus of atom itself.

Total energy of a system may be given as summation of different forms of energy at a moment. Mathematically;

T.E (Total energy) = K.E + P.E + I.E

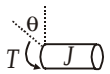
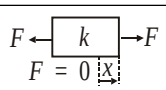
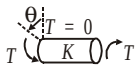
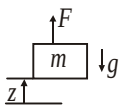
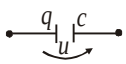
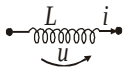
where K.E = Kinetic energy

P.E = Potential energy

I.E = Internal energy

Some different forms of energy interaction and associated work interactions with block diagram are given in table 1.4.

Table 1.4 *Some forms of energy and the associated work interactions*

S. No.	*Macroscopic form of energy*	*Governing equation*	*Energy interaction*	*Work interaction*	*Block diagram*
1.	Kinetic energy (translation)	$F = m \cdot \frac{dV}{dt}$	$\Delta E = \frac{1}{2} m \cdot (V_2^2 - V_1^2)$	$= - F \cdot dx$	
2.	Kinetic energy (rotational)	$T = J \cdot \frac{d\omega}{dt}$	$\Delta E = \frac{1}{2} \mathrm{J} \cdot (\omega_2^2 - \omega_1^2)$	$= - T \cdot d\theta$	
3.	Spring stored energy (translational)	$F = kx$	$\Delta E = \frac{1}{2} k \cdot (x_2^2 - x_1^2)$	$= - F \cdot dx$	
4.	Spring stored energy (rotational)	$T = K \cdot \theta$	$\Delta E = \frac{1}{2} \mathrm{K} \cdot (\theta_2^2 - \theta_1^2)$	$= - T \cdot d\theta$	
5.	Gravitational energy	$F = mg$	$\Delta E = mg \cdot (Z_2 - Z_1)$	$= - F \cdot dz$	
6.	Electrical energy (capacitance)	$u = \frac{q}{c}$	$\Delta E = \frac{1}{2} \frac{q^2}{c} = \frac{1}{2} cu^2$	$= - u \cdot dq$	
7.	Electrical energy (inductance)	$\phi = L \cdot i$	$\Delta E = \frac{1}{2} Li^2 = \frac{1}{2} \frac{\phi^2}{L}$	$= - i \cdot d\phi$	

1.12. HEAT AND WORK

When two systems at different temperatures are brought into contact there are observable changes in some of their properties and changes continue till the two don't attain the same temperature if contact is prolonged. Thus, there is some kind of energy interaction at the boundary which causes change in temperatures. This form of energy interaction is called heat. *Thus 'heat' may be termed as the energy interaction at the system boundary which occurs due to temperature difference only.* Heat is observable in transit at the interface i.e. boundary, it can not be contained in a system. In general the heat transfer to the system is assigned with positive (+) sign while the heat transfer from the system is assigned with negative (–) sign. Its units are Calories.

In thermodynamics the work can be defined as follows:

"*Work shall be done by the system if the total effect outside the system is equivalent to the raising of weight and this work shall be positive work*".

In above definition the work has been defined as positive work and says that there need not be actual raising of weight but the effect of the system behaviour must be reducible to the raising of a weight and nothing else. Its units are N. m or Joule. Heat and work are two transient forms of energy.

Let us look at a piston cylinder mechanism (closed system), where high pressure air is filled inside the cylinder fitted with a piston exerting force against some resistance. As the piston moves a distance say '*l*', the work would be done. It can be reduced to the raising of weight by replacing this resisting system by a frictionless pulley and lever such that a weight W is raised, Fig. 1.8.

For example, if an electrical battery and resistance is considered as a system, then this system shall do work when electric current flows in an external resistance as this resistance could be replaced by an ideal frictionless motor driving a frictionless pulley and raising a weight.

Here, also in reference to work it is obvious that the work is the entity which is available at the boundary of system, thus work can not be stored rather it is energy interaction in transit at the boundary. From the thermodynamic definition of work the sign convention established as positive work shall be the one which is done by the system while the negative work shall be the one that is done upon the system.

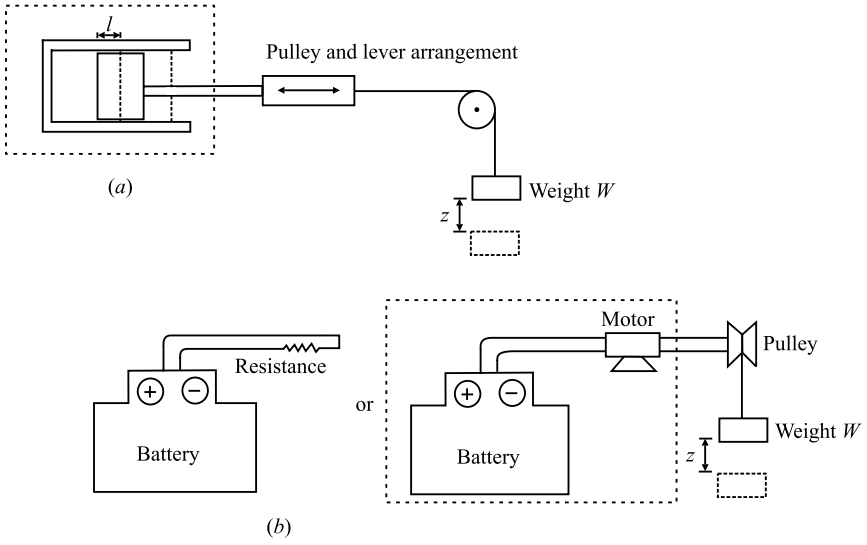


Fig. 1.8 *Thermodynamic work*

1.13 GAS LAWS

Thermodynamic analysis relies largely upon the gas laws, which are known as Boyle's law (1662) and Charle's law (1787). Boyle's law says that if temperature of a gas is held constant then its molar volume is inversely proportional to the pressure. Mathematically it can be related as $p\bar{v}$ = constant. Here p is the pressure and $\bar{v}$ is the molar volume of gas, i.e. volume per mole.

Charle's law says that for the pressure of gas held constant the volume of gas is directly proportional to the temperature of gas. Mathematically it can be given as $\bar{v}/T$ = constant, where T is the temperature of the gas. It also says that if the molar volume of gas is held constant, the pressure of gas is directly proportional to temperature, i.e. p/T = constant. Figure 1.9 shows the graphical representation.

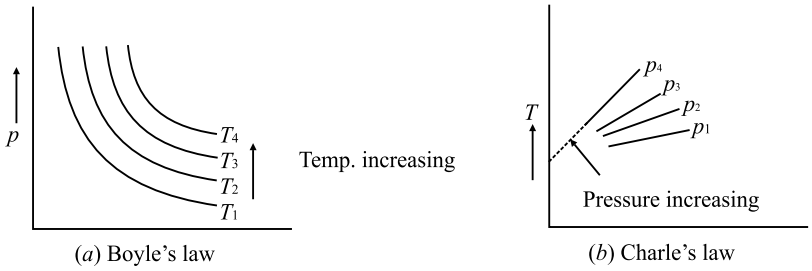


Fig 1.9 *Graphical representations of gas laws at constant temperature and at constant pressure*

Boyle's and Charle's law when combined together result in,

$$p\bar{v}/T = \text{constant}$$

or $p\bar{v} = \bar{R}T$, where $\bar{R}$ is the universal gas constant.

1.14 IDEAL GAS

Engineering thermodynamics deals with different systems having gaseous working fluids. Some gases behave as ideal gas and some as non-ideal gas. Based on the experimental methods various equations of state of gases have been developed.

For perfect gas the ideal gas equation shows that

$p\bar{v} = \bar{R}T$, where $\bar{R}$ is the universal gas constant and can be related as $R = \bar{R}/M$, here R is the characteristic gas constant and M is the molar mass or molecular weight of the substance, $\bar{v}$ is volume per mole. Universal gas constant has value given as 8.31441 kJ/k mol.K. or $pV = mRT$, where m is mass of the substance, V is the volume of substance,

i.e. $V = n \cdot \bar{v}$

$m = n \cdot M$, where 'n' is no. of moles.

Gas constant is also related to specific heats at constant pressure and volume as follows,

$$R = c_p - c_v$$

Upon plotting the variables P, V, T for an ideal gas on three mutually perpendicular axes, the three dimensional entity generated is called P-V-T surface and can be used for studying the thermodynamic properties of ideal gas. Figure 1.10 shows the typical P-V-T surface for an ideal gas.

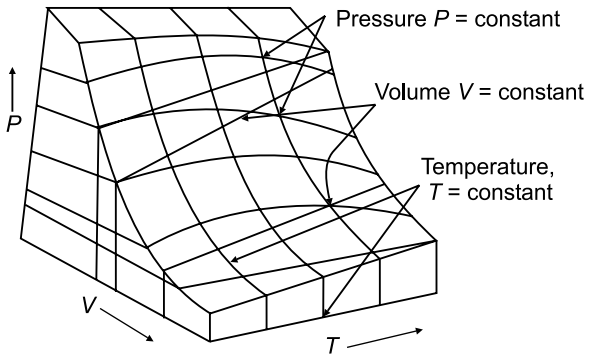


Fig. 1.10 *P-V-T surface for ideal gas*

For certain gases the molecular weight and gas constant are given in table 1.5.

Table 1.5

Gas	*Molecular weight, kg/kmol*	*Gas constant, kJ/kg.K*
Air	28.97	0.287
Carbon dioxide	44.01	0.189
Hydrogen	2.016	4.124
Helium	4.004	2.077
Nitrogen	28.01	0.297
Oxygen	32.00	0.260
Steam	18.02	0.461

1.15 DALTON'S LAW, AMAGAT'S LAW AND PROPERTY OF MIXTURE OF GASES

Dalton's law of partial pressures states that the "total pressure of a mixture of gases is equal to the sum of partial pressures of constituent gases." Partial pressure of each constituent can be defined as the pressure exerted by the gas if it alone occupied the volume at the same temperature.

Thus, for any mixture of gases having '*j*' gases in it, the mathematical statement of Dalton's law says,

$$p = p_1 + p_2 + p_3 + ... + p_j$$

if $$V = V_1 = V_2 = V_3 = ... = V_j$$

and $$T = T_1 = T_2 = T_3 = ... T_j$$

Dalton's law can be applied to both mixture of real gases and ideal gases.



constituent gases Mixture

Fig. 1.11 *(a) Dalton's law of partial pressures, (b) Amagat's law*

Let us take mixture of any three, perfect gases, say, 1, 2, 3 in a container having volume '*V*' and temperature *T*.

Equation of state for these gases shall be,

$$p_1V = m_1R_1T \; ; \; p_2 \; V = m_2 \cdot R_2 \cdot T, \; p_3V = m_3 \cdot R_3 \cdot T$$

The partial pressures of three gases shall be,

$$p_1 = \frac{m_1R_1T}{V} \; , \; p_2 = \frac{m_2 \cdot R_2 \cdot T}{V} \; , \; p_3 = \frac{m_3 \cdot R_3 \cdot T}{V}$$

From Dalton's law;

$$p = p_1 + p_2 + p_3 = (m_1R_1 + m_2R_2 + m_3R_3) \cdot \frac{T}{V}$$

or, it can be given in general form as,

$$pV = T \cdot \sum_{i=1}^{j} m_i \cdot R_i$$

where i refers to constituent gases

Amagat's law of additive volumes states that volume of a gas mixture is equal to the sum of volumes each gas would occupy at the mixture pressure and temperature.

$$V = V_1 + V_2 + V_3 \; ..... \; + V_j$$

$$p = p_1 = p_2 = p_3 \; ..... \; p_j$$

$$T = T_1 = T_2 = T_3 = ... \; T_j$$

Mass balance upon mixture yields $m = m_1 + m_2 + m_3$

or

$$m = \sum_{i=1}^{j} m_i$$

From above the gas constant for the mixture can be given as;

$$R = \frac{m_1R_1 + m_2R_2 + m_3R_3}{(m_1 + m_2 + m_3)}$$

or, in general form,

$$R = \frac{\sum_{i=1}^{j} m_i \cdot R_i}{\sum_{i=1}^{j} m_i}$$

Mole fraction x_i of a constituent gas can be defined as the ratio of number of moles of that constituent to the total number of moles of all constituents.

Thus mole fractions of three gases, if number of moles of three gases are n_1, n_2 and n_3;

$$x_1 = \frac{n_1}{n_1 + n_2 + n_3}$$

$$x_2 = \frac{n_2}{n_1 + n_2 + n_3}$$

$$x_3 = \frac{n_3}{n_1 + n_2 + n_3}$$

or, in general $x_i = \dfrac{n_i}{\sum n_i}$

Total no. of moles,

$$n = n_1 + n_2 + n_3 \text{ or, } n = \sum_{i=1}^{j} n_i$$

Sum of mole fractions of all constituent equals to 1,

$$\sum x_i = \frac{\sum n_i}{n} = 1$$

Number of moles of any constituent gas,

$$n_i = n \cdot x_i$$

For M_i being the molecular weight of a constituent gas, the mass m_i of that constituent shall be

$$m_i = n_i \cdot M_i$$

or, $$m_i = n \cdot x_i \cdot M_i$$

and the total mass m, shall be

$$m = \sum m_i = n. \sum x_i \cdot M_i$$

Molecular weight of mixture shall be:

$$M = \frac{m}{n} = \sum x_i \cdot M_i$$

1.16 REAL GAS

When a gas is found to disobey the perfect gas law, i.e. the equation of state for ideal gas, then it is called 'real gas'. Real gas behaviour can also be shown by a perfect gas at the changed thermodynamic states such as high pressure etc.

Deviation of real gas from ideal gas necessitates the suitable equation of state which can be used for interrelating the thermodynamic properties P, V, and T.

From the kinetic theory of gases it is obvious that the ideal gas equation of state suits the gas behaviour when intermolecular attraction and volume occupied by the molecules themselves is negligibly small in reference to gas volume. At high pressures intermolecular forces and volume of molecules both increase and so the gas behaviour deviates from ideal gas to real gas.

A number of mathematical equations of state suggested by Van der-Waals, Berthelot, Dieterici, Redlich-Kwong, Beattie-Bridgeman and Martin-Hou etc. are available for analysing the real gas behaviour.

Dalton's law and Amagat's law can also be used for real gases with reasonable accuracy in conjunction with modified equations of state.

As the ideal gas equation does not conform to the real gas behaviour in certain ranges of pressures and temperatures, so the perfect gas equation should be modified using compressibility factor for the gas at given pressure and temperature.

Such modified form of equations shall be;

$$P\bar{v} = Z \cdot \bar{R} \cdot T$$

Here Z is the compressibility factor, a function of pressure and temperature.

Thus, compressibility factor is like a correction factor introduced in ideal equation of state for suiting the real gas behaviour. Compressibility factor is an indication of deviation of a gas from ideal gas behaviour and can be mathematically given as;

$$Z = f(P, T)$$

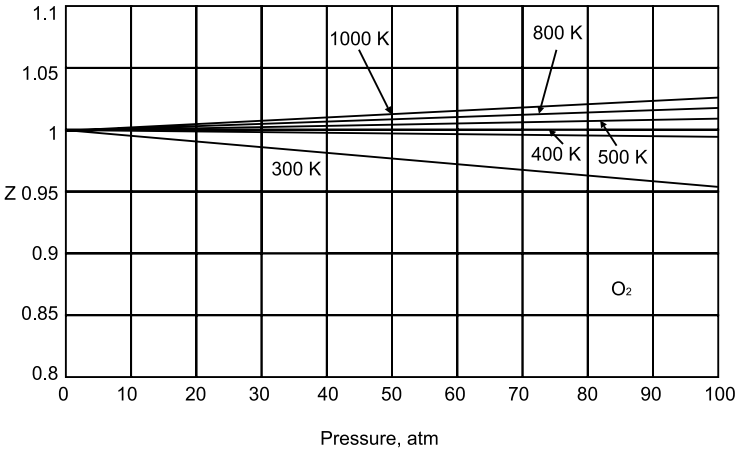
or $$Z = \frac{v_{actual}}{v_{ideal}}$$

Here, $v_{ideal} = \frac{RT}{P}$ i.e. $Z = 1$ for ideal gases while Z can be greater than or less than unity.

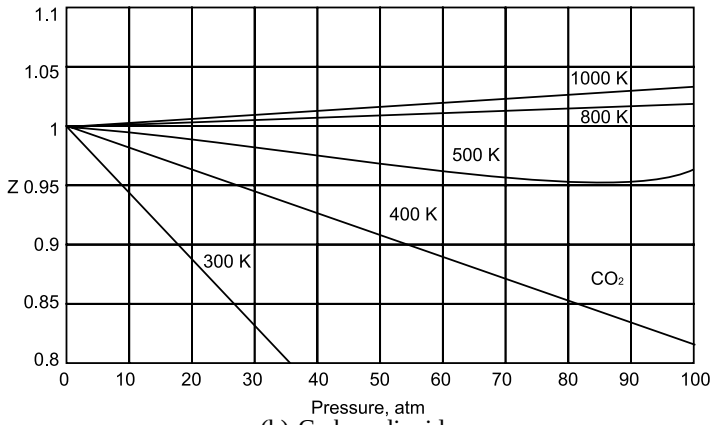
Individual graphical representations are available for getting the compressibility factor as shown in Fig 1.12. Compressibility factor charts are available for different substances. Compressibility factors for various substances can also be shown on a generalized compressibility chart using reduced properties. Reduced properties are non-dimensional properties given as ratio of existing property to critical property of substance. Such as reduced pressure is ratio of pressure of gas to critical pressure of gas. Similarly, reduced temperature can be given by ratio of temperature of gas to critical temperature of gas.

Reduced pressure, $p_R = \frac{p}{p_c}$

Reduced temperature, $T_R = \frac{T}{T_c}$



(a) Oxygen



(b) Carbon dioxide

Fig. 1.12 *Compressibility factors,* Z

where P_c and T_c denote critical pressure and critical temperature respectively. These reduced pressure and reduced temperature are used for getting the generalized compressibility chart of the form, $Z = f(p_R, T_R)$ where Z for all gases is approximately same. This similar behaviour of compressibility correction factor for different gases in respect to reduced pressures and temperatures is called "principle of corresponding states." Fig. 1.13 shows a generalized compressibility chart. In generalized compressibility chart a set of curves is fitted through a set of experimentally determined Z values plotted against reduced

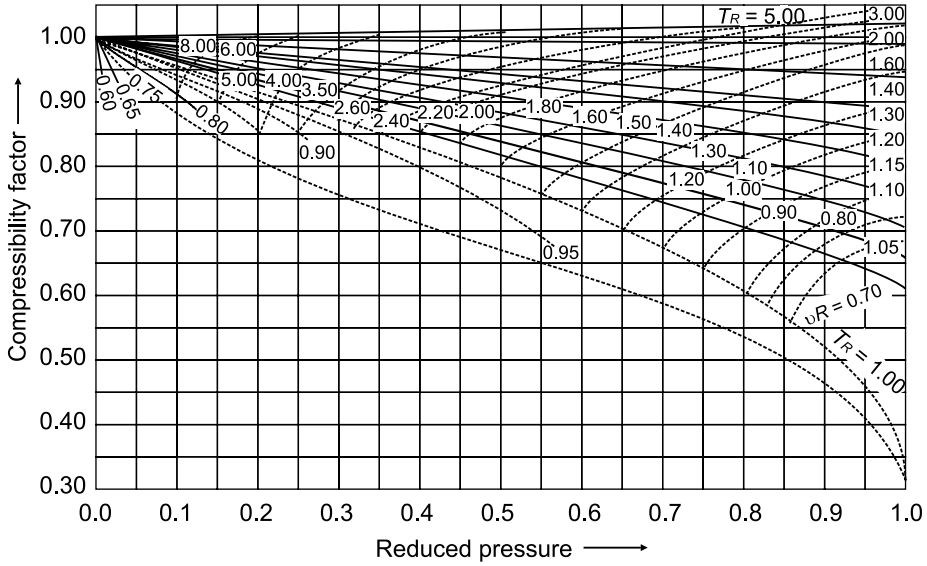


Fig. 1.13 *(a) Generalized compressibility chart, $p_R \leq 1.0$*

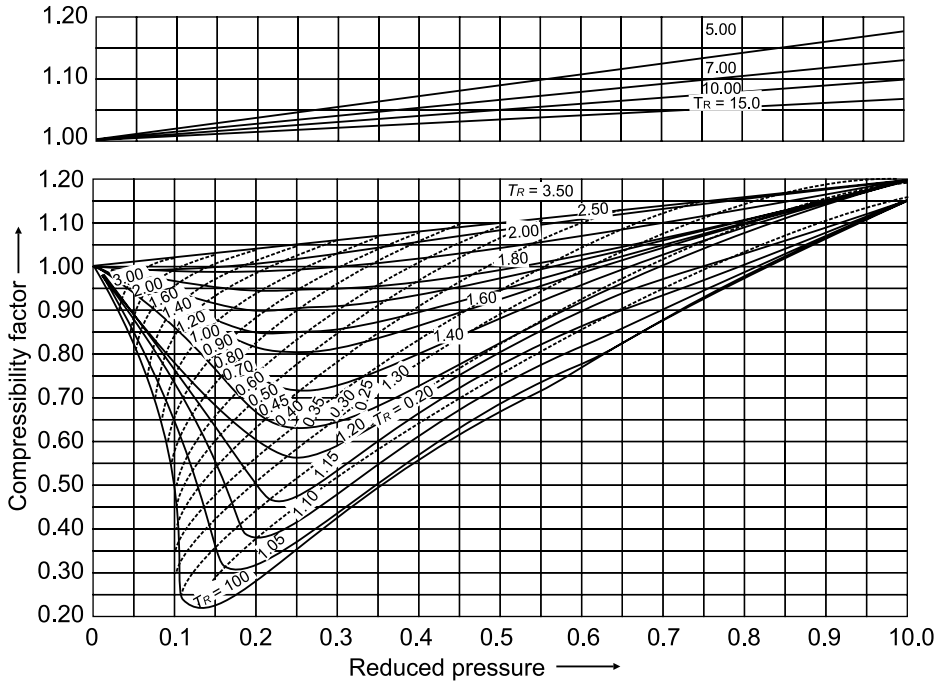


Fig. 1.13 *(b) Generalized compressibility chart, $p_R \leq 10.0$*

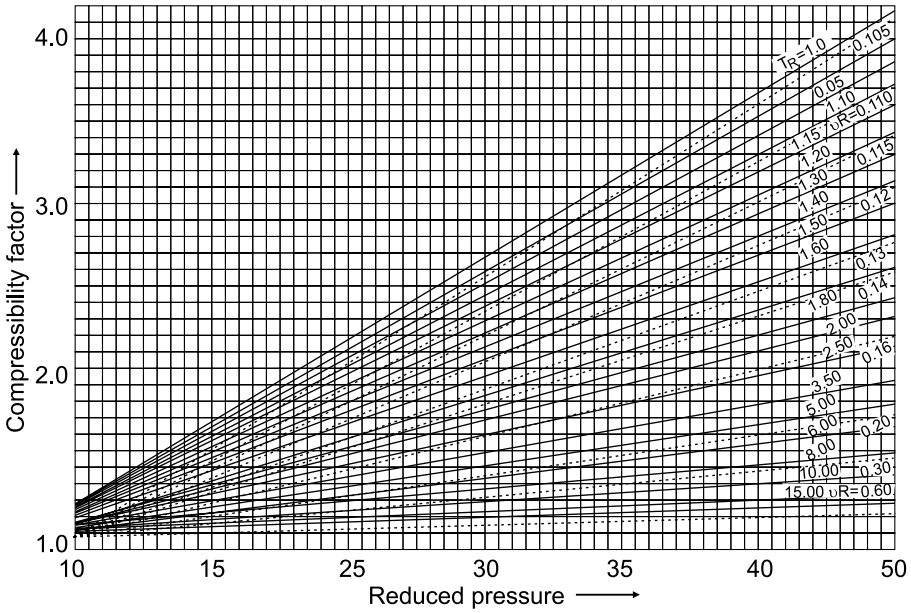


Fig. 1.13 *(c) Generalized compressibility chart, $10 \leq p_R \leq 40$*

pressure and reduced temperatures for several gases. On the generalized compressibility chart it could be seen that at "very small pressures the gases behave as an ideal gas irrespective of its temperature" and also at "very high temperatures the gases behave as ideal gas irrespective of its pressure".

1.17 VANDER' WAALS AND OTHER EQUATIONS OF STATE FOR REAL GAS

Vander' Waals suggested the equation of state for real gas in 1873. The equation has been obtained applying the laws of mechanics at molecular level and introducing certain constants in the equation of state for ideal gas. This equation agrees with real gas behaviour in large range of gas pressures and temperatures.

Vander' Waals equation of state for real gas is as follows,

$$\left(p + \frac{a}{\bar{v}^2}\right)(\bar{v} - b) = \bar{R}T$$

where '*a*' is the constant to take care of the mutual attraction amongst the molecules and thus $\left(\frac{a}{\bar{v}^2}\right)$ accounts for cohesion forces.

Table 1.6 *Vander' Waals constant*

Gas	*Constant a, $N.m^4/(kg.\ mol)^2$*	*Constant b, $m^3/kg.mol$*
Helium	34176.2×10^2	2.28×10^{-2}
Hydrogen	251.05×10^2	2.62×10^{-2}
Oxygen	1392.5×10^2	3.14×10^{-2}
Air	1355.22×10^2	3.62×10^{-2}
Carbon dioxide	3628.50×10^2	3.14×10^{-2}

Constant '*b*' accounts for the volumes of molecules which are neglected in perfect gas equation, thus it denotes "co-volume", Mathematically,

$$a = \frac{27\bar{R}^2 T_c^2}{64 p_c}, \quad b = \left(\bar{R} \cdot T_c\right) / \left(8 p_c\right)$$

Here, p_c, T_c are critical point pressures and temperatures having values as given in appendix.

Thus these constants '*a*' & '*b*' are determined from behaviour of substance at the critical point.

In general it is not possible to have a single equation of state which conforms to the real gas behaviour at all pressures and temperatures.

A few more equations of state for real gas as suggested by various researchers are as follows.

Redlich-Kwong equation of state for real gas,

$$p = \frac{\bar{R}T}{(\bar{v} - b)} - \frac{a}{\bar{v} \cdot (\bar{v} + b) \cdot \sqrt{T}}$$

where $a = 0.4278\left(\frac{\bar{R}^2 \cdot T_c^{2.5}}{p_c}\right)$ and $b = 0.08664\left(\frac{\bar{R} \cdot T_c}{p_c}\right)$

Berthelot equation of state for real gas,

$$p = \frac{\bar{R}T}{(\bar{v} - b)} - \frac{a}{T \cdot \bar{v}^2},$$

where $a = \left(\frac{27 \cdot \bar{R}^2 \cdot T_c^3}{64 \cdot p_c}\right)$ and $b = \left(\frac{\bar{R} \cdot T_c}{8\ p_c}\right)$

Here *a* and *b* refer to the constants as suggested in respective equations.

Beattie-Bridgeman equation of state given in 1928, for real gas has five constants determined experimentally. It is,

$$p = \frac{\bar{R} \cdot T}{(\bar{v})^2}\left(1 - \frac{C}{\bar{v} \cdot T^3}\right)(\bar{v} + B) - \frac{A}{(\bar{v})^2}$$

where, $A = A_0\left(1 - \frac{a}{\bar{v}}\right)$ and $B = B_0\left(1 - \frac{b}{\bar{v}}\right)$

Constants used in Beattie – Bridgeman equation are given in Table 1.7 when *p* is in *k* p_a, $\bar{v}$ is in m^3/k mol, T is in K, and $\bar{R}$ = 8.314 *k* p_a m^3/k *mol.K*.

Table 1.7. *Beattie -Bridgeman constants*

Gas	A_0	*a*	B_0	*b*	*c*
Helium	2.1886	0.05984	0.01400	0.0	40
Hydrogen	20.0117	–0.00506	0.02096	–0.04359	504
Oxygen	151.0857	0.02562	0.04624	0.004208	4.80×10^4
Air	131.8441	0.01931	0.04611	–0.001101	4.34×10^4
Carbon dioxide	507.2836	0.07132	0.10476	0.07235	6.60×10^5

Virial equations of state propose a form of equation which can be suitably modified and used for real gases.

These equations of state are in the form,

$$\frac{p\bar{v}}{\bar{R}T} = A_0 + A_1 \,.\, p + A_2 \,.\, p^2 + A_3 \,.\, p^3 + ...$$

or

$$\frac{p\bar{v}}{\bar{R}T} = B_0 + \frac{B_1}{\bar{v}} + \frac{B_2}{\bar{v}^2} + \frac{B_3}{\bar{v}^3} + ...$$

Where A_0, A_1, A_2, A_3, ...and B_0, B_1, B_2, B_3.....

are called the "virial coefficients" and depend upon temperature alone. Virial equations of state can be suitably modified based on experimental *P, v, T* data for the real gas analysis. Virial constants can be calculated if the suitable model for describing the forces of interaction between the molecules of gas under consideration is known.

EXAMPLES

1. *Find out the pressure difference shown by the manometer deflection of 30 cm of Mercury. Take local acceleration of gravity as 9.78 m/s^2 and density of mercury at room temperature as 13,550 kg/m^3.*

Solution:

From the basic principles of fluid statics,

Pressure difference = $\rho \cdot gh$

= $13550 \times 30 \times 10^{-2} \times 9.78$

= **39755.70 Pa Ans.**

2. *An evacuated cylindrical vessel of 30 cm diameter is closed with a circular lid. Estimate the effort required for lifting the lid, if the atmospheric pressure is 76 cm of mercury column (Take g = 9.78 m/s^2)*

Solution:

Effort required for lifting the lid shall be equal to the force acting upon the lid. Thus, effort required = Pressure × Area

= $(76 \times 10^{-2} \times 13550 \times 9.78) \times (3.14 \times (30 \times 10^{-2})^2/4)$

= **7115.48 N Ans.**

3. *Calculate the actual pressure of air in the tank if the pressure of compressed air measured by manometer is 30 cm of mercury and atmospheric pressure is 101 kPa. (Take g = 9.78 m/s^2)*

Solution:

Pressure measured by manometer on the tank is gauge pressure, which shall be

= $\rho.g.h$

= $(13550 \times 9.78 \times 30 \times 10^{-2})$

= 39755.70 Pa

= 39.76 kPa

Actual pressure of air = Gauge pressure + atmospheric pressure

= 39.76 + 101

= **140.76 kPa Ans.**

4. *Determine gauge pressure at a depth of 1 m in a tank filled with oil of specific gravity 0.8. Take density of water as 1000 kg/m^3 and g = 9.81 m/s^2.*

Solution:

Density of oil = Specific gravity × Density of water

$\rho_{oil} = 0.8 \times 1000$

$\rho_{oil} = 800 \text{ kg/m}^3$

Gauge pressure $= (\rho_{oil} \times g \times h)$

$= 800 \times 9.81 \times 1$

or $= 7848 \text{ N/m}^2$

Gauge pressure = **7.848 kPa. Ans.**

5. *Calculate the gas pressure using a mercury manometer with one limb open to atmosphere as shown in Fig. 1.14. Barometer reading is 76 cm and density of mercury is 13.6 × 10³ kg/m³. Take g = 9.81 m/s².*

Solution:

Figure shows that the difference of height in mercury columns is 40 cm.

In reference to level *AB* the pressure exerted by gas, p_{gas} can be written as sum of atmospheric pressure and pressure due to mercury column at *AB*

$p_{gas} = (\rho_{mercury} \times 9.81 \times 40 \times 10^{-2}) + \text{Atmospheric pressure}$

$= (13.6 \times 10^3 \times 9.81 \times 40 \times 10^{-2}) + (13.6 \times 10^3 \times 9.81 \times 76 \times 10^{-2})$

$= 154762.56 \text{ N/m}^2$

or P_{gas} = **154.76 kPa Ans.**

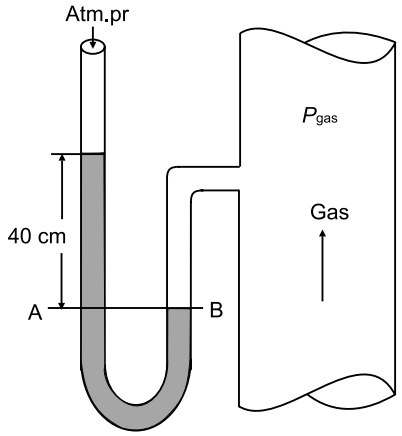


Fig. 1.14

6. *1 kg of water falls from an altitude of 1000 m above ground level. What will be change in the temperature of water at the foot of fall, if there are no losses during the fall. Take specific heat of water as 1 kcal/kg·K*

Solution:

Initially when water is at 1000 m, it shall have potential energy in it. This potential energy shall get transformed upon its fall and change the temperature of water.

By law of energy conservation

Potential energy = Heat required for heating water

$$= \frac{1 \times 9.81 \times 1000}{4.18} = 1 \times 1 \times 10^3 \times \Delta T$$

or $\Delta T = 2.35^\circ\text{C}$

Change in temperature of water = **2.35°C Ans.**

7. *A spring balance is used for measurement of weight. At standard gravitational acceleration it gives weight of an object as 100 N. Determine the spring balance reading for the same object when measured at a location having gravitational acceleration as 8.5 m/s^2.*

Solution:

At standard gravitational acceleration, mass of object = $\frac{100}{9.81}$

= 10.194 kg

Spring balance reading = Gravitational force in mass

= 10.194 × 8.5

= 86.649 N

= **86.65 N Ans.**

8. *An incompressible gas in the cylinder of 15 cm diameter is used to support a piston, as shown. Manometer indicates a difference of 12 cm of Hg column for the gas in cylinder. Estimate the mass of piston that can be supported by the gas. Take density of mercury as 13.6 × 10^3 kg/m^3.*

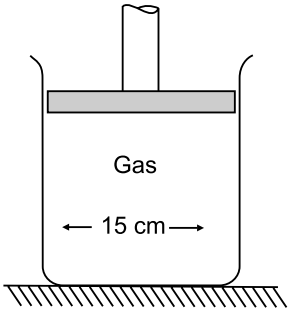


Fig. 1.15

Solution:

Piston shall be supported by the gas inside, therefore, let mass of piston be '*m*' kg.

Weight of piston = Upward thrust by gas

$$m.g = p \times \pi \times \frac{d^2}{4}$$

$$m \times 9.81 = (12 \times 10^{-2} \times 13.6 \times 10^3 \times 9.81) \times \frac{\pi}{4} \times (15 \times 10^{-2})^2$$

$$m = 28.84 \text{ kg}$$

Mass of piston = **28.84 kg. Ans.**

9. *Determine pressure of steam flowing through a steam pipe when the U-tube manometer connected to it indicates as shown in figure 1.16. During pressure measurement some steam gets condensed in manometer tube and occupies a column of height 2 cm (AB) while mercury gets raised by 10 cm (CD) in open limb. Consider barometer reading as 76 cm of Hg, density of mercury and water as 13.6 × 10^3 kg/m^3 and 1000 kg/m^3 respectively.*

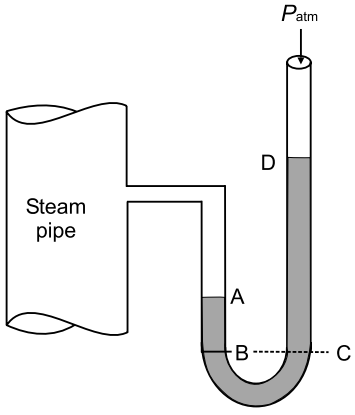


Fig. 1.16

Solution:

Let us make pressure balance at plane *BC*.

$$p_{steam} + p_{water,\ AB} = p_{atm} + p_{Hg\ ,\ CD}$$

$$p_{steam} = p_{atm} + p_{Hg,\ CD} - p_{water,\ AB}$$

$$p_{atm} = (13.6 \times 10^3 \times 76 \times 10^{-2} \times 9.81)$$

$$p_{atm} = 101396.16 \text{ N/m}^2$$

$$p_{water,\ AB} = (1000 \times 2 \times 10^{-2} \times 9.81)$$

$$p_{water,\ AB} = 196.2 \text{ N/m}^2$$

$$p_{\text{Hg, } CD} = (13.6 \times 10^3 \times 10 \times 10^{-2} \times 9.81)$$
$$p_{\text{Hg, } CD} = 13341.6 \text{ N/m}^2$$

Substituting for getting steam pressure,

$$p_{\text{steam}} = 101396.16 + 13341.6 - 196.2$$
$$p_{\text{steam}} = 114541.56 \text{ N/m}^2$$

or $p_{\text{steam}} =$ **114.54 kPa Ans.**

10. *A vessel has two compartments 'A' and 'B' as shown with pressure gauges mounted on each compartment. Pressure gauges of A and B read 400 kPa and 150 kPa respectively. Determine the absolute pressures existing in each compartment if the local barometer reads 720 mm Hg.*

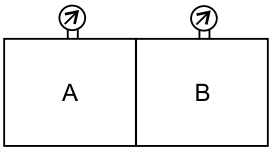


Fig. 1.17

Solution:

Atmospheric pressure from barometer

$$= (9810) \times (13.6) \times (0.720)$$
$$= 96060 \text{ Pa}$$
$$= 96.06 \text{ kPa}$$

Absolute pressure in compartment *A*,

$$P_{\text{abs, } A} = P_{\text{gauge, } A} + P_{\text{atm}}$$
$$= 400 + 96.06 = 496.06 \text{ kPa}$$

Absolute pressure in compartment *B*,

$$P_{\text{abs, } B} = P_{\text{gauge, } B} + P_{\text{atm}}$$
$$= 150 + 96.06 = 246.06 \text{ kPa}$$

Absolute pressure in compartments *A* & *B*

= **496.06 kPa & 246.06 kPa Ans.**

11. *Determine the air pressure in a tank having multifluid manometer connected to it, with the tube open to atmosphere as shown in figure. Tank is at an altitude where atmospheric pressure is 90 kPa. Take densities of water, oil and mercury as 1000 kg/m^3, 850 kg/m^3 and 13600 kg/m^3 respectively.*

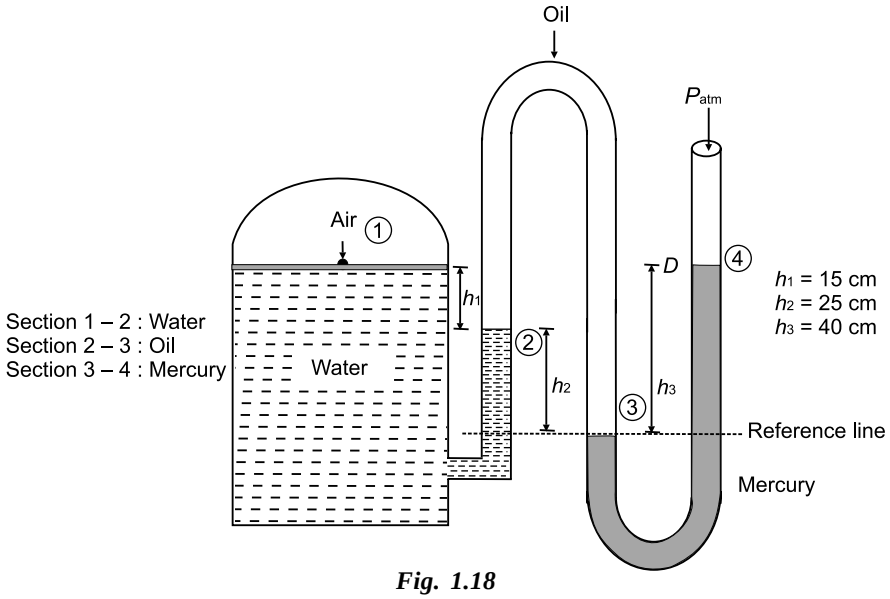


Fig. 1.18

Solution:

It is obvious that the lengths of different fluids in U-tube are due to the air pressure and the pressure of air in tank can be obtained by equalizing pressures at some reference line. Strating from point (1) the pressure can be given as under for equilibrium,

$$p_1 + \rho_{water} \cdot g \cdot h_1 + \rho_{oil} \cdot g \cdot h_2 = p_{atm} + \rho_{mercury} \cdot g \cdot h_3$$

Given : $\rho_{water} = 1000$ kg/m^3, $\rho_{oil} = 850$ kg/m^3, $\rho_{mercury} = 13600$ kg/m^3

$h_1 = 0.15$ m,

$h_2 = 0.25$ m,

$h_3 = 0.40$ m,

$p_{atm} = 90$ kPa

Substituting we get $p_1 = 139.81$ kPa

Air pressure = **139.81 kPa Ans.**

12. *Estimate the kinetic energy associated with space object revolving around earth with a relative velocity of 750 m/s and subjected to gravitational force of 4000 N. Gravitational acceleration may be taken as 8 m/s^2.*

Solution:

$$\text{Mass of object} = \frac{\text{Gravitational force}}{\text{Gravitational acceleration}}$$

$$= \frac{4000}{8}$$

$$= 500 \text{ kg}$$

$$\text{Kinetic energy} = \frac{1}{2} \times 500 \times (750)^2$$

$$= 140625000 \text{ J}$$

Kinetic energy = **1.4×10^8 J Ans.**

13. *Determine the molecular weight of a gas if its specific heats at constant pressure and volume are $c_p = 2.286$ kJ/kg K and $c_v = 1.768$ kJ/kg K.*

Solution:

Gas constant shall be,

$$R = c_p - c_v$$

$$= 0.518 \text{ kJ/kg.K}$$

Molecular weight of gas

$$= \frac{\bar{R}}{R} = \frac{\text{Universal gas constant}}{\text{Characteristic gas constant}}$$

$$= \frac{8.3143}{0.518}$$

= **16.05 kg/k mol Ans.**

14. *A perfect gas at pressure of 750 kPa and 600 K is expanded to 2 bar pressure. Determine final temperature of gas if initial and final volume of gas are 0.2 m^3 and 0.5 m^3 respectively.*

Solution:

Initial states = 750×10^3 Pa, 600 K, 0.2 m^3

Final states = 2 bar or 2×10^5 Pa, 0.5 m^3.

Using perfect gas equation,

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\frac{750 \times 10^3 \times 0.2}{600} = \frac{2 \times 10^5 \times 0.5}{T_2}$$

$$T_2 = 400 \text{ K}$$

Final temperature = **400 K or 127° C Ans.**

15. *A vessel of 5 m^3 capacity contains air at 100 kPa and temperature of 300K. Some air is removed from vessel so as to reduce pressure and temperature to 50 kPa and 7°C respectively. Find the amount of of air removed and volume of this mass of air at initial states of air. Take R = 287 J/kg.K for air.*

Solution:

Initial states : 100×10^3 Pa, 300 K, 5 m^3

Final states : 50×10^3 Pa, 280 K, 5 m^3

Let initial and final mass of air be m_1 and m_2. From perfect gas equation of air,

$$m_1 = \frac{p_1 V_1}{RT_1}; \quad m_2 = \frac{p_2 V_2}{RT_2}$$

$$m_1 = \frac{100 \times 10^3 \times 5}{287 \times 300} \quad ; \quad m_2 = \frac{50 \times 10^3 \times 5}{287 \times 280}$$

Mass of removed, $(m_1 - m_2)$

$$= \left(\frac{100 \times 10^3 \times 5}{287 \times 300} \right) - \left(\frac{50 \times 10^3 \times 5}{287 \times 280} \right)$$

$$m_1 - m_2 = 2.696 \text{ kg}$$

Volume of this mass of air at initial states i.e 100 kPa and 300 K;

$$V = \frac{(m_1 - m_2) . RT_1}{p_1}$$

$$= \frac{2.696 \times 287 \times 300}{100 \times 10^3}$$

Volume = 2.32 m^3

Mass of air removed = **2.696 kg**

Volume of air at initial states = **2.32 m^3 Ans.**

16. *A cylindrical vessel of 1 m diameter and 4 m length has hydrogen gas at pressure of 100 kPa and 27°C. Determine the amount of heat to be supplied so as to increase gas pressure to 125 kPa. For hydrogen take C_p = 14.307 kJ/kg.K, C_v = 10.183 kJ/kg K.*

Solution:

Assuming hydrogen to be perfect gas let initial and final states be denoted by subscript 1 and 2.

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}, \text{ Here } V_2 = V_1$$

$$T_2 = \frac{p_2 V_2 \cdot T_1}{p_1 V_1} = \frac{p_2 \ T_1}{p_1} = \frac{125 \times 10^3 \times 300}{100 \times 10^3}$$

$$T_2 = 375 \text{ K}$$

As it is constant volume heating so, heat supplied,

$$Q = m \cdot C_v (T_2 - T_1)$$

From perfect gas characteristics, $R = C_p - C_v$

$$R = 4.124 \text{ kJ/kg} \cdot K$$

Mass of hydrogen, $m = \frac{p_1 V_1}{RT_1} = \frac{100 \times 10^3 \times \pi \times (0.5)^2 \times 4}{4.124 \times 10^3 \times 300}$

$$m = 0.254 \text{ kg}$$

Heat added, $Q = 0.254 \times 10.183 \times (375 - 300)$

Heat to be supplied = **193.99 kJ Ans.**

17. *Two cylindrical vessels of 2 m^3 each are inter connected through a pipe with valve in-between. Initially valve is closed and one vessel has 20 kg air while 4 kg of air is there in second vessel. Assuming the system to be at 27ºC temperature initially and perfectly insulated, determine final pressure in vessels after the valve is opened to attain equilibrium.*

Solution:

When the valve is opened then the two vessels shall be connected through pipe and transfer of air shall take place in order to attain equilibrium state. After attainment of equilibrium total mass of air shall be 24 kg.

Final total volume $= 2 \times 2 = 4 \text{ m}^3$

Using perfect gas equation.

$$pV = mRT$$

$$p = \frac{mRT}{V}$$

For air, $R = 287 \text{ J/kg K}$

Substituting values, $p = \frac{24 \times 287 \times 300}{4}$

$$= 516600 \text{ N/m}^2$$

Final pressure = **516.6 kPa Ans.**

18. *Determine the pressure of 5 kg carbon dixoide contained in a vessel of 2 m^3 capacity at 27º C, considering it as*

(i) perfect gas

(ii) real gas.

Solution:

Given : Volume, $V = 2\ m^3$, Universal gas constt. = 8.314 kJ/kg . K

Temperature, $T = 27^oC$

$= (273 + 27)\ K$

$T = 300\ K$

Mass, $m = 5\ kg$

Let pressure exerted be 'p'.

(i) Considering it as perfect gas,

$$pV = mR_{CO_2}T$$

$$R_{CO_2} = \frac{\text{Universal gas constt.}}{\text{Molecular weight of } CO_2}$$

$$R_{CO_2} = \frac{8.314\times10^3}{44.01}$$

$$R_{CO_2} = 188.9\ J/kg \cdot K$$

Substituting in perfect gas equation,

$$p = \frac{5\times188.9\times300}{2} = 141675\ N/m^2$$

Pressure = **$1.417 \times 10^5\ N/m^2$ Ans.**

(ii) Considering it as real gas let us use Vander-Waals equation;

$$\left(p + \frac{a}{\bar{v}^2}\right)(\bar{v} - b) = \bar{R}T$$

where '$\bar{v}$' is molar specific volume and constants 'a' and 'b' can be seen from Table 1.6.

$$\bar{R} = 8.314 \times 10^3$$

Molar specific volume, $\bar{v} = \frac{2\times44.01}{5}$

$$\bar{v} = 17.604\ m^3/kg \cdot mol$$

Vander-Waals Constant,

$a = 3628.5 \times 10^2\ N \cdot m^4/(kg \cdot mol)^2$

$b = 3.14 \times 10^{-2}\ m^3/kg \cdot mol$

Substituting values in Vander Waals equation,

$$\left(p + \frac{3628.5\times10^2}{(17.604)^2}\right)(17.604 - 3.14 \times 10^{-2}) = (8.314 \times 10^3 \times 300)$$

$$p + 1170.86 = 141936.879$$

$$p = 140766.019\ N/m^2$$

Pressure $= 1.408 \times 10^5\ N/m^2$

For CO_2 as perfect gas $= 1.417 \times 10^5\ N/m^2$

For CO_2 as real gas = **$1.408 \times 10^5\ N/m^2$ Ans.**

(using Vander-Waals equation)

19. *Determine the specific volume of steam at 17672 kPa and 712 K considering it as (a) perfect gas, (b) considering compressibility effects. Take critical pressure = 22.09 MPa, critical temperature =647.3 K,* R_{steam} *= 0.4615 kJ/kg·K.*

Solution:

(*a*) Considering steam as perfect gas,

$$\text{Sp. volume} = \frac{R_{\text{steam}} \cdot T}{p}$$

$$= \frac{0.4615 \times 712}{17672}$$

Specific volume = **0.0186 m³/kg Ans.**

(*b*) Considering compressibility effects, the specific volume can be given by product of compressibility factor '*Z*' and "specific volume when perfect gas".

$$\text{Reduced pressure} = \frac{p}{\text{Critical pressure}}$$

$$= \frac{17672}{22.09 \times 10^3}$$

Reduced pressure = 0.8

$$\text{Reduced temperature} = \frac{T}{\text{Critical temperature}}$$

$$= \frac{712}{647.3}$$

Reduced temperature = 1.1

From generalized compressibility chart compressibility factor '*Z*' can be seen for reduced pressure and reduced temperatures of 0.8 and 1.1. We get,

Z = 0.785

Actual specific volume = 0.785 × 0.0186

= **0.0146 m³/kg. Ans.**

20. *A spherical balloon of 5 m diameter is filled with Hydrogen at 27ºC and atmospheric pressure of 1.013 bar. It is supposed to lift some load if the surrounding air is at 17ºC. Estimate the maximum load that can be lifted.*

Solution:

Balloon filled with H_2 shall be capable of lifting some load due to buoyant forces.

$$\text{Volume of balloon} = \frac{4}{3} . \pi . \left(\frac{5}{2}\right)^3 = 65.45 \text{ m}^3$$

Mass of H_2 in balloon can be estimated considering it as perfect gas.

$$\text{Gas constant for } H_2 = \frac{8.314 \times 10^3}{2} = 4.157 \times 10^3 \text{ J/kg} \cdot \text{K}$$

$$\text{Mass of } H_2 \text{ in balloon} = \frac{P_{\text{balloon}} \cdot V_{\text{balloon}}}{R_{H_2} \cdot T_{H_2}}$$

$$= \frac{1.013 \times 10^5 \times 65.45}{4.157 \times 10^3 \times 300}$$

$$m_{H_2} = 5.316 \text{ kg}$$

Volume of air displaced = Volume of balloon
= 65.45 m^3

$$\text{Mass of air displaced} = \frac{1.013 \times 10^5 \times 65.45}{R_{\text{air}} \times (17 + 273)}$$

$$R_{\text{air}} = 0.287 \text{ kJ/kg . K}$$

$$m_{\text{air}} = \frac{1.013 \times 10^5 \times 65.45}{0.287 \times 10^3 \times 290}$$

$$m_{\text{air}} = 79.66 \text{ kg}$$

Load lifting capacity due to buoyant force = $m_{\text{air}} - m_{H_2}$
= 79.66 – 5.316
= **74.344 kg Ans.**

21. *A pump draws air from large air vessel of 20 m^3 at the rate of 0.25 m^3/min. If air is initially at atmospheric pressure and temperature inside receiver remains constant then determine time required to reduce the receiver pressure to* $\frac{1}{4}$ *th of its original value.*

Solution:

Let volume of receiver be *V, m*^{3} and volume sucking rate of pump be *v* m^3/min, then theoretically problem can be modelled using perfect gas equation. Here *p* is pressure in receiver and *T* is temperature in vessel.

$$pV = mRT$$

Here pressure '*p*' and temperature '*T*' shall change with respect to time *t*. Differentiating perfect gas equation with respect to time.

$$V \cdot \frac{dp}{dt} = RT \frac{dm}{dt}$$

Here $\left(\frac{dm}{dt}\right)$ is mass extraction rate from receiver in kg/min. This mass rate can be given using perfect gas equation when volume flow rate (m^3/min) is given as *v*. So.

$$\frac{dm}{dt} = -\frac{pv}{RT} \text{ (–ve as mass gets reduced with time)}$$

Substituting,

$$V \cdot \frac{dp}{dt} = -RT \cdot \frac{pv}{RT}$$

$$V \cdot \frac{dp}{dt} = -pv$$

$$\int_0^t dt = -\frac{V}{v}\int_0^t \frac{dp}{p}$$

$$t = -\frac{V}{v} \ln\left(\frac{p_2}{p_1}\right)$$

Here final pressure, $p_2 = \frac{p_1}{4}$, $V = 20\ m^2$, $v = 0.25\ m^3/min$

So time, $$t = \frac{V}{v} \ln\left(\frac{1}{4}\right)$$

$$\text{time} = \frac{20}{0.25} \ln(4) = 110.9 \text{ minutes}$$

$$= \textbf{110.9 minutes} \quad \textbf{Ans.}$$

22. *In 5 kg mixture of gases at 1.013 bar and 300 K the various constituent gases are as follows, 80% N_2, 18% O_2, 2% CO_2.*

Determine the specific heat at constant pressure, gas constant for the constituents and mixture and also molar mass of mixture taking $\gamma = 1.4$ for N_2 and O_2 and $\gamma = 1.3$ for CO_2.

Universal gas constant = 8314 J/kg · K

Solution:

Gas constants for constituent gases shall be,

$$R_{N_2} = \frac{8314}{\text{mol. wt. of } N_2} = \frac{8314}{28} = \textbf{296.9 J/kg · K}$$

$$R_{O_2} = \frac{8314}{\text{mol. wt. of } O_2} = \frac{8314}{32} = \textbf{259.8 J/kg · K}$$

$$R_{CO_2} = \frac{8314}{\text{mol. wt. of } CO_2} = \frac{8314}{44} = \textbf{188.9 J/kg . K}$$

Gas constant for mixture, R_{mixture}

$$= \left(\frac{m_{N_2}}{M}\right).R_{N_2} + \left(\frac{m_{O_2}}{M}\right).R_{O_2} + \left(\frac{m_{CO_2}}{M}\right).R_{CO_2}$$

$$= (0.80 \times 296.9) + (0.18 \times 259.8) + (0.02 \times 188.9)$$

$$R_{\text{mixture}} = \textbf{288.06 J/kg . K}$$

Specific heat at constant pressure for constituent gases.

$$C_{p,\,N_2} = \left(\frac{\gamma}{\gamma - 1}\right).R_{N_2} = \left(\frac{1.4}{0.4}\right) \times 296.9 = \textbf{1.039 kJ/kg . K}$$

$$C_{p,O_2} = \left(\frac{\gamma}{\gamma-1}\right).R_{O_2} = \left(\frac{1.4}{0.4}\right) \times 259.8 = \textbf{0.909 kJ/kg . K}$$

$$C_{p,CO_2} = \left(\frac{\gamma}{\gamma-1}\right).R_{CO_2} = \left(\frac{1.3}{0.3}\right) \times 188.9 = \textbf{0.819 kJ/kg . K}$$

$$C_{p,\text{mixture}} = \left(\frac{m_{N_2}}{M}\right).C_{P,N_2} + \left(\frac{m_{O_2}}{M}\right).C_{P,O_2} + \left(\frac{m_{CO_2}}{M}\right).C_{P,CO_2}$$

$$C_{p,\text{mixture}} = (0.80 \times 1.039) + (0.18 \times 0.909) + (0.02 \times 0.819)$$

$$= \textbf{1.0276 kJ/kg . K}$$

Molar mass of mixture $= \sum x_i \,.\, M_i = \dfrac{\text{Total mass of mixture}}{\text{Total no. of moles}}$

$$x_i = \frac{n_i}{\sum n_i}, \text{ here } M_i = \text{mol. wt. of } i\text{th constituent.}$$

No. of moles of constituent gases,

$$n_{N_2} = \frac{m_{N_2}}{\text{Mol. wt. } N_2} = \frac{0.8 \times 5}{28} = 0.143$$

$$n_{O_2} = \frac{m_{O_2}}{\text{Mol. wt. } O_2} = \frac{0.18 \times 5}{32} = 0.028$$

$$n_{CO_2} = \frac{m_{CO_2}}{\text{Mol. wt. } CO_2} = \frac{0.02 \times 5}{44} = 0.0023$$

Total mole numbers in mixture $= n_{N_2} + n_{O_2} + n_{CO_2}$

$= (0.143 + 0.028 + 0.0023)$

$\sum n_i = 0.1733$

Mole fractions of constituent gases,

$$x_{N_2} = \frac{n_{N_2}}{\sum n_i} = \frac{0.143}{0.1733} = 0.825$$

$$x_{O_2} = \frac{n_{O_2}}{\sum n_i} = \frac{0.028}{0.1733} = 0.162$$

$$x_{CO_2} = \frac{n_{CO_2}}{\sum n_i} = \frac{0.0023}{0.1733} = 0.0133$$

Molecular wt. of mixture = Molar mass of mixture $= \sum x_i \,.\, M_i$

$= (0.825 \times 28) + (0.162 \times 32) + (0.0133 \times 44)$

$= \textbf{28.87 kg/kmol}$ **Ans.**

23. *A gas mixture comprises of 18% O_2, 75% N_2 and 7% CO_2 by volume at 0.5 MPa and 107ºC. For 5 kg mass of mixture carry out gravimetric analysis and determine the partial pressure of gases in mixture.*

Solution:

Mole fraction of constituents $\Rightarrow x_i = \dfrac{n_i}{n} = \dfrac{V_i}{V}$

where 'n_i' and 'V_i' are no. of moles and volume fraction of constituent while 'n' and V are total no. of moles and total volume of mixture.

$$x_{O_2} = \frac{0.18}{1} = 0.18$$

$$x_{N_2} = \frac{0.75}{1} = 0.75$$

$$x_{CO_2} = \frac{0.07}{1} = 0.07$$

Molecular weight of mixture = Molar mass
$= (0.18 \times 32) + (0.75 \times 28) + (0.07 \times 44)$
$= 29.84$

Gravimetric analysis refers to the mass fraction analysis.

Mass fraction of constituents

$$= \frac{m_i}{m} = \frac{(V_i / V) \times \text{Mol. wt. of constituent}}{\text{Mol. wt. of mixture}}$$

Mole fraction of O_2 $= \dfrac{0.18 \times 32}{29.84} =$ **0.193**

Mole fraction of N_2 $= \dfrac{0.75 \times 28}{29.84} =$ **0.704** } **Ans.**

Mole fraction of CO_2 $= \dfrac{0.07 \times 44}{29.84} =$ **0.104**

Partial pressures of constituents = Volume fraction × Pressure of mixture

Partial pressure of O_2 = 0.18 × 0.5 = **0.09 MPa**
Partial pressure of N_2 = 0.75 × 0.5 = **0.375 MPa** } **Ans.**
Partial pressure of CO_2 = 0.07 × 0.5 = **0.35 MPa**

24. *A steel insulated tank of 6 m^3 volume is equally divided into two chambers using a partition. The two portions of tank contain N_2 gas at 800 kPa and 480 K and CO_2 gas at 400 kPa and 390 K. Determine the equilibrium temperature and pressure of mixture after removing the partition. Use $\gamma = 1.4$ for N_2, $\gamma = 1.3$ for CO_2.*

Solution:

Since tank is insulated so adiabatic mixing can be considered. Let us consider N_2 and CO_2 to behave as perfect gas.

No. of moles of N_2

$$n_{N_2} = \frac{p_{N_2} \cdot V_{N_2}}{\bar{R} \cdot T_{N_2}} = \frac{800 \times 10^3 \times 3}{8314 \times 480} = 0.601$$

No of moles of CO_2

$$n_{CO_2} = \frac{p_{CO_2} \cdot V_{CO_2}}{\bar{R} \cdot T_{CO_2}} = \frac{400 \times 10^3 \times 3}{8314 \times 390} = 0.370$$

Total no. of moles of mixture,

$$\begin{aligned} n &= n_{N_2} + n_{CO_2} \\ &= 0.601 + 0.370 \\ &= 0.971 \end{aligned}$$

Specific heat for N_2 at constant volume,

$$C_{v,\,N_2} = \frac{R_{N_2}}{(\gamma_{N_2} - 1)} = \frac{(8314/28)}{(1.4 - 1)}$$

$$C_{v,\,N_2} = 742.32 \text{ J/kg . K}$$

Specific heat for CO_2 at constant volume,

$$C_{v,\,CO_2} = \frac{R_{CO_2}}{(\gamma_{CO_2} - 1)} = \frac{(8314/44)}{(1.3 - 1)}$$

$$C_{v,\,CO_2} = 629.85 \text{ J/kg . K}$$

Mass of N_2 = n_{N_2} × Mol. wt. of N_2 = 0.601 × 28 = 16.828 kg

Mass of CO_2 = n_{CO_2} × Mol. wt. of CO_2 = 0.370 × 44 = 16.28 kg.

Let us consider the equilibrium temperature of mixture after adiabatic mixing at *T.* Applying energy conservation principle :

$$m_{N_2} . C_{v,\,N_2} . (T - T_{N_2}) + m_{CO_2} . C_{v,\,CO_2} . (T - T_{CO_2}) = 0$$

$\{16.828 \times 742.32 (T - 480)\} + \{16.28 \times 629.85 (T - 390)\} = 0$

$22745.7 . T = 9995088.881$

Equilibrium temperature, **T = 439.4 K Ans.**

Equilibrium pressure of mixture, $T_{mixture}$ = 439.4 K, $V_{mixture}$ = 6 m^3

$$p_{mixture} = \frac{n.\bar{R}.T_{mixture}}{V_{mixture}} = \frac{0.971 \times 8314 \times 439.4}{6}$$

Equilibrium pressure = **591.205 kPa Ans.**

25. *2 kg of Hydrogen and 3 kg of Helium are mixed together in an insulated container at atmospheric pressure and 100 K temperature. Determine the specific heat of final mixture if specific heat at constant pressure is 11.23 kJ/kg. K and 5.193 kJ/kg . K for H_2 and He respectively.*

Solution:

Two gases are non reacting, therefore, specific heat of final mixture can be obtained by following

for adiabatic mixing.

$$C_{p,\text{ mixture}} = \frac{c_{p,H_2}.m_{H_2} + c_{p,He}.m_{He}}{(m_{H_2} + m_{He})}$$

Substituting values,

$$= \frac{(2\times 11.23) + (3\times 5.193)}{(3+2)}$$

$$C_{p,\text{ mixture}} = \textbf{7.608 kJ/kg . K Ans.}$$

26. *A mixture of 18 kg hydrogen, 10 kg nitrogen and 2 kg of carbon dioxide is put into a vessel at atmospheric conditions. Determine the capacity of vessel and the pressure in vessel if it is heated upto twice of initial temperature. Take ambient temperature as 27ºC.*

Solution:

Gas constant for mixture can be obtained as;

$$R_{\text{mixture}} = \frac{(m_{H_2}.R_{H_2} + m_{N_2}.R_{N_2} + m_{CO_2}.R_{CO_2})}{(m_{H_2} + m_{N_2} + m_{CO_2})}$$

$$R_{H_2} = \frac{8.314}{2} \text{ kJ/kg . K}$$

$$R_{N_2} = \frac{8.314}{28} \text{ kJ/kg . K}$$

$$R_{CO_2} = \frac{8.314}{44} \text{ kJ/kg . K}$$

$$R_{H_2} = 4.15 \text{ kJ/kg . K}$$

$$R_{N_2} = 0.297 \text{ kJ/kg . K}$$

$$R_{CO_2} = 0.189 \text{ kJ/kg . K}$$

$$R_{\text{mixture}} = \frac{(18\times 4.15 + 10\times 0.297 + 2\times 0.189)}{30}$$

$$R_{\text{mixture}} = 2.606 \text{ kJ/kg . K}$$

Considering mixture to be perfect gas;

Capacity of vessel $V_{\text{mixture}} = \dfrac{m_{\text{mixture}}.R_{\text{mixture}}.T}{p}$

Here, $p = 101.325$ kPa

$$V_{\text{mixture}} = \frac{30\times 2.606\times 300.15}{101.325}$$

Capacity of vessel = **231.58 m^3 Ans.**

For constant volume heating, final pressure shall be,

$$p_{final} = p_{initial} \times \frac{T_{final}}{T_{initial}}$$

$$p_{final} = 101.325 \times 2 = \textbf{202.65 kPa} \quad \textbf{Ans.}$$

27. *Determine the ratio of exit to inlet diameter of a duct having heating element in the mid of duct. Atmospheric air enters the duct at 27°C and gets heated up to 500 K before leaving the duct. The kinetic and potential energy changes during passage of air through duct are negligible.*

Solution:

Said air heating process is the case of constant pressure process. Let inlet state be '1' and exit state '2'.

Therefore, by Charle's law volume and temperature can be related as;

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}$$

or
$$\frac{\left(\frac{\pi}{4} \times d_2^2\right) \times \text{Velocity at 2}}{\left(\frac{\pi}{4} \times d_1^2\right) \times \text{Velocity at 1}} = \frac{T_2}{T_1}$$

Since $\Delta \text{K.E} = 0$, so $\frac{d_2^2}{d_1^2} = \frac{T_2}{T_1}$

or
$$\frac{d_2}{d_1} = \sqrt{\frac{T_2}{T_1}}$$

Exit to inlet diameter ratio $= \sqrt{\frac{500}{300.15}} = 1.29 = \textbf{1.29}$ **Ans.**

28. *A vessel of 2 m^3 volume contains hydrogen at atmospheric pressure and 27°C temperature. An evacuating pump is connected to vessel and the evacuation process is continued till its pressure becomes 70 cm of Hg vacuum.*

Estimate the mass of hydrogen pumped out. Also determine the final pressure in vessel if cooling is carried up to 10°C. Take atmospheric pressure as 76 cm of Hg and universal gas constant as 8.314 kJ/kg. K

Solution:

For hydrogen, gas constant, $R = \frac{8.314}{2}$

$$R = 4.157 \text{ kJ/kg . K}$$

Say initial and final states are given by '1' and '2'.

Mass of hydrogen pumped out shall be difference of initial and final mass inside vessel.

Final pressure of hydrogen = Atm. pr. – Vacuum pr.

$= 76 - 70$

$= 6$ cm of Hg.

Therefore, pressure difference $= 76 - 6$

$= 70$ cm of Hg.

$$= \frac{70}{76} \times 101.325 \text{ kPa}$$

$= 93.33$ kPa

$$\text{Mass pumped out} = \frac{p_1V_1}{RT_1} - \frac{p_2V_2}{RT_2}\text{; here } V_1 = V_2 = V \text{ and } T_1 = T_2 = T.$$

$$= \frac{V}{RT}(p_1 - p_2)$$

$$= \frac{2 \times 93.33 \times 10^3}{4.157 \times 300.15 \times 10^3}$$

$=$ **0.15 kg. Ans.**

During cooling upto 10ºC, the process may be considered as constant volume process. Say the state before and after cooling are denoted by suffix 2 and 3.

Therefore, $$p_3 = \frac{T_3}{T_2} \cdot p_2$$

$$= \frac{283.15}{300.15} \times \frac{6 \times 101.325}{76}$$

Final pressure after cooling $=$ **7.546 kPa. Ans.**

EXERCISE

1.1 Define thermodynamics and discuss different approaches to study of thermodynamics.

1.2 Write short notes on the following:

Thermodynamic properties, state, path, process, closed system, isolated system, open system, extensive and intensive properties.

1.3 What is meant by quasi-static process? Also discuss its physical significance.

1.4 Describe thermodynamic equilibrium of a system.

1.5 State thermodynamic definition of work. Also differentiate between heat and work.

1.6 What is energy? What are different forms of it?

1.7 Explain the concept of continuum.

1.8 Define perfect gas.

1.9 Differentiate between characteristic gas constant and universal gas constant.

1.10 State the Dalton's law of partial pressures and assumptions for it.

1.11 What is meant by real gas? Why ideal equation of state cannot be used for it?

1.12 Write equations of state for real gas.

1.13 Define conpressibility factor.

1.14 Write Boyle's law and Charle's law.

1.15 Determine the absolute pressure of gas in a tank if the pressure gauge mounted on tank reads 120 kPa pressure. [221.3 kPa]

1.16 What shall be the volume of a fluid having its specific gravity as 0.0006 and mass as 10 kg? [16.67 m^3]

1.17 Determine the pressure of compressed air in an air vessel, if the manometer mounted on it shows a pressure of 3 m of mercury. Assume density of mercury to be 13.6×10^3 kg/m^3 and atmospheric pressure as 101 kPa. [501.25 kPa]

1.18 Calculate the kinetic energy of a satellite revolving around the earth with a speed of 1 *km/s*. Assume acceleration due to gravity as 9.91 m/s^2 and gravitational force of 5 kN. [254.8 MJ]

1.19 If the gauge pressure of oil in a tube is 6.275 kPa and oil's specific gravity is 0.8, then determine depth of oil inside tube. [80 cm]

1.20 Determine the work required for displacing a block by 50 m and a force of 5 kN. [250 kJ]

1.21 Determine the barometer reading in millimetres of Hg if the vacuum measured on a condenser is 74.5 cm of Hg and absolute pressure is 2.262 kPa. [760 mm]

1.22 Determine the absolute pressures for the following;

(i) Gauge pressure of 1.4 MPa

(ii) Vacuum pressure of 94.7 kPa

Take barometric pressure as 77.2 cm of Hg and density of mercury as 13.6×10^3 kg/m^3. [1.5 MPa, 8.3 kPa]

1.23 Determine the pressure acting upon surface of a vessel at 200 m deep from surface of sea. Take barometric pressure as 101 kPa and specific gravity of sea water as 1.025. [2.11 MPa]

1.24 A vacuum gauge gives pressure in a vessel as 0.1 bar, vacuum. Find absolute pressure within vessel in bars. Take atmospheric pressure as 76 cm of mercury column, g = 9.8 m/s^2, density of mercury = 13.6 g/cm^3. [0.91 bar]

1.25 Determine the work done upon a spring having spring constant of 50 kN/m. Spring is stretched to 0.1 m from its unstretched length of 0.05 m. [0.0625 kJ]

1.26 Determine the mass of oxygen contained in a tank of 0.042 m^3 at 298 K and 1.5×10^7 Pa considering it as perfect gas. Also determine the mass using compressibility charts. [8.25, 8.84]

1.27 What will be specific volume of water vapour at 1 MPa and 523 K, if it behaves as ideal gas? Also determine the same considering generalized compressibility chart. [0.241 m^3/kg, 0.234 m^3/kg]

1.28 Calculate the pressure of CO_2 gas at 27ºC and 0.004 m^3/kg treating it as ideal gas. Also determine the pressure using Van der Waals equation of state. [14.17 MPa, 6.9 MPa]

1.29 Determine molecular weight and gas constant for a mixture of gases having 65% N_2, 35% CO_2 by mole. [33.6 kg/k mol. 0.247 kJ/kg . K]

1.30 Considering air as a mixture of 78% N_2, 22% O_2 by volume determine gas constant, molecular weight, C_p and C_v for air at 25ºC. [0.2879 kJ/kg . K, 28.88 kg/K mol, 1.0106 kJ/kg . K, 0.722 kJ/kg . K]

1.31 What minimum volume of tank shall be required to store 8 kmol and 4 kmol of O_2 and CO_2 respectively at 0.2 MPa, 27ºC ? [149.7 m^3]

1.32 Two tanks *A* and *B* containing O_2 and CO_2 have volumes of 2 m^3 and 4 m^3 respectively. Tank *A* is at 0.6 MPa, 37ºC and tank *B* is at 0.1 MPa and 17ºC. Two tanks are connected through some pipe so as to allow for adiabatic mixing of two gases. Determine final pressure and temperature of mixture. [0.266 MPa, 30.6ºC]

1.33 Determine the molecular weight and gas constant for some gas having C_p = 1.968 kJ/kg . K, C_v = 1.507 kJ/kg . K. [18.04 kg/kmol, 0.461 kJ/kg . K]

2

Zeroth Law of Thermodynamics

2.1 INTRODUCTION

Thermodynamics is the branch of science which deals with the energy interactions. In order to find whether energy interactions are taking place or not some measurable mathematical parameters are needed. These parameters are called thermodynamic properties. Out of number of thermodynamic properties discussed earlier the 'temperature' is one property.

One is well familiar with the qualitative statement of the state of a system such as cold, hot, too cold, too hot etc. based on the day to day experience. The degree of hotness or coldness is relative to the state of observer. For example, let us take an iron bar. Obviously the bar shall have intial temperature equal to the room temperature. Now let us heat this metal bar. Observations at the molecular level show that upon heating the molecular activity inside the bar gets increased. This may be attributed to the more agitated state of molecules as energy is given to them in the form of heating of the bar. From the physiological sensations it can be felt that this has resulted in increase in the degree of hotness of the bar. This qualitative indication of the relative hotness can be exactly defined by using thermodynamic property known as *temperature*. If this hot bar is brought in contact with another bar at room temperature one can feel that after some time the two bars which were initially at high and low temperatures attain the same temperature which is lying between the two temperatures. It is indicative of the fact that there has been exchange of some entity between two bars resulting in the attainment of final equilibrium temperature. This state of attainment of common equilibrium temperature is also termed as the state of thermal equilibrium. Thus, the temperature becomes a potential indicator of the energy interactions in the systems.

A look at the history shows that for quantitative estimation of temperature a German instrument maker Mr. Gabriel Daniel Fahrenheit (1686-1736) came up with idea of instrument like thermometer and developed mercury in glass thermometer. Here he observed that height of mercury column used to change as the bulb of thermometer was brought in the environments having different degrees of hotness. In the year 1742, a Swedish astronomer Mr. Anders Celsius described a scale for temperature measurement. This scale later on became very popular and is known as *Centigrade Scale*. For caliberation of these measuring instruments some reference states of different substances were used initially and the relative state of temperature of the substance could be quantified. Later on with the passage of time things were standardised and internationally acceptable temperature scales and instruments were developed.

2.2 PRINCIPLE OF TEMPERATURE MEASUREMENT AND ZEROTH LAW OF THERMODYNAMICS

After the identification of 'Temperature' as a thermodynamic property for quantification of the energy interactions the big question was its estimation. Based on the relative degree of coldness/hotness concept it was concluded that the absolute value of temperature is difficult to be described. Hence it was mooted

to make temperature estimations in reference to certain widely acceptable known thermal states of the substances. Temperature is thus the intensive parameter and requires reference states. These acceptable known thermal states are such as the boiling point of water commonly called steam point, freezing point of water commonly called ice point etc. These easily reproducible and universally acceptable states of the substance are known as *reference states* and the temperature values assigned to them are called *reference temperatures.* Since these reference points and reference temperatures maintain their constant value, therefore these are also called *fixed points* and *fixed temperatures* respectively. A list of these fixed points is given in Table 2.1.

Table 2.1 *Some fixed points used for International Practical Temperature Scale*

Sl. No.	*Reference State*	*Temperature* °C
1.	Ice point	0
2.	Steam point	100
3.	Triple point of water	0.010
4.	Triple point of hydrogen	–259.34
5.	Triple point of oxygen	–218.79
6.	Oxygen point (normal boiling point)	–182.96
7.	Silver point (normal freezing point)	961.93
8.	Gold point (normal freezing point)	1064.43
9.	Zinc point (normal freezing point)	419.58
10.	Neon point (normal boiling point)	–246.05
11.	Sulphur point (normal boiling point)	444.60

The methodology adopted was to first develop a temperature measurement system which could show some change in its characteristics (property) due to heat interactions taking place with it. Such systems are called *thermometers*, the characteristics of property which shows change in its value is termed *thermometric property* and the substance which shows change in its thermometric property is called *thermometric substance.* Science that deals with the temperature and its measurement is called *thermometry.* For example in case of *clinical thermometer* the *mercury* in glass is the *thermometric substance* and since there is change in length of mercury column due to the heat interactions taking place between the thermometer and the body whose temperature is to be measured, therefore the *length* is the *thermometric property*. Thus, the underlying principle of temperature measurement is to bring the thermometer in thermal equilibrium with the body whose temperature is to be measured, i.e. when there is no heat interaction or the state when two (thermometer and body) attain same temperature. In this process it is to be noted that thermometer is already caliberated using some standard reference points by bringing thermometer in thermal equilibrium with reference states of the substance.

Zeroth law of thermodynamics states that if the bodies A and B are in thermal equilibrium with a third body C separately then the two bodies A and B shall also be in thermal equilibrium with each other. This is the principle of temperature measurement. Block diagram shown in Fig. 2.1*a* and 2.1*b* show the zeroth law of thermodynamics and its application for temperature measurement respectively.

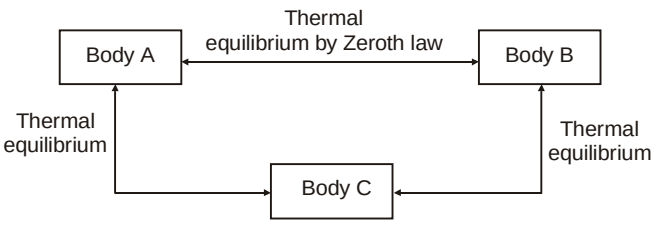


Fig. 2.1a *Zeroth law of thermodynamics*

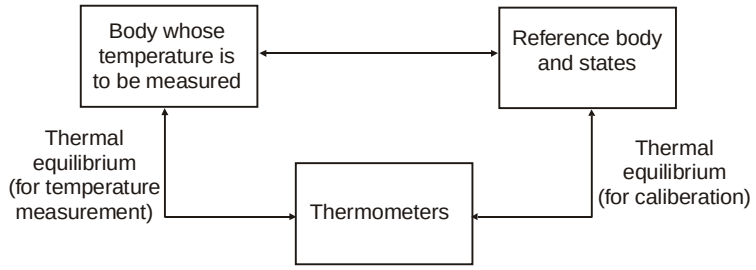


Fig. 2.1b *Application of Zeroth law for temperature measurement*

2.3 TEMPERATURE SCALES

Number of temperature measuring scales came up from time to time. The text ahead gives a brief idea of the different temperature scales used in thermometry. Different temperature scales have different names based on the names of persons who originated them and have different numerical values assigned to the reference states.

(*a*) Celsius Scale or Centigrade Scale

Anders Celsius gave this Celsius or Centigrade scale using ice point of 0°C as the lower fixed point and steam point of 100ºC as upper fixed point for developing the scale. It is denoted by letter *C*. Ice point refers to the temperature at which freezing of water takes place at standard atmospheric pressure. Steam point refers to the temperature of water at which its vaporization takes place at standard atmospheric pressure. The interval between the two fixed points was equally divided into 100 equal parts and each part represented 1ºC or 1 degree celsius.

(*b*) Fahrenheit Scale

Fahrenheit gave another temperature scale known as Fahrenheit scale and has the lower fixed point as 32 F and the upper fixed point as 212 F. The interval between these two is equally divided into 180 part. It is denoted by letter F. Each part represents 1 F.

(*c*) Rankine Scale

Rankine scale was developed by William John MacQuorn Rankine, a Scottish engineer. It is denoted by letter *R*. It is related to Fahrenheit scale as given below.

$$T_R = T_F + 459.67$$

(*d*) Kelvin Scale

Kelvin scale proposed by Lord Kelvin is very commonly used in thermodynamic analysis. It also defines the absolute zero temperature. Zero degree Kelvin or absolute zero temperature is taken as –273.15ºC. It is denoted by letter K.

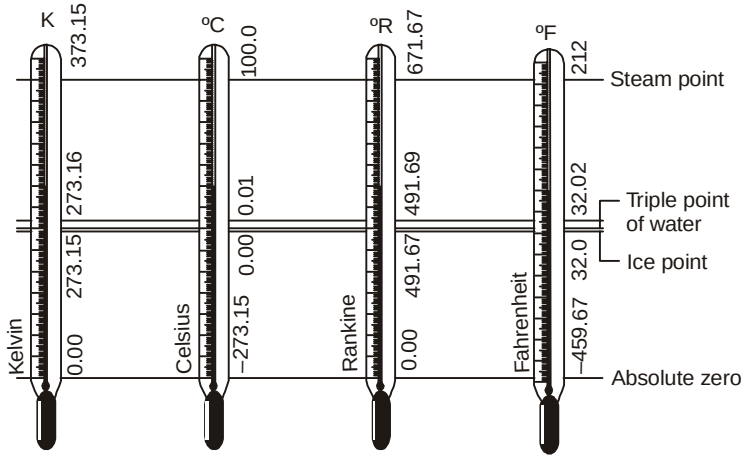


Fig. 2.2 *Different temperature scales*

Detailed discussion on Kelvin scale has been done in chapter 4 along with absolute thermodynamic temperature scale. Mathematically, it is related to the different temperature scales as follows,

$$\frac{T_C}{100} = \frac{T_F - 32}{180} = \frac{T_K - 273.15}{100} = \frac{T_R - 491.67}{180}$$

$$\frac{T_K}{100} = \frac{T_R}{180}$$

2.4 TEMPERATURE MEASUREMENT

For measurement of temperature number of thermometers are available using different thermometric properties of the thermometric substances. Length, volume, pressure, resistance, e.m.f. etc. are the commonly used thermometric properties for thermometers. Different thermometers developed using these thermometric properties are given below.

(*a*) Liquid Thermometer

Liquid thermometers are those thermometers that employ liquids as the thermometric substance and the change in volume of liquid with heat interaction is the characteristics used for temperature measurement. Commonly used liquids in such thermometers are Mercury and Alcohol. Fig. 2.3 shows the mercury in glass thermometer. In this the change in volume of the mercury results in the rise or fall in the level of mercury column in the glass tube. Out of the two liquids mercury is preferred over alcohol as it has low specific heat and hence absorbs little heat from body. Mercury is comparatively a good conductor of heat. Mercury can be seen in a fine capillary tube conveniently. Mercury does not wet the wall of the tube. Mercury has a uniform coefficient of expansion over a wide range of temperature and remains liquid over a large range as its freezing and boiling points are –39ºC and 357°C respectively.

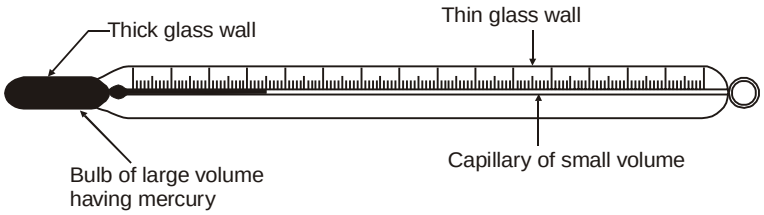


Fig. 2.3 *Mercury in glass thermometer*

(*b*) Gas Thermometers

Thermometers using gaseous thermometric substance are called gas thermometers. Gas thermometers are advantageous over the liquid thermometers as the coefficient of expansion of gases is more compared to liquids therefore these are more sensitive. Also thermal capacity of a gas is low compared to liquid so even a small change can also be recorded accurately. Gas thermometers are not suitable for routine work as they are large, cumbersome and can be used only in certain fixed conditions. These are used mainly for calibration and standardization purpose. Main types of gas thermometers are discussed ahead.

(*i*) *Constant volume gas thermometer* : Fig. 2.4 shows a typical constant volume gas thermometer having a glass bulb '*B*' connected to glass tube. Other end of glass tube is connected to mercury reservoir through a rubber tube. There is a fixed marking '*M*' over the glass tube. Difference in levels of mercury in reservoir with reference to mark '*M*' is seen on the scale. Bulb '*B*' is generally filled with 1/7th of its volume by mercury so as to compensate for expansion of bulb '*B*'. This is done so as to keep volume of air in bulb upto the fixed mark '*M*'.

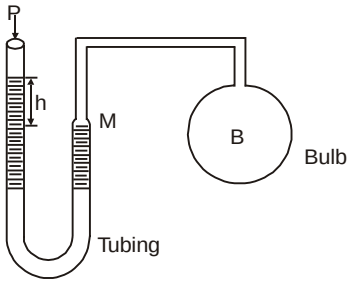


Fig. 2.4 *Constant volume gas thermometer*

Initially the bulb '*B*' is kept in melting ice and reservoir level is suitably adjusted so that mercury level is at mark '*M*'. Corresponding to this difference in level of reservoir and mark '*M*' of h_i height, the ice point pressure shall be,

$$P_i = P + (h_i \cdot \rho \cdot g) = P_0$$

Bulb is kept at the boiling water (steam point) and again the reservoir is adjusted so as to keep mercury at the fixed mark. For difference in mercury levels between mark '*M*' and reservoir level being h_s the pressure corresponding to steam point shall be

$$P_s = P + (h_s \cdot \rho \cdot g) = P_{100}$$

Now for the bulb '*B*' kept in the bath whose temperature is to be measured, again the reservoir is to be adjusted so as to keep mercury level at mark '*M*'. At this state if the difference in mercury levels is h_t, then the pressure shall be,

$$P_t = P + (h_t \cdot \rho \cdot g)$$

For a fixed volume, the pressure variation with respect to temperature can be given as,

$$P = P_0 (1 + \alpha \cdot t)$$

Similarly making appropriate substitutions one can give the temperature t as follows,

$$t = \frac{(h_t - h_i) \times 100}{(h_s - h_i)}$$

(*ii*) *Constant pressure gas thermometer* : These thermometers are based on the principle that, pressure remaining constant the volume of a given mass of gas is directly proportional to its absolute temperature. Fig. 2.5 shows a constant pressure gas thermometer having a silica bulb 'B' connected to

the reservoir '*R*' containing mercury through a *connecting tube* '*A*', *compensating bulb* '*C*' having a compensating tube with volume equal to the connecting tube. Manometer tube contains sulphuric acid.

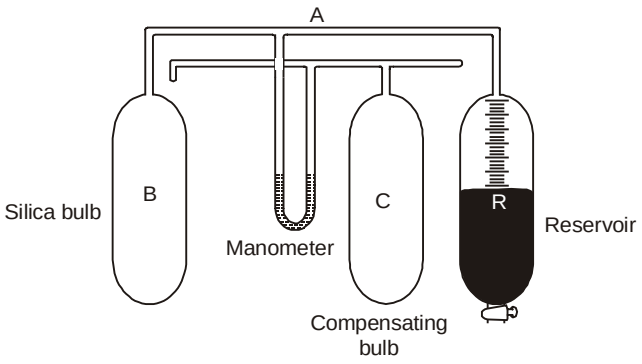


Fig. 2.5 *Constant pressure gas thermometer*

Initially the reservoir is filled with mercury upto zero marking and the stop cock is closed. The bulbs 'B' 'R' and 'C' are immersed in melting ice. Tubes are sealed when the pressure on the two sides as shown by manometer is the same i.e. the pressure in silica bulb 'B' and compensating bulb 'C' are same. When the pressure on two sides of the manometer containing sulphuric acid is same the acid level in two limbs shall be same. This way the pressure of gas and air can be maintained same. Now let us assume the silica bulb 'B' to have definite number of molecules of air. Also, the compensating bulb and compensating tube contain the same number of molecules of air. If the silica bulb is immersed in the environment whose temperature is to be measured and compensating bulb being kept in melting ice. Both connecting tube and compensating tubes are at the room temperature and the air in silica bulb attains temperature equal to the temperature to be measured.

(*c*) Electrical resistance thermometer

Electrical resistance thermometer first developed by Siemen in 1871, also known as 'Platinum Resistance Thermometer' works on the principle of change in resistance of the thermometric substance (platinum) with temperature. Thus resistance is the thermometric property used in these thermometers. It consists of a pure platinum wire wound in a double spiral on a mica plate. Two ends of the platinum wire are connected to the copper leads (for low temperatures) or platinum leads (for high temperatures). Principle of Wheatstone bridge is employed in these thermometers, as shown in Fig. 2.6. It has a set of compensating leads having exactly similar resistance as leads used. Platinum wire and the compensating leads are enclosed in a sealed glazed porcelain tube having binding terminals at the top. The resistance of wire can be mathematically related as

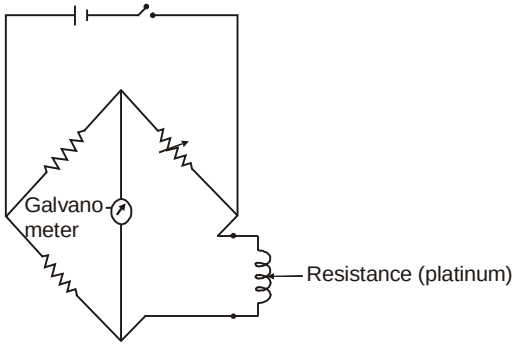


Fig. 2.6 *Electrical resistance thermometer, (principle of wheatstone bridge)*

$$R_t = R_0 \cdot (1 + a \cdot t + b \cdot t^2)$$

where a and b are the constants having their values depending upon the nature of material used.

Using fixed points of ice point and steam point the temperature can be mathematically obtained by substituting the different parameters in the following,

$$t = \frac{(R_t - R_i)\times 100}{(R_s - R_i)},$$

where R_i & R_s are resistance values for ice and steam points

(*d*) Thermoelectric Thermometer

Thermo electric thermometer works on the principle of Seebeck effect. *Seebeck effect* says that a current flows or e.m.f. is produced in a circuit of two dissimilar metals having one junction as hot while other as cold junction. Current produced in this way is called thermo electric current while the e.m.f. produced is called thermo e.m.f. Measurement of temperature is being done by knowing the e.m.f. produced which is the thermometric property here.

In such type of thermometer a sensitive galvanometer is connected with thermocouple as shown in Fig. 2.7. One junction is kept at ice point and other in oil bath having any temperature. Upon heating the oil bath it is seen that the thermal e.m.f. is produced by Seebeck effect. Temperature of the oil bath is measured by some calibrated thermometer of any other type. Further the temperature of oil bath is changed to known temperatures and for different temperatures the e.m.f. is noted and a graph is plotted between the temperature of bath and e.m.f.

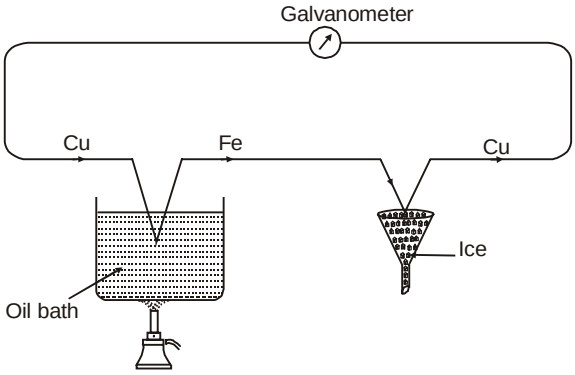


***Fig. 2.7** Thermoelectric thermometer using a galvanometer*

Now for using this thermocouple the cold junction shall still be maintained at the ice point while the hot junction may be kept in contact with the bath whose temperature is to be measured. To get the temperature depending upon the e.m.f. available the caliberated graph is used and corresponding temperature noted from there.

In these thermometers the potentiometers may also be used as shown in Fig. 2.8. Here also the one junction is maintained at ice point while the other junction is put at the temperature to be measured. The potentiometer wire is directly calibrated to measure temperature.

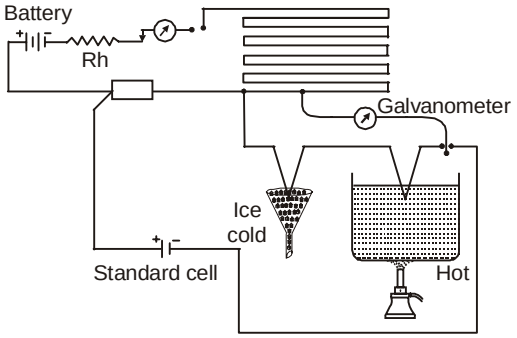


***Fig. 2.8** Thermoelectric thermometer using a potentiometer*

Here the length of the potentiometer wire at which the balance point is obtained is used for getting temperature.

EXAMPLES

***1.** Determine the human body temperature in degree celsius (°C) if the temperature in Fahrenheit is 98.6°F.*

Solution:

Degree Celsius and Fahrenheit are related as below,

$$T\,(°C) = \frac{T^{o}(F) - 32}{1.8}$$

Substituting values.

$$T\,(°C) = \frac{98.6 - 32}{1.8} = 37°\text{C}$$

Temperature in degree celsius shall be **37°C**. **Ans.**

2. *A temperature scale is being developed using the following relation.*

$$t = a \cdot \ln(p) + \left(\frac{b}{2}\right)$$

where 'p' is thermometric property and 'a' and 'b' are constants. Determine celsius temperature corresponding to thermometric property of 6.5, if ice point and steam point give thermometric property value of 3 and 8.

Solution:

For Ice point; $t = 0°\text{C}$ and $p = 3$

For Steam point; $t = 100°\text{C}$ and $p = 8$

Using thermometric relation,

$$0 = a \ln(3) + \left(\frac{b}{2}\right)$$

$$100 = a \ln(8) + \frac{b}{2}$$

Solving the above two equations, we get

$$a = 101.95$$

$$b = 224$$

Thus, $$t = 101.95.\ \ln(p) + \left(\frac{224}{2}\right)$$

$$t = 101.95 \ln(p) + 112$$

For $p = 6.5$, $t =$ **302.83°C Ans.**

3. *In a thermoelectric thermometer for t°C temperature, the emf is given as;*

$$E = 0.003 \cdot t - 5 \times 10^{-7} \cdot t^2 + 0.5 \times 10^{-3},\ \textit{volts}$$

Thermometer is having reference junction at ice point and is calibrated at ice point and steam points. What temperature shall be shown by the thermometer for a substance at 30°C?

Solution:

At ice point; $t = 0°\text{C}$, $E_0 = 0.5 \times 10^{-3}$, volts

At steam point, $t = 100°\text{C}$, $E_{100} = 0.0265$, volts

When $t = 30°\text{C}$

$E_{30} = 9.14 \times 10^{-3}$ volts

Thus temperature shown by this thermometer;

$$t = \left(\frac{E_{30} - E_0}{E_{100} - E_0}\right) \times (T_{100} - T_0)$$

$$= \left(\frac{9.14\times10^{-3} - 0.5\times10^{-3}}{0.0265 - 0.5\times10^{-3}} \right) \times 100$$

$$= \mathbf{33.23°C\ Ans.}$$

4. *Estimate the % variation in temperature reading from a thermocouple having its test junction in gas and other reference junction at ice point. The temperature of gas using gas thermometer is found 50°C. Thermocouple is caliberated with emf varying linearly between ice point and steam point. When thermocouple's test junction is kept in gas at t°C and reference junction at ice point, the e.m.f. produced in millivolts is,*

$$e = 0.18 \cdot t - 5.2 \times 10^{-4} \times t^2, \text{ millivolts.}$$

Solution:

As ice point and steam points are two reference points, so
at ice point having t = 0°C, e.m.f. = 0
at steam point having t = 100°C, e.m.f. = 12.8 mV
at gas temperature of 50°C, e.m.f. = 7.7 mV
Since e.m.f. variation is linear so, temperature at e.m.f. of 7.7 mV;

$$= \frac{(100-0)\times7.7}{(12.8-0)}$$

$$= 60.16°C$$

Temperature of gas using thermocouple = 60.16°C
% variation in temperature reading with respect to gas thermometer reading of 50°C.

$$= \frac{60.16-50}{50} \times 100$$

$$= \mathbf{20.32\%\ Ans.}$$

5. *In an unknown temperature scale freezing point of water is 0°X and boiling point of water is 1000°X. Obtain a conversion relation between degrees X and degree celsius. Also determine the absolute zero in degree X.*

Solution:

Let the conversion relation be $X = aC + b$
where C is temperature in degree celsius, a & b are constants and X is temperature in °X.
At freezing point, temperature = 0°C, 0°X

or, $0 = a \,.\, 0 + b$

$\Rightarrow$ $b = 0$

At boiling point, temperature = 100°C, 1000°X

$1000 = a \cdot 100 + b$

$\Rightarrow$ $a = 10$

Conversion relation

$$\boxed{\boldsymbol{X = 10\,.\,C}}\ \textbf{Ans.}$$

Absolute zero temperature in °C = – 273.15°C
Absolute zero temperature in °X = – 2731.5°X

$$\boxed{\boldsymbol{-2731.5°X}}\ \textbf{Ans.}$$

EXERCISE

2.1 State Zeroth law of thermodynamics.

2.2 Explain, how the Zeroth law of thermodynamics can be used for temperature measurement.

2.3 Write short notes on the following:

Thermometry, thermometric substance, thermometric property, Constant volume gas thermometer.

2.4 Sketch and explain the working of constant pressure thermometer.

2.5 Write equivalence amongst different temperature scales. Also write brief note on each of them.

2.6 Obtain triple point of water in Fahrenheit, Rankine and Kelvin scale.

2.7 Heating of a body causes its temperature to change by 30°F. Find out the increase in temperature in °R and °C.

2.8 Temperature of an object changes by 10°C. What is the change in temperature in °R, °F.

(–18°R, –18F)

2.9 Prove that the difference between the two temperatures in Celsius scale is same as that in Kelvin scale.

2.10 On some temperature scale 0°C is equivalent to 100°B and 100°C is equivalent to 300°B. Determine the temperature in °C corresponding to 200°B. (50°C)

2.11 During temperature measurement of a body it is seen that the same numerical reading is obtained in Celsius and Fahrenheit scales. What is the temperature in degree Rankine? (419.67°R)

2.12 Write a generic computer program for conversion of temperature in °C, K, °F and °R into one another.

3

First Law of Thermodynamics

3.1 INTRODUCTION

Let us take water in a container and heat it from the bottom. What will happen? Container and the water inside shall start getting heated up. This heating is being sensed by either touching it or by measuring its initial and final temperatures. What has caused it to happen so?

Let us take bicycle wheel and paddle it very fast, Chained wheel starts rotating very fast. Now let us apply the brake. The wheel gets stopped. What has made it to stop? Also, if we touch the brake shoe and observe its temperature it shall show that the brake shoe has got heated up. Why has it happened so?

Answer for the above question lies in the energy interactions.

The heating up of the container and water has been caused by the heat being added through the burner flame. Heat available in the flame is being transferred to the container resulting into temperature rise of the container and water.

The fast rotation of the bicycle wheel by paddling has been due to work done in the form of paddling and causing wheel to rotate. Subsequent application of the brake has brought wheel to rest as the driving potential with the wheel is gradually dissipated due to the friction between the brake shoe and wheel. Thus the energy of wheel gets transferred to brake shoe bringing it to rest and heating up of the brake shoe (observed by the rise in temperature of brake shoe).

Thus, it is obvious that there is some entity which is responsible for the above phenomenon. This entity is called the energy which is available in different forms, some times causing the wheel rotation, heating up of water etc. Similar to the cases discussed above one may look at other real life systems and understand phenomenon occurring in them. It can be concluded that it is the energy interaction in some form which is responsible for occurrence of such phenomenon. These energy interactions only permit the transformation of energy from one form to other while making the creation and destruction of energy impossible. Hence, it is true that “energy can neither be produced nor destroyed, it can only change it’s form”. The prevailing law of “energy conservation” also states the same thing.

Here in this chapter we shall look into the first law of thermodynamics, heat, work and its definition, flow and non-flow work, their estimation, steady and unsteady flow processes, their analysis and limitations of the first law of thermodynamics.

3.2 THERMODYNAMIC PROCESSES AND CALCULATION OF WORK

Thermodynamic processes can be precisely categorized as cyclic process and non-cyclic process. The cyclic process is the one in which the initial and final states are identical i.e. system returns to its initial states after occurrence of process. The non cyclic process is the one in which the initial and final states are different i.e. the occurrence of process is accompanied by the state change. Thermodynamic work and its explanation has already been given in Article 1.12.

Let us consider a system consisting of a tank filled with water and fitted with a stirrer at room temperature, Fig. 3.1. Work can be transferred to the system by the stirrer and the temperature of water shall rise. When stirring stops, the system shall cool down till it reaches to the room temperature. Thus, the process is cyclic as the initial and final states are identical.

Let us now take a cylinder having piston and gas filled inside. If the gas is made to expand due to heating, the piston shall undergo displacement and say the piston displacement is dx. If the force exerted by gas on face of piston is F and the cross section area of piston is A, then the displacement work done may be given by :

$$dW = F \cdot dx$$

For the gas pressure being p, the force may be given by $F = p \cdot A$. Substituting for F,

$$dW = p \cdot A \cdot dx$$

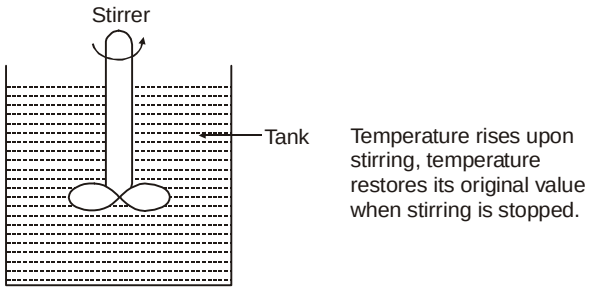


Fig. 3.1 *Cyclic process*

or, $dW = p \cdot dV$, where dV is the elemental change in volume or the volumetric displacement.

If the total displacement of piston is given by L then the total work can be had by integrating the above dW with respect to x for displacement L, or with respect to volume for volume change.

$$W = \int p \cdot dV = \int p \cdot A \cdot dx$$

Now, let us examine whether the work estimated above is in conformity to thermodynamic definition of work or not. If the piston displacement is transferred to a suitable link then the weight can be raised, thus it satisfies thermodynamic definition of work. What about the nature of process? cyclic or non cyclic. It is obvious that the initial and final states are not identical therefore, it is a non-cyclic process.

Thus, the work W as defined above refers to thermodynamic work for a non-cyclic process.

Thermodynamic processes can be further classified based on the thermodynamic constraints under which they occur. Different types of thermodynamic processes are as detailed below.

(*i*) *Constant pressure process or isobaric process*: It refers to the thermodynamic process in which there is no change in pressure during the process. Such type of processes are also known as isobaric processes. To understand let us take a cylindrical vessel having gas in it. It has a piston above it. Piston is free to reciprocate in the cylinder. Under normal situation piston shall be subjected to atmospheric pressure. Now, let heat be added to cylinder from bottom of cylinder. Due to heat addition, presuming energy transfer taking place reversibly and system always remaining in equilibrium, the gas shall try to expand. Expansion of gas results in raising up of the piston and it attains a new state say 2. Process is shown on p-V diagram in Fig. 3.2.

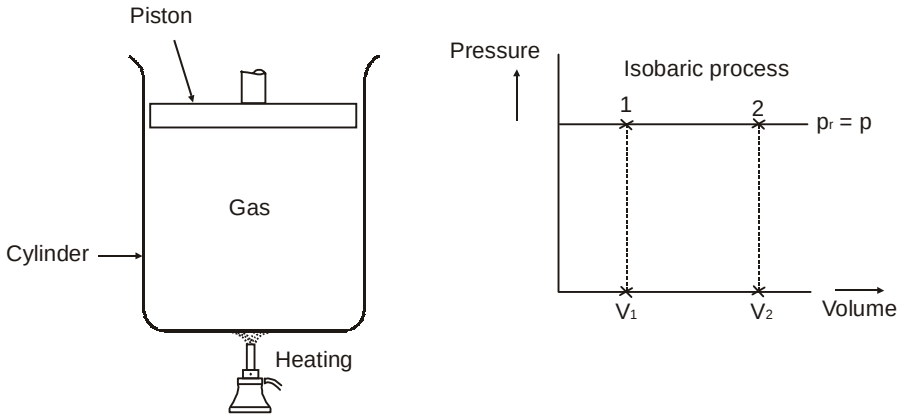


Fig. 3.2 *Isobaric process*

The work involved in the raising of piston shall be given by,

$$W_{1-2} = \int_{V_1}^{V_2} P \cdot dV = P \cdot (V_2 - V_1)$$

Mathematically from the first law of thermodynamics, it can be given that,

$$dQ = dU + dW$$

$$\int_1^2 dQ = \int_1^2 dU + \int_1^2 dW$$

$$Q_{1-2} = m\, c_v\, (T_2 - T_1) + P(V_2 - V_1)$$
$$= m\, c_v\, (T_2 - T_1) + mR(T_2 - T_1)$$

Substituting for c_v, i.e. $\quad c_v = \dfrac{R}{(\gamma - 1)}$

$$Q_{1-2} = mR\,(T_2 - T_1)\left\{\frac{1}{(\gamma - 1)} + 1\right\}$$

(*ii*) *Constant volume process or isochoric process*: When a fluid undergoes a thermodynamic process in a fixed enclosed space such that the process occurs at constant volume, then the process is called constant volume process or isochoric process. Let us consider heating of a gas in fixed enclosure at constant volume. On *p–V* diagram this process is represented by a vertical line as shown in Fig. 3.3. Area under the process line is zero which indicates that there is rise in pressure but there is no work done as there is no change in volume. Work involved shall be,

$$W_{1-2} = \int_{V_1=V}^{V_2=V} P.\,dV = 0$$

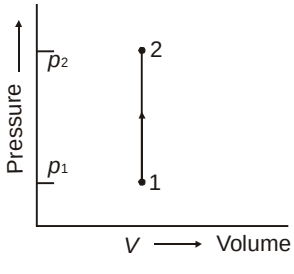


Fig. 3.3 *Isochoric process*

From first law of thermodynamics,

$$dQ = dU + dW$$

$$\int_1^2 dQ = \int_1^2 dU + \int_1^2 dW = \int_1^2 dU + 0$$

or $$Q_{1-2} = U_2 - U_1 = mc_v (T_2 - T_1)$$

Thus, it indicates that the effect of heat addition in constant volume process is to increase the temperature and consequently the internal energy of system.

(*iii*) *Constant temperature process or isothermal process*: Thermodynamic process in which the temperature remains constant is called constant temperature or isothermal process. In this case the gas or vapour may be heated at constant temperature and there shall be no change in internal energy. The work done will be equal to the amount of heat supplied, as shown ahead. For a perfect gas during isothermal process;

$$p_1V_1 = p_2V_2 = \text{Constant, or, } P = \frac{P_1V_1}{V}$$

so work involved, $$W_{1-2} = \int_{V_1}^{V_2} P \cdot dV$$

$$W_{1-2} = \int_{V_1}^{V_2} \frac{p_1V_1}{V} dV = P_1V_1 \ln \left(\frac{V_2}{V_1}\right)$$

$$W_{1-2} = P_1V_1 \ln r$$

where r = ratio of final and initial volumes.

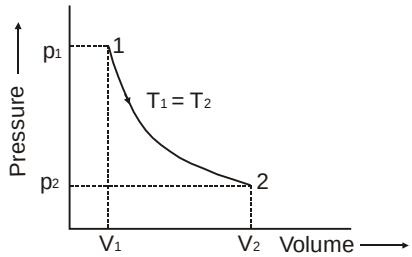


Fig. 3.4 *Isothermal process*

By first law of thermodynamics

$$\int_1^2 dQ = \int_1^2 dW + \int_1^2 dU$$

$$Q_{1-2} = W_{1-2} + (U_2 - U_1) = W_{1-2} + 0$$

as $$U_2 - U_1 = mc_v (T_2 - T_1), \text{ and } T_1 = T_2$$

(*iv*) *Adiabatic process*: An adiabatic process is the thermodynamic process in which there is no heat interaction during the process, i.e. during the process, $Q = 0$. In these processes the work interaction is there at the expense of internal energy. If we talk of adiabatic expansion then it shall mean that work is done at the cost of its own internal energy. The adiabatic process follows the law PV^γ = constant where γ is called adiabatic index and is given by the ratio of two specific heats. Thus, it is obvious that adiabatic expansion shall be accompanied by the fall in temperature while temperature will rise during adiabatic compression. The adiabatic expansion process is shown on Fig. 3.5. Work done during expansion shall be,

$W_{1-2} = \int_{V_1}^{V_2} P \cdot dV$, where PV^γ = constant, therefore solving after substitution. Work shall be,

$$W_{1-2} = \frac{P_1V_1 - P_2V_2}{(\gamma - 1)}$$

From first law of thermodynamics $\int_1^2 dQ = \int_1^2 dU + \int_1^2 dW$

$$Q_{1-2} = (U_2 - U_1) + \frac{p_1V_1 - p_2V_2}{(\gamma - 1)}$$

$$0 = mc_v\,(T_2 - T_1) + \frac{p_1V_1 - p_2V_2}{(\gamma - 1)}$$

$$\Rightarrow \qquad W_{1-2} = mc_v\,(T_1 - T_2)$$

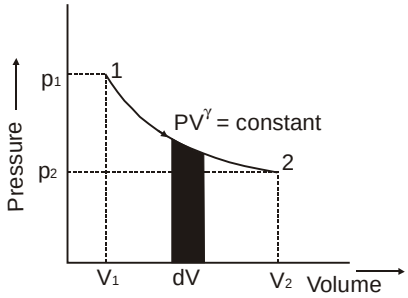


Fig. 3.5 *Adiabatic expansion*

(*v*) *Polytropic process*: Polytropic process is the most commonly used process in practice. In this, the thermodynamic process is said to be governed by the law PV^n = constant where n is the index which can vary from $-\infty$ to $+\infty$. Figure 3.6 shows some typical cases in which the value of n is varied and the type of process indicated for different values of n. Thus the various thermodynamics processes discussed above are special cases of polytropic process. Work interaction in case of polytropic process can be given as,

$$W_{1-2} = \int_{V_1}^{V_2} p \cdot dV$$

where $P_1V_1^{\,n} = P_2V_2^{\,n}$ = constant

Solving the above, we get

$$W_{1-2} = \frac{p_1V_1 - p_2V_2}{(n-1)}$$

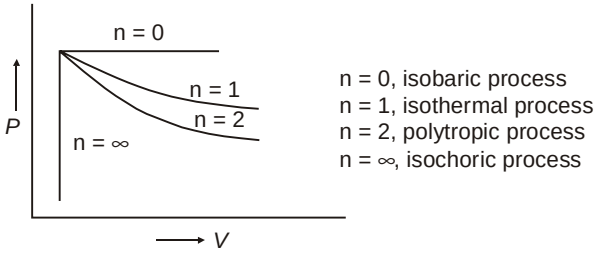


Fig. 3.6 *Polytropic process*

From first law of thermodynamics,

$$\int_1^2 dQ = \int_1^2 dU + \int_1^2 dW$$

or $$Q_{1-2} = (U_2 - U_1) + \frac{p_1V_1 - p_2V_2}{n-1}$$

or $$Q_{1-2} = mc_v\,(T_2 - T_1) + \frac{mR(T_1 - T_2)}{(n-1)}$$

also $$c_v = \frac{R}{\gamma - 1} \text{ or } R = c_v\,(\gamma - 1)$$

$$Q_{1-2} = mc_v\,\{(T_2 - T_1) + \frac{(\gamma - 1)}{(n-1)} \cdot (T_1 - T_2)\}$$

Thus heat transfer during a polytropic process for a perfect gas;

$$Q_{1-2} = mc_v\,(T_2 - T_1) \times \left(\frac{\gamma - n}{1 - n}\right)$$

or also, substituting for c_v

$$Q_{1-2} = \left(\frac{\gamma - n}{\gamma - 1}\right) \times W$$

(*vi*) *Hyperbolic process*: Hyperbolic process is the one in which product of pressure and volume remains constant during the process. The curve for such an expansion process is a rectangular hyperbola and hence this is known as hyperbolic expansion.

For a perfect gas $\frac{PV}{T}$ = Constant, if T is also constant then it means that for a perfect gas the hyperbolic process shall also be isothermal process. Figure 3.7 shows hyperbolic expansion process between 1 and 2. Work done during process shall be

$$W_{1-2} = \int_1^2 p \cdot dV \text{ and } p_1V_1 = p_2V_2 = \text{constant}$$

$$W_{1-2} = \int_{V_1}^{V_2} \frac{p_1V_1}{V}\, dV = p_1V_1 \ln \frac{V_2}{V_1}$$

or, $W_{1-2} = p_1V_1 \ln r$, where $r = \frac{V_2}{V_1}$, ratio of final and initial volumes from first law of thermodynamics,

$$\int_1^2 dQ = \int_1^2 dU + \int_1^2 dW = (U_2 - U_1) + p_1V_1 \ln \left(\frac{V_2}{V_1}\right)$$

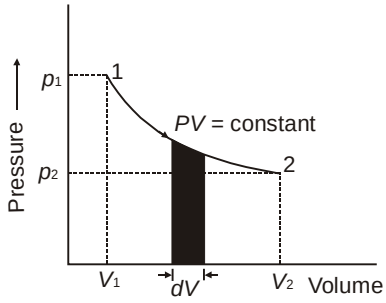


Fig. 3.7 *Hyperbolic expansion*

(*vii*) *Free Expansion*: Free expansion, as the name implies refers to the unrestrained expansion of a gas. Let as take an insulated tank having two compartments separated by a partition, say *A* and *B*. Let us assume that compartment *A* is filled with gas while *B* is having vacuum. If now the partition is removed and gas allowed to occupy the whole volume of tank, then the gas expands to fill the complete volume space. New pressure of gas will be lesser as compared to initial pressure of gas occupying the compartment *A*. A close look at the expansion process shows that the expansion due to removal of partition is unresisted expansion due to gas expanding in vacuum. This is also known as free expansion. The reverse of free expansion is impossible and so it is an irreversible process.

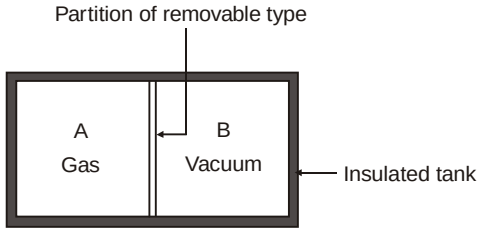


Fig. 3.8 *Free expansion*

During free expansion no work shall be done by the gas or on the gas due to no boundary displacement in the system.

$$W_{\text{free expansion}} = 0$$

Also in the above there shall be no heat interaction as tank is insulated. From first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$
$$O = \Delta U + O$$

or, $U_{A+B} = U_A$, i.e. initial and final internal energies are same, which means for a perfect gas initial and final temperatures of gas are same.

Table 3.1 *Thermodynamic processes*

Sl. No.	*Process*	*Governing equations*	*Heat interaction*	*Displacement work or non flow work during state change from 1 to 2* $W = \int_1^2 p.dV$
1.	Isobaric process	p = constant $\frac{T_2}{T_1} = \frac{v_2}{v_1}$ index $n = 0$	$q = c_p \times (T_2 - T_1)$	$W = p(V_2 - V_1)$
2.	Isochoric process	V = constant $\frac{T_1}{T_2} = \frac{p_1}{p_2}$ index, $n = \infty$	$q = c_v \times (T_2 - T_1)$	$W = 0$
3.	Isothermal process	T = constant $p_1V_1 = p_2V_2$ index, $n = 1$	$q = p_1V_1 \times \ln\left(\frac{V_2}{V_1}\right)$	$W = P_1V_1 \ln \frac{V_2}{V_1}$
4.	Adiabatic process	$p_1V_1^{\gamma} = p_2V_2^{\gamma}$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$ $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$ index, $n = \gamma$	$q = 0$	$W = \frac{p_1V_1 - p_2V_2}{\gamma - 1}$

5.	Polytropic process	$p_1V_1^n = p_2V_2^n$ $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{n-1}$ $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$ $C_n = c_v\left(\frac{\gamma - n}{1-n}\right)$ where C_n is specific heat for polytropic process.	$q = c_v\left(\frac{\gamma - n}{1-n}\right)$ $\times (T_2 - T_1)$ or, $q = \left(\frac{\gamma - n}{\gamma - 1}\right)$ $\times$ work	$W = \frac{p_1V_1 - p_2V_2}{n-1}$
6.	Hyperbolic process	pV = constant but not necessarily T = constant	$q = c_v\,(T_2 - T_1)$ $+ RT_1 \ln\left(\frac{V_2}{V_1}\right)$	$W = RT_1 \ln\left(\frac{V_2}{V_1}\right)$
7.	Free expansion in adiabatic conditions	Unresisted expansion	$q = 0$	$W = 0$

3.3 NON-FLOW WORK AND FLOW WORK

Work interaction taking place in a system can be classified as flow work or non-flow work based on the nature of process.

Two basic types of processes are

(*i*) Flow process

(*ii*) Non flow process

3.3.1 Flow Process

Flow process is the one in which fluid enters the system and leaves it after work interaction, which means that such processes occur in the systems having open boundary permitting mass interaction across the system boundary.

Figure 3.9 shows the flow process occurring in a system. Flow processes can be further classified into steady flow and non-steady flow processes. Examples of engineering systems having steady flow processes are flow through nozzles, turbines, compressors etc. and the examples of nonsteady flow processes are the filling or emptying of any vessel. Flow process shown indicates various energy and mass interactions taking place across the system boundary.

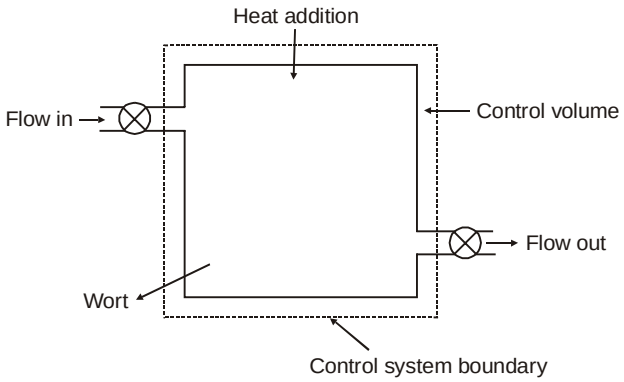


Fig. 3.9 *Flow process*

As the mass interaction takes place here so for every mass fraction crossing the boundary, work is done upon or by the system. Thus, a portion of work is always required to push the fluid mass into or out of the system. This amount of work is called flow work, or, "work required for causing flow of fluid to or from the system is called flow work".

Here in the control volume shown say, some mass of fluid element is to be pushed into the control volume.

Fluid mass can be injected into the control volume with certain force, say F. The force required for pushing (F) owing to the pressure P of fluid element may be quantified as; $F = P.A$, where A is cross-section area of the passage. For injecting entire mass of fluid, the force F must act through a distance L. Thus, work done in injecting the fluid element across the boundary will be,

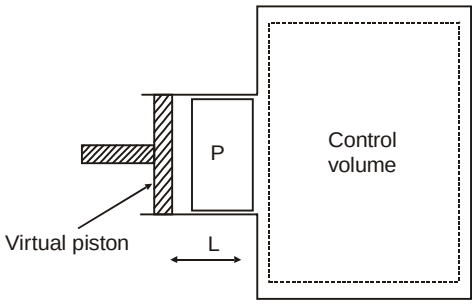


Fig. 3.10 *Flow work*

$$W = F \cdot L = P \cdot A \cdot L, \ (kJ)$$

or $$w = P \cdot v, \ (kJ/kg)$$

This work is the flow work. Thus, flow work per unit mass can be given as the product of pressure and specific volume. It is also referred to as flow energy or convected energy or transport energy.

3.3.2 Non-Flow Processes

Non-flow process is the one in which there is no mass interaction across the system boundaries during the occurrence of the process.

Figure 3.11 shows block diagram of a piston-cylinder arrangement in which a constant mass of fluid inside the cylinder is shown to undergo compression. Thus, during compression the type of process shall be non-flow process and the work interaction shall be non-flow work.

Say, the force exerted by piston is F, and cross-section area of piston being A, the elemental work done in compressing along the length dL shall be

$$dW = F \cdot dL$$

If pressure of fluid is P then $F = P \cdot A$.

so $$dW = P \cdot A \cdot dL$$

The total work done in piston displacement, from 1 to 2 shall be,

$$\int_1^2 dW = \int_1^2 P \cdot A \cdot dL$$

$$\int_1^2 dW = \int_1^2 P \cdot dV$$

or $$W_{1-2} = \int_1^2 P \cdot dV$$

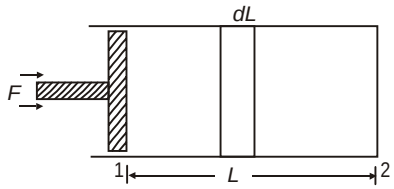


Fig. 3.11 *Non-flow process*

Thus, this is called the non-flow work or displacement work.

3.4 FIRST LAW OF THERMODYNAMICS

Benjamin Thompson (Count Runsford) 1753-1814 discovered the equivalence of work and heat in the course of manufacturing canon (1797) by boring solid metal submerged in the water. He was intrigued by the water boiling because of mechanical work of boring, as no heat had been added to the water. In his words, "is it possible that such a quantity of heat as would have caused five pounds of ice cold water to boil could have been furnished by so inconsiderable a quantity of metallic dust merely in consequence of a change in its capacity for heat?" Other experiments later discovered more evidence until some fifty years after the above experiment.

Let as take a bicycle, tyre pump and use it for inflating the bicycle tyre. It is observed that the pump becomes hotter during use. This phenomenon of heating of pump is obviously not from heat transfer but because of the work done. Although the heating of pump could also be realized by heat transfer. It indicates that some effects can be caused equivalently by heat or work and that there exist some relationship between heat and work.

James Prescott Joule (1818-1889) an English scientist and one time student assistant to John Dalton (1766-1844) with assistance from Lord Kelvin showed conclusively that mechanical work and heat are equivalent.

For example, let us take a closed system which permits work interaction and heat interaction both, as in case of stirring in a container, fig. 3.12. As a result of stirring it is seen that the temperature of water gets raised up. This rise in temperature can be accounted by quantifying the amount of heat supplied for raising this temperature. Thus, it is obvious that for any closed system undergoing a cycle $\oint W = J \cdot \oint Q$, where J is Joule's constant.

i.e., the net heat interaction is proportional to the work interaction. Also the constant is known as "Joule's mechanical equivalent of heat". Joule's constant is described as;

$$\frac{W}{Q} = J = 4.18 \ \frac{\text{Joules}}{\text{Calories}}$$

Thus, J is a numerical conversion factor which could be unity if the heat is also given in joules.

For any cyclic process in the closed system the relationship between heat and work shall be, (if the consistent units are used)

$$\oint \delta q = \oint \delta W.$$

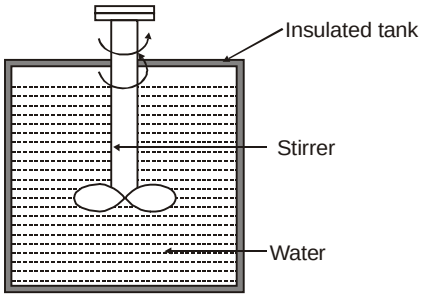


Fig. 3.12 *Closed system*

Thus first law of thermodynamics states that "in a closed system undergoing a cyclic process, the net work done is proportional to the net heat taken from the surroundings" or "for any cycle of a closed system the net heat transfer equals the net work".

First law of thermodynamics can't be proved but it is supported by a large number of experiments and no exceptions have been observed. It is therefore termed as the law of nature.

Mathematical expression for the first law of thermodynamics can be rearranged and it shall be,

$$\oint (\delta q - \delta W) = 0$$

which shows that the quantity $(\delta q - \delta W)$ is a thermodynamic property.

***For non-cyclic process*:** Let us now take up a system undergoing a non-cyclic process where transfer of heat and work take place and there is some change in the state of system i.e. initial and final states are different.

Figure 3.13 shows the non-cyclic process occurring between states 1 and 2. The change in state is accomplished by the energy interactions. If we assume the system to have the heat interaction ΔQ and work interaction ΔW, then from the basic principles it can be said that :

Energy lost = Energy gained

as the energy can neither be created nor destroyed.

Therefore, between states 1–2 one can write energy balance as,

$$Q_{1-2} - W_{1-2} = U_{1-2}$$

where Q_{1-2}, W_{1-2} and U_{1-2} are the heat, work and stored energy values. *This stored energy is called as internal energy for a system having negligible electrical, magnetic, solid distortion and surface tension effects.*

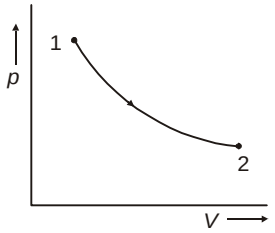


***Fig. 3.13** Non cyclic process*

General expression based on above can be given as follows :

$$\Delta Q - \Delta W = \Delta U$$

or

$$\int_1^2 dQ - \int_1^2 dW = \int_1^2 dU$$

or, for elemental interactions; $dQ - dW = dU$

$$dQ = dU + dW$$

Thus, the first law of thermodynamics for non-cyclic processes can be given by

$$\int dQ = \int dU + \int dW$$

Above equations make it obvious that the internal energy change in the closed system during any non-cyclic process is obtained by subtracting the net amount of work done by the system from the net amount of heat added to the system i.e. $\Delta U = Q - W$.

Actually, there is no absolute value of internal energy of any system. Therefore its value may be taken to be zero for any particular state of the system and absolute value in reference to arbitarily assumed state may be easily defined.

Mathematically, it can be shown that the internal energy is a thermodynamic property, as explained ahead. Let us consider the non-cyclic process following paths *A*, *B* and *C* in the directions as shown in Fig. 3.14.

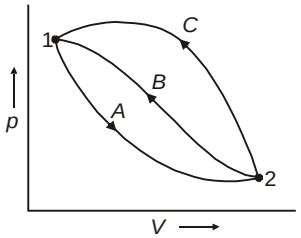


***Fig. 3.14** Two different thermodynamic cycles.*

As the processes *A* & *B* and *A* & *C* constitute a thermodynamic cycle starting and finishing at state 1, the first law of thermodynamics for cyclic process can be employed,

$$\oint (\delta Q - \delta W) = 0$$

For the cycle following path 1–*A*–2–*B*–1, the first law of thermodynamics says,

$$\oint (\delta Q - \delta W) = 0$$

$$1\text{–}A\text{–}2\text{–}B\text{–}1$$

or $$\int_{1,\,A}^{2} (\delta Q - \delta W) + \int_{2,\,B}^{1} (\delta Q - \delta W) = 0$$

which can be rewritten as,

$$\int_{1,\,A}^{2} (\delta Q - \delta W) = -\int_{2,\,B}^{1} (\delta Q - \delta W)$$

or $$\int_{1,\,A}^{2} (\delta Q - \delta W) = \int_{1,\,B}^{2} (\delta Q - \delta W) \qquad \text{(i)}$$

Also, for the cycle following path 1–*A*–2–*C*–1, the first law of thermodynamics can be applied as,

$$\oint (\delta Q - \delta W) = 0$$

$$1\text{–}A\text{–}2\text{–}C\text{–}1$$

or $$\int_{1,\,A}^{2} (\delta Q - \delta W) + \int_{2,\,C}^{1} (\delta Q - \delta W) = 0$$

or $$\int_{1,\,A}^{2} (\delta Q - \delta W) = -\int_{2,\,C}^{1} (\delta Q - \delta W)$$

or $$\int_{1,\,A}^{2} (\delta Q - \delta W) = \int_{1,\,C}^{2} (\delta Q - \delta W) \qquad \text{(ii)}$$

From equations (*i*) & (*ii*) it is obvious that

$$\int_{1,\,A}^{2} (\delta Q - \delta W) = \int_{1,\,B}^{2} (\delta Q - \delta W) = \int_{1,\,C}^{2} (\delta Q - \delta W)$$

which shows that $(\delta Q - \delta W)$ is some property as it is independent of the path being followed.

Also, it can be rewritten as,

$$\int_{1,\,A}^{2} \delta U = \int_{1,\,B}^{2} \delta U = \int_{1,\,C}^{2} \delta U$$

or $$\Delta U_{1\text{–}2,\,A} = \Delta U_{1\text{–}2,\,B} = \Delta U_{1\text{–}2,\,C}$$

which means the change in internal energy is independent of the path followed and therefore *internal energy is a thermodynamic property*.

3.5 INTERNAL ENERGY AND ENTHALPY

Let us take a mass at certain elevation in earth's gravitational field and make it move with certain velocity. Energy considerations say that the mass shall have the potential energy (P.E = mgz) and kinetic energy (K.E = (1/2) . mC^2) stored in it. Similarly, several other forms of energy such as due to magnetic, electrical, solid distortion and surface tension effects can be estimated as the contributory components of stored energy.

Difference of heat and work interactions yield the stored energy as given below; $E = Q - W$.

If the energy at macroscopic level as discussed above could be separated from the total stored energy E, then the amount of energy left shall be called internal energy.

Mathematically,

Internal energy, U = (Stored energy) – (Kinetic energy) – (Potential energy) – (Magnetic energy) – (Electrical energy) – (Surface tension energy) – (Solid distortion energy).

Therefore, stored energy is summation of internal energy, potential energy, kinetic energy, magnetic, electrical, surface tension, solid distortion etc. types of energy.

For the situation when magnetic, electric, surface tension, solid distortion effects are negligible, the stored energy shall be;

$$E = U + \text{KE} + \text{PE}$$

or,
$$E = U + \frac{mC^2}{2} + mgz$$

or, on unit mass basis; $\boxed{e = u + \frac{C^2}{2} + gz}$

and the change in stored energy relative to some reference state shall be given as,

$$\Delta E = \Delta U + \Delta\text{KE} + \Delta\text{PE}.$$

Enthalpy (H) of a substance at any point is quantification of energy content in it, which could be given by summation of internal energy and flow energy. Enthalpy is very useful thermodynamic property for the analysis of engineering systems.

Mathematically, it is given as,

$$\boxed{H = U + PV}$$

On unit mass basis, the specific enthalpy could be given as,

$$\boxed{h = u + pv}$$

A look at expression of enthalpy shows that as we can't have absolute value of internal energy, the absolute value of enthalpy can not be obtained. Therefore only change in enthalpy of substance is considered. For certain frequently used substances such as steam, the enthalpy values of steam are available in tabulated form in Steam Tables at different thermodynamic states.

From the definition of enthalpy;

$$h = u + pv$$

or
$$dh = du + p \cdot dv + v \cdot dp.$$

For a constant pressure process, $dp = 0$.

$$dh = du + pdv$$

or,
$$dh = dq_{p = \text{constt}}$$ (From first law of thermodynamics)

3.6 SPECIFIC HEATS AND THEIR RELATION WITH INTERNAL ENERGY AND ENTHALPY

Specific heats of the substance refer to the amount of heat interaction required for causing unit change in temperature of the unit mass of substance. This unit change in temperature may be realized under constant volume and constant pressure conditions separately.

Therefore, the above heat value obtained with heat interaction occurring under constant volume conditions is called specific heat at constant volume, denoted as c_v. Whereas the above heat value obtained with heat interaction occurring under constant pressure conditions is called specific heat at constant pressure, denoted as c_p.

Mathematically, the heat interaction causing ΔT temperature change in m mass of substance can be given as,

For isochoric conditions;

$$Q_v = m \cdot c_v \cdot \Delta T$$

and for isobaric conditions

$$Q_p = m \cdot c_p \cdot \Delta T$$

or $$c_v = \frac{Q_v}{m \cdot \Delta T}$$

or, $$c_p = \frac{Q_p}{m \cdot \Delta T}$$

For getting the specific heat values, substituting $m = 1$, $\Delta T = 1$,

$$\boxed{c_v = Q_v} \text{ and } \boxed{c_p = Q_p}$$

The specific heat at constant volume can also be given as the partial derivative of internal energy with respect to temperature at constant volume.

Thus $$c_v = \left(\frac{\partial u}{\partial T}\right)_v$$

or, $$c_v = \left(\frac{du}{dT}\right)_v$$

Also from first law of thermodynamics, on unit mass basis

$$dq = du + pdv$$

at constant volume, $dv = 0$

$$dq = du$$

or $$dq = c_v \cdot dT = du$$

$$\boxed{c_v = \frac{dq}{dT}}, \text{ for } v = \text{constant}$$

Specific heat at constant pressure can be given as the partial derivative of enthalpy with respect to temperature at constant pressure.

Mathematically: $$c_p = \left(\frac{\partial h}{\partial T}\right)_p$$

or $$c_p = \left(\frac{dh}{dT}\right)_p$$

From definition of enthalpy, at unit mass basis.

$$h = u + pv$$

or $$dh = du + pdv + vdp$$

at constant pressure, $dp = 0$

$$dh = du + pdv$$

substituting from first law of thermodynamics $dq = du + pdv$

$$dh = dq$$

or $$dq = c_p \cdot dT = dh$$

$$\boxed{c_p = \frac{dq}{dT}}, \text{ for } p = \text{constant}$$

Let us try to establish relationship between c_p and c_v.

From enthalpy definition, at unit mass basis

$$h = u + pv$$

or $$h = u + RT \{\text{for ideal gas}\}$$

Taking partial derivative,

$$dh = du + RdT$$

Also we know for an ideal gas, c_p

$$dh = c_p \cdot dT;\ du = c_v \cdot dT$$

Substituting dh and du

$$c_p \cdot dT = c_v \cdot dT + R \cdot dT$$

or $$c_p = c_v + R$$

or $$\boxed{c_p - c_v = R}$$

Difference of specific heats at constant pressure and volume is equal to the gas constant for an ideal gas.

Also the ratio of specific heats at constant pressure and volume could be given as γ,

$$\frac{c_p}{c_v} = \gamma$$

Combining above two relations of c_p and c_v we get,

$$c_p = \frac{\gamma . R}{(\gamma - 1)} \text{ and } c_v = \frac{R}{(\gamma - 1)}$$

3.7 FIRST LAW OF THERMODYNAMICS APPLIED TO OPEN SYSTEMS

Let us consider an open system as shown in Fig. 3.15 having inlet at section 1–1 and outlet at section 2–2. The cross-section area, pressure, specific volume, mass flow rate, energy at section 1–1 and 2–2 are

Section 1–1 = A_1, p_1, v_1, m_1, e_1

Section 2–2 = A_2, p_2, v_2, m_2, e_2

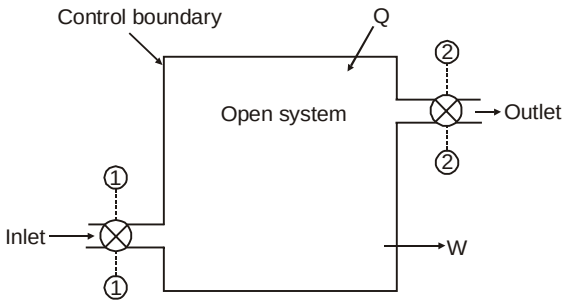


Fig. 3.15

Open system is also having heat and work interactions Q, W as shown in figure above.

Applying the energy balance at the two sections, it can be given as,

Energy added to the system + Stored energy of the fluid at inlet

= Stored energy of the fluid at outlet

Quantifying the various energies;

Energy of fluid at inlet shall comprise of stored energy and flow energy as given here.

$$= m_1(e_1 + p_1v_1)$$

Similarly, energy of fluid at outlet shall comprise of stored energy and flow energy,

= Stored energy + Flow energy

$$= m_2 (e_2 + p_2v_2)$$

The energy added to the system shall be the net energy interaction due to heat and work interactions.

$$= Q - W$$

Writing energy balance, mathematically;

$$Q - W + m_1 (e_1 + p_1v_1) = m_2 (e_2 + p_2v_2)$$

or
$$Q + m_1(e_1 + p_1v_1) = W + m_2(e_2 + p_2v_2)$$

If the mass flow rates at inlet and exit are same, then

$$Q + m(e_1 + p_1v_1) = W + m(e_2 + p_2v_2)$$

On unit mass basis

$$q + e_1 + p_1v_1 = w + e_2 + p_2v_2$$

Thus,

Heat + (Stored energy + Flow energy)$_1$ = Work + (Stored energy + Flow energy)$_2$

Stored energy at inlet and outlet can be mathematically given as,

$$e_1 = u_1 + \frac{C_1^2}{2} + gz_1$$

and
$$e_2 = u_2 + \frac{C_2^2}{2} + gz_2$$

where C_1 and C_2 are velocities at inlet and exit, u_1 and u_2 are internal energy at inlet and outlet, z_1 and z_2 are elevations of inlet and exit.

3.8 STEADY FLOW SYSTEMS AND THEIR ANALYSIS

Steady flow refers to the flow in which its properties at any point remain constant with respect to time. Steady system is the system whose properties are independent of time, i.e. any property at a point in system shall not change with time.

Let us take an open system having steady flow. Figure 3.16 shows steady flow system having inlet at section 1–1, outlet at section 2–2, heat addition Q and work done by the system W.

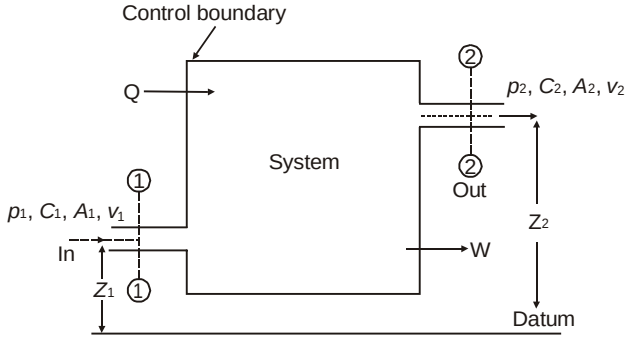


Fig. 3.16 *Steady flow system*

	At section 1–1	*At section 2–2*
Pressure, (N/m^2)	p_1	p_2
Sp volume, (m^3/kg)	v_1	v_2
Velocity, (m/s)	C_1	C_2
Elevation, (m)	z_1	z_2
Cross-section area, (m^2)	A_1	A_2
Mass flow rate, (kg/s)	m_1	m_2
Internal energy, (J/kg)	u_1	u_2

As described in earlier article the energy balance when applied to open system results in

$$Q + m_1(e_1 + p_1v_1) = W + m_2(e_2 + p_2v_2)$$

Substituting for e_1 and e_2

$$Q + m_1\left(u_1 + \frac{C_1^2}{2} + gz_1 + p_1v_1\right) = W + m_2\left(u_2 + \frac{C_2^2}{2} + gz_2 + p_2v_2\right)$$

and from definition of enthalpy,

$$h_1 = u_1 + p_1v_1$$
$$h_2 = u_2 + p_2v_2$$

therefore,

$$\boxed{Q + m_1\left(h_1 + \frac{C_1^2}{2} + gz_1\right) = W + m_2\left(h_2 + \frac{C_2^2}{2} + gz_2\right)}$$

Above equation is known as steady flow energy equation (S.F.E.E.). If the mass flow rates at inlet and exit are same, i.e. $m_1 = m_2 = m$

then,
$$Q + m\left(h_1 + \frac{C_1^2}{2} + gz_1\right) = W + m\left(h_2 + \frac{C_2^2}{2} + gz_2\right)$$

or, on unit mass basis the S.F.E.E. shall be;

$$\boxed{q + h_1 + \frac{C_1^2}{2} + gz_1 = w + h_2 + \frac{C_2^2}{2} + gz_2}$$

where
$$q = \frac{Q}{m},\ w = \frac{W}{m}$$

The steady flow energy equation can be used as a tool for carrying out thermodynamic analysis of engineering system with suitable modifications.

Special Case : Such as for any system of perfectly insulated type, $Q = 0$

The steady flow energy equation gets modified to;

$$h_1 + \frac{C_1^2}{2} + gz_1 = w + h_2 + \frac{C_2^2}{2} + gz_2$$

Application of Continuity equation results in,

$$m_1 = m_2$$

or,
$$\frac{A_1 C_1}{v_1} = \frac{A_2 C_2}{v_2}$$

For any system having more than one inlets, outlets and energy interactions the example is shown below.

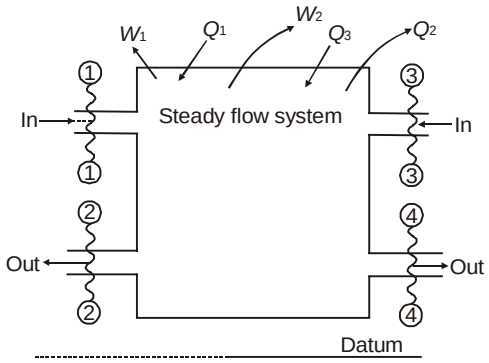


Fig. 3.17 *Example of steady flow system*

Salient properties at different sections are tabulated as under

	Section 1–1	*Section* 2–2	*Section* 3–3	*Section* 4–4
Pressure, (N/m^2)	p_1	p_2	p_3	p_4
Sp. volume, (m^3/kg)	v_1	v_2	v_3	v_4
Mass flow rate, (kg/s)	m_1	m_2	m_3	m_4
Internal energy, (J/kg)	u_1	u_2	u_3	u_4
Velocity, (m/s)	C_1	C_2	C_3	C_4
Elevation, (m)	z_1	z_2	z_3	z_4
Cross-section area, (m^2)	A_1	A_2	A_3	A_4

Net heat added, $Q = Q_1 - Q_2 + Q_3$
Net work done, $W = W_1 + W_2$
Applying steady flow energy equation on the system as shown in Fig 3.17;

$$Q + m_1\left(u_1 + \frac{C_1^2}{2} + gz_1 + p_1v_1\right) + m_3\left(u_3 + \frac{C_3^2}{2} + gz_3 + p_3v_3\right)$$

$$= W + m_2\left(u_2 + \frac{C_2^2}{2} + gz_2 + p_2v_2\right) + m_4\left(u_4 + \frac{C_4^2}{2} + gz_4 + p_4v_4\right)$$

Substituting enthalpy values, h_1, h_2, h_3, h_4 and for Q and W;

$$\boxed{\begin{aligned} &Q_1 - Q_2 + Q_3 + m_1\left(h_1 + \frac{C_1^2}{2} + gz_1\right) + m_3\left(h_3 + \frac{C_3^2}{2} + gz_3\right) \\ &= (W_1 + W_2) + m_2\left(h_2 + \frac{C_2^2}{2} + gz_2\right) + m_4\left(h_4 + \frac{C_4^2}{2} + gz_4\right) \end{aligned}}$$

Case 1
If the inlet and exit velocities are negligible, then

$$KE_1 = KE_2 = KE_3 = KE_4 = 0$$

and S.F.E.E. is modified to

$$Q_1 - Q_2 + Q_3 + m_1(h_1 + gz_1) + m_3(h_3 + gz_3) = W_1 + W_2 + m_2(h_2 + gz_2) + m_4(h_4 + gz_4)$$

Case 2

If there is no change in elevation and mass flow rates at all inlets and outlets are same, then, $m_1 = m_2 = m_3 = m_4 = m$

$$Q_1 - Q_2 + Q_3 + m \,.\, h_1 + m\, h_3 = W_1 + W_2 + m\, h_2 + m\, h_4$$

or, on unit mass basis

$$q_1 - q_2 + q_3 + h_1 + h_3 = w_1 + w_2 + h_2 + h_4$$

3.9 FIRST LAW APPLIED TO ENGINEERING SYSTEMS

Here the first law of thermodynamics applied to different engineering systems is discussed. It is assumed in general that the processes are of steady flow type and so the steady flow energy equation can be directly used with modifications in it.

(*a*) *Turbine*: It is the device in which the high temperature and high pressure fluid is expanded to low temperature and pressure resulting in generation of positive work at turbine shaft. Thus, turbine is a work producing device.

Turbines using gas as working fluid are called gas turbine where as turbines using steam are called steam turbines.

Expansion in turbine is assumed to be of adiabatic type so that the maximum amount of work is produced.

Assuming change in kinetic energy, potential energy to be negligible, the steady flow energy equation can be modified and written between 1 and 2 as,

$$O + mh_1 = W_T + mh_2$$
$$W_T = m(h_1 - h_2)$$

i.e., $Q = 0$ and total energy interaction is available in the form of work

Turbine work = $m(h_1 - h_2) = m\, c_p(T_1 - T_2)$

Here m is mass flow rate and T_1, T_2 are temperatures at inlet and outlet.

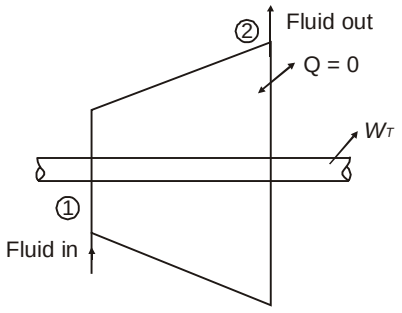


Fig. 3.18 *Turbine*

(*b*) *Compressor*: Compressor is a work absorbing device used for increasing the pressure of a fluid. Pressure of a fluid is increased by doing work upon it, which is accompanied by increase in temperature depending on the gas properties.

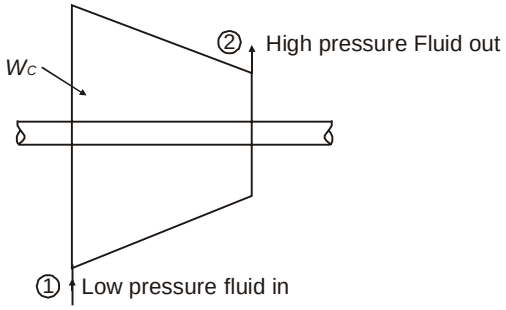


Fig. 3.19 *Compressor*

For compression of a gas adiabatic process is used as in this there is no heat loss and so minimum work requirement.

Let us assume change in kinetic energy and potential energy to be negligible between 1 & 2 and also flow to be of steady type.

Applying steady flow energy equation in modified form:

$$Q = 0$$
$$\Delta KE = 0$$
$$\Delta PE = 0$$
$$W_c = (-ve) \text{ work for compression}$$
$$mh_1 = -W_c + mh_2$$

or
$$W_c = m(h_2 - h_1)$$

Adiabatic compression work = $m(h_2 - h_1) = mc_p (T_2 - T_1)$

Here T_1, T_2 are temperatures at inlet and outlet and m is mass flow rate.

(*c*) *Pump*: A pump is used for pumping liquid or suction of liquid. In case of pump the following assumptions can be made for using S.F.E.E.

(*i*) Heat transfer is zero, $Q = 0$

(*ii*) Change in internal energy is zero, $\Delta U = 0$

Therefore

$$m\left(p_1v_1 + \frac{C_1^2}{2} + gz_1\right) = m\left(p_2v_2 + \frac{C_2^2}{2} + gz_2\right) - W_{pump}$$

or,
$$W_{pump} = m\{(p_2v_2 - p_1v_1) + \frac{C_2^2 - C_1^2}{2} + g(z_2 - z_1)\}$$

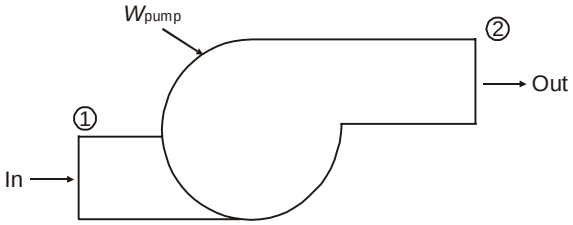


Fig. 3.20 *Pump*

(*d*) *Boiler*: Boiler is the engineering device used for steam generation at constant pressure. Heat is supplied externally to the boiler for steam generation depending upon state of steam desired.

Boiler may be assumed similar to a closed vessel having no work interaction, no change in kinetic energy, no change in potential energy.

i.e. $W = 0,\ \Delta KE = 0,\ \Delta PE = 0.$

Applying steady flow energy equation

$$Q_{boiler} + m(h_1) = m(h_2)$$

or $$Q_{boiler} = m(h_2 - h_1) = m\, c_p\, (T_2 - T_1)$$

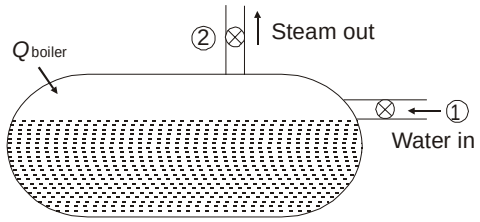


Fig. 3.21 *Boiler*

(*e*) *Condenser*: Condenser is the device used for condensing vapour into liquid at constant pressure. It is a type of heat exchanger in which another cool fluid is used for condensing the vapours into liquid. Heat exchange between the hot fluid and cold fluid takes place indirectly as cold fluid passes through the tubes and hot vapours are outside tubes in the shell.

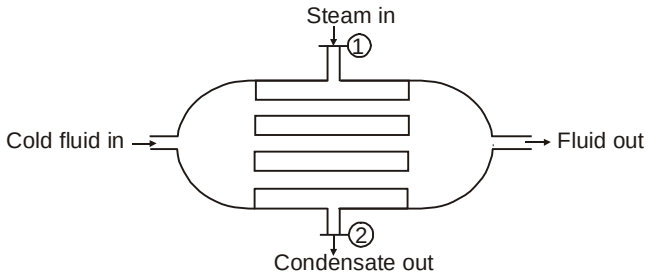


Fig. 3.22 *Condenser*

Steady flow energy equation can be applied with the following assumptions :

(*i*) No work interaction, $W = 0$

(*ii*) No change in kinetic energy, $\Delta KE = 0$

(*iii*) No change in potential energy, $\Delta PE = 0$

Heat lost by steam,

$$Q = m\,(h_1 - h_2)$$

(*f*) *Nozzle*: Nozzle is the engineering device in which expansion of fluid takes place and pressure drops simultaneously. Thus in nozzle the velocity of fluid increases from inlet to exit. In case of subsonic flow the nozzle has converging cross-section area in the duct where as in supersonic flow the nozzle has diverging cross-section area in the duct.

Let us take a converging cross-section area duct as shown in Fig. 3.23.

Flow through the nozzle may be analysed with following assumptions:

(*i*) No heat interaction, i.e. $Q = 0$, during passage through duct.

(*ii*) No work interaction, i.e. $W = 0$, during passage through duct.

(*iii*) No change in elevation from 1 to 2, i.e. $\Delta PE = 0$.

Applying S.F.E.E on nozzle,

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

or, $$\frac{C_2^2 - C_1^2}{2} = h_1 - h_2$$

$$C_2 = \sqrt{C_1^2 + 2(h_1 - h_2)}$$

or, $$C_2 = \sqrt{C_1^2 + 2c_p(T_1 - T_2)}$$

In case, the velocity at inlet to nozzle is very small, then C_1 may be neglected and velocity at nozzle exit shall be:

$$C_2 = \sqrt{2c_p(T_1 - T_2)}$$

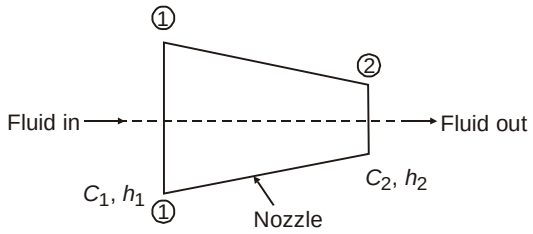


Fig. 3.23 *Nozzle*

(*g*) *Throttling*: Throttling refers to passage of a fluid through some restricted opening under isenthalpic conditions. Thus in the figure shown below the fluid passes through a restriction from section 1 to 2 and undergoes drop in its pressure and increase in volume, but during this passage enthalpy remains constant, such that $h_1 = h_2$.

Based on above throttling process the device called "throttle valve" has been developed in which pressure drop is realized without involving any work and heat interaction, change in kinetic energy and potential energy. Temperature may drop or increase during the throttling process and shall depend upon the Joule-Thomson coefficient, a property based on characteristic of substance.

Joule-Thomson coefficient $\mu = \left(\frac{\partial T}{\partial p}\right)_{h=\text{constt.}}$

and if $\mu = 0$, Temperature remains constant

$\mu > 0$, Temperature decreases.

$\mu < 0$ Temperature increases.

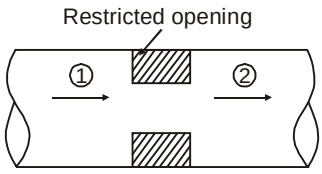


Fig. 3.24 *Throttling process*

(*h*) *Combustion chamber*: Combustion chambers are commonly used in gas turbine installations, in which fuel is injected at high pressure into a chamber having high pressure, high temperature air in it and ignited for heat release at constant pressure.

Mass balance yields:

$$m_2 = m_1 + m_f$$

where m_1, m_2, m_f are mass flow rates at 1, 2, and *f* shown in figure.

S.F.E.E. may be applied with assumptions of $\Delta KE = 0$,

$$\Delta PE = 0,$$

$$W = 0$$

Here, $Q = m_f \times$ Calorific value of fuel

or $$Q = m_f \times CV$$

$$\boxed{Q + m_1 h_1 + m_f h_f = m_2 h_2}$$

substituting for Q

$$m_f CV + m_1 h_1 + m_f h_f = m_2 h_2$$

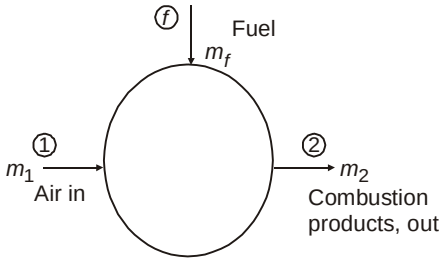


Fig. 3.25 *Combustion chamber*

(*i*) *Adiabatic mixing*: Adiabatic mixing refers to mixing of two or more streams of same or different fluids under adiabatic conditions.

Let us consider two streams of same fluid with mass flow rates m_1 and m_2 to get mixed together adiabatically.

Assumptions for applying S.F.E.E shall be;

(*i*) No heat interaction, $Q = 0$

(*ii*) No work interaction, $W = 0$

(*iii*) No change in kinetic energy, $\Delta KE = 0$

(*iv*) No change in potential energy, $\Delta PE = 0$.

Thus,

$$m_1 \cdot h_1 + m_2 \cdot h_2 = m_3 \cdot h_3.$$

or $$m_1 \cdot c_p \cdot T_1 + m_2 \cdot c_p \cdot T_2 = m_3 \cdot c_p \cdot T_3$$

or $$\boxed{T_3 = \frac{m_1 T_1 + m_2 T_2}{m_3}}$$

By mass balance,

$$m_1 + m_2 = m_3$$

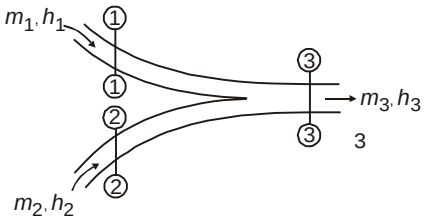


Fig. 3.26 *Adiabatic mixing*

3.10 UNSTEADY FLOW SYSTEMS AND THEIR ANALYSIS

In earlier discussions, for a steady flow system, it has been assumed that the properties do not change with time.

However, there exist a number of systems such as filling up of a bottle or emptying of a vessel etc. in which properties change continuously as the process proceeds. Such systems can not be analysed with the steady state assumptions. Unsteady flow processes are also known as transient flow processes or variable flow processes.

Let us take example of filling up of the bottle.

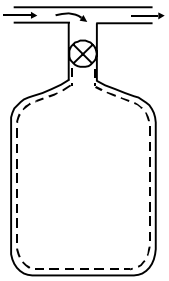


Fig. 3.27 *Filling of the bottle*

The bottle is filled up gradually, therefore it is case of an unsteady system. By conservation of mass, the unsteady process over a period of time '*dt*' can be expressed as following in generic form.

(Mass entering the control volume in time *dt*)

– (Mass leaving the control volume in time *dt*)

= Net change in mass in control volume in time *dt.*

If the mass flow rate at inlet and exit are given as m_i, m_e then

$$\frac{dm_i}{dt} - \frac{dm_e}{dt} = \frac{dm_{cv}}{dt}$$

and also, $$\sum m_i - \sum m_e = (m_{\text{final}} - m_{\text{initial}})_{cv}$$

By the conservation of energy principle applied on control volume for time '*t*', energy balance yields;

Net energy interaction across the boundary in time *dt*

+ Energy entering into control volume in time *dt*

– Energy leaving out of control volume in time *dt*

= Change in energy in control volume in time *dt*

Mathematically, it can be given as:

$$\boxed{(Q - W) + \sum E_i - \sum E_e = \Delta E_{cv}}$$

where $$E_i = \int_0^t m_i \left(h_i + \frac{C_i^2}{2} + gz_i\right) . dt$$

$$E_e = \int_0^t m_e \left(h_e + \frac{C_e^2}{2} + gz_e\right) . dt$$

Thus, the above mass balance and energy balance can be used for analysing the unsteady flow systems with suitable assumptions.

It may be assumed that the control volume state is uniform and fluid properties are uniform and steady at inlet and exit.

Simplified form of energy balance written above can be given as;

$$Q - W + \sum m_i \left(h_i + \frac{C_i^2}{2} + gz_i\right) - \sum m_e \left(h_e + \frac{C_e^2}{2} + gz_e\right)$$

$$= (m_{\text{final}} \,.\, u_{\text{final}} - m_{\text{initial}} \,.\, u_{\text{initial}})_{cv}$$

If the changes in kinetic energy and potential energy are negligible, then energy balance gets modified as;

$$Q - W + \sum m_i \cdot h_i - \sum m_e \cdot h_e = (m_{\text{final}} \cdot u_{\text{final}} - m_{\text{initial}} \cdot u_{\text{initial}})_{cv}$$

Case 1: Let us now use the energy and mass balance to the unsteady flow process of filling up a bottle as shown in Figure 3.27. Bottle is initially empty and connected to a pipe line through valve for being filled.

Let us denote initial state of system by subscript 1 and final state by 2.

Initially as bottle is empty, so $m_1 = 0$

From mass balance

$$\sum m_i - \sum m_e = (m_2 - 0)_{cv}$$

Here there is no exit from the bottle so $m_e = 0$

hence, $$\sum m_i = m_2$$

or, $$m_i = m_2$$

Mass entered into bottle = Final mass inside the bottle

Applying the energy balance assuming change in kinetic and potential energy to be negligible, treating bottle filling process to be occurring in insulated environment, and no work interaction, we get

$Q \approx 0$, $W \approx 0$, $\Delta KE \approx 0$, $\Delta PE \approx 0$,

Initial internal energy in bottle = 0

Mass leaving = 0

$$0 = -\sum m_i \cdot h_i + (m_2 \cdot u_2)_{cv}$$

or $$m_i \cdot h_i = m_2 u_2$$

also $$\boxed{h_i = u_2 \text{ as } m_i = m_2}$$

Enthalpy of fluid entering bottle = Final internal energy of fluid in bottle.

If fluid is ideal gas, then $c_p \cdot T_i = c_v \cdot T_2$

or $$T_2 = \gamma \cdot T_i$$

where $$\frac{c_p}{c_v} = \gamma$$

Case 2: Let us now take a case of emptying of bottle. Arrangement is shown in Fig. 3.28.

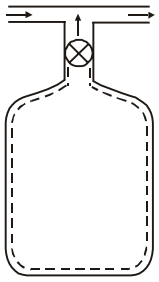


Fig. 3.28 *Emptying of bottle*

Initially bottle has mass m_1 and finally as a result of emptying, say mass left is m_2 after some time.

Applying mass balance, (as mass entering is zero),

or, $$0 - \sum m_e = (m_2 - m_1)_{cv}$$

or $$\sum m_e = (m_1 - m_2)_{cv}$$

or $$\boxed{m_e = (m_1 - m_2)_{cv}}$$

Total mass leaving the bottle = (Mass reduced in bottle)

Applying energy balance, with the assumptions given below;

(*i*) No heat interaction i.e. $Q = 0$

(*ii*) No work interaction i.e. $W = 0$

(*iii*) No change in kinetic energy i.e. $\Delta KE = 0$

(*iv*) No change in potential energy i.e. $\Delta PE = 0$

$$-\sum m_e \cdot h_e = (m_2 u_2 - m_1 u_1)_{cv}$$

or, $$(-m_e \cdot h_e) = (m_2 u_2 - m_1 u_1)_{cv}$$

Substituting for 'm_e' we get $(m_2 - m_1)_{cv} \cdot h_e = (m_2 u_2 - m_1 u_1)_{cv}$

In case of complete emptying, $m_2 = 0$

and so, $\boxed{h_e = u_1}$

3.11 LIMITATIONS OF FIRST LAW OF THERMODYNAMICS

First law of thermodynamics based on law of energy conservation has proved to be a powerful tool for thermodynamic analysis. But over the period of time when it was applied to some real systems, it was observed that theoretically first law stands valid for the processes which are not realizable practically. It was then thought that there exist certain flaws in first law of thermodynamics and it should be used with certain limitations.

Say for example let us take a bicycle wheel and paddle it to rotate. Now apply brake to it. As a result of braking wheel comes to rest upon coming in contact with brake shoe. Stopping of wheel is accompanied by heating of brake shoe. Examining the situation from Ist law of thermodynamics point of view it is quite satisfying that rotational energy in wheel has been transformed into heat energy with shoe, thus causing rise in its temperature:

Now, if we wish to introduce the same quantity of heat into brake shoe and wish to restore wheel motion then it is not possible simply, whereas theoretically first law permits the conversion from heat to work (rotation of wheel in this case) as well.

Therefore, it is obvious that Ist law of thermodynamics has certain limitations as given below:

(*i*) First law of thermodynamics does not differentiate between heat and work and assures full convertibility of one into other whereas full conversion of work into heat is possible but the vice-versa is not possible.

(*ii*) First law of thermodynamics does not explain the direction of a process. Such as theoretically it shall permit even heat transfer from low temperature body to high temperature body which is not practically feasible. Spontaneity of the process is not taken care of by the first law of thermodynamics.

Perpetual motion machine of the first kind (PMM-I) is a hypothetical device conceived, based on violation of First law of thermodynamics. Let us think of a system which can create energy as shown below.

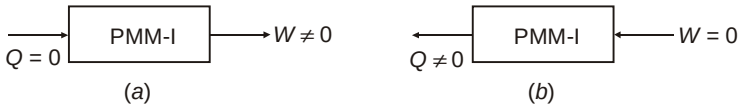


Fig. 3.29 *PMM-I, based on violation of Ist law of thermodynamics*

Here a device which is continuously producing work without any other form of energy supplied to it has been shown in (*a*), which is not feasible.

Similarly a device which is continuously emitting heat without any other form of energy supplied to it has been shown in (*b*), which is again not feasible.

Above two imaginary machines are called Perpetual Motion Machines of 1st kind.

EXAMPLES

1. *Figure shows a system comprising of gas in cylinder at pressure of 689 kPa.*

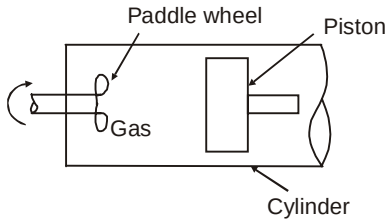


Fig. 3.30

Fluid expands from a volume of 0.04 m^3 to 0.045 m^3 while pressure remains constant. Paddle wheel in the system does a work of 4.88 kJ on the system. Determine (a) work done by system on the piston (b) the net amount of work done on or by the system.

Solution:

(*a*) It is a closed system. If the pressure on face of piston is uniform, then the work done on piston can be obtained as,

$$W = p \int_1^2 dV$$

$$= 689 \times 10^3 (0.045 - 0.04)$$

Work done on piston, $W = 3445$ J or 3.445 kJ

Work done on piston = **3.445 kJ Ans.**

(*b*) Paddle work done on the system = – 4.88 kJ

Net work of system $\Rightarrow W_{net} = W_{piston} + W_{paddle}$

$$= 3445 - 4880$$

$$W_{net} = -1435 \text{ J}$$

Work done on system = 1435 J or 1.435 kJ.

Work done on system = **1.435 kJ Ans.**

2. *A gas at 65 kPa, 200°C is heated in a closed, rigid vessel till it reaches to 400°C. Determine the amount of heat required for 0.5 kg of this gas if internal energy at 200°C and 400°C are 26.6 kJ/kg and 37.8 kJ/kg respectively.*

Solution:

Given $m = 0.5$ kg

$u_1 = 26.6$ kJ/kg

$u_2 = 37.8$ kJ/kg

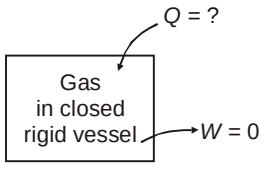


Fig. 3.31

As the vessel is rigid therefore work done shall be zero.

$W = 0$

From first law of thermodynamics;

$$Q = U_2 - U_1 + W = m(u_2 - u_1) + 0$$

$$Q = 0.5 (37.8 - 26.6)$$

$$Q = 5.6 \text{ kJ}$$

Heat required = **5.6 kJ Ans.**

3. *Carbon dioxide passing through a heat exchanger at a rate of 50 kg/hr is to be cooled down from 800°C to 50°C. Determine the rate of heat removal assuming flow of gas to be of steady and constant pressure type. Take* c_p *= 1.08 kJ/kg K.*

Solution:

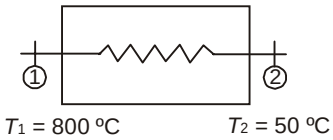


Fig. 3.32

Given, $m = 50$ kg/hr

Writing down the steady flow energy equation.

$$q + h_1 + \frac{C_1^2}{2} + gz_1 = h_2 + \frac{C_2^2}{2} + gz_2 + w$$

Here let us assume changes in kinetic and potential energy to be negligible. During flow the work interaction shall also be zero.

Hence $q = h_2 - h_1$

or $Q = m\,(h_2 - h_1)$

$= m \cdot c_p \cdot (T_2 - T_1)$

$= 50 \times 1.08 \times (750)$

$= 40500$ kJ/hr

Heat should be removed at the rate of **40500 kJ/hr Ans.**

4. *A completely evacuated cylinder of 0.78* m^3 *volume is filled by opening its valve to atmosphere and air rushing into it. Determine the work done by the air and by surroundings on system.*

Solution : Total work done by the air at atmospheric pressure of 101.325 kPa,

$$W = \int_{\text{cylinder}} p.\,dv + \int_{\text{air}} p.\,dv$$

$= 0 + p.\Delta v$, it is –ve work as air boundary shall contract

Work done by air $= -101.325 \times 0.78 =$ **– 79.03 kJ**

Work done by surroundings on system = **+ 79.03 kJ** } **Ans.**

5. *A system comprising of a gas of 5 kg mass undergoes expansion process from 1 MPa and 0.5* m^3 *to 0.5 MPa. Expansion process is governed by,* $p.v^{1.3}$ *= constant. The internal energy of gas is given by, u = 1.8 pv + 85, kJ/kg. Here 'u' is specific internal energy, 'p' is pressure in kPa, 'v' is specific volume in* m^3*/kg. Determine heat and work interaction and change in internal energy.*

Solution:

Given mass of gas, $m = 5$ kg, $pv^{1.3}$ = constant

Assuming expansion to be quasi-static, the work may be given as,

$$W = m\int p.dv$$

$$= \frac{p_2 V_2 - p_1 V_1}{(1-n)}$$

From internal energy relation, change in specific internal energy,

$$\Delta u = u_2 - u_1 = 1.8\,(p_2 v_2 - p_1 v_1),\ \text{kJ/kg}$$

Total change, $\Delta U = 1.8 \times m \times (p_2 v_2 - p_1 v_1)$, kJ

$\Delta U = 1.8 \times (p_2 V_2 - p_1 V_1)$

Between states 1 and 2,

$p_1 V_1^{1.3} = p_2 V_2^{1.3}$

or $p_1 V_1^{1.3} = p_2 V_2^{1.3}$

$$\Rightarrow \quad V_2 = (0.5) \cdot \left(\frac{1}{0.5}\right)^{1/1.3}$$

$V_2 = 0.852 \text{ m}^3$

Total change in internal energy, $\Delta U = -133.2$ kJ

Work, $$W = \frac{(0.5 \times 0.852 - 1 \times 0.5) \times 10^3}{(1 - 1.3)}$$

$W = 246.67$ kJ

From first law,

$\Delta Q = \Delta U + W$

$= -133.2 + 246.7$

$\Delta Q = 113.5$ kJ

Heat interaction = **113.5 kJ**
Work interaction = **246.7 kJ**
Change in internal energy = **–133.2 kJ** } **Ans.**

6. *A gas contained in a cylinder is compressed from 1 MPa and 0.05 m^3 to 2 MPa. Compression is governed by $pV^{1.4}$ = constant. Internal energy of gas is given by;*

$$U = 7.5\ pV - 425, \text{ kJ.}$$

where p is pressure in kPa and V is volume in m^3.

Determine heat, work and change in internal energy assuming compression process to be quasi-static.

Also find out work interaction, if the 180 kJ of heat is transferred to system between same states. Also explain, why is it different from above.

Solution:

Final state, volume $$V_2 = \left(\frac{p_1}{p_2}\right)^{1/1.4} \cdot V_1$$

$$= \left(\frac{1}{2}\right)^{1/1.4} \cdot 0.05$$

$V_2 = 0.03 \text{ m}^3$

Change in internal energy,

$\Delta U = U_2 - U_1 = (7.5\ p_2 V_2 - 7.5\ p_1 V_1)$

$= 7.5 \times 10^3 (2 \times 0.03 - 1 \times 0.05)$

$\Delta U = 75$ kJ

For quasi-static process,

Work, $$W = \int_1^2 p \cdot dV$$

$$= \frac{p_2V_2 - p_1V_1}{1-n}$$

$$= \frac{(2\times 0.03 - 1\times 0.05)\times 10^3}{(1-1.4)}$$

$$W = 25 \text{ kJ, (–ve)}$$

From first law of thermodynamics,

Heat interaction $\Delta Q = \Delta U + W$

$= 75 + (-25)$

$= 50$ kJ

Heat = **50 kJ**

Work = **25 kJ** (–ve) } **Ans.**

Internal energy change = **75 kJ**

If 180 kJ heat transfer takes place, then from Ist law,

$$\Delta Q = \Delta U + W$$

Since end states remain same, therefore ΔU, i.e. change in internal energy remains unaltered.

$$180 = 75 + W$$

or $$W = \mathbf{105\ kJ}$$

This work is different from previous work because the process is not quasi-static in this case. **Ans.**

7. *Determine the heat transfer and its direction for a system in which a perfect gas having molecular weight of 16 is compressed from 101.3 kPa, 20°C to a pressure of 600 kPa following the law $pV^{1.3}$ = constant. Take specific heat at constant pressure of gas as 1.7 kJ/kg.K.*

Solution:

Characteristic gas constant, $$R = \frac{\text{Universal gas constant}}{\text{Molecular weight}}$$

$$= \frac{8.3143\times 10^3}{16}, \text{ J/kg.K}$$

$$= 519.64, \text{ J/kg.K}$$

or $$= 0.51964, \text{ kJ/kg.K}$$

$$R = 0.520, \text{ kJ/kg.K}$$

$$C_v = C_p - R$$

$$= 1.7 - 0.520$$

$$C_v = 1.18, \text{ kJ/kg.K}$$

or $$\gamma = \frac{C_p}{C_v} = \frac{1.7}{1.18} = 1.44$$

For polytropic process, $$V_2 = \left(\frac{p_1}{p_2}\right)^{1/1.3} \cdot V_1$$

or $$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{\frac{1.3-1}{1.3}}$$

$$T_2 = 293 \cdot \left(\frac{600}{101.3} \right)^{0.231}$$

$$T_2 = 441.9 \text{ K}$$

Work, $$W = \frac{R(T_1 - T_2)}{(1.3-1)}$$

$$W = 258.1 \text{ kJ/kg}$$

For polytropic process

Heat, $$Q = \left(\frac{\gamma - n}{\gamma - 1} \right) \cdot W = \left(\frac{1.44 - 1.3}{1.44 - 1} \right) \times 258.1$$

= 82.12, kJ/kg (+ve) Ans.

8. *In a nozzle air at 627°C and twice atmospheric pressure enters with negligible velocity and leaves at a temperature of 27°C. Determine velocity of air at exit, assuming no heat loss and nozzle being horizontal. Take C_P = 1.005 kJ/kg.K for air.*

Solution: Applying steady flow energy equation with inlet and exit states as 1, 2 with no heat and work interaction and no change in potential energy.

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

Given that, $C_1 \approx 0$, negligible inlet velocity

$$C_2 = \sqrt{2(h_1 - h_2)}$$

Exit velocity, $$C_2 = \sqrt{2 \cdot C_p (T_1 - T_2)}$$

Given, T_1 = 900 K, T_2 = 300 K

or $$C_2 = \sqrt{2 \times 1.005 \times 10^3 (900 - 300)}$$

$$C_2 = 1098.2 \text{ m/s}$$

Exit velocity = **1098.2 m/s. Ans.**

9. *An air compressor requires shaft work of 200 kJ/kg of air and the compression of air causes increase in enthalpy of air by 100 kJ/kg of air. Cooling water required for cooling the compressor picks up heat of 90 kJ/kg of air. Determine the heat transferred from compressor to atmosphere.*

Solution:

Work interaction, W = – 200 kJ/kg of air

Increase in enthalpy of air = 100 kJ/kg of air

Total heat interaction, Q = Heat transferred to water + Heat transferred to atmosphere.

Writing steady flow energy equation on compressor, for unit mass of air entering at 1 and leaving at 2.

$$h_1 + \frac{C_1^2}{2} + gZ_1 + Q = h_2 + \frac{C_2^2}{2} + gZ_2 + W$$

Assuming no change in potential energy and kinetic energy;

$$\Delta KE = \Delta PE = 0$$
$$Q = (h_2 - h_1) + W$$
$$= 100 + (-200)$$
$$Q = -100 \text{ kJ/kg of air}$$

Also

$$Q = -100 = \text{Heat transferred to water} + \text{Heat transferred to atmosphere}$$

or

$$-100 = (-90) + \text{Heat transferred to atmosphere}$$

Heat transferred to atmosphere = **– 10 kJ/kg** of air **Ans.**

10. *In a cinema hall with seating capacity of 500 persons the comfort conditions are created by circulating hot water through pipes in winter season. Hot water enters the pipe with enthalpy of 80 kcal/kg and leaves the pipe with enthalpy of 45 kcal/kg. The difference in elevation of inlet pipe and exit pipe is 10 m with exit pipe being higher than inlet pipe. Heat requirement per person is 50 kcal/hr. Estimate the quantity of water circulated per minute, neglecting changes in velocity.*

Solution:

Above problem can be solved using steady flow energy equations upon hot water-flow,

$$Q + m_1\left(h_1 + \frac{C_1^2}{2} + gz_1\right) = W + m_2\left(h_2 + \frac{C_2^2}{2} + gz_2\right)$$

Here total heat to be supplied = 500 × 50

Heat lost by water (–ve), Q = – 25000 kcal/hr.

There shall be no work interaction and change in kinetic energy, so, steady flow energy equation shall be,

$$Q + m_1(h_1 + gz_1) = m_2(h_2 + gz_2)$$

Here $m_1 = m_2 = m$, $h_1 = 80$ kcal/kg, $h_2 = 45$ kcal/kg

$$Q + m(h_1 - h_2) = m(gz_2 - gz_1)$$
$$-(25000 \times 10^3 \times 4.18) + m(80 - 45) \times 10^3 \times 4.18 = m \times (9.81 \times 10)$$
$$m = 714.76 \text{ kg/hr}$$

or, Water circulation rate = **11.91 kg/min. Ans.**

11. *Figure 3.33 shows steam injector for lifting water from a depth of 2 m from axis of injector. Determine the rate at which steam should be supplied for pumping unit mass of water. Enthalpy of steam entering and water entering injector are 720 kcal/kg and 24.6 kcal/kg respectively.*

Enthalpy of steam and water mixture leaving injector is 100 kcal/kg. Velocity of steam entering injector is 50 m/s while mixture leaves with 25 m/s. Heat loss from injector to surroundings is 12 kcal/kg of steam sent through injector.

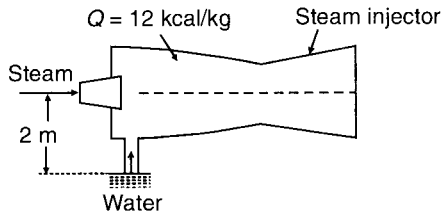


Fig. 3.33

Solution:

Let mass of steam to be supplied per kg of water lifted be '*m*' kg. Applying law of energy conservation upon steam injector, for unit mass of water lifted.

Energy with steam entering + Energy with water entering = Energy with mixture leaving + Heat loss to surroundings.

$$m\left[\frac{(50)^2}{2} + (720\times10^2\times4.18)\right] + 1 \times [(24.6 \times 10^3 \times 4.18) + (9.81 \times 2)]$$

K.E. Enthalpy Enthalpy P.E

$$= (1 + m)\left[(100\times10^3\times4.18) + \frac{(25)^2}{2}\right] + [m \times 12 \times 10^3 \times 4.18]$$

Enthalpy K.E Heat loss

m [3010850] + [102847.62] = (1 + m) . (418312.5) + m[50160]

Upon solving, m = 0.124 kg steam/kg of water

Steam supply rate = **0.124 kg/s** per kg of water. **Ans.**

12. *An inelastic flexible balloon is inflated from initial empty state to a volume of 0.4 m^3 with H_2 available from hydrogen cylinder. For atmospheric pressure of 1.0313 bar determine the amount of work done by balloon upon atmosphere and work done by atmosphere.*

Solution:

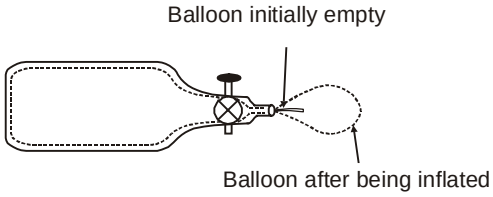


Fig. 3.34

Here let us assume that the pressure is always equal to atmospheric pressure as balloon is flexible, inelastic and unstressed and no work is done for stretching balloon during its filling. Figure 3.34 shows the boundary of system before and after filling balloon by firm line and dotted line respectively.

Displacement work, $$W = \int_{\text{cylinder}} p.dV + \int_{\text{balloon}} p.dV$$

$\int p.dV$ = 0 as cylinder shall be rigid.

$= 0 + p \cdot \Delta V$

$= 0 + 1.013 \times 10^5 \times 0.4$

= 40.52 kJ

Work done by system upon atmosphere = **40.52 kJ**

Work done by atmosphere = **– 40.52 kJ** } **Ans.**

13. *In a steam power plant 5 kW of heat is supplied in boiler and turbine produces 25% of heat added while 75% of heat added is rejected in condenser. Feed water pump consumes 0.2% of this heat added*

for pumping condensate to boiler. Determine the capacity of generator which could be used with this plant.

Solution:

Given, $Q_{add} = 5000$ J/s

so, $W_T = 0.25 \times 5000 = 1250$ J/s

$Q_{rejected} = 0.75 \times 5000 = 3750$ J/s

$W_p = (-)\ 0.002 \times 5000 = 10$ J/s

Capacity of generator $= W_T - W_P$

$= 1250 - 10$

$= 1240$ J/s or 1240 W

$=$ **1.24 kW Ans.**

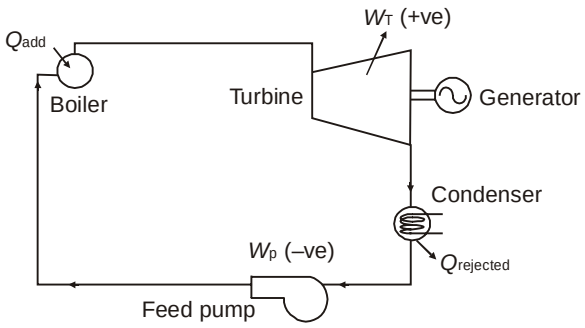


Fig. 3.35

14. *In a gas turbine installation air is heated inside heat exchanger upto 750°C from ambient temperature of 27°C. Hot air then enters into gas turbine with the velocity of 50 m/s and leaves at 600°C. Air leaving turbine enters a nozzle at 60 m/s velocity and leaves nozzle at temperature of 500°C. For unit mass flow rate of air determine the following assuming adiabatic expansion in turbine and nozzle,*

(a) heat transfer to air in heat exchanger

(b) power output from turbine

(c) velocity at exit of nozzle.

Take c_p for air as 1.005 kJ/kg°K.

Solution:

In heat exchanger upon applying S.F.E.E. with assumptions of no change in kinetic energy, no work interaction, no change in potential energy, for unit mass flow rate of air,

$$h_1 + Q_{1-2} = h_2$$

$$Q_{1-2} = h_2 - h_1$$

$$Q_{1-2} = C_p \cdot (T_2 - T_1)$$

Heat transfer to air in heat exchanger $Q_{1-2} =$ **726.62 kJ Ans.**

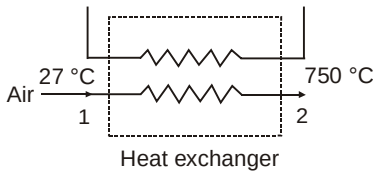


Fig. 3.36

In gas turbine let us use S.F.E.E., assuming no change in potential energy, for unit mass flow rate of air

$$h_2 + \frac{C_2^2}{2} = h_3 + \frac{C_3^2}{2} + W_T$$

$$W_T = (h_2 - h_3) + \left(\frac{C_2^2 - C_3^2}{2}\right)$$

$$= C_p(T_2 - T_3) + \left(\frac{C_2^2 - C_3^2}{2}\right)$$

$$= 1.005\ (750 - 600) + \left(\frac{50^2 - 60^2}{2}\right) \times 10^{-3}$$

Power output from turbine = **150.2 kJ/s Ans.**

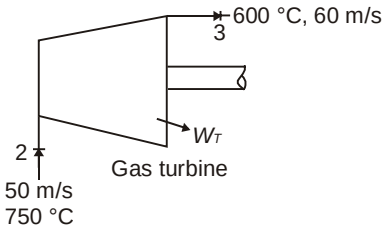


Fig. 3.37

Applying S.F.E.E. upon nozzle assuming no change in potential energy, no work and heat interactions, for unit mass flow rate,

$$h_3 + \frac{C_3^2}{2} = h_4 + \frac{C_4^2}{2}$$

$$\frac{C_4^2}{2} = (h_3 - h_4) + \frac{C_3^2}{2}$$

$$= C_p(T_3 - T_4) + \frac{C_3^2}{2}$$

$$= 1.005\ (600 - 500) + \left(\frac{60^2}{2}\right) \times 10^{-3}$$

$$\frac{C_4^2}{2} = 102.3$$

$$C_4 = 14.3 \text{ m/s}$$

Velocity at exit of nozzle = **14.3 m/s Ans.**

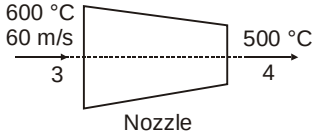


Fig. 3.38

15. *One mol of air at 0.5 MPa and 400 K, initially undergoes following processes, sequentially*

(a) heating at constant pressure till the volume gets doubled.

(b) expansion at constant temperature till the volume is six times of initial volume.

Determine the work done by air.

Solution:

For constant pressure heating, say state changes from 1 to 2

$$W_a = \int_1^2 p_1 \, dV$$

$$W_a = p_1 (V_2 - V_1)$$

It is given that $V_2 = 2V_1$

so $W_a = p_1 V_1$

$$W_a = RT_1$$

For subsequent expansion at constant temperature say state changes from 2 to 3.

Also given that $\frac{V_3}{V_1} = 6$, so $\frac{V_3}{V_2} = 3$

Work, $W_b = \int_2^3 pdV$

$$= \int_2^3 \frac{RT}{V} dV = RT_2 \ln \frac{V_3}{V_2}$$

$$W_b = RT_2 \ln (3)$$

Temperature at 2 can be given by perfect gas considerations as,

$$\frac{T_2}{T_1} = \frac{V_2}{V_1}$$

or $T_2 = 2 T_1$

Total work done by air, $W = W_a + W_b$

$= RT_1 + RT_2 \ln (3)$

$= RT_1 + 2RT_1 \ln (3)$

$= RT_1 (1 + 2 \ln 3) = 8.314 \times 400 (1 + 2 \ln 3)$

Work done = **10632.69 kJ Ans.**

16. *Determine the work done by gas for the arrangement shown in Fig. 3.39. Here spring exerts a force upon piston which is proportional to its deformation from equilibrium position. Spring gets deflected due to heating of gas till its volume becomes thrice of original volume. Initial states are 0.5 MPa and 0.5 m^3 while final gas pressure becomes 1 MPa. Atmospheric pressure may be taken as 1.013 × 10^5 Pa.*

Solution:

Let stiffness of spring be k and it undergoes a deflection by 'x' along x-axis. Force balance at any equilibrium position of piston shall be,

$$p. A = p_{atm} \cdot A + kx,$$

here x shall be linear displacement of piston due to expansion of gas. Let volume of gas change from V_0 to some value V. Then, $x = \frac{V - V_0}{A}$, V_0 is volume of gas when spring is at its natural length.

$$p \cdot A = p_{atm} \cdot A + k\left(\frac{V - V_0}{A}\right)$$

or $$(p - p_{atm}) = \frac{k(V - V_0)}{A^2}$$

Work done by gas between initial and final states, $W = \int_i^f p.dV$

$$W = p_{atm}(V_f - V_i) + \frac{k}{A^2}\left\{\frac{V^2}{2} - V_0.V\right\}_i^f$$

$$= p_{atm}(V_f - V_i) + \frac{k}{A^2}\left\{\frac{V_f^2 - V_i^2}{2} - V_0 V_f + V_0 . V_i\right\}$$

$$= (V_f - V_i)\, p_{atm} + \frac{k}{A^2}\left\{\left(\frac{V_f^2 - V_i^2 - 2V_0 V_f + 2V_0 . V_i}{2}\right)\right\}$$

$$= (V_f - V_i)\, p_{atm} + \left\{\frac{k}{2A^2}((V_f - V_0) + (V_i - V_0))(V_f - V_i)\right\}$$

$$= (V_f - V_i)\left\{P_{atm}\frac{k}{2A^2}((V_f - V_0) + (V_i - V_0))\right\}$$

from above force balance, $(p_f - p_{atm}) = \frac{k}{A^2}(V_f - V_0)$

$$(p_i - p_{atm}) = \frac{k}{A^2}(V_i - V_0)$$

or $$W = (V_f - V_i)\left\{p_{atm} + \left(\frac{p_f - p_{atm}}{2}\right) + \left(\frac{p_i - p_{atm}}{2}\right)\right\}$$

$$= (V_f - V_i)\left\{\frac{p_i + p_f}{2}\right\}$$, substituting pressure and volume values, $V_f = 3V_i$

$W =$ **0.75×10^6 J Ans.**

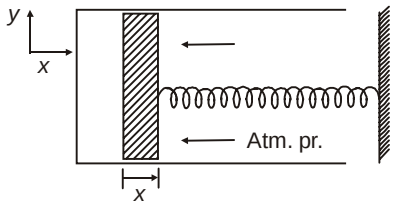


Fig. 3.39

17. *A closed insulated container has frictionless and smooth moving insulated partition as shown in Fig. 3.40 such that it equally divides total 1 m^3 of volume, when both the gases are at initial pressure of 0.5 MPa and ambient temperature of 27°C. Subsequently the nitrogen is heated using electrical heating element such that volume of N_2 becomes 3/4 of total volume of container. Determine, (i) final pressure of hydrogen, (ii) Work done by partition, (iii) Workdone by N_2 and H_2 (iv) Heat added to N_2 by electric heater.*

Take $C_{p,N_2} = 1.039$ *kJ/kg. K,* $C_{p,H_2} = 14.307$ *kJ/kg . K,*

$R_{N_2} = 0.2968$ *kJ/kg . K,* $R_{H_2} = 4.1240$ *kJ/kg . K*

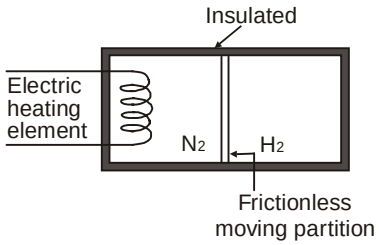


Fig. 3.40

Solution:

With the heating of N_2 it will get expanded while H_2 gets compressed simultaneously. Compression of H_2 in insulated chamber may be considered of adiabatic type.

Adiabatic Index of compression for H_2 can be obtained as,

$$C_{p,H_2} = R_{H_2}\left(\frac{\gamma_{H_2}}{\gamma_{H_2}-1}\right)$$

$$14.307 = 4.124\left(\frac{\gamma_{H_2}}{\gamma_{H_2}-1}\right)$$

$$\gamma_{H_2} = 1.405$$

Adiabatic Index of expansion for N_2, $C_{p,N_2} = R_{N_2}\left(\frac{\gamma_{N_2}}{\gamma_{N_2}-1}\right)$

$$1.039 = 0.2968\left(\frac{\gamma_{N_2}}{\gamma_{N_2}-1}\right)$$

$$\gamma_{N_2} = 1.399$$

(*i*) For hydrogen, $p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$

Here $\gamma = \gamma_{H_2} = 1.405$, $V_1 = 0.5$ m^3

$p_1 = 0.5 \times 10^6$ Pa, $V_2 = 0.25$ m^3

Final pressure of $H_2 = 0.5 \times 10^6 \left(\frac{0.5}{0.25}\right)^{1.405}$

= **1.324 MPa Ans.**

(*ii*) Since partition remains in equilibrium throughout hence no work is done by partition. It is a case similar to free expansion.

Partition work = 0. Ans.

(*iii*) Work done upon H_2,

$$W_{H_2} = \frac{P_1V_1 - p_2V_2}{(\gamma_{H_2}-1)}$$

Here $p_1 = 0.5 \times 10^6$ Pa, $p_2 = 1.324 \times 10^6$ Pa, $V_1 = 0.5$ m^3, $V_2 = 0.25$ m^3.

Work done by hydrogen, $W_{H_2} = \dfrac{(-)0.081\times10^6}{0.405}$

$= \mathbf{(-)\ 2 \times 10^5\ J\ Ans.}$

Work done by N_2 = Work done upon H_2

Work done by nitrogen = **+ 2 × 10^5 J Ans.**

(*iv*) Heat added to N_2 can be obtained using first law of thermodynamics as

$$Q_{N_2} = \Delta U_{N_2} + W_{N_2} \Rightarrow Q_{N_2} = mc_v(T_2 - T_1) + W_{N_2}$$

Final temperature of N_2 can be obtained considering it as perfect gas.

Therefore, $$T_2 = \frac{p_2V_2T_1}{p_1V_1}$$

p_2 = Final pressure of N_2 which will be equal to that of H_2 as the partition is free and frictionless.

$$p_2 = 1.324 \times 10^6 \text{ Pa.}$$

T_2 = Final temperature of $N_2 = \dfrac{1.324\times10^6\times0.75\times300}{0.5\times10^6\times0.5}$

$= 1191.6$ K

mass of N_2, $m = \dfrac{p_1V_1}{RT_1} = \dfrac{0.5\times10^6\times0.5}{0.2968\times10^3\times300} = 2.8$ kg.

Specific heat at constant volume, $C_v = C_p - R \Rightarrow C_{v,N_2} = 0.7422$ kJ/kg . K.

Heat added to N_2, $Q_{N_2} = \{2.8 \times (1191.6 - 300) \times 0.7422 \times 10^3\} + 2 \times 10^5$

$= \mathbf{2052.9\ kJ\ Ans.}$

18. *A cylinder of 2 m^3 has air at 0.5 MPa and temperature of 375°K. Air is released in atmosphere through a valve on cylinder so as to run a frictionless turbine. Find the amount of work available from turbine assuming no heat loss and complete kinetic energy being used for running turbine. Take $C_{p,\,air}$ = 1.003 kJ/kg . K, $C_{v,\,air}$ = 0.716 kJ/kg . K, R_{air} = 0.287 kJ/kg . K.*

Solution:

Let initial states and final states of air inside cylinder be given by m_1, p_1, V_1, T_1, and m_2, p_2, V_2, T_2 respectively. It is a case of emptying of cylinder.

Initial mass of air, $$m_1 = \frac{p_1V_1}{P_{\text{air}}.T_1} = 9.29 \text{ kg.}$$

For adiabatic expansion during release of air through valve from 0.5 MPa to atmospheric pressure.

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$= 375\left(\frac{1.013\times10^5}{0.5\times10^6}\right)^{\frac{1.4-1}{1.4}}$$

$$T_2 = 237.65 \text{ K}$$

Final mass of air left in tank, $m_2 = \dfrac{p_2 V_2}{R T_2}$

$$m_2 = \frac{1.013 \times 10^5 \times 2}{(0.287 \times 10^3 \times 237.65)} = 2.97 \text{ kg}$$

Writing down energy equation for unsteady flow system

$$(m_1 - m_2)\left(h_2 + \frac{C^2}{2}\right) = m_1 u_1 - m_2 u_2$$

$$(m_1 - m_2)\frac{C^2}{2} = (m_1 u_1 - m_2 u_2) - (m_1 - m_2)h_2$$

Kinetic energy available for running turbine

$= (m_1 C_v T_1 - m_2 C_v T_2) - (m_1 - m_2) \cdot C_p \cdot T_2$

$= (9.29 \times 0.716 \times 10^3 \times 375) - (2.97 \times 0.716 \times 10^3 \times 237.65) - \{(9.29 - 2.97) \times 1.003 \times 10^3 \times 237.65\}$

$= 482.54 \times 10^3$ J

Amount of work available = **482.54 kJ Ans.**

19. *A rigid and insulated tank of 1 m^3 volume is divided by partition into two equal volume chambers having air at 0.5 MPa, 27°C and 1 MPa, 500 K. Determine final pressure and temperature if the partition is removed.*

Solution:

Using perfect gas equation for the two chambers having initial states as 1 and 2 and final state as 3.

$p_1 = 0.5 \times 10^6$ Pa, $V_1 = 0.5$ m^3, $T_1 = 300$ K

$$n_1 = \frac{p_1 V_1}{\bar{R} T_1}$$

$$= \frac{0.5 \times 10^6 \times 0.5}{8314 \times 300}$$

$$n_1 = 0.1002$$

and $$n_2 = \frac{p_2 V_2}{\bar{R} T_2}$$ where $p_2 = 1 \times 10^6$ Pa, $V_2 = 0.5$ m^3, $T_2 = 500$ K

$$= \frac{1 \times 10^6 \times 0.5}{8314 \times 500}$$

$$n_2 = 0.1203$$

For tank being insulated and rigid we can assume, $\Delta U = 0$, $W = 0$, $Q = 0$, so writing ΔU,

$$\Delta U = n_1 C_v (T_3 - T_1) + n_2 C_v (T_3 - T_2) = 0$$

or $$T_3 = 409.11 \text{ K}$$

Using perfect gas equation for final mixture,

$$p_3 = \frac{(n_1 + n_2)\bar{R} T_3}{(V_1 + V_2)} = 0.75 \text{ MPa}$$

Final pressure and temperature = **0.75 MPa, 409.11 K Ans.**

20. *An evacuated bottle of 0.5 m^3 volume is slowly filled from atmospheric air at 1.0135 bars until the pressure inside the bottle also becomes 1.0135 bar. Due to heat transfer, the temperature of air inside the bottle after filling is equal to the atmospheric air temperature. Determine the amount of heat transfer.* *[U.P.S.C., 1994]*

Solution:

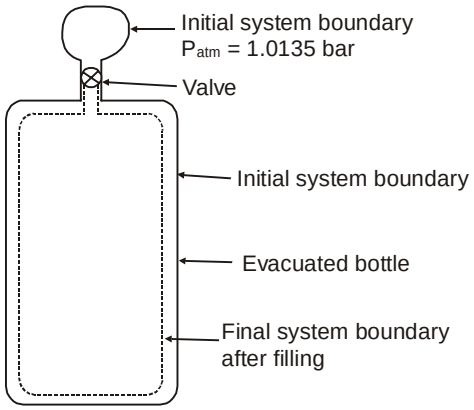


Fig. 3.41

Displacement work; $W = 1.0135 \times 10^5 \times (0 - 0.5)$

$W = -0.50675 \times 10^5$ Nm

Heat transfer, $Q = 0.50675 \times 10^5$ Nm

Heat transfer = **0.50675×10^5 Nm** **Ans.**

21. *A compressed air bottle of 0.3 m^3 volume contains air at 35 bar, 40°C. This air is used to drive a turbogenerator sypplying power to a device which consumes 5 W. Calculate the time for which the device can be operated if the actual output of the turbogenerator is 60% of the maximum theoretical output. The ambient pressure to which the tank pressure has fallen is 1 bar. For air, $\frac{C_p}{C_v} = 1.4$.*

[U.P.S.C. 1993]

Solution:

Here turbogenerator is fed with compressed air from a compressed air bottle. Pressure inside bottle gradually decreases from 35 bar to 1 bar. Expansion from 35 bar to 1 bar occurs isentropically. Thus, for the initial and final states of pressure, volume, temperature and mass inside bottle being given as P_1, V_1, T_1 & m_1 and P_2, V_2, T_2 & m_2 respectively. It is transient flow process similar to emptying of the bottle.

$$\left(\frac{P_2}{T_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1}$$, Given: $P_1 = 35$ bar, $T_1 = 40$°C or 313 K

$V_1 = 0.3$ m^3; $V_2 = 0.3$ m^3

$P_2 = 1$ bar.

$$T_2 = T_1 \left(\frac{P_2}{T_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = 113.22 \text{ K}$$

By perfect gas law, initial mass in bottle, $m_1 = \frac{P_1V_1}{RT_1} = \frac{35\times10^2\times0.3}{0.287\times313}$

$$m_1 = 11.68 \text{ kg}$$

Final mass in bottle, $m_2 = \frac{P_2V_2}{RT_2} = \frac{1\times10^2\times0.3}{0.287\times113.22}$

$$m_2 = 0.923 \text{ kg}$$

Energy available for running turbo generator or work;

$$W + (m_1 - m_2)\, h_2 = m_1\, u_1 - m_2\, u_2$$

$$W = (m_1u_1 - m_2u_2) - (m_1 - m_2)\, h_2$$

$$= (m_1\, c_v\, T_1 - m_2\, c_v\, T_2) - (m_1 - m_2) \cdot c_p \cdot T_2$$

Taking $c_v = 0.718$ kJ/kg . K and $c_P = 1.005$ kJ/kg · K

$$W = \{(11.68 \times 0.718 \times 313) - (0.923 \times 0.718 \times 113.22)\}$$
$$- \{(11.68 - 0.923) \times 1.005 \times 113.22\}$$

$$W = 1325.86 \text{ kJ}$$

This is the maximum work that can be had from the emptying of compressed air bottle between given pressure limits.

Turbogenerator's actual output = 5 kJ/s

Input to turbogenerator = $\frac{5}{0.6}$ = 8.33 kJ/s.

Time duration for which turbogenerator can be run;

$$\Delta t = \frac{1325.86}{8.33}$$

$$\Delta t = 159.17 \text{ sec.}$$

Duration ≈ **160 seconds** **Ans.**

22. *3 kg of air at 1.5 bar pressure and 77°C temperature at state 1 is compressed polytropically to state 2 at pressure 7.5 bar, index of compression being 1.2. It is then cooled at constant temperature to its original state 1. Find the net work done and heat transferred.* *[U.P.S.C. 1992]*

Solution:

Different states as described in the problem are denoted as 1, 2 and 3 and shown on *p-V* diagram.

Process 1-2 is polytropic process with index 1.2

So, $$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

or, $$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

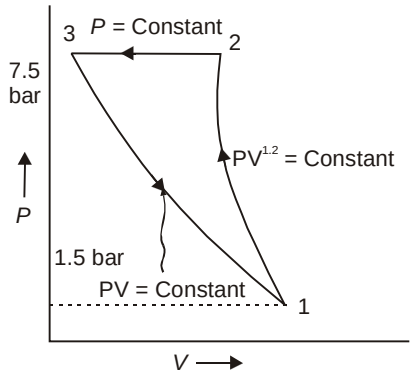


Fig. 3.42

$$= 350 \cdot \left(\frac{7.5}{1.5}\right)^{\frac{1.2-1}{1.2}}$$

$$T_2 = 457.68 \text{ K}$$

At state 1, $P_1V_1 = mRT_1$

$$\frac{1.5\times10^5}{10^3} \cdot V_1 = 3 \times 0.287 \times 350$$

or, $V_1 = 2.009 \approx 2.01 \text{ m}^3$

For process 1-2, $V_2^{1.2} = \frac{P_1V_1^{1.2}}{P_2}$, $V_2 = \left\{\frac{1.5\times(2.01)^{1.2}\times10^5}{7.5\times10^5}\right\}^{\frac{1}{1.2}}$

or, $V_2 = 0.526 \text{ m}^3$

Process 2-3 is constant pressure process, so $\frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3}$ gets modified as,

$\Rightarrow$ $V_3 = \frac{V_2 \cdot T_3}{T_2}$

Here process 3-1 is isothermal process, so $T_1 = T_3$

or, $V_3 = \frac{0.526\times350}{457.68}$

or, $V_3 = 0.402 \text{ m}^3$

During process 1-2 the compression work;

$$W_{1-2} = \frac{m.R.(T_1 - T_2)}{1-n}$$

$$= \frac{3\times0.287(457.68-350)}{(1-1.2)}$$

$$W_{1-2} = -463.56 \text{ kJ}$$

Work during process 2-3,

$$W_{2-3} = P_2 (V_3 - V_2)$$
$$= 7.5 \times 10^5 (0.402 - 0.526)$$
$$= - 93 \text{ kJ}$$

Work during process 3-1,

$$W_{3-1} = P_3 V_3 \ln \left(\frac{V_1}{V_3} \right) = 7.5 \times 10^5 \times 0.402 \times \ln \left(\frac{2.01}{0.402} \right)$$

$$W_{3-1} = 485.25 \text{ kJ}$$

Net work, $W_{net} = W_{1-2} + W_{2-3} + W_{3-1}$

$$= - 463.56 - 93 + 485.25$$

Network = **– 71.31 kJ** **Ans.**

–ve work shows work done upon the system. Since it is the cycle, so

$$W_{net} = Q_{net}$$

$$\oint dW = \oint dQ = - 71.31 \text{ kJ}$$

Heat transferred from system = **71.31 kJ** **Ans.**

23. *A compressed air bottle of volume 0.15 m^3 contains air at 40 bar and 27°C. It is used to drive a turbine which exhausts to atmosphere at 1 bar. If the pressure in the bottle is allowed to fall to 2 bar, determine the amount of work that could be delivered by the turbine.* *[U.P.S.C. 1998]*

Solution:

$$c_p = 1.005 \text{ kJ/kg . K}, \; c_v = 0.718 \text{ kJ/kg K}, \; \gamma = 1.4$$

Initial mass of air in bottle $\Rightarrow m_1 = \dfrac{p_1 V_1}{R T_1} = \dfrac{40 \times 10^2 \times 0.15}{0.287 \times 300}$

$$m_1 = 6.97 \text{ kg}$$

Final mass of air in bottle $\Rightarrow m_2 = \dfrac{p_2 V_2}{R T_2}$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}}, \; m_2 = \frac{2 \times 10^2 \times 0.15}{0.287 \times 127.36}$$

$$= \left(\frac{2}{40} \right)^{\frac{1.4-1}{1.4}}, \; m_2 = 0.821 \text{ kg.}$$

$$T_2 = 127.36 \text{ K}$$

Energy available for running of turbine due to emptying of bottle,

$$= (m_1 \, c_v \, T_1 - m_2 \, c_v \, T_2) - (m_1 - m_2) \cdot c_p \cdot T_2$$
$$= \{(6.97 \times 0.718 \times 300) - (0.821 \times 0.718 \times 127.36)\}$$
$$- \{(6.97 - 0.821) \times 1.005 \times 127.35\}$$
$$= 639.27 \text{ kJ.}$$

Work available from turbine = **639.27 kJ** **Ans.**

EXERCISE

3.1 Define the first law of thermodynamics. Also give supporting mathematical expression for it.

3.2 How the first law of thermodynamics is applied to a closed system undergoing a non-cyclic process?

3.3 Show that internal energy is a property.

3.4 Explain the following :

(*a*) Free expansion

(*b*) Polytropic process

(*c*) Hyperbolic process

Also obtain expressions for work in each case.

3.5 Show that for a polytropic process.

$$Q = \left(\frac{\gamma - n}{\gamma - 1}\right) W$$

where Q and W are heat and work interactions and n is polytropic index.

3.6 Derive the steady flow energy equation.

3.7 Explain a unsteady flow process.

3.8 Show that for a given quantity of air supplied with a definite amount of heat at constant volume, the rise in pressure shall be directly proportional to initial absolute pressure and inversely proportional to initial absolute temperature.

3.9 How much work is done when 0.566 m^3 of air initially at a pressure of 1.0335 bar and temperature of 7°C undergoes an increase in pressure upto 4.13 bar in a closed vessel? [0]

3.10 An ideal gas and a steel block are initially having same volumes at same temperature and pressure. Pressure on both is increased isothermally to five times of its initial value. Show with the help of *P–V* diagram, whether the quantities of work shall be same in two processes or different. If different then which one is greater. Assume processes to be quasi-static.

3.11 An inventor has developed an engine getting 1055 MJ from fuel and rejecting 26.375 MJ in exhaust and delivering 25 kWh of mechanical work. Is this engine possible? [No]

3.12 For an ideal gas the pressure is increased isothermally to '*n*' times its initial value. How high would the gas be raised if the same amount of work were done in lifting it? Assume process to be quasi-static.

3.13 A system's state changes from *a* to *b* as shown on *P–V* diagram

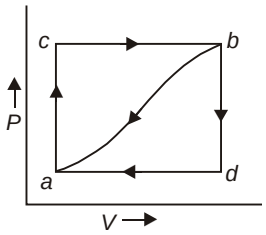


Fig. 3.43

Along path '*acb*' 84.4 kJ of heat flows into the system and system does 31.65 kJ of work. Determine heat flow into the system along path '*adb*' if work done is 10.55 kJ. When system returns from '*b*' to '*a*' following the curved path then work done on system is 21.1 kJ. How much heat is absorbed or rejected? If internal energy at '*a*' and '*d*' are 0 and 42.2 kJ, find the heat absorbed in processes '*ad*' and '*db*'. [63.3 kJ, – 73.85 kJ, 52.75 kJ, 10.55 kJ]

3.14 A tank contains 2.26 m^3 of air at a pressure of 24.12 bar. If air is cooled until its pressure and temperature becomes 13.78 bar and 21.1°C respectively. Determine the decrease of internal energy.

[– 5857.36 kJ]

3.15 Water in a rigid, insulating tank is set in rotation and left. Water comes to rest after some time due to viscous forces. Considering the tank and water to constitute the system answer the following.
(*i*) Is any work done during the process of water coming to rest?
(*ii*) Is there a flow of heat?
(*iii*) Is there any change in internal energy (*U*)?
(*iv*) Is there any change in total energy (*E*)? [No, No, Yes, No]

3.16 Fuel-air mixture in a rigid insulated tank is burnt by a spark inside causing increase in both temperature and pressure. Considering the heat energy added by spark to be negligible, answer the following :
(*i*) Is there a flow of heat into the system?
(*ii*) Is there any work done by the system?
(*iii*) Is there any change in internal energy (*U*) of system?
(*iv*) Is there any change in total energy (*E*) of system? [No, No, No, No]

3.17 Calculate the work if in a closed system the pressure changes as per relation $p = 300 \, . \, V + 1000$ and volume changes from 6 to 4 m^3. Here pressure '*p*' is in Pa and volume '*V*' is in m^3. [– 5000J]

3.18 Hydrogen from cylinder is used for inflating a balloon to a volume of $35m^3$ slowly. Determine the work done by hydrogen if the atmospheric pressure is 101.325 kPa. [3.55 MJ]

3.19 Show that the work done by an ideal gas is mRT_1, if gas is heated from initial temperature T_1 to twice of initial temperature at constant volume and subsequently cooled isobarically to initial state.

3.20 Derive expression for work done by the gas in following system. Piston-cylinder device shown has a gas initially at pressure and volume given by P_1, V_1. Initially the spring does not exert any force on piston. Upon heating the gas, its volume gets doubled and pressure becomes P_2.

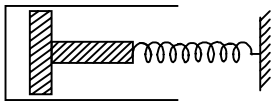


Fig. 3.44 *Piston-cylinder arrangement*

3.21 An air compressor with pressure ratio of 5, compresses air to $\frac{1}{4}$th of the initial volume. For inlet temperature to be 27°C determine temperature at exit and increase in internal energy per kg of air. [101.83°C, 53.7 kJ/kg]

3.22 In a compressor the air enters at 27°C and 1 atm and leaves at 227°C and 1 MPa. Determine the work done per unit mass of air assuming velocities at entry and exit to be negligible. Also determine the additional work required, if velocities are 10 m/s and 50 m/s at inlet and exit respectively. [200.9 kJ/kg, 202.1 kJ/kg]

3.23 Turbojet engine flies with velocity of 270 m/s at the altitude where ambient temperature is –15°C. Gas temperature at nozzle exit is 873 K and fuel air ratio is 0.019. Corresponding enthalpy values for air and gas at inlet and exit are 260 kJ/kg and 912 kJ/kg respectively. Combustion efficiency is 95% and calorific value of fuel is 44.5 MJ/kg. For the heat losses from engine amounting to 21 kJ/kg of air determine the velocity of gas jet at exit. [613.27 m/s]

3.24 Oxygen at 3MPa and 300°C flowing through a pipe line is tapped out to fill an empty insulated rigid tank. Filling continues till the pressure equilibrium is not attained. What shall be the temperature of the oxygen inside the tank? If $\gamma = 1.39$. [662.5°C]

3.25 Determine work done by fluid in the thermodynamic cycle comprising of following processes :
(*a*) Unit mass of fluid at 20 atm and 0.04 m^3 is expanded by the law $PV^{1.5}$ = constant, till volume gets doubled.
(*b*) Fluid is cooled isobarically to its original volume.
(*c*) Heat is added to fluid till its pressure reaches to its original pressure, isochorically. [18.8 kJ]

3.26 An air vessel has capacity of 10 m^3 and has air at 10 atm and 27°C. Some leakage in the vessel causes air pressure to drop sharply to 5 atm till leak is repaired. Assuming process to be of reversible adiabatic type determine the mass of air leaked. [45.95 kg]

3.27 Atmospheric air leaks into a cylinder having vacuum. Determine the final temperature in cylinder when inside pressure equals to atmospheric pressure, assuming no heat transferred to or from air in cylinder. [144.3°C]

3.28 Determine the power available from a steam turbine with following details;
Steam flow rate = 1 kg/s
Velocity at inlet and exit = 100 m/s and 150 m/s
Enthalpy at inlet and exit = 2900 kJ/kg, 1600 kJ/kg
Change in potential energy may be assumed negligible. [1293.75 kW]

3.29 Determine the heat transfer in emptying of a rigid tank of 1m^3 volume containing air at 3 bar and 27°C initially. Air is allowed to escape slowly by opening a valve until the pressure in tank drops to 1 bar pressure. Consider escape of air in tank to follow polytropic process with index n = 1.2 [76.86 kJ]

3.30 A pump is used for pumping water from lake at height of 100 m consuming power of 60 kW. Inlet pipe and exit pipe diameters are 150 mm and 180 mm respectively. The atmospheric temperature is 293 K. Determine the temperature of water at exit of pipe. Take specific heat of water as 4.18 kJ/kg.K [293.05K]

3.31 Air at 8 bar, 100°C flows in a duct of 15 cm diameter at rate of 150 kg/min. It is then throttled by a valve upto 4 bar pressure. Determine the velocity of air after throttling and also show that enthalpy remains constant before and after throttling. [37.8 m/s]

3.32 Determine the power required by a compressor designed to compress atmospheric air (at 1 bar, 20°C) to 10 bar pressure. Air enters compressor through inlet area of 90cm^2 with velocity of 50 m/s and leaves with velocity of 120 m/s from exit area of 5 cm^2. Consider heat losses to environment to be 10% of power input to compressor. [50.4 kW]

4

Second Law of Thermodynamics

4.1 INTRODUCTION

Earlier discussions in article 3.11 throw some light on the limitations of first law of thermodynamics. A few situations have been explained where first law of thermodynamics fails to mathematically explain non-occurrence of certain processes, direction of process etc. Therefore, need was felt to have some more law of thermodynamics to handle such complex situations. Second law came up as embodiment of real happenings while retaining the basic nature of first law of thermodynamics. Feasibility of process, direction of process and grades of energy such as low and high are the potential answers provided by IInd law. Second law of thermodynamics is capable of indicating the maximum possible efficiencies of heat engines, coefficient of performance of heat pumps and refrigerators, defining a temperature scale independent of physical properties etc.

4.2 HEAT RESERVOIR

Heat reservoir is the system having very large heat capacity i.e. it is a body capable of absorbing or rejecting finite amount of energy without any appreciable change in its' temperature. Thus in general it may be considered as a system in which any amount of energy may be dumped or extracted out and there shall be no change in its temperature. Such as *atmosphere* to which large amount of heat can be rejected without measurable change in its temperature. Large river, sea etc. can also be considered as reservoir, as dumping of heat to it shall not cause appreciable change in temperature.

Heat reservoirs can be of two types depending upon nature of heat interaction i.e. heat rejection or heat absorption from it. Heat reservoir which rejects heat from it is called *source.* While the heat reservoir which absorbs heat is called *sink.* Some times these heat reservoirs may also be called Thermal Energy Reservoirs (TER).

4.3 HEAT ENGINE

Heat engine is a device used for converting heat into work as it has been seen from nature that conversion from work to heat may take place easily but the vice-versa is not simple to be realized. Heat and work have been categorized as two forms of energy of low grade and high grade type. Conversion of high grade of energy to low grade of energy may be complete (100%), and can occur directly whereas complete conversion of low grade of energy into high grade of energy is not possible. For converting low grade of energy (heat) into high grade of energy (work) some device called heat engine is required.

Thus, heat engine may be precisely defined as "a device operating in cycle between high temperature source and low temperature sink and producing work". Heat engine receives heat from source, transforms

some portion of heat into work and rejects balance heat to sink. All the processes occurring in heat engine constitute cycle.

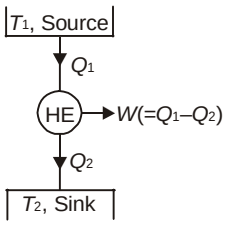


Fig. 4.1 *Heat engine*

Block diagram representation of a heat engine is shown above. A practical arrangement used in gas turbine plant is also shown for understanding the physical singnificance of heat engine.

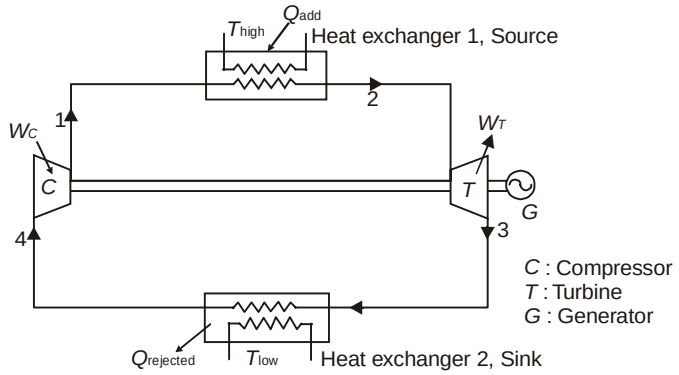


Fig. 4.2 *Closed cycle gas turbine power plant*

Gas turbine installation shows that heat is added to working fluid from 1–2 in a 'heat exchanger 1' and may be treated as heat supply by source. Working fluid is expanded in turbine from 2–3 and produces positive work. After expansion fluid goes to the 'heat exchanger 2' where it rejects heat from it like heat rejection in sink. Fluid at state 4 is sent to compressor for being compressed to state 1. Work required for compression is quite small as compared to positive work available in turbine and is supplied by turbine itself.

Therefore, heat engine model for it shall be as follows,

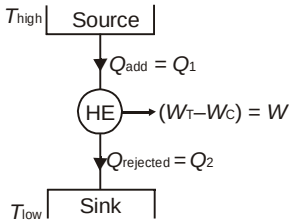


Fig. 4.3 *Heat engine representation for gas turbine plant*

Efficiency of heat engine can be given by the ratio of net work and heat supplied.

$$\eta_{\text{heat engine}} = \frac{\text{Net work}}{\text{Heat supplied}} = \frac{W}{Q_1}$$

For gas turbine plant shown

$$W = W_T - W_C$$

and $$Q_1 = Q_{\text{add}}$$

Also since it is operating in cycle, so;

$$W_T - W_C = Q_{\text{add}} - Q_{\text{rejected}}$$

therefore, efficiency of heat engine can be given as;

$$\eta_{\text{heat engine}} = \frac{W_T - W_C}{Q_{\text{add}}}$$

$$= \frac{Q_{\text{add}} - Q_{\text{rejected}}}{Q_{\text{add}}}$$

$$\eta_{\text{heat engine}} = 1 - \frac{Q_{\text{rejected}}}{Q_{\text{add}}}$$

4.4 HEAT PUMP AND REFRIGERATOR

Heat pump refers to a device used for extracting heat from a low temperature surroundings and sending it to high temperature body, while operating in a cycle. In other words heat pump maintains a body or system at temperature higher than temperature of surroundings, while operating in cycle. Block diagram representation for a heat pump is given below:

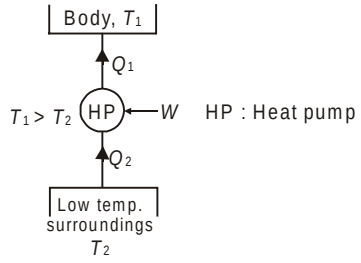


Fig. 4.4 *Heat pump*

As heat pump transfers heat from low temperature to high temperature, which is non spontaneous process, so external work is required for realizing such heat transfer. Heat pump shown picks up heat Q_2 at temperature T_2 and rejects heat $Q_1$for maintaining high temperature body at temperature T_1.

For causing this heat transfer heat pump is supplied with work W as shown.

As heat pump is not a work producing machine and also its objective is to maintain a body at higher temperature, so its performance can't be defined using efficiency as in case of heat engine. Performance of heat pump is quantified through a parameter called coefficient of performance (C.O.P). *Coefficient of performance* is defined by the ratio of desired effect and net work done for getting the desired effect.

$$\boxed{\text{C.O.P.} = \frac{\text{Desired effect}}{\text{Net work done}}}$$

For heat pump :

Net work = W

Desired effect = heat transferred Q_1 to high temperature body at temperature, T_1.

$$(COP)_{HP} = \frac{Q_1}{W}$$

also $$W = Q_1 - Q_2$$

so $$\boxed{(COP)_{HP} = \frac{Q_1}{Q_1 - Q_2}}$$

Refrigerator is a device similar to heat pump but with reverse objective. It maintains a body at temperature lower than that of surroundings while operating in a cycle. Block diagram representation of refrigerator is shown in Fig 4.5.

Refrigerator also performs a non spontaneous process of extracting heat from low temperature body for maintaining it cool, therefore external work W is to be done for realizing it.

Block diagram shows how refrigerator extracts heat Q_2 for maintaining body at low temperature T_2 at the expense of work W and rejects heat to high temperature surroundings.

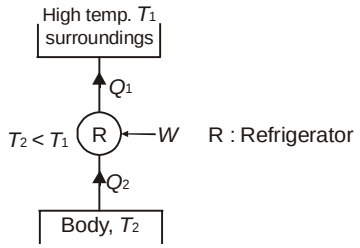


Fig. 4.5 *Refrigerator*

Performance of refrigerator is also quantified by coefficient of performance, which could be defined as:

$$(COP)_{refrigerator} = \frac{\text{Desired effect}}{\text{Net work}} = \frac{Q_2}{W}$$

Here $$W = Q_1 - Q_2$$

or $$(COP)_{refrigerator} = \frac{Q_2}{Q_1 - Q_2}$$

COP values of heat pump and refrigerator can be interrelated as:

$$(COP)_{HP} = (COP)_{refrigerator} + 1$$

4.5 STATEMENTS FOR IIND LAW OF THERMODYNAMICS

Rudolph Julius Emmanuel Clausius, a German physicist presented a first general statement of second law of thermodynamics in 1850 after studying the work of Sadi Carnot. It was termed as Clausius statement of second law. Lord Kelvin and Max Planck also came up with another statement of second law which was termed as Kelvin-Planck statement for second law of thermodynamics. Thus, there are two statements of second law of thermodynamics, (although they are equivalent as explained ahead).

***Clausius statement of second law of thermodynamics*:** "It is impossible to have a device that while operating in a cycle produces no effect other than transfer of heat from a body at low temperature to a body at higher temperature."

Above statement clearly indicates that if a non spontaneous process such as transferring heat from low temperature body to high temperature body is to be realized then some other effects such as

external work requirement is bound to be there. As already seen in case of refrigerator the external work is required for extracting heat from low temperature body and rejecting it to high temperature body.

***Kelvin-Planck statement of second law of thermodynamics*:** "It is impossible for a device operating in a cycle to produce net work while exchanging heat with bodies at single fixed temperature".

It says that in order to get net work from a device operating in cycle (i.e. heat engine) it must have heat interaction at two different temperatures or with body/reservoirs at different temperatures (i.e. source and sink).

Thus, above two statements are referring to feasible operation of heat pump/refrigerator and heat engine respectively. Devices based on violation of IInd law of thermodynamics are called Perpetual motion machines of 2nd kind (PMM-II). Fig 4.6 shows such PMM-II.

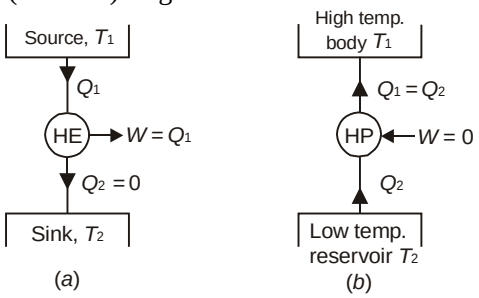


***Fig. 4.6** Perpetual Motion Machine of IInd kind*

PMM-II shown in Fig. 4.6*a*, refers to a heat engine which produces work while interacting with only one reservoir. PMM-II shown in Fig. 4.6*b*, refers to the heat pump which transfers heat from low temperature to high temperature body without spending work.

4.6 EQUIVALENCE OF KELVIN-PLANCK AND CLAUSIUS STATEMENTS OF IIND LAW OF THERMODYNAMICS

Kelvin-Planck and Clausius statements of IInd law of thermodynamics are actually two different interpretations of the same basic fact. Here the equivalence of two statements has been shown. For establishing equivalence following statements may be proved.

(*a*) System based on violation of Kelvin-Planck statement leads to violation of Clausius statement.

(*b*) System based on violation of Clausius statement leads to violation of Kelvin-Planck statement.

The exaplanation for equivalence based on above two is explained ahead.

(*a*) Let us assume a heat engine producing net work while exchanging heat with only one reservoir at temperature T_1, thus based on violation of Kelvin Planck statement. Let us also have a perfect heat pump operating between two reservoirs at temperatures T_1 and T_2. Work requirement of heat pump may be met from the work available from heat engine. Layout shown ahead explains the proposed arrangement.

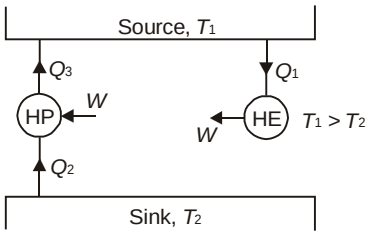


***Fig. 4.7** System based on violation of Kelvin Planck statement*

If heat pump takes input work from output of heat engine then,

$$Q_3 = Q_2 + W$$

and $$W = Q_1$$

or $$Q_3 = Q_1 + Q_2$$

Combination of heat engine and heat pump shall thus result in an equivalent system working as heat pump transferring heat from low temperature T_2 to high temperature T_1 without expense of any external work. This heat pump is based on violation of Clausius statement and therefore not possible.

Hence, it shows that violation of Kelvin Planck statement leads to violation of Clausius statement.

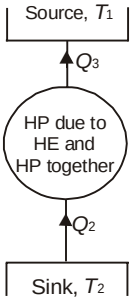


Fig. 4.8 *Equivalent system*

(*b*) Let us assume a heat pump which operating in cycle transfers heat from low temperature reservoir to high temperature reservoir without expense of any work, thus based on violation of Clausius statement.

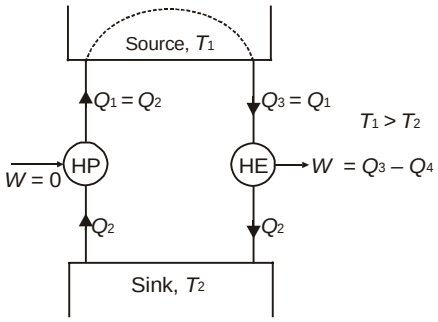


Fig. 4.9 *System based on violation of Clausius statement*

Heat pump transfers heat Q_1 to high temperature reservoir while extracting heat Q_2 from low temperature reservoir. Mathematically, as no work is done on pump, so

$$Q_2 = Q_1$$

Let us also have a heat engine between same temperature limits of T_1 and T_2 and produce net work W. Heat engine receives heat Q_3 from source which may be taken equal to Q_1. Let us now devise for heat rejected from heat pump be given directly to heat engine. In such a situation the combination of heat pump and heat engine results in equivalent heat engine which produces work 'W' while exchanging heat with only one reservoir at temperature T_2. Arrangement is shown by dotted lines. This type of equivalent system is producing work as a result of only one heat interaction and thus violation of Kelvin Planck statement.

Thus, it shows that violation of Clausius statement also causes violation of Kelvin Planck statement.

Hence from (*a*) and (*b*) proved above it is obvious that the Clausius and Kelvin-Planck statements are equivalent. Conceptually the two statements explain the basic fact that,

(*i*) net work can't be produced without having heat interactions taking place at two different temperatures.

(*ii*) non spontaneous process such as heat flow from low temperature body to high temperature body is not possible without spending work.

4.7 REVERSIBLE AND IRREVERSIBLE PROCESSES

Reversible processes as described in chapter 1 refer to "the thermodynamic processes occurring in the manner that states passed through are always in thermodynamic equilibrium and no dissipative effects are present." Any reversible process occurring between states 1–2 upon reversal, while occurring from 2–1 shall not leave any mark of process ever occurred as states traced back are exactly similar to those in forward direction. Reversible processes are thus very difficult to be realized and also called ideal processes. All thermodynamic processes are attempted to reach close to the reversible process in order to give best performance.

Thermodynamic process which does not fulfil conditions of a reversible process are termed irreversible processes. Irreversibilities are the reasons causing process to be irreversible. Generally, the irreversibilities can be termed as internal irreversibility and external irreversibility. Internal irreversibility is there because of internal factors whereas external irreversibility is caused by external factors at the system-surrounding interface. Generic types of irreversibilities are due to;

(*i*) Friction,

(*ii*) Electrical resistance,

(*iii*) Inelastic solid deformations,

(*iv*) Free expansion

(*v*) Heat transfer through a finite temperature difference,

(*vi*) Non equilibrium during the process, etc.

(*i*) ***Friction:*** Friction is invariably present in real systems. It causes irreversibility in the process as work done does not show equivalent rise in kinetic or potential energy of the system. Fraction of energy wasted due to frictional effects leads to deviation from reversible states.

(*ii*) ***Electrical resistance*:** Electrical resistance in the system also leads to presence of dissipation effects and thus irreversibilities. Due to electric resistance dissipation of electrical work into internal energy or heat takes place. The reverse transformation from heat or internal energy to electrical work is not possible, therefore leads to irreversibility.

(*iii*) ***Inelastic solid deformation:*** Deformation of solids, when of inelastic type is also irreversible and thus causes irreversibility in the process. If deformation occurs within elastic limits then it does not lead to irreversibility as it is of reversible type.

(*iv*) ***Free expansion:*** Free expansion as discussed earlier in chapter 3, refers to the expansion of unresisted type such as expansion in vacuum. During this unresisted expansion the work interaction is zero and without expense of any work it is not possible to restore initial states. Thus, free expansion is irreversible.

(*v*) ***Heat transfer through a finite temperature difference:*** Heat transfer occurs only when there exist temperature difference between bodies undergoing heat transfer. During heat transfer if heat addition is carried out in finite number of steps then after every step the new

state shall be a non-equilibrium state. In order to have equilibrium states in between, the heat transfer process may be carried out in infinite number of steps. Thus, infinitesimal heat transfer every time causes infinitesimal temperature variation. These infinitesimal state changes shall require infinite time and process shall be of quasi-static type, therefore reversible. Heat transfer through a finite temperature difference which practically occurs is accompanied by irreversible state changes and thus makes processes irreversible.

(*vi*) ***Non equilibrium during the process:*** Irreversibilities are introduced due to lack of thermodynamic equilibrium during the process. Non equilibrium may be due to mechanical inequilibrium, chemical inequilibrium, thermal inequilibrium, electrical inequilibrium etc. and irreversibility are called mechanical irreversibility, chemical irreversibility, thermal irreversibility, electrical irreversibility respectively. Factors discussed above are also causing non equilibrium during the process and therefore make process irreversible.

Comparative study of reversible and irreversible processes shows the following major differences.

Difference between reversible and irreversible processes

Reversible process	*Irreversible process*
(*i*) Reversible process can not be realized in practice	(*i*) All practical processes occurring are irreversible processes
(*ii*) The process can be carried out in the reverse direction following the same path as followed in forward direction	(*ii*) Process, when carried out in reverse direction follows the path different from that in forward direction.
(*iii*) A reversible process leaves no trace of occurrence of process upon the system and surroundings after its' reversal.	(*iii*) The evidences of process having occurred are evident even after reversal of irreversible process.
(*iv*) Such processes can occur in either directions without violating second law of thermodynamics.	(*iv*) Occurrence of irreversible processes in either direction is not possible, as in one direction it shall be accompanied with the violation of second law of thermodynamics.
(*v*) A system undergoing reversible processes has maximum efficiency. So the system with reversible processes are considered as reference systems or bench marks.	(*v*) System having irreversible processes do not have maximum efficiency as it is accompanied by the wastage of energy.
(*vi*) Reversible process occurs at infinitesimal rate i.e. quasi-static process.	(*vi*) Irreversible processes occur at finite rate.
(*vii*) System remains throughout in thermodynamic equilibrium during occurrence of such process.	(*vii*) System does not remain in thermodynamic equilibrium during occurrence of irreversible processes.
(*viii*) Examples; Frictionless motion, controlled expansion and compression, Elastic deformations, Electric circuit with no resistance, Electrolysis, Polarization and magnetisation process etc.	(*viii*) Examples; Viscous fluid flow, inelastic deformation and hysteresis effect, Free expansion, Electric circuit with resistance, Mixing of dissimilar gases, Throttling process etc.

4.8 CARNOT CYCLE AND CARNOT ENGINE

Nicholas Leonard Sadi Carnot, an engineer in French army originated use of cycle (Carnot) in thermodynamic analysis in 1824 and these concepts provided basics upon which second law of thermodynamics was stated by Clausius and others.

Carnot cycle is a reversible thermodynamic cycle comprising of four reversible processes.

Thermodynamic processes constituting Carnot cycle are;

(*i*) Reversible isothermal heat addition process, (1–2, Q_{add})

(*ii*) Reversible adiabatic expansion process (2–3, W_{expn} +ve)

(*iii*) Reversible isothermal heat release process (3–4, $Q_{rejected}$)

(*iv*) Reversible adiabatic compression process (4–1, W_{compr} –ve)

Carnot cycle is shown on *P–V* diagram between states 1, 2, 3 4, and 1. A reciprocating piston-cylinder assembly is also shown below *P–V* diagram.

Process 1 –2 is isothermal heat addition process of reversible type in which heat is transferred to system isothermally. In the piston cylinder arrangement heat Q_{add} can be transferred to gas from a constant temperature source T_1 through a cylinder head of conductor type.

First law of thermodynamics applied on 1–2 yields;

$$Q_{add} = U_2 - U_1 + W_{1-2}$$

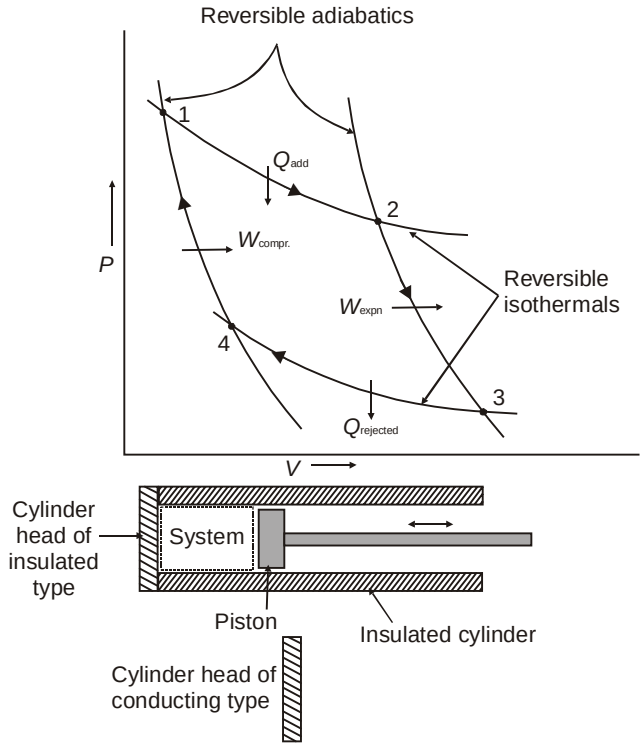


Fig. 4.10 *Carnot cycle*

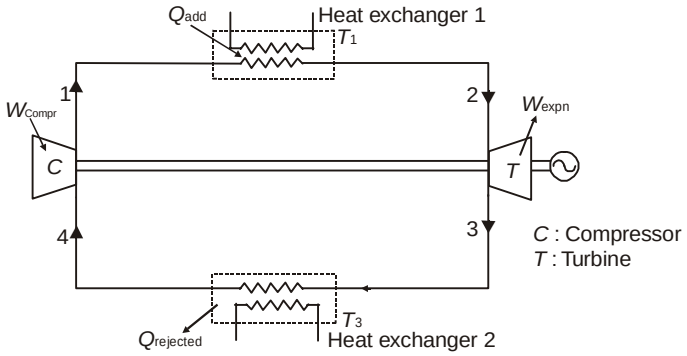


Fig. 4.11 *Gas turbine plant: Carnot heat engine*

For the perfect gas as working fluid in isothermal process no change in internal energy occurs, therfore $U_2 = U_1$

and $$Q_{add} = W_{1-2}$$

Process 2–3 is reversible adiabatic expansion process which may be had inside cylinder with cylinder head being replaced by insulating type cylinder head so that complete arrangement is insulated and adiabatic expansion carried out.

During adiabatic expansion say work W_{expn} is available,

$$Q_{2-3} = 0$$

From first law of thermodynamics;

$$0 = (U_3 - U_2) + W_{expn}$$

or $$W_{expn} = (U_2 - U_3)$$

Process 3–4 is reversible isothermal heat rejection for which cylinder head of insulating type may be replaced by conducting type as in 1–2 and heat ($Q_{rejected}$) be extracted out isothermally.

From first law of thermodynamics applied on process 3–4,

$$-Q_{rejected} = (U_4 - U_3) + (-W_{3-4})$$

for perfect gas internal energy shall remain constant during isothermal process. Thus, $U_3 = U_4$

$$-Q_{rejected} = -W_{3-4}$$

or $$Q_{rejected} = W_{3-4}$$

Process 4–1 is the reversible adiabatic compression process with work requirement for compression. In the piston cylinder arrangement cylinder head of conducting type as used in 3–4 is replaced by insulating type, so that the whole arrangement becomes insulated and adiabatic compression may be realized,

From first law applied on process 4–1

For adiabatic process; $$Q_{4-1} = 0$$

$$\Rightarrow 0 = (U_1 - U_4) + (-W_{compr})$$

or $$\boxed{W_{compr} = (U_1 - U_4)}$$

Efficiency of reversible heat engine can be given as;

$$\eta_{rev,\,HE} = \frac{\text{Net work}}{\text{Heat supplied}}$$

Here, Net work = $W_{expn} - W_{compr}$

and heat is supplied only during process 1–2, therefore

$$\text{heat supplied} = Q_{add}$$

Substituting in the expression for efficiency.

$$\eta_{rev,\,HE} = \frac{W_{expn} - W_{compr}}{Q_{add}}$$

Also for a cycle

$$\sum_{cycle} W = \sum_{cycle} Q$$

so $$W_{net} = Q_{add} - Q_{rejected}$$

Hence

$$\eta_{rev,\,HE} = 1 - \frac{Q_{rejected}}{Q_{add}}$$

As the heat addition takes place at high temperature, while heat rejection takes place at low temperature, so writing these heat interactions as $Q_{high,}$ Q_{low} we get,

$$\boxed{\eta_{rev,\,HE} = 1 - \frac{Q_{low}}{Q_{high}}}$$

$$\boxed{\eta_{Carnot} = 1 - \frac{Q_{low}}{Q_{high}}}$$

Piston-cylinder arrangement shown and discussed for realizing Carnot cycle is not practically feasible as;

(*i*) Frequent change of cylinder head i.e. of insulating type and diathermic type for adiabatic and isothermal processes is very difficult.

(*ii*) Isothermal heat addition and isothermal-heat rejection are practically very difficult to be realized

(*iii*) Reversible adiabatic expansion and compression are not possible.

(*iv*) Even if near reversible isothermal heat addition and rejection is to be achieved then time duration for heat interaction should be very large i.e. infinitesimal heat interaction occurring at dead slow speed. Near reversible adiabatic processes can be achieved by making them to occur fast. In a piston-cylinder reciprocating engine arrangement such speed fluctuation in a single cycle is not possible.

Carnot heat engine arrangement is also shown with turbine, compressor and heat exchangers for adiabatic and isothermal processes. Fluid is compressed in compressor adiabatically, heated in heat exchanger at temperature T_1, expanded in turbine adiabatically, cooled in heat exchanger at temperature T_3 and sent to compressor for compression. Here also following practical difficulties are confronted;

(*i*) Reversible isothermal heat addition and rejection are not possible.

(*ii*) Reversible adiabatic expansion and compression are not possible.

Carnot cycle can also operate reversibly as all processes constituting it are of reversible type. Reversed Carnot cycle is shown below;

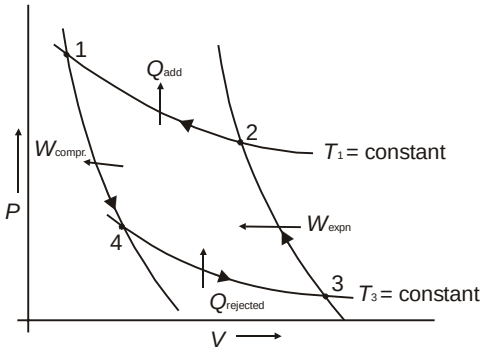


Fig. 4.12 *Reversed Carnot cycle*

Heat engine cycle in reversed form as shown above is used as ideal cycle for refrigeration and called "Carnot refrigeration cycle".

4.9 CARNOT THEOREM AND ITS COROLLARIES

Carnot theorem states that "any engine cannot have efficiency more than that of reversible engine operating between same temperature limits."

Different corollaries of Carnot theorem are,

(*i*) Efficiency of all reversible engines operating between same temperature limits is same.

(*ii*) Efficiency of a reversible engine does not depend on the working fluid in the cycle.

Using Clausius and Kelvin Planck statements, the Carnot theorem can be proved easily. Let us take two heat engines HE_I and HE_{II} operating between same temperature limits T_1, T_2 of source and sink as shown in Fig. 4.13*a*.

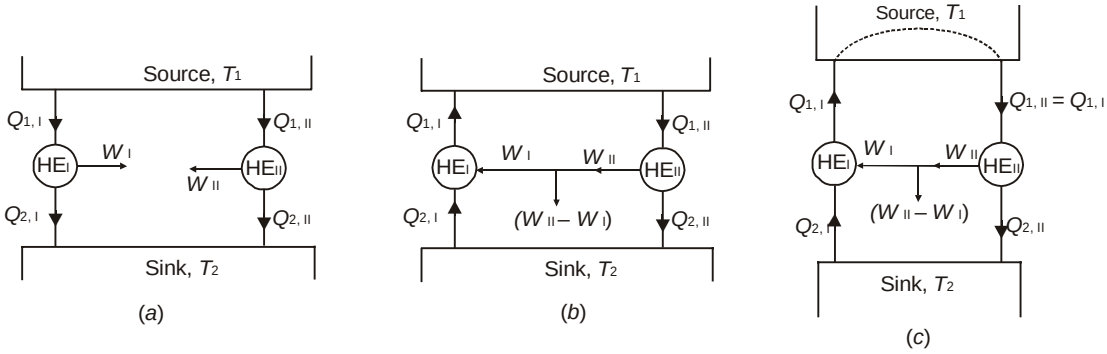


Fig. 4.13 *Proof of Carnots theorem*

Arrangement shown has heat engine, HE_1 getting $Q_{1,I}$ from source, rejecting $Q_{2,I}$ and producing work W_I. Heat engine, HE_{II} receives $Q_{1,II,}$ rejects $Q_{2,II}$ and produces work W_{II}.

$$W_I = Q_{1,I} - Q_{2,I}$$

$$W_{II} = Q_{1,II} - Q_{2,II}$$

Efficiency of engines HE_I, HE_{II}

$$\eta_{HE_I} = \frac{W_I}{Q_{1,I}}$$

$$\eta_{HE_{II}} = \frac{W_{II}}{Q_{1,II}}$$

Now let us assume that engine HE_1 is reversible engine while HE_{II} is any engine.

As per Carnot's theorem efficiency of HE_I (reversible engine) is always more than that of HE_{II}. Let us start with violation of above statement, i.e., efficiency of HE_{II} is more than that of HE_I

$$\eta_{HE_I} < \eta_{HE_{II}}$$

or
$$\frac{W_I}{Q_{1,I}} < \frac{W_{II}}{Q_{1,II}}$$

Let us take the heat addition to each engine to be same i.e.

$$Q_{1,\,I} = Q_{1,\,II}$$

Hence
$$W_I < W_{II}$$

Also we have assumed that engine HE_1 is of reversible type, so let us operate it in reversed manner, as shown in Figure 4.13*b*.

Let us also assume that the work requirement of reversed heat engine, HE_1 be fed by work output W_{II} of the heat engine HE_{II}. Since W_{II} is more than W_1, a net work ($W_{II} - W_I$) shall also be available as output work after driving HE_1. Also since $Q_{1,\,I}$ and $Q_{1,\,II}$ are assumed to be same, the heat rejected by reversed HE_1 may be supplied to heat engine, HE_{II} as shown in figure 4.13c by dotted lines. Thus, it results into an equivalent heat engine which produces net work ($W_{II} - W_I$) while heat interaction takes place with only one reservoir at temperature, T_2. This is a violation of Kelvin Planck statement, so the assumption made in beginning that efficiency of reversible engine is less than that of other engine, is not correct.

Hence, it is established that out of all heat engines operating within same temperature limits, the reversible engine has highest efficiency.

Similarly for showing the correctness of corollaries of Carnot theorem the heat engines and their combinations be considered like above and proved using Kelvin-Planck and Clausius statements.

4.10 THERMODYNAMIC TEMPERATURE SCALE

After the Carnot's theorem and its corollary were stated and verified, it was thought to have a thermodynamic temperature scale, independent of thermometric substance and principles of thermometry. Such a temperature scale can be developed with the help of reversible heat engine concept and is called thermodynamic temperature scale. Defining thermodynamic temperature scale refers to the assigning of numerical values to different temperatures using reversible heat engines.

From the previous discussions on heat engines it is obvious that the efficiency of a reversible heat engine depends on the temperatures of reservoir with which heat interaction takes place.

Mathematically, it can be easily given by any unknown function 'f';

$$\eta_{rev,\,HE} = f\,(T_{high},\ T_{low})$$

where T_{high} and T_{low} are the two temperatures of high temperature source and low temperature sink.

or
$$\eta_{rev,\,HE} = 1 - \frac{Q_{low}}{Q_{high}} = f\,(T_{high},\ T_{low})$$

Unknown function 'f' may be substituted by another unknown function, say ϕ

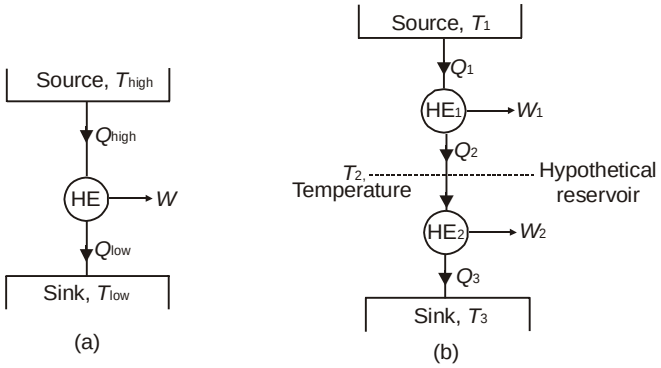


Fig. 4.14 *Reversible heat engine and its combinations*

$$\frac{Q_{high}}{Q_{low}} = \phi\ (T_{high},\ T_{low})$$

Thus, some functional relationship as defined by 'ϕ' is established between heat interactions and temperatures.

Let us now have more than one reversible heat engines operating in series as shown in Figure 4.14b, between source and sink having T_1 and T_3 temperatures. In between an imaginary reservoir at temperature T_2 may be considered.

From the above for two reversible heat engines;

$$\frac{Q_1}{Q_2} = \phi\ (T_1,\ T_2)$$

and

$$\frac{Q_2}{Q_3} = \phi\ (T_2,\ T_3)$$

Combination of two heat engines may be given as shown here,

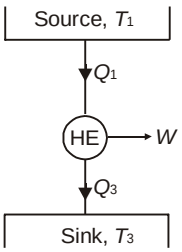


Fig. 4.15 *Equivalent heat engine for two reversible heat engines operating in series.*

$$\frac{Q_1}{Q_3} = \frac{Q_1 / Q_2}{Q_2 / Q_3}$$

or

$$\frac{Q_1}{Q_3} = \phi\ (T_1,\ T_3)$$

$$\frac{Q_1}{Q_3} = \frac{Q_1}{Q_2}.\frac{Q_2}{Q_3}$$

or $$\phi\ (T_1,\ T_3) = \phi\ (T_1,\ T_2)\ .\ \phi\ (T_2,\ T_3)$$

Above functional relation is possible only if it is given by another function ψ as follows.

$$\phi\ (T_1,\ T_2) = \frac{\psi(T_1)}{\psi(T_2)}$$

$$\phi\ (T_2,\ T_3) = \frac{\psi(T_2)}{\psi(T_3)}$$

$$\phi\ (T_1,\ T_3) = \frac{\psi(T_1)}{\psi(T_3)}$$

Thus,

$$\frac{Q_1}{Q_2} = \frac{\psi(T_1)}{\psi(T_2)}$$

$$\frac{Q_2}{Q_3} = \frac{\psi(T_2)}{\psi(T_3)}$$

$$\frac{Q_1}{Q_3} = \frac{\psi(T_1)}{\psi(T_3)}$$

Lord Kelvin based upon his observations proposed that the function $\psi\,(T)$ can be arbitrarily chosen based on Kelvin scale or absolute thermodynamic temperature scale as;

$$\psi\ (T) = \text{Temperature } T \text{ in Kelvin Scale}$$

Therefore,

$$\frac{Q_1}{Q_2} = \frac{\psi(T_1)}{\psi(T_2)} = \frac{T_1}{T_2}$$

$$\frac{Q_2}{Q_3} = \frac{\psi(T_2)}{\psi(T_3)} = \frac{T_2}{T_3}$$

$$\frac{Q_1}{Q_3} = \frac{\psi(T_1)}{\psi(T_3)} = \frac{T_1}{T_3}$$

where T_1, T_2, T_3 are temperatures in *absolute thermodynamic scale.*

Here heat absorbed and heat rejected is directly proportional to temperatures of reservoirs supplying and accepting heats to heat engine. For a Carnot heat engine or reversible heat engine operating between reservoirs at temperature T and triple point of water, T_t;

$$\frac{Q}{Q_t} = \frac{T}{T_t} = \frac{T}{273.16}$$

or $$\boxed{T = 273.16.\frac{Q}{Q_t}}$$

Here for a known Q and Q_t values the temperature T can be defined. Thus, heat interaction acts as thermometric property in thermodynamic temperature scale, which is independent of thermometric substance. It may be noted that negative temperatures cannot exist on thermodynamic temperature scale.

Let us now have a large number of reversible heat engines (Carnot engines) operating in series as shown in Figure 4.16.

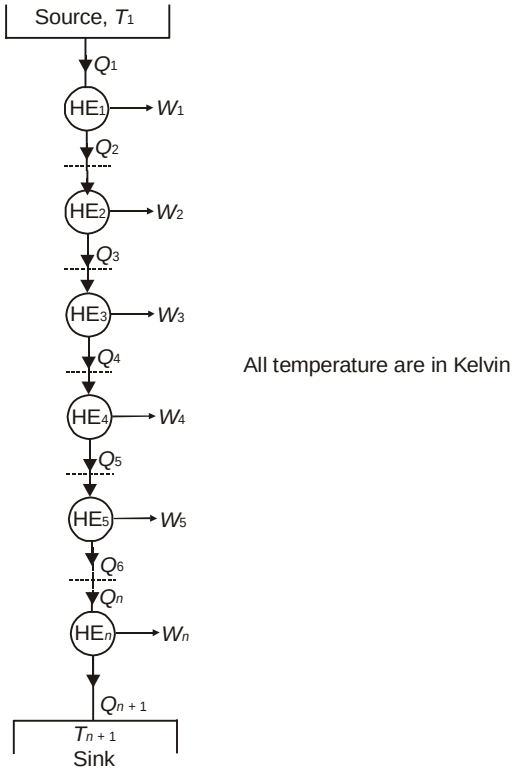


Fig. 4.16 *Series of reversible heat engines*

From thermodynamic temperature scale for different engines,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\frac{Q_2}{Q_3} = \frac{T_2}{T_3}$$

$$\frac{Q_3}{Q_4} = \frac{T_3}{T_4}$$

For nth engine

$$\frac{Q_n}{Q_{n+1}} = \frac{T_n}{T_{n+1}}$$

Here work output from each engine shall continuously diminish the heat supplied to subsequent heat engine. Let us assume work outputs from '*n*' engines to be same; i.e.

$$W_1 = W_2 = W_3 = W_4 = ... = W_n$$

or $$(Q_1 - Q_2) = (Q_2 - Q_3) = (Q_3 - Q_4) = ... = (Q_n - Q_{n+1})$$

or $$(T_1 - T_2) = (T_2 - T_3) = (T_3 - T_4) = ... = (T_n - T_{n+1})$$

It is obvious that for a large number of heat engines the heat rejected by *n*th engine shall be negligible i.e for very large value of n, $Q_{n+1} \to 0$

or for Lim $n \to \infty$, Lim $Q_{n+1} \to 0$

Thus, from thermodynamic temperature scale when heat rejection approaches zero, the temperature of heat rejection also tends to zero as a limiting case. But in such a situation when heat rejection is zero, the heat engine takes form of a perpetual motion machine of 2nd kind, where work is produced with only heat supplied to it. Thus, it leads to violation of Kelvin-Planck statement. Hence it is not possible.

Also it can be said that "it is impossible to attain absolute zero temperature in finite number of operations." There exists absolute zero temperature on thermodynamic temperature scale, but cannot be attained without violation of second law of thermodynamics. This fact is popularly explained by *third law of thermodynamics.*

Carnot cycle efficiency can now be precisely defined as function of source and sink temperatures.

$$\eta_{carnot} = 1 - \frac{Q_{low}}{Q_{high}}$$

$$\eta_{carnot} = 1 - \frac{T_{low}}{T_{high}}$$

Thus, it is seen that Carnot cycle efficiency depends only upon lower and higher temperatures. Carnot cycle efficiency is high for small values of sink temperature (T_{low}) and larger values of source temperature (T_{high}).

Therefore for maximum efficiency, Carnot cycle must operate between maximum possible source and minimum possible sink temperatures.

EXAMPLES

1. *Using IInd law of thermodynamics show that the following are irreversible*

(i) Free expansion.

(ii) Heat transfer through finite temperature difference.

Solution:

(*i*) Let us consider a perfectly insulated tank having two compartments divided by thin wall. Compartment I has gas while II has vacuum. When wall is punctured then gas in I expands till pressure in I and II gets equalised. Let us assume that free expansion is reversible i.e. the gas in II returns into I and original states are restored.

When gas is allowed to expand, say it produces work W from a device D due to expansion. This work W is available due to change in internal energy of gas. Internal energy of gas can be restored by adding equivalent heat Q to it from a source as shown. This whole arrangement if consolidated can be

treated as a device which is producing work while exchanging heat with single body. Thus, it is violation of IInd law of thermodynamics, therefore the assumption that free expansion is reversible is incorrect. Free expansion is irreversible.

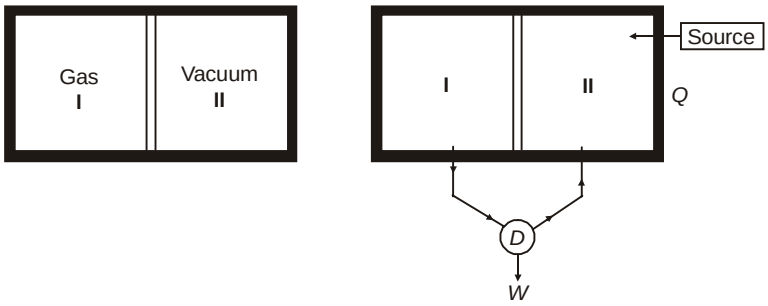


Fig. 4.17 *Free expansion*

(*ii*) For showing that the heat transfer through finite temperature difference is irreversible, let us start with the fact that such heat transfer is reversible. Let us take a heat source (T_1) and sink (T_2) and assume that a heat Q_{1-2} flows from T_1 to T_2.

Let us have a heat engine operating between T_1 and T_2 as shown and producing work *W*. Let us reverse heat transfer process from T_2 to T_1 i.e. Q_{2-1}, as assumed. Let us assume $Q_2 = Q_{2-1}$.

This assumption paves the way for eliminating sink. Let us now remove sink and directly supply Q_2 as Q_{2-1} (= Q_2). This results in formation of a heat engine which produces work while exchanging heat with single reservoir, the violation of IInd law of thermodynamics. (Kelvin Planck statement).

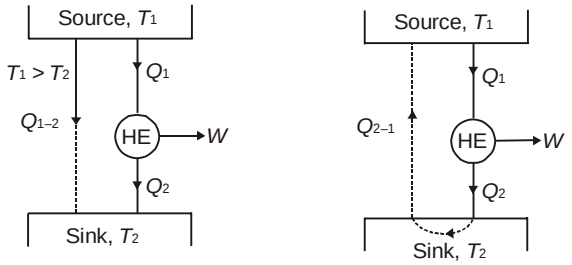


Fig. 4.18 *Heat transfer through a finite temperature difference*

Hence, assumption that heat transfer through finite temperature is reversible, stands incorrect. Therefore, heat transfer through finite temperature difference is irreversible.

2. *Determine the heat to be supplied to a Carnot engine operating between 400ºC and 15ºC and producing 200 kJ of work.*

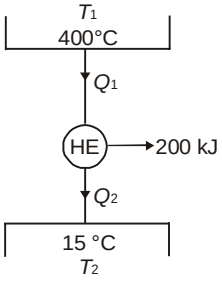


Fig. 4.19

Solution :

To find out $Q_1 = ?$

In Carnot engine from thermodynamic temperature scale;

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

and work $W = Q_1 - Q_2$

Thus $$\frac{Q_1}{Q_2} = \frac{673}{288} \quad (1)$$

and $$Q_1 - Q_2 = 200 \text{ kJ} \quad (2)$$

From equations 1 and 2, upon solving

$$Q_1 = 349.6 \text{ kJ}$$

and $$Q_2 = 149.6 \text{ kJ}$$

Heat to be supplied = **349.6 kJ Ans.**

3. *A refrigerator operates on reversed Carnot cycle. Determine the power required to drive refrigerator between temperatures of 42ºC and 4ºC if heat at the rate of 2 kJ/s is extracted from the low temperature region.*

Solution:

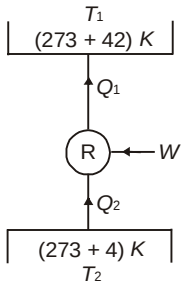


Fig. 4.20

To find out, W = ?

Given : $T_1 = 315$ K, $T_2 = 277$ K

and $Q_2 = 2$ kJ/s

From thermodynamic temperature scale;

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

or $$\frac{Q_1}{2} = \frac{315}{277}$$

or $$Q_1 = 2.274 \text{ kJ/s}$$

Power/Work input required = $Q_1 - Q_2$

$= 2.274 - 2$

Power required = 0.274 kJ/s

Power required for driving refrigerator = **0.274 kW Ans.**

4. *A reversible heat engine operates between two reservoirs at 827ºC and 27ºC. Engine drives a Carnot refrigerator maintaining –13ºC and rejecting heat to reservoir at 27ºC. Heat input to the engine is 2000 kJ and the net work available is 300 kJ. How much heat is transferred to refrigerant and total heat rejected to reservoir at 27ºC?*

Solution:

Block diagram based on the arrangement stated;

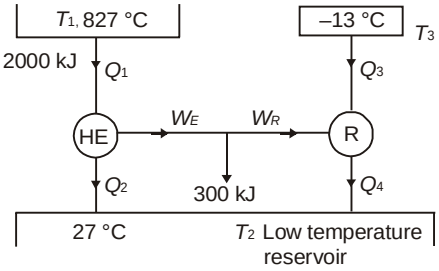


Fig. 4.21

We can write, for heat engine, $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

$$\frac{Q_1}{Q_2} = \frac{1100}{300}$$

Substituting Q_1 = 2000 kJ, we get Q_2 = 545.45 kJ

Also $W_E = Q_1 - Q_2 = 1454.55$ kJ

For refrigerator,

$$\frac{Q_3}{Q_4} = \frac{260}{300} \quad (1)$$

Also, $W_R = Q_4 - Q_3$ (2)

and $W_E - W_R = 300$

or $W_R = 1154.55$ kJ

Equations (1) & (2) result in,

$$Q_4 - Q_3 = 1154.55 \quad (3)$$

From equations (1) & (3),

$Q_3 = 7504.58$ kJ

$Q_4 = 8659.13$ kJ

Total heat transferred to low temperature reservoir

$= Q_2 + Q_4 = 9204.68$ kJ

Heat transferred to refrigerant = **7504.58 kJ**

Total heat transferred to low temperature reservoir = **9204.68 kJ** **Ans.**

5. *In a winter season when outside temperature is –1°C, the inside of house is to be maintained at 25°C. Estimate the minimum power required to run the heat pump of maintaining the temperature. Assume heating load as 125 MJ/h.*

Solution:

$$COP_{HP} = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} = \frac{1}{\left(1 - \frac{Q_2}{Q_1}\right)}$$

Also we know

$$\frac{Q_1}{Q_2} = \frac{298.15}{272.15}$$

Thus $COP_{HP} = 11.47$

Also $COP_{HP} = \frac{Q_1}{W}$, Substituting Q_1

therefore $W = 10.89$ MJ/h

or, $W = 3.02$ kW

Minimum power required = **3.02 kW Ans.**

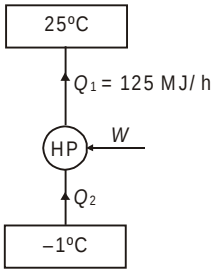


Fig. 4.22

6. *A cold storage plant of 40 tonnes of refrigeration capacity runs with its performance just* $\frac{1}{4}$ *th of its Carnot COP. Inside temperature is –15ºC and atmospheric temperature is 35ºC. Determine the power required to run the plant. [Take : One ton of refrigeration as 3.52 kW]*

Solution:

Cold storage plant can be considered as a refrigerator operating between given temperatures limits.

Capacity of plant = Heat to be extracted = 140.8 kW

$$\text{Carnot COP of plant} = \frac{1}{\left(\frac{308}{258.15}\right) - 1}$$

$$= 5.18$$

Actual $\text{COP} = \frac{5.18}{4} = 1.295$

Also actual $\text{COP} = \frac{Q_2}{W}$, hence W = 108.73 kW.

Power required = **108.73 kW Ans.**

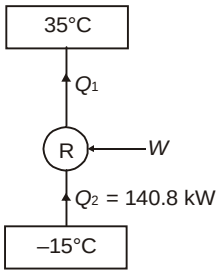


Fig. 4.23

7. *What would be maximum efficiency of engine that can be had between the temperatures of 1150ºC and 27ºC ?*

Solution:

Highest efficiency is that of Carnot engine, so let us find the Carnot cycle efficiency for given temperature limits.

$$\eta = 1 - \left(\frac{273 + 27}{273 + 1150}\right)$$

η = **0.7891 or 78.91% Ans.**

8. *A domestic refrigerator maintains temperature of – 8ºC when the atmospheric air temperature is 27ºC. Assuming the leakage of 7.5 kJ/min from outside to refrigerator determine power required to run this refrigerator. Consider refrigerator as Carnot refrigerator.*

Solution:

Here heat to be removed continuously from refrigerated space shall be 7.5 kJ/min or 0.125 kJ/s.

For refrigerator, C.O.P. shall be,

$$\frac{0.125}{W} = \frac{265}{(300-265)}$$

or $W = 0.0165$ kJ/s.

Power required = **0.0165 kW Ans.**

9. *Three reversible engines of Carnot type are operating in series as shown between the limiting temperatures of 1100 K and 300 K. Determine the intermediate temperatures if the work output from engines is in proportion of 3 : 2 : 1.*

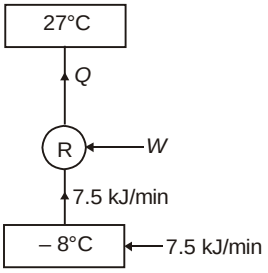


Fig. 4.23

Solution:

Here, $W_1 : W_2 : W_3 = 3 : 2 : 1$

Efficiency of engine, HE_1,

$$\frac{W_1}{Q_1} = \left(1-\frac{T_2}{1100}\right) \Rightarrow Q_1 = \frac{1100.W_1}{(1100-T_2)}$$

for HE_2 engine,

$$\frac{W_2}{Q_2} = \left(1-\frac{T_3}{T_2}\right)$$

for HE_3 engine,

$$\frac{W_3}{Q_3} = \left(1-\frac{300}{T_3}\right)$$

From energy balance on engine, HE_1

$$Q_1 = W_1 + Q_2 \Rightarrow Q_2 = Q_1 - W_1$$

Above gives,

$$Q_1 = \left\{\frac{1100\,W_1}{(1100-T_2)} - W_1\right\} = W_1\left\{\frac{T_2}{1100-T_2}\right\}$$

Substituting Q_2 in efficiency of HE_2

$$\frac{W_2}{W_1\left(\frac{T_2}{1100-T_2}\right)} = \left(1-\frac{T_3}{T_2}\right)$$

or $$\frac{W_2}{W_1} = \left(\frac{T_2}{1100-T_2}\right)\left(\frac{T_2-T_3}{T_2}\right) = \left(\frac{T_2-T_3}{1100-T_2}\right)$$

or $$\left\{\frac{2}{3} = \left(\frac{T_2-T_3}{1100-T_2}\right)\right\}$$

or $2200 - 2T_2 = 3T_2 - 3T_3$

$$\boxed{5T_2 - 3T_3 = 2200}$$

Energy balance on engine HE_2 gives,

$$Q_2 = W_2 + Q_3$$

Substituting in efficiency of HE_2,

$$\frac{W_2}{(W_2 + Q_3)} = \left(\frac{T_2 - T_3}{T_2}\right)$$

or $$W_2 . T_2 = (W_2 + Q_3)(T_2 - T_3)$$

or $$Q_3 = \frac{W_2 \ T_3}{(T_2 - T_3)}$$

Substituting Q_3 in efficiency of HE_3,

$$\frac{W_3}{\left(\frac{W_2 T_3}{T_2 - T_3}\right)} = \frac{T_3 - 300}{T_3}$$

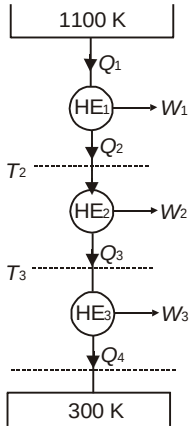


Fig. 4.25

$$\frac{W_3}{W_2} = \left(\frac{T_3}{T_2 - T_3}\right)\left(\frac{T_3 - 300}{T_3}\right)$$

$$\frac{1}{2} = \frac{T_3 - 300}{T_2 - T_3}$$

$$3T_3 - T_2 = 600$$

Solving, equations of T_2 and T_3, $T_3 = 433.33$ K

$$T_2 = 700 \text{ K}$$

Intermediate temperatures: **700 K** and **433.33 K Ans.**

10. *A Carnot engine getting heat at 800 K is used to drive a Carnot refrigerator maintaining 280 K temperature. Both engine and refrigerator reject heat at some temperature, T, when heat given to engine is equal to heat absorbed by refrigerator. Determine efficiency of engine and C.O.P. of refrigerator.*

Solution:

Efficiency of engine,

$$\frac{W}{Q_1} = \left(\frac{800 - T}{800}\right)$$

For refrigerator, COP

$$\frac{Q_3}{W} = \frac{280}{(T - 280)}$$

It is given that $Q_1 = Q_3 = Q$

so, from engine

$$\frac{W}{Q} = \left(\frac{800 - T}{800}\right)$$

From refrigerator,

$$\frac{Q}{W} = \frac{280}{T - 280}$$

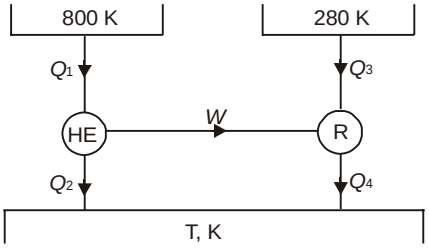


Fig. 4.26

From above two $\left(\frac{Q}{W}\right)$ may be equated,

$$\frac{T-280}{280} = \frac{800-T}{800}$$

Temperature, T = 414.8 K

$$\text{Efficiency of engine} = \left(\frac{800-414.8}{800}\right) = \mathbf{0.4815\ Ans.}$$

$$\text{C.O.P. of refrigerator} = \left(\frac{280}{414.8-280}\right) = \mathbf{2.077\ Ans.}$$

11. *0.5 kg of air executes a Carnot power cycle having a thermal efficiency of 50%. The heat transfer to the air during isothermal expansion is 40 kJ. At the beginning of the isothermal expansion the pressure is 7 bar and the volume is 0.12 m^3. Determine the maximum and minimum temperatures for the cycle in Kelvin, the volume at the end of isothermal expansion in m^3, and the work and heat transfer for each of the four processes in kJ. For air c_P = 1.008 kJ/kg . K, c_v= 0.721 kJ/kg. K. [U.P.S.C. 1993]*

Solution:

Given: $\eta_{carnot} = 0.5$, $m = 0.5$ kg

$P_2 = 7$ bar, $V_2 = 0.12\ m^3$

Let thermodynamic properties be denoted with respect to salient states;

$$\text{Carnot efficiency } \eta_{Carnot} = 1 - \frac{T_1}{T_2}$$

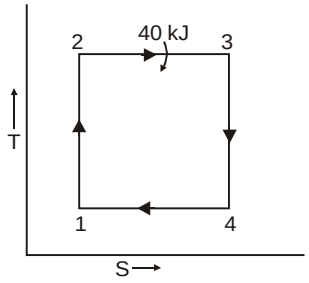


Fig. 4.27

or, $$\frac{T_1}{T_2} = 0.5$$

or, $$T_2 = 2T_1$$

Corresponding to state 2, $P_2 V_2 = mRT_2$

$$7 \times 10^5 \times 0.12 = 0.5 \times 287 \times T_2$$

$$T_2 = 585.36 \text{ K}$$

Heat transferred during process 2-3 (isothermal expansion), $Q_{23} = 40$ kJ

$$Q_{23} = W_{23} = P_2 V_2 \ln \left(\frac{V_3}{V_2}\right)$$

$$40 = mRT_2 \ln \times \left(\frac{V_3}{V_2}\right) = 0.5 \times 0.287 \times 585.36 \ln \left(\frac{V_3}{0.12}\right)$$

$$V_3 = 0.1932 \text{ m}^3$$

Temperature at state 1, $$T_1 = \frac{T_2}{2}$$

$$T_1 = 292.68 \text{ K}$$

During process 1–2,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\gamma = \frac{c_p}{c_v} = \frac{1.008}{0.721}, \quad \gamma = 1.398$$

Thus, $$P_1 = 0.613 \text{ bar}$$

$$P_1 V_1 = mRT_1$$

$$0.613 \times 10^5 \times V_1 = 0.5 \times 287 \times 292.68$$

$$V_1 = 6.85 \times 10^{-4} \text{ m}^3$$

Heat transferred during process 4 – 1 (isothermal compression) shall be equal to the heat transferred during process 2 – 3 (isothermal expansion).

For isentropic process, $dQ = 0$

$$dW = dU$$

During process 1 – 2, isentropic process, $W_{12} = -mc_v (T_2 - T_1)$

$$Q_{12} = 0, \; W_{12} = -0.5 \times 0.721 (585.36 - 292.68)$$

$$W_{12} = -105.51 \text{ kJ, (–ve work)}$$

During process 3 – 4, isentropic process, $W_{34} = -mc_v (T_4 - T_3)$

$$Q_{34} = 0, \; W_{34} = +0.5 \times 0.721 \times (585.36 - 292.68)$$

$$W_{34} = +105.51 \text{ kJ (+ve work)}$$

Ans.

Process	*Heat transfer*	*Work interaction*
1 – 2	0	– 105.51, kJ
2 – 3	40 kJ	40 kJ
3 – 4	0	+ 105.51, kJ
4 – 1	– 40 kJ	– 40 kJ

Maximum temperature of cycle = 585.36 kJ
Minimum temperature of cycle = 292.68 kJ
Volume at the end of isothermal expansion = 0.1932 m^3

12. *A reversible engine as shown in figure during a cycle of operation draws 5 mJ from the 400 K reservoir and does 840 kJ of work. Find the amount and direction of heat interaction with other reservoirs.* *[U.P.S.C. 1999]*

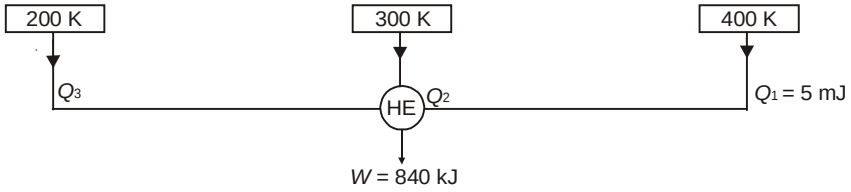


Fig. 4.28

Solution:

Let us assume that heat engine rejects Q_2 and Q_3 heat to reservoir at 300 K and 200 K respectively. Let us assume that there are two heat engines operating between 400 K and 300 K temperature reservoirs and between 400 K and 200 K temperature reservoirs. Let each heat engine receive Q_1' and Q_1'' from reservoir at 400 K as shown below:

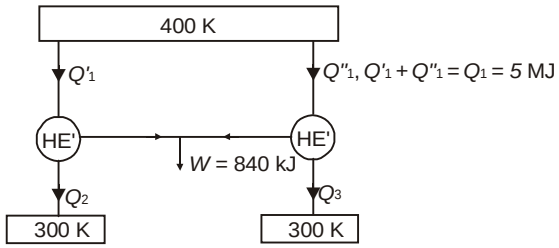


Fig. 4.29 *Assumed arrangement*

Thus, $$Q'_1 + Q''_1 = Q_1 = 5 \times 10^3 \text{ kJ}$$

also, $$\frac{Q_1'}{Q_2} = \frac{400}{300}, \text{ or, } Q'_1 = \frac{4}{3} Q_2$$

and $$\frac{Q_1''}{Q_3} = \frac{400}{200} \text{ or, } Q''_1 = 2Q_3$$

Substituting Q'_1 and Q''_1

$$\frac{4}{3} Q_2 + 2Q_3 = 5000$$

Also from total work output, $Q'_1 + Q''_1 - Q_2 - Q_3 = W$

$$5000 - Q_2 - Q_3 = 840$$

$$Q_2 + Q_3 = 4160$$

$$Q_3 = 4160 - Q_2$$

Substituting Q_3, $\frac{4}{3}Q_2 + 2(4160 - Q_2) = 5000$

$$\frac{4}{3}Q_2 - 2\,Q_2 = 5000 - 8320$$

$$\frac{-2Q_2}{3} = -3320$$

$$Q_2 = 4980 \text{ kJ}$$

and $$Q_3 = -820 \text{ kJ}$$

Negative sign with Q_3 shows that the assumed direction of heat Q_3 is not correct and actually Q_3 heat will flow from reservoir to engine. Actual sign of heat transfers and magnitudes are as under:

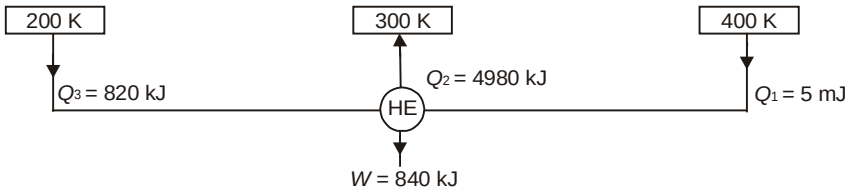


Fig 4.30

$Q_2 = 4980$ kJ, from heat engine
$Q_3 = 820$ kJ, to heat engine **Ans.**

13. *A heat pump working on a reversed Carnot cycle takes in energy from a reservoir maintained at 3ºC and delivers it to another reservoir where temperature is 77ºC. The heat pump drives power for it's operation from a reversible engine operating within the higher and lower temperature limits of 1077ºC and 77ºC. For 100 kJ/s of energy supplied to the reservoir at 77ºC, estimate the energy taken from the reservoir at 1077ºC.* *[U.P.S.C. 1994]*

Solution:

Arrangement for heat pump and heat engine operating together is shown here. Engine and pump both reject heat to the reservoir at 77ºC (350 K).

For heat engine.

$$\eta_E = 1 - \frac{350}{1350} = \frac{W}{Q_1}$$

$$0.7407 = \frac{Q_1 - Q_2}{Q_1}$$

$$0.7407 = 1 - \frac{Q_2}{Q_1}$$

$$Q_2 = 0.2593\ Q_1$$

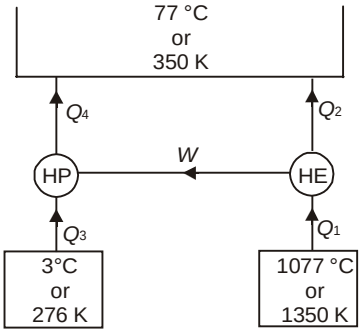


Fig. 4.31

For heat pump

$$COP_{HP} = \frac{Q_4}{Q_4 - Q_3}$$

$$\text{COP}_{HP} = \frac{350}{350 - 276} = \frac{Q_4}{Q_4 - Q_3}$$

$$\Rightarrow \quad Q_4 = 1.27 Q_3$$

Work output from engine = Work input to pump

$$Q_1 - Q_2 = Q_4 - Q_3 \Rightarrow Q_1 - 0.2593\ Q_1 = Q_4 - \frac{Q_4}{1.27}$$

Also it is given that $Q_2 + Q_4 = 100$

Substituting Q_2 and Q_4 as function of Q_1 in following expression,

$$Q_2 + Q_4 = 100$$

$$0.2593\ Q_1 + \frac{Q_1}{0.287} = 100$$

$$Q_1 = 26.71 \text{ kJ}$$

Energy taken by engine from reservoir at 1077°C = **26.71 kJ** **Ans.**

14. *A reversible engine is used for only driving a reversible refrigerator. Engine is supplied 2000 kJ/s heat from a source at 1500 K and rejects some energy to a low temperature sink. Refrigerator is desired to maintain the temperature of 15°C while rejecting heat to the same low temperature sink. Determine the temperature of sink if total 3000 kJ/s heat is received by the sink.*

Solution:

Let temperature of sink be T_{sink} K.

Given: $Q_{sink,\ HE} + Q_{sink,\ R} = 3000$ kJ/s

Since complete work output from engine is used to run refrigerator so,

$$2000 - Q_{sink,\ HE} = Q_{sink,\ R} - Q_R$$

$$Q_R = 3000 - 2000 = 1000 \text{ kJ/s}$$

Also for engine,

$$\frac{2000}{1500} = \frac{Q_{sink,\ HE}}{T_{sink}} \Rightarrow Q_{sink,\ HE} = \frac{4}{3} T_{sink,}$$

For refrigerator,

$$\frac{Q_R}{288} = \frac{Q_{sink,R}}{T_{sink}} \Rightarrow Q_{sink,\ R} = \frac{1000\ T_{sink}}{288}$$

Substituting $Q_{sink,\ HE}$ and $Q_{sink,\ R}$ values.

$$\frac{4}{3} T_{sink} + \frac{1000\ T_{sink}}{288} = 3000$$

$$\Rightarrow \quad T_{sink} = 624.28 \text{ K}$$

Temperature of sink = **351.28°C Ans.**

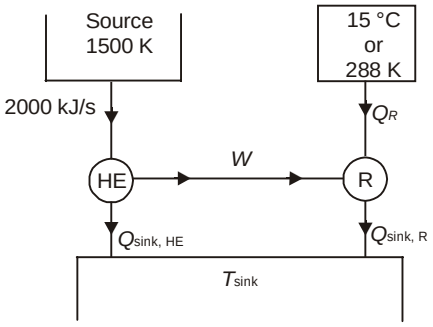


Fig. 4.32

15. *A reversible heat engine runs between 500°C and 200°C temperature reservoirs. This heat engine is used to drive an auxiliary and a reversible heat pump which runs between reservoir at 200°C and the body at 450°C. The auxiliary consumes one third of the engine output and remaining is consumed for driving heat pump. Determine the heat rejected to the body at 450°C as fraction of heat supplied by reservoir at 500°C.*

Solution:

Let the output of heat engine be W. So $\frac{W}{3}$ is consumed for driving auxiliary and remaining $\frac{2W}{3}$ is consumed for driving heat pump for heat engine,

$$\eta = \frac{W}{Q_1} = 1 - \frac{473}{773}$$

$$\frac{W}{Q_1} = 0.3881$$

$$\text{COP of heat pump} = \frac{723}{723 - 473} = \frac{Q_3}{2W/3}$$

$$\Rightarrow \qquad 2.892 = \frac{3Q_3}{2W}$$

Substituting W,

$$\frac{Q_3}{Q_1} = 0.7482$$

Ratio of heat rejected to body at 450ºC to the heat supplied by the reservoir = **0.7482** **Ans.**

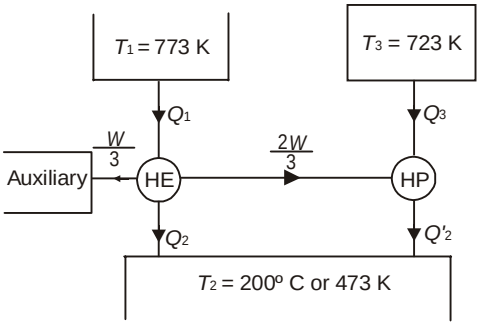


Fig. 4.33

16. *A reversible heat engine operates between a hot reservoir at T_1 and a radiating surface at T_2. Heat radiated from the surface is proportional to the surface area and temperature of surface raised to power 4. Determine the condition for minimum surface area for a given work output.*

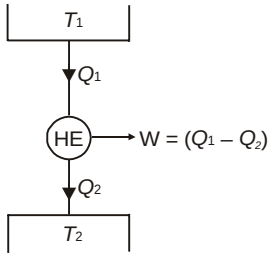


Fig. 4.34

Solution:

Heat rejected = Heat radiated from surface at $T_2 = K \,.\, A \,.\, T_2^4$, where A is surface area and K is proportionality constant.

$$\frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$

$$\frac{K \cdot A \cdot T_2^4}{W} = \frac{T_2}{T_1 - T_2}$$

$$\Rightarrow \quad A = \frac{W}{T_2^3 (T_1 - T_2) \cdot K}$$

In order to have minimum surface area the denominator in above expression of A should be maximum i.e. T_2^3 $(T_1 - T_2)$ should be maximum. Differentiating with respect to T_2.

$$\frac{d}{dT_2} \{(T_2^3 (T_1 - T_2))\} = 0$$

$$3T_1 . \ T_2^2 - 4T_2^3 = 0$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{3}{4} \Rightarrow T_2 = T_1 \cdot \frac{3}{4}$$

Taking second differential

$$\frac{d^2}{dT_2^2} \{T_2^3 . (T_1 - T_2)\} = 6T_1 . T_2 - 12T_2^2$$

Upon substitution it is –ive so $\left(\frac{T_2}{T_1} = \frac{3}{4}\right)$ is the condition for $\{T_2^3 (T_1 - T_2)\}$ to be maximum and so the minimum surface area

$$\boxed{\frac{T_2}{T_1} = \frac{3}{4}}$$ **Ans.**

17. *A cold body is to be maintained at low temperature T_2 when the temperature of surrounding is T_3. A source is available at high temperature T_1. Obtain the expression for minimum theoretical ratio of heat supplied from source to heat absorbed from cold body.*

Solution:

Let us consider a refrigerator for maintaining cold body and also a reversible heat engine for driving refrigerator to operate together, Fig 4.35.

To obtain; $\frac{Q_1}{Q_3}$

For heat engine, $\frac{W}{Q_1} = \frac{T_1 - T_3}{T_1}$

For refrigerator, $\frac{Q_3}{W} = \frac{T_2}{T_3 - T_2}$

Combining the above two:

$$\frac{Q_1}{Q_3} = \frac{T_1 \times (T_3 - T_2)}{T_2 \times (T_1 - T_3)}$$

Ratio of heat supplied from source to heat absorbed from cold body = $\frac{T_1.(T_3 - T_2)}{T_2.(T_1 - T_3)}$ **Ans.**

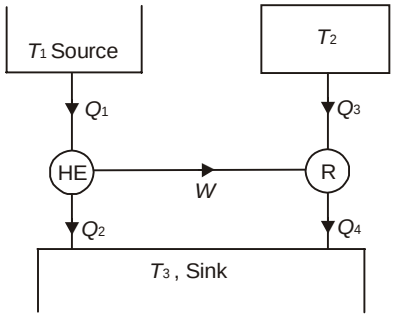


Fig. 4.35

18. *A heat pump is run by a reversible heat engine operating between reservoirs at 800°C and 50°C. The heat pump working on Carnot cycle picks up 15 kW heat from reservoir at 10°C and delivers it to a reservoir at 50°C. The reversible engine also runs a machine that needs 25 kW. Determine the heat received from highest temperature reservoir and heat rejected to reservoir at 50°C.*

Solution:

Schematic arrangement for the problem is given in figure.

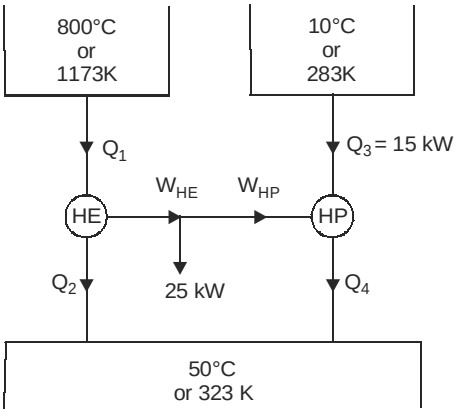


Fig. 4.36

For heat engine,

$$\eta_{HE} = \frac{W_{HE}}{Q_1} = 1 - \frac{323}{1173}$$

$$\Rightarrow \quad \frac{W_{HE}}{Q_1} = 0.7246$$

For heat pump,

$$W_{HP} = Q_4 - Q_3 = Q_4 - 15$$

$$COP = \frac{T_4}{T_4 - T_3} = \frac{Q_4}{Q_4 - Q_3}$$

$$\Rightarrow \quad \frac{323}{(323-283)} = \frac{Q_4}{Q_4 - 15}$$

$$\Rightarrow \quad Q_4 = 17.12 \text{ kW}$$

$$\Rightarrow \quad W_{HP} = 17.12 - 15 = 2.12 \text{ kW}$$

Since, $W_{HE} = W_{HP} + 25$

$$\Rightarrow \quad W_{HE} = 27.12 \text{ kW}$$

$$\eta_{HE} = 0.7246 = \frac{W_{HE}}{Q_1}$$

$$\Rightarrow \quad Q_1 = 37.427 \text{ kW}$$

$$\Rightarrow \quad Q_2 = Q_1 - W_{HE}$$

$$= 37.427 - 27.12$$

$$Q_2 = 10.307 \text{ kW}$$

Hence heat rejected to reservoir at 50°C

$$= Q_2 + Q_4$$

$$\Rightarrow \quad = 10.307 + 17.12$$

$$= \mathbf{27.427\ kW} \quad \mathbf{Ans.}$$

Heat received from highest temperature reservoir = 37.427kW **Ans.**

19. *Two insulated tanks are connected through a pipe with closed valve in between. Initially one tank having volume of 1.8m³ has argon gas at 12 bar, 40°C and other tank having volume of 3.6m³ is completely empty. Subsequently valve is opened and the argon pressure gets equalized in two tanks. Determine,* (a) *the final pressure & temperature* (b) *the change of enthalpy and* (c) *the work done considering argon as perfect gas and gas constant as 0.208 kJ/kg. K*

Solution:

Total volume, $V = V_1 + V_2 = 5.4\ m^3$

By perfect gas law, $p_1V_1 = mRT_1$

$12 \times 10^2 \times 1.8 = m \times 0.208 \times 313$

$$\Rightarrow \quad m = 33.18 \text{ kg}$$

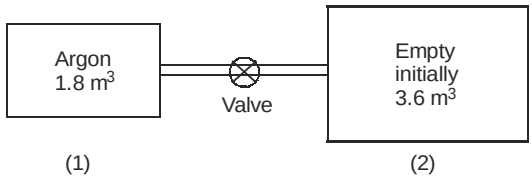


Fig. 4.37

By gas law for initial and final state,

$$p_1V_1 = p_{final} \times V_{final}$$

$$12 \times 10^2 \times 1.8 = p_{final} \times 5.4$$

Final pressure $\Rightarrow p_{final} = 400$ kPa or 4 bar **Ans.**

Here since it is insulated system and it has no heat transfer so, there will be no change in internal energy, hence there will be no change in temperature. Also by Ist law of thermodynamics, since there is no heat transfer due to system being insulated and no work due to frictionless expansion;

Final temperature = 313K.

$$dq = du + dw \Rightarrow du = 0$$

i.e. $T_{initial} = T_{final}$

Change in enthalpy = 0 **Ans.**

Work done = 0 **Ans.**

EXERCISE

4.1 State the Kelvin Planck and Clausius statements of 2nd law of thermodynamics.

4.2 Show the equivalence of two statements of 2nd law of thermodynamics.

4.3 Write short notes on the following:
Heat reservoir, Heat engine, Heat pump and refrigerator.

4.4 Explain the reversible and irreversible processes.

4.5 Describe Carnot cycle and obtain expression for its efficiency as applied to a heat engine.

4.6 Why Carnot cycle is a theoretical cycle? Explain.

4.7 Show that coefficient of performance of heat pump and refrigerator can be related as;

$$COP_{Ref} = COP_{HP-1}$$

4.8 State Carnot theorem. Also prove it.

4.9 Show that the efficiencies of all reversible heat engines operating between same temperature limits are same.

4.10 Show that efficiency af an irreversible engine is always less than the efficiency of reversible engine operating between same temperature limits.

4.11 Assume an engine to operate on Carnot cycle with complete reversibility except that 10% of work is required to overcome friction. For the efficiency of reversible cycle being 30%, what shall be the efficiency of assumed engine.

For same magnitude of energy required to overcome friction, if machine operated as heat pump, then what shall be ratio between refrigerating effect and work required. [27%, 2.12]

4.12 A Carnot engine operating between certain temperature limits has an efficiency of 30%. Determine the ratio of refrigerating effect and work required for operating the cycle as a heat pump between the same temperature limits. [2.33]

4.13 An inventor claims to have developed an engine that takes in 1055 mJ at a temperature of 400K and rejects 42.2 MJ at a temperature of 200 K while delivering 15kWh of mechanical work. Check whether engine is feasible or not. [Engine satisfies Ist law but violates 2nd law]

4.14 Determine which of the following is the most effective way to increase Carnot engine efficiency

(i) To increase T_2 while keeping T_1 fixed.

(ii) To decrease T_1 while keeping T_2 fixed. [If T_1 is decreased]

4.15 A refrigerator has COP one half as great as that of a Carnot refrigerator operating between reservoirs at temperatures of 200 K and 400 K, and absorbs 633 KJ from low temperature reservoir. How much heat is rejected to the high temperature reservoir? [1899 kJ]

4.16 Derive a relationship between COP of a Carnot refrigerator and the efficiency of same refrigerator when operated as an engine.

Is a Carnot engine having very high efficiency suited as refrigerator?

4.17 Calculate COP of Carnot refrigerator and Carnot heat pump, if the efficiency of the Carnot engine between same temperature limits is 0.17. [5, 6]

4.18 For the reversible heat engines operating in series, as shown in figure 4.36. Show the following, if work output is twice that of second.

$$3T_2 = T_1 + 2T_3$$

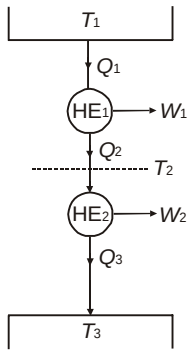


Fig. 4.36

4.19 A domestic refrigerator is intended to freeze water at 0ºC while water is available at 20ºC. COP of refrigerator is 2.5 and power input to run it is 0.4 kW. Determine capacity of refrigerator if it takes 14 minutes to freeze. Take specific heat of water as 4.2 kJ/kg. ºC. [10 kg]

4.20 A cold storage plant of 49.64 hp power rating removes 7.4 MJ/min and discharges heat to atmospheric air at 30ºC. Determine the temperature maintained inside the cold storage. [–40ºC]

4.21 A house is to be maintained at 21ºC from inside during winter season and at 26ºC during summer. Heat leakage through the walls, windows and roof is about 3×10^3 kJ/hr per degree temperature difference between the interior of house and environment temperature. A reversible heat pump is proposed for realizing the desired heating/cooling. What minimum power shall be required to run the heat pump in reversed cycle if outside temperature during summer is 36ºC? Also find the lowest environment temperature during winter for which the inside of house can be maintained at 21ºC. [0.279 kW, 11ºC]

4.22 Estimate the minimum power requirement of a heat pump for maintaining a commercial premises at 22ºC when environment temperature is –5ºC. The heat load on pump is 1×10^7 kJ/day.

4.23 A reversible engine having 50% thermal efficiency operates between a reservoir at 1527ºC and a reservoir at some temperature *T*. Determine temperature *T* in *K*.

4.24 A reversible heat engine cycle gives output of 10 kW when 10 kJ of heat per cycle is supplied from a source at 1227ºC. Heat is rejected to cooling water at 27ºC. Estimate the minimum theoretical number of cycles required per minute. [75]

4.25 Some heat engine *A* and a reversible heat engine *B* operate between same two heat reservoirs. Engine *A* has thermal efficiency equal to two-third of that of reversible engine *B*. Using second law of thermodynamics show that engine *A* shall be irreversible engine.

4.26 Show that the COP of a refrigeration cycle operating between two reservoirs shall be, $COP_{ref} = \left(\frac{1}{\eta_{max}} - 1\right)$, if η_{max} refers to thermal efficiency of a reversible engine operating between same temperature limits.

4.27 A heat pump is used for maintaining a building at 20ºC. Heat loss through roofs and walls is at the rate of 6×10^4 kJ/h. An electric motor of 1 kW rating is used for driving heat pump. On some day when environment temperature is 0ºC, would it be possible for pump to maintain building at desired temperature? [No]

4.28 Three heat engines working on carnot cycle produce work output in proportion of 5 : 4 : 3 when operating in series between two reservoirs at 727°C and 27°C. Determine the temperature of intermediate reservoirs. [435.34°C, 202°C]

4.29 Determine the power required for running a heat pump which has to maintain temperature of 20°C when atmospheric temperature is –10°C. The heat losses through the walls of room are 650 W per unit temperature difference of inside room and atmosphere. [2 kW]

4.30 A heat pump is run between reservoirs with temperatures of 7°C and 77°C. Heat pump is run by a reversible heat engine which takes heat from reservoir at 1097°C and rejects heat to reservoir at 77°C. Determine the heat supplied by reservoir at 1097°C if the total heat supplied to reservoir at 77°C is 100 kW. [25.14 kW]

4.31 A refrigerator is used to maintain temperature of 243K when ambient temperature is 303K. A heat engine working between high temperature reservoir of 200°C and ambient temperature is used to run this refrigerator. Considering all processes to be reversible, determine the ratio of heat transferred from high temperature reservoir to heat transferred from refrigerated space. [0.69]

5
Entropy

5.1 INTRODUCTION

Till now the detailed explanation of Zeroth law, first law and second law of thermodynamics have been made. Also we have seen that the first law of thermodynamics defined a very useful property called internal energy. For overcoming the limitations of first law, the second law of thermodynamics had been stated. Now we need some mathematical parameter for being used as decision maker in respect of feasibility of process, irreversibility, nature of process etc. Here in this chapter a mathematical function called 'entropy' has been explained. 'Entropy' is the outcome of second law and is a thermodynamic property. Entropy is defined in the form of calculus operation, hence no exact physical description of it can be given. However, it has immense significance in thermodynamic process analysis.

5.2 CLAUSIUS INEQUALITY

Let us take any reversible process 1–2 as shown on *P–V* diagram. Let us also have a reversible adiabatic process 1–1′ followed by reversible isothermal process 1′–2′ and a reversible adiabatic process 2′ – 2, as approximation to the original process 1 – 2 such that area under 1 – 2 equals to that under 1–1′–2′ –2. By first law of thermodynamics for process shown by 1–2.

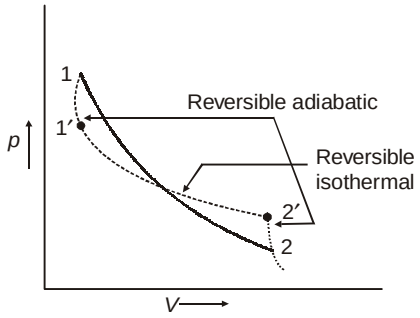


Fig. 5.1 *Reversible adiabatic, isothermal and reversible process*

$$Q_{1-2} = (U_2 - U_1) + W_{1-2}$$

First law on 1–1′–2′–2 processes; (Heat and work are path functions and internal energy is point function)

$$Q_{1-1'-2'-2} = (U_2 - U_1) + W_{1-1'-2'-2}$$

As already assumed that $W_{1-2} = W_{1-1'-2'-2}$

so $Q_{1-1'-2'-2} = Q_{1-2}$

In the path 1 – 1′ –2′ –2 during adiabatic processes 1 – 1′ & 2 –2′ there is no heat interaction so the total heat interaction in 1 –2 is getting occurred during isothermal process 1′ – 2′ only.

Hence, it is always possible to replace any reversible process by a series of reversible adiabatic, reversible isothermal and reversible adiabatic processes between the same end states provided the heat interaction and work involved remains same.

If the number of reversible adiabatic and reversible isothermal processes is quite large then the series of such processes shall reach close to the original reversible process.

Let us undertake this kind of substitution for the processes in a reversible cycle

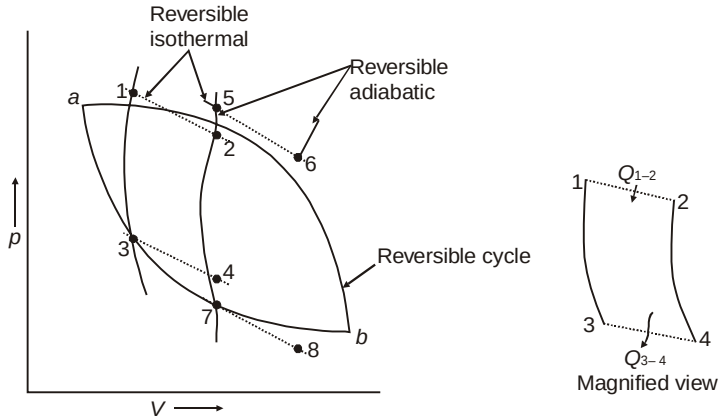


Fig. 5.2 *A reversible cycle replaced by reversible adiabatics and reversible isotherms*

Figure 5.2 shows replacement of original processes in cycle $a - b - a$ by adiabatic and isothermals. This shall result in a number of Carnot cycles appearing together in place of original cycle. Two Carnot cycles thus formed are shown by 1 –2 – 4 –3 and 5 – 6 – 8 – 7. Magnified view of first Carnot cycle is also shown separately where heat supplied at high temperature is Q_{1-2} and heat rejected at low temperature is Q_{3-4}.

From thermodynamic temperature scale;

For Carnot cycle 1 – 2 – 4 – 3,

$$\frac{Q_{1-2}}{Q_{3-4}} = \frac{T_{1-2}}{T_{3-4}}$$

or

$$\frac{Q_{1-2}}{T_{1-2}} = \frac{Q_{3-4}}{T_{3-4}}$$

For Carnot cycle 5 – 6 – 8 – 7,

$$\frac{Q_{5-6}}{T_{5-6}} = \frac{Q_{7-8}}{T_{7-8}}$$

Now taking sign conventions for heat added and rejected;

$$\frac{Q_{1-2}}{T_{1-2}} + \frac{Q_{3-4}}{T_{3-4}} = 0$$

and

$$\frac{Q_{5-6}}{T_{5-6}} + \frac{Q_{7-8}}{T_{7-8}} = 0$$

Hence, if there are '*n*' number of Carnot cycles replacing the original reversible cycle, then

$$\left(\frac{Q_{1-2}}{T_{1-2}}+\frac{Q_{3-4}}{T_{3-4}}\right)+\left(\frac{Q_{5-6}}{T_{5-6}}+\frac{Q_{7-8}}{T_{7-8}}\right)+\ldots=0$$

or, it can be given as summation of the ratio of heat interaction (Q) to the temperature (T) at which it occurs, being equal to zero.

$$\sum\frac{Q}{T}=0$$

If number of Carnot cycles is very large, then the zig-zag path formed due to replacing adiabatics and isotherms shall reach very close to original cycle.

In such situation the cyclic integral of $\frac{Q}{T}$ may be given in place of above.

or $$\oint\left(\frac{dQ}{T}\right)_{\text{rev}}=0$$

Here it indicates that $\left(\frac{dQ}{T}\right)$ is some thermodynamic property. Above expression developed for a reversible heat engine cycle also remains valid for internally reversible engines. In case of internally reversible engines T shall be temperature of working fluid at the time of heat interaction.

$$\oint\left(\frac{dQ}{T}\right)_{\text{int, rev}}=0$$

Let us now try to find out what happens to $\oint\frac{dQ}{T}$ when we have an irreversible engine cycle. Let there be a reversible and irreversible heat engine operating between same temperature limits, such that heat added to them is same.

From Carnot's theorem for both reversible and irreversible heat engine cycles,

$$\eta_{\text{rev}}>\eta_{\text{irrev}}$$

or $$\left(1-\frac{Q_{\text{rejected}}}{Q_{\text{add}}}\right)_{\text{rev}}>\left(1-\frac{Q_{\text{rejected}}}{Q_{\text{add}}}\right)_{\text{irrev}}$$

or $$\left(\frac{Q_{\text{rejected}}}{Q_{\text{add}}}\right)_{\text{irrev}}>\left(\frac{Q_{\text{rejected}}}{Q_{\text{add}}}\right)_{\text{rev}}$$

For same heat added, i.e. $Q_{\text{add, rev}}=Q_{\text{add, irrev}}=Q_{\text{add}}$

$$Q_{\text{rejected, irrev}}>Q_{\text{rejected, rev}}$$

or $$\frac{Q_{\text{rejected, irrev}}}{Q_{\text{rejected, rev}}}>1$$

For absolute thermodynamic temperature scale,

$$\left(\frac{Q_{\text{add}}}{Q_{\text{rejected}}}\right)_{\text{rev}}=\frac{T_{\text{add}}}{T_{\text{rejected}}},\ \text{upon substitution we get,}$$

$$\frac{Q_{\text{rejected, irrev}}}{Q_{\text{add}}} > \frac{T_{\text{rejected}}}{T_{\text{add}}}$$

or $$\frac{Q_{\text{rejected, irrev}}}{T_{\text{rejected}}} > \frac{Q_{\text{add}}}{T_{\text{add}}}$$

Upon substituting sign convention, we get

$$\frac{Q_{\text{add}}}{T_{\text{add}}} + \frac{Q_{\text{rejected, irrev}}}{T_{\text{rejected}}} < 0$$

or $$\frac{Q_{\text{add, irrev}}}{T_{\text{add}}} + \frac{Q_{\text{rejected, irrev}}}{T_{\text{rejected}}} < 0$$

If it is given in the form of cyclic integral.

or $$\boxed{\oint \left(\frac{dQ}{T}\right)_{\text{irrev}} < 0}$$

Now combining for reversible and irreversible paths it can be given as;

$$\oint \left(\frac{dQ}{T}\right) \leq 0$$

This is called *Clausius inequality*.

here, $$\oint \left(\frac{dQ}{T}\right) = 0 \text{ for reversible cycle}$$

$$\oint \left(\frac{dQ}{T}\right) < 0, \text{ for irreversible cycle}$$

$$\oint \left(\frac{dQ}{T}\right) > 0, \text{ for impossible cycle}$$

5.3 ENTROPY–A PROPERTY OF SYSTEM

From Clausius inequality mathematically it is shown that for a reversible cycle.

$$\oint \left(\frac{dQ}{T}\right)_{\text{rev}} = 0$$

Let us take a reversible cycle comprising of two processes *A* and *B* as shown and apply Clausius inequality.

$$\int_{a\ \text{path}\ A}^{b} \frac{dQ}{T} + \int_{b\ \text{path}\ B}^{a} \frac{dQ}{T} = 0$$

$$\int_{a\ \text{path } A}^{b} \frac{dQ}{T} = -\int_{b\ \text{path } B}^{a} \frac{dQ}{T}$$

or

$$\int_{a\ \text{path } A}^{b} \frac{dQ}{T} = \int_{a\ \text{path } A}^{b} \frac{dQ}{T}$$

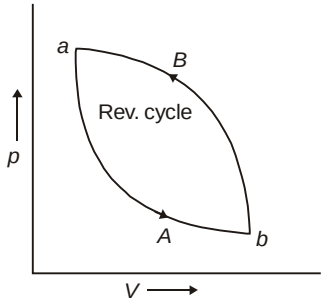


Fig. 5.3 *Reversible cycle*

Hence, it shows that $\left(\frac{dQ}{T}\right)$ is some property and does not depend upon path followed. This thermodynamic property is called "entropy". Entropy is generally denoted by '*S*' or 'ϕ'.

Thus, the energy interactions in the form of heat are accompanied by entropy changes.

Writing it as function of entropy change.

$$\int_{a\ \text{path } A}^{b} \frac{dQ}{T} = \int_{a\ \text{path } A}^{b} \frac{dQ}{T} = S_b - S_a$$

or

$$\int \left(\frac{dQ}{T}\right)_{\text{rev}} = \int dS$$

Since entropy is point function and depends only upon end states therefore entropy change for any process following reversible or irreversible path shall be same.

$$(S_b - S_a)_{\text{rev, path}} = (S_b - S_a)_{\text{irrev, path}}$$

$$\boxed{\int_{a}^{b} \frac{dQ_{\text{rev}}}{T} = \Delta S_{\text{rev, path}} = \Delta S_{\text{irrev,path}}}$$

Entropy is an extensive property and has units J/K. Specific entropy may be given on unit mass basis;

$$s = \frac{S}{m} \text{ (J/kg K)}$$

Entropy, as obvious from definition is defined for change in entropy value, therefore absolute value of entropy cannot be defined. Entropy is always given as change, i.e. relative values can be expressed.

Let us now have two thermodynamic cycles $a - b - a$ following paths ($a - R - b$, $b - R - a$) and ($a - R - b$, $b - I - a$).

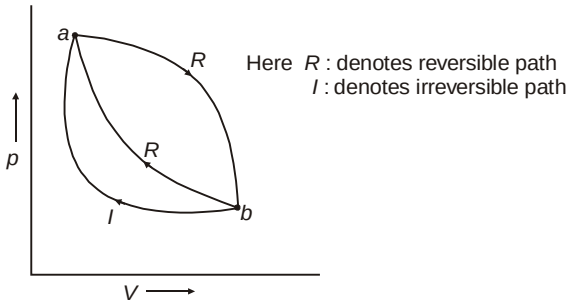


Fig. 5.4 *Reversible and irreversible cycle*

We have from Clausius inequality.

For reversible cycle $a - R - b - R - a$

$$\oint \left(\frac{dQ}{T}\right)_{rev} = 0$$

$$\int_{a,R}^{b} \frac{dQ}{T} + \int_{b,R}^{a} \frac{dQ}{T} = 0$$

For irreversible cycle $a - R - b - I - a$

$$\oint \left(\frac{dQ}{T}\right) < 0$$

or $$\int_{a,R}^{b} \frac{dQ}{T} + \int_{b,I}^{a} \frac{dQ}{T} < 0$$

or $$\int_{a,R}^{b} \frac{dQ}{T} < - \int_{b,I}^{a} \frac{dQ}{T}$$

Also from definition of entropy.

$$\int \left(\frac{dQ}{T}\right)_{rev} = \int dS$$

so from above

$$\int_{a,R}^{b} \frac{dQ}{T} = \int_{a}^{b} dS$$

or $$\left(\frac{dQ}{T}\right)_{rev} = dS$$

For reversible cycle.

$$\int_{a,\,R}^{b} \frac{dQ}{T} = -\int_{b,\,R}^{a} \frac{dQ}{T}$$

Substituting it in expression for irreversible cycle.

$$-\int_{b,\,R}^{a} \frac{dQ}{T} < -\int_{b,\,I}^{a} \frac{dQ}{T}$$

also from definition of entropy.

$$-\int_{b}^{a} dS < -\int_{b,\,I}^{a} \frac{dQ}{T}$$

or
$$\int_{b,\,I}^{a} \frac{dQ}{T} < \int_{b}^{a} dS$$

or, in general it can be given as,

$$\left(\frac{dQ}{T}\right)_{irrev} < dS$$

Combining the above two we get inequality as following,

$$\boxed{dS \geq \frac{dQ}{T}}$$

where
$$dS = \frac{dQ}{T} \text{ for reversible process}$$

$$dS > \frac{dQ}{T} \text{ for irreversible process}$$

Mathematical formulation for entropy ($dQ_{rev} = T \cdot dS$) can be used for getting property diagrams between "temperature and entropy" ($T - S$), "enthalpy and entropy" ($h - S$).

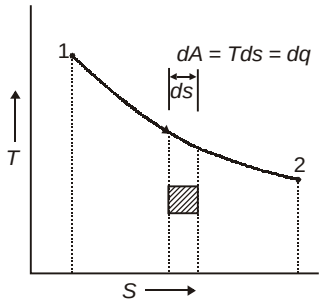


Fig. 5.5 *T-S diagram*

Area under process curve on T–S diagram gives heat transferred, for internally reversible process

$$dQ_{int,\,rev} = T \cdot dS$$

$$\boxed{Q_{int,\,rev} = \int_{1}^{2} T \cdot dS}$$

5.4 PRINCIPLE OF ENTROPY INCREASE

By second law, entropy principle has been obtained as,

$$\boxed{dS \geq \frac{dQ}{T}}$$

For an isolated system:

$$dQ = 0, \text{ therefore } dS_{\text{isolated}} \geq 0$$

for a reversible process $dS_{\text{isolated}} = 0$

i.e. $S_{\text{isolated}} = \text{constant}$

for an irreversible process $dS_{\text{isolated}} > 0$

which means the entropy of an isolated system always increases, if it has irreversible processes,

In general form $\boxed{dS_{\text{isolated}} \geq 0}$

It may be concluded here that the entropy of an isolated system always increases or remains constant in case of irreversible and reversible processes respectively. This is known as "Principle of entropy increase" of "entropy principle".

Universe which comprises of system and surroundings may also be treated as isolated system and from entropy principle;

$$dS_{\text{Universe}} \geq 0$$

Which means that entropy of universe either increases continuously or remains constant depending upon whether processes occurring in it are of "irreversible" or "reversible" type respectively.

Since, Universe = System + Surrounding.

therefore

$$dS_{\text{system}} + dS_{\text{surrounding}} \geq 0$$

or

$$\Delta S_{\text{system}} + \Delta S_{\text{surrounding}} \geq 0$$

Since most of the processes occurring generally are of irreversible type in universe, so it can be said that in general entropy of universe keeps on increasing and shall be maximum after attaining the state of equilibrium, which is very difficult to attain.

In the above expression, system and surroundings are treated as two parts of universe (an isolated system). So the total entropy change during a process can be given by sum of "entropy change in system" and "entropy change in surroundings". This total entropy change is also called "entropy generation" or "entropy production". Entropy generation will be zero in a reversible process.

Therefore

$$\Delta S_{\text{total}} = S_{\text{gen}} = \Delta S_{\text{system}} + \Delta S_{\text{surrounding}}$$

For closed systems

In case of closed systems there is no mass interaction but heat and work interactions are there. Entropy change is related to heat interactions occurring in system and surroundings.

Total entropy change.

or

Entropy generated

$$\Delta S_{\text{total}} = S_{\text{gen}} = \Delta S_{\text{system}} + \Delta S_{\text{surrounding}}$$

For system changing it's state from 1 to 2 i.e. initial and final state.

$$\Delta S_{\text{system}} = S_2 - S_1$$

For surroundings, entropy change depends upon heat interactions.

$$\Delta S_{\text{surrounding}} = \frac{Q_{\text{surrounding}}}{T_{\text{surrounding}}}$$

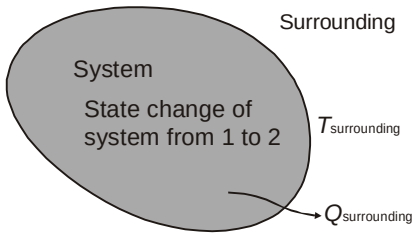


Fig. 5.6 *Closed system*

or $$-\Delta S_{\text{total}} = S_{\text{gen}} = (S_2 - S_1) + \frac{Q_{\text{surrounding}}}{T_{\text{surrounding}}}$$

or $$S_2 - S_1 = m(s_2 - s_1)$$

where m is mass in system and s_1 and s_2 are specific entropy values at initial and final state, then

$$\boxed{S_{\text{gen}} = m(s_2 - s_1) + \frac{Q_{\text{surrounding}}}{T_{\text{surrounding}}}}$$

For open systems

In case of open systems the mass interactions also take place along with energy interactions. Here mass flow into and out of system shall also cause some entropy change, so a control volume as shown in figure is to be considered. Entropy entering and leaving at section $i - i$ and $o - o$ are considered. Mass flow carries both energy and entropy into or out of control volume. Entropy transfer with mass flow is called "entropy transport". It is absent in closed systems.

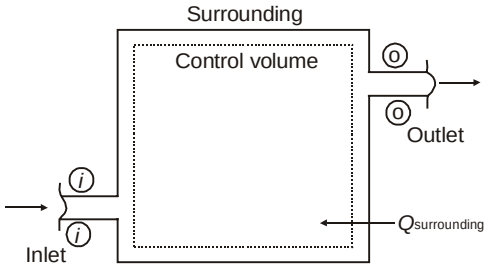


Fig. *5.7 Open system*

If control volume undergoes state change from 1 to 2, then entropy change in control volume shall be $(S_2 - S_1)$ while entropy entering and leaving out may be given as S_i and S_o respectively.

By principle of entropy increase, total entropy change shall be,

$$\Delta S_{\text{total}} = S_{\text{gen}} = (S_2 - S_1) + (S_o - S_i) + \frac{Q_{\text{surrounding}}}{T_{\text{surrounding}}}$$

Entropy entering and leaving out may be given as sum of entropy of all mass flows into and out of system in case of *uniform flow process.*

Therefore $S_i = \sum m_i \,.\, s_i$

$S_o = \sum m_o \,.\, s_o$

where m_i and m_o are mass flows into and out of system, and s_i and s_o are specific entropy associated with mass entering and leaving.

Substituting,

$$\boxed{\Delta S_{\text{total}} = S_{\text{gen}} = (S_2 - S_1) + \left(\sum m_o \,.\, s_o - \sum m_i \,.\, s_i\right) + \frac{Q_{\text{surrounding}}}{T_{\text{surrounding}}}}$$

In case of steady flow process since properties do not change with respect to time during any process, therefore within control volume there shall be no change in entropy.

i.e. $S_1 = S_2$

Total entropy change or entropy generation for this case shall be;

$$\Delta S_{\text{total}} = S_{\text{gen}} = (S_o - S_i) + \frac{Q_{\text{surrounding}}}{T_{\text{surrounding}}}$$

$$\boxed{\Delta S_{\text{total}} = S_{\text{gen}} = \left(\sum m_o \,.\, s_o - \sum m_i \,.\, s_i\right) + \frac{Q_{\text{surrounding}}}{T_{\text{surrounding}}}}$$

In all the cases discussed above $\Delta S_{\text{total}} \geq 0$ or $S_{\text{gen}} \geq 0$.

Entropy generated can be taken as criterion to indicate feasibility of process as follows;

* If S_{gen} or $\Delta S_{\text{total}} = 0$ then process is reversible.
* If S_{gen} or $\Delta S_{\text{total}} > 0$ then process is irreversible.
* If S_{gen} or $\Delta S_{\text{total}} < 0$ then process is impossible.

One thing is very important about entropy generated that S_{gen} is not a thermodynamic property and it's value depends on the path followed whereas entropy change of system $(S_2 - S_1)$ is a point function and so thermodynamic property. It is because of the fact that entropy change is cumulative effect of entropy transfer/change in system and surroundings.

5.5 ENTROPY CHANGE DURING DIFFERENT THERMODYNAMIC PROCESSES

Isothermal process

Let us find out entropy change for *isothermal heat addition* process. As isothermal process can be considered internally reversible, therefore entropy change shall be;

$$\Delta S_{a-b} = \int_a^b \left(\frac{dQ}{T}\right)$$

or $$\Delta S_{a-b} = \frac{1}{T} \int_a^b dQ$$

or $$\boxed{\Delta S_{a-b} = \frac{Q_{a-b}}{T}}$$

where Q_{a-b} is total heat interaction during state change $a - b$ at temperature T.

Isentropic process

It is the process during which change in entropy is zero and entropy remains constant during process.

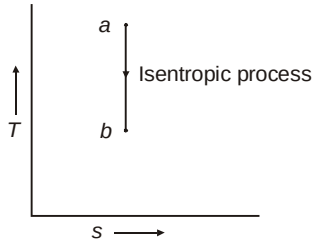


Fig. 5.8 *Isentropic process*

It indicates that when $\Delta S_{a-b} = 0$.

then $Q_{a-b} = 0$

which means there is no heat interaction during such process and this is adiabatic process.

Hence, it can be said that "a reversible isentropic process shall be adiabatic, where as if isentropic process is adiabatic then it may or may not be reversible''.

Thus, adiabatic process may or may not be reversible. It means in reversible adiabatic process all states shall be in equilibrium and no dissipative effects are present along with no heat interaction whereas in adiabatic process there is no heat interaction but process may be irreversible.

Finally, it can be concluded that an adiabatic process may or may not be isentropic whereas a reversible adiabatic process is always isentropic.

An adiabatic process of non isentropic type is shown below where irreversibility prevails, say due to internal friction.

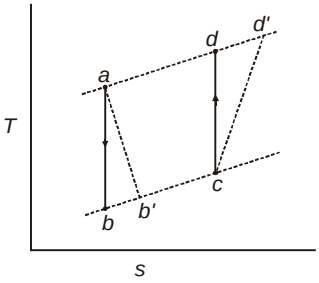


Fig. 5.9 *Isentropic and non-isentropic processes*

Here $a - b$ is reversible adiabatic expansion of isentropic type.

Non-isentropic or adiabatic expansion is shown by $a - b'$.

Isentropic expansion efficiency may be defined as ratio of actual work to ideal work available during expansion.

$$\eta_{\text{isen, expn}} = \frac{\text{Actual work in expansion}}{\text{Ideal work in expansion}}$$

$$\boxed{\eta_{\text{isen,expn}} = \frac{h_a - h_{b'}}{h_a - h_b}}$$

Similarly, isentropic and non-isentropic compression process are shown as $c - d$ and $c - d'$ respectively.

Isentropic compression efficiency can be defined on same lines as,

$$\eta_{\text{isen, compr}} = \frac{\text{Ideal work in expansion}}{\text{Actual work in expansion}}$$

$$\boxed{\eta_{\text{isen, compr}} = \frac{h_d - h_c}{h_{d'} - h_c}}$$

For ideal gases

Combination of first and second law yields;

$$Tds = du + pdv$$

also we know $du = c_v \cdot dT$ and for perfect gas $p = \frac{RT}{v}$

substituting for du and R

$$T \cdot ds = c_v \cdot dT + \frac{RT}{v} \cdot dv$$

or $$ds = c_v \cdot \frac{dT}{T} + \frac{R dv}{v}$$

or $$s_2 - s_1 = \int_1^2 c_v \cdot \frac{dT}{T} + \int_1^2 \frac{R dv}{v}$$

If c_v is function of temperature then,

$$\boxed{s_2 - s_1 = \int_1^2 c_v(T) \cdot \frac{dT}{T} + R \cdot \ln \frac{v_2}{v_1}}$$

If specific heat is constant then,

$$\boxed{s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \cdot \ln \frac{v_2}{v_1}}$$

Also combination of Ist and IInd law yields following using; $h = u + pv$,

or $$dh = du + pdv + vdp$$

$$T \cdot ds = dh - v\, dp$$

substituting $dh = c_p \cdot dT$, and $v = \frac{RT}{p}$

$$T \cdot ds = c_p \cdot dT - v \cdot dp$$

or $$ds = c_p \cdot \frac{dT}{T} - \frac{R dp}{p}$$

entropy change

$$s_2 - s_1 = \int_1^2 \frac{c_p\, dT}{T} - \int_1^2 \frac{R dp}{p}$$

If specific heat is function of temperature then

$$\boxed{s_2 - s_1 = \int_1^2 \frac{c_p\,(T)dT}{T} - R \ln \frac{p_2}{p_1}}$$

If specific heat is constant, then

$$\boxed{s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}}$$

Above expressions given in enclosed box may be suitably used for getting the change in entropy.

Polytropic Process

Entropy change in a polytropic process having governing equation as pv^n = constant, can be obtained as below,

For polytropic process between 1 and 2, $p_1 v_1^n = p_2 v_2^n$

or $$\left(\frac{p_1}{p_2}\right) = \left(\frac{v_2}{v_1}\right)^n$$

Also, from gas laws,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

$$\frac{p_1}{p_2} = \left(\frac{v_2}{v_1} \times \frac{T_1}{T_2}\right)$$

Above two pressure ratios give,

$$\frac{v_2}{v_1} = \left(\frac{T_1}{T_2}\right)^{\left(\frac{1}{n-1}\right)}$$

Substituting $\left(\frac{v_2}{v_1}\right)$ in the entropy change relation derived earlier.

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

$$= c_v \ln \left(\frac{T_2}{T_1}\right) + R \ln \left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}$$

$$s_2 - s_1 = c_v \ln \left(\frac{T_2}{T_1}\right) + \left(\frac{R}{n-1}\right) \ln \frac{T_1}{T_2}$$

For perfect gas $R = c_p - c_v$

$R = \gamma \,.\, c_v - c_v$

$$R = c_v (\gamma - 1)$$

Substituting R in entropy change

$$s_2 - s_1 = c_v \ln \left(\frac{T_2}{T_1}\right) + \frac{c_v(\gamma-1)}{(n-1)} \ln \left(\frac{T_1}{T_2}\right)$$

$$= c_v \ln \frac{T_2}{T_1} \left\{1 - \left(\frac{\gamma-1}{n-1}\right)\right\}$$

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} \left(\frac{n-\gamma}{n-1}\right)$$

Entropy change in polytropic process.

$$\boxed{s_2 - s_1 = \left(c_v \cdot \ln \frac{T_2}{T_1}\right)\left(\frac{n-\gamma}{n-1}\right)}$$

5.6 ENTROPY AND ITS RELEVANCE

Entropy has been introduced as a property based on the concept of IInd law of thermodynamics and derived from the thermodynamics involved in heat engines. A large number of definitions are available for entropy. To understand entropy let us take some gas in a closed vessel and heat it. Upon heating of gas the motion of gas molecules inside the vessel gets increased. State of molecular motion inside vessel depends upon the quantum of heat supplied. If we measure new kinetic energy of gas molecules, it is found to be larger than that initially. Also, the rate of intermolecular collision and randomness in molecular motion gets increased. In nutshell it could be said that heating has caused increase in energy level of gas molecules and thus resulting in increased disorderness inside the vessel. Higher is the energy with molecules, higher shall be the degree of disorderness. Entropy is closely defined using the degree of disorderness. It is said that greater is the molecular disorderness in the system greater shall be entropy. Mathematically, it can be supported by greater entropy value due to large heat supplied (dQ/T).

Thus "entropy can be defined as a parameter for quantifying the degree of molecular disorderness in the system". "Entropy is a measure of driving potential available for occurrence of a process".

Entropy is also an indicator of the direction of occurrence of any thermodynamic process. Mathematically, it has been seen from second law of thermodynamics that entropy of an isolated system always increases. Therefore, a process shall always occur in such a direction in which either entropy does not change or increases. In general almost all real processes are of irreversible type so entropy tends to increase. As entropy cannot be defined absolutely so the change in entropy should always have a positive or zero value.

5.7 THERMODYNAMIC PROPERTY RELATIONSHIP

Different thermodynamic properties such as *P, V, T, U, H, S* etc. can be related to one another using the combination of mathematical forms of first law, second law of thermodynamics and definitions of properties. Here specific values of properties are related.

For a non-flow process in closed system.

$$dq = du + dw$$

or

$$dq = du + p \cdot dv$$

Also, for a reversible process from definition of entropy, by second law we can write

$$dq = Tds$$

Combining above two,

$$\boxed{Tds = du + p \cdot dv}$$

From definition of enthalpy, specific enthalpy

$$h = u + pv$$

or $$dh = du + p \cdot dv + v \cdot dp$$

substituting from above

$$\boxed{dh = T \cdot ds + vdp}$$

Above relations may be used for getting the variation of one property with the other, such as for constant pressure process, $dh = T \cdot ds$

or $$\left(\frac{dh}{ds}\right)_{p=\text{constt.}} = T$$

which means slope of constant pressure line on enthalpy – entropy diagram ($h - s$) is given by temperature.

Also from above two relations

$$\boxed{T \cdot ds = c_v \cdot dT + p\, dv} \qquad \{\text{as } du = c_v \cdot dT\}$$

Substituting for dh and rearranging, $dh = T \cdot ds + v \cdot dp$ $\{\text{as } dh = c_p \cdot dT\}$

or $$\boxed{Tds = c_p \cdot dT - vdp}$$

For a constant pressure process above yields

$$\left(\frac{dT}{ds}\right)_{p=\text{constt.}} = \frac{T}{c_p}$$

It gives the slope of constant pressure line on $T - s$ diagram.

Similarly, for a constant volume process,

$$\left(\frac{dT}{ds}\right)_{v=\text{const.}} = \frac{T}{c_v}$$

It gives the slope of constant volume line on $T - s$ diagram.

It can be concluded from the above mathematical explanations for slope that *slope of constant volume line is more than the slope of constant pressure line as* $c_p > c_v$.

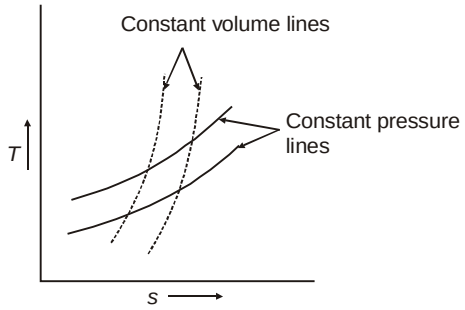


Fig. 5.10 *T-s diagram showing isobaric and isochoric process.*

5.8 THIRD LAW OF THERMODYNAMICS

'Third law of thermodynamics', an independent principle uncovered by 'Nernst' and formulated by 'Planck', states that the "Entropy of a pure substance approaches zero at absolute zero temperature." This fact can also be corroborated by the definition of entropy which says it is a measure of molecular disorderness. At absolute zero temperature substance molecules get frozen and do not have any activity, therefore it may be assigned zero entropy value at crystalline state. Although the attainment of absolute zero temperature is impossible practically, however theoretically it can be used for defining absolute entropy value with respect to zero entropy at absolute zero temperature. Second law of thermodynamics also shows that absolute zero temperature can't be achieved, as proved earlier in article 4.10. Third law of thermodynamics is of high theoretical significance for the sake of absolute property definitions and has found great utility in thermodynamics.

EXAMPLES

1. *Calculate the change in entropy of air, if it is throttled from 5 bar, 27ºC to 2 bar adiabatically.*

Solution:

Here $p_1 = 5$ bar, $T_1 = 300$ K.

$p_2 = 2$ bar, $c_{p\ \text{air}} = 1.004$ kJ/kg.K

$R = 0.287$ kJ/kg.K

Entropy change may be given as;

$$s_2 - s_1 = \left\{ c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \right\}$$

for throttling process $h_1 = h_2$

i.e. $c_p T_1 = c_p T_2$

or $T_1 = T_2$

Hence,

$$\text{Change in entropy} = 1.004 \ln (1) - 0.287 \ln \left(\frac{2}{5} \right)$$

$$= 0.263 \text{ kJ/kg.K}$$

Change in entropy = **0.263 kJ/kg.K Ans.**

2. *Find the change in entropy of steam generated at 400ºC from 5 kg of water at 27ºC and atmospheric pressure. Take specific heat of water to be 4.2 kJ/kg.K, heat of vaporization at 100ºC as 2260 kJ/kg and specific heat for steam given by;* $c_p = R\,(3.5 + 1.2T + 0.14T^2)$*, J/kg.K*

Solution:

Total entropy change = Entropy change during water temperature rise (ΔS_1).

+ Entropy change during water to steam change (ΔS_2)
+ Entropy change during steam temperature rise (ΔS_3)

$$\Delta S_1 = \frac{Q_1}{T_1} \text{ where } Q_1 = m\, c_p \cdot \Delta T$$

Heat added for increasing water temperature from 27ºC to 100ºC.

$= 5 \times 4.2 \times (100 - 27)$

$= 1533$ kJ

therefore, $$\Delta S_1 = \frac{1533}{300} = 5.11 \text{ kJ/K}$$

Entropy change during phase transformation;

$$\Delta S_2 = \frac{Q_2}{T_2}$$

Here Q_2 = Heat of vaporization = 5 × 2260 = 11300 kJ

Entropy change, $\Delta S_2 = \frac{11300}{373.15} = 30.28$ kJ/K.

Entropy change during steam temperature rise;

$$\Delta S_3 = \int_{373.15}^{673.15} \frac{dQ}{T}$$

Here $dQ = mc_p \cdot dT$; for steam $R = \frac{8.314}{18} = 0.462$ kJ/kg.K

Therefore, c_p for steam = $0.462\ (3.5 + 1.2 \cdot T + 0.14T^2) \times 10^{-3}$

$= (1.617 + 0.5544\ T + 0.065\ T^2) \times 10^{-3}$

or $$\Delta S_3 = \int_{373.15}^{673.15} 5 \times 10^{-3} \times \left(\frac{1.617}{T} + 0.5544 + 0.065\,T \right) dT$$

$= 51843.49 \times 10^{-3}$ kJ/K

$\Delta S_3 = 51.84$ kJ/K

Total entropy change = 5.11 + 30.28 + 51.84

= **87.23 kJ/K Ans.**

3. *Oxygen is compressed reversibly and isothermally from 125 kPa and 27ºC to a final pressure of 375 kPa. Determine change in entropy of gas?*

Solution:

Gas constant for oxygen:

$$R = \frac{8.314}{32} = 0.259 \text{ kJ/kg.K}$$

For reversible process the change in entropy may be given as;

$$\Delta s = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

Substituting values of initial & final states

$$\Delta s = -R \ln \left(\frac{375}{125} \right) = -0.285 \text{ kJ/kg.K}$$

Entropy change = – **0.285 kJ/kg. K Ans.**

4. *Determine the change in entropy of universe if a copper block of 1 kg at 150ºC is placed in a sea water at 25ºC. Take heat capacity of copper as 0.393 kJ/kg K.*

Solution:

Entropy change in universe

$$\Delta S_{universe} = \Delta S_{block} + \Delta S_{water}$$

where $\Delta S_{block} = mC. \ln \frac{T_2}{T_1}$

Here hot block is put into sea water, so block shall cool down upto sea water at 25ºC as sea may be treated as sink.

Therefore, $T_1 = 150ºC$ or 423.15 K

and $T_2 = 25ºC$ or 298.15 K

$$\Delta S_{block} = 1 \times 0.393 \times \ln \left(\frac{298.15}{423.15} \right)$$

$$= – 0.1376 \text{ kJ/K}$$

Heat lost by block = Heat gained by water

$$= – 1 \times 0.393 \times (423.15 – 298.15)$$

$$= – 49.125 \text{ kJ}$$

Therefore, $\Delta S_{water} = \frac{49.125}{298.15} = 0.165$ kJ/k

Thus, $\Delta S_{universe} = – 0.1376 + 0.165$

$= 0.0274$ kJ/k or 27.4 J/K

Entropy change of universe = **27.4 J/K Ans.**

5. *Determine change in entropy of universe if a copper block of 1 kg at 27ºC is dropped from a height of 200 m in the sea water at 27ºC. (Heat capacity for copper= 0.393 kJ/kg.K)*

Solution:

$$\Delta S_{universe} = \Delta S_{block} + \Delta S_{sea\ water}$$

Since block and sea water both are at the same temperature so,

$$\Delta S_{universe} = \Delta S_{sea\ water}$$

Conservation of energy equation yields;

$$Q – W = \Delta U + \Delta PE + \Delta KE$$

Since in this case, $W = 0$, $\Delta KE = 0$, $\Delta U = 0$

$$Q = \Delta PE$$

Change in potential energy $= \Delta PE = mgh = 1 \times 9.81 \times 200$

$$= 1962 \ J$$

$$Q = 1962 \ J$$

$$\Delta S_{universe} = \Delta S_{sea\ water} = \frac{1962}{300} = 6.54 \text{ J/kg K}$$

Entropy change of universe = **6.54 J/kg.K Ans.**

6. *Determine entropy change of universe, if two copper blocks of 1 kg & 0.5 kg at 150ºC and 0ºC are joined together. Specific heats for copper at 150ºC and 0ºC are 0.393 kJ/kg K and 0.381 kJ/kg K respectively.*

Solution:

Here, $\Delta S_{universe} = \Delta S_{block\ 1} + \Delta S_{block\ 2}$

Two blocks at different temperatures shall first attain equilibrium temperature. Let equilibrium temperature be T_f.

Then from energy conservation.

$$1 \times 0.393 \times (423.15 - T_f) = 0.5 \times 0.381 \times (T_f - 273.15)$$

$$T_f = 374.19\ K$$

Hence, entropy change in block 1, due to temperature changing from 423.15 *K* to 374.19 *K*.

$$\Delta S_1 = 1 \times 0.393 \times \ln\left(\frac{374.19}{423.15}\right) = -0.0483 \text{ kJ/K}$$

Entropy change in block 2

$$\Delta S_2 = 0.5 \times 0.381 \times \ln\left(\frac{374.19}{273.15}\right) = 0.0599 \text{ kJ/K}$$

Entropy change of universe = 0.0599 – 0.0483

= 0.0116 kJ/K

Entropy change of universe = **0.0116 kJ/K Ans.**

7. *A cool body at temperature T_1 is brought in contact with high temperature reservoir at temperature T_2. Body comes in equilibrium with reservoir at constant pressure. Considering heat capacity of body as C, show that entropy change of universe can be given as;*

$$C\left[\left(\frac{T_1 - T_2}{T_2}\right) - \ln\frac{T_1}{T_2}\right]$$

Solution:

Since body is brought in contact with reservoir at temperature T_2, the body shall come in equilibrium when it attains temperature equal to that of reservoir, but there shall be no change in temperature of the reservoir.

Entropy change of universe $\Delta S_{universe} = \Delta S_{body} + \Delta S_{reservoir}$

$$\Delta S_{body} = C \ln \frac{T_2}{T_1}$$

$$\Delta S_{reservoir} = \frac{-C(T_2 - T_1)}{T_2}$$

as, heat gained by body = Heat lost by reservoir

$$= C\ (T_2 - T_1)$$

Thus, $$\Delta S_{universe} = C \ln \frac{T_2}{T_1} - \frac{C(T_2 - T_1)}{T_2}$$

or, rearranging the terms,

$$\Delta S_{universe} = \frac{C(T_1 - T_2)}{T_2} - C \ln\left(\frac{T_1}{T_2}\right)$$

Hence proved.

8. *Determine the rate of power loss due to irreversibility in a heat engine operating between temperatures of 1800 K and 300 K. Engine delivers 2 MW of power when heat is added at the rate of 5 MW.*

Solution:

For irreversible operation of engine

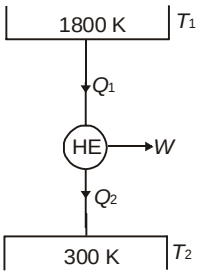


Fig. 5.11

$$\text{Rate of entropy generation} = \frac{Q_1}{T_1} + \frac{Q_2}{T_2}$$

$$= \frac{-5}{1800} + \frac{Q_2}{T_2}$$

Also, $W = Q_1 - Q_2 = 5 \times 10^6 - Q_2$

given $W = 2 \text{ MW} = 2 \times 10^6 \text{ W}$

so $Q_2 = 3 \times 10^6 \text{ W}$

Therefore,

$$\text{entropy generated} = \left(\frac{-5}{1800} + \frac{3}{300}\right) \times 10^6$$

$$\Delta S_{gen} = 7222.22 \text{ W/K}$$

Work lost $= T_2 \times \Delta S_{gen}$

$= 300 \times 7222.22$

$= 2.16 \times 10^6 \text{ W}$

or $= 2.16 \text{ MW}.$

Work lost = **2.16 MW Ans.**

9. *A system at 500 K and a heat reservoir at 300 K are available for designing a work producing device. Estimate the maximum work that can be produced by the device if heat capacity of system is given as;*

$$C = 0.05\ T^2 + 0.10T + 0.085, \text{ J/K}$$

Solution:

System and reservoir can be treated as source and sink. Device thought of can be a Carnot engine operating between these two limits. Maximum heat available from system shall be the heat rejected till it's temperature drops from 500 K to 300 K.

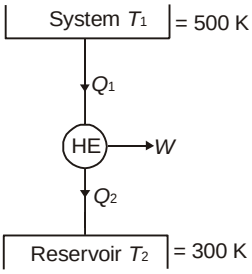


Fig. 5.12

Therefore,

Maximum heat $Q_1 = \int_{T_1}^{T_2} C \cdot dT$

$$Q_1 = \int_{500}^{300} (0.05\ T^2 + 0.10T + 0.085)\ dT$$

$$Q_1 = 1641.35 \times 10^3 \text{ J}$$

Entropy change of system, $\Delta S_{\text{system}} = \int_{500}^{300} C \frac{dT}{T}$

$$= -4020.043 \text{ J/K}$$

$$\Delta S_{\text{reservoir}} = \frac{Q_2}{T_2} = \frac{Q_1 - W}{T_2}$$

$$= \left(\frac{1641.35 \times 10^3 - W}{300} \right)$$

Also, we know from entropy principle

$$\Delta S_{\text{universe}} \geq 0$$

and $$\Delta S_{\text{universe}} = \Delta S_{\text{system}} + \Delta S_{\text{reservoir}}$$

Thus, upon substituting

$$(\Delta S_{\text{system}} + \Delta S_{\text{reservoir}}) \geq 0$$

$$\left\{ -4020.043 + \left(\frac{1641.35 \times 10^3 - W}{300} \right) \right\} \geq 0$$

or $$1451.123 - \frac{W}{300} \geq 0$$

$$1451.123 \geq \frac{W}{300}$$

$$W \leq 435337.10$$

or $$W \leq 435.34 \text{ kJ}$$

Hence Maximum work = **435.34 kJ**

For the given arrangement, device can produce maximum **435.34 kJ** of work. **Ans.**

10. *Determine the change in enthalpy and entropy if air undergoes reversible adiabatic expansion from 3MPa, 0.05 m^3 to 0.3 m^3.*

Solution:

For reversible adiabatic process, governing equation for expansion,

$$PV^{1.4} = \text{Constt.}$$

Also, for such process entropy change = 0.

Initial state : 3MPa, 0.05 m^3

Final state : 0.3 m^3

$$\text{Using } \frac{p_2}{p_1} = \left(\frac{V_1}{V_2} \right)^{1.4} \text{ or } V = \left(\frac{p_1 V_1^{1.4}}{p} \right)^{\frac{1}{1.4}}$$

we get $$p_2 = 0.244\ MPa$$

From first law, second law and definition of enthalpy;

$$dH = T \cdot dS + V dp$$

or, for adiabatic process of reversible type, $dS = 0$.

$$dH = V \cdot dp$$

$$\int_1^2 dH = \int_1^2 V \cdot dp$$

Substituting V, and actual states

$$H_2 - H_1 = \int_{3000}^{244} \left(\frac{3000 \times 0.05^{1.4}}{p} \right)^{\frac{1}{1.4}} dp$$

or $\Delta H = 268.8$ kJ

Enthalpy change = **268.8 kJ.**

Entropy change = **0 Ans.**

11. *During a free expansion 2 kg air expands from 1 m^3 to 10m^3 volume in an insulated vessel. Determine entropy change of (a) the air (b) the surroundings (c) the universe.*

Solution:

During free expansion temperature remains same and it is an irreversible process. For getting change in entropy let us approximate this expansion process as a reversible isothermal expansion.

(*a*) Change in entropy of air

$$\Delta S_{air} = m.R \ln \frac{V_2}{V_1}$$

$$= 2 \times 287 \ln \left(\frac{10}{1} \right)$$

$$\Delta S_{air} = 1321.68 \text{ J/K}$$

= 1321.68 J/K Ans.

(*b*) During free expansion on heat is gained or lost to surroundings so,

$$\Delta S_{surroundings} = 0$$

Entropy change of surroundings,

= **0 Ans.**

(*c*) Entropy change of universe

$$\Delta S_{universe} = \Delta S_{air} + \Delta S_{surroundings}$$

$$= 1321.68 \text{ J/K}$$

= 1321.68 J/K Ans.

12. *Determine the change in entropy of 0.5 kg of air compressed polytropically from 1.013 × 10^5 P_a to 0.8 MPa and 800 K following index 1.2. Take C_v = 0.71 kJ/kg . K.*

Solution:

Let initial and final states be denoted by 1 and 2.

For polytropic process pressure and temperature can be related as,

$$\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} = \frac{T_2}{T_1}$$

or $$T_2 = 800 \times \left(\frac{0.8\times10^6}{1.013\times10^5}\right)^{\frac{1.2-1}{1.2}}$$

Temperature after compression = 1128.94 K

Substituting in entropy change expression for polytropic process,

$$(s_2 - s_1) = C_v \left(\frac{n-\gamma}{n-1}\right).\ln\frac{T_2}{T_1}$$

$$= 0.71 \times 10^3\left(\frac{1.2-1.4}{1.2-1}\right)\ln\left(\frac{1128.94}{800}\right)$$

$$= -244.54,\ \text{kJ/kg . K}$$

Total entropy change $= m\,(s_2 - s_1)$

$$= 0.5 \times 244.54$$

$\Delta S =$ **122.27 J/K Ans.**

13. *A heat engine is working between the starting temperature limits of T_1 and T_2 of two bodies. Working fluid flows at rate 'm' kg/s and has specific heat at constant pressure as C_p. Determine the maximum obtainable work from engine.*

Solution:

In earlier discussions we have seen that in order to have highest output from engine, it should operate in reversible cycle and satisfy following relation,

$$\oint \frac{dQ}{T} = 0$$

Let us assume that the two bodies shall attain final temperature of T_f and engine shall then get stopped. so,

$$\int_{T_1}^{T_f} mC_p\frac{dT}{T} + \int_{T_2}^{T_f} mC_p\frac{dT}{T} = 0$$

or $$mC_p\left\{\ln\left(\frac{T_f}{T_1}\right) + \ln\left(\frac{T_f}{T_2}\right)\right\} = 0$$

or $$mC_p \cdot \ln\left(\frac{T_f^2}{T_1 \cdot T_2}\right) = 0$$

Here, $mC_p \neq 0$, so, $\ln\left(\frac{T_f^2}{T_1 \cdot T_2}\right) = 0$

or $$T_f = \sqrt{T_1 \cdot T_2}$$

Maximum work $= Q_{\text{supplied}} - Q_{\text{rejected}}$

$$= mC_p(T_1 - T_f) - mC_p(T_f - T_2)$$

$$= mC_p\{T_1 - 2T_f + T_2\}$$

$$= mC_p\{T_1 - 2 \cdot \sqrt{T_1 \cdot T_2} + T_2\}$$

$$\text{Maximum work} = \boldsymbol{mC_p \left\{\sqrt{T_1} - \sqrt{T_2}\right\}^2} \textbf{ Ans.}$$

14. *A heat engine operates between source at 600 K and sink at 300 K. Heat supplied by source is 500 kcal/s. Evaluate feasibility of engine and nature of cycle for the following conditions. (i) Heat rejected being 200 kcal/s, (ii) Heat rejected being 400 kcal/s (iii) Heat rejected being 250 kcal/s.*

Solution:

Clausius inequality can be used for cyclic process as given below; consider '1' for source and '2' for sink.

$$\oint \frac{dQ}{T} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2}$$

(*i*) For Q_2 = 200 kcal/s

$$\oint \frac{dQ}{T} = \frac{500}{600} - \frac{200}{300} = 0.1667$$

As $\oint \frac{dQ}{T} \gtrless 0$, therefore under these conditions engine is *not possible*. **Ans.**

(*ii*) For Q_2 = 400 kcal/s

$$\oint \frac{dQ}{T} = \frac{500}{600} - \frac{400}{300} = -0.5$$

Here $\oint \frac{dQ}{T} < 0$, so engine is *feasible* and cycle is *irreversible* **Ans.**

(*iii*) For Q_2 = 250 kcal/s

$$\oint \frac{dQ}{T} = \frac{500}{600} - \frac{250}{300} = 0$$

Here, $\oint \frac{dQ}{T} = 0$, so engine is *feasible* and cycle is *reversible.* **Ans.**

15. *Along a horizontal and insulated duct the pressure and temperatures measured at two points are 0.5 MPa, 400 K and 0.3 MPa, 350 K. For air flowing through duct determine the direction of flow.*

Solution:

Let the two points be given as states 1 and 2, so,

p_1 = 0.5 MPa, T_1 = 400 K

p_2 = 0.3 MPa, T_2 = 350 K

Let us assume flow to be from 1 to 2

So entropy change $\Delta s_{1-2} = s_1 - s_2 = C_p \ln \left(\frac{T_1}{T_2}\right) - R \ln \left(\frac{p_1}{p_2}\right)$

For air, R = 0.287 kJ/kg . K

C_p = 1.004 kJ/kg . K

Hence $s_1 - s_2 = 1.004 \ln \left(\frac{400}{350}\right) - 0.287 \ln \left(\frac{0.5}{0.3}\right)$

$$= -0.01254 \text{ kJ/kg . K}$$

or $$s_1 - s_2 = 0.01254 \text{ kJ/kg .K}$$

It means $s_2 > s_1$ hence the assumption that flow is from 1 to 2 is correct as from second law of thermodynamics the entropy increases in a process i.e. $s_2 \geq s_1$.

Hence flow occurs from 1 to 2 i.e. from **0.5 MPa, 400 K** to **0.3 MPa & 350 K Ans.**

16. *An ideal gas is heated from temperature T_1 to T_2 by keeping its volume constant. The gas is expanded back to it's initial temperature according to the law pv^n = constant. If the entropy change in the two processes are equal, find the value of 'n' in terms of adiabatic index γ.* *[U.P.S.C. 1997]*

Solution:

During constant volume process change in entropy $\Delta S_{12} = mc_v \cdot \ln \frac{T_2}{T_1}$

Change in entropy during polytropic process, $\Delta S_{23} = mc_v \left(\frac{\gamma - n}{n-1}\right) \ln \frac{T_2}{T_1}$

Since the entropy change is same, so

$$\Delta S_{12} = \Delta S_{23}$$

$$mc_v \ln \frac{T_2}{T_1} = mc_v \left(\frac{\gamma - n}{n-1}\right) \ln \frac{T_2}{T_1}$$

or $$\boxed{n = \frac{\gamma + 1}{2}}$$ **Ans.**

17. *A closed system executed a reversible cycle 1–2–3–4–5–6–1 consisting of six processes. During processes 1–2 and 3–4 the system receives 1000 kJ and 800 kJ of heat, respectively at constant temperatures of 500 K and 400 K, respectively. Processes 2–3 and 4–5 are adiabatic expansions in which the steam temperature is reduced from 500 K to 400 K and from 400 K to 300 K respectively. During process 5–6 the system rejects heat at a temperature of 300 K. Process 6–1 is an adiabatic compression process. Determine the work done by the system during the cycle and thermal efficiency of the cycle.*

[U.P.S.C. 1995]

Solution:

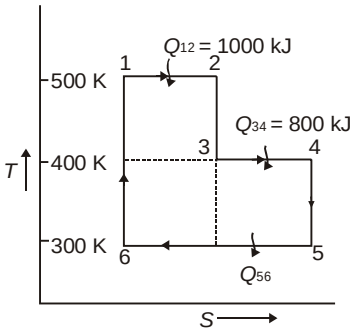


Fig. 5.13

Heat added = $Q_{12} + Q_{14}$

Total heat added = 1800 kJ

For heat addition process 1–2,

$$Q_{12} = T_1 \cdot (S_2 - S_1)$$
$$1000 = 500 \cdot (S_2 - S_1)$$

or, $S_2 - S_1 = 2$

For heat addition process 3–4, $Q_{34} = T_3 \cdot (S_4 - S_3)$

$$800 = 400 \cdot (S_4 - S_3)$$

or, $S_4 - S_3 = 2$

Heat rejected in process 5–6

$$Q_{56} = T_5 \cdot (S_5 - S_6)$$
$$= T_5 \cdot \{(S_2 - S_1) + (S_4 - S_3)\}$$
$$Q_{56} = 300 \cdot \{2 + 2\} = 1200 \text{ kJ}$$

Net work done = Net heat

$$= (Q_{12} + Q_{34}) - Q_{56}$$
$$W_{net} = 1800 - 1200$$
$$W_{net} = 600 \text{ kJ}$$

$$\text{Thermal efficiency of cycle} = \frac{W_{net}}{\text{Heat added}}$$

$$= \frac{600}{1800}$$

$$= 0.3333 \text{ or } 33.33\%$$

Work done = **600 kJ**
Thermal efficiency = **33.33%** **Ans.**

18. *A reversible heat engine has heat interaction from three reservoirs at 600 K, 700 K and 800 K. The engine rejects 10 kJ/s to the sink at 320 K after doing 20 kW of work. The heat supplied by reservoir at 800 K is 70% of the heat supplied by reservoir at 700 K then determine the exact amount of heat interaction with each high temperature reservoir.*

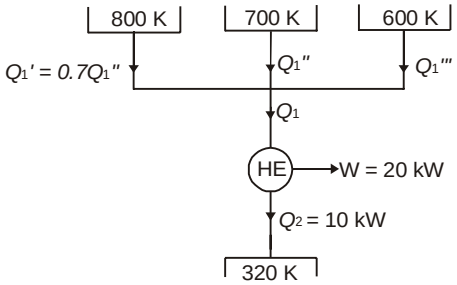


Fig. 5.14

Solution:

Let heat supplied by reservoir at 800 K, 700 K and 600 K be Q'_1, Q''_1, Q'''_1.

Here,

$$Q_1 - Q_2 = W$$

$\Rightarrow$ $Q_1 = 30$ kJ/s

Also given that, $Q'_1 = 0.7\ Q''_1$

$$Q'''_1 = Q_1 - (0.7\ Q''_1 + Q''_1)$$

$$Q'''_1 = Q_1 - 1.7\ Q''_1$$

For reversible engine

$$\Rightarrow \quad \frac{Q_1'}{800} + \frac{Q_1''}{700} + \frac{Q_1'''}{600} - \frac{Q_2}{320} = 0$$

$$\Rightarrow \quad \frac{0.7Q_1'}{800} + \frac{Q_1''}{700} + \frac{(Q_1 - 1.7Q_1'')}{600} - \frac{10}{320} = 0$$

$$\Rightarrow \quad Q''_1 = 88.48 \text{ kJ/s}$$

$$Q'_1 = 61.94 \text{ kJ/s}$$

$$Q'''_1 = -\ 120.42 \text{ kJ/s}$$

Heat supplied by reservoir at 800 K = **61.94 kJ/s**

Heat supplied by reservoir at 700 K = **88.48 kJ/s**

Heat supplied to reservoir at 600 K = **120.42 kJ/s Ans.**

19. *A rigid insulated tank is divided into two chambers of equal volume of 0.04 m^3 by a frictionless, massless thin piston, initially held at position with a locking pin. One chamber is filled with air at 10 bar & 25°C and other chamber is completely evacuated, Subsequently pin is removed and air comes into equilibrium. Determine whether the process is reversible or irreversible. Consider, R = 0.287 kJ/kg.K and c_v = 0.71 kJ/kg.K.*

Solution:

Let us assume process to be adiabatic and so the heat interaction would not be there. Also in view of this expansion being frictionless expansion there would be no work done, *i.e.*, $W = 0$, $Q = 0$.

Let initial and final states be indicated by subscripts 1 and 2.

Using first law of thermodynamics;

$$dQ = dW + dU$$

$$\Rightarrow \quad 0 = 0 + mc_v(T_2 - T_1)$$

$$\Rightarrow \quad T_1 = T_2 = 298 \text{ K}$$

Volume changes are, $V_2 = 2V_1 = 0.08 \text{ m}^3$

Using gas laws, $p_1V_1 = p_2V_2 \Rightarrow p_2 = 0.5p_1 = 5$ bar

Initial Mass of air, $m_1 = \frac{p_1V_1}{RT_1} \Rightarrow m_1 = \frac{10 \times 10^2 \times 0.04}{0.287 \times 298} = 0.4677 \text{ kg}$

Change of entropy, $(S_2 - S_1) = mRln\left(\frac{V_2}{V_1}\right) + mc_v ln\left(\frac{T_2}{T_1}\right)$

$$(S_2 - S_1) = 0.4677\ \{0.287 \times \ln 2 + 0.71 \ln 1)$$

$$= 0.09304 \text{ kJ/K}$$

From reversibility/irreversibility considerations the entropy change should be compared with $\int_1^2 \left(\frac{dQ}{T}\right)_{rev}$.

In this case; $\int_1^2 \left(\frac{dQ}{T}\right)_{rev}$ = 0, while entropy change = 0.09304 kJ/K.

Here $(S_2 - S_1) > \int_1^2 \left(\frac{dQ}{T}\right)_{rev}$, which means the **process is irreversible. Ans.**

20. *Two tanks A and B are connected through a pipe with valve in between. Initially valve is closed and tanks A and B contain 0.6 kg of air at 90°C, 1 bar and 1 kg of air at 45°C, 2 bar respectively. Subsequently valve is opened and air is allowed to mix until equilibrium. Considering the complete system to be insulated determine the final temperature, final pressure and entropy change.*

Solution:

In this case due to perfectly insulated system, $Q = 0$, Also $W = 0$

Let the final state be given by subscript f' and initial states of tank be given by subscripts 'A' and 'B'. p_A = 1 bar, T_A = 363 K, m_A = 0.6 kg; T_B = 318K, m_B = 1kg, p_B = 2 bar

$$\Delta Q = \Delta W + \Delta U$$

$$0 = 0 + \{(m_A + m_B) + C_v.T_f - (m_A.C_vT_A) - (m_B.C_v.T_B)\}$$

$$T_f = \frac{(m_A.C_v.T_A + m_B.C_v.T_B)}{(m_A + m_B).C_v} = \frac{(0.6\times 363 + 1\times 318)}{(0.6+1)}$$

T_f = 334.88 K, Final temperature = **334.88 K Ans.**

Using gas law for combined system after attainment of equilibrium,

$$p_f = \frac{(m_A + m_B).RT_f}{(V_A + V_B)}$$

$$V_A = \frac{m_A RT_A}{p_A};\ V_B = \frac{m_B RT_B}{p_B}$$

$$V_A = 0.625\ m^3;\ V_B = 0.456\ m^3$$

$$\Rightarrow \quad p_f = \frac{(1+0.6)\times 0.287\times 334.88}{(0.625+0.456)} = 142.25 \text{ kPa}$$

Final pressure = **142.25 kPa Ans.**

Entropy change;

$$\Delta S = \{((m_A + m_B).s_f) - (m_A.s_A + m_B s_B)\}$$

$$\Delta S = \{m_A(s_f - s_A) + m_B\,(s_f - s_B)\}$$

$$= \left\{m_A\left(C_p \ln\frac{T_f}{T_A} - R\ln\frac{p_f}{p_A}\right) + m_B\left(C_p \ln\frac{T_f}{T_B} - R\ln\frac{p_f}{p_B}\right)\right\}$$

Considering C_p = 1.005 kJ/kg.K

$$\Delta S = \left\{0.6\left(1.005\ln\frac{334.88}{363} - 0.287\ln\frac{142.25}{100}\right) + 1\left(1.005\ln\frac{334.88}{318} - 0.287\ln\frac{142.25}{200}\right)\right\}$$

$$\Delta S = \{ -0.1093 + 014977\}$$

$$= 0.04047 \text{ kJ/K}$$

Entropy produced = **0.04047 kJ/K Ans.**

21. *Three tanks of equal volume of $4m^3$ each are connected to each other through tubes of negligible volume and valves in between. Tank A contains air at 6 bar, 90°C, tank B has air at 3 bar, 200°C and tank C contains nitrogen at 12 bar, 50°C. Considering adiabatic mixing determine (i) the entropy change when valve between tank A and B is opened until equilibrium, (ii) the entropy change when valves between tank C; tank A and tank B are opened until equilibrium. Consider R_{Air} = 0.287 kJ/kg.K, g_{Air} = 1.4, $R_{Nitrogen}$ = 0.297 kJ/kg. K and $g_{Nitrogen}$ = 1.4.*

Solution:

Let states in tanks A, B & C be denoted by subscripts A, B & C respectively.

(*i*)When tank A and B are connected;

$$C_{v,\text{Air}} = \frac{R_{\text{Air}}}{(r_{Air} - 1)} = 0.718\text{kJ/kg.K}$$

After adiabatic mixing let the states be denoted by subscript 'D'.

Internal energy before mixing = Internal energy after mixing

$$m_A.C_{v_{Air}}.T_A + m_B.C_{v,Air}.T_B = (m_A + m_B).C_{V,Air}.T_D$$

$$\Rightarrow \quad T_D = \frac{(m_A.T_A + m_B.T_B)}{(m_A + m_B)}$$

Using gas laws, $$m_A = \frac{p_A.V_A}{R_{Air}T_A} = \frac{6\times10^2\times4}{0.287\times363} = 23.04\text{kg}$$

$$m_B = \frac{p_B.V_B}{R_{Air}T_B} = \frac{3\times10^2\times4}{0.287\times573} = 7.29\text{kg}$$

Final temperature $\Rightarrow T_D = \left\{\frac{(23.04\times363)+(7.29\times573)}{(23.04+7.29)}\right\}$ = 413.47 K

Final pressure, $p_D = \frac{R_{Air}T_D.m_D}{V_D} = \frac{R_{Air}T_D.(m_A+m_B)}{(V_A+V_B)}$

$$p_D = \frac{0.287\times413.47\times(23.04+7.29)}{(4+4)} = 449.89 \text{ kPa}$$

$$p_D = 4.4989 \text{ bar}$$

Entropy change, $\Delta_{\Delta S} = (S_D - S_A) + (S_D - S_B)$

$$\Delta S = \left\{(m_A.C_{p,Air} \ln\frac{T_D}{T_A} - m_A.R_{Air} \ln\frac{p_D}{p_A}) + (m_B.C_{p,Air} \ln\frac{T_D}{T_B} - m_B.R_{Air} \ln\frac{p_D}{p_B})\right\}$$

$$\Delta S = \left\{\left(23.04\times1.005\ln\left(\frac{413.47}{363}\right)\right) - \left(23.04\times0.287\ln\left(\frac{449.89}{600}\right)\right) + \left(7.29\times1.005\ln\left(\frac{413.47}{573}\right)\right) - \left(7.29\times0.287\ln\left(\frac{449.89}{300}\right)\right)\right\}$$

$$= \{3.014 + 1.904 + (-2.391) - 0.848\}$$

Entropy chnage, ΔS = **1.679 kJ/K** **Ans.**

(*ii*) After the three tanks A, B, and C are interconnected then the equilibrium will be attained amongst three. Equilibrium between A & B will result in state D as estimated in part (*i*) above. Thus it may be considered as the mixing of state D and nitrogen in tank C. Let the final state attained be 'F'.

After adiabatic mixing the final gas properties (as a result of mixing of air, state D and nitrogen, state C) may be estimated as under

$$m_c = \frac{p_c.V_c}{R_{Nitrogen}.T_c} = \frac{12\times10^2\times4}{0.297\times323}$$

m_c = 50.04 kg ; $m_D = m_A + m_B$ = 30.33 kg

$$C_{v,\text{ Nitrogen}} = \frac{R_{Nitrogen}}{\gamma_{Nitrogen}-1} = \frac{0.297}{(1.4-1)} = 0.7425 \text{ kJ/kgK}$$

$C_{p,\text{Nitrogen}} = \gamma_{\text{Nitrogen}} .C_{v,\text{ Nitrogen}} = 1.4 \times 0.7425$

$= 1.0395$ kJ/kg.K

$m_F = (m_D + m_c) = (23.04 + 7.29 + 50.04) = 80.37$ kg

$C_{v,F} = \{(m_D.C_{v,\text{ Air}} + m_c.C_{v,\text{ Nitrogen}})/(m_D + m_C)\}$

$= 0.733$ kJ/kg.K

$$R_F = \frac{m_D.R_{Air} + m_c.R_{Nitrogen}}{(m_D + m_C)}$$

$$= \left\{\frac{(30.33\times 0.287 + 50.04\times 0.297}{80.37}\right\}$$

$$R_F = 0.293 \text{ kJ/kg.K}$$

By first law of thermodynamics,

$$\Delta Q = \Delta W + \Delta U; \qquad \text{here } \Delta Q = 0,\ \Delta W = 0$$

Internal energy before mixing = Internal energy after mixing

$$m_D.C_{v,\text{Air}}.T_D + m_c.C_{v,\text{Nitrogen}}.T_c = (m_D + m_C)C_{v_F}.T_F$$

$$\Rightarrow (30.33 \times 0.718 \times 413.47) + (50.04 \times 0.7425 \times 323) = 80.37 \times 0.733 \times T_F$$

$\Rightarrow T_F =$ 356.55 K; Final temperature after mixing = 356.55 K

Final pressure after mixing;

$$p_F = \frac{m_F.R_F.T_F}{V_F} = \frac{(80.37\times 0.293\times 356.55)}{(4+4+4)}$$

$$p_F = \textbf{699.68 kPa;} \text{ Final pressure.}$$

Entropy change after mixing, $\Delta S = (S_F - S_D) + (S_F - S_C)$

$$\Delta S = m_D\left\{C_{p,Air}\ln\frac{T_F}{T_D} - R_{Air}\ln\frac{p_F}{p_D}\right\} + m_C\left\{C_{p,Nitrogen}\ln\frac{T_F}{T_C} - R_{Nitrogen}\ln\frac{p_F}{p_C}\right\}$$

$$= 30.33\left\{1.005\ln\frac{356.55}{413.47} - 0.287\ln\frac{699.68}{449.89}\right\} + 50.04\left\{1.0395\ln\frac{356.55}{323} - 0.297\ln\frac{699.68}{1200}\right\}$$

$$= -8.359 + 13.158$$

$$\Delta S = \textbf{4.799 kJ/kg.K} \quad \textbf{Ans.}$$

EXERCISE

5.1 Discuss the significance of Clausius inequality.
5.2 Define the 'entropy'. Also explain how it is a measure of irreversibility?
5.3 Explain the difference between isentropic process and adiabatic process.
5.4 How does the second law of thermodynamics overcome limitations of first law of thermodynamics?
5.5 Show that entropy of universe is increasing.
5.6 Is the adiabatic mixing of fluids irreversible ? If yes, explain.
5.7 Why does entropy generally increase ? Explain.
5.8 Explain the entropy principle and apply it to a closed system.
5.9 How the feasibility of any process can be ensured?
5.10 Give the third law of thermodynamics.
5.11 Explain why the slope of constant volume line is more than the slope of constant pressure line on *T*–*S* diagram.
5.12 Explain, whether the arrangement shown below for a reversible engine is feasible. If no then why? Give the correct arrangement.

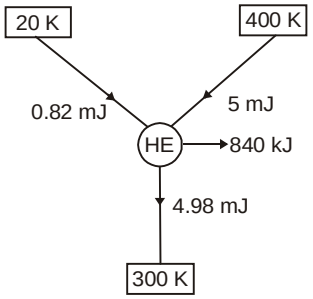


Fig. 5.15

5.13 Using second law of thermodynamics check the following and also indicate nature of cycle.

(*i*) Heat engine receiving 1000 kJ of heat from a reservoir at 500 K and rejecting 700 kJ heat to a sink at 27ºC.

(*ii*) Heat engine receiving 1000 kJ of heat from a reservoir at 500 K and rejecting 600 kJ of heat to a sink at 27ºC.

(*i*) Possible, irreversible cycle
(*ii*) Possible, reversible cycle

5.14 Determine the change in entropy of air during it's heating in a perfectly insulated rigid tank having 5 kg of air at 2 atm. Air is heated from 40ºC to 80ºC temperature.

5.15 Calculate change in entropy of air during the process in which a heat engine rejects 1500 kJ of heat to atmosphere at 27ºC during its operation. [5 kJ/K]

5.16 Determine the final temperature and total entropy change during a process in which metal piece of 5 kg at 200ºC falls into an insulated tank containing 125 kg of water at 20ºC. Specific heat of metal = 0.9 kJ/kg.K, Specific heat of water = 4.184 kJ/kg.K. [21.53ºC, 0.592 kJ/K]

5.17 Show that for air undergoing isentropic expansion process;

$$ds = c_p \frac{dv}{v} + c_v \frac{dp}{p}$$

5.18 Determine the change in entropy of air, if it is heated in a rigid tank from 27ºC to 150ºC at low pressure. [246.8 J/kg.K]

5.19 An electrical resistance of 100 ohm is maintained at constant temperature of 27ºC by a continuously flowing cooling water. What is the change in entropy of the resistor in a time interval of one minute ? [0]

5.20 A water tank of steel is kept exposed to sun. Tank has capacity of 10 m^3 and is full of water. Mass of steel tank is 50 kg and during bright sun temperature of water is 35ºC and by the evening water cools down to 30ºC. Estimate the entropy change during this process. Take specific heat for steel as 0.45 kJ/kg.K and water as 4.18 kJ/kg.K. [5.63 kJ/K]

5.21 Heat engine operating on Carnot cycle has a isothermal heat addition process in which 1 MJ heat is supplied from a source at 427ºC. Determine change in entropy of (i) working fluid, (ii) source, (iii) total entropy change in process. [1.43 kJ/K, – 1.43 kJ/K, 0]

5.22 A system operating in thermodynamic cycle receives Q_1 heat at T_1 temperature and rejects Q_2 at temperature T_2. Assuming no other heat transfer show that the net work developed per cycle is given as,

$$W_{\text{cycle}} = Q_1 \frac{Q_1}{T_1} + \left(1 - \frac{T_2}{T_1}\right) - T_2 \cdot S_{\text{gen}}$$

where S_{gen} is amount of entropy produced per cycle due to irreversibilities in the system.

5.23 A rigid tank contains 5 kg of ammonia at 0.2 MPa and 298 K. Ammonia is then cooled until its pressure drops to 80 kPa. Determine the difference in entropy of ammonia between initial and final state. [–14.8 kJ/K]

5.24 Determine the change in entropy in each of the processes of a thermodynamic cycle having following processes;

(*i*) Constant pressure cooling from 1 to 2, P_1 = 0.5 MPa, V_1 = 0.01 m^3

(*ii*) Isothermal heating from 2 to 3, P_3 = 0.1 MPa, T_3 = 25ºC, V_3 = 0.01 m^3

(*iii*) Constant volume heating from 3 to 1.

Take C_p = 1 kJ/kg . K for perfect gas as fluid.

[–0.0188 kJ/kg . K, 0.00654 kJ/kg . K, 0.0134 kJ/kg . K]

5.25 Conceptualize some toys that may approach close to perpetual motion machines. Discuss them in detail.

5.26 Heat is added to air at 600 kPa, 110°C to raise its temperature to 650°C isochorically. This 0.4 kg air is subsequently expanded polytropically up to initial temperature following index of 1.32 and finally compressed isothermally up to original volume. Determine the change in entropy in each process and pressure at the end of each process. Also show processes on p-V and T-s diagram, Assume C_v = 0.718 kJ/kg.K, R = 0.287 kJ/kg.K [0.2526 kJ/K, 0.0628 kJ/K, 0.3155 kJ/K 1445 kPa, 38.45 kPa]

5.27 Air expands reversibly in a piston-cylinder arrangement isothermally at temperature of 260°C while its volume becomes twice of original. Subsequently heat is rejected isobarically till volume is similar to original. Considering mass of air as 1 kg and process to be reversible determine net heat interaction and total change in entropy. Also show processes on T-s diagram. [– 161.8 kJ/kg, – 0.497 kJ/kg.K]

5.28 Ethane gas at 690 kPa, 260°C is expanded isentropically up to pressure of 105 kPa, 380K. Considering initial volume of ethane as 0.06 m^3 determine the work done if it behaves like perfect gas.

Also determine the change in entropy and heat transfer if the same ethane at 105 kPa, 380K is compressed up to 690 kPa following $p.V.^{1.4}$ = constant. [0.8608 kJ/K, 43.57 kJ]

5.29 Determine the net change in entropy and net flow of heat from or to the air which is initially at 105 kPa, 15°C. This 0.02 m^3 air is heated isochorically till pressure becomes 420 kPa and then cooled isobarically back up to original temperature. [– 0.011kJ/K, – 6.3 kJ]

5.30 Air initially at 103 kPa, 15°C is heated through reversible isobaric process till it attains temperature of 300°C and is subsequently cooled following reversible isochoric process up to 15°C temperature. Determine the net heat interaction and net entropy change. [101.9 kJ, 0.246 kJ/K]

5.31 Calculate the entropy change when 0.05 kg of carbon dioxide is compressed from 1 bar, 15°C to 830 kPa pressure and 0.004m^3 volume. Take C_p = 0.88 kJ/kg.K. This final state may be attained following isobaric and isothermal process. [0.0113 kJ/K]

5.32 Two insulated tanks containing 1 kg air at 200 kPa, 50°C and 0.5 kg air at 100 kPa, 80°C are connected through pipe with valve. Valve is opened to allow mixing till the equilibrium. Calculate the amount of entropy produced. [0.03175 kJ/K]

6

Thermodynamic Properties of Pure Substance

6.1 INTRODUCTION

Engineering systems have an inherent requirement of some substance to act as working fluid i.e. transport agent for energy and mass interactions. Number of working fluids are available and are being used in different systems suiting to the system requirements. Steam is also one of such working fluids used exhaustively because of its favourable properties. In thermal power plants steam is being extensively used. Water has capability to retain its chemical composition in all of its' phases i.e. steam and ice, and also it is almost freely available as gift of nature.

Pure substance refers to the "*substance with chemical homogeneity and constant chemical composition.*" H_2O is a pure substance as it meets both the above requirements. Any substance, which undergoes a chemical reaction, cannot be pure substance.

6.2 PROPERTIES AND IMPORTANT DEFINITIONS

Pure substance as defined earlier is used for operating various systems, such as steam is used for power generation in steam power plants. Hence, for thermodynamic analysis thermodynamic properties are required. Pressure and temperature are the properties that can be varied independently over wide range in a particular phase. Therefore, the behaviour of properties of pure substance have to be studied and mathematical formulations be made for their estimation.

Various dependent properties discussed ahead shall be enthalpy, internal energy, specific volume, entropy etc.

Some of terms used in discussion ahead are given as under.

(*a*) *Sensible heating*: It refers to the heating of substance in single phase. It causes rise in temperature of substance. In case of cooling in above conditions it shall be called sensible cooling.

(*b*) *Latent heating*: It is the heating of substance for causing its phase change without any change in it's temperature. If heat is extracted for causing phase change without any change in its temperature it will be called latent cooling.

(*c*) *Normal boiling point*: It is the temperature at which vapour pressure equals to atmospheric pressure and at this temperature phase change from liquid to gas begins.

(*d*) *Melting point*: It is the temperature at which phase change from solid to liquid takes place upon supplying latent heat.

(*e*) *Saturation states*: Saturation state of a substance refers to the state at which its phase transformation takes place without any change in pressure and temperature. These can be saturated solid state, saturated liquid state and saturated vapour state. For example saturated

vapour state refers to the state of water at which its phase changes to steam without varying pressure and temperature.

(*f*) *Saturation pressure*: It is the pressure at which substance changes its phase for any given temperature. Such as at any given temperature water shall get converted into steam at a definite pressure only, this pressure is called saturation pressure corresponding to given temperature. For water at 100°C the saturation pressure is 1 atm pressure.

(*g*) *Saturation temperature*: It refers to the temperature at which substance changes its phase for any given pressure. For water at 1 atm pressure the saturation temperature is 100°C.

(*h*) *Triple point*: Triple point of a substance refers to the state at which substance can coexist in solid, liquid and gaseous phase in equilibrium. For water it is 0.01°C i.e. at this temperature ice, water and steam can coexist in equilibrium. Table 6.1 given below gives triple point data for number of substances.

Table 6.1 *Triple point and critical point*

	Triple point		*Critical point*	
Substance	*Pressure, kPa*	*Temperature, °C*	*Pressure, MPa*	*Temperature, °C*
Water	0.611	0.01	22.12	374.15
Helium	5.1	– 271	0.23	– 268
Hydrogen	7.0	– 259	1.3	– 213
Oxygen	0.15	– 219	5.0	– 119
Nitrogen	12.5	– 210	3.4	– 147
Ammonia	6.1	– 78	11.3	132
Carbondioxide	517	– 57	7.39	31
Mercury	1.65×10^{-7}	– 39	18.2	899

(*i*) *Critical states*: "Critical state refers to that state of substance at which liquid and vapour coexist in equilibrium." In case of water at 22.12 MPa, and 374.15°C the water and vapour coexist in equilibrium, thus it is the highest pressure and temperature at which distinguishable water and vapour exist together. Data for critical state of many substances is given in the table 6.1. Specific volume at critical point for water is 0.00317 m^3/kg.

(*j*) *Dryness fraction*: It is the mass fraction of vapour in a mixture of liquid and vapour at any point in liquid-vapour mixture region. It is generally denoted by '*x*'. It is also called quality of steam.

(*k*) *Compressed liquid or subcooled liquid*: Liquid at temperature less than saturation temperature corresponding to a given pressure is called compressed liquid or subcooled liquid. Degree of subcooling is given by the temperature difference between liquid temperature and saturation temperature of liquid at given pressure.

Degree of subcooling = Saturation temperature at given pressure – Temperature of liquid.

(*l*) *Superheated steam*: Steam having temperature more than the saturation temperature corresponding to given pressure is called superheated steam. Amount of superheating is quantified by degree of superheating. Degree of superheating is given by difference between temperature of steam and saturation temperature at given pressure.

Degree of superheating = Temperature of steam – Saturation temperature at given pressure.

6.3 PHASE TRANSFORMATION PROCESS

Let us study phase transformation from ice to steam by taking ice at –20°C in an open vessel i.e. at atmospheric pressure, and heat it from bottom. Salient states passed through the phase change are as given under. Melting point of ice is 0°C and boiling point of water is 100°C for water at 1 atmospheric pressure.

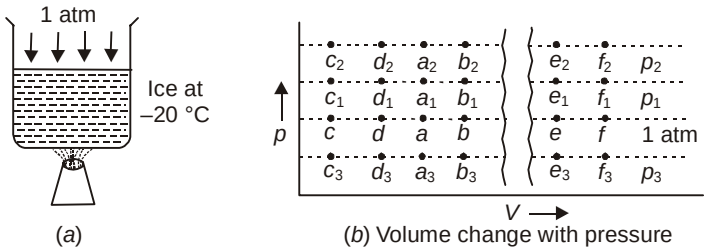


Fig. 6.1 *Phase transformation process*

Say initial state is given by '*a*' at –20°C and 1 atmospheric pressure.

(*i*) Upon heating the ice its temperature increases from –20°C to 0°C while being in solid phase. Temperature increase is accompanied by increase in volume and new state '*b*' is attained. This heating is sensible heating as heating causes increase in temperature in same phase.

(*ii*) After ice reaches to 0°C, the melting point, it is ready for phase transformation into water. Further heat addition to it causes melting and now water at 0°C is available. This heating is called latent heating and heat added is called latent heat. New state attained is '*c*' and volume gets reduced due to typical characteristic of water. As defined earlier state '*b*' is called *saturation solid state* as phase can change here without any change in pressure and temperature. State '*c*' is called *saturated liquid state* with respect to solidification.

(*iii*) Further heating of water at 0°C shall cause increase in its temperature upto 100°C. This heat addition is accompanied by increase in volume and state changes from '*c*' to '*d*' as shown on *p-V* diagram. Here typical behaviour of water from 0 to 4°C is neglected. This heating is sensible heating in liquid phase. State '*d*' is called saturated liquid state with respect to vaporization. Thus, there are two saturated liquid states '*c*' and '*d*' depending upon direction of transformation.

(*iv*) Water at 100°C and 1 atmosphere is ready for getting vaporized with supply of latent heat of vaporization. Upon adding heat to it the phase transformation begins and complete liquid gradually gets transformed into steam at state '*e*'. This phase change is accompanied by large increase in volume. Heating in this zone is called latent heating. State '*e*' is called *saturated vapour state* or saturated steam state.

(*v*) Steam at 100°C upon heating becomes hotter and its temperature rises. Say, the heating causes temperature rise upto 200°C. This increase in temperature is also accompanied by increase in volume up to state '*f*' as shown on *p-V* diagram. This heating is sensible heating in gaseous phase.

Similar phase transformations can be realized at different pressures and such salient states be identified and marked. Joining all saturated solid states at different pressures results in a locii, which is called "saturated solid line." Similarly, joining all saturated liquid states with respect to solidification and saturated liquid states with respect to vaporization results in two 'saturated liquid lines'. Locii of all saturated vapour states at different pressure generates 'saturated vapour line'. The lines thus obtained are shown in Fig. 6.2 detailing *p-V* diagram for water. Point at which "saturated liquid line" with respect

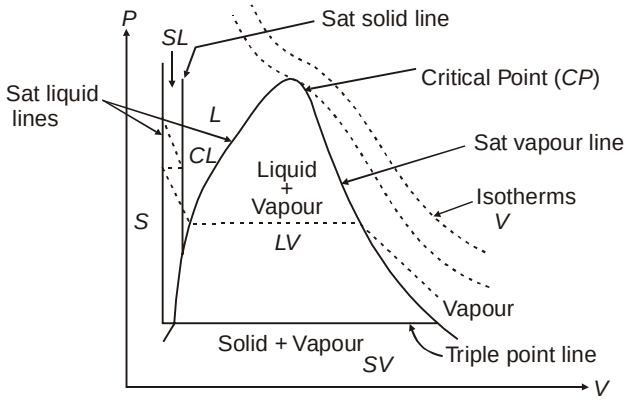


Fig. 6.2 *P-V diagram for water*

to vaporization meets with "saturated vapour line" is called "Critical point". Critical point is also sometimes referred to as "Critical state".

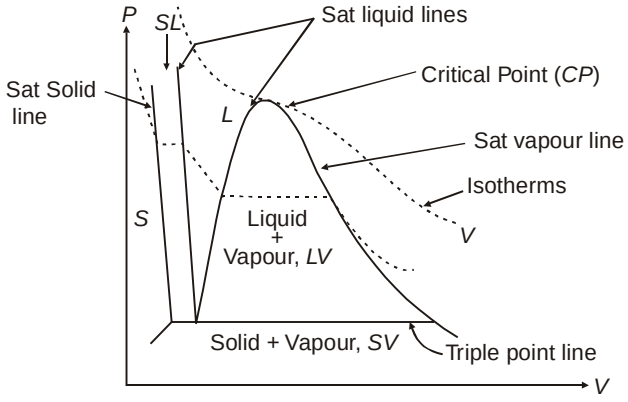


Fig. 6.3 *P-V diagram for Carbon dioxide*

On *P-V* diagram region marked *S* shows the solid region, mark *SL* shows solid-liquid mixture region, mark *LV* shows liquid-vapour mixture region, mark *CL* shows compressed liquid region. Triple point line is also shown and it indicates coexistence of solid, liquid and gas in equilibrium. Region marked *SV* and lying below triple point line is sublimation region where solid gets transformed directly into vapour upon supply of latent heat of sublimation. *P-V* diagram for substance which has different characteristics of contraction upon freezing is also shown here, Fig. 6.3. Carbon dioxide is such substance. Difference in *p-V* diagrams for two different substances (water and CO_2) may be understood from here.

6.4 GRAPHICAL REPRESENTATION OF PRESSURE, VOLUME AND TEMPERATURE

Graphical representations based on variation of thermodynamic properties can be obtained from the study of actual phase transformation process. In earlier discussion the variation of pressure and volume

during phase transformation from ice to steam has been shown and explained. On these lines the standard graphical representations in terms of *p-V*, *T-V*, *p-T* and *p-V-T* for water can be obtained. *p-V* diagram as obtained has already been discussed in article 6.2.

(*i*) ***T-V* diagram:** It gives variation of temperature with volume. Let us look at different steps involved in phase transformation and how are the temperature variations.

a – b: Temperature rises from –20°C to 0°C with volume increase, (ice)

b – c: Temperature remains constant at 0°C due to phase change and volume decreases, (ice to water)

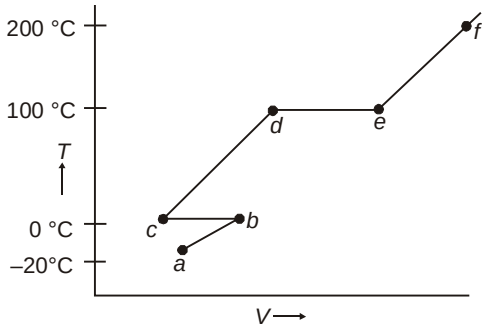


Fig. 6.4 *Temperature-Volume variation at 1 atm pressure*

c – d: Temperature increases from 0°C to 100°C and volume increases, (water).

d – e: Temperature remains constant at 100°C, phase changes from liquid to vapour and volume increases, (water to steam).

e – f: Temperature increases from 100°C to 200°C and volume increases, (steam).

Similar to above, the *T-V* variations at different pressures can be obtained and identical states i.e. saturation states be joined to get respective saturation lines as shown in Fig. 6.5 ahead.

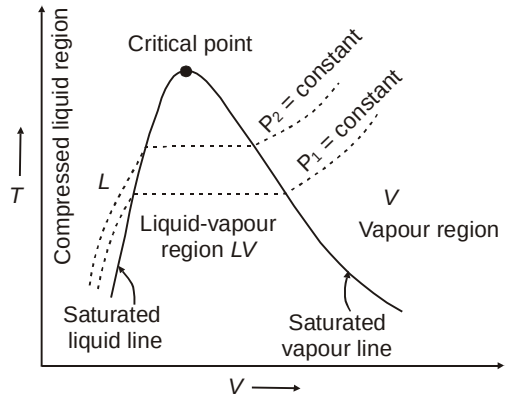


Fig. 6.5 *T-V diagram for water*

(*ii*) ***P-T* diagram:** It is the property diagram having pressure on Y-axis and temperature on X-axis. This can also be obtained by identifying and marking salient states during phase transformation and subsequently generating locii of identical states at different pressures.

For the phase change discussed in article 6.3, pressure and temperature variation shall be as described ahead. This phase change occurs at constant pressure of 1 atm.

a – b: Temperature rises from –20°C to 0°C, phase is solid.
b – c: Temperature does not rise, phase changes from ice to water (solid to liquid)
c – d: Temperature rises from 0°C to 100°C, phase is liquid
d – e: Temperature does not rise, phase changes from liquid to gas (water to steam)
e – f: Temperature rises from 100°C to 200°C, phase is gas (steam).

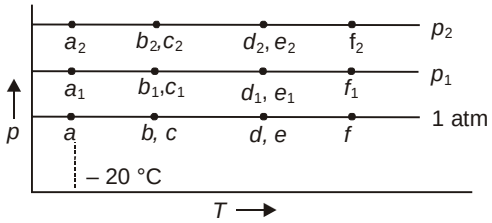


Fig. 6.6 *P-T variation for phase transformation at constant pressures*

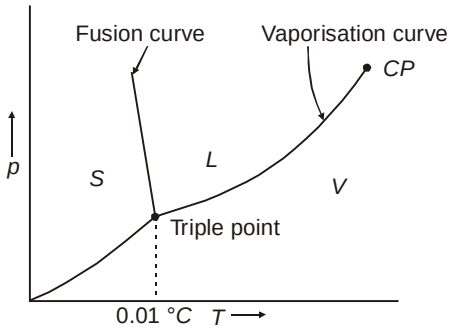
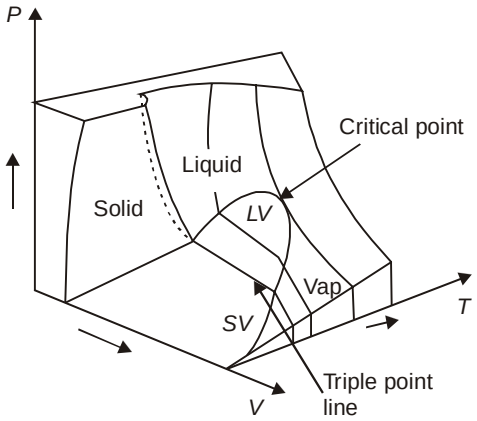
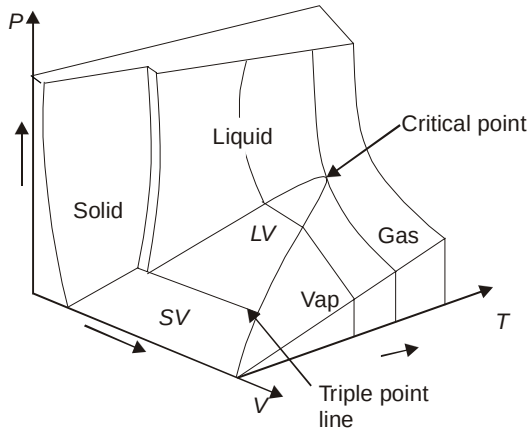


Fig. 6.7 *P-T diagram for water*

(*iii*) ***P-V-T* surface:** This is the three dimensional variation describing three thermodynamic properties *P*, *V* and *T*. As we know that for defining a state at least two properties are needed, therefore on *P-V-T* surface also, by knowing any two the third can be seen and continuous variation of these properties is available. Here pressure, volume and temperature are taken on mutually perpendicular axis and surface obtained is depicted below with all salient points. *P-V-T* surface shall be different for different substances depending on their characteristics. Here *P-V-T* surface for two substances having opposite characteristics are given in Fig. 6.8.



(A) P-V-T surface for water (which expands upon freezing.)



(B) P-V-T surface for Carbon dioxide (which contracts upon freezing.)

Fig. 6.8 *P-V-T surface for water and CO_2*

6.5 THERMODYNAMIC RELATIONS INVOLVING ENTROPY

Entropy change during phase transformation process can be studied with temperature at any given pressure, based on discussions in article 6.3. Entropy changes for every state change are estimated and plotted on *T-S* diagram for 1 atm. pressure and '*m*' mass of ice.

a – b: Temperature changes from –20°C to 0°C. Phase is solid

Entropy change $\Delta S_{a-b} = S_b - S_a = \int_{T_a}^{T_b} \frac{dQ_{a-b}}{T}$

Here $dQ_{a-b} = m \cdot c_{p,\,\text{ice}} \cdot dT$

and $T_a = 253$ K, $T_b = 273$ K.

or $\Delta S_{a-b} = \int_{T_a}^{T_b} \frac{m\, c_{p,\text{ice}}.dT}{T}$

b – c: Temperature does not change. It is constant at 0°C.

Entropy change,

$$\Delta S_{b-c} = S_c - S_b = \frac{\Delta Q_{b-c}}{T_b}$$

Here, ΔQ_{b-c} = Latent heat of fusion/melting of ice at 1 atm

$T_b = 273$ K

c – d: Temperature rises from 0°C to 100°C. Phase is liquid.

Entropy change,

$$\Delta S_{c-d} = S_d - S_c = \int_{T_c}^{T_d} \frac{dQ_{c-d}}{T}$$

Here $dQ_{c-d} = m \cdot c_{p,\text{water}} \cdot dT$ and $T_c = 273$ K, $T_d = 373$ K.

or, $$\Delta S_{c-d} = \int_{T_c}^{T_d} \frac{m\, c_{p,\text{water}}\, dT}{T}$$

d – e: Temperature does not change. It is constant at 100°C.
Entropy change,

$$\Delta S_{d-e} = S_e - S_d = \frac{\Delta Q_{d-e}}{T_d}$$

Here, ΔQ_{d-e} = latent heat of vaporization at 1 atm.
and $T_d = 373$ K.

e – f: Temperature rises from 100°C to 200°C. Phase is gas (steam)
Entropy change,

$$\Delta S_{e-f} = S_f - S_e = \int_{T_e}^{T_f} \frac{dQ_{e-f}}{T}$$

Here, $dQ_{e-f} = m \cdot c_{p,\text{steam}} \cdot dT$ and $T_e = 373$ *K*, $T_f = 473$ K

or, $$\Delta S_{e-f} = \int_{T_e}^{T_f} \frac{m\, c_{p,\text{steam}} \,.\, dT}{T}$$

Above entropy change when plotted on *T–S axis* result as below.

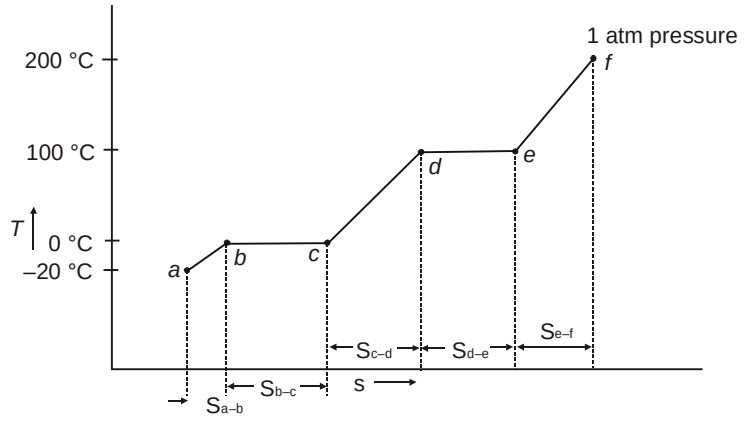


Fig. 6.9 *Temperature-entropy variation for phase change at 1 atm.*

On the similar lines, (as discussed above) the *T–S* variation may be obtained for water at other pressures. In the *T–S* diagram different important zones and lines are earmarked. Out of whole *T–S* diagram, generally the portion detailing liquid, liquid-vapour zone, vapour zone are of major interest due to steam undergoing processes and subsequently condensed and evaporated. Both *T–S* diagrams are shown here.

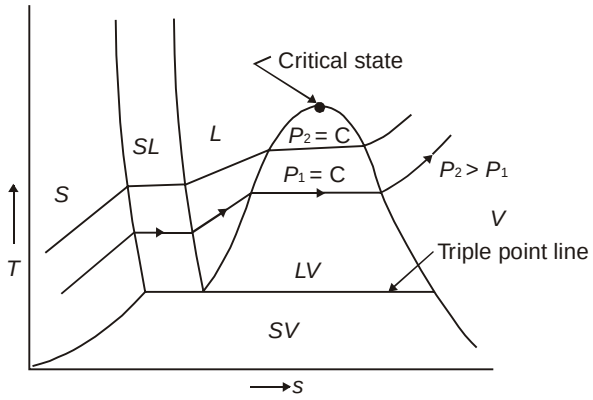


Fig. 6.10 *T–s diagram for water*

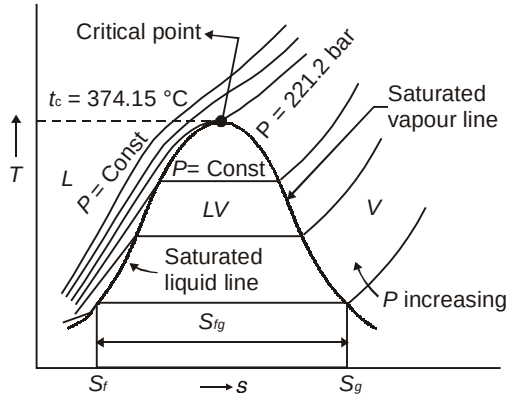


Fig. 6.11 *T–s diagram showing liquid and vapour regions*

6.6 PROPERTIES OF STEAM

For thermodynamic analysis the following thermodynamic properties of steam are frequently used.

"Saturation temperature (T), saturation pressure (P), specific volume (ν), enthalpy (h), entropy (s), internal energy (u)"

Let us look at *T–S* diagram below.

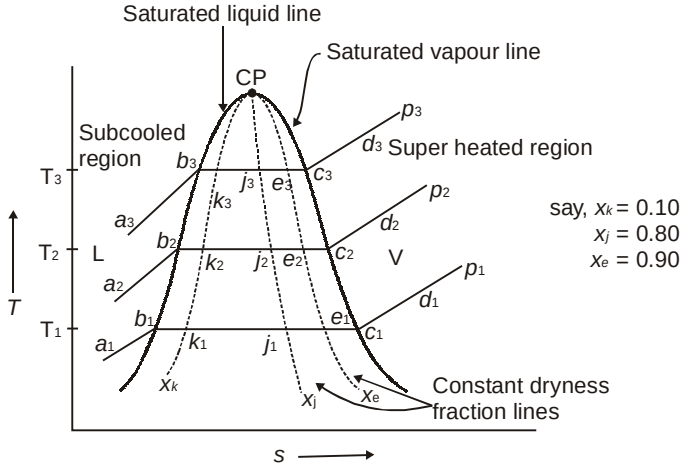


Fig. 6.12 *T–s diagram*

Thermodynamic properties and nomenclature used is indicated inside bracket along with property. Discussion is based on unit mass of steam/mixture. *T–S* diagram for 2-phases i.e. liquid and vapour has saturated liquid line and saturated vapour line meeting at critical point. Three constant pressure lines corresponding to pressure P_1, P_2 and P_3 are shown. Let us take a constant pressure line for pressure p_1 which has states a_1, b_1, k_1, j_1, e_1, c_1, d_1 shown upon it. Region on the left of saturated liquid line is liquid region. Region enclosed between saturated liquid line and saturated vapour line is liquid-vapour mixture

region or also called wet region. Region on the right of saturated vapour line is vapour region. All the states lying on saturated liquid line are liquid (water) states shown as b_1, b_2, b_3 at different pressures. States a_1, a_2 and a_3 are the states lying in subcooled region. Compressed liquid or subcooled liquid exists at a_1, a_2 and a_3 at pressures p_1, p_2 and p_3.

Degree of sub cooling at a_1 = Saturation temperature for pressure p_1 – Temperature at a_1

$= (T_1 - T_{a1})$

where T_1, T_2, T_3 are saturation temperatures at pressures p_1, p_2 and p_3.

At constant pressure p_1 when we move towards right of state b_1 then the phase transformation of water into steam (vapour) just begins. This conversion from liquid to vapour takes place gradually till whole liquid gets converted to vapour. Phase transformation into vapour gets completed at c_1, c_2, c_3 at pressures p_1, p_2 and p_3 respectively. States c_1, c_2, c_3 are called saturated vapour states and substance is completely in vapour phase at these points. Beyond state c_1, at pressure p_1 it is vapour phase and sensible heating shall cause increase in temperature. State d_1 is the state of steam called superheated steam. Superheated steam exists at d_1, d_2, and d_3 at pressures p_1, p_2 and p_3. Degree of superheat at $d_1 = T_{d1} - T_1$

Similarly, degree of superheat at $d_2 = T_{d2} - T_2$,

degree of superheat at $d_3 = T_{d3} - T_3$.

As pressure is increased upto critical pressure then constant pressure line is seen to become tangential to critical point (CP) at which water instantaneously flashes into vapour.

In the wet region states k_1, j_1 and e_1 are shown at pressure p_1.

At state b_1 mixture is 100% liquid

At state k_1 mixture has larger liquid fraction, say 90% liquid fraction and 10% vapour fraction.

At state j_1 mixture has say 20% liquid fraction and 80% vapour fraction.

At state e_1 mixture has say 10% liquid fraction and 90% vapour fraction.

At state c_1 mixture is 100% vapour.

Similarly, from explanation given above the states b_2, k_2, j_2, e_2, c_2 and b_3, k_3, j_3, e_3 and c_3 can be understood at pressures p_2 and p_3 respectively.

At any pressure for identifying the state in wet region, fraction of liquid and vapour must be known, for identifying state in subcooled region, degree of subcooling is desired and for identifying state in superheated region, degree of superheating is to be known.

For wet region a parameter called dryness fraction is used. Dryness fraction as defined earlier can be given as, x:

Dryness fraction; $$x = \frac{\text{Mass of vapour}}{\text{Mass of liquid} + \text{Mass of vapour}}$$

Dryness fraction values can be defined as follows;

Dryness fraction at state $b_1 = 0$

Dryness fraction at state $k_1 = 0.10$

$$\left\{\text{as assumed} = \frac{0.10\,(\text{m})}{(0.9 + 0.1)\text{m}}\right\}$$

Dryness fraction at state $j_1 = 0.80$

Dryness fraction at state $e_1 = 0.90$

Dryness fraction at state $c_1 = 1.00$

Thus, saturated liquid line and saturated vapour line are locii of all states having 0 and 1 dryness fraction values.

At critical point dryness fraction is either 0 or 1.

For different pressures the locii of constant dryness fraction points may be obtained and it yields constant dryness fraction lines corresponding to $x_k(0.10)$, $x_j(0.80)$, $x_e(0.90)$, as shown by dotted lines.

Let us use subscript 'f' for liquid states and 'g' for vapour states.

Therefore, enthalpy corresponding to saturated liquid state = h_f,

Enthalpy corresponding to saturated vapour state = h_g.

Similarly, entropy may be given as s_f and s_g.

Specific volume may be given as v_f and v_g.

Internal energy may be given as u_f and u_g.

At any pressure for some dryness fraction x, the total volume of mixture shall comprise of volume occupied by liquid and vapour both.

$$\text{Total volume, } V = V_f + V_g.$$

Similarly,

Total mass, $m = m_f + m_g$, i.e. mass of liquid and mass of vapour put together substituting for volume,

$$m \cdot v = m_f \cdot v_f + m_g . v_g.$$

where m is total mass and v is specific volume of mixture

$$v = \left(1 - \frac{m_g}{m}\right) \cdot v_f + \left(\frac{m_g}{m}\right) v_g$$

or

$$v = \left(1 - \frac{m_g}{m}\right) \cdot v_f + \left(\frac{m_g}{m}\right) \cdot v_g$$

or

$$v = \left(1 - \frac{m_g}{m}\right) \cdot v_f + \left(\frac{m_g}{m}\right) \cdot v_g$$

From definition,

$$x = \frac{m_g}{m_f + m_g} = \frac{m_g}{m}$$

or,

$$v = (1 - x) \cdot v_f + x \cdot v_g$$

or,

$$v = v_f + x \cdot (v_g - v_f)$$

or,

$$v = v_f + x \cdot v_{fg}.$$

here v_{fg} indicates change in specific volume from liquid to vapour

Similarly, enthalpy, entropy and internal energy may be defined for such states in wet region.

i.e.

$$h = h_f + x\,(h_g - h_f)$$

or

$$h = h_f + x \cdot h_{fg}$$

here h_{fg} is difference in enthalpy between saturated liquid and vapour states. Actually h_{fg} is energy or heat required for vaporization or heat to be extracted for condensation i.e. latent heat.

$$\text{Similarly, } s = s_f + x \cdot s_{fg} \text{ where } s_{fg} = s_g - s_f$$

$$u = u_f + x \cdot u_{fg} \text{ where } u_{fg} = u_g - u_f.$$

6.7 STEAM TABLES AND MOLLIER DIAGRAM

Steam being pure substance has its unique and constant properties at different pressures and temperatures. Therefore, thermodynamic properties can be estimated once and tabulated for future use. Steam table is a tabular presentation of properties such as specific enthalpy, entropy, internal energy and specific volume at different saturation pressures and temperatures. Steam table may be on pressure basis or on temperature basis. The table on pressure basis has continuous variation of pressure and corresponding to it : saturation temperature (T_{sat}), enthalpy of saturated liquid (h_f), enthalpy of saturated vapour (h_g), entropy of saturated liquid (s_f), entropy of saturated vapour (s_g), specific volume of saturated liquid (v_f), specific volume of saturated vapour (v_g), internal energy of saturated liquid (u_f), internal energy of saturated vapour (u_g) are given on unit mass basis, i.e. as shown in table 6.2.

Similar to above the temperature based table which gives continuous variation of temperature and corresponding to it saturation pressure and other properties as h_f, h_g, h_{fg}, s_f, s_g, s_{fg}, v_f, v_g, u_f, u_g and u_{fg} are given.

Similarly, steam properties for superheated steam are also estimated and tabulated at some discrete pressures for varying degree of superheat. Super heated steam table are available for getting enthalpy, entropy, specific volume and internal energy separately. Example of superheated steam table for enthalpy is given here:

Table 6.2 *Pressure based steam table*

Pressure	*Sat. Temp.*	*Enthalpy*			*Entropy*			*Specific volume*		*Internal energy*		
	T_{sat}	h_f	h_g	h_{fg}	s_f	s_g	s_{fg}	v_f	v_g	u_f	u_g	u_{fg}
	°C	kJ/kg	kJ/kg	kJ/kg	kJ/kg°K	kJ/kg°K	kJ/kg°K	m^3/kg	m^3/kg	kJ/kg	kJ/kg	kJ/kg

Table 6.3 *Temperature based steam table*

Tempe-rature	*Sat. Pressure*	*Enthalpy*			*Entropy*			*Specific volume*		*Internal energy*		
°C	kPa	h_f	h_g	h_{fg}	s_f	s_g	s_{fg}	v_f	v_g	u_f	u_g	u_{fg}
		kJ/kg	kJ/kg	kJ/kg	kJ/kg°K	kJ/kg°K	kJ/kg°K	m^3/kg	m^3/kg	kJ/kg	kJ/kg	kJ/kg

Table 6.4 *Superheated steam table for enthalpy*

Pressure	*Sat. temp.* °C	*Enthalpy values for varying degree of superheat (kJ/kg)*						
kPa	(T_{sat})	T_1	T_2	T_3	T_4	T_5	T_6	T_7

Here T_1, T_2, T_3, T_4, T_5, T_6, T_7 ... are more than T_{sat} and have increasing value of degree of superheat. Steam tables as discussed above are available in appendix in this text book.

Mollier diagram is the enthalpy-entropy (h–s) diagram for steam. This diagram is obtained on the basis of following equation depending upon the phase transformation as discussed earlier.

$Tds = dh - vdp$. (First and second law combined)

For constant pressure $\left(\frac{dh}{ds}\right)_p = T$

Enthalpy entropy diagram as obtained for all phases of water is as given in Figure 6.13 here.

Generally, liquid and vapour region is only of interest in engineering systems, so mostly used portion of *h–s* diagram is taken out and shown in Fig. 6.14. It is popularly known as *mollier diagram* or *mollier chart.*

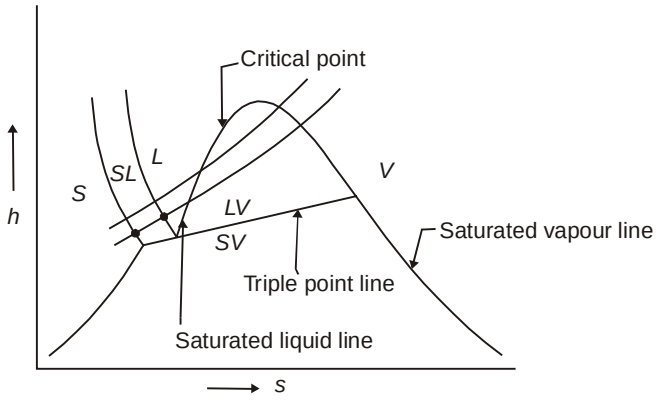


Fig. 6.13 *Enthalpy-entropy diagram for all phases*

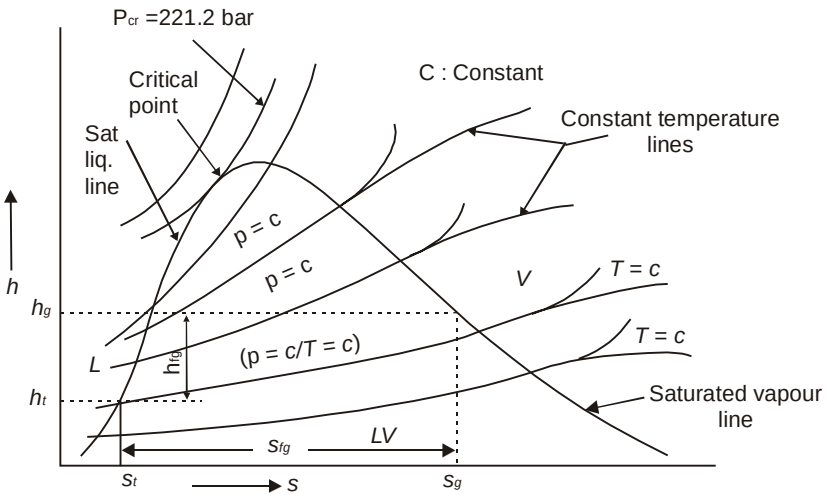


Fig. 6.14 *h–s diagram (Mollier diagram)*

Different significant lines such as saturated liquid line, saturated vapour line, isobaric lines, isothermal lines, constant specific volume lines, constant dryness fraction lines are shown upon Mollier diagram. Nature of variation of different lines can be explained from the real behaviour of substance and mathematical expression based on combination of first and second law.

Such as, why isobaric lines diverge from one another? This is due to the increase in saturation temperature with increase in pressure. Slope of isobar is equal to saturation temperature as shown in the beginning, therefore it also increases with increasing pressure.

Why isothermal lines are not visible in wet region? It is because constant temperature lines and constant pressure lines coincide upon in wet region. For every pressure there shall be definite saturation temperature which remains constant in wet region.

Mollier chart is also given in appendix, at the end of this book.

6.8 DRYNESS FRACTION MEASUREMENT

Dryness fraction is the basic parameter required for knowing the state of substance in liquid-vapour mixture region (wet region). For any pressure the dryness fraction varies from 0 to 1 in the wet region i.e. from saturated liquid to saturated vapour.

Dryness fraction being ratio of mass of vapour and total mass of substance can be conveniently estimated if these two mass values are known. It may also be termed as 'quality' of steam or 'dryness factor'.

$$\text{Dryness fraction} = \frac{\text{Mass of vapour}}{\text{Total mass i.e. (mass of vapour + mass of liquid)}}$$

Here we shall be looking into standard methods available for dryness fraction measurement. These are;

(*i*) Throttling calorimeter
(*ii*) Separating calorimeter
(*iii*) Separating and throttling calorimeter
(*iv*) Electrical calorimeter.

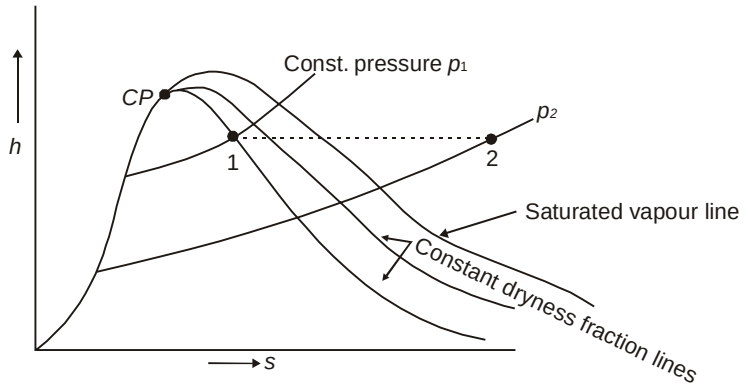


Fig. 6.15 *Throttling process on h–s diagram*

(*i*) ***Throttling calorimeter:*** In this the throttling action is utilised for getting dryness fraction. If a mixture is throttled, then upon throttling its enthalpy remains constant before and after throttling. Let us look upon states of substance on *h–s* diagram, before and after throttling.

Wet mixture being at state '1' initially, attains a new state '2' upon being throttled upto pressure p_2. This state at the end of throttling lies in the superheated region such that,

$$h_1 = h_2$$

Say, dryness fraction at state 1 is *x*, then enthalpy at this point can be given as

$$h_1 = h_{f \text{ at } p_1} + x \times h_{fg \text{ at } p_1}$$

In the above expression $h_{f \text{ at } p_1}$ and $h_{fg \text{ at } p_1}$ can be seen from steam table if pressure of wet steam is known. Also the enthalpy at state 2 (end of throttling) can be seen from superheated steam table if pressure and temperature at '2' are known.

Substituting in, $h_1 = h_2$

$$h_{f \text{ at } p_1} + x \times h_{fg \text{ at } p_1} = h_2$$

Here h_2, $h_{f \text{ at } p_1}$, $h_{fg \text{ at } p_1}$ are all known as explained above.

$$\text{Therefore, } x = \frac{h_2 - h_{f \text{ at } p_1}}{h_{fg \text{ at } p_1}}$$

Now the arrangements are to be made for, (*a*) measurement of pressure of wet steam in the begining, (*b*) throttling of wet mixture such that state at the end of throttling lies in superheated region, (*c*) measurement of pressure, temperature of throttled steam. Arrangement used in throttling calorimeter is as shown in Fig. 6.16.

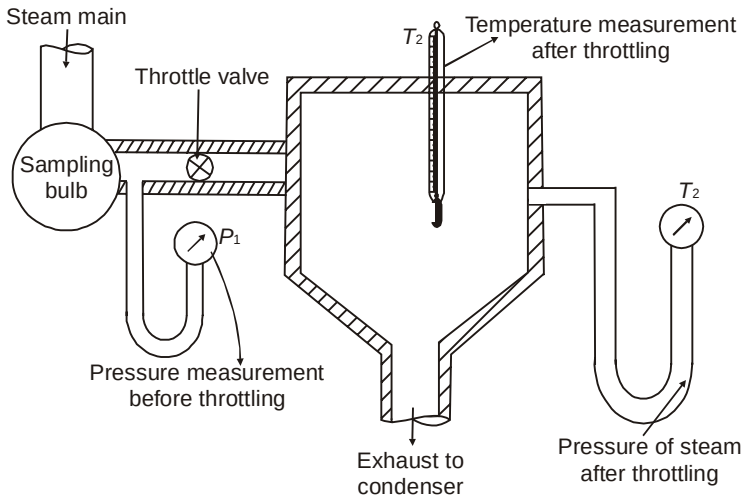


Fig. 6.16 *Throttling calorimeter*

(*ii*) ***Separating calorimeter:*** In this type of calorimeter the known mass of wet mixture is collected through a sampling bulb and sent to a separating chamber. Separating chamber has the series of obstacles, and zig-zag path inside it so that when mixture passes through them the liquid particles get separated due to sudden change in direction of flow and gravity action. Liquid thus separated out is collected in a collection tank and is measured. Thus by knowing the two mass values dryness fraction can be estimated as;

$$\text{Dryness fraction} = \frac{\{(\text{Total mass}) - (\text{Mass of liquid})\}}{\text{Total mass}}$$

Layout of separating calorimeter is given in Fig. 6.17.

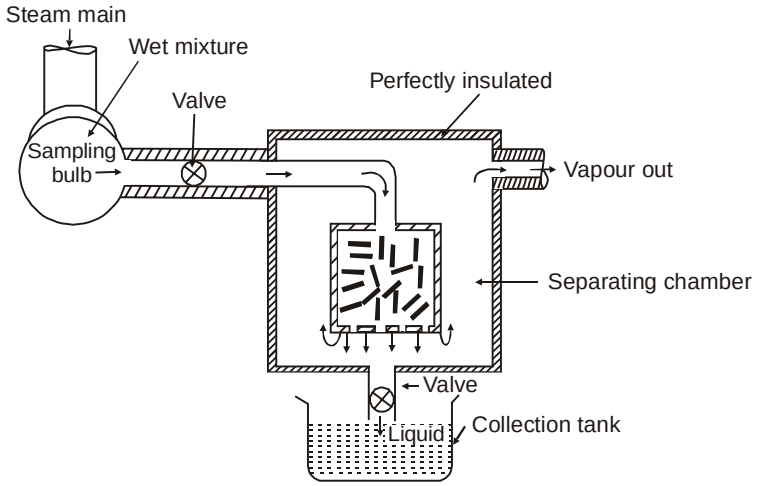


Fig. 6.17 *Separating calorimeter*

(*iii*) ***Separating and throttling calorimeter:*** Some times when the wet mixture is extremely wet then upon throttling state of steam is unable to become superheated. In such situations mixture is first passed through separating calorimeter to reduce liquid fraction in it and subsequently this less wet mixture is passed through throttling calorimeter. In such calorimeters the arrangement is as shown below.

Here excessively wet steam is first sent to separating calorimeter where its wetness is reduced by separating out some liquid fraction, say mass m_{f_1}. Less wet steam is sent to throttling calorimeter and its dryness fraction estimated as x_2. Then at the end this throttled steam (of superheated type) is made to pass through condenser. Mass of condensate is measured from that collected in condensate tank, say m_2. Thus m_2 is total mass of steam sent from separating calorimeter to throttling calorimeter. As mass of liquid collected in collection tank of separating calorimeter is m_{f_1} then total mass of wet steam under examination is $(m_{f_1} + m_2)$. Dryness fraction at section 1–1 shall be;

$$x_1 = \frac{(\text{Mass of vapour at } 1-1)}{\text{Total mass}}$$

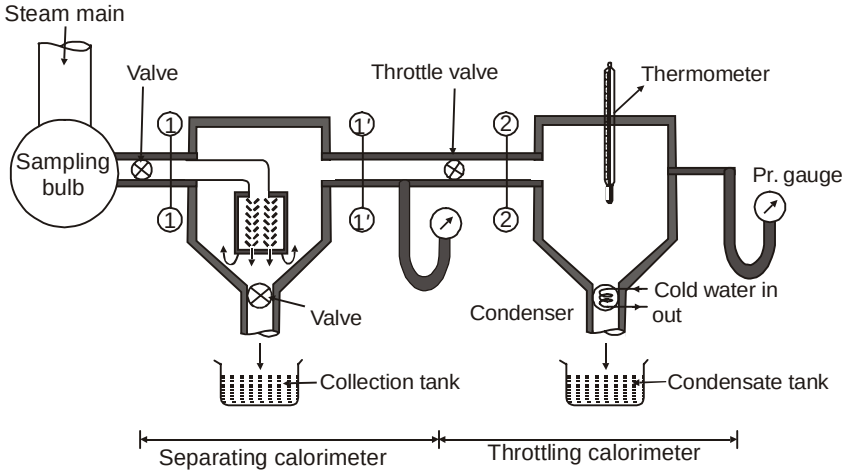


Fig. 6.18 *Separating and throttling calorimeter*

Mass of vapour at 1–1 shall be similar to mass of vapour entering at 2–2.

Mass of vapour at 2–2 = $x_2 \times m_2$

Hence, dryness fraction at 1–1, $x_1 = \dfrac{x_2 \cdot m_2}{m_{f1} + m_2}$

Separating and throttling processes occurring are also shown on *h*–*s* diagram in Fig. 6.19.

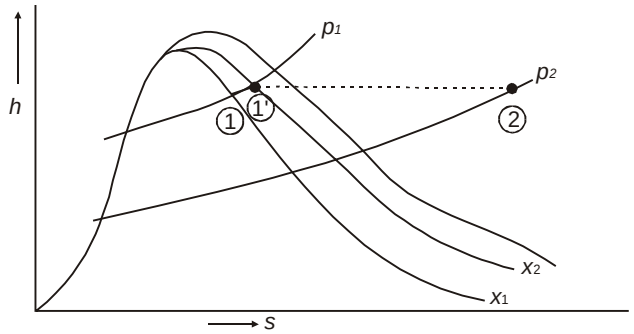


Fig. 6.19 *Separating and throttling together on h–s diagram*

(iv) ***Electrical Calorimeter:*** In electrical calorimeter too the principle employed is similar to that of throttling calorimeter. Here also wet mixture is brought to the superheated state by heating and not by throttling. For known amount of heat added and the final enthalpy for superheated steam being known, one can find out the initial enthalpy.

For mass *m* of mixture, heat Q_{add} added by heater, and the enthalpies before and after heating being h_1, h_2, steady flow energy equation may be written as; $mh_1 + Q_{add} = mh_2$.

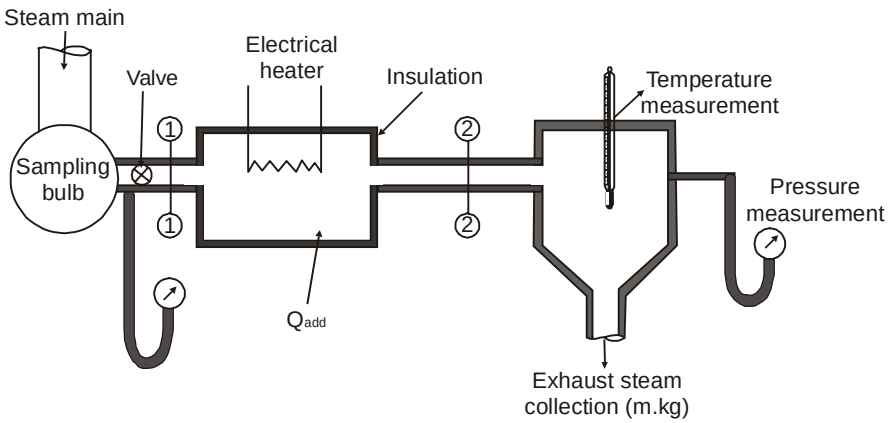


Fig. 6.20 *Electrical calorimeter*

Here, for electrical heater Q_{add} = V.I, where V and I are voltage and current.

h_2 is known, as mixture is brought to superheated state and pressure and temperature measured, locate enthalpy from superheated steam table, also $h_1 = h_{f \text{ at } p1} + x \,.\, h_{fg \text{ at } p1}$, here $h_{f \text{ at } p1}$ & $h_{fg \text{ at } p1}$ can be seen from steam table.

Now h_1 being known, for known *m*, h_2, Q_{add}, dryness fraction '*x*' can be easily obtained.

EXAMPLES

1. Derive the expressions for the following :

(a) Work of evaporation or external work of evaporation
(b) True latent heat
(c) Internal energy of steam
(d) Entropy of water
(e) Entropy of evaporation
(f) Entropy of wet steam
(g) Entropy of superheated steam

Solution:

(*a*) *Work of evaporation*: This is the work done due to evaporation of water to steam as phase transformation from water to steam is accompanied by increase in volume. Work of evaporation can be estimated as; = $p\ (v_g - v_f)$, for unit mass

For very low pressures of steam generation, where $v_f <<< v_g$.

Work of evaporation = $p \cdot v_g$

Work of evaporation for wet steam with dryness fraction 'x'

$= p \cdot x \cdot v_g$.

(*b*) *True latent heat*: Latent heat causing the phase transformation from water to steam is accompanied with change in volume as well. Therefore, latent heat shall have two components i.e. (i) true latent heat causing phase change and (ii) work of evaporation due to volume increase. Mathematically, for unit mass, True latent heat = $h_{fg} - p(v_g - v_f)$.

(*c*) *Internal energy of steam*: For a given mass of steam the total heat energy with steam can be said to comprise of

(*i*) Sensible heat
(*ii*) True latent heat
(*iii*) Work of evaporation

Out of above three the third component gets consumed in doing work. The internal energy of steam shall consist of first two components. For the enthalpy 'h' of steam,

(for unit mass) Internal energy, $u = h - p(v_g - v_f)$

neglecting v_f for low pressures,

$$u = h - p \cdot v_g.$$

for wet steam $u = h - p \cdot x v_g$

for super heated steam,

$$h = h_g + c_{p\ \text{superheat}}\ (T_{\text{superheat}} - T_{\text{sat}})$$

Hence

$$u = \{h_g + c_{p\ \text{superheat}} \cdot (T_{\text{superheat}} - T_{\text{sat}})\} - pv_{\text{superheat}}$$

(*d*) *Entropy of water*: Entropy change of water as discussed in article 6.5 can be given as :

$$\int_1^2 ds = \int_{T_1}^{T_2} c_{p\ \text{water}} \frac{dT}{T}$$

for constant specific heat of water

$$(s_2 - s_1) = c_{p\ \text{water}} \ln \left(\frac{T_2}{T_1} \right)$$

Absolute entropy may be given in reference to absolute zero temperature

$$s = c_{p\ \text{water}} \cdot \ln\left(\frac{T}{273.15}\right)$$

(*e*) *Entropy of evaporation*: During evaporation heat absorbed is equal to latent heat of evaporation. Therefore, for unit mass

$$s_{\text{evaporation}} = \frac{h_{fg}}{T_{\text{sat}}}$$

During incomplete evaporation (for wet steam)

$$s_{\text{evaporation}} = \frac{x \cdot h_{fg}}{T_{\text{sat}}}$$

(*f*) *Entropy of wet steam*:

Entropy of wet steam = Entropy of water + Entropy of evaporation

For unit mass,

$$s_{\text{wet}} = c_{p\ \text{water}} \ln \frac{T_2}{T_1} + x \cdot \frac{h_{fg}}{T_2}$$

(*g*) *Entropy of super heated steam*: Entropy change during constant pressure heating for superheating unit mass of the steam.

$$= c_{p\ \text{steam}} \ln\left(\frac{T_{\text{sup}}}{T_{\text{sat}}}\right)$$

Total entropy of super heated steam, starting with water at temperature T_1.

$$s_{\text{superheat}} = c_{p\ \text{water}} \ln\left(\frac{T_{\text{sat}}}{T_1}\right) + \frac{h_{fg}}{T_{\text{sat}}} + c_{p\ \text{steam}} \cdot \ln\left(\frac{T_{\text{super heat}}}{T_{\text{sat}}}\right).$$

2. Throttling calorimeter has steam entering to it at 10 MPa and coming out of it at 0.05 MPa and 100°C. Determine dryness fraction of steam.

Solution:

During throttling, $h_1 = h_2$

At state 2, enthalpy can be seen for superheated steam at 0.05 MPa and 100°C.

Thus, $h_2 = 2682.5$ kJ/kg

At state 1, before throttling

$$h_{f\ \text{at10MPa}} = 1407.56 \text{ kJ/kg}$$

$$h_{fg\ \text{at10MPa}} = 1317.1 \text{ kJ/kg}$$

$$h_1 = h_{f\ \text{at10MPa}} + x_1\, h_{fg\ \text{at10MPa}} = h_2$$

$$2682.5 = 1407.56 + (x_1 \cdot 1317.1)$$

$$x_1 = 0.968$$

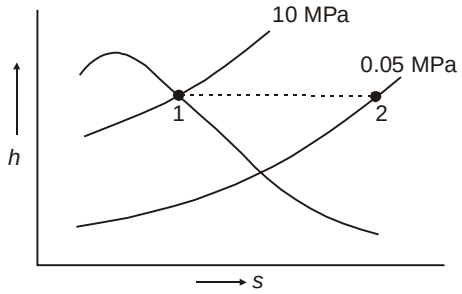


Fig. 6.21

Dryness fraction is **0.968.** **Ans.**

3. Determine internal energy of steam if its enthalpy, pressure and specific volumes are 2848 kJ/kg, 12 MPa and 0.017 m^3/kg.

Solution:

Internal energy, $u = h - pv$

$= (2848 - 12 \times 10^3 \times 0.017)$ kJ/kg

$= 2644$ kJ/kg

Internal energy = **2644 kJ/kg** **Ans.**

4. Determine entropy of 5 kg of steam at 2 MPa and 300°C. Take specific heat of super heated steam as 2.1 kJ/kg.K.

Solution:

Steam state 2 MPa and 300°C lies in superheated region as saturation temperature at 2 MPa is 212.42°C and $h_{fg} = 1890.7$ kJ/kg.

Entropy of unit mass of superheated steam with reference to absolute zero.

$$= c_{p\ \text{water}} \ln\left(\frac{T_{\text{sat}}}{273.15}\right) + \frac{h_{fg,2\text{MPa}}}{T_{\text{sat}}} + c_{p\ \text{superheat}} \ln\frac{T_{\text{super heat}}}{T_{\text{sat}}}$$

Substituting values

$$= 4.18 \ln\left(\frac{485.57}{273.15}\right) + \left(\frac{1890.7}{485.57}\right) + \left[2.1 \ln\left(\frac{573.15}{485.57}\right)\right]$$

$= 6.646$ kJ/kg.K.

Entropy of 5 kg of steam = 33.23 kJ/K

Entropy of steam = **33.23 kJ/K** **Ans.**

5. Water in a pond boils at 110°C at certain depth in water. At what temperature the water shall boil if we intend to boil it at 50 cm depth from above mentioned level.

Solution:

Boiling point = 110°C, pressure at which it boils = 143.27 kPa (from steam table, sat. pressure for 110°C)

At further depth of 50 cm the pressure

$= 143.27 - ((10^3 \times 9.81 \times 0.50) \times 10^{-3})$

$= 138.365$ kPa.

Boiling point at this depth = $T_{\text{sat, } 138.365}$

From steam table this temperature = 108.866 = 108.87°C

Boiling point = **108.87°C** **Ans.**

6. Water-vapour mixture at 100°C is contained in a rigid vessel of 0.5 m^3 capacity. Water is now heated till it reaches critical state. What was the mass and volume of water initially?

Solution:

In a rigid vessel it can be treated as constant volume process.

$$v_1 = v_2$$

Since final state is given to be critical state, then specific volume at critical point,

$$v_2 = 0.003155 \text{ m}^3/\text{kg}$$

At 100°C saturation temperature, from steam table

$$v_f = 0.001044 \text{ m}^3/\text{kg}, \; v_g = 1.6729 \text{ m}^3/\text{kg}$$

Thus for initial quality being x_1

$$v_1 = v_{f\,100°C} + x_1 \cdot v_{fg100°C}$$

or $$0.003155 = 0.001044 + x_1 \times 1.671856$$

$$x_1 = 0.0012627$$

$$\text{Mass of water initially} = \text{Total mass} \cdot (1 - x_1)$$

$$\text{Total mass of fluid} = \frac{V}{v_2} = \frac{0.5}{0.003155} = 158.48 \text{ kg}$$

$$\text{Mass of water} = 158.48 \text{ kg}$$

$$\text{Volume of water} = 158.48 \times 0.001044$$

$$= 0.1652 \text{ m}^3$$

Mass of water = **158.48 kg,**
Volume of water = **0.1652 m^3** **Ans.**

7. Determine slope of an isobar at 2 MPa and 500°C on mollier diagram.

Solution:

On mollier diagram ($h - s$ diagram) the slope of isobaric line may be given as

$$\left(\frac{dh}{ds}\right)_{p = \text{const}} = \text{Slope of isobar}$$

From Ist and IInd law combined;

$$Tds = dh - vdp$$

for constant pressure

$$\left(\frac{dh}{ds}\right)_{p = \text{const}} = T$$

Here temperature $T = 773.15$ K

$$\text{hence slope} = \left(\frac{dh}{ds}\right)_{p = \text{const}} = 773.15$$

Slope = **773.15** **Ans.**

8. Determine enthalpy, specific volume, entropy for mixture of 10% quality at 0.15 MPa.

Solution:

Given, $x = 0.10$

At 0.15 MPa, from steam table;

$h_f = 467.11$ kJ/kg, $h_g = 2693.6$ kJ/kg

$v_f = 0.001053$ m^3/kg, $v_g = 1.1593$ m^3/kg

$s_f = 1.4336$ kJ/kg.K, $s_g = 7.2233$ kJ/kg.K

Enthalpy at $x = 0.10$

$h = h_f + x.h_{fg} = 467.11 + \{0.10 \times (2693.6 - 467.11)\}$

$h = 689.759$ kJ/kg

Specific volume, $v = v_f + x.v_{fg}$

$= 0.001053 + \{0.10 \times (1.1593 - 0.001053)\}$

$v = 0.116877$ m^3/kg

Entropy, $s = s_f + x.s_{fg}$

$= 1.4336 + \{0.10 \times (7.2233 - 1.4336)\}$

$s = 2.01257$ kJ/kg.K

$h =$ **689.759 kJ/kg**
$v =$ **0.116877 m^3/kg**
$s =$ **2.01257 kJ/kg.K** **Ans.**

9. In a piston-cylinder arrangement the steam at 1.0 MPa, 80% dryness fraction, and 0.05 m^3 volume is heated to increase its volume to 0.2 m^3. Determine the heat added.

Solution:

Given;

Initial states, 1: $P_1 = 1.0$ MPa, $V_1 = 0.05$ m^3, $x_1 = 0.80$

Final state, 2: $V_2 = 0.2$ m^3, $P_2 = 1$ MPa

Work done during constant pressure process, $W = P_1(V_2 - V_1)$

$W = 1000\ (0.2 - 0.05)$

$W = 150$ kJ

$$\text{Mass of steam} = \frac{V_1}{v_1}$$

From steam table at P_1; $v_f = 0.001127$ m^3/kg

$v_g = 0.19444$ m^3/kg

$u_f = 761.68$ kJ/kg

$u_{fg} = 1822$ kJ/kg

so $v_1 = v_f + x_1\, v_{fg}$

$v_1 = 0.15578$ m^3/kg

$$\text{Hence, mass of steam} = \frac{0.05}{0.15578} = 0.32097 \text{ kg}$$

$$\text{Specific volume at final state} = \frac{V_2}{\text{mass of steam}}$$

$v_2 = 0.62311$ m^3/kg

Corresponding to this specific volume the final state is to be located for getting the internal energy at final state at 1 MPa

$$v_2 > v_{g1\ \text{MPa}}$$

hence state lies in superheated region, from the steam table by interpolation we get temperature as;

State lies between temperature of 1000°C and 1100°C

so exact temperature at final state

$$= 1000 + \frac{100 \times (0.62311 - 0.5871)}{(0.6335 - 0.5871)}$$

$$= 1077.61°C$$

Thus internal energy at final state, 1 MPa, 1077.61°C;

$$u_2 = 4209.6 \text{ kJ/kg}$$

Internal energy at initial state,

$$u_1 = u_f + x_1.u_{fg} = 761.68 + 0.8 \times 1822$$

$$u_1 = 2219.28 \text{ kJ/kg}$$

From first law of thermodynamics;

$$Q - W = \Delta U$$

$$Q = (U_2 - U_1) + W$$

$$= m(u_2 - u_1) + W$$

$$= 0.32097\ (4209.6 - 2219.28) + 150$$

$$Q = 788.83 \text{ kJ}$$

Heat added = **788.83 kJ** **Ans.**

10. Steam at 800 kPa and 200°C in a rigid vessel is to be condensed by cooling. Determine pressure and temperature corresponding to condensation.

Solution:

Here steam is kept in rigid vessel, therefore its' specific volume shall remain constant.

Specific volume at initial state, 800 kPa, 200°C, v_1,

It is superheated steam as T_{sat} = 170.43°C at 800 kPa.

From superheated steam table; $v_1 = 0.2404 \text{ m}^3/\text{kg}$.

At the begining of condensation, specific volume = $0.2404 \text{ m}^3/\text{kg}$.

$$v_2 = 0.2404 \text{ m}^3/\text{kg}$$

This v_2 shall be specific volume corresponding to saturated vapour state for condensation.

Thus $v_2 = v_g = 0.2404 \text{ m}^3/\text{kg}$

Looking into steam table $v_g = 0.2404 \text{ m}^3/\text{kg}$ shall lie between temperatures 175°C ($v_g = 0.2168 \text{ m}^3/\text{kg}$) and 170°C ($v_g = 0.2428 \text{ m}^3/\text{kg}$) and pressures 892 kPa (175°C) and 791.7 kPa (170°C).

By interpolation, temperature at begining of condensation

$$T_2 = 175 - \frac{(175 - 170) \times (0.2404 - 0.2168)}{(0.2428 - 0.2168)}$$

$$T_2 = 170.46°C$$

$$\text{Similarly, pressure, } p_2 = 892 - \frac{(892 - 791.7) \times (0.2404 - 0.2168)}{(0.2428 - 0.2168)}$$

$$p_2 = 800.96 \text{ kPa}$$

Pressure and temperature at condensation = **800.96 kPa** and **170.46°C** **Ans.**

11. Feed water pump is used for pumping water from 30°C to a pressure of 200 kPa. Determine change in enthalpy assuming water to be incompressible and pumping to be isentropic process.

Solution:

From Ist and IInd law;

$$Tds = dh - vdp$$

for isentropic process, $ds = 0$

hence $$dh = vdp$$

i.e. $$(h_2 - h_1) = v_1 (p_2 - p_1)$$

Corresponding to initial state of saturated liquid at 30°C; from steam table;

$$p_1 = 4.25 \text{ kPa}, \; v_f = v_1 = 0.001004 \text{ m}^3/\text{kg}$$

Therefore

$$(h_2 - h_1) = 0.001004 \; (200 - 4.25)$$

$$(h_2 - h_1) = 0.197 \text{ kJ/kg}$$

Enthalpy change = **0.197 kJ/kg** **Ans.**

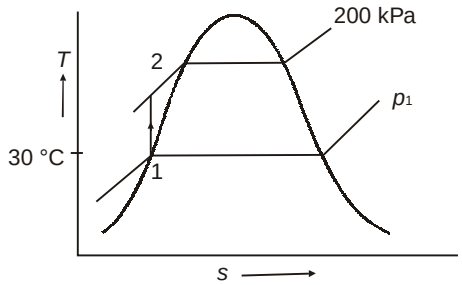


Fig. 6.22

12. A rigid vessel contains liquid-vapour mixture in the ratio of 3:2 by volume. Determine quality of water vapour mixture and total mass of fluid in vessel if the volume of vessel is 2 m^3 and initial temperature is 150°C.

Solution:

From steam table at 150°C,

$$v_f = 0.001091 \text{ m}^3/\text{kg}, \; v_g = 0.3928 \text{ m}^3/\text{kg}$$

Volume occupied by water = 1.2 m^3

Volume of steam = 0.8 m^3

$$\text{Mass of water} \Rightarrow m_f = \frac{1.2}{0.001091} = 1099.91 \text{ kg}$$

$$\text{Mass of steam} \Rightarrow m_g = \frac{0.8}{0.3928} = 2.04 \text{ kg}$$

Total mass in tank = $m_f + m_g = 1103.99$ kg

$$\text{Quality or Dryness fraction, } x = \frac{2.04}{1103.99} = 0.001848$$

Mass = **1103.99 kg,** Quality = **0.001848** **Ans.**

13. Steam turbine expands steam reversibly and adiabatically from 4 MPa, 300°C to 50°C at turbine exit. Determine the work output per kg of steam.

Solution:

From SFEE on steam turbine;

$$W = (h_1 - h_2)$$

Initially at 4 MPa, 300°C the steam is super heated so enthalpy from superheated steam table or Mollier diagram.

$$h_1 = 2886.2 \text{ kJ/kg}, \; s_1 = 6.2285 \text{ kJ/kg.K}$$

Reversible adiabatic expansion process has entropy remaining constant. On Mollier diagram the state 2 can be simply located at intersection of constant temperature line for 50°C and isentropic expansion line.

Else from steam tables at 50°C saturation temperature;

$$h_f = 209.33 \text{ kJ/kg}, \; s_f = 0.7038 \text{ kJ/kg.K}$$

$$h_{fg} = 2382.7 \text{ kJ/kg}, \; s_{fg} = 7.3725 \text{ kJ/kg.K}$$

Here $s_1 = s_2$, let dryness fraction at 2 be x_2,

$$6.2285 = 0.7038 + x_2 \times 7.3725$$

$$x_2 = 0.7494$$

Hence enthalpy at state 2,

$$h_2 = h_f + x_2.h_{fg} = 209.33 + 0.7494 \times 2382.7$$

$$h_2 = 1994.93 \text{ kJ/kg}$$

$$\text{Steam turbine work} = (2886.2 - 1994.93)$$

$$= 891.27 \text{ kJ/kg}$$

Turbine output = **891.27 kJ/kg** **Ans.**

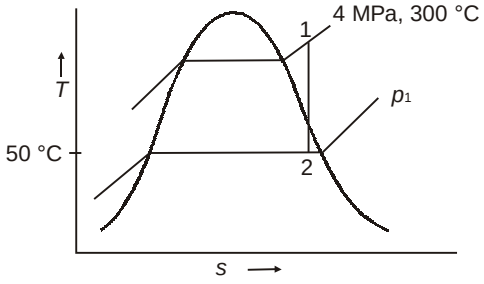


Fig. 6.23

14. In a closed vessel the 100 kg of steam at 100 kPa, 0.5 dry is to be brought to a pressure of 1000 kPa inside vessel. Determine the mass of dry saturated steam admitted at 2000 kPa for raising pressure. Also determine the final quality.

Solution:

It is a constant volume process.

Volume of vessel V = (Mass of vapour) × (Specific volume of vapour)

Initial specific volume, v_1

$$v_1 = v_{f,\ 100\text{kPa}} + x_1.v_{fg,\ 100\text{kPa}}$$

at 100 kPa from steam table;

$$h_{f,\ 100\ \text{kPa}} = 417.46 \text{ kJ/kg}$$

$u_{f,\ 100\ kPa} = 417.36$ kJ/kg
$v_{f,\ 100\ kPa} = 0.001043$ m^3/kg
$h_{fg,\ 100\ kPa} = 2258$ kJ/kg,
$u_{fg,\ 100\ kPa} = 2088.7$ kJ/kg,
$v_{g,\ 100\ kPa} = 1.6940$ m^3/kg

given $x_1 = 0.5$, $v_1 = 0.8475$ m^3/kg

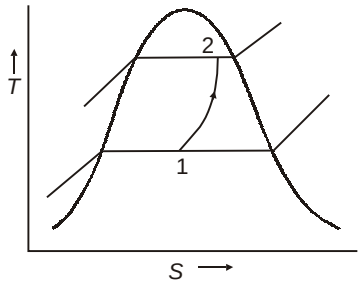


Fig. 6.24

Enthalpy at 1, $h_1 = 417.46 + 0.5 \times 2258 = 1546.46$ kJ/kg

Thus, volume of vessel $= (100 \times 0.5) \times (0.8475)$

$V = 42.375$ m^3

Internal energy in the beginning $= U_1 = m_1 \times u_1$

$= 100\ (417.36 + 0.5 \times 2088.7)$

$U_1 = 146171$ kJ

Let the mass of dry steam added be 'm', Final specific volume inside vessel, v_2

$$v_2 = v_{f,\ 1000\ kPa} + x_2.v_{fg,\ 1000\ kPa}$$

At 2000 kPa, from steam table,

$v_{g,\ 2000\ kPa} = 0.09963$ m^3/kg
$u_{g,\ 2000\ kPa} = 2600.3$ kJ/kg
$h_{g,\ 2000\ kPa} = 2799.5$ kJ/kg

Total mass inside vessel = Mass of steam at 2000 kPa + Mass of mixture at 100 kPa

$$\frac{V}{v_2} = \frac{V}{v_{g\ 2000\ kPa}} + \frac{V}{v_1}$$

$$\frac{42.375}{v_2} = \frac{42.375}{0.09963} + \frac{42.375}{0.8475}$$

$$v_2 = 0.089149$$

substituting $v_2 = v_{f,\ 1000\ kPa} + x_2.v_{fg,\ 1000\ kPa}$

At 1000 kPa from steam table,

$h_{f,\ 1000\ kPa} = 762.81$ kJ/kg,
$h_{fg,\ 1000\ kPa} = 2015.3$ kJ/kg $v_{f,\ 1000\ kPa} = 0.001127$ m^3/kg
$v_{g,\ 1000\ kPa} = 0.19444$ m^3/kg

it gives $x_2 = 0.455$

For adiabatic mixing

$$(100 + m) \cdot h_2 = 100 \times h_1 + m \times h_{g,\ 2000\ \text{kPa}}$$

$$(100 + m) \cdot (762.81 + 0.455 \times 2015.3) = \{100 \times (1546.46)\} + \{m \times 2799.5\}$$

It gives upon solving;

$$(100 + m)\ (1679.77) = 154646 + (2799.5)m$$

$$m = 11.91 \text{ kg}$$

Mass of dry steam at 2000 kPa to be added = **11.91 kg**
Quality of final mixture = **0.455** **Ans.**

15. In a condenser the following observations were made,

Recorded condenser vacuum = 71.5 cm of Mercury
Barometer reading = 76.8 cm of Mercury
Temperature of condensation = 35°C
Temperature of hot well = 27.6°C
Mass of condensate per hour = 1930 kg
Mass of cooling water per hour = 62000 kg
Inlet temperature = 8.51°C
Outlet temperature 26.24°C
Determine the state of steam entering condenser.

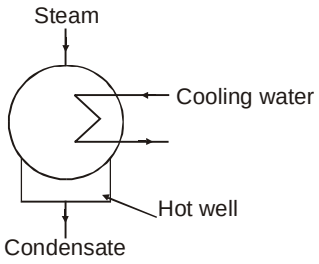


Fig. 6.25

Solution:

From Dalton's law of partial pressure the total pressure inside condenser will be sum of partial pressures of vapour and liquid inside.

$$\text{Condenser pressure} = \left\{\frac{76.8 - 71.5}{(73.55)} \times 101.325\right\} \text{kPa}$$

$$= 7.30 \text{ kPa}$$

Partial pressure of steam corresponding to 35°C from steam table;

$$= 5.628 \text{ kPa}$$

Enthalpy corresponding to 35°C from steam table,

$$h_f = 146.68 \text{ kJ/kg}$$

$$h_{fg} = 2418.6 \text{ kJ/kg}$$

Let quality of steam entering be 'x'.

From energy balance;

$$m_w(26.24 - 8.51) \times 4.18 = 1930\ (146.68 + x \times 2418.6 - 4.18 \times 27.6)$$

or $62000\ (17.73 \times 4.18) = 1930\ (31.31 + x \times 2418.6)$

which gives $x = 0.97$

Dryness fraction of steam entering = **0.97** **Ans.**

16. In a vertical vessel of circular cross section having diameter of 20 cm water is filled upto a depth of 2 cm at a temperature of 150°C. A tight fitting frictionless piston is kept over the water surface and a force of 10 kN is externally applied upon the piston. If 600 kJ of heat is supplied to water determine the dryness fraction of resulting steam and change in internal energy. Also find the work done.

Solution:

Heating of water in vessel as described above is a constant pressure heating. Pressure at which process occurs

$$= \frac{\text{Force}}{\text{area}} + \text{atmospheric pressure}$$

$$= \left(\frac{10}{\frac{\pi}{4}(0.2)^2} + 101.3 \right) \text{kPa}$$

$$= 419.61 \text{ kPa}$$

Volume of water contained $= \frac{\pi}{4} \times (0.2)^2 \times (0.02)$

Volume $= 6.28 \times 10^{-4}\ m^3$

Mass of water $= 6.28 \times 10^{-4} \times 10^3 = 0.628$ kg

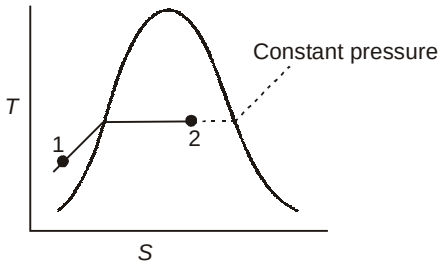


Fig. 6.26

Heat supplied shall cause sensible heating and latent heating.

Hence, Enthalpy change = Heat supplied

$$600 = \{(h_{f \text{ at } 419.6 \text{ kPa}} + x.h_{fg \text{ at } 419.6 \text{ kPa}}) - (4.18 \times 150)\} \times 0.628$$

$$600 = \{(612.1 + x.2128.7) - 627\} \times 0.628$$

Dryness fraction $x = 0.456$

Dryness fraction of steam produced = **0.456** **Ans.**

Internal energy of water, initially

$$U_1 = mh_1 - p_1V_1$$

$$= (0.628 \times 4.18 \times 150) - (419.61 \times 6.28 \times 10^{-4})$$

$$U_1 = 393.5 \text{ kJ}$$

Finally, internal energy of wet steam

$$U_2 = mh_2 - p_2V_2$$

Here $V_2 = m \cdot x \cdot v_{g \text{ at } 419.61 \text{ kPa}}$

$= 0.456 \times 0.4435 \times 0.628$

$= 0.127 \text{ m}^3$

Hence $U_2 = (0.628 \times 1582.8) - (419.61 \times 0.127)$

$U_2 = 940.71 \text{ kJ}$

Change in internal energy = $U_2 - U_1$.

Change in internal energy = **547.21 kJ Ans.**

Work done = $P \cdot (V_2 - V_1)$

$= 419.61 \times (0.127 - 6.28 \times 10^{-4})$

Work done = 53.03 kJ

Work done = **53.03 kJ** **Ans.**

17. In a separating and throttling calorimeter the total quantity of steam passed was 40 kg and 2.2 kg of water was collected from separator. Steam pressure before throttling was 1.47 MPa and temperature and pressure after throttling are 120°C and 107.88 kPa. Determine the dryness fraction of steam before entering to calorimeter. Specific heat of superheated steam may be considered as 2.09 kJ/kg.K.

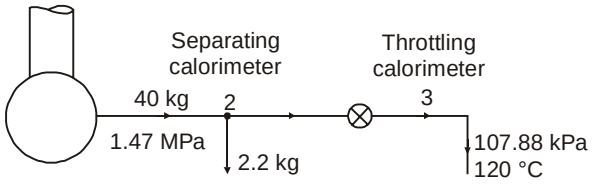


Fig. 6.27

Solution:

Consider throttling calorimeter alone,

Degree of superheat = 120 – 101.8 = 18.2°C

Enthalpy of superheated steam = (2673.95 + (18.2 × 2.09))

= 2711.988 kJ/kg

Enthalpy before throttling = Enthalpy after throttling

$840.513 + x_2.1951.02 = 2711.988$

or $x_2 = 0.9592$

For separating calorimeter alone, dryness fraction, $x_1 = \dfrac{40 - 2.2}{40}$

$x_1 = 0.945$

Overall dryness fraction = $(x_1.x_2) = (0.945 \times 0.9592)$

$= 0.9064$

Dryness fraction : **0.9064** **Ans.**

18. A rigid vessel is divided into two parts A and B by means of frictionless, perfectly conducting piston. Initially, part A contains 0.4 m^3 of air (ideal gas) at 10 bar pressure and part B contains 0.4 m^3 of wet steam at 10 bar. Heat is now added to both parts until all the water in part B is evaporated. At this condition the pressure in part B is 15 bar. Determine the initial quality of steam in part B and the total amount of heat added to both parts. *[U.P.S.C. 1995]*

Solution:

Here heat addition to part *B* shall cause evaporation of water and subsequently the rise in pressure. Final, part *B* has dry steam at 15 bar. In order to have equilibrium the part *A* shall also have pressure of 15 bar. Thus, heat added

$$Q = V(P_2 - P_1)$$
$$= 0.4(15 - 10) \times 10^2$$
$$Q = 200 \text{ kJ}$$

Final enthalpy of dry steam at 15 bar, $h_2 = h_{g \text{ at } 15 \text{ bar}}$

$$h_2 = 2792.2 \text{ kJ/kg}$$

Let initial dryness fraction be x_1. Initial enthalpy,

$$h_1 = h_{f \text{ at } 10\text{bar}} + x_1 \cdot h_{fg \text{ at } 10 \text{ bar}}$$
$$h_1 = 762.83 + x_1 \cdot 2015.3$$

Heat balance yields,

$$h_1 + Q = h_2$$
$$(762.83 + x_1 \cdot 2015.3) + 200 = 2792.2$$
$$x_1 = 0.907$$

Heat added = **200 kJ**
Initial quality = **0.907** **Ans.**

19. A piston-cylinder contains 3 kg of wet steam at 1.4 bar. The initial volume is 2.25 m^3. The steam is heated until its' temperature reaches 400°C. The piston is free to move up or down unless it reaches the stops at the top. When the piston is up against the stops the cylinder volume is 4.65 m^3. Determine the amounts of work and heat transfer to or from steam. *[U.P.S.C. 1994]*

Solution:

From steam table, specific volume of steam at 1.4 bar = 1.2455 m^3/kg

$$= v_{g \text{ at } 1.4 \text{ bar}}$$

Specific volume of wet steam in cylinder, $v_1 = \dfrac{2.25}{3} = 0.75 \text{ m}^3/\text{kg}$

Dryness fraction of initial steam, $x_1 = \dfrac{0.75}{1.2455} = 0.602$

Initial enthalpy of wet steam, $h_1 = h_{f \text{ at } 1.4 \text{ bar}} + x_1 \cdot h_{fg \text{ at } 1.4 \text{ bar}}$

$$= 457.99 + (0.602 \times 2232.3) \Rightarrow h_1 = 1801.83 \text{ kJ/kg}$$

At 400°C specific volume of steam, $v_2 = \dfrac{4.65}{3} = 1.55 \text{ m}^3/\text{kg}$

For specific volume of 1.55 m^3/kg at 400°C the pressure can be seen from the steam table. From superheated steam tables the specific volume of 1.55 m^3/kg lies between the pressure of 0.10 MPa (specific volume 3.103 m^3/kg at 400°C) and 0.20 MPa (specific volume 1.5493 m^3/kg at 400°C). Actual pressure can be obtained by interpolation;

$$P_2 = 0.10 + \left(\frac{0.20 - 0.10}{(1.5493 - 3.103)} \right) \times (1.55 - 3.103)$$

$$P_2 = 0.199 \text{ MPa} \approx 0.20 \text{ MPa}$$

Saturation pressure at 0.20 MPa = 120.23°C

Finally the degree of superheat = 400 – 120.23

= 279.77°C

Final enthalpy of steam at 0.20 MPa and 400°C, h_2= 3276.6 kJ/kg

Heat added during process = $m\ (h_2 - h_1)$

= 3 × (3276.6 – 1801.83)

ΔQ = 4424.31 kJ

Internal energy of initial wet steam, $u_1 = u_{f \text{ at } 1.4 \text{ bar}} + x_1.u_{fg \text{ at } 1.4 \text{ bar}}$

u_1 = 457.84 + (0.607 × 2059.34)

u_1 = 1707.86 kJ/kg

Internal energy of final state,

$u_2 = u_{\text{at } 0.2 \text{ MPa, } 400°C}$

u_2 = 2966.7 kJ/kg

Change in internal energy $\Rightarrow \Delta U = m(u_2 - u_1)$

= 3 × (2966.7 – 1707.86)

ΔU = 3776.52 kJ

From first law of thermodynamics,

Work done $\Delta W = \Delta Q - \Delta U$

= 4424.31 – 3776.52

Work done, ΔW = 647.79 kJ

Heat transfer = **4424.31 kJ**
Work transfer = **647.79 kJ** **Ans.**

20. *An insulated vessel is divided into two compartments connected by a valve. Initially, one compartment contains steam at 10 bar, 500°C, and the other is evacuated. The valve is opened and the steam is allowed to fill the entire volume, achieving a final pressure of 1 bar. Determine the final temperature, in °C, the percentage of the vessel volume initially occupied by steam and the amount of entropy produced, in kJ/kg. K.* *[U.P.S.C. 1993]*

Solution:

Here throttling process is occurring therefore enthalpy before and after expansion remains same. Let initial and final states be given by 1 and 2. Initial enthalpy, from steam table.

$h_{1 \text{ at } 10 \text{ bar and } 500°C}$ = 3478.5 kJ/kg

$s_{1 \text{ at } 10 \text{ bar and } 500°C}$ = 7.7622 kJ/kg.*K*

$v_{1 \text{ at } 10 \text{ bar and } 500°C}$ = 0.3541 m^3/kg

Finally pressure becomes 1 bar so the final enthalpy at this pressure (of 1 bar) is also 3478.5 kJ/kg which lies between superheat temperature of 400°C and 500°C at 1 bar. Let temperature be T_2,

$h_{\text{at 1 bar, } 400°C}$ = 3278.2 kJ/kg

$h_{\text{at 1 bar, } 500°C}$ = 3488.1 kJ/kg

$$h_2 = 3478.5 = h_{\text{at 1 bar, } 400°C} + \frac{(h_{\text{at 1 bar,}500°C} - h_{\text{at 1 bar,}400°C})}{(500 - 400)} (T_2 - 400)$$

$$3478.5 = 3278.2 + \left\{\frac{3488.1 - 3278.2}{100}\right\} (T_2 - 400)$$

$$T_2 = 495.43°C,$$

Final temperature = **495.43°C Ans.**

Entropy for final state,

$$s_2 = s_{\text{at 1 bar, 400°C}} + \frac{(s_{\text{at 1 bar,500°C}} - s_{\text{at 1 bar,400°C}})}{(500 - 400)} (495.43 - 400)$$

$$s_2 = 8.5435 + \left\{\frac{8.8342 - 8.5435}{100}\right\} \times (95.43)$$

$$s_2 = 8.8209 \text{ kJ/kg. K}$$

$$\text{Change in entropy, } \Delta s = 8.8209 - 7.7622$$

$$= 1.0587 \text{ kJ/kg . K}$$

Change in entropy = **1.0587 kJ/kg K Ans.**

$$\text{Final specific volume, } v_2 = v_{\text{at 1 bar, 400°C}} + \frac{(v_{\text{at 1 bar,500°C}} - v_{\text{at 1 bar,400°C}})}{(500 - 400)} \times (95.43)$$

$$= 3.103 + \left\{\frac{3.565 - 3.103}{100}\right\} \times 95.43$$

$$v_2 = 3.544 \text{ m}^3/\text{kg}$$

$$\text{Percentage volume occupied by steam} = \frac{0.3541}{3.544} \times 100 = 9.99\%$$

Percentage of vessel volume initially occupied by steam = **9.99% Ans.**

21. *Determine the maximum work per kg of steam entering the turbine and the irreversibility in a steam turbine receiving steam at 2.5 MPa, 350°C and rejecting steam at 20 kPa, 0.92 dry. During the expansion the one-quarter of initial steam is bled at 30 kPa, 200°C. Consider the heat loss during expansion as 10kJ/s and atmospheric temperature as 30°C.*

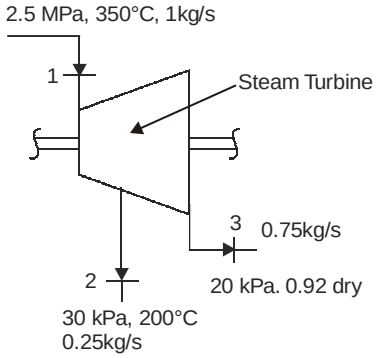


Fig. 6.28

Solution:

For the states shown on turbine in the figure, the steam table can be used to get following values:

At 2.5 MPa, 350°C,

$$h_1 = 3126.3 \text{ kJ/kg}, \; s_1 = 6.8403 \text{ kJ/kg.K}$$

At 30 MPa, 200°C,

$$h_2 = 2878.6 \text{ kJ/kg.}$$
$$s_2 = 8.5309 \text{ kJ/kg.K}$$

At 20 kPa, 0.92 dry,

$$h_f = 251.40\text{kJ/kg},\ h_g = 2609.7\text{kJ/kg}$$
$$s_f = 0.8320 \text{ kJ/kg.K},\ s_{fg} = 7.0766 \text{ kJ/kg.K}$$
$$\Rightarrow \quad h_3 = 251.40 + 0.92 \times (2609.7 - 251.40)$$
$$h_3 = 2421.04 \text{ kJ/kg}$$
$$s_3 = 0.8320 + (0.92 \times 7.0766)$$
$$s_3 = 7.3425 \text{ kJ/kg.K}$$

At atmospheric temperature,

$$h_{f \text{ at } 30°C} = h_0 = 125.79 \text{ kJ/kgs};\ s_{\text{fat } 30°C} = s_0 = 0.4369\text{kJ/kg.K}$$

Availability of steam entering turbine,

$$A_1 = (h_1 - h_0) - T_0(s_1 - s_0)$$
$$= (3126.3 - 125.79) - 303(6.8403 - 0.4369)$$
$$= 1060.28 \text{ kJ/kg}$$

Availability of steam leaving turbine at state 2 & 3,

$$A_2 = (h_2 - h_0) - T_0(s_2 - s_0) = (2878.6 - 125.79) - 303(8.5309 - 0.4369)$$
$$A_2 = 300.328 \text{ kJ/kg}$$
$$A_3 = (h_3 - h_0) - T_0(s_3 - s_0)$$
$$= (2421.04 - 125.79) - 303(7.3425 - 0.4369)$$
$$A_3 = 202.85 \text{ kJ/kg}$$

Maximum work per kg of steam entering turbine for

$$W_{\max} = 1 \times A_1 - m_2A_2 - m_3A_3$$
$$W_{\max} = A_1 - (0.25)A_2 - 0.75A_3$$
$$= 1060.28 - (0.25 \times 300.328) - (0.75 \times 202.85)$$
$$W_{\max} = \mathbf{833.06 \text{ kJ/kg}} \quad \textbf{Ans.}$$

Irreversibility, $I = T_0 (m_2 \times s_2 + m_3 \times s_3 - m_1s_1) - Q.$

$$= 303 (0.25 \times 8.5309 + 0.75 \times 7.3425 - 6.8403) - (-10)$$

Irreversibility, $I = \mathbf{252.19 \text{ kJ/s}}$ **Ans.**

22. *Determine the change in availability due to throttling of steam from 6 MPa and 400°C to 5 MPa when surroundings are at 100 kPa and 20°C. The changes in KE and PE may be considered negligible.*

Solution:

From steam tables

Initially at 6 MPa, 400°C, $h_1 = 3177.2$ kJ/kg

$$s_1 = 6.5408 \text{ kJ/kg.K}$$

After throttling at 5 Mpa,

$$h_2 = h_1; \text{ in view of throttling process.}$$

Hence at 5 MPa and

$$h_2 = 3177.2 \text{ kJ/kg}$$

Superheated Steam table gives,

at 5 MPa, $h_{\text{at } 350°C} = 3068.4$ kJ/kg

at 5 MPa, $h_{\text{at } 400°C} = 3195.7$ kJ/kg

Hence by interpolation at 5 MPa, enthalpy of 3177.2 kJ/kg will be at

$$T_2 = 350 + \frac{(400-350)\times(3177.2-3068.4)}{(3195.7-3068.4)}$$

$$T_2 = 392.7°C$$

After throttling steam will be at 5 MPa, 392.7°C.

By interpolation Entropy, $s_2 = 6.6172$ kJ/kg.K

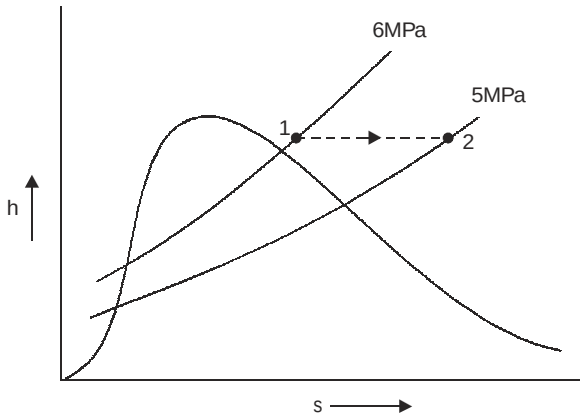


Fig. 6.29

At dead state of 20°C,

$$h_{f \text{ at } 20°C} = h_0 = 83.96 \text{ kJ/kg}$$

$$s_{f \text{ at } 20°C} = s_0 = 0.2966 \text{ kJ/kg}$$

Hence availability at state 1,

$$A_1 = (h_1 - h_0) - T_0 (s_1 - s_0) + \frac{1}{2}(C_1^2) + g(z_1 - z_0)$$

$$\Rightarrow \quad A_1 = (3177.2 - 83.96) - 293(6.5408 - 0.2966) + 0 + 0$$

$$A_1 = 1263.68 \text{ kJ/kg}$$

Availability of steam after throttling,

$$A_2 = (h_2 - h_0) - T_0 (s_2 - s_0) + (\frac{1}{2}C_2^2) + g(z_2 - z_0)$$

$$A_2 = (3177.2 - 83.96) - 293\ (6.6172 - 0.2966)$$

$$A_2 = 1241.30 \text{ kJ/kg}$$

Change in availability = $A_2 - A_1$

$$= (1241.30 - 1263.68)$$

$$= -22.5 \text{ kJ/kg}$$

= 22.5 kJ/kg, decrease. Ans.

23. *A parallel flow heat exchanger has hot water flowing at 95°C for heating cold water at 15°C to 45°C. Hot water flows at the rate of 800 gm/sec and the temperature of this hot water stream should not be less than 50°C at exit. Estimate the second law efficiency and rate of exergy destruction considering dead state temperature of 25°C.*

Solution:

Let hot stream and cold stream be shown to enter at section 1–1 and leave at 2 – 2

Given; $T_{H_1} = 95°C$, $T_{H_2} = 50°C$, $m_H = 800$ gm/s

$T_{C_1} = 15°C$, $T_{C_2} = 45°C$ $m_H = 0.8$kg/s

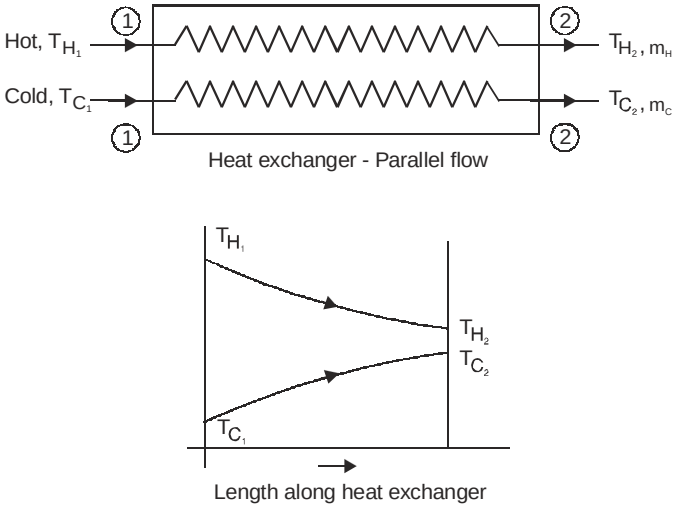


Fig. 6.30

For parallel flow heat exchanger as shown in figure;

$m_H \times C_{p,H}.(T_{H_1} - T_{H_2}) = m_c \times C_{p,c}.(T_{C_2} - T_{C_1})$

$\Rightarrow \quad 0.8 \times (95 - 50) = m_c\ (45 - 15)$

$\Rightarrow \quad m_c = 1.2$ kg/s

$$\text{Second law efficiency} = \frac{\text{(Rate exergy increase in cold stream)}}{\text{(Rate of exergy input to heat exchanger through hot stream)}}$$

The exergy entering (input) through hot water stream,

$$A_{H_1} = m_H\{(h_{H_1} - h_0) - T_0(s_{H_1} - s_0)\}$$

Using steam tables

At 25°C $h_0 = 104.89$ kJ/kg, $s_0 = 0.3674$ kJ/kg.K saturated liquid yields.

At 95°C, $h_{H_1} = h_{f \text{ at } 95°C} = 397.96$ kJ/kg

(for hot stream)

$s_{H_1} = s_{f \text{ at } 95°C} = 1.2500$ kJ/kg.K

At 50°C , $h_{H_2} = h_{f \text{ at } 50°C} = 209.33$ kJ/kg.K

(for hot stream)

$s_{H_2} = s_{f \text{ at } 50°C} = 0.7038$ kJ/kg.K

At 45°C, $h_{c_2} = h_{f \text{ at } 45°C} = 188.45$ kJ/kg.K

(for cold stream)

$s_{c_2} = s_{f \text{ at } 45°C} = 0.6387$ kJ/kg.K

At 15°C, $h_{c_1} = h_{f \text{ at } 15°C} = 62.99$ kJ/kg.K

(for cold stream)

$s_{c_1} = s_{f \text{ at } 15°C} = 0.2245$ kJ/kg.K

Rate of exergy input through hot water stream,

$$A_{H_1} = 0.8 \times \{(397.96 - 104.89) - 298(1.25 - 0.3674)\}$$
$$A_{H_1} = 24.04 \text{ kJ/s}$$

Rate of exergy increase in cold stream;

$$\Delta A_c = m_c\{(h_{c_2} - h_{c_1}) - T_0(s_{c_2} - s_{c_1})\}$$
$$\Delta A_c = 1.2\{(188.45 - 62.99) - 298(0.6387 - 0.2245)\}$$
$$\Delta A_c = 2.43 \text{ kJ/s}$$

$$\text{Second law efficiency} = \frac{\Delta A_c}{A_{H_1}} = \frac{2.43}{24.04} = 0.1011$$

or **10.11%** **Ans.**

Rate of exergy loss in hot stream,

$$\Delta A_H = m_H\{(h_{H_1} - h_{H_2}) - T_0(s_{H_1} - s_{H_2})\}$$
$$\Delta A_H = 0.8\{(397.96 - 209.33) - 298(1.25 - 0.7038)\}$$
$$\Delta A_H = 20.69 \text{ kJ/s}$$

Hence exergy destruction

$$= \Delta A_H - \Delta A_C$$
$$= 20.69 - 2.43$$
$$= \mathbf{18.26 \text{ kJ/s}} \quad \textbf{Ans.}$$

EXERCISE

6.1 Discuss generation of steam from ice at –5°C at 1 atm with the help of *T–S* and *P–V* diagrams.

6.2 What is meant by mollier diagram? Explain.

6.3 Write short notes on the following;
Sensible heating, Latent heating, Critical point, Triple point

6.4 Discuss different zones on *T–V* diagram for steam.

6.5 Derive the expression for enthalpy change during steam generation from feed water to superheated steam.

6.6 Discuss the throttling calorimeter for dryness fraction measurement.

6.7 Give a neat sketch of "separating and throttling calorimeter" for dryness fraction measurement.

6.8 Sketch the throttling and superheating processes on *h–s* and *T–S* diagrams.

6.9 Determine the final condition of steam if it is passed through a reducing valve which lowers the pressure from 2 MPa to 1 MPa. Assume initial state of steam to be 15% wet. [0.87]

6.10 Determine the final condition of steam, workdone, heat transferred and change in entropy if 0.5 kg of steam at 1 MPa and 0.8 dry is heated at constant pressure until its volume gets doubled.
[408.6°C, 77.5 kJ, 453.5 kJ, 0.895 kJ/K]

6.11 Determine the state of substance if 3346 kJ of heat is added to wet steam in a closed rigid vessel of $3m^3$ volume containing 5 kg of wet steam at a pressure of 200 kPa till its pressure become 304 kPa. [Dry]

6.12 Complete the following table from steam table.

	Pressure (MPa)	*Temperature (°C)*	*Enthalpy (kJ/kg)*	*Quality (x)*	*Specific volume (m^3/kg)*	*Entropy (kJ/kg.K)*
(*a*)	1	–	–	–	–	6.5865
(*b*)	–	250.4	–	0	–	–
(*c*)	10	–	–	0.8	–	–
(*d*)	20	700	–	–	–	–
(*e*)	15	800	–	–	–	–

(*a*) 179.9°C, 762.8 kJ/kg, 1, 0.1944 m^3/kg.

(*b*) 4 MPa, 1087.31 kJ/kg, 1.252 m^3/kg, 2.7964 kJ/kg.K

(*c*) 311.06°C, 2461.33 kJ/kg, 0.01442 m^3/kg, 5.1632 kJ/kg.K

(*d*) 3809 kJ/kg, 1, 0.02113 m^3/kg, 6.7993 kJ/kg.

(*e*) 4092.4 kJ/kg, 1, 0.0321 m^3/kg, 7.204 kJ/kg.K.

6.13 Determine the pressure in a rigid vessel and volume of rigid vessel if it contains 500 kg of water at 65°C.
[25 kPa, 0.51 m^3]

6.14 Estimate the change in volume of water and the total heat required for its' vaporization in a boiler producing saturated steam at 75 kPa. One kg feed water is supplied to boiler as saturated water.
[2.22 m^3, 2.28 MJ]

6.15 Determine enthalpy, entropy and specific volume for following cases

(*i*) Steam at 4 MPa and 80% wet. (*ii*) Steam at 10 MPa and 550°C.

(*iii*) Steam at 8 MPa and 295°C.

Also estimate the above properties using Mollier diagram and quantify the percentage variation

[1430.13 kJ/kg, 3.45 kJ/kg.K, 0.011 m^3/kg]

[3500.9 kJ/kg, 6.76 kJ/kg.K, 0.036 m^3/kg]

[2758 kJ/kg, 5.74 kJ/kg.K, 0.024 m^3/kg]

6.16 Determine the temperature of steam at 20 MPa if its specific volume is $0.0155m^3$/kg. [520°C]

6.17 Steam undergoes reversible adiabatic expansion in steam turbine from 500 kPa, 300°C to 50 kPa. Determine the work output per kg of steam turbine and quality of steam leaving steam turbine.
[357.64 kJ/kg, 0.98]

6.18 Steam flowing through two pipelines at 0.5 MPa are mixed together so as to result in a mixture flowing at 2.2 kg/s and mass flow ratio of two is 0.8. One stream has quality of 0.8. Determine the temperature of second stream so as to result in the final mixture having dryness fraction of 0.994.
[300°C approx.]

6.19 A steam turbine operates with isentropic efficiency of 90%. Turbine handles 6 kg/s of steam at 0.980 MPa and 200°C and leaves at 0.294 MPa. Determine the power developed in hp and change of entropy from inlet to exit. [1660 hp, 0.050 kJ/kg.K]

6.20 A boiler is fed with water velocity of 2m/s, 1.96 MPa, 100°C. Steam is produced at 400°C temperature and comes out with velocity of 50 m/s. Determine the rate at which heat should be supplied per kg of steam for above operation of boiler. [2824.8 kJ/kg]

6.21 A steam nozzle is supplied steam at 1 MPa, 200°C and 100 m/s. Expansion upto 0.3 MPa occurs in the nozzle. Assuming isentropic efficiency of nozzle to be 0.9 determine final steam velocity.

6.22 Combined separating and throttling calorimeter is used to determine quality of steam. Following observations are made;

Steam inlet pressure = 1.4 MPa

Pressure after throttling = 0.1 MPa

Temperature after throttling = 120°C

Water collected in separator = 0.45 kg

Steam condensed after throttling = 6.75 kg

Take specific heat of superheated steam = 2.1 kJ/kg.K

Also find the limiting quality of steam to be measured by above throttling calorimeter alone assuming that separating calorimeter is not there. [0.90, 0.94]

6.23 Steam at 400kPa, dryness fraction of 0.963 is isentropically compressed till it becomes dry saturated. This one kg steam is then heated isobarically till the initial volume is attained and subsequently steam is restored to initial state following isochoric cooling. Determine the net work and net heat interactions. Also show processes on T-s diagram. [29.93 kJ/kg, 29.93 kJ/kg]

6.24 Wet steam at 1 MPa, 0.125m^3 volume and enthalpy of 1814 kJ is throttled up to 0.7 bar pressure. Determine the final state of steam, initial mass and dryness fraction considering c_p = 2.1 kJ/kg.K

[101.57°C, 0.675kg, 0.953]

6.25 Steam initially at 5 bar, 0.6 dry is isochorically heated till its pressure becomes 10 bar. This 15 kg steam is expanded up to 3 bar following $pv^{1.3}$ = constant. Subsequently steam is cooled at constant pressure till its dryness fraction becomes half of that existed after second process. Determine the heat, work and entropy change in three processes.

[I process: 13.38 MJ, 0, 30,285 kJ/K.

II process: – 1.25MJ, 2.73 MJ, – 2.99kJ/K

III process: – 15.22 MJ, – 1.28 MJ, 37.4 kJ/K]

6.26 Determine the heat transfer and change in entropy in each process when steam at 20 bar, 250°C expands till it reaches 4 bar following $pv^{1.35}$ = constant and subsequently heated at constant volume till its pressure becomes 8 bar. [– 319.36 kJ/kg, & – 0.725 kJ/kg.K

764.95kJ/kg & 1.65 kJ/kg.K]

6.27 A closed vessel of 0.6 m^3 initially has steam at 15 bar, 250°C. Steam is blown off till pressure drops up to 4 bar. Subsequently vessel is cooled at constant pressure till it becomes 3 bar. Considering the expansion of gas to be isentropic during blow-off determine heat transferred during cooling process.

[– 620.38 kJ]

6.28 Determine the heat transferred when steam is taken out isobarically from a boiler tank till boiler is left with 80% water only. Volume of boiler tank is 10m^3 and initially it has equal volumes of steam and water at 10 bar. [1.75×10^6 kJ]

6.29 Determine the temperature of steam at 1.5 MPa having mass of 50 gm and stored in vessel with volume of 0.0076 m^3. Vessel is cooled until pressure in vessel becomes 1.1 MPa. Determine the temperature at which steam will be just dry saturated during cooling process. Also determine the final dryness fraction and total heat rejected. [250°C, 191.6°C, 0.85, 18.63 kJ]

6.30 Calculate the dryness fraction of steam after throttling when it is throttled from 1.4 MPa to 1 MPa & 423K.

Also determine the final condition of steam if this pressure drop takes place in closed vessel of 0.56 m^3 volume and heat is lost by conduction and radiation. [0.98, 0.298]

7

Availability and General Thermodynamic Relations

7.1 INTRODUCTION

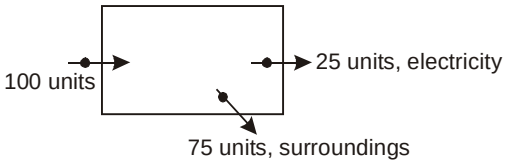
In the present civilization the use of energy resources has increased tremendously. Fast depleting fossil fuel reserves have inevitably gathered the attention of one and all to think and devise for optimum energy utilization. In order to optimally use energy, the efforts are required for identification and elimination of the sources of inefficiency during it's use, which obviously requires in depth study and analysis. A look into the laws of thermodynamics shows that the first law of thermodynamics bases upon the series of experiments done by James Joules, demonstrating the bidirectional numerical equivalence of converting work into heat while second law of thermodynamics exhibits a unidirectional equivalence between work and heat, i.e. for a given amount of heat the equivalent amount of work cannot be obtained whereas vice-a-versa may be there. Thus, the concept of quality of energy came into existence and work is considered as high grade of energy and heat as low grade of energy. Other forms of high grade energy are electrical energy, wind energy, tidal energy etc. and low grade energy may be heat from nuclear reactions, heat from combustion of fuel etc. Engineers have been using the first law of thermodynamics stating the energy conservation, therefore it could be concluded that energy can not be destroyed and exists with matters in all forms everywhere. It is now quite convincing to understand that the scarcity of energy resources and energy crisis is a paradox. Still in real life we find scarcity of energy, as in practice one is interested in the ability to feed, drive machines and occurrence of energy processes etc. Such discussions gave birth to the concept of 'available energy' and 'unavailable energy' or a concept of 'maximum work'.

This concept became very important in phenomenological thermodynamics, as it referred to the possibilities of performing work in real conditions. G. Gouy and A. Stodola pioneered in the studies pertaining to effect of ambient temperature upon the obtainable work and law of the loss of maximum work. The law of the loss of maximum work says that the work obtained is always less than the maximum obtainable work due to the irreversibility in thermal processes. Available energy concept came out of these propositions. Quality of energy, its convertibility into other forms and capability to perform work etc. are quantitatively defined using availability analysis. New term 'exergy' was introduced by Z. Rant in 1956 so as to differentiate it from energy. 'Exergy' analysis or 'availability' analysis has capability to identify and quantify the causes of thermodynamic imperfections in thermodynamic processes and thus indicate about the possibilities of improving the processes. It is preferred over energy analysis as energy analysis can not detect majority of thermodynamic imperfections. Such as the irreversible heat transfer, throttling and adiabatic combustion etc. do not have any energy loss but make the quality of energy inferior. Energy entering with fuel, electricity, flowing streams of matter and so on can be

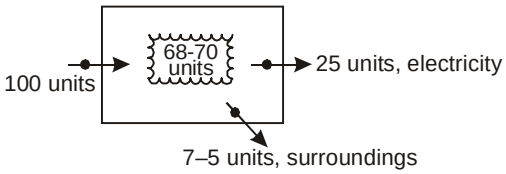
accounted for in products and by products. Energy cannot be destroyed. The idea that something can be destroyed is useful but should not be applied to 'energy', however it could be applied to another variable 'exergy'. Moreover, it is exergy and not energy that properly gauges the quality (utility), say one kJ of electricity generated by a power plant versus one kJ in plant cooling water stream. Electricity obviously has greater quality and the greater economic value.

These phenomenon can be evaluated by second law analysis easily. Exergy analysis could be integrated with principles of engineering economics to determine the potential for cost effective improvement of existing systems. Exergy and costing principles can also be used at initial design stage to develop systems that are 'optimized in annualized cost', 'sparing in use of fossil fuels' and 'environmentally friendly'.

Let us see a electricity generating power cycle as shown.



(*a*) Energy basis



(*b*) Exergy basis

Fig. 7.1 *Energy and exergy basis*

Here Fig. 7.1(*a*) represents energy basis indicating that out of 100 energy units entering with fuel, 25 energy units are obtained as electricity and the remaining 75 units are discharged to surroundings. On exergy basis it may also be considered that 100 units of exergy enter with fuel are 25 units of exergy exit along with the electricity. For remaining 75 units it is seen that 68-70 units of exergy are destroyed within the plant due to irreversibilities and only 5-7 units are discharged to surroundings. Here it is worth noting from exergy basis that out of 75 units of energy considered to be discharged to surroundings in energy basis actually only 5-7 units are discharged to surroundings and the majority 68-70 units are lost due to irreversibilities. The loss of energy due to irreversibilities can be minimized by minimizing or eliminating the causes of irreversibility. Exergy analysis thus shows that significant performance improvement can come only by identifying and correcting the sources of inefficiency within system as the discharge is a minor area of concern.

7.2 AVAILABILITY OR EXERGY

From earlier discussions, it is obvious that energy can be conveniently categorised as low grade energy and high grade energy. Also, the second law of thermodynamics prohibits the complete conversion of low grade energy into high grade energy. The portion of low grade energy that can be converted is called 'available energy' or 'exergy' or 'availability' and the portion of energy not available for conversion is called 'unavailable energy' or 'anergy'. Mathematically;

$$\text{Anergy} = \text{Energy} - \text{Exergy}.$$

"Exergy can be quantified as the amount of work obtainable by bringing some matter to the state of thermodynamic equilibrium with common components of natural surroundings through reversible processes, thus involving interaction only with above mentioned components of nature."

As per Moran and Sciubba (1994), the "exergy refers to the maximum theoretical work that can be extracted from a combined system comprising of 'system' and 'environment' as the system passes from a given state to equilibrium with the environment—that is, system changes its' state to the dead state at which combined system possesses energy but no exergy."

Rickert defined "exergy as the shaft work or electrical energy necessary to produce a material in its specified state from materials common in the environment in a reversible way, heat being exchanged only with environment."

Exergy is an extensive property whose value is fixed by the state of system once the environment has been specified. Exergy can also be represented on an intensive basis i.e. per unit mass or per mole basis. For all states of the system exergy shall be numerically greater than or equal to zero.

$$\text{Exergy} \geq 0$$

Exergy as defined above is a measure of departure of the state of a system from that of environment. For state at emperature T and environment at temperature T_0 the difference $(T \sim T_0)$ shall decide the value of exergy i.e. greater the difference, the greater shall be exergy value. This exergy can be of basically two types i.e. chemical exergy and thermomechanical exergy. Thermomechanical exergy can be further classified as physical, kinetic and potential exergy. Physical exergy is the work obtainable by taking the substance by reversible physical processes from its initial states pressure 'p' and temperature 'T' to the state determined by the temperature and pressure of environment. Kinetic exergy is equal to the kinetic energy, when the velocity is considered relative to the surface of the earth. Potential exergy is equal to the potential energy when it is evaluated with respect to the average level of the surface of the earth in the locality of the process under consideration.

Chemical exergy refers to the work that can be obtained by taking a substance state at environmental pressure and temperature to the state of thermodynamic equilibrium with environment and bring system to restricted dead state.

Thermomechanical exergy refers to the maximum theoretical work obtainable as system passes from some given state to the restricted dead state.

Thermal exergy is defined as the sum of 'physical exergy' and 'chemical exergy'.

Rant defined, "exergy as that part of energy which could be fully converted into any other kind of energy". Exergy is function of state parameters of matter under consideration and of the state parameters of common components of environment as exergy results from the possibility of interaction between matter under consideration and common components of environment.

'Environment' here refers to the region or part of surroundings whose intensive properties do not change significantly with the occurrence of processes under consideration, while 'surroundings' comprise of everything that is not included in system. Environment is considered to be large and homogeneous in terms of pressure and temperature. Environment is regarded free of irreversibilities. All significant irreversibilities are present in the system and its' immediate surroundings. Irreversibilities present within system are called 'internal irreversibilities' while 'external irreversibilities' are those present in its' immediate surroundings.

'*Dead state*' refers to the state at which system and the environment are at mechanical, thermal and chemical equilibrium. Thus neither there can be any spontaneous change within the system or within the environment, nor any spontaneous interaction between the two. Dead state being a limiting state is also called '*restricted dead state*'. At dead state the system is at same temperature and pressure as that of its' surroundings and shall have no kinetic energy or potential energy relative to surroundings. A system

shall thus have zero exergy (availability) at dead state and yield maximum possible work only when it follows a reversible process from its' state to the state of its' surroundings (dead state). Exergy or availability thus quantifies the maximum theoretical work available without violation of any laws of thermodynamics.

An engine operating with heat reservoir at T_1 and supplying Q_1 amount of heat and the environment temperature being T_0 shall give maximum amount of work when it operates between T_1 and T_0

$$\text{Maximum efficiency, } \eta_{max} = 1 - \frac{T_0}{T_1} = \eta_{rev}$$

$$\text{Maximum work } = Q_1 \,.\, \eta_{max} = \text{Availability}$$

Let us consider a 'combined system' and find work done. Combined system comprises of control system and environment. Contents of control system do not mix with environment or have any reaction with environment. Maximum work is available when control system changes its state from initial state to dead state.

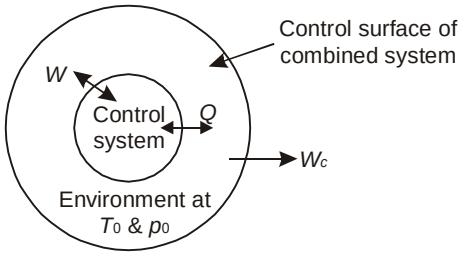


Fig. 7.2 *Combined system*

Let the control system have heat and work interaction Q and W with environment. Let us have only work interactions, W_c across the control surface of combined system. Let us use following nomenclature,

A = availability or exergy
E = energy, KE = kinetic energy,
PE = potential energy
p = pressure
Q = heat
S = entropy
T = temperature
U = internal energy
W = work

Subscript:
c = combined system
e = environment
s = control system
0 = dead state
i = initial state

Here for combined system

$\Delta V_c = 0$, where $V_c = V_s + V_e$, although V_s of system or V_e of environment may change but total volume of combined system shall remain constant.

Total work interaction of combined system can be given by total energy change of combined system.

$$W_c = -\Delta E_c$$

Energy change of combined system = Energy change in control system + Energy change in environment

$$\boxed{\Delta E_c = \Delta E_s + \Delta E_e}$$

Energy change in control system, ΔE_s = Energy of system at dead state i.e., final state – Energy of system at initial state

Energy of system at dead state,

as energy $E = U + KE + PE$

$E_{s,\text{ at dead state}} = U_0$, as $KE = PE = 0$

Energy of system at initial state, $E_{s,\text{initially}} = E_{s,i}$

$\Delta E_s = U_0 - E_{s,i}$

Energy change in environment, shall be due to heat interaction and the work associated with its' volume change (*pdv* work). For example expansion inside a piston cylinder arrangement shall have piston also displacing the volume of environment (*pdV* work is boundary work). Change in extensive properties, internal energy, entropy, volume of environment can be given by first law of thermodynamics.

$\Delta E_e = \Delta U_e = T_0 \Delta S_e - p_0 . \Delta V_e$

Hence, work interaction of combined system,

$W_c = - \{(U_0 - E_{s,i}) + (T_0 \Delta S_e - p_0 \Delta V_e)\}$

Also, we have seen that for combined system

$\Delta V_c = 0$

$\Delta V_s + \Delta V_e = 0$

or, $\Delta V_s = - \Delta V_e$

Here, ΔV_s = change in volume of system = (Final volume of system at dead state – Initial volume)

$\Delta V_s = V_0 - V_{s,i}$

Substituting in W_c,

$W_c = \{(E_{s,i} - U_0) + p_0(V_{s,i} - V_0) - T_0 \Delta S_e\}$

Total entropy change of combined system shall be due to irreversibilities within the combined system;

$\Delta S_c = \Delta S_s + \Delta S_e$

$\Delta S_c = (S_0 - S_{s,i}) + \Delta S_e$

or, $\Delta S_e = (- S_0 + S_{s,i}) + \Delta S_c$

Substituting ΔS_e in work, W_c,

$$\boxed{W_c = \{(E_{s,i} - U_0) + p_0(V_{s,i} - V_0) - T_0 ((- S_0 + S_{s,i}) + \Delta S_c)\}}$$

For the combined system total entropy change shall be either zero for reversible process or more than zero for irreversible process;

Mathematically, $\Delta S_c \geq 0$

Hence,

$W_c \leq \{(E_{s,i} - U_0) + p_0(V_{s,i} - V_0) - T_0(S_{s,i} - S_0)\}$

For reversible processes there will be no entropy generation, i.e. $\Delta S_c = 0$ and work shall be maximum only when the combined system is internally reversible from all respects.

Thus, $W_{c,\text{ max}} = \{(E_{s,i} - U_0) + p_0(V_{s,i} - V_0) - T_0(S_{s,i} - S_0)\}$

In general terms for any initial state of system, which is having all reversible processes.

$W_{c,\text{ max}} = \{(E - U_0) + p_0(V - V_0) - T_0(S - S_0)\}$

$$\boxed{\text{Availability or exergy, } A = W_{c,\text{ max}} = \{(E - U_0) + p_0(V - V_0) - T_0(S - S_0)\}}$$

Availability or exergy cannot be less than zero as the maximum work interaction can not be less than zero.

Also it can be given as,

$$W_c = A - T_0 \,.\, \Delta S_c$$

Above expression shows that some work done by combined system gets lost i.e. the irreversibilities causing entropy production keep work below its' maximum value.

Availability or exergy is not conserved like energy. Exergy gets destroyed by irreversibilities when the control system changes to dead state and no work is done by combined system as in case of spontaneous change.

Availability destruction is proportional to entropy generation due to irreversibilities in processes. Irreversibility can be given as the product of dead state temperature and entropy generation due to irreversible process.

$$I = T_0 \,.\, \Delta S_c \quad \text{and} \quad W_c = A - I \quad \text{and} \quad I = A - W_c$$

Above discussion indicates that the *maximum work shall be obtained when a process takes place in reversible manner.* But in fact almost all the processes in real life occur in irreversible manner, so some portion of energy is always unavailable. As irreversible processes are continuously increasing therefore unavailable energy is also gradually increasing. This phenomenon is also called *principle of degradation of energy or law of degradation of energy*.

$$\text{Availability, } A = \{(E - U_0) - T_0(S - S_0) + p_0(V - V_0)\},\ \text{kJ}$$

Availability per unit mass,

$$\omega = \{(e - u_0) - T_0(s - s_0) + p_0(v - v_0)\},\ \text{kJ/kg}$$

Availability or exergy is thus a measure of departure of state of system from that of environment. Thus, it is an attribute of system and environment together. However, once the environment is specified, a numerical value can be assigned to availability in terms of property values for system only. Hence, *exergy can be regarded as property of the system.*

7.3 AVAILABILITY ASSOCIATED WITH HEAT AND WORK

Let us consider a reversible heat engine having heat transfer from environment to control system and vice-a-versa.

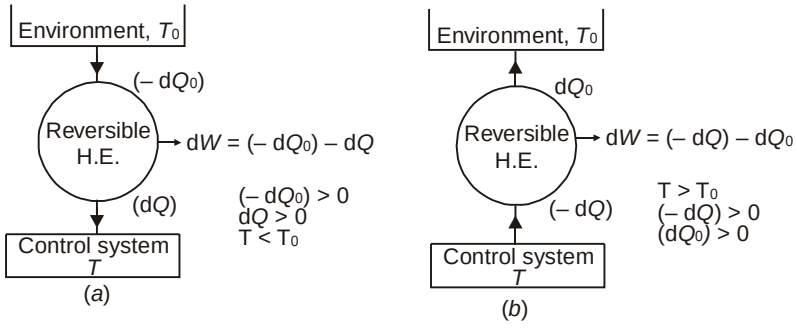


Fig. 7.3

Let us consider a reversible heat engine transferring heat δQ to the control system at temperature T from environment at temperature T_0. From second law of thermodynamics,

$$\frac{\delta Q}{T} = \left(\frac{-\delta Q_0}{T_0}\right)$$

work, $$\delta W = \left(\frac{T_0}{T} - 1\right).\delta Q$$

Let us now consider a reversible heat engine transferring heat δQ from control system to environment at T_0. From second law of thermodynamics,

$$\frac{\delta Q_0}{T_0} = \left(\frac{-\delta Q}{T}\right)$$

so work, $$\delta W = \left(1 - \frac{T_0}{T}\right)(-\delta Q)$$

$$= \left(\frac{T_0}{T} - 1\right).\delta Q$$

Availability associated with heat transfer : Let us consider a control system at dead state interacting with other system and there is heat interaction Q in control system. Let the final state of control system be given by 'f'. Due to heat interaction Q the control system may get heated up or cooled so that the final state is different from that of environment. Control system's temperature may increase from T_0 to T_f or decrease from T_f to T_0 but in every case availability shall increase. Availability of control system at final state gives maximum work that will be available from the combined system (control system + environment) as control system returns to dead state.

Work available from a reversible heat engine when control system gets heated or cooled by environment,

$$W_{\max} = \int_f^0 \left(\frac{T_0}{T} - 1\right)\delta Q$$

Availability associated with heat transfer = $\int_0^f \left(1 - \frac{T_0}{T}\right)\delta Q$

$$\boxed{A_Q = \int \left(1 - \frac{T_0}{T}\right)\delta Q}$$

If there is irreversibility present within control system due to internal irreversibilities, then availability change from initial to final state can be given as

$$\boxed{A_Q = \int \left(1 - \frac{T_0}{T}\right)\delta Q - I}$$

Since, here control system's initial state was dead state having zero availability so the change in availability

$$\boxed{\Delta A_Q = \int \left(1 - \frac{T_0}{T}\right)\delta Q - I}$$

Availability associated with work : Let us consider a control system initially at dead state. Control system has adiabatic compression occurring in it due to work interaction with some other system. – *W* work is done on control system and it attains some final state, '*f* '. Availability in this case shall be the maximum work available from the combined system of control system and environment as control system returns to the dead state.

If the work *W* is done by the control system as it returns from final state '*f* ' to dead state and the change in volume V_f to V_0 takes place by displacing the environment (*pdV* work), then availability associated with work,

$$\boxed{A_w = \Delta A_w = [W - p_0(V_f - V_0)]}$$

In case no boundary work is there, then $V_f = V_0$

$$\boxed{A_w = W = \Delta A_w}$$

Here it is also the availability change as system is returning to dead state. In case there is availability loss due to internal irreversibilities then change in availability,

$$\boxed{A_w = [W - p_0(V_f - V_0)] - I = \Delta A_w}$$

Similarly, availability associated with kinetic energy and potential energy can be given as,

$\boxed{A_{KE} = \frac{1}{2} mV^2}$; availability with K.E.

and $\boxed{A_{PE} = mgz}$; availability with P.E.

Generalized availability equation : A general availability equation for a control system having heat and work interactions with other systems can be obtained using earlier formulations. Let us consider a control system interacting with other systems and also having irreversibilities causing availability destruction in it. For elemental change during a process the energy balance can be given as, $dE = \delta Q - \delta W$.

Total entropy change, $dS = \frac{\delta Q}{T} + \delta S_{\text{irrev}}$

where *T* is temperature on control surface having δQ heat transfer and δS_{irrev} is entropy generated due to irreversibilities

Energy equation can be rewritten as,

$$dE + p_0 dV = \delta Q - \delta W + p_0 dV$$

Entropy equation can be rewritten as,

$$T_0\, dS = \frac{\delta Q \cdot T_0}{T} + T_0 . \delta S_{\text{irrev}}$$

Combining modified forms of energy and entropy equations by subtracting one from other,

$$dE + p_0 dV - T_0 \cdot dS = \delta Q - \delta W + p_0 dV - \frac{\delta Q \cdot T_0}{T} - T_0 \cdot \delta S_{\text{irrev}}$$

$$dE + p_0 dV - T_0 \cdot dS = \left(1 - \frac{T_0}{T}\right) \cdot \delta Q - (\delta W - p_0 \cdot dV) - T_0 \cdot \delta S_{\text{irrev}}$$

We have already seen in earlier article that the change in availability can be given as,

$$dA = dE + p_0.dV - T_0.dS$$

Hence,

$$dA = \left(1 - \frac{T_0}{T}\right).\delta Q - (\delta W - p_0.dV) - T_0.\delta S_{\text{irrev}}$$

Here, $T_0.\delta S_{\text{irrev}} = I$

For any process in control system between states 1 and 2, availability change can be given as,

$$\Delta A_{1-2} = \underbrace{\int_1^2 \left(1 - \frac{T_0}{T}\right) dQ}_{\substack{\text{Availability associated} \\ \text{with heat transfer}}} - \underbrace{(W_{1-2} - p_0 \Delta V_{1-2})}_{\substack{\text{Availability associated} \\ \text{with work interaction}}} - \underbrace{I}_{\text{Irreversibility}}$$

Generally, for any control mass in control system the availability change can be given as,

$$\boxed{\Delta A_{1-2} = \int_1^2 \left(1 - \frac{T_0}{T}\right) dQ - (W - p_0 \Delta V) - I}$$

Above availability change can also be given on per unit time basis.

7.4 EFFECTIVENESS OR SECOND LAW EFFICIENCY

Performance of engineering systems are generally measured using efficiency as defined by first law of thermodynamics. Efficiency as defined by first law uses energy for its' quantification. *Second law efficiency* or *effectiveness* or *exergetic efficiency* is an analogous parameter defined using availability.

Energy balance for a system with steady state yields,

Energy in = Energy output + Energy loss

Availability equation shall yield,

Availability in = (Availability output + Availability loss + Availability destruction due to irreversibility)

Mathematically,

(by first law), Efficiency $\eta = \dfrac{\text{Energy out in product (= Output)}}{\text{Energy in}}$

$$\eta = \frac{\text{Energy input} - \text{Energy loss}}{\text{Energy input}}$$

$$\boxed{\eta = 1 - \frac{\text{Energy loss}}{\text{Energy input}}}$$

(by second law), Effectiveness, $\varepsilon = \dfrac{\text{Availability output}}{\text{Availability in}}$

$$\varepsilon = 1 - \left[\frac{\text{Availability loss + Availability destruction due to irreversibility}}{\text{Availability in}}\right]$$

Effectiveness can also be given as the ratio of thermal efficiency to the maximum possible thermal efficiency (reversible processes) under same conditions.

$$\varepsilon = \frac{\eta_{th}}{\eta_{th,rev}}$$

For work producing systems, effectiveness = $\left(\frac{\text{Useful work}}{\text{Reversible work or maximum work}}\right)$

For work absorbing systems, effectiveness = $\left(\frac{\text{Reversible work or maximum work}}{\text{Useful work}}\right)$

For refrigerators and heat pumps, effectiveness = $\left(\frac{\text{COP}}{\text{COP}_{rev.}}\right)$

In general terms, Second law efficiency or effectiveness = $\left(\frac{\text{Availability used}}{\text{Availability supplied}}\right)$

7.5 SECOND LAW ANALYSIS OF STEADY FLOW SYSTEMS

Let us consider a steady flow system as shown,

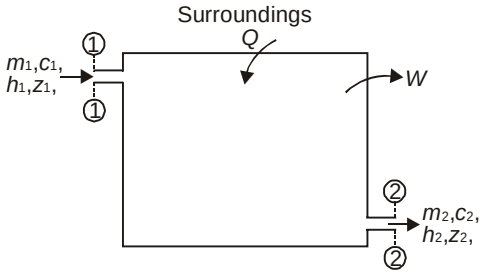


Fig. 7.4 *Steady flow system*

Section 1–1 and 2–2 refer to inlet and exit respectively.

Steady flow system interacts with surroundings at P_0 and T_0. Steady flow energy equation can be given as,

$$Q + m_1\left(h_1 + \frac{c_1^2}{2} + gz_1\right) = W + m_2\left(h_2 + \frac{c_2^2}{2} + gz_2\right)$$

Entropy generated,

$$S_{gen} = m_2 s_2 - m_1 s_1 + \frac{Q_{surr}}{T_0}$$

Heat transfer from control system, (–) = Heat gained by surrounding (+); $-Q = Q_{surr}$, from above two equations, substitution for Q yields,

$$-T_0 S_{gen} + m_1\left(h_1 + \frac{c_1^2}{2} + gz_1 - s_1 T_0\right) = W + m_2\left(h_2 + \frac{c_2^2}{2} + gz_2 - s_2 T_0\right)$$

or, $W = m_1\left(h_1 + \frac{c_1^2}{2} + gz_1 - T_0 s_1\right) - m_2\left(h_2 + \frac{c_2^2}{2} + gz_2 - T_0 s_2\right) - T_0 \cdot S_{gen}$

W can be quantified as above. This W shall be actual work available from system. Here entropy generation due to irreversibilities in processes reduce W and so for fully reversible processes $S_{gen} = 0$ and we get maximum available work;

$$W_{max} = m_1\left(h_1 + \frac{c_1^2}{2} + gz_1 - T_0 s_1\right) - m_2\left(h_2 + \frac{c_2^2}{2} + gz_2 - T_0 s_2\right)$$

In general terms, actual work available in this kind of systems where boundary work ($p.dV$) is absent can be given as under,

$$W = \sum m_i\left(h_i + \frac{c_i^2}{2} + gz_i - T_0 \cdot s_i\right) - \sum m_e\left(h_e + \frac{c_e^2}{2} + gz_e - T_0 \cdot s_e\right) - T_0 \cdot S_{gen}$$

where subscript 'i' and 'e' refer to inlet and exit in system.

For no irreversibilities present or for reversible processes,

$$W_{max} = \sum m_i\left(h_i + \frac{c_i^2}{2} + gz_i - T_0 \cdot s_i\right) - \sum m_e\left(h_e + \frac{c_e^2}{2} + gz_e - T_0 \cdot s_e\right)$$

Expression 1

Change in availability,

$$\Delta A = \sum m_i\left\{(h_i - h_0) + \frac{c_i^2}{2} + gz_i - T_0(s_i - s_0)\right\} - \sum m_e \cdot \left\{(h_e - h_0) + \frac{c_e^2}{2} + gz_e - T_0(s_e - s_0)\right\}$$

on unit mass basis,

$$\Delta\omega = \{T_0 \cdot \Delta s - \Delta h - \Delta K.E. - \Delta P.E.\} \text{ kJ/kg}$$

where $\Delta K.E.$ and $\Delta P.E.$ refer to kinetic energy and potential energy changes in system.

It indicates that the change in availability can be given by the difference of fluid stream availability at inlet and exit. "Fluid stream availability" can be defined in respect to dead state as, $c_0 = 0$, $z_0 = 0$

$$\psi = (h - h_0) + \frac{c^2}{2} + gz - T_0(s - s_0) \text{ kJ/kg}$$

$$\psi = \underline{(u - u_0) + p_0(v - v_0) - T_0(s - s_0)} + \frac{c^2}{2} + gz, \text{ kJ/kg.}$$

Here underlined terms are called physical exergy, $c^2/2$ is kinetic exergy and gz is potential exergy

$$\Delta A = W_{max} = \sum m_i \psi_i - \sum m_e \cdot \psi_e$$

Change of availability can be obtained using stream availability as described above. "Stream availability" is quantification of availability at a point.

Irreversibility rate in steady flow process can be given as, $I = T_0.S_{gen}$, (kW)

Exergy (Availability) and energy can be compared based upon their characteristics as given below.

Exergy (availability)	*Energy*
1. Exergy does not follow the law of conservation.	1. Energy follows the law of conservation.
2. It is function of states of the matter under consideration and the 'environment'.	2. It is function of the state of matter under consideration.
3. It is estimated with respect to the state of reference imposed by environment.	3. It may be calculated based upon the assumed state of reference.
4. Exergy always depends upon pressure.	4. In case of ideal gas energy does not depend upon pressure.
5. Exergy increases with temperature drop at low temperatures. For constant pressure processes exergy attains minimum value at the temperature of environment.	5. Energy increases with rise of temperature.
6. Exergy has positive value for ideal vacuum.	6. Energy is zero for an ideal vacuum.

7.6 GENERAL THERMODYNAMIC RELATIONS

Objective of this section is to develop mathematical relations for estimation of various thermodynamic properties such as *u, h, s* etc. for a compressible system. Thermodynamic properties such as pressure, volume and temperature (*P, V, T*) etc. can be directly measured experimentally while some other properties can not be measured directly and require thermodynamic relations for their determination. These thermodynamic relations are the basis for getting useful thermodynamic properties.

Important mathematical relations : To define state of a simple compressible system of known mass and composition one requires minimum two independent intensive properties. Thus, all intensive properties can be determined through functions of the two independent intensive properties such as,

$$p = p(T, v),\ u = u(T, v),\ h = h(T, v)...$$

Above are functions of two independent variables and can be given in general as, $z = z(x, y)$, where x, y are independent variables.

Exact differential : In earlier discussions we have seen that the differential of any property should be exact. Therefore, let us review calculus briefly.

Exact differential of any function z shall be as given below for z being continuous function of x and y.

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy$$

or, $$dz = M \cdot dx + N \cdot dy$$

where, $M = \left(\frac{\partial z}{\partial x}\right)_y$, $N = \left(\frac{\partial z}{\partial y}\right)_x$ i.e. M is partial derivative of z with respect to x when variable y is held constant and N is partial derivative of z with respect to y when variable x is held constant.

Here, since M and N have continuous first partial derivative therefore, order of differentiation is immaterial for properties and second partial derivative can be given as,

$$\frac{\partial}{\partial y}\left[\left(\frac{\partial z}{\partial x}\right)_y\right]_x = \frac{\partial}{\partial x}\left[\left(\frac{\partial z}{\partial y}\right)_x\right]_y$$

or, $$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$$

Thus, the test of exactness for any property shall be,

$$\boxed{\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y}$$

Reciprocity relation and cyclic relation : Let us consider three variables x, y, z such that any two of these are independent variables. Thus, we can write

$$x = x(y, z);\ y = y(x, z)$$

In differential form, $$dx = \left(\frac{\partial x}{\partial y}\right)_z \cdot dy + \left(\frac{\partial x}{\partial z}\right)_y \cdot dz;\ dy = \left(\frac{\partial y}{\partial x}\right)_z dx + \left(\frac{\partial y}{\partial z}\right)_x dz$$

Combining above two relations we get

$$\left[1 - \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial x}\right)_z\right] dx = \left[\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x + \left(\frac{\partial x}{\partial z}\right)_y\right] dz$$

As x and z are independent variables so let us keep z constant and vary x, i.e. $dz = 0$ and $dx \neq 0$ which yields reciprocity relation as,

$$\left[1 - \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial x}\right)_z\right] = 0$$

or, $$\boxed{\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial x}\right)_z = 1}$$

or, $$\boxed{\left(\frac{\partial x}{\partial y}\right)_z = \frac{1}{\left(\frac{\partial y}{\partial x}\right)_z}}$$ *Reciprocity relation*

Similarly, let us keep x constant and vary z i.e. $dx = 0$, $dz \neq 0$ which shall be possible only when;

$$\left[\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x + \left(\frac{\partial x}{\partial z}\right)_y\right] = 0$$

or $$\boxed{\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1}$$ *Cyclic relation*

7.6.1 Gibbs and Helmholtz Functions

For a simple compressible system of fixed chemical composition thermodynamic properties can be given from combination of first law and second law of thermodynamics as,

$$du = T \cdot ds - pdv$$
$$dh = T \cdot ds + vdp$$

Gibbs function (g) and Helmholtz function (f) are properties defined as below.

Gibbs function,

$\boxed{g = h - T \cdot s}$, on unit mass basis i.e. specific Gibb's function

also, $\boxed{G = H - T \cdot S}$

Helmholtz function,

$\boxed{f = u - T \cdot s}$, on unit mass basis i.e. specific Helmholtz function

also, $\boxed{F = U - T \cdot S}$

In differential form Gibbs function can be given as below for an infinitesimal reversible process

$$dg = dh - T \cdot ds - s \cdot dT$$

$\boxed{dg = vdp - sdT}$ for a reversible isothermal process, $\int_1^2 dg = \int_1^2 vdp$

or, also $\boxed{dG = Vdp - SdT}$ for reversible isothermal process; $\int_1^2 dG = \int_1^2 Vdp$

For a "reversible isobaric and isothermal process", $dp = 0$, $dT = 0$ $dG = 0$

i.e. $G =$ constant

'Gibbs function' is also termed as 'Gibbs free energy'. For a reversible isobaric and isothermal process Gibbs free energy remains constant or Gibbs function of the process remains constant. Such reversible thermodynamic processes may occur in the processes involving change of phase, such as sublimation, fusion, vaporization etc., in which Gibbs free energy remains constant.

'Helmholtz function' is also called 'Helmholtz free energy'. For any infinitesimal reversible process Helmholtz function can be given in differential form as,

$$df = du - T \cdot ds - sdT$$

or, $\boxed{df = -pdv - sdT}$

or, $\boxed{dF = -pdV - SdT}$

For a reversible isothermal process

$$df = -pdv$$

or, $$\int_1^2 df = -\int_1^2 p \cdot dv$$

or, $$\int_1^2 dF = -\int_1^2 pdV$$

For a reversible isothermal and isochoric process, $dT = 0$, $dV = 0$

$$df = 0$$

or, $$dF = 0$$

or, $\boxed{F = \text{Constant}}$

Above concludes that the Helmholtz free energy remains constant during a reversible isothermal and isochoric process. Such processes may occur during chemical reactions occurring isothermally and isochorically.

7.6.2 Maxwell Relations

Differential equations of thermodynamic properties, *u, h, f* and *g* can be given as function of *p, T, v, s* as below:

$$du = T \cdot ds - pdv$$
$$dh = T \cdot ds + vdp$$
$$df = -pdv - s \cdot dT$$
$$dg = vdp - s \cdot dT$$

Above equations can be used for defining the functions *u, h, f, g* based upon analogy with,

$dz = M \cdot dx + N \cdot dy$, for $z = z(x, y)$ and $\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$.

From above four equations for properties to be exact differentials, we can write functions;

$$u = u(s, v)$$
$$h = h(s, p)$$
$$f = f(v, T)$$
$$g = g(p, T)$$

For differential of function '*u*' to be exact;

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v$$

For differential of function '*h*' to be exact; $\left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p$

For differential of function '*f*' to be exact; $\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T$

For differential of function '*g*' to be exact; $\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T$

Above four conditions for exact differentials of thermodynamic properties result into "Maxwell relations".

Thus, "Maxwell relations" are :

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v$$

$$\left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T$$

$$\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T$$

Maxwell relations have large significance as these relations help in estimating the changes in entropy, internal energy and enthalpy by knowing p, v and T. Some applications of these equations are discussed in subsequent articles.

7.6.3 Clapeyron Equation

Let us look upon phase change at fixed temperature and pressure and estimate changes in specific entropy, internal energy and enthalpy during phase change. Let us start with one of Maxwell relations;

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T$$

From earlier discussions on pure substances we have seen that during phase transformation at some temperature the pressure is saturation pressure. Thus pressure is also independent of specific volume and can be determined by temperature alone. Hence, $p_{sat} = f(T_{sat})$

or $$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{dp}{dT}\right)_{sat}$$

Here $\left(\frac{dp}{dT}\right)_{sat}$ is the slope of saturation curve on pressure-temperature ($p - T$) diagram at some point determined by fixed constant temperature during phase transformation and is independent of specific volume.

Substituting in the Maxwell relation.

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{dp}{dT}\right)_{sat}$$

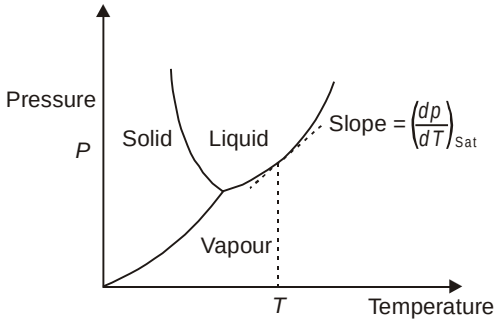


Fig. 7.5 *Pressure-temperature diagram for pure substance*

Thus, during vaporization i.e. phase transformation from liquid to vapour state, above relation can be given as,

$$\int_{\text{sat. liquid}}^{\text{dry vapour}} ds = \int_{\text{sat. liquid}}^{\text{dry vapour}} \left(\frac{dp}{dT}\right)_{sat} \cdot dv$$

Using notations for dry vapour and saturated liquid it can be given as,

$$(s_g - s_f) = \left(\frac{dp}{dT}\right)_{sat} \cdot (v_g - v_f)$$

or,
$$\left(\frac{dp}{dT}\right)_{sat} = \left(\frac{s_g - s_f}{v_g - v_f}\right)$$

$$\boxed{\left(\frac{dp}{dT}\right)_{sat} = \left(\frac{s_{fg}}{v_{fg}}\right)}$$

From differential form of specific enthalpy,

$$dh = T \cdot ds + v \cdot dp$$

for phase change occurring at constant pressure and temperature,

$$dh = T \cdot ds$$

for saturated liquid to dry vapour transformation,

$$(h_g - h_f) = T \cdot (s_g - s_f)$$
$$h_{fg} = T \cdot s_{fg}$$

Substituting $\frac{h_{fg}}{T}$ in place of entropy s_{fg} in $\left(\frac{dp}{dT}\right)_{sat}$

$$\boxed{\left(\frac{dp}{dT}\right)_{sat} = \left(\frac{h_{fg}}{T \cdot v_{fg}}\right)}$$ *Clapeyron equation*

Above equation is called Clapeyron equation. It can be used for determination of change in enthalpy during phase change i.e. h_{fg} from the p, v and T values which can be easily measured. Thus, Clapeyron equation can also be used for "sublimation process" or " melting occurring at constant temperature and pressure" just by knowing slope of saturation curve on p-T diagram, temperature and change in specific volume.

Hence, for initial state '1' getting transformed into final state '2' due to phase transformation at constant pressure and temperature, general form of Clapeyron equation:

$$\left(\frac{dp}{dT}\right)_{sat} = \left(\frac{h_{12}}{T \cdot v_{12}}\right)$$

At low pressure during liquid-vapour transformation it is seen that specific volume of saturated liquid state is very small as compared to dry vapour state, i.e. $v_f <<< v_g$. Also at low pressure the substance in vapour phase may be treated as perfect gas. Therefore, Clapeyron equation can be modified in the light of two approximations of "v_f being negligible compared to v_g at low pressures" and "ideal gas equation of state during vapour phase at low pressure, $\left(v_g = \frac{RT}{p}\right)$".

Clapeyron equation thus becomes, Clausius-Clapeyron equation as given here,

$$\left(\frac{dp}{dT}\right)_{sat} = \frac{(h_g - h_f)}{(T \cdot v_g)}$$

or, $$\left(\frac{dp}{dT}\right)_{sat} = \frac{h_g - h_f}{T \cdot (RT / p)}$$

or, $$\left(\frac{dp}{dT}\right)_{sat} = \left(\frac{h_{fg} \cdot p}{RT^2}\right)$$

or, $$\boxed{\left(\frac{dp}{p}\right)_{sat} = h_{fg} \cdot \left(\frac{dT}{RT^2}\right)_{sat}}$$ *Clausius-Clapeyron equation*

Integrating between two states 1 and 2

$$\boxed{\ln\left(\frac{p_2}{p_1}\right)_{sat} = \frac{h_{fg}}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)_{sat}}$$

Clausius-Clapeyron equation is thus a modified form of Clapeyron equation based upon certain approximations and is valid for low pressure phase transformations of liquid-vapour or solid-vapour type.

7.6.4 General Relations for Change in Entropy, Enthalpy, Internal Energy and Specific Heats

Let us now derive expressions for changes in entropy, enthalpy, internal energy and specific heats as a function of thermodynamic properties, p, v and T. For defining a state any two of the properties amongst the p, v, and T may be regarded as independent properties. Let us take (T, p) and (T, v) as two sets of independent properties for defining other dependent properties.

Temperature and Pressure (T, p) as Independent Properties : By considering T and p as independent properties, dependent property say entropy can be given as,

$$s = s(T, p)$$

Writing differential form of entropy function,

$$ds = \left(\frac{\partial s}{\partial T}\right)_p \cdot dT + \left(\frac{\partial s}{\partial p}\right)_T \cdot dp$$

From Maxwell relations the partial derivative $\left(\frac{\partial s}{\partial p}\right)_T$ can be substituted by $-\left(\frac{\partial v}{\partial T}\right)_p$ as,

$$\boxed{ds = \left(\frac{\partial s}{\partial T}\right)_p \cdot dT - \left(\frac{\partial v}{\partial T}\right)_p \cdot dp}$$

Similarly, specific enthalpy can be given as function of T and p; $h = h(T, p)$

Writing differential form; $dh = \left(\frac{\partial h}{\partial T}\right)_p dT + \left(\frac{\partial h}{\partial p}\right)_T \cdot dp$

We have already seen that specific heat at constant pressure can be given as function of specific enthalpy and temperature at constant pressure.

$$C_p = \left(\frac{\partial h}{\partial T}\right)_P$$

Substituting C_p in *dh*,

$$\boxed{dh = C_p \cdot dT + \left(\frac{\partial h}{\partial p}\right)_T \cdot dp}$$

From definition of enthalpy, first and second law combined,

$$dh = T \cdot ds + vdp$$

Substituting *dh* and *ds* from above,

$$C_p \cdot dT + \left(\frac{\partial h}{\partial p}\right)_T \cdot dp = T \cdot \left(\frac{\partial s}{\partial T}\right)_p dT - T\left(\frac{\partial v}{\partial T}\right)_p \cdot dp + v \cdot dp$$

or,

$$\underline{\left\{\left(\frac{\partial h}{\partial p}\right)_T + T\left(\frac{\partial v}{\partial T}\right)_p - v\right\} dp = \left\{T \cdot \left(\frac{\partial s}{\partial T}\right)_p - C_p\right\} dT}$$

Here *T* and *p* are considered to be independent variables so let us keep pressure constant and vary temperature i.e. $dp = 0$, $dT \neq 0$. It yields in modified form of above underlined equation as,

$$\left\{T \cdot \left(\frac{\partial s}{\partial T}\right)_p - C_p\right\} dT = 0$$

or, $$C_P = T \cdot \left(\frac{\partial s}{\partial T}\right)_p$$

or, $$\left(\frac{\partial s}{\partial T}\right)_p = \frac{C_p}{T}$$

Substituting $\left(\frac{\partial s}{\partial T}\right)_p$ in *ds* expression we get, $\boxed{ds = \frac{C_p dT}{T} - \left(\frac{\partial v}{\partial T}\right)_p \cdot dp}$

Similarly, temperature can be kept constant and pressure varied independently as,

$$dT = 0,\ dp \neq 0$$

Above underlined equation gets modified as,

$$\left\{\left(\frac{\partial h}{\partial p}\right)_T + T\left(\frac{\partial v}{\partial T}\right)_p - v\right\} dp = 0$$

or, $$\boxed{\left(\frac{\partial h}{\partial p}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_p}$$

Substituting $\left(\frac{\partial h}{\partial p}\right)_T$ in expression for *dh* we get

or, $$\boxed{dh = C_p\, dT + \left\{v - T\left(\frac{\partial v}{\partial T}\right)_p\right\} dp}$$

$$h_2 - h_1 = \int_{T_1}^{T_2} C_p dT + \int_{p_1}^{p_2} \left\{ v - T\left(\frac{\partial v}{\partial T}\right)_p \right\} dp$$

Temperature and Specific Volume (T, v) as independent property: Considering T and v as independent properties the dependent properties can be given as,

$$u = u(T, v)$$

Writing differential form of specific internal energy.

$$\boxed{du = \left(\frac{\partial u}{\partial T}\right)_v \cdot dT + \left(\frac{\partial u}{\partial v}\right)_T dv}$$

From definition of specific heat at constant volume, $C_v = \left(\frac{\partial u}{\partial T}\right)_v$

or,
$$\boxed{du = C_v \cdot dT + \left(\frac{\partial u}{\partial v}\right)_T dv}$$

Writing specific entropy as function of T and v,

$$s = s(T, v)$$

$$\boxed{ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv}$$

From Maxwell relations, $\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v$; substituting in above we get

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial p}{\partial T}\right)_v dv$$

From I and II law combined, $du = T \cdot ds - pdv$

Substituting du and ds in above equation,

$$C_v\, dT + \left(\frac{\partial u}{\partial v}\right)_T dv = T\left(\frac{\partial s}{\partial T}\right)_v dT + T \cdot \left(\frac{\partial p}{\partial T}\right)_v dv - pdv$$

Rearranging terms,

$$\underline{\left\{\left(\frac{\partial u}{\partial v}\right)_T - T\left(\frac{\partial p}{\partial T}\right)_v + p\right\} dv = \left\{T \cdot \left(\frac{\partial s}{\partial T}\right)_v - C_v\right\} dT}$$

As T and v are considered independent variables therefore let us keep T as constant and v as variable, i.e. $dT = 0$, $dv \neq 0$. It yields,

$$\left\{\left(\frac{\partial u}{\partial v}\right)_T - T\left(\frac{\partial p}{\partial T}\right)_v + p\right\} dv = 0$$

or,
$$\boxed{\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p}$$

Similarly, let us keep v constant and T as variable i.e $dv = 0$ $dT \neq 0$. It yields,

$$\left\{T\left(\frac{\partial s}{\partial T}\right)_v - C_v\right\} dT = 0$$

or,
$$\left\{T\left(\frac{\partial s}{\partial T}\right)_v - C_v\right\} = 0$$

or,
$$\boxed{C_v = T\cdot\left(\frac{\partial s}{\partial T}\right)_v} \text{ or, } \boxed{\left(\frac{\partial s}{\partial T}\right)_v = \frac{C_v}{T}}$$

Let us now substitute $\left(\frac{\partial u}{\partial v}\right)_T$ in the differential function du which yields,

$$\boxed{du = C_v \cdot dT + \left\{T\left(\frac{\partial p}{\partial T}\right)_v - p\right\} dv}$$

For any state change from 1 to 2 we can get change in internal energy, as

$$\boxed{u_2 - u_1 = \int_{T_1}^{T_2} C_v dT + \int_{v_1}^{v_2} \left\{T\cdot\left(\frac{\partial p}{\partial T}\right)_v - p\right\} dv}$$

Also, let us substitute $\left(\frac{\partial s}{\partial T}\right)_v$ in the expression of entropy change 'ds'. It results in,

$$\boxed{ds = \frac{C_v dT}{T} + \left(\frac{\partial p}{\partial T}\right)_v dv}$$

Thus, number of expressions are available for getting the change in h, u and s, which are summarized as under.

$$\boxed{dh = C_p dT + \left\{v - T\left(\frac{\partial v}{\partial T}\right)_p\right\} dp}$$

$$\boxed{du = C_v\, dT + \left\{T\left(\frac{\partial p}{\partial T}\right)_v - p\right\} dv}$$

$$\boxed{ds = \frac{C_p .dT}{T} - \left(\frac{\partial v}{\partial T}\right)_p \cdot dp}$$

$$\boxed{ds = \frac{C_v dT}{T} + \left(\frac{\partial p}{\partial T}\right)_v \cdot dv}$$

From above two expressions of entropy change, the difference between C_p and C_v values can be obtained as,

$$(C_p - C_v)dT = T \cdot \left(\frac{\partial p}{\partial T}\right)_v dv + T\left(\frac{\partial v}{\partial T}\right)_p dp$$

Let us write from equation of state, the function $p = p(T, v)$

Differential, $$dp = \left(\frac{\partial p}{\partial T}\right)_v dT + \left(\frac{\partial p}{\partial v}\right)_T dv$$

Substituting dp in above equation of $(C_p - C_v)\, dT$ we get,

$$(C_p - C_v)\, dT = T\left(\frac{\partial p}{\partial T}\right)_v dv + T\left(\frac{\partial v}{\partial T}\right)_p \cdot \left(\frac{\partial p}{\partial T}\right)_v dT + T\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial p}{\partial v}\right)_T dv$$

or,

$$\left[(C_p - C_v) - T\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial p}{\partial T}\right)_v\right] dT = \left[T\left(\frac{\partial p}{\partial T}\right)_v + T\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial p}{\partial v}\right)_T\right] dv$$

Since T and v are independent so let us keep T as constant and v as variable i.e $dT = 0$ $dv \neq 0$.

$$\left[T\left(\frac{\partial p}{\partial T}\right)_v + T\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial p}{\partial v}\right)_T\right] dv = 0$$

or,
$$\left(\frac{\partial p}{\partial T}\right)_v = -\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial p}{\partial v}\right)_T$$

Similarly keeping v as constant and T variable i.e., $dv = 0$, $dT \neq 0$

$$\left\{(C_p - C_v) - T\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial p}{\partial T}\right)_v\right\} dT = 0$$

or,
$$\boxed{(C_p - C_v) = T\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial p}{\partial T}\right)_v}$$

Substituting for $\left(\frac{\partial p}{\partial T}\right)_v$ from above in $(C_p - C_v)$ we get,

$$\boxed{(C_p - C_v) = -T\left(\frac{\partial v}{\partial T}\right)_p^2 \cdot \left(\frac{\partial p}{\partial v}\right)_T}$$

In single phase region the specific volume can also be given as function of T & p and the differential of function v shall be,

$$v = v\ (T, p)$$

or, $$dv = \left(\frac{\partial v}{\partial T}\right)_p dT + \left(\frac{\partial v}{\partial p}\right)_T dp$$

The above differential form of specific volume indicates that it depends upon partial derivatives of specific volume with respect to temperature and pressure. Partial derivatives of v with respect to temperature can be related to "volume expansivity" or "coefficient of volume expansion" as below,

$$\boxed{\text{Volume expansivity, } \beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p}$$

Partial derivative of specific volume with respect to pressure can be related to "isothermal compressibility", α as below.

$$\boxed{\text{Isothermal compressibility, } \alpha = \frac{-1}{v}\left(\frac{\partial v}{\partial p}\right)_T}$$

Inverse of isothermal compressibility is called "isothermal bulk modulus",

$$B_T = -v\left(\frac{\partial p}{\partial v}\right)_T$$

Thus, volume expansivity gives the change in volume that occurs when temperature changes while pressure remains constant. Isothermal compressibility gives change in volume when pressure changes while temperature remains constant. These volume expansivity and isothermal compressibility are thermodynamic properties.

Similarly, the change in specific volume with change in pressure isentropically is also called "isentropic compressibility" or "adiabatic compressibility", Mathematically $\alpha_s = \frac{-1}{v}\left(\frac{\partial v}{\partial p}\right)_s$. Reciprocal of isentropic compressibility is called "isentropic bulk modulus," $B_s = -v\left(\frac{\partial p}{\partial v}\right)_s$

Substituting β and α in $(C_p - C_v)$ expression;

$$C_p - C_v = -T(\beta^2 \cdot v^2)\left(\frac{-1}{\alpha \cdot v}\right)$$

or,

$$\boxed{C_p - C_v = \frac{v \cdot T \cdot \beta^2}{\alpha}} \quad \textit{Mayer relation}$$

Above difference in specific heat expression is called "Mayer relation" and helps in getting significant conclusion such as,

- The difference between specific heats is zero at absolute zero temperature i.e. specific heats at constant pressure and constant volume shall be same at absolute zero temperature (T = 0 K).
- Specific heat at constant pressure shall be generally more than specific heat at constant volume i.e., $C_p \geq C_v$. It may be attributed to the fact that 'α' the isothermal compressibility shall always be

+ve and volume expansivity 'β' being squared in $(C_p - C_v)$ expression shall also be +ve. Therefore $(C_p - C_v)$, shall be either zero or positive value depending upon magnitudes of v, T, β and α.

- For incompressible substances having $dv = 0$, the difference $(C_p - C_v)$ shall be nearly zero. Hence, specific heats at constant pressure and at constant volume are identical.

Let us obtain expression for ratio of specific heats. Earlier we have obtained C_p and C_v as below,

$$C_p = T \cdot \left(\frac{\partial s}{\partial T}\right)_p \quad \text{or,} \quad \frac{C_p}{T} = \left(\frac{\partial s}{\partial T}\right)_p$$

and

$$C_v = T \cdot \left(\frac{\partial s}{\partial T}\right)_v \quad \text{or} \quad \frac{C_v}{T} = \left(\frac{\partial s}{\partial T}\right)_v$$

By cyclic relation we can write for p, T and s properties;

$$\left(\frac{\partial s}{\partial T}\right)_p \cdot \left(\frac{\partial T}{\partial p}\right)_s \cdot \left(\frac{\partial p}{\partial s}\right)_T = -1$$

or,

$$\left(\frac{\partial s}{\partial T}\right)_p = \frac{-1}{\left(\frac{\partial T}{\partial p}\right)_s \cdot \left(\frac{\partial p}{\partial s}\right)_T}$$

Similarly for s, T and v properties we can write using cyclic relation;

$$\left(\frac{\partial s}{\partial T}\right)_v \cdot \left(\frac{\partial T}{\partial v}\right)_s \cdot \left(\frac{\partial v}{\partial s}\right)_T = -1$$

or,

$$\left(\frac{\partial s}{\partial T}\right)_v = \frac{-1}{\left(\frac{\partial T}{\partial v}\right)_s \cdot \left(\frac{\partial v}{\partial s}\right)_T}$$

Substituting in the relation for (C_p/T) and $\left(\frac{C_v}{T}\right)$.

$$\left(\frac{C_p}{T}\right) = \frac{-1}{\left(\frac{\partial T}{\partial p}\right)_s \cdot \left(\frac{\partial p}{\partial s}\right)_T}$$

and

$$\left(\frac{C_v}{T}\right) = \frac{-1}{\left(\frac{\partial T}{\partial v}\right)_s \cdot \left(\frac{\partial v}{\partial s}\right)_T}$$

Taking ratio of two specific heats,

$$\frac{C_p}{C_v} = \frac{\left(\frac{\partial T}{\partial v}\right)_s \cdot \left(\frac{\partial v}{\partial s}\right)_T}{\left(\frac{\partial T}{\partial p}\right)_s \cdot \left(\frac{\partial p}{\partial s}\right)_T}$$

or, $$\left(\frac{C_p}{C_v}\right) = \left(\frac{\partial T}{\partial v}\right)_s \cdot \left(\frac{\partial v}{\partial s}\right)_T \cdot \left(\frac{\partial p}{\partial T}\right)_s \cdot \left(\frac{\partial s}{\partial p}\right)_T$$

$$\left(\frac{C_p}{C_v}\right) = \left\{\left(\frac{\partial v}{\partial s}\right)_T \left(\frac{\partial s}{\partial p}\right)_T\right\} \cdot \left\{\left(\frac{\partial p}{\partial T}\right)_s \left(\frac{\partial T}{\partial v}\right)_s\right\}$$

By chain rule of calculus we can write,

$$\left(\frac{\partial v}{\partial p}\right)_T = \left(\frac{\partial v}{\partial s}\right)_T \cdot \left(\frac{\partial s}{\partial p}\right)_T$$

and $$\left(\frac{\partial p}{\partial v}\right)_s = \left(\frac{\partial p}{\partial T}\right)_s \cdot \left(\frac{\partial T}{\partial v}\right)_s$$

Upon substitution in specific heat ratio we get,

$$\left(\frac{C_p}{C_v}\right) = \left(\frac{\partial v}{\partial p}\right)_T \cdot \left(\frac{\partial p}{\partial v}\right)_s$$

or, $$= \left\{\frac{-1}{v}\left(\frac{\partial v}{\partial p}\right)_T\right\} \cdot \left\{\frac{1}{\frac{-1}{v}\left(\frac{\partial v}{\partial p}\right)_s}\right\}$$

$$\frac{C_p}{C_v} = \frac{\alpha}{\alpha_s} = \frac{\text{Isothermal compressibility}}{\text{Isentropic compressibility}}$$

Thus, ratio of specific heats at constant pressure and constant volume can be given by the ratio of isothermal compressibility and isentropic compressibility.

7.6.5 Joule-Thomson Coefficient

Joule-Thomson coefficient is defined as the rate of change of temperature with pressure during an isenthalpic process or throttling process. Mathematically, Joule-Thomson coefficient (μ) can be given as, $\mu = \left(\frac{\partial T}{\partial p}\right)_h$

It is defined in terms of thermodynamic properties and is itself a property. Joule-Thomson coefficient gives slope of constant enthalpy lines on temperature—pressure diagram. Thus, it is a parameter for characterizing the throttling process. Slope of isenthalpic line may be positive, zero or negative, i.e. $\mu > 0$, $\mu = 0$ and $\mu < 0$ respectively. Mathematically evaluating the consequence of μ we see,

– for $\mu > 0$, temperature decreases during the process.

– for $\mu = 0$, temperature remains constant during the process.

– for $\mu < 0$, temperature increases during the process.

Joule-Thomson expansion can be shown as in Fig. 7.6. Here gas or liquid is passed through porous plug for causing isenthalpic process. Valve put near exit is used for regulating pressure after constant enthalpy process i.e. p_2.

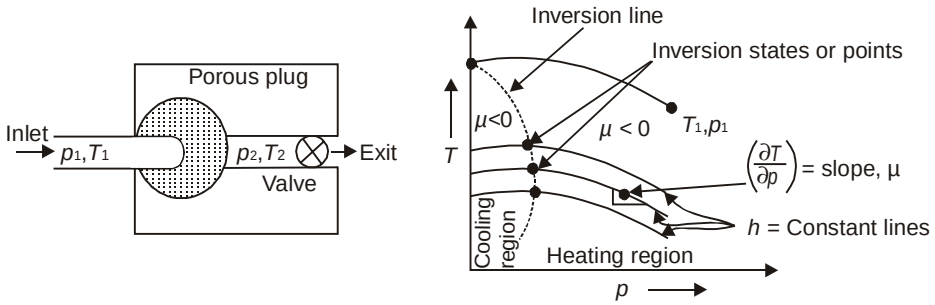


Fig. 7.6 *Joule-Thomson expansion*

If pressure p_2 is varied then the temperature variation occurs in the isenthalpic manner as shown in *T-p* diagram. This graphical representation of isenthalpic curve gives the Joule-Thomson coefficient by its slope at any point. Slope may be positive, negative or zero at different points on the curve. The points at which slope has zero value or Joule-Thomson coefficient is zero are called "inversion points" or "inversion states". Temperature at these inversion states is called "inversion temperature". Locii of these inversion states is called "inversion line". Thus, inversion line as shown divides *T-p* diagram into two distinct region i.e. one on the left of line and other on the right of line. For the states lying on left of the inversion line temperature shall decrease during throttling process while for the states on right of inversion line throttling shall cause heating of fluid being throttled. Temperature at the intersection of inversion line with zero pressure line is called "maximum inversion temperature".

7.6.6 Chemical Potential

In case of multicomponent systems such as non-reacting gas mixtures the partial molal properties are used for describing the behaviour of mixtures and solutions. Partial molal properties are intensive properties of the mixture and can be defined as,

$$X_i = \left(\frac{\partial X}{\partial n_i}\right)_{T,\, p,\, n_k}$$

where X is extensive property for multi component system in single phase.

$X = X(T, p, n)$ i.e. function of temperature, pressure and no. of moles of each component n_k refers to the all n values with varing k values and are kept constant except for n_i.

In multicomponent systems the partial molal Gibbs function for different constituents are called "chemical potential" for particular constituent. Chemical potential, μ can be defined for ith component as,

$$\mu_i = \left(\frac{\partial G}{\partial n_i}\right)_{T,\, p,\, n_k}$$

where G, n_i, n_k, T and P have usual meanings. Chemical potential being a partial molal property is intensive property.

Also, it can be given as,

$$G = \sum_{i=1}^{j} (n_i \cdot \mu_i)$$

Thus, for non reacting gas mixture the expression for internal energy, enthalpy, Helmholtz function can be given using G defined as above,

$$\text{Internal energy, } U = TS - pV + \sum_{i=1}^{j} n_i \ \mu_i$$

$$\text{Enthalpy, } H = TS + \sum_{i=1}^{j} n_i \cdot \mu_i$$

$$\text{Helmholtz function, } F = -pV + \sum_{i=1}^{j} n_i \cdot \mu_i$$

Writing differential of G considering it as function of (T, p, n_1, n_2, ... n_j)

$$G = G(T, p, n_1, n_2, n_3 \ldots n_j)$$

$$dG = \left(\frac{\partial G}{\partial p}\right)_{T,n} dp + \left(\frac{\partial G}{\partial T}\right)_{p,\,n} dT + \sum_{i=1}^{j}\left(\frac{\partial G}{\partial n_i}\right)_{T,\,p,\,n_k} dn_i$$

From definition of Gibbs function $dG = Vdp - SdT$, for T = constant,

$$V = \left(\frac{\partial G}{\partial p}\right)_{T,\,n}$$

$$\text{for pressure as constant, } -S = \left(\frac{\partial G}{\partial T}\right)_{p,\,n}$$

Therefore,

$$\boxed{dG = Vdp - SdT + \sum_{i=1}^{j} (\mu_i \cdot dn_i)}$$

Also from $G = \sum_{i=1}^{j} (n_i \cdot \mu_i)$ we can write differential as,

$$\boxed{dG = \sum_{i=1}^{j} (n_i \cdot d\mu_i) + \sum_{i=1}^{j} (\mu_i \cdot dn_i)}$$

From two differential of function G we get,

$$\boxed{Vdp - SdT = \sum_{i=1}^{j} (n_i \cdot d\mu_i)}$$

Above equation is also called Gibbs-Duhem equation.

7.6.7 Fugacity

From earlier discussions for a single component system one can write,

$$G = n \cdot \mu$$

or, $\mu = \dfrac{G}{n} \Rightarrow$ Chemical potential for pure substance = Gibbs function per mole.

or $$\bar{g} = \frac{G}{n} = \mu$$

For Gibbs function written on unit mole basis,

For constant temperature $$\bar{v} = \left(\frac{\partial \mu}{\partial p}\right)_T$$

If single component system is perfect gas then, $\bar{v} = \dfrac{\bar{R}T}{p}$

or, $$\left(\frac{\partial \mu}{\partial p}\right)_T = \frac{\bar{R}T}{p}$$

or $$\boxed{\mu_T = \bar{R}T \ln p + \text{constant}}$$

Here chemical potential may have any value depending upon the value of pressure. Above mathematical formulation is valid only for perfect gas behaviour being exhibited by the system. For a real gas above mathematical equation may be valid if pressure is replaced by some other property called 'fugacity'.

Fugacity was first used by Lewis.

Fugacity denoted by '$\mathcal{F}$' can be substituted for pressure in above equation,

$$\mu = \bar{R}T \ln \mathcal{F} + \text{Constant}$$

For constant pressure using $\bar{v} = \left(\dfrac{\partial \mu}{\partial p}\right)_T$ and above equation, we get

$$\bar{R}T\left(\frac{\partial \ln \mathcal{F}}{\partial p}\right)_T = \bar{v}$$

Thus, for a limiting case when ideal gas behaviour is approached the fugacity of a pure component shall equal the pressure in limit of zero pressure.

$$\lim_{p \to 0}\left(\frac{\mathcal{F}}{p}\right) = 1$$

For an ideal gas $$\mathcal{F} = p$$

For real gas, equation of state can be given using compressibility factor as,

$$p\bar{v} = Z\bar{R}T$$

or, $$\bar{v} = \frac{Z\bar{R}T}{p}$$

Substituting the fugacity function,

$$\frac{Z\bar{R}T}{p} = \bar{R}T\left(\frac{\partial \ln \mathcal{F}}{\partial p}\right)_T$$

$$\left(\frac{\partial \ln \mathcal{F}}{\partial p}\right)_T = \frac{Z}{p}$$

or, $$\left(\frac{\partial \ln \mathcal{F}}{\partial \ln p}\right)_T = Z.$$ Here as $p \to 0$ the $Z \to 1$

Also we have seen

$$\mu_T = \bar{R}\, T \ln \mathcal{F} + \text{constant}$$

or, $$(d\mu)_T = \bar{R}\, T\, d\,(\ln \mathcal{F})_T$$

or, $$dg_T = \bar{R}\, Td\,(\ln \mathcal{F})_T$$

Integrating between very low pressure p^* and high pressure p.

$$\int_g^{g^*} dg_T = \int_{\mathcal{F}}^{\mathcal{F}^*} \bar{R}T\ d(\ln \mathcal{F})_T$$

or $$g = g^* + RT \ln \left(\frac{\mathcal{F}}{\mathcal{F}^*}\right)$$

Here for very low pressure, $\mathcal{F}^* = p^*$

or, $$g = g^* + RT \ln \left(\frac{\mathcal{F}}{p^*}\right)$$

When low pressure is 1 atm then the ratio $\left(\frac{\mathcal{F}}{\mathcal{F}^*}\right)$ is called “activity”.

EXAMPLES

1. *Steam at 1.6 MPa, 300ºC enters a flow device with negligible velocity and leaves at 0.1 MPa, 150ºC with a velocity of 150 m/s. During the flow heat interaction occurs only with the surroundings at 15ºC and steam mass flow rate is 2.5 kg/s. Estimate the maximum possible output from the device.*

Solution:

Let us neglect the potential energy change during the flow.

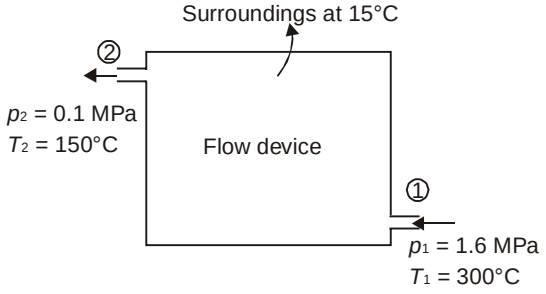


Fig. 7.7

Applying S.F.E.E., neglecting inlet velocity and change in potential energy,

$$W_{max} = (h_1 - T_0\, s_1) - \left(h_2 + \frac{C_2^2}{2} - T_0 s_2 \right)$$

$$W_{max} = (h_1 - h_2) - T_0(s_1 - s_2) - \frac{C_2^2}{2}$$

From steam tables,

$$h_1 = h_{\text{at 1.6 MPa, 300°C}} = 3034.8 \text{ kJ/kg}$$
$$s_1 = s_{\text{at 1.6 MPa, 300°C}} = 6.8844 \text{ kJ/kg} \cdot \text{K}$$
$$h_2 = h_{\text{at 0.1 MPa, 150°C}} = 2776.4 \text{ kJ/kg}$$
$$s_2 = s_{\text{at 0.1 MPa, 150°C}} = 7.6134 \text{ kJ/kg} \cdot \text{K}$$

Given; $T_0 = 288\ K$

$$W_{max} = (3034.8 - 2776.4) - 288(6.8844 - 7.6134) - \frac{(150)^2}{2} \times 10^{-3}$$

$$= 457.1 \text{ kJ/kg}$$

Maximum possible work $= 2.5 \times 457.1$ kJ/s $= 1142.75$ kW

Maximum possible work = **1142.75 kW** **Ans.**

2. *Two tanks A and B contain 1 kg of air at 1 bar, 50°C and 3 bar, 50°C when atmosphere is at 1 bar, 15°C. Identify the tank in which stored energy is more. Also find the availability of air in each tank.*

Solution:

In these tanks the air stored is at same temperature of 50°C. Therefore, for air behaving as perfect gas the internal energy of air in tanks shall be same as it depends upon temperature alone. But the availability shall be different.

Both the tanks have same internal energy **Ans.**

Availability of air in tank

$$A = \{E - U_0\} + p_0(V - V_0) - T_0(S - S_0)$$
$$= m\,\{(e - u_0) + p_0(v - v_0) - T_0(s - s_0)\}$$
$$= m\left\{c_v(T - T_0) + p_0\left(\frac{RT}{p} - \frac{RT_0}{p_0}\right) - T_0\left(c_p \ln\frac{T}{T_0} - R\ln\frac{p}{p_0}\right)\right\}$$

$$A = m\left\{c_v(T - T_0) + R\left(\frac{P_0}{p}T - T_0\right) - T_0 c_p \ln\frac{T}{T_0} + T_0 R \ln\frac{p}{p_0}\right\}$$

For tank A, $m = 1$ kg, $c_v = 0.717$ kJ/kg · K, $T = 323$ K, $R = 0.287$ kJ/kg · K, $T_0 = 288$ K, $p_0 = 1$ bar, $c_p = 1.004$ kJ/kg · K

$$\text{Availability}_A = 1\,\{0.717\,(323 - 288) + 0.287\,(1 \times 323 - 288) - (288 \times 1.004)\ln\left(\frac{323}{288}\right)$$
$$+ 288 \times 0.287 \ln 1\}$$
$$= 1.98 \text{ kJ}$$

For tank B, T = 323 K, p = 3 bar

$$\text{Availability}_B = 1\left\{0.717\,(323-288)+0.287\left(\frac{1}{3}\times 323-288\right)-\right.$$

$$\left(288\times 1.004\ln\frac{323}{288}\right)+\left(288\times 0.287\ln\frac{3}{1}\right) = 30.98 \text{ kJ}$$

Availability of air in tank *B* is more than that of tank *A*.

Availability of air in tank A = **1.98 kJ** Availability of air in tank B = **30.98 kJ**	**Ans.**

3. *15 kg/s steam enters a perfectly insulated steam turbine at 10 bar, 300ºC and leaves at 0.05 bar, 0.95 dry with velocity of 160 m/s. Considering atmospheric pressure to be 1 bar, 15ºC. Determine (a) power output, (b) the maximum power for given end states, (c) the maximum power that could be obtained from exhaust steam. Turbine rejects heat to a pond having water at 15ºC.*

Solution:

From steam tables,

Enthalpy at inlet to turbine, $h_1 = 3051.2$ kJ/kg

$s_1 = 7.1229$ kJ/kg · K

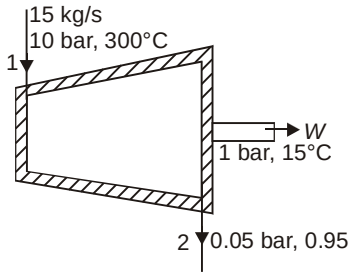


Fig. 7.8

Enthalpy at exit of turbine, $h_2 = h_{\text{at 0.05 bar, 0.95 dry}}$

$s_2 = s_{f \text{ at 0.05 bar}} + (0.95 \times s_{fg \text{ at 0.05 bar}})$

$s_2 = 0.4764 + (0.95 \times 7.9187)$

$s_2 = 7.999$ kJ/kg · K

Similarly, $h_2 = h_{f \text{ at 0.05 bar}} + (0.95 \times h_{fg \text{ at 0.05 bar}})$

$= 137.82 + (0.95 \times 2423.7)$

$h_2 = 2440.34$ kJ/kg

Neglecting the change in potential energy and velocity at inlet to turbine, the steady flow energy equation may be written as to give work output.

$$w = (h_1 - h_2) - \frac{V_2^2}{2}$$

$$w = (3051.2 - 2440.34) - \left(\frac{(160)^2}{2} \times 10^{-3}\right)$$

$$w = 598.06 \text{ kJ/kg}$$

Power output = m.w = 15 × 598.06 = 8970.9 kW

Power output = **8970.9 kW** **Ans.**

Maximum work for given end states,

$$w_{max} = (h_1 - T_0 \cdot s_1) - \left(h_2 + \frac{V_2^2}{2} - T_0 \cdot s_2\right)$$

$$w_{max} = (3051.2 - 288 \times 7.1229) - \left(2440.34 + \frac{(160)^2 \times 10^{-3}}{2} - 288 \times 7.999\right)$$

$$w_{max} = 850.38 \text{ kJ/kg}$$

$$W_{max} = m \cdot w_{max} = 15 \times 850.38 = 12755.7 \text{ kW}$$

Maximum power output = **12755.7 kw** **Ans.**

Maximum power that could be obtained from exhaust steam shall depend upon availability with exhaust steam and the dead state. Stream availability of exhaust steam,

$$A_{exhaust} = \left(h_2 + \frac{V_2^2}{2} - T_0 s_2\right) - (h_0 - T_0\, s_0)$$

$$= (h_2 - h_0) + \frac{V_2^2}{2} - T_0\,(s_2 - s_0)$$

Approximately the enthalpy of water at dead state of 1 bar, 15ºC can be approximated to saturated liquid at 15ºC.

$$h_0 = h_{f \text{ at } 15^\circ C} = 62.99 \text{ kJ/kg}$$

$$s_0 = s_{f \text{ at } 15^\circ C} = 0.2245 \text{ kJ/kg} \cdot \text{K}$$

Maximum work available from exhaust steam

$$= (2440.34 - 62.99) + \left(\frac{(160)^2}{2} \times 10^{-3}\right) - 288\,(7.999 - 0.2245)$$

$$A_{exhaust} = 151.1 \text{ kJ/kg}$$

Maximum power that could be obtained from exhaust steam

$$= m \times A_{exhaust}$$

$$= 15 \times 151.1 = 2266.5 \text{ kW}$$

Maximum power from exhaust steam = **2266.5 kW** **Ans.**

4. *5 kg of steam, initially at elevation of 10 m and velocity of 25 m/s undergoes some process such that finally it is at elevation of 2m and velocity of 10 m/s. Determine the availability corresponding to the initial and final states. Also estimate the change in availability assuming environment temperature and pressure at 25°C and 100 kPa respectively. Thermodynamic properties u, v, s are as under.*

Dead state of water

$u_0 = 104.86$ kJ/kg
$v_0 = 1.0029 \times 10^{-3}$ m^3/kg
$s_0 = 0.3673$ kJ/kg · K

Initial state

$u_1 = 2550$ kJ/kg
$v_1 = 0.5089$ m^3/kg
$s_1 = 6.93$ kJ/kg · K

Final state

$u_2 = 83.94$ kJ/kg
$v_2 = 1.0018 \times 10^{-3}$ m^3/kg
$s_2 = 0.2966$ kJ/kg · K

Solution:

Availability at any state can be given by,

$$A = m\left[(u-u_0)+P_0(v-v_0)-T_0(s-s_0)+\frac{V^2}{2}+gz\right]$$

Availability at initial state,

$$A_1 = 5\left[(2550-104.86)\times10^3+100\times10^3(0.5089-1.0029\times10^{-3})\right.$$
$$\left.-298(6.93-0.3673)\times10^3+\frac{(25)^2}{2}+(9.81\times10)\right]$$

$A_1 = 2704.84$ kJ.

= **2704.84 kJ** Availability at initial state **Ans.**

Availability at final state

$$A_2 = 5\left[(83.93-104.86)\times10^3+100\times10^3(1.0018\times10^{-3}-1.0029\right.$$
$$\left.\times10^{-3})-298(0.2966-0.3673)\times10^3+\frac{(10)^2}{2}+(9.81\times2)\right]$$

$A_2 = 1.09$ kJ

Availability at final state = **1.09 kJ** **Ans.**

Change in availability: $A_2 - A_1 = 1.09 - 2704.84$
$= -2703.75$ kJ

Hence availability decreases by **2703.75 kJ** **Ans.**

5. *For a steady flow process as shown below, prove that irreversibility, $I = T_0 S_{gen}$, where T_0 and P_0 are temperature and pressure at dead state.*

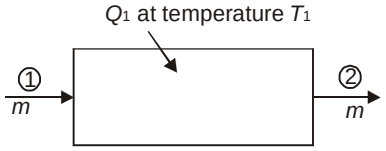


Fig. 7.9

Solution:

Let us assume changes in kinetic and potential energy to be negligible. Let us use subscript 1 for inlet and 2 for outlet.

From first law of thermodynamics;

$$Q_1 + m_1h_1 = m_2h_2, \text{ here } m_1 = m_2 = m$$

or $$Q_1 = m(h_2 - h_1)$$

From second law of thermodynamics,

$$S_{\text{gen}} + \frac{Q_1}{T_1} + ms_1 = ms_2;\ m(s_2 - s_1) = \frac{Q_1}{T_1} + S_{\text{gen}}$$

From availability considerations in control volume,

$$\left(1 - \frac{T_0}{T_1}\right) Q_1 + ma_1 = m \cdot a_2 + I$$

or, $$\left(1 - \frac{T_0}{T_1}\right) Q_1 + m\{(h_1 - h_0) - T_0(s_1 - s_0)\} = m\{(h_2 - h_0) - T_0(s_2 - s_0)\} + I$$

or,

Upon substituting from above equations,

$$m(s_2 - s_1) = \frac{Q_1}{T_1} + \frac{I}{T_0}$$

or it can be given that

$$\frac{I}{T_0} = S_{\text{gen}}$$

or, $$\boxed{I = T_0 \cdot S_{\text{gen}}}$$ Hence proved.

6. *Exhaust gases leave an internal combustion engine at 800ºC and 1 atmosphere, after having done 1050 kJ of work per kg of gas in engine. (c_p of gas = 1.1 kJ/kg · K). The temperature of surrounding is 30ºC*

(i) How much available energy per kg of gas is lost by throwing away the exhaust gases?

(ii) What is the ratio of the lost available exhaust gas energy to engine work?

Solution:

Loss of available energy = Irreversibility = $T_0 \cdot \Delta S_c$

Here, T_0 = 303 K = Temperature of surroundings

$\Delta S_c = \Delta S_s + \Delta S_e$

$$\text{Change in entropy of system} = \frac{1050}{(273+800)} = 0.9786 \text{ kJ/kg} \cdot \text{K}$$

$$\text{Change in entropy of surroundings} = \frac{-c_p \cdot (800-30)}{(273+30)}$$

$$= \frac{-1.1 \times 770}{303}$$

$$= -2.7954 \text{ kJ/kg} \cdot \text{K}$$

$$\text{Loss of available energy} = 303\ (-2.7954 + 0.9786)$$

$$= -550.49 \text{ kJ/kg}$$

$$\text{Loss of available energy} = 550.49 \text{ kJ/kg}$$

$$\text{Ratio of lost available exhaust gas energy to engine work} = \frac{550.49}{1050}$$

$$= \boxed{\frac{\mathbf{0.524}}{\mathbf{1}}} \textbf{ Ans.}$$

7. *10 kg of water undergoes transformation from initial saturated vapour at 150ºC, velocity of 25 m/s and elevation of 10 m to saturated liquid at 20ºC, velocity of 10 m/s and elevation of 3 m. Determine the availability for initial state, final state and change of availability considering environment to be at 0.1 MPa and 25ºC and g = 9.8 m/s^2.*

Solution:

Let us consider velocities and elevations to be given in reference to environment. Availability is given by

$$A = m\left[(u - u_0) + P_0(v - v_0) - T_0(s - s_0) + \frac{C^2}{2} + gz\right]$$

Dead state of water, $u_0 = 104.88$ kJ/kg

$v_0 = 1.003 \times 10^{-3}$ m^3/kg

$s_0 = 0.3674$ kJ/kg · K

For initial state of saturated vapour at 150ºC.

$u_1 = 2559.5$ kJ/kg, $v_1 = 0.3928$ m^3/kg, $s_1 = 6.8379$ kJ/kg · K

For final state of saturated liquid at 20ºC,

$u_2 = 83.95$ kJ/kg, $v_2 = 0.001002$ m^3/kg, $s_2 = 0.2966$ kJ/kg · K

Substituting in the expression for availability

Initial state availability,

$$A_1 = 10 \times [(2559.5 - 104.88) + (0.1 \times 10^3 \times (0.3928 - 0.001003) - (298.15 \times (6.8379 - 0.3674)) + \left(\frac{(25)^2}{2} \times 10^{-3}\right) + (9.81 \times 10 \times 10^{-3})]$$

$$A_1 = 5650.28 \text{ kJ}$$

Final state availability

$$A_2 = 10[(83.95 - 104.88 + (0.1 \times 10^3 \times (0.001002 - 0.001003)) - (298.15 \times (0.2966 - 0.3674)) + \left(\frac{(10)^2}{2} \times 10^{-3}\right) + (9.81 \times 3 \times 10^{-3})\Big]$$

$$A_2 = 2.5835 \text{ kJ} \simeq 2.58 \text{ kJ}$$

$$\text{Change in availability, } \Delta A = A_2 - A_1$$
$$= 2.58 - 5650.28$$
$$= -5647.70 \text{ kJ}$$

Initial availability = **5650.28 kJ**
Final availability = **2.58 kJ**
Change in availability = Decrease by **5647.70 kJ** **Ans.**

8. *A steam turbine has steam flowing at steady rate of 5 kg/s entering at 5 MPa and 500ºC and leaving at 0.2 MPa and 140ºC. During flow through turbine a heat loss of 600 kJ/s occurs to the environment at 1 atm and 25ºC. Determine*

(i) the availability of steam at inlet to turbine,
(ii) the turbine output
(iii) the maximum possible turbine output, and
(iv) the irreversibility

Neglect the changes in kinetic energy and potential energy during flow.

Solution:

Let inlet and exit states of turbine be denoted as 1 and 2.

At inlet to turbine,

$p_1 = 5$ MPa, $T_1 = 500$ºC, $h_1 = 3433.8$ kJ/kg, $s_1 = 6.9759$ kJ/kg · K

At exit from turbine.

$p_2 = 0.2$ MPa, $T_2 = 140$ºC, $h_2 = 2748$ kJ/kg, $s_2 = 7.228$ kJ/kg · K

At dead state,

$p_0 = 101.3$ kPa, $T_0 = 25$ºC, $h_0 = 104.96$ kJ/kg, $s_0 = 0.3673$ kJ/kg · K

Availability of steam at inlet, $A_1 = m[(h_1 - h_0) - T_0 (s_1 - s_0)]$

$$A_1 = 5 [(3433.8 - 104.96) - 298.15 (6.9759 - 0.3673)]$$
$$A_1 = 6792.43 \text{ kJ}$$

Availability of steam at inlet = **6792.43 kJ** **Ans.**

Applying first law of thermodynamics

$$Q + mh_1 = mh_2 + W.$$
$$W = m(h_1 - h_2) - Q$$
$$= 5(3433.8 - 2748) - 600$$
$$W = 2829 \text{ kJ/s}$$

Turbine output = **2829 kW** **Ans.**

Maximum possible turbine output will be available when irreversibility is zero.

$$W_{rev} = W_{max} = A_1 - A_2$$
$$= m [(h_1 - h_2) - T_0(s_1 - s_2)]$$
$$= 5[(3433.8 - 2748) - 298.15 (6.9759 - 7.228)]$$
$$W_{max} = 3053.18 \text{ kJ/s}$$

Maximum output = **3053.18 kW** **Ans.**

Irreversibility can be estimated by the difference between the maximum output and turbine output.

$$I = W_{max} - W = 224.18 \text{ kJ/s}$$

Irreversibility = **224.18 kW** **Ans.**

9. *Show that the sublimation line and vaporization lines have different slopes at triple point on the phase diagram of water.*

Solution:

It is desired to show that slope of sublimation line shown by 0–1 is different than vaporization line 1 – 2.

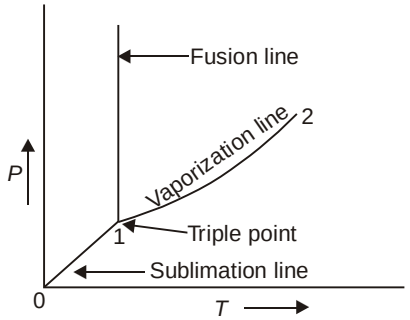


Fig 7.10

To show the slope let us find $\frac{dp}{dT}$ values at triple point 1. Here *i, f, g* subscripts refer to ice, water and steam states,

By Clapeyron equation.

$$\left(\frac{dp}{dT}\right)_{0-1} - \left(\frac{dp}{dT}\right)_{1-2} = \frac{s_{ig}}{v_{ig}} - \frac{s_{fg}}{v_{fg}}$$

For triple point state $s_{ig} = s_{if} + s_{fg}$

and $v_{ig} = v_{if} + v_{fg}$

Substituting in above slope difference expression

$$\left(\frac{dp}{dT}\right)_{0-1} - \left(\frac{dp}{dT}\right)_{1-2} = \left(\frac{s_{if} + s_{fg}}{v_{if} + v_{fg}}\right) - \frac{s_{fg}}{v_{fg}}$$

$$= \frac{s_{if} \cdot v_{fg} + s_{fg} \cdot v_{fg} - s_{fg} v_{if} - s_{fg} v_{fg}}{(v_{if} + v_{fg}) \cdot v_{fg}}$$

$$= \frac{s_{if} \cdot v_{fg} - s_{fg} \cdot v_{if}}{(v_{if} + v_{fg}) \cdot v_{fg}}$$

It is seen that $v_{if} <<< v_{fg}$ but the order of s_{if} being less than s_{fg} is not very small as compared to $v_{if} <<< v_{fg}$. Neglecting smaller terms by order of magnitude

$$\left(\frac{dp}{dT}\right)_{0-1} - \left(\frac{dp}{dT}\right)_{1-2} = \frac{s_{if}}{v_{fg}}$$

Here, s_{if} and v_{fg} both are positive quantities so the ratio $\left(\frac{s_{if}}{v_{fg}}\right)$ is also positive and hence difference of slopes between sublimation line and vaporization line is positive. Thus, it shows that slope of sublimation line and vaporization line are different.

10. *Obtain the expression for change in internal energy of gas obeying the Vander Waals equation of state.*

Solution:

Van der Waals equation of state can be given as under,

$$\left(p + \frac{a}{v^2}\right)(v - b) = \text{RT} \Rightarrow p = \frac{RT}{v-b} - \frac{a}{v^2} \Rightarrow \frac{RT}{v-b} = p + \frac{a}{v^2}$$

Differentiating this equation of state, partially w.r.t. T at constant volume,

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b}$$

General expression for change in internal energy can be given as under,

$$du = C_v\, dT + \left\{T\left(\frac{\partial p}{\partial T}\right)_v - p\right\} dv$$

Substituting in the expression for change in internal energy

$$du = C_v \cdot dT + \left\{T \cdot \frac{R}{(v-b)} - p\right\} dv$$

Substituting for $\left(\frac{RT}{v-b}\right)$ is expression of du,

$$du = C_v \cdot dT + \left\{p + \frac{a}{v^2} - p\right\} dv$$

$$du = C_v \cdot dT + \frac{a}{v^2}\, dv$$

The change in internal energy between states 1 and 2,

$$\int_1^2 du = u_2 - u_1 = \int_1^2 C_v\, dT - a\left(\frac{1}{v_2} - \frac{1}{v_1}\right)$$

$$\boxed{u_2 - u_1 = \int_1^2 C_v .dT - a\left(\frac{1}{v_2} - \frac{1}{v_1}\right)} \quad \textbf{Ans.}$$

11. *500 kJ of heat is removed from a constant temperature heat reservoir maintained at 835 K. Heat is received by a system at constant temperature of 720 K. Temperature of the surroundings, the lowest available temperature is 280 K. Determine the net loss of available energy as a result of this irreversible heat transfer.* *[U.P.S.C. 1992]*

Solution:

Here, T_0 = 280 K, i.e surrounding temperature.

Availability for heat reservoir = $T_0 \cdot \Delta S_{reservoir}$

$$= 280 \times \frac{500}{835}$$

$$= 167.67 \text{ kJ/kg} \cdot \text{K}$$

Availability for system = $T_0 \cdot \Delta S_{system}$

$$= 280 \times \frac{500}{720}$$

$$= 194.44 \text{ kJ/kg} \cdot \text{K}$$

Net loss of available energy = (167.67 – 194.44)

$$= -26.77 \text{ kJ/kg} \cdot \text{K}$$

Loss of available energy = **26.77 kJ/kg · K** **Ans.**

12. *Steam flows through an adiabatic steady flow turbine. The enthalpy at entrance is 4142 kJ/kg and at exit 2585 kJ/kg. The values of flow availability of steam at entrance and exit are 1787 kJ/kg and 140 kJ/kg respectively, dead state temperature T_0 is 300 K, determine per kg of steam, the actual work, the maximum possible work for the given change of state of steam and the change in entropy of steam. Neglect changes in kinetic and potential energy.* *[U.P.S.C. 1993]*

Solution:

Here dead state is given as 300 K and the maximum possible work for given change of state of steam can be estimated by the difference of flow availability as given under:

W_{max} = 1787 – 140 = 1647 kJ/kg

Actual work from turbine, W_{actual} = 4142 – 2585

Actual work = 1557 kJ/kg

Actual work = **1557 kJ/kg**
Maximum possible work = **1647 kJ/kg** **Ans.**

13. *What shall be second law efficiency of a heat engine having efficiency of 0.25 and working between reservoirs of 500ºC and 20ºC?*

Solution:

$$\text{Reversible engine efficiency, } \eta_{rev} = 1 - \frac{T_{min}}{T_{max}} = 1 - \frac{293}{773} = 0.6209$$

$$\text{Second law efficiency} = \frac{\eta}{\eta_{rev}} = \frac{0.25}{0.6209} = 0.4026 \text{ or } 40.26\%$$

$$= \mathbf{40.26\%} \quad \textbf{Ans.}$$

14. *An adiabatic cylinder of volume 10 m^3 is divided into two compartments A and B each of volume 6 m^3 and 4 m^3 respectively, by a thin sliding partition. Initially the compartment A is filled with air at 6 bar and 600 K, while there is a vacuum in the compartment B. Air expands and fills both the compartments.Calculate the loss in available energy. Assume atmosphere is at 1 bar and 300 K.*

[U.P.S.C. 1997]

Solution:

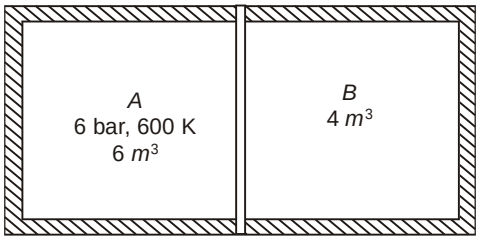


Fig 7.11

Here $T_0 = 300$ K, $P_0 = 1$ bar

$V_A = 6\ m^3$, $V_B = 4\ m^3$

$P_1 = 6$ bar, $T_1 = 600$ K

Initially, $V_1 = V_A = 6m^3$, and finally, $V_2 = V_A + V_B = 10m^3$

Expansion occurs in adiabatic conditions.

Temperature after expansion can be obtained by considering adiabatic expansion.

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$T_2 = 600\left(\frac{6}{10}\right)^{(1.4-1)} = 489.12 \text{ K}$$

$$\text{Mass of air, } m = \frac{P_1V_1}{RT_1} = \frac{6\times10^5\times6}{287\times600} = 20.91 \text{ kg}$$

Change in entropy of control system, $(S_2 - S_1) = mc_v \ln \frac{T_2}{T_1} + mR \ln \frac{V_2}{V_1}$

$$\Delta S_S = (S_2 - S_1) = 20.91\left\{0.718\times\ln\left(\frac{489.12}{600}\right) + 0.287\ln\left(\frac{10}{6}\right)\right\}$$

$$= -2.01 \times 10^{-3} \text{ kJ/K}$$

Here, there is no change in entropy of environment, $\Delta S_e = 0$

Total entropy change of combined system = $\Delta S_c = \Delta S_s + \Delta S_e$

$$= -2.01 \times 10^{-3} \text{ kJ/K}$$

Loss of available energy = Irreversibility = $T_0 \cdot \Delta S_c$

$$= 300 \times (-2.01 \times 10^{-3})$$

$$= -0.603 \text{ kJ}$$

Loss of available energy = **0.603 kJ** **Ans.**

15. *Prove that ideal gas equation satisfies the cyclic relation.*

Solution:

Ideal gas equation, $Pv = RT$

Let us consider two variable (v, T) to be independent and P as dependent variable.

$$P = P(v, T) = \frac{RT}{v}$$

By cyclic relation,

$$\boxed{\left(\frac{\partial P}{\partial v}\right)_T \cdot \left(\frac{\partial v}{\partial T}\right)_p \cdot \left(\frac{\partial T}{\partial P}\right)_v = -1}$$

Let us find the three partial derivatives separately and then substitute.

$$\left(\frac{\partial P}{\partial v}\right)_T = \frac{-RT}{v^2}, \left(\frac{\partial v}{\partial p}\right)_p = \frac{R}{p}, \left(\frac{\partial T}{\partial P}\right)_v = \frac{v}{R}$$

Substituting

$$\left(\frac{-RT}{v^2}\right)\left(\frac{R}{P}\right)\left(\frac{v}{R}\right) = \frac{-RT}{Pv} = -1 \text{ Hence proved.}$$

16. *A heat engine is working between 700° C and 30°C. The temperature of surroundings is 17°C. Engine receives heat at the rate of 2×10^4 kJ/min and the measured output of engine is 0.13 MW. Determine the availability, rate of irreversibility and second law efficiency of engine.*

Solution:

$$\text{Availability or reversible work, } W_{rev} = \eta_{rev} \cdot Q_1 = \left(1 - \frac{303}{573}\right) \times 2 \times 10^4$$

$$= 1.38 \times 10^4 \text{ kJ/min}$$

$$\text{Rate of irreversibility, } I = W_{rev} - W_{useful}$$

$$= \left(\frac{1.38\times10^4}{60} - 0.13\times10^3\right) = 100 \text{ kJ/s}$$

$$\text{Second law efficiency} = \frac{W_{useful}}{W_{rev}}$$

$$= \frac{0.13\times10^3}{\left(\frac{1.38\times10^4}{60}\right)} = 0.5652 \text{ or } 56.52\%$$

Availability = **1.38×10^4 kJ/min,**
Rate of irreversibility = **100 kW,** Second law efficiency = **56.52%** **Ans.**

17. *A rigid tank contains air at 1.5 bar and 60°C. The pressure of air is raised to 2.5 bar by transfer of heat from a constant temperature reservoir at 400°C. The temperature of surroundings is 27°C. Determine per kg of air, the loss of available energy due to heat transfer.* *[U.P.S.C. 1998]*

Solution:

Loss of available energy = Irreversibility = $T_0 \cdot \Delta S_c$

$T_0 = 300$ K, $\Delta S_c = \Delta S_s + \Delta S_e$

Change in entropy of system = ΔS_s

Change in entropy of environment/surroundings $= \Delta S_e$

Here heat addition process causing rise in pressure from 1.5 bar to 2.5 bar occurs isochorically. Let initial and final states be given by subscript 1 and 2.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2},\ T_1 = 333\text{ K},\ T_2 = ?,\ P_1 = 1.5\text{ bar},\ P_2 = 2.5\text{ bar}$$

$$T_2 = \frac{2.5 \times 333}{1.5} = 555\text{ K}$$

Heat addition to air in tank,

$$Q = m \cdot c_p \cdot \Delta T = 1 \times 1.005 \times (555 - 333)$$

$$Q = 223.11\text{ kJ/kg}$$

$$\Delta S_s = \frac{Q}{T_1} = \frac{223.11}{333} = 0.67\text{ kJ/kg} \cdot \text{K}$$

$$\Delta S_e = \frac{-Q}{T_{\text{reservoir}}} = \frac{-223.11}{673} = -0.332\text{ kJ/kg} \cdot \text{K}$$

$$\Delta S_c = 0.67 - 0.332$$

$$\Delta S_c = 0.338\text{ kJ/kg} \cdot \text{K}$$

$$\text{Loss of available energy} = 300 \times (0.338)$$

$$= 101.4\text{ kJ/kg}$$

Loss of available energy = **101.4 kJ/kg** **Ans.**

18. *Using the Maxwell relation derive the following $T \cdot ds$ equation, $T \cdot ds = C_p \cdot dT - T \cdot \left(\frac{\partial v}{\partial T}\right)_p dp$*

[U.P.S.C. 1998]

Solution:

Let $s = s(T, p)$

$$ds = \left(\frac{\partial s}{\partial T}\right)_p \cdot dT + \left(\frac{\partial s}{\partial p}\right)_T \cdot dp$$

or, $$T \cdot ds = T \cdot \left(\frac{\partial s}{\partial T}\right)_p \cdot dT + T \cdot \left(\frac{\partial s}{\partial p}\right)_T \cdot dp$$

Using Maxwell's relation, $\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$

and $$T \cdot \left(\frac{\partial s}{\partial T}\right)_p = C_p$$

Substitution yields,

$$\boxed{T \cdot ds = C_p \cdot dT - T \cdot \left(\frac{\partial v}{\partial T}\right)_p \cdot \partial p}$$ Hence proved

19. *Determine the enthapy of vaporization of water at 200ºC using Clapeyron equation. Compare it with tabulated value.*

Solution:

Clapeyron equation says, $h_{fg} = T \cdot v_{fg} \cdot \left(\frac{dp}{dT}\right)_{sat}$

From steam tables

$$v_{fg} = (v_g - v_f)_{\text{at } 200^oC} = (0.12736 - 0.001157) = 0.126203 \text{ m}^3/\text{kg}$$

Let us approximate,

$$\left(\frac{dp}{dT}\right)_{\text{sat, } 200^oC} = \left(\frac{\Delta p}{\Delta T}\right)_{\text{sat, } 200^oC} = \frac{P_{\text{sat at } 205^oC} - P_{\text{sat at } 195^oC}}{(205 - 195)}$$

$$= \frac{(1.7230 - 1.3978)}{10} = 0.03252 \text{ MPa/}^oC$$

Substituting in Clapeyron equation,

$$h_{fg} = (273 + 200) \times 0.126203 \times 0.03252 \times 10^3$$
$$= 1941.25 \text{ kJ/kg}$$

Calculated enthalpy of vaporization = **1941.25 kJ/kg.**
Enthalpy of vaporization from steam table = **1940.7 kJ/kg.** **Ans.**

20. *Determine h_{fg} of R –12 refrigerant at – 10ºC using both Clapeyron equation and the Clapeyron-Clausius equation. Give the deviation in %. Take*

$P_{\text{sat at } -5^oC} = 260.96$ kPa
$P_{\text{sat at } -15^oC} = 182.60$ kPa.
$v_{g \text{ at } -10^oC} = 0.07665 \text{ m}^3/\text{kg}$
$v_{f \text{ at } -10^oC} = 0.00070 \text{ m}^3/\text{kg}$
$R = 0.06876 \text{ kJ/kg} \cdot \text{K}$
$h_{fg \text{ at } -10^oC} = 156.3$ kJ/kg from tables.

Solution:

By Clapeyron equation

$$h_{fg} = T \cdot v_{fg} \left\{\frac{dP}{dT}\right\}_{sat}$$

$$= T \cdot (v_g - v_f) \left\{\frac{\Delta P}{\Delta T}\right\}$$

$$= (-5 + 273) \times (0.07665 - 0.0007) \times \left(\frac{P_{\text{sat at } -5^oC} - P_{\text{sat at } -15^oC}}{(-5 - (-15))}\right)$$

$$= 268 \times 0.07595 \times \frac{(260.96 - 182.60)}{10}$$

$$h_{fg} = 159.49 \text{ kJ/kg}$$

By Clapeyron-Clausius equation,

$$\ln\left(\frac{P_2}{P_1}\right)_{sat} = \frac{h_{fg}}{R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]_{sat}$$

$$\ln\left(\frac{P_{\text{sat at}-5^\circ C}}{P_{\text{sat at}-15^\circ C}}\right) = \frac{h_{fg}}{R}\left[\frac{1}{T_{\text{sat at}-15^\circ C}}-\frac{1}{T_{\text{sat at}-5^\circ C}}\right]$$

$$\ln\left(\frac{260.96}{182.60}\right) = \frac{h_{fg}}{0.06876}\times\left[\frac{1}{(-15+273)}-\frac{1}{(-5+273)}\right]$$

$\Rightarrow \quad h_{fg} = 169.76$ kJ/kg

% deviation from Clapeyron equation

$$= \left(\frac{169.76-159.49}{159.49}\right)\times 100$$

$= 6.44\%$

h_{fg} by Clapeyron equation = **159.49 kJ/kg**
h_{fg} by Clapeyron-Clausius equation = **169.76 kJ/kg**
% deviation in h_{fg} value by Clapeyron-Clausius equation compared to the value from Clapeyron equation = **6.44%** **Ans.**

21. *Determine the volume expansivity and isothermal compressibility of steam at 300 kPa and 300ºC.*

Solution:

$$\text{Volume expansivity} = \frac{1}{v}\left[\frac{\partial v}{\partial T}\right]_P$$

$$\text{Isothermal compressibility} = \frac{-1}{v}\left[\frac{\partial v}{\partial P}\right]_T$$

Let us write $\frac{\partial v}{\partial T} = \frac{\Delta v}{\Delta T}$ and $\frac{\partial v}{\partial P} = \frac{\Delta v}{\Delta P}$. The differences may be taken for small pressure and temperature changes.

Volume expansivity,

$$= \frac{1}{v}\left[\frac{\partial v}{\partial T}\right]_{300\text{ kpa}}$$

$$= \frac{1}{v_{\text{at 300 kpa, 300°C}}}\left[\frac{v_{350^\circ C}-v_{250^\circ C}}{(350-250)}\right]_{300\text{ kpa}}$$

$$= \frac{1}{0.8753}\left[\frac{0.9534-0.7964}{100}\right]_{300\text{ kpa}}$$

Volume expansivity = **1.7937×10^{-3} K^{-1}** **Ans.**

$$= \frac{-1}{v_{\text{at } 300\text{kpa},\ 300^\circ\text{C}}} \left[\frac{v_{350\text{kpa}} - v_{250\text{kpa}}}{(350 - 250)} \right]_{300^\circ\text{C}}$$

$$= \frac{-1}{0.8753} \left[\frac{0.76505 - 1.09575}{100} \right]$$

$$= 3.778 \times 10^{-3}\ \text{KPa}^{-1}$$

Isothermal compressibility = **3.778×10^{-3} kPa^{-1}** **Ans.**

22. *An evacuated tank of 0.5 m^3 is filled by atmospheric air at 1 bar till the pressure inside tank becomes equal to atmospheric temperature. Considering filling of tank to occur adiabatically determine final temperature inside the tank and also the irreversibility and change in entropy. Take atmospheric temperature as 25ºC.*

Solution:

Filling of the tank is a transient flow (unsteady flow) process. For the transient filling process, considering subscripts '*i*' and '*f*' for initial and final states,

$$h_i = u_f$$

$$c_p T_i = c_v T_f$$

$$T_f = \frac{c_p}{c_v} T_i$$

$$T_f = \frac{1.005}{0.718} \times 298.15$$

Inside final temperature, T_f = **417.33 · K** **Ans.**

Change in entropy $\Delta S_{\text{gen}} = (S_f - S_i) + \Delta S_{\text{surr}}$

$$= c_p \ln \frac{T_f}{T_i} + 0$$

$$= 1.005 \times \ln \frac{417.33}{298.15}$$

Change in entropy ΔS_{gen} = **0.3379 kJ/kg · K** **Ans.**

Irreversibility, $I = T_0 \cdot \Delta S_{\text{gen}}$

$$= 298.15 \times 0.3379$$

Irreversibility, I = **100. 74 kJ/kg** **Ans.**

23. *A closed vessel stores 75 kg of hot water at 400ºC. A heat engine transfers heat from the hot water to environment which is maintained at 27ºC. This operation of heat engine changes temperature of hot water from 400ºC to 27ºC over a finite time. Determine the maximum possible work output from engine. Take specific heat of water as 4.18 kJ/kg· K.*

Solution:

Here the combined closed system consists of hot water and heat engine. Here there is no thermal reservoir in the system under consideration. For the maximum work output, irreversibility = 0

Therefore, $\frac{d}{dt}(E - T_0\ S) = W_{max}$

or $W_{max} = (E - T_0\ S)_1 - (E - T_0\ S)_2$

Here $E_1 = U_1 = m\ c_p\ T_1$, $E_2 = U_2 = m\ c_p\ T_2$

$T_1 = 400 + 273 = 673$ K, $T_2 = 27 + 273 = 300$ K $= T_0$

Therefore,

$$W_{max} = mc_p\ (T_1 - T_2) - T_0(S_1 - S_2)$$

$$= mc_p\ (T_1 - T_2) - T_0 \cdot m \cdot c_p \left(\ln \frac{T_1}{T_2}\right)$$

$$= 75 \times 4.18 \times \left\{(673 - 300) - 300 \times \ln\left(\frac{673}{300}\right)\right\}$$

$$= 40946.6 \text{ kJ}$$

Maximum work = **40946.6 kJ** **Ans.**

24. *In a steam turbine the steam enters at 50 bar, 600ºC and 150 m/s and leaves as saturated vapour at 0.1 bar, 50 m/s. During expansion, work of 1000 kJ/kg is delivered. Determine the inlet stream availability, exit stream availability and the irreversibility. Take dead state temperature as 25ºC.*

Solution:

$h_1 = h_{\text{at 50 bar, 600ºC}} = 3666.5$ kJ/kg, $s_1 = s_{\text{at 50 bar, 600ºC}} = 7.2589$ kJ/kg · K

$h_2 = h_{g \text{ at 0.1 bar}} = 2584.7$ kJ/kg, $s_2 = s_{g \text{ at 0.1 bar}} = 8.1502$ kJ/kg · K

$$\text{Inlet stream availability} = \left(h_1 + \frac{c_1^2}{2}\right) - T_0 s_1$$

$$= \left(3666.5 + \frac{(150)^2 \times 10^{-3}}{2}\right) - (298 \times 7.2589)$$

$$= 1514.59 \text{ kJ/kg}$$

Input stream availability is equal to the input absolute availability.

$$\text{Exit stream availability} = \left(h_2 + \frac{c_2^2}{2}\right) - T_0\ s_2$$

$$= \left(2584.7 + \frac{(50)^2 \times 10^{-3}}{2}\right) - (298 \times 8.1502)$$

$$= 157.19 \text{ kJ/kg}$$

Exit stream availability is equal to the exit absolute availability.

$$W_{rev} = 1514.59 - 157.19 = 1357.4 \text{ kJ/kg}$$

$$\text{Irreversibility} = W_{rev} - W = 1357.4 - 1000 = 357.4 \text{ kJ/kg}$$

This irreversibility is in fact the availability loss.

Inlet stream availability = **1514.59 kJ/kg**
Exit stream availability = **157.19 kJ/kg**
Irreversibility = **357.4 kJ/kg** **Ans.**

EXERCISE

7.1 Define 'available energy' and 'unavailable energy'.

7.2 What do you understand by second law efficiency? How does it differ from first law efficiency?

7.3 What is meant by a dead state? Discuss its' importance.

7.4 Define availability. Obtain an expression for availability of closed system.

7.5 Differentiate between useful work and maximum useful work in reference to the availability.

7.6 What do you understand by Gibbs function? How does it differ from the availability function?

7.7 Describe the Helmholtz function.

7.8 What are Maxwell relations? Discuss their significance?

7.9 Describe Clapeyron equation.

7.10 What do you understand by Joule-Thomson coefficient? Explain.

7.11 Describe chemical potential.

7.12 Write short notes on the following:

(*i*) Clapeyron-Clausius equation,

(*ii*) Volume expansivity

(*iii*) Fugacity,

(*iv*) Second law analysis of engineering systems.

7.13 Determine the loss of availability when 1 kg air at 260ºC is reversibly and isothermally expanded from 0.145 m^3 initial volume to 0.58 m^3 final volume. [70.56 kJ/kg]

7.14 Determine the entropy generation and decrease in available energy when a heat source of 727ºC transfers heat to a body at 127ºC at the rate of 8.35 MJ/min. Consider the temperature of sink as 27ºC. [12.54 kJ/K · min, 3762 kJ]

7.15 Determine the available energy of furnace having the gases getting cooled from 987ºC to 207ºC at constant temperature while the temperature of surroundings is 22ºC. [–518.1 kJ/kg]

7.16 Determine the available amount of energy removed and the entropy increase of universe when 5 kg air at 1.38 bar, 500 K is cooled down to 300 K isobarically. The temperature of surroundings may be taken as 4ºC. [–268.7 kJ. 3.316 kJ/K]

7.17 Determine the entropy change, unavailable energy and available energy for the process in which 2 kg air is heated isobarically so as to cause increase in its temperature from 21ºC to 315ºC. Take T_0 = 10ºC. [1.393 kJ/K, 394.2 kJ, 196.6 kJ]

7.18 Steam enters in a steam turbine at 60 bar, 500ºC and leaves at 0.1 bar, 0.89 dry with a flow rate of 3.2652 × 10^4 kg/hr. Determine the loss of available energy. [1286.2 kJ/s]

7.19 Determine the available portion of heat removed from 2.5 kg air being cooled from 2.1 bar, 205ºC to 5ºC at constant volume. The heat is rejected to surroundings at – 4ºC. [– 97.2 kJ]

7.20 Prove that heat is an inexact differential. {Q (T, s)}.

7.21 Derive an expression for change in entropy of a gas obeying Vander Waals equation of state.

7.22 Determine the coefficient of thermal expansion and coefficient of isothermal compressibility for a gas obeying Vander Waals equation of state.

7.23 Determine the second law efficiency of a heat engine operating between 700ºC and 30ºC. The heat engine has efficiency of 0.40. [55.74%]

7.24 Determine the amount of heat that can be converted to the useful work if total heat at 1000 kJ is available at 500ºC. The temperature of environment is 17ºC. [624.84 kJ]

7.25 Determine the change in availability of air contained in an insulated vessel of 20×10^3 cm^3. The initial state of air is 1 bar. 40ºC. The air is heated so as to arrive at temperature of 150ºC. The temperature of surrounding environment may be considered as 20ºC. [0.027 kJ]

7.26 Determine the enthalpy of vaporization of water at 50ºC using the Clapeyron equation.

[2396.44 kJ/kg]

7.27 Determine the % variation in the enthalpy of vaporization of steam at 500 kPa using Clapeyron equation as compared to value in steam table. [0.201%]

7.28 Air enters a compressor at 40ºC, 500 kPa for being compressed upto 2000 kPa. Consider the compression to be at constant temperature without internal irreversibilities. Air flows into compressor at 6 kg/min. Neglecting the changes in kinetic energy and potential energy determine the availability transfers accompanying heat and work and irreversibility. Take T_0 = 25ºC, P_0 = 1 bar. The control volume may be taken as under,

(*i*) Control volume comprises of compressor alone

(*ii*) Control volume comprises of compressor and its immediate surroundings so that heat transfer occurs at T_0.

[(*i*) – 0.597 kJ/s, –12.46 kJ/s, 0.597 kJ/s

(*ii*) 0 kJ/s, –12.46 kJ/s, 0.597 kJ/s]

7.29 Steam expands in a cylinder from 22 bar, 450ºC to 4.5 bar, 250ºC. The environment may be considered to be at 1 bar, 25ºC. Determine

(*i*) the availability of steam at inlet, exit and change in availability.

(*ii*) the irreversibilities in the expansion process. [196 kJ, 132 kJ, –64 kJ, 5.5 kJ]

7.30 In a steam power cycle steam enters at 60 bar, 500ºC into turbine and leaves at 0.04 bar. The isentropic efficiency of turbine is 85% and that of pump is 70%. Considering the environment to have T_0 = 25ºC, P_0 =1 bar, determine the second law efficiency of constituent components i.e. boiler, turbine, condenser and pump. [67.6%, 99%, 84.4%, 65%]

8
Vapour Power Cycles

8.1 INTRODUCTION

Thermodynamic cycles can be primarily classified based on their utility such as for power generation, refrigeration etc. Based on this thermodynamic cycles can be categorized as;

(*i*) Power cycles,

(*ii*) Refrigeration and heat pump cycles.

(*i*) ***Power cycles:*** Thermodynamic cycles which are used in devices producing power are called power cycles. Power production can be had by using working fluid either in vapour form or in gaseous form. When vapour is the working fluid then they are called vapour power cycles, whereas in case of working fluid being gas these are called gas power cycles. Thus, power cycles shall be of two types,

(*a*) Vapour power cycle,

(*b*) Gas power cycle.

Vapour power cycles can be further classified as,

1. Carnot vapour power cycle
2. Rankine cycle
3. Reheat cycle
4. Regenerative cycle.

Gas power cycles can be classified as,

1. Carnot gas power cycle
2. Otto cycle
3. Diesel cycle
4. Dual cycle
5. Stirling cycle
6. Ericsson cycle
7. Brayton cycle

Here in the present text Carnot, Rankine, reheat and regenerative cycles are discussed.

(*ii*) ***Refrigeration and heat pump cycles:*** Thermodynamic cycles used for refrigeration and heat pump are under this category. Similar to power cycles, here also these cycles can be classified as “air cycles” and “vapour cycles” based on type of working fluid used.

8.2 PERFORMANCE PARAMETERS

Some of commonly used performance parameters in cycle analysis are described here.

Thermal efficiency: Thermal efficiency is the parameter which gauges the extent to which the energy input to the device is converted to net work output from it.

$$\text{Thermal efficiency} = \frac{\text{Net work in cycle}}{\text{Heat added in cycle}}$$

Heat rate: Heat rate refers to the amount of energy added by heat transfer to cycle to produce unit net work output. Usually energy added may be in kcal, unit of net work output in kW h and unit of heat rate may be in kcal/kW h. It is inverse of thermal efficiency.

Back work ratio: Back work ratio is defined as the ratio of pump work input (–ve work) to the work produced (+ve work) by turbine.

$$\text{Back work ratio} = \frac{W_{\text{pump}}}{W_{\text{turbine}}}$$

Generally, back work ratio is less than one and as a designer one may be interested in developing a cycle which has smallest possible back-work ratio. Small back-work ratio indicates smaller pump work (–ve work) and larger turbine work (+ve work).

Work ratio: It refers to the ratio of net work to the positive work.

$$\text{Mathematically, work ratio} = \frac{W_{\text{net}}}{W_{\text{turbine}}}$$

Specific steam consumption: It indicates the steam requirement per unit power output. It is generally given in kg/kW. h and has numerical value lying from 3 to 5 kg/kW. h

$$\text{Specific steam consumption} = \frac{3600}{W_{\text{net}}}, \text{ kg/kW.h}$$

8.3 CARNOT VAPOUR POWER CYCLE

Carnot cycle has already been defined earlier as an ideal cycle having highest thermodynamic efficiency. Let us use Carnot cycle for getting positive work with steam as working fluid. Arrangement proposed for using Carnot vapour power cycle is as follows.

1 – 2 = Reversible isothermal heat addition in the boiler

2 – 3 = Reversible adiabatic expansion in steam turbine

3 – 4 = Reversible isothermal heat rejection in the condenser

4 – 1 = Reversible adiabatic compression or pumping in feed water pump

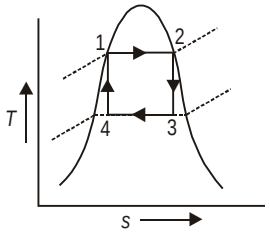


Fig 8.1 *Carnot vapour power cycle*

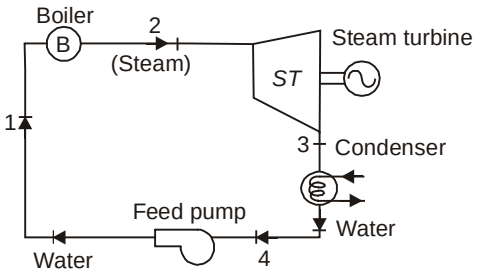


Fig 8.2 *Arrangement for Carnot cycle*

Assuming steady flow processes in the cycle and neglecting changes in kinetic and potential energies, thermodynamic analysis may be carried out.

$$\text{Thermal efficiency} = \frac{\text{Net work}}{\text{Heat added}}$$

Net work = Turbine work – Compression/Pumping work

For unit mass flow.

$$W = (h_2 - h_3) - (h_1 - h_4)$$

$$\text{Heat added, } Q_{add} = (h_2 - h_1)$$

$$\eta_{Carnot} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$$

$$= 1 - \frac{h_3 - h_4}{h_2 - h_1}$$

Here heat rejected, $Q_{rejected} = (h_3 - h_4)$

or $$\eta_{Carnot} = 1 - \frac{Q_{rejected}}{Q_{add}}$$

Also, heat added and rejected may be given as function of temperature and entropy as follows:

$$Q_{add} = T_1 \times (s_2 - s_1)$$

$$Q_{rejected} = T_3 \times (s_3 - s_4)$$

Also, $s_1 = s_4$ and $s_2 = s_3$

Therefore, substituting values: $\eta_{Carnot} = 1 - \frac{T_3}{T_1}$

or $$\eta_{Carnot} = 1 - \frac{T_{minimum}}{T_{maximum}}$$

Let us critically evaluate the processes in Carnot cycle and see why it is not practically possible.

1–2: Reversible Isothermal Heat Addition

Isothermal heat addition can be easily realised in boiler within wet region as isothermal and isobaric lines coincide in wet region. But the superheating of steam can't be undertaken in case of Carnot cycle as beyond saturated steam point isothermal heat addition can't be had inside boiler. This fact may also be understood from *T–S* diagram as beyond 2 the constant pressure line and constant temperature lines start diverging. It may be noted that boiler is a device which generates steam at constant pressure.

2–3: Reversible Adiabatic Expansion

Saturated steam generated in boiler at state '2' is sent for adiabatic expansion in steam turbine upto state 3. During this expansion process positive work is produced by steam turbine and a portion of this work available is used for driving the pump.

3–4: Reversible Isothermal Heat Rejection

Heat release process is carried out from state 3 to 4 in the condenser. Condenser is a device in which constant pressure heat rejection can be realized. Since expanded steam from steam turbine is available in wet region at state 3. Therefore, constant temperature heat rejection can be had as constant temperature and constant pressure lines coincide in wet region.

Heat rejection process is to be limited at state 4 which should be vertically below state 1. Practically it is very difficult to have such kind of control.

4–1: Reversible Adiabatic Compression (Pumping)

Carnot cycle has reversible adiabatic compression process occurring between 4 and 1, which could be considered for pumping of water into boiler.

In fact it is very difficult for a pump to handle wet mixture which undergoes simultaneous change in its phase as its pressure increases.

Above discussion indicates that Carnot vapour power cycle is merely theoretical cycle and cannot be used for a practical working arrangement. Also the maximum efficiency of Carnot cycle is limited by maximum and minimum temperatures in the cycle. Highest temperature attainable depends upon metallurgical limits of boiler material.

8.4 RANKINE CYCLE

Rankine cycle is a thermodynamic cycle derived from Carnot vapour power cycle for overcoming its limitations. In earlier discussion it has been explained that Carnot cycle cannot be used in practice due to certain limitations. Rankine cycle has the following thermodynamic processes.

1 – 2 = Isobaric heat addition (in boiler)

2 – 3 = Adiabatic expansion (in turbine)

3 – 4 = Isobaric heat release (in condenser)

4 – 1 = Adiabatic pumping (in pump)

T – S, *h – S* and *P – V* representations are as shown below.

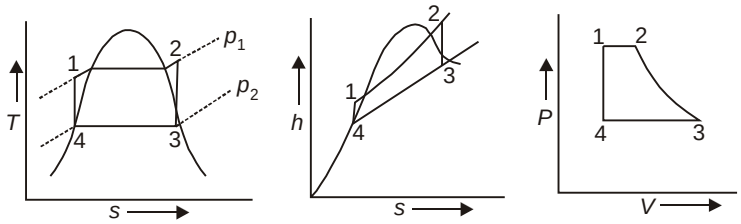


Fig. 8.3 *T–s, h–S and P–V representations of Rankine cycle*

Practical arrangement in a simple steam power plant working on Rankine cycle is shown ahead.

Thus in Rankine cycle, isothermal heat addition and heat rejection processes have been replaced by isobaric processes. Realization of 'isobaric heat addition' and 'heat rejection' in 'boiler' and 'condenser' respectively is in conformity with nature of operation of these devices. Isobaric heat addition can be had in boiler from subcooled liquid to superheated steam without any limitations.

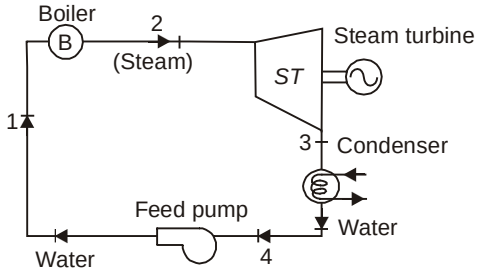


Fig. 8.4 *Simple steam power plant layout*

Let us understand the arrangement.

1 – 2: High pressure water supplied by feed pump is heated and transformed into steam with or without superheat as per requirement. This high pressure and temperature steam is sent for expansion in steam turbine. Heat added in boiler, for unit mass of steam.

$$Q_{add} = (h_2 - h_1)$$

2 – 3: Steam available from boiler is sent to steam turbine, where it's adiabatic expansion takes place and positive work is available. Expanded steam is generally found to lie in wet region. Expansion of steam is carried out to the extent of wet steam having dryness fraction above 85% so as to avoid condensation of steam on turbine blades and subsequently the droplet formation which may hit hard on blade with large force.

Turbine work, for unit mass, $W_{turbine} = (h_2 - h_3)$.

3 – 4: Heat rejection process occurs in condenser at constant pressure causing expanded steam to get condensed into saturated liquid at state 4.

Heat rejected in condenser for unit mass, $Q_{rejected} = (h_3 - h_4)$

4 – 1: Condensate available as saturated liquid at state 4 is sent to feed pump for being pumped back to boiler at state 1.

For unit mass, Pump work $W_{pump} = h_1 - h_4$.

Here pumping process is assumed to be adiabatic for the sake of analysis whereas it is not exactly adiabatic in the pump.

From first and second law combined together;

$$dh = T \cdot ds + v \cdot dp.$$

Here in this adiabatic pumping process. $ds = 0$

Therefore $$dh = v \cdot dp.$$

or $$(h_1 - h_4) = v_4 (p_1 - p_4)$$

or $$(h_1 - h_4) = v_4 (p_1 - p_3). \{\text{as } p_3 = p_4\}$$

$$W_{pump} = v_4(p_1 - p_3)$$

Rankine cycle efficiency can be mathematically given by the ratio of net work to heat added.

$$\eta_{Rankine} = \frac{W_{turbine} - W_{pump}}{Q_{add}}$$

$$\eta_{Rankine} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$$

In the above expression, the enthalpy values may be substituted from steam table, mollier charts and by analysis for getting efficiency value.

Rankine cycle efficiency may be improved in the following ways:

(*a*) By reducing heat addition in boiler, which could be realized by preheating water entering into it.

(*b*) By increasing steam turbine expansion work, i.e. by increasing expansion ratio within limiting dryness fraction considerations.

(*c*) By reducing feed pump work.

(*d*) By using heat rejected in condenser for feed water heating. etc.

Irreversibilities and losses in Rankine cycle: In actual Rankine cycle there exist various irreversibilities and losses in its' constituent components and processes in them. In Rankine cycle the major irreversibility is encountered during the expansion through turbine. Irreversibilities in turbine significantly reduce the

expansion work. Heat loss from turbine to surroundings, friction inside turbine and leakage losses contribute to irreversibilities. Due to this irreversible expansion there occurs an increase in entropy as compared to no entropy change during reversible adiabatic expansion process. This deviation of expansion from ideal to actual process can be accounted for by isentropic turbine efficiency. Ideal expansion in steam turbine is shown by 2–3 on *T–S* representation. Actual expansion process is shown by 2–3′.

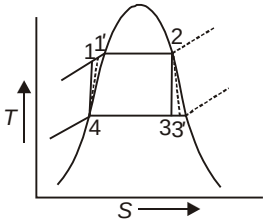


Fig. 8.5 *Rankine cycle showing non-ideal expansion and pumping process*

$$\text{Isentropic turbine efficiency, } \eta_{\text{isen, } t} = \frac{W_{\text{t,actual}}}{W_{\text{t,ideal}}} = \frac{W_{2-3'}}{W_{2-3}}\text{; Actually, } W_{2-3'} < W_{2-3}$$

or,
$$\eta_{\text{isen, } t} = \left(\frac{h_2 - h_{3'}}{h_2 - h_3}\right)$$

Another important location for irreversibilities is the pump. During pumping some additional work is required to overcome frictional effects. Ideally pumping is assumed to take place with no heat transfer during pumping whereas actually it may not be so. Thus the pumping process as shown by ideal process 4–1 gets' modified to 4–1′ which is accompanied by increase in entropy across the pump. Isentropic efficiency of pump is a parameter to account for non-idealities of pump operation. Isentropic efficiency of pump is defined by;

$$\eta_{\text{isen, } p} = \frac{W_{\text{p,ideal}}}{W_{\text{p,actual}}} = \frac{W_{4-1}}{W_{4-1'}}\text{; Actually, } W_{4-1} < W_{4-1'}$$

or,
$$\eta_{\text{isen, } p} = \left(\frac{h_1 - h_4}{h_{1'} - h_4}\right)$$

Thus, it indicates that actually pump work required shall be more than ideal pump work requirement.

Apart from the turbine and pump irreversibilities explained above there may be other sources of inefficiency too. These turbine and pump irreversibilities accounted for by isentropic efficiency of turbine and pump are called external irreversibility. Sources of internal irreversibilities are heat transfer from system to surroundings, frictional pressure loss in rest of components etc. There also occurs the steam pressure drop due to friction between pipe surface and working fluid.

8.5 DESIRED THERMODYNAMIC PROPERTIES OF WORKING FLUID

Working fluid being used in vapour power cycles must have following desirable properties. Generally water is used as working fluid in vapour power cycles as it is easily available in abundance and satisfies most of requirements. Other working fluids may be mercury, sulphur dioxide and hydrocarbons etc.

(*i*) Working fluid should be cheap and easily available.

(*ii*) Working fluid should be non-toxic, non-corrosive and chemically stable.

(*iii*) Fluid must have higher saturation temperature at moderate pressures as it shall yield high efficiency because most of heat will be added at high temperature. Thus, mean temperature of heat addition shall be high even at moderate pressure.

(*iv*) Working fluid should have smaller specific heat so that sensible heat supplied is negligible and Rankine cycle approaches to Carnot cycle. In case of fluid having small specific heat hatched portion shown in Fig. 8.6 will be absent or minimum.

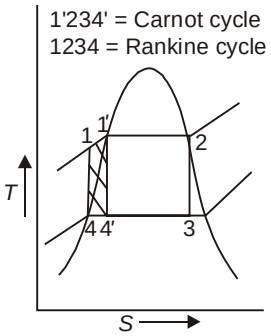


Fig. 8.6 *Carnot cycle and Rankine cycle emphasizing for small specific heat of fluid*

(*v*) Saturated vapour line should be steep enough so that state after expansion has high dryness fraction.

(*vi*) Working fluid density should be high so that the size of plant becomes smaller.

(*vii*) Working fluid should have its' critical temperature within metallurgical limits.

(*viii*) It should show significant decrease in volume upon condensation.

(*ix*) Working fluid should have its' freezing point much below atmospheric pressure so that there is no chance of freezing in condenser.

8.6 PARAMETRIC ANALYSIS FOR PERFORMANCE IMPROVEMENT IN RANKINE CYCLE

Let us carry out study of the influence of thermodynamic variables upon Rankine cycle performance.

(*i*) *Pressure of steam at inlet to turbine:* Steam pressure at inlet to turbine may be varied for same temperature of steam at inlet. Two different pressures of steam at inlet to turbine, also called throttle pressure are shown in Fig. 8.7. Comparative study shows that for back pressure and steam inlet temperatures being same the increase in steam inlet pressure from p'_1 to p_1 is accompanied by the reduction in net heat added as shown by hatched area $A_{2'3'37}$ and increase in net heat added by the amount shown by area $A_{1'1271'}$. Generally two areas $A_{2'3'37}$ and $A_{1'1271'}$ are nearly same which means that the increment in net heat added due to increasing throttle pressure from $p_{1'}$ to p_1 is accompanied by decrease in net heat addition and the net heat added remains same as at lower throttle pressure $p_{1'}$. But increase in throttle pressure to p_1 also causes reduction in the heat rejected. At pressure $p_{1'}$ heat rejected is given by area $A_{43'6'54}$ while at pressure p_1 heat rejected is given by area A_{43654}

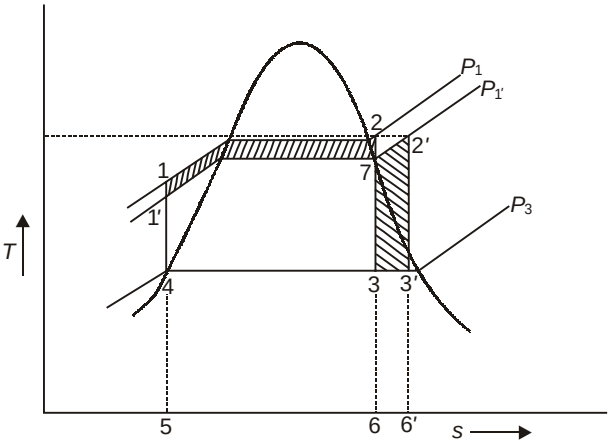


Fig. 8.7 *Rankine cycle showing two different throttle pressures*

$$A_{43654} < A_{43'6'54}$$

$$(\text{Heat rejected})_{\text{cycle } 1234} < (\text{Heat rejected})_{\text{cycle } 1'2'3'4}$$

Cycle efficiency is given by $\eta_{\text{cycle}} = 1 - \dfrac{\text{Heat rejected}}{\text{Heat added}}$

Hence, it is obvious that increasing steam pressure at inlet to steam turbine is accompanied by increase in cycle thermal efficiency. But this increase in pressure increases wetness of steam as shown by states 3 and 3′ i.e. $x_3 < x_3$, where x is dryness fraction. This increase in wetness of expanding steam decreases the adiabatic efficiency of turbine and also increases the chances of erosion of steam turbine blades. Therefore, as there are two contrary consequences of increasing throttle pressure so a compromise is to be had. Normally, to avoid erosion of turbine blades the minimum dryness fraction at turbine exhaust should not go below 0.88.

(*ii*) *Temperature of steam at inlet to turbine:* Increasing temperature of steam at inlet to turbine may also be called as superheating of steam at inlet to turbine. The two Rankine cycles having different degree of superheating are shown in Fig. 8.8.

For two different temperatures of steam at inlet to turbine i.e. T_2 and $T_{2'}$ while $T_2 < T_{2'}$ the comparison of two cycles 12341 and 12′3′41 shows the effect of increasing temperature at inlet to turbine.

Rankine cycle represented on *T–S* diagram show that increasing temperature from T_2 to $T_{2'}$ causes increase in net work by amount as shown by area $A_{22'3'32}$. This increase in steam temperature is also accompanied by increased heat addition as shown by area $A_{22'6'62}$. It is seen that this ratio of increase in net work to increase in heat addition is more than similar ratio for rest of cycle so the net effect is to improve the cycle thermal efficiency. It may also be stated that this increase in steam temperature from

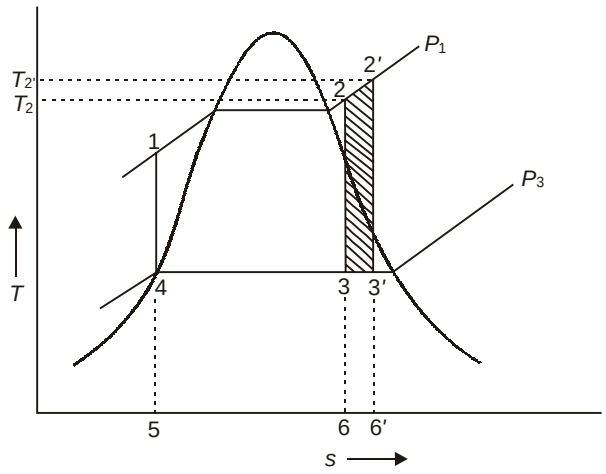


Fig. 8.8 *Effect of increasing temperature at inlet to turbine*

T_2 to $T_{2'}$ i.e. increase in degree of superheat increases mean temperature of heat addition, which increases thermal efficiency. With increased steam temperature the state of steam after expansion becomes more dry i.e. increase in temperature from 2 to 2' makes steam more dry after expansion i.e. from 3 to 3' having dryness fraction $x_{3'} > x_3$. This hotter steam supply to turbine is also advantageous from the specific work point of view. The work done per unit mass gets increased by superheating steam at inlet to turbine. Therefore, one is always interested in realizing highest possible temperature of steam, provided it is within metallurgical temperature limits. Highest practical steam temperature at turbine inlet presently is 650ºC.

(*iii*) *Pressure at the end of expansion:* Let us see the influence of pressure at the end of expansion from steam turbine. This pressure may also be called exhaust pressure or back pressure or condenser pressure. Rankine cycle with two different exhaust pressures is shown in Fig. 8.9, while the maximum pressure and temperature remains same.

With the lowering of back pressure from p_3 to $p_{3'}$ Rankine cycles get modified from 12341 to 1′23′4′1′. This reduction in back pressure causes increment in net work as shown by area $A_{1'1433'4'51'}$ and also the heat addition increases by the amount shown by area $A_{1'166'1'}$. It is seen that the two areas are such that the thermal efficiency of cycle increases by lowering back pressure as increase in heat addition is more than increase in heat rejection. This lowering back pressure is accompanied by increase in wetness of steam from 3 to 3′ i.e. dryness fraction $x_3 > x_{3'}$. Practically there exists limitation of dryness fraction after expansion to avoid erosion of turbine blades so lowering exhaust pressure is limited by it, inspite of showing improvement in thermal efficiency.

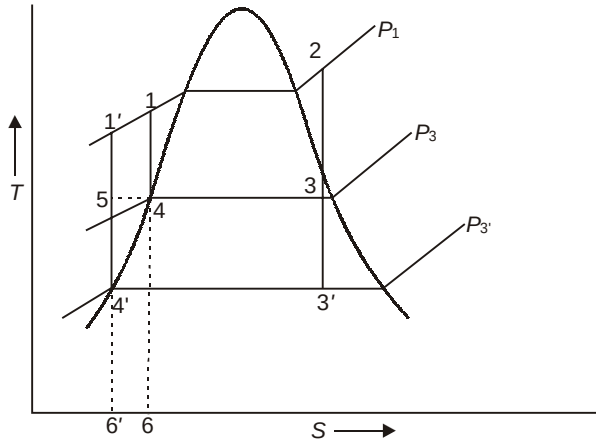


Fig. 8.9 *Effect of varying exhaust pressure*

(*iv*) *Temperature of feed water at inlet to boiler:* Temperature of feed water at inlet to boiler may be increased by employing some means such as feed water heating. This increase in feed water temperature reduces the heat requirement in boiler for getting desired state at inlet to steam turbine. Thus, with the reduced heat addition the thermal efficiency gets increased.

Different approaches practically used for improving the Rankine cycle performance have resulted into modified forms of Rankine cycle also called as Reheat cycle, Regenerative cycle etc.

8.7 REHEAT CYCLE

Schematic of reheat cycle is as shown in Fig. 8.10. Reheat cycle is based on the simple fact of realizing high efficiency with higher boiler pressure and yet avoid low quality of steam at turbine exhaust.

Here steam generated in boiler is supplied to high pressure steam turbine at state 2 and is expanded upto state 3. This steam is sent to boiler for being reheated so that its temperature gets increased, normally this temperature after reheating may be equal to temperature at inlet of high pressure steam turbine. Steam after reheating is supplied to subsequent turbine at state 4, say to low pressure steam turbine. Steam is now expanded upto the exhaust pressure say state ‘5’. Expanded steam is subsequently sent to condenser and condensate at state ‘6’ is pumped back to the boiler employing feed pump at state ‘1’. Thus, it is possible to take advantage of high steam pressure at inlet to steam turbine as the problem of steam becoming excessively wet with increasing steam pressure could be regulated by reheating during the expansion. Expansion occurs in two stages one begining at high pressure and other occurring at low pressure with reheating in between. The principal advantage of reheat is to increase the quality of steam at turbine exhaust.

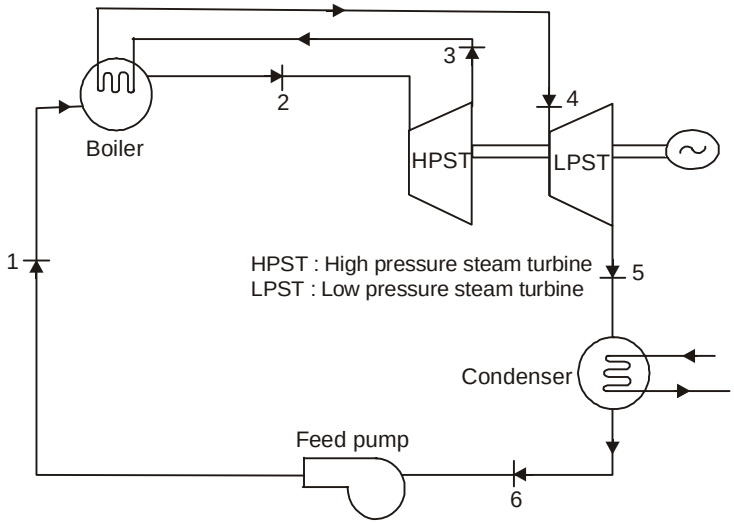


Fig 8.10 *Reheat cycle*

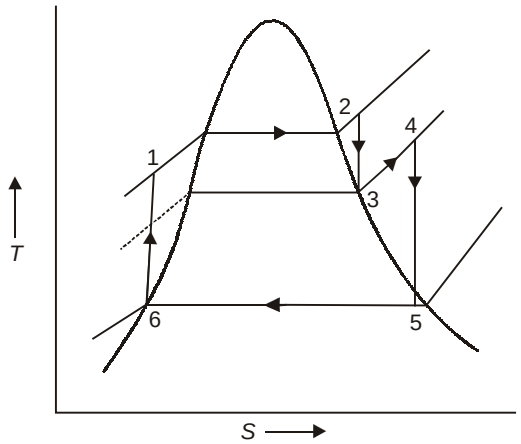


Fig 8.11 *T–S representation for reheat cycle*

Secondary advantage of reheating is marginal improvement in thermal efficiency when steam pressure is above 100 bar. At low steam pressure reheating does not show gain in cycle thermal efficiency and even the efficiency may be less than that of Rankine cycle due to mean temperature of heat addition being lower. Generally, with modern high pressure boilers and supercritical boilers reheating is essentially employed. Reheating is disadvantageous from economy of plant perspective as the cost of plant increases due to arrangement for reheating and increased condensation requirements due to increased dryness fraction of steam after expansion.

Thermodynamic analysis of reheat cycle as shown on *T–S* diagram may be carried out for estimation of different parameters as below,

$$\text{Total turbine work output} = W_{\text{HPST}} + W_{\text{LPST}}$$

$$\text{Net work, } W_{\text{net}} = \text{(Total turbine work output)} - \text{(Pump work)}$$

$$W_{\text{net}} = W_{\text{HPST}} + W_{\text{LPST}} - W_p$$

where different works for m_s mass of steam are,

$$\text{HP steam turbine, } W_{\text{HPST}} = m_s \cdot (h_2 - h_3)$$

LP steam turbine, $W_{LPST} = m_s \cdot (h_4 - h_5)$

Feed Pump, $W_p = (h_1 - h_6) \cdot m_s$

$W_{net} = \{(h_2 - h_3) + (h_4 - h_5) - (h_1 - h_6)\} \cdot m_s$

Heat supplied for m_s mass of steam; $Q_{add} = (h_2 - h_1) \cdot m_s + m_s \cdot (h_4 - h_3)$

Cycle thermal efficiency, $\eta_{Reheat} = \dfrac{W_{net}}{Q_{add}}$

$$\boxed{\eta_{Reheat} = \frac{\{(h_2 - h_3) + (h_4 - h_5) - (h_1 - h_6)\}}{\{(h_2 - h_1) + (h_4 - h_3)\}}}$$

Specific work output, $\boxed{W_{reheat} = \{(h_2 - h_3) + (h_4 - h_5) - (h_1 - h_6)\}}$

Generally not more than two stages of reheat are practically employed. Theoretically, the improvement in efficiency due to second reheat is about half of that which results from single reheat. Also more number of reheat stages shall result into superheated steam at turbine exhaust. Thus, mean temperature of heat rejection gets raised and efficiency drops.

8.8 REGENERATIVE CYCLE

Regenerative cycle is a modified form of Rankine cycle in which it is devised to increase mean temperature of heat addition so that cycle gets close to Carnot cycle in which all heat addition occurs at highest possible temperature. In regenerative cycle the feed water is heated up so as to reduce the heat addition in boiler and heat addition occur at hotter feed water temperature. Theoretically regenerative cycle arrangement is as shown in Fig. 8.12.

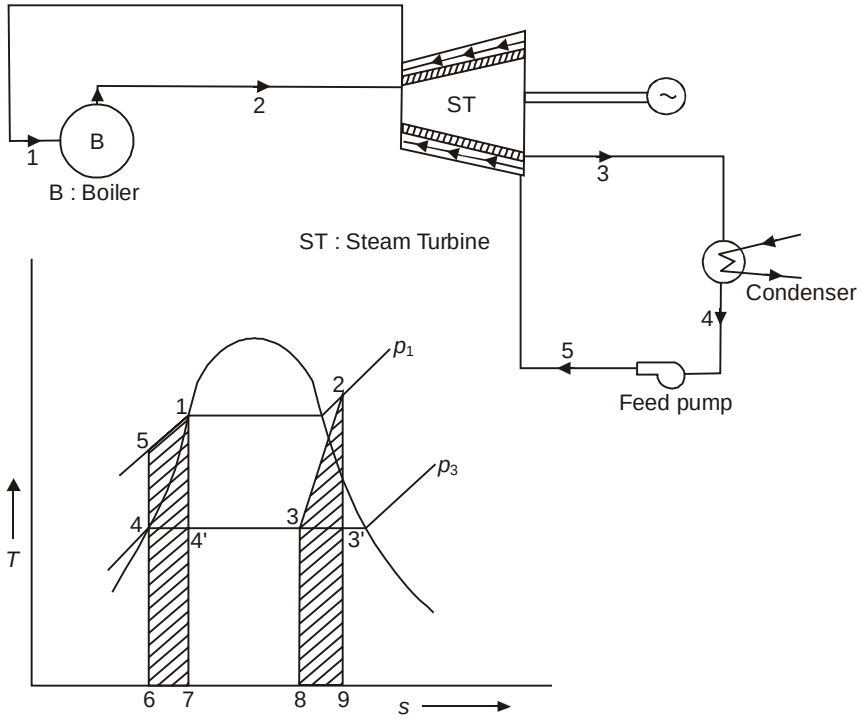


Fig 8.12 *Schematic for theoretical regenerative cycle and T-s representation.*

Theoretical arrangement shows that the steam enters the turbine at state 2 (temperature T_2) and expands to (temperature T_3) state 3. Condensate at state 5 enters the turbine casing which has annular space around turbine. Feed water enters turbine casing at state 5 and gets infinitesimally heated upto state 1 while flowing opposite to that of expanding steam. This hot feed water enters into boiler where steam generation occurs at desired state, say 2. Feed water heating in steam turbine casing is assumed to occur reversibly as the heating of feed water occurs by expanding steam with infinitesimal temperature difference and is called "regenerative heating". This cycle is called regenerative cycle due to regenerative heating employed in it. Regenerative heating refers to the arrangement in which working fluid at one state is used for heating itself and no external heat source is used for this purpose. Here feed water picks up heat from steam expanding in steam turbine, thus the expansion process in steam turbine shall get modified from 2-3′ ideally to 2-3. Heat picked up by feed water for getting heated up from state 5 to 1 is shown by hatched area A_{17651} on *T-S* diagram. Under ideal conditions for cent per cent heat exchange effectiveness the two areas i.e. A_{29832} indicating heat extraction from steam turbine and A_{17651} indicating heat recovered by feed water shall be same. Thus, *T-S* representation of regenerative cycle indicates that the cycle efficiency shall be more than that of Rankine cycle due to higher average temperature of heat addition.

But there exists serious limitation regarding realization of the arrangement described above. Limitations are due to impossibility of having a steam turbine which shall work as both expander for getting work output and heat exchanger for feed water heating. Also with the heat extraction from steam turbine the state of expanded steam at exhaust pressure shall be extremely wet, hence not desired. Due to these limitations the regenerative cycle is realized employing the concept of bleeding out steam from turbine and using it for feed water heating in feed water heaters.

Feed Water Heaters: The feed water heater refers to the device in which heat exchange occurs between two fluids i.e. steam and feed water either in direct contact or indirect contact. Direct contact feed water heater is the one in which bled steam and feed water come in direct contact. These are also called open feed water heater.

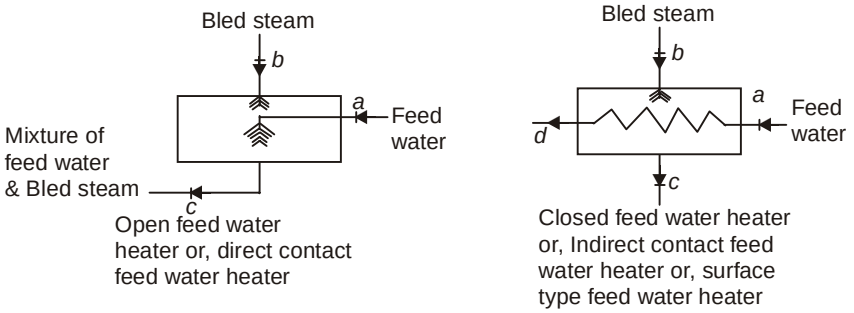


Fig. 8.13 *Feed water heaters*

In open feed water heater two fluids i.e., bled steam and feed water are at same pressure and adiabatic mixing is assumed to take place. Normally, it is considered that the mixture leaves open feed water heater as saturated liquid. Energy balance upon it shall be as follows,

$$m_a \cdot h_a + m_b \cdot h_b = (m_a + m_b) \cdot h_c$$

where subscripts *a*,*b* and *c* are for feed water, bled steam and mixture of the two as shown in Figure 8.13.

Indirect contact feed water heater as shown in Figure 8.13 is the one in which two fluid streams i.e. bled steam and feed water do not come in direct contact, but the heat exchange between two streams

occurs indirectly through metal interface. These are also called closed feed water heaters. In these feed water heaters since two fluids do not contact each other so they may be at different pressures. In these the arrangement comprises of steel, copper or brass tubes of solid drawn type placed in a shell. The heat transfer takes place through tube surface. Feed water flows inside the tube and is heated by extracted steam from outside. Steam enters in the shell and comes in contact with the tubes and then condenses over tubes. Steam condenses and trickles down and is collected in shell. Figure 8.14 gives schematic of surface type feed water heater. Performance of feed water heater is quantified using a parameter called "terminal temperature difference". Terminal temperature difference (T.T.D.) refers to the difference of temperature between temperature of feed water outlet and saturation temperature of steam entering the heater.

Terminal temperature difference = (Feed water outlet temperature – Saturation temperature of steam entering heater)

T.T.D. has its value lying around 5–8ºC. T.T.D. shall be zero in desuperheater type heaters where superheated steam is used for feed water heating upto saturation temperature of steam.

In feed water heaters where steam pressure is quite high, the condensate from heater is expanded in an expander called 'flash tank' or 'drain expander' or 'drain cooler'. In this flash tank some portion of condensate gets converted into steam which is further used for heating feed water.

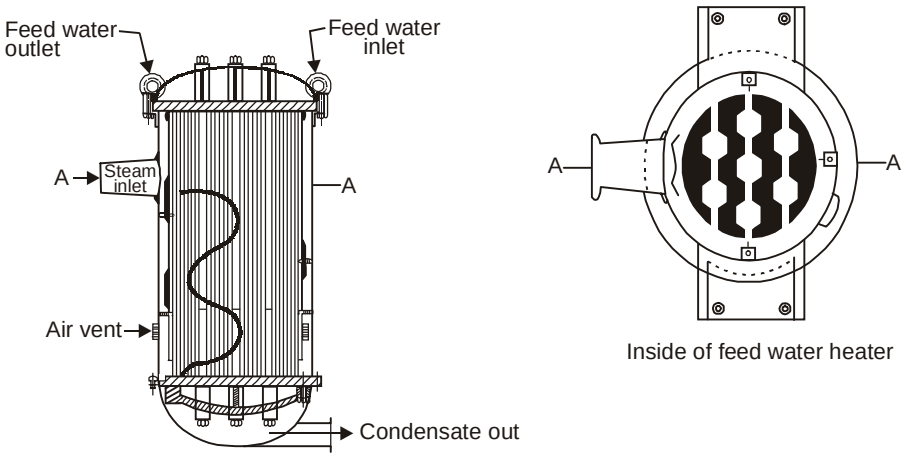


Fig. 8.14 *Surface type feed heater*

Direct contact heaters or open type heaters are more efficient than indirect contact type due to direct contact between two fluids. Feed water can achieve saturation temperature corresponding to the pressure of heating steam. In this case the terminal temperature difference is zero. During heating the non-condensable gases dissolved in water get released and are thrown out through vent passage. Deaerator is a type of open feed water heater. Schematic is shown in subsequent article on deaerator.

Deaerator: Deaerator is a type of open type feed water heater employed for the removal of dissolved oxygen and carbon dioxide from the feed water. The dissolved oxygen when not removed gets disintegrated into nascent oxygen at high temperature and pressure and forms iron oxide upon coming in contact with metal. This iron oxide formed causes pitting on the metal surface. At high temperature and pressure, dissolved CO_2 combines with metal and forms carbolic acid which causes mild pitting on

metal surfaces. Therefore, it becomes necessary to remove dissolved gases from the feed water. Although feed water treatment plant is there but still the impurities may creep in along with the make up water added to compensate for loss of water from system due to leakages at valve, pipe flanges, steam valve spindles and boiler blow down etc. Normally added water is 3-5% of total boiler feed.

Deaeration of water is based on the principle of decreasing partial pressure of gas for removal of dissolved gases. Henery's law and Dalton's law of partial pressure may be considered in this reference for understanding the phenomenon. Henery's law states that, "the mass of gas dissolved in a definite mass of liquid at a given temperature is directly proportional to partial pressure of gas in contact with liquid". It is good for the gases having no chemical reaction with water.

Decrease in partial pressure of gas in water is achieved by increasing the vapour pressure by heating the water. Here feed water is heated by low pressure steam for heating it upto its saturation temperature. Feed water entering deaerator is broken into small particles so as to increase contact area for better heat exchange with high temperature steam. Constructional detail of deaerator is shown in Fig. 8.15 which has basically deaerator head and storage tank. Water enters deaerator head from top on to a distributor plate. The water trickles down from upper most tray to the bottom trays through tiny holes in these trays. Steam enters storage tank from one end and enters deaerator head after passing through water collected in tank. Steam gradually heats feed water flowing downward with its' portion getting condensed and remaining steam flowing along with liberated gases out of deaerator head. Steam and gas mixture vented out from the top of deaerator head may be used for preheating make up water or feed water entering deaerator, if economical. The deaerated water is collected in storage tank below the deaerator head. A bubbler line is also provided in deaerator tank for fast heating of deaerator system during start up of unit. Bubbler line is a perforated pipe laid at bottom of storage tank through which steam for heating the water is supplied.

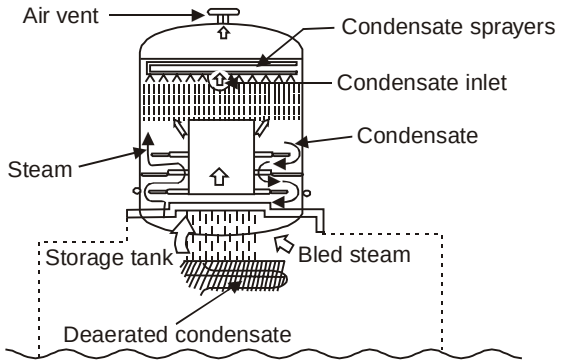


Fig. 8.15 *Deaerator*

Feed heater arrangements: In regenerative cycle, feed water heaters of different types are employed. There are some generic arrangements frequently used in these cycles. The arrangements are discussed below with two bleed points from where m_1 and m_2 masses of steam are bled out at pressures p_1 and p_2 and expansion occurs upto pressure p_3. Total steam flowing is taken as 1 kg.

(*i*) *Surface type heaters method:* This employs surface type feed water heaters and the arrangement for them is as shown in Fig. 8.16.

Here two surface heaters are used for showing the arrangement. Condensate of the bled steam is drained out using drain pump and sent in the main line at high pressure. This arrangement is also called as drain pump method.

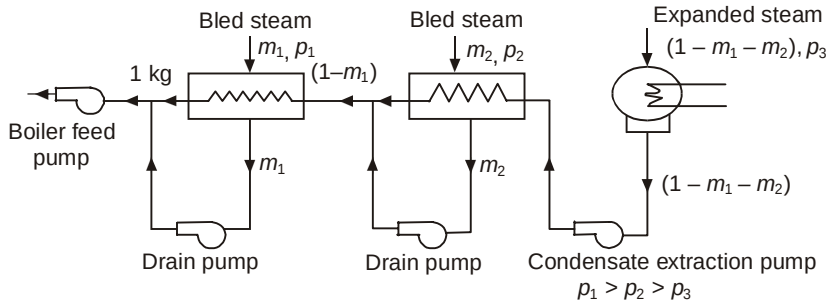


Fig. 8.16 *Arrangement in drain pump method or surface type feed water heaters method*

(*ii*) *Open type heater method:* In this arrangement the open type feed water heaters are employed as shown. Here due to contact of two fluids, there occurs mixing of bled steam with water and is taken out using pump for being sent to next open feed water heater.

(*iii*) *Surface type heaters with hot well:* This arrangement employs a hot well with surface type heaters. Bled steam condensate leaving surface type heaters is sent to hot well from where it is picked up by pump and flown through heaters for getting heated up. Arrangement is shown in Fig. 8.18.

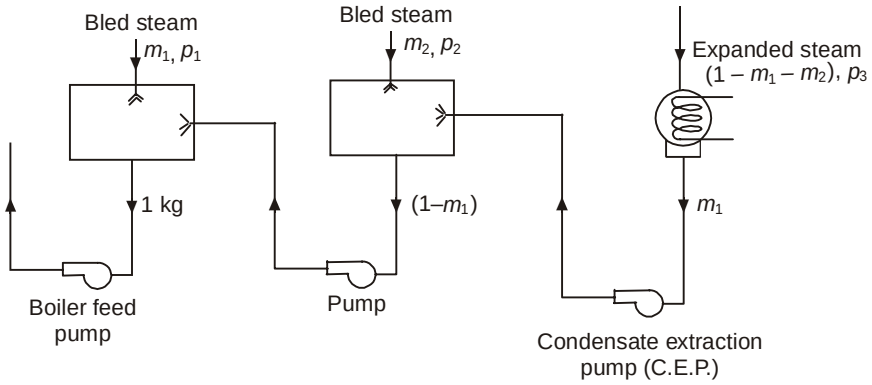


Fig. 8.17 *Arrangement with open feed water heaters*

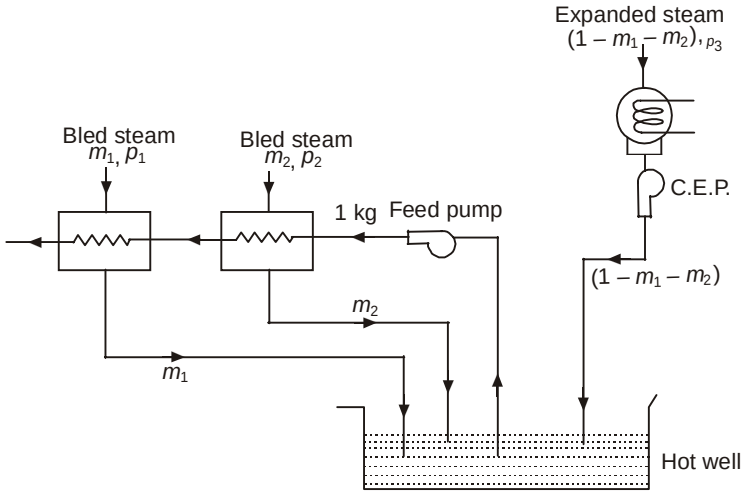


Fig 8.18 *Arrangement with surface type feed water heaters and hotwell*

(*iv*) *Cascade method:* This arrangement is shown in Fig. 8.19. Here bled steam condensate is throttled and cascaded into low pressure surface heaters. Bled steam condensate from last heater is sent to hotwell from where it is picked up and pumped through surface type heaters. For lowering the pressure of condensate before mixing the traps may also be used. A trap allows the liquid to be throttled to a lower pressure and also traps the vapour.

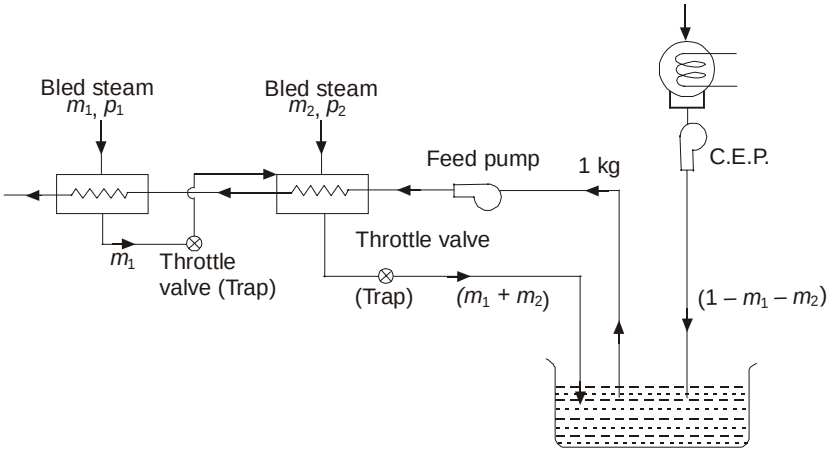


Fig. 8.19 *Cascade method*

Thermodynamics of regenerative cycle: Schematic of regenerative cycle with single feed water heater of open type is shown in Fig. 8.20. Arrangement shows that steam is bled out from turbine at state 6 and fed into feed heater.

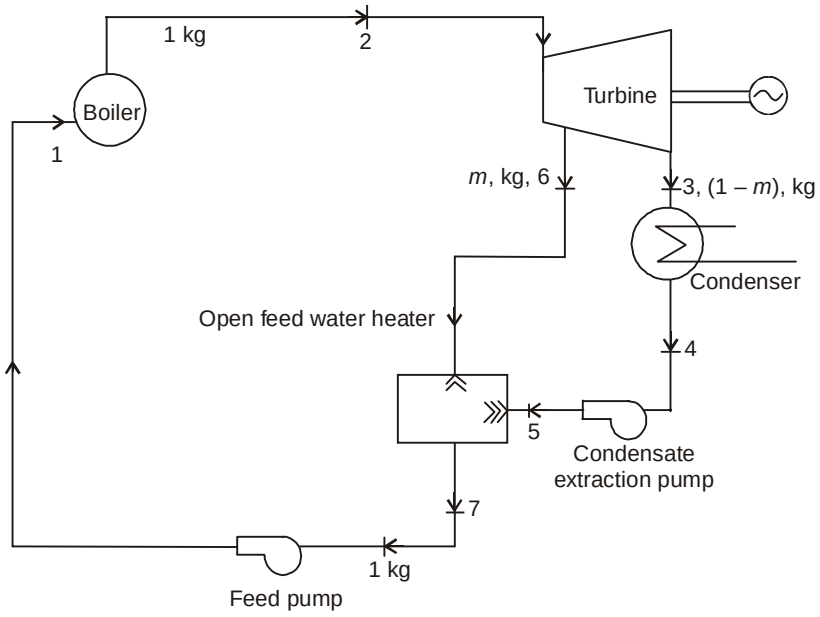


Fig. 8.20 *Schematic for regenerative cycle with one open feed water heater*

Feed water leaving at state 7 as shown on *T-S* diagram is being pumped upto boiler pressure through feed pump. *T-S* diagram indicates that the amount of heat picked up by feed water is shown by hatched area $A_{1751'}$. In case of absence of bleeding and feed heater the feed water will enter into boiler at state 1′ as compared to state 1 when regenerative heating is employed. Thus, advantage of hotter feed water to boiler can be realized by bleeding expanding steam from turbine. Regeneration can be seen in the feed water heating as the bled steam gets mixed with feed water at state 5 thus resulting into hot feed water.

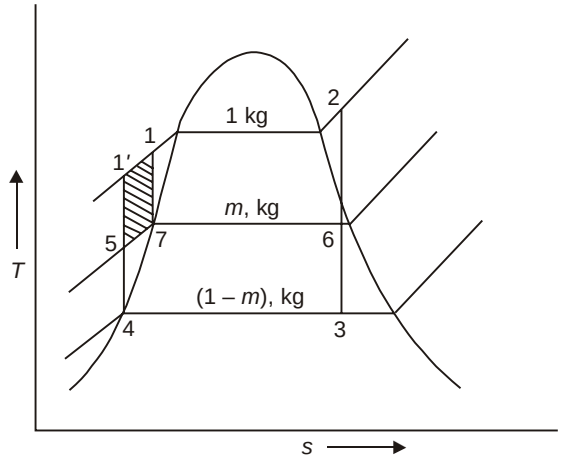


Fig. 8.21 *T-s representation for regenerative cycle with one open feed water heater*

As described earlier the bleeding offers advantage in terms of increased cycle efficiency due to increased mean temperature of heat addition. Hotter feed water also offers advantage in terms of reduced thermal stresses in boiler due to reduced temperature difference and less tendency of condensation of sulphur dioxide. Bleeding of steam causes reduced mass flow in condenser thereby reducing size of condenser. Bleeding is also disadvantageous because the work done per unit mass of steam gets reduced, thus increasing cost of the plant. Boiler capacity is to be increased for a given output.

Here it can be concluded that if the number of feed heaters be increased then their could be substantial increase in feed water temperature, thus offering a cycle having high mean temperature of heat addition, close to Carnot cycle. But it shall be accompanied by reduced work output and increased cost of the plant. Generally, the number of feed water heaters employed lies between 3 to 8 with average temperature rise in each heater being 10–15ºC. For example, if there are six heaters then first two may be surface type or indirect contact type followed by open type or direct contact feed water heater which shall also act as deaerator followed by three surface type feed water heaters.

For the regenerative cycle considered, with unit mass of steam leaving boiler and 'm' kg of steam bled out for feed water heating:

$$\text{Steam turbine work} = (h_2 - h_6) + (1 - m) \cdot (h_6 - h_3)$$

$$\text{Pump work} = (1 - m) \cdot (h_5 - h_4) + 1 \cdot (h_1 - h_7)$$

$$\text{Net work} = \{(h_2 - h_6) + (1 - m) \cdot (h_6 - h_3)\} - \{(1 - m) \cdot (h_5 - h_4) + (h_1 - h_7)\}$$

$$\text{Heat added} = 1 \cdot (h_2 - h_1)$$

$$\text{Hence, regenerative cycle efficiency} = \frac{\text{Net work}}{\text{Heat added}}$$

$$\eta_{\text{regenerative}} = \frac{\{(h_2 - h_6) + (1-m)(h_6 - h_3)\} - \{(1-m)(h_5 - h_4) + (h_1 - h_7)\}}{(h_2 - h_1)}$$

Example: Regenerative cycle with two surface type heaters, (Fig. 8.22)

Let us carry out thermodynamic analysis for 1 kg of steam generated in boiler at pressure p_1 and masses of bled steam be m_6 and m_7 at pressure p_6 and p_7.

Steam turbine work = $\{1 \cdot (h_2 - h_6) + (1 - m_6) \cdot (h_6 - h_7) + (1 - m_6 - m_7) \cdot (h_7 - h_3)\}$

Total pumping work = $\{(1 - m_6 - m_7) \cdot (h_5 - h_4) + m_7 \cdot (h_{10} - h_9) + m_6 \cdot (h_{14} - h_8)\}$

Bled steam leaving surface heaters 1 and 2 are at state 8 and 9 which are saturated liquid states at respective pressure, i.e., $h_8 = h_{f \text{ at } p_6}$, $h_9 = h_{f \text{ at } p_7}$

$$\text{Heat added in boiler} = 1 \cdot (h_2 - h_1)$$

Applying heat balance on two surface heaters we get,

On surface heater 1,

$$m_6 \cdot h_6 + (1 - m_6) \cdot h_{12} = m_6 \cdot h_8 + (1 - m_6) \cdot h_{13}$$

On surface heater 2,

$$m_7 \cdot h_7 + (1 - m_6 - m_7) \cdot h_5 = m_7 \cdot h_9 + (1 - m_6 - m_7) \cdot h_{11}$$

At the point of mixing of output from surface heater and bled condensate the heat balance yields.

$$1 \cdot h_1 = (1 - m_6) \cdot h_{13} + m_6 \cdot h_{14}$$

$$h_1 = (1 - m_6) \cdot h_{13} + m_6 \cdot h_{14}$$

and

$$(1 - m_6 - m_7) \cdot h_{11} + m_7 \cdot h_{10} = (1 - m_6) \cdot h_{12}$$

Upon the pumps 1, 2 and 3;

On pump 1,

$$v_4(p_1 - p_3) = h_5 - h_4$$

and

$$h_5 = v_4 (p_1 - p_3) + h_4$$

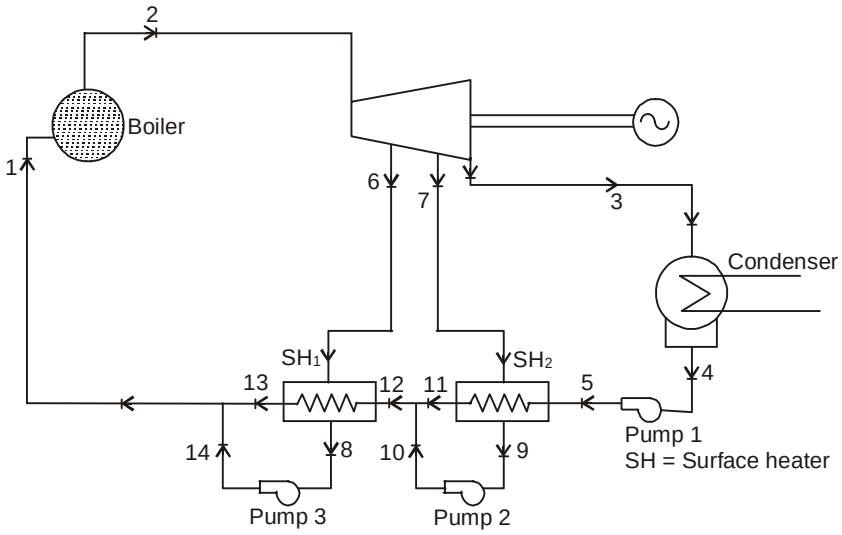
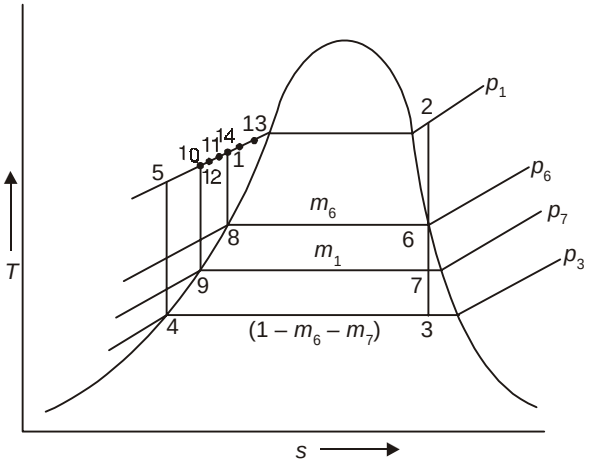


Fig. 8.22 *(a) Schematic of regenerative cycle with two surface type heaters*



(b) T-s diagram

Fig. 8.22 *Schematic and T-S representation for regenerative cycle with two surface type heaters*

On pump 2,

$$v_9(p_1 - p_7) = h_{10} - h_9$$
$$h_{10} = v_9 (p_1 - p_7) + h_9$$

On pump 3,

$$v_8(p_1 - p_6) = h_{14} - h_8$$
$$h_{14} = v_8 (p_1 - p_6) + h_8$$

Above different enthalpy expressions can be used for getting enthalpy values at salient points of interest in order to get the net work and cycle efficiency of this arrangement.

8.9 BINARY VAPOUR CYCLE

Generally, water is used as working fluid in vapour power cycle as it is found to be better than any other fluid if looked from the point of view of desirable characteristics of working fluid. Water is poor in respect to the following desired characteristics of working fluid.

Fluid should have critical temperature well above the highest temperature set by metallurgical limits of construction material. Fluid should have a saturation pressure at the maximum cycle temperature that poses no strength problems and a saturation pressure at the minimum cycle temperature that posses no difficulty of leakage from atmosphere.

In respect to above properties water is found to exhibit poor characteristics as; Water has critical temperature of 374ºC which is about 300 ºC less than the temperature limits set by metallurgical properties. The saturation pressure of water is quite high even at moderate temperatures so it does not have desirable properties at higher temperatures. Therefore in high temperature region a substance which has low saturation pressure should be used and the fluid should have its' critical temperature well above metallurgical limits of about 600ºC.

Therefore, it can be concluded that no single working fluid satisfies all the desirable requirements of working fluid, different working fluids may have different attractive features in them, but not all. So let us think of striking a combination of any two working fluids which are well suited together such as mercury and water. In such cases two vapour cycles operating on two different working fluids are put together and the arrangement is called binary vapour cycle.

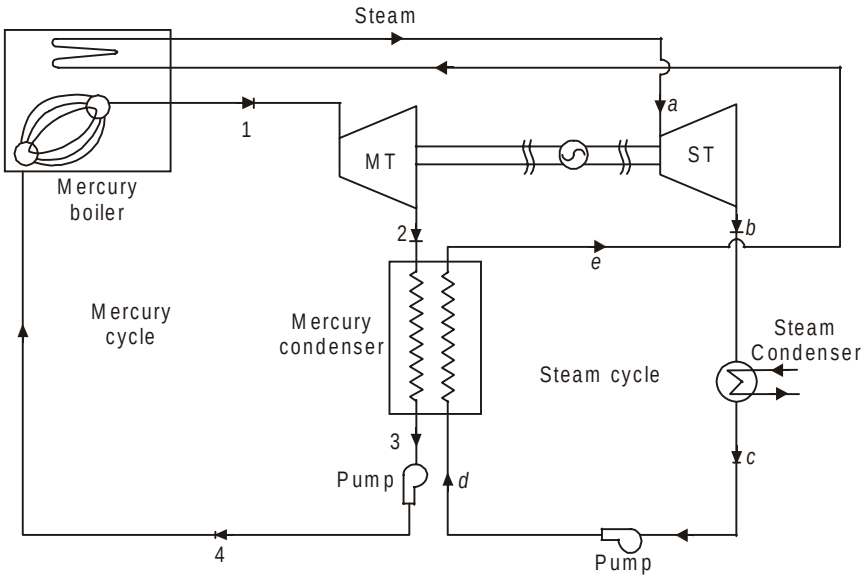
Mercury has comparatively small saturation pressures at high temperature values but due to excessively low pressure values and large specific volume values at low temperatures it cannot be used alone

as working fluid. Mercury also does not wet the surface in contact so there is inefficient heat transfer although 0.002% of solution of magnesium and potassium is added to give it wetting property of steel. Steam is used with mercury for overcoming some limitations of mercury. Thus in combination of mercury-steam, the mercury is used for high pressures while steam is used for low pressure region.

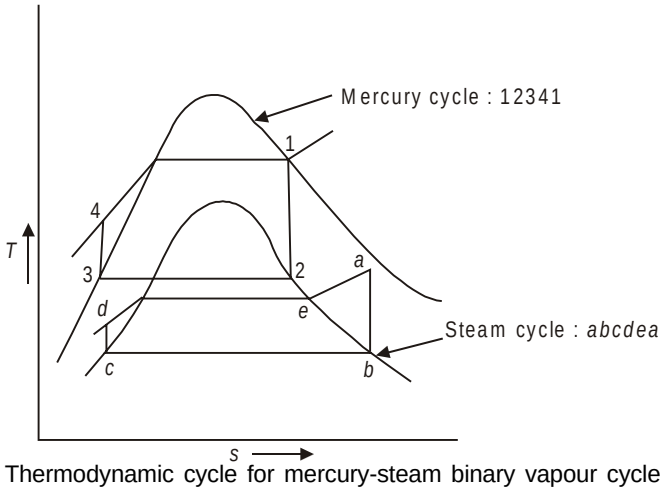
Layout for mercury-steam binary vapour cycle is shown on Fig. 8.23 along with it's depiction on *T-S* diagram. Here, mercury vapour are generated in mercury boiler and sent for expansion in mercury turbine and expanded fluid leaves turbine and enters into condenser. From condenser the mercury condensate is pumped back into the mercury boiler. In mercury condenser the water is used for extracting heat from mercury so as to condense it. The amount of heat liberated during condensation of mercury is too large to evaporate the water entering mercury condenser. Thus, mercury condenser also acts as steam boiler. For superheating of steam an auxilliary boiler may be employed or superheating may be realized in the mercury boiler itself.

MT : Mercury turbine

ST : Steam turbine



Mercury-steam binary vapour cycle



Thermodynamic cycle for mercury-steam binary vapour cycle

Fig. 8.23 *Schematic of binary vapour cycle*

$$\text{Net work from cycle shall be, } W_{net} = W_{MT} + W_{ST} - \Sigma W_{pump}$$

$$\text{Work from mercury turbine, } W_{MT} = m_{MT} \cdot (h_1 - h_2)$$

$$\text{Work from steam turbine, } W_{ST} = m_{ST} \cdot (h_a - h_b)$$

$$\text{Pump work} = m_{MT} \cdot (h_4 - h_3) + m_{ST} \cdot (h_d - h_c)$$

$$\text{Heat added to the cycle, } Q_{add} = m_{MT} \cdot (h_1 - h_4) + m_{ST} \cdot (h_a - h_e)$$

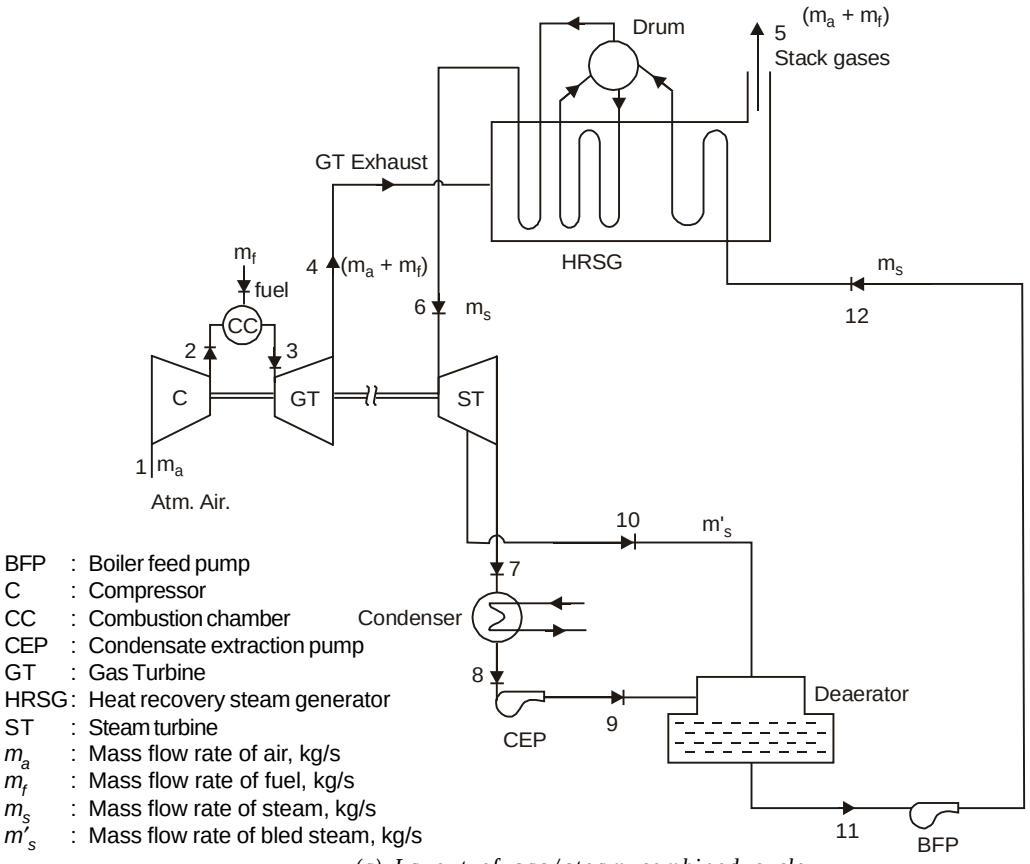
$$\text{Binary Cycle efficiency, binary} = \frac{W_{net}}{Q_{add}}$$

$$= \frac{\{m_{MT}(h_1 - h_2) + m_{ST}(h_a - h_b) - m_{MT}(h_4 - h_3) - m_{ST}(h_d - h_c)\}}{\{m_{MT}(h_1 - h_4) + m_{ST}(h_a - h_e)\}}$$

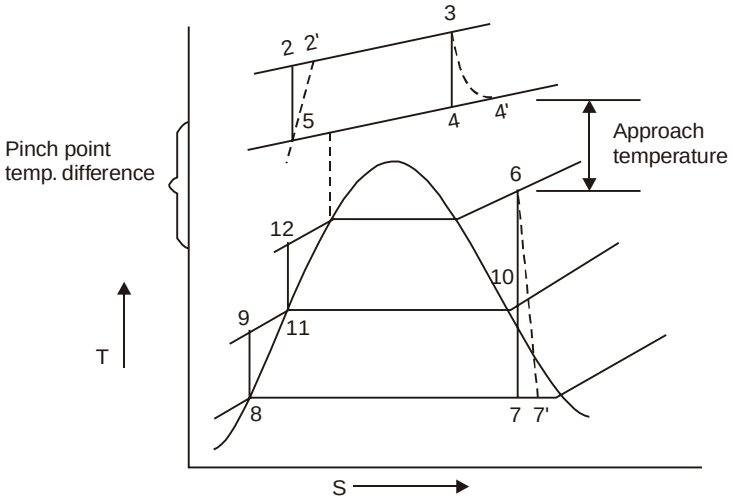
8.10 COMBINED CYCLE

Combined cycle refers to the combination of two cycles operating in synergy. The thermodynamic cycles operating together in the form of combined cycles have capability to operate in isolation too for producing work output. These different cycles have to operate on different fluids. Among different combined cycles the gas/steam combination is popular. The gas/steam combined cycles have combination of Brayton cycle and Rankine cycle. Exhaust gases from gas turbine in Brayton cycle are sent to heat recovery steam generator (HRSG) or waste heat recovery boiler (WHRB) for generation of steam to be expanded in steam turbine in Rankine cycle. High temperature cycle in combined cycle is called topping cycle and low temperature cycle is called bottoming cycle. Thus, in combined cycle the heat rejected by higher temperature cycle is recovered, in lower temperature cycle such as in heat recovery steam generator for generation of steam which subsequently runs steam turbine and augments the work output. In different combined cycles the topping cycles could be Otto cycle, Brayton cycle and Rankine cycle while Rankine cycle is generally used as bottoming cycle.

Fig. 8.24 shows the layout of a typical gas/steam combined cycle. Combined cycle could have various arrangements depending upon the alterations in topping cycle and bottoming cycle arrangements. In the shown layout there is simple gas turbine cycle, compression of air occurs between states 1 and 2. Subsequently heat addition and expansion occurs in combustion chamber and gas turbine through processes 2-3 and 3-4 respectively. Exhaust gases from gas turbine enter into heat recovery steam generator (HRSG) at state 4 and leave at state 5. Steam generated at state 6 from HRSG is sent to steam turbine for expansion and thus steam turbine work output augments work output of gas turbine. Expanded steam enters condenser at state 7 and condensate is sent back to HRSG at state 12 after passing it through deaerator. For m_a, m_f and m_s being flow rates of air, fuel and steam respectively thermodynamic analysis is carried out as under.



(a) Layout of gas/steam combined cycle



(b) T-s diagram representation for combined cycle.

Fig. 8.24 *(a) Layout of gas/steam combined cycle (b) T-S diagram representation for combined cycle*

Thus the work requirement in compressor,

$$W_c = m_a (h_2 - h_1)$$

Heat addition in combustion chamber, for fuel having calorific value *CV*

$$Q_{\text{add}} = m_f \times CV$$

Energy balance upon combustion chamber yields,

$$m_a \times h_2 + m_f \times CV = (m_a + m_f)\, h_3$$

Work available from gas turbine

$$\text{W}_{\text{GT}} = (m_a + m_f)\,(h_3 - h_4)$$

Net work from topping cycle, $\text{W}_{\text{topping}} = W_{GT} - W_c$

Work available from steam turbine, for bled steam mass flow rate for deaeration being m_s.

$$W_{ST} = \{m_s(h_6 - h_7) + (m_s - m'_s)\,(h_7 - h_{10})\}$$

Pump works

$$W_{CEP} = (m_s - m_s')\,(h_9 - h_8)$$

$$W_{BFP} = m_s.(h_{12} - h_{11})$$

Net work available from bottoming cycle

$$W_{\text{bottoming}} = W_{ST} - W_{CEP} - W_{BFP}$$

Hence total work output from combined cycle

$$\text{W}_{\text{combined}} = \text{W}_{\text{topping}} + \text{W}_{\text{bottoming}}$$

Thermal efficiency of combined cycle,

$$\eta_{\text{combined}} = \frac{W_{\text{combined}}}{Q_{\text{add}}}$$

Thermal efficiency of topping cycle (gas turbine cycle),

$$\eta_{\text{topping}} = \frac{W_{\text{topping}}}{Q_{\text{add}}}$$

We can see that work output of gas turbine cycle is less than combined cycle work output, while the heat addition remains same. Thus, thermal efficiency of combined cycle is more than gas turbine cycle (topping cycle),

As, $W_{\text{combined}} > W_{\text{topping}} \Rightarrow \eta_{\text{combined}} > \eta_{\text{topping}}$

8.11 COMBINED HEAT AND POWER

Combined heat and power refers to the arrangement in which cycle produces work (power) along with heat utilization for process heating. There exist number of engineering applications where both power and process heat are simultaneously required. Such arrangement is also called cogeneration. Cogeneration may be defined as the arrangement of producing more than one useful form of energy. Food processing

industry and chemical industry are the industries where steam is required for different processes and cogeneration is an attractive option for getting electricity alongwith process steam. Schematic of a cogeneration plant is shown in Fig. 8.25.

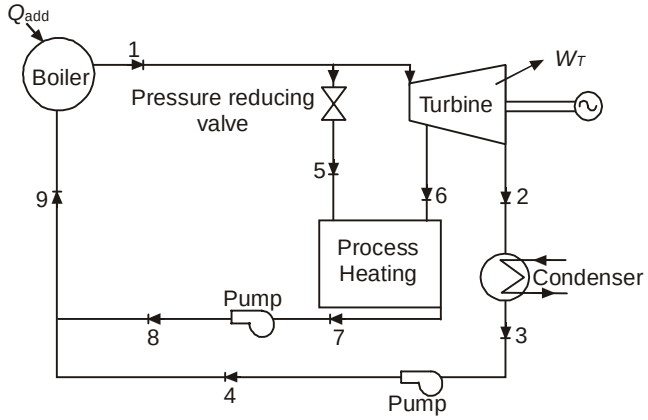


Fig. 8.25 *Schematic for cogeneration*

Cogeneration arrangement is popularly used in cold countries for district heating where, in this arrangement the power plant supplies electricity along with steam for process needs, such as space heating and domestic water heating.

Thermodynamic analysis of the cogeneration arrangement shows;

$$\text{Heat added in boiler } Q_{add} = m_1 (h_1 - h_9)$$

$$\text{Heat used in process heating } Q_{process} = \{m_5 (h_5 - h_7) + m_6 (h_6 - h_7)\}$$

$$\text{Turbine work: } W_T = \{(m_1 - m_5) \cdot (h_1 - h_6) + (m_1 - m_5 - m_6) \cdot (h_6 - h_2)\}$$

$$\text{Pump work: } W_P = \{(m_5 + m_6) \cdot (h_8 - h_7) + (m_1 - m_5 - m_6) \cdot (h_4 - h_3)\}$$

When there is no process heating then; $m_5 = 0$ and $m_6 = 0$

8.12 DIFFERENT STEAM TURBINE ARRANGEMENT

In certain applications simple steam turbines are unable to meet specific requirements. Back pressure turbine, pass out or extraction turbine and mixed pressure turbines are such special purpose turbines whose details are given ahead.

(*a*) *Back pressure turbine:* Back pressure turbine is the one in which steam is not expanded upto lowest pressure in steam turbine, instead steam leaves the turbine at higher pressure which is appropriate for the process steam/heating requirement. Thus, in back pressure turbine expansion is limited to high back pressure and steam leaving turbine goes for process heating. Generally, steam leaving turbine at high back pressure will be superheated. Since steam is to be used for process heating so the rate of heat transfer should be high. Superheated steam is not suitable for heating because of small rate of heat transfer therefore superheated steam should be desuperheated and brought to saturated steam state as saturated steam has high rate of heat transfer and also the control of temperature is convenient.

Thus, back pressure turbine has the provision of desuperheating as shown in Fig. 8.26. The steam leaving tubine enters into desuperheater where it is transformed into saturated steam. Saturated steam is subsequently sent for process heating where it gets condensed and condensate is sent back to the boiler through pump. A by pass valve is also provided so that if there is no power requirement then whole steam may be sent for process heating through desuperheater by closing turbine valve and opening by pass valve.

(*b*) *Pass out or extraction turbine:* Pass out turbine refers to the steam turbine having provision for extraction of steam during expansion. Such provision is required because in combined heat and power requirement the steam available from back pressure turbine may be more than required one or the power produced may be less than the required value. Pass out turbine has arrangement for continuous extraction of a part of steam at the desired pressure for process heating and left out steam goes into low pressure section of turbine through a pressure control valve. In the low pressure section of turbine, a control mechanism is provided so that the speed of turbine and pressure of steam extracted remains constant irrespective of the variations in power produced and process heating.

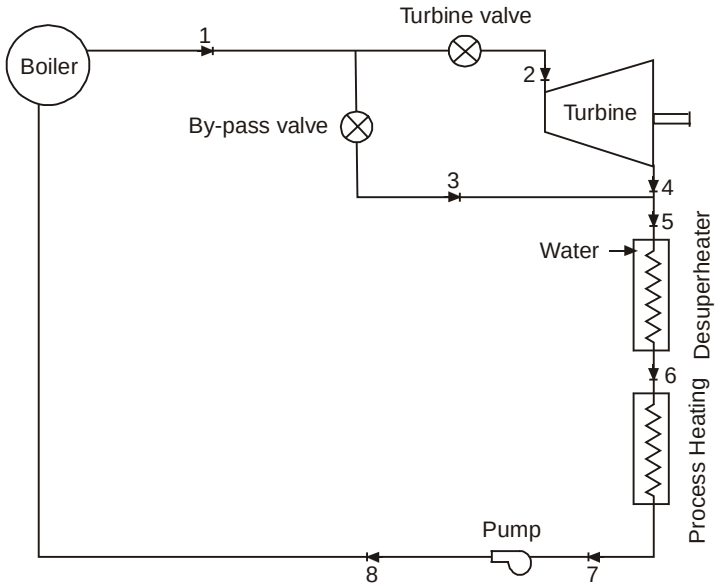


Fig. 8.26 *Back pressure turbine*

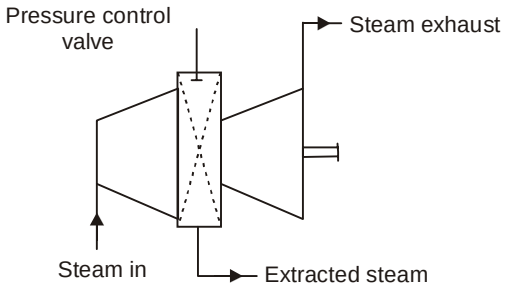


Fig. 8.27 *Pass out turbine*

The pass out turbines have to operate under widely varying load so its efficiency is quite poor. For facilitating the operation of pass out turbine from no extraction to full steam extraction conditions, nozzle control governing or throttle control governing are used.

(*c*) *Mixed pressure turbine:* These are the turbines which have capability of admitting steam at more than one pressures and subjecting multiple pressure steam streams to expand. Generally, mixed pressure turbines utilize high pressure steam from a boiler and also low pressure steam from exhaust of a non-condensing engine or some auxiliary of the plant.

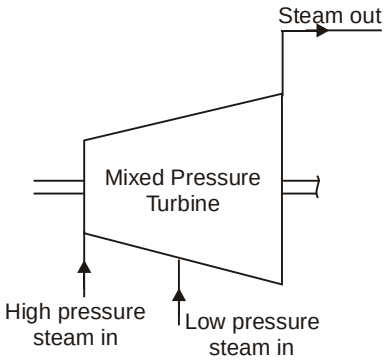


Fig 8.28 *Mixed pressure turbine*

Mixed pressure turbines are preferred when steam at single pressure is not available in desired quantity for producing required power. These mixed pressure turbines actually have more than one turbines in one cylinder.

EXAMPLES

1. *A Carnot cycle works on steam between the pressure limits of 7 MPa and 7 kPa. Determine thermal efficiency, turbine work and compression work per kg of steam.*

Solution:

T-s representation for the Carnot cycle operating between pressure of 7 MPa and 7 kPa is shown in Fig. 8.29

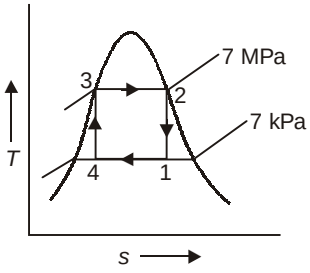


Fig. 8.29

Enthalpy at state 2, $h_2 = h_{\text{g at 7 MPa}}$

$h = 2772.1$ kJ/kg

Entropy at state 2, $s_2 = s_{\text{g at 7 MPa}}$

$s_2 = 5.8133$ kJ/kg · K

Enthalpy and entropy at state 3,

$h_3 = h_{f \text{ at 7 MPa}} = 1267$ kJ/kg

$s_3 = s_{f \text{ at 7 MPa}} = 3.1211$ kJ/kg ·K

For process 2 –1, $s_1 = s_2$· Let dryness fraction at state 1 be x_1.

$$s_1 = s_2 = s_{f \text{ at 7 kPa}} + x_1 \cdot s_{fg \text{ at 7 kPa}}$$

$$5.8133 = 0.5564 + x_1 \cdot 7.7237$$

$$x_1 = 0.6806$$

Enthalpy of state 1, $h_1 = h_{f \text{ at 7 kPa}} + x_1 \cdot h_{fg \text{ at 7 kPa}}$

$= 162.60 + (0.6806 \times 2409.54)$

$h_1 = 1802.53$ kJ/kg

Let dryness fraction at state 4 be x_4,

For process 4–3, $s_4 = s_3 = s_{f \text{ at 7 kPa}} + x_4 \cdot s_{fg \text{ at 7 kPa}}$

$3.1211 = 0.5564 + x_4 \cdot 7.7237$

$$x_4 = 0.3321$$

$$\text{Enthalpy at state 4, } h_4 = h_{f \text{ at 7 kPa}} + x_4 \cdot h_{fg \text{ at 7 kPa}}$$
$$= 162.60 + (0.3321 \times 2409.54)$$
$$h_4 = 962.81 \text{ kJ/kg}$$

$$\text{Thermal efficiency} = \frac{\text{Net work}}{\text{Heat added}}$$

$$\text{Expansion work per kg} = h_2 - h_1 = (2772.1 - 1802.53) = 969.57 \text{ kJ/kg}$$

$$\text{Compression work per kg} = h_3 - h_4 = (1267 - 962.81)$$
$$= 304.19 \text{ kJ/kg (+ve)}$$

$$\text{Heat added per kg} = h_2 - h_3 = (2772.1 - 1267)$$
$$= 1505.1 \text{ kJ/kg (–ve)}$$

$$\text{Net work per kg} = (h_2 - h_1) - (h_3 - h_4) = 969.57 - 304.19$$
$$= 665.38 \text{ kJ/kg}$$

$$\text{Thermal efficiency} = \frac{665.38}{1505.1} = 0.4421 \text{ or } 44.21\%$$

Thermal efficiency = **44.21%**
Turbine work = **969.57 kJ/kg (+ve)**
Compression work = **304.19 kJ/kg (–ve)** **Ans.**

2. *A steam power plant uses steam as working fluid and operates at a boiler pressure of 5 MPa, dry saturated and a condenser pressure of 5 kPa. Determine the cycle efficiency for (a) Carnot cycle (b) Rankine cycle. Also show the T-s representation for both the cycles.*

Solution:

From steam tables:

At 5 MPa $h_{f,\ 5MPa} = 1154.23$ kJ/kg, $s_{f,\ 5\ MPa} = 2.92$ kJ/kg · K

$h_{g,\ 5MPa} = 2794.3$ kJ/kg, $s_{g,\ 5\ MPa} = 5.97$ kJ/kg · K

At 5 kPa

$h_{f,\ 5kPa} = 137.82$ kJ/kg, $s_{f,\ 5kPa} = 0.4764$ kJ/kg · K

$h_{g,\ 5kPa} = 2561.5$ kJ/kg, $s_{g,\ 5kPa} = 8.3951$ kJ/kg · K

$v_{f,\ 5kPa} = 0.001005\ m^3/kg$

As process 2-3 is isentropic, so $s_2 = s_3$

and

$$s_3 = s_{f,\ 5kPa} + x_3 \cdot s_{fg,\ 5kPa} = s_2 = s_{g,\ 5MPa}$$
$$x_3 = 0.694$$

Hence enthalpy at 3,

$$h_3 = h_{f,\ 5kPa} + x_3 \cdot h_{fg,\ 5kPa}$$
$$h_3 = 1819.85 \text{ kJ/kg}$$

Enthalpy at 2, $h_2 = h_{g,\ 5MPa} = 2794.3$ kJ/kg

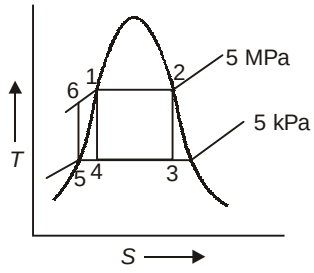


Fig. 8.30

Carnot cycle : 1–2–3–4–1

Rankine cycle : 1–2–3–5–6–1

Process 1-4 is isentropic, so $s_1 = s_4$

$$s_1 = 2.92 = 0.4764 + x_4 \cdot (8.3951 - 0.4764)$$
$$x_4 = 0.308$$

Enthalpy at 4,

$$h_4 = 137.82 + (0.308 \times (2561.5 - 137.82))$$
$$h_4 = 884.3 \text{ kJ/kg}$$

Enthapy at 1,

$$h_1 = h_{f \text{ at } 5 \text{ MPa}}$$
$$h_1 = 1154.23 \text{ kJ/kg}$$

Carnot cycle (1-2-3-4-1) efficiency:

$$\eta_{\text{carnot}} = \frac{\text{Net work}}{\text{Heat added}}$$
$$= \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$$
$$= \frac{\{(2794.3 - 1819.85) - (1154.23 - 884.3)\}}{(2794.3 - 1154.23)}$$
$$\eta_{\text{carnot}} = 0.4295$$

or

$$\eta_{\text{carnot}} = \mathbf{42.95\%} \quad \textbf{Ans.}$$

In Rankine cycle, 1-2-3-5-6-1

Pump work, $h_6 - h_5 = v_{f,\,5}(p_6 - p_5)$
$= 0.001005\ (5000 - 5)$
$h_6 - h_5 = 5.02$
$h_5 = h_{f \text{ at } 5\text{kPa}} = 137.82$ kJ/kg
Hence $h_6 = 137.82 + 5.02 = 142.84$ kJ/kg
$h_6 = 142.84$ kJ/kg
Net work in Rankine cycle $= (h_2 - h_3) - (h_6 - h_5)$
$= 974.45 - 5.02$
$= 969.43$ kJ/kg
Heat added $= h_2 - h_6$
$= 2794.3 - 142.84$
$= 2651.46$ kJ/kg

$$\text{Rankine cycle efficiency} = \frac{969.43}{2651.46}$$
$$\eta_{\text{Rankine}} = 0.3656$$

or

$$\eta_{\text{Rankine}} = \mathbf{36.56\%} \quad \textbf{Ans.}$$

3. *A steam turbine plant operates on Rankine cycle with steam entering turbine at 40 bar, 350ºC and leaving at 0.05 bar. Steam leaving turbine condenses to saturated liquid inside condenser. Feed pump pumps saturated liquid into boiler. Determine the net work per kg of steam and the cycle efficiency assuming all processes to be ideal. Also show cycle on T-s diagram. Also determine pump work per kg of steam considering linear variation of specific volume.*

Solution:

From steam table

$$h_2 = h_{\text{at 40 bar, 350°C}} = 3092.5 \text{ kJ/kg}$$
$$s_2 = s_{\text{at 40 bar, 350°C}} = 6.5821 \text{ kJ/kg·K}$$
$$h_4 = h_{f \text{ at 0.05 bar}} = 137.82 \text{ kJ/kg}$$
$$s_4 = s_{f \text{ at 0.05 bar}} = 0.4764 \text{ kJ/kg}$$
$$v_4 = v_{f \text{ at 0.05 bar}} = 0.001005 \text{ m}^3\text{/kg}$$

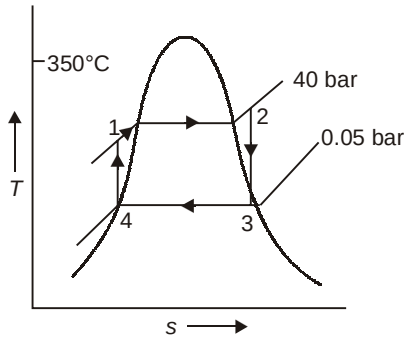


Fig. 8.31

Let dryness fraction at state 3 be x_3,

For ideal process, 2-3, $s_2 = s_3$

$$s_2 = s_3 = 6.5821 = s_{f \text{ at 0.05 bar}} + x_3 \cdot s_{fg \text{ at 0.05 bar}}$$
$$6.5821 = 0.4764 + x_3 \cdot 7.9187$$
$$x_3 = 0.7711$$
$$h_3 = h_{f \text{ at 0.05 bar}} + x_3 \cdot h_{fg \text{ at 0.05 bar}}$$
$$= 137.82 + (0.7711 \times 2423.7)$$
$$h_3 = 2006.74 \text{ kJ/kg}$$

For pumping process

$$h_1 - h_4 = v_4 \cdot \Delta p = v_4 \times (p_1 - p_4)$$
$$h_1 = h_4 + v_4 \times (p_1 - p_4)$$
$$= 137.82 + (0.001005 \times (40 - 0.05) \times 10^2)$$
$$h_1 = 141.84 \text{ kJ/kg}$$

Pump work per kg of steam $= (h_1 - h_4) = 4.02$ kJ/kg

Net work per kg of steam = (Expansion work – Pump work) per kg of steam
$= (h_2 - h_3) - (h_1 - h_4)$

$$= 1081.74 \text{ kJ/kg}$$

$$\text{Cycle efficiency} = \frac{\text{Net work}}{\text{Heat added}}$$

$$= \frac{1081.74}{(h_2 - h_1)}$$

$$= \frac{1081.74}{(3092.5 - 141.84)}$$

$$= 0.3667 \text{ or } 36.67\%$$

Net work per kg of steam = **1081.74 kJ/kg**
Cycle efficiency = **36.67%**
Pump work per kg of steam = **4.02 kJ/kg** **Ans.**

4. *A steam power plant running on Rankine cycle has steam entering HP turbine at 20 MPa, 500ºC and leaving LP turbine at 90% dryness. Considering condenser pressure of 0.005 MPa and reheating occurring upto the temperature of 500ºC determine,*

(a) the pressure at wich steam leaves HP turbine

(b) the thermal efficiency

Solution:

Let us assume that the condensate leaves condenser as saturated liquid and the expansion in turbine and pumping processes are isentropic.

From steam tables,

$$h_2 = h_{\text{at 20 MPa, 500ºC}} = 3238.2 \text{ kJ/kg}$$

$$s_2 = 6.1401 \text{ kJ/kg} \cdot \text{K}$$

$$h_5 = h_{\text{at 0.005 MPa, 0.90 dry}}$$

$$h_5 = h_{f \text{ at 0.005 MPa,}} + 0.9 \times h_{fg \text{ at 0.005 MPa}}$$

$$= 137.82 + (0.9 \times 2423.7)$$

$$h_5 = 2319.15 \text{ kJ/kg}$$

$$s_5 = s_{f \text{ at 0.005 MPa,}} + 0.9 \times s_{fg \text{ at 0.005 MPa}}$$

$$= 0.4764 + (0.9 \times 7.9187)$$

$$s_5 = 7.6032 \text{ kJ/kg} \cdot \text{K}$$

$$h_6 = h_{f \text{ at 0.005 MPa}} = 137.82 \text{ kJ/kg}$$

It is given that temperature at state 4 is 500ºC and due to isentropic process $s_4 = s_5 = 7.6032$ kJ/kg ·K. The state 4 can be conveniently located on Mollier chart by the intersection of 500ºC constant temperature line and entropy value of 7.6032 kJ/kg · K and the pressure and enthalpy obtained. But these shall be approximate.

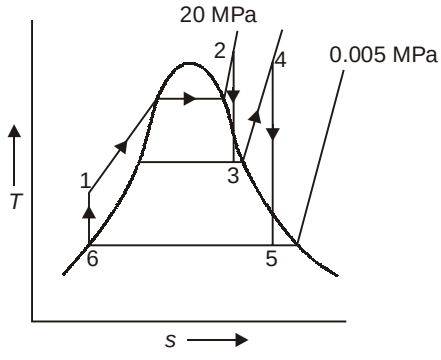


Fig. 8.32

The state 4 can also be located by interpolation using steam table. The entropy value of 7.6032 kJ kg · K lies between the superheated steam states given under, p = 1.20 *MP*a, $s_{\text{at 1.20 MPa and 500°C}}$ = 7.6759 kJ/kg · K

p = 1.40 *MP*a, $s_{\text{at 1.40 MPa and 500°C}}$ = 7.6027 kJ/kg · K

By interpolation state 4 lies at pressure

$$= 1.20 + \frac{(1.40 - 1.20)}{(7.6027 - 7.6759)} (7.6032 - 7.6759)$$

$$= 1.399 \text{ MPa} \approx 1.40 \text{ MPa}$$

Thus, steam leaves HP turbine at 1.4 MPa

Enthalpy at state 4, h_4 = 3474.1 kJ/kg

For process 2-3, $s_2 = s_3$ = 6.1401 kJ/kg · K. The state 3 thus lies in wet region as $s_3 < s_{g \text{ at 1.40 MPa.}}$ Let dryness fraction at state 3 be x_3.

$$s_3 = s_{f \text{ at 1.4 MPa}} + x_3 \cdot s_{fg \text{ at 1.4 MPa}}$$

$$6.1401 = 2.2842 + x_3 \cdot 4.1850$$

$$x_3 = 0.9214$$

$$h_3 = h_{f \text{ at 1.4 MPa}} + x_3 \cdot h_{fg \text{ at 1.4 MPa}}$$

$$= 830.3 + (0.9214 \times 1959.7) = 2635.97 \text{ kJ/kg}$$

Enthalpy at 1, $h_1 = h_6 + v_6(p_1 - p_6)$

$$= h_{f \text{ at 0.005 MPa}} + v_{f \text{ at 0.005 MPa}} (20 - 0.005) \times 10^3$$

$$= 137.82 + (0.001005 \times 19.995 \times 10^3)$$

$$h_1 = 157.91 \text{ kJ/kg}$$

Net work per kg of steam $= (h_2 - h_3) + (h_4 - h_5) - (h_1 - h_6)$

$$= 1737.09 \text{ kJ/kg}$$

Heat added per kg of steam $= (h_2 - h_1) = 3080.29$ kJ/kg

$$\text{Thermal efficiency} = \frac{\text{Net work}}{\text{Heat added}} = \frac{1737.09}{3080.29} = 0.5639 \text{ or } 56.39\%$$

Pressure of steam leaving HP turbine = **1.40 MPa**
Thermal efficiency = **56.39%**

Ans.

5. *In a steam turbine installation running on ideal Rankine cycle steam leaves the boiler at 10 MPa and 700°C and leaves turbine at 0.005 MPa. For the 50 MW output of the plant and cooling water entering and leaving condenser at 15°C and 30°C respectively determine*

(a) the mass flow rate of steam in kg/s

(b) the mass flow rate of condenser cooling water in kg/s

(c) the thermal efficiency of cycle

(d) the ratio of heat supplied and rejected (in boiler and condenser respectively).

Neglet K.E. and P.E. changes.

Solution:

From steam table

$$\text{At inlet to turbine, } h_2 = h_{\text{at 10 MPa, 700°C}}$$

$$h_2 = 3870.5 \text{ kJ/kg}$$

$$s_2 = 7.1687 \text{ kJ/kg} \cdot \text{K}$$

For process 2-3, $s_2 = s_3$ and $s_3 < s_{f \text{ at } 0.005 \text{ MPa}}$ so state 3 lies in wet region. Let dryness fraction at state 3 be x_3.

$$s_3 = 7.1687 = s_{f \text{ at } 0.005 \text{ MPa}} + x_3 \cdot s_{fg \text{ at } 0.005 \text{ MPa}}$$

$$7.1687 = 0.4764 + (x_3 \times 7.9187)$$

$$x_3 = 0.845$$

$$h_3 = h_{f \text{ at } 0.005 \text{ MPa}} + x_3 \cdot h_{fg \text{ at } 0.005 \text{ MPa}}$$

$$= 137.82 + (0.845 \times 2423.7)$$

$$h_3 = 2185.85 \text{ kJ/kg}$$

$$h_4 = h_{f \text{ at } 0.005 \text{ MPa}} = 137.82 \text{ kJ/kg}$$

$$\text{For pumping process, } (h_1 - h_4) = v_4 \times (p_1 - p_4)$$

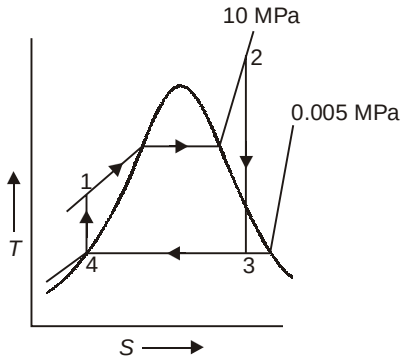


Fig. 8.33

$$v_4 = v_{f \text{ at } 0.005 \text{ MPa}} = 0.001005 \text{ m}^3/\text{kg}$$

$$h_1 = 137.82 + (0.001005 \times (10 - 0.005)) \times 10^2$$

$$h_1 = 138.82 \text{ kJ/kg}$$

$$\text{Net output per kg of steam, } w_{\text{net}} = (h_2 - h_3) - (h_1 - h_4)$$

$$= (3870.5 - 2185.85) - (138.82 - 137.82)$$

$$w_{\text{net}} = 1683.65 \text{ kJ/kg}$$

$$\text{Mass flow rate of steam, } m_s = \frac{50 \times 10^3}{1683.65} = 29.69 \text{ kg/s}$$

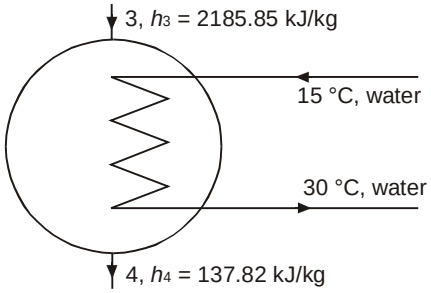


Fig. 8.34

By heat balance on condenser, for mass flow rate of water being m_w kg/s.

$$(h_3 - h_4) \times m_s = m_w \cdot C_{p,\,w}\,(T_{w,\,\text{out}} - T_{w,\,\text{in}})$$

$$29.69 \times (2185.85 - 137.82) = m_w \times 4.18\ (15)$$

$$m_w = 969.79 \text{ kg/s}$$

The heat added per kg of steam $q_{\text{add}} = (h_2 - h_1) = 3731.68$ kJ/kg

$$\text{Thermal efficiency} = \frac{w_{\text{net}}}{q_{\text{add}}} = \frac{1683.65}{3731.68} = 0.4512 \text{ or } 45.12\%$$

$$\text{Ratio of heat supplied and rejected} = \frac{(h_2 - h_1)}{(h_3 - h_4)} = 1.822$$

Mass of flow rate of steam	= **29.69 kg/s**
Mass flow rate of condenser cooling water	= **969.79 kg/s**
Thermal efficiency	= **45.12%**
Ratio of heat supplied and rejected	= **1.822**

Ans.

6. *A regenerative Rankine cycle has steam entering turbine at 200 bar, 650ºC and leaving at 0.05 bar. Considering feed water heaters to be of open type determine thermal efficiency for the following conditions;*

(a) there is no feed water heater

(b) there is only one feed water heater working at 8 bar

(c) there are two feed water heaters working at 40 bar and 4 bar respectively.

Also give layout and T-s representation for each of the case described above.

Solution:

Case (a) When there is no feed water heater

$$\text{Thermal efficiency of cycle} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$$

From steam tables,

$$h_2 = h_{\text{at 200 bar, 650°C}} = 3675.3 \text{ kJ/kg}$$

$$s_2 = s_{\text{at 200 bar, 650°C}} = 6.6582 \text{ kJ/kg} \cdot \text{K}$$

$$h_4 = h_{f \text{ at 0.05 bar}} = 137.82 \text{ kJ/kg}$$

$$v_4 = v_{f \text{ at 0.05 bar}} = 0.001005 \text{ m}^3\text{/kg}.$$

$$h_{f \text{ at 0.05 bar}} = 137.82 \text{ kJ/kg}, \; h_{fg \text{ at 0.05 bar}} = 2423.7 \text{ kJ/kg}$$

$$s_{f \text{ at 0.05 bar}} = 0.4764 \text{ kJ/kg} \cdot \text{K}, \; s_{fg \text{ at 0.05 bar}} = 7.9187 \text{ kJ/kg} \cdot \text{K}$$

For process 2 – 3, $s_2 = s_3$. Let dryness fraction at 3 be x_3.

$$s_3 = 6.6582 = s_{f \text{ at 0.05 bar}} + x_3 \cdot s_{fg \text{ at 0.05 bar}}$$

$$6.6582 = 0.4764 + x_3 \cdot 7.9187$$

$$x_3 = 0.781$$

$$h_3 = h_{f \text{ at 0.05 bar}} + x_3 \cdot h_{fg \text{ at 0.05 bar}}$$

$$= 2030.73 \text{ kJ/kg}$$

For pumping process 4-1,

$$h_1 - h_4 = v_4 \cdot \Delta p$$

$$h_1 - 137.82 = 0.001005 \times (200 - 0.05) \times 10^2$$

$$h_1 = 157.92 \text{ kJ/kg}$$

Thermal efficiency of cycle = $\dfrac{(3675.3 - 2030.73) - (157.92 - 137.82)}{(3675.3 - 157.92)}$

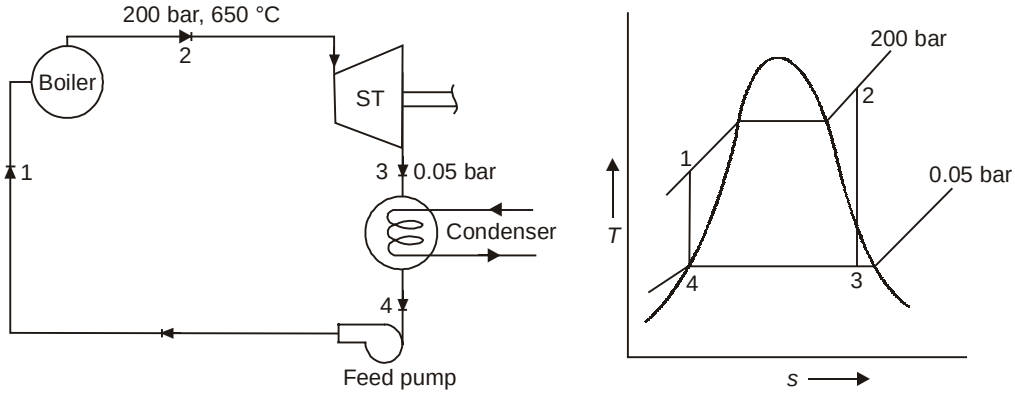


Fig. 8.35 *Layout and T-s diagram, (Q 6.a)*

= 0.4618 or 46.18%

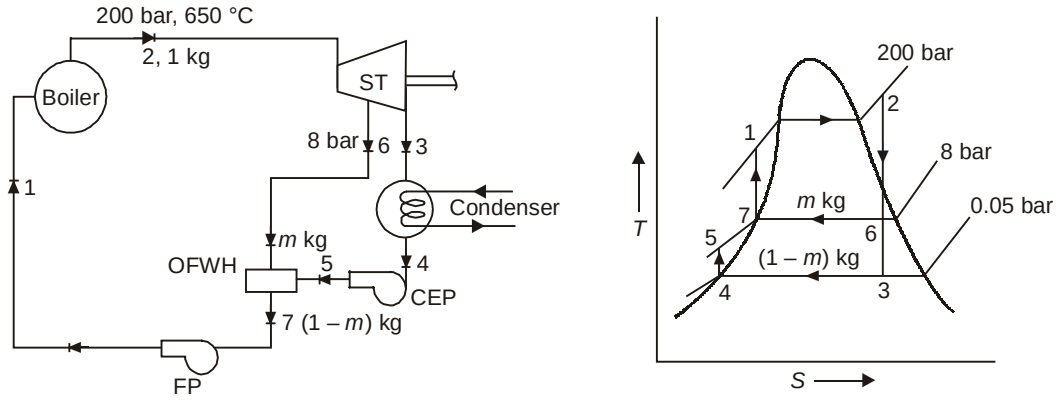
Case (b) When there is only one feed water heater working at 8 bar

Here, let mass of steam bled for feed heating be *m* kg

For process 2-6, $s_2 = s_6 = 6.6582 \text{ kJ/kg} \cdot \text{K}$

Let dryness fraction at state 6 be x_6

$$s_6 = s_{f \text{ at 8 bar}} + x_6 \cdot s_{fg \text{ at 8 bar}}$$



CEP = Condensate Extraction Pump
FP = Feed Pump
ST = Steam Turbine
OFWH = Open Feed Water Heater

Fig. 8.36 *Layout and T-s diagram, (Q 6,b)*

From steam tables, $h_{f \text{ at } 8 \text{ bar}} = 721.11$ kJ/kg

$v_{f \text{ at } 8 \text{ bar}} = 0.001115\ m^3/\text{kg}$, $h_{fg \text{ at } 8 \text{ bar}} = 2048$ kJ/kg

$s_{f \text{ at } 8 \text{ bar}} = 2.0462$ kJ/kg · K, $s_{fg \text{ at bar}} = 4.6166$ kJ/kg · K

Substituting entropy values, $x_6 = 0.999$

$h_6 = h_{f \text{ at } 8 \text{ bar}} + x_6 \cdot h_{fg \text{ at } 8 \text{ bar}} = 721.11 + (0.999 \times 2048) = 2767.06$ kJ/kg

Assuming the state of fluid leaving open feed water heater to be saturated liquid at 8 bar.

$$h_7 = h_{f \text{ at } 8 \text{ bar}} = 721.11 \text{ kJ/kg.}$$

For process 4-5, $h_5 = h_4 + v_4 \times (8 - 0.05) \times 10^2 = 137.82 + (0.001005 \times 7.95 \times 10^2) = 138.62$ kJ/kg

Applying energy balance at open feed water heater,

$$m \times h_6 + (1 - m) \times h_5 = 1 \times h_7$$

$$(m \times 2767.06) + ((1 - m) \times 138.62) = 721.11$$

$$m = 0.2216 \text{ kg}$$

For process 7-1, $h_1 = h_7 + v_7\ (200 - 8) \times 10^2$; here $h_7 = h_{f \text{ at } 8 \text{ bar}}$, $v_7 = v_{f \text{ at } 8 \text{ bar}}$

$$h_1 = h_7 + v_{f \text{ at } 8 \text{ bar}}\ (200 - 8) \times 10^2 = 721.11 + (0.001115 \times 192 \times 10^2)$$

$$h_1 = 742.518 \text{ kJ/kg}$$

$$\text{Thermal efficiency of cycle} = \frac{(h_2 - h_6) + (1 - m) \cdot (h_6 - h_3) - \{(1 - m)(h_5 - h_4) + (h_1 - h_7)\}}{(h_2 - h_1)}$$

$$= \frac{(3675.3 - 2767.06) + (1 - 0.2216) \times (2767.06 - 2030.73) - \{(1 - 0.2216) \times (138.62 - 137.82) + (742.518 - 721.11)\}}{(3675.3 - 742.518)}$$

Thermal efficiency of cycle = 0.4976 or 49.76%

Case (c) When there are two feed water heaters working at 40 bar and 4 bar

Here, let us assume the mass of steam at 40 bar, 4 bar to be m_1 kg, and m_2 kg respectively.

For process 2–10–9–3, $s_2 = s_{10} = s_9 = s_3 = 6.6582$ kJ/kg ·K

At state 10. $s_{10} > s_{g \text{ at } 40 \text{ bar}}$ (6.0701 kJ/kg · K) so state 10 lies in superheated region at 40 bar pressure. From steam table by interpolation, $T_{10} = 370.36^{o}$C so, $h_{10} = 3141.81$ kJ/kg

Let dryness fraction at state 9 be x_9 so,

$$s_9 = 6.6582 = s_{f \text{ at } 4 \text{ bar}} + x_9 \cdot s_{fg \text{ at } 4 \text{ bar}}$$

$$6.6582 = 1.7766 + x_9 \times 5.1193$$

$$x_9 = 0.9536$$

$$h_9 = h_{f \text{ at } 4 \text{ bar}} + x_9 \times h_{fg \text{ at } 4 \text{ bar}} = 604.74 + 0.9536 \times 2133.8$$

$$h_9 = 2639.53 \text{ kJ/kg}$$

Assuming the state of fluid leaving open feed water heater to be saturated liquid at respective pressures i.e.

$$h_{11} = h_{f \text{ at } 4 \text{ bar}} = 604.74 \text{ kJ/kg}, \; v_{11} = 0.001084 \; m^3/\text{kg} = v_{f \text{ at } 4 \text{ bar}}$$

$$h_{13} = h_{f \text{ at } 40 \text{ bar}} = 1087.31 \text{ kJ/kg}, \; v_{13} = 0.001252 \; m^3/\text{kg} = v_{f \text{ at } 40 \text{ bar}}$$

For process 4–8, i.e. in CEP.

$$h_8 = h_4 + v_4 \times (4 - 0.05) \times 10^2$$

$$= 137.82 + (0.001005 \times 3.95 \times 10^2)$$

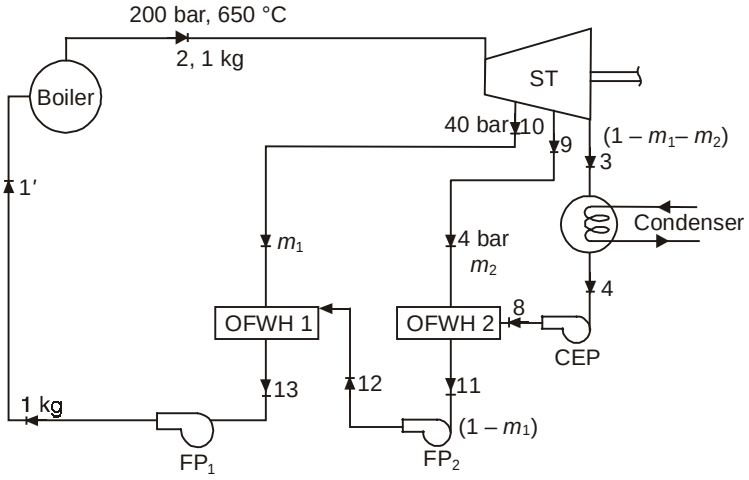
$$h_8 = 138.22 \text{ kJ/kg}$$

For process 11-12, i.e. in FP_2,

$$h_{12} = h_{11} + v_{11}(40–4) \times 10^2$$

$$= 604.74 + (0.001084 \times 36 \times 10^2)$$

$$h_{12} = 608.64 \text{ kJ/kg}$$



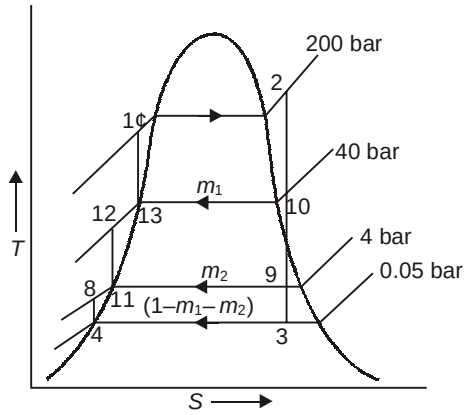


Fig. 8.37 *Layout and T-s diagram. (Q6.c)*

For process 13-1′ i.e. in FP_1, $h'_1 = h_{13} + v_{13}(200 - 40) \times 10^2$

$= 1087.31 + (0.001252 \times 160 \times 10^2)$

$h'_1 = 1107.34$ kJ/kg

$(m_1 \times 3141.81) + (1 - m_1) \times 608.64 = 1087.31$

$m_1 = 0.189$ kg

Applying energy balance an open feed water heater 1 (OFWH1)

$(m_1 \times h_{10}) + (1 - m_1) \times h_{12} = 1 \times h_{13}$

$(m_1 \times 3141.81) + (1 - m_1) \times 608.64 = 1087.31$

$m_1 = 0.189$ kg

Applying energy balance an open feed water heater 2 (OFWH2)

$m_2 \times h_9 + (1 - m_1 - m_2)\, h_8 = (1 - m_1) \times h_{11}$

$(m_2 \times 2639.53) + (1 - 0.189 - m_2) \times 138.22 = (1 - 0.189) \times 604.74$, $m_2 = 0.151$ kg

Thermal efficiency of cycle,

$$\eta = \frac{\{(h_2 - h_{10}) + (1 - m_1)(h_{10} - h_9) + (1 - m_1 - m_2)(h_9 - h_3)\} - \{W_{CEP} + W_{FP_1} + W_{FP_2}\}}{(h_2 - h_{1'})}$$

$W_{CEP} = (1 - m_1 - m_2)\,(h_8 - h_4) = 0.264$ kJ/kg steam from boiler

$W_{FP_1} = (h_{1'} - h_{13}) = 20.03$ kJ/kg of steam from boiler

$W_{FP_2} = (1 - m_1)\,(h_{12} - h_{11}) = 3.16$ kJ/kg of steam from boiler

$W_{CEP} + W_{FP_1} + W_{FP_2} = 23.454$ kJ/kg of steam from boiler

$$\eta = \frac{\{(3675.3 - 3141.81) + (1 - 0.189)(3141.81 - 2639.53) + (1 - 0.189 - 0.151)(2639.53 - 2030.73)\} - \{23.454\}}{(3675.3 - 1107.34)}$$

$= 0.5137$ or 51.37%

Cycle thermal efficiency η_a = **46.18%**

η_b = **49.76%**

η_c = **51.37%** **Ans.**

Hence it is obvious that efficiency increases with increase in number of feed heaters.

7. *A reheat cycle has steam generated at 50 bar, 500ºC for being sent to high pressure turbine and expanded upto 5 bar before supplied to low pressure turbine. Steam enters at 5 bar, 400ºC into low pressure turbine after being reheated in boiler. Steam finally enters condenser at 0.05 bar and subsequently feed water is sent to boiler. Determine cycle efficiency, specific steam consumption and work ratio.*

Solution:

From steam table,

$$h_2 = h_{\text{at 50 bar, 500ºC}} = 3433.8 \text{ kJ/kg}$$

$$s_2 = s_{\text{at 50 bar, 500ºC}} = 6.9759 \text{ kJ/kg} \cdot \text{K}$$

$$s_3 = s_2 = 6.9759 \text{ kJ/kg} \cdot \text{K}$$

Since $s_3 > s_{\text{g at 5 bar}}$ so state 3 lies in superheated region at 5 bar,

By interpolation from steam tables,

$$T_3 = 183.14^\circ\text{C at 5 bar}, \; h_3 = 2818.03 \text{ kJ/kg}$$

$$h_4 = h_{\text{ at 5 bar, 400ºC}} = 3271.9 \text{ kJ/kg}$$

$$s_4 = s_{\text{ at 5 bar, 400ºC}} = 7.7938 \text{ kJ/kg} \cdot \text{K}$$

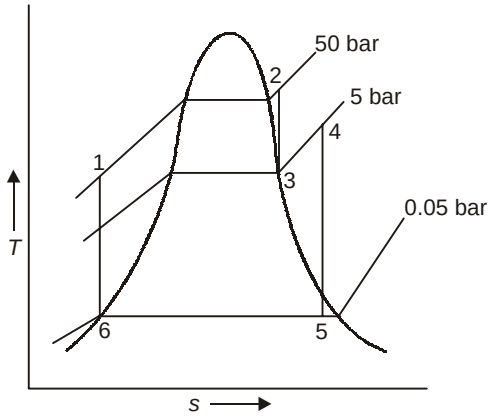


Fig. 8.38

For expansion process 4-5, $s_4 = s_5 = 7.7938 \text{ kJ/kg} \cdot \text{K}$

Let dryness fraction at state 5 be x_5.

$$s_5 = s_{f \text{ at 0.05 bar}} + x_5 \times s_{fg \text{ at 0.05 bar}}$$

$$7.7938 = 0.4764 + x_5 \times 7.9187$$

$$x_5 = 0.924$$

$$h_5 = h_{f \text{ at 0.05 bar}} + x_5 \times h_{fg \text{ at 0.05 bar}}$$

$$h_5 = 137.82 + 0.924 \times 2423.7 = 2377.32 \text{ kJ/kg}$$

$$h_6 = h_{f \text{ at 0.05 bar}} = 137.82 \text{ kJ/kg}$$

$$h_6 = v_{f \text{ at 0.05 bar}} = 0.001005 \text{ m}^3\text{/kg}$$

For process 6-1 in feed pump, $h_1 = h_6 + v_6 \times (50 - 0.05) \times 10^2$

$$h_1 = 137.82 + 0.001005 \times (49.95 \times 10^2)$$

$$h_1 = 142.84 \text{ kJ/kg}$$

$$\text{Cycle efficiency} = \frac{W_{net}}{Q_{add}}$$

$$W_T = (h_2 - h_3) + (h_4 - h_5)$$
$$= (3433.8 - 2818.03) + (3271.9 - 2377.32)$$
$$= 1510.35 \text{ kJ/kg}$$
$$W_{pump} = (h_1 - h_6) = 142.84 - 137.82 = 5.02 \text{ kJ/kg}$$
$$W_{net} = W_T - W_{pump} = 1505.33 \text{ kJ/kg}$$
$$Q_{add} = (h_2 - h_1) = 3433.8 - 142.84 = 3290.96 \text{ kJ/kg}$$

$$\text{Cycle efficiency} = \frac{1505.33}{3290.96} = 0.4574 \text{ or } 45.74\%$$

We know, 1 hp = 0.7457 kW

$$\text{Specific steam consumption} = \frac{0.7457 \times 3600}{1505.33} = 1.78 \text{ kg/hp} \cdot \text{hr}$$

$$\text{Work ratio} = \frac{\text{Net work}}{\text{Positive work}} = \frac{W_{net}}{W_T} = \frac{1505.33}{1510.35} = 0.9967$$

Cycle efficiency = **45.74%,** Specific steam consumption = **1.78 kg/hp. hr.**
Work ratio = **0.9967.** **Ans.**

8. *In a steam power plant the high pressure turbine is fed with steam at 60 bar, 450°C and enters low pressure turbine at 3 bar with a portion of steam bled out for feed heating at this intermediate pressure. Steam finally leaves low pressure turbine at 0.05 bar for inlet to condenser. Closed feed heater raises the condensate temperature to 115°C. Bled steam leaving closed feed heater is passed through trap to mix with condensate leaving condenser. Consider actual alternator output to be 30 MW, boiler efficiency as 90% and alternator efficiency of 98%. Determine,*

(a) the mass of steam bled for feed heating,
(b) the capacity of boiler in kg/hr.
(c) the overall thermal efficiency of plant

Also give layout and T-s diagram.

Solution:

From steam tables,

At state 2, $h_2 = 3301.8$ kJ/kg, $s_2 = 6.7193$ kJ/kg · K

$h_5 = h_{f \text{ at } 0.05 \text{ bar}} = 137.82$ kJ/kg, $v_5 = v_{f \text{ at } 0.05 \text{ bar}} = 0.001005$ m^3/kg

Let mass of steam bled for feed heating be m kg/kg of steam generated in boiler. Let us also assume that condensate leaves closed feed water heater as saturated liquid i.e.

$$h_8 = h_{f \text{ at } 3 \text{ bar}}$$
$$h_8 = 561.47 \text{ kJ/kg}$$

For process 2–3–4, $s_2 = s_3 = s_4 = 6.7193$ kJ/kg · K

Let dryness fraction at state 3 and state 4 be x_3 and x_4 respectively.

$$s_3 = 6.7193 = s_{f \text{ at } 3 \text{ bar}} + x_3 \cdot s_{fg \text{ at } 3 \text{ bar}}$$

$$= 1.6718 + x_3 \times 5.3201$$

$$x_3 = 0.949$$

$$s_4 = 6.7193 = s_{f \text{ at } 0.05 \text{ bar}} + x_4 \cdot s_{fg \text{ at } 0.05 \text{ bar}}$$

$$= 0.4764 + x_4 \times 7.9187$$

$$x_4 = 0.788$$

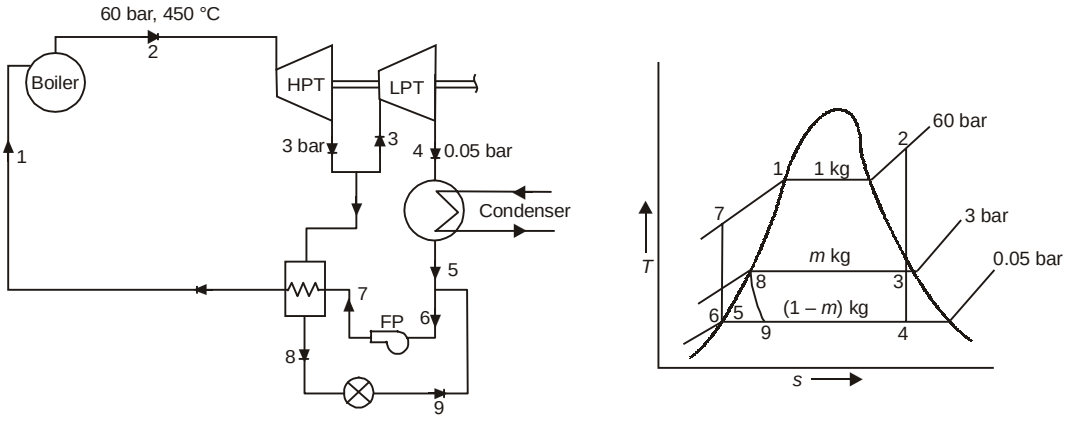


Fig. 8.39 *Layout and T-s diagram*

Thus, $h_3 = h_{f \text{ at } 3 \text{ bar}} + x_3 \cdot h_{fg \text{ at } 3 \text{ bar}} = 561.47 + (0.949 \times 2163.8) = 2614.92$ kJ/kg

$h_4 = h_{f \text{ at } 0.05 \text{ bar}} + x_4 \cdot h_{fg \text{ at } 0.05 \text{ bar}} = 137.82 + (0.788 \times 2423.7) = 2047.6$ kJ/kg

Assuming process across trap to be of throttling type so, $h_8 = h_9 = 561.47$ kJ/kg. Assuming $v_5 = v_6$,

Pumping work $(h_7 - h_6) = v_5 \cdot (60 - 0.05) \times 10^2$

$(h_7 - h_6) = 6.02$ kJ/kg

For mixing process between condenser and feed pump,

$$(1 - m) \cdot h_5 + m \cdot h_9 = 1 \cdot h_6$$

$$(1 - m) \times 137.82 + m \times 561.47 = h_6$$

$$h_6 = 137.82 + m \times 423.65$$

Therefore, $h_7 = h_6 + 6.02 = 143.84 + m \times 423.65$

Applying energy balance at closed feed water heater;

$$m \times h_3 + (1 - m)h_7 = m \cdot h_8 + (4.18 \times 115)$$

$(m \times 2614.92) + (1 - m)\ (143.84 + m \times 423.65) = m \times 561.47 + 480.7$, $m = 0.144$ kg

Steam bled for feed heating = **0.144 kg/kg steam generated.** **Ans.**

The net power output $W_{net} = (h_2 - h_3) + (1 - m)\ (h_3 - h_4) - (1 - m) \times (h_7 - h_6)$

$= 1167.28$ kJ/kg steam generated

Mass of steam required to be generated = $\dfrac{30 \times 10^3}{0.98 \times W_{net}}$

$$= \frac{30\times10^3}{0.98\times1167.28} = 26.23 \text{ kg/s}$$

$$= 94428 \text{ kg/hr}$$

Capacity of boiler required = **94428 kg/hr** **Ans.**

$$\text{Overall thermal efficiency} = \frac{W_{net}}{Q_{add}}$$

$$Q_{add} = \frac{(h_2 - h_1)}{0.90} = \frac{(3301.8 - 4.18\times115)}{0.90}$$

$$= 3134.56 \text{ kJ/kg}$$

$$\text{Overall thermal efficiency} = \frac{1167.28}{3134.56} = 0.3724 \text{ or } 37.24\%$$

Overall thermal efficiency = **37.24%** **Ans.**

9. *A steam power plant has expansion occurring stages in three stages with steam entering first stage at 30 bar, 400°C and leaving first stage at 6 bar for being sent to second stage with some steam being bled out for feed heating in closed feed water heater. Steam leaves second stage at 1 bar and enters third stage with some more steam being bled out for feed heating in closed feed water heater. Steam finally leaves third stage at 0.075 bar after complete expansion and enters condenser. Condensate temperature is 38ºC at inlet to second heater, temperature of feed water after first heater and second heater is 150ºC and 95ºC respectively. Bled steam is condensed to saturated liquid with no undercooling in each of feed heater. Drain from first heater is passed through steam trap into second feed heater and combined drain from second heater is pumped by drain pump into feed line after second heater. Considering efficiency ratio of turbine as 0.8 and turbine output of 15 MW determine the capacity of drain pump. Neglect drain pump work.*

Solution:

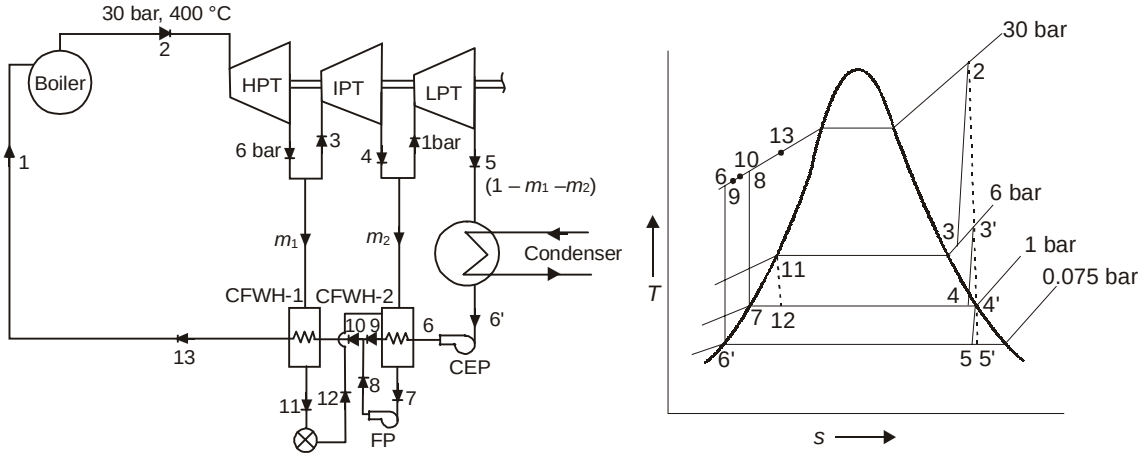


Fig. 8.40 *Layout and T-s diagram*

At inlet to first turbine stage, $h_2 = 3230.9$ kJ/kg, $s_2 = 6.9212$ kJ/kg ·K

For ideal expansion process $s_2 = s_3$

By interpolation, $T_3 = 190.97^oC$ from superheated steam tables at 6 bar

$$h_3 = 2829.63 \text{ kJ/kg}$$

actual state at exit of first stage $h'_3 = h_2 - 0.8 \times (h_2 - h_3)$

$$h'_3 = 2909.88 \text{ kJ/kg}$$

Actual state 3′ shall be at 232.78ºC, 6 bar, so $s'_3 = 7.1075$ kJ/kg · K

For second stage $s'_3 = s_4$; By interpolation, $s_4 = 7.1075$

$$= s_{f \text{ at 1 bar}} + x_4 \cdot s_{fg \text{ at 1 bar}}$$

$$7.1075 = 1.3026 + x_4 \cdot 6.0568$$

$$x_4 = 0.958$$

$$h_4 = h_{f \text{ at 1 bar}} + x_4 \cdot h_{fg \text{ at 1 bar}}$$

$$= 417.46 + (0.958 \times 2258)$$

$$h_4 = 2580.62 \text{ kJ/kg}$$

Actual enthalpy at exit from second stage, $h_{4'} = h_{3'} - 0.8\ (h_{3'} - h_4)$

$$h_{4'} = 2646.47 \text{ kJ/kg}$$

Actual dryness fraction, $x_{4'} \Rightarrow h_{4'} = h_{f \text{ at 1 bar}} + x_{4'} \cdot h_{fg \text{ at 1 bar}}$

$x_{4'} = 0.987$, Actual entropy, $s_{4'} = 7.2806$ kJ/kg · K

For third stage, $s_{4'} = s_5 = 7.2806 = s_{f \text{ at 0.075 bar}} + x_5 \cdot s_{fg \text{ at 0.075 bar}}$

$$x_5 = 0.8735$$

$$h_5 = 2270.43 \text{ kJ/kg}$$

Actual enthalpy at exit from third stage, $h_{5'} = h_{4'} - 0.8(h_{4'} - h_5)$

$$h_{5'} = 2345.64 \text{ kJ/kg}$$

Let mass of steam bled out be m_1 and m_2 kg at 6 bar, 1 bar respectively.

By heat balance on first closed feed water heater, (see schematic arrangement)

$$h_{11} = h_{f \text{ at 6 bar}} = 670.56 \text{ kJ/kg}$$

$$m_1 \times h_{3'} + h_{10} = m_1 \cdot h_{11} + 4.18 \times 150$$

$$(m_1 \times 2829.63) + h_{10} = (m_1 \cdot 670.56) + 627$$

$$h_{10} + (2159.07)\ m_1 = 627$$

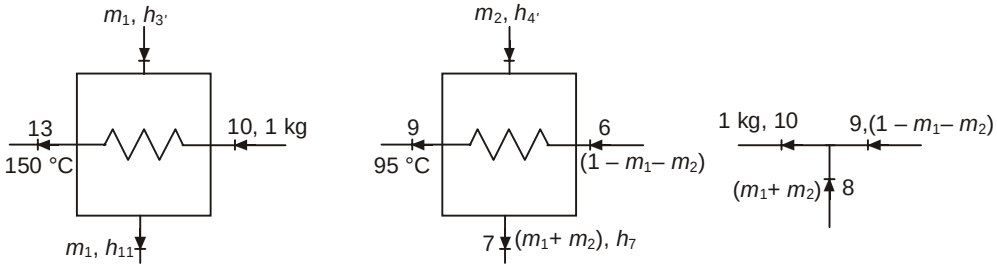
By heat balance on second closed feed water heater, (see schematic arrangement)

$$h_7 = h_{f \text{ at 1 bar}} = 417.46 \text{ kJ/kg}$$

$$m_2 \cdot h_{4'} + (1 - m_1 - m_2) \times 4.18 \times 38 = (m_1 + m_2) \cdot h_7 + 4.18 \times 95 \times (1 - m_1 - m_2)$$

$$(m_2 \cdot 2646.47) + (1 - m_1 - m_2) \times 158.84 = ((m_1 + m_2) \cdot 417.46) + (397.1 \times (1 - m_1 - m_2))$$

$$m_2 \times 2467.27 - m_1 \times 179.2 - 238.26 = 0$$

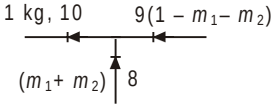


Heat balance at point of mixing.

$$h_{10} = (m_1 + m_2) \cdot h_8 + (1 - m_1 - m_2) \times 4.18 \times 95$$

Neglecting pump work, $h_7 = h_8$

$$h_{10} = m_2 \times 417.46 + (1 - m_1 - m_2) \times 397.1$$



Substituting h_{10} and solving we get, $m_1 = 0.1293$ kg

$m_2 = 0.1059$ kg/kg of steam generated.

Turbine output per kg of steam generated,

$$w_T = (h_2 - h_{3'}) + (1 - m_1)(h_{3'} - h_{4'}) + (1 - m_1 - m_2) \cdot (h_{4'} - h_{5'})$$

$w_T = 780.446$ kJ/kg of steam generated.

$$\text{Rate of steam generation required} = \frac{15 \times 10^3}{780.446} = 19.22 \text{ kg/s}$$

or $= 69192$ kg/hr

Capacity of drain pump i.e. FP shown in layout $= (m_1 + m_2) \times 69192$

$= 16273.96$ kg/hr

Capacity of drain pump = **16273.96 kg/hr** **Ans.**

10. *A steam power plant has steam entering at 70 bar, 450ºC into HP turbine. Steam is extracted at 30 bar and reheated upto 400ºC before being expanded in LP turbine upto 0.075 bar. Some portion of steam is bled out during expansion in LP turbine so as to yield saturated liquid at 140ºC at the exit of open feed water heater. Considering HP and LP turbine efficiencies of 80% and 85% determine the cycle efficiency. Also give layout and T-s diagram.*

Solution:

At inlet to HP turbine, $h_2 = 3287.1$ kJ/kg, $s_2 = 6.6327$ kJ/kg.K

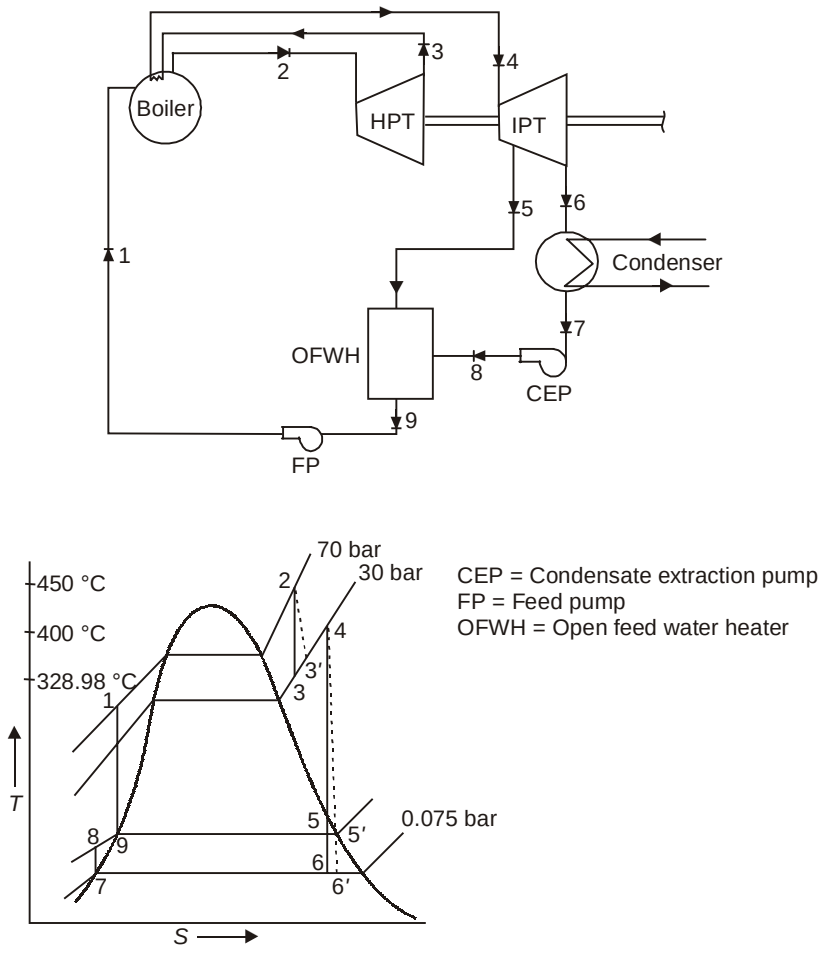


Fig. 8.41 *Layout and T-S diagram*

By interpolation state 3 i.e. for insentropic expansion between 2 – 3 lies at 328.98ºC at 30 bar.

$$h_3 = 3049.48 \text{ kJ/kg.}$$

Actual enthalpy at 3', $h_{3'} = h_2 - 0.80\ (h_2 - h_3)$

$$h_{3'} = 3097 \text{ kJ/kg}$$

Enthalpy at inlet to LP turbine, $h_4 = 3230.9$ kJ/kg, $s_4 = 6.9212$ kJ · K

For ideal expansion from 4-6, $s_4 = s_6$. Let dryness fraction at state 6 be x_6.

$$s_6 = 6.9212 = s_{f \text{ at } 0.075 \text{ bar}} + x_6 \cdot s_{fg \text{ at } 0.075 \text{ bar}}$$

$$x_6 = 0.827$$

$$h_6 = h_{f \text{ at } 0.075 \text{ bar}} + x_6 \cdot h_{fg \text{ at } 0.075 \text{ bar}} = 2158.55 \text{ kJ/kg}$$

For actual expansion process in LP turbine.

$$h_{6'} = h_4 - 0.85\ (h_4 - h_6)$$

$$h_{6'} = 2319.4 \text{ kJ/kg}$$

Ideally, enthalpy at bleed point can be obtained by locating state 5 using $s_5 = s_4$. The pressure at bleed point shall be saturation pressure corresponding to the 140°C i.e. from steam table $p_5 = 3.61$ bar. Let dryness fraction at state 5 be x_5.

$$s_5 = 6.9212 = s_{f \text{ at } 140^\circ C} + x_5 \cdot s_{fg \text{ at } 140^\circ C} \Rightarrow x_5 = 0.99$$

$$h_5 = h_{f \text{ at } 140^\circ C} + x_5 \cdot h_{fg \text{ at } 140^\circ C} \Rightarrow h_5 = 2712.38 \text{ kJ/kg}$$

Actual exthalpy $h_{5'} = h_4 - 0.85\ (h_4 - h_5) = 2790.16$ kJ/kg

Enthalpy at exit of open feed water heater, $h_9 = h_{f \text{ at } 30 \text{ bar}} = 1008.42$ kJ/kg

Specific volume at inlet of CEP, $v_7 = 0.001008\ m^3/kg$,

Enthalpy at inlet of CEP, $h_7 = 168.79$ kJ/kg

For pumping process 7-8

$$h_8 = h_7 + v_7\ (3.61 - 0.075) \times 10^2$$

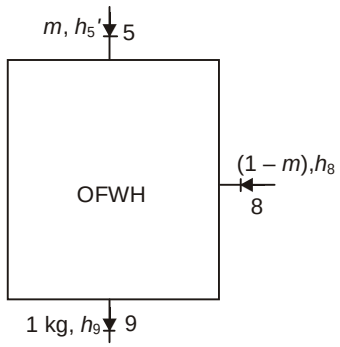
$$h_8 = 169.15 \text{ kJ/kg}$$

Applying energy balance at open feed water heater. Let mass of bled steam be m kg per kg of steam generated.

$$m \times h_{5'} + (1 - m) \cdot h_8 = h_9$$

$$(m \times 2790.16) + ((1 - m) \cdot 169.15) = 1008.42$$

$$m = 0.32 \text{ kg/kg of steam generated}$$



For process on feed pump, 9 – 1, $v_9 = v_{f \text{ at } 140^\circ C} = 0.00108$

$$h_1 = h_9 + v_9 \times (70 - 3.61) \times 10^2$$

$$h_1 = 1015.59 \text{ kJ/kg}$$

Net work per kg of steam generated,

$$W_{net} = (h_2 - h_{3'}) + (h_4 - h_{5'}) + (1 - m) \cdot (h_{5'} - h_{6'}) - \{(1 - m)\ (h_8 - h_7) + (h_1 - h_9)\}$$

$$= 181.1 + 440.74 + 320.117 - \{0.2448 + 7.17\}$$

$$W_{net} = 934.54 \text{ kJ/kg steam generated}$$

Heat added per kg of steam generated,

$$q_{add} = (h_2 - h_1) + (h_4 - h_{3'})$$

$$q_{add} = 2262.51 + 133.9 = 2396.41 \text{ kJ/kg of steam generated}$$

Thermal efficiency, $\eta = \dfrac{W_{net}}{q_{add}} = \dfrac{934.54}{2396.41}$

$$\eta = 0.3899 \text{ or } 38.99\%$$

Thermal efficiency = **38.99%** **Ans.**

11. *A steam power plant works on regenerative cycle with steam entering first turbine stage at 150 bar, 500°C and getting expanded in three subsequent stages upto the condenser pressure of 0.05 bar. Some steam is bled out between first and second stage for feed heating in closed feed water heater at 10 bar with the saturated liquid condensate being pumped ahead into the boiler feed water line. Feed water leaves closed feed water heater at 150 bar, 150°C. Steam is also taken out between second and third stages at 1.5 bar for being fed into an open feed water heater working at that pressure. Saturated liquid at 1.5 bar leaves open feed water heater for being sent to closed feed water heater. Considering mass flow rate of 300 kg/s into the first stage of turbine determine cycle thermal efficiency and net power developed in kW. Also give lay out and T-s representation.*

Solution:

Enthalpy of steam entering ST_1, $h_2 = 3308.6$ kJ/kg, $s_2 = 6.3443$ kJ/kg · K

For isentropic expansion 2-3-4-5, $s_2 = s_3 = s_4 = s_5$

Let dryness fraction of states 3, 4 and 5 be x_3, x_4 and x_5

$$s_3 = 6.3443 = s_{f \text{ at } 10 \text{ bar}} + x_3 \cdot s_{fg \text{ at } 10 \text{ bar}}$$

$$\Rightarrow \quad x_3 = 0.945$$

$$h_3 = 2667.26 \text{ kJ/kg}$$

$$s_4 = 6.3443 = s_{f \text{ at } 1.5 \text{ bar}} + x_4 \cdot s_{fg \text{ at } 1.5 \text{ bar}}$$

$$\Rightarrow \quad x_4 = 0.848 \Rightarrow h_4 = 2355.18 \text{ kJ/kg}$$

$$s_5 = 6.3443 = s_{f \text{ at } 0.05 \text{ bar}} + x_5 \cdot s_{fg \text{ at } 0.05 \text{ bar}}$$

$$\Rightarrow \quad x_5 = 0.739 \cdot h_5 = 1928.93 \text{ kJ/kg}$$

$$h_6 = h_{f \text{ at } 0.05 \text{ bar}} = 137.82 \text{ kJ/kg}$$

$$v_6 = 0.001005 \ m^3/\text{kg}$$

$$= v_{f \text{ at } 0.05 \text{ bar}}$$

$$h_7 = h_6 + v_6 (1.5 - 0.05) \times 10^2 = 137.96 \text{ kJ/kg}$$

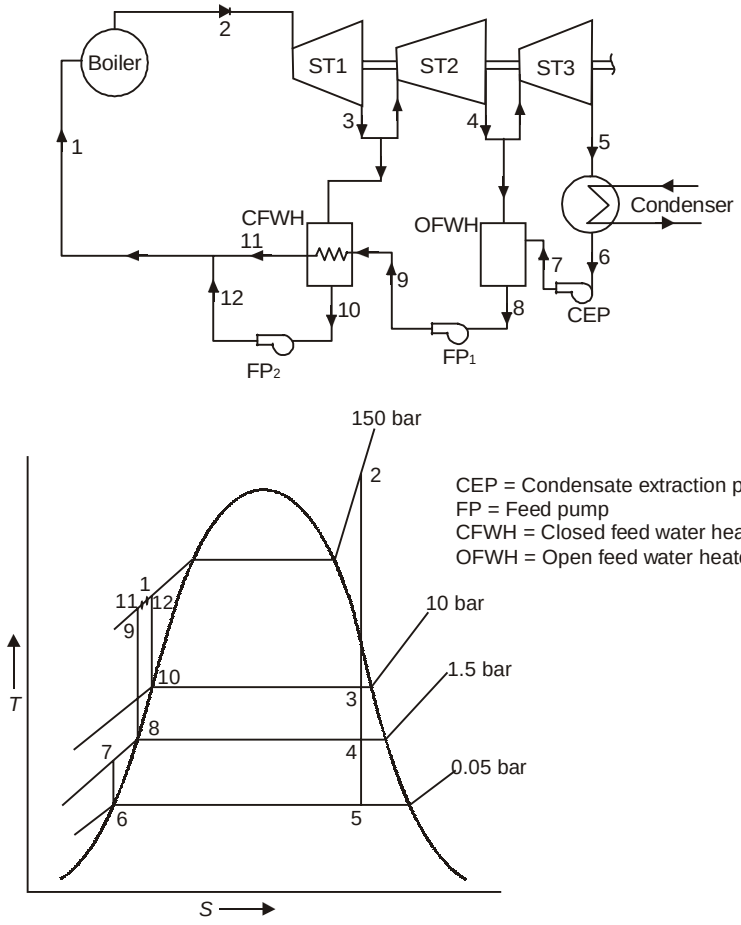


Fig. 8.42 *Layout and T-s diagram*

$h_8 = h_{f \text{ at } 1.5 \text{ bar}} = 467.11$ kJ/kg,

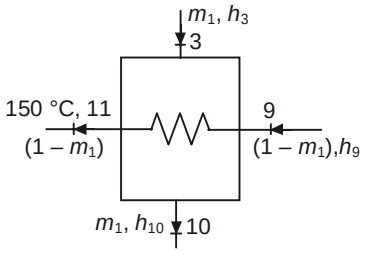
$v_8 = 0.001053 \text{ m}^3/\text{kg} = v_{f \text{ at } 1.5 \text{ bar}}$

$h_9 = h_8 + v_8(150 - 1.5) \times 10^2 = 482.75$ kJ/kg

$h_{10} = h_{f \text{ at } 150 \text{ bar}} = 1610.5$ kJ/kg

$v_{10} = 0.001658 \text{ m}^3/\text{kg} = v_{f \text{ at } 150 \text{ bar}}$

$h_{12} = h_{10} + v_{10}\ (150 - 10) \times 10^2 = 1633.71$ kJ/kg



Let mass of steam bled out at 10 bar, 1.5 bar be m_1 and m_2 per kg of steam generated.

Heat balance on closed feed water heater yields,

$$m_1 \cdot h_3 + (1 - m_1)\ h_9 = m_1 \cdot h_{10} + (1 - m_1) \times 4.18 \times 150$$

$$(m_1 \times 2667.26) + (1 - m_1) \times 482.75 = (m_1 \times 1610.5) + (627 \cdot (1 - m_1))$$

$$m_1 = 0.12 \text{ kg/kg of steam generated.}$$

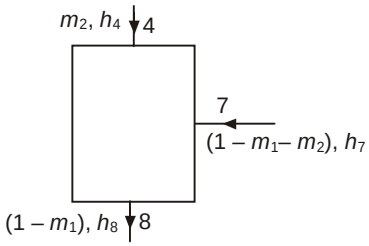
Heat balance on open feed water can be given as under

$$m_2 \cdot h_4 + (1 - m_1 - m_2) \cdot h_7 = (1 - m_1) \cdot h_8$$

$$(m_2 \times 2355.18) + (1 - m_1 - m_2) \times 137.96 = (1 - m_1) \times 467.11$$

$$(m_2 \times 2355.18) + (1 - 0.12 - m_2) \times 137.96 = (1 - 0.12) \times 467.11$$

$$m_2 = 0.13 \text{ kg/kg of steam}$$



For mass flow rate of 300 kg/s $\Rightarrow m_1 = 36$ kg/s, $m_2 = 39$ kg/s

For mixing after closed feed water heater,

$$h_1 = (4.18 \times 150) \cdot (1 - m_1) + m_1 \times h_{12} = 747.81 \text{ kJ/kg}$$

Net work output per kg of steam generated = $w_{ST_1} + w_{ST_2} + w_{ST_3} - \{w_{CEP} + w_{FP} + w_{FP_2}\}$

$$w_{net} = (h_2 - h_3) + (1 - m_1)\ (h_3 - h_4) + (1 - m_1 - m_2)\ (h_4 - h_5) - \{(1 - m_1 - m_2) \cdot (h_7 - h_6) + (1 - m_1) \cdot (h_9 - h_8) + (m_1 \cdot (h_{12} - h_{10}))\}$$

$$w_{net} = 641.34 + 274.63 + 319.69 = \{0.105 + 13.76 + 2.7852\}$$

$$w_{net} = 1219.00 \text{ kJ/kg of steam generated.}$$

Heat added per kg of steam generated. $q_{add} = (h_2 - h_1) = 2560.79$ kJ/kg

$$\text{Cycle thermal efficiency, } \eta = \frac{w_{net}}{q_{add}} = 0.4760 \text{ or } 47.6\%$$

Net power developed in kW = $1219 \times 300 = 365700$ kW

Cycle thermal efficiency = **47.6%**
Net power developed = **365700 kW** **Ans.**

12. *A steam power plant has expansion of steam leaving boiler at 100 bar, 500ºC occurring in three stages i.e. HPT, IPT and LPT (high pressure, intermediate pressure and low pressure turbine) upto condenser pressure of 0.075 bar. At exit of HPT some steam is extracted for feed heating in closed feed water heater at 20 bar and remaining is sent to IPT for subsequent expansion upto 4 bar. Some more quantity of steam is extracted at 4 bar for feed heating in open feed water heater and remaining steam is allowed to expand in low pressure turbine upto condenser pressure. Feed water leaves closed feed water heater at 100 bar and 200ºC. The condensate leaving as saturated liquid at 20 bar is trapped into open feed water heater. The state of liquid leaving open feed water heater may be considered saturated liquid at 4 bar. For a net power output of 100 MW determine thermal efficiency and steam generation rate in boiler.*

(a) Modify the above arrangement by introducing reheating of steam entering IPT at 20 bar upto 400ºC. Obtain thermal efficiency of modified cycle and compare it with non-reheat type arrangement.

Solution:

At inlet to HPT, $h_2 = 3373.7$ kJ/kg, $s_2 = 6.5966$ kJ/kg · K

For isentropic expansion between 2-3-4-5, $s_2 = s_3 = s_4 = s_5$

State 3 lies in superheated region as $s_3 > s_{g \text{ at } 20 \text{ bar}}$. By interpolation from superheated steam table, $T_3 = 261.6$ºC. Enthalpy at 3. $h_3 = 2930.57$ kJ/kg.

Since $s_4 < s_{g \text{ at } 4 \text{ bar}}$ so states 4 and 5 lie in wet region.

Let dryness fraction at state 4 and 5 be x_4 and x_5.

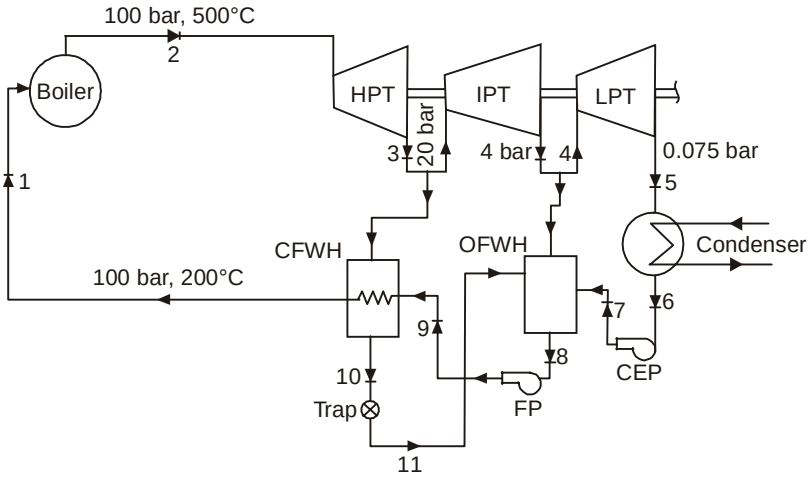
$$s_4 = 6.5966 = s_{f \text{ at } 4 \text{ bar}} + x_4 \cdot s_{fg \text{ at } 4 \text{ bar}}$$

$$x_4 = 0.941$$

$$h_4 = h_{f \text{ at } 4 \text{ bar}} + x_4 \cdot h_{fg \text{ at } 4 \text{ bar}} = 2612.65 \text{ kJ/kg}$$

for state 5, $$s_5 = 6.5966 = s_{f \text{ at } 0.075 \text{ bar}} + x_5 \cdot s_{fg \text{ at } 0.075 \text{ bar}}$$

$$x_5 = 0.784$$



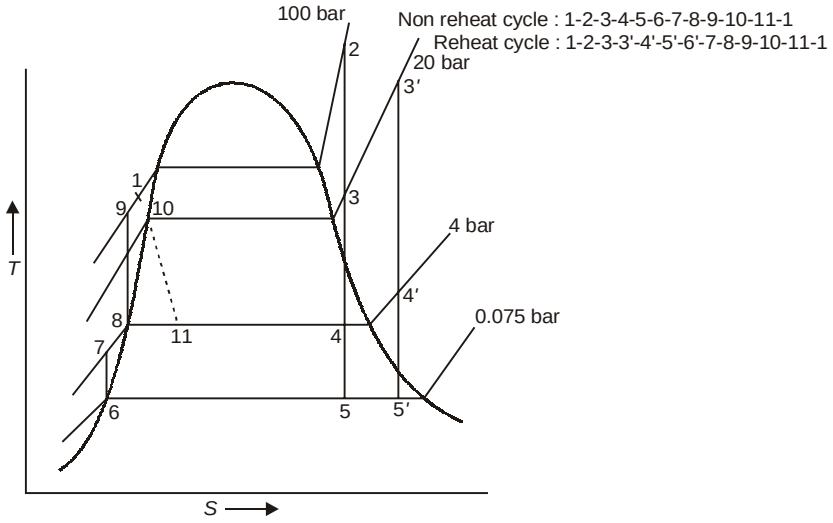


Fig. 8.43 *Layout and T-s diagram*

$$h_{5} = h_{f \text{ at } 0.075 \text{ bar}} + x_5 \cdot h_{fg \text{ at } 0.075 \text{ bar}} = 2055.09 \text{ kJ/kg}$$

Let mass of steam bled at 20 bar, 4 bar be m_1 and m_2 per kg of steam generated respectively.

$$h_{10} = h_{f \text{ at } 20 \text{ bar}} = 908.79 \text{ kJ/kg}, \; h_8 = h_{f \text{ at } 4 \text{ bar}} = 604.74 \text{ kJ/kg}$$

$$\text{At trap } h_{10} = h_{11} = 908.79 \text{ kJ/kg}$$

At condensate extraction pump, (CEP), $h_7 - h_6 = v_6 (4 - 0.075) \times 10^2$

$$v_6 = v_{f \text{ at } 0.075 \text{ bar}} = 0.001008 \text{ m}^3/\text{kg}$$

$$h_6 = h_{f \text{ at } 0.075 \text{ bar}} = 168.79 \text{ kJ/kg} \Rightarrow h_7 = 169.18 \text{ kJ/kg}$$

At feed pump, (FP), $h_9 - h_8 = v_8 (20 - 4) \times 10^2$

$$h_8 = h_{f \text{ at } 4 \text{ bar}} = 604.74 \text{ kJ/kg}$$

$$v_8 = v_{f \text{ at } 4 \text{ bar}} = 0.001084 \text{ m}^3/\text{kg}$$

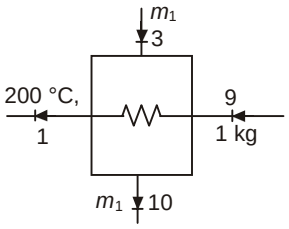
$$h_9 = 604.74 + (0.001084 \times 16 \times 10^2) = 606.47 \text{ kJ/kg}$$

Let us apply heat balance at closed feed water heater,

$$m_1 \cdot h_3 + h_9 = m_1 \cdot h_{10} + 4.18 \times 200$$

$$(m_1 \times 2930.57) + 606.47 = (m_1 \times 908.79) + 836$$

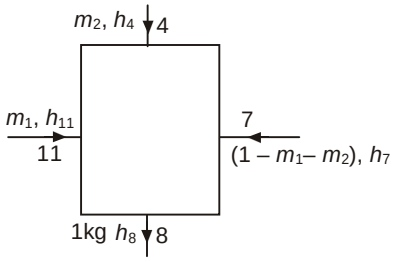
$$m_1 = 0.114 \text{ kg}$$



Applying heat balance at open feed water,

$$m_1 h_{11} + m_2 \cdot h_4 + (1 - m_1 - m_2) \cdot h_7 = h_8$$

$$(m_1 \cdot 908.79) + (m_2 \times 2612.65) + ((1 - m_1 - m_2) \cdot 169.18) = 604.74,\ m_2 = 0.144 \text{ kg}$$



Net work per kg steam generated,

$$w_{net} = (h_2 - h_3) + (1 - m_1) \cdot (h_3 - h_4) + (1 - m_1 - m_2)$$
$$(h_4 - h_5) - \{(1 - m_1 - m_2) \cdot (h_7 - h_6) + (h_9 - h_8)\}$$
$$= 443.13 + 281.67 + 413.7 - \{0.288 + 1.73\}$$
$$w_{net} = 1136.48 \text{ kJ/kg}$$

Heat added per kg steam generated,

$$q_{add} = (h_2 - h_1) = (3373.7 - 4.18 \times 200) = 2537.7 \text{ kJ/kg}$$

$$\text{Thermal efficiency} = \frac{w_{net}}{q_{add}} = 0.4478 \text{ or } 44.78\%$$

$$\text{Steam generation rate} = \frac{100 \times 10^3}{w_{net}} = 87.99 \text{ kg/s}$$

Thermal efficiency = **44.78%**
Steam generation rate = **87.99 kg/s** **Ans.**

(*a*) *For the reheating introduced at 20 bar up to 400ºC:* The modified cycle representation is shown on T-s diagram by 1-2-3-3'-4'-5'-6-7-8-9-10-11

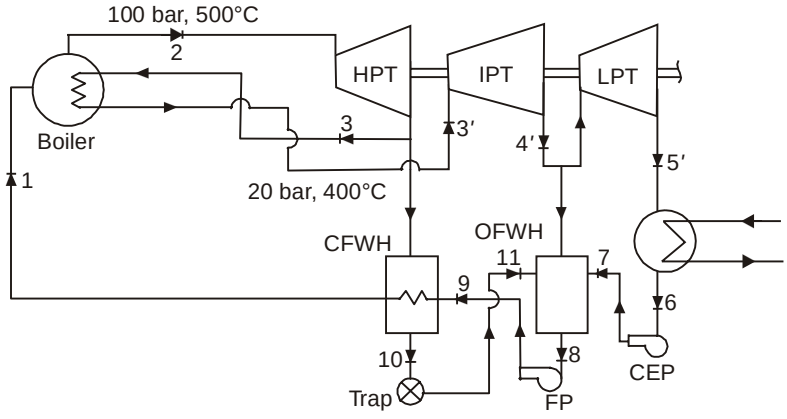


Fig. 8.44 *Reheat cycle*

At state 2,

$$h_2 = 3373.7 \text{ kJ/kg},$$
$$s_2 = 6.5966 \text{ kJ/kg} \cdot \text{K}$$

At state 3,

$$h_3 = 2930.57 \text{ kJ/kg}$$

At state 3',

$$h_{3'} = 3247.6 \text{ kJ/kg}$$
$$s_{3'} = 7.1271 \text{ kJ/kg} \cdot \text{K}$$

At state 4' and 5' $s_{3'} = s_{4'} = s_{5'} = 7.1271$ kJ/kg · K

From steam tables by interpolation state 4' is seen to be at 190.96°C at 4 bar

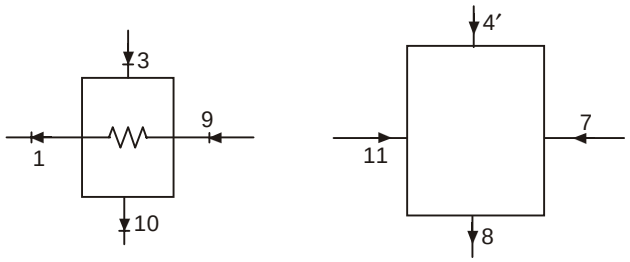
$$h_{4'} = 2841.02 \text{ kJ/kg}$$

Let dryness fraction at state 5' be $x_{5'}$.

$$s_{5'} = 7.1271 = s_{f \text{ at } 0.075 \text{ bar}} + x_{5'} \cdot s_{fg \text{ at } 0.075 \text{ bar}}$$
$$\Rightarrow \quad x_{5'} = 0.853$$
$$h_{5'} = h_{f \text{ at } 0.075 \text{ bar}} + x_{5'} \cdot h_{fg \text{ at } 0.075 \text{ bar}}$$
$$h_{5'} = 2221.11 \text{ kJ/kg}$$

Let mass of bled steam at 20 bar and 4 bar be $m_{1'}, m_{2'}$ per kg of steam generated. Applying heat balance at closed feed water heater.

$$m_{1'} \cdot h_3 + h_9 = m_{1'} \cdot h_{10} + 4.18 \times 200$$
$$\Rightarrow \quad m_{1'} = 0.114 \text{ kg}$$



Applying heat balance at open feed water heater

$$m_{1'} \cdot h_{11} + m_{2'} \cdot h_{4'} + (1 - m_{1'} - m_{2'}) \cdot h_7 = h_8$$
$$(0.114 \times 908.79) + (m_{2'} \cdot 2841.02) + (1 - 0.114 - m_{2'}) \cdot 169.18 = 604.74$$
$$m_{2'} = 0.131 \text{ kg}$$

Net work per kg steam generated

$w_{net} = (h_2 - h_3) + (1 - m_{1'}) \cdot (h_{3'} - h_{4'}) + (1 - m_{1'} - m_{2'}) (h_{4'} - h_{5'}) - \{(1 - m_{1'} - m_{2'}) \cdot (h_7 - h_6) + (h_9 - h_8)\}$

$$w_{net} = 443.13 + 360.22 + 468.03 - \{0.293 + 1.73\}$$
$$w_{net} = 1269.36 \text{ kJ/kg}$$

Heat added per kg steam generated, $q_{add} = (h_2 - h_1) + (1 - m_{1'}) (h_{3'} - h_3)$

$$= 2537.7 + 280.88$$
$$q_{add} = 2818.58 \text{ kJ/kg}$$

Thermal efficiency $\eta = \dfrac{w_{net}}{q_{add}} = 0.4503$ or 45.03%

% Increase in thermal efficiency due to reheating $= \left(\dfrac{0.4503 - 0.4478}{0.4478}\right) \times 100$

$= 0.56\%$

Thermal efficiency of reheat cycle = **45.03%**
% Increase in efficiency due to reheating = **0.56%** **Ans.**

13. *In a binary vapour cycle working on mercury and steam, the mercury vapour is generated dry saturated at 8.45 bar and expanded upto 0.07 bar in mercury turbine. The condenser or mercury cycle is used for generating steam at 40 bar, 0.98 dry. The steam is superheated separately upto 450ºC and then supplied into steam turbine for being expanded upto 0.075 bar. Two closed feed water heaters are used by bleeding out steam at 8 bar and 1 bar so as to provide feed water leaving at 150ºC and 90ºC respectively. Condensate leaves feed water heater as saturated liquid at respective pressures and is mixed with the hot feed water leaving the respective feed heater. The turbine running on mercury has capability of converting 85% of available heat into work. The enthalpies of mercury may be taken as,*

enthalpy of dry saturated vapour at 8.45 bar = 349 kJ/kg
enthalpy after isentropic expansion to 0.07 bar = 234.5 kJ/kg
enthalpy of saturated liquid at 0.07 bar = 35 kJ/kg

Assume feed water to enter at 150ºC into mercury condenser. Neglect pump work for getting efficiency. Determine the steam generation rate per kg of mercury and efficiency of cycle.

Solution:

For mercury cycle,

Insentropic heat drop = 349 – 234.5 = 114.5 kJ/kg Hg
Actual heat drop = 0.85 × 114.5 = 97.325 kJ/kg Hg
Heat rejected in condenser = [349 – 97.325 – 35]
= 216.675 kJ/kg
Heat added in boiler = 349 – 35 = 314 kJ/kg

For steam cycle,

Enthalpy of steam generated = $h_{\text{at 40 bar, 0.98 dry}}$ = 2767.13 kJ/kg
Enthalpy of steam at inlet to steam turbine h_2 = $h_{\text{at 40 bar, 450°C}}$ = 3330.3 kJ/kg
Entropy of steam at inlet to steam turbine, s_2 = 6.9363 kJ/kg · K

Therefore, heat added in condenser of mercury cycle

$= h_{\text{at 40 bar, 0.98 dry}} - h_{\text{feed at 40 bar}}$
$= 2767.13 - 4.18 \times 150 = 2140.13$ kJ/kg

Therefore, mercury required per kg of steam $= \dfrac{2140.13}{\text{Heat rejected in condenser}}$

$= \dfrac{2140.13}{216.675} = 9.88$ kg per kg of steam

For isentropic expansion, $s_2 = s_3 = s_4 = s_5 = 6.9363$ kJ/kg · K

State 3 lies in superheated region, by interpolation the state can be given by, temperature 227.07ºC at 8 bar, h_3 = 2899.23 kJ/kg

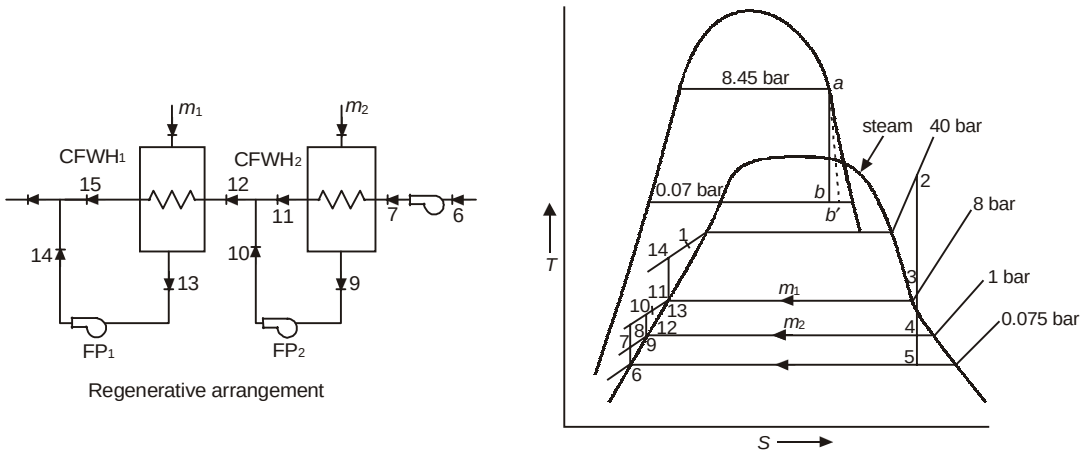


Fig. 8.45 *T-s diagram showing expansion and bleeding*

State 4 lies in wet region, say with dryness fraction $x_{4,}$

$$s_4 = 6.9363 = s_{f \text{ at } 1 \text{ bar}} + x_4 \times s_{fg \text{ at } 1 \text{ bar}} \Rightarrow x_4 = 0.93$$

$$h_4 = h_{f \text{ at } 1 \text{ bar}} + x_4 \cdot h_{fg \text{ at } 1 \text{ bar}} = 2517.4 \text{ kJ/kg}$$

Let state 5 lie in wet region with dryness fraction x_5,

$$s_5 = 6.9363 = s_{f \text{ at } 0.075 \text{ bar}} + x_5 \cdot s_{fg \text{ at } 0.075 \text{ bar}}$$

$$x_5 = 0.828$$

$$h_5 = 2160.958 \text{ kJ/kg}$$

Let mass of steam bled at 8 bar and 1 bar be m_1 and m_2 per kg of steam generated.

$$h_7 = h_6 + v_6(1 - 0.075) \times 10^2$$

$$= h_{f \text{ at } 0.075 \text{ bar}} + v_{f \text{ at } 0.075 \text{ bar}} (1 - 0.075) \times 10^2$$

$$= 168.79 + 0.001008 \times (1 - 0.075) \times 10^2$$

$$h_7 = 168.88 \text{ kJ/kg}$$

$$h_9 = h_{f \text{ at } 1 \text{ bar}} = 417.46 \text{ kJ/kg}$$

$$h_{13} = h_{f \text{ at } 8 \text{ bar}} = 721.11 \text{ kJ/kg}$$

Applying heat balance on $CFWH_1$, $T_1 = 150^oC$ and also $T_{15} = 150^oC$

$$m_1 \times h_3 + (1 - m_1) \times h_{12} = m_1 \times h_{13} + (4.18 \times 150) \times (1 - m_1)$$

$$(m_1 \times 2899.23) + (1 - m_1) \times h_{12} = (m_1 \times 721.11) + 627 \cdot (1 - m_1)$$

Applying heat balance on $CFWH_2$, $T_{11} = 90^oC$

$$m_2 \times h_4 + (1 - m_1 - m_2) \times h_7 = m_2 \times h_9 + (1 - m_1 - m_2) \times 4.18 \times 90$$

$$(m_2 \times 2517.4) + (1 - m_1 - m_2) \times 168.88 = (m_2 \times 417.46) + 376.2 (1 - m_1 - m_2)$$

Heat balance at mixing between $CFWH_1$ and $CFWH_2$,

$$(1 - m_1 - m_2) \times 4.18 \times 90 + m_2 \times h_{10} = (1 - m_1) \times h_{12}$$

$$376.2 (1 - m_1 - m_2) + m_2 \times h_{10} = (1 - m_1) \times h_{12}$$

For pumping process 9–10, $h_{10} = h_9 + v_9(8 - 1) \times 10^2$

$$h_{10} = h_{f \text{ at bar}} + v_{f \text{ at 1 bar}} \ (7 \times 10^2)$$
$$h_{10} = 417.46 + 0.001043 \times 700 = 418.19 \text{ kJ/kg}$$

Solving above equations, we get

$$m_1 = 0.102 \text{ kg per kg steam generated}$$
$$m_2 = 0.073 \text{ kg per kg steam generated.}$$

Pump work in process 13–14, $h_{14} - h_{13} = v_{13} \times (40 - 8) \times 10^2 = 0.001252 \times 32 \times 10^2$

$$h_{14} - h_{13} = 4.006 \text{ kJ/kg}$$
$$\text{Total heat supplied} = (9.88 \times 314) + (3330.3 - 2767.13)$$
$$= 3665.49 \text{ kJ/kg of steam}$$

Net work per kg of steam,

$$w_{net} = w_{mercury} + w_{steam}$$
$$= \{9.88 \times 97.325\} + \{(h_2 - h_3) + (1 - m_1) \cdot (h_3 - h_4) + (1 - m_1 - m_2) \cdot (h_4 - h_5) - (1 - m_1 - m_2)(h_4 - h_6) - m_2 (h_{10} - h_9) - m_1 (h_{14} - h_{13})\}$$
$$= \{961.571\} + \{431.07 + 342.88 + 294.06 - 0.074 - 0.053 - 0.408\}$$
$$w_{net} = 2029.046 \text{ kJ/kg}$$

$$\text{Thermal efficiency of binary vapour cycle} = \frac{2029.046}{3665.49} = 0.5536 \text{ or } 55.36\%$$

Thermal efficiency = **55.36%** **Ans.**

14. *A steam power plant has mixed pressure turbine of output 2500 hp with high pressure steam entering at 20 bar, 300°C and low pressure steam entering at 2 bar and dry saturated. The steam leaves turbine at 0.075 bar and efficiency ratio of both HP and LP stages are 0.8. The Willan's line for both are straight line and steam consumption at no load is 10% of full load. Determine the HP steam required for producing 1000 hp if low pressure steam is available at the rate of 1.5 kg/s.*

Solution:

This is a mixed pressure turbine so the output of turbine shall be sum of the contributions by HP and LP steam streams.

For HP: At Inlet of HP steam, $\Rightarrow h_1 = 3023.5$ kJ/kg, $s_1 = 6.7664$ kJ/kg · K

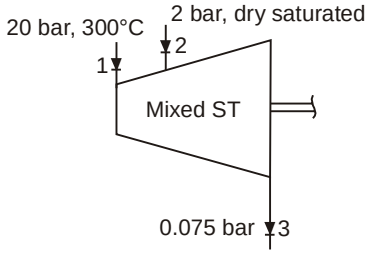
Ideally, $s_2 = s_1 = 6.7664$

$$s_2 = 6.7664 = s_{f \text{ at 0.075 bar}} + x_3 \times s_{fg \text{ at 0.075 bar}}$$

$\Rightarrow$ $x_3 = 0.806$

$$h_{3HP} = h_{f \text{ at 0.075 bar}} + x_3 \cdot h_{fg \text{ at 0.075 bar}} = 2108.03 \text{ kJ/kg}$$

Actual enthalpy drop in HP = $(h_1 - h_{3HP}) \times 0.8 = 732.38$ kJ/kg



For LP: At inlet of LP steam

$h_2 = 2706.7$ kJ/kg, $s_2 = 7.1271$ kJ/kg · K

Enthalpy at exit, $h_{3LP} = 2222.34$ kJ/kg

Actual enthalpy drop in LP = $(h_1 - h_{3LP}) \times 0.8 = 387.49$ kJ/kg

$$\text{HP steam consumption at full load} = \frac{2500 \times 0.7457}{732.38} = 2.54 \text{ kg/s}$$

$$\text{HP steam consumption at no load} = 0.1 \times 2.54 = 0.254 \text{ kg/s}$$

$$\text{LP steam consumption at full load} = \frac{2500 \times 0.7457}{387.49} = 4.81 \text{ kg/s}$$

$$\text{LP steam consumption at no load} = 0.1 \times 4.81 = 0.481 \text{ kg/s}$$

The problem can be solved geometrically by drawing Willan's line as per scale on graph paper and finding out the HP stream requirement for getting 1000 hp if LP steam is available at 1.5 kg/s.

or,

Analytically the equation for Willan's line can be obtained for above full load and no load conditions for HP and LP separately.

Willan's line for HP: $y = mx + C$, here y = steam consumption, kg/s

x = load, hp

$$y_{HP} = m_{HP} \cdot x + C_{HP}$$

$$2.54 = m_{HP} \cdot 2500 + C_{HP}$$

$$0.254 = m_{HP} \cdot 0 + C_{HP} \Rightarrow C_{HP} = 0.254 \text{ and } m_{HP} = 9.144 \times 10^{-4}$$

$$\Rightarrow \quad y_{HP} = 9.144 \times 10^{-4} \cdot x_{HP} + 0.254$$

Willan's line for LP: $y_{LP} = m_{LP} \cdot x_{LP} + C_{LP}$

$$4.81 = m_{LP} \cdot 2500 + C_{LP}$$

$$0.481 = m_{LP} \cdot C_{LP} \Rightarrow C_{LP} = 0.481, \; m_{LP} = 1.732 \times 10^{-3}$$

$$y_{LP} = 1.732 \times 10^{-3} \cdot x_{LP} + 0.481$$

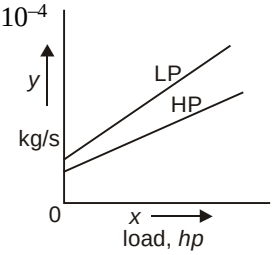


Fig. 8.46 *Tentative representation of Willan's line*

Total output (load) from mixed turbine, $x = x_{HP} + x_{LP}$

For load of 1000 hp to be met by mixed turbine, let us find out the load shared by LP for steam flow rate of 1.5 kg/s

$$1.5 = 1.732 \times 10^{-3} \cdot x_{LP} + 0.481$$

$$x_{LP} = 588.34 \text{ hp}$$

Since by 1.5 kg/s of LP steam only 588.34 hp output contribution is made so remaining (1000 – 588.34 = 411.66 hp), 411.66 hp should be contributed by HP steam. By Willan's line for HP turbine,

$$y_{HP} = (9.144 \times 10^{-4} \times 411.66) + 0.254 = 0.63 \text{ kg/s}$$

So, HP steam requirement = 0.63 kg/s

HP steam required = **0.63 kg/s** **Ans.**

15. *A steam power plant installation has steam leaving boiler at 40 bar, 300°C and expanding in HP turbine upto 2 bar. Half of steam leaving HP turbine is sent for process heating and remaining enters a separator where all the moisture is removed. Dry steam from separator is sent to low pressure LP turbine at 2 bar and gets expanded upto 0.075 bar.*

The drain (moisture) of separator gets mixed with condensate from process heater and combined flow enters the hot well at 90°C. Trap is provided at exit of both process heater and separator. Condensate extraction pump extracts condensate from condenser and sends it to hot well at 40°C. Neglecting pump work and radiation losses etc. determine temperature of water leaving hotwell and heat transferred per kg in process heater. Also find out thermal efficiency of installation and give layout.

Solution:

Let us carry out analysis for 1 kg of steam generated in boiler.

Enthalpy at inlet to HPT, $h_2 = 2960.7$ kJ/kg, $s_2 = 6.3615$ kJ/kg · K

State at 3 i.e. exit from HPT can be identified by $s_2 = s_3 = 6.3615$ kJ/kg · K

Let dryness fraction be x_3, $s_3 = 6.3615 = s_{f \text{ at } 2 \text{ bar}} + x_3 \cdot s_{fg \text{ at } 2 \text{ bar}}$

$\Rightarrow \quad x_3 = 0.863 \qquad h_3 = 2404.94$ kJ/kg

If one kg of steam is generated in boiler then at exit of HPT, 0.5 kg goes into process heater and 0.5 kg goes into separator

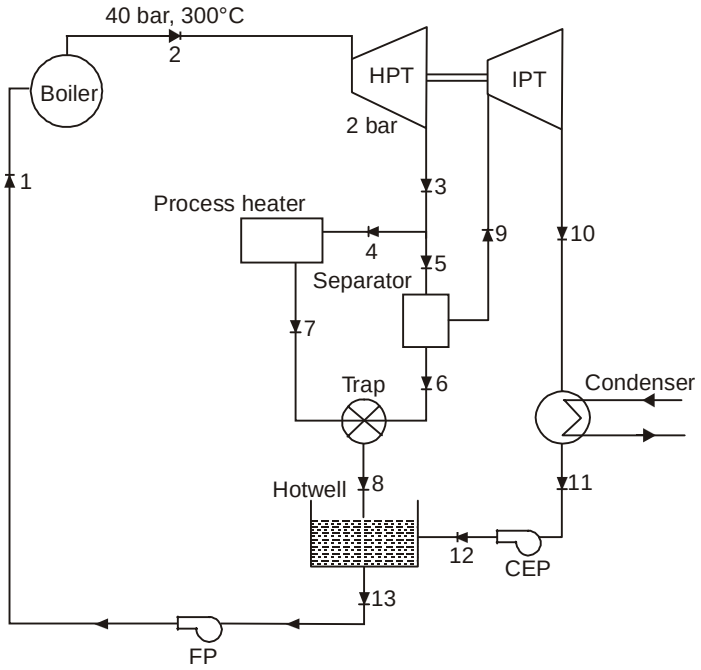


Fig. 8.47 *Layout*

Mass of moisture retained in separator = (1 – 0.863) × 0.5 = 0.0685 kg

Therefore, mass of steam entering LPT = 0.5 – 0.0685 = 0.4315 kg

Total mass of water entering hot well at 8 (i.e. from process heater and drain from separator) = 0.5685 kg

Let us assume the temperature of water leaving hotwell be T°C. Applying heat balance for mixing;

$(0.5685 \times 4.18 \times 90) + (0.4315 \times 4.18 \times 40) = (1 \times 4.18 \times T)$, $T = 68.425$°C

Temperature of water leaving hotwell = **68.425°C** **Ans.**

Applying heat balanced on trap

$$0.5 \times h_7 + 0.0685 \times h_{f \text{ at } 2 \text{ bar}} = (0.5685 \times 4.18 \times 90)$$

$$h_7 = 358.59 \text{ kJ/kg}$$

Therefore, heat transferred in process heater = $0.5 \times (h_3 - h_7)$

$= 1023.175$ kJ/kg steam generated

Heat transferred per kg steam generated = **1023.175 kJ/kg** steam generated **Ans.**

For state 10 at exit of LPT, $s_{10} = s_3 = s_2 = 6.3615$ kJ/kg · K

Let dryness fraction be x_{10}

$$s_{10} = 6.3615 = s_{f \text{ at } 0.075 \text{ bar}} + x_{10} \cdot s_{fg \text{ at } 0.075 \text{ bar}}$$

$$\Rightarrow \quad x_{10} = 0.754$$

$$\Rightarrow \quad h_{10} = h_{f \text{ at } 0.075 \text{ bar}} + x_{10} \cdot h_{\text{fg at } 0.075 \text{ bar}}$$

$$h_{10} = 1982.91$$

Net work output, neglecting pump work per kg of steam generated,

$$w_{net} = (h_2 - h_3) \times 1 + 0.4315 \times (h_3 - h_{10})$$

$$= 555.76 + 182.11$$

$$w_{net} = 737.87 \text{ kJ/kg steam generated}$$

Heat added in boiler per kg steam generated, $q_{add} = (h_2 - h_1)$

$$= (2960.7 - 4.18 \times 68.425)$$

$$q_{add} = 2674.68 \text{ kJ/kg}$$

$$\text{Thermal efficiency} = \frac{w_{net}}{q_{add}} = 0.2758 \text{ or } 27.58\%$$

Thermal efficiency = **27.58%** **Ans.**

16. *In a steam power plant operating on Rankine cycle, the steam enters the turbine at 70 bar and 550°C with a velocity of 30 m/s. It discharges to the condenser at 0.20 bar with a velocity 90 m/s. If the steam flow rate is 35 kg/s, find the thermal efficiency and net power produced.* *[U.P.S.C. 1992]*

Solution:

From steam tables, $h_1 = 3530.9$ kJ/kg, $s_1 = 6.9486$ kJ/kg · K

Assuming isentropic expansion in nozzle, $s_1 = s_2 = 6.9486$ kJ/kg · K

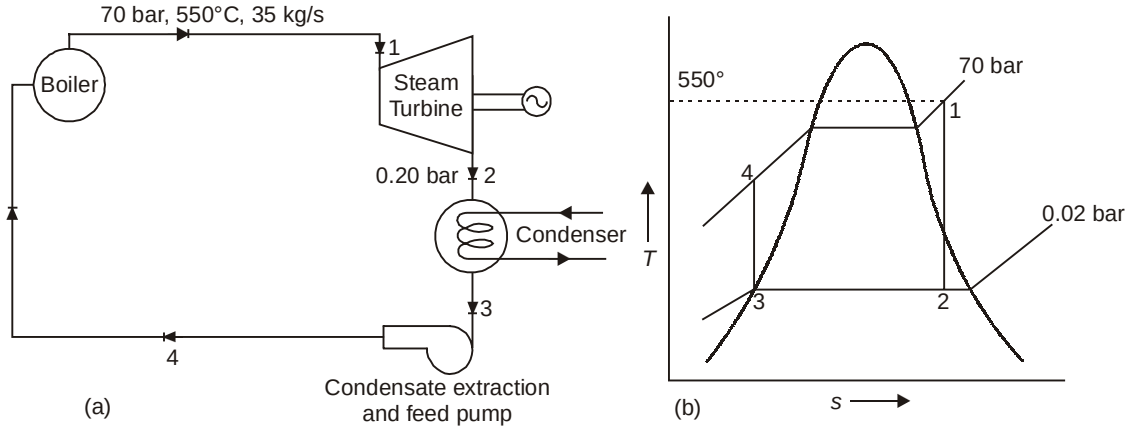


Fig. 8.48 *Schematic and representation on T-s diagram*

Let dryness fraction at state 2 be x_2 then;

$$s_2 = s_{f \text{ at } 0.2 \text{ bar}} + x_2 \times s_{fg \text{ at } 0.2 \text{ bar}}$$
$$6.9486 = 0.8320 + x_2 \cdot (7.0766)$$

Dryness fraction at state 2, $x_2 = 0.864$

Hence, $h_2 = h_{f \text{ at } 0.2 \text{ bar}} + x_2 \cdot h_{fg \text{ at } 0.2 \text{ bar}}$
$= 251.40 + (0.864 \times 2358.3)$

$h_2 = 2288.97$ kJ/kg

Considering pump work to be of isentropic type, $\Delta h_{34} = v_3 \times \Delta p_{34}$

From steam stable, $v_3 = v_{f \text{ at } 0.2 \text{ bar}} = 0.001017\ m^3$ /kg

or

$$\Delta h_{34} = 0.001017 \times \frac{(70 - 0.20) \times 10^5}{10^3}$$

Pump work, $W_P = \Delta h_{34} = 7.099$ kJ/kg

Turbine work, $W_T = \Delta h_{12} = (h_1 - h_2) = (3530.9 - 2288.97)$

$W_T = 1241.93$ kJ/kg

Net work $= W_T - W_P$
$= 1241.93 - 7.099$

$W_{net} = 1234.831$ kJ/kg

Power produced = mass flow rate $\times W_{net}$
$= 35 \times 1234.831$
$= 43219.085$ kJ/s

Net Power = **43.22 MW** **Ans.**

Heat supplied in boiler $= (h_1 - h_4)$, kJ/kg

Enthalpy at state 4, $h_4 = h_3 + \Delta h_{34}$
$= h_{f \text{ at } 0.2 \text{ bar}} + \Delta h_{34}$
$= 251.40 + 7.099$

$h_4 = 258.49$ kJ/kg

Total Heat supplied to boiler $= 35 \times (3530.9 - 258.49)$
$= 114534.35$ kJ/s

$$\text{Thermal efficiency} = \frac{\text{Net work}}{\text{Heat supplied}}$$
$$= 0.3773$$

Thermal efficiency = **37.73%** **Ans.**

17. *The following data refers to a steam turbine power plant employing one stage of regenerative feed heating:*
State of steam entering HP stage : 10 MPa, 600ºC
State of steam entering LP stage: 2 MPa, 400ºC
Condenser pressure: 10 KPa

The correct amount of steam is bled for feed heating at exit from the HP stage. Calculate the mass of steam bled per kg of steam passing through the HP stage and the amount of heat supplied in the boiler per second for an output of 10 MW. Neglect pump work, *[U.P.S.C. 1993]*

Solution:

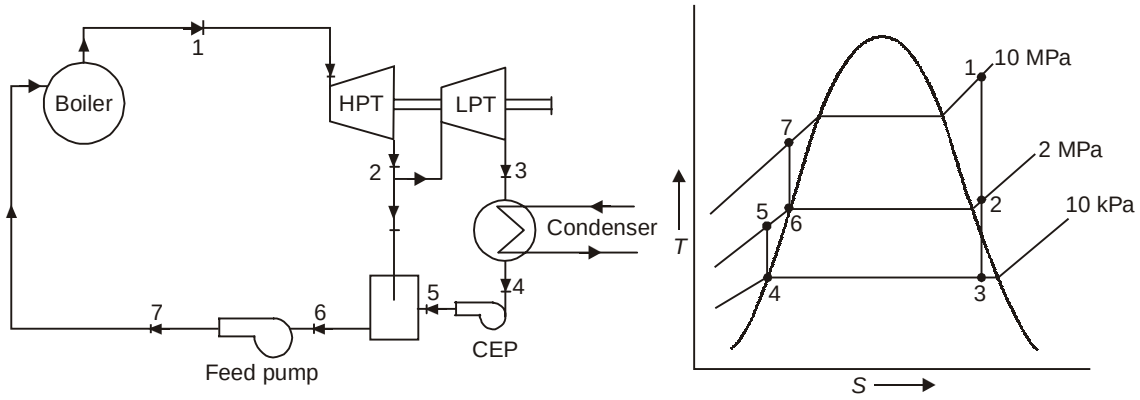


Fig. 8.49 *Possible arrangement and T-s representation*

From steam tables:

$h_1 = 3625.3$ kJ/kg, $s_1 = 6.9029$ kJ/kg · K

Due to isentropic expansion, $s_1 = s_2 = s_3 = 6.9029$ kJ/kg · K

At state 2, i.e. at pressure of 2 MPa and entropy 6.9029 kJ/kg · K.

By interpolating state for s_2 between 2 MPa, 300ºC and 2 MPa, 350ºC from steam tables,

$$h_2 = 3105.08 \text{ kJ/kg}$$

For state 3, i.e. at pressure of 0.01 MPa entropy, s_3 lies in wet region as $s_3 < s_g$ at 0.01 MPa. Let dryness fraction be x_3 at this state

$$s_3 = s_{f \text{ at } 0.01 \text{ MPa}} + x_3 \cdot s_{fg \text{ at } 0.01 \text{ MPa}}$$

$$6.9029 = 0.6493 + x_3 \times 7.5009$$

$$x_3 = 0.834$$

Enthalpy at state 3, $h_3 = h_{f \text{ at } 0.01 \text{ MPa}} + x_3 \cdot h_{fg \text{ at } 0.01 \text{ MPa}}$

$$= 191.83 + (0.834 \times 2392.8)$$

$$h_3 = 2187.43 \text{ kJ/kg}$$

Let the mass of steam bled be m_b per kg of steam from exit of HP for regenerative feed heating.

Considering state at exit from feed heater being saturated liquid the enthalpy at exit of feed heater will be, $h_{f \text{ at } 2 \text{ MPa}}$.

$$h_6 = h_{f \text{ at } 2 \text{ MPa}} = 908.79 \text{ kJ/kg}$$

For adiabatic mixing in feed heater, for one kg of steam leaving boiler, the heat balance yields,

$$(1 - m_b) \cdot h_5 + m_b \cdot h_2 = h_6$$

While neglecting pump work,

$$h_5 = h_4 = h_{f \text{ at } 0.01 \text{ MPa}} = 191.83 \text{ kJ/kg}$$

Substituting in heat balance on the feed heater,

$$(1 - m_b) \cdot 191.83 + m_b \cdot 3105.08 = 908.79$$

m_b = 0.246 kg per kg of steam entering HP turbine

Steam bled per kg of steam passing through HP stage = **0.246 kg** **Ans.**

Let mass of steam leaving boiler be m kg/s.

$$\text{Output} = 10 \times 10^3 = m(h_1 - h_2) + m(1 - m_b)(h_2 - h_3)$$

$$10 \times 10^3 = m\{(3625.3 - 3105.08) + (1 - 0.246)(3105.08 - 2187.43)\}$$

$$m = 8.25 \text{ kg/s}$$

Neglecting pump work, $h_7 = h_6 = 908.79$ kJ/kg

Heat supplied to boiler, $Q_{7-1} = m(h_1 - h_7)$

$$Q_{7-1} = 8.25\ (3625.3 - 908.79)$$

$$= 22411.208 \text{ kJ/s}$$

Heat added = **22411.21 kJ/s** **Ans.**

18. *Steam enters the first stage of a turbine at 100 bar, 500ºC and expands isentropically to 10 bar. It is then reheated to 500ºC and expanded in the second stage to the condenser pressure of 0.1 bar. Steam is bled from the first stage at 20 bar and fed to a closed feed water heater. Feed water leaves the closed heater at 100 bar, 200ºC (enthalpy = 856.8 kJ/kg), while the condensate is supplied to the open heater into which steam is bled at 4 bar pressure. Saturated liquid at 4 bar exits from the open heater and enters the closed heater. The net output of the turbine is 50 MW. Assuming the turbine and pump processes to be isentropic, determine the mass of steam bled at each feed water heater per kg of steam entering the first stage, the mass of steam entering the first stage per second, and the thermal efficiency.*

[U.P.S.C. 1995]

Solution:

From steam table, at inlet to first stage of turbine,

$h_1 = h_{\text{at 100 bar, 500ºC}} = 3373.7$ kJ/kg

$s_1 = s_{\text{at 100 bar, 500ºC}} = 6.5966$ kJ/kg · K

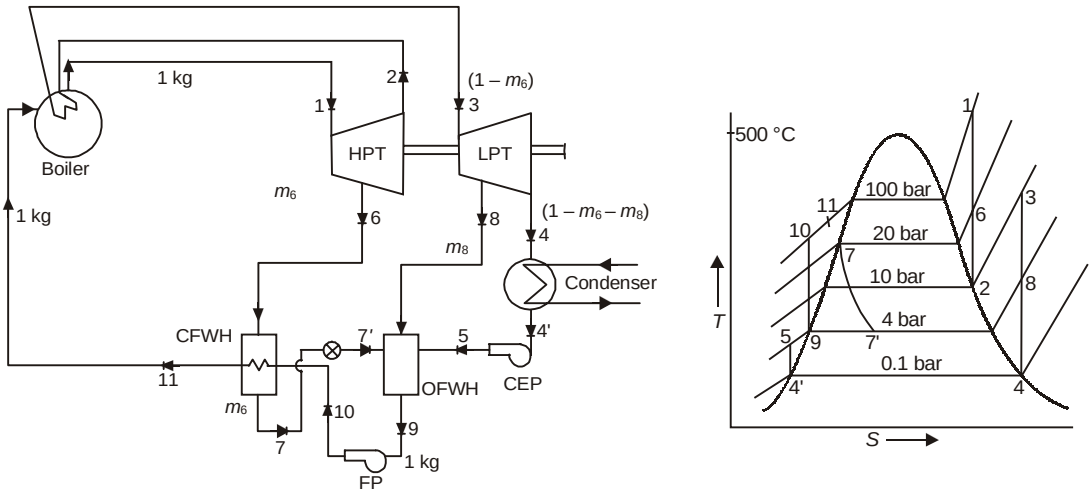


Fig. 8.50 *Arrangement and T-s representation*

Due to isentropic expansion, $s_1 = s_6 = s_2$ and $s_3 = s_8 = s_4$

State at 6 i.e. bleed state from HP turbine, Temperature by interpolation from steam table = 261.6ºC

At inlet to second stage of turbine, h_6 = 2930.572 kJ/kg

$$h_3 = h_{\text{at 10 bar, 500ºC}} = 3478.5 \text{ kJ/kg}$$

$$s_3 = s_{\text{at 10 bar, 500ºC}} = 7.7622 \text{ kJ/kg} \cdot \text{K}$$

At exit from first stage of turbine i.e. at 10 bar and entropy of 6.5966 kJ/kg · K Temperature by interpolation from steam table at 10 bar and entropy of 6.5966 kJ/kg · K

$$T_2 = 181.8ºC$$

$$h_2 = 2782.8 \text{ kJ/kg}$$

State at 8, i.e. bleed state from second stage of expansion, i.e. at 4 bar and entropy of 7.7622 kJ/kg · K Temperature by interpolation from steam table, $T_8 = 358.98ºC \simeq 359ºC$

$$h_8 = 3188.7 \text{ kJ/kg}$$

State at 4 i.e. at condenser pressure of 0.1 bar and entropy of 7.7622 kJ/kg. K the state lies in wet region. So let the dryness fraction be x_4.

$$s_4 = s_{f \text{ at 0.1 bar}} + x_4 \cdot s_{fg \text{ at 0.1 bar}}$$

$$7.7622 = 0.6493 + x_4 \cdot 7.5009$$

$$x_4 = 0.95$$

$$h_4 = h_{f \text{ at 0.1 bar}} + x_4 \cdot h_{fg \text{ at 0.1 bar}}$$

$$= 191.83 + (0.95 \times 2392.8)$$

$$h_4 = 2464.99 \text{ kJ/kg}$$

Given, $h_{11} = 856.8$ kJ/kg, $h_9 = h_{f \text{ at 4 bar}}$

$$h_9 = 604.74 \text{ kJ/kg}$$

Considering pump work, the net output can be given as,

$$W_{net} = W_{HPT} + W_{LPT} - (W_{CEP} + W_{FP})$$

where $W_{HPT} = \{(h_1 - h_6) + (1 - m_6)(h_6 - h_2)\}$ per kg of steam from boiler.

$W_{LPT} = \{(1 - m_6) + (h_3 - h_8)(1 - m_6 - m_8)(h_8 - h_4)\}$ per kg of steam from boiler.

For closed feed water heater, energy balance yields;

$$m_6 \cdot h_6 + h_{10} = m_6 \cdot h_7 + h_{11}$$

Assuming condensate leaving closed feed water heater to be saturated liquid,

$$h_7 = h_{f \text{ at 20 bar}} = 908.79 \text{ kJ/kg}$$

Due to throttline, $h_7 = h_{7'} = 908.79$ kJ/kg

For open feed water heater, energy balance yields,

$$m_6 \cdot h_{7'} + m_8 \cdot h_8 + (1 - m_6 - m_8) \cdot h_5 = h_9$$

For condensate extraction pump, $h_5 - h_{4'} = v_{4'} \cdot \Delta p$

$$h_5 - h_{f \text{ at 0.1 bar}} = v_{f \text{ at 0.1 bar}} \cdot (4 - 0.1) \times 10^2$$

$$h_5 - 191.83 = (0.001010) \times (3.9 \times 10^2)$$

$$h_5 = 192.224 \text{ kJ/kg}$$

For feed pump, $h_{10} - h_9 = v_9 \cdot \Delta p$

$$h_{10} - 604.74 = v_{f \text{ at 4 bar}} \times (100 - 4) \times 10^2$$

$$h_{10} - 604.74 = 0.001084 \times 96 \times 10^2$$

$$h_{10} = 615.15 \text{ kJ/kg}$$

Substituting in energy balance upon closed feed water heater,

$$m_6 \times 2701.2 + 615.15 = m_6 \times 908.79 + 856.8$$

$$m_6 = 0.135 \text{ kg per kg of steam from boiler.}$$

Substituting in energy balance upon feed water heater,

$$m_6 \cdot h_{7'} + m_8 \cdot h_8 + (1 \cdot m_6 - m_8) \cdot h_5 = h_9$$

$$(0.135 \times 908.79) + (m_8 \times 3188.7) + (1 - 0.135 - m_8) \times 192.224 = 604.74$$

$$m_8 = 0.105 \text{ kg per kg of steam from boiler}$$

Let mass of steam entering first stage of turbine be m kg, then

$$W_{HPT} = m\{(h_1 - h_6) + (1 - m_6)(h_6 - h_2)\}$$

$$= m\{(3373.7 - 2930.572) + (1 - 0.135)\}(2930.572 - 2782.8)$$

$$W_{HPT} = m\{570.95\}, \text{ kJ}$$

Also,

$$W_{LPT} = \{(1 - m_6)(h_3 - h_8) + (1 - m_6 - m_8) \cdot (h_8 - h_4)\}, \text{ per kg of steam from boiler}$$

$$W_{LPT} = \{(1 - 0.135)(3478.5 - 3188.7) + (1 - 0.135 - 0.105) \cdot (3188.7 - 2464.99)\}$$

$$W_{LPT} = m\{800.69\} \text{ kJ}$$

Pump works (negative work)

$$W_{CEP} = m \cdot (1 - m_6 - m_8)(h_5 - h_{4'})$$

$$= m \cdot (1 - 0.135 - 0.105) \cdot (192.224 - 191.83)$$

$$W_{CEP} = \{0.299\ m\}$$

$$W_{FP} = m\{h_{10} - h_9\}$$

$$= m\{615.45 - 604.74\}$$

$$W_{FP} = \{10.71\ m\}$$

Net output

$$W_{net} = W_{HPT} + W_{LPT} - W_{CEP} - W_{FP}$$

$$50 \times 10^3 = \{570.95\ m + 800.69\ m - 0.299\ m - 10.71\ m\}$$

$$m = 36.75 \text{ kg/s}$$

Heat supplied in boiler, $Q_{add} = m(h_1 - h_{11})$

$$= 36.75\ (3373.7 - 856.8)$$

$$= 92496.075 \text{ kJ/s}$$

$$\text{Thermal efficiency} = \frac{W_{net}}{Q_{add}}$$

$$= \frac{50 \times 10^3}{92496.075}$$

$$= 0.54056 \text{ or } 54.06\%$$

Mass of steam bled at 20 bar = **0.135 kg per kg** of steam entering first stage
Mass of steam bled at 4 bar = **0.105 kg per kg** of steam entering first stage
Mass of steam entering first stage = **36.75 kg/s**
Thermal efficiency = **54.06%**

Ans.

19. *A steam power plant installation has reheating and regenerative feed water heating employing a surface type feed heater and other contact type feed heater on high pressure side and low pressure side respectively. Steam enters HP turbine at 100 bar, 803 K and leaves high pressure turbine at 25 bar from where some steam is bled for feed heating in high pressure surface type heater and remaining is reheated up to 823 K and then expanded in low pressure turbine up to 0.05 bar pressure. The contact type feed heater is supplied with steam bled at 6 bar from LP steam turbine. There occurs throttling pressure loss of 3 bar in reheater. Surface type feed heater sends the drain to contact type feed heater from where the total feed is sent to surface type feed heater employing a boiler feed water pump as saturated water at pressure of 100 bar. Determine the amounts of steam bled off, overall thermal efficiency and specific steam consumption in kg/kwh. Considering tubring efficiency pump efficiency, generator efficiency, and mechanical efficiency as 0.85, 0.90 & 0.95 respectively and plant output as 120 MW. Consider discharges of drains at saturated liquid state at respective pressures in feed heaters. Also show how the processes on T-s and h-s diagrams along with line sketch of arrangement.*

Solution: From steam table,

At 100 bar, 803 K the state of inlet steam

$$h_1 = 3450.02 \text{ kJ/kg}, \; s_1 = 6.6923 \text{ kJ/kgK}$$

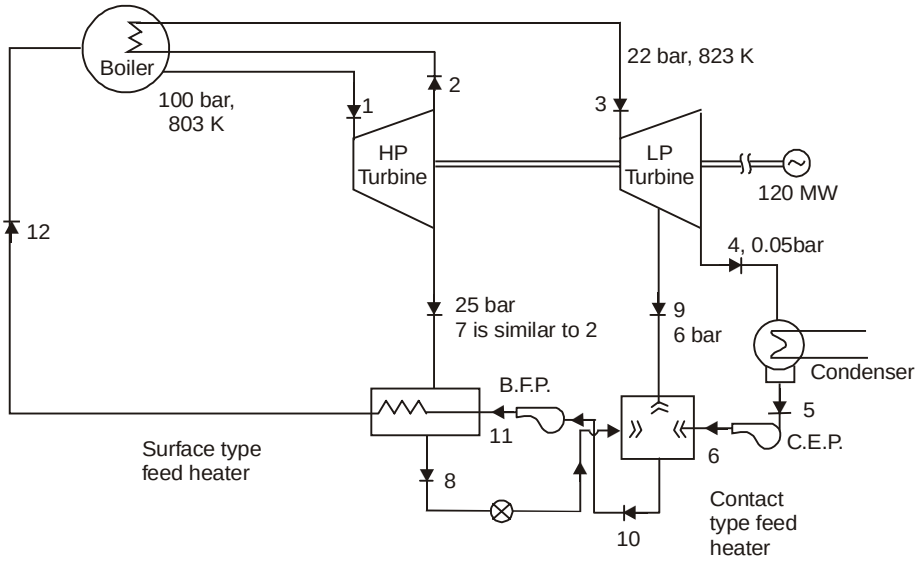
At inlet to LP steam turbine at 22 bar, 823K

$$h_3 = 3576.99 \text{ kJ/kg}, \; s_3 = 7.52411 \text{ kJ/kg.K}$$

For exit from HP turbine, $s_1 = s_2$

Using Mollier diagram

$$h_2 = 3010, \text{ kJ/kg.K.}$$



BFP : Boiler feed pump
CEP : Condensate extraction pump

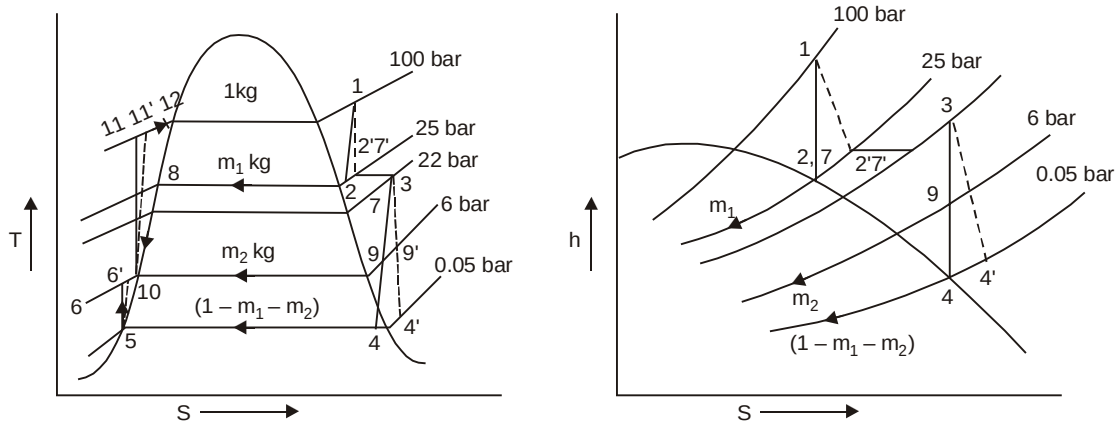


Fig. 8.51

Let us consider 1 kg of steam generated in boiler and bled fractions be m_1 & m_2.

Considering turbine efficiency, $\eta_{turb} = \dfrac{h_1 - h_2'}{h_1 - h_2}$

$$0.85 = \frac{3450.02 - h_2'}{(3450.02 - 3010)}$$

$$\Rightarrow \quad h_2' = 3076.003 \text{ kJ/kg}$$

From Mollier diagram, considering isentropic expansion in LP turbine

$$h_9 = 3175 \text{ kJ/kg}$$

$$h_4 = 2300 \text{kJ/kg}$$

Considering turbine efficiency, $0.85 = \dfrac{h_3 - h_9'}{h_3 - h_9}$

$$0.85 = \frac{(3576.99 - h_9')}{(3576.99 - 3175)}$$

$$\Rightarrow \quad h_9' = 3235.29 \text{ kJ/kg}$$

Also $$0.85 = \frac{h_3 - h_4'}{h_3 - h_4} = \frac{(3576.99 - h_4')}{(3576.99 - 2300)}$$

$$\Rightarrow \quad h_4' = 2491.5 \text{ kJ/kg}$$

From steam table

$$h_5 = h_{f \text{ at } 0.05 \text{ bar}} = 137.82 \text{ kJ/kg},$$

$$v_s = v_{f \text{ at } 0.05 \text{ bar}} = 0.001005 \text{ m}^3/\text{kg}$$

$$h_8 = h_{f \text{ at } 25 \text{ bar}} = 962.11 \text{ kJ/kg},$$

$$h_{12} = h_{f \text{ at } 100 \text{ bar}} = 1407.56 \text{ kJ/kg},$$

Energy balance at surface type feed heater yields;

$$m_1 \times h_7 + h_{11}' = h_{12} + m_1 h_8 \Rightarrow m_1 = \frac{(h_{11}' - h_{12})}{(h_8 - h_7)}$$

In this arrangement, $h_2 = h_7 \Rightarrow m_1 = \dfrac{(h_{11}' - h_{12})}{(h_8 - h_2')}$

Energy balance at contact type heater yields

$$m_1 \times h_8 + m_2 \times h_9 + (1 - m_1 - m_2)h_6' = 1 \times h_{10}$$

for process 5–6 in condensate extraction pump,

$$h_6 - h_5 = v_5 . \Delta p = v_5 \times (p_6 - p_5)$$

$$h_6 - 137.82 = 0.001005 \times (6 - 0.05) \times 10^2$$

$$\Rightarrow \quad h_6 = 138.42 \text{ kJ/kg}$$

$$\text{Considering pump efficiency } \eta_{\text{pump}} = \frac{(h_6 - h_5)}{(h_6' - h_5)}$$

$$0.90 = \frac{(138.42 - 137.82)}{(h_6' - 137.82)}$$

$$h_6' = 138.49 \text{ kJ/kg}$$

From steam tables,

$$h_{10} = h_{f \text{ at } 6 \text{ bar}} = 670.56 \text{ kJ/kg}$$

$$v_{10} = v_{f \text{ at } 6 \text{ bar}} = 0.001101 \text{ m}^3\text{/kg}$$

$$h_8 = h_{f \text{ at } 25 \text{ bar}} = 962.11 \text{ kJ/kg}$$

Substituting in heat balance of contact type feed heater,

$$(m_1 \times 962.11) + (m_2 \times 3175) + ((1 - m_1 - m_2) \times 138.49) = 670.56$$

$$(m_1 \times 823.62) + (m_2 \times 3036.51) = 532.07$$

For process 10-11 in boiler feed pump;

$$(h_{11} - h_{10}) = v_{10}.\Delta p = 0.001101 \times (100 - 6) \times 10^2$$

$$h_{11} = 670.56 + (0.001101 \times 94 \times 10^2)$$

$$h_{11} = 680.91$$

Considering pump efficiency, $0.90 = \dfrac{h_{11} - h_{10}}{h_{11}' - h_{10}}$

$$h_{11}' = 682.06 \text{ kJ/kg}$$

Substituting values, $m_1 = \dfrac{(682.06 - 1407.56)}{(962.11 - 3076.003)}$

$= 0.343$ kg/kg of inlet steam to HP turbine

Also substitution yields;

$(0.343 \times 823.62) + (m_2 \times 3036.51) = 532.07$

$m_2 = 0.082$ kg/kg of inlet steam to HP turbine

Work from HP turbine, $W_{HP} = (h_1 - h_2')$

$W_{HP} = (3450.02 - 3076.003) = 374.017$ KJ/kg

Work from LP turbine,

$W_{LP} = (1 - m_1)(h_3 - h_9') + (1 - m_1 - m_2)(h_9' - h_4')$

$W_{LP} = (1 - 0.343)(3576.99 - 3235.29) + (1 - 0.343 - 0.082) \times (3235.29 - 2491.5)$

$W_{LP} = 652.18$ kJ/kg

Pump work $= W_{CEP} + W_{BFP} = (1 - m_1 - m_2)(h_6' - h_5) + (h_{11}' - h_{10})$

$W_{pump} = (1 - 0.343 - 0.082)(138.49 - 137.82) + (682.06 - 670.56)$

$= 0.38525 + 11.5 = 11.885$ kJ/kg

Net work, $W_{net} = W_{HP} + W_{LP} - W_{pump}$

$= 374.017 + 652.18 - 11.885$

$= 1014.312$ kJ/kg

$$\text{Specific steam consumption} = \frac{3600}{W_{net} \times \eta_{gen} \times \eta_{mech}}$$

$$= \frac{3600}{(1014.312 \times 0.95 \times 0.95)}$$

Specific steam consumption = 3.93 kg/kw.h **Ans.**

$$\text{Overall thermal efficiency} = \frac{W_{net} \times \eta_{mech} \times \eta_{gen}}{(h_1 - h_{12}) + (1 - m_1)(h_3 - h_2')}$$

$$= \frac{1014.312 \times 0.95 \times 0.95}{(345.02 - 1407.56) + (1 - 0.343)(3576.99 - 3076.003)}$$

Overall thermal efficiency, $\eta_{overall}$ = 0.3656 or 36.56% **Ans.**

Mass of steam required = specific steam consumption × 120 × 103

= 471600 kg/hr

Mass of steam bled from HP turbine = 0.343 × 471600 = 161758.8 kg/hr **Ans.**

Mass of steam held from LP turbine = 0.082 × 471600 = 38671.2 kg/hr **Ans.**

20. *In an installation where the heating of 14000 kW and electrical power both are required, a back pressure turbine is employed. Back pressure turbine has initial steam state of 20 bar, 350°C and exhaust steam leaves at 1.5 bar. The efficiency ratio of turbine is 0.75 and in this installation the condensate drain from heater is fed back to boiler at condensing temperature. Determine the power available to generator.*

Solution: The states of steam at inlet and exit of back pressure turbine are shown here.

From steam tables:

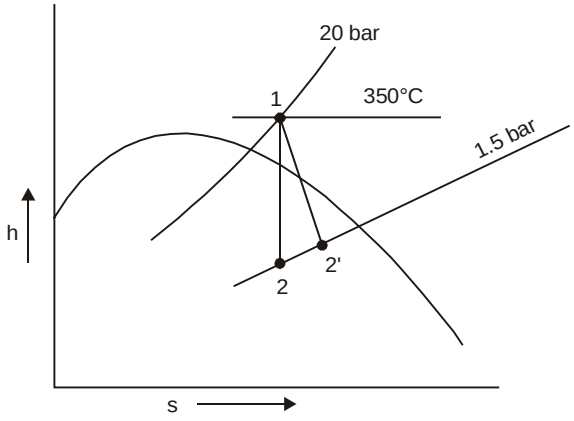


Fig. 8.52

At 20 bar, 350°C

h_1 = 3137.0 kJ/kg, s_1 = 6.9563 kJ/kg.K

$s_1 = s_2$ = 6.9563 kJ/kg.K, let dryness fraction be x_2 at '2'

s_2 = 6.9563 kJ/kg.K = $s_{f \text{ at } 1.5 \text{ bar}} + x_2\, s_{fg \text{ at } 1.5 \text{ bar}}$

$\Rightarrow$ 6.9563 = 1.4336 + x_2 × 7.2233

$\Rightarrow$ x_2 = 0.765, Enthalpy at '2',

$h_2 = h_{f \text{ at } 1.5 \text{ bar}} + x_2.h_{fg \text{ at } 1.5 \text{ bar}}$

h_2 = 467.11 + (0.765 × 2226.5)

h_2 = 2170.38 kJ/kg.K

For given efficiency ratio of turbine; $0.75 = \dfrac{(h_1 - h_{2'})}{(h_1 - h_2)}$

$h_{2'}$ = 2412.04 kJ/kg.K.

Enthalpy of condensed steam = $h_{f \text{ at } 1.5 \text{ bar}}$ = 467.11 kJ/kg.K

In this back pressure turbine the steam at state $h_{2'}$ will be available for heaitng till it becomes condensed steam with enthalpy $h_{f \text{ at } 1.5}$ bar

Thus mass of steam required for heating action.

$$= \frac{(\text{Heating to be done})}{(h_{2'} - h_{f \text{ at } 1.5 \text{ bar}})}$$

$$= \frac{14000}{(2412.04 - 467.11)} = 7.198 \text{ kg/s}$$

Hence power available to generator

= mass flow rate of steam × $(h_1 - h_2')$

= 7.198 × (3137 – 2412.04)

= 5218.26 kW

21. *In a back pressure turbine installation the steam is supplied at pressure of 20 bar, 350°C and the exhaust steam from turbine at pressure of 2 bar is used for process heating. Condensate from heater is returned to boiler at 120°C. Considering turbine isentropic efficiency of 0.8, boiler efficiency of 80% and heat required for heating being 9000 kW determine the power generated and boiler heat requirement.*

Solution: From steam table,

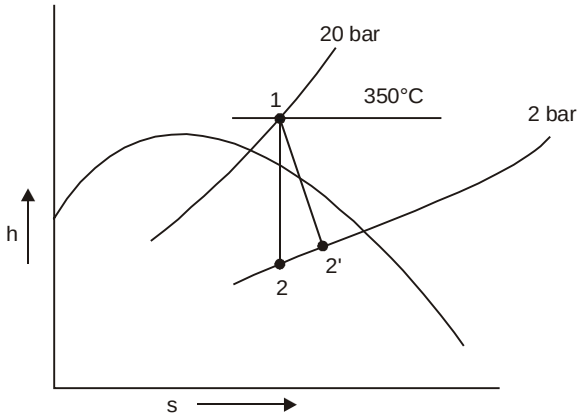


Fig. 8.53

h_1 = 3137 kJ/kg

s_1 = 6.9563 kJ/kg.K.

At 2 bar pressure, let dryness fractions fraction be x_2.

$s_2 = s_1 = 6.9563 = s_{f \text{ at 2 bar}} + x_2.s_{fg \text{ at 2 bar}}$

$6.9563 = 1.5301 + (x_2 \times 5.597)$

$\Rightarrow \quad x_2 = 0.969$

hence, $h_2 = h_{f \text{ at 2 bar}} + x_2.h_{fg \text{ at 2 bar}}$

$= 504.70 + (0.969 \times 2201.9)$

$h_2 = 2638.34$ kJ/kg

Considering turbine insentripic efficiency

$$0.8 = \frac{h_1 - h_{2'}}{h_1 - h_2} = \frac{3137 - h_{2'}}{(3137 - 2638.34)}$$

$\Rightarrow \quad h_{2'} = 2738.07$ kJ/kg

Let us consider the condensate going to boiler to be saturated liquid at 120°C. Let the mass flow rate of steam be m_s kg/s.

Thus, heat supplied for process heating = 9000 kW

$$9000 = m_s\,(h_{2'} - h_{f \text{ at } 120°C})$$

$$9000 = m_s(2738.07 - 503.71)$$

$$\Rightarrow \quad m_s = 4.03 \text{ kg/s}$$

Thus, power developed = $m_s \times (h_1 - h_{2'})$ = 4.03(3137 – 2738.07)

Power develped = 1607.68 kW

Total heat consumption in in boiler = $(h_1 - h_{f \text{ at } 120°C}) \times m_s$

= (3137 – 503.71) × 4.03

= 10612.16 kW

Actual heat consumption in boiler = $\dfrac{10612.16}{0.8}$ = 13265.2 kJ/s

22. *A pass out turbine is fed with steam at 20 bar, 250°C. The pass out steam pressure is 5 bar and the exhaust pressure is 0.075 bar. In this installation the total power required is 4500 kW and heat load is 15000 kW. The efficiency of HP and LP turbine stages is 0.8 in each stage. Considering the reduction in total steam consumption to result in steam consumption rate of 10 kg/s, determine the new power output if the heat load remains same. Also assume the nozzle control governing and throttle governing at the inlets of HP turbine and LP turbine respectively.*

Solution:

For first part of expanssion in HP stage.

From steam table

h_1 = 3137 kJ/kg

s_1 = 6.9563 kJ/kg.K.

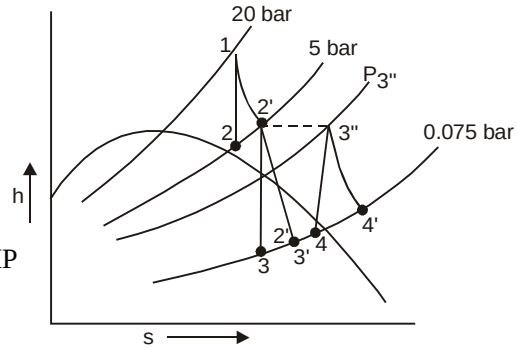


Fig. 8.54

It indicates that state of steam after expansion in HP stage lies in superheated region.

From superheated steam table,

degree of superheat may be estimated.

At 5 bar,

$s_{g \text{ at 5 bar, } 151.86°C}$ = 6.8213 kJ/kg.K

$s_{\text{at 5 bar, } 200°C}$ = 7.0592 kJ/kg.K

$$T_2 = \frac{(200 - 151.86) \times (6.9563 - 6.8213)}{(7.0592 - 6.8213)} + 151.86$$

$$T_2 = 179.18°C$$

$$h_{g \text{ at 5 bar, } 151.86°C} = 2748.7 \text{ kJ/kg},$$

$$h_{\text{at 5 bar, } 200°C} = 2855.4 \text{ kJ/kg},$$

$$h_2 = 2748.7 + \frac{(2885.4 - 2748.7)}{(200 - 151.86)} \times (179.18 - 151.86)$$

$$h_2 = 2813.41 \text{ kJ/kg}.$$

Considering turbine efficiency, $0.8 = \dfrac{(h_1 - h_{2'})}{(h_1 - h_2)}$

$$0.8 = \frac{(3137 - h_{2'})}{(3137 - 2813.41)}$$

$$h_{2'} = 2878.13 \text{ kJ/kg}$$

Heat available for process heating = $h_{2'} - h_{f \text{ at 5 bar}}$

$$= 2878.13 - 640.23$$

$$= 2237.9 \text{ kJ/kg}$$

Let mass flow rate of steam be $m_{s,\text{ HP}}$ to HP turbine and $m_{s,\text{heat}}$ for process heating after HP.

Thus, mass flow rate of steam required for process heating of 15000 kW

$$m_{s,\text{ heat}} = \frac{15000}{2237.9} = 6.70 \text{ kg/s}.$$

The h-s diagram shows two different arrangements. One arrangement shown by (1 – 2′ – 3′) indicates the case of no extraction in turbine. While (1 – 2′ – 3″ – 4′) shows the case of passout turbine.

For the first case

Using turbine efficiency, $0.8 = \dfrac{(T_1 - T_{2'})}{(T_1 - T_2)}$

$\Rightarrow$ $T_2' = T_1 - 0.8\ (T_1 - T_2) = 350 - 0.8(350 - 179.18)$

$$T_2' = 213.34°C$$

$\Rightarrow$ $$s_{2'} = 6.8213 + \frac{(7.0592 - 6.8213)}{(200 - 151.86)} \times (213.34 - 151.86)$$

$$s_2' = 7.125 \text{ kJ.kg.K}$$

$$s_3 = s_2' = 7.125 \text{ kJ/kg.K}$$

Let dryness fraction at 3 be x_3

$$s_3 = s_{f \text{ at 0.075 bar}} + x_3 \times s_{fg \text{ at 0.075 bar}}$$

$$7.125 = 0.5754 + (x_3 \times 7.675)$$

$\Rightarrow$ $$x_3 = 0.853$$

$$\Rightarrow \quad h_3 = h_{f \text{ at } 0.075 \text{ bar}} + x_3.h_{fg \text{ at } 0.075 \text{ bar}}$$
$$= 168.79 + (0.853 \times 2406)$$
$$h_3 = 2221.11 \text{ kJ/kg}$$

Using LP turbine stage efficiency,

$$0.8 = \frac{(h_{2'} - h_{3'})}{(h_{2'} - h_3)}$$
$$0.8 = \frac{(2878.13 - h_{3'})}{(2878.13 - 2221.11)}$$
$$h_{3'} = 2352.5 \text{ kJ/kg}$$

Power produced by HP stage, $= m_{s,HP} \times (h_1 - h_{2'})$
$$= m_{s,HP} (3137 - 2878.13)$$
$$P_{,HP} = 258.87\ m_{s,HP}$$

Power produced by LP stage, $= (m_{s,HP} - m_{s,heat})\ (h_{2'} - h_{3'})$
$$= (m_{s,HP} - 6.7)\ (2878.13 - 2352.51)$$
$$P_{,LP} = (m_{s,HP} - 6.7)525.62$$

Total power produced $= P_{HP} + P_{LP}$
$$= 258.87\ m_{s,HP} + 525.62\ (m_{s,HP} - 6.7)$$
$$4500 = (784.49\ m_{s,HP} - 3521.65)$$
$$m_{s,HP} = 10.23 \text{ kg/s}$$

For the IInd case when steam consumption changes

Since total steam consumption is reduced to 10 kg/s, i.e., $m_{s,HP} = 10$kg/s, but heat load remains save, so steam required for process heating would not change and $m_{s,heat}$ remains as 6.7 kg/s.

Thus $m_{s,HP,\ new} = 10$ kg/s & $m_{s,\ heat,\ new} = 6.7$ kg/s.

Also it is given in problem that the nozzle control governing employed at entrance of HP turbine stage therefore pressure at inlet to turbine remains same as 20 bar.

Hence power produced by HP turbine $= 10 \times (h_1 - h_{2'})$
$$= 10 \times (3137 - 2878.13)$$
$$= \text{Power}_{,\ HP,\ new} = 2588.7 \text{ kW}$$

Also the problem says that LP steam turbine stage is having throttle control governing, so by symmetry of triangles on pressure – steam consumption graph.

$$\frac{p_{3''}}{M_{3''}} = \frac{p_{2'}}{M_{2'}}$$

Here, $M_3'' = (m_{s,HP,new} - m_{s,heat,\ new}) = 10 - 6.7 = 3.3$ kg/s.

$$M_2' = (m_{s,HP} - m_{s,heat}) = 10.23 - 6.7 = 3.53 \text{ kg/s.}$$

$$p_3'' = \frac{5 \times 3.3}{3.53} = 4.67 \text{ bar}$$

Also, in view of throttling, $h_2' = h_3'' = 2878.13$ kJ/kg.

From steam table this state of steam at 4.67 bar pressure and enthalpy of 2878.13 kJ/kg can be located as it lies in super heated state by interpolation. Also it can be directly seen from mollier diagram, which would however yield approximate value. After locating this state on mollier diagram considering iseentropic expansion the state 4 can be located and enthalpy value be known.

At 4.67 bar, 2878.13 kJ/kg let temperature be T_3''

By interpolation,

At 4.67 bar, 200°C,h = 2857.08 kJ/kg

$$s = 7.096 \text{kJ/kg.K}$$

At 4.67 bar, 250°C, h = 2961.86 kJ/kg,

$$s = 7.30654$$

$$T_3'' = \frac{(250 - 200) \times (2878.13 - 2857.08)}{(2961.86 - 2857.08)}$$

$$T_3'' = 210.04°\text{C}$$

$$\Rightarrow \quad s_3'' = 7.096 + \frac{(7.30654 - 7.096) \times (210.04 - 200)}{(250 - 200)}$$

$$s_3'' = 7.138 \text{ kJ/kg.K.}$$

For isentropic expansion in LP turbine stage, $s_3'' = s_4$

At 0.075 bar and entropy value of 7.138 kJ/kg.K the state of steam lies in wet region, hence let us consider dryness fraction at state 4 as x_4.

$$s_4 = 7.138 = s_{f \text{ at } 0.075 \text{ bar}} + x_4 \cdot s_{fg \text{ at } 0.075 \text{ bar}}$$

$$7.138 = 0.5764 + (x_4 \cdot 7.6750)$$

$$\Rightarrow \quad x_4 = 0.855$$

Enthalpy at state 4

$$h_4 = h_{f \text{ at } 0.075 \text{ bar}} + x_4 . h_{\text{fg at } 0.075 \text{ bar}}$$

$$h_4 = 168.79 + (0.855 \times 2406) = 2225.92 \text{ kJ/kg}$$

Considering isentropic efficiency of LP turbine,

$$0.8 = \left(\frac{h_{3''} - h_{4'}}{h_{3''} - h_4} \right)$$

$$\Rightarrow \quad 0.8 = \left(\frac{2878.13 - h_{4'}}{2878.13 - 2225.92} \right) \Rightarrow h_{4'} = 2356.36 \text{ kJ/kg}$$

Power produced by LP turbine = $M_{3''} \times (h_{3''} - h_{4'})$

$= 3.3 \times (2878.13 - 2356.36)$

$\text{Power}_{\text{LP, new}} = 1721.84$ kW.

Total power produced = Power, new = 2588.7 + 1721.84 = 4310.54 kw

23. *Determine the steam consumption rate and the heat available for process heating from the pass out turbine installation having steam entering HP turbine stage at 20 bar, 350°C and the steam pressure before first stage nozzles is 17.5 bar. Steam is available for process heating at 5 bar pressure, the pressure before LP turbine's first stage nozzles is 4 bar and the exhaust pressure is 0.075 bar. Condensed steam temperature after process heating is 90°C. Electric power developed is 8 mW, alternator efficiency is 97.5%, turbine's mechanical losses are 50 kW, HP & LP turbine efficiency are 80% each and condenser discharge is 8 kg/s.*

Solution: All the salient states are tentatively shown on h-s diagram. Steam table can be used to get various enthalpy values at salient states.

Also mollier diagram can be used for quick but approximate analysis.

$h_1 = h_{20\text{bar, }350°C} = 3137$ kJ/kg.

$s_1 = 6.9563$ kJ/kg.K

$h_1' = h_1 = 3137$ kJ/kg.

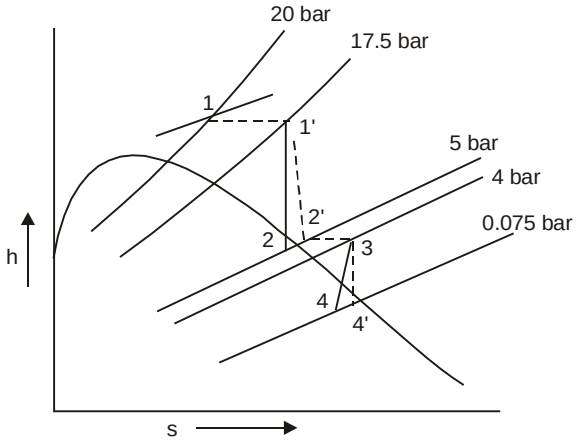


Fig. 8.55

Let us assume total steam consumption rate be m_1 kg/s and m_h be the mass flowrate of steam for process heating and m_3 kg/s be steam flow rate to LP stage for expansion.

Let us find out remaining enthalpy and other properties through mollier diagram.

entropy $s_1' = s_2 = 7$ kJ/kg.K.

$h_2 = 2830$ kJ/kg.

Considering HP stage efficiency,

$$0.8 = \frac{h_1' - h_2'}{h_1' - h_2}$$

$$0.8 = \frac{(3137 - h_{2'})}{(3137 - 2830)} \Rightarrow h_{2'} = 2891.4 \text{ kJ/kg}$$

$$h_3 = h_{2'} = 2891.4 \text{ kJ/kg}$$

From mollier diagram, $s_3 = s_4 = 7.1$ kJ/kg.K

$$h_4 = 2210 \text{ kJ/kg}$$

Considering LP stage efficiency,

$$0.8 = \frac{(h_3 - h_{4'})}{(h_3 - h_4)}$$

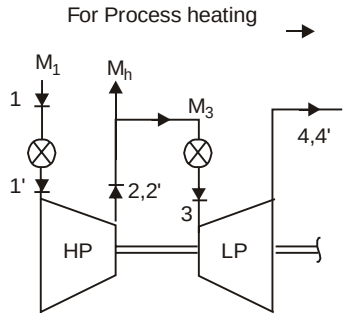


Fig. 8.56 *Schematic for pass out turbine*

$$0.8 = \frac{(2891.4 - h_{4'})}{(2891.4 - 2210)} \Rightarrow h_{4'} = 2346.28 \text{ kJ/kg}$$

Consider discharge of 8 kg/s to be the steam flow rate through LP turbine, *i.e.*, $m_3 = 8$ kg/s.

The power made available to alternator can be estimated by considering electric power developed.

$$\text{Power available to alternator} = \frac{8}{0.975} = 8.2051 \text{ MW}$$

$$\text{Total power produced} = 8.2051 \times 10^3 + 50 = 8255.1 \text{ kW}$$

$$\text{Power produced by LP turbine} = m_3 \times (h_3 - h_{4'}) = 5451.2 \text{ kW.}$$

$$\text{Power produced by HP turbine} = 8255.1 - 5451.2$$

$$= 2803.9 \text{ kW}$$

$$\text{Mass flow rate through HP turbine} \Rightarrow m_1 = \frac{2803.9}{(h_{1'} - h_{2'})}$$

$$m_1 = \frac{2803.9}{(3137 - 2891.4)}$$

Total steam consumption rate. $m_1 = 11.42$ kg/s

$$\text{Heat available for process heating} = m_h \times (h_2 - h_{f \text{ at } 90°C})$$

$$= (m_1 - m_3)\,(2830 - 376.92)$$

$$= (11.42 - 8)\,(2830 - 376.92)$$

$$= 8389.53 \text{ kW}$$

24. *A mixed pressure turbine has high pressure steam entering at 12 bar, 300°C, low pressure steam entering at 2 bar, dry and saturated and exhausts steam at 0.075 bar Turbine delivers 720 kW and mechanical efficiency is 90%. The efficiency of HP and LP turbine stages is 80% each. Determine the steam consumption rate in kg/kW.h at inlet of both HP and LP modes.*

Solution: The expansion processes are shown on h-s diagram.

from steam table, for HP steam,

$$h_1 = h_{\text{at 12 bar,300°C}} = 3045.8 \text{ kJ/kg}$$

$$s_1 = 7.0317 \text{ kJ/kg.K.}, \; s_4 = s_1$$

$$s_4 = s_{f \text{ at 0.075 bar}} + x_4.s_{fg \text{ at 0.075 bar}}$$

$$7.0317 = 0.5764 + x_4.7.6750$$

$$\Rightarrow \quad x_4 = 0.841$$

$$h_4 = h_{f \text{ at 0.075 bar}} + x_4.\, h_{fg \text{ at 0.075 bar}}$$

$$= 168.79 + (0.841 \times 2406)$$

$$h_4 = 2192.24 \text{ kJ/kg}$$

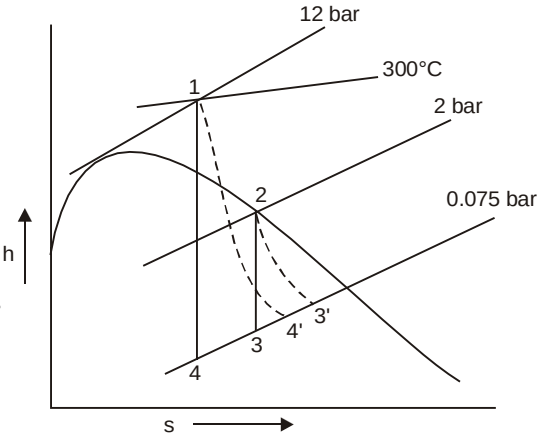


Fig. 8.57

Considering efficiency, $0.8 = \dfrac{h_1 - h_{4'}}{h_1 - h_4}$

$$\Rightarrow \quad h_4' = 2362.95 \text{ kJ/kg}$$

From steam turbine, for LP steam,

$$h_2 = h_{g \text{ at 2 bar}} = 2706.7 \text{ kJ/kg}$$

$$s_2 = s_{g \text{ at 2 bar}} = 7.1271 \text{ kJ/kg.K}$$

Let dryness fraction at 3 be x_3.

$$s_3 = s_2 = s_{f \text{ at 0.075 bar}} + x_3.\, s_{fg \text{ at 0.075 bar}}$$

$$= 7.1271 = 0.5764 + (x_3 \times 7.6750)$$

$$x_3 = 0.854$$

$$h_3 = h_{f \text{ at 0.075 bar}} + x_3.h_{g \text{ at 0.075 bar}}$$

$$= 168.79 + (0.854 \times 2406)$$

$$h_3 = 2223.51 \text{ kJ/kg}$$

Considering LP efficiency, $0.8 = \dfrac{(h_2 - h_{3'})}{(h_2 - h_3)}$

$$0.8 = \frac{(2706.7 - h_{3'})}{(2706.7 - 2223.51)}$$

$$h_3' = 2320.15 \text{ kJ/kg}$$

For given turbine output of 720 kW and mechanical efficiency of 90% the power developed in turbine = $\dfrac{720}{0.9}$ = 800 kW

Let the steam consumption rate (in kg/s) of HP and LP steam be m_{HP} & m_{LP} and power produced

(in kW) be P_{HP} and P_{LP}.

$$P_{HP} = m_{HP} \times (h_1 - h_{4'})$$

$$\text{HP steam consumption, kg/kWh} = \frac{m_{HP}}{P_{HP}} = \frac{3600}{(h_1 - h_{4'})} = 5.27 \text{ kg/kWh}$$

$$\text{LP steam consumption, kg/kWh} = \frac{m_{LP}}{P_{LP}} = \frac{3600}{(h_2 - h_{3'})} = 9.31 \text{ kg/kWh.}$$

25. *Determine the power output from a mixed pressure turbine fed with 2 kg/s of HP steam at 16 bar, 300°C and 1.5 kg/s of LP steam at 2 bar dry saturated. The expansion efficiency is 90% for both and the exhaust pressure is 0.075 bar. Consider that low pressure steam is admitted through a throttle value and there is no throttling of high pressure steam. This mixed pressure turbine develops 3 MW power full output when dry and saturated steam at 2 bar is supplied and leaves at 0.075 bar. Neglect the throttling losses.*

Solution:

Let us assume that pressure after mixing of high pressure and low pressure streams is p_2 bar.

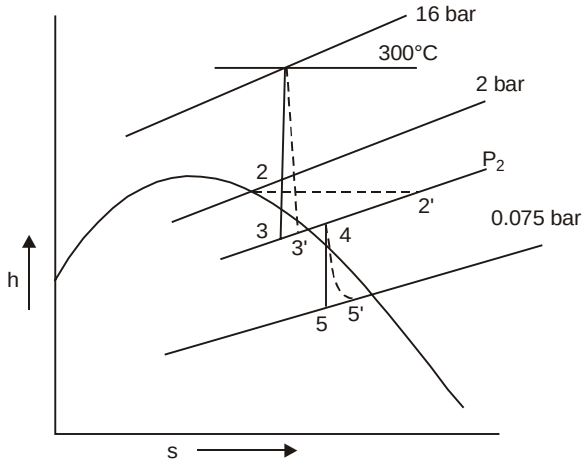


Fig. 8.58

$$m_{HP} = 2\text{kg/s}, \; m_{LP} = 1.5 \text{ kg/s}$$

From steam tables;

$$h_1 = h_{\text{at 15 bar, 300°C}} = 3034.8\text{kJ/kg}$$
$$s_1 = s_{\text{at 10 bar, 300°C}} = 6.8844 \text{ kJ/kg.K} = s_3$$

Using expansion efficiency;

$$0.9 = \frac{h_1 - h_{3'}}{h_1 - h_3}$$

Let dryness fraction at state 3 be x_3 so.

$$s_3 = 6.8844 = s_{f\text{at 2bar}} + x_3 . s_{fg \text{ at 2bar}}$$
$$6.8844 = 1.5301 + (x_3 \times 5.597)$$

$$x_3 = 0.9566$$

$$h_3 = h_{f \text{ at } 2 \text{ bar}} + x_3 . h_{fg \text{ at } 2 \text{ bar}}$$

$$= 504.7 + (0.9566 \times 2201.9)$$

$$h_3 = 2611.04 \text{ kJ/kg}$$

$$h_3' = h_1 - (0.9 \times (h_1 - h_3)) = 2653.42 \text{ kJ/kg}$$

In this case it is given that this turbine develops 3 MW when LP steam is supplied at 2 bar, dry saturated and expanded upto 0.075 bar.

Let mass flow rate of steam in this case be m_s kg/s.

In this case the enthalpy, $h_{in} = h_{g \text{ at } 2 \text{ bar}} = 2706.7 \text{ kJ/kg} = h_2 = h_2$

$$s_{in} = s_{g \text{ at } 2 \text{ bar}} = 7.1271 \text{ kJ/kg.K.}$$

Let dryness fraction at exit state be x_{out}.

$$s_{out} = s_{in} = s_{f \text{ at } 0.075 \text{ bar}} + x_{out} \, s_{fg \text{ at } 0.075 \text{ bar}}$$

$$7.1271 = 0.5764 + x_{out} . 7.6750.$$

$$x_{out} = 0.8535$$

$$h_{out} = h_{f \text{ at } 0.075 \text{ bar}} + x_{out} . h_{fg \text{ at } 0.075 \text{ bar}}$$

$$= 168.79 + (0.8535 \times 2406)$$

$$h_{out} = 2222.31 \text{ kJ/kg}$$

Considering expansion efficiency

$$0.9 = \frac{h_{in} - h_{out'}}{h_{in} - h_{out}} = \frac{(2706.7 - h_{out'})}{(2706.7 - 2222.31)}$$

$$h_{out'} = 2270.75 \text{ kJ/kg}$$

$$m_s \times (h_{in} - h_{out'}) = 3 \times 10^3$$

$$m_s = \frac{3000}{(2706.7 - 2270.75)} = 6.88 \text{ kg/s.}$$

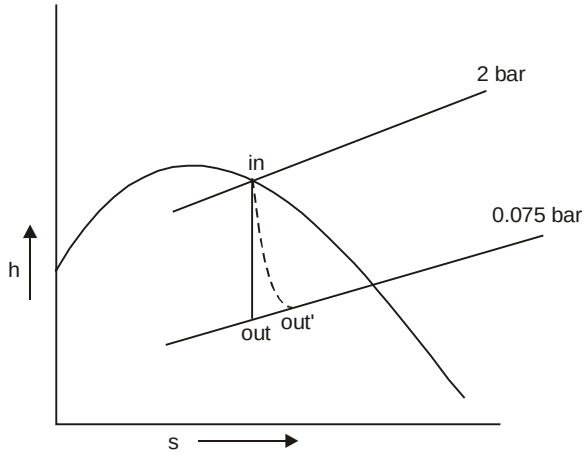


Fig. 8.59

Mixed pressure turbine is being fed with HP steam too along with LP steam. This is normally done when power available cannot be met by LP steam. During this mixed pressure operation it is quite possible that pressure of steam at exit of HP turbine may not be same as LP steam pressure of 2 bar and let us assume that the steam pressure at exit from HP steam be p_2. In this case LP steam has to be throttled up to pressure p_2 while maintaining speed constant. Thus, for throttle governing considerations.

$$\frac{p_2}{(m_{HP} + m_{LP})} = \frac{2 \times 10^2}{m_s}$$

$$p_2 = \frac{2 \times 10^2 \times (2 + 1.5)}{6.88} = 1.017 \times 10^2 \text{ kPa}$$

For state after mixing;

$$h_4 = \frac{(m_{HP} \times h_{3'}) + (m_{LP} \times h_{2'})}{(m_{HP} \times m_{LP})} = 2676.25 \text{ kJ/kg}$$

Power output from mixed pressure turbine

$$\text{Power} = m_{\text{HP}}(h_1 - h_{3'}) + (m_{\text{HP}} + m_{\text{LP}})(h_4 - h_{5'})$$

For pressure of 1.017 bar and enthalpy of 2676.25 kJ/kg the iseutropic expansion upto 0.075 bar can be seen.

$$h_5 = 2290 \text{ kJ/kg}$$

$$\text{Considering expansion efficiency, } 0.9 = \frac{h_4 - h_{5'}}{h_4 - h_5}$$

$$h_{5'} = 2328.63 \text{ kJ/kg}$$

$$\text{Power} = 2(3034.8 - 2653.42) + (2 + 1.5)(2676.25 - 2328.63)$$

$$= 1979.43 \text{ kW.}$$

EXERCISE

8.1 Explain Carnot vapour power cycle and its relevance.

8.2 Give limitations of Carnot vapour power cycle and explain how Rankine cycle helps in overcoming them.

8.3 Discuss the limitations of maximum and minimum temperatures in a steam power cycle.

8.4 What should be the desired properties of a working fluid in a Rankine cycle?

8.5 Compare the efficiency and specific steam consumption of ideal Carnot cycle with Rankine cycle for given pressure limits.

8.6 Discuss the effect of pressure of steam at inlet to turbine, temperature at inlet to turbine and pressure at exit from turbine upon Rankine cycle performance.

8.7 Describe reheat cycle and compare it with simple Rankine cycle.

8.8 What do you understand by ideal regenerative cycle? Why is it not possible in practice? Also give actual regenerative cycle.

8.9 Compare performance of regenerative cycle with simple Rankine cycle.

8.10 What do you understand by binary vapour power cycles?

8.11 Discuss advantages of binary vapour cycles over single vapour cycle.

8.12 What is mixed pressure turbine?

8.13 Describe pass out turbines and back pressure turbines.

8.14 Discuss briefly the factors limiting thermal efficiency of a steam power plant.

8.15 Describe combined heat and power cycles.

8.16 A steam engine working on Rankine cycle operates between 1.96 MPa, 250ºC and 13.7 kPa. If engine consumes steam at the rate of 0.086 kg per second, determine Rankine cycle efficiency, neglecting pump work. Also, find Rankine cycle efficiency considering pump work. [58.8%]

8.17 In a steam power plant working on boiler pressure of 80 bar and condenser pressure of 0.075 bar determine cycle efficiency considering it to work on Carnot cycle. [44.9%]

8.18 A reheat cycle operates between 80 bar and 0.075 bar. Temperature of steam entering turbine is 600ºC and first stage of expansion is carried out till the steam becomes dry saturated. Subsequently steam is reheated upto the initial temperature at inlet. Neglecting pump work determine efficiency and specific steam consumption in kg/hp·hr. [42.5%, 141 kg/hp-hr]

8.19 A steam turbine installation of 60 MW capacity has steam entering turbine at 7 MPa, 500ºC with steam bleeding at 2 MPa and 0.2 MPa for feed heating. Remaining steam at 2 MPa is reheated upto 480ºC. Steam finally leaves turbine at 36 mm Hg (absolute). Give layout, mass fraction of steam bled out per kg of steam generated, cycle efficiency and mass flow rate of steam entering turbine.

[0.159, 0.457, 46.58 kg/s]

8.20 In a steam turbine steam enters at 1.4 MPa, 320ºC and exhausts at 175 kPa. Determine the steam flow rate considering turbine internal efficiency of 70% and load requirement of 800 kW. [2.6 kg/s]

8.21 During trial of turbine in steam power plant the steam is found to enter turbine at 2.1 MPa, 260ºC with flow rate of 226 kg/s. Mechanical efficiency is 95% and turbine output is 100 MW. The exhaust pressure is 140 kPa and condensate temperature is 70ºC.

Unfortunately the old boiler fails and is to be replaced by another boiler generating steam at 4.9 MPa, 320ºC. This unit may drive an additional turbine with an internal efficiency of 90% which exhausts at 2.1 MPa. A part of exhaust is reheated to above trial conditions, including flow rate and remainder heats feed water to 205ºC. Determine (*a*) total plant output, (*b*) new total steam flow rate, kg/s, (*c*) old turbine internal efficiency, (*d*) the overall thermal efficiency. [14.9 MW, 293 kg/s, 94.3%, 22.9%]

8.22 A steam turbine has steam entering at 40 bar, 450ºC and leaving at 0.06 bar. Turbine develops 4000 hp at 3000 rpm with expansion occurring in two stages and reheating in between at 2 bar and both stages give equal output. The stages have efficiency ratio of 0.8. Determine thermal efficiency of cycle.

[32.2%]

8.23 A regenerative cycle has steam supplied at 28 bar, 300ºC in first stage. A steam fraction is bled out at 3 bar for feed heating. The feed heater drains are pumped into feed line downstream of heater at the same temperature as the bled steam. The steam expansion may be considered isentropic throughout. Condenser works at 0.15 bar. Determine mass of steam bled per kg of steam generated and thermal efficiency. Also give layout and *T*-*s* diagram. [0.143 kg/kg steam generated, 31.2%]

8.24 A steam power plant has steam generated at 140 bar, 400ºC and throttled to 70 bar before supplied to turbine. Steam expands isentropically to the pressure of 15 bar and at this pressure some quantity of steam is bled out for feed heating which leaves as condensate at 40ºC. Remaining steam is reheated at constant pressure by mixing it with the throttled steam taken from boiler in such amount that the resultant mixture has temperature of 250ºC. This mixture then expands isentropically upto condenser

pressure of 0.075 bar. Determine horse power developed by turbine for steam generation rate of 50 ton/hr. Also give layout and *T-s* diagram. [14400 hp]

8.25 Steam is required for process heating at 3 bar, dry saturated along with power output of 1 MW. A back pressure turbine with 70% internal efficiency is used for this purpose. For the steam supplied at the rate of 2.78 kg/s determine the pressure and temperature of steam to be generated from boiler.

[37.3 bar, 344ºC]

8.26 A steam turbine has steam being generated at 30 bar, 400ºC for being expanded upto 0.04 bar with bleeding of some steam at 3 bar for feed heating in closed feed water heater. Feed water leaves feed heater at 130ºC and the condensate leaving feed heater as saturated liquid is sent to drain cooler where it gets cooled to 27ºC and finally discharged in hotwell. Condensate from condenser is also discharged in hotwell from where feed water is picked by a pump and passed through drain cooler and subsequently closed feed water heater. Assuming no losses and negligible pump work determine the thermal efficiency of cycle and mass of steam bled out per kg of steam generated. Draw layout and *T-s* diagram. [38.82%, 0.167 kg/kg steam]

8.27 In a steam turbine plant steam enters turbine at 20 bar, 250ºC and leaves at 0.05 bar. During expansion some amount of steam is bled out at 5 bar, 1.5 bar and 0.3 bar for feed heating in closed feed water heaters. Condensate leaves each feed heater as saturated liquid and is passed through traps into next feed heater at lower pressure. Combined drain at 0.3 bar is cooled in drain cooler upto condenser temperature and then put into hot well which also collects condensate coming from condenser, Determine,

(*a*) the masses of steam bled per kg of steam generated

(*b*) the thermal efficiency of cycle

(*c*) the net work output per kg of steam generated

(*d*) the layout and *T-s* diagram. Neglect pump work.

[0.088 kg, 0.0828 kg, 0.046 kg, 35.63%, 807 kJ/kg]

8.28 A binary vapour power cycle works on mercury and steam such that dry saturated mercury vapour at 4.5 bar is supplied to mercury turbine and leaves at 0.04 bar. Steam is generated as dry saturated at 15 bar and supplied to steam turbine for being expanded upto condenser pressure of 0.04 bar. Determine thermal efficiency of cycle. For mercury take,

$h_{f \text{ at } 4.5 \text{ bar}} = 62.93$ kJ/kg, $h_{g \text{ at } 4.5 \text{ bar}} = 355.98$ kJ/kg,

$v_{\text{f at } 0.04 \text{ bar}} = 0.0000765 \text{ m}^3/\text{kg}$

$h_{f \text{ at } 0.04 \text{ bar}} = 29.98$ kJ/kg, $h_{\text{g at } 0.04 \text{ bar}} = 329.85$ kJ/kg

$s_{g \text{ at } 4.5 \text{ bar}} = 0.5397 \text{ kJ/kg} \cdot \text{K}$, $s_{g \text{ at } 0.04 \text{ bar}} = 0.6925 \text{ kJ/kg} \cdot \text{K}$,

$s_{f \text{ at } 0.04 \text{ bar}} = 0.0808 \text{ kJ/kg} \cdot \text{K}$

9
Gas Power Cycles

9.1 INTRODUCTION

In the preceding chapter brief description is made for thermodynamic cycles comprising of both power and refrigeration cycles. In the present chapter the thermodynamic cycles operating on gas as working fluid and producing power are studied. Gas power cycles are commonly used for power production as in gas turbine plants, internal combustion engines and other applications. Here performance prediction based on thermodynamic analysis is presented for ideal cycles of different types used for gas turbine installations and internal combustion engines.

9.2 AIR-STANDARD CYCLE

Air standard cycle refers to thermodynamic cycle being studied with certain assumptions, so as to use the principles of thermodynamics conveniently. It is the most simplified form of thermodynamic cycle under consideration. General assumptions made for a cycle to be air-standard cycle are as follows:

(*i*) Air is the working fluid and behaves as a perfect gas.
(*ii*) Working fluid does not get changed in its mass and composition.
(*iii*) Thermodynamic processes constituting cycle are reversible.
(*iv*) There is no heat loss from system to surrounding and vice-versa.
(*v*) During heat addition process, heat is assumed to be supplied from a high temperature source.
(*vi*) During heat rejection process, heat is assumed to be rejected to a low temperature sink.
(*vii*) Specific heats of working fluid do not change throughout the cycle.

After incorporating above assumptions in thermodynamic cycle analysis, the analysis done is called air standard cycle analysis. Results obtained from such analysis disagree quite largely from the actual performances due to over simplification, but to begin with, such analysis is necessary. Gradually the assumptions made may be removed and analysis becomes realistic.

Air standard or ideal efficiency can be given as follows:

$$= \frac{\text{Ideal work done i.e. (heat supplied – heat rejected)}}{\text{Heat supplied}}$$

$$\eta_{\text{air}} = 1 - \frac{\text{heat rejected}}{\text{heat supplied}}$$

Mean effective pressure (m.e.p.) is the parameter which indicates the average pressure throughout the cycle. Mathematically, it is given as follows:

$$\text{m.e.p.} = \frac{\text{work done per cycle}}{\text{stroke volume}}$$

Graphically it is given by mean height of $P - V$ diagram.

Air standard cycle analysis for Carnot gas power cycle has already been discussed in chapter 4.

Otto Cycle : This is a modified form of Carnot cycle in order to make it a realistic cycle. Otto cycle has two constant volume and two adiabatic processes as shown below.

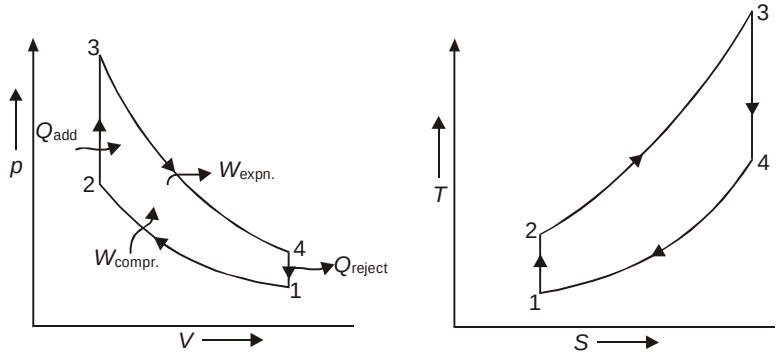


Fig 9.1 *p-V and T-S representations of Otto cycle*

Thermodynamic processes constituting Otto cycle are

1 – 2 = Adiabatic compression process, (–ve work, W_{compr})

2 – 3 = Constant volume heat addition process (+ve heat, Q_{add})

3 – 4 = Adiabatic expansion process, (+ve work, W_{expan})

4 – 1 = Constant volume heat rejection process (–ve heat, Q_{reject})

In order to have an engine based on Otto cycle let us find out the relevance of above processes. Spark ignition type internal combustion engines are based on this cycle.

Process 1 – 2, adiabatic compression process can be realized by piston moving from volume V_1 to V_2 and therefore compressing air.

Process 2 – 3, heat addition process can be undertaken in constant volume manner with piston at volume V_2 and heat added to woking fluid.

Heat addition is practically realized by combustion of fuel and air. As a result of heat addition the compressed air attains state 3 and it is allowed to expand from 3–4 adiabatically. After expansion air is brought back to original state 1 by extracting heat from it at volume V_1.

Internal combustion engine based on Otto cycle is explained ahead. Let us find air-standard thermal efficiency of Otto cycle.

Compression ratio for the cycle shown can be given by the ratio of volumes of air before and after compression. It is generally denoted by r. For unit mass of air and properties at states given with subscript 1, 2, 3, 4, we can write,

$$r = \frac{V_1}{V_2} = \frac{V_4}{V_3}$$

Heat added during 2–3, constant volume process

$$q_{add} = c_v \times (T_3 - T_2)$$

Heat rejected during 4–1, constant volume process

$$q_{rejected} = c_v \times (T_4 - T_1)$$

Air standard efficiency of Otto cycle

$$\eta_{otto} = \frac{\text{Net work}}{\text{Heat added}}$$

For a cycle,

Net work = Heat added – Heat rejected

$$= c_v \{(T_3 - T_2) - (T_4 - T_1)\}$$

Substituting in the expression for efficiency;

$$\eta_{otto} = \frac{c_v\{(T_3 - T_2) - (T_4 - T_1)\}}{c_v(T_3 - T_2)}$$

or

$$\eta_{otto} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

For perfect gas, by gas laws,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = r^{\gamma - 1}$$

and

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma - 1} = r^{\gamma - 1}$$

from above

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \Rightarrow \frac{T_2}{T_3} = \frac{T_1}{T_4}$$

or

$$1 - \frac{T_2}{T_3} = 1 - \frac{T_1}{T_4}$$

or

$$\frac{T_3 - T_2}{T_4 - T_1} = \frac{T_3}{T_4} = r^{\gamma - 1}$$

Substituting in the expression for η_{otto}

$$\eta_{otto} = 1 - \frac{1}{r^{\gamma - 1}}$$

Air standard efficiency of Otto cycle thus depends upon compression ratio (r) alone.

For $r = 1$, $\eta_{otto} = 0$

As compression ratio increases, the Otto cycle efficiency increases as shown in Fig. 9.2.

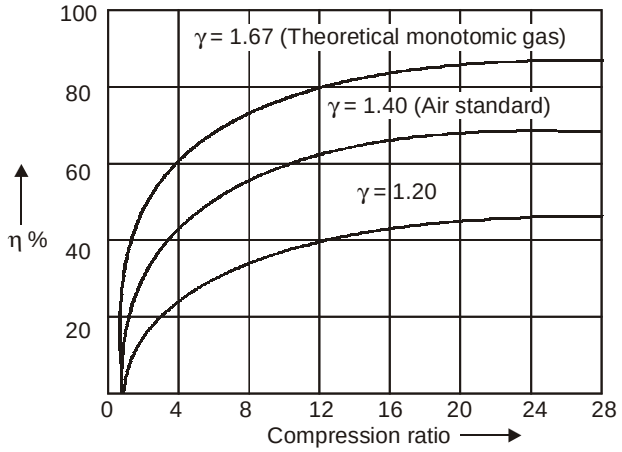


Fig. 9.2 *Otto cycle efficiency vs. compression ratio*

Mean effective pressure for Otto cycle can be estimated by knowing the area of diagram (work) and length of diagram (volume).

Mean effective pressure (m.e.p.)

$$= \frac{\text{Net Work (W)}}{\text{Volume change } (\Delta V)}$$

Net work can be obtained from the area enclosed by cycle on *P–V* diagram.

Thus, Net work = Area of diagram

$$= \left\{\frac{p_3 V_3 - p_4 V_4}{(\gamma - 1)}\right\} - \left\{\frac{p_2 V_2 - p_1 V_1}{(\gamma - 1)}\right\}$$

or
$$= \frac{1}{(\gamma - 1)}\left\{p_4 V_4\left(\frac{p_3 V_3}{p_4 V_4} - 1\right) - p_1 V_1\left(\frac{p_2 V_2}{p_1 V_1} - 1\right)\right\}$$

If $V_2 = V_3$ 1, i.e. unity clearance volume,

Then $V_1 = r$ and $V_4 = r$

Therefore, Net work

$$= \frac{1}{(\gamma - 1)}\left\{p_4 r\left(\frac{p_3}{p_4 \cdot r} - 1\right) - p_1 \cdot r\left(\frac{p_2}{p_1 \cdot r} - 1\right)\right\}$$

Also, for adiabatic expansion and compression process;

$$\frac{p_2}{p_1} = \frac{p_3}{p_4} = (r)^{\gamma}$$

Substituting in expression for Net work,

$$\text{Net work} = \frac{1}{(\gamma - 1)} \{p_4 \cdot r \cdot (r^{\gamma - 1} - 1) - p_1 \cdot r\,(r^{\gamma - 1} - 1)\}$$

or
$$= \frac{r}{(\gamma - 1)}\,(r^{\gamma - 1} - 1) \cdot (p_4 - p_1)$$

Writing expression for m.e.p. as ratio of area of diagram and length of diagram $(V_1 - V_2)$.

$$(\text{m.e.p.})_{\text{Otto}} = \frac{r(r^{\gamma - 1} - 1) \cdot (p_4 - p_1)}{(V_1 - V_2) \cdot (\gamma - 1)}$$

or
$$= \frac{r(r^{\gamma - 1} - 1) \cdot (p_4 - p_1)}{(r - 1)\,(\gamma - 1)}$$

Let us write ratio $\frac{p_4}{p_1} = \alpha$,

Therefore

$$(\text{m.e.p.})_{\text{Otto}} = \frac{r(r^{\gamma - 1} - 1) \cdot p_1 \cdot (\alpha - 1)}{(r - 1) \cdot (\gamma - 1)}$$

Otto cycle has limitations such as isochoric heat addition and rejection is difficult in piston cylinder arrangement. Also, adiabatic compression and expansions are difficult to be realized in practice.

Diesel cycle : Diesel cycle is modified form of Otto cycle. Here heat addition process is replaced from constant volume type to constant pressure type. In a piston cylinder arrangement heat addition with piston at one position allows very little time for heat supply in Otto cycle. By having heat addition at constant pressure the sufficient time is available for heat supply in Diesel cycle. Compression ignition engines work based on Diesel cycles.

Thermodynamic processes constituting Diesel cycle are as given below.

1 – 2 = Adiabatic compression, (–ve work, W_{compr})

2 – 3 = Heat addition at constant pressure (+ve heat, Q_{add})

3 – 4 = Adiabatic expansion, (+ve work, W_{expn})

4 – 1 = Heat rejection at constant volume (–ve heat, $Q_{rejected}$)

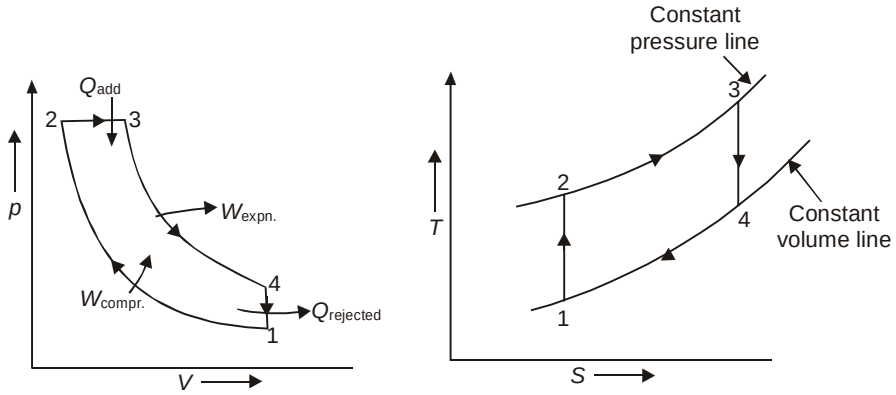


Fig 9.3 *p –V and T–S representation for Diesel cycle*

$P - V$ and $T - S$ representations for the cycle are shown in Fig. 9.3.

Thermodynamic analysis of the cycle for unit mass of air shows;

$$\text{Heat added} = c_p\,(T_3 - T_2)$$

$$\text{Heat rejected} = c_v\,(T_4 - T_1)$$

Let us assume; Compression ratio, $r = \dfrac{V_1}{V_2}$

$$\text{Cut off ratio, } \rho = \frac{V_3}{V_2}$$

$$\text{Expansion ratio} = \frac{V_4}{V_3}$$

Air standard efficiency for Diesel cycle may be given as,

$$\eta_{diesel} = \frac{\text{heat added} - \text{heat rejected}}{\text{heat added}}$$

$$= \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{c_p(T_3 - T_2)}$$

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma}\left\{\frac{(T_4 - T_1)}{(T_3 - T_2)}\right\}$$

Using perfect gas equation and governing equation for thermodynamic process 1 –2;

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

and $$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

Combining above two, we get $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

$$\frac{T_2}{T_1} = (r)^{\gamma-1}$$

$$\Rightarrow \quad T_2 = T_1 \cdot r^{\gamma-1}$$

also as $$\frac{V_3}{V_2} = \frac{T_3}{T_2}$$

or $$\frac{T_3}{T_2} = \rho$$

or $$T_3 = T_2 \cdot \rho$$

or $$T_3 = T_1 \cdot r^{\gamma-1} \cdot \rho$$

Also for adiabatic process 3 – 4 combining the following:

$$\frac{p_3 V_3}{T_3} = \frac{p_4 V_4}{T_4} \cdot \text{and } p_3 V_3^{\gamma} = p_4 V_4^{\gamma}$$

We get, $$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$$

or $$\frac{T_3}{T_4} = \left(\frac{V_4}{V_2} \times \frac{V_2}{V_3}\right)^{\gamma-1}$$

$$= \left(\frac{V_1}{V_2} \times \frac{V_2}{V_3}\right)^{\gamma-1}$$

$$\frac{T_3}{T_4} = \left(\frac{r}{\rho}\right)^{\gamma-1}$$

$$T_4 = T_1 \cdot \rho \cdot r^{\gamma-1} \times \frac{\rho^{\gamma-1}}{r^{\gamma-1}}$$

$$T_4 = T_1 \cdot \rho^{\gamma}$$

Substituting T_2, T_3 and T_4 as function of T_1, r, ρ, γ in the expression for air standard efficiency of diesel cycle

$$\eta_{diesel} = 1 - \frac{1}{\gamma}\left\{\frac{\rho^{\gamma}-1}{r^{\gamma-1}(\rho-1)}\right\}$$

on

$$\eta_{diesel} = 1 - \frac{1}{\gamma \cdot r^{\gamma-1}}\left\{\frac{\rho^{\gamma}-1}{(\rho-1)}\right\}$$

Thus, Diesel cycle efficiency depends upon r, ρ and γ.

For air as working fluid the variation of Diesel cycle efficiency with compression ratio for varying cut-off ratios is given in Figure 9.4.

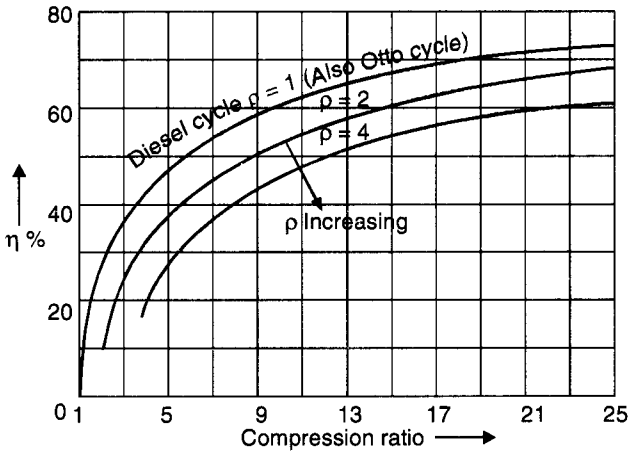


***Fig. 9.4** Diesel cycle efficiency vs. compression ratio*

Mean effective pressure for Diesel cycle can be obtained by ratio of area enclosed on $P - V$ diagram (work) and length of $P - V$ diagram (volume). Assuming clearance volume to be unity i.e. $V_2 = 1$

Net Work = Area of diagram

$$= \frac{p_3 V_3 - p_4 V_4}{\gamma - 1} + p_2 (V_3 - V_2) - \frac{p_2 V_2 - p_1 V_1}{\gamma - 1}$$

Substituting in terms of cut off ratio and compression ratio:

$$= \frac{p_2 \rho - p_4 \cdot r - (p_2 - p_1 r)}{\gamma - 1} + p_2 (\rho - 1)$$

or

$$\text{Net work} = \frac{p_2}{\gamma - 1}\{\gamma(\rho - 1) - r^{1-\gamma} \cdot (r^{\gamma} - 1)\}$$

$$\text{Mean effective pressure} = \frac{\text{Area of } P-V \text{ diagram}}{\text{Length of } P-V \text{ diagram}}$$

$$= \left(\frac{p_2}{\gamma - 1}\right)\{\gamma \cdot (\rho - 1) - r^{1-\gamma}(\rho^{\gamma} - 1)\} \times \frac{1}{(r-1)}$$

or

$$(\text{m.e.p.})_{\text{Diesel}} = \frac{p_1 \cdot r^{\gamma}\left[\gamma(\rho - 1) - r^{1-\gamma} \cdot (\rho^{\gamma} - 1)\right]}{(\gamma - 1)(r - 1)}$$

Dual cycle : It is also called 'mixed cycle' or 'limited pressure cycle.' Dual cycle came up as a result of certain merits and demerits associated with Otto cycle and Diesel cycle due to heat addition occurring at constant volume and constant pressure respectively.

Dual cycle is the combination of Otto cycle and Diesel cycle in which heat addition takes place partly at constant volume and partly at constant pressure.

Thermodynamic processes involved in Dual cycle are given as under.

1 – 2 = Adiabatic compression (–ve work, W_{compr})

2 – 3 = Heat addition at constant volume (+ve heat, Q_{add}, v)

3 – 4 = Heat addition at constant pressure (+ve heat, Q_{add}, ρ)

4 – 5 = Adiabatic expansion (+ve work, W_{expn})

5 – 1 = Heat rejection at constant volume (–ve heat, Q_{rejected})

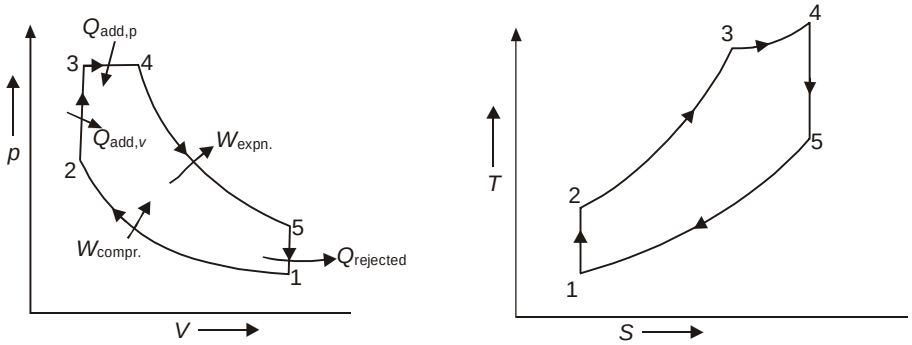


Fig 9.5 *P–V and T–S representations of dual cycle*

Let us assume the following for thermodynamic analysis:

Clearance volume = Unity

$$\text{Compression ratio, } r = \frac{V_1}{V_2}$$

$$\text{Cut-off ratio, } \rho = \frac{V_4}{V_3}$$

$$\text{Pressure ratio during heat addition, } \alpha = \frac{p_3}{p_2}$$

For unit mass of air as working fluid throughout the cycle.

Total Heat added = Heat added at constant volume (2 –3) + Heat added at constant pressure (3 – 4)

$$Q_{\text{add}} = c_v\,(T_3 - T_2) + c_p\,(T_4 - T_3)$$

$$\text{Heat rejected, } Q_{\text{rejected}} = c_v\,(T_5 - T_1)$$

Air standard efficiency for Dual cycle can be given as;

$$\eta_{\text{dual}} = \frac{\text{Heat added} - \text{Heat rejected}}{\text{Heat added}}$$

$$= \frac{c_v(T_3 - T_2) + c_p(T_4 - T_3) - c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)}$$

$$= 1 - \frac{c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)}$$

or
$$= 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)}$$

From gas laws applied to process 2–3,

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

or
$$T_2 = T_3 \times \frac{p_2}{p_3}$$

$$T_2 = \frac{T_3}{\alpha}$$

For process 3–4,

$$\frac{V_4}{T_4} = \frac{V_3}{T_3}$$

or
$$T_4 = T_3 \times \frac{V_4}{V_3}$$

$$T_4 = \rho \cdot T_3$$

For adiabatic process 4–5

$$\frac{T_4}{T_5} = \left(\frac{V_5}{V_4}\right)^{\gamma-1}$$

or
$$T_5 = \frac{T_4}{\left(\frac{V_5}{V_4}\right)^{\gamma-1}}$$

$$T_5 = \frac{T_4 \cdot \rho^{\gamma-1}}{r^{\gamma-1}}$$

Substituting T_4

or
$$T_5 = \frac{T_3 \cdot \rho^{\gamma}}{r^{\gamma-1}}$$

For adiabatic process 1–2

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

or $$T_1 = \frac{T_2}{r^{\gamma-1}}$$

Substituting for T_2

$$T_1 = \frac{T_3}{\alpha \cdot r^{\gamma-1}}$$

Substituting for T_1, T_2, T_4, T_5 in expression for efficiency.

$$\eta_{\text{dual}} = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{\alpha \cdot \rho^{\gamma} - 1}{(\alpha - 1) + \alpha \cdot \gamma(\rho - 1)}\right]$$

For unity cut off ratio i.e. absence of 3 – 4 process, cycle becomes equal to Otto cycle.

i.e. for $$\rho = 1,\ \eta_{\text{dual}} = 1 - \frac{1}{r^{\gamma-1}} = \eta_{\text{otto}}$$

For the pressure ratio α being unity, cycle gets modified to Diesel cycle.

i.e. for $$\alpha = 1, \qquad \eta_{\text{dual}} = 1 - \frac{1}{\gamma \cdot r^{\gamma-1}}\left(\frac{\rho^{\gamma} - 1}{(\rho - 1)}\right) = \eta_{\text{diesel}}$$

Mean effective pressure of dual cycle can be obtained as follows:
Net work = Area enclosed in $P - V$ diagram

$$= p_3 (V_4 - V_3) + \frac{p_4 V_4 - p_5 V_5}{\gamma - 1} - \frac{p_2 V_2 - p_1 V_1}{\gamma - 1}$$

or $$= p_3(\rho - 1) + \frac{p_3 \rho - p_5 r - p_2 + p_1 r}{(\gamma - 1)}$$

Making suitable substitutions for p_1, p_2 and p_5, it gets modified as

$$\text{Net work} = \frac{p_3\left[\alpha \cdot \gamma(\rho - 1) + (\alpha - 1) - r^{1-\gamma} \cdot (\alpha \cdot \rho^{\gamma} - 1)\right]}{\alpha(\gamma - 1)}$$

Substituting for mean effective pressure

$$(\text{m.e.p.})_{\text{dual}} = \frac{\text{Area of diagram}}{\text{Length of diagram}}$$

$$= \frac{1}{(r-1)} \times \left\{\frac{p_3\left[\alpha \cdot \gamma(\rho - 1) + (\alpha - 1) - r^{1-\gamma} \cdot (\alpha \cdot \rho^{\gamma} - 1)\right]}{\alpha(\gamma - 1)}\right\}$$

Substituting p_3 as function of p_1

or

$$(\text{m.e.p.})_{\text{dual}} = \frac{p_1 \cdot r^{\gamma}\left[\alpha\,\gamma(\rho - 1) + (\alpha - 1) - r^{1-\gamma} \cdot (\alpha \cdot \rho^{\gamma} - 1)\right]}{(\gamma - 1)(r - 1)}$$

9.3 BRAYTON CYCLE

Brayton cycle, popularly used for gas turbine power plants comprises of adiabatic compression process, constant pressure heat addition, adiabatic expansion process and constant pressure heat release process. A schematic diagram for air-standard Brayton cycle is shown in Fig. 9.6. Simple gas turbine power plant working on Brayton cycle is also shown here.

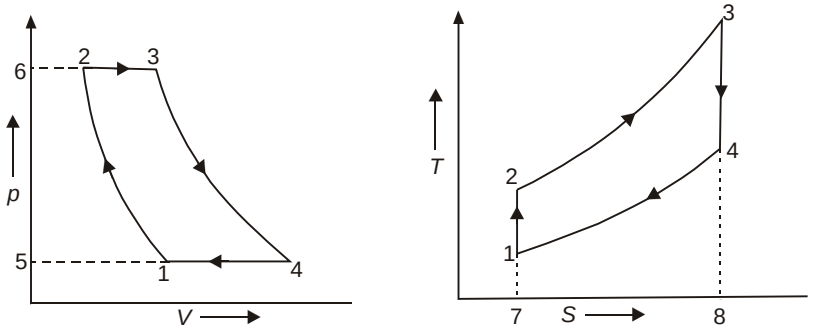
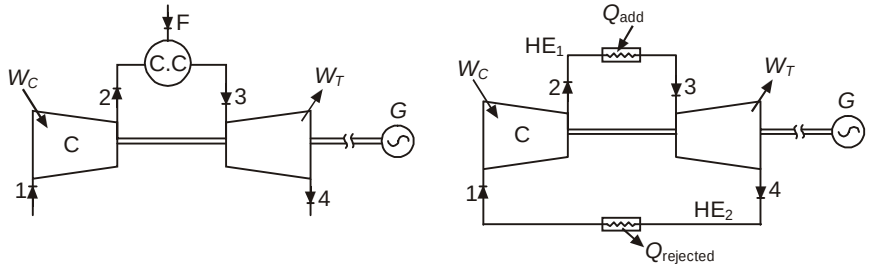


Fig. 9.6 *Brayton cycle on P–V and T–S diagram*



C : Compressor
CC : Combustion chamber

(a) Open type

F : Fuel
G : Generator
HC : Heat exchanger

(b) Closed type

Fig. 9.7 *Simple gas turbine plant*

Thermodynamic cycle shows following processes:

1-2 : Adiabatic compression, involving (–ve) work, W_C in compressor.

2-3 : Constant pressure heat addition, involving heat Q_{add} in combustion chamber or heat exchanger.

3-4 : Adiabatic expansion, involving (+ve) work, W_T in turbine.

4-1 : Constant pressure heat rejection, involving heat, $Q_{rejected}$ in atmosphere or heat exchanger.

In the gas turbine plant layout shown process 1–2 (adiabatic compression) is seen to occur in compressor, heat addition process 2–3 occurs in combustion chamber having open type arrangement and in heat exchanger in closed type arrangement. Process 3–4 of adiabatic expansion occurs in turbine. In open type arrangement exhaust from turbine is discharged to atmosphere while in closed type, heat rejection occurs in heat exchanger. In gas turbine plant of open type, air entering compressor gets compressed and subsequently brought up to elevated temperature in combustion chamber where fuel is added to high pressure air and combustion occurs. High pressure and high temperature combustion products are sent for expansion in turbine where its' expansion yields positive work. Expanded combustion

products are subsequently discharged to atmosphere. Negative work required for compression is drawn from the positive work available from turbine and residual positive work is available as shaft work for driving generator.

In gas turbine plant of closed type the working fluid is recycled and performs different processes without getting contaminated. Working fluid is compressed in compressor and subsequently heated up in heat exchanger through indirect heating. High pressure and high temperature working fluid is sent for getting positive work from turbine and the expanded working fluid leaving turbine is passed through heat exchanger where heat is picked up from working fluid. Thus, the arrangement shows that even costly working fluids can also be used in closed type as it remains uncontaminated and is being recycled.

Air standard analysis of Brayton cycle gives work for compression and expansion as;

$$W_C = m_1 \cdot (h_2 - h_1)$$

$$W_T = m_3 \cdot (h_3 - h_4)$$

for air standard analysis, $m_1 = m_3$, where as in actual cycle

$m_3 = m_1 + m_f$, in open type gas turbine

$m_3 = m_1$, in closed type gas turbine

For the fuel having calorific value *CV* the heat added in air standard cycle;

$Q_{add} = m_1(h_3 - h_2)$, whereas $Q_{add} = m_f \times CV$ for actual cycle.

$$\text{Net work} = W_T - W_C$$

$$W_{net} = \{m_3 (h_3 - h_4) - m_1(h_2 - h_1)\}$$

$$\text{Air standard cycle efficiency} = \frac{W_{net}}{Q_{add}}$$

$$= \frac{m_1\{(h_3 - h_4) - (h_2 - h_1)\}}{m_1(h_3 - h_2)}$$

$$\eta_{Brayton} = \left\{1 - \frac{(h_4 - h_1)}{(h_3 - h_2)}\right\}$$

$$\eta_{Brayton} = \left\{1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}\right\} \text{ as, } (h_4 - h_1) = c_p\,(T_4 - T_1)$$

$$\text{and } (h_3 - h_2) = c_p\,(T_3 - T_2)$$

For processes 1–2 and 3–4, which are of isentropic type,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{(\gamma-1)}{\gamma}} \text{ and } \frac{T_3}{T_4} = \left(\frac{p_2}{p_1}\right)^{\frac{(\gamma-1)}{\gamma}}$$

Let the pressure ratio be, $\frac{p_2}{p_1} = r$,

$$\frac{T_2}{T_1} = (r)^{\frac{(\gamma-1)}{\gamma}} \text{ and } \frac{T_3}{T_4} = (r)^{\frac{(\gamma-1)}{\gamma}}$$

so, $$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$\frac{T_4}{T_1} = \frac{T_3}{T_2}$$

$$\frac{T_4 - T_1}{T_1} = \frac{T_3 - T_2}{T_2}$$

$$\frac{(T_4 - T_1)}{(T_3 - T_2)} = \frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}$$

Making substitution for $\left\{\frac{(T_4 - T_1)}{(T_3 - T_2)}\right\}$ in cycle efficiency:

$$\eta_{\text{Brayton}} = \left(1 - \frac{T_1}{T_2}\right) = 1 - \left(\frac{p_1}{p_2}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = 1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}}$$

Air standard Brayton cycle efficiency: $\boxed{\eta_{\text{Brayton}} = 1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}}}$

Thus, it is obvious from the expression of efficiency that it depends only on pressure ratio (r) and nature of gas (γ). For pressure ratio of unity, efficiency shall be zero. For a particular gas the cycle efficiency increases with increasing pressure ratio. Here the variation of efficiency with pressure ratio is shown for air ($\gamma = 1.4$) and monoatomic gas as argon ($\gamma = 1.66$).

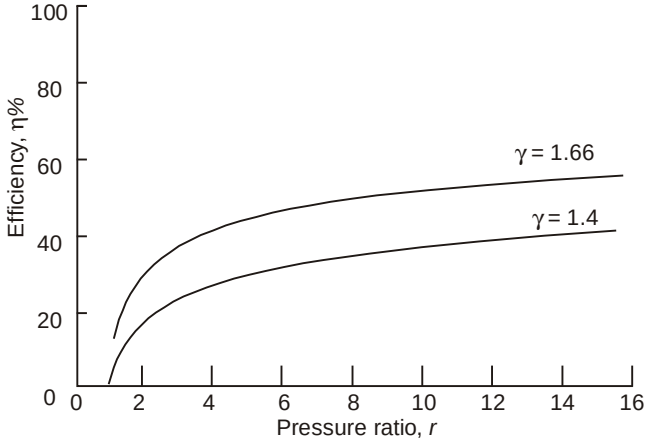


Fig. 9.8 *Efficiency vs. pressure ratio in simple cycle*

Specific work output for the plant can be given as below:

$$w = \{(h_3 - h_4) - (h_2 - h_1)\}, \text{ as } m_1 = 1$$

$$w = c_p \{(T_3 - T_4) - (T_2 - T_1)\}$$

or,

$$\frac{w}{c_p T_1} = \left\{\left(\frac{T_3}{T_1} - \frac{T_4}{T_1}\right) - \left(\frac{T_2}{T_1} - 1\right)\right\}$$

$$= \left\{\left(\frac{T_3}{T_1}\right)\left(1-\frac{T_4 \cdot T_1}{T_3 \cdot T_1}\right)-\left(\frac{T_2}{T_1}-1\right)\right\}$$

In the cycle, states 3 and 1 refer to the states having highest temperature and pressure and lowest temperature and pressure states. T_3 is also referred as turbine inlet temperature while T_1 is ambient temperature in case of open type arrangement. Let the ratio $\frac{T_3}{T_1}$ be given by some constant '*t*' i.e. $\frac{T_3}{T_1} = t$.

Therefore, upon making substitutions,

$$\frac{w}{c_p T_1} = \left\{t\left(1-\frac{T_4}{T_3}\right)-\left(\frac{T_2}{T_1}-1\right)\right\}$$

as,
$$\frac{T_4}{T_3} = \frac{1}{r^{\frac{\gamma-1}{\gamma}}} \text{ and } \frac{T_2}{T_1} = r^{\frac{\gamma-1}{\gamma}}$$

$$\boxed{\frac{w}{c_p T_1} = \left\{t \cdot \left(1-\frac{1}{r^{\frac{\gamma-1}{\gamma}}}\right)-\left(r^{\frac{\gamma-1}{\gamma}}-1\right)\right\}}$$

Here $\left(\frac{w}{c_p T_1}\right)$ is a non-dimensional form of specific work output.

In general for a given arrangement T_1 being ambient temperature remains fixed (mostly atmospheric temperature). So specific work output may be said to depend upon '*t*' (temperature ratio) and pressure ratio '*r*'. Specific work output shall be zero for unity pressure ratio value i.e. for $r = 1$, $\frac{w}{c_p T_1} = 0$.

For certain value of temperature ratio specific work output shall increase with increasing pressure ratio value. Graphical pattern showing variation of specific work output with pressure ratio is shown below for certain values of '*t*'.

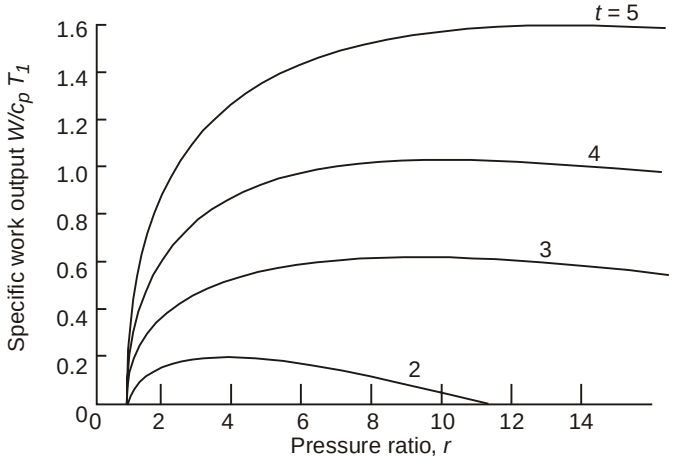


Fig. 9.9 *Specific work output vs. pressure ratio for simple cycle*

The temperature ratio '*t*' is generally used as design parameter. Higher is the '*t*' value higher shall be specific work output. Maximum value of '*t*' depends upon the maximum temperature which the highly stressed parts of turbine can withstand for the required working life (also called metallurgical limit).

In the modern gas turbine value of '*t*' ranges from 3.5 to 6. Further higher values of turbine inlet temperature and thus '*t*' value can be realized with improved gas turbine blade cooling.

Specific work output vs. pressure ratio variation shows that there exist some optimum pressure ratio value at which specific work output is maximum at given '*t*' value. Let us differentiate expression for specific work output with respect to $r^{\left(\frac{\gamma-1}{\gamma}\right)}$.

$$\text{Let } r^{\left(\frac{\gamma-1}{\gamma}\right)} = a,\ t = \text{constant}$$

$$w = c_p T_1 \left\{ t \left(1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}} \right) - \left(r^{\frac{\gamma-1}{\gamma}} - 1 \right) \right\}$$

$$w = c_p T_1 \left\{ t \left(1 - \frac{1}{a} \right) - (a-1) \right\}$$

$$\frac{dw}{da} = c_p T_1 \left\{ t \left(\frac{1}{a^2} \right) - 1 \right\}$$

$$\text{Equating } \frac{dw}{da} = 0, \text{ so } \frac{t}{a^2} - 1 = 0$$

$$t = a^2$$

$$a = \sqrt{t}, \text{ or } \boxed{r = t^{\gamma/2(\gamma-1)}}$$

Thus,
$$r^{\frac{\gamma-1}{\gamma}} = \sqrt{t} \text{ or, } r^{\frac{\gamma-1}{\gamma}} \cdot r^{\frac{\gamma-1}{\gamma}} = t$$

Substituting for $r^{\frac{\gamma-1}{\gamma}}$ and *t*;

$$\frac{T_2}{T_1} \times \frac{T_3}{T_4} = \frac{T_3}{T_1}$$

or,
$$\boxed{T_2 = T_4}$$

Thus, the specific work output is seen to be maximum for a given '*t*' value when the pressure ratio is such that temperature at compressor outlet and turbine outlet are equal.

For all values of pressure ratio '*r*' lying between 1 and $t^{\gamma/2(\gamma-1)}$ temperature T_4 is greater than T_2. In order to reduce heat transfer from external source for improving efficiency a heat exchanger can be incorporated. This concept of using heat exchanger gives birth to modified form of simple gas turbine cycle with heat exchange. This modified form of cycle is also called regenerative gas turbine cycle. Simple gas turbine cycles with modifications of reheat and intercooling are also used for improved performance.

9.4 REGENERATIVE GAS TURBINE CYCLE

In earlier discussions it is seen that for the maximization of specific work output the gas turbine exhaust temperature should be equal to compressor exhaust temperature. The turbine exhaust temperature is normally much above the ambient temperature. Thus, their exist potential for tapping the heat energy getting lost to surroundings with exhaust gases. Here it is devised to use this potential by means of a heat exchanger called regenerator, which shall preheat the air leaving compressor before entering the combustion chamber, thereby reducing the amount of fuel to be burnt inside combustion chamber (combustor).

Regenerative air standard gas turbine cycle shown ahead in Fig. 9.10 has a regenerator (counter flow heat exchanger) through which the hot turbine exhaust gas and comparatively cooler air coming from compressor flow in opposite directions. Under ideal conditions, no frictional pressure drop occurs in either fluid stream while turbine exhaust gas gets cooled from 4 to 4′ while compressed air is heated from 2 to 2′. Assuming regenerator effectiveness as 100% the temperature rise from 2–2′ and drop from 4 to 4′ is shown on *T-S* diagram.

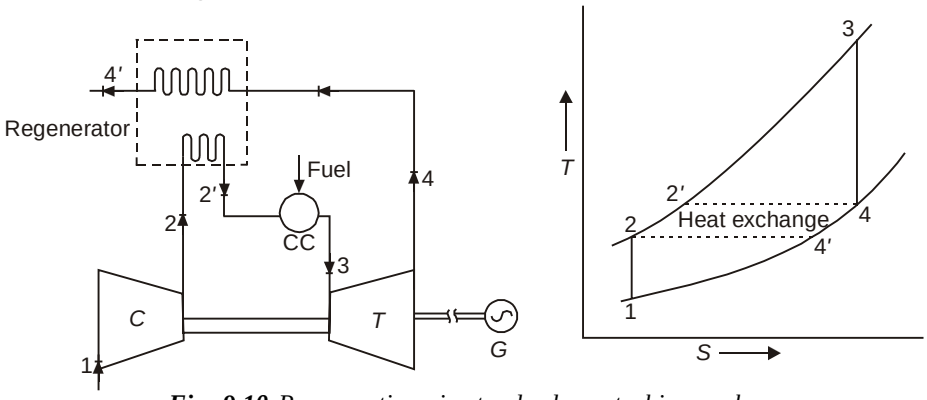


Fig. 9.10 *Regenerative air standard gas turbine cycle.*

Regenerator effectiveness, $\varepsilon = \dfrac{h_{2'} - h_2}{h_4 - h_2}$, where '*h*' refers to specific enthalpy values.

Thus, thermodynamically the amount of heat now added shall be

$$Q_{\text{add, regen}} = m(h_3 - h_{2'})$$

where as without regenerator the heat added; $Q_{\text{add}} = m(h_3 - h_2)$

Here it is obvious that, $Q_{\text{add, regen}} < Q_{\text{add}}$.

This shows an obvious improvement in cycle thermal efficiency as every thing else remains same. Net work produced per unit mass flow is not altered by the use of regenerator.

Air standard cycle thermal efficiency, $\eta_{\text{regen}} = \dfrac{(h_3 - h_4) - (h_2 - h_1)}{(h_3 - h_{2'})}$

$$\eta_{\text{regen}} = \frac{c_p \cdot (T_3 - T_4) - c_p (T_2 - T_1)}{c_p (T_3 - T_{2'})}$$

Thus, regenerative gas turbine cycle efficiency approaches to Carnot cycle efficiency at $r = 1$ which is because in this limiting situation the Carnot cycle requirement of total heat addition and rejection at maximum and minimum cycle temperature is satisfied. Variation of efficiency with cycle pressure ratio for regenerative cycle is shown in Fig. 9.11 for fixed '*t*' values.

Cycle efficiency is seen to decrease with increasing pressure ratio till pressure ratio becomes equal to $r = t^{\left(\frac{\gamma-1}{2\gamma}\right)}$. In earlier discussions it has been shown that at this pressure ratio value the specific work output becomes maximum. Same specific work output variation is true for this case too. For further higher values of pressure ratio the heat exchanger shall cool the air leaving compressor and lower the cycle efficiency than simple cycle efficiency.

This is because of which the constant '*t*' lines are not extended beyond the point where they meet efficiency curve for simple cycle.

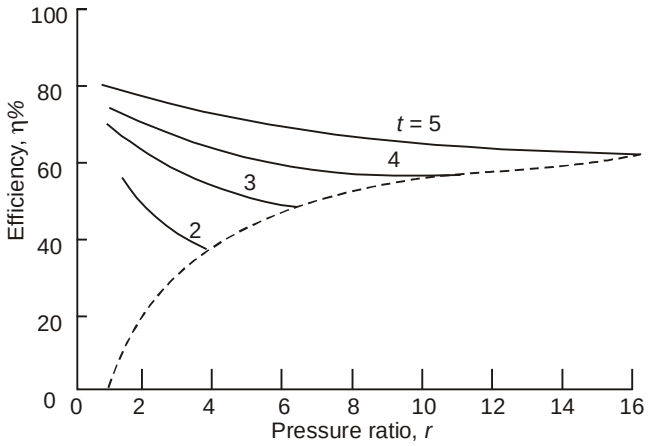


Fig. 9.11 *Efficiency vs. pressure ratio for regenerative gas turbine cycle.*

Thus, it can be concluded that for getting improvement in efficiency by incorporating regenerator the pressure ratio '*r*' appreciably less than the optimum one for maximum specific work output should be used. Also maximum turbine inlet temperature should be used.

For ideal heat exchange in regenerator with 100% effectiveness,

Upon substitution it yields $T_{2'} = T_4$

$$\eta_{\text{regen}} = \frac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_4)}$$

$$\eta_{\text{regen}} = 1 - \frac{(T_2 - T_1)}{(T_3 - T_4)}$$

From thermodynamic processes

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = r^{\left(\frac{\gamma-1}{\gamma}\right)}$$

or,
$$\frac{T_2 - T_1}{T_3 - T_4} = \frac{T_1}{T_4}$$

or,
$$\frac{T_2 - T_1}{T_3 - T_4} = \frac{T_1}{T_3} \times \frac{T_3}{T_4}$$

Also we know

$$\frac{T_3}{T_1} = t;$$

or,
$$\frac{T_2 - T_1}{T_3 - T_4} = \frac{r^{\left(\frac{\gamma-1}{\gamma}\right)}}{t}$$

Substituting in expression for efficiency,

$$\eta_{regen} = 1 - \frac{r^{\left(\frac{\gamma-1}{\gamma}\right)}}{t}$$

Thus, the expression for regenerative cycle efficiency shows that efficiency of regenerative gas turbine cycle depends upon maximum temperature (T_3) in cycle and efficiency increases with increasing 't' value or turbine inlet temperature (T_3) at constant cycle pressure ratio. Also the efficiency decreases with increasing pressure ratio for fixed 't' value, contrary to the behaviour of efficiency shown in case of simple gas turbine cycle where efficiency increases with increasing pressure ratio.

At
$$r = 1,\ \eta_{regen} = 1 - \frac{1}{t} = 1 - \frac{T_1}{T_3} = 1 - \frac{T_{min}}{T_{max}}$$

i.e.
$$\eta_{regen} = \eta_{Carnot} \text{ at } r = 1$$

9.5 REHEAT GAS TURBINE CYCLE

Reheat gas turbine cycle arrangement is shown in Fig. 9.12. In order to maximize the work available from the simple gas turbine cycle one of the option is to increase enthalpy of fluid entering gas turbine and extend its expansion upto the lowest possible enthalpy value.

C : Compressor
CC : Combustion chamber
G : Generator
f : Fuel
HPT : High pressure turbine
LPT : Low pressure turbine
RCC : Reheat combustion chamber

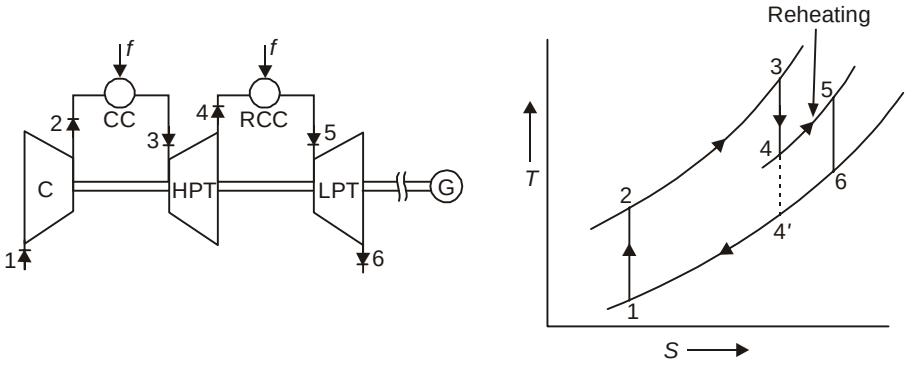


Fig. 9.12 *Reheat gas turbine cycle*

This can also be said in terms of pressure and temperature values i.e. inject fluid at high pressure and temperature into gas turbine and expand upto lowest possible pressure value. Upper limit at inlet to turbine is limited by metallurgical limits while lower pressure is limited to near atmospheric pressure in case of open cycle. For further increasing the net work output the positive work may be increased by using multistage expansion with reheating in between. In multistage expansion the expansion is divided into parts and after part expansion working fluid may be reheated for getting larger positive work in left out expansion. For reheating another combustion chamber may be used.

Here in the arrangement shown ambient air enters compressor and compressed air at high pressure leaves at 2. Compressed air is injected into combustion chamber for increasing its temperature upto desired turbine inlet temperature at state 3. High pressure and high temperature fluid enters high pressure turbine (HPT) for first phase of expansion and expanded gases leaving at 4 are sent to reheat combustion chamber (reheater) for being further heated. Thus, reheating is a kind of energising the working fluid. Assuming perfect reheating (in which temperature after reheat is same as temperature attained in first combustion chamber), the fluid leaves at state 5 and enters low pressure turbine (LPT) for remaining expansion upto desired pressure value. Generally, temperature after reheating at state 5 is less than temperature at state 3. In the absence of reheating the expansion process within similar pressure limits goes upto state 4′. Thus, reheating offers an obvious advantage of work output increase since constant pressure lines on *T*-*S* diagram diverge slightly with increasing entropy, the total work of the two stage turbine is greater than that of single expansion from state 3 to state 4′. i.e.,

$$(T_3 - T_4) + (T_5 - T_6) > (T_3 - T_{4'}).$$

Here it may be noted that the heat addition also increases because of additional heat supplied for reheating. Therefore, despite the increase in net work due to reheating the cycle thermal efficiency would not necessarily increase. Let us now carry out air standard cycle analysis.

Net work output in reheat cycle, $W_{\text{net, reheat}} = W_{\text{HPT}} + W_{\text{LPT}} - W_{\text{C}}$

$$W_{\text{HPT}} = m(h_3 - h_4),\ W_{\text{LPT}} = m(h_5 - h_6),\ W_{\text{C}} = m(h_2 - h_1)$$

$$W_{\text{net, reheat}} = m\{(h_3 - h_4) + (h_5 - h_6) - (h_2 - h_1)\}$$

$$W_{\text{net, reheat}} = m\, c_p\, \{(T_3 - T_4) + (T_5 - T_6) - (T_2 - T_1)\}$$

assuming $T_3 = T_5$ i.e. perfect reheating

$$W_{\text{net, reheat}} = m\, c_p\, \{(2T_3 - T_4) - T_6 - (T_2 - T_1)\}$$

$$W_{\text{net, reheat}} = m\, c_p\, T_1 \left\{\left(\frac{2T_3}{T_1} - \frac{T_4}{T_1}\right) - \left(\frac{T_6}{T_1}\right) - \left(\frac{T_2}{T_1} - 1\right)\right\}$$

Here $\dfrac{T_3}{T_1} = t$; pressure ratio, $r = \left(\dfrac{T_2}{T_1}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}$

Let pressure ratio for HP turbine be r_1 and for LP turbine be r_2 then;

$$r = r_1 \times r_2$$

$$r_1 = \left(\frac{T_3}{T_4}\right)^{\left(\frac{\gamma}{\gamma-1}\right)},\ r_2 = \left(\frac{T_5}{T_6}\right)^{\left(\frac{\gamma}{\gamma-1}\right)} = \frac{r}{r_1};$$

or, $$\left(\frac{T_5}{T_6}\right) = \left(\frac{r}{r_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = \left(\frac{T_3}{T_6}\right)$$

$$\left(\frac{W_{\text{net, reheat}}}{mc_p\,T_1}\right) = \left\{\left(2t - \frac{T_4}{T_3}.\frac{T_3}{T_1}\right) - \left(\frac{T_6}{T_3}\times\frac{T_3}{T_1}\right) - \left(r^{\frac{\gamma-1}{\gamma}} - 1\right)\right\}$$

Let $\left(\frac{W_{\text{net, reheat}}}{mc_p\,T_1}\right) = w$

then $$w = \left\{\left(2t - \frac{t}{(r_1)^{\left(\frac{\gamma-1}{\gamma}\right)}}\right) - \left(\left(\frac{r_1}{r}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}\times t\right) - \left(r^{\left(\frac{\gamma-1}{\gamma}\right)} - 1\right)\right\}$$

$$\frac{dw}{dr_1} = 0 - \left(\left(\frac{1-\gamma}{\gamma}\right)\times\frac{t}{r_1^{\left(\frac{2\gamma-1}{\gamma}\right)}}\right) - \left(\left(\frac{\gamma-1}{\gamma}\right)\frac{t}{r_1^{1/\gamma}}\times\frac{1}{r^{\left(\frac{\gamma-1}{\gamma}\right)}}\right) - 0$$

Equating $\frac{dw}{dr_1}$ to zero, $\frac{1}{r_1^{2\gamma-1/\gamma}} = \frac{1}{r^{\left(\frac{\gamma-1}{\gamma}\right)}.r_1^{1/\gamma}} \Rightarrow r^{\left(\frac{\gamma-1}{\gamma}\right)} = r_1^{2\left(\frac{\gamma-1}{\gamma}\right)}$

or, $$r_1 = \sqrt{r} = r_2$$

Thus, expansion pressure ratio for high pressure turbine and low pressure turbine should be same for maximum work output from the cycle. With such optimum division the expression for work output and efficiency can be given as,

$$w = \left\{\left(2t - \frac{t}{(r)^{\left(\frac{\gamma-1}{2\gamma}\right)}}\right) - \left(\frac{t}{(r)^{\frac{\gamma-1}{2\gamma}}}\right) - \left(r^{\left(\frac{\gamma-1}{\gamma}\right)} - 1\right)\right\}$$

Let, $r^{\left(\frac{\gamma-1}{\gamma}\right)} = c$ then,

$$\left(\frac{W_{\text{net, reheat}}}{mc_p\,T_1}\right) = \left\{\left(2t - \frac{2t}{\sqrt{c}}\right) - (c-1)\right\}$$

$$\boxed{\frac{W_{\text{net, reheat}}}{mc_pT_1} = \left\{2t - \frac{2t}{\sqrt{c}} - c + 1\right\}}$$

Efficiency for reheat cycle

$$\eta_{\text{reheat}} = \frac{W_{\text{net, reheat}}}{m(h_3 - h_2) + m(h_5 - h_4)}$$

$$= \frac{W_{\text{net, reheat}}}{mc_p\{(T_3 - T_2) + (T_5 - T_4)\}}$$

$$\eta_{\text{reheat}} = \frac{W_{\text{net, reheat}}}{mc_pT_1\left\{\left(\frac{T_3}{T_1} - \frac{T_2}{T_1}\right) + \left(\frac{T_5}{T_1} - \frac{T_4}{T_1}\right)\right\}}$$

Let us simplify denominator first,

$$= m\, c_p\, T_1 \left\{\left(\frac{T_3}{T_1} - \frac{T_2}{T_1}\right) + \left(\frac{T_5}{T_1} - \frac{T_4}{T_1}\right)\right\}$$

We know $T_5 = T_3$ and $\frac{T_3}{T_1} = t$

$$= m\, c_p\, T_1 \left\{\left(t - \frac{T_2}{T_1}\right) + \left(t - \frac{T_4}{T_1}\right)\right\}$$

Also $\frac{T_2}{T_1} = r^{\left(\frac{\gamma-1}{\gamma}\right)}, \frac{T_3}{T_4} = r^{\left(\frac{\gamma-1}{2\gamma}\right)}$

Substituting,

$$= m\, c_p\, T_1 \left\{\left(t - r^{\left(\frac{\gamma-1}{\gamma}\right)}\right) + \left(t - \frac{T_4}{T_3} \times \frac{T_3}{T_1}\right)\right\}$$

$$= m\, c_p\, T_1 \left\{2t - r^{\frac{\gamma-1}{\gamma}} - \frac{t}{r^{\left(\frac{\gamma-1}{2\gamma}\right)}}\right\}$$

Thus, Denominator $= m\, c_p\, T_1 \left\{2t - c - \frac{t}{\sqrt{c}}\right\}$

Substituting in expression for efficiency for optimum division of pressure ratio;

$$\eta_{\text{reheat}} = \frac{W_{\text{net, reheat}}}{mc_pT_1\left\{2t - c - \frac{t}{\sqrt{c}}\right\}}$$

$$\boxed{\eta_{\text{reheat}} = \frac{\left\{2t - \frac{2t}{\sqrt{c}} - c + 1\right\}}{\left\{2t - c - \frac{t}{\sqrt{c}}\right\}}}$$

A plot showing variation of efficiency with pressure ratio '*r*' is shown in Fig. 9.13 along with simple cycle efficiency variation. It indicates that reheating offers increase in specific work output at the

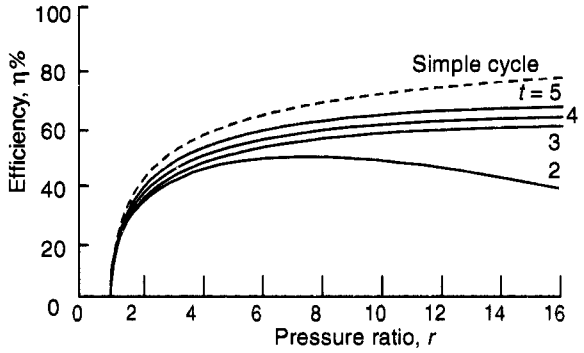


Fig. 9.13 *Reheat cycle efficiency vs. cycle pressure ratio*

cost of cycle efficiency. This reduction in efficiency may be attributed to the addition of a less efficient cycle 4564′ to a simple cycle. 4564′ is a less efficient cycle since it operates over a smaller temperature range. Variation of specific work output with pressure ratio is shown in Fig. 9.14. It shows how specific work output shows increase with increasing pressure ratio upto optimum pressure ratio. It may also be noted that in reheat cycle, the temperature of exhaust gases at exit of gas turbine gets increased as compared to simple cycle within similar limits. Therefore, reheat cycle offers potential for use of regenerator for harnessing the hotter exhaust from gas turbine.

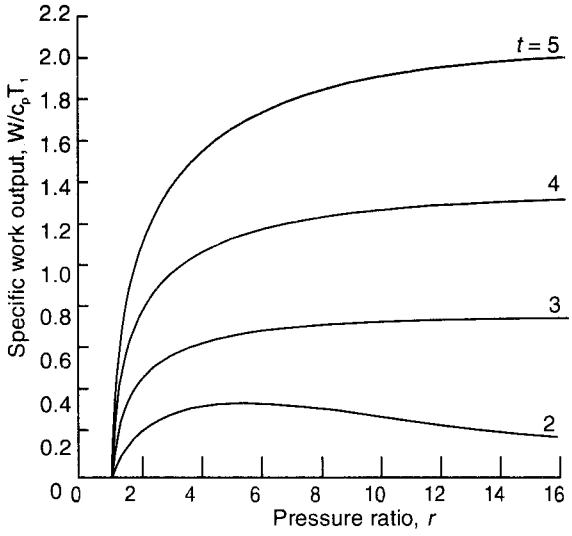


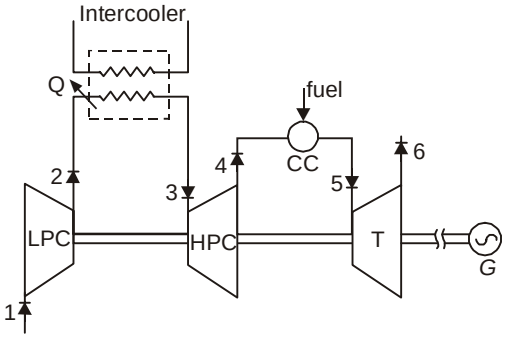
Fig. 9.14 *Reheat cycle specific work output vs. cycle pressure ratio*

9.6 GAS TURBINE CYCLE WITH INTERCOOLING

Net work output from gas turbine cycle can also be increased by reducing negative work i.e. compressor work. Multistaging of compression process with intercooling in between is one of the approach for

reducing compression work. It is based on the fact that for a fixed compression ratio higher is the inlet temperature higher shall be compression work requirement and vice-a-versa. Schematic for intercooled gas turbine cycle is given in Fig. 9.15.

Thermodynamic processes involved in multistage intercooled compression are shown in Figs. 9.16, 9.17. First stage compression occurs in low pressure compressor (LPC) and compressed air leaving LPC at '2' is sent to intercooler where temperature of compressed air is lowered down to state 3 at constant pressure. In case of perfect intercooling the temperatures at 3 and 1 are same. Intercooler is a kind of heat exchanger where heat is picked up from high temperature compressed air. The amount of compression work saved due to intercooling is obvious from *p-V* diagram and shown by area 2342′. Area 2342′ gives the amount of work saved due to intercooling between compression.



CC : Combustion chamber
LPC : Low pressure compressor
HPC : High pressure compressor
T : Turbine
G : Generator

Fig. 9.15 *Gas turbine cycle with intercooling*

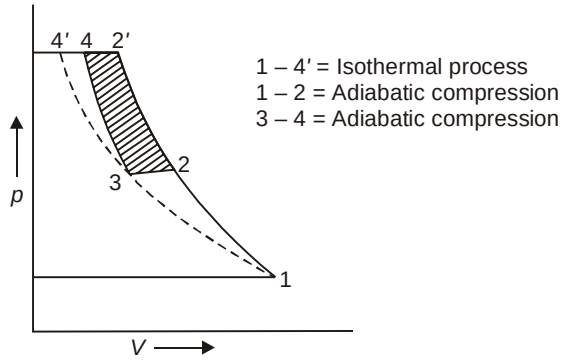


Fig. 9.16 *Intercooled compression*

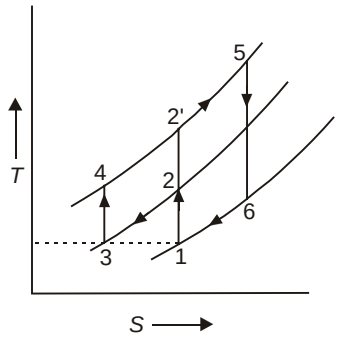


Fig. 9.17 *T-S diagram for gas turbine cycle with intercooling*

Some large compressors have several stages of compression with intercooling between stages. Use of multistage compression with intercooling in a gas turbine power plant increases the net work produced because of reduction in compressor work. Intercooled compression results in reduced temperature at the end of final compression. *T-S* diagram for gas turbine cycle with intercooling shows that in the absence of intercooling within same pressure limits the state at the end of compression would be 2′ while with perfect intercooling this state is at 4 i.e., $T_{2'} > T_4$. The reduced temperature at compressor

exit leads to additional heat requirement in combustion chamber i.e. more amount of fuel is to be burnt for attaining certain turbine inlet temperature as compared to simple cycle without intercooling.

Thus, intercooled cycle thermal efficiency may not increase with intercooling because of simultaneous increase in heat addition requirement. The lower temperature at compressor exit enhances the potential for regeneration so when intercooling is used in conjunction with regeneration an appreciable increase in thermal efficiency can result.

Net work output in gas turbine cycle with intercooling;

$$W_{\text{net, intercool}} = m\{(h_5 - h_6) - (h_4 - h_3) - (h_2 - h_1)\}$$
$$W_{\text{net, intercool}} = mc_p\{(T_5 - T_6) - (T_4 - T_3) - (T_2 - T_1)\}$$

Cycle thermal efficiency;

$$\eta_{\text{intercool}} = \frac{\{(h_5 - h_6) - (h_4 - h_3) - (h_2 - h_1)\}}{\{h_5 - h_4\}}$$

9.7 GAS TURBINE CYCLE WITH REHEAT AND REGENERATION

During discussion of reheat cycle it is seen that because of hotter exhaust from gas turbine, there exists possibility for performance improvement with regeneration. Figs. 9.18, 9.19 give schematic and cycle representation for gas turbine cycle with reheat and regeneration.

Here the reduction in thermal efficiency due to reheat is overcome by adding regeneration. Higher gas turbine exhaust gas temperature is thus fully utilized in regenerator (heat exchanger). Additional heat supplied for reheating in order to increase work output is supplemented to some extent by regenerator. It is seen that in this kind of combustion thermal efficiency is increased.

Net work output,

$$W_{\text{net}} = m\{(h_6 - h_7) + (h_4 - h_5) - (h_2 - h_1)\}$$

Heat added,

$$Q_{\text{add}} = m\{(h_4 - h_3) + (h_6 - h_5)\}$$

Thermal efficiency;

$$\eta_{\text{reheat and regenerator}} = \frac{\{(h_6 - h_7) + (h_4 - h_5) - (h_2 - h_1)\}}{\{(h_4 - h_3) + (h_6 - h_5)\}}$$

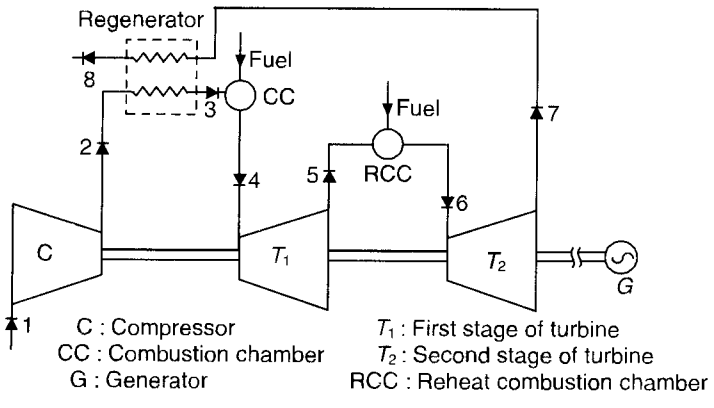


Fig. 9.18 Schematic for gas turbine cycle with reheat and regeneration

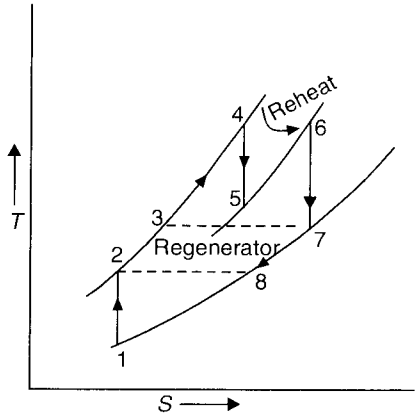
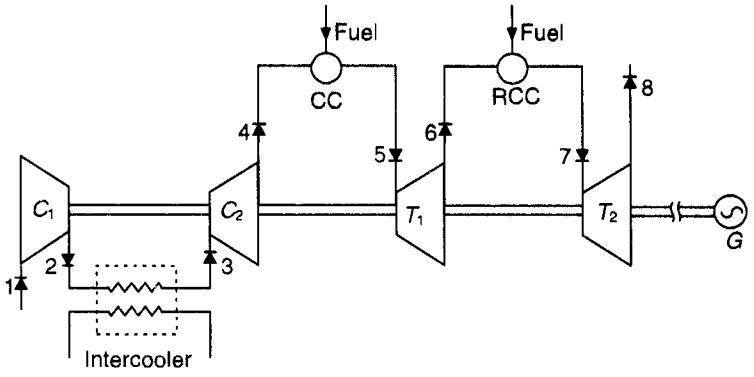


Fig. 9.19 Thermodynamic representation of gas turbine cycle with reheat and regenerator

9.8 GAS TURBINE CYCLE WITH REHEAT AND INTERCOOLING

Schematic arrangement for gas turbine cycle with reheating during expansion and intercooling during compression is given in Fig. 9.20 along with *T-S* diagram. Here reheating and intercooling offer appreciable increase in specific work output but the thermal efficiency deteriorates as reheating and intercooling both have additional heat requirement. Although due to higher temperature the gas turbine exhaust offers potential for regeneration.



C_1 : Low pressure compressor stage
C_2 : High pressure compressor stage
CC : Combustion chamber
G : Generator
RCC : Reheat combustion chamber
T_1 : High pressure turbine stage
T_2 : Low pressure turbine stage

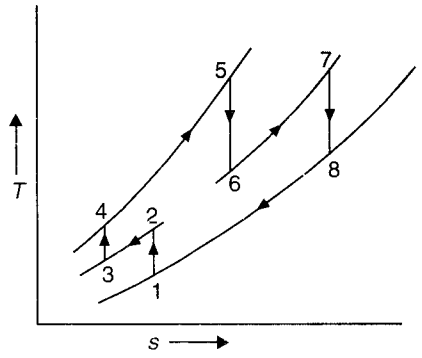


Fig. 9.20 *Schematic for gas turbine cycle with reheat and intercooling and T-S diagram representation*

Air standard cycle analysis for this arrangement gives,

$$W_{net} = m\,\{(h_5 - h_6)\} + (h_7 - h_8) - (h_4 - h_3) - (h_2 - h_1)\}$$

Heat added

$$Q_{add} = m\{(h_5 - h_4) + (h_7 - h_6)\}$$

Cycle thermal efficiency,

$$\eta_{cycle} = \frac{\{(h_5 - h_6) + (h_7 - h_8) - (h_4 - h_3) - (h_2 - h_1)\}}{\{(h_5 - h_4) + (h_7 - h_6)\}}$$

9.9 GAS TURBINE CYCLE WITH REGENERATION, REHEAT AND INTERCOOLING

Regenerative gas turbine employing reheating during expansion and intercooling during compression is considered here as shown in Fig. 9.21. This combination offers considerable increase in net work output and thermal efficiency.

C_1 : Low pressure compressor stage
C_2 : High pressure compressor stage
CC : Combustion chamber
RCC : Reheat combustion chamber
T_1 : High pressure turbine stage
T_2 : Low pressure turbine stage
G : Generator

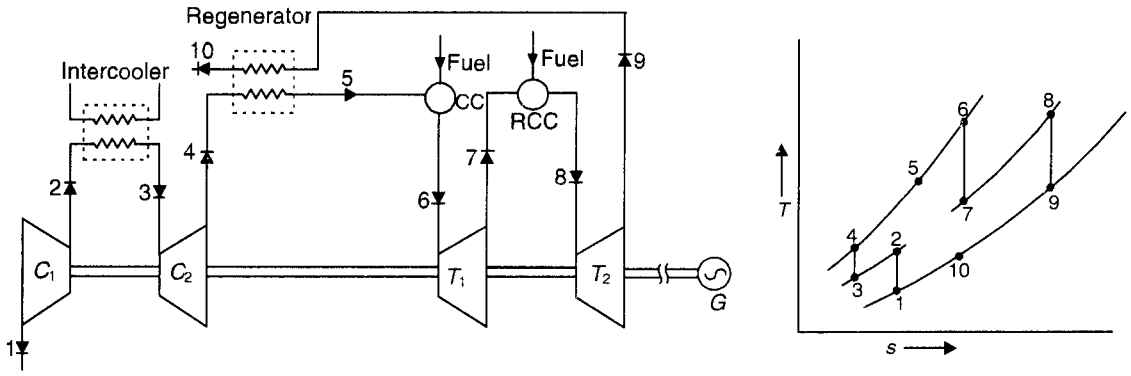


Fig. 9.21 Schematic for gas turbine cycle with regeneration, reheat and intercooling and T-S diagram

Based upon air standard cycle considerations thermodynamic analysis gives the Net work output from cycle,

$$W_{net} = m\,\{(h_6 - h_7) + (h_8 - h_9) - (h_4 - h_3) - (h_2 - h_1)\}$$

Heat added

$$Q_{add} = m\{(h_6 - h_5) + (h_8 - h_7)\}$$

Cycle thermal efficiency,

$$\eta_{cycle} = \frac{\{(h_6 - h_7) + (h_8 - h_9) - (h_4 - h_3) - (h_2 - h_1)\}}{\{(h_6 - h_5) + (h_8 - h_7)\}}$$

9.10 GAS TURBINE IRREVERSIBILITIES AND LOSSES

Till now the discussions have been confined to air standard Brayton cycle. But the realistic gas turbine cycle has deviations from air standard cycle due to,

(*i*) frictional effects within compressor and turbine which causes increase in specific entropy of working fluid across these components.

(*ii*) friction which shall cause drop in pressure of working fluid across the constant pressure processes.

Apart from above irreversibilities of the gas turbine power plant the irreversibilities of combustion chamber are quite significant.

Salient state points of realistic gas turbine Brayton cycle with above irreversibilities and losses are shown below:

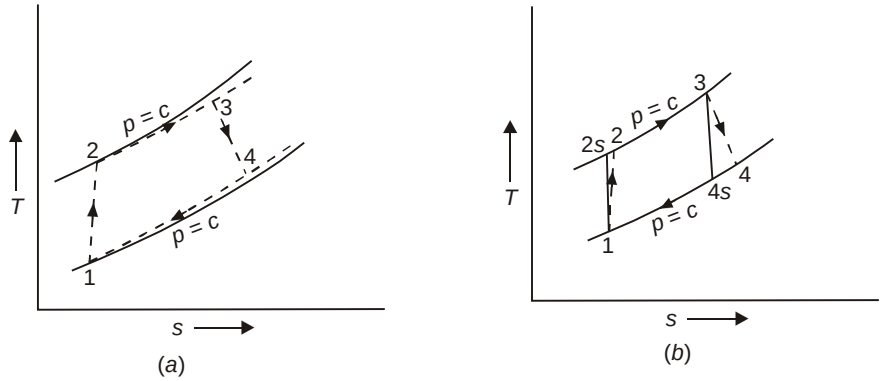


Fig. 9.22 *Effect of irreversibilities and losses in gas turbine cycle.*

Isentropic efficiency of turbine and compressor can be mathematically given as below:

Isentropic efficiency of turbine, $\eta_{\text{isen, } t} = \left\{ \dfrac{h_3 - h_4}{h_3 - h_{4s}} \right\}$

$$\text{i.e. } \eta_{\text{isen, } t} = \frac{\text{Actual expansion work}}{\text{Ideal expansion work}}$$

Isentropic efficiency of compressor, $\eta_{\text{isen, } c} = \left\{ \dfrac{h_{2s} - h_1}{h_2 - h_1} \right\}$

$$\text{i.e. } \eta_{\text{isen, } c} = \frac{\text{Ideal compressor work}}{\text{Actual compressor work}}$$

Other factors causing the real cycle to be different from ideal cycle are as given below:

(*i*) Fluid velocities in turbomachines are very high and there exists substantial change in kinetic energy between inlet and outlet of each component. In the analysis carried out earlier the changes in kinetic energy have been neglected whereas for exact analysis it cannot be.

(*ii*) In case of regenerator the compressed air cannot be heated to the temperature of gas leaving turbine as the terminal temperature difference shall always exist.

(*iii*) Compression process shall involve work more than theoretically estimated value in order to overcome bearing and windage friction losses.

Different factors described above can be accounted for by stagnation properties, compressor and turbine isentropic efficiency and polytropic efficiency.

Stagnation properties: Stagnation properties are properties corresponding to stagnation states. Stagnation state of a fluid refers to the state at which fluid is adiabatically brought to the state of rest without work transfer. By using stagnation properties for themodynamic analysis the effect of kinetic energy variation is taken care and analysis becomes realistic. Let us write down the steady flow energy equation considering no heat and work interactions between states 1 and state 0. Let state '2' be the stagnation state. Generally stagnation states are denoted by using subscript '0'.

At stagnation state velocity of fluid shall be zero.

$$h_1 + \frac{c_1^2}{2} = h_0 + 0, \text{ or, } h_0 = h_1 + \frac{c_1^2}{2}$$

or, $$\boxed{h_0 = h + \frac{c^2}{2}}$$

When the fluid is perfect gas, then

$$c_p T_0 = c_p T + \frac{c^2}{2}$$

$$\boxed{T_0 = T + \frac{c^2}{2c_p}}$$

where T_0 = stagnation temperature or total temperature
T = static temperature, c = velocity

$\frac{c^2}{2c_p}$ = dynamic temperature

When a gas stream is slowed down and the temperature rises then there is simultaneous rise in pressure. The stagnation pressure p_0 is defined similar to T_0 with certain modification that the gas is brought to rest isentropically Stagnation pressure can be mathematically given as,

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\gamma/(\gamma-1)}$$

Let us use stagnation properties for an isentropic compression process between states 1 and 2. Stagnation pressure ratio,

$$\frac{p_{02}}{p_{01}} = \frac{p_{02}}{p_2} \times \frac{p_1}{p_{01}} \times \frac{p_2}{p_1}$$

$$= \left(\frac{T_{02}}{T_2} \times \frac{T_1}{T_{01}} \times \frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}$$

$$\boxed{\frac{p_{02}}{p_{01}} = \left(\frac{T_{02}}{T_{01}}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}}$$

Thus, it shows p_0 and T_0 can be used similar to static values. Stagnation pressures and temperatures can be used with static values to determine combined thermodynamic and mechanical state of stream. Representation of compression process on *T-S* diagram using static and stagnation properties gives better conceptual information.

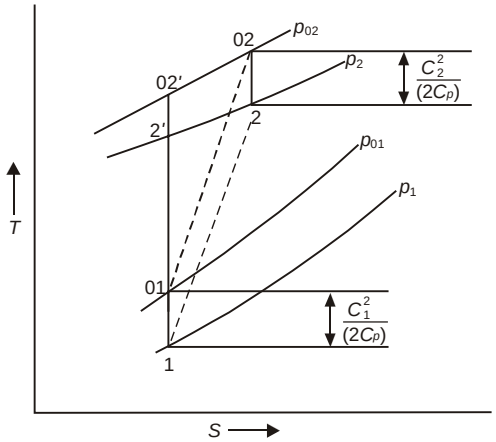


Fig. 9.23 *Compression process representation using static and stagnation states*

9.11 COMPRESSOR AND TURBINE EFFICIENCY

Isentropic efficiency of compressor and turbine as defined earlier can also be given in terms of stagnation properties.

$$\text{Isentropic compressor efficiency, } \eta_{\text{isen, c}} = \frac{W_{\text{ideal}}}{W_{\text{actual}}} = \frac{\Delta h_{0'}}{\Delta h_0}$$

In terms of temperature values, assuming mean specific heat value over a range of temperature, the efficiency can be given as,

$$\eta_{\text{isen, c}} = \frac{T_{02'} - T_{01}}{T_{02} - T_{01}}$$

$$\text{Isentropic turbine efficiency, } \eta_{\text{isen, t}} = \frac{W_{\text{actual}}}{W_{\text{ideal}}}$$

$$\eta_{\text{isen, t}} = \frac{T_{03} - T_{04}}{T_{03} - T_{04'}}$$

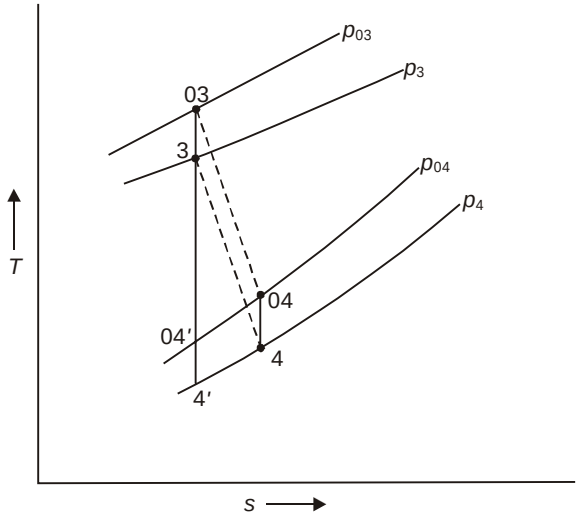


Fig. 9.24 *Expansion process representation using static and stagnation states*

Polytropic efficiency: In case of multistage compression and expansion the isentropic efficiency of complete machine as defined earlier may vary with pressure ratio. Actually it is found that compressor isentropic efficiency decreases with pressure ratio while turbine isentropic efficiency increases with pressure ratio, therefore use of fixed typical values of these efficiencies is not proper.

Let us consider an axial flow compressor comprising of number of stages having equal stage efficiency for all constituent stages as η_s.

Let isentropic temperature rise in a stage be $\Delta T_{s'}$ while actual temperature rise is ΔT_s. Let isentropic temperature rise in whole compressor be $\Delta T'$ while actual temperature rise be ΔT.

$$\text{For a stage, } \eta_s = \frac{\Delta T_{s'}}{\Delta T_s}$$

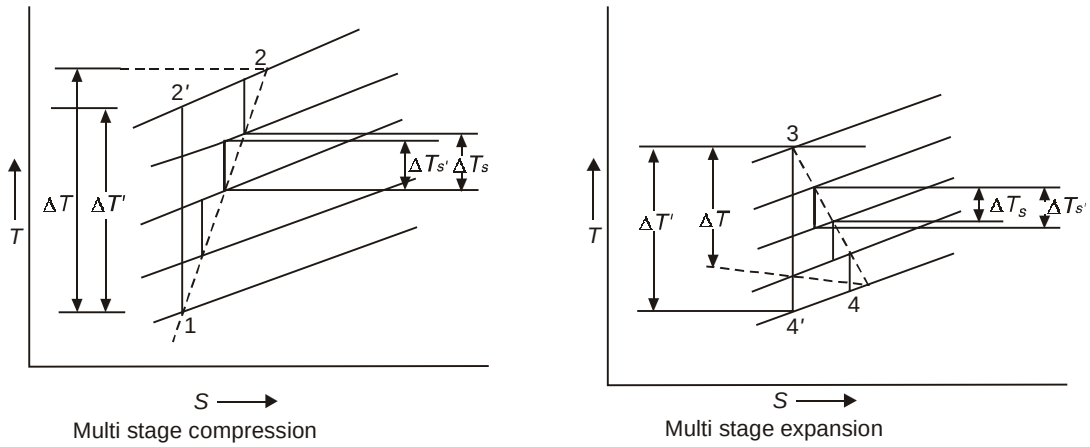


Fig. 9.25

Isentropic efficiency of compression, $\eta_{\text{isen, } c} = \dfrac{\Delta T'}{\Delta T}$

From stage efficiency expression,

$$\sum \Delta T_s = \sum\left(\frac{\Delta T_{s'}}{\eta_s}\right)$$

or

$$\sum \Delta T_s = \frac{\sum \Delta T_{s'}}{\eta_s}$$

It is obvious that, $\sum \Delta T_s = \Delta T$ for multistage compressor

so,

$$\Delta T = \frac{\sum \Delta T_{s'}}{\eta_s}$$

Substituting from compressor isentropic efficiency in stage efficiency expression

$$\frac{\Delta T'}{\eta_{\text{isen, c}}} = \frac{\sum \Delta T_{s'}}{\eta_s}$$

or

$$\boxed{\eta_{\text{isen, c}} = \eta_s\left(\frac{\Delta T'}{\sum \Delta T_s'}\right)}$$

From the nature of constant pressure lines on *T-S* diagram it could be concluded that constant pressure lines are of diverging type.

Therefore, $\Delta T' < \sum \Delta T_s'$

Hence, $\boxed{\eta_{\text{isen, c}} < \eta_s}$

With the increasing pressure ratio this difference between $\sum \Delta T_s'$ and $\Delta T'$ goes on increasing and so with increasing pressure ratio the $\eta_{\text{isen, c}}$ goes on reducing despite constant stage efficiency. This

phenomenon may be attributed to the fact that the increase in temperature due to friction in a stage results in more work requirement on next stage. This effect is also termed as the 'preheat' effect.

For multistage expansion in turbine: Let expansion in turbine be of multistage tpye with each stage having equal stage efficiency of η_s and stage isentropic temperature drop as $\Delta T'_s$ and actual stage temperature drop as ΔT_s. Let overall temperature drop in turbine be $\Delta T'$ for isentropic expansion and ΔT for actual expansion while turbine isentropic efficiency is $\eta_{\text{isen, t.}}$

$$\text{Stage efficiency of turbine, } \eta_s = \frac{\Delta T_s}{\Delta T'_s}$$

Cumulatively

$$\sum \Delta T'_s = \frac{\sum \Delta T_s}{\eta_s}$$

It is obvious from Fig. 9.25 showing multistage expansion that,

$$\sum \Delta T_s = \Delta T$$

So,
$$\sum \Delta T'_s = \frac{\Delta T}{\eta_s}$$

or,
$$\eta_s = \frac{\Delta T}{\sum \Delta T'_s}$$

$$\text{Turbine isentropic efficiency, } \eta_{\text{isen, t.}} = \frac{\Delta T}{\Delta T'}$$

Combining two expressions for turbine isentropic efficiency and stage efficiency, it yields,

$$\eta_{\text{isen, t}} = \eta_s \cdot \frac{\sum \Delta T'_s}{\Delta T'}$$

It may also be seen that due to diverging nature of constant pressure lines on *T-S* diagram, $\Delta T' < \sum \Delta T'_s$.

Therefore, $\boxed{\eta_{\text{isen, t}} > \eta_s}$

With increasing pressure ratio this difference between $\Delta T'$ and $\sum \Delta T'_s$ goes on increasing and thus with increasing pressure ratio $\eta_{\text{isen, t}}$ goes on increasing despite constant stage efficiency.

This may be attributed to the fact that frictional reheating in one stage is partially recovered as work in next stage. Graphical pattern showing behaviour of $\eta_{\text{isen, c}}$ and $\eta_{\text{isen, t}}$ is shown in Fig. 9.26.

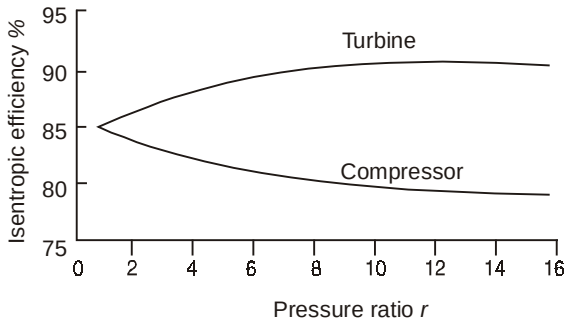


Fig. 9.26 *Isentropic efficiency of compressor and turbine with pressure ratio*

Thus, it is seen that isentropic efficiency of turbine and compressor are not suitable parameters to account for non-idealities in turbines and compressors. Hence the polytropic efficiency, also called small-stage efficiency is used for turbines and compressors. "Polytropic efficiency is defined as the isentropic efficiency of an elemental stage in the compression or expansion process such that it remains constant throughout the whole process." Polytropic efficiency for compressor and turbine may be denoted as $\eta_{\text{poly, c}}$ and $\eta_{\text{poly, t}}$. In an elemental stage let the isentropic temperature rise be dT' while actual temperature rise is dT.

For compressor, $$\eta_{\text{poly, c}} = \frac{dT'}{dT}$$

Isentropic compression processes shall follow; $\dfrac{T}{p^{\gamma-1/\gamma}}$ = constant

Taking log of both sides, $\log T = \left(\dfrac{\gamma-1}{\gamma}\right) \log p + \log (\text{constant})$

Differentiating partially,

$$\frac{dT'}{T} = \left(\frac{\gamma-1}{\gamma}\right)\frac{dp}{p}$$

Substituting dT' from polytropic efficiency expression,

$$\eta_{\text{poly, c}}\ \frac{dT}{T} = \left(\frac{\gamma-1}{\gamma}\right)\frac{dp}{p}$$

Integrating between states 1 and 2;

$$\eta_{\text{poly, c}} \ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}$$

$$\eta_{\text{poly, c}} = \frac{\ln\left(\dfrac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}}{\ln\left(\dfrac{T_2}{T_1}\right)}$$

or $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma\cdot\eta_{\text{poly,c}}}\right)}$$

Let us write $\left(\dfrac{\gamma-1}{\gamma\,\eta_{\text{poly, c}}}\right) = \left(\dfrac{n-1}{n}\right)$ then, $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{(n-1)/n}$

Hence '*n*' is new index of non isentropic compression process and may be called polytropic index. Thus, polytropic efficiency of compressor yields modified index of compression to account for non-isentropic compression.

$$\frac{n-1}{n} = \left(\frac{\gamma - 1}{\gamma \cdot \eta_{\text{poly, c}}} \right)$$

Similarly for polytropic efficiency of expansion in turbine. In elemental stage of turbine the isentropic temperature rise dT' and actual temperature rise dT occurs.

$$\eta_{\text{poly, t}} = \left(\frac{dT}{dT'} \right)$$

Expansion in elemental stage is governed by $\frac{T}{p^{\frac{\gamma-1}{\gamma}}}$ = Constant

Taking log and differentiating partially,

$$\frac{dT'}{T} = \left(\frac{\gamma - 1}{\gamma} \right) \frac{dp}{p}$$

Substituting from polytropic efficiency of turbine,

$$\frac{dT}{\eta_{\text{poly,t}} \cdot T} = \left(\frac{\gamma - 1}{\gamma} \right) \frac{dp}{p}$$

$$\frac{dT}{T} = \left(\frac{(\gamma - 1) \cdot \eta_{\text{poly, t}}}{\gamma} \right) \cdot \frac{dp}{p}$$

Integrating between states 3 and 4

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4} \right)^{\left(\frac{\eta_{\text{poly,t}} \cdot (\gamma - 1)}{\gamma} \right)}$$

Here also the polytropic index for non-isentropic expansion process may be given as,

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4} \right)^{(n-1)/n}$$

$$\boxed{\frac{n-1}{n} = \left(\frac{\eta_{\text{poly,t}} \cdot (\gamma - 1)}{\gamma} \right)}$$

Thus, the polytropic efficiency may be assumed constant over a range of pressure ratio as it takes care of variation of isentropic efficiency with pressure ratio.

9.12 ERICSSON CYCLE

From earlier discussions, we have seen that the thermal efficiency of gas turbine power plant may be increased by regeneration, reheat and intercooling. But there exists limit to the number of stages practically. If a large number of multistages of compression with intercooling and large number of multistages of expansion with reheating and regeneration are employed then the cycle is called Ericsson cycle. Ericsson

cycle is shown below in Fig. 9.27. Here 1-2 is isothermal compression while 2-3 is constant pressure heat addition process. 3-4 is isothermal expansion and 4-1 is constant pressure heat rejection process. 3-4 is isothermal expansion and 4-1 is constant pressure heat rejection process.Heat addition and heat rejection processes may be considered to occur in heat exchanger having 100% effectiveness.

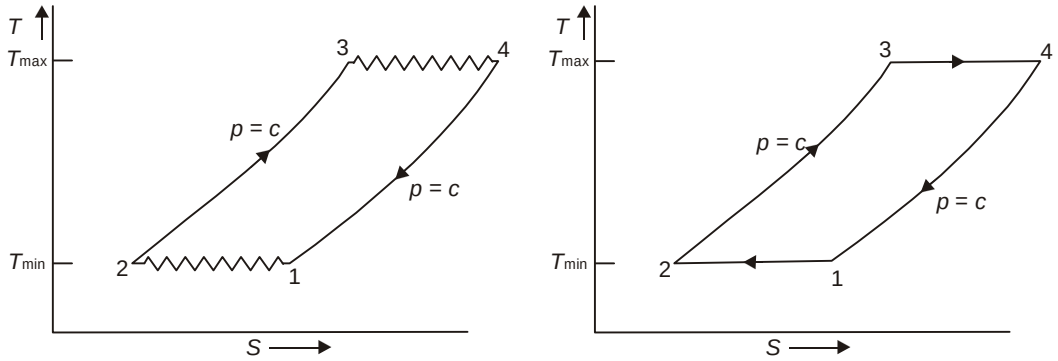


Fig. 9.27 *Ericsson cycle*

The Ericsson engine involves;

(*i*) a turbine through which gas expands isothermally doing work and absorbing heat from an energy reservoir at T_{max}. (process 3-4).

(*ii*) a compressor that compresses gas isothermally while heat is rejected from the gas at T_{min} (process 1-2) to the energy reservoir at T_{min}.

(*iii*) a counter flow heat exchanger which is used as regenerator and the gas coming from compressor is heated in process 2-3 while the gas leaving turbine is cooled in process 4-1. In this type of regenerator the desired heat exchange is possible only when an infinitesimal temperature difference exists between two gas streams at any section and regenerator should operate reversibly. Although such regenerator is not practically possible.

Thus, all processes in Ericsson cycle are reversible processes and cycle approaches to Carnot cycle.

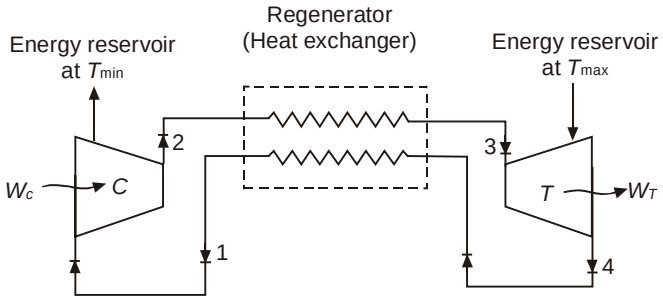


Fig. 9.28 *Schematic arrangement for Ericsson cycle*

$$\text{Heat added, } Q_{add} = RT_{max} \cdot \ln\frac{V_4}{V_3} = RT_{max} \cdot \frac{p_3}{p_4}$$

Heat rejected, $$Q_{\text{rejected}} = -RT_{\min} \cdot \ln \frac{V_2}{V_1} = RT_{\min} \cdot \ln \frac{p_2}{p_1}$$

Here,

$$p_2 = p_3 \,\&\, p_1 = p_4, \text{ so } \frac{p_3}{p_4} = \frac{p_2}{p_1}$$

Also, $$W_{\text{net}} = Q_{\text{add}} - Q_{\text{rejected}}$$

Ericsson cycle efficiency, $$\eta_{\text{Ericsson}} = \frac{Q_{\text{add}} - Q_{\text{rejected}}}{Q_{\text{add}}}$$

Substitution yields,

$$\eta_{\text{Ericsson}} = \frac{T_{\max} - T_{\min}}{T_{\max}}$$

$$\eta_{\text{Ericsson}} = 1 - \frac{T_{\min}}{T_{\max}} = \eta_{\text{Carnot}}$$

9.13 STIRLING CYCLE

Stirling cycle consists of four reversible processes as given below:

(*i*) Isothermal compression from state 1 to 2 at temperature $T_{\min}$.
(*ii*) Constant volume heat addition from state 2 to 3.
(*iii*) Isothermal expansion from state 3 to 4 at temperature $T_{\max}$.
(*iv*) Constant volume heat rejection from state 4 to 1.

Here also similar to Ericsson cycle a regenerator with 100% effectiveness is employed. Regenerator facilitates for the heat rejected during process 4–1 to be used as heat input in process 2-3. Efficiency of Stirling cycle is similar to that of Carnot cycle.

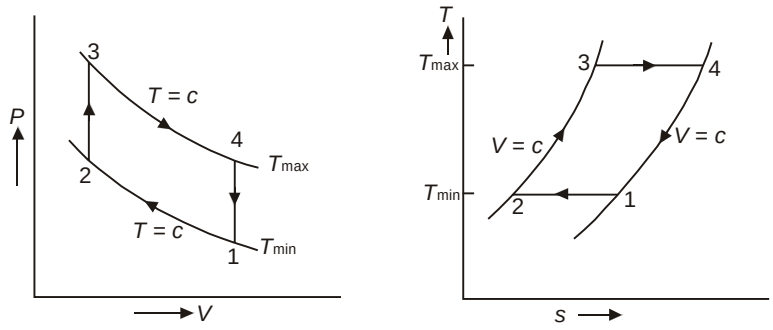


Fig. 9.29 *Stirling cycle*

For a regenerator having 100% effectiveness the heat added in process 2-3 and heat rejected in process 4-1 will be same. If we consider regenerator as part of system then heat interactions with surroundings shall be taking place at temperatures $T_{\min}$ and $T_{\max}$ only. Heat added in cycle will be during process 3–4 at temperature $T_{\max}$ and heat rejected in cycle will be during process 1-2 at temperature $T_{\min}$.

Heat added, $$Q_{add} = RT_{max} \cdot \ln\left(\frac{V_4}{V_3}\right)$$

Heat rejected, $$Q_{rejected} = RT_{min} \cdot \ln\left(\frac{V_1}{V_2}\right)$$

Here, $$V_4 = V_1 \text{ and } V_3 = V_2 \text{ so, } \frac{V_4}{V_3} = \frac{V_1}{V_2} = r$$

Stirling cycle thermal efficiency, $$\eta_{stirling} = \frac{W_{net}}{Q_{add}} = \frac{Q_{add} - Q_{rejected}}{Q_{add}}$$

$$= 1 - \frac{RT_{min} \cdot \ln\left(\frac{V_1}{V_2}\right)}{RT_{max} \cdot \ln\left(\frac{V_4}{V_3}\right)}$$

$$\eta_{stirling} = 1 - \frac{T_{min}}{T_{max}} = \eta_{Carnot}$$

In case of regenerator operating with effectiveness less than 100% the heat added and rejected in processes 2-3 and 4-1 will not be same and some heat would be lost. Let us consider regenerator effectiveness as '$\in$'.

Heat added, $$Q_{add} = RT_{max} \cdot \ln\left(\frac{V_4}{V_3}\right) + (1 - \in) \cdot C_v \left(T_{max} - T_{min}\right)$$

Heat rejected, $$Q_{rejected} = RT_{min} \cdot \ln\left(\frac{V_1}{V_2}\right) + (1 - \in) \cdot C_v \left(T_{max} - T_{min}\right)$$

Modified value of Stirling cycle efficiency, $$\eta'_{stirling} = \frac{R(T_{max} - T_{min}) \cdot \ln r}{R\,T_{max} \cdot \ln r + (1 - \in) C_v (T_{max} - T_{min})}$$

EXAMPLES

1. *A four stroke SI engine has the compression ratio of 6 and swept volume of 0.15 m^3. Pressure and temperature at the beginning of compression are 98 kPa and 60°C. Determine the pressure, volume and temperatures at all salient points if heat supplied to it is 150 kJ/kg. Also find out entropy change, work done, efficiency and mean effective pressure of cycle assuming c_p = 1 kJ/kg · K, c_v = 0.71 kJ/kg · K. Also plot the cycle on T-S diagram.*

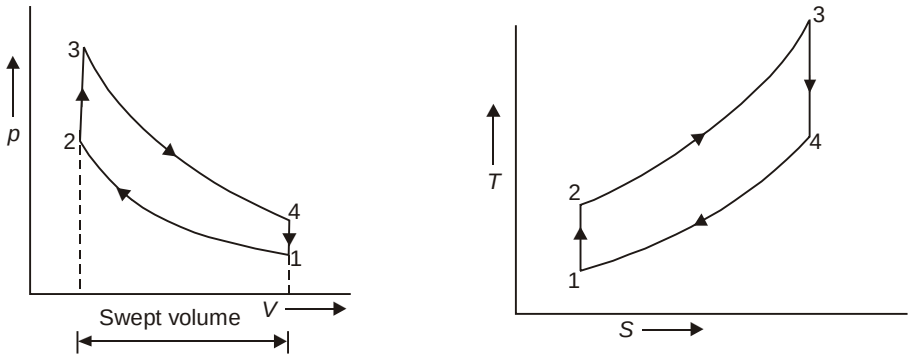


Fig. 9.30

Solution:
SI engines operate on Otto cycle. Consider working fluid to be perfect gas.

Here $\gamma = \dfrac{c_p}{c_v} = 1.4$

$c_p - c_v = R = 0.29$ kJ/kg · K.

Given: $P_1 = 98$ kPa

$T_1 = 60 + 273.15 = 333.15$ K

$Q_{2\text{-}3} = 150$ kJ/kg

Compression ratio, $r = \dfrac{V_1}{V_2} = \dfrac{0.15 + V_2}{V_2} = 6$

Therefore, $V_2 = 0.03$ m^3

Total cylinder volume $= 0.18$ m$^3 = V_1$

From perfect gas law, $PV = mRT$

$$m = \frac{P_1 V_1}{RT_1} = 0.183 \text{ kg.}$$

From state 1 to 2 by PV^{γ} = constant

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \text{ or } P_2 = P_1 \times \left(\frac{V_1}{V_2}\right)^{\gamma} = 1204.03 \text{ kPa}$$

Also $\dfrac{P_1 V_1}{T_1} = \dfrac{P_2 V_2}{T_2}$ yields, $T_2 = 682.18$ K

From heat addition process 2 – 3

$Q_{2\text{-}3} = mc_v (T_3 - T_2)$

$150 = 0.183 \times 0.71 (T_3 - 682.18)$

$T_3 = 1836.65$ K

Also from

$$\frac{P_3V_3}{T_3} = \frac{P_2V_2}{T_2},\ P_3 = 3241.64 \text{ kPa}$$

For adiabatic expansion 3-4, $P_3V_3^{\gamma} = P_4V_4^{\gamma}$

and $V_4 = V_1$

Hence $$P_4 = \frac{P_3 \cdot V_3^{\gamma}}{V_1^{\gamma}} = 263.85 \text{ kPa}$$

and from $$\frac{P_3V_3}{T_3} = \frac{P_4V_4}{T_4};\ T_4 = 896.95 \text{ K}$$

Entropy change from 2-3 and 4-1 are same, and can be given as,

$$S_3 - S_2 = S_4 - S_1 = mc_v \ln\frac{T_4}{T_1} = 0.183 \times 0.71 \times \ln\left(\frac{896.95}{333.15}\right)$$

Entropy change $\Delta S_{3\text{-}2} = \Delta S_{4\text{-}1} = 0.1287$ kJ/K.

Heat rejected, $Q_{4\text{-}1} = mc_v \times (T_4 - T_1)$

$= 0.183 \times 0.71\ (896.95 - 333.15)$

$Q_{4-1} = 73.25$ kJ

Net Work $= (Q_{2\text{-}3}) - (Q_{4\text{-}1}) = 150 - 73.25$

Net Work $= 76.75$ kJ

$$\text{Efficiency} = \frac{\text{Net work}}{\text{Heat added}}$$

$$= \frac{76.75}{150} = 0.5117 \text{ or } \boxed{\eta = 51.17\%}$$

$$\text{Mean effective pressure} = \frac{\text{Work}}{\text{Volume change}} = \frac{76.75}{0.15} = 511.67 \text{ kPa}$$

$\boxed{\text{m.e.p.} = \textbf{511.67 kPa}}$ **Ans.**

2. *In a Diesel engine during the compression process, pressure is seen to be 138 kPa at* $\frac{1}{8}$ *th of stroke and 1.38 MPa at* $\frac{7}{8}$ *th of stroke. The cut-off occurs at* $\frac{1}{15}$ *th of stroke. Calculate air standard efficiency and compression ratio assuming indicated thermal efficiency to be half of ideal efficiency, mechanical efficiency as 0.8, calorific value of fuel = 41800 kJ/kg and* $\gamma = 1.4$. *Also find fuel consumption bhp/hr.*

Solution:

As given

$$V_A = V_2 + \frac{7}{8}(V_1 - V_2)$$

$$V_B = V_2 + \frac{1}{8}(V_1 - V_2)$$

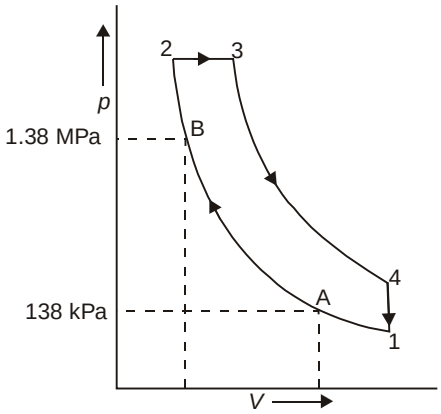


Fig. 9.31

and also $$p_A \cdot V_A^{\gamma} = p_B \cdot V_B^{\gamma}$$

or $$\left(\frac{V_A}{V_B}\right) = \left(\frac{P_B}{P_A}\right)^{1/\gamma}$$

$$= \left(\frac{1380}{138}\right)^{1/1.4}$$

$$\frac{V_A}{V_B} = 5.18$$

Also substituting for V_A & V_B

$$\frac{V_2 + \frac{7}{8}(V_1 - V_2)}{V_2 + \frac{1}{8}(V_1 - V_2)} = 5.18$$

It gives $$r = 19.37$$

$$\frac{V_1}{V_2} = 19.37, \text{ Compression ratio} = 19.37$$

As given; Cut off occurs at $\left\{\frac{V_1 - V_2}{15}\right\}$ volume

or $$V_3 = \left\{\frac{V_1 - V_2}{15}\right\} + V_2$$

Cut off ratio, $$\rho = \frac{V_3}{V_2} = 2.22$$

$$\text{Air standard efficiency for Diesel cycle} = 1 - \frac{1}{r^{\gamma-1}\gamma}\left[\frac{\rho^{\gamma}-1}{\rho-1}\right]$$

$$= 0.6325$$

$$\eta_{\text{air standard}} = 63.25\%$$

$$\text{Overall efficiency} = \text{air standard efficiency} \times 0.5 \times 0.8$$

$$= 0.6325 \times 0.5 \times 0.8$$

$$= 0.253 \text{ or } 25.3\%$$

$$\text{Fuel consumption, } bhp/hr = \frac{75\times60\times60}{0.253\times41800\times10^2}$$

$$= 0.255 \text{ kg}$$

Compression ratio = **19.37**
Air standard efficiency = **63.25%**
Fuel consumption, *bhp/hr* = **0.255 kg** **Ans.**

3. *In an IC engine using air as working fluid, total 1700 kJ/kg of heat is added during combustion and maximum pressure in cylinder does not exceed 5 MPa. Compare the efficiency of following two cycles used by engine:*

(a) cycle in which combustion takes place isochorically.

(b) cycle in which half of heat is added at constant volume and half at constant pressure.

Temperature and pressure at the beginning of compression are 100°C and 103 kPa. Compression and expansion processes are adiabatic. Specific heat at constant pressure and volume are 1.003 kJ/kg · K and 0.71 kJ/kg · K.

Solution:

1-2-3-4 = cycle (*a*)

1-2′-3′-4′-5 = cycle (*b*)

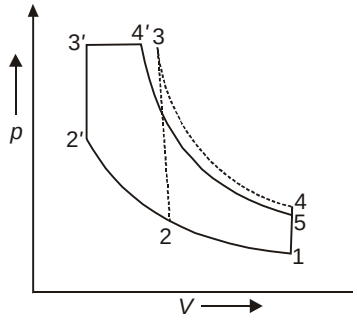


Fig. 9.32

Here $\frac{c_p}{c_v} = \gamma = 1.4$

and $R = 0.293$ kJ/kg · K

Let us consider 1 kg of air for perfect gas,

$$PV = mRT$$

or $$V_1 = \frac{mRT_1}{p_1}$$

$$= \frac{1\times0.293\times373.15}{103}$$

$$V_1 = 1.06 \text{ m}^3$$

at state 3, $p_3V_3 = mRT_3$ or $T_3 = \frac{5000\times V_3}{1\times0.293} = 17064.8\ V_2$

for cycle (*a*) and also for cycle (*b*)

$$T'_3 = 17064.8\ V'_2$$

(a) For Otto cycle

$$Q_{23} = c_v(T_3 - T_2); \text{ given } Q_{23} = 1700 \text{ kJ/kg}$$

$$1700 = 0.71(T_3 - T_2)$$

or $$T_3 - T_2 = 2328.77, \text{ or } T_2 = T_3 - 2328.77$$

From gas law

$$\frac{p_2 V_2}{T_2} = \frac{p_3 V_3}{T_3}$$

$$\frac{p_2 \cdot V_2}{(T_3 - 2328.77)} = \frac{5000 \times V_2}{T_3}, \{\text{as } V_2 = V_3\}$$

or $$\frac{p_2}{(T_3 - 2328.77)} = \frac{5000}{T_3}$$

Substituting T_3 as function of V_2

$$\frac{p_2}{(17064.8V_2 - 2328.77)} = \frac{5000}{(17064.8V_2)}$$

or $$p_2 = \frac{5000(17064.8V_2 - 2328.77)}{(17064.8V_2)}$$

Also $$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$$

or $$103 \times (1.06)^{1.4} = \frac{5000(17064.8V_2 - 2328.77)}{(17064.8V_2)} \cdot (V_2^{1.4})$$

upon solving it yields

$$381.4V_2 = 17064.8\ V_2^{2.4} - 2328.77\ V_2^{1.4}$$

$$V_2 = 44.7\,V_2^{2.4} - 6.1\,V_2^{1.4}$$

$$44.7\,V_2^{1.4} - 6.1\,V_2^{0.4} - 1 = 0$$

or $$V_2^{1.4} - 0.136\,V_2^{0.4} - 0.022 = 0$$

By hit and trial it yields $$V_2 = 0.18 \text{ m}^3$$

Thus, compression ratio $$r = \frac{V_1}{V_2} = \frac{1.06}{0.18} = 5.89$$

Otto cycle efficiency, $$\eta_{\text{otto}} = 1 - \frac{1}{r^{\gamma - 1}} = 0.5083$$

or $$\eta_{\text{otto}} = 50.83\%$$

(b) For mixed or dual cycle

Given:

$$c_p\,(T_{4'} - T_{3'}) = c_v\,(T_{3'} - T_{2'}) = \frac{1700}{2} = 850$$

or $T_{3'} - T_{2'} = 1197.2$

or $T_{2'} = T_{3'} - 1197.2$

Also $\dfrac{P_{2'}V_{2'}}{T_{2'}} = \dfrac{P_{3'}V_{3'}}{T_{3'}}$

$\Rightarrow$ $\dfrac{P_{2'}V_{2'}}{(T_{3'} - 1197.2)} = \dfrac{5000 \cdot V_{2'}}{T_{3'}}$

or $\dfrac{P_{2'}}{(T_{3'} - 1197.2)} = \dfrac{5000}{T_{3'}}$

Also we had seen earlier that $T_{3'} = 17064.8\ V_{2'}$

$$\frac{P_{2'}}{(17064.8V_{2'} - 1197.2)} = \frac{5000}{(17064.8\,V_{2'})}$$

or $$P_{2'} = \frac{5000(17064.8\,V_{2'} - 1197.2)}{(17064.8\,V_{2'})}$$

For adiabatic process 1-2′

$$P_1V_1^{\gamma} = P_{2'}V_{2'}^{\gamma}$$

or $$103 \times (1.06)^{1.4} = (V_{2'})^{1.4} \cdot \left(5000 - \frac{350.78}{V_{2'}}\right)$$

$$111.75\ V_{2'} = 5000\ V_{2'}^{2.4} - 350.78\ V_{2'}^{1.4}$$

$$1 = 44.74\,V_{2'}^{1.4} - 3.14\,V_{2'}^{0.4}$$

or $$V_{2'}^{1.4} - 0.07\,V_{2'}^{0.4} - 0.022 = 0$$

By hit and trial

$$V_{2'} = 0.122\ \text{m}^3$$

Therefore upon substituting $V_{2'}$

$$P_{2'} = 2124.75\ \text{kPa}$$

$$T_{3'} = 2082\ \text{K}$$

$$T_{2'} = 884.8\ \text{K}$$

From constant pressure heat addition

$$c_p\,(T_{4'} - T_{3'}) = 850$$

$$1.003\,(T_{4'} - 2082) = 850$$

$$T_{4'} = 2929.5\ \text{K}$$

Also $\dfrac{P_{4'}\,V_{4'}}{T_{4'}} = \dfrac{P_{3'}\,V_{3'}}{T_{3'}}$ {here $p_{3'} = p_{4'}$ and $V_{2'} = V_{3'}$}

or $$V_{4'} = \frac{0.122 \times 2929.5}{2082}$$

$$V_{4'} = 0.172\ \text{m}^3$$

Using adiabatic formulations

$$\left(\frac{V_5}{V_{4'}}\right)^{\gamma-1} = \left(\frac{T_{4'}}{T_5}\right) \{\text{here } V_5 = V_1\}$$

or $$T_5 = 2929.5 \times \left(\frac{0.172}{1.06}\right)^{0.4}$$

$$T_5 = 1415.4 \text{ K}$$

Heat rejected in process 5 – 1, $Q_{5-1} = c_v (T_5 - T_1)$

$$= 0.71 (1415.4 - 373.15)$$

$$Q_{5-1} = 739.99 \text{ kJ}$$

Efficiency of mixed cycle = $\frac{1700 - 739.99}{1700}$ = 0.5647 or 56.47%

$$\boxed{\eta_{\text{Otto cycle}} = \mathbf{50.83\%};\ \eta_{\text{mixed cycle}} = \mathbf{56.47\%}} \quad \textbf{Ans.}$$

4. *In an air standard Brayton cycle the minimum and maximum temperature are 300 K and 1200 K, respectively. The pressure ratio is that which maximizes the net work developed by the cycle per unit mass of air flow. Calculate the compressor and turbine work, each in kJ/kg air, and thermal efficiency of the cycle.* *[U.P.S.C. 1994]*

Solution:

Maximum temperature, $T_3 = 1200$ K

Minimum temperature, $T_1 = 300$ K

Optimum pressure ratio for maximum work output,

$$r_p = \left(\frac{T_{\max}}{T_{\min}}\right)^{\frac{\gamma}{2(\gamma-1)}}$$

$$r_p = \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)}}$$

$$r_p = 11.3$$

$$\frac{p_2}{p_1} = r_p = 11.3, \text{ For process 1–2, } \frac{T_2}{T_1} = \left(\frac{P_2}{T_1}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}$$

$$T_2 = 300 \cdot (11.3)^{\frac{1.4-1}{1.4}}$$

$$T_2 = 600.21 \text{ K}$$

For process 3–4,

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}, \Rightarrow T_4 = 599.79 \text{ K}$$

Heat supplied, $Q_{23} = c_p\,(T_3 - T_2) = 602.79$ kJ/kg

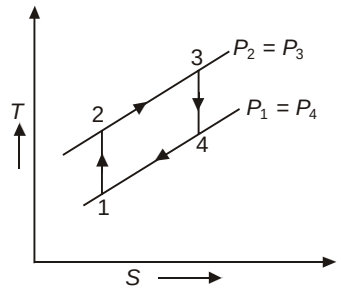


Fig. 9.33

Compressor work, $W_C = c_p \cdot (T_2 - T_1) = 1.005 \times (600.21 - 300) = 301.71$ kJ/kg

Turbine work $W_T = c_p \cdot (T_3 - T_4) = 1.005\,(1200 - 599.79) = 603.21$ kJ/kg

Thermal efficiency $= \dfrac{\text{Net work}}{\text{Heat supplied}} = \dfrac{W_T - W_c}{Q_{23}} = \dfrac{603.21 - 301.71}{602.79} = 0.5002$

Compressor work = **301.71 kJ/kg,** Turbine work = **603.21 kJ/kg,**
Thermal efficiency = **50.02%** **Ans.**

5. *A gas turbine unit receives air at 1 bar, 300 K and compresses it adiabatically to 6.2 bar. The compressor efficiency is 88%. The fuel has a heating value of 44186 kJ/kg and the fuel-air ratio is 0.017 kg fuel/kg of air. The turbine internal efficiency is 90%. Calculate the work of turbine and compressor per kg of air compressed and thermal efficiency. For products of combustion $c_p = 1.147$ kJ/kg K, $\gamma = 1.33$.* *[U.P.S.C. 1992]*

Solution:

Gas turbine cycle is shown by 1–2–3–4 on T-S diagram,

Given:

$P_1 = 1$ bar, $P_2 = P_3 = 6.2$ bar, F/A ratio = 0.017

$T_1 = 300$ K, $\eta_{\text{compr.}} = 88\%$, $\eta_{\text{turb}} = 90\%$

Heating value of fuel = 44186 kJ/kg

For process 1–2 being isentropic,

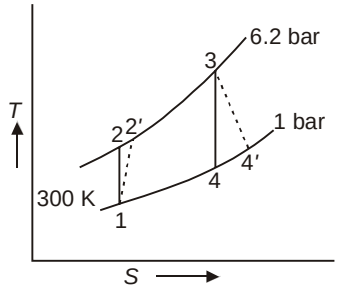


Fig. 9.34

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = 505.26 \text{ K}$$

Considering compressor efficiency, $\eta_{\text{compr}} = \dfrac{T_2 - T_1}{T_{2'} - T_1}$, $0.88 = \dfrac{(505.26 - 300)}{(T_{2'} - 300)}$

Actual temperature after compression, $T'_2 = 533.25$ K

During process 2–3 due to combustion of unit mass of air compressed the energy balance shall be as under,

Heat added = $m_f \times$ Heating value

$$= ((m_a + m_f) \cdot c_{p,\,\text{comb}} \cdot T_3) - (m_a \cdot c_{p,\,\text{air}} \cdot T_{2'})$$

or $$\left(\frac{m_f}{m_a}\right) \times 44186 = \left(\left(1+\frac{m_f}{m_a}\right) c_{p,\,\text{comb}}\, T_3\right) - (c_{p,\,\text{air}} \times 533.25)$$

Here, $\dfrac{m_f}{m_a} = 0.017$, $c_{p,\,\text{comb}} = 1.147$ kJ/kg · K, $c_{p,\,\text{air}} = 1.005$ kJ/kg · K

Upon substitution

$$(0.017 \times 44186) = ((1 + 0.017) \times 1.147 \times T_3) - (1.005 \times 533.25)$$

$$T_3 = 1103.37 \text{ K}$$

For expansion 3–4 being

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{n-1}{n}}$$

$$T_4 = 1103.37 \times \left(\frac{1}{6.2}\right)^{\frac{0.33}{1.33}}$$

$$T_4 = 701.64 \text{ K}$$

Actual temperature at turbine inlet considering internal efficiency of turbine,

$$\eta_{\text{turb}} = \frac{T_3 - T_{4'}}{T_3 - T_4};\ 0.90 = \frac{(1103.37 - T_{4'})}{(1103.37 - 701.64)}$$

$$T_{4'} = 741.81 \text{ K}$$

Compressor work, per kg of air compressed = $c_{p,\,\text{air}} \cdot (T_{2'} - T_1)$

$$W_C = 1.005 \times (533.25 - 300)$$

$$W_C = 234.42 \text{ kJ/kg or air}$$

Compressor work = **234.42 kJ/kg of air** **Ans.**

Turbine work, per kg of air compressed = $c_{p,\,\text{comb}} \cdot (T_3 - T_{4'})$

$$= 1.147 \times (1103.37 - 741.81)$$

Turbine work = **414.71 kJ/kg of air** **Ans.**

$$W_T = 414.71, \text{ kJ/kg of air}$$

$$\text{Net work} = W_T - W_C = (414.71 - 234.42)$$

$$W_{\text{net}} = 180.29, \text{ kJ/kg of air}$$

$$\text{Heat supplied} = 0.017 \times 44186 = 751.162 \text{ kJ/kg of air.}$$

$$\text{Thermal efficiency} = \frac{W_{\text{net}}}{\text{Heat supplied}} = \frac{180.29}{751.162} = 24\%$$

Thermal efficiency = **24%** **Ans.**

6. *In a Brayton cycle gas turbine power plant the minimum and maximum temperature of the cycle are 300 K and 1200 K. The compression is carried out in two stages of equal pressure ratio with intercooling*

of the working fluid to the minimum temperature of the cycle after the first-stage of compression. The entire expansion is carried out in one stage only. The isentropic efficiency of both compressors is 0.85 and that of the turbine is 0.9. Determine the overall pressure ratio that would give the maximum net work per kg working fluid. Take $\gamma = 1.4$. *[U.P.S.C. 1995]*

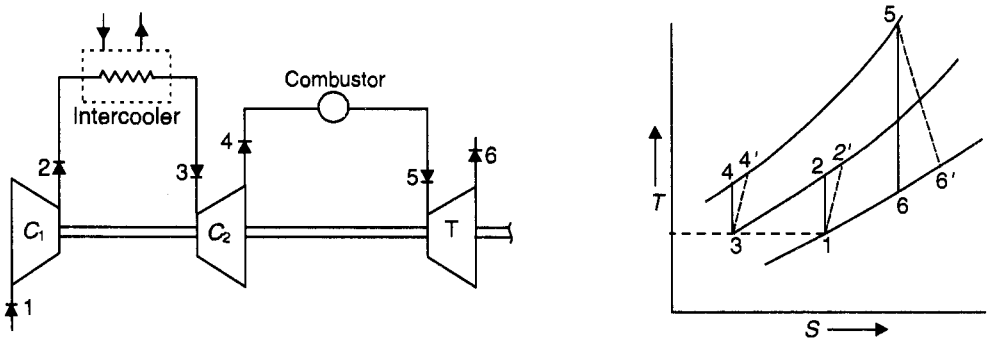


Fig. 9.35 *Arrangement and T-S representation*

Given: $T_5 = 1200$ K, $T_1 = 300$ K, $\dfrac{P_2}{P_1} = \dfrac{P_4}{P_3}$, $T_3 = 300$ K, $\eta_{\text{isen, c}} = 0.85$

$$\eta_{\text{isen, T}} = 0.9, \text{ As } \frac{P_2}{P_1} = \frac{P_4}{P_3}, \text{ so } \frac{P_4}{P_3} = \frac{P_2}{P_1} = \sqrt{\frac{P_4}{P_1}} = \sqrt{r_p}$$

For process, 1–2,
$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_2}{T_1} = \left(r_p\right)^{\frac{\gamma-1}{2\gamma}}$$

For process 3–4,
$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(r_p\right)^{\frac{\gamma-1}{2\gamma}}$$

$$\frac{T_4}{T_1} = \left(r_p\right)^{\frac{\gamma-1}{2\gamma}}$$

For process 5–6,
$$\frac{T_5}{T_6} = \left(\frac{P_4}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_5}{T_6} = \left(r_p\right)^{\frac{\gamma-1}{\gamma}} \text{ or, } T_6 = \frac{T_5}{\left(r_p\right)^{\frac{\gamma-1}{\gamma}}}$$

Compressor work, $W_C = W_{C1} + W_{C2}$

$= c_p (T_{2'} - T_1) + c_p(T_{4'} - T_3)$

$= c_p (T_{2'} - T_1) + c_p(T_{4'} - T_1)$

Using Isentropic efficiency of compressor,

$$\eta_{\text{isen, c}} = \frac{T_2 - T_1}{T_{2'} - T_1} = \frac{T_4 - T_3}{T_{4'} - T_3} = \frac{T_4 - T_1}{T_{4'} - T_1}$$

Thus,

$$W_C = c_p \cdot \frac{(T_2 - T_1)}{\eta_{\text{isen, c}}} + c_p \frac{(T_4 - T_1)}{\eta_{\text{isen, c}}}$$

$$= \frac{c_p}{\eta_{\text{isen, c}}} \left\{ \left(r_p^{\frac{(\gamma-1)}{2\gamma}} \cdot T_1 - T_1 \right) + \left(r_p^{\frac{(\gamma-1)}{2\gamma}} \cdot T_1 - T_1 \right) \right\}$$

$$W_C = \frac{2c_p \cdot T_1 \left(r_p^{\frac{(\gamma-1)}{2\gamma}} - 1 \right)}{\eta_{\text{isen, c}}}$$

Turbine work, $W_T = c_p \cdot (T_5 - T_{6'})$

Using isentropic efficiency of turbine, $\eta_{\text{isen, T}} = \frac{T_5 - T_{6'}}{T_5 - T_6}$

$$W_T = c_p \cdot (T_5 - T_6)\ \eta_{\text{isen, T}} = c_p \cdot T_5 \cdot \eta_{\text{isen, T}} \left(1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} \right)$$

Net work $W_{\text{net}} = W_T - T_C$

$$= c_p \cdot T_5 \cdot \eta_{\text{isen, T}} \cdot \left(1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} \right) - \frac{2c_p \cdot T_1 \left(r_p^{\frac{\gamma-1}{2\gamma}} - 1 \right)}{\eta_{\text{isen, c}}}$$

Differentiating new work with respect to r_p.

$$\frac{d\, W_{\text{net}}}{d\, r_p} = c_p \cdot T_5 \cdot \eta_{\text{isen, T}} \cdot \left(\frac{\gamma - 1}{\gamma} \right) \cdot r_p^{\frac{1-2\gamma}{\gamma}} - \frac{2c_p \cdot T_1}{\eta_{\text{isen, c}}} \cdot \left(\frac{\gamma - 1}{2\gamma} \right) \cdot r_p^{\frac{-1-\gamma}{2\gamma}}$$

Putting, $\frac{d\, W_{\text{net}}}{d\, r_p} = 0$

$$c_p \cdot T_5 \cdot \eta_{\text{isen, T}}\ r_p^{\frac{1-2\gamma}{\gamma}} = \frac{c_p \cdot T_1}{\eta_{\text{isen, c}}} \cdot r_p^{\frac{(-1-\gamma)}{2\gamma}}$$

$$r_p^{\frac{3(1-\gamma)}{2\gamma}} = \frac{T_1}{T_5 \cdot \eta_{\text{isen, c}}\ \eta_{\text{isen, T}}} \Rightarrow r_p = \left(\frac{T_1}{T_5 \cdot \eta_{\text{isen, c}}.\ \eta_{\text{isen, T}}} \right)^{\frac{2\gamma}{3(1-\gamma)}}$$

$$r_{p,\ opt} = \left(\frac{T_1}{T_5 \cdot \eta_{\text{isen, c}}\ \eta_{\text{isen, T}}} \right)^{\frac{2\gamma}{3(1-\gamma)}}$$

Substituting known values,

$$r_{p,\ opt} = \left(\frac{300}{1200 \times 0.85 \times 0.90} \right)^{\frac{2\times 1.4}{3(1-1.4)}} = 13.6$$

Overall optimum pressure ratio = **13.6** **Ans.**

7. *An air compressor has eight stages of equal pressure 1.35. The flow rate through the compressor and its' overall efficiency are 50 kg/s and 82% respectively. If the air enters the compressor at a pressure of 1 bar and temperature of 313 K, determine;*

(i) State of air at the exit of compressor
(ii) Polytropic or small stage efficiency
(iii) Efficiency of each stage
(iv) Power required to drive the compressor assuming overall efficiency as 90%.

[U.P.S.C. 1992]

Solution:

(*i*) Theoretically state of air at exit can be determined by the given stage pressure ratio of 1.35. Let pressure at inlet to first stage be P_1 and subsequent intermediate pressure be P_2, P_3, P_4, P_5, P_6, P_7, P_8, and exit pressure being P_9.

Therefore,

$$\frac{P_2}{P_1} = \frac{P_3}{P_2} = \frac{P_4}{P_3} = \frac{P_5}{P_4} = \frac{P_6}{P_5} = \frac{P_7}{P_6} = \frac{P_8}{P_7} = \frac{P_9}{P_8} = 1.35$$

or
$$\frac{P_9}{P_1} = (1.35)^8 = 11.03$$

Theoretically, the temperature at exit of compressor can be predicted considering isentropic compression of air ($\gamma = 1.4$)

$$\frac{T_9}{T_1} = \left(\frac{P_9}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = (11.03)^{\frac{0.4}{1.4}}$$

$$T_9 = 621.9 \text{ K}$$

Considering overall efficiency of compression 82% the actual temperature at compressor exit can be obtained

$$\frac{T_9 - T_1}{T_{9,\text{ actual}} - T_1} = 0.82$$

$$T_{9,\ \text{actual}} = 689.71\ \text{K}$$

Let the actual index of compression be 'n', then

$$\left(\frac{T_{9,\text{actual}}}{T_1}\right) = \left(\frac{P_9}{P_1}\right)^{\frac{n-1}{n}}$$

$$\left(\frac{689.71}{313}\right) = (11.03)^{\frac{n-1}{n}}$$

$$n = 1.49$$

State of air at exit of compressor, Pressure = **11.03 bar**
Temperature = **689.71 K** **Ans.**

(*ii*) Let polytropic efficiency be $\eta_{\text{polytropic}}$ for compressor, then;

$$\frac{n-1}{n} = \left(\frac{\gamma-1}{\gamma}\right)\times\frac{1}{\eta_{\text{polytropic}}}$$

$$\frac{(1.49-1)}{1.49} = \left(\frac{1.4-1}{1.4}\right)\times\frac{1}{\eta_{\text{polytropic}}}$$

$$\eta_{\text{polytropic}} = 0.8688 \text{ or } 86.88\%$$

Polytropic efficiency = **86.88%** **Ans.**

(*iii*) Stage efficiency can be estimated for any stage. Say first stage.

Ideal state at exit of compressor stage $\Rightarrow \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$

$$T_2 = 341.05\ \text{K}$$

Actual temperature at exit of first stage can be estimated using polytropic index 1.49.

$$\frac{T_{2,\text{actual}}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$T_{2,\ \text{actual}} = 345.47\ \text{K}$$

Stage efficiency for first stage, $\eta_{s,\ 1} = \dfrac{(T_2 - T_1)}{(T_{2,\text{actual}} - T_1)}$

$$= \frac{(341.05-313)}{(345.47-313)} = 0.8638 \text{ or } 86.38\%$$

Actual temperature at exit of second stage,

$$\frac{T_{3,\text{actual}}}{T_{2,\text{actual}}} = \left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}}$$

$$T_{3,\ \text{actual}} = 381.30 \text{ K}$$

Ideal temperature at exit of second stage

$$\frac{T_3}{T_{2,\ \text{actual}}} = \left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_3 = 376.43 \text{ K}$$

Stage efficiency for second stage, $\eta_{s,2} = \dfrac{T_3 - T_{2,\ \text{actual}}}{T_{3,\ \text{actual}} - T_{2,\ \text{actual}}} = \dfrac{376.43 - 345.47}{381.3 - 345.47}$

$$\eta_{s,2} = 0.8641 \text{ or } 86.41\%$$

Actual temperature at exit of third stage,

$$\frac{T_{4,\ \text{actual}}}{T_{3,\ \text{actual}}} = \left(\frac{P_4}{P_3}\right)^{\frac{n-1}{n}}$$

$$T_{4,\ \text{actual}} = 420.85 \text{ K}$$

Ideal temperature at exit of third stage,

$$\frac{T_4}{T_{3,\ \text{actual}}} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}}, \quad T_4 = 415.47 \text{ K}$$

Stage efficiency for third stage, $\eta_{s,\ 3} = \dfrac{T_4 - T_{3,\ \text{actual}}}{T_{4,\ \text{actual}} - T_{3,\ \text{actual}}} = \dfrac{(415.47 - 381.3)}{(420.85 - 381.3)}$

$$= 0.86396 \text{ or } 86.40\%$$

Stage efficiency = **86.4%** **Ans.**

(*iv*) From steady flow energy equation,

$$W_C = \int_1^9 dw = \int_1^9 dh, \text{ and } dh = du + pdv + vdp$$

$$dh = dq + vdp$$

for $dq = 0$ in adiabatic process

$$dh = vdp$$

$$W_C = \int_1^9 vdp$$

Here for polytropic compression

$$Pv^{1.49} = \text{constant i.e. } n = 1.49$$

$$W_C = \left(\frac{n}{n-1}\right) mRT_1 \left[\left(\frac{P_9}{T_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \left(\frac{1.49}{0.49}\right) \times 50 \times 0.287 \times 313 \left[\left(\frac{11.03}{1.0}\right)^{\frac{1.49-1}{1.49}} - 1\right]$$

$$W_C = 16419.87 \text{ kJ/s}$$

Due to overall efficiency being 90% the actual compressor work = $\frac{W_c}{0.90}$

$$W_{C,\text{ actual}} = 18244.30 \text{ kJ/s}$$

Power required to drive compressor = **18244.30 kJ/s** **Ans.**

8. *A thermodynamic cycle has following processes occurring sequentially;*

- *Adiabatic compression from 1 to 2*
- *Constant volume heat addition from 2 to 3 (explosion)*
- *Adiabatic expansion from 3 to 4*
- *Isothermal heat rejection from 4 to 1*

Obtain expression for air standard cycle efficiency considering compression ratio of 'r' and expansion ratio of 'e'.

Solution:

Let us analyze for 1 kg of air,

Given,

$$\frac{V_1}{V_2} = r, \frac{V_4}{V_3} = e; V_2 = V_3, T_1 = T_4$$

$$\frac{V_1}{V_4} = \frac{r}{e}, \text{ For process 4–1, } P_1V_1 = P_4V_4 = RT_1$$

$$\text{Heat added} = c_v(T_3 - T_2)$$

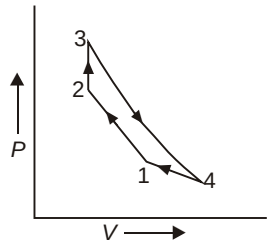


Fig. 9.36

Heat rejected in isothermal process, 4–1 $= P_4V_4 \ln\left(\frac{V_4}{V_1}\right)$

$$\text{Net work} = \text{Heat added} - \text{Heat rejected}$$

$$W_{\text{net}} = c_v(T_3 - T_2) - P_4V_4 \ln\left(\frac{e}{r}\right)$$

$$\text{Efficiency, } \eta = \frac{\text{Net work}}{\text{Heat added}} = \frac{c_v(T_3 - T_2) - P_4V_4 \ln\left(\frac{e}{r}\right)}{c_v(T_3 - T_2)}$$

$$\eta = 1 - \frac{RT_1 \ln\left(\frac{e}{r}\right)}{c_v(T_3 - T_2)}$$

As, $c_v = \frac{R}{(\gamma - 1)}$, substituting in expression for efficiency,

$$\eta = 1 - \frac{(\gamma - 1)\cdot T_1 \ln\left(\frac{e}{r}\right)}{(T_3 - T_2)}$$

$$\eta = 1 - \frac{(\gamma - 1)\ln\left(\frac{e}{r}\right)}{\left(\frac{T_3}{T_1} - \frac{T_2}{T_1}\right)} = 1 - \frac{(\gamma - 1)\ln\left(\frac{e}{r}\right)}{\left(\frac{T_3}{T_4} - \frac{T_2}{T_4}\right)}$$

For process 3–4, $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = e^{\gamma-1}$

For process 1–2, $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow \frac{T_2}{T_4} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r^{\gamma-1}$

Substituting in expression of η

$$\boxed{\eta = 1 - \frac{(\gamma - 1)\ln\left(\frac{e}{r}\right)}{(e^{\gamma-1} - r^{\gamma-1})}}$$ **Ans.**

9. *In a gas turbine installation air is supplied at 1 bar, 27°C into compressor having compression ratio of 8. The air leaving combustion chamber is heated upto 1100 K and expanded upto 1 bar. A heat exchanger having effectiveness of 0.8 is fitted at exit of turbine for heating the air before its inlet into combustion chamber. Assuming polytropic efficiency of the compressor and turbine as 0.85 and 0.90 determine cycle efficiency, work ratio and specific work output of plant. Take c_p = 1.0032 kJ/kg ·K for air.*

Solution:

γ = 1.4, $\eta_{\text{poly, c}}$ = 0.85, $\eta_{\text{poly, T}}$ = 0.90

Using polytropic efficiency the index of compression and expansion can be obtained as under,

Let compression index be n_c

$$\left(\frac{n_c - 1}{n_c}\right) = \left(\frac{\gamma - 1}{\gamma \cdot \eta_{\text{poly, c}}}\right)$$

$$\eta_c = 1.506$$

Let expansion index be n_T.

$$\left(\frac{n_T - 1}{n_T}\right) = \left(\frac{\eta_{\text{poly,T}} \cdot (\gamma - 1)}{\gamma}\right)$$

$$\Rightarrow \quad n_T = 1.346$$

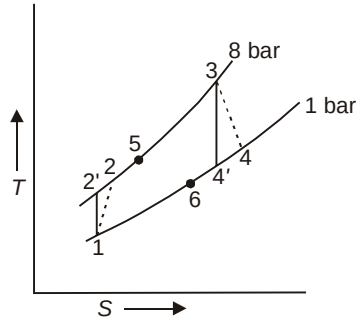


Fig. 9.37 *T-S representation*

For process 1–2

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n_c-1}{n_c}}$$

$$\frac{T_2}{300} = (8)^{\frac{1.506-1}{1.506}}$$

$$T_2 = 603.32 \text{ K}$$

$$T_3 = 1100 \text{ K}$$

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{n_T-1}{n_T}}$$

$$\frac{T_4}{1100} = \left(\frac{1}{8}\right)^{\frac{1.346-1}{1.346}}$$

$$T_4 = 644.53 \text{ K}$$

HE = Heat exchanger
C = Compressor
CC = Combustion chamber
T = Turbine

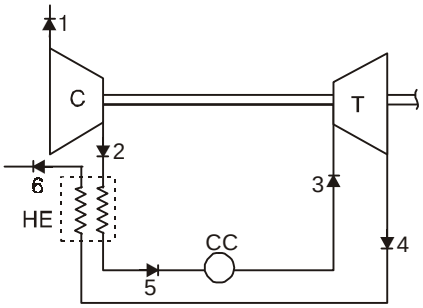


Fig. 9.38 *Layout of regenerative cycle*

Using heat exchanger effectiveness,

$$\varepsilon = 0.8 = \frac{T_5 - T_2}{T_4 - T_2}$$

$$0.8 = \frac{T_5 - 603.32}{(644.53 - 603.32)}$$

$$T_5 = 636.28 \text{ K}$$

Heat added in combustion chambers, $q_{\text{add}} = c_p (T_3 - T_5)$

$$q_{\text{add}} = 1.0032 (1100 - 636.28)$$

$$q_{\text{add}} = 465.204 \text{ kJ/kg}$$

Compressor work $W_C = c_p (T_2 - T_1) = 1.0032 (603.32 - 300) = 304.29$ kJ/kg

Turbine work, $W_T = c_p (T_3 - T_4) = 1.0032 (1100 - 644.53) = 456.93$ kJ/kg

$$\text{Cycle efficiency} = \frac{W_T - W_C}{q_{\text{add}}} = \frac{456.93 - 304.29}{465.204} = 0.3281 \text{ or } 32.81\%$$

$$\text{Work ratio, } \frac{W_T - W_C}{W_T} = 0.334$$

Specific work output $= W_T - W_C = 152.64$ kJ/kg

Cycle efficiency = **32.81%,** Work ratio = **0.334,** **Ans.**
Specific work output = **152.64 kJ/kg**

10. *A gas turbine plant has air being supplied at 1 bar, 27ºC to compressor for getting compressed upto 5 bar with isentropic efficiency of 85%. Compressed air is heated upto 1000 K in combustion chamber where also occurs a pressure drop of 0.2 bar. Subsequently expansion occurs to 1 bar in turbine. Determine isentropic efficiency of turbine, if thermal efficiency of plant is 20%.*

Neglect the air property variation throughout cycle. Take γ = 1.4

Solution:

T_1 = 300 K, P_1 = 1 bar, P_2 = 5 bar, η_c = 0.85, T_3 = 1000 K

For process 1–2′

$$\frac{T_{2'}}{T_1} = \left(\frac{P_{2'}}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_{2'} = 300\ (5)^{\frac{(1.4-1)}{1.4}}$$

$$T_{2'} = 475.15 \text{ K}$$

$$\eta_c = 0.85 = \frac{T_{2'} - T_1}{T_2 - T_1}$$

$$T_2 = 506.06 \text{ K}$$

For process 3–4′

$$\frac{T_{4'}}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{4.8}\right)^{\frac{1.4-1}{1.4}}$$

$$T_{4'} = 1000 \times \left(\frac{1}{4.8}\right)^{\frac{1.4-1}{1.4}}$$

$$T_{4'} = 638.79 \text{ K}$$

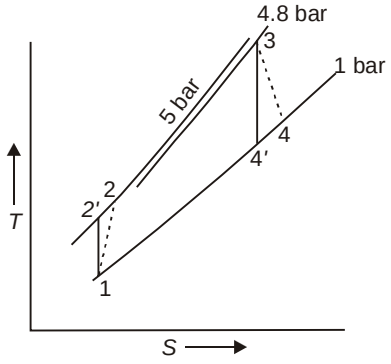


Fig. 9.39 *T-S representation*

Compressor work per kg $W_C = c_p\ (T_2 - T_1)$ = 1.0032 × (506.06 – 300) = 206.72 kJ/kg

Turbine work per kg, $W_T = c_p(T_3 - T_4)$ = 1.0032 × (1000 – T_4), kJ/kg

Net output, $W_{net} = W_T - W_C$ = {206.72 – (1.0032 (1000 – T_4))}, kJ/kg

Heat added, $q_{add} = c_p\ (T_3 - T_2)$ = 1.0032 (1000 – 506.06) = 495.52 kJ/kg

Thermal efficiency, $\eta = \dfrac{W_{net}}{q_{add}} \Rightarrow 0.20 = \dfrac{206.71 - (1.0032 \times (1000 - T_4))}{495.52}$

$$T_4 = 892.73 \text{ K}$$

Therefore, isentropic efficiency of turbine, $\eta_T = \dfrac{T_3 - T_4}{T_3 - T_{4'}} = \dfrac{(1000 - 892.73)}{(1000 - 638.79)}$

$$\eta_T = 0.2969 \text{ or } 29.69\%$$

Turbine isentropic efficiency = **29.69%** **Ans.**

11. *A gas turbine plant has air supplied at 1 bar, 27ºC for being compressed through pressure ratio of 10. Compression of air is achieved in two stages with perfect intercooling in between at optimum pressure. The maximum temperature in cycle is 1000 K and compressed air at this temperature is sent for expansion in two stages of gas turbine. First stage expansion occurs upto 3 bar and is subsequently reheated upto 995 K before being sent to second stage. Fuel used for heating in combustion chamber has calorific value of 42,000 kJ/kg. Considering c_p = 1.0032 kJ/kg. K throughout cycle determine, net output, thermal efficiency and air fuel ratio when air flows into compressor at 30 kg/s. Take isentropic efficiency of compression and expansion to be 85% and 90% respectively.*

Solution:

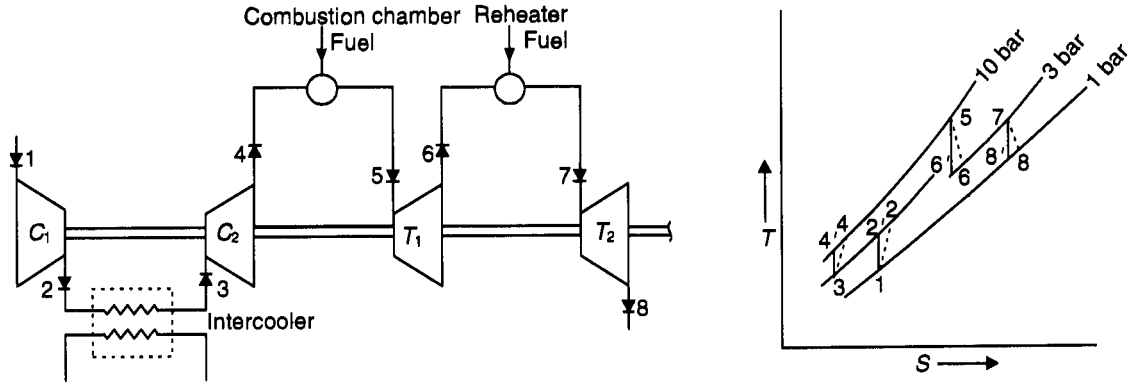


Fig. 9.40 Layout and T-S diagram

For perfect intercooling the pressure ratio of each compression stage = $\sqrt{10}$ = 3.16

For process 1–2', $\frac{T_{2'}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_{2'} = (3.16)^{\frac{1.4-1}{1.4}} \times 300 = 416.76$ K

Considering isentropic efficiency of compression,

$$\eta_c = 0.85 = \frac{T_{2'} - T_1}{T_2 - T_1} = \frac{416.76 - 300}{T_2 - 300}$$

For perfect intercooling, $T_3 = T_1 = 300$ K, $T_2 = 437.36$ K

For process 3–4,

$$\frac{T_{4'}}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{1.4-1}{1.4}} \Rightarrow T_{4'} = 300 \times (3.16)^{\frac{1.4-1}{1.4}} = 416.76 \text{ K}$$

Again due to compression efficiency, $\eta_c = 0.85 = \frac{T_{4'} - T_3}{T_4 - T_3}$

$$T_4 = 437.36 \text{ K}$$

Total compressor work, $W_C = 2 \times c_p \times (437.36 - 300) = 275.59$ kJ/kg

$T_5 = 1000$ K, For Expansion process 5–6'

$$\frac{T_{6'}}{T_5} = \left(\frac{P_6}{P_5}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{6'} = 1000 \times \left(\frac{3}{10}\right)^{\frac{1.4-1}{1.4}} = 708.93 \text{ K}$$

Considering expansion efficiency, $0.90 = \frac{T_5 - T_6}{T_5 - T_{6'}}$

$\Rightarrow$ $T_6 = 738.04$ K

$T_7 = 995$ K

For expansion in 7–8′,

$$\frac{T_{8'}}{T_7} = \left(\frac{P_8}{P_7}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow \quad T_{8'} = 995\left(\frac{1}{3}\right)^{\frac{1.4-1}{1.4}} = 726.95 \text{ K}$$

Considering expansion efficiency, $0.90 = \dfrac{T_7 - T_8}{T_7 - T_{8'}} \Rightarrow T_8 = 753.75$ K

Expansion work output per kg air = $c_p\,(T_5 - T_6) + c_p\,(T_7 - T_8)$

$W_T = 1.0032\ \{(1000 - 738.04) + (995 - 753.75)\}$

$W_T = 514.85$ kJ/kg

Heat added per kg air $= c_p(T_5 - T_4) + c_p(T_7 - T_6)$

$q_{\text{add}} = 1.0032\ \{(1000 - 437.36) + (995 - 738.04)\}$

$q_{\text{add}} = 822.22$ kJ/kg

Fuel required per kg of air, $m_f = \dfrac{822.22}{42000} = 0.01958$

$$\text{Air-fuel ratio} = \frac{1}{0.01958} = 51.07$$

$$\text{Net output} = W_T - W_C = 239.26 \text{ kJ/kg}$$

Output for air flowing at 30 kg/s, = 239.26 × 30 = 7177.8 kW

$$\text{Thermal efficiency} = \frac{W_T - W_C}{q_{\text{add}}} = \frac{239.26}{822.22} = 0.2909 \text{ or } 29.09\%$$

Thermal efficiency = **29.09%**, Net output = **7177.8** kW, A/F ratio = **51.07** **Ans.**

12. *A regenerative-reheat cycle has air entering at 1 bar, 300 K into compressor having intercooling in between the two stages of compression. Air leaving first stage of compression is cooled upto 290 K at 4 bar pressure in intercooler and subsequently compressed upto 8 bar. Compressed air leaving second stage compressor is passed through a regenerator having effectiveness of 0.80. Subsequent combustion chamber yields 1300 K at inlet to turbine having expansion upto 4 bar and then reheated upto 1300 K before being expanded upto 1 bar. Exhaust from turbine is passed through regenerator before discharged out of cycle. For the fuel having heating value of 42000 kJ/kg determine fuel-air ratio in each combustion chamber, total turbine work and thermal efficiency. Consider compression and expansion to be isentropic and air as working fluid throughout the cycle.*

Solution:

$T_1 = 300$ K, $P_1 = 1$ bar, $P_2 = P_3 = 4$ bar, $T_3 = 290$ K, $T_6 = 1300$ K,

$P_6 = P_4 = 8$ bar, $T_8 = 1300$ K, $P_8 = 4$ bar

For process 1–2,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = 445.79 \text{ K}$$

For process 3–4,

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_4 = 353.51 \text{ K}$$

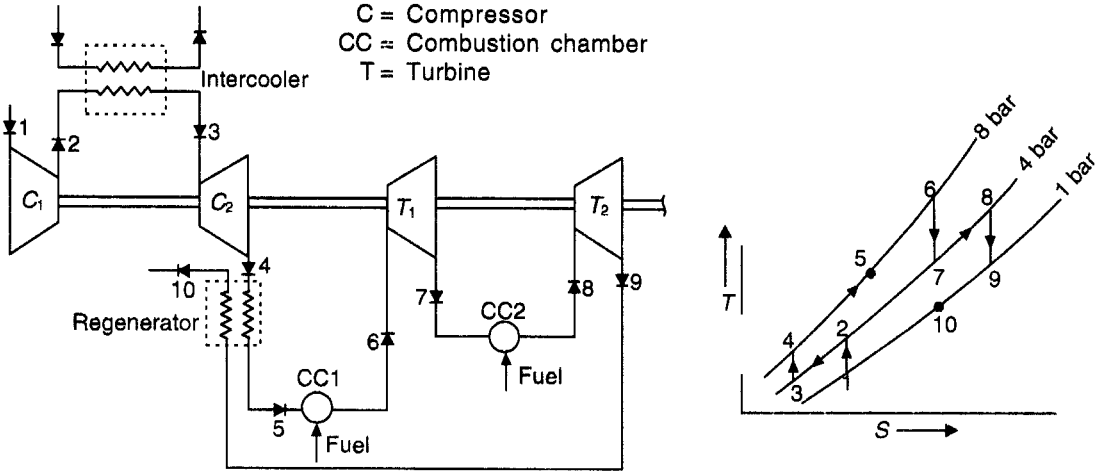


Fig. 9.41 Layout and T-S diagram

For process 6–7,

$$\frac{T_7}{T_6} = \left(\frac{P_7}{P_6}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4}{8}\right)^{\frac{1.4-1}{1.4}} \Rightarrow T_7 = 1066.43 \text{ K}$$

For process 8–9,

$$\frac{T_9}{T_8} = \left(\frac{P_9}{P_8}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_9 = 1300 \times \left(\frac{1}{4}\right)^{\frac{1.4-1}{1.4}} = 874.83 \text{ K}$$

In regenerator, effectiveness $= 0.80 = \dfrac{T_5 - T_4}{T_9 - T_4} = \dfrac{T_5 - 353.51}{(874.83 - 353.51)}$

$\Rightarrow$ $T_5 = 770.56$ K

Compressor work per kg air, $W_c = c_p (T_2 - T_1) + c_p (T_4 - T_3)$

$W_c = 209.97$ kJ/kg

Turbine work per kg air, $W_T = c_p (T_6 - T_7) + c_p (T_8 - T_9)$

$$W_T = 660.85 \text{ kJ/kg}$$

$$\text{Heat added per kg air, } q_{add} = c_p (T_6 - T_5) + c_p (T_8 - T_7)$$

$$q_{add} = 765.45 \text{ kJ/kg}$$

$$\text{Total fuel required per kg of air} = \frac{765.45}{42000} = 0.0182$$

$$\text{Net work } W_{net} = W_T - W_C = 450.88 \text{ kJ/kg}$$

$$\text{Cycle thermal efficiency, } \eta = \frac{W_{net}}{q_{add}} = \frac{450.88}{765.45} = 0.5890 \text{ or } 58.9\%$$

$$\text{Fuel required per kg air in combustion chamber 1, } = \frac{c_p (T_6 - T_5)}{42000} = 0.0126$$

$$\text{Fuel required per kg air in combustion chamber 2, } = \frac{c_p (T_8 - T_7)}{42000} = 0.0056$$

Fuel-air ratio in two combustion chambers = **0.0126, 0.0056**
Total turbine work = **660.85 kJ/kg**
Cycle thermal efficiency = **58.9%** **Ans.**

13. *A Stirling engine operates between temperature limits of 700 K and 300 K with compression ratio of 3. During heat addition process total 30 kJ/s heat is added. The regenerator efficiency is 90% and the pressure at beginning of compression is 1 bar. The number of cycles per minute is 100. Considering specific heat at constant volume as 0.72 kJ/kg·K, determine the brake output. Take R = 29.27 kJ/kg · K. Also determine stroke volume.*

Solution:

Work done per kg of air

$$= R(T_2 - T_1). \ln r$$

$$= 29.27 (700 - 300) \text{ In } 3$$

$$W = 12862.55 \text{ kJ/kg}$$

Heat added per kg of air

$$= RT_2 \ln r + (1 - \varepsilon) c_v \cdot (T_2 - T_1)$$

$$= (29.27 \times 700 \times \ln 3) + \{(1 - 0.9) \times 0.72 \times (700 - 300)\}$$

$$q = 22538.27 \text{ kJ/kg}$$

$$\text{For 30 kJ/s heat supplied, the mass of air/s} = \frac{30}{22538.27}$$

$$= 1.33 \times 10^{-3} \text{ kg/s}$$

$$\text{Mass of air per cycle} = \frac{1.33 \times 10^{-3}}{100} = 1.33 \times 10^{-5} \text{ kg/cycle}$$

$$\text{Brake output} = 12862.55 \times 1.33 \times 10^{-3} = 17.11 \text{ kW}$$

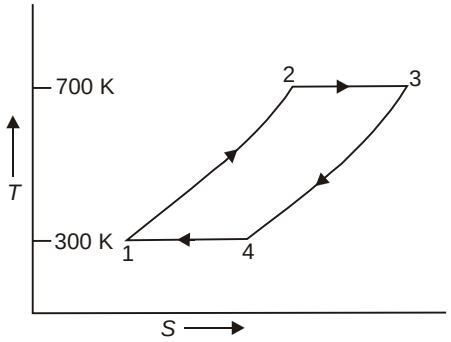


Fig. 9.42 *Stirling cycle*

$$\text{Stroke volume,} \quad V = \frac{mRT}{P} = \frac{\left(1.33\times10^{-5}\right)\times29.27\times300}{1\times10^{2}} = 0.00116 \text{ m}^3$$

Brake output = **17.11 kW**
Stroke volume = **0.00116 m³** **Ans.**

14. *A gas turbine installation operates with fixed maximum and minimum temperatures T_3 and T_1 respectively. Show that the optimum specific work output shall be obtained at same overall pressure ratio for each of following arrangement. Also give expression for this pressure ratio for air as working fluid throughout.*

(i) there is single stage compression followed by two stages of expansion in turbine. The expansion ratio in two stages is equal and reheating is done upto the maximum temperature at inlet of second stage of expansion.

(ii) there occurs compression in two stages of equal compression ratio with intercooling upto the minimum cycle temperature at inlet to second stage of compression followed by single stage expansion in turbine.

Take isentropic efficiency of compressor and turbine stages as η_c and η_T respectively.

Solution:

Let the overall pressure ratio be *r*.

For arrangement (*i*)

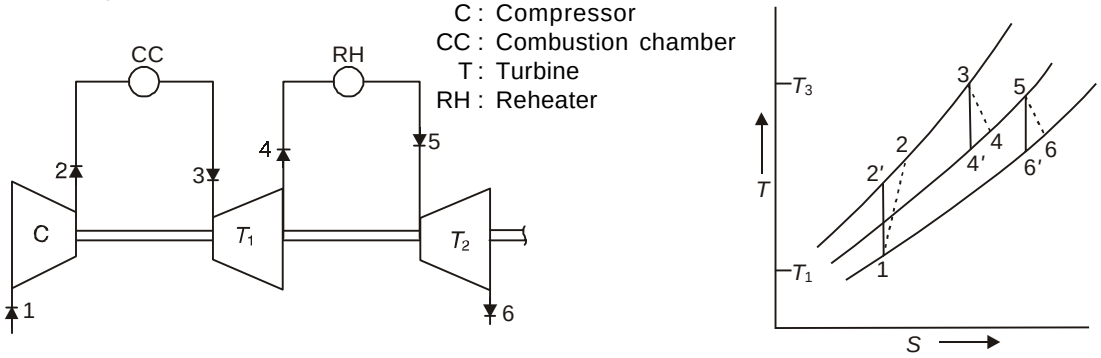


Fig. 9.43 *Layout and T-S diagram for (i) arrangement*

In compressor,

$$\frac{T_{2'}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = r^{\frac{\gamma-1}{\gamma}}$$

$$T_{2'} = T_1 \cdot r^{\frac{\gamma-1}{\gamma}}$$

$$\eta_c = \frac{T_{2'} - T_1}{T_2 - T_1} \Rightarrow T_2 = T_1 \left\{1 + \frac{r^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c}\right\}$$

Let expansion ratio in turbine stages be $r_T = \sqrt{r}$

For first stage expansion

$$\frac{T_{4'}}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{4'} = T_3 \left(\frac{1}{r_T}\right)^{\frac{\gamma-1}{\gamma}}$$

Isentropic efficiency of turbine,

$$\eta_T = \frac{T_3 - T_4}{T_3 - T_{4'}} \Rightarrow T_4 = T_3 - \eta_T (T_3 - T_{4'})$$

$$\Rightarrow \qquad T_4 = T_3 \left\{1 - \eta_T \left(1 - \frac{1}{r_T^{\frac{\gamma-1}{\gamma}}}\right)\right\}$$

For second stage expansion, $T_5 = T_3$

$$\frac{T_5}{T_{6'}} = \left(\frac{P_5}{P_6}\right)^{\frac{\gamma-1}{\gamma}}, \; T_{6'} = T_3 \cdot \left(\frac{1}{r_T}\right)^{\frac{\gamma-1}{\gamma}}$$

Isentropic efficiency, $\eta_T = \dfrac{T_5 - T_6}{T_5 - T_{6'}}$

$$T_6 = T_5 - \eta_T (T_5 - T_{6'})$$

$$T_6 = T_5 \left\{1 - \eta_T \left(1 - \frac{1}{r_T^{\frac{\gamma-1}{\gamma}}}\right)\right\} = T_3 \left\{1 - \eta_T \left(1 - \frac{1}{r_T^{\frac{\gamma-1}{\gamma}}}\right)\right\}$$

Net work per kg of air

$$W_{net} = \{c_p(T_3 - T_4) + c_p(T_5 - T_6)\} - \{c_p(T_2 - T_1)\}$$

$$W_{net} = c_p \left[T_3 - T_3 \left\{ 1 - \eta_T \left(1 - \frac{1}{r_T^{\frac{\gamma-1}{\gamma}}} \right) \right\} + T_3 - T_3 \right.$$

$$\left. \left\{ 1 - \eta_T \left(1 - \frac{1}{r_T^{\frac{\gamma-1}{\gamma}}} \right) \right\} - T_1 \left\{ 1 + \frac{r^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c} \right\} + T_1 \right]$$

$$W_{net} = c_p \left[2T_3 \cdot \eta_T \left(1 - \frac{1}{r^{\frac{\gamma-1}{2\gamma}}} \right) - T_1 \cdot \frac{r^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c} \right]$$

Differentiating with respect to r.

$$\frac{d W_{net}}{dr} = -T_3 \cdot \eta_T \left(\frac{1-\gamma}{2\gamma} \right) r^{\frac{1-3\gamma}{2\gamma}} - \frac{T_1}{\eta_c} \left(\frac{\gamma-1}{\gamma} \right) r^{\frac{-1}{\gamma}}$$

Substituting it equal to zero. $\frac{d W_{net}}{dr} = 0$

$$\Rightarrow \qquad \boxed{r = \left(\frac{\eta_c \cdot \eta_T \cdot T_3}{T_1} \right)^{\frac{2}{3}\left(\frac{\gamma}{\gamma-1}\right)}}$$

Optimum pressure ratio
For arrangement *(ii)*

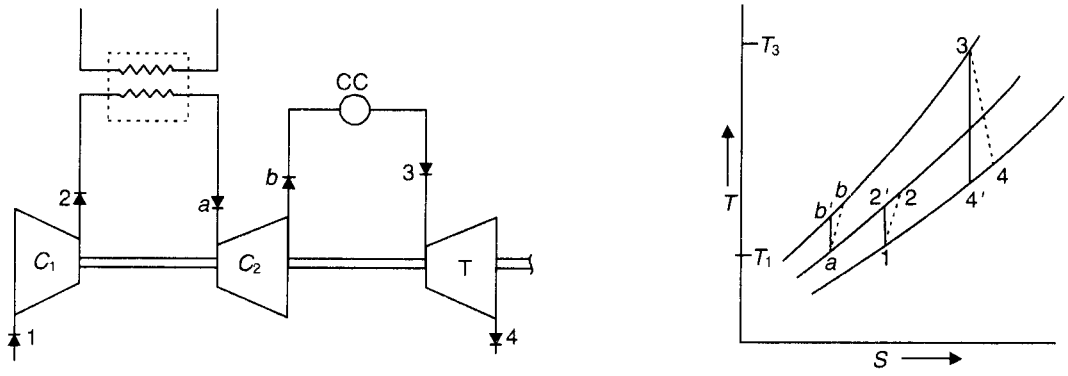


Fig. 9.44 *Layout and T-S representation for (ii) arrangement*

In compressor the stage pressure ratio, $r_c = \sqrt{r}$

For first stage compressor, $\frac{T_{2'}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$, $T_{2'} = T_1 \cdot (r_c)^{\frac{\gamma-1}{\gamma}}$

Isentropic compression efficiency, $\eta_c = \frac{T_{2'} - T_1}{T_2 - T_1}$, $\Rightarrow T_2 = T_1 \left\{1 + \frac{r_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c}\right\}$

For second stage compressor, $T_a = T_1$

$$\frac{T_{b'}}{T_a} = \left(\frac{P_b}{P_a}\right)^{\frac{\gamma-1}{\gamma}}, \; T_{b'} = T_1 \; (r_c)^{\frac{\gamma-1}{\gamma}}$$

Isentropic efficiency of compression, $\eta_c = \frac{T_{b'} - T_a}{T_b - T_a} \Rightarrow T_b = T_1 \left\{1 + \frac{r_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c}\right\}$

For expansion in turbine,

$$\frac{T_3}{T_{4'}} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_{4'} = \frac{T_3}{(r)^{\frac{\gamma-1}{\gamma}}}$$

Isentropic expansion efficiency, $\eta_T = \frac{T_3 - T_4}{T_3 - T_{4'}}$

$$\Rightarrow \qquad T_4 = T_3 \left\{1 - \eta_T \left(1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}}\right)\right\}$$

Net work per kg of air,

$$W_{\text{net}} = \{c_p (T_2 - T_1) + c_p(T_b - T_a)\} - \{c_p(T_3 - T_4)\}$$

$$= c_p \left\{T_1 \left(1 + \frac{r_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c}\right) - T_1 + T_1 \left(1 + \frac{r_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c}\right) - T_1\right\} - c_p \left\{T_3 - T_3 \left(1 - \eta_T \left(1 - \frac{1}{r^{\frac{\gamma-1}{\gamma}}}\right)\right)\right\}$$

Differentiating W_{net} with respect to 'r' and equating it to zero.

$$\frac{dW_{\text{net}}}{dr} = 0, \text{ yields, } (r)^{\frac{3(\gamma-1)}{2\gamma}} = \left(\frac{T_3 \cdot \eta_T \cdot \eta_c}{T_1}\right)$$

$$\Rightarrow \qquad \boxed{r = \left(\frac{T_3 \cdot \eta_T \cdot \eta_c}{T_1}\right)^{\frac{2}{3}\left(\frac{\gamma}{\gamma-1}\right)}} \quad \textbf{Ans.}$$

Hence it is proved that the optimum pressure ratio remains same for both arrangements as shown here.

15. *In a gas/steam combined cycle power plant exhaust from gas turbine leaves finally through HRSG at 420 K and generates steam in heat recovery steam generator (HRSG) at 6 MPa and 400ºC. Ambient air at 1 bar, 17ºC enters the compressor operating at pressure ratio of 10. Turbine inlet temperature of gas turbine is 1400 K. The steam turbine operates with condenser pressure of 15 kPa. Consider air as working fluid throughout gas turbine cycle and combined cycle output to be 37.3 MW. Determine the overall efficiency, mass flow rate of steam per kg of air. Take* $c_{p,\,air} = 1.0032$ *kJ/kg · K*

Solution:

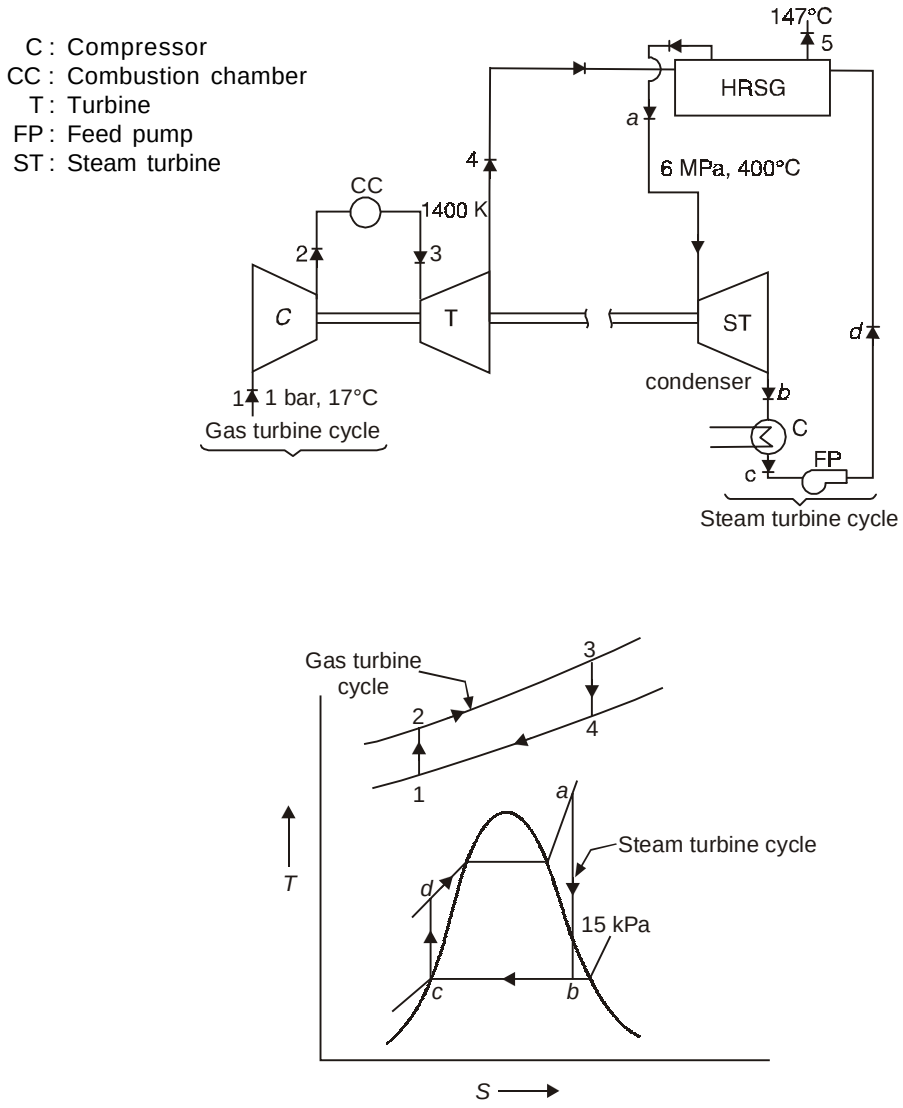


Fig. 9.45 *Layout and combined T-S diagram (approximate representation)*

In gas turbine cycle, $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = 290 \times (10)^{\frac{1.4-1}{1.4}} = 559.9$ K

$T_3 = 1400$ K,

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_4 = 1400 \left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}} = 725.13 \text{ K}$$

Compressor work per kg, $W_C = c_p(T_2 - T_1) = 1.0032 \times (559.9 - 290) = 270.76$ kJ/kg

Turbine work per kg, $W_T = c_p(T_3 - T_4) = 677.03$ kJ/kg

Heat added in combustion chamber per kg, $q_{add} = c_p(T_3 - T_2) = 842.79$ kJ/kg

Net gas turbine output, $W_{net, GT} = W_T - W_C = 406.27$ kJ/kg air

Heat recovered in HRSG for steam generation per kg of air

$$q_{HRSG} = c_p(T_4 - T_5) = 306.11 \text{ kJ/kg}$$

At inlet to steam turbine,

$$h_a = 3177.2 \text{ kJ/kg}, \; s_a = 6.5408 \text{ kJ/kg} \cdot \text{K}$$

For expansion in steam turbine, $s_a = s_b$

Let dryness fraction at state *b* be *x*.

$$s_b = 6.5408 = s_{f \text{ at } 15 \text{ kPa}} + x \cdot s_{fg \text{ at } 15 \text{ kPa}}$$

$$x = 0.7976$$

$$h_b = h_{f \text{ at } 15 \text{ kPa}} + x \cdot h_{fg \text{ at } 15 \text{ kPa}} = 2118.72 \text{ kJ/kg}$$

At exit of condenser, $h_c = h_{f \text{ at } 15 \text{ kPa}} = 225.94$ kJ/kg, $v_c = 0.001014$ m^3/kg

At exit of feed pump, $h_d \Rightarrow h_d - h_c = v_c \cdot (6 \times 10^3 - 15) \times 10^2$

$$h_d = 606.88 \text{ kJ/kg}$$

Heat added per kg of steam = $h_a - h_d = 2570.32$ kJ/kg

Mass of steam generated per kg of air = $\frac{q_{HRSG}}{(h_a - h_d)}$

= 0.119 kg steam per kg air.

Net steam turbine cycle output, $W_{net, ST} = (h_a - h_b) - (h_d - h_c)$

$$W_{net, ST} = 677.54 \text{ kJ/kg}$$

Steam cycle output per kg of air = $W_{net, ST} \times 0.119 = 80.63$ kJ/kg air

Total combined cycle output = $(W_{net, GT} + W_{net, ST}) = 486.9$ kJ/kg air

Combined cycle efficiency, $\eta_{cc} = \frac{(W_{net,GT} + W_{net,ST})}{q_{add}} = 0.5777$ or 57.77%

In the absence of steam cycle, Gas turbine cycle efficiency, $\eta_{GT} = \frac{W_{net,GT}}{q_{add}}$

$$\eta_{GT} = 0.4821 \text{ or } 48.21\%$$

Thus, efficiency is seen to increase in combined cycle upto 57.77% as compared to gas turbine offering 48.21% efficiency.

Overall efficiency = **57.77%**
Steam per kg of air = **0.119 kg** steam/kg air

Ans.

16. *In an I.C. engine operating on the dual cycle (limited pressure cycle), the temperature of the working fluid (air) at the beginning of compression is 27ºC. The ratio of the maximum and minimum pressures of the cycle is 70 and the compression ratio is 15. The amount of heat added at constant volume and at constant pressure are equal. Compute the air standard thermal efficiency of the cycle. State three main reasons why the actual thermal efficiency is different from the theoretical value.*

[U.P.S.C. 1993]

Solution:

$$T_1 = 27^{\circ}C = 300 \text{ K}$$

$$\frac{P_4}{P_1} = \frac{P_3}{P_1} = 70$$

Compression ratio, $\frac{V_1}{V_2} = 15 = \frac{V_1}{V_3}$

Heat added at constant volume = Heat added at constant pressure

$$Q_{23} = Q_{34}$$

$$m \cdot c_v(T_3 - T_2) = m \cdot c_p \cdot (T_4 - T_3)$$

$$(T_3 - T_2) = \gamma \cdot (T_4 - T_3)$$

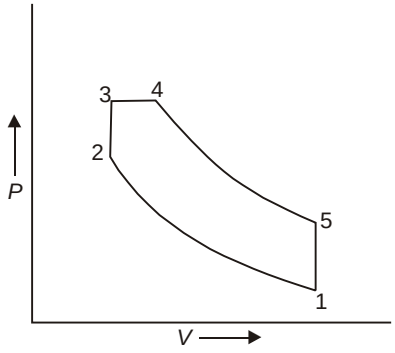


Fig. 9.46

For process 1–2;

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\frac{T_2}{300} = (15)^{0.4}$$

$$T_2 = 886.25$$

and

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma} = (15)^{1.4}$$

$$P_2 = P_1 \cdot (15)^{1.4}$$

$$P_2 = 44.3\ P_1$$

For process 2–3

$$\frac{P_2}{P_3} = \frac{T_2}{T_3}$$

$$\frac{44.3P_1}{70P_1} = \frac{886.25}{T_3}$$

or, $T_3 = 1400.39$

Using equal heat additions for processes 2–3 and 3–4,

$$(T_3 - T_2) = \gamma\,(T_4 - T_3)$$

$$(1400.39 - 886.25) = 1.4(T_4 - 1400.39)$$

$$T_4 = 1767.63$$

For process 3–4,

$$\frac{V_3}{V_4} = \frac{T_3}{T_4}$$

$$\frac{V_3}{V_1} \times \frac{V_1}{V_4} = \frac{T_3}{T_4}$$

$$\frac{1}{15} \times \left(\frac{V_1}{V_4}\right) = \left(\frac{1400.39}{1767.63}\right)$$

$$\frac{V_1}{V_4} = 11.88$$

$$\frac{V_5}{V_4} = 11.88$$

For process 4–5,

$$\frac{P_4}{P_5} = \left(\frac{V_5}{V_4}\right)^{\gamma} \text{ or, } \frac{T_4}{T_5} = \left(\frac{V_5}{V_4}\right)^{\gamma-1}$$

$$\frac{P_4}{P_5} = (11.88)^{1.4} \text{ or, } \frac{T_4}{T_5} = (11.88)^{0.4}$$

$$T_5 = \frac{1767.63}{(11.88)^{0.4}} = 656.85$$

Air standard thermal efficiency $= 1 - \dfrac{\text{Heat rejected}}{\text{Heat added}}$

$$= 1 - \frac{m\,c_v(T_5 - T_1)}{m\,c_p(T_4 - T_3) + m\,c_v(T_3 - T_2)}$$

$$= 1 - \frac{(T_5 - T_1)}{\gamma\cdot(T_4 - T_3) + (T_3 - T_2)}$$

$$= 1 - \frac{(656.85 - 300)}{1.4\times(1767.63 - 1400.39) + (1400.39 - 886.25)}$$

Air standard thermal efficiency = 0.6529

Air standard thermal efficiency = **65.29%** **Ans.**

Actual thermal efficiency may be different from theoretical efficiency due to following reasons;

(*a*) Air standard cycle analysis considers air as the working fluid while in actual cycle it is not air throughout the cycle. Actual working fluid which are combustion products do not behave as perfect gas.

(*b*) Heat addition does not occur isochorically in actual process. Also combustion is accompanied by inefficiency such as incomplete combustion, dissociation of combustion products, etc.

(*c*) Specific heat variation occurs in actual processes where as in air standard cycle analysis specific heat variation is neglected. Also during adiabatic process theoretically no heat loss occurs while actually these processes are accompanied by heat losses.

EXERCISE

9.1 What do you mean by air standard cycles? Discuss its' significance.

9.2 Derive the expression for Carnot cycle efficiency.

9.3 Discuss limitations of Carnot cycle and explain Otto cycle, Diesel cycle and Dual cycle in light of these limitations.

9.4 Derive expression for efficiency of Otto cycle, Dual cycle and Diesel cycle.

9.5 What is meant by mean effective pressure? Discuss its' relevance.

9.6 Obtain expressions for mean effective pressure of Otto cycle, Diesel cycle and Dual cycle.

9.7 Discuss gas turbine cycles. Obtain efficiency of Brayton cycle.

9.8 What are possible modifications in gas turbine cycle? Explain.

9.9 Explain different irreversibilities and losses in gas turbine cycle.

9.10 Define polytropic efficiency and its significance mathematically for compressors.

9.11 Obtain optimum pressure ratio condition for minimum compressor work requirement in two stage perfect intercooled compression.

9.12 Compare the influence of reheating, regeneration and intercooling on performance of gas turbine cycle.

9.13 Discuss Ericsson cycle and obtain its efficiency.

9.14 Obtain efficiency of Stirling cycle.

9.15 Write short notes on compressor and turbine efficiencies.

9.16 An Otto cycle operates between maximum and minimum pressures of 600 *kPa* and 100 *kPa*. The minimum and maximum temperatures in the cycle are 27ºC and 1600 K. Determine thermal efficiency of cycle and also show it on T-s and P-V diagram. [48%]

9.17 In an air standard Carnot cycle operating between temperatures of 57ºC and 1327ºC, determine the heat added if cycle efficiency is 79.4% and the minimum pressure of cycle is 100 kPa. [300 kJ/kg]

9.18 Determine air standard efficiency for a diesel engine having L/D ratio of 1.6 and bore as 25 cm. Take the clearance volume and pressure at end of suction as 1.2 litres and 1 atm. respectively. Also obtain mean effective pressure. Volume after combustion is 1.8 times clearance volume. [63.5, 6 bar]

9.19 An engine operates on Dual cycle with a compression ratio of 15. At the end of suction the air is available at 1 atm and 27ºC. Total heat added is 430 kJ/kg. Heat supply is in ratio of (0.536:1) for heat supply at constant volume and constant pressure. Determine cycle efficiency and mean effective pressure. [65%, 0.342 MPa]

9.20 A regenerative cycle has heat exchanger effectiveness of 75% and pressure ratio of 4 with compression occurring in two stages of equal pressure ratio with intercooling back to initial temperature of 15ºC. Maximum temperature in cycle is 650º C and expansion occurs with efficiency of 0.88. Compression process has isentropic efficiency of 0.85 in each stage. Considering air as working fluid throughout the cycle determine efficiency of cycle. [34.6%]

9.21 A gas turbine installation has intercooling, regeneration and reheating with air entering compressor at 100 kPa, 290 K and compressed to 0.41 MPa. Subsequently it is cooled until temperature drops to 13ºC in intercooler and finally compressed to 0.75 MPa. Regenerator effectiveness is 0.70 and turbine inlet temperature (TIT) is 1350 K and expands to 0.41 MPa where it is reheated upto 1350 K. The exhaust pressure is 0.1 MPa. Determine overall efficiency considering expansion and compression processes to be of isentropic type. Take air as working fluid throughout. [54.3%]

9.22 In an actual gas turbine the compressor requires 300 kJ/kg of work to quadruple the inlet pressure. For inlet air temperature of 100ºC determine, the compressor exit air temperature and compressor efficiency. [671.6 K, 60.6%]

9.23 In a gas turbine installation the exhaust from gas turbine is sent for process heating at 200 kPa. Turbine produces power just sufficient to drive the compressor. Turbine inlet temperature is 815ºC and air is supplied to compressor at 1 bar, 17ºC. Determine compressor pressure ratio for air as working fluid throughout. [3]

9.24 A Brayton cycle producing 75 kW is designed for maximum work. The compressor inlet conditions are 100 kPa and 27ºC. Compression ratio is 5.5. For air standard Brayton cycle determine the turbine inlet temperature, cycle efficiency and air flow rate. [794 K, 38.6%, 0.631 kg/s]

9.25 A gas turbine cycle has reheating and heat exchanger employed. The air is supplied at 1 bar, 15ºC into compressor where it is compressed upto 4 bar with isentropic efficiency of 82%. Turbine inlet temperature is 700ºC and expansion occurs in two stages with equal pressure ratio upto 1 bar. The reheating between two stages occurs upto 700ºC. Isentropic efficiency of expansion is 85%. Exhaust from low pressure turbine is passed into a heat exchanger having 0.75 effectiveness. Heat exchanger heats the discharge from compressor before being supplied to combustion chamber. Considering air as woking fluid throughout determine cycle efficiency. [32.1%]

9.26 In a gas turbine plant air is supplied at 1 bar, 290 K to compressor having isentropic efficiency of 0.84 and pressure ratio of 5. The air flow rate to compressor is 111600 kg/hr. Compressed air is passed through a heat exchanger and it leaves exchanger at 360ºC, from where it is supplied to combustion chamber. Fuel having calorific value of 41800 kJ/kg is supplied at the rate of 890 kg/hr in combustion chamber. The combustion products expand from 4.7 bar to 1.95 bar, 410ºC in turbine. Take $c_{p\ \text{air}} = 1.0032$ kJ/kg K, $\gamma_{\text{air}} = 1.4$, $c_{p\ \text{comb, products}} = 1.1286$ kJ/kg K, $\gamma_{\text{comb, products}} = 1.34$. Determine *hp* developed. [3000 hp]

9.27 Show that the necessary condition for positive work output from an open cycle gas turbine can be given by,

$$\eta_c \times \eta_T \times T_{\max} > T_{\min} \cdot r^{\frac{\gamma-1}{\gamma}}$$

The simple gas turbine cycle has compressor with isentropic efficiency of η_c and pressure ratio of '*r*'. Turbine has isentropic efficiency of η_T and expansion ratio similar to that of compressor. The maximum and minimum temperature are $T_{\max}$ and $T_{\min}$ and air is working fluid throughout with γ as ratio of specific heat.

9.28 In a gas turbine installation based on Brayton cycle the highest and lowest temperatures are T_H and T_L respectively. The pressure ratio and heat input from high temperature reservoir vary such that temperature T_H and T_L remain constant. Show that the expression for net maximum work output is, $W_{\max} = C_P.T_L\left(\sqrt{\frac{T_H}{T_L}} - 1\right)^2$.

9.29 A gas turbine installation has double stage compressor and double stage turbine having pressure ratio across each stage as 8:1 in both compressor and turbine. Pressure at inlet to compressor is 1 bar and temperature entering each compressor is 293K and temperature

entering each turbine is 1373K. Considering ideal regenerator in cycle determine cycle thermal efficiency and turbine work output.

[0.613, 1.235 MJ/kg]

9.30 An ideal Stirling cycle running on hydrogen works between temperature of 723°C and 23°C while highest and lowest pressure are 30 bar and 5 bar respectively. Considering it to be closed cycle determine the heat transfer to regenerator per cycle, net work per cycle and cycle efficiency.

[3.67 MJ, 0.85 MJ, 70%]

9.31 An ideal Eriscsson cycle is running on helium. Helium is at 1.5 bar, 500°C at the start of compression process and expansion process occurs at a pressure of 35 bar. Temperature of low temperature reservoir is 298 K. Determine the thermal efficiency and net work of cycle.

[61.5%, 31.075MJ]

9.32 Determine thermal efficiency of cycle, backwork ratio and net power produced from a gas turbine installation having compressor admitting air at 1 bar, 300K & 5 m^3/s volume flow rate. Air is compressed in two stages up to 12 bar with intercooling to 300K between stages at a pressure of 350 kPa. Turbine inlet temperature is 1127 °C and expanssion occurs in two stages with reheat to 1340K between stages at a pressure of 3.5 bar. Compressor and turbine stage efficiencies are 87% and 85% respectively. Also find the % gain in efficiency if a regenerator with effectiveness of 80% is employed in plant. Show the process on T-S diagram.

[29.8%, 0.441, 2.17 MW, 59.66%]

9.33 A gas turbine runs at pressure ratio of 7 and maximum temperature of 1000K. Air enters compressor at temperature of 288 K. The isentropic efficiency of turbine and compressor are 90% & 85% respectively. Find the percentage reduction in efficiency compared to ideal Brayton cycle. Also find net work output and back work ratio of gas turbine. Show the processes on T-S diagram. [32.85%, 132.5 kJ/kg, 0.656]

10
Fuels and Combustion

10.1 INTRODUCTION

Every real life system requires energy input for its' performance. Energy input may be in the form of heat. Now question arises from where shall we get heat? Traditionally heat for energy input can be had from the heat released by fuel during combustion process. Fuels have been provided by nature and the combustion process provides a fluid medium at elevated temperature. During combustion the energy is released by oxidation of fuel elements such as carbon C, hydrogen H_2 and sulphur S, i.e. high temperature chemical reaction of these elements with oxygen O_2 (generally from air) releases energy to produce high temperature gases. These high temperature gases act as heat source.

In this chapter the detailed study of fuels and their combustion is being made.

Air fuel ratio: It refers to the ratio of amount of air in combustion reaction with the amount of fuel. Mathematically, it can be given by the ratio of mass of air and mass of fuel.

$$AF = \frac{\text{Mass of air}}{\text{Mass of fuel}} = \left(\frac{\text{Molecular wt. of air} \times \text{no. of moles of air}}{\text{Molecular wt. of fuel} \times \text{no. of moles of fuel}} \right)$$

Fuel-air ratio is inverse of Air-fuel ratio. Theoretical air-fuel ratio can be estimated from stoichiometric combustion analysis for just complete combustion.

Equivalence ratio: It is the ratio of actual fuel-air ratio to the theoretical fuel-air ratio for complete combustion. Fuel-air mixture will be called lean mixture when equivalence ratio is less than unity while for equivalence ratio value being greater than unity the mixture will be rich mixture.

Theoretical air: Theoretical amount of air refers to the minimum amount of air that is required for providing sufficient oxygen for complete combustion of fuel. Complete combustion means complete reaction of oxygen present in air with C, H_2, S etc. resulting into carbon dioxide, water, sulphur dioxide, nitrogen with air as combustion products. At the end of complete reaction there will be no free oxygen in the products. This theoretical air is also called "stoichiometric air".

Excess air: Any air supplied in excess of "theoretical air" is called excess air. Generally excess air is 25 to 100% to ensure better and complete combustion.

Flash point and Fire point: Flash point refers to that temperature at which vapour is given off from liquid fuel at a sufficient rate to form an inflammable mixture but not at a sufficient rate to support continuous combustion.

Fire point refers to that temperature at which vaporization of liquid fuel is sufficient enough to provide for continuous combustion.

These temperatures depend not only on the fuel characteristics but also on the rate of heating, air movement over fuel surface and means of ignition. These temperatures are specified in reference to certain standard conditions. Although flash point and fire point temperatures are defined in relation with ignition but these temperatures are not measure of ignitability of fuel but of the initial volatility of fuel.

Adiabatic flame temperature: Adiabatic flame temperature refers to the temperature that could be attained by the products of combustion when the combustion reaction is carried out in limit of adiabatic operation of combustion chamber. Limit of adiabatic operation of combustion chamber means that in the absence of work, kinetic and potential energies the energy released during combustion shall be carried by the combustion products with minimum or no heat transfer to surroundings. This is the maximum temperature which can be attained in a combustion chamber and is very useful parameter for designers. Actual temperature shall be less than adiabatic flame temperature due to heat transfer to surroundings, incomplete combustion and dissociation etc.

Wet and dry analysis of combustion: Combustion analysis when carried out considering water vapour into account is called "wet analysis" while the analysis made on the assumption that vapour is removed after condensing it, is called "dry analysis".

Volumetric and gravimetric analysis: Combustion analysis when carried out based upon percentage by volume of constituent reactants and products is called volumetric analysis.

Combustion analysis carried out based upon percentage by mass of reactants and products is called gravimetric analysis.

Pour point: It refers to the lowest temperature at which liquid fuel flows under specified conditions.

Cloud point: When some petroleum fuels are cooled, the oil assumes cloudy appearance. This is due to paraffin wax or other solid substances separating from solution. The temperature at which cloudy appearance is first evident is called cloud point.

Composition of air: Atmospheric air is considered to be comprising of nitrogen, oxygen and other inert gases. For combustion calculations the air is considered to be comprising of nitrogen and oxygen in following proportions. Molecular weight of air is taken as 29.

Composition of air by mass = Oxygen (23.3%) + Nitrogen (76.7%)

Composition of air by volume = Oxygen (21%) + Nitrogen (79%)

Enthalpy of combustion: Enthalpy of combustion of fuel is defined as the difference between the enthalpy of the products and enthalpy of reactants when complete combustion occurs at given temperature and pressure. It may be given as higher heating value or lower heating value. Higher heating value (HHV) of fuel is the enthalpy of combustion when all the water (H_2O) formed during combustion is in liquid phase. Lower heating value (LHV) of fuel refers to the enthalpy of combustion when all the water (H_2O) formed during combustion is in vapour form. The lower heating value will be less than higher heating value by the amount of heat required for evaporation of water.

HHV = LHV + (Heat required for evaporation of water)

It is also called calorific value of fuel and is defined as the number of heat units liberated when unit mass of fuel is burnt completely in a calorimeter under given conditions.

Enthalpy of formation: Enthalpy of formation of a compound is the energy released or absorbed when compound is formed from its elements at standard reference state. Thus enthalpy of formation shall equal heat transfer in a reaction during which compound is formed from its' elements at standard reference state. Enthalpy of formation will have positive (+ ve) value if formation is by an endothermic reaction and negative (– ve) value if formation is by an exothermic reaction.

Standard reference state: It refers to thermodynamic state at which the enthalpy datum can be set for study of reacting systems. At standard reference state, zero value is assigned arbitrarily to the enthalpy of stable elements. Generally, standard reference state is taken as 25°C and 1 atm,

i.e. $T_{ref} = 25°C = 298.15$ K, $p_{ref} = 1$ atm

Dissociation: It refers to the combustion products getting dissociated and thus absorbing some of energy. Such as, the case of carbon dioxide getting formed during combustion and subsequently getting dissociated can be explained as below,

Combustion: $C + O_2 \rightarrow CO_2 + \text{Heat}$

Dissociation: Heat + $CO_2 \rightarrow C + O_2$

Thus generally, dissociation has inherent requirement of high temperature and heat.

10.2 TYPES OF FUELS

'Fuel' refers to a combustible substance capable of releasing heat during its combustion. In general fuels have carbon, hydrogen and sulphur as the major combustible chemical elements. Sulphur is found to be relatively less contributor to the total heat released during combustion. Fuels may be classified as solid, liquid and gaseous fuel depending upon their state.

Solid fuel: Coal is the most common solid fuel. Coal is a dark brown/black sedimentary rock derived primarily from the unoxidized remains of carbon-bearing plant tissues. It can be further classified into different types based upon the composition. Composition can be estimated using either "proximate analysis" or by "ultimate analysis". Proximate analysis is the one in which the individual constituent element such as C, H_2, S, N_2 etc. are not determined rather only fraction of moisture, volatile matter, ash, carbon etc. are determined. Thus proximate analysis is not exact and gives only some idea about the fuel composition. Proximate analysis of coal gives, various constituents in following range, Moisture 3–30%, Volatile matter 3–50%, Ash 2–30% and Fixed carbon 16–92%.

In "ultimate analysis" the individual elements such as C, H_2, N_2, S and ash etc. present in the fuel are determined on mass basis. Thus, it gives relative amounts of chemical elements constituting fuel. In general the percentage by mass of different elements in coal lies in the following range:

Carbon	:	50 to 95%
Hydrogen	:	2 to 50%
Oxygen	:	2 to 40%
Nitrogen	:	0.5 to 3%
Sulphur	:	0.5 to 7%
Ash	:	2 to 30%

Different types of coal available are listed in the table hereunder. These are typical values for particular types of coal samples and may change for different types of coal.

Table 10.1 *Different types of coal*

Sl. No.	*Type*	*% by mass*		*Ultimate analysis, % by mass*					*Lower calorific value, kcal/kg*
		Moisture	*Volatile matter in dry coal*	*C*	H_2	O_2	$N_2 + S_2$	*Ash*	
1.	Peat	20	65	43.70	6.42	44.36	1.52	4.00	3200
2.	Lignite	15	50	56.52	5.72	31.89	1.62	4.25	2450
3.	Bituminous coal	2	25	74.00	5.98	13.01	2.26	4.75	7300
4.	Anthracite coal	1	4	90.27	3.30	2.32	1.44	2.97	7950

Liquid fuels: Fuels in liquid form are called liquid fuels. Liquid fuels are generally obtained from petroleum and its by-products. These liquid fuels are complex mixture of different hydrocarbons, and obtained by refining the crude petroleum oil. Commonly used liquid fuels are petrol, kerosene diesel, aviation fuel, light fuel oil, heavy fuel oil etc.

Various liquid fuels of hydrocarbon family lie in Paraffin (C_nH_{2n+2} – chain structure), Olefins (C_nH_{2n} – chain structure), Napthalene (C_nH_{2n} – ring structure), Benzene (C_nH_{2n-6} – ring structure),

Nepthalene (C_nH_{2n-12} – ring structure) category. Percentage by volume composition of some of liquid fuels is given below.

Table 10.2 *Composition of liquid fuels*

Sl.No.	*Fuel*	*% by volume*		
		Carbon	*Hydrogen*	*Sulphur*
1.	Petrol	85.5	14.4	0.1
2.	Kerosene	86.3	13.6	0.1
3.	Diesel	86.3	12.8	0.9
4.	Benzole	91.7	8.0	0.3
5.	Light fuel oil	86.2	12.4	1.4
6.	Heavy fuel oil	86.1	11.8	2.1

Liquid fuels offer following advantages over solid fuel.

(i) Better mixing of fuel and air is possible with liquid fuel.
(ii) Liquid fuels have no problem of ash formation.
(iii) Storage and handling of liquid fuels is easy compared to solid fuels.
(iv) Processing such as refining of liquid fuels is more convenient.

Gaseous fuels: These are the fuels in gaseous phase. Gaseous fuels are also generally hydrocarbon fuels derived from petroleum reserves available in nature. Most common gaseous fuel is natural gas. Gaseous fuels may also be produced artificially from burning solid fuel (coal) and water. Some of gaseous fuels produced artificially are coal gas, producer gas etc. Volumetric analysis of gaseous fuels is presented in Table 10.3. Gaseous fuels offer all advantages as there in liquid fuels except ease of storage.

Table 10.3 *Composition of gaseous fuels*

Sl.No.	*Fuel*	*% by volume*								
		H_2	O_2	N_2	CO	CH_4	C_2H_4	C_2H_6	C_4H_8	CO_2
1.	Natural gas	—	—	3	1	93	—	3	—	—
2.	Coal gas	53.6	0.4	6	9	25	—	—	3	3
3.	Producer gas	12	—	52	29	2.6	0.4	—	—	4

10.3 CALORIFIC VALUE OF FUEL

During combustion, the chemical energy of fuel gets transformed into molecular kinetic or molecular potential energy of products. This energy associated with combustion also called calorific value of fuel is very important to be known for thermodynamic design and calculations of combustion systems. "Bomb calorimeter" is one of the ways to get the heating value of solid and liquid fuels when burnt at constant volume. Different types of bomb calorimeters as given by At water, Davis, Emerson, Mahler, Parr, Peters and Williams are available. Bomb calorimeter as given by Emerson is discussed here. For getting the heating value of gaseous fuel the gas calorimeter is also discussed here.

10.4 BOMB CALORIMETER

Emerson's bomb calorimeter is shown in Fig. 10.1 here. Its major components are bomb, bucket, stirrer, crucible or fuel pan, jacket, thermometer etc. A known quantity of fuel under investigation is kept in the crucible. Crucible has a electric coil with d.c. supply in it. Bomb is charged with oxygen under

pressure. Bomb is surrounded by a bucket containing water to absorb the heat released as fuel burns. Bomb also has an outer jacket with dead air space surrounding the bucket to minimise heat loss to surroundings. When electricity is flown into coil the fuel gets ignited. Bomb is actually a strong shell capable of withstanding about 100 atmosphere pressure. Inner bomb wall surface is lined with enamel and external wall surface is plated so as to prevent corrosion due to high temperature combustion products. Different operations while using it for calorific value measurement are as given ahead. First weigh the empty calorimeter bucket and fill it with a definite quantity of water at temperature about 2–3°C less than jacket water temperature. Charge bomb with oxygen at high pressure without disturbing the fuel kept in crucible. Ensure that there are no leaks in bomb and after ensuring it as leak proof place it in bomb jacket. Install thermometer and stirrer. Start stirrer for 3 to 4 minutes for temperature uniformity in bucket. Take temperature readings at definite interval, say after every five minutes.

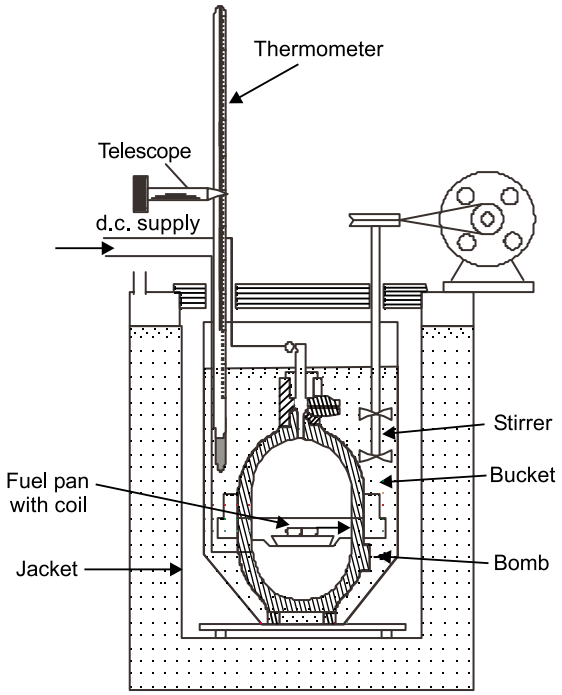


Fig. 10.1 *Bomb calorimeter*

Active the coil so as to fire fuel. Record temperature every 30 seconds until maximum temperature is reached and after attaining maximum temperature read temperature after every 5 minutes. These temperatures are required to account for the heat exchange with jacket water.

Later on remove bomb from calorimeter, release the gases and dismantle the bomb. Collect and weigh the iron fuse wire which remains. For getting accurate result the bomb should be washed with distilled water and washings titrated to obtain the amount of nitric acid and sulphuric acid formed. Corrections are made for small amount of heat transfer which occurs.

For making the calculations a curve of temperature is plotted with time. Determine the rate of temperature rise before firing.

Also, determine the rate of temperature drop after the maximum temperature is reached. Cooling correction may be added to the measured temperature rise. Heat balance may be applied as,

Heat released by fuel during combustion + Heat released by combustion of fuse wire
= Heat absorbed by water and calorimeter

For getting heat absorbed by calorimeter the water equivalent of calorimeter is used which can be determined by burning a fuel of known calorific value. Generally benzoic acid and napthalene having calorific value of 6325 kcal/kg and 9688 kcal/kg are being used for finding water equivalent of calorimeter.

Mathematically,

$$m_{\text{fuel}} \times \text{CV}_{\text{fuel}} + m_{fw} \times \text{CV}_{fw} = (m_w + m_c) \cdot R$$

$$\text{CV}_{\text{fuel}} = \left\{ \frac{\left(m_w + m_c\right) \cdot R - m_{fw} \cdot \text{CV}_{fw}}{m_{\text{fuel}}} \right\}$$

where m_{fuel} and m_{fw} are mass of fuel and mass of fuse wire, m_w and m_c are mass of water in calorimeter and water equivalent of calorimeter, R is correct temperature rise, CV_{fuel} is higher calorific value of fuel, CV_{fw} is calorific value of fuse wire. Calorific value of fuel estimated is higher calorific value as the water formed during combustion is condensed.

10.5 GAS CALORIMETER

Schematic of gas calorimeter is shown in Fig.10.2 It is used for estimating the heating value of gaseous fuels. It has burner with arrangement to regulate and measure flow rate and pressure of gaseous fuel. Combustion products pass through tubes which are surrounded by flowing water. Here the volume flow rate of gas flowing through calorimeter is measured. Water conditions are adjusted so as to cool the products of combustion to ambient air temperature. Rate of water flow through the calorimeter is measured and its' temperature rise is determined. If the heat exchange between the calorimeter and its' surroundings is neglected then the heat received by water shall be equal to the heating value of fuel. For precise estimation of heating value of gaseous fuels the procedure as specified by ASTM is to be followed.

10.6 COMBUSTION OF FUEL

Combustion of fuel refers to the chemical reaction that occurs between fuel and air to form combustion products with energy release i.e. oxidation of combustible fuel results into energy and products of combustion.

In other words, during chemical reaction the bonds within fuel molecules get broken and atoms and electrons rearrange themselves to yield products. During complete combustion carbon present in fuel transforms into carbon dioxide, hydrogen into water, sulphur into sulphur dioxide and nitrogen into nitrogen oxides. Thus it is obvious that the total mass before combustion and after combustion remains constant although the elements exist in form of different chemical compounds in reactants and products. Generic form of combustion equations shall be as follows;

(i) $$\text{C} + \text{O}_2 \rightarrow \text{CO}_2$$

On mass basis, (12) + (32) → (44)

$$1 \text{ kg (C)} + \frac{8}{3} \text{ kg}(\text{O}_2) \rightarrow \frac{11}{3} \text{ kg}(\text{CO}_2)$$

$$1 \text{ mol (C)} + 1 \text{ mol } (\text{O}_2) \rightarrow 1 \text{ mol } (\text{CO}_2)$$

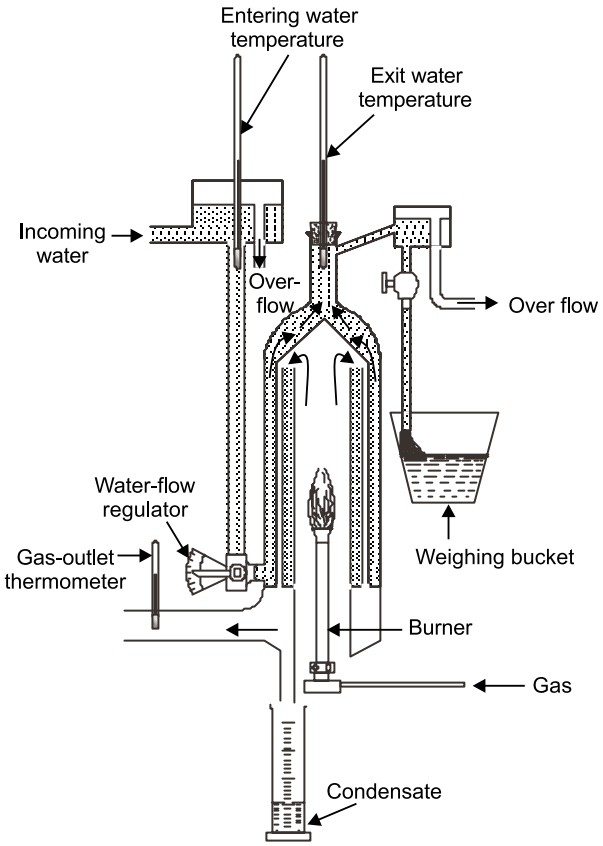


Fig. 10.2 *Gas calorimeter*

(ii) $2C + O_2 \rightarrow 2CO$

On mass basis, (24) + (32) → (56)

$$1 \text{ kg (C)} + \frac{4}{3} \text{ kg } (O_2) \rightarrow \frac{7}{3} \text{ kg (CO)}$$

On mole basis, 2 mol (C) + 1 mol (O_2) → 2 mol (CO)

(iii) $2CO + O_2 \rightarrow 2CO_2$

On mass basis, 56 + 32 → 88

$$1 \text{ kg (CO)} + \frac{4}{7} \text{ kg } (O_2) \rightarrow \frac{11}{7} \text{ kg } (CO_2)$$

On mole basis, 2 mol (CO) + 1 mol (O_2) → 2 mol (CO_2)

(iv) $2H_2 + O_2 \rightarrow 2H_2O$

On mass basis, 4 + 32 → 36

1 kg (H_2) + 8 kg (O_2) → 9 kg (H_2O)

On mole basis, 2 mol (H_2) + 1 mol (O_2) → 2 mol (H_2O)

(v) $S + O_2 \rightarrow SO_2$

On mass basis, 32 + 32 → 64

1 kg (S) + 1 kg (O_2) → 2 kg (SO_2)

On mole basis, 1 mol (S) + 1 mol (O_2) → 1 mol (SO_2)

(vi) $$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

On mass basis, 16 + 64 → 44 + 36

$$1 \text{ kg } (CH_4) + 4 \text{ kg } (O_2) \rightarrow \frac{11}{4} \text{ kg } (CO_2) + \frac{9}{4} \text{ kg } (H_2O)$$

On mole basis,

1 mol (CH_4) + 2 mol (O_2) → 1 mol (CO_2) + 2 mol (H_2O)

(vii) $$2C_2H_6 + 7O_2 \rightarrow 4CO_2 + 6H_2O$$

On mass basis, 60 + 224 → 176 + 108

$$1 \text{ kg } (C_2H_6) + \frac{56}{15} \text{ kg } (O_2) \rightarrow \frac{44}{15} \text{ kg } (CO_2) + \frac{27}{15} \text{ kg } (H_2O)$$

On mole basis,

2 mol (C_2H_6) + 7 mol (O_2) → 4 mol (CO_2) + 6 mol (H_2O)

(viii) In general, for any hydrocarbon's complete combustion,

$$a \cdot C_8H_{18} + b \cdot O_2 \rightarrow d \cdot CO_2 + e \cdot H_2O$$

Equating C, H and O on both sides of above equation,

$a \times 8 = d$ or, $d = 8a$

$a \times 18 = e \times 2$ or, $e = 9a$

$b \times 2 = (d \times 2) + e$ or, $2b = 2d + e$, or $2b = 16a + 9a$

Above yield, $b = 12.5a$, $d = 8a$, $e = 9a$

In order to avoid fraction let us round it off by multiplying by 2,

$$2a\ C_8H_{18} + 25a \cdot O_2 \rightarrow 16a \cdot CO_2 + 18a \cdot H_2O$$

Combustion equation shall now be,

$$2C_8H_{18} + 25O_2 \rightarrow 16CO_2 + 18H_2O$$

On mass basis, 228 + 800 → 704 + 324

$$1 \text{ kg } (C_8H_{18}) + \frac{200}{57} \text{ kg } (O_2) \rightarrow \frac{176}{57} \text{ kg } (CO_2) + \frac{81}{57} \text{ kg } (H_2O)$$

On mole basis,

$$2 \text{ mol } (C_8H_{18}) + 25 \text{ mol } (O_2) \rightarrow 16 \text{ mol } (CO_2) + 18 \text{ mol } (H_2O)$$

Theoretically, the calorific value of fuel can be determined based upon the fuel constituents and the heat evolved upon their oxidation.

Heat evolved during oxidation of some of the fuel constituents are as under,

Table 10.4

Fuel constituent	*Higher calorific value, kcal/kg*	*Lower calorific value, kcal/kg*	*Products of oxidation*
C	8100	—	CO_2
C	2420	—	CO
CO	2430	—	CO_2
H_2	34400	29000	H_2O
S	2200	—	SO_2

In any fuel containing carbon (C), hydrogen (H_2), oxygen (O) and sulphur (S) the higher calorific value of fuel can be estimated using respective calorific values for constituents. In the fuel oxygen is not present in free form but is associated with hydrogen.

$$\underset{(2)}{H_2} + \underset{(16)}{O} \rightarrow \underset{(18)}{H_2O}$$

On mass basis, $1 \text{ kg } (H_2) + 8 \text{ kg } (O) \rightarrow 9 \text{ kg } (H_2O)$

$$\frac{1}{8} \text{ kg } (H_2) + 1 \text{ kg } (O) \rightarrow \frac{9}{8} \text{ kg } (H_2O)$$

Above chemical equation indicates that with every unit mass of oxygen, $\frac{1}{8}$ of oxygen mass shall be the mass of hydrogen associated with it. Thus, the free hydrogen available for oxidation (combustion) shall be only $\left(H - \frac{O}{8}\right)$ where H refers to hydrogen mass and O refers to oxygen mass.

The higher calorific value of fuel can thus be given as under using the mass fractions of constituent elements known. If percentage mass fractions of fuel constituents are given by C, H, O and S then.

$$\text{H.C.V. of fuel} = \frac{1}{100} \left[8100 \cdot C + 34{,}400 \left(H - \frac{O}{8}\right) + 2220 \cdot S\right] \text{ kcal/kg}$$

Lower calorific value of fuel can be given by,

L.C.V. of fuel = (H.C.V. of fuel) – (Heat carried by water vapour formed per kg of fuel burnt)

The amount of latent heat carried depends upon the pressure at which evaporation takes place and quantity of water vapour formed. Generally, the evaporation is considered to take place at saturation temperature of 15°C and the latent heat of water vapour at this saturation temperature is 588.76 kcal/kg. During combustion of fuel the water shall be formed due to hydrogen present in fuel, therefore mass of water vapour can be given by the mass fraction of hydrogen. Thus,

$$\text{L.C.V. of fuel} = \left\{(\text{H.C.V. of fuel}) - \left(\frac{9 \times H \times 588.76}{100}\right)\right\}, \text{ kcal/kg}$$

10.7 COMBUSTION ANALYSIS

From earlier discussions it is seen how the combustion of a fuel can be written in the form of chemical reactions for the oxidation of different elements constituting fuel. Based on chemical reactions the mass of oxygen required per kg of element can be estimated. From these the oxygen requirement per kg of fuel constituents and subsequently mass of air required for the estimated oxygen requirement can be known. Let us see how air required for complete burning of a fuel having 85.5% carbon, 12.3% hydrogen and 2.2% ash is calculated;

Mass of oxygen required for constituents;

1 kg of carbon shall require $\frac{8}{3}$ kg of oxygen

1 kg of hydrogen shall require 8 kg of oxygen

Ash shall not undergo oxidation.

Table 10.5 *Air requirement for fuel (on mass basis)*

Mass of constituents per kg of fuel (A)	*Oxygen required per kg of constituent (B)*	*Oxygen required per kg of fuel (C) = (A) × (B)*	*Mass of air required, (theoretical, for air having 23% oxygen, 77% nitrogen)* $D = \frac{\Sigma(C) \times 100}{23}$
C = 0.855	8/3	2.28	
H_2 = 0.123	8	0.984	$= \frac{3.264 \times 100}{23} = 14.19$
Ash = 0.022	—	—	
		ΣC = 3.264	

Above calculations show that for the given fuel composition 14.19 kg of minimum air shall be theoretically required for complete combustion of one kg of fuel. Combustion analysis can be carried out either on mass basis or volume basis. Similar kind of analysis can be made on volume basis. Let us take heptane (C_7H_{16}) as the fuel and consider its' complete combustion.

Table 10.6 *Air required for fuel (on volume basis)*

Constituent	*Proportional mass, A*	*Proportional volume,* $B = \frac{A}{mol.wt.}$	*Percentage volume,* $C = \frac{B}{\Sigma B}$	*O_2 required per m^3 of D × 100*	*O_2 required per 100 m^3 of element,* $E = D \times C$ *element*	*Air required for 100 m^3 of fuel (theoretical for air having 21% O_2 and 79% N_2), F* $= \frac{\Sigma E}{21} \times 100$
C	12 × 7 = 84	7	46.67	1	46.67	$= \frac{73.335}{21} \times 100$
H_2	2 × 8 = 16	8	53.33	0.5	26.665	= 349.21
		ΣB = 15			ΣE = 73.335	

Hence volume of air required is 349.21 m^3 for 100 m^3 of fuel.

Volumetric analysis of combustion can be transformed into gravimetric analysis and vice versa. Such conversion is required because the volumetric analysis is available experimentally. Let us take an exhaust gas sample containing CO, CO_2, N_2 and O_2. Let their volume fractions be X_1, X_2, X_3 and X_4 respectively.

Table 10.7 Conversion from volume based analysis to mass based analysis

Constituent	*Volume of Constituent per m^3, V*	*Molecular weight, M*	*Proportional masses, X = M.V*	*Mass of constituent per kg of dry exhaust gases,* $m = \frac{M.V}{\Sigma X}$
CO	X_1	28	$28.X_1$	$\left(\frac{28 \cdot X_1}{\Sigma X}\right)$
CO_2	X_2	44	$44.X_2$	$\left(\frac{44 \cdot X_2}{\Sigma X}\right)$
N_2	X_3	28	$28.X_3$	$\left(\frac{28 \cdot X_3}{\Sigma X}\right)$
O_2	X_4	32	$32.X_4$	$\left(\frac{32 \cdot X_4}{\Sigma X}\right)$

Total of proportional masses $\Sigma X = 28X_1 + 44X_2 + 28X_3 + 32X_4$

10.8 DETERMINATION OF AIR REQUIREMENT

(a) From combustion equations of fuel constituents (percentage basis) such as C, H, O and S as described earlier the total oxygen required per kg of fuel for complete combustion can be given as;

$$= \frac{1}{100}\left[\frac{8}{3}\,\mathrm{C} + 8\left(\mathrm{H} - \frac{\mathrm{O}}{8}\right) + \mathrm{S}\right]$$

As air is taken to have 23% of oxygen by mass, therefore total air requirement per kg of fuel shall be;

$$= \frac{1}{100}\left[\frac{8}{3}\,\mathrm{C} + 8\left(\mathrm{H} - \frac{\mathrm{O}}{8}\right) + \mathrm{S}\right] \times \frac{100}{23}$$

Minimum air required per kg of fuel for complete combustion

$$= \frac{1}{23}\left[\frac{8}{3}\,\mathrm{C} + 8\left(\mathrm{H} - \frac{\mathrm{O}}{8}\right) + \mathrm{S}\right]$$

(b) Air requirement per kg of fuel can also be determined if the volumetric analysis of dry flue gases and mass fraction of carbon per kg of fuel is known. Let us take the dry flue gases having mass fractions as defined earlier as,

1 kg of flue gas has $\left(\frac{28X_1}{\Sigma X}\right)$ kg of CO and $\left(\frac{44X_2}{\Sigma X}\right)$ kg of CO_2.

Now the total mass of carbon present in CO and CO_2 present in 1 kg of flue gas can be estimated and equated with mass of carbon present per kg of fuel.

Mass of carbon per kg of flue gas = Mass of carbon in CO per kg flue gas
+ Mass of carbon in CO_2 per kg flue gas

Mass of carbon in CO per kg flue gas = $\left(\frac{28X_1}{\Sigma X}\right) \times \frac{12}{28} = \left(\frac{12X_1}{\Sigma X}\right)$

Mass of carbon in CO_2 per kg flue gas = $\left(\frac{44X_2}{\Sigma X}\right) \times \frac{12}{44} = \left(\frac{12X_2}{\Sigma X}\right)$

Total carbon per kg of flue gas = $\left\{\frac{12X_1}{\Sigma X} + \frac{12X_2}{\Sigma X}\right\}$

$= \frac{12}{\Sigma X}$ $(X_1 + X_2)$kg per kg of flue gas.

Now if we look at the fact that from where carbon is coming in flue gases then it is obvious that carbon is available only in fuel. Let us assume that carbon present in fuel completely goes into flue gases. Let us also assume that fuel does not contain nitrogen so what ever nitrogen is there in flue gas it will be because of nitrogen present in air. Let the mass of flue gases formed per kg of fuel after combustion be 'm_g'. The mass of carbon in per kg fuel say C can be equated to total mass of carbon calculated above.

Mass fraction of carbon in fuel = Total mass of carbon in m_g mass of flue gases.

$$C = \left\{\frac{12}{\Sigma X}(X_1 + X_2)\right\} \times m_g$$

$$\text{Mass of flue gases, } m_g = \left\{\frac{C \cdot \Sigma X}{12(X_1 + X_2)}\right\}$$

Mass of air supplied per kg of fuel can be known from the nitrogen fraction present in flue gases as only source of nitrogen is air. 0.77 kg of N_2 is available in 1 kg of air. From flue gas analysis $\left(\frac{28X_3}{\Sigma X}\right)$ is the mass of nitrogen available in per kg of flue gas so total mass of nitrogen present in flue gases (m_g) due to combustion of unit mass of fuel shall be $\left\{\left(\frac{28X_3}{\Sigma X}\right) \cdot m_g\right\}$.

From air composition 0.77 kg N_2 is present per kg of air so the mass of air supplied per kg of flue gas formed,

$= \left(\frac{28X_3}{\Sigma X}\right) \times \frac{1}{0.77}$, kg air per kg of flue gas

Mass of air supplied per kg of fuel $= \left(\frac{28X_3}{\Sigma X}\right) \times \frac{m_g}{0.77}$

Substituting for m_g, $= \left(\frac{28X_3}{\Sigma X}\right) \times \frac{1}{0.77} \times \left\{\frac{C \cdot \Sigma X}{12(X_1 + X_2)}\right\}$

$$\text{Mass of air supplied per kg of fuel} = \frac{28X_3 \cdot C}{9.24(X_1 + X_2)}$$

where X_1, X_2, X_3 are volume fractions of CO, CO_2 and N_2 present in unit volume of flue gas and C is mass fraction of carbon present in unit mass of fuel.

For complete combustion of fuel there shall be no CO and only CO_2 shall contain carbon i.e. $X_1 = 0$.

(c) Generally for ensuring complete combustion of fuel, excess air is supplied. In case of combustion if there is incomplete combustion of carbon resulting into formation of carbon monoxide, then

additional oxygen shall be required for converting carbon monoxide (CO) into carbon dioxide (CO_2). From earlier discussions let us consider unit mass of flue gas containing $\left(\frac{28X_1}{\Sigma X}\right)$ kg of CO and $\left(\frac{32X_4}{\Sigma X}\right)$ kg of O_2. The oxygen present in fuel remains in association with hydrogen. For combustion of CO into CO_2 the mass of oxygen required is $\frac{4}{7}$ kg per kg of CO. Thus oxygen required for CO present in 1 kg flue gas

$$= \left(\frac{28X_1}{\Sigma X}\right) \times \frac{4}{7}$$

$$= \left(\frac{16X_1}{\Sigma X}\right) \text{ kg of oxygen}$$

Therefore, excess oxygen available per kg of flue gas formed,

$$= \left(\frac{32X_4}{\Sigma X}\right) - \left(\frac{16X_1}{\Sigma X}\right)$$

$$= \frac{16}{\Sigma X}\ (2X_4 - X_1)$$

Excess oxygen supplied per kg of fuel burnt $= \left\{\frac{16}{\Sigma X}(2X_4 - X_1)m_g\right\}$

$$= \frac{16}{\Sigma X}(2X_4 - X_1)\left\{\frac{C\Sigma X}{12(X_1 + X_2)}\right\}$$

$$= \frac{4}{3}\frac{(2X_4 - X_1)\cdot \mathrm{C}}{(X_1 + X_2)}$$

Hence excess air supplied per kg of fuel

$$= \frac{4C(2X_4 - X_1)}{3(X_1 + X_2)} \times \frac{1}{0.23}$$

Excess air supplied per kg of fuel burnt $= \dfrac{4C\cdot(2X_4 - X_1)}{0.69(X_1 + X_2)}$

10.9 FLUE GAS ANALYSIS

Flue gas analysis refers to the determination of composition of exhaust gases. Flue gas analysis can be done theoretically and experimentally. Here experimental method of flue gas analysis is described. Various devices available for measuring the composition of products of combustion (flue gas) are Orsat Analyzer, Gas chromatograph. Infrared analyzer and Flame ionisation detector etc. Data from these devices can be used to determine the mole fraction of flue gases. Generally this analysis is done on dry basis which may also be termed as “dry product analysis” and it refers to describing mole fractions for all gaseous products except water vapour.

Orsat analyzer: It is also called as Orsat apparatus and is used for carrying out volumetric analysis of dry products of combustion. Schematic of apparatus is shown in Fig.10.3 It has three flasks containing different chemicals for absorption of CO_2, O_2 and CO respectively and a graduated eudiometer tube connected to an aspirator bottle filled with water.

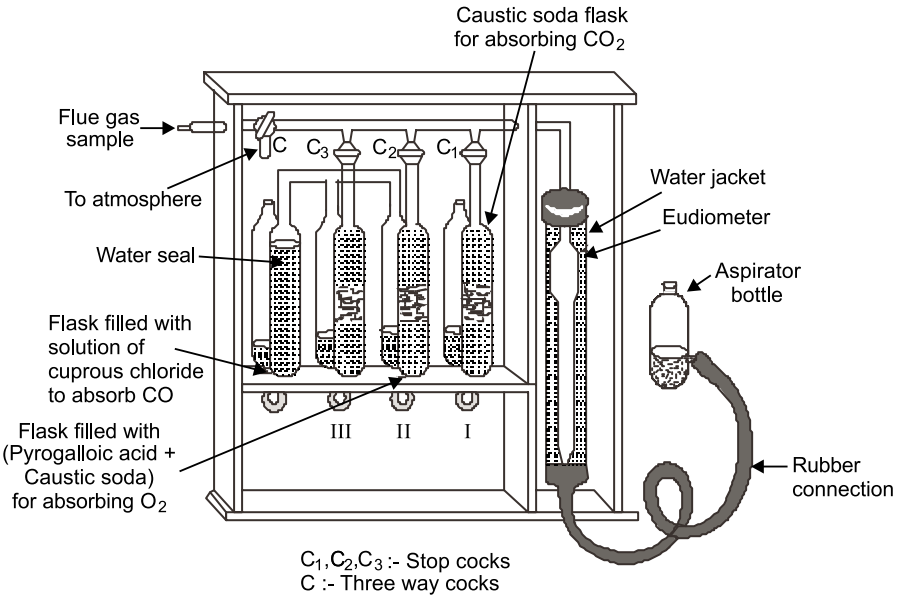


Fig. 10.3 *Orsat analyzer*

Flask I is filled with NaOH or KOH solution (about one part of KOH and 2 parts of water by mass). This 33% KOH solution shall be capable of absorbing about fifteen to twenty times its own volume of CO_2. Flask II is filled with alkaline solution of pyrogallic acid and above KOH solution. Here 5 gm of pyrogallic acid powder is dissolved in 100 cc of KOH solution as in Flask I. It is capable of absorbing twice its own volume of O_2. Flask III is filled with a solution of cuprous chloride which can absorb CO equal to its' volume. Cuprous chloride solution is obtained by mixing 5 mg of copper oxide in 100 cc of commercial HCl till it becomes colourless. Each flask has a valve over it and C_1, C_2, C_3 valves are put over flasks I, II and III. All the air or any other residual gas is removed from eudiometer by lifting the aspirator bottle and opening main value. The flue gas for analysis is taken by opening the main valve (three way valve) while valves C_1, C_2 and C_3 are closed. 100 cc of flue gas may be taken into eudiometer tube by lowering aspirator bottle until the level is zero and subsequently forced into flasks for absorbing different constituents. Aspirator bottle is lifted so as to inject flue gas into flask I with only valve C_1 in open state where CO_2 present shall be absorbed. Aspirator bottle is again lowered and reading of eudiometer tube taken. Difference in readings of eudiometer tube initially and after CO_2 absorption shall give percentage of CO_2 by volume. Similar steps may be repeated for getting O_2 and CO percentage by volume for which respective flask valve shall be opened and gas passed into flask. Thus Orsat analyzer directly gives percentage by volume of constituents. In case of other constituents to be estimated the additional flasks with suitable chemical may be used. The remaining volume in eudiometer after absorption of all constituents except N_2 shall give percentage volume of N_2 in flue gas.

As in combustion of hydrocarbon fuel the H_2O is present in flue gases but in orsat analysis dry flue gases are taken which means H_2O will be condensed and separated out. Therefore the percentage

by volume of constituents estimated shall be on higher side as in actual product H_2O is there but in dry flue gas it is absent. Orsat analyzer does not give exact analysis.

10.10 FUEL CELLS

Fuel cell refers to a device having fuel and oxidizer in it which undergoes controlled chemical reaction to produce combustion products and provide electric current directly. In fuel cells the fuel and oxidizer react in stages on two separate electrodes i.e. anode (+ ive electrode) and cathode (– ive electrode). Two electrodes are separated by electrolyte in between. The chemical reaction is carried out to produce electric power without moving parts or the use of intermediate heat transfers as in power cycles. Thus fuel cell does not work on any cycle. Fuel cells based on hydrogen-oxygen fuel cells have been used to provide power to the spacecrafts. Fuel cells based on natural gas are also under process of development. Fuel cells are also being developed to power automobiles. Let us look at hydrogen-oxygen fuel cell in detail.

Figure 10.4 shows the schematic of hydrogen-oxygen fuel cell. Here H_2 supplied diffuses through the porous anode and reacts on anode surface with OH^-ions resulting into H_2O and free electrons, the reaction for it is given in figure.

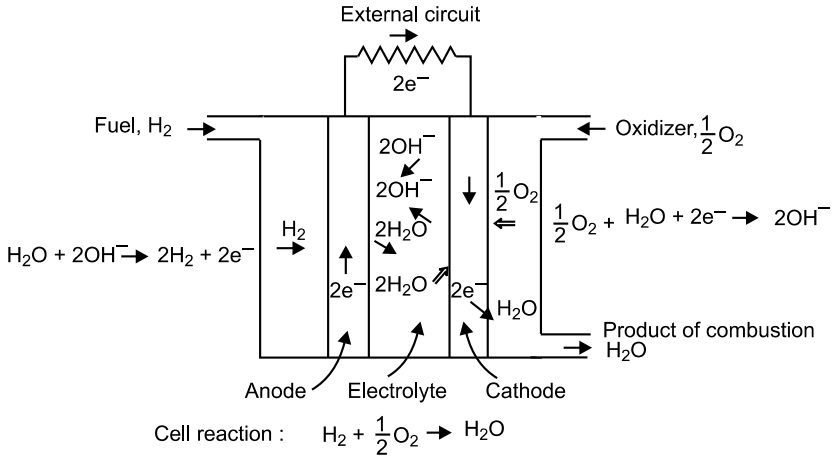


Fig. 10.4 *Hydrogen-Oxygen fuel cell*

Free electrons liberated enter the circuit while water goes into electrolyte. Oxygen supplied combines with water in electrolyte and electrons coming from electric circuit to produce OH^- ions and H_2O as per chemical reaction given in figure. OH^- ions are transported through the electrolyte. Overall fuel cell has chemical reaction as

$$H_2 + \frac{1}{2}O_2 \rightarrow H_2O$$

Thus in hydrogen-oxygen fuel cell electricity and water are produced.

EXAMPLES

1. *Coal having following composition by mass is burnt with theoretically correct amount of air.*

86% C, 6% H, 5% O, 2% N, 1% S

Determine the air-fuel ratio.

Solution:

Combustion equation for the coal (100 kg of coal) can be given as under;

$$\left(\frac{86}{12}\cdot C+\frac{6}{1}\cdot H+\frac{5}{16}\cdot O+\frac{2}{14}\cdot N+\frac{1}{32}\cdot S\right)+n(O_2+3.76\ N_2)\rightarrow$$

$$a\cdot CO_2+b\cdot SO_2+d\cdot N_2+e\cdot H_2O$$

From above equation C, H, O, S, N can be equated on both the sides as under,

$$\text{C; } 7.16 = a$$
$$\text{H; } 6 = 2e$$
$$\text{O; } (0.3125 + 2n) = 2a + 2b + e$$
$$\text{N; } (0.1429 + 3.76 \times 2n) = 2d$$
$$\text{S; } 0.03125 = b$$

Solving we get $a = 7.16$

$b = 0.03125$

$d = 32.163$

$e = 3$

$n = 8.535$

Amount of air required shall be $[8.535 \times (4.76)]$ kg mol per 100 kg of coal.

$$\text{Air-fuel ratio} = \frac{8.535\times 4.76\times 28.97}{100}$$

= 11.77 kg air per kg of fuel

Air-fuel ratio = 11.77 ***Ans.***

2. *One kg C_8H_{18} fuel is supplied to an engine with 13 kg of air. Determine the percentage by volume of CO_2 in dry exhaust gas considering exhaust gas to consist of CO_2, CO and N_2.*

Solution:

Combustion equation in mol. basis for one kg of fuel supplied shall be as under.

$$\frac{1}{114}(C_8H_{18})+n(0.21\ O_2+0.79\ N_2)\rightarrow a\cdot CO_2+b\cdot CO+d\cdot N_2+e\cdot H_2O$$

Equating the coefficients on both sides,

$$\text{C; } \frac{8}{114} = 0.0702 = a + b$$

$$O_2\text{; } 0.21\ n = a + \frac{b}{2} + \frac{e}{2}$$

$$\text{H; } \frac{18}{114} = 0.1579 = 2e$$

N_2; $0.79\ n = d$. Also it is given that 13 kg of air per kg of fuel is supplied, therefore

$$n = \frac{13}{28.97} = 0.4487$$

Solving above following are available,

$a = 0.0393$

$b = 0.0309$

$d = 0.3545$

$e = 0.07895$

$n = 0.4487$

Constituents of dry exhaust gas shall be CO_2, CO and N_2 as indicated,

$$\text{Therefore dry exhaust gas} = (a + b + d)$$

$$= (0.0393 + 0.0309 + 0.3545) = 0.4247$$

$$\text{Percentage by volume of } CO_2 \text{ in dry exhaust gas} = \frac{a}{(a+b+d)} \times 100$$

$$= \frac{0.0393 \times 100}{0.4247}$$

$$= 9.25\%$$

% by volume of CO_2 = 9.25% ***Ans.***

3. *In a boiler the coal having 88% C, 3.8% H_2, 2.2% O_2 and remaining ash is burnt in the furnace. It is found that CO_2 going with flue gases constitute to be 12% and temperature of flue gases is 260°C. The flue gas sample is analyzed using Orsat apparatus at room temperature. Determine the percentage of CO_2 that would be there for complete combustion of fuel.*

Solution:

Actual percentage of CO_2 in flue gases = 12%

Constituent mass per kg of fuel	*Combustion product (x)*		*Molecular weight (y)*		*Volume fraction z = (x)/(y)*	
	CO_2	H_2O	CO_2	H_2O	CO_2	H_2O
C = 0.88	$\left(0.88 \times \frac{44}{12}\right)$		44		0.0733	
H_2 = 0.038		$\left(0.038 \times \frac{18}{2}\right)$		18		0.019

Exhaust gases shall comprise of CO_2, H_2O, O_2 and N_2

$$\text{Actual percentage of } CO_2 = \frac{CO_2 \times 100}{(CO_2 + H_2O + O_2 + N_2)}$$

$$12 = \frac{0.0733 \times 100}{(0.0733 + 0.019 + (O_2 + N_2))}$$

$$(O_2 + N_2) = 0.5185$$

Orsat apparatus analyzes only dry flue gas so the percentage of CO_2 can be obtained as,

$$\%\ CO_2 = \frac{CO_2 \times 100}{(CO_2 + O_2 + N_2)} = \frac{0.0733 \times 100}{(0.0733 + 0.5185)} = 12.38\%$$

% CO_2 = 12.38% ***Ans.***

4. *Determine the percentage analysis of combustion products by mass and by volume when gravimetric analysis of a hydrocarbon fuel indicates 86% C and 14% H_2. Excess air supplied is 50% for combustion. Consider air to have 23.2% of oxygen by mass and remaining as nitrogen, molecular weights of carbon, oxygen, nitrogen and hydrogen are 12, 16, 14 and 1 respectively. Also estimate the change in internal energy of per kg of products when cooled from 2100°C to 900°C when the internal energies of combustion products are as under,*

T°C		*Internal energy, kJ/kg*		
	CO_2	H_2O	N_2	O_2
2100	2040	3929	1823	1693
900	677	1354	694	635

Solution:

Theoretical mass of oxygen required per kg of fuel depending upon its' constituents

$$C + O_2 \rightarrow CO_2$$
$$12 \text{ kg} + 32 \text{ kg} \rightarrow 44 \text{ kg}$$

or, $$1 \text{ kg} + \frac{8}{3} \text{ kg} \rightarrow \frac{11}{3} \text{ kg}$$

or, $$0.86 \text{ kg} + 2.29 \text{ kg} \rightarrow 3.15 \text{ kg}$$

For hydrogen,

$$2H_2 + O_2 \rightarrow 2H_2O$$
$$4 \text{ kg} + 32 \text{ kg} \rightarrow 36 \text{ kg}$$
$$1 \text{ kg} + 8 \text{ kg} \rightarrow 9 \text{ kg}$$
$$0.14 \text{ kg} + 1.12 \text{ kg} \rightarrow 1.26 \text{ kg}$$

Hence, minimum mass of oxygen required kg per kg of fuel = $\left(0.86 \times \frac{8}{3} + 0.14 \times 8\right)$ = 3.41 kg

As it is given that 50% excess air is supplied so the excess oxygen = $3.41 \times 0.5 = 1.705$ kg

Total oxygen supplied = 3.41 + 1.705 = 5.115 kg

Constituents of exhaust gases per kg of fuel, $CO_2 = \frac{11}{3} \times 0.86 = 3.15$ kg

$H_2O = 9 \times 0.14 = 1.26$ kg

$N_2 = \left(\frac{5.115 \times 76.8}{23.2}\right) = 16.932$ kg

Excess $O_2 = 1.705$ kg

For one kg of fuel the composition of combustion products is as under,

Exaust gas constituents (x)	*Gravimetric analysis %* $g = \frac{(x \times 100)}{\Sigma(x)}$	*Molecular weight, y*	*Proportional volume,* $z = \frac{g}{y}$	*Volumetric analysis %,* $v = \frac{(z \times 100)}{\Sigma(z)}$
CO_2 = 3.15 kg	13.67	44	0.311	8.963
H_2O = 1.26 kg	5.47	18	0.304	8.761
N_2 = 16.932 kg	73.47	28	2.624	75.619
O_2 = 1.705 kg	7.39	32	0.231	6.657
Total = 23.047	100%		3.47	100%

Change in internal energy for given temperature variation shall be estimated as under, on gravimetric basis

Mass fraction of constituents of exhaust gases (g)	*Change in internal energy per kg of constituents, (b) kJ/kg*	*Change in internal energy (c) = (g × b), kJ*
CO_2 = 0.1367	2040 – 677 = 1363	186.32
H_2O = 0.0547	3929 – 1354 = 2575	140.85
N_2 = 0.7347	1823 – 694 = 1129	829.48
O_2 = 0.0739	1693 – 635 = 1058	78.19

Total change in internal energy = 1234.84 kJ/kg of exhaust gases

Change in internal energy = 1234.84 kJ/kg of exhaust gases ***Ans.***

5. *Determine the percentage excess air supplied to boiler for burning the coal having following composition on mass basis,*

C: 0.82
H_2: 0.05
O_2: 0.08
N_2: 0.03
S: 0.005
moisture: 0.015

Volumetric analysis of dry flue gases shows the following composition,

CO_2 = 10%
CO = 1%
N_2 = 82%
O_2 = 7%

Solution:

For constituent components combustion reactions are as under. Out of given constituents only C, H_2, and S shall require oxygen for burning reaction.

$$C + O_2 \rightarrow CO_2$$
$$12 \text{ kg} + 32 \text{ kg} \rightarrow 44 \text{ kg}$$
$$1 \text{ kg} + \frac{8}{3} \text{ kg} \rightarrow \frac{11}{3} \text{ kg}$$
$$2H_2 + O_2 \rightarrow 2H_2O$$
$$4 \text{ kg} + 32 \text{ kg} \rightarrow 36 \text{ kg}$$
$$1 \text{ kg} + 8 \text{ kg} \rightarrow 9 \text{ kg}$$
$$S + O_2 \rightarrow SO_2$$
$$32 \text{ kg} + 32 \text{ kg} \rightarrow 64 \text{ kg}$$
$$1 \text{ kg} + 1 \text{ kg} \rightarrow 2 \text{ kg}$$

Net mass of oxygen required per kg of coal

$$= \{(0.82 \times \frac{8}{3}) + (0.05 \times 8) + (0.005 \times 1) - (0.08)\}$$
$$= 2.512 \text{ kg}$$

Mass of air required per kg of coal, considering oxygen to be 23% in the air by mass,

$$= \frac{100}{23} \times 2.512$$

= 10.92 kg air per kg of coal

Volume of constituent per mol. of dry flue gas % (a)	*Molecular wt. (b)*	*Proportional mass of constituents* $c = (a \times b)$	*Mass per kg of flue gas* $d = \frac{c}{\Sigma(c)}$	*Mass of carbon per kg of dry flue gas*
$CO_2 = 10$	44	440	0.147	$\frac{0.147 \times 12}{44} = 0.04009$
$CO = 1$	28	28	0.0094	$\frac{0.0094 \times 12}{28} = 0.00403$
$N_2 = 82$	28	2296	0.768	
$O_2 = 7$	32	224	0.075	Total carbon per kg of dry flue gas = 0.04412
		$\Sigma(c) = 2988$		

Carbon is given to be 0.82 kg in per unit mass of coal and the mass of carbon per unit mass of dry flue gas is 0.04412 kg so the mass of dry flue gases per kg of coal shall be

$$= \frac{\text{Carbon mass per kg of dry flue gases}}{\text{Carbon mass per kg of coal}}$$

$$= \frac{0.82}{0.04412}$$

= 18.59 kg

Therefore, the mass of CO per kg of coal = 18.59 × 0.0094

= 0.1747 kg

The mass of excess O_2 per kg of coal (i.e. unutilized O_2) = 18.59 × 0.075

= 1.394 kg

The CO produced in combustion products per kg of coal shall further require O_2 for its' complete burning to CO_2.

$$2CO + O_2 \rightarrow 2CO_2$$

$$56 \text{ kg} + 32 \text{ kg} \rightarrow 88 \text{ kg}$$

$$1 \text{ kg} + \frac{4}{7} \text{ kg} \rightarrow \frac{11}{7} \text{ kg}$$

The mass of O_2 required for complete burning of 0.1747 kg CO per kg of coal shall be = $0.1747 \times \frac{4}{7} = 0.998$ kg.

Out of excess O_2 coming out with dry flue gases 0.0998 kg of O_2 shall be utilized for complete burning of CO. Thus, the net excess O_2 per kg of coal = 1.394 – 0.0998 = 1.2942 kg O_2

Hence, excess air required for 1.2942 kg O_2 = $\frac{1.2942 \times 100}{23} = 5.627$ kg air

$$\% \text{ excess air} = \frac{5.627}{10.92} \times 100 = 51.53\%$$

$\boxed{\% \text{ excess air} = 51.53\%}$ ***Ans.***

6. *C_2H_6 burns completely with air when the air-fuel ratio is 18 on mass basis. Determine the percent excess or percent deficiency of air as appropriate and the dew point temperature of combustion products when cooled at 1 atm.*

Solution:

Given air-fuel ratio on mass basis is first transformed into the air-fuel ratio on molar basis.

$$\text{A/F ratio on molar basis} = \frac{\text{No. of moles of air}}{\text{No. of moles of fuel}} = \frac{\left(\frac{\text{Mass of air}}{\text{Mol. wt. of air}}\right)}{\left(\frac{\text{Mass of fuel}}{\text{Mol. wt. of fuel}}\right)}$$

$$= \frac{\text{Mass of air}}{\text{Mass of fuel}} \times \frac{\text{Mol. wt. of fuel}}{\text{Mol. wt. of air}}$$

$$= (\text{A/F ratio on mass basis}) \times \frac{\text{Mol. wt. of } C_2H_6}{\text{Mol. wt. of air}}$$

$$= 18 \times \frac{30}{29}$$

A/F ratio on molar basis = 18.62

Chemical reaction for complete combustion may be given as,

$$C_2H_6 + n(O_2 + 3.76\ N_2) \rightarrow aCO_2 + bN_2 + dO_2 + eH_2O$$

Also, for complete combustion there will be $\left(\frac{18.62}{4.76}\right)$ moles of air for each mole of fuel,

So, $n = 3.912$

Equating coefficient

C: $2 = a$	Solving we get,	$a = 2$
H: $6 = 2e$		$b = 14.71$
O: $2n = 2a + 2d + e$		$d = 0.412$
N: $(3.76 \times 2n) = 2b$		$e = 3$

Complete combustion reaction may be given as,

$$C_2H_6 + 3.912\ (O_2 + 3.76\ N_2) \rightarrow 2CO_2 + 14.71N_2 + 0.412O_2 + 3H_2O$$

or, $$C_2H_6 + 3.5(O_2 + 3.76\ N_2) \rightarrow 2CO_2 + 13.16\ N_2 + 3H_2O$$

Hence air-fuel ratio (theoretical) on mol. basis

$$(\text{A/F}) = \frac{3.5 \times 4.76}{1} = 16.66$$

Since theoretical air-fuel ratio is less than actual so it means excess air is supplied.

$$\text{Percentage of excess air} = \frac{(\text{A/F})_{\text{actual}} - (\text{A/F})_{\text{theoretical}}}{(\text{A/F})_{\text{theoretical}}} \times 100$$

$$= \left(\frac{18.62 - 16.66}{16.66}\right) \times 100$$

$= 11.76\%$

Percentage of excess air = 11.76% ***Ans.***

Total amount of mixture = $(a + b + d + e)$

$= (2 + 14.71 + 0.412 + 3)$

$= 20.122$ kg/mol. of fuel

Partial pressure of water vapour $P = \left(\frac{e}{a+b+d+e}\right) \times 1.013$

$= 0.151$ bar

From steam table saturation temperature corresponding to 0.151 bar is seen to be 54°C.

Dew point temperature = 54°C ***Ans.***

7. *Obtain the volumetric composition of combustion products obtained after combustion of C_7H_{16} (heptane) being burnt with 50% excess air. Also obtain the average molecular weight and specific volume of combustion products at S.T.P. Consider the volume per kg mol. at S.T.P. to be 22.4 m^3 and air to have 21% O_2 and 79% N_2 by volume.*

Solution:

Here excess air supplied shall result in unutilized O_2 with combustion products.

*Excess O_2 supplied = $0.5 \times 73.33 = 36.665$ m^3

Total O_2 supplied = $73.33 + 36.665 = 109.995 m^3$

**Total N_2 supplied = $(109.995) \times \frac{79}{21} = 413.79 m^3$

Total volume of combustion products = $\{46.67 + 53.33 + 36.665 + 413.79\}$

$= 550.455$ m^3

Average molecular weight of combustion products can be estimated by summation of mass of constituents per mol. of combustion products.

Constituents of combustion products	*Fraction of constituents per unit products* $b = \frac{a}{\Sigma(a)}$	*Molecular weight (c)*	*Mass of constituents per mol. of combustion* $(d = b \times c)$	*Average molecular weight of combustion products* $c = \Sigma(d)$
CO_2(= 46.67)	$\frac{46.67}{550.455} = 0.0848$	44	3.7312	
H_2O(= 53.33)	$\frac{53.33}{550.455} = 0.0969$	18	1.7442	28.65
O_2(= 36.665)	$\frac{36.665}{550.455} = 0.0666$	32	2.1312	
N_2(= 413.79)	$\frac{413.79}{550.455} = 0.7517$	28	21.0476	

Constituents of C_7H_{16}	*Proportional mass* (a)	*Proportional volume* $(b) = \frac{a}{mol.wt}$	*% Volume* $(c) = \frac{b \times 100}{\Sigma(b)}$	O_2 *required per* m^3 *of fuel* (d)	O_2 *required for* 100 m^3 *of fuel* (e) = c × d	*% Combustion product per*100 m^3 *of fuel*			
						CO_2	H_2O	O_2	N_2
C (mol. wt. = 12)	12 × 7 = 84	7	46.67	1 ($C + O_2 \rightarrow CO_2$) (1 mol 1 mol → 1 mol.)	46.67	46.67			
H_2 (mol. wt. = 2)	2 × 8 = 16	8	53.33	0.5 ($2H_2 + O_2 \rightarrow 2H_2O$) 2 mol 1 mol 2 mol	26.66		53.33		
		Σ(b) = 15			73.33	46.67	53.33	36.665*	413.79

Specific volume = $\frac{22.4}{28.65}$ = 0.7818 m^3/kg

Average mol. wt. = 28.65 Specific volume = 0.7818 m^3/kg	***Ans.***

Mass of fuel having 0.864 kg of carbon = $\frac{0.864}{\left(\frac{\text{Mass of carbon}}{\text{per kg of fuel}}\right)} = \frac{0.864}{0.847}$ = 1.02kg

It shows that one mol of dry flue gases shall require burning of 1.02 kg of fuel and 29.89 kg of air.

So the air-fuel ratio = $\frac{29.89}{1.02}$ = 29.3

% Excess air = $\frac{(29.3-15)\times 100}{15}$ = 95.33%

Air fuel ratio = 29.3 % excess air = 95.33%	***Ans.***

Here the calculations above have been based on balancing the carbon present. These can also be done based on oxygen-hydrogen balance as under;

Alternate approach (Not very accurate)

Mass of oxygen present in one mol of dry flue gas = Mass of oxygen in CO_2
+ Mass of O_2 appearing with dry flue gas

$$= \left(\frac{3.168\times 32}{44}\right) + (0.108\times 32)$$

$$= 5.76 \text{ kg}$$

Mass of oxygen present with nitrogen in air = $\left(\frac{22.96\times 23.2}{76.8}\right)$ = 6.94 kg

Here, the mass of oxygen supplied in air (6.94 kg) is more than oxygen present in dry flue gases (5.76 kg) which indicates that some oxygen gets consumed in formation of water. Orsat analysis analyzes dry flue gas.

Mass of oxygen forming water (H_2O) = 6.94 – 5.76 = 1.18 kg

$$2H_2 + O_2 \rightarrow 2H_2O$$
$$4 \text{ kg} + 32 \text{ kg} \rightarrow 36 \text{ kg}$$
$$1 \text{ kg} + 8 \text{ kg} \rightarrow 9 \text{ kg}$$

Mass of hydrogen in fuel = $\left(\frac{1.18}{8}\right)$ = 0.1475 kg

Mass of fuel = $\frac{0.1475}{0.153}$ = 0.964 kg

Thus one mol of dry flue gases are generated when 0.964 kg of fuel is burnt with 29.89 kg of air.

8. *During Orsat analysis of the combustion products of an engine running on diesel ($C_{12}H_{26}$) the CO_2 and O_2 are found to be 7.2% and 10.8% respectively and rest is N_2 by volume. Determine the air-fuel ratio and percentage excess air considering air to have O_2 and N_2 in proportion of 23.2% and 76.8% respectively by mass.*

Solution:

There are two approaches for getting the air-fuel ratio and subsequently excess air. One is by carbon-balancing and other by balancing of oxygen-hydrogen.

Minimum air required for complete combustion of 1 kg of fuel can be obtained considering the composition of fuel i.e. $C_{12}H_{26}$.

Mass of constituent per kg of fuel, $C_{12}H_{26}$ *(a)*	*Oxygen required per kg of constituents* *(b)*	*Oxygen required per kg of fuel* *(c) = a × b*	*Minimum air required per kg of fuel for complete combustion*
$C = \frac{12 \times 12}{(12 \times 12 + 26 \times 1)} = 0.847$	$\frac{8}{3}$	$\frac{8}{3} \times 0.847 = 2.259$	$\frac{100}{23.2} \times 3.483$
$H_2 = \frac{26 \times 1}{(12 \times 12 + 26 \times 1)} = 0.153$	8	$8 \times 0.153 = 1.224$	= 15 kg
		Total = 3.483	

Dry flue gases contain 7.2% of CO_2, 10.8% O_2 and 82% N_2 by volume. This volumetric composition is to be converted to gravimetric (mass basis) i.e. mol fractions may be estimated as mol is unit of volume.

Mass of constituent in one mol of dry flue gases = Volume fraction × Mol. wt. of constituent

Mass of CO_2 in one mol of dry flue gases = $0.072 \times 44 = 3.168$ kg

Mass of N_2 in one mol of dry flue gases = $0.82 \times 28 = 22.96$ kg

Corresponding to N_2 the mass of air per mol of dry flue gases can be obtained as,

$$= \frac{22.96 \times 100}{76.8} = 29.89 \text{ kg}$$

Carbon present in one mol of dry flue gases = $\left(3.168 \times \frac{12}{44}\right) = 0.864$ kg

Therefore, from the carbon present in one mol of dry flue gases the amount of fuel containing this much of carbon can be calculated.

So, air fuel ratio = $\frac{29.89}{0.964} = 31.0$

% excess air = $\frac{(31-15)}{15} \times 100 = 106.67\%$

Air-fuel ratio and % excess air calculated differ from those estimated using balancing of carbon. Generally, carbon balancing is used for getting accurate results.

9. *Fuel having 88% C and 12% H_2 is burnt using 98% of air compared to theoretical air requirement for combustion. Determine the following considering that H_2 is completely burnt and carbon burns to CO and CO_2 leaving no free carbon.*

(*a*) *% analysis of dry flue gases by volume*

(*b*) *% heat loss due to incomplete combustion. Consider gross calorific values in kJ/kg as, C to CO_2 = 34694 kJ/kg, C to CO = 10324 kJ/kg, H_2 = 143792 kJ/kg*

Solution:

Mass of constituent per kg of fuel (a)	*Oxygen required per kg of constituent* (b)	*Oxygen required per kg of fuel* $c = (a \times b)$	*Minimum air required per kg of fuel for complete combustion* $\frac{100}{23.2} \times \Sigma(c)$
C = 0.88	$\frac{8}{3}$	2.35	$\frac{3.30 \times 100}{23.2}$ = 14.27 kg
H_2 = 0.12	8	0.95	
		Total 3.30	

Actual air supplied for burning = $0.98 \times 14.27 = 13.98$ kg

Less amount of air supplied = 14.27 – 13.98 = 0.29 kg. Due to this amount of air complete carbon could not get burnt into CO_2 and CO is there.

Let us write burning of carbon to yield CO and CO_2

$C + O_2 \rightarrow CO_2$ $\qquad$ $2C + O_2 \rightarrow 2CO$

12 kg + 32 kg $\rightarrow$ 44 kg $\qquad$ 24 kg + 32 kg $\rightarrow$ 56 kg

1 kg + 2.67 kg $\rightarrow$ 3.67 kg $\qquad$ 1kg + 1.33 kg $\rightarrow$ 2.33 kg

Thus the amount of air saved by burning C to CO instead of CO_2, per kg of carbon

$$= (2.67 - 1.33) \times \frac{100}{23.2}$$

$$= 5.78 \text{ kg}$$

Thus the amount of carbon burnt to CO per kg of fuel shall be

$$= \frac{0.29 \times 0.88}{5.78} = 0.044 \text{ kg}$$

Out of 0.88 kg C per kg of fuel only 0.044 kg C gives CO while remaining (0.88 – 0.044 = 0.836) 0.836 kg C shall yield CO_2.

Volumetric analysis of combustion products is present as under.

Constituents of dry flue gas	*Mass of constituent per kg of fuel* (a)	*Proportional Volume in mol* $(b) = \left(\frac{a}{Mol.wt}\right)$	*Percentage volume* $(c) = \frac{b}{\Sigma(b)} \times 100$
CO_2	$0.836 \times \frac{44}{12}$ = 3.06	$\frac{3.06}{44}$ = 0.069	15.11%
CO	$0.044 \times \frac{28}{12}$ = 0.103	$\frac{0.103}{28}$ = 0.0037	0.81%
N_2	$0.768 \times 13.98 = 10.74$	$\frac{10.74}{28}$ = 0.384	84.08%
		$\Sigma(b)$ = 0.4567	100%

Volumetric analysis of dry flue gas
= 15.11% CO_2, 0.81% CO, 84.08 % N_2 ***Ans.***

Calorific value of fuel in actual case due to incomplete burning of carbon.

$$= (0.836 \times 34694) + (0.044 \times 10324)$$
$$= 29458.44 \text{ kJ}$$

Theoretical calorific value of fuel with complete burning of carbon

$$= (0.88 \times 34694) = 30530.7 \text{ kJ}$$

Theoretical calorific value of fuel $= \{(0.88 \times 34694) + (0.12 \times 143792)\}$

$$= 47785.76 \text{ kJ}$$

Percentage heat loss due to incomplete combustion

$$= \frac{(30530.72 - 29458.44)}{47785.76} \times 100$$
$$= 2.24\%$$

% Heat loss = 2.24% ***Ans.***

10. *During production of gas the air and steam are passed through an incandescent coal bed. The coal is seen to have 95% of carbon and remaining as incombustible. The gas produced has hydrogen, nitrogen and carbon monoxide. Determine, the steam required per kg of coal and total air required per kg of coal when the heat of formation for steam is 147972 kJ/kg of hydrogen and for carbon monoxide it is 10324 kJ/kg of carbon. Also obtain volumetric analysis of gas. Take temperature of water as 20°C at 1 atm pressure. Take air to have 23.2% O_2 and 76.8% N_2 by mass.*

Solution:

Combustion equation can be written, separately for reaction with air and with steam as both are passed through.

Reaction with air

$$2C + O_2 \rightarrow 2CO$$
$$24 \text{ kg} + 32 \text{ kg} \rightarrow 56 \text{ kg}$$
$$1 \text{ kg of C} + \frac{4}{3} \text{ kg of } O_2 \rightarrow \frac{7}{3} \text{ kg of CO} + 10324 \text{ kJ} \quad ...(i)$$

Reaction with steam

$$C + H_2O \rightarrow CO + H_2$$
$$12 \text{ kg} + 18 \text{ kg} \rightarrow 28 \text{ kg} + 2 \text{ kg}$$
$$1 \text{ kg C} + \frac{3}{2} \text{ kg } H_2O \rightarrow \frac{7}{3} \text{ kg CO} + \frac{1}{6} \text{ kg } H_2 + \{10324 - \frac{1}{6} \times 147972 - \frac{3}{2} \underbrace{((100 - 20) \times 4.18 + 2253)\}}_{\text{Sensible heat + Latent heat}}$$
$$1 \text{ kg C} + \frac{3}{2} \text{ kg } H_2O \rightarrow \frac{7}{3} \text{ kg CO} + \frac{1}{6} \text{ kg } H_2 - 182191 \text{ kJ} \quad ...(ii)$$

Two equations for burning with air and reaction with steam can be balanced in terms of heat released provided equation (ii) is multiplied by $\left(\frac{10324}{182191}\right)$ throughout, so equation (ii) gets modified into,

0.057 kg C + 0.085 kg H_2O (steam) $\rightarrow$ 0.1322 kg CO + 0.0094 kg H_2 – 10324 kJ ...(*iii*)

Thus, total carbon used from reaction (i) and (iii) with no heat release can be given as,

$= (1 + 0.057)$ kg C = 1.057 kg carbon.

$$\text{Steam required per kg of coal} = \frac{0.085 \times 0.95}{1.057} = 0.0764 \text{ kg}$$

$$\text{Total air required per kg of coal} = \left(\frac{4}{3} \times \frac{0.95}{1.057}\right) \times \frac{100}{23.2} = 5.16 \text{ kg}$$

Steam required per kg of coal = 0.0764 kg
Air required per kg of coal = 5.16 kg **Ans.**

Total carbon monoxide = (2.333 + 0.132)
= 2.465 kg

Total hydrogen = 0.0094 kg

$$\text{Total nitrogen} = \frac{4}{3} \times \frac{76.8}{23.2} = 4.414 \text{ kg}$$

Volumetric analysis

Constituents (*a*)	*Mol. wt.* (*b*)	*Proportional volume in mol.* (*c*) = $\frac{a}{b}$	*% volume of gas* (*d*) = $\frac{c}{\Sigma(c)} \times 100$
CO = 2.465 kg	28	0.088	35.16%, CO
H_2 = 0.0094 kg	2	0.0047	1.88%, H_2
N_2 = 4.414 kg	28	0.1576	62.96%, N_2
		$\Sigma(c) = 0.2503$	100%

% by volume = 35.16% CO, 1.88% H_2, 62.96% N_2 **Ans.**

11. *Determine the higher and lower calorific values of coal for which following observations are made in bomb calorimeter.*

Mass of coal sample = 1 gm
Mass of water in bomb calorimeter = 2.5 kg
Initial temperature of water = 20°C
Maximum recorded temperature of water = 22.6°C
Water equivalent of apparatus = 750 gm
Cooling correction = + 0.018°C

Consider coal to have 5% H_2 in it.

Solution:

Using the cooling correction the corrected rise in temperature of water can be obtained as,

$= (22.6 – 20) + 0.018 = 2.618°C$

Heat supplied to calorimeter and water = $(2500 + 750) \times 2.618$
= 8508.5 Cal/gm
Higher calorific value of coal = 8508.5 Cal/gm

In order to get lower calorific value the heat carried by the steam formation is to be excluded from higher calorific value.

$$2H_2 + O_2 \rightarrow 2H_2O$$
$$1 \text{ kg } H_2 + 8 \text{ kg } O_2 \rightarrow 9 \text{ kg } H_2O$$

(Mass of steam formed per kg of coal) = 0.05×9
= 0.45 gm steam per gm of coal.

From steam table h_{fg} at 20°C = 586 kcal/kg

$$\text{Lower heating value of coal} = 8508.5 - \left(0.45 \times \frac{586 \times 10^3}{10^3}\right)$$
= 8244.8 cal/gm

Higher heating value = 8508.5 kcal/kg
Lower heating value = 8244.8 kcal/kg ***Ans.***

12. *Composition of a fuel by volume is as follows:*

H_2 – 52%
CH_4 – 20%
CO – 16%
CO_2 – 3%
O_2 – 2%
N_2 – 7%

Higher calorific value of fuel constituents H_2, CO and CH_4 may be taken as 28424 kJ/m^3, 27463 kJ/m^3, 87780 kJ/m^3 at 1 bar and 0°C respectively. The latent heat of one kg of water for transformation to steam at 1 bar, 0°C may be taken as 2445 kJ. Air may be considered to have 21% O_2 by volume. Determine

(*a*) *the volume of air required for complete combustion of 1 m^3 of fuel.*
(*b*) *the volumetric analysis of dry flue gases considering 20% excess air.*
(*c*) *the lower calorific value of fuel.*

Solution:

Combustion reactions for constituents are,

$2H_2 + O_2 \rightarrow 2H_2O$,
1 mol 0.5 mol 1 mol

$CH_4 + 2O_2 \rightarrow 2H_2O + CO_2$,
1 mol 2 mol 2 mol 1 mol

$2CO + O_2 \rightarrow 2CO_2$,
1 mol 0.5 mol 1 mol

Minimum air required = $0.72 \times \frac{100}{21} = 3.43$ m^3

Volume of air required for complete combustion = 3.43 m^3 ***Ans.***

Volumetric analysis

Constituents per m^3 of fuel (a)	O_2 *required per m^3 of constituents of fuel (b)*	O_2 *required per m^3 of fuel (c) = a × b*	*Combustion products per m^3 of fuel*			
			CO_2	H_2O	O_2	N_2
H_2 = 0.52	0.5	0.26	—	0.52	—	—
CH_4 = 0.20	2	0.40	0.20	0.40	—	—
CO = 0.16	0.5	0.08	0.16	—	—	—
CO_2 = 0.03	—	—	0.03	—	—	—
O_2 = 0.02	—	(– 0.02)	—	—	0.144	—
N_2 = 0.07	—	—	—	—	—	3.322
			ΣCO_2 = 0.39	ΣH_2O = 0.92	ΣO_2 = 0.144	ΣN_2 = 3.322

Total combustion product per m^3 of fuel = 4.776 m^3

Total dry flue gases per m^3 of fuel = 3.856 m^3

Excess air supplied is 20% so total volume of air supplied = Minimum air required + excess air supplied.

Excess air supplied = $0.2 \times 3.43 = 0.686$ m^3

Total air supplied = $3.43 + 0.686 = 4.116$ m^3 air per m^3 of fuel

**Oxygen present in excess air = $0.72 \times 0.2 = 0.144$ m^3, This will be with combustion products

Nitrogen present in total air = $\frac{4.116 \times 79}{21} = 3.252$ m^3

**Since 0.07 m^3 N_2 per m^3 of fuel is already present in fuel so total nitrogen available in combustion products = $3.252 + 0.07 = 3.322$ m^3 per m^3 of fuel.

$$\% \text{ by volume of } CO_2 = \frac{(0.20 + 0.16 + 0.03) \times 100}{3.856} = 10.12\%$$

$$\% \text{ by volume of } O_2 = \frac{0.144 \times 100}{3.856} = 3.73\%$$

$$\% \text{ by volume of } N_2 = \frac{3.322 \times 100}{3.856} = 86.15\%$$

Volumetric composition of dry flue gases = 10.12% CO_2, 3.73% O_2, 86.15% N_2 ***Ans.***

Higher calorific value may be obtained from the values given for constituents H_2, CH_4 and CO.

$= \{(0.52 \times 28424) + (0.20 \times 27463) + (0.16 \times 87780)\}$

$= 34317.88$ kJ/m^3

Steam produced during combustion of 1 m^3 of fuel.

$$m_{steam} = \frac{0.92}{22.4} = 0.0411 \text{ kg}$$

Lower calorific value = Higher calorific value – ($m_{steam} \times$ latent heat)

$= \{34317.88 - (0.0411 \times 2445)\}$

$= 34217.39$ kJ/m^3

Lower calorific value = 34217.39 kJ/m^3 ***Ans.***

13. *Determine the volume of air supplied for complete combustion of one m^3 of fuel gas. If 40% excess air is supplied then find the percentage contraction in volume after the products of combustion have been cooled. The composition of fuel gas by volume is, H_2 = 20%, CH_4 = 3%, CO = 22%, CO_2 = 8%, N_2 = 47%.Consider air to have 21% O_2 and 79% N_2 by volume.*

Solution:

Combustion reactions for constituents are

$$2H_2 + O_2 \rightarrow 2H_2O$$

1 mol 0.5 mol 1 mol

$$CH_4 + 2O_2 \rightarrow 2H_2O + CO_2$$

1 mol 2 mol 2 mol 1 mol

$$2CO + O_2 \rightarrow 2CO_2$$

1 mol 0.5 mol 1 mol

Volumetric analysis (without excess air)

Constituents per m^3 of fuel gas (a)	*O_2 required per m^3 of fuel constituents* (b)	*O_2 required per m^3 of fuel* (c) = a × b	*Combustion products per m^3 of fuel*			
			CO_2	H_2O	O_2	N_2
H_2 = 0.20	0.5	0.1	—	0.2	—	—
CH_4 = 0.03	2	0.06	0.03	0.06	—	—
CO = 0.22	0.5	0.11	0.22	—	—	—
CO_2 = 0.08	—	—	0.08	—	—	
N_2 = 0.47	—	—	—	—	—	0.47
		ΣO_2 = 0.27	ΣCO_2 = 0.33	ΣH_2O = 0.26	—	ΣN_2 = 0.47

Minimum volume of air required = $\frac{0.27 \times 100}{21}$ = 1.286 m^3 air per m^3 of fuel.

Minimum air required =1.286 m^3/m^3 of fuel ***Ans.***

With excess air there shall be additional amount of O_2 and N_2 present in combustion products.

With 40% excess air the dry flue gas shall comprise of following:

(*i*) CO_2 = 0.33 m^3 per m^3 of fuel.

(*ii*) N_2 = 0.47 + nitrogen from total air (minimum air + excess air)

$$= 0.47 + \left(1.286 \times 1.4 \times \frac{79}{100}\right)$$

= 1.89 m^3 per m^3 of fuel

(*iii*) O_2 = oxygen from excess air

$$= \left(\frac{1.286 \times 0.4 \times 21}{100}\right)$$

= 0.108 m^3 per m^3 of fuel

Total volume of dry flue gas formed = (0.33 + 1.89 + 0.108) m^3 per m^3 of fuel

= 2.328 m^3 per m^3 of fuel.

Volume before combustion = Volume of fuel gas + Volume of air supplied

= (1 + 1.286 ´ 1.5)m^3 = 2.929 m^3

% Contraction in volume = $\left(\frac{2.929 - 2.328}{2.929}\right) \times 100 = 20.52\%$

% contraction in volume = 20.52% ***Ans.***

14. *A hydrocarbon fuel when burned with air gave the following Orsat analysis, CO_2: 11.94%, O_2: 2.26%, CO: 0.41%, N_2: 83.39%*

Determine,

(i) the air-fuel ratio on mass basis

(ii) the percent of carbon and hydrogen in the fuel on mass basis, and

(iii) percentage of theoretical air supplied.

Assume air to have 21% oxygen.

Solution:

Orsat analysis gives, CO_2 = 11.94%, O_2 = 2.26%, CO = 0.41%, N_2 = 83.39%

$H_2O = 100 - (11.94 + 2.26 + 0.41 + 83.39)$

$H_2O = 2\%$

Let hydrogen fuel be 'C_aH_b'.

$$C_aH_b + d \cdot O_2 + \left(\frac{79}{21} \cdot d\right) N_2 \rightarrow e \cdot CO + f \cdot CO_2 + \frac{b}{2} H_2O + g \cdot O_2 + h \cdot N_2$$

On mass basis,

$\rightarrow 28e + 44f + 9b + 32g + 28h$

Mass of constituents of exhaust gas, $CO_2 = 44f$, $CO = 28e$, $H_2O = 9b$, $O_2 = 32g$, $N_2 = 28h$

Exhaust gas mass = $(28e + 44f + 9b + 32g + 28h)$

As per analysis,

$44f = 0.1194\ (28e + 44f + 9b + 32g + 28h)$

$28e = 0.0041\ (28e + 44f + 9b + 32g + 28h)$

$9b = 0.02\ (28e + 44f + 9b + 32g + 28h)$

$32g = 0.0226\ (28e + 44f + 9b + 32g + 28h)$

$28b = 0.8339\ (28e + 44f + 9b + 32g + 28h)$

Also,

$12 \cdot a = 12e + 12f$, or, $a = e + f$

$$\frac{79}{21} \cdot d \cdot 28 = 28h$$

$h = 3.76d$

Solving above we get

$a = 19.53e$

$b = 15.18e$

$d = 54.09e$

$f = 18.53e$

$g = 4.82e$

$h = 203.37e$

$$\text{Air-fuel ratio} = \frac{\left(32 \times d + \frac{79}{21} \times d \times 28\right)}{(12 \times a + b)} = 29.77$$

Air-fuel ratio = 29.77 ***Ans.***

$$\text{Carbon fraction} = \frac{12a}{(12a + b)} = 0.9392$$

$$\text{Hydrogen fraction} = \frac{b}{(12a + b)} = 0.0608$$

% Carbon = 93.92% ***Ans.***

% Hydrogen = 6.08%

$$\text{Percentage theoretical air supplied} = \frac{\left(32d + \frac{79}{21} \times d \times 28\right) \times 100}{\left(32d + \frac{79}{21} d \times 28 + 12a + b\right)}$$

$$= 96.75$$

% Theoretical air supplied = 96.75% ***Ans.***

15. *Obtain the stoichiometric reaction equation for CH_4 being burnt with*

(i) 100% theoretical air

(ii) 200% excess air

(iii) 20% less than theoretical air requirement

Consider air to have 21% oxygen and 79% nitrogen by mass.

Solution:

Let number of air molecules required for one molecule of CH_4 be '*n*'. Combustion equation for CH_4,

$$CH_4 + n\left(O_2 + \frac{79}{21} \cdot N_2\right) \rightarrow a \cdot CO_2 + b \cdot N_2 + d \cdot H_2O$$

From above equation the C, H_2 and O_2 can be equated as under;

C, $1 = a$, or $a = 1$

O_2, $n = a + \frac{d}{2}$, or $2n = 2a + d$

H_2, $2 = d$, or $d = 2$

N_2, $n \times \frac{79}{21} = b$, or $b = 3.76\ n$

It gives

$n = 2$, $a = 1$

$b = 7.52$, $d = 2$

Stoichiometric combustion equation shall be,

For 100% theoretical air

$CH_4 + 2(O_2 + 3.76\ N_2) \rightarrow CO_2 + 7.52N_2 + 2H_2O$ ***Ans.***

When 200% of excess air is added then total air supplied will be 300% i.e. (200% excess + 100% theoretical). Due to this excess O_2 shall be there in combustion products.

Thus modified form of combustion equation shall be,

$CH_4 + 3 \times 2\ (O_2 + 3.76\ N_2) \rightarrow a \cdot CO_2 + b \cdot N_2 + d \cdot H_2O + e \cdot O_2$

Equating C, H_2, O_2 and N_2 we get

C; $a = 1$

O_2; $a + \frac{d}{2} + e = 6$

H_2; $d = 2$

N_2; $b = 6 \times 3.76$

It yields,

$a = 1$

$b = 22.56$

$d = 2$

$e = 4$

Now the combustion equation with 200% excess air shall be;

With 200% excess air;

$$CH_4 + 6(O_2 + 3.76N_2) \rightarrow CO_2 + 22.56N_2 + 2H_2O + 4O_2$$

Ans.

When there is 20% less air then actual air supplied will be 20% less than 100% theoretical air. Due to less than theoretical air supplied there will be incomplete combustion and CO will be present with combustion products.

$$CH_4 + 0.8 \times 2\ (O_2 + 3.76N_2) \rightarrow aCO_2 + bCO + dN_2 + eH_2O$$

Equating C, H_2, O_2 and N_2 we get

C; $a + b = 1$

O_2; $a + \frac{b}{2} + \frac{e}{2} = 1.6$

H_2; $e = 2$

N_2; $d = 1.2 \times 3.76 = 4.512$

It yields,

$a = 0.2$

$b = 0.8$

$d = 4.512$

$e = 2$

Hence for 20% less air the combustion equation shall be as under

For 20% less air.

$$CH_4 + 1.6\ (O_2 + 3.76N_2) \rightarrow 0.2CO_2 + 0.8CO + 4.512N_2 + 2H_2O$$

Ans.

16. *Determine the adiabatic flame temperature for the combustion of carbon monoxide (CO) with 150 percent theoretical amount of oxygen to form CO_2. The reactants enter the steady flow reactor at 25°C, 1 atm and the products are CO_2 and excess O_2. Enthalpy of O_2, CO and CO_2 are 0 kJ/kg mole, – 110418 kJ/kg and – 393,137 kJ/kg mole respectively. The constant pressure specific heats of CO_2 and O_2 at 1 atm, may be assumed to be 56.43 and 36.5 kJ/kg mole K, respectively.[U.P.S.C. 1993]*

Solution:

Combustion equation of carbon monoxide yields,

$$2CO + \frac{3}{2}O_2 \rightarrow 2CO_2 + \frac{1}{2}O_2$$

Since the process is adiabatic so the total enthalpy of reactants and products shall remain same,

Enthalpy of reactants = $2\ (-\ 110{,}418) + \frac{1}{2}(0)$

$= -\ 220{,}836$ kJ/kg · mole

Enthalpy of products for adiabatic flame temperature to be T°K.

$$= [2\ (-393{,}137) + (56.43 \times (T - 298)) + (\frac{1}{2} \times 36.5 \times (T - 298))]$$

Enthalpy of reactants = Enthalpy of products

$$-220{,}836 = [2 \times (-393{,}137) + (56.43 \times (T - 298)) + (\frac{1}{2} \times 36.5 \times (T - 298))\}$$

$$T = 7869.48 \text{ K}$$

. Adiabatic flame temperature = 7869.48 K ***Ans***

EXERCISE

10.1 What do you understand by fuel?

10.2 Describe different types of fuel.

10.3 What do you mean by endothermic and exothermic reactions? Explain.

10.4 Describe proximate analysis and ultimate analysis and their relevance.

10.5 What is meant by stoichiometric air-fuel ratio?

10.6 Describe calorific values of fuel and its' measurement.

10.7 What is excess air?

10.8 Explain working of Orsat analyzer for flue gas analysis.

10.9 Describe fuel cells.

10.10 Define adiabatic flame temperature.

10.11 A coal sample has 90% carbon, 3.3% hydrogen, 3% oxygen, 0.8% nitrogen. 0.9% sulphur and 2% ash. For 50% excess air supply determine percentage composition of dry flue gases by volume and minimum oxygen required.

[13% CO_2, 7.1.% O_2, 80% N_2, minimum oxygen = 2.64 kg]

10.12 A coal sample has 85% carbon, 6% hydrogen, 6% oxygen and 3% ash by mass. What will be the minimum quantity of air required per kg of coal? Take air to have 23% O_2 and 77% N_2 by mass. [11.70 kg]

10.13 For a hydrocarbon fuel having 84.8% carbon, 15.2% hydrogen by mass determine the stoichiometric air-fuel ratio. Also obtain the volumetric composition of combustion products with 15% excess air being supplied. [15.11, 12.51% CO_2, 2.89% O_2, 84.60% N_2]

10.14 For a coal fuel sample having 86% carbon, 4% hydrogen, 4.5% oxygen and 5.5% ash by mass determine minimum air required per kg of coal. Also find volumetric composition of combustion products when the actual air supplied for combustion is 1.5 times the minimum air required for complete combustion.

[11.16 kg air/kg of fuel, 12.52% CO_2, 80.47% N_2, 7.01% O_2]

10.15 During the combustion of coal having 22% carbon, 3.5% hydrogen, 6.5% oxygen and remaining ash by mass, the dry flue gas has volumetric composition of 8% CO_2, 1.5% CO, 80.5% N_2 and 10% O_2. Calculate the minimum air required per kg of coal and % excess air. Consider air to have 23% and 77% of oxygen and nitrogen respectively by mass. [3.48 kg air, 43.3% excess air]

10.16 A liquid fuel has 86% C and 14% H_2 by mass. During combustion actual air supplied is 10% less than the theoretically required for complete combustion. Obtain the volumetric composition of dry flue gases considering that all hydrogen gets burnt and carbon gets burnt to form carbon monoxide and carbon dioxide with no free carbon left. [11.39% CO_2, 4.78% CO, 83.82% N_2]

10.17 A producer gas sample has 2% CH_4, 2% O_2, 14% H_2, 22% CO, 5% CO_2 and 55% N_2 by volume. Determine the minimum air requirement per m^3 of gas and the volumetric analysis of combustion products for 40% excess air supply. Take air to have 21% oxygen by volume.
[0.952 m^3 air, 14.7% CO_2, 81.25% N_2, 4.05% O_2]

10.18 Determine air fuel ratio and percentage excess air for a liquid fuel sample whose ultimate analysis shows 85% carbon and 15% hydrogen. It produces dry exhaust gases having 11.5% CO_2, 1.2% CO, 0.9% O_2, 86% N_2 by volume after combustion. [15.07, 12.47% excess air]

10.19 C_{10} H_{22} is burnt with air fuel ratio of 13 by mass. Combustion is such that complete of hydrogen burns into water and there is no free oxygen and no free carbon. Also the carbon dioxide and carbon monoxide are present in combustion products. Determine volumetric analysis of combustion products. [7.68% CO_2, 6.25% CO, 15.32% H_2O, 70.75% N_2]

10.20 A hydrocarbon fuel has 86% carbon and 14% hydrogen. The heat evolved by 1 kg carbon burning to CO_2 is 33800 kJ and 1 kg hydrogen burning is 142000 kJ. For 25% of excess air supply determine the mass of air required and higher calorific value. [18.52 kg air, 48948 kJ/kg fuel]

10.21 Determine air fuel ratio for combustion of C_3H_8 (Propane) with 200 percent theoretical air supplied. [31.62]

10.22 Transform the volumetric analysis as given into gravimetric analysis. 15% CO, 2.2% CO, 1.6% O_2, 81.2% N_2. [21.6% CO_2, 2.2% CO, 1.7% O_2, 74.5% N_2]

10.23 A coal sample has 66% C, 5.9% H_2, 1% S, 19.9% O_2, 1.5% N_2, 5.6% ash by mass and calorific value of 29000 kJ/kg. When this coal is burnt with 30% excess air the temperature of flue gases leaving is 300°C and ambient temperature is 17°C. Considering combustion to be complete and partial pressure of moisture to be 0.07 bar in flue gases and energy accompanied being 3075 kJ/kg of steam, determine,

(i) the air-fuel ratio by mass.
(ii) the volumetric analysis of combustion products.
(iii) the heat carried away by flue gases as percentage of heat released by coal.

[11.55, 19.37% O_2, 4.25% H_2O, 0.16% SO_2, 4.91% O_2, 71.31% N_2, 17.36%]

10.24 Determine the higher and lower calorific values of gas at atmospheric pressure and 15°C using the following observations from a gas calorimeter for any fixed time.

Atmospheric pressure = 76 cm Hg
Gas burnt = 0.015 m^3
Cooling water collected = 11 kg
Condensate collected = 0.01 kg
Cooling water temperature rise = 6°C
Gas pressure above atmosphere = 4.2 cm of water
Gas temperature = 17°C
Latent heat of steam at atmospheric pressure = 2440 kJ/kg
Density of mercury = 13600 kg/m^3.

[Higher calorific value = 18470 kJ/m^3, Lower calorific value = 18445.6 kJ/m^3]

10.25 Determine the net and gross calorific values per kg of mixture at constant pressure for stoichiometric mixture of air and C_6H_6 (benzene) vapour at 25°C. Enthalpy of combustion for C_6H_6 at 25°C is – 3169500 kJ/kmol and the water is present in vapour phase in the combustion products. [2861 kJ/kg, 2981 kJ/kg]

11
Boilers and Boiler Calculations

11.1 INTRODUCTION

Steam is extensively used for various applications such as power production, industrial processes, work interaction, heating etc. With the increasing use of steam in different engineering systems the steam generation technology has also undergone various developments starting from 100 B.C. when Hero of Alexandria invented a combined reaction turbine and boiler.

Boiler, also called steam generator is the engineering device which generates steam at constant pressure. It is a closed vessel, generally made of steel in which vaporization of water takes place. Heat required for vaporization may be provided by the combustion of fuel in furnace, electricity, nuclear reactor, hot exhaust gases, solar radiations etc.

Earlier boilers were closed vessels made from sheets of wrought iron which were lapped, riveted and formed into shapes of simple sphere type or complex sections such as the one shown in Fig. 11.1. It is the 'Wagon boiler' of Watt developed in 1788.

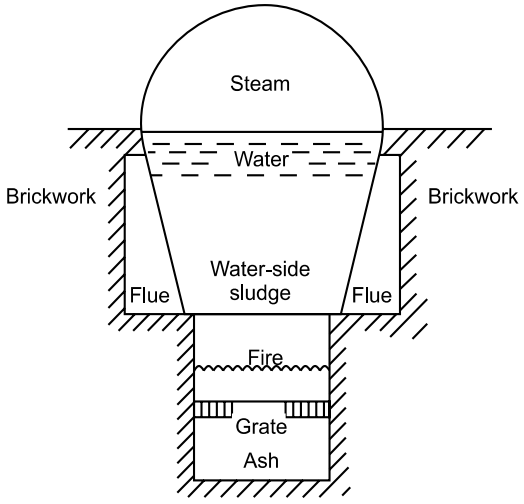


Fig. 11.1 *Wagon boiler of Watt,* (1788)

According to A.S.M.E. (American Society of Mechanical Engineers, U.S.A.) code a boiler is defined as a combination of apparatus for producing, furnishing or recovering heat together with the apparatus for transferring the heat so made available to water which could be heated and vaporised to steam form.

Boiler technology got revolutionized during second world war, when the need arose for the boilers to supply steam to field installations. Field requirements were critical as the boiler installation and commissioning should take place in minimum time. Therefore the 'Package boilers' which were complete with all auxiliaries as one unit came up and gradually transformed into modern boiler having lot of accessories and mountings. Thus in a boiler other than heat supplying unit, shell and tubes, a number of other devices are used for its control, safe and efficient operation. Devices which are mounted on boiler for its control and safe operation are called "mountings" while devices which are mounted on boiler for improving its performance are called "accessories". Thus boiler mountings are necessary while boiler accessories are optional.

11.2 TYPES OF BOILERS

Boilers are of many types. Depending upon their features they can be classified as given under:

(a) Based upon the orientation/axis of the shell: According to the axis of shell boiler can be classified as vertical boiler and horizontal boiler.
   (i) *Vertical boiler* has its shell vertical.
   (ii) *Horizontal boiler* has its shell horizontal.
   (iii) *Inclined boiler* has its shell inclined.

(b) Based upon utility of boiler: Boilers can be classified as
   (i) *Stationery boiler*, such boilers are stationery and are extensively used in power plants, industrial processes, heating etc.
   (ii) *Portable boiler*, such boilers are portable and are of small size. These can be of the following types,

   Locomotive boiler, which are exclusively used in locomotives.
   Marine boiler, which are used for marine applications.

(c) Based on type of firing employed: According to the nature of heat addition process boilers can be classified as,
   (i) Externally fired boilers, in which heat addition is done externally i.e. furnace is outside the boiler unit. Such as Lanchashire boiler, Locomotive boiler etc.
   (ii) Internally fired boilers, in which heat addition is done internally i.e. furnace is within the boiler unit. Such as Cochran boiler, Bobcock Wilcox boiler etc.

(d) Based upon the tube content: Based on the fluid inside the tubes, boilers can be,
   (i) *Fire tube boilers*, such boilers have the hot gases inside the tube and water is outside surrounding them. Examples for these boilers are, Cornish boiler, Cochran boiler, Lancashire boiler, Locomotive boiler etc.
   (ii) *Water tube boilers*, such boilers have water flowing inside the tubes and hot gases surround them. Examples for such boilers are Babcock-Wilcox boiler, Stirling boiler, La-Mont boiler, Benson boiler etc.

(e) Based on type of fuel used: According to the type of fuel used the boilers can be,
   (i) *Solid fuel fired boilers*, such as coal fired boilers etc.
   (ii) *Liquid fuel fired boilers*, such as oil fired boilers etc.
   (iii) *Gas fired boilers*, such as natural gas fired boilers etc.

(f) Based on circulation: According to the flow of water and steam within the boiler circuit the boilers may be of following types,
   (i) *Natural circulation boilers*, in which the circulation of water/steam is caused by the density difference which is due to the temperature variation.

(ii) *Forced circulation boilers*, in which the circulation of water/steam is caused by a pump i.e. externally assisted circulation.

(g) Based on extent of firing: According to the extent of firing the boilers may be,
   (i) *Fired boilers*, in which heat is provided by fuel firing.
   (ii) *Unfired boilers*, in which heat is provided by some other source except fuel firing such as hot flue gases etc.
   (iii) *Supplementary fired boilers*, in which a portion of heat is provided by fuel firing and remaining by some other source.

11.3 REQUIREMENTS OF A GOOD BOILER

Different requirements of a good boiler are given below. In general boiler is supposed to generate large quantity of steam at desired pressure and temperature quickly and efficiently.

(a) It should be capable of generating steam at desired rate at desired pressure and temperature with minimum fuel consumption and cost.
(b) It should have sufficient steam and water storage capacity to meet fluctuation in demand and to prevent fluctuation in steam pressure or water level.
(c) Boiler should have a constant and thorough circulation of water.
(d) It should be equipped with all necessary mountings.
(e) Boiler should have capability to get started quickly from cold.
(f) Its construction should be simple and have good workmanship for the ease of inspection and repairs i.e. easily accessible parts.
(g) Boiler should have its heating surface nearly at right angle to the current of hot gases for good heat transfer.
(h) There should be minimum frictional power loss during flow of hot gases and water/steam i.e. pressure drop throughout the system should be minimum.
(i) Tubes should be so designed so as to have minimum soot deposition and good strength against wear. Boiler should have a mud drum to receive all impurities.
(j) Boiler should have strength to withstand excessive thermal stresses.
(k) Boiler should occupy less floor area and space.

Boilers may be selected for a particular applications considering above general requirements and constraints, if any. For deciding the boiler for any application, generally following criterion are made;

(i) Steam pressure requirement
(ii) Steam temperature requirement
(iii) Steam generation rate
(iv) Initial cost and constraints
(v) Running and maintenance costs
(vi) Availability of fuel and water
(vii) Inspection and maintenance requirements.

11.4 FIRE TUBE AND WATER TUBE BOILERS

Fire tube boilers are those boilers in which hot gases (combustion products) flow inside the tubes and water surrounds them. Water extracts heat for its phase transformation from the hot gases flowing inside the tubes, thus heat is indirectly transferred from hot gas to water through a metal interface.

Such boilers came up in eighteenth century and were extensively used for steam generation in variety of applications. With the passage of time and coming up of another types of boilers the fire tube

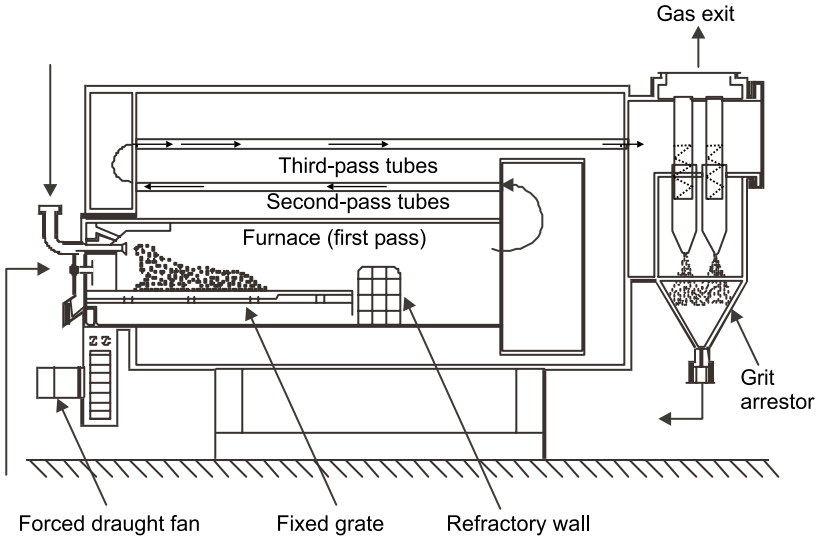


Fig. 11.2 *Fire tube boiler*

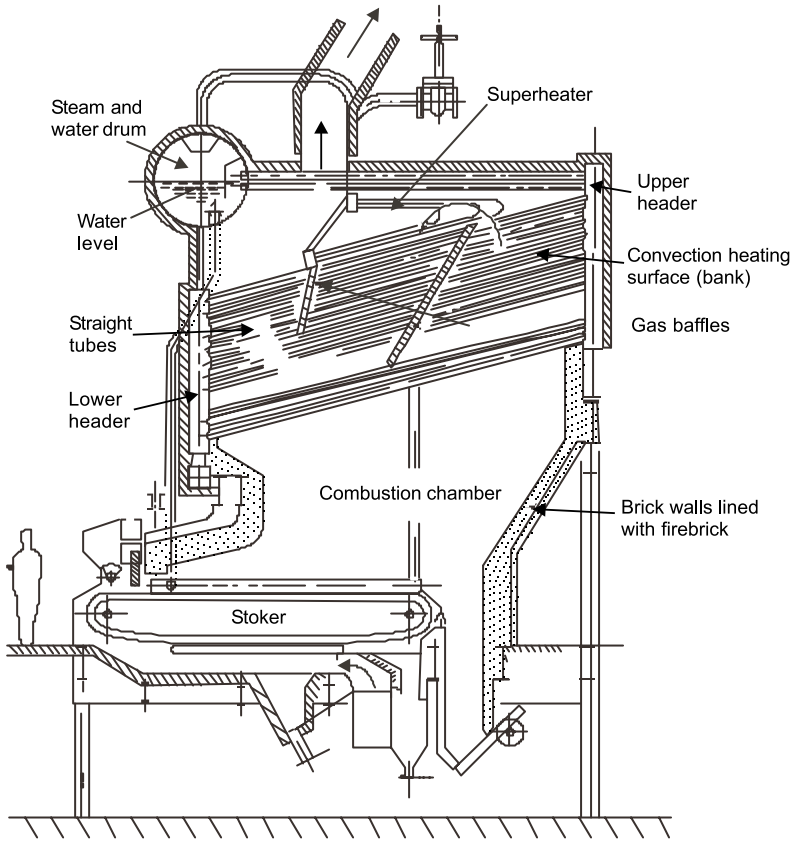


Fig. 11.3 *Water tube boiler*

boilers have lost their charm to some extent due to limitations in terms of steam pressure. Fire tube boilers are used for applications having small steam requirement. Different types of fire tube boilers have been discussed ahead.

Water tube boilers are those boilers in which water flows inside the tubes and hot gases surround them. This type of boilers came up as a solution to the problem of explosion faced in fire tube boilers when the pressure and steam generation capacity were increased. In such boilers the shell behaved as heated pressure vessel subjected to internal pressure which set up tensile stresses (hoop stress) in walls. Mathematically, this stress can be given as,

Hoop stress = $\frac{P \times D}{2t}$ where P is internal working pressure, D is diameter of shell and t is thickness of shell wall.

Above expression shows that if 'P' (pressure) increases then either 'D' (diameter) should be decreased or 't' (thickness) be increased to keep stress within acceptable limits. While increasing thickness the mass of boiler and cost of manufacturing both increase therefore the reduction of 'D' (diameter) is an attractive option. This became the basis for water tube boilers in which small diameter of tube facilitated quite high pressure steam generation.

Such boilers came up in late eighteenth and nineteenth century. George Babcock and Stephen Wilcox gave straight-tube boiler of water tube boiler type in 1867 which was subsequently modified and developed as present 'Babcock and Wilcox boiler'.

Water tube boilers may be further classified based on type of tubes employed. These can be *Straight water tube boilers* and *Bent water tube* boilers. Straight water tube boilers are those in which tubes carrying water are straight from one end to the other end. At the two ends headers are provided. In general water comes down from drum into down header and after passing through tubes get heated and evaporated to steam which is carried back to drum through upcomer header or riser. Circulation of water is caused by the density difference as density of feed water is more than density of hot water/wet/dry steam due to lower temperature of feed water.

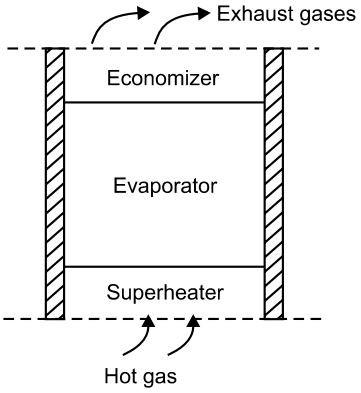


Fig. 11.4 *Relative position of boiler sections*

Bent water tube boilers are those in which bent tubes are employed for carrying water. Bent water tubes are advantageous over straight water tubes in many respects. Bent tubes offer better access into boiler and ease of inspection and maintenance. Also tube arrangement can be modified so as to maximize heating surface and exposure of tubes to hot gases.

Circulation is better in case of bent tube boilers as compared to straight tube, since the orientation of tubes in case of former is generally at inclination from vertical while for later it is horizontal. Stirling

boiler is one such boiler. In water tube boilers the heat distribution generally occurs amongst economiser tubes, evaporator tubes, superheater tubes. Hottest gases are designed to come in contact with superheater tubes. The evaporator tubes are in between superheater and economizer tubes. Relative position of three sections shall be as shown here.

A comparative study between fire tube and water tube boiler is presented below to understand relative merits of one over the other.

Advantages of fire tube boilers

(a) Fire tube boilers are more flexible and can meet sudden steam demand without much pressure fluctuations. It is because of the large volume of water contained by these boilers and heat energy stored in hot water. It may be noted that energy stored in a definite volume of water at given pressure and temperature shall be more than that stored in same volume of steam at same thermodynamic states.

(b) Fire tube boilers are less sensitive to the failure of feed water supply as they have large capacity of water stored. Such feed water supply failure is very damaging in water tube boilers due to small storage capacity.

(c) Fire tube boilers are rigid and simple in construction, therefore have great reliability and less initial cost. Number of parts in fire tube boilers is less than those in water tube boilers so maintenance cost is also small. Since thickness of boiler shell is large enough so the problems of pitting and erosion are less. Also the large drum of boiler provides ample water space and desired conditions for dry steam generation.

Advantages of water tube boilers

(a) Steam generation rate is large in water tube boilers as compared to fire tube boilers due to small quantity of water contained, large heating surface, better circulation of water etc. Water tube boilers are made in bigger sizes with very high limit to maximum output due to smaller drum, circulation etc.

(b) Maximum pressure of steam generation is quite high in water tube boilers (125 bar and above) compared to fire tube boilers (up to 20 bar) due to fluid flowing through tubes of small diameter and diameter of drum being relatively small.

(c) In case of explosion the steam generation may not stop in water tube boilers as the place of explosion in tubes can be plugged easily. While in fire tube boilers the explosion is very dangerous due to large quantity of water flashing into steam.

(d) Water tube boilers are easy to fabricate and transport due to the small size of drum. The shell of fire tube boiler shall be nearly twice or thrice of the shell of water tube boiler for same power.

(e) Water tube boilers are generally externally fired and various parts of boiler are more readily accessible for cleaning, inspection and maintenance, compared to fire tube boilers.

Characteristics of fire tube and water tube boilers are tabulated as under

Table 11.1

Characteristics	*Fire tube boiler*	*Water tube boiler*
(a) Steam Pressure	It is limited to 20–30 bar. In case of waste heat boilers, it can be more.	It is virtually unlimited within metallurgical and design limits.
(b) Unit output	Limited to about 20 MW. within design limits.	It is virtually unlimited
(c) Fuel	All commercial fuels and treated waste can be used.	Any fuel can be used. Also the furnace size is large.

Characteristics	Fire tube boiler	Water tube boiler
(d) Erection	It is packaged ready for work site.	It is to be shop assembled or erected at site.
(e) Efficiency	Normally 80–85%, gross calorific value, but can be further increased using accessories.	Normally 85–90%, gross calorific but can be further increased using accessories.
(f) Application	Generally for heat supply.	Generally for power and heat supply together.
(g) Inspection requirement.	Frequent inspection requirement. It is more than in water tube	Inspection requirement is less than in fire tube boiler. boilers.

Composite boilers: These are the boilers developed as combination of fire tube and water tube boilers so as to derive advantages of both designs. In these boilers there is no restriction in pressure and output capabilities. Such boilers are usually transported to site in sections/modules and assembled at site. Generic arrangement in composite boiler is shown in Fig. 11.5.

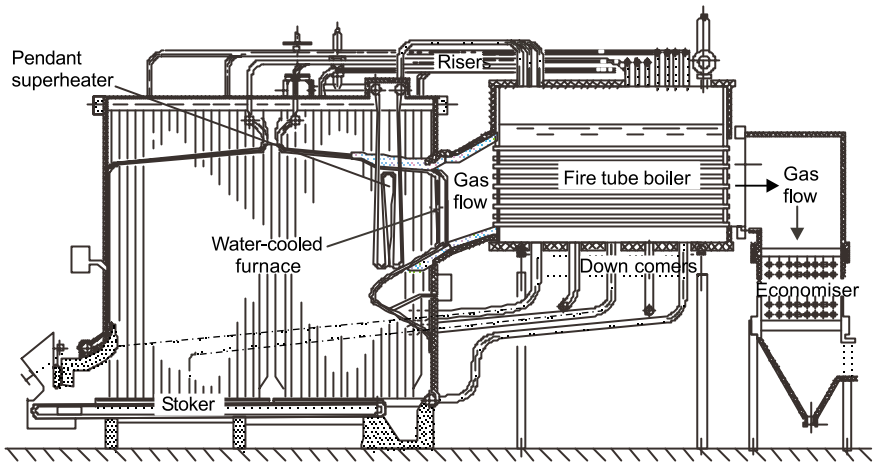


Fig. 11.5 *Composite boiler*

11.5 SIMPLE VERTICAL BOILER

Simple vertical boiler shown in Fig. 11.6 has a vertical boiler shell of cylindrical shape. It has fire box of cylindrical type inside the shell. Vertical passage of tubular type called uptake is provided over fire box for exhaust of flue gases. Cross tubes are provided for improving water circulation and increasing heating surface. At the bottom of fire box a fire grate is provided for burning fuel. Total heating surface area is about 7–10 times grate area. Man hole and hand holes are provided in the shell for access to inside of shell. Hot gases raising from fire grate go upwardly and heat the water contained in shell and tubes. Steam generated in shell can be tapped through a steam stop valve placed on the crown of shell. Such boilers have steam generation capacity up to 1000 kg per hour and maximum steam pressure up to 10 bar. Size of the boiler ranges from 0.6 m diameter to 2 m diameter and height from 1.2 m to 4 m high. Boiler efficiency is nearly 50%.

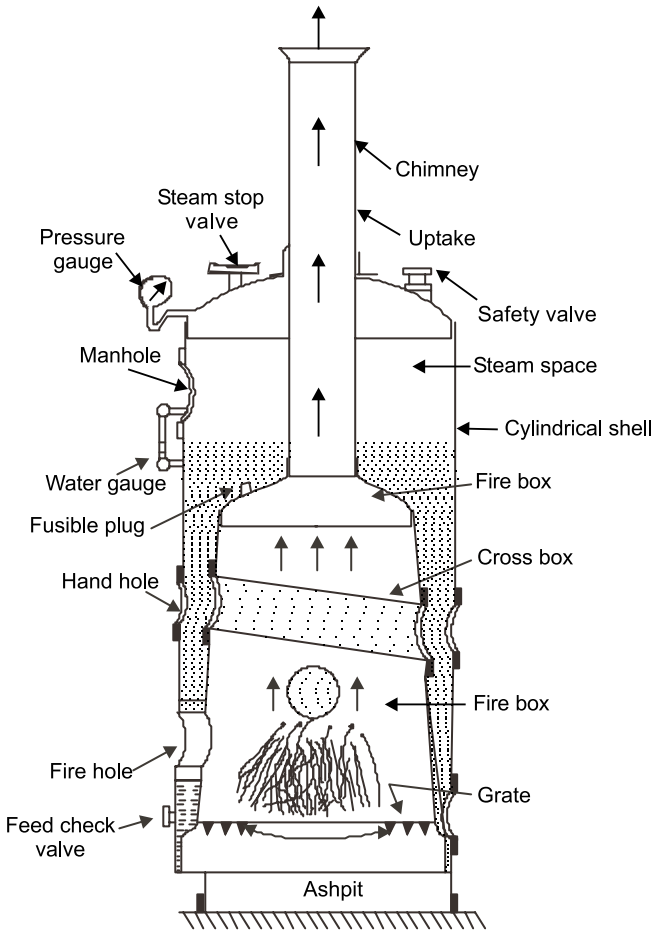


Fig. 11.6 *Simple vertical boiler*

11.6 COCHRAN BOILER

This is a fire tube boiler of vertical type and came up as a modification over the simple vertical boiler in order to maximize heating surface. Total heating surface area is 10–25 times the grate area. It has cylindrical shell with hemispherical crown. Hemispherical geometry offers maximum volume space for given mass of material and is also very good for strength and maximization of radiant heat absorption. Figure. 11.7 shows the schematic of Cochran boiler with various mountings upon it. Fire box is also of hemispherical form. Flue gases flow from fire box to refractory material lined combustion chamber through a flue pipe. Incomplete combustion if any can get completed in combustion chamber and hot gases subsequently enter into tubes. After coming out of fire tubes hot gases enter into smoke box having chimney upon it. As the fire box is separately located so any type of fuel such as wood, paddy husk, oil fuel etc. can be easily burnt.

These boilers are capable of generating steam up to pressure of 20 bar and steam generating capacity from 20 kg/hr to 3000 kg/hr. Boilers have dimensions ranging from 1 m diameter and 2 m height to 3 m diameter and 6 m height. Efficiency of such boilers ranges between 70 and 75%.

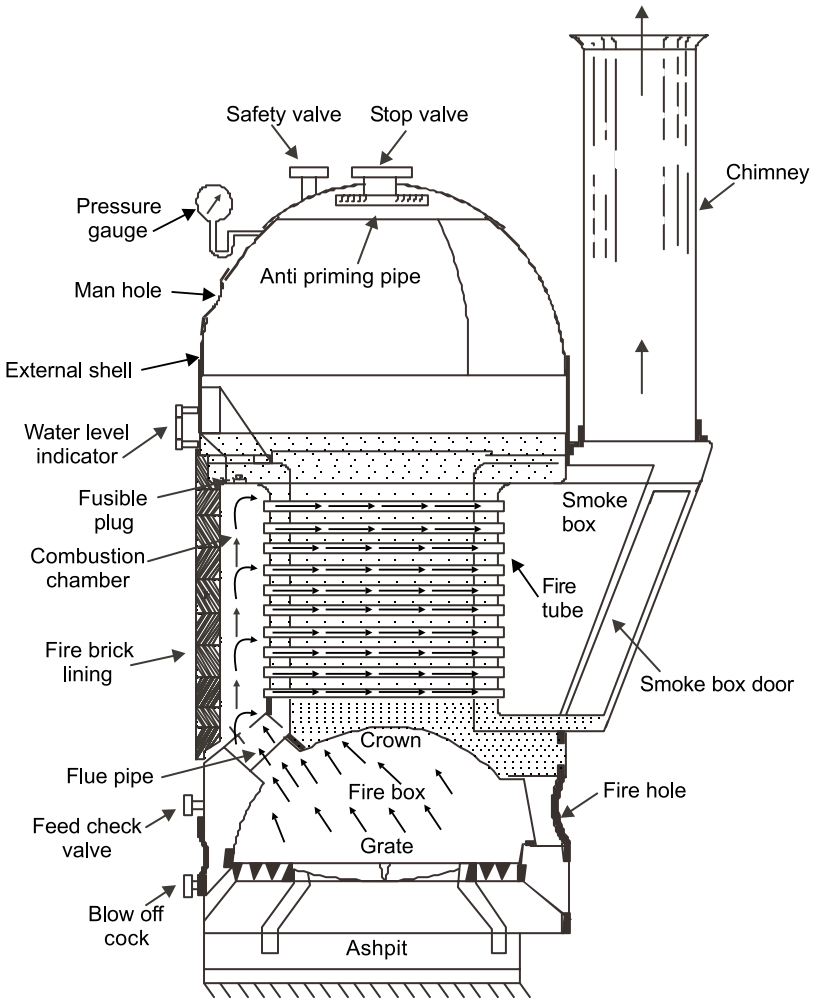


Fig. 11.7 *Cochran boiler*

11.7 LANCASHIRE BOILER

It is a horizontal fire tube boiler. General arrangement in the boiler is shown in Fig. 11.8.

Boiler is mounted on a *brickwork setting* with front end of shell sloping about 1 : 250 for emptying the shell. It has a circular shell connected *to end plates* supported by gusset plates. Two *fire tubes* run throughout the length of the boiler. Fire tubes are of diameter less than half the diameter of shell and diameter of fire tubes is reduced as shown to have access to lower side of boiler.

Fire bridge is provided to prevent fuel from falling over the end of furnace. Fire bridge also helps in producing a better mixture of air and gases for perfect combustion by partly enveloping the combustion space. Hot gases start from grate area, enter into fire tubes and come out at back of boiler from where these gases flow towards the front of boiler through *bottom flue*. Upon reaching the front these hot gases flow through the *side flues* and enter the *main outlet*. Outlet passage may also be used commonly by more than one boilers. About 85% of actual heat transferred is transferred through surface of fire tubes while 15% is transferred through bottom and side flues.

Plan, elevation and side views of Lancashire boiler shown in figure explain the furnace, different firetubes, bottom flues, side flues etc. Dampers are provided at the end of side flues for regulating the pressure difference (draught) for exit of burnt gases. Other mountings and accessories are shown in the elevation of Lancashire boiler.

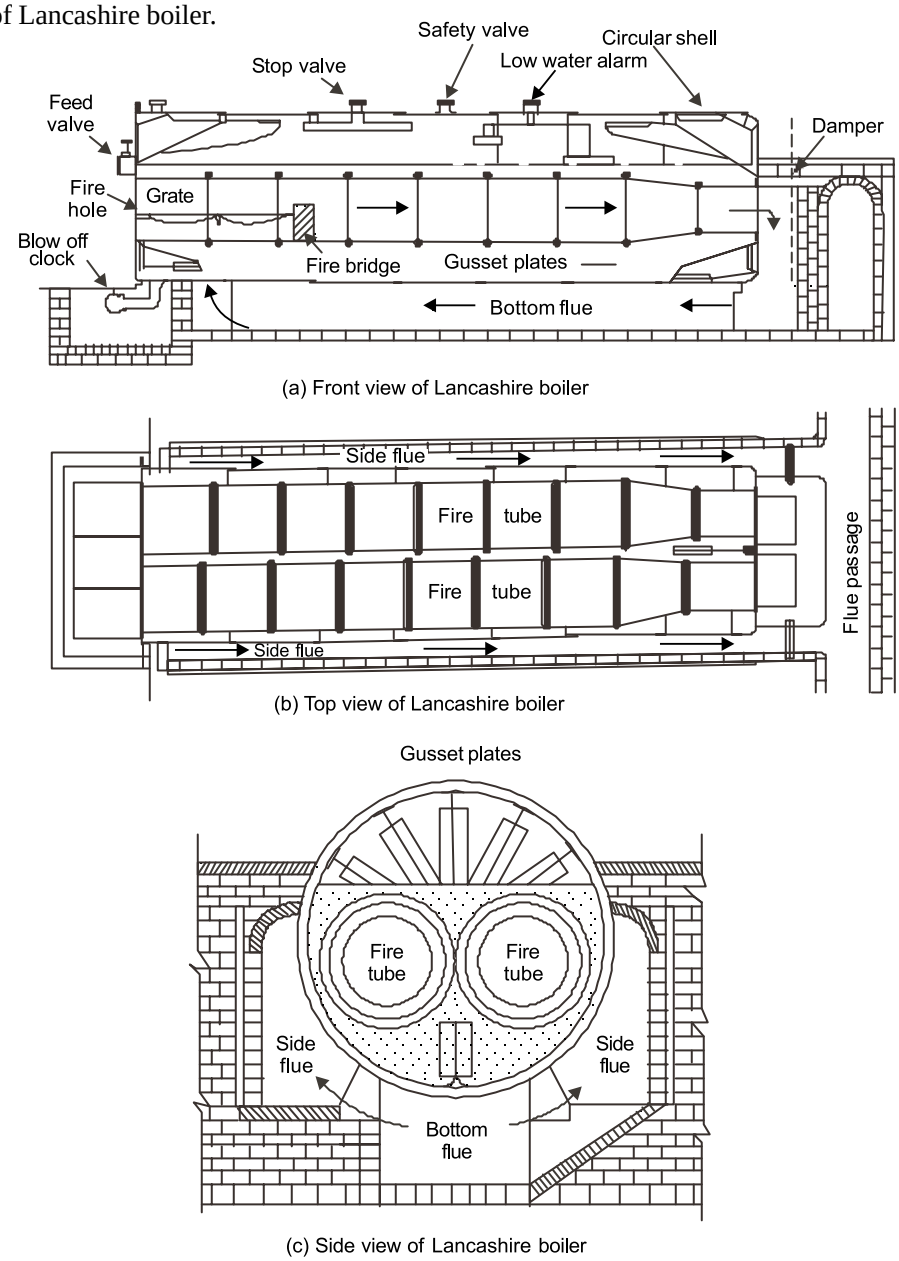


Fig. 11.8 *Lancashire boiler*

Working pressure in these boilers are in the range of 0.7 MPa to 2 MPa and efficiency of the boiler is about 65%–70%. Size of these boiler depends upon size of shell which may be 2 m to 3 m in diameter and 6m to 10m in length.

11.8 CORNISH BOILER

This is a horizontal fire tube boiler having single flue gas tube. General arrangement is very similar to Lancashire boiler. Water surrounds the flue gas tube in the shell. Hot flue gases after passing through the tube are divided into two portions at the end of boiler and pass through side flue passages to reach upto the front of boiler and then enter into bottom flue gas passage for escaping out through chimney after traversing the entire length of bottom passage. Hot gases thus traverse complete length of passage from end to end of boiler thrice i.e. through main flue gas tube, side flues and bottom flues. Heat transfer is more from side flues than bottom flue due to sedimentation in bottom. These boilers are generally capable of producing steam up to the rate of 1350 kg/hr and maximum steam pressure up to 12 bar. Shell is generally of length 4 to 7 m and diameter 1.2 to 1.8 m.

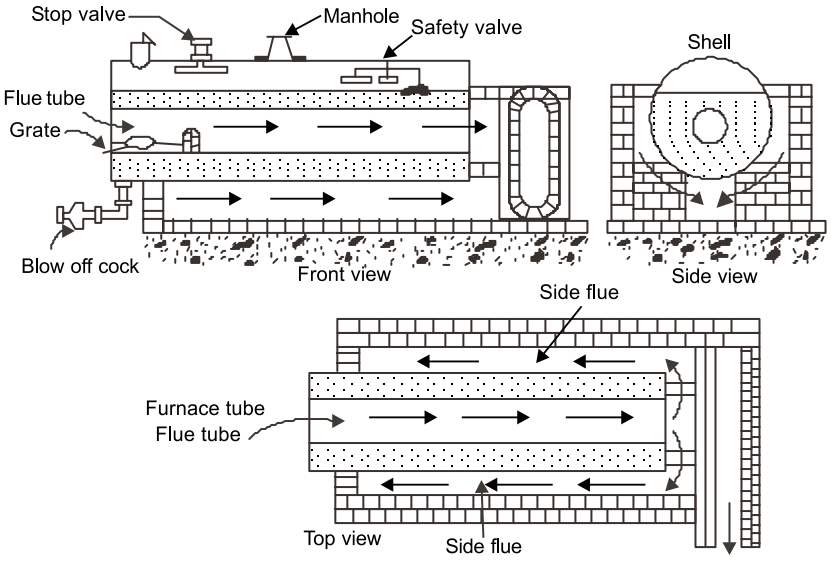


Fig. 11.9 *Cornish boiler*

11.9 LOCOMOTIVE BOILERS

These boilers were invented for getting steam to run a steam engine used in locomotives. These are fire tube type of boilers. It has basically three parts i.e. smoke box, shell and fire box. Figure 11.10 shows a general arrangement in locomotive boiler.

Inside *fire box* the fuel (coal) is burnt over the *grate*. For feeding fuel the *fire* hole is used. Hot gases produced in fire box are diverted by *fire brick arch* and enter into the *fire tubes* surrounded with water. Steam produced gets collected in a steam drum fitted on top of the shell. Arrangement for super heating is there in these boilers.

As shown the wet steam goes through inlet headers of superheater and after passing through tubes, it returns to the outlet header of superheater and is taken out for steam engine.

A very large door is provided at the end of smoke box so as to facilitate cleaning and maintenance of complete boiler.

As it is a moving boiler, therefore, its chimney is completely eliminated. For expelling the burnt gases (draught) the exhaust steam coming out from steam engine is being used. Thus it is an artificial draught used in these boilers for expelling burnt gases.

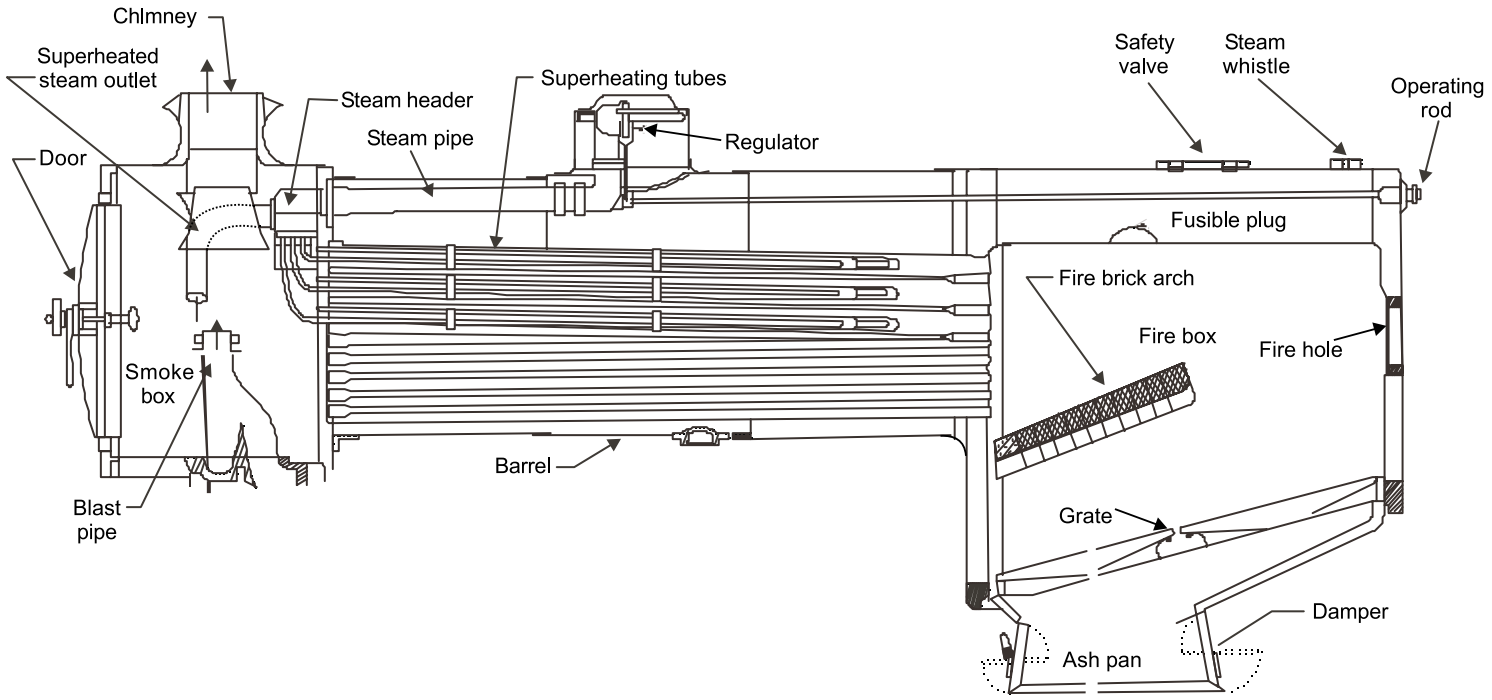


Fig. 11.10 *Locomotive boiler*

11.10 NESTLER BOILERS

This is fire tube type of fired horizontal axis boiler. Figure 11.11 shows the schematic arrangement in Nestler boiler. The boiler shell comprising of two thick mild steel plates with large number of fire tubes fitted between two plates is shown here. A bigger diameter furnace tube extending from burner end to other end is used for carrying hot flue gases from one smoke box to the other smoke box. At the rear end smoke box chimney is provided for rejection of exhaust gases.

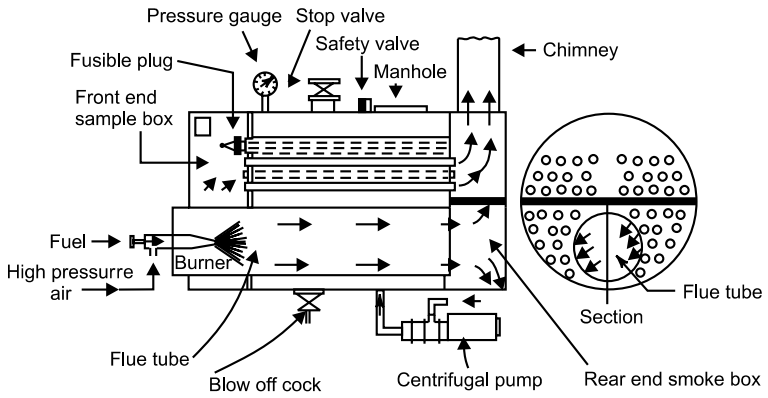


Fig. 11.11 *Nestler boiler*

Hot gases pass through furnace tube and enter into rear end smoke box and pass through fire tubes to front end smoke box for final discharge through chimney. Water surrounding tubes get transformed into steam and gets collected in steam space. Here furnace oil, a dark coloured highly viscous oil is used for firing in furnace. Oil is first heated up to 80°C by electric heater before being supplied to burner for injection into furnace tube. Blower is employed for atomization of furnace oil into furnace. Such boilers are capable of generating steam upto pressure of 10–11 bar.

11.11 BABCOCK AND WILCOX BOILER

It is a water tube boiler suitable for meeting demand of increased pressure and large evaporation capacity or large sized boiler units. Figure 11.12 shows the Babcock and Wilcox boiler. It has three main parts:

(i) Steam and water drum
(ii) Water tubes
(iii) Furnace.

Steam and water drum is a long drum fabricated using small shells riveted together. End *cover plates* can be opened as and when required. Mountings are mounted on drum as shown. Drum is followed by water tubes which are arranged below drum and connected to one another and drum through headers. Header in which water flows from drum to tubes is called *down take header* while headers in which flow is from tubes to drum is called *uptake header*.

Soot deposition takes place in mud box which is connected to downtake header. “Blow off cock” for blowing out the sediments settled in *mud box* is shown in figure. Superheater tubes are also shown in the arrangement, which are U-shape tubes placed horizontally between drum and water tubes. Superheating of steam is realized in superheater tubes.

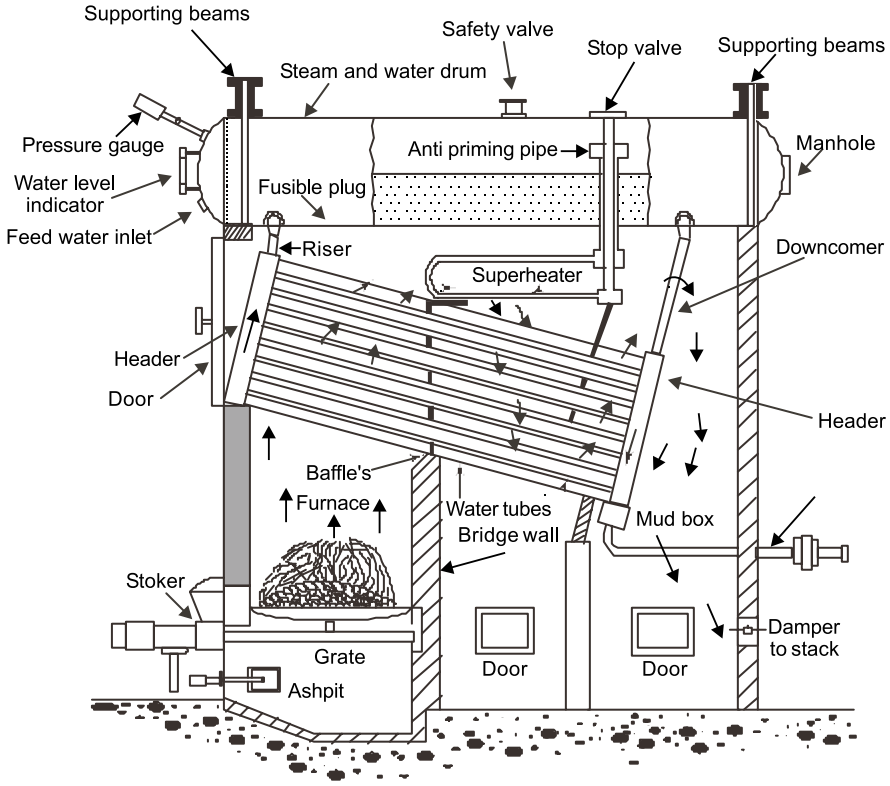


Fig. 11.12 *Babcock and Wilcox boiler*

Below the superheater and water tubes is the *furnace*, at the front of which *fuel feed hopper* is attached. *Mechanical stoker* is arranged below the hopper for feeding fuel. Bridge wall and baffles made of fire resistant bricks are constructed so as to facilitate hot gases moving upward from the *grate* area, then downwards and again upwards before escaping to the chimney. A *smoke box* is put at the back of furnace through which smoke goes out via *chimney*, put at top of smoke box. A *damper* is used for regulating pressure difference (draught) causing expulsion of hot gases.

The complete boiler unit with all mountings and accessories is suspended by steel slings from girders resting on steel columns. It is done so as to permit free expansion and contraction of boiler parts with temperature.

11.12 STIRLING BOILER

This is a water tube boiler in which bent tubes are connected to three or four drums together. These bent tubes are inclined as shown in Fig. 11.13. Bent tubes are advantageous in respect to flexibility in maximizing heating surface and no requirement of headers. For deflecting the hot gases the baffles are provided. Here three steam drums and one mud drum is shown. Feed water enters the first steam drum located near the exit passage and pass through tubes to mud drum and subsequently gets raised through tubes into other steam drum. Thus the water is circulated so as to pick maximum heat from hot gases. Steam generated is collected in upper portion of steam drums from where it can be extracted out. Hot gases coming from furnace area travel across the boiler and go out of exit passage after transferring heat contained by them.

Such boilers are capable of generating steam upto maximum pressure of 60 bar and steam generation rate up to 50,000 kg/hr.

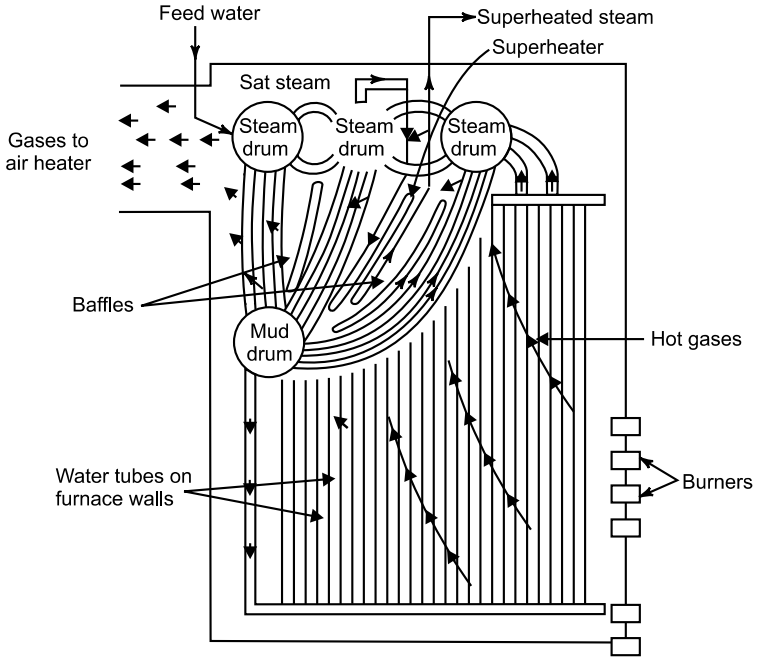


Fig. 11.13 *Stirling boiler*

11.13 HIGH PRESSURE BOILER

High pressure boilers generally operate in supercritical range. Need of such boilers is felt because high pressure and temperature of steam generated in boiler improves plant efficiency. These boilers have

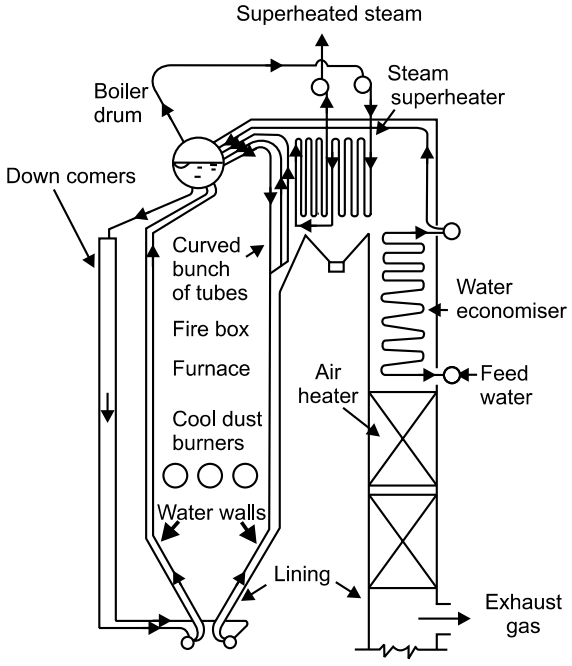


Fig. 11.14 *High pressure boiler with natural circulation*

forced circulation of water/steam in the boiler. This forced circulation is maintained by employing suitable pump. The steam drum is of very small size and in some cases it may be even absent too. This is because of using forced circulation. In case of natural circulation drum size has to be large. Schematic of high pressure boiler is shown in figure 11.14. In fact the high pressure boilers have been possible because of availability of high temperature resistant materials. Here direct heating of water tubes is done by the excessively hot gases present in fire box. The fire box has large volume as otherwise exposed water tubes shall melt. Heat is picked by number of parallel tubes containing water. These parallel tubes appear as if it is a wall due to close spacing of tubes. Water circulation circuit is shown in line diagram.

High pressure boilers may have natural circulation in case the steam pressure desired lies between 100 and 170 bar and size is not constraint. High pressure boilers have capability of generating larger quantity of steam per unit of furnace volume.

High pressure boilers are disadvantageous from safety point of view and therefore, stringent reliability requirements of mountings is there.

11.14 BENSON BOILER

It is a water tube boiler capable of generating steam at supercritical pressure. Figure 11.15. shows the schematic of Benson boiler. Mark benson, 1992 conceived the idea of generating steam at supercritical pressure in which water flashes into vapour without any latent heat requirement. Above critical point the water transforms into steam in the absence of boiling and without any change in volume i.e. same density. Contrary to the bubble formation on tube surface impairing heat transfer in the normal pressure boilers, the supercritical steam generation does not have bubble formation and pulsations etc. due to it. Steam generation also occurs very quickly in these boilers. As the pressure and temperatures have to be more than critical point, so material of construction should be strong enough to withstand thermal stresses. Feed pump has to be of large capacity as pressure inside is quite high, which also lowers the plant efficiency due to large negative work requirement. Benson boilers generally have steam generation pressure more than critical pressure and steaming rate of about 130–135 tons/hr. Thermal efficiency of these boilers is of the order of 90%.

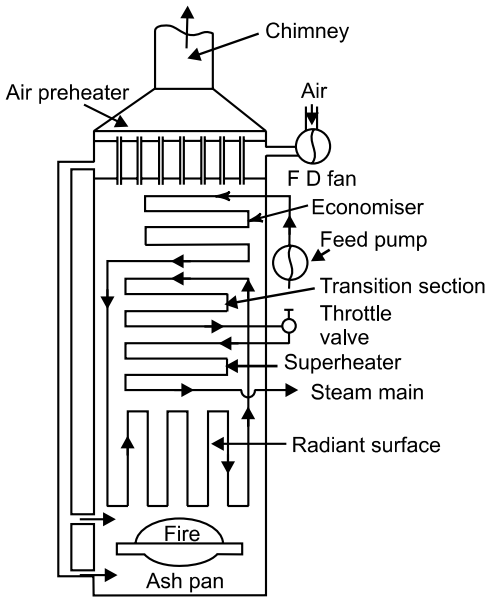


Fig. 11.15 *Benson boiler*

11.15 LOEFFLER BOILER

This a forced circulation type boiler having both direct and indirect type of heat exchange between superheated steam/water and hot gases/steam respectively. Schematic arrangement of a Loeffler boiler is shown in Fig. 11.16. Here the hot combustion gases emerging out of furnace are firstly used for superheating of steam and secondly for reheating/economiser sections. Steam generation is realized through the superheated steam being injected into evaporator drum. Saturated steam thus generated in evaporator drum as a result of mixing of superheated steam and water is picked up by steam circulation pump. This pump forces saturated steam at high pressure through superheater tubes where the hottest flue gases from furnace superheat steam coming from evaporator. Flue gases subsequently pass through reheater/economiser sections as shown. Superheated steam coming out of superheater section is partly taken out through steam main and remaining is injected into evaporator drum. Generally superheated steam is divided in proportion of 1 : 2 for steam main and evaporator drum respectively.

Feed water to the boiler is pumped by feed pump through the economiser section to evaporator drum. Generally steam generated is at pressure of about 120 bar and temperature of 500°C.

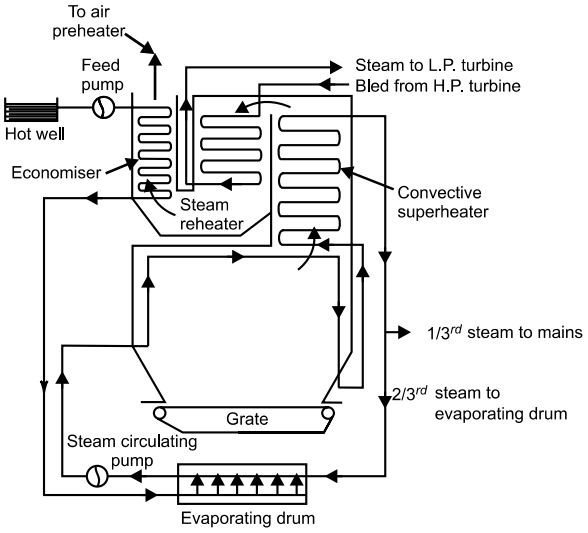


Fig. 11.16 *Schematic arrangement in Loeffler Boiler*

Loeffler boiler is advantageous in many respects such as there is no possibility of soot deposition in evaporator section. Also by the use of higher pressure steam the heat transfer rate gets improved. This boiler is much compact as compared to other natural circulation boilers.

11.16 VELOX BOILER

Velox boiler is a fire tube boiler having forced circulation. Boiler has gas turbine, compressor, generator, feed pump, circulation pump etc. as its integral components. Thus Velox boiler unit is a compact steam generating plant. Figure 11.17 shows the line diagram of Velox boiler unit.

Boiler unit has a compressor supplying high pressure air at about 3 bar into the oil burner so as to produce combustion products at high pressure and thus have hot flue gases flowing through fire tubes at very high velocity of the order of supersonic velocity. Flue gases flowing at supersonic velocity

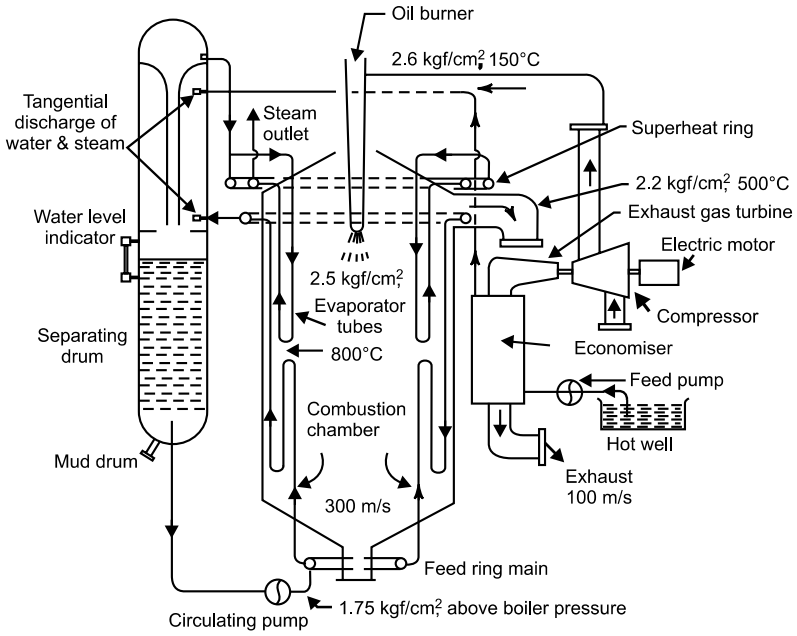


Fig. 11.17 *Velox boiler unit*

facilitate very high rate of heat interaction between two fluids, generally of the order of 2×10^7 kcal/m^3 of combustion volume. Combustion space is lined with concentric vertical tubes having hot flue gases passing through the inner tube and water surrounding it in outer tube. Hot flue gases pass through superheater section and subsequently enter into gas turbine for its expansion. Gas turbine drives the compressor used for producing compressed air. Expanded gases coming out of gas turbine at about 100–125 m/sec enter into economiser where feed water picks up heat from gas turbine exhaust. Hot feed water coming out of economiser is sent into steam/water drum from where water is circulated through vertical concentric tubes using a circulating pump. During the water flow in combustion volume space it partially gets transformed into steam and the mixture is injected tangentially into drum. Tangential discharge of mixture forms a circulatory flow (vortex) causing steam release due to centrifugal action, thus separation of water/steam. Steam is subsequently passed through superheater section while water is again circulated using circulation pump. Steam passes through steam headers after superheating. Surplus energy, if any in gas turbine is used by alternator attached to it which supplements the electricity requirement in various auxiliary devices.

Velox boilers are very flexible and capable of quick starting. Overall efficiency of the boiler unit is about 55–60%. Boiler is capable of handling maximum of 100 tons/hr water which is limited by the limitation of maximum power requirement in compressor.

11.17 LA MONT BOILER

This is a water tube boiler having forced circulation. Schematic showing the arrangement inside boiler is given in Fig. 11.18. Boiler has vertical shell having three distinct zones having water tubes in them, namely evaporator section, superheater section and economiser section.

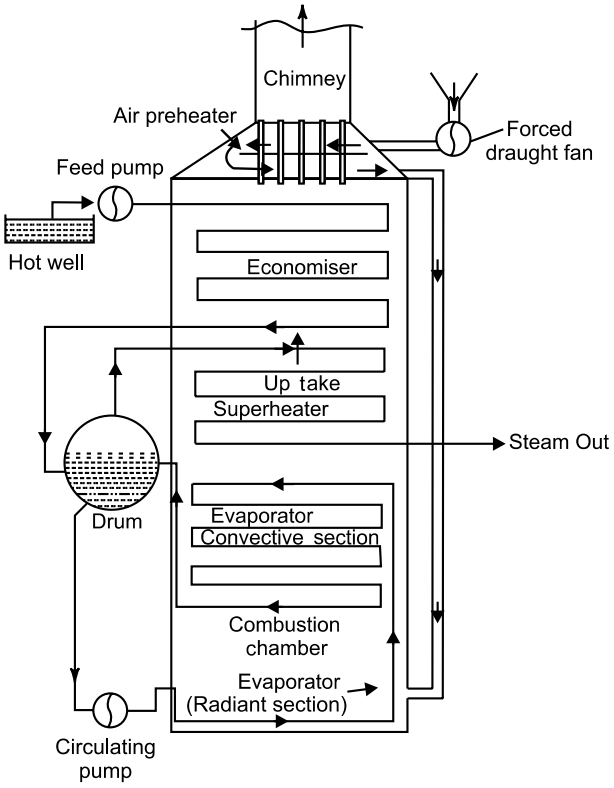


Fig. 11.18 *La Mont boiler*

Feed water is fed from feed pump to pass through economiser tubes. Hot water from economiser goes into drum from where hot feed water is picked up by a circulating pump. Centrifugal pump may be steam driven or of electric driven type. Pump increases pressure and water circulates through evaporation section so as to get converted into steam and enters back to drum. Steam available in drum enters into superheater tubes and after getting superheated steam leaves through steam main.

11.18 FLUIDIZED BED BOILER

Fluidized bed combustion is the recent development and large number of boilers are coming up with this arrangement. Schematic of fluidized bed combustion is shown in Fig. 11.19 for explaining the principle of fluidized combustion. Here a bed of inert, refractory sand type material is forced to get fluidized by the air passing through it. Air used for fluidization is heated before being sent into the bed. Auxiliary fuel which is generally gas gets burnt above or within the bed so as to cause bed temperature to go up to about 650°C. When suitable temperature level is attained then coal is fed on it or into it for being burnt. The burning of auxiliary fuel is stopped at the moment when burning of coal becomes self sustainable.

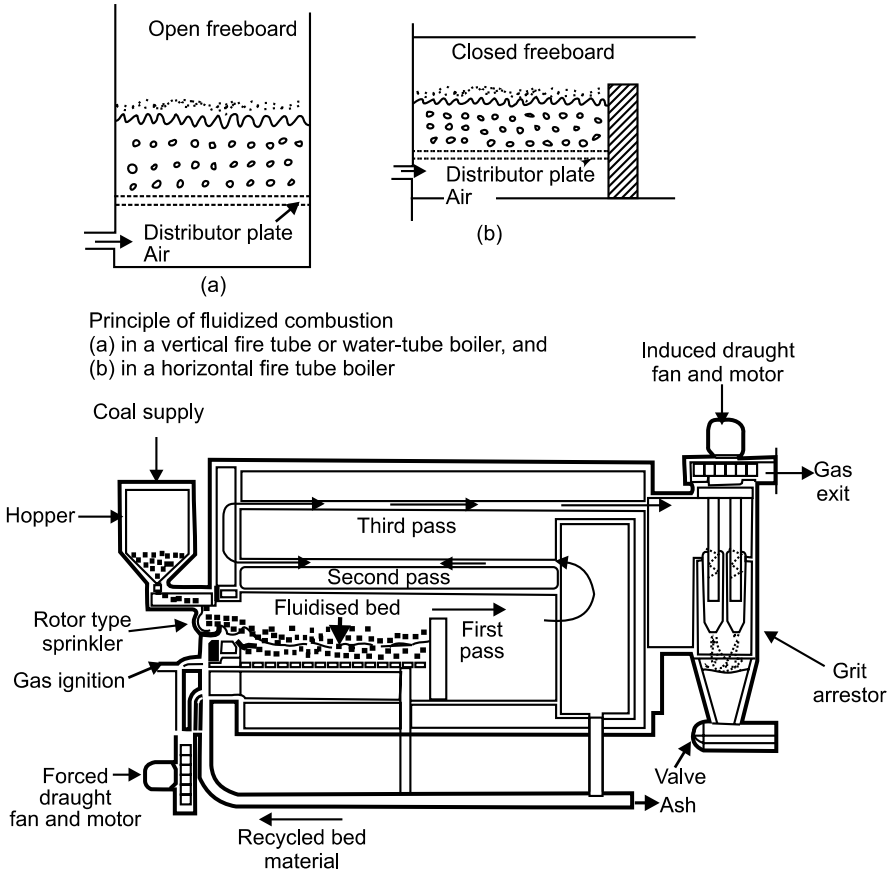


Fig. 11.19 *Horizontal fire tube boiler with fluidized bed*

The supply of coal and air are governed by the demand on the boiler. Maximum bed temperature generally reaches up to 950°C as this temperature control avoids clinker formation and emission of undesirable salts. For maintaining temperature of bed the arrangement is made for cooling of bed by water tubes and also by supplying excess air for cooling. Sometimes the low temperature flue gases leaving boiler are recirculated for bed cooling. Fluidized bed combustion offers advantage of using any kind of fuel i.e. solid, liquid or gaseous fuel. Also in this type of combustion the use of dolomite or lime stone as bed material helps in retaining sulphur in fuel. The clinker formation and emission of undesired substances is also avoided as the combustion can be controlled up to 950°C. Due to large quantity of both combustible and incombustible material present on the bed there occurs the problems of erosion in bed tubes and surroundings and also large burden on bed etc.

Fluidized bed combustion is used in both fire tube and water tube boilers but the water tube boiler offers advantage of greater flexibility in design of furnace shape and allowing for greater freeboard in which entrained particles can drop back into bed. Air velocity is generally limited to 2.5 m/s as beyond this the possibility of incomplete combustion increases. Fluidized bed boilers may have different types of fluidized beds such as,

(i) *Shallow beds,* which have bed depth up to about 30 cm. Due to such small depth the tubes for cooling can't be used in bed and excess air or recirculated flue gases are used for cooling.

(ii) *Deep beds*, which have depth of bed by about 1 m.

(iii) *Rotating fluidized bed,* which is caused to rotate about horizontal axis. It has advantage of fast mixing of fuel into bed.

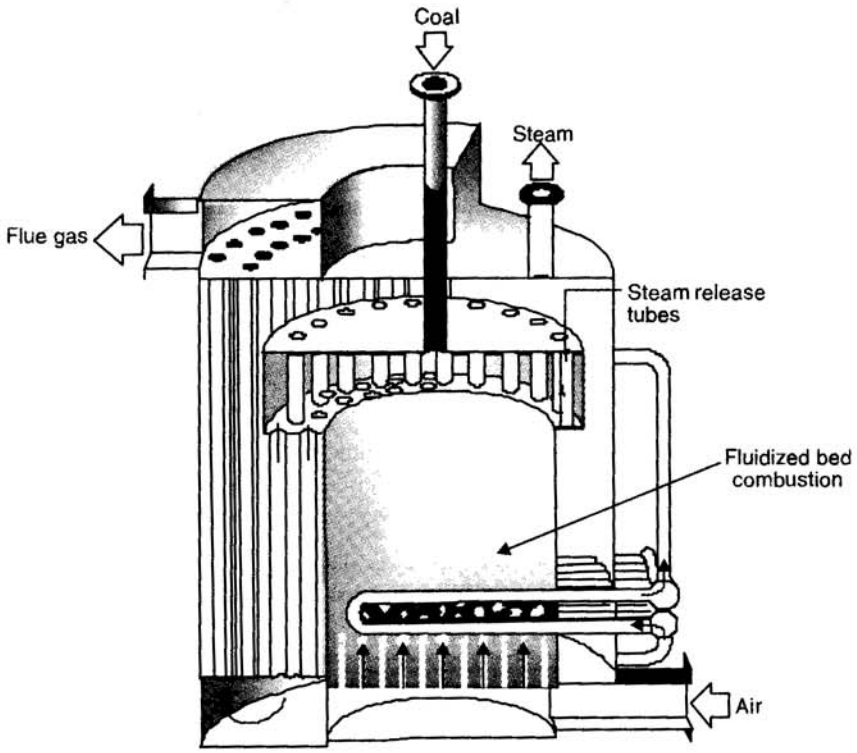


Fig. 11.20 *Vertical firetube boiler with fluidized bed*

Bed material is generally sand in sizes of 1–2 mm. Other materials such as dolomite and alumina may also be used. For removing the sulphur content from flue gases suitably sized limestone is mixed in bed material.

Figures 11.19 and 11.20 show the horizontal axis fludized bed boiler and vertical axis fludized bed boiler of fire tube type respectively.

11.19 WASTE HEAT BOILER

Waste heat boilers also called Heat recovery steam generators (HRSG) are the simple water tube boilers having the hot exhaust gases as the heat source instead of furnace. These boilers are outcome of the efforts for harnessing the heat going waste along with exhaust gases. Hot exhaust gases may be available from gas turbine exhaust, incinerators etc. Here water flowing through water tubes picks up the heat from hot gases flowing over the tubes and gets transformed into steam. Water gets transformed into steam within tubes and stored in drum from where it is drawn for superheating or for use, as the case may be.

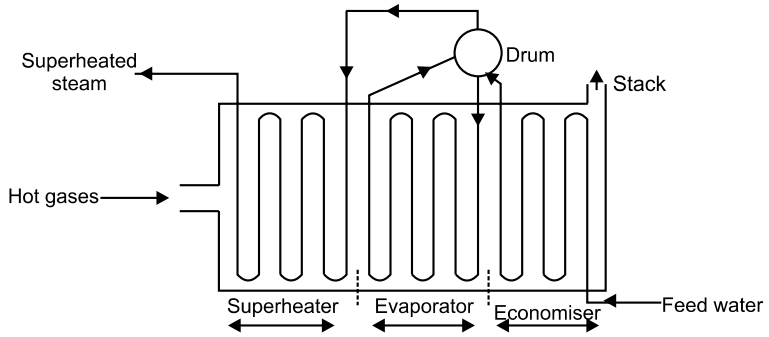
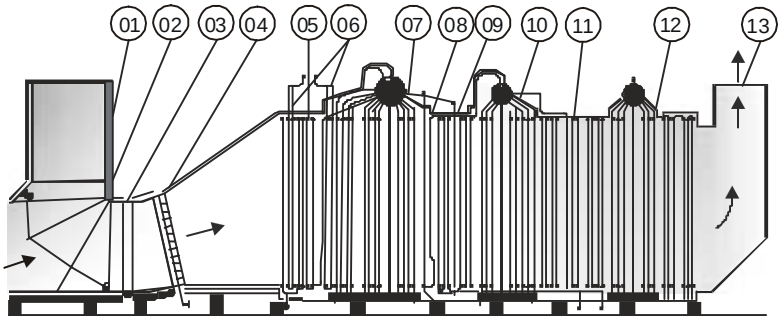


Fig. 11.21 *Line diagram for single pressure HRSG*



01 Bypass stack
02 Divertor damper
03 Expansion joint
04 Auxiliary burner /Distribution grid
05 HP superheater
06 Reheater
07 HP evaporator
08 IP superheater
09 HP economiser
10 IP evaporator
11 IP economiser
12 LP evaporator
13 Outlet stack

Fig. 11.22 *Multi pressure HRSG*

Heat recovery steam generators may generate steam at single pressure or at multiple pressures. The schematic for single pressure and multi pressure steam generation are shown in Figs. 11.21, 11.22. Circulation system in such boilers may be natural circulation or forced circulation. Natural circulation HRSG typically consist of vertical tubes and horizontal flow arrangement. Circulation is maintained by the density difference between cold water in downcomer and hot steam-water mixture in evaporator tubes. Forced circulation HRSG are characterized by horizontal tubes with vertical gas flow and use of pumps to circulate steam-water mixture inside the tubes. Due to vertical arrangement of forced circulation HRSG's, the plan area required is less. Therefore, where available floor space is limited, vertical waste heat boilers are suitable. Different aspects like performance, start up time and field erection requirements are generally same for both forced and natural circulation boilers but the operation and maintenance costs are higher for forced circulation HRSG due to presence of circulation pump. Generally horizontal natural circulation HRSG's are preferred choice, not withstanding their disadvantage in respect of space requirements. However, in applications where space constraints exist, vertical HRSG's with natural circulation have been recently developed.

HRSG's generating steam at multipressure are attractive as they extract heat effectively and efficiently. Each pressure level of steam generation requires an economiser, an evaporator and a super-

heater as shown. The positioning of these various heat exchangers in the gas stream is critical as general design philosophy is to exchange heat from gas to fluid at the highest temperature difference available. This is best accomplished by making gas and steam/water temperature gradient nearly parallel to each other. In single pressure HRSG the superheater, evaporator and economiser are placed in descending order along the gas path while in multi pressure HRSG this general order is maintained but various sections may be interchanged so that a nearly parallel temperature gradient may be achieved. Figure. 11.23 details temperature variation along the length of HRSG.

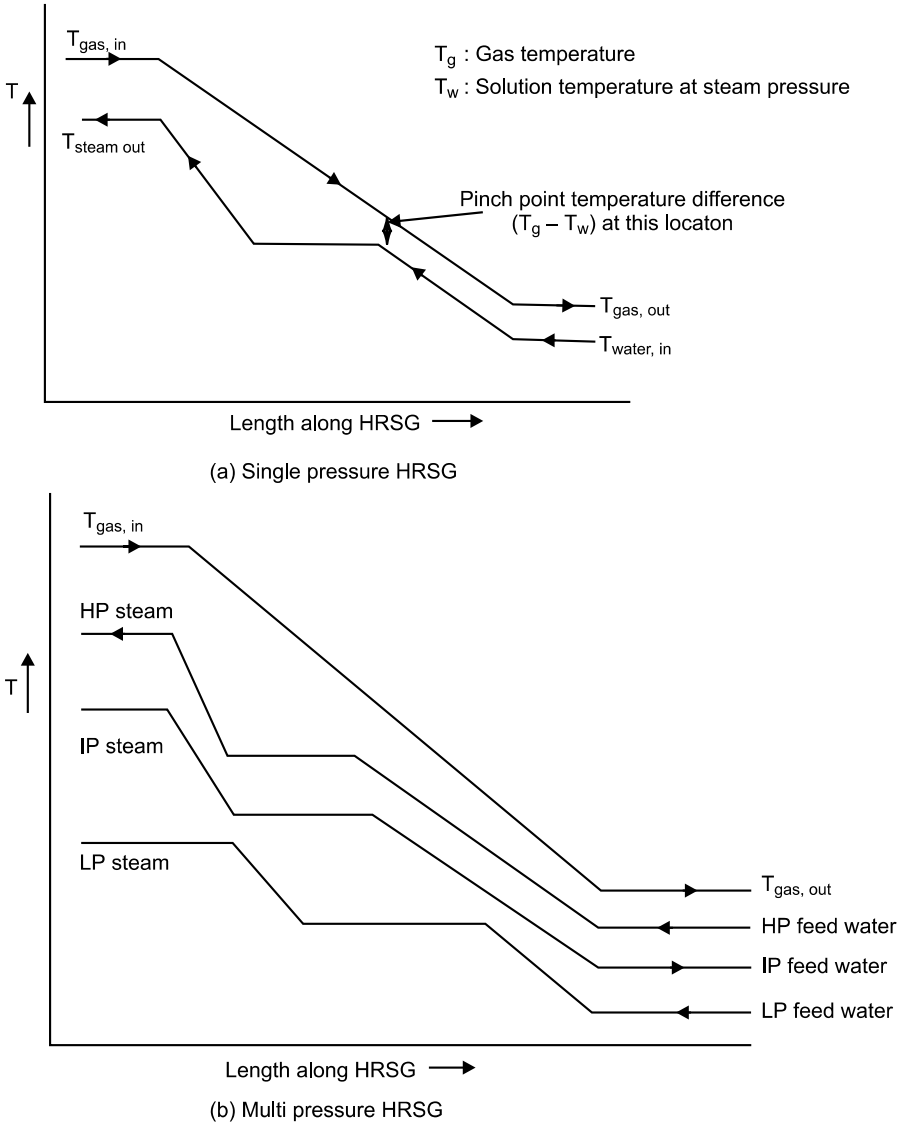


Fig. 11.23 *Temperature variation in HRSG*

HRSG has three basic parameters of pinch point, approach temperature and allowable gas side pressure drop through heat recovery system which effect the effectiveness of heat exchange. Pinch

point is the difference between the gas temperature leaving the evaporator section of system and the saturation temperature corresponding to the steam pressure in that section. Approach temperature is the difference between the saturation temperature of fluid and inlet temperature of fluid. In general it is seen that lowering the pinch point shows an increase in total heat recovered in system. However, lowering pinch point shall require more heat exchange surface and result in increase in cost and gas side draught losses. Unit investment cost is higher at low pinch point. Thus optimum design is obtained based upon it. Generally pinch point of 8–10°C is used in view of above aspects.

Lowering the approach temperature can result in more steam production at that pressure level but high approach temperature offers high level of stability. Higher approach temperatures in economiser section will eliminate the probability of steam formation in economiser at lower loads or during start up.

The allowable gas side pressure drop through HRSG influences the design and cost of unit. Very low pressure drop results in large heat exchange surface and low gas velocities. Very small gas velocity produces higher uncertainty of design with consequence of unstable performances. Generally allowable pressure drop is 250 mm to 300 mm of H_2O.

Stack gas temperature (temperature of exhaust gases leaving HRSG) selection depends upon the need to effectively recover the heat and also prevent corrosion etc. in stack.

The HRSG may also be of supplementary fired type when heat input by exhaust gases is insufficient for steam generation of desired quantity and quality. In supplementary fired HRSG burners are also put in HRSG for increasing its steam generation capacity.

11.20 BOILER MOUNTINGS AND ACCESSORIES

Boiler mountings and accessories have been defined earlier and shown on the different boilers. Different mountings are

(i) Water level indicator
(ii) Safety valves
(iii) High steam and low water safety valves
(iv) Fusible plug
(v) Pressure gauge
(vi) Stop valve
(vii) Feed check valve
(viii) Blow off cock
(ix) Manhole and mud box

Various boiler accessories are:

(i) Superheater
(ii) Economiser
(iii) Air preheater
(iv) Feed pump

Water level indicator: It is used for knowing the level of water in boiler as water level inside boiler should not go below a certain limit. General arrangement is shown in Fig. 11.24 with the different parts in it.

It has two tubes one is front glass tube while other is metal tube. Water level is seen through glass tube which is made strong enough to withstand high steam pressure and temperature. Two control cocks

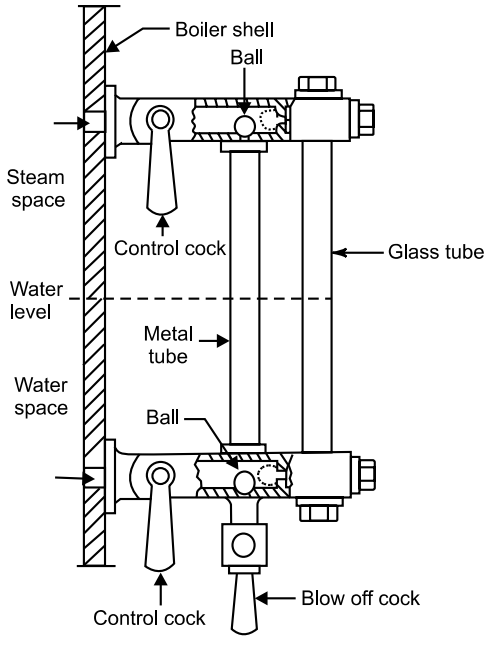


Fig. 11.24 *Water level indicator*

are provided for regulating steam and water passage from boiler to glass tube. For blow off purpose a blowing cock is also provided as shown. In case of breakage of glass tube the possibility of accident is prevented by providing two balls. As glass tube breaks the rush of water and steam carries the two balls with it and closes the openings for glass tube, thus water and steam flowing out can be prevented.

Number of other types of water level indicators are also available.

Safety valve: Its function is to prevent the steam pressure from exceeding a limiting maximum pressure value. Safety valve should operate automatically by releasing excess steam and bring pressure down within safe limits. These are of different types such as 'dead weight safety valve', 'lever safety valve' 'spring loaded safety valve' etc. Figure 11.25 gives the general description of 'dead weight safety valve'.

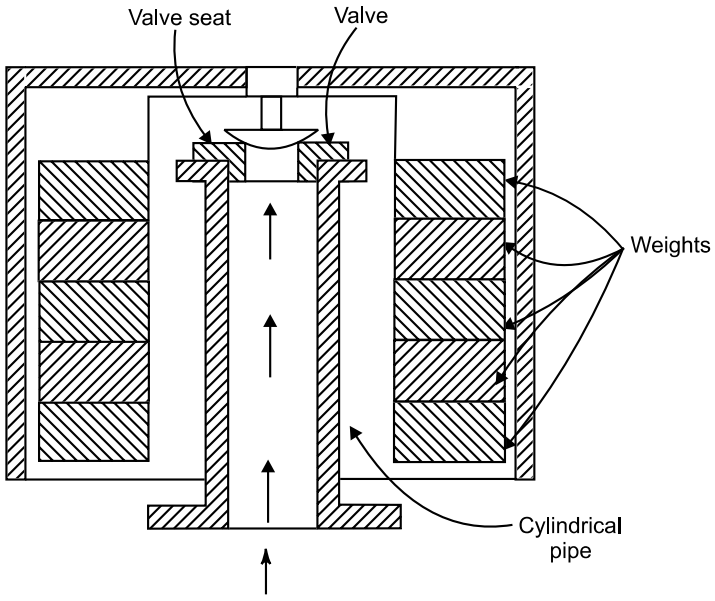


Fig. 11.25 *Dead weight safety valve*

It has a large vertical pipe on the top of which a valve seat is fixed. Valve rests upon this valve seat. A weight carrier is hung on the top of valve upon which cast iron rings enclosed in cast iron cover are placed in weight carrier as dead weight.

When the pressure of steam exceeds the total weight of valve, it is lifted and falls back as steam pressure gets reduced.

High steam and low water safety valve: This is a combined form of safety valve and low water level indicator. Figure. 11.26 shows the high steam and low water safety valve. It prevents from excessive pressure as it has a simple lever safety valve loaded by two weights as shown. Low water safety arrangement is activated through float put in boiler shell and prevents from overheating due to low water. It has two distinct valves as shown in Fig. 11.26. When the pressure inside goes beyond limiting value then high steam valve gets lifted up and excess pressure of steam is released. When the water level goes below critical level then low water valve gets raised up as it is fulcrumed and is linked to float on one end and balance weight on other end. With raising of low water valve the hemispherical valve shifts up from valve seat and steam escapes out with hissing sound. This hissing sound is loud enough to warn

attendant about the low water level in boiler. When the water level is correct then high steam valve simply acts as the dead weight safety valve. A drain pipe is also provided so that the steam getting condensed can be drained out. Condensation of steam is possible due to throttle of steam during its release from valves.

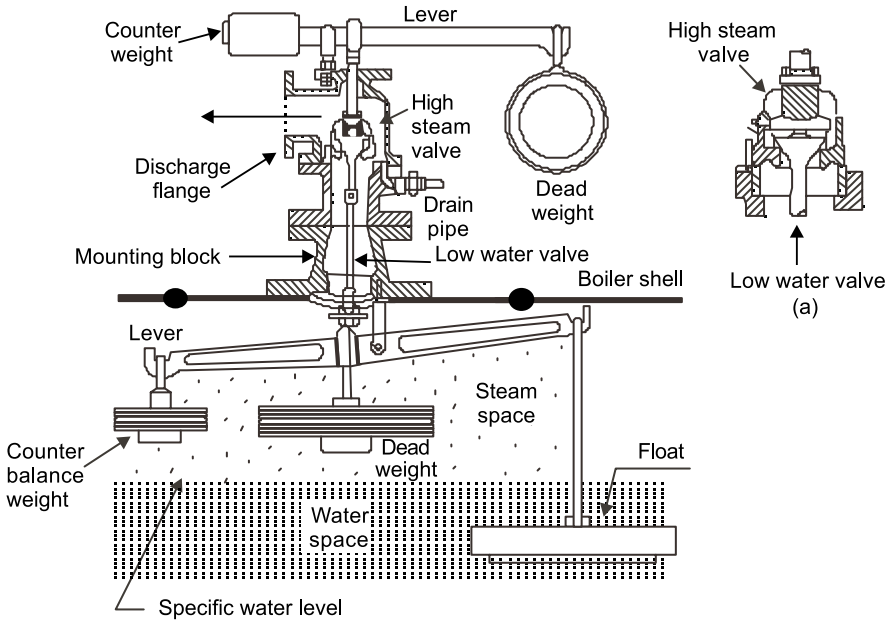


Fig. 11.26 *High steam and low water safety valve*

Fusible plug: It is a safety device used for preventing the level of water from going down below a critical point and thus avoid overheating. Fusible plug is mounted at crown plate of combustion chamber.

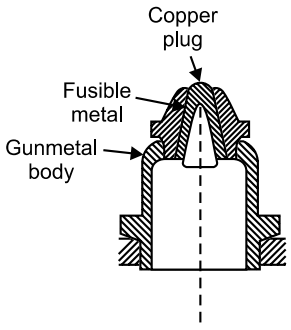


Fig. 11.27 *Fusible plug*

Fusible plug has gun metal body and a copper plug put with fusible metal at interface of copper plug and gun metal body. As water level goes down the heat available from furnace could not be completely utilized for steam formation and so the overheating may cause melting of fusible metal. Fusible metal is a low melting point metal. Thus upon melting of lining the copper plug falls down and

water falls from this opening onto furnace and thus quenches fire.

Pressure gauge: It is mounted at front top. Generally Bourdon type pressure gauge are being used for pressure measurement. Pressure is continuously monitored so as to avoid occurrence of over shooting of boiler pressure. Although safety devices to protect boiler against pressure rising beyond a limit are provided but pressure gauges are also used for monitoring pressure.

Stop valve: It regulates the flow of steam from the boiler as shown in Fig 11.28. This is generally mounted on highest part of boiler shell and performs function of regulating the flow of steam from boiler. Stop valve generally has main body of cast steel, valve, valve seat and nut etc. are of brass. Stop valve can be easily operated by rotating the hand wheel which causes lifting or lowering of spindle, thus causing opening or closing of valve.

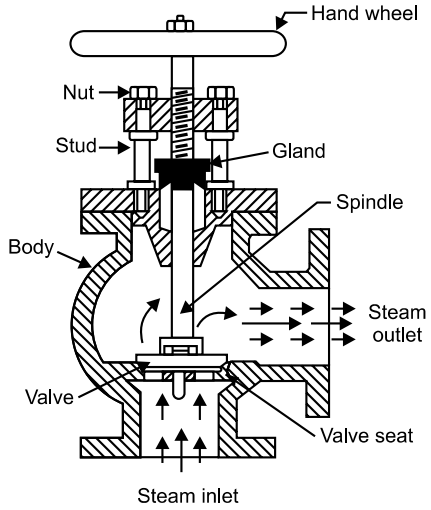


Fig. 11.28 *Stop valve*

Feed check valve: It is a non return valve at the end of delivery pipe from feed water pump and is placed on boiler shell slightly below normal water level. Figure 11.29 shows the arrangement in a feed check valve. It has a check valve whose opening and closing are regulated by the position of spindle. By

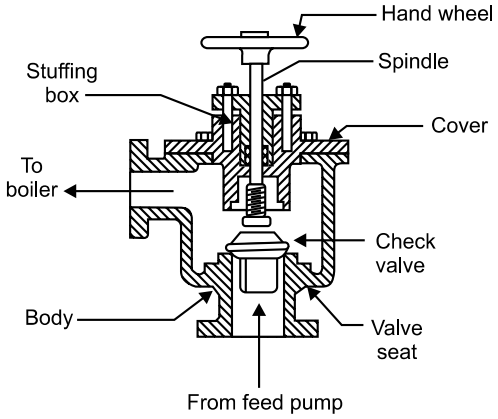


Fig. 11.29 *Feed check valve*

hand wheel rotation the position of spindle can be altered suitably. Feed check valve permits unidirec-

tional flow of water from feed pump to be boiler shell. Under normal running the pressure of feed water coming from pump is more than pressure inside the boiler and so the feed water continues to enter the shell. While during the non working of feed pump the pressure in boiler shell is more and so the check valve gets closed.

Blow off cock: It is used for periodical cleaning by discharging the water and sediments from bottom of boiler. Figure 11.29 shows the blow off cock. Blow off cock is fitted to the bottom of boiler shell. Blow off cock has a plug of conical type put into the mating casing. Plug position is altered for opening and closing the flow. Plug has rectangular opening which when comes in line with inlet and outlet passage then blow off cock is open and when opening is not in line then cock is closed. Plug is rotated by spindle.

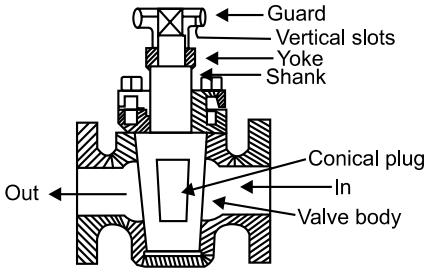
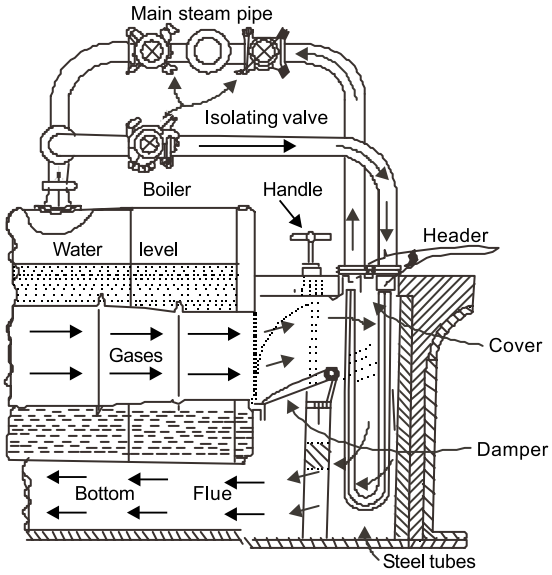


Fig. 11.30 *Blow off cock*

Blow off cock also helps in regulating the salt concentration as frequent draining helps in throwing out the salt deposited over period of time. Opening blow off cock removes deposited sediments in boiler.

Manhole and mud box: Manhole provides opening for cleaning, inspection and maintenance purpose. Mud box is a collection chamber (as shown in Babcock and Wilcox boiler) for collecting the mud.

Superheater: Its purpose is to super heat steam and is a type of heat exchanger in which steam flows inside tubes and hot gases surround it. Figure 11.31 shows the smooth tube hairpin type superheater (Sudgen's superheater) and convective and radiant superheater.



(a) Smooth tube hairpin type superheater

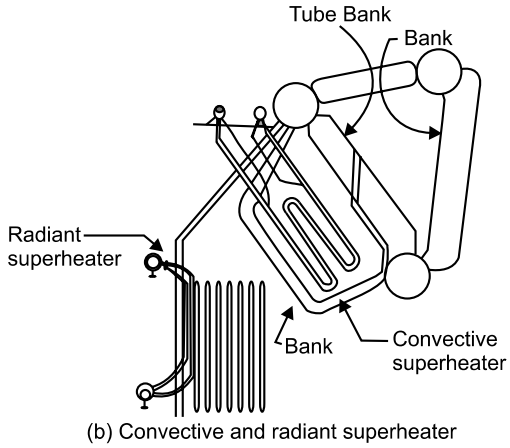


Fig. 11.31 *Superheater*

In hair pin superheater the steam generated is passed through isolating valve to U-shaped steel tubes. Superheated steam leaves superheater through tube connected to steam stop valve. Hot gases from fire tube are diverted over superheater tubes by damper as shown. These hot gases upon passing over steel tubes leave boiler through bottom flue. The convective and radiant superheater as shown has two set of tubes picking up heat through convection and radiation.

Economizer: It is also a heat recovery device in which feed water is heated from heat available with exhaust gases. Thus hot feed water available from economizer lowers the fuel requirement in

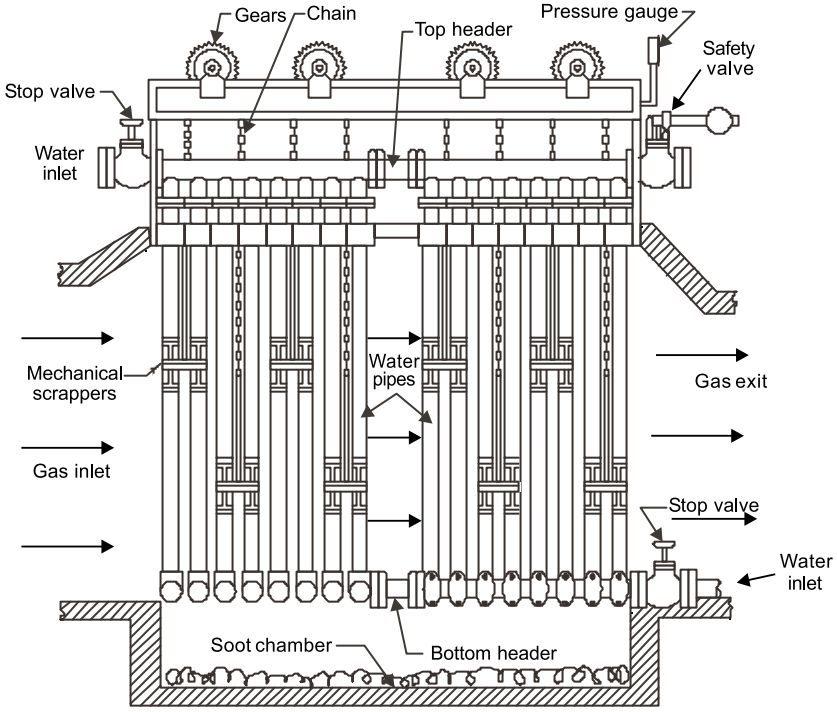


Fig. 11.32 *Economizer*

combustion. It is also a type of heat exchanger having exhaust gas and feed water as two fluids. General arrangement in economizer is shown in Fig. 11.32. Economizer also helps in removal of dissolved gases by preheating of water and thus minimizes tendency of corrosion and pitting. Hotter feed water also reduces thermal strain in boiler parts.

Economizer is located in the boiler structure so as to expose the economizer surface to hot gases. Its location varies with the boiler designs. Typical economizer called Green's economizer as shown in Fig. 11.32 has vertical pipes of cast iron fitted with two headers at bottom and top respectively. Feed water passes through bottom header, economizer tubes and top header to boiler. Thus economizer is simply a heat exchanger where heat is transferred from hot flue gases to water inside the tubes through metal interface. Top header is also provided with a safety valve so as to avoid explosion due to excessive pressure of water developing inside economizer tubes. Bottom header is also provided with a blow off valve so as to throw out the sediments deposited in feed water. Economizer is also provided with scrapers fitted to clean pipes from the deposition of soot carried by the flue gases. Continuous scrapping is always desired so as to maximize heat transfer rate. Economizer also has a by pass provided so that flue gases can be diverted when economizer is out of full or part operation due to failure or cleaning purpose or feed water temperature control.

Air preheater: It is used for recovering the heat going along with exhaust gases by the air before being sent to furnace. Heat is recovered by passing exhaust gases through an air to air heat exchanger as shown in Fig. 11.33. Air preheaters are generally placed after economizer and before chimney. Air when preheated before supply to furnace/combustion chamber helps in achieving 'faster rate of combustion', 'possibility of burning inferior quality coal/fuel' and 'increased rate of evaporation from boiler' etc.

Air preheaters are of tubular type, plate type and regenerative type. This classification of air preheaters bases upon the kind of arrangement used for heat exchange between two fluids. Generally, tubular type air preheater are generally used in small boilers. Tubular air preheater has hot flue gases passing inside tubes and air blown over these tubes.

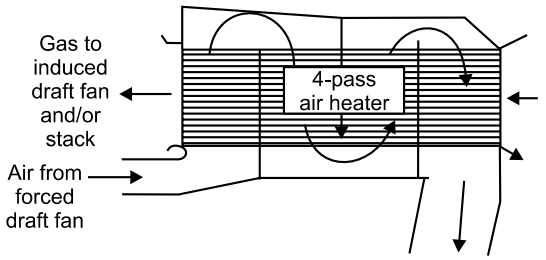


Fig. 11.33 *Tubular air preheater*

In case of plate type air preheater there are number of plates having air and flue gases flowing through alternative spacings. In regenerative type air preheater there is a wire mesh rotor which is alternatively heated and cooled by the hot flue gases and air to be used for combustion.

Feed pump: Feed pump is used for sending water into boiler at the pressure at which steam generation takes place. It is generally of three types i.e. centrifugal pump, reciprocating pump and injectors.

A reciprocating type feed pump is shown in Fig. 11.34. In boilers the pumps raise feed water pressure to the value more than the highest operating pressure of boiler. Pumps also have capability to deliver feed water in excess to the maximum evaporation rate of boiler. This excess capacity of feed pump is generally 15–20% of maximum continuous rating and is required to meet one or more of following situations.

(i) Sometimes excessive steam demand may occur.

(ii) Since boilers are to be blown out frequently to remove depositions and salts, therefore excess capacity is required.

(iii) Malfunctioning of boiler may cause carrying away of water with steam, thereby causing water shortage in boiler.

(iv) Over a period of time pump capacity decreases and so excess pump capacity is desired.

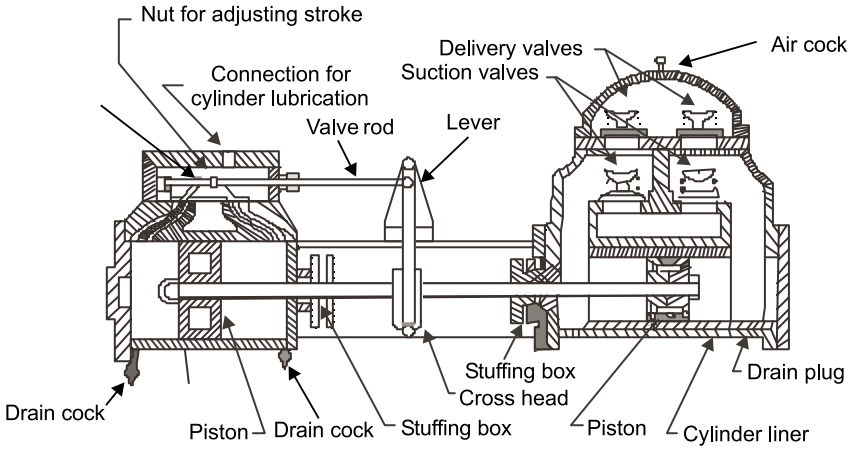


Fig. 11.34 *Reciprocating type pump, Duplex feed pump*

Now a days multistage centrifugal type pumps are used, which have favourable pressure/volume characteristics.

Steam trap: Steam traps perform function of catching steam getting condensed in the form of condensate (water) due to partial condensation of steam in pipes, steam jackets etc. Figure 11.35 shows a bucket type steam trap. Water available due to partial condensation enters steam trap at inlet *A*. Steam

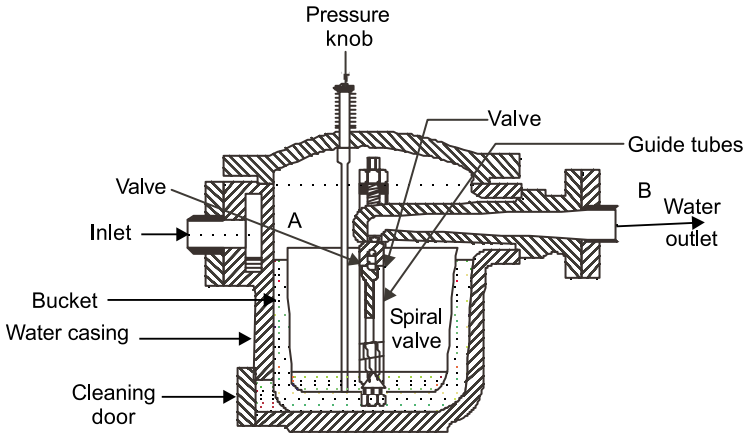


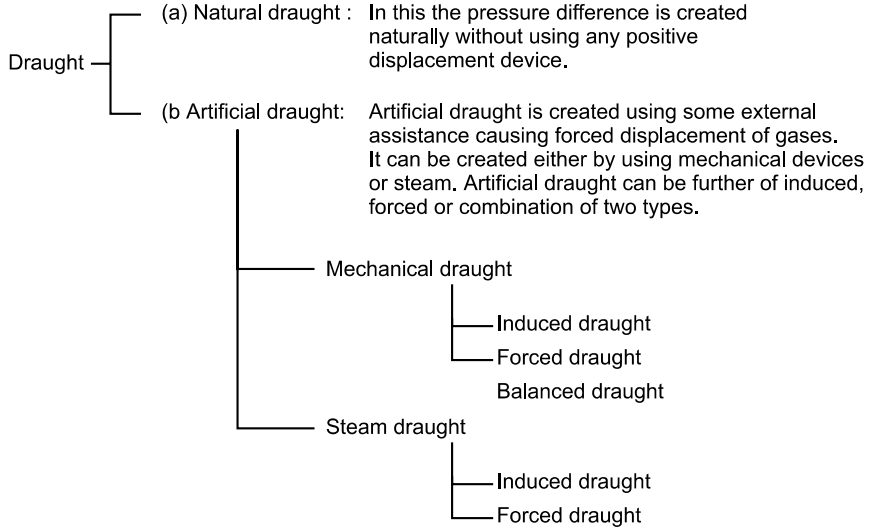
Fig. 11.35 *Bucket type steam trap*

trap casing already has water in it and bucket keeps on floating. As the water level in steam trap casing rises to the extent that water overflows from the bucket, due to excess weight of water in bucket the bucket sinks down and discharge valve opens causing water to leave through outlet passage *B*. After sufficient water is drained out the weight of water in bucket reduces and the bucket starts floating again. This rise of bucket closes the discharge valve again.

11.21 BOILER DRAUGHT

Draught refers to the pressure difference created for the flow of gases inside the boiler. Boiler unit has a requirement of the expulsion of combustion products and supply of fresh air inside furnace for continuous combustion. The obnoxious gases formed during combustion should be discharged at such an height as will render the gases unobjectionable. A chimney or stack is generally used for carrying these combustion products from inside of boiler to outside, i.e. draught is created by use of chimney. Draught may be created naturally or artificially by using some external device. Draught can be classified as below:

- In this the pressure difference is created naturally without using any positive displacement device.
- Artificial draught is created using some external assistance causing forced displacement of gases. It can be created either by using mechanical devices or steam. Artificial draught can be of induced type, forced type or combination of two types.



Thus the draught in boiler may be said to be required for, 'providing and maintaining the supply of sufficient air for combustion', 'expulsion of combustion products from furnace region' and 'discharge of burnt gases to atmosphere'. The amount of draught required shall depend upon, 'type of boiler', 'rate of fuel burning', 'rate at which combustion products are produced' and 'the air requirement rate'. As the pressure difference is very small so draught is measured in 'mm' of water. Mathematically, pressure due to 1 mm of water column is equivalent to 1 kgf/m^2.

11.22 NATURAL DRAUGHT

It is produced employing chimney. The natural draught is produced by a chimney due to the fact that the hot gases inside the chimney are lighter than the outside cold air i.e. density difference of hot gases inside chimney and cold atmospheric air. Thus in a boiler unit the combustion products (hot) rise from fuel bed through chimney, and are replaced by fresh air (cold) entering the grate. It means that amount of draught produced by a chimney depends upon flue-gas temperature. Intensity of draught produced by chimney also depends upon height of chimney. Draught produced by a taller chimney is large as the difference in weight between the column of air inside and that of air outside increases with height. Generally draught is less than 12 kgf/m^2 in chimneys.

In stricter terms the word 'chimney' is used for brick or concrete structure and 'stack' is used for metallic one. Chimneys are generally made of steel, brick or reinforced concrete. Steel chimneys or stacks are most desirable for smaller boiler units due to small initial cost, ease of construction and erection. On account of small space requirement as compared to other stacks, self sustaining steel stacks are used in some large power plants. Steel stacks have problem of rust and corrosion, so painting requirements are quite stringent. Brick chimneys are required where permanent chimney with longer life is required. Such chimneys have inherent disadvantages of leakages etc. across the construction, therefore careful construction is required. Leakage of air across chimney wall effects intensity of draught. Brick chimneys are constructed of round, octagonal, or square section. Generally brick chimney has two walls with air space between them and inner wall having fire brick lining. Concrete chimneys are used due to the absence of joints, light weight and space economy as compared with brick chimneys. Also the reinforced concrete chimney is less expensive compared to brick chimney along with minimum chances of leakage across walls.

Calculations: As it is obvious from earlier discussion that the vertical duct called chimney creates natural draught so estimation of height of chimney is very important. Figure 11.36 shows the schematic of chimney in a boiler unit. During no working of boiler the pressure inside boiler is atmospheric pressure. Pressure at outlet of chimney will be less than atmospheric pressure due to altitude difference.

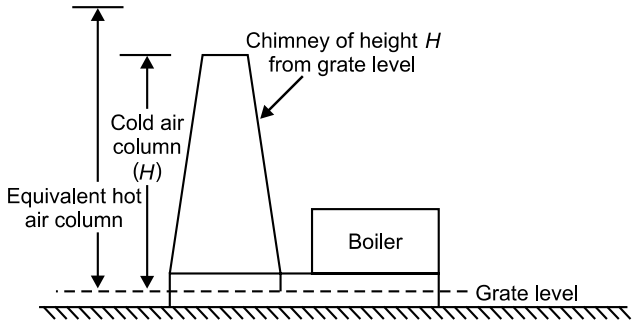


Fig. 11.36 *Schematic of chimney*

During boiler operation the chimney shall be filled with hot gases and the pressure at bottom of chimney (p_b) shall be summation of 'pressure at chimney outlet', (p_0) and 'pressure due to hot gas column of height H'.

Pressure at bottom of chimney = Pressure at outlet + Pressure due to hot gas column

$$p_b = p_0 + \rho_g \cdot g \cdot H$$

where ρ_g is density of hot gases.

However, the pressure at grate level remains unchanged. Pressure difference between the grate level pressure and bottom of chimney causes flow of gases. This pressure difference is also called static draught.

Let us consider the combustion of fuel in furnace. Combustion products are released as a result of this combustion process. Fuel may be considered to be comprising of hydrocarbons.

$$\text{Fuel} + \text{Air} \rightarrow \text{Combustion products} + \text{Heating value}$$

In the hydrocarbon fuel major constituents are carbon, hydrogen, nitrogen etc. As a result of complete combustion carbon gets transformed into carbon dioxide and hydrogen yields steam. The combustion products have major fraction of carbon dioxide and steam, but the volume of steam is negligible compared to volume of combustion product. The volume of combustion products can be taken equal to volume of air supplied, measured at same temperature and pressure.

Let us assume various properties as,

T_a = Atmospheric temperature, K

T_g = Average temperature of hot gases inside chimney, K

T_0 = Absolute zero temperature, 273.15 K

ρ_a = Density of air at absolute temperature, 1.293 kg/m^3

ρ_g = Density of hot gases inside chimney, kg/m^3

Δp = Pressure difference, draught in Pa

$\Delta p'$ = Pressure difference, draught in kgf/m^2

h_g = Equivalent height of hot gas column to produce draught, Δp in 'metres'.

h_w = Equivalent height of water column in 'mm' to produce draught, Δp

m = Mass of air supplied per kg of fuel

C = Hot gas velocity in chimney, m/s

M_g = Discharge rate through chimney, kg/s

Total mass of combustion products (hot gases) for one kg of fuel = (m + 1), kg

Using the assumption regarding combustion products,

Volume of hot gases at temperature, T_g = Volume of air supplied at temperature, T_g

$$= \frac{\text{Mass of air}}{\text{Density of air at } T_g}$$

$$= \frac{m}{\left(\frac{\rho_a \cdot T_0}{T_g}\right)}$$

$$= \frac{m \cdot T_g}{\rho_a \cdot T_0}$$

Pressure of hot gases in chimney at grate level = Density of hot gases × height of chimney × gravitational acceleration

$$= \left(\frac{\text{Mass of hot gases}}{\text{Volume of hot gases at } T_g}\right) \times H \times g$$

$$= \left\{\frac{(m+1)}{\left(\frac{m \cdot T_g}{\rho_a \cdot T_0}\right)}\right\} \times H \times g$$

$$= \left\{\left(\frac{m+1}{m}\right)\frac{\rho_a \cdot T_0}{T_g}\right\} H \cdot g$$

Pressure due to cool air (outside) column of height H at grate level

$$= \text{Density of air at } T_a \times \text{Height} \times g$$

$$= \left(\frac{\rho_a \cdot T_0}{T_a}\right) \times H \times g$$

$$= \frac{\rho_a \cdot T_0 \cdot H \cdot g}{T_a}$$

Natural Draught produced = Difference of pressures due to cool air column and hot gas column of height '*H*'.

$$\Delta p = \left\{\frac{\rho_a \cdot T_0 \cdot H \cdot g}{T_a}\right\} - \left\{\frac{(m+1) \cdot \rho_a \cdot T_0 \cdot H \cdot g}{m \cdot T_g}\right\}$$

$$\boxed{\Delta p = \rho_a \cdot T_0 \cdot H \cdot g \left\{\frac{1}{T_a} - \left(\frac{m+1}{m}\right) \cdot \frac{1}{T_g}\right\}, \text{ Pa}}$$

Or

Draught in kgf/m^2, $\Delta p' = \rho_a \cdot T_0 \cdot H \left\{\frac{1}{T_a} - \left(\frac{m+1}{m}\right) \cdot \frac{1}{T_g}\right\}$, kgf/m^2

Substituting values for T_0, ρ_a and rounding off values we get,

$$\boxed{\Delta p' = 353 \cdot H \left\{\frac{1}{T_a} - \left(\frac{m+1}{m}\right) \cdot \frac{1}{T_g}\right\}, \text{ kgf/m}^2}$$

Height of hot gas column equivalent to draught produced,

$$h_g = \frac{\Delta p', \text{ kgf}/\text{m}^2}{\text{Density of hot gases, kg}/\text{m}^3 \text{ at } T_g}$$

$$= \frac{\rho_a \cdot T_0 \cdot H \cdot \left\{\frac{1}{T_a} - \left(\frac{m+1}{m}\right) \cdot \frac{1}{T_g}\right\}}{\left\{\frac{\rho_a \cdot T_0}{T_g} \cdot \left(\frac{m+1}{m}\right)\right\}}$$

$$h_g = H \cdot \left\{\left(\frac{m}{m+1}\right) \cdot \frac{T_g}{T_a} - 1\right\}, \text{ metres}$$

As the 1 mm of water column exerts a pressure of 1 kgf/m^2 so the draught in terms of water column can be given by,

$$h_w = \rho_a \cdot T_0 \cdot H \left\{\frac{1}{T_a} - \left(\frac{m+1}{m}\right) \cdot \frac{1}{T_g}\right\}, \text{ mm of water.}$$

Thus natural draught can be mathematically given by Δp, $\Delta p'$, h_g and h_w.

Actually in boilers this draught requirement is different from that theoretically estimated due to the draught losses. The magnitude of these losses varies from boiler to boiler due to different arrangements within them. Some of generic losses shall be because of:

- frictional losses due to resistance offered by passage surface roughness, different equipments as grate, superheater, air preheater, economiser etc. through which gas passes.

- pressure losses in bends, baffles, supports etc.
- kinetic energy required with gases for moving at certain velocity throughout.

Therefore while designing the chimney due considerations should be made for overcoming above losses. These losses constitute about 20% of the static draught produced.

Hot gas velocity in chimney: Assuming chimney to be frictionless the hot gas velocity in chimney could be given using the equivalent hot gas column height;

$$C = \sqrt{2g \cdot h_g}\ , \text{ m/s}$$

For the chimney having friction losses, the hot gas velocity shall be lesser. If the equivalent pressure loss due to friction in hot gas column is given by h_f then the velocity of hot gases,

$$C = \sqrt{2g \cdot \left(h_g - h_f\right)}\ , \text{ m/s}$$

$$= \sqrt{2g \cdot h_g \left(1 - \frac{h_f}{h_g}\right)}$$

Substituting values for 'g'

$$C = 4.43 \sqrt{h_g \left(1 - \frac{h_f}{h_g}\right)}$$

or $$\boxed{C = K \cdot \sqrt{h_g}}$$

where K is a constant and its value is available for different types of chimneys as given below. It depends upon the friction loss fraction.

$K = 0.825$ for brick chimney, and

$K = 1.1$ for steel chimney

Diameter of chimney: Diameter of chimney could be estimated from the mass flow rate of hot gases through chimney and its velocity.

$$\text{Chimney cross-sectional area, } A = \frac{\text{Discharge rate}}{\text{Velocity of hot gases} \times \text{Density of hot gas}}$$

$$\text{Diameter of chimney} = \sqrt{\frac{4}{\pi} \times \frac{M_g}{C \cdot \rho_g}}$$

Discharge through chimney: Mass flow rate of hot gases through chimney could be obtained as,

M_g = Cross-sectional area × Velocity of hot gas × Density

$$M_g = A \times C \times \rho_g$$

$$\boxed{M_g = A \times \rho_g \times K \cdot \sqrt{h_g}}$$

For hot gas pressure p_g, the density of gas ρ_g can be given using perfect gas approximation,

$\rho_g = \dfrac{p_g}{R\,T_g}$. Here R is gas constant

so $$M_g = \frac{A \cdot p_g \cdot K \cdot \sqrt{h_g}}{R\,T_g}$$

Discharge through chimney can be mathematically maximized for certain conditions. A look at expression of discharge given earlier shows that for a particular chimney.

$$M_g \propto \left\{\left(\sqrt{h_g}\right) \times \left(\frac{1}{T_g}\right)\right\}$$

or $$M_g = \text{Constant} \cdot \left[\frac{H}{T_g^2} \cdot \left\{\left(\frac{m}{m+1}\right) \cdot \frac{T_g}{T_a} - 1\right\}\right]^{1/2}$$

Differentiating discharge with respect to hot gas temperature and equating it to zero for optimum condition,

$$\frac{d\,M_g}{d\,T_g} = 0$$

we get, $$\boxed{\frac{T_g}{T_a} = \frac{2(m+1)}{m}}$$

or $$T_g = \frac{2(m+1)}{m} \cdot T_a$$

Upon substituting T_g value in $\frac{d^2 M_g}{d\,T_g^2} = 0$ we see that it is condition for maxima of M_g.

Thus discharge through chimney is maximum for the hot gas temperature given by

$$\boxed{T_g = 2\left(\frac{m+1}{m}\right) \cdot T_a}$$

This hot gas temperature shall be slightly more than twice of ambient temperature for maximum discharge rate through chimney.

For the condition of maximum discharge, draught can be obtained as,

$$\Delta p_{\text{for max. discharge}} = \rho_a \cdot T_0 \cdot H \cdot g \left\{\frac{1}{2\,T_a}\right\}$$

$$\boxed{\Delta p_{\text{for max. discharge}} = \frac{\rho_a \cdot T_0 \cdot H \cdot g}{2\,T_a}}$$

In terms of water column,

$$\boxed{h_{w,\text{ for max. discharge}} = \frac{\rho_a \cdot T_0 \cdot H}{2\,T_a}}$$ mm of water column.

Maximum discharge could be obtained upon substituting optimum temperature, T_g

$$M_{g,\,\max} = \frac{K \cdot A \cdot p_g \cdot m \cdot \sqrt{H}}{2R \cdot T_a (m+1)}$$

For a smooth and frictionless chimney,

$$M_{g,\,\max} = \frac{\sqrt{2g} \cdot A \cdot p_g \cdot m \cdot \sqrt{H}}{2R \cdot T_a (m+1)},\ \text{kg/s}$$

Above expression for maximum discharge rate can also be used for getting the height of chimney for a given gas flow rate.

Efficiency of chimney: It has been explained in natural draught that the pressure difference is created due to density difference caused by temperature difference. Hence it is obvious that the flue gases should leave at quite high temperature for creating required density difference. Thus the flue gases leave with sufficient heat energy, which could be used in boiler if some other mechanism is employed for exhaust of flue gases such as artificial draught. Therefore, efficient chimney should have such a design so that flue gases leave at lowest possible temperature. Efficiency of chimney is quantification of the cost of natural draught in terms of energy, i.e. the large amount of usable energy going along waste hot gases. Normally this efficiency of chimney is less than 1 percent.

Chimney efficiency is defined as the ratio of "energy with unit mass of gas in natural draught" and "the extra heat carried by same mass of gas due to high temperature in natural draught as compared to that in artificial draught".

$$\text{Chimney efficiency} = \frac{\text{Energy with unit mass of gas in natural draught}}{\text{Extra heat carried away in natural draught compared to artifical draught by unit mass of gas}}$$

Let us assume that the temperature of flue gases in artificial draught is $T_{g,a}$ which will be less than T_g in natural draught. Let the specific heat of hot flue gases be $C_{p,g}$, J/kg · K.

So, energy with unit mass of hot flue gas in nature draught = $1 \times h_g \times g$, joule

$$= 9.81 \times H \left\{ \left(\frac{m}{m+1} \right) \cdot \frac{T_g}{T_a} - 1 \right\}$$

Extra heat carried by unit mass of hot gas in natural draught compared to artificial draught

$$= C_{p,\,g} \times 1 \times (T_g - T_{g,\,a})$$
$$= C_{p,\,g} \times 1 \times (T_g - T_{g,\,a}),\ \text{joule}$$

$$\eta_{\text{chimney}} = \frac{9.81 \times H \left\{ \left(\frac{m}{m+1} \right) \cdot \frac{T_g}{T_a} - 1 \right\}}{C_{p,g} (T_g - T_{g,a})}$$

Expression for efficiency of chimney shows that it is directly proportional to the height of chimney, but the efficiency is found to be very small even for very tall chimney. It is seen that about more than 20% of total heat released gets lost along with hot flue gases in case of natural draught. Therefore the artificial draught becomes more economical in the situation where cost of harnessing the extra heat carried with flue gases is less than the cost of energy lost in natural draught.

11.23 ARTIFICIAL DRAUGHT

Artificial draught refers to the externally created draught employing some equipments for it. Its requirement is felt, when the natural draught becomes insufficient for exhaust of flue gases. In general it is seen that for draught requirements being more than 40 mm of water, the natural draught does not work and becomes highly uneconomical. In the modern large power plants this draught produced by chimney is insufficient and requires some artificial method. Also the size of boiler units in use today forbid the use zof natural draught as the flue gas handling capacity is limited. In case of natural draught the fuel rate upto (20 kg/hr per m^2 of grate area could be handled while with artificial draught it goes up to 300 kg/hr per m^2 of grate area. Apart from these limitations the economy of using artificial draught over natural draught beyond a limit also make it attractive. For same steam generation the fuel consumption gets reduced by up to 15% with use of artificial draught in a boiler.

Artificial draught may be produced either by mechanical means such as fans, blowers etc. or by using steam jet for producing draught. Thus artificial draught can be classified as,

(i) Mechanical draught
(ii) Steam jet draught.

Artificial draught systems do not require tall chimney/stack, but small stack is always required for discharge of flue gases to certain height in atmosphere for minimizing pollution.

11.23.1 Mechanical Draught

Mechanical draught produced using fans, blowers etc. could be of forced type, induced type or the combination of the two. Line diagram showing the arrangements is shown in Fig. 11.37.

(i) *Forced draught:* It is the arrangement in which high pressure air is delivered to the furnace so as to force flue gases out through stack. Air under pressure may be fed to stokers or grate for which a fan/blower is put at the bottom of furnace. As due to pressurised air the pressure inside furnace becomes more than atmospheric pressure so it should be properly sealed, otherwise gas may leak through the cracks in setting into the boiler unit. Also the flames from furnace may flare out upon opening the fire door, so it should be equipped with dampers to shut off air supply when furnace doors are opened. It is obvious from here that the fan in case of forced draught shall handle fresh atmospheric air.

(ii) *Induced draught:* Induced draught is the one in which the suction created on furnace side draws flue gases and throws them out through small chimney/stack. Fan is located at base of chimney in induced draught so as to reduce pressure at fuel bed below atmospheric pressure. The fan in induced draught shall handle hot flue gases. Power required to drive the fan/blower in case of induced draught is less than that in case of forced draught fan. Mathematically it can be given as below.

For volume of fluid handled being V (m^3/s) at pressure of p, the power required shall be,

$$= \frac{p \cdot V}{\eta_{\text{mech}}}$$

In case of *induced draught* fan, as hot flue gases are to be handled, so,

total mass handled by fan for m_f kg of fuel burnt per unit time $= m_f + m \cdot m_f$

$$= m_f (1 + m), \text{ kg/s}$$

From earlier article, density of hot flue gases $= \dfrac{\rho_a \cdot (m+1) \cdot T_0}{m \cdot T_g}$

Therefore, volume handled by fan = $\frac{\text{Mass}}{\text{Density}}$

$$= \frac{m_f (1+m) \cdot m \cdot T_g}{\rho_a \cdot (1+m) \cdot T_0}$$

$$V_{\text{induced}} = \frac{m \cdot m_f \cdot T_g}{\rho_a \cdot T_0}$$

Power required = $\frac{p \cdot V_{\text{induced}}}{\eta_{\text{mech}}}$

$$\boxed{\text{Induced draught fan power requirement} = \frac{p \cdot m \cdot m_f \cdot T_g}{\rho_a \cdot T_0 \cdot \eta_{\text{mech}}}}$$

In case of *forced draught* the fan handles air at atmospheric temperature so,
total mass of air handled = $m_f \cdot m$, kg/s

Volume of air at absolute zero temperature = $\frac{m_f \cdot m}{\rho_a}$

Volume of air at atmospheric temperature = $\frac{m_f \cdot m \cdot T_a}{\rho_a \cdot T_0}$ m^3/s

Power required in forced draught fan = $\frac{p \cdot m_f \cdot m \cdot T_a}{\rho_a \cdot T_0 \cdot \eta_{\text{mech}}}$

$$\boxed{\text{Forced draught fan power requirement} = \frac{p \cdot m_f \cdot m \cdot T_a}{\rho_a \cdot T_0 \cdot \eta_{\text{mech}}}}$$

Comparing the two power requirements,

$$\frac{\text{Power required in induced draught}}{\text{Power required in forced draught}} = \frac{T_g}{T_a} = \text{More than 1}$$

As $T_g > T_a$ so the power requirement in induced draught is more than that of forced draught.

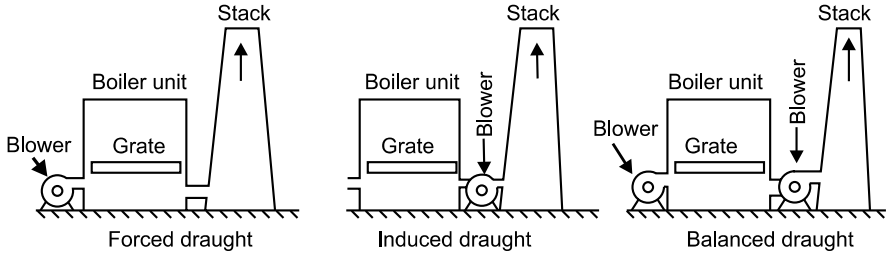


Fig. 11.37 *Mechanical draught*

(iii) ***Balanced draught:*** Sometimes it is seen that forced draught or induced draught alone is not suitable for boiler unit due to their own inherent limitations. In these situations a combination of forced draught and induced draught is being used. Such combined arrangement for

draught is called balanced draught. Here both forced draught fan/blower and induced draught fan/blower are employed. Forced draught fan ensures complete supply of air for proper combustion after overcoming all resistances while induced draught fan takes care of post combustion resistances, thus ensuring complete removal of flue gases. Pressure variation in balanced draught shows the pressure values through out boiler unit.

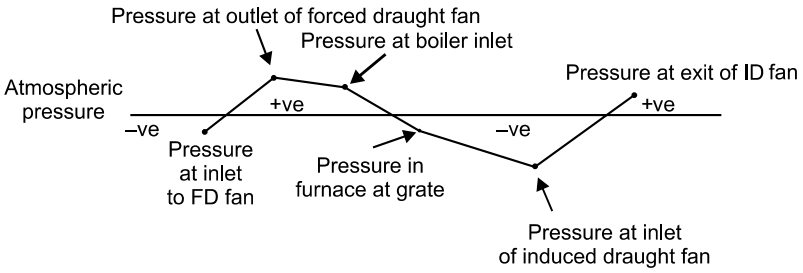


Fig. 11.38 *Pressure variation in balanced draught*

(iv) ***Comparison of forced and induced draught:*** A comparison of forced and induced draught system shows that,

- Power requirement is more in case of induced draught fan blower as compared to forced draught because of hot flue gases being handled by induced draught fan and atmospheric air being handled by forced draught fan. Induced draught fan size may be upto twice of the fan size in forced draught. Material of induced draught fan should be capable of handling hot gases laden with solid particles (ash).
- As the induced draught fan/blower withstands high temperature, so the fan cooling requirements are very stringent as compared to forced draught.
- Leakage of atmospheric air into furnace due to sub atmospheric pressure inside the induced draught system causes dilution of flue gases.

  In case of forced draught, the pressure inside is more than atmospheric pressure so chances of leakage of furnace gases out to atmosphere are there. This may cause blow out upon opening of furnace doors in case of forced draught.
- Combustion rate is better in case of forced draught system due to better air dispersion across the grate, as compared to induced draught system.
- Maintenance is easy in forced draught fan as compared to induced draught due to their locations.

(v) *Advantages of mechanical draught:* Mechanical draught has various advantages in comparison with natural draught. Such as,

- Better fuel economy due to complete utilization of heat in boiler, i.e. flue gases could be cooled to lowest possible temperature before exhaust.
- Better combustion rate as compared to natural draught system, because of better distribution and mixing of air and fuel.
- Combustion and evaporation rates can be easily regulated in case of mechanical draught by varying operating states of fan/blower.

- Inferior quality fuel may also be used in mechanical draught.
- Height of chimney/stack used in mechanical draught is much smaller as compared to natural draught.
- Boiler plant efficiency gets improved with mechanical draught.
- Efficiency of mechanical draught is much more than that of natural draught.
- Fuel burning per unit grate area per unit time is up to 300 kg/m^2·hr in mechanical draught as compared to upto 100 kg/m^2·hr in natural draught.
- Mass of flue gases handled is more in mechanical draught as compared to natural draught.

11.23.2 Steam Jet Draught

Artificial draught may also be produced by steam jets. Draught produced using steam jets may also be of induced type or forced type. In case of forced steam jet draught, the steam jet is located so as to force air and steam up through the bed, i.e. steam-jet is placed before the grate. In case of induced draught the steam jet is placed near stack with direction to smoke box so as to induce air across the grate and throw it out from stack. Steam jet draught is simple, cheap and requires minimum attention. With forced draught, the use of steam jet also prevents the formation of clinkers, thus permitting for low grade fuels in furnace. Schematic for steam jet draught is shown in Fig. 11.39. Steam jet draught is disadvantageous because of the fact that steam is not available initially when boiler unit just starts. Steam for draught is taken from boiler unit itself. Initially high pressure steam may be supplied externally. Steam jet draught is used in locomotive boilers.

Various merits of steam jet draught are listed below,

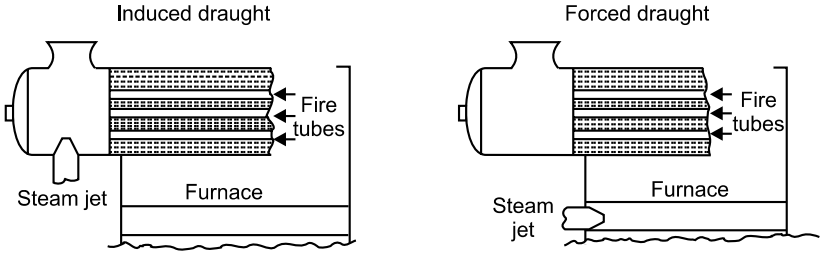


Fig. 11.39 *Schematic for steam jet draught.*

- For operation of steam jet draught steam is required, which is available without much investment.
- Low grade fuels can also be used in furnace with steam jet draught.
- Steam jet draught is simple, economical, requires least attention and minimum space.
- Use of steam and its mixing with ash/burnt residuals prevent formation of clinkers.

11.24 EQUIVALENT EVAPORATION

From earlier discussions it is seen that there exists a large variety of the boilers in terms of their arrangement, efficiency, steam generation rate, steam condition, type of fuel used, firing method and draught etc. For comparing one boiler with other any of the above parameters can not be considered as they are interdependent. Therefore, for comparing the capacity of boilers working at different pressures, temperatures, different final steam conditions etc. a parameter called “equivalent evaporation” can be

used. Equivalent evaporation actually indicates the amount of heat added in the boiler for steam generation. Equivalent evaporation refers to the quantity of dry saturated steam generated per unit time from feed water at 100°C to steam at 100°C at the saturation pressure corresponding to 100°C. Sometimes it is also called equivalent evaporation from and at 100°C. Thus, mathematically it could be given as,

$$\text{Equivalent evaporation} = \frac{\text{Mass of steam generated per hour} \times \left(\text{Heat supplied to generate steam in boiler}\right)}{\text{Heat supplied for steam generation at 100°C from water at 100°C (i.e. Latent heat)}}$$

Heat supplied for generating steam at 100°C from water at 100°C at respective saturation pressure is 538.9 or 539 kcal/kg.

For a boiler generating steam at 'm' kg/hr at some pressure 'p' and temperature 'T', the heat supplied for steam generation = $m \times (h - h_w)$, where h is enthalpy of final steam generated and h_w is enthalpy of feed water. Enthalpy of final steam shall be;

$h = h_f + h_{fg} = h_g$, for final steam being dry saturated steam h_f, h_{fg} and h_g are used for their usual meanings.

$h = h_f + x \times h_{fg}$, for wet steam as final steam.

$h = h_g + c_{p,\ \text{sup. steam}} \cdot (T_{\text{sup}} - T_{\text{sat}})$, for superheated final steam.

$$\text{Equivalent evaporation, (kg/kg of fuel)} = \left\{\frac{m(h - h_w)}{538.9}\right\}$$

Equivalent evaporation is thus a parameter which could be used for comparing the capacities of different boilers.

11.25 BOILER EFFICIENCY

Boiler efficiency quantifies, how effectively the heat is being used in boiler. Thus it could be given by the ratio of heat actually used for steam generation and total heat available due to combustion of fuel in boiler.

$$\text{Boiler efficiency} = \frac{\text{Heat used in steam generation}}{\text{Total heat available due to fuel burning}}$$

$$= \frac{m\left(h - h_w\right)}{m_f \times \text{C.V.}}$$

here m_f is the mass of fuel burnt per hour, C.V. is calorific value of fuel used (kcal/kg), m is mass of steam generated per hour and enthalpies h and h_w are that of final steam and feed water, kcal/kg.

Generally high heating value of fuel is used as calorific value of fuel.

11.26 HEAT BALANCE ON BOILER

Heat balance on boiler refers to the accounting for total heat released inside boiler and its distribution. Total heat available inside boiler is due to burning of fuel and can be quantified by the product of mass of fuel and heating value of fuel. Heat distribution can be given comprising of the following, based on unit mass of fuel burnt.

(a) *Heat used for generation of steam:* Heat taken for generation of steam per kg of fuel burnt shall be;

$$Q_{\text{steam}} = m_{\text{steam}} \cdot (h - h_w)$$

Here m_{steam} is mass of steam generated per kg of fuel burnt, h is enthalpy of final steam produced and h_w is enthalpy of feed water.

Q_{steam} = Equivalent evaporation × 539, kcal/kg of fuel burnt

(b) *Heat lost due to incomplete combustion:* Combustion is said to be complete when the carbon and hydrogen present in fuel get burnt into carbon dioxide and water vapour respectively. If the carbon burns into carbon monoxide instead of carbon dioxide then combustion is said to be incomplete. This incomplete combustion can be easily made complete by supplying excess air and devising for its fast mixing with fuel inside combustion chamber. Heat loss due to incomplete combustion can be quantified by knowing the additional heat that would be available with complete combustion.

Heat loss due to incomplete combustion = Heat released when carbon burns into CO_2 – Heat released when carbon burns into CO.

During complete combustion of carbon into CO_2, 3.38×10^4 kJ of heat is released while with incomplete combustion of carbon into CO, 1.012×10^4 kJ of heat is released by burning one kg, of carbon. Hence heat loss due to incomplete combustion of one kg of carbon shall be

$$= (3.38 \times 10^4 - 1.012 \times 10^4),\ \text{kJ}$$

$$= 2.368 \times 10^4 \text{ kJ per kg of carbon}$$

In order to get the estimate of total heat loss, it is first required to get the mass of carbon burnt to CO instead of CO_2. For which the mass fraction of CO present in flue gas mixture comprising of CO and CO_2 is required.

$$\text{Fraction of CO in flue gases} = \frac{f_{\text{CO}}}{f_{\text{CO}} + f_{\text{CO}_2}}$$

where f_{CO} and f_{CO_2} are percentage by volume of CO and CO_2 present in flue gases.

$$\text{Mass of carbon burnt into CO instead of } CO_2 = \left(\frac{f_{\text{CO}}}{f_{\text{CO}} + f_{\text{CO}_2}} \right) \times f_c$$

where f_C is fraction of carbon present in per kg of fuel.

$Q_{\text{incomplete}}$ = Heat loss due to incomplete combustion

$$= \left(\frac{f_{\text{CO}}}{f_{\text{CO}} + f_{\text{CO}_2}} \right) \times f_c \times 2.368 \times 10^4,\ \text{kJ/kg of fuel}$$

(c) *Heat loss to dry flue gases:* A large portion of heat getting lost goes along with flue gases. Flue gases leaving boiler comprises of dry flue gases and steam. Heat loss with dry flue gases can be given by,

$$Q_{\text{dry flue}} = m_{\text{dfg}} \times Cp_{\text{g}} \times (T_g - T_a)$$

where m_{dfg} is mass of dry flue gas per kg of fuel, Cp_g is specific heat of dry flue gas and T_g and T_a are temperature of flue gas and air entering combustion chamber.

(d) *Heat loss to steam in flue gases:* Steam is produced due to burning of hydrogen present in fuel into water vapour. Heat lost with steam in flue gases shall be

$$Q_{\text{steam in flue}} = m_s \times (h_{s1} - h_{f1})$$

where m_s is mass of steam produced per kg of fuel, h_{s1} and h_{f1} are "enthalpy values of steam at gas temperature T_g and partial pressure of vapour in flue gas" and "enthalpy of water at mean boiler temperature" respectively.

(e) *Heat lost in unburnt fuel:* Some portion of heat may get lost in unburnt fuel, which could be given by the product of mass of unburnt fuel per kg of fuel and its calorific value, as

$$Q_{\text{unburnt}} = m_{\text{ubf}} \times \text{CV}$$

where m_{ubf} is mass of unburnt fuel per kg of fuel and CV is calorific value of fuel.

(f) *Heat loss due to moisture in fuel:* Moisture present in fuel shall also cause the loss of heat. This moisture shall get evaporated and superheated as fuel is burnt. For evaporation and superheating of moisture latent and sensible heat requirement shall be met from heat available in boiler due to burning of fuel. Mathematically, for unit mass of fuel burnt, it can be given as,

$$Q_{\text{moisture}} = m_{\text{moist}} \times (h_{s2} - h_{f2})$$

where m_{moist} is mass of moisture per kg of fuel burnt, h_{s2} is enthalpy of final steam produced and h_{f2} is enthalpy of water at boiler furnace temperature.

(g) *Heat loss due to convection, radiation and other unaccountable losses:* In a boiler heat also gets lost due to convection, radiation from the boiler's surface exposed to atmosphere. The heat loss may also be there due to unconsumed hydrogen and hydrocarbon etc. Exact quantification of these losses is not possible, therefore these can be estimated by the difference of total heat available and cumulative heat loss described from (a) to (f).

Heat loss due to convection, radiation and other unaccounted losses = Heat released by per kg fuel burnt – Σ Heat loss components described from (a) to (f).

$Q_{\text{unaccounted}} = (m_f \times CV) - (Q_{\text{steam}} + Q_{\text{incomplete}} + Q_{\text{dry flue}} + Q_{\text{steam in flue}} + Q_{\text{unburnt}} + Q_{\text{moisture}})$

Heat balance sheet can be drawn after the above different components are quantified. It gives a quick account of heat released and its distribution for unit mass of fuel burnt or unit time. Heat balanced sheet shall be as given below.

Table 11.2 *Heat balance sheet on per minute basis (unit time basis)*

Total Heat (*kcal*)	*Supplied percentage*	*Heat distribution* (*kcal*)	*percentage, %*
$Q = m_f \times \text{CV}$	100%	(a) Heat used for generation $Q_{\text{steam}} = m_{\text{steam}} \cdot (h - h_w)$	$\frac{Q_{\text{stream}} \times 100}{Q}$
		(b) Heat lost due to incomplete combustion, $Q_{\text{incomplete}} = \left(\frac{f_{co}}{f_{co} + f_{co_2}}\right) \times f_c \times 2.368 \times 10^4 \times 4.18$	$\frac{Q_{\text{incomplete}} \times 100}{Q}$
		(c) Heat lost due to dry flue gas, $Q_{\text{dry flue}} = m_{\text{dfg}} \cdot C_{pg} \cdot (T_g - T_a)$	$\frac{Q_{\text{dry flue}} \times 100}{Q}$
		(d) heat loss of steam in flue gas,	$\frac{Q_{\text{steam in flue}} \times 100}{Q}$

(contd.)

Total Heat (*kcal*)	*Supplied percentage*	*Heat distribution kcal*	*percentage, %*
		$Q_{\text{steam in flue}} = m_s \cdot (h_{s1} - h_{f1})$ (e) Heat lost in unburnt fuels $Q_{\text{unburnt}} = m_{\text{ubf}} \cdot CV$ (f) Heat loss due to moisture in fuel $Q_{\text{moisture}} = m_{\text{moist}} \cdot (h_{s2} - h_{f2})$ (g) Heat loss due to convection, radiation and other unaccounted losses, $Q_{\text{unaccounted}} = Q - Q_{\text{steam}} - Q_{\text{incomplete}} - Q_{\text{dry flue}} - Q_{\text{steam in flue}} - Q_{\text{unburnt}} - Q_{\text{moisture}}$	$\frac{Q_{\text{unbrunt}} \times 100}{Q}$ $\frac{Q_{\text{moisture}} \times 100}{Q}$ $\frac{Q_{\text{unaccounted}} \times 100}{Q}$

11.27 BOILER TRIAL

Boiler trial refers to running the boiler under test conditions for its performance estimation. It gives the steam generation capacity of boiler, thermal efficiency of plant and heat balance sheet of the boiler. Under trial the boiler is run for quite long durations so as to attain steady state. Generally the boilers are run for 4 to 6 hours duration for the boilers of oil fired type and coal fired types. Duration of boiler run for attaining steady state changes from boiler to boiler. Observations are taken after the boiler attains steady state for a duration ranging from 10–15 minutes. Measurements are made for fuel supply, combustion analysis, steam generation rate and its quality/state, flue gas and their analysis, temperature and pressure at salient locations and all other measurements as required for heat balance sheet preparation.

EXAMPLES

1. *A boiler has chimney of 30 m height to produce natural draught of 12 mm of water column. Ambient air temperature is 27°C and boiler furnace requires 20 kg of air per kg of fuel for complete combustion. Determine minimum temperature of burnt gases leaving chimney.*

Solution:

Given: m = 20 kg air per kg of fuel, H = 30 m, T_a = 273 + 27 = 300 K

Draught in terms of water column,

$$h_w = \rho_a \cdot T_0 \cdot H \cdot \left\{ \frac{1}{T_a} - \left(\frac{m+1}{m} \right) \cdot \frac{1}{T_g} \right\}, \text{ mm of water}$$

Substituting ρ_a, T_0 and rounding off

$$h_w = 353\ H \left\{ \frac{1}{T_a} - \left(\frac{m+1}{m} \right) \cdot \frac{1}{T_g} \right\}, \text{ mm of water}$$

$$12 = 353 \times 30 \left\{ \frac{1}{300} - \left(\frac{20+1}{20} \right) \cdot \frac{1}{T_g} \right\}$$

$$T_g = 432.86 \text{ K}$$

Temperature of burnt gases = 432.86 K ***Ans.***

2. *Determine the height of chimney required in a boiler having natural draught equivalent to 20 mm of water. The flue gases are at temperature of 300°C, atmospheric air temperature is 27°C and 18 kg air per kg of fuel is required in boiler.*

Solution:

Given: m = 18 kg air per kg of fuel, h_w = 20 mm, T_a = 300K, T_g = 573 K

Draught in terms of water column,

$$h_w = 353\ H \left\{ \frac{1}{T_a} - \left(\frac{m+1}{m} \right) \cdot \frac{1}{T_g} \right\} \text{mm of water}$$

$$20 = 353\ H \left\{ \frac{1}{300} - \left(\frac{18+1}{18} \right) \cdot \frac{1}{573} \right\}$$

$$H = 37.99 \text{ m}$$

Height of chimney = 37.99 m ***Ans.***

3. *A boiler house has natural draught chimney of 20 m height. Flue gases are at temperature of 380°C and ambient temperature is 27°C. Determine the draught in mm of water column for maximum discharge through chimney and also the air supplied per kg of fuel.*

Solution:

Given: Height of chimney, H = 20 m, T_g = 653 K, T_a = 300 K

For maximum discharge condition,

$$\frac{T_g}{T_a} = 2\left(\frac{m+1}{m} \right)$$

$$2\left(1 + \frac{1}{m} \right) = \frac{653}{300}$$

$$m = 11.32 \text{ kg air per kg of fuel}$$

Air supplied = 11.32 kg/kg of fuel ***Ans.***

Draught in mm of water column

$$h_w = 353 \times 20 \left[\frac{1}{300} - \left(\frac{11.32+1}{11.32} \right) \cdot \frac{1}{653} \right]$$

$$h_w = 11.77 \text{ mm of water}$$

Draught = 11.77 mm of water ***Ans.***

4. A *boiler may have waste gases leaving the installation when artificial draught is used at 150°C. The natural draught chimney is of 60 m height. The hot gases within chimney are at temperature of 300°C and air requirement is 19 kg per kg of fuel burnt. The atmospheric air is at 17°C temperature and mean specific heat of hot gases is 1.0032 kJ/kg · K. The calorific value of fuel burnt is 32604 kJ/kg. Determine*

(i) the draught produced in mm of water

(ii) the efficiency of chimney

(iii) the extra heat carried away by flue gases per kg of fuel.

Solution:

Given: $T_g = 300 + 273 = 573$ K, $T_a = 17 + 273 = 290$ K, $T_{g,\,a} = 150 + 273$, $H = 60$ m, $m = 19$ kg air per kg of fuel, $T_{g,\,a} = 423$ K

Draught in mm of water column

$$h_w = 353 \cdot H \cdot \left[\frac{1}{T_a} - \left(\frac{m+1}{m}\right) \cdot \frac{1}{T_g}\right]$$

$$= 353 \times 60 \times \left[\frac{1}{290} - \left(\frac{19+1}{19}\right) \cdot \frac{1}{573}\right]$$

$$= 34.13 \text{ mm}$$

Draught = 34.13 mm of water column ***Ans.***

$$\text{Chimney efficiency} = \frac{9.81 \times H \times \left\{\left(\frac{m}{m+1}\right) \cdot \frac{T_g}{T_a} - 1\right\}}{c_{pg} \cdot (T_g - T_{g,a})}$$

$$= \frac{9.81 \times 60 \times \left\{\left(\frac{19}{19+1}\right) \cdot \frac{573}{290} - 1\right\}}{1.0032 \times (573 - 423) \times 10^3}$$

$$= 3.431 \times 10^{-3} \text{ or } 0.3431\%$$

Chimney efficiency = 0.3431% ***Ans.***

Extra heat carried by flue gases per kg of fuel

$$= (19 + 1)\, C_{p,\,g} \times (T_g - T_{g,\,a})$$

$$= 20 \times 1.0032 \times (573 - 423)$$

$$= 3009.6 \text{ kJ per kg of fuel}$$

Extra heat carried away by flue gases per kg of fuel burnt = 3009.6 kJ ***Ans.***

5. *Determine the temperature of hot flue gases, natural draught produced and efficiency of chimney for maximum discharge through chimney having height of 80 m. Boiler furnace is supplied with 20 kg air per kg of fuel. The minimum temperature of hot gases with artificial draught is 110°C. Temperature of surroundings is 27°C and specific heat of flue gases is 1.0032 kJ/kg K.*

Solution:

Given: $T_a = 27 + 273 = 300$ K, $T_{g,\,a} = 110 + 273 = 383$ K, $m = 20$, $H = 80$m, $c_{p,\,g} = 1.0032$ kJ/kg · K

For maximum discharge through chimney, $\frac{T_g}{T_a} = 2\left(\frac{m+1}{m}\right)$

$$T_g = 300 \times 2 \times \left(\frac{20+1}{20}\right) = 630 \text{ K}$$

Natural draught produced in mm of water column,

$$h_w = 353 \cdot H \cdot \left\{\frac{1}{T_a} - \left(\frac{m+1}{m}\right)\cdot\frac{1}{T_g}\right\}$$

$$= 353 \times 80 \times \left\{\frac{1}{300} - \left(\frac{20+1}{20}\right)\cdot\frac{1}{630}\right\}$$

$$h_w = 47.07 \text{ mm of water}$$

$$\text{Efficiency of chimney} = \frac{9.81 \times H \times \left\{\left(\frac{m}{m+1}\right)\times\frac{T_g}{T_a} - 1\right\}}{c_{p,g} \times \left(T_g - T_{g,a}\right)}$$

$$= \frac{9.81 \times 80 \times \left\{\left(\frac{20}{21}\right)\times\frac{630}{300} - 1\right\}}{1.0032 \times (630 - 383) \times 10^3}$$

$$= 3.167 \times 10^{-3} \text{ or } 0.3167\%$$

Hot gas temperature in chimney = 630 K
Natural draught = 47.07 mm of water
Chimney efficiency = 0.3167%

Ans.

6. *In a boiler installation coal is burnt at the rate of 2.5 $\times 10^3$ kg/hr and hot gases are generated at the rate of 20 kg per kg of coal burnt. Hot gases are at the temperature of 327°C and ambient air temperature is 27°C. Different pressure losses in furnace grate, flues, bends and chimney are measured to be 7 mm, 6 mm, 3 mm, and 2 mm of water respectively. Hot gases leaving chimney have velocity equivalent to 1.2 mm of water column. Considering the actual natural draught to be 90% of theoretical draught determine the height and diameter of chimney.*

Solution:

Given: $T_g = 327 + 273 = 600$ K, $T_a = 27 + 273 = 300$ K, $m + 1 = 20$, $m = 19$ kg air/kg fuel

Pressure head required to overcome different losses and velocity head shall be sum of all losses, given $h = 7 + 6 + 3 + 2 = 18$ mm of water.

Theoretically the draught of 18 mm of water is to be produced but as mentioned, the actual draught will be more than theoretical draught produced.

$$\text{Actual natural draught, } h_w = \frac{18}{0.9} = 20 \text{ mm of water.}$$

$$\text{Let height of chimney be } H,\ h_w = 353 \cdot H \left\{\frac{1}{T_a} - \left(\frac{m+1}{m}\right)\cdot\frac{1}{T_g}\right\}$$

$$20 = 353 \times H \left\{\frac{1}{300} - \left(\frac{20}{19}\right)\times\frac{1}{600}\right\}$$

$$\therefore \quad H = 35.88 \text{ m}$$

Height of chimney = 35.88 m ***Ans.***

$$\text{Density of hot gases, } \rho_g = \frac{\rho_a \cdot T_0}{T_g}\left(\frac{m+1}{m}\right)$$

$$= \frac{353}{T_g}\left(\frac{m+1}{m}\right)$$

$$\rho_g = \frac{353}{600}\times\left(\frac{19+1}{19}\right) = 0.619 \text{ kg/m}^3$$

Velocity of hot gases, $C = \sqrt{2g \cdot h_g}$

Height of hot gases column, $h_g = H\left\{\left(\frac{m}{m+1}\right)\cdot\frac{T_g}{T_a}-1\right\}$

$$= 35.88 \times \left\{\left(\left(\frac{19}{20}\right)\times\frac{600}{300}\right)-1\right\}$$

$$h_g = 32.29 \text{ m}$$

Mass flow rate of hot gases, $M_g = \frac{2.5\times10^3\times20}{3600} = 13.89$ kg/s

Velocity, $C = \sqrt{2gh_g} = \sqrt{2\times9.81\times32.29} = 25.17$ m/s

$$M_g = \frac{\pi}{4}D^2\times C\times\rho_g$$

$$D = \sqrt{\frac{4\times M}{\pi\times c\times\rho_g}}$$

Diameter of chimney, $D = \sqrt{\frac{4\times13.89}{\pi\times25.17\times0.619}}$

$$D = 1.06 \text{ m}$$

Diameter of chimney = 1.06 m ***Ans***.

7. *Determine the capacity of motor required for running induced draught fan and forced draught fan required for maintaining draught of 50 mm of water column. Consider hot gases to leave boiler at 300°C, coal being burnt per hour at the rate of 2000 kg/hr, air supplied at the rate of 19 kg/kg of coal, ambient air temperature of 27°C and mechanical efficiency as 90%.*

Solution:

Power required in *FD* fan $= \frac{P\cdot m\cdot M\cdot T_1}{\rho\, T_0\cdot\eta_{\text{mech}}}$

Power required in 1D fan $= \frac{P\cdot m\cdot M\cdot T}{\rho\, T_0\cdot\eta_{\text{mech}}}$

Given, $T_1 = 300$ K, $T_0 = 273$ K, $T = 573$ K, $m = 19$ kg/kg coal,
$M = 2000$ kg/hr $= 0.556$ kg/s, $\rho = 1.293$ kg/m^3,

$h_{\text{water}} = 50$ mm or 50 kgf/m^2, $\eta_{\text{mech}} = 0.90$

$P = 50$ kgf/m^2

or

$$P = 490.5 \text{ N/m}^2$$

Substituting in expression for power outputs,

$$\text{For FD fan, power} = \frac{490 \cdot 5 \times 19 \times 0.556 \times 300}{1.293 \times 273 \times 0.90}$$

$$= 4893.11 \text{ W}$$

$$= 4.89 \text{ kW}$$

$$\text{For 1D fan, power} = \frac{490 \cdot 5 \times 19 \times 0.556 \times 573}{1.293 \times 273 \times 0.90} = 9345.84 \text{ W}$$

$$= 9.35 \text{ kW}$$

Power for *FD* fan = 4.89 kW
Power for 1*D* fan = 9.35 kW ***Ans.***

8. *A boiler installation is seen to have operating parameters as given under when natural draught system, forced draught system and induced draught systems are used. Considering specific heat of hot gases as 1.0032 kJ/kg · K determine,*

(i) the ratio of power required for induced and forced draught system.

(ii) the ratio of heat carried away with flue gases in artificial draught and natural draught.

	Natural draught	*Artificial draught*	
		Forced	*Induced*
Hot gas temperature, ℃	*327*	*27*	*177*
Temperature of surroundings, ℃	*27*	*27*	*27*
Mass of air required per kg of fuel	*25*	*20*	*20*

Solution:

$$\frac{\text{Brake power for induced draught}}{\text{Brake power for forced draught}} = \frac{T_g}{T_a} = \frac{(177 + 273)}{(27 + 273)} = 1.5$$

Heat carried by hot flue gases in artificial draught for unit mass of fuel burnt

$$Q_{g,\ ad} = (20 + 1) \times 1.0032 \times (177 - 27)$$

$$= 3160.08 \text{ kJ per kg of fuel burnt}$$

Heat carried by hot flue gases in natural draught for unit mass of fuel burnt

$$Q_{g,\ ad} = (25 + 1) \times 1.0032 \times (327 - 27)$$

$$= 7824.96 \text{ kJ per kg of fuel burnt}$$

Thus it shows that significantly large amount of heat is lost in natural draught as compared to artificial draught. Ratio of heat carried away in artificial and natural draught = $\frac{3160.08}{7824.96} = 0.404$

Ratio of power required = 1.5
Ratio of heat carried away = 0.404 ***Ans.***

9. *Determine the actual evaporation per kg of coal and the equivalent evaporation if during boiler trial of one hour duration following observations are made:*

Feed water supply temperature: 27℃

Mean steam generation pressure: 10 bar,

Dryness fraction of steam generated: 0.95
Feed water supplied: 2500 kg/hr
Coal burnt: 275 kg/hr
Mass of water in boiler after trial = 300 kg less than that at commencenent of trial.

Solution:

From steam table at 10 bar, $h_f = 762.81$ kJ/kg, $h_g = 2778.1$ kJ/kg

$h_{fg} = 2015.29$ kJ/kg

Enthalpy of steam steam generated, $h = h_f + x \cdot h_{fg}$

$= 762.81 + 0.95 \times 2015.29$

$h = 2677.34$ kJ/kg

Mass of water evaporator per hour = 2500 + 300 = 2800 kg/hr

Water evaporated per kg of coal = $\frac{2800}{275}$ = 10.18 kg per kg of coal.

Actual evaporation = 10.18 kg per kg of coal ***Ans.***

Equivalent evaporation = $\frac{10.18 \times 2677.34}{2257}$ = 12.08 kg per kg of coal

Equivalent evaporation = 12.08 kg per kg of coal ***Ans.***

10. *A boiler is being tested for 24 hours and during this trial steam at average pressure of 10 bar, dry saturated is produced from 15 ton of water consuming 1.5 ton of coal. Composition of coal has 3% moisture and 4% ash. Feed water is added at 35°C. Determine,*

(i) the boiler efficiency,
(ii) the equivalent evaporation per kg of dry coal, and
(iii) the equivalent evaporation per kg of combustible present in coal.

Solution:

Enthalpy of steam generated = $h_{g \text{ at } 10 \text{ bar}} = 2778.1$ kJ/kg

Heat supplied per kg of water for steam generation = $2778.1 - 4.18 \times 35$

$= 2631.8$ kJ/kg

Steam generated per kg of coal = $\frac{15 \times 10^3}{1.5 \times 10^3} = 10$ kg

Boiler efficiency = $\frac{10 \times 2631.8}{30.1 \times 10^3} = 0.8744$ or 87.44%

Boiler efficiency = 87.44% ***Ans.***

Equivalent evaporation per kg of dry coal = $\frac{10 \times 2631.8}{2257 \times (1 - 0.03)}$

$= 12.02$ kg

Equivalent evaporation per kg of combustible present in coal

$= \frac{12.02 \times 0.97}{0.93}$

$= 12.53$ kg

Equivalent evaporation per kg of dry coal = 12.02 kg
Equivalent evaporation per kg of combustible = 12.53 kg

Ans.

11. *During the boiler trial for 24 hours following observations were made.*
Steam generation: 16 bar, dry saturated.
Coal consumed: 10,000 kg
Rate of steam generation: 2500 kg/hr
Feed water temperature: 27°C
Total heating surface area: 3000 m^2
Total grate area: 4m^2
Calorific value of coal: 28000 kJ/kg
Determine:

(i) the mass of coal burnt per m^2 of grate per hour
(ii) the equivalent evaporation from and at 100°C per kg of coal
(iii) the equivalent evaporation from and at 100°C per m^2 of total heating surface per hour
(iv) the boiler efficiency

Solution:

Coal burnt per hour $= \frac{10{,}000}{24} = 416.67$ kg/hr

Coal burnt per m^2 of grate per hour $= \frac{416.67}{4}$

$= 104.17$ kg/m^2 grate surface per hr

Equivalent evaporation from and at 100°C per kg of coal can be obtained by the ratio of heat added to steam per kg of coal burnt and latent heat from and at 100°C

$$= \frac{\text{Heat added to steam per kg of coal burnt}}{\text{Latent heat from and at } 100^\circ\text{C}}$$

Rate of steam generated per kg of coal $= \frac{2500}{416.67} = 5.99 \approx 6$ kg steam/kg coal

Heat added to steam per kg of coal $= 6\,[h_{g \text{ at } 16 \text{ bar}} - 4.18 \times 27\}$

$= 6\{2794 - 112.86\}$

$= 16086.84$ kJ

Latent, heat from and at 100°C = 2257 kJ/kg

Equivalent evaporation from and at 100°C per kg of coal $= \frac{16086.84}{2257}$

$= 7.13$ kg

Equivalent evaporation from and at 100°C per m^2 of total surface per hour

$= \frac{7.13 \times 416.67}{3000}$

$= 0.99$ kg

Boiler efficiency $= \frac{7.13 \times 2257}{28000} = 0.5747$ or 57.47%

Mass of coal burnt per m^2 of grate per hour = 104.17 kg
Equivalent evaporation from and at 100°C per kg of coal = 7.13 kg
Equivalent evaporation from and at 100°C per m^2 of total heating surface per hour = 0.99 kg
Boiler efficiency = 57.47% ***Ans.***

12. *Steam is generated in a boiler at 30 bar 300°C at the rate of 11 kg/s with feed water entering economiser at 100°C. During one hour test 5000 kg fuel is used in boiler. Calorific value of fuel is 35000 kJ/kg. For the feed water being supplied to boiler to be at 27°C determine;*

(i) the equivalent evaporation per kg of fuel
(ii) the boiler efficiency
(iii) the percentage of fuel energy utilised in economiser

Solution:

Mass of steam generated per kg of fuel = 7.92 kg/kg fuel $\left(= \frac{11 \times 3600}{5000}\right)$

Heat added to per kg steam per kg of fuel = (Enthalpy of steam at 30 bar, 300°C – Enthalpy of feed water)
= (2993.5 – (4.18 × 27))
= 2880.64 kJ

$$\text{Equivalent evaporation per kg of fuel} = \frac{7.92 \times 2880.64}{2257} = 10.11 \text{ kg}$$

$$\text{Boiler efficiency} = \frac{10.11 \times 2257}{35000} = 0.6519 \text{ or } 65.19\%$$

Heat utilised in economiser per kg of fuel = 7.92 × 4.18 × (100 – 27)
= 2416.71 kJ

$$\text{Percentage of energy utilised in economiser} = \frac{2416.71}{35000} = 0.069, \text{ or } 6.9\%$$

Equivalent evaporation per kg of fuel = 10.11 kg
Boiler efficiency = 65.19%
Percentage of energy utilised in economiser = 6.9% ***Ans.***

13. *A boiler is capable of generating 8 kg steam per kg of fuel at 400°C. Feed water is supplied at pressure of 30 bar, 40°C and leaves economiser at 150°C for entering to evaporator. Steam leaves evaporator with dryness fraction of 0.98 and enters the superheater. Fuel used has calorific value of 29000 kJ/kg. Considering no pressure loss inside the boiler determine boiler efficiency and fraction of heat given to steam in each section of boiler.*

Solution:

From steam table; h_f at 30 bar = 1008.42 kJ/kg, $h_{g \text{ at } 30 \text{ bar}}$ = 2804.2 kJ/kg
Enthalpy of final steam generated = $h_{\text{at 30 bar, 400°C}}$
= 3230.9 kJ/kg

Heat supplied per kg of fuel = 1 × 29000 kJ/kg = 29000 kJ/kg

Heat to be added for per kg steam generation = $h_{\text{at 30 bar, 400°C}} - h_{\text{water at 40°C}}$
= 3230.9 – 4.18(40)

$= 3063.7$ kJ/kg

$$\text{Boiler efficiency} = \frac{8 \times 3063.7}{29000} = 0.8452 \approx 84.52\%$$

Heat added in economiser per kg of steam generated $= 4.18 \times (150 - 40)$

$= 459.8$ kJ/kg

$$\% \text{ Fraction of heat in economiser} = \frac{459.8}{3063.7} \times 100$$

$= 15.01\%$

Heat added in evaporator per kg of steam generated

$= (h_{f \text{ at } 30 \text{ bar}} + 0.98 \times h_{fg \text{ at } 30 \text{ bar}}) - (4.18 \times 150)$

$= 1008.42 + (0.98 \times 1795.78) - 627$

$= 2141.28$ kJ/kg

$$\% \text{ Fraction of heat in evaporator} = \frac{2141.28}{3063.7} \times 100 = 69.89\%$$

Heat added in super heater per kg of steam generated, by difference

$= (3063.7 - 459.8 - 2141.28)$

$= 462.62$ kJ/kg

$$\% \text{ Fraction of heat in superheater} = \frac{462.62}{3063.7} \times 100 = 15.1\%$$

Boiler efficiency = 84.52%
% fraction of heat in economiser, evaporator and superheater = 15.01%, 69.89% and 15.1% ***Ans.***

14. *During a boiler trial the volumetric analysis of flue gases (combustion products) going across the economiser section are 8.4% CO_2, 11.3% O_2 and 80.3% N_2 and 8% CO_2, 11.4% O_2 and 80.6% N_2 respectively at inlet and exit of economiser section of boiler. Feed water enters economiser at 20°C and leaves at 125°C at the rate of 3 kg/s. Temperature of flue gases at inlet and exit of economiser are 425°C and 300°C respectively. Coal is supplied to boiler at the rate of 18 kg/min and the coal used has 80% of carbon by mass in it. Considering atmospheric air to be at 15°C and specific heat of flue gases as 1.05 kJ/kg · °C determine,*

(i) the heat released by flue gases in economiser section,
(ii) the amount of air leaked into economiser,
(iii) the heat gained by feed water in economiser.

Solution:

Given: 0.8 kg of C in 1 kg of coal.

Table 11.3 gives the mass of dry flue gas at inlet and exit of economiser as 23.65 kg and 24.78 kg respectively. Thus, there is increase in mass of dry flue gases from inlet to exit. This increase in flue gases may be attributed to the leakage of air.

Therefore, air leakage in economiser per kg of coal

$= 24.78 - 23.65$

$= 1.13$ kg air per kg coal

Heat rejected by flue gases during its' passage through economiser shall be given by the difference of heat entering with flue gas and air leakage and heat leaving with flue gases at exit of economiser.

Table 9.3 *Volumetric and gravimetric analysis of flue gas in economiser*

Constituents	*Mol. Wt.*	*Volume per m^3, (b)*		*Proportional mass* ($c = a \times b$)		*C per kg of flue gases,* $d = \left(\frac{c}{\Sigma c}\right) \times \frac{12}{44}$	
	(a)	*At inlet*	*At exit*	*At inlet*	*At exit*	*At inlet*	*At exit*
CO_2	44	0.084	0.08	3.696	3.52	0.03383	0.03228
O_2	32	0.113	0.114	3.616	3.648	Dry flue gas per kg of coal = $\frac{0.8}{0.03383}$ = 23.65 kg	Dry flue gas per kg of coal = $\frac{0.8}{0.03228}$ = 24.78 kg
N_2	28	0.803	0.806	22.484	22.568		
Total		1.00	1.00	29.796	29.736		

It may be assumed that the specific heat of leakage and flue gases are same.

Heat entering economiser with flue gases and leakage

$= (23.65 \times 1.05 \times 425) + (1.13 \times 1.05 \times 15)$

$= 10571.61$ kJ

Heat leaving economiser with flue gases $= (24.78 \times 1.05 \times 300)$

$= 7805.7$ kJ

Thus, heat lost in economiser per kg of coal $= 10571.61 - 7805.7$

$= 2765.91$ kJ

Heat picked by feed water in economiser per kg of coal

$= m_w \times C_{p,w} \times \Delta T$

$= \left(\frac{3 \times 60}{18}\right) \times 4.18 \times (125 - 20)$

$= 3971$ kJ per kg of coal

Heat released by flue gases = 2765.91 kJ per kg of coal
Air leakage = 1.13 kg air per kg of coal
Heat gained by feed water = 3971 kJ per kg of coal

Ans.

15. *Draw up a heat balance sheet for the boiler in kJ per kg of dry coal and also determine the boiler efficiency and effectiveness of air heater for the following data obtained during boiler trial. The boiler has economiser and air preheater as two accessories in it.*

Atmospheric air temperature: 15°C
Steam generation: 40 bar, 400°C

Steam generated per kg of coal = 8 kg

Feed water temperature at inlet to economiser = 27°C

Feed water temperature at exit of economiser = 137°C

Moisture in coal burnt = 1.5%

Flue gas temperature entering air heater =300°C

Flue gas temperature leaving air heater and entering chimney = 150°C

Temperature of air entering boiler furnace = 120°C

Dry coal composition by mass = 84% C, 4% H_2, 7% O_2 and remainder ash
Dry flue gas composition by volume = 12.5% CO_2, 7.5% O_2, 80% N_2

Datum temperature = 15°C

Calorific value of coal = 32600 kJ/kg

For air and dry flue gas, c_p =1.0032 kJ/kg · K

Partial pressure of vapour in flue gas = 0.075 bar

Specific pressure of vapour = 2.0064 kJ/kg · K

Determine the boiler efficiency and the efficiency of heat exchange in air heater.
Also prepare heat balance sheet in kJ per kg of dry coal.

Solution:

Here the schematic of economiser, boiler and air heater are as shown below:

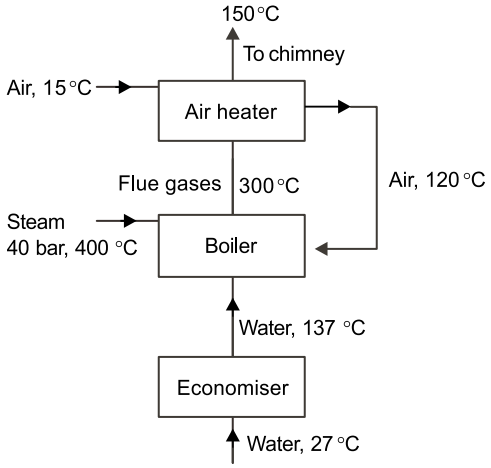


Fig. 11.40

Let us now determine the mass of dry flue gas per kg of dry coal for which the mass of carbon per kg of dry flue gas can be obtained by the following analysis.

Composition by volume of dry flue gas	*Molecular weight*	*Mass of constituents (a)*	*Mass per kg of dry flue gas,* $b = \frac{a}{\Sigma a}$	*Mass of carbon per kg of dry flue gas*
$CO_2 = 0.125$ $O_2 = 0.075$ $N_2 = 0.80$	44 32 28	$0.125 \times 44 = 5.5$ $0.075 \times 32 = 2.4$ $0.80 \times 28 = 22.4$	0.1815 0.0792 0.7393	$\frac{0.1815 \times 12}{44} = 0.0495$
		$\Sigma a = 30.3$		

The dry coal is given to have C as 0.84 kg per kg of coal.

Therefore, the mass of dry flue gas per kg of coal = $\frac{\text{carbon present per kg of coal}}{\text{carbon present per kg of dry flue gas}}$

$= \frac{0.84}{0.0495} = 16.97$ kg dry flue gas per kg of coal

For the given coal composition the H_2O produced during combustion = $0.04 \times 9 = 0.36$

Given coal has 0.05 kg ash per kg of coal.

Amount of air supplied for combustion of one kg of dry coal

$= 16.97 - (1 - \text{ash content} - H_2O \text{ formed})$

$= 16.97 - (1 - 0.05 - 0.36)$

$= 16.38$ kg

Moisture per kg of dry coal = $\frac{\text{Moisture in coal burnt per kg of coal}}{\text{Mass of dry coal per kg of coal fired}}$

$= \frac{0.015}{(1 - 0.015)}$

$= 0.0152$ kg

Hence, the total moisture per kg of dry coal = 0.36 + 0.0152 = 0.3752 kg

Steam generated per kg of dry coal = $\frac{8}{(1-0.015)}$ = 8.12 kg steam

From steam tables

Enthalpy of steam generated = $h_{\text{at 40 bar, 400°C}}$ = 3213.6 kJ/kg

At partial pressure of vapour i.e. 0.075 bar, saturation temperature is 40.29°C

and $h_{g \text{ at } 0.075 \text{ bar}}$ = 1168.79 kJ/kg

For the given datum temperature of 15°C the heat available with steam, dry flue gas, moisture in flue gas, feed water etc. can be estimated as under.

Heat available with steam = 8.12 (3213.6 – (4.18 × 15)) = 25585.31 kJ

Heat carried by dry flue gases = (16.97 × 1.0032 × (150 – 15)

= 2298.28 kJ

Heat carried by moisture in flue gas

= 0.3752 × {(168.79 – (4.18 × 15)) + 2.0064 × (150 – 40.29)}

= 122.39 kJ

Heat available with feed water = 8.12 × (27 – 15) × 4.18 = 407.29 kJ

Heat available with one kg of coal = 32600 kJ/kg

The heat balance sheet may be prepared for datum state of 15°C as under per kg of dry coal.

Heat Supplied/kg of coal			*Heat Utilized/kg of coal*		
	Quantity,	*kJ%*		*Quantity,*	*kJ%*
Heat available with coal	32600	98.77%	(a) Heat utilized by steam	25585.31	77.51%
			(b) Heat carried by dry flue gas	2298.28	6.96%
Heat available with feed water	407.29	1.23%	(c) Heat carried by moisture in flue gas	122.39	0.37
			(d) Heat loss due to radiation etc.	5001.31	15.16
Total	33007.29	100%		33007.29	100%

Boiler efficiency can be obtained as under,

Heat utilized by steam per kg of coal = 8.12 (3213.6 – (4.18 × 27))

= 25178.01 kJ

Heat supplied by fuel = 32600 kJ/kg coal

Boiler efficiency = $\frac{25178.01}{32600}$ = 77.23%

For getting the efficiency of heat exchange in air heater the accounting may be made for total heat available and heat utilized.

Heat utilized by air = 16.38 × 1.0032 × (120 – 15)

= 1725.4 kJ/kg coal

Heat available in air heater = Heat available with dry flue gas + Heat supplied by the moisture.

= {16.97 × 1.0032 × (300 – 137)} + {0.3752 × 2.0064 × (300 – 137)}

= 2897.67 kJ/kg coal

Efficiency of heat exchange in air heater = $\frac{1725.4}{2897.67}$ = 59.54%

Boiler efficiency = 86.75%
Efficiency of heat exchange in air heater = 59.54% ***Ans.***

16. *In a boiler the coal utilized has composition by mass as 85% C, 5% H_2, 6% ash and remaining oxygen. The combustion results in flue gases at 200°C temperature and composition by volume of dry flue gas as 11% CO_2, 1% CO, 8% O_2 and 80% N_2. The temperature of air is 20°C and the pressure of flue gas is 1.5 bar. Consider the specific heat of dry flue gas as 1.0032 kJ/kg, specific heat of super-heated steam 2.05 kJ/kg.K, air to have 23% O_2 by mass and calorific value of 1 kg CO getting burnt to CO_2 as 10,000 kJ/kg.*

Determine,

(a) total air supplied per kg of coal,
(b) heat carried away by moist flue gas per kg of coal, and
(c) the partial pressure of steam in hot flue gas.

Solution:

Here for getting the mass of air supplied per kg of coal the combustion analysis may be carried out as under.

Composition by volume of dry flue	*Molecular weight*	*Mass of constituents (a)*	*Mass per kg of dry flue gas (b)* = $\frac{a}{\Sigma a}$	*Mass of carbon per kg of dry flue gas*
CO_2 = 0.11	44	0.11 × 44 = 4.84	0.1611	$0.1611 \times \frac{12}{44}$ = 0.0439
CO = 0.01	28	0.01 × 28 = 0.28	0.0093	$0.0093 \times \frac{12}{28}$ = 0.0039
O_2 = 0.08	32	0.08 × 32 = 2.56	0.0085	———
N_2 = 0.80	28	0.80 × 28 = 22.4	0.7457	Total mass of carbon = 0.0478
Total = 1.00		Σa = 30.04	Σb = 0.9246	

Mass of dry flue gas per kg of coal

$$= \frac{\text{Mass of carbon per kg of coal (given as 0.85)}}{\text{Mass of carbon per kg of dry flue gas}}$$

$$= \frac{0.85}{0.0478} = 17.78 \text{ kg/kg of coal}$$

H_2O generated during combustion = 0.05 × 9 = 0.45 kg/kg of coal

Mass of air supplied per kg of coal = 17.78 – (1 – ash – H_2O)

= 17.78 – (1 – 0.06 – 0.45)

= 17.29 kg/kg of coal

Total air supplied per kg of coal = 17.29 kg ***Ans.***

For the pressure of flue gas being 1.5 bar the partial pressure of steam can be given as

$$\frac{\text{No. of } H_2O \text{ mol in flue gas}}{\text{Total no. of mol in flue gas}} \times \text{Total pressure}$$

$$= \frac{\left(\frac{0.45}{18}\right)}{\left(\frac{17.78}{30.04} + \frac{0.45}{18}\right)} \times 1.05 = 0.04 \text{ bar}$$

The enthalpy of this steam from steam table,

$$h_{g \text{ at } 0.04\text{bar}} = 2554.4 \text{ kJ/kg}, \; T_{sat} = 28.96°\text{C}$$

Therefore, heat in vapour = $0.45 \{(2554.4 - 4.18 \times 20) + 2.05 \times (200 - 28.96)\}$

$$= 1269.64 \text{ kJ/kg of coal}$$

Heat in dry flue gas = $1.0032 \times (17.78 \times (200 - 20)) = 3210.64$ kJ/kg of coal

Heat in CO = $\frac{0.28}{30.04} \times 17.78 \times 10000 = 1657.26$ kJ/kg of coal

Total heat carried by most flue gas = 1269.64 + 3210.64 + 1657.26

$$= 6137.54 \text{ kJ/kg of coal}$$

Partial pressure of steam = 0.04 bar
Heat carried by moist flue gas per kg of coal = 6137.54 kJ ***Ans.***

17. *A boiler unit generates steam at 20 bar, 300°C from feed water supplied to boiler at 50°C. Coal used in boiler has calorific value of 30,000 kJ/kg and is used at rate of 600 kg/hr for steam generation rate of 5000 kg/hr. Determine the overall efficiency of boiler and the equivalent evaporation of boiler unit at 100°C in kg/hr.*

Determine the saving of coal in kg/hr if an economiser fitted to boiler to raise feed water temperature up to 75°C increases the overall efficiency of boiler unit by 5% for all other things remaining same.

Solution:

Steam generation per unit coal burnt per hour = $\frac{5000}{600}$

$$m_{steam} = 8.33 \text{kg steam/kg of coal}$$

At 20 bar, 300°C, the enthalpy of final steam,

$$h_{final} = 3023.5 \text{ kJ/kg}$$

Enthalpy of feed water ,

$$h_{water} = 209.33 \text{ kJ/kg}$$

Overall efficiency of boiler = $\frac{8.33(3023.5 - 209.33)}{30000}$

$$= 0.7814 = 78.14\%$$

Equivalent evaporation of boiler unit

$$= \frac{m_{\text{steam}}(h_{\text{final}} - h_{\text{water}})}{2257}$$

$$= \frac{8.33(3023.5 - 209.33)}{2257}$$

= 10.386 kg steam per kg of coal

Equivalent evaporation of boiler unit at 100°C in kg/hr

= (10.386 × 600) kg/hr = 6231.6 kg/hr

After fitting economiser the enthalpy of feed water,

h_{water} = 313.93 kJ/kg

Modified overall efficiency of boiler unit = 78.14 + 5 = 83.14%

Let the coal consumption be m_{coal} kg per hour.

Modified overall efficiency of boiler unit = 0.8314 = $\frac{(3023.5 - 313.93) \times 5000}{m_{\text{coal}} \times 30,000}$

m_{coal} = 543.17 kg/hr

Saving of coal = 600 – 543.17 = 56.83 kg/hr ***Ans.***

18. *A boiler generates 5000 kg/hr steam at 20 bar, 0.98 dry from feed water supplied to it at 60°C. Boiler sums on coal supplied at the rate of 600 kg/hr and air supplied at the rate of 16 kg per kg coal. The calorific value of coal is 30000 kJ/kg and boiler room temperature is 20°C. Considering 86% of heat being lost with flue gases. Determine temperature of flue gases leaving boiler. Take specific heat of flue gases as 1.005 kJ/kg·K.*

Solution: Mass of steam generated per kg of coal

$$= \frac{5000}{600} = 8.33 \text{ kg steam per kg coal}$$

Enthalpy of final steam produced at 20 bar, 0.98 dry

$h_{\text{final}} = h_{f \text{ at } 20 \text{ bar}} + 0.98 \times h_{fg \text{ at } 20 \text{ bar}}$

= 908.79 + (0.98 × 1890.7)

= 2761.67 kJ/kg

Enthalpy of feed water,

$h_{\text{water}} = h_{f \text{ at } 60° \text{ C}}$ = 251.13 kJ/kg

Heat utilized for steam generation = 8.33(2761.67 – 251.13)

= 20912.8 kJ per kg of coal

For the given coal, the heat lost per kg of coal

= 30,000 – 20912.8

= 9087.2 kJ per kg coal

Heat lost with flue gases = 0.86 × 9087.2 = 7814.9 kJ per kg coal

Let the temperature of flue gases leaving boiler be T_{gas}.

Heat lost with flue gases = 7814.9 = $(m_{flue} + m_{air}) \times c_{p\ gas}\ (T_{gas} - 293)$

$$7814.19 = (16 + 1) \times 1.005 \times (T_{gas} - 293)$$

$$\boxed{T_{gas} = 750.37 \text{ K} = 477.37°\text{C}} \quad \textbf{\textit{Ans.}}$$

19. *In a boiler unit forced draught fan delivers ambient air at 20°C with velocity of 20 m/s. The draught lost through grate is 30 mm of water column. Determine the power required to drive the fan if fan's mechanical efficiency is 80% and coal is burnt at the rate of 1000 kg per hour and air is supplied at the rate of 16 kg per kg of coal. Ambient pressure and density of air may be taken as 1.01325 bar and 1.29 kg/m^3.*

Solution:

Total draught loss = Pressure equivalent to velocity head + Draught loss through grate.

Pressure equivalent to velocity head

$$= \frac{1}{2}\rho V^2 = \frac{1}{2} \times 1.29 \times (20)^2 = 258 \text{ N/m}^2$$

Since 1 mm of water column is equal to 9.81 N/m^2;

so pressure equivalent to velocity head = $\frac{258}{9.81}$ = 26.29 mm of water

Hence total draught loss = 26.29 + 30 = 56.29 mm of water column

Pressure required, p = 56.29 × 9.81, N/m^2 = 552.21 N/m^2

$$\text{Forced draught fan power requirement} = \frac{p \cdot m_f \cdot m_a \cdot T_a}{\rho_a \cdot T_o \cdot \eta_{mech}}$$

$$= \frac{552.21 \times 1000 \times 16 \times 293}{1.29 \times 273 \times 0.80 \times 3600}, \text{ W}$$

$$\boxed{\text{F.D. fan, power} = 2552.39 \text{ W or } 2.55 \text{ kW}} \quad \textbf{\textit{Ans.}}$$

20. *A boiler unit has 45 m high chimney through which flue gases at 630K flow. Air requirement is 15 kg air per kg of fuel burnt and ambient temperature is 300 K. Determine the draught produced in mm of water column, equivalent draught in metre of hot gas column and temperature of chimney gases for maximum discharge in a given time and the draught produced. Also find the efficiency of chimney if the minimum temperature of artificial draught is 150°C and mean specific heat of flue gas is 1.005 kJ/kg.K. If net calorific value of fuel is 30,000 kJ/kg then determine the percentage of extra heat spent in natural draught.*

Solution: Given T_g = 630K, T_a = 300K, $T_{g,a}$ = 150 + 273 = 423K, H = 45m, m = 15 kg air/kg fuel

$$\text{Draught in mm of water column} = h_w = 353H\left[\frac{1}{T_a} - \left(\frac{m+1}{m}\right).\frac{1}{T_g}\right]$$

$$h_w = 353 \times 45\left[\frac{1}{300} - \left(\frac{15+1}{15}\right).\frac{1}{630}\right]$$

h_w = 26.06 mm of water column ***Ans.***

Draught in metres of hot gas column = $h_g = H\left\{\left(\frac{m}{m+1}\right).\frac{T_g}{T_a}-1\right\}$

$$h_g = 45\left\{\left(\frac{15}{15+1}\right).\frac{630}{300}-1\right\}$$

h_g = 43.59 metres of hot gas column ***Ans.***

Temperature of chimney gases for maximum discharge,

$$T_{g,\max} = T_a \cdot 2\left(\frac{m+1}{m}\right)$$

$$\Rightarrow \quad T_{g,\max} = 300\times 2\times\left(\frac{15+1}{15}\right) = 640\text{K}. \quad \textbf{\textit{Ans.}}$$

Draught produced for condition of maximum discharge.

$$h_w = 353H\left\{\frac{1}{T_a}-\left(\frac{m+1}{m}\right).\frac{1}{T_g}\right\}$$

$$= 353\times 45\left\{\frac{1}{300}-\left(\frac{15+1}{15}\right)\times\frac{1}{630}\right\} = 26.05 \text{ mm}$$

= 26.05 mm of water column ***Ans.***

$$\text{Efficiency of chimney} = \frac{9.81\times H\times\left\{\left(\frac{m}{m+1}\right)\frac{T_g}{T_a}-1\right\}}{c_{p,g}(T_g - T_{g,a})}$$

$$= \frac{9.81\times 45\times\left\{\left(\frac{15}{15+1}\right)\times\frac{630}{300}-1\right\}}{1.005\times(630-423)\times 10^3}$$

$= 2.0556 \times 10^{-3}$

Efficiency of chimney = 0.2056% ***Ans.***

Extra heat carried away by the flue gases per kg of fuel

$= (15 + 1) \times c_{p.g} \times (T_g - T_{g,a})$

$= (15 + 1) \times 1.005 \times (630 - 423)$

= 3328.56 kJ

% Heat spent in natural draught $= \dfrac{3328.56}{30000} \times 100$

percentage extra heat carried in natural draught = 11.09% ***Ans.***

21. *In a boiler installation dry flue gases are formed at the mean temperature of 630 K when outside air temperature is 300 K. Air is consumed at the rate of 15 kg air per kg of coal and coal is required at the rate of 1600 kg per hour. Actual draught may be taken as 60% of theoretical draught. Determine height of chimney, if the various draught losses are 14 mm of water column.*

Solution: Theoretical draught $= \dfrac{14}{0.6} = 23.33$ mm of water column

We know $$h_w = 353 \times H \left\{ \frac{1}{T_a} - \left(\frac{m+1}{m} \right) \frac{1}{T_g} \right\}$$

$$\Rightarrow \quad 23.33 = 353H \left\{ \frac{1}{300} - \left(\frac{15+1}{15} \right) \frac{1}{630} \right\}$$

$$\Rightarrow \quad H = 40.29 \text{ m}$$

Height of chimney = 40.29 m ***Ans.***

22. *A boiler unit has forced draught fan maintaining draught of 100 mm of water column while discharging 30 m^3/s through outlet section of 1.8m^2 area. Ambient temperature is 300 K and assume mass of 1 m^3 of air at NTP as 1.293 kg to find out power of motor of forced draught fan if fan efficiency is 85%. Determine the power consumption if FD fan is substituted by ID fan of similar efficiency considering flue gas temperature of 150°C.*

Solution: Considering the discharge rate, the velocity of air through outlet

$$= \frac{30}{1.8} = 16.67 \text{m/s} = \frac{m.m_f}{\rho_a}$$

Pressure created due to the gases flowing at 16.67 m/s

$$= \frac{1}{2} \rho V^2 = \frac{1}{2} \times 1.293 \times (16.67)^2$$

$$= 179.66 \text{ N/m}^2$$

or 18.31 mm of water

Total draught = Static draught + Draught due to discharge

= (100 + 18.31)mm of water

= 118.31 mm of water column

Power of motor of forced draught fan $= \dfrac{P \cdot m \cdot m_f T_a}{\rho_a T_o \cdot \eta_{mech}}$

$$= \frac{118.31 \times 9.81 \times 300 \times 30}{0.85 \times 273 \times 10^3} = 45.01 \text{ kW}$$

Power consumption of induced draught (ID) fan = $\text{Power}_{FD} \times \frac{T_{g,a}}{T_a}$

$$= 45.01 \times \frac{423}{300} = 63.46 \text{ kW} \quad \textbf{\textit{Ans.}}$$

23. *In a boiler installation feed water enters at 30°C and leaves economiser section at 110°C for being fed into boiler. Steam generated in boiler at 20 bar, 0.98 dry and fed to super heater where its' temperature is raised up to 300°C. For the coal with calorific value of 30,500 kJ/kg and steam generation rate of 10kg/kg of coal burnt determine the energy received per kg of water and steam in economiser, boiler and superheater section as fraction of energy supplied by coal. Take* $C_{p,feed\ water}$ *= 4.18 kJ/kg·K,* $C_{p,superheated\ steam}$ *= 2.093 kJ/kg·K.*

Solution:

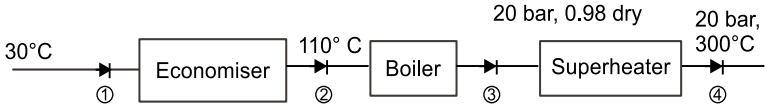


Fig. 11.41

From steam table, h_4 = 3023.5 kJ/kg

$h_{f \text{ at } 20 \text{ bar}}$ = 908.79 kJ/kg

$h_{fg \text{ at } 20 \text{ bar}}$ = 1890.7 kJ/kg

h_3 = 908.79 + (0.98 × 1890.7) = 2761.7 kJ/kg

For feed water, $h_1 = h_{f \text{ at } 30°C}$ = 125.79 kJ/kg

Total heat supplied = $(h_4 - h_1)$ = (3023.5 – 125.79) = 2897.71 kJ/kg

Heat consumed in economiser = $C_{p,\text{ feed water}} \times (110 - 30)$

= 4.18 × (110 – 30) = 334.4 kJ/kg steam

= 334.4 × 8 = 2678.4 kJ/kg coal

Heat consumed in boiler = $h_3 - h_2 = 2761.7 - (C_{p \text{ feed water}} \times 110)$

= 2301.9 kJ/kg steam

= 2301.9 × 8 = 18415.2 kJ/kg coal

Heat consumed in super heater = $h_4 - h_3$ = 3023.5 – 2761.7

= 261.8 kJ/kg steam

= 261.8 × 8 = 2094.4 kJ/kg coal

Heat provided by burning of coal = 30,500 kJ/kg coal

Fraction of energy consumed in economiser

$$= \frac{2678.4}{30,500} = 0.0878 \text{ or } 8.78\% \quad \textbf{\textit{Ans.}}$$

Fraction of energy consumed in boiler = $\frac{18415.2}{30500}$ = 0.6038 or 60.38% ***Ans.***

Fracton of energy consumed in superheater = $\frac{2094.4}{30500}$ = 0.0687 or 6.87%

EXERCISE

11.1 Define boiler.
11.2 Classify the boilers and briefly describe each type of them.
11.3 Enlist the requirements of a good boiler.
11.4 Differentiate between fire tube and water tube boilers.
11.5 Describe briefly a vertical boiler. Also give its' neat sketch.
11.6 Sketch and completely label a Lancashire boiler. Also explain its' working.
11.7 Sketch and describe working of Locomotive boilers.
11.8 What do you understand by high pressure boilers?
11.9 Explain working of Bacock and Wilcox boiler.
11.10 Sketch and describe a Stirling boiler.
11.11 Sketch and describe working of Loeffler boiler.
11.12 Differentiate between mountings and accessories.
11.13 Classify mountings into safety fittings and control fittings.
11.14 Describe superheater, economiser and air preheater with neat sketches. Also indicate suitable location of these on a boiler with line diagram.
11.15 Write short notes on, water level indicator, safety valves, fusible plug, feed check valve, pressure gauge, stop valve and blow off cock.
11.16 Define boiler draught and also classify it.
11.17 Describe functions of chimney in a boiler.
11.18 Obtain the expression for the natural draught in terms of height of water column. Also state the assumption made.
11.19 Derive the condition for maximum discharge through a chimney in natural draught.
11.20 Compare natural draught with artificial draught.
11.21 Describe briefly different types of mechanical draught.
11.22 Compare the power requirements for forced draught and induced draught.
11.23 Define equivalent evaporation and also give its significance.
11.24 Describe balanced draught.
11.25 What is meant by boiler trial? Explain.
11.26 Determine the height of chimney required to produce draught equivalent to 16.7 mm of water column for the flue gases at 300°C and ambient temperature of 20°C. Take the air requirement to be 20 kg/kg of fuel. [30 m]
11.27 Calculate the draught produced in *mm* of water by chimney of 35 m height, flue gas temperature of 643 K, boiler house temperature of 307 K and air supplied at 18.8 kg per kg of coal. [20 mm]
11.28 Show that the height of chimney required for producing a draight equivalent to 15 mm of water can not be less than 30 m, if the flue gas temperature is 250°C, ambient temperature is 20°C and minimum 18 kg air per kg of fuel is required.

11.29 In a boiler installation the height of chimney is 38 m and the hot gas temperature is 570 K and ambient air temperature is 300 K. Considering the diameter of chimney to be 2m, air/fuel ratio being 19 determine the mass flow rate of flue gases flowing out. [3012 kg/min]

11.30 Determine the percentage of excess air supplied in a boiler where coal having 80% C, 6% moisture, 14% ash by mass is burnt inside. Hot gas temperature is 570 K and ambient air temperature is 300 K. The height of chimney is 30 m to produce draught equivalent to 1.5 cm of water column. [20.1%]

11.31 For the maximum discharge condition through a chimney having height of 15 m determine the draught in mm of water when ambient air temperature is 15°C. [9.2 mm]

11.32 Determine the efficiency of chimney which operates with hot gas temperature of 370°C in natural draught and if it is operated with artificial draught the hot gas temperature drops down to 150°C. The height of chimney is 45 m, air/fuel ratio is 24 and ambient temperature is 35°C, Take C_p for hot gases as 1.004 kJ/kg · K. [0.2%]

11.33 In a boiler chimney the different pressure losses are measured to be as under,
Pressure loss on grate and boiler flues = 6 mm of water in each.
Pressure loss in bends and chimney duct = 2 mm of water in each.
Pressure head causing flow of hot gases = 1.2 mm of water.
Determine the height and diameter of chimney if air/coal ratio is 14, average coal consumption is 2000 kg/hr, hot gas temperature is 300°C, ambient air temperature is 310 K and actual draught is 0.8 times theoretical draught. [44.8 m, 1.6 m]

11.34 Determine the power required for a forced draught and induced draught fan in a boiler having coal consumption of 25 kg/min and air/fuel ratio of 19 by mass. The ambient air temperature and hot gas temperature at exit are 30°C and 140°C respectively. Draught required is equivalent to 32.2 mm of water and mechanical efficiency of motor/fan is merely 70%. [4.16 hp, 5.97 hp]

11.35 Determine the capacity of motor required for running forced draught fan to maintain draught of 50 mm of water when the hot flue gas temperature and boiler house temperature are 175°C and 30°C respectively. The boiler has coal requirement of 25.4 kg/hr and the flue gases are produced at the rate of 19 kg per kg of coal burnt. Considering mechanical efficiency of 78% compare the power requirement of forced draught and induced draught arrangement.
[50.5 hp, FD requires 33% less power than 1D]

11.36 A boiler has equivalent evaporation of 1300 kg/hr from and at 100°C. The fuel consumption per hour is 159.5 kg/hr and the boiler efficiency is 72%. Determine actual equivalent evaporation if feed is supplied at 110°C to generate steam at 100 kg/m^2 per hour and 15 bar, 200°C. Also find calorific value of coal burnt and the grate area.
[1255 kg/hr, 25498 kJ/kg, 12.6 m^2]

11.37 Determine equivalent evaporation per kg of coal and efficiency of boiler when it consumes coal at the rate of 52 kg/hr and steam is generated at 7 bar, 0.907 dry, 541.7 kg/hr from feed water supplied at 40°C.
[11.075 kg per kg of coal, 83.3%]

11.38 During a boiler trial for 24 hour the following is observed;
Steam generated = 160000 kg
Mean steam pressure = 12 bar
State of steam generated = 0.85
Feed water temperature = 30°C
Coal burnt = 16000 kg

C.V. of coal = 33400 kJ/kg

Determine equivalent evaporation from and at 100°C and efficiency of boiler.

[10.45 kg/kg of coal, 70.65%]

11.39 A boiler has air supplied at 20°C and flue gas temperature of 220°C. The coal burnt has composition by mass of 85% C, 3.9% H, 1.4% O and remaining ash. Volumetric composition of dry flue gas is 12.7% CO_2, 1.4% CO, 4.1.% O_2, 81.8% N_2. The dew point temperature of wet flue gases is 50°C. Calorific value of coal may be taken as 10157.4 kJ/kg. Take C_p of dry flue gas as 1.0032 kJ/kg · K, C_p of superheated steam as 1.5048 kJ/kg · K.

Determine the heat carried away by dry flue gas, heat carried by moisture from combustion and heat loss due to incomplete combustion of carbon for per kg of coal burnt.

[3080.7 kJ/kg, 969.8 kJ/kg, 2027.3 kJ/kg]

11.40 Draw heat balance sheet for a boiler whose 24 hour trial yields following;

Ambient temperature = 35°C

Coal burnt = 19200 kg, Ash collected = 1440 kg,

Combustion in ash = 54 kg, Steam generated = 198000 kg,

Moisture in coal = 2.16%,

Steam temperature = 198.9°C, Feed water temperature = 150°C, Flue gas temperature = 315°C, Coal composition by mass = 83.39% C, 4.56% H_2, 5.05% O_2, 0.64% S, 1.03% N_2, 5.33% ash

Flue gas analysis by volume

= 12.8% CO_2, 6.4% O_2, 0.2% CO, 80.6% N_2

The partial pressure of steam = 0.07 bar,

Specific heat of steam = 2.0064 kJ/kg · K

[Heat supplied in steam generation = 66.62%
Heat carried by dry flue gas = 13.38%,
Heat carried by moisture = 3.76%,
Heat carried by ash = 0.28%,
Unaccounted heat = 15.96%]

11.41 Steam is generated at 100000 kg/hr from a boiler at 100 bar, 500°C with feed water supplied at 160°C. The fuel burnt in boiler has calorific value of 21000 kJ/kg and boiler efficiency is 88%. Determine rate of fuel burnt per hour and percentage of total heat absorbed in economiser, evaporator and superheater. [14592 kg/hr, 27.09%, 48.84%, 24.07%]

11.42 Compare the steam generation capacities of two boilers *A* and *B* for which operating parameters are, Steam generation pressure = 14 bar, feed water temperature = 27°C, Specific heat of feed water = 41.8 kJ/kg · K, Specific heat of steam = 2.1 kJ/kg · K,

For boiler A:
State of steam = 0.9 dry
Boiler efficiency = 73%
Calorific value of coal = 34000 kJ/kg

For boiler B:
State of steam = 240°C
Boiler efficiency = 79.5%
Calorific value of coal = 46000 kJ/kg

[For boiler *A* 10 kg/kg of coal burnt, for boiler *B* 14 kg/kg of coal burnt]

11.43 A boiler generates steam at 15 bar, 0.95 dry from feed water entering at 65°C, 2040 kg/hr. Dry coal burnt is 232 kg/hr and mean boiler house temperature is 25.3°C and flue gas temperature is 440°C. Consider partial pressure of steam as 0.07 bar.

Composition of dry coal by mass: 83% C, 6% H_2, 5% O_2, 6% ash.

Composition of dry flue gas by volume: 10.50% CO_2, 1.3% CO, 7.67% O_2, 80.53% N_2

Higher calorific value of dry coal = 34276 kJ/kg

Specific heat of dry flue gas = 0.9948 kJ/kg · K

Determine (a) total mass of flue gases per kg of coal, (b) percentage of excess air supplied, (c) heat balance sheet per kg of dry coal

[17.58 kg/kg of coal, excess air = 49.51%, Heat taken by steam = 62.02%,Heat taken by dry flue gas = 21.1%, Heat by moisture = 5.32%, Unaccounted = 11.56%]

12
Steam Engine

12.1 INTRODUCTION

Steam engine is a device which is especially designed to transform energy, originally dormant or potential, into active and usefully available kinetic energy. Origin of reciprocating steam engine dates back to BC 200. Steam engine is an exceedingly ingenious, but unfortunately, still very imperfect device for transforming heat obtained by chemical combination of combustible substance with air into mechanical energy. In steam engine the mechanical effect is seen due to the expansion of steam which is generated in boiler and supplied to steam engine. Steam engines have been successfully used in the mill, driving locomotive or steam boat, pumps, fans, blowers, small electricity generators, road rollers etc.

A steam engine plant shall have boiler, condenser, feed pump along with steam engine. Steam generated in boiler is sent to steam engine where it is expanded upto certain pressure. Steam leaving engine are fed to condenser where steam gets converted into condensate (water) which is sent back to boiler through feed pump. Figure 12.1 shows the schematic of simple steam engine plant.

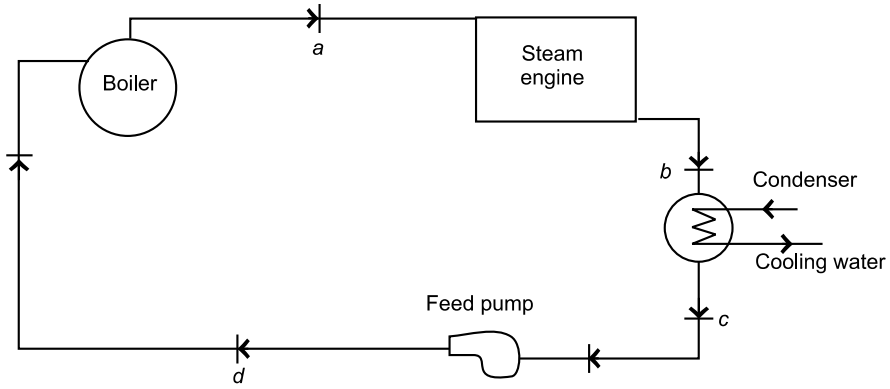


Fig. 12.1 *Schematic for simple steam engine plant*

Here in this chapter the philosophy of steam engine, its construction and working, compound steam engine etc. are described in detail.

12.2 CLASSIFICATION OF STEAM ENGINES

Steam engines can be classified in the following ways. Selection of particular steam engine depends upon the requirements and constraints of an application and features of the engine

(i) ***Based on axis of engine:*** Steam engine can be classified as horizontal or vertical steam engine based upon the orientation of cylinder axis. Engine having vertical axis cylinder is called vertical steam engine and engine having horizontal axis cylinder is called horizontal steam engine.

(ii) ***Based on speed of engine:*** Speed of steam engine can be considered for classifying steam engine as below:

(a) High speed engine—having speed of 250 rpm and above.

(b) Medium speed engine having speed lying between 100 to 250 rpm.

(c) Slow speed engine having speed less than 100 rpm.

(iii) ***Based on type of steam action:*** Depending upon whether the steam acts on both sides of piston or only on one side of piston the engine can be classified as double acting steam engine or single acting steam engine. In case of single acting engine out of two piston strokes i.e. forward stroke and backward stroke, only one stroke is used for work output. While in double acting engine both the forward and backward strokes of piston are used for work output alternatively i.e. expansion of steam occurs from both sides of piston alternatively. Theoretically, it may be said that power developed from double acting engine will be nearly double of power developed from single acting steam engine.

Power output from double acting engine is not exactly twice of output from single acting engine because only one side complete piston face is available for action on other side piston face area available for action is less due to presence of connecting rod, pin etc.

(iv) ***Based on expansive or non expansive type of working:*** Steam engines can be of non expansive type in which steam is injected throughout the piston stroke i.e. piston movement is caused due to high pressure steam forcing piston from one end to other end throughout and not due to expansion of steam. Such type of steam engines are called non-expansive steam engine. *P-V* diagram showing expansive and non expansive engine is given in Fig. 12.2. Expansive type steam engine has steam injection for only a part of piston stroke and the expansion occurs in remaining part of stroke. The cut off ratio in case of non expansive engine is unity while for expansive engine it is less than unity. Expansive engines offer greater efficiency as compared to non expansive engine but the work out put per stroke from expansive engine is less as compared to non expansive engine. Expansive engines are used where efficiency is important or engine operation is required on intermittent basis. For large power requirements such as in case of boiler feed pumps, hoist engines etc. the non-expansive engines are required.

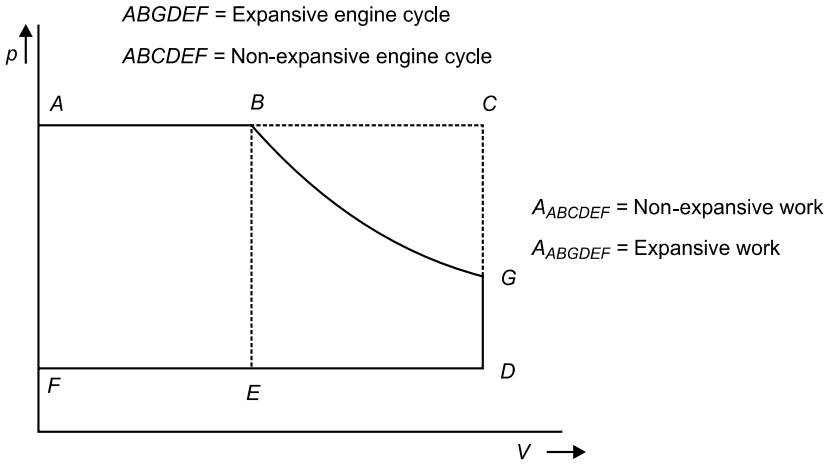


Fig. 12.2 *Expansive and non-expansive engine on* P-V *diagram*

(v) ***Based on type of exhaust from engine:*** Exhaust from steam engine may either go to atmosphere or to condenser. In certain applications where expansion in steam engine is

extended up to sub atmospheric pressure then the exhaust steam from engine is sent to condenser. Such type of engine whose exhaust goes to condenser are termed as 'Condensing engine'. The steam engines whose exhaust steam is delivered to atmosphere is called 'non-condensing engine'. Steam engine of condensing type generally deliver steam at 0.05 bar inside condenser. Steam transforms into condensate in condenser and condensate is sent again to boiler through feed pump. Due to lower back pressure in condensing engine the work output from such engine is more. Non condensing engine delivers steam at pressure more than atmospheric pressure or near atmospheric pressure and this steam can not be re-used for the engine cycle.

(vi) ***Based on number of stages:*** Depending upon the number of stages of expansion in steam engine, it may be termed as single stage, two stage, three stage, four stage engine etc.

(vii) ***Based on number of cylinders:*** Steam engine may have steam expanding in single cylinder and engine is called "simple steam engine". Steam engine may have the expansion of steam occuring in more than one cylinder such as expansion occurring in high pressure cylinder and low pressure cylinder etc. Such steam engine having expansion in more than one cylinders sequentially are termed as "Compound steam engine". Compound steam engine having three cylinders may have one cylinder called as high pressure cylinder, second cylinder as intermediate pressure cylinder and third cylinder as low pressure cylinder.

(viii) ***Based on type of governing:*** Steam engine may have different types of governing as throttle governing and cut-off governing. Engine may be called as throttle governed engine or cut-off governed engine.

(ix) ***Based on type of application:*** Steam engine may be of following types depending upon applications as;

(a) Stationary engine
(b) Marine engine
(c) Locomotive engine

12.3 WORKING OF STEAM ENGINE

Schematic of simple steam engine is shown in Fig. 12.3 along with important components labelled upon it.

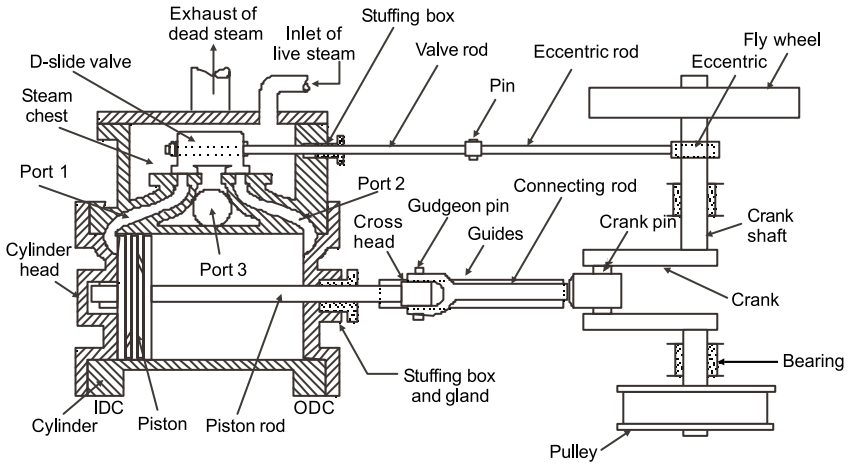


Fig.12.3 *Schematic of simple steam engine*

Simple steam engine shown is a horizontal double acting steam engine having cylinder fitted with cylinder cover on left side of cylinder. Cylinder cover has stuffing box and gland through which the piston rod reciprocates. One end of piston rod which is inside cylinder has piston attached to it. Piston has piston rings upon it for preventing leakage across the piston. Other end of piston rod which is outside cylinder has cross head attached to it. Cross head slides in guide ways so as to have linear motion in line with engine axis. Cross head is connected to the small end of connecting rod by the gudgeon pin. Big end of connecting rod is mounted over crank pin of the crank. Reciprocating motion of piston rod is transformed into rotary motion of crankshaft by cross head, connecting rod and crank. Cross head transmits the motion of piston rod to connecting rod. Cross head guide ways bear the reaction force.

Crank is integral part of crank shaft mounted on bearings. Crankshaft has fly wheel mounted on one end and pulley mounted on other end. Crankshaft also has an eccentric with eccentric rod mounted on it. Eccentric performs function of converting rotary motion of crankshaft into reciprocating motion of valve rod. Other end of eccentric rod transmits motion to valve rod which passes through stuffing box fitted in steam chest. Valve rod controls the movement of D-slide valve inside the seam chest. D-slide valve opens and closes the exhaust and inlet passages from steamchest to engine cylinder. Steam chest has two openings one for inlet of live steam and other for exit of dead or expanded steam. Live steam refers to the steam having sufficient enthalpy with it for doing work in steam engine. Dead steam refers to the steam having insufficient enthalpy with it and does not have capability to produce work. High pressure and high temperature steam (live steam) enters from main inlet passage into steam chest. D-slide valve occupies such a position that passage (port 1) from the steam chest to engine cylinder gets opened. High pressure steam enters cylinder and forces piston towards other dead centre.

Linear motion of piston is transformed into rotation of crankshaft through crosshead, connecting rod, gudgeon pin and crank.

When piston reaches other dead centre then the corresponding displacement of valve rod causes shifting of D-slide valve such that other passage (port 2) from steam chest to cylinder gets opened and passage 1 comes in communication with the exhaust passage. Thus the live steam enters from steam chest to cylinder through passage 1 and dead steam leaves from cylinder to exhaust passage through passage 2. Steam forces piston from inner dead centre to outer dead centre and from other side of piston dead steam leaves from exhaust passage simultaneously. Since it is double acting steam engine so both strokes produce shaft work due to steam being injected on both sides of piston alternatively. If it is a single acting steam engine then steam injection takes place on one side of piston only for producing power and return stroke occurs due to inertia of flywheel. Flywheel mounted on crankshaft overcomes the fluctuations in speed, if any due to its high inertia. Pulley mounted on shaft is used for transmitting power.

Apart from the components shown in figure the engine has throttle valve, governor mechanism, oil pump, relief valves etc. for its proper functioning.

Different major components of steam engine are described as below.

(i) ***Piston and Piston rod:*** Piston is a cylindrical part made of cast iron, cast steel or forged steel depending upon application. Piston has rectangular grooves upon it to prevent leakage across the piston-cylinder interface. Piston is fitted on the piston rod by nut. Generally piston rod is tapered for its' portion inside piston and has threaded portion at end with provision of tightening nut upon it as shown in Fig. 12.4.

(ii) ***Piston rings:*** Piston rings are fitted over piston in circumferential grooves. Piston rings are generally two or three in number so as to prevent leakage. Rings are split and in case of single ring leakage may occur across the point of split of ring. Hence more than one rings are put upon piston with staggered split ends so that leakage may not occur across these openings. Rings have sufficient elasticity in them so as to exert continuous force on cylinder wall for preventing leakage. Rings may be of cast iron and are turned slightly more than bore of cylinder and split at some point.

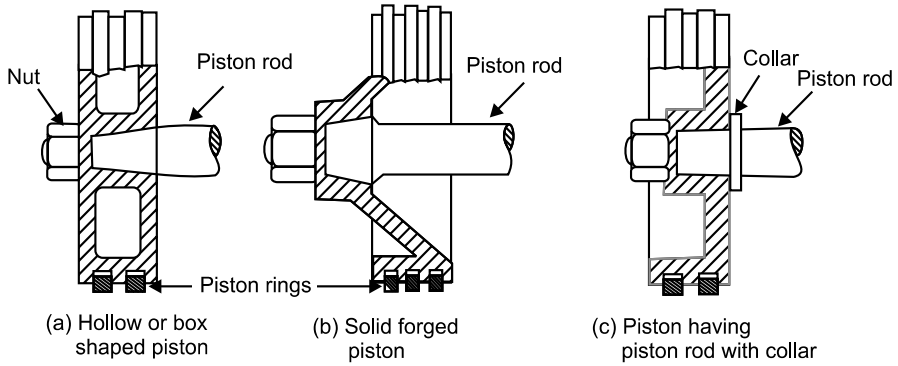


Fig. 12.4 *Piston with piston rod*

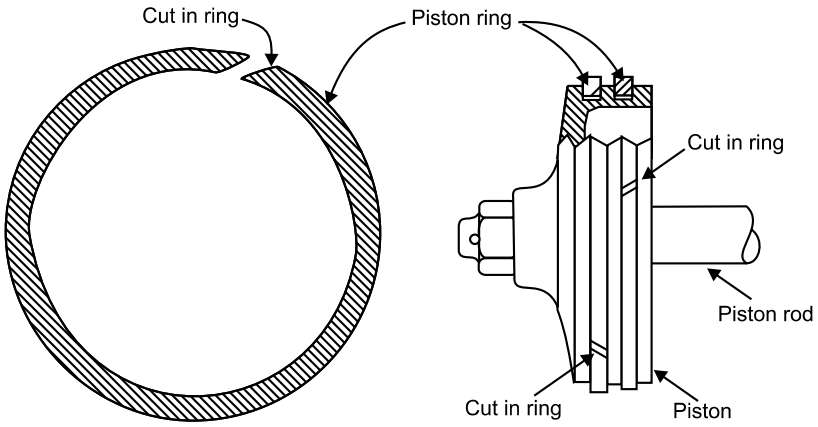


Fig. 12.5 *Piston rings*

(iii) ***Connecting rod:*** Connecting rod as shown in Fig. 12.6 has big end and small end fitted upon crank pin and cross head respectively. It generally has I-section or circular section and is made of steel by forging. Connecting rod has relative motion occurring at big end and small end so there is requirement of lubrication at these locations. There is suitable passage inside connecting rod for carrying lubricant under pressure to the respective locations. Lubricant generally passes from main bearing section to the crankshaft and subsequently to crank pin. Crank pin has holes over it from where lubricant oozes out and lubricates the mating parts of big end and pin. Crank pin also forces lubricant through connecting rod to the gudgeon pin on the cross head. Connecting rod has a through hole from big end to small end for this purpose.

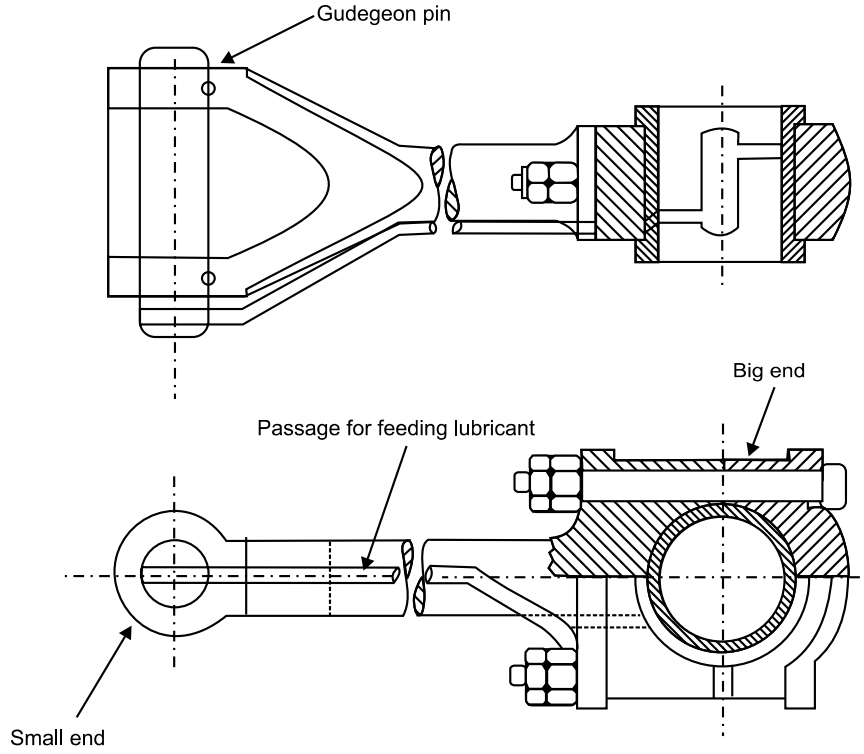


Fig. 12.6 *Connecting rod*

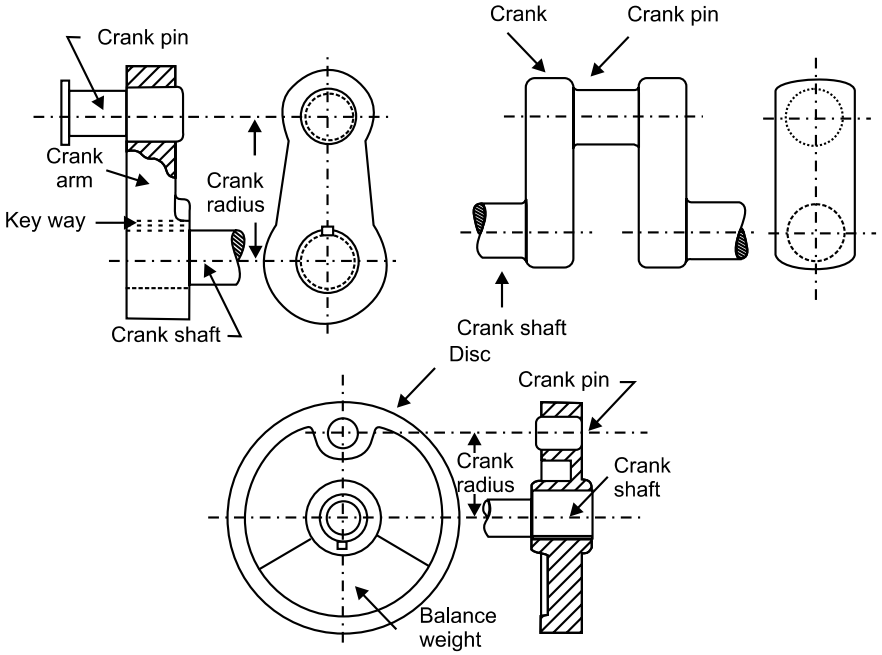


Fig. 12.7 *Crank pin, crank and crank shaft*

(iv) ***Crank and Crank shaft:*** Figure 12.7 shows the crank pin, crank and crank shaft as integral part. Reciprocating motion of cross head is transmitted to crank pin through connecting rod and can be transformed into rotary motion of crank shaft due to typical geometry. Connecting rod has its big end mounted on crank pin and the size of stroke depends upon the distance between the shaft and crank pin. Distance between the crank pin axis and axis of crankshaft is called radius of crank. Stroke of piston is double of the radius of crank as one complete rotation of crank shaft shall occur due to displacement of piston between two dead centres. Different types of crank arrangements used in steam engines are of overhung crank and disc crank as shown in Fig. 12.7. The overhung crank has crank pin mounted on crank as a cantilever supported element. Crank pin is shrunk fit on the crank i.e. heating for expanding the hole and subsequent cooling for shrunk fit.

Disc crank has a cast iron disc in place of crank, upon which the crank pin is mounted. In case of crank shaft of heavier engines having bearing support on both ends their are two discs having pin in between.

(v) ***Stuffing box:*** Stuffing box is a kind of packing box through which the reciprocating shaft/ rod passes and leakage across the reciprocating member is prevented. Stuffing box and different parts of it are shown in Fig. 12.8. Body of stuffing box is integral casting with cylinder cover and has gland to be tightened upon it with packing in between. Subsequently the packing is compressed by tightening gland by nuts and studs.

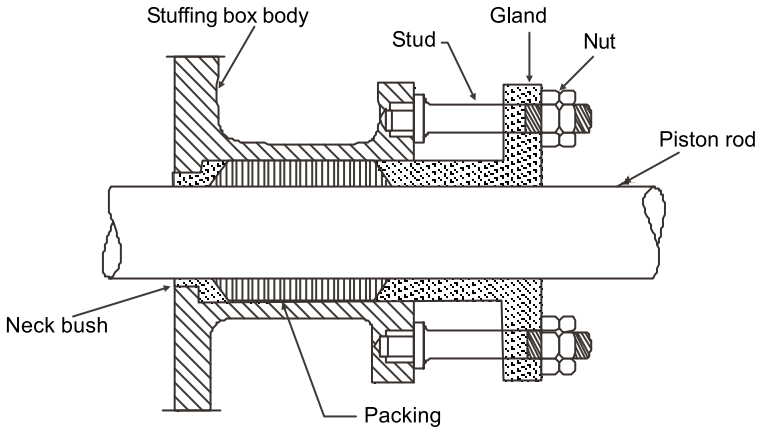


Fig. 12.8 *Stuffing box*

Leakage is prevented due to tightening of gland causing packing to be under pressure. Generally greased hempen rope, or asbestos or rubber ropes are used as packing material. Pressure exerted by packing can be changed by degree of tightening of studs and nuts. Metal packings, are also sometimes used in case of high pressure steam engines.

Stuffing box are placed at the location where piston rod passes through cylinder head and where the valve rod passes through the steam chest.

(vi) *Crosshead and guideways:* Piston rod is connected to connecting rod through cross head in between. Piston rod always does linear motion while connecting rod has continuously varying positions depending upon the position of crankshaft. Thus inspite of oblique position of connecting rod at any instant the piston rod has to be always in line with cylinder axis. Cross head and guides are the intermediate members which permit this transfer of motion from piston rod to connecting rod.

Since directions of forces acting in linearly moving piston rod and oblique moving connecting rod are different so they result into a third resultant force which is countered by guides. During both forward stroke and backward stroke these resultant forces are of different types thereby requiring guides on both sides of cross head.

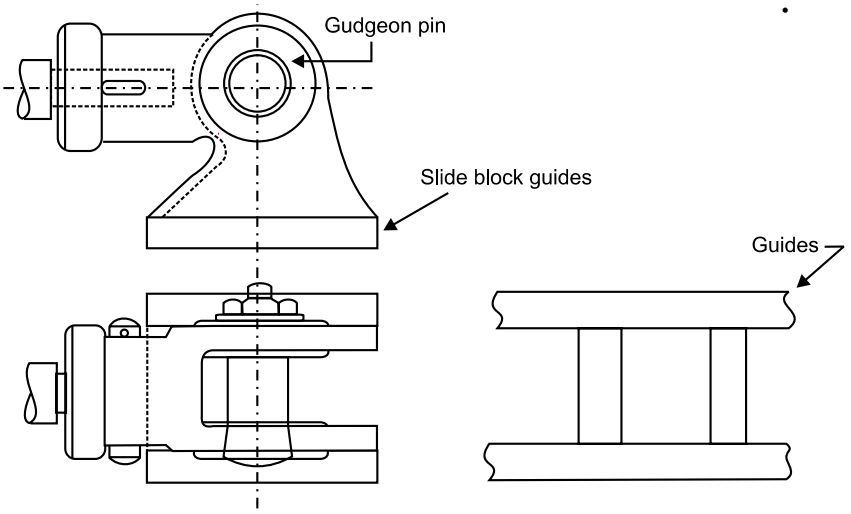


Fig. 12.9 *Crosshead and guides*

(vii) *Eccentric:* Eccentric performs the function of transforming only rotary motion of crankshaft into reciprocating motion of valve rod. Operating principle of eccentric is quite similar to that of crank as both transform one type of motion into other. Important difference between the eccentric and crank is that eccentric can not convert reciprocating motion into rotary motion as done by crank. Figure 12.10 shows the eccentric which has three basic parts called eccentric sheave, eccentric strap and eccentric rod. There is relative motion between the sheave and strap. Arrangement of lubrication is also provided at the interface of sheave and strap. Sheave is fixed on the shaft using a key. Sheave may be of solid (single piece) type or of split type. Split sheaves are preferred when eccentric is to be used between two cranks and single piece eccentric can not be slided on the shaft. Sheaves are made out of cast iron and steel. After placing sheave upon shaft the eccentric strap is tightened through bolts and nuts. Distance between centre of sheave to centre of crankshaft is called "radius of eccentric" and the total linear displacement of valve is called "throw of eccentric". Throw of eccentric is equal to twice the radius of eccentric.

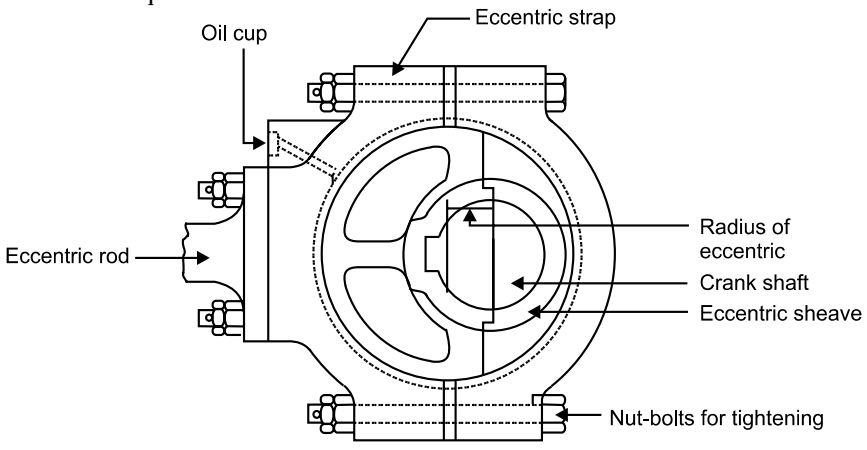


Fig. 12.10 *Eccentric*

(viii) *Slide valve:* Slide valve is provided in steam chest for quantitative regulation of supply of live steam into cylinder and exhaust of dead steam from cylinder. Slide valve is operated by the valve rod connected to eccentric rod and eccentric. Slide valves are of two types namely D-slide valve and Piston valve.

Figure 12.11 shows the D-slide valve having shape of alphabet-D. Valve has its' two ends resting upon the steam chest having three openings comprising of two between cylinder and chest and one between chest and exhaust port.

Amongst three ports two outer ports are called steam ports at head end and crank end respectively. Third port in middle is called exhaust port. In one position of D-slide valve as shown in figure both passages connecting steam chest with cylinder are closed due to overlap of valve over passages. Distance by which valve overlaps the ports in its middle position is called 'lap'. Every valve while resting on port shall have overlaps on both sides and are called as 'steam lap' or 'outside lap' on one side and on other side it is called 'exhaust lap' or 'inside lap'. When valve moves towards cylinder head then the port 2 is open for live steam to enter into cylinder from crank end. Also port 1 and port 2 are enclosed in D-slide valve and dead steam coming out from cylinder port 1 leaves out through exhaust port 3. When valve moves from cylinder head end to crank end then the port 1 is open for live steam to enter into cylinder while dead steam shall leave cylinder from port 2 to port 3. The cut off of steam occurs till the valve keeps ports in open state, allowing entry of live steam (high pressure and high temperature steam). The size of lap decides the point of cut off. In case of no lap the admission of steam and cut off shall occur at the dead centres. Also the exhaust of steam shall occur at dead centres. Due to non linear motion of connecting rod and eccentric rod the steam lap and exhaust laps are not equal. Ports are made open somewhat earlier than the stipulated position of piston so as to reduce throttling losses. Thus valve opens the port even when piston is at dead centre. This distance by which valve covers the steam port when piston is at dead centre is called 'lead'. Similarly the 'angle of advance' can be obtained for lead. Amount of valve opening/closing can be modified by the valve gear mechanism and thus allowing for variable expansion in cylinder.

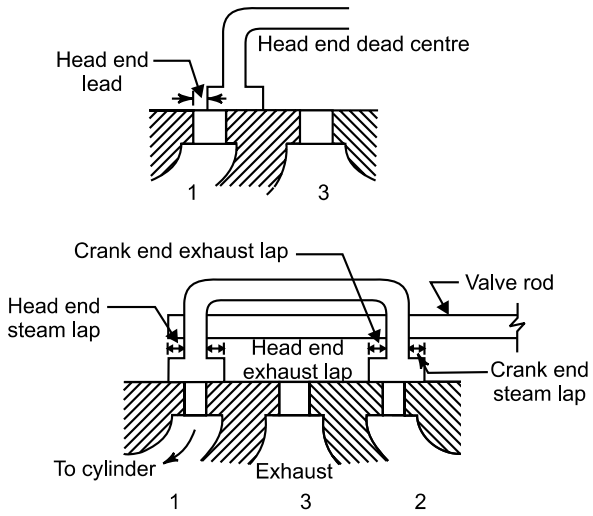


Fig. 12.11 *D-slide valve*

D-slide valve has simple construction and ease of repair and maintenance. But these valves have significant amount of frictional losses and excessive wear of valve seat. There occurs large amount of throttling due to gradual opening/closing of ports. Also there are large condensation losses as the ports 1 and 3 are alternatively used by live steam (high pressure and temperature steam) and dead steam (low pressure and temperature steam) thereby causing alternate heating and cooling of steam chest ports. These valves are not suited for super- heated steam as due to valve and seat being of flat type can not be properly lubricated and so high temperature steam may cause excessive thermal stresses and distortion of valve/valve seat.

Piston valve: Piston valve has piston-cylinder kind of arrangement for regulating the opening/ closing of different ports. These piston valves have lesser frictional resistance compared to D-slide valve as D-slide valves are highly pressed down against valve seat in steam chest and have larger frictional losses. Schematic of piston valve for double acting steam engine is shown in Fig. 12.12.

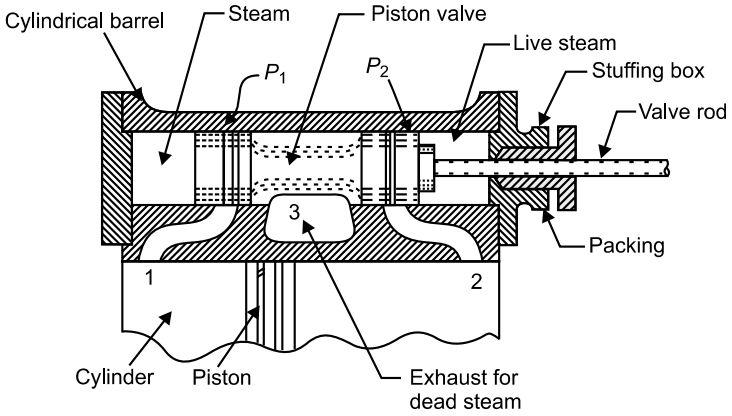


Fig. 12.12 *Piston valve*

Here similar to D-slide valve having D-shape, a piston valve with neck in between performs the similar function. Two pistons (P_1 and P_2) on same valve rod cover and uncover the ports 1 and 3 for the purpose of injecting high pressure steam and rejecting low pressure steam alternately. Working principle is similar to that of D-slide valve. For example when piston is near the cylinder head end then it shall uncover port 2 for live steam to enter into cylinder. Expanded steam or low pressure steam on other side of piston shall find its way out to exhaust through port 1 and 3. Similarly during piston valve's position near crank end the port 1 acts as inlet port and port 2 acts as exhaust port. These piston valves also suffer from disadvantages of throttling of steam due to slow opening/closing of ports. Also the points of admission of steam, cut-off and exhaust etc. are not independently adjustable.

12.4 THERMODYNAMIC CYCLE

Steam engine works on the modified Rankine cycle. P-V representation of cycle is shown in Fig. 12.13 for steam engine with and without clearance volume.

Let us take steam engine without clearance in which live steam enters the cylinder at state 1 and steam injection continues up to state 2. Point 2 is showing the state at cut-off. Subsequently steam is expanded up to state 3 theoretically and stroke gets completed. The expansion process is of hyperbolic type. Hyperbolic expansion process is one having the P-V = constant. In actual practice this expansion is not continued upto 3 due to the fact that positive work available from engine in later part of stroke is

much less than negative work required for maintaining piston movement. Hence expansion process is terminated even before this piston reaches the extreme position (dead centre).

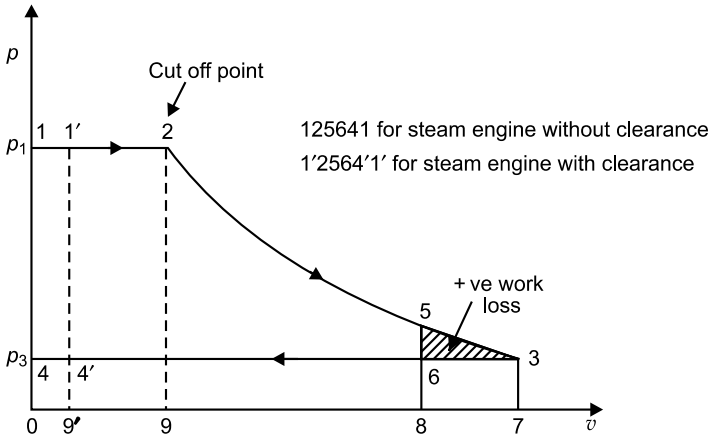


Fig. 12.13 *Modified Rankine cycle on P-V diagram for steam engines*

Here expansion process is shown to be terminated at state 5 and exhaust port is opened at this point, thereby causing sudden drop of pressure as shown in 5-6. Exhaust of dead steam occurs from 6 to 4. It is obvious from *P-V* diagram that the terminating expansion process before piston reaching dead centre shortens the expansion stroke length although actual work output from engine does not get affected. This modified form of cycle is called as 'modified Rankine cycle'. In case of steam engine with clearance volume the modified Rankine cycle is as shown by 1′2564′1′ while for engine without clearance volume it is given by 125641. Practically steam engine always has clearance volume but for the ease of mathematical analysis it may be considered to be without clearance volume. Therefore such cycle without clearance is also termed as hypothetical cycle. *T-S* representation of the modified Rankine cycle is shown in Fig. 12.14.

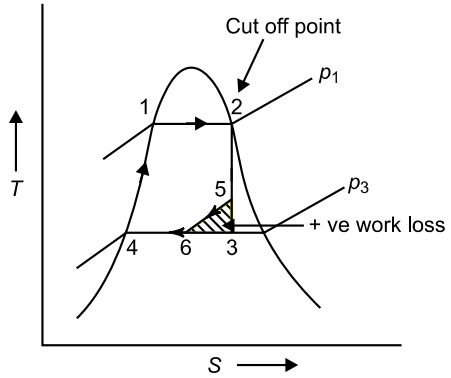


Fig. 12.14 *T-S representation of modified Rankine cycle*

From *p-V* diagram the work output from steam engine can be mathematically given by knowing the area enclosed by cycle. Theoretical work in case of hypothetical cycle, i.e. without clearance.

W = Area enclosed by 125641 on *P-V* diagram

W = Area 1290 + Area 25689 – Area 4680

$$W = p_1(V_2 - V_1) + p_1 V_2 \log\left(\frac{V_5}{V_2}\right) - p_3 (V_5 - V_1)$$

Here expansion ratio, $r = \dfrac{V_5}{V_2}$ and $V_1 = 0$ as zero clearance is there.

$$W = (p_1 \times V_2) + (p_1 \times V_2 \ln r) - p_3 \cdot V_5$$

Work output neglecting clearance volume;

$$\boxed{W_{\text{without CV}} = p_1 V_2 (1 + \ln r) - p_3 V_5}$$

For steam engine cycle considering clearance volume cycle work shall be,

$$W = \text{Area } 1'2564'1' \text{ on } p\text{-}V \text{ diagram}$$

$$W = \text{Area } 1290 + \text{Area } 2589 - \text{Area } 4680 - \text{Area } 11'4'4$$

$$W = (p_1 V_2) + p_1 V_2 \ln \left(\frac{V_5}{V_2}\right) - (p_3 \cdot V_6) - (p_1 - p_3) V_1'$$

$$W = (p_1 V_2) + p_1 V_2 \ln r - (p_3 V_6) - (p_1 - p_3) V_1'$$

As $V_5 = V_6$

$$\boxed{W_{\text{with CV}} = (p_1 V_2) (1 + \ln r) - (p_3 V_5) - (p_1 - p_3) V_1'}$$

Let us denote pressure p_3 as back pressure p_b and clearance volume as V_C then expressions for cycle work without and with clearance volume are as follows;

$$\boxed{W_{\text{without CV}} = p_1 V_2 (1 + \ln r) - (p_b \cdot rV_2)}$$

$$\boxed{W_{\text{with CV}} = p_1 V_2 (1 + \ln r) - (p_b \cdot rV_2) - (p_1 - p_b) V_C}$$

Mean effective pressure for the hypothetical cycle can be obtained using the work obtained earlier.

$$\text{Mean effective pressure} = \frac{\text{Cycle work}}{\text{Stroke volume}}$$

In case of without clearance volume

$$mep_{\text{without CV}} = \left(\frac{W_{\text{without CV}}}{V_5}\right)$$

or,

$$mep_{\text{without CV}} = \frac{\{p_1 V_2 (1 + \ln r) - (p_b \cdot r \cdot V_2)\}}{r \cdot V_2}$$

$$\boxed{mep_{\text{without CV}} = \frac{p_1 (1 + \ln r)}{r} - p_b}$$

In case of cycle with clearance volume,

$$mep_{\text{with CV}} = \frac{W_{\text{with CV}}}{\text{Stroke volume}}$$

$$= \frac{p_1 V_2 (1 + \ln r) - (p_b \cdot r \cdot V_2) - (p_1 - p_b) \cdot V_c}{(V_5 - V_C)}$$

$$\boxed{mep_{\text{with CV}} = \frac{p_1 V_2 (1 + \ln r) - (p_b \cdot r V_2) - ((p_1 - p_b) \cdot V_c)}{(rV_2 - V_c)}}$$

12.5 INDICATOR DIAGRAM

Indicator diagrams for steam engine are shown in Fig. 12.15. Hypothetical indicator diagram shown by cycle *abcde* and the actual indicator diagram for an engine are given here. Hypothetical indicator diagram is obtained considering all processes in cycle to be ideal and assuming no heat loss and pressure drop etc.

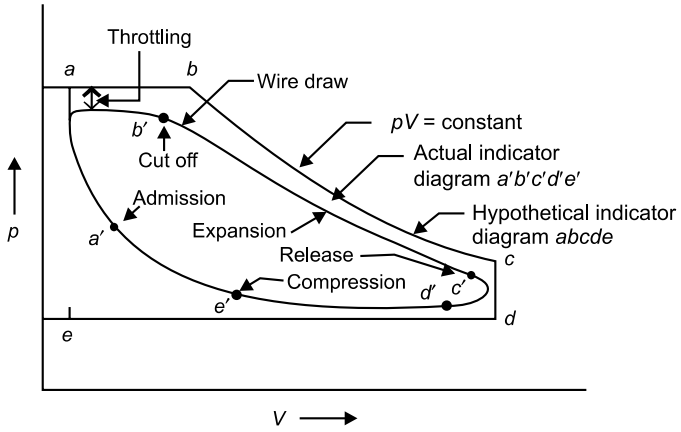


Fig. 12.15 *Hypothetical and actual indicator diagram*

Hypothetical indicator diagram shows the indicator diagram having steam admission beginning at *a* and continuing up to *b*. Expansion occurs between *b* and *c* in hyperbolic manner. At *c* there occurs sudden pressure drop due to opening of exhaust valve up to *d*. Piston travels from *d* to *e* and steam injection begins at *a* and thus cycle gets completed. In this hypothetical diagram all processes are considered to occur as per their theoretical assumptions. While actual diagram is based upon the actual occurrence of all processes. This is a reason why actual diagram shown by *a′b′c′d′e′* differs from hypothetical indicator diagram. In hypothetical diagram it is assumed that there occurs no pressure drop, valves open and close instantaneously, expansion occurs following hyperbolic process and admission of steam and its exhaust occur at end of strokes etc.

The actual indicator diagram differs from hypothetical indicator diagram because of the following factors.

(i) Practically there shall be pressure drop due to friction, throttling and wire drawing etc. Friction, throttling and wire drawing occur in valves and ports connecting steam chest and cylinder thereby causing pressure drop.

(ii) Inlet and exit valves (ports 1, 2 and 3) can never be opened or closed instantaneously, which means that there shall always be some time required for completely opening or closing the valves. Therefore, in order to ensure entry of maximum amount of high pressure and temperature (live steam) steam it is desired to advance the opening of inlet valve and suitably modify the operation of valves for remaining processes. Thus it may be said that actually the inlet, cut off and release occur gradually.

(iii) Expansion in steam engine does not occur hyperbolically in actual process due to varying heat interactions. This causes shift in expansion line from *bc* in hypothetical engine to the actual expansion line as shown in actual indicator diagram (*b′c′*). It may be understood that due to condensation of steam the expansion follows some other law.

(iv) Exhaust of steam begins at *c′* in order to overcome limitation of gradual opening of exhaust valve. Also the sudden pressure drop is not possible. Actual pressure drop occurs during *c′d′* in non-instantaneous manner upto back pressure value.

(v) In actual cycle the exhaust pressure will always be above the back pressure in order to overcome the friction and throttling losses during exhaust.

(vi) In actual indicator diagram exit of steam is stopped at e', thereby causing trapping of some steam which gets subsequently compressed in cylinder till piston moves towards inner dead centre and inlet port is opened. This residual steam getting compressed offers cushioning effect. This compression of residual steam is represented on actual indicator diagram by $e'a'$.

(vii) Actual engine shall always have clearance volume contrary to hypothetical diagram based on no clearance volume assumption. Clearance volume is required, because, the piston can not be allowed to collide with cylinder head.

Clearance volume offers cushioning effect due to steam occupying it and 'clearance volume also allows for accumulation of condensate (water) with excessive pressure rise which may otherwise cause damage to cylinder head'.

Thus it is seen that the actual indicator diagram is modified form of hypothetical indicator diagram due to practical limitations. The amount by which actual indicator diagram differs from hypothetical indicator diagram is quantified by parameter called 'diagram factor'. Diagram factor is the ratio of area enclosed in actual indicator diagram to the area enclosed in hypothetical indicator diagram. Diagram factor has value less than unity. Designer shall always intend to have engine having diagram factor close to unity.

$$\text{Diagram factor} = \frac{\text{Area enclosed in actual indicator diagram}}{\text{Area enclosed in hypothetical indicator diagram}}.$$

12.6 SATURATION CURVE AND MISSING QUANTITY

In a steam engine it could be seen that the same passage acts as the passage for inlet of live steam (high pressure and high temperature steam) and exit of dead steam (low pressure and low temperature steam). During the steam admission stroke passage walls, valve face/port and cylinder walls become hotter and in exhaust stroke these surfaces become cooler due to low temperature steam passing through passages. Subsequently when high temperature steam again enters the engine cylinder then the hot steam comes in contact with cool surfaces which lead to condensation of a portion of fresh steam entering. Condensation is visible till the temperature of contact surfaces equals the hot steam temperature. Later on during exhaust stroke when cooler steam exits through valve then cold steam comes in contact with hot surfaces (surfaces are hot due to hot steam admission). Due to cold steam contact with hot surfaces reevaporation of condensed steam occurs. This reevaporation of steam slightly increases work and reduces wetness of steam. During admission of steam the condensation causes loss of steam without doing work. Condensation during intake generally causes increase in steam consumption by up to 40%.

Condensation of steam can be prevented to some extent by one or more of the following ways.

(a) Superheated steam may be supplied to engine thereby offering great margin upto the condensation state. It will allow for only reduction in degree of superheat due to contact with low temperature surfaces and thus condensation is prevented.

(b) Condensation can also be controlled by providing steam jacket around cylinder wall so as to maintain engine's contact surfaces at high temperatures thereby, preventing condensation.

(c) Valve timings can be modified so as to result in greater degree of compression prior to admission of steam. This increased compression yields increased temperature of residual steam therefore, causing increased temperature of engine surfaces. Thus during the

subsequent admission of steam condensation of admitting steam shall be limited due to less temperature difference.

(d) Condensation can be reduced by increasing the speed of engine for definite output because for particular power output higher speed shall reduce size of engine, thereby reducing the contact surface area. Hence with smaller contact surface area the condensation gets reduced.

If the multiple expansion is employed in place of single stage expansion then also condensation gets reduced as the temperature range in every expansion stage gets lowered. Smaller temperature range in any expansion causes lower temperature variations and so reduced condensation.

Thus it is seen that condensation and reevaporation are processes which shall always be there whether in large quantity or small quantity in an actual engine. Because of this condensation the actual volume occupied by steam will be less than hypothetical (theoretical) volume. This difference between the actual volume of steam and theoretical volume of steam at any point is known as 'missing quantity'. If we assume all steam states during expansion to be dry and saturated and obtain the expansion curve throughout the stroke, then such curve is called saturation curve. Figure 12.16 shows the saturation curve and missing quantity of steam.

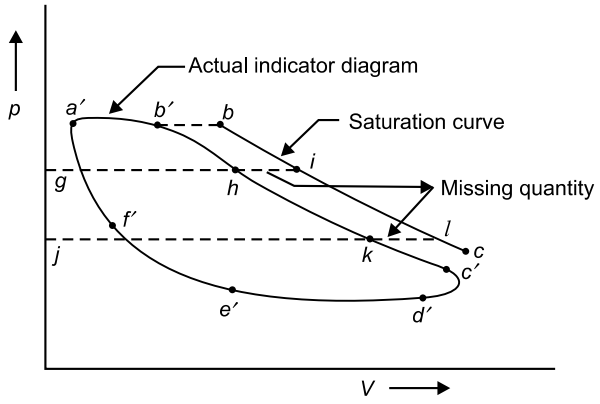


Fig. 12.16 *Saturation curve*

Here $b'c'$ shows the actual expansion curve and bc is the expansion curve considering all states of steam during expansion to be dry and saturated. At any point during expansion the amount of steam condensed can be accounted by missing quantity. At some pressure, line ghi shows the volume occupied by steam in actual engine as V_{gh} and theoretical volume occupied being V_{gi}. Length 'gh' and 'gi' indicate these volumes. Using these volume values dryness fraction neglecting leakage and other losses can be given by, $x_h = \dfrac{gh}{gi}$. Similarly dryness fraction at 'k', $x_k = \dfrac{jk}{jl}$.

During return stroke there shall occur reevaporation of expanded steam leaving cylinder and this reduces the missing quantity of steam in this stroke. This missing quantity of steam causes the loss of work as evident from p-V diagram. Loss of work can be quantified by the area between saturation curve (bc) and actual expansion curve ($b'c'$). Missing quantity can be quantified as,

At point 'h' the volume of missing quantity = 'hi' $\times$ SF_V

where SF_V is scaling factor for volume axis of indicator diagram.

Similarly at other pressure, the volume of missing quantity of steam at point 'k' = 'kl' $\times$ SF_V

12.7 HEAT BALANCE AND OTHER PERFORMANCE PARAMETERS

Heat balance for steam engine is the accounting of the total heat supplied to engine. Heat balance sheet is documentation of heat supplied to engine and the different items in which this heat supplied gets consumed. For preparing heat balance sheet the engine is run for known time at constant load and constant steam supply rate and observations taken for different parameters.

Table 12.1 *Heat balance sheet for steam engine*

Heat supplied	*Heat utilization*
1. Heat added in steam, Q_1, 100%	(a) Heat equivalent to bhp, Q_a (b) Heat lost in condensate, Q_b (c) Heat lost in cooling water, Q_c (d) Unaccounted heat lost in radiation, friction etc. $Q_d = Q_1 - (Q_a + Q_b + Q_c)$

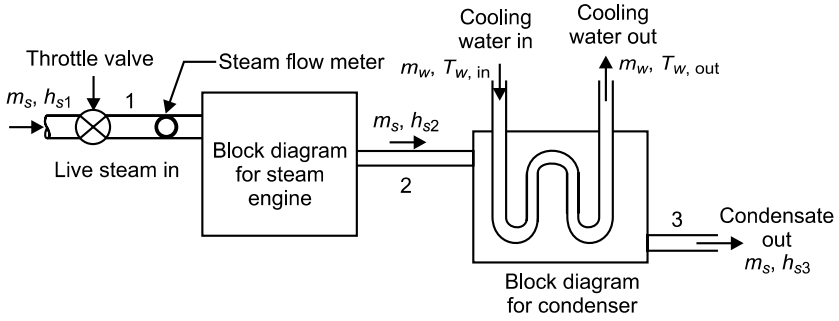


Fig. 12.17 *Schematic for trial of steam engine*

1. Heat supplied with steam; $Q_1 = m_s \cdot h_{s_1}$, where m_s is mass flow rate of steam in kg/min and h_{s1} is enthalpy of steam at inlet to engine.
   (a) *Heat equivalent to B.H.P.:* Brake power of engine is the power available at crank shaft of engine. Brake power of engine is always less than indicated power by the amount of power lost in overcoming friction in moving parts. Brake power of engine is generally measured employing dynamometers which are of many types. Power output of steam engines was determined originally by use of brake, hence the power output of steam engines was termed as brake power. This term 'brake power' has persisted since then and is now used for all kind of engines. A mechanical friction dynamometer of 'Rope brake' type is shown in Fig. 12.18.

   Schematic of rope brake dynamometer has a rope wound around the flywheel of steam engine as shown.

   Rope is hung from a spring balance suspended from rigid support. On the other end of rope a known weight 'W' is suspended. Rope wound round engine flywheel absorbs energy from flywheel. Friction between rope and flywheel can be increased by increasing the weight suspended. For the different parameters having values as denoted in Fig. 12.18 the torque acting on flywheel can be given by

   T = Net force or frictional force × Radius.

   where, Net force = $(W - S) \times g$ newton

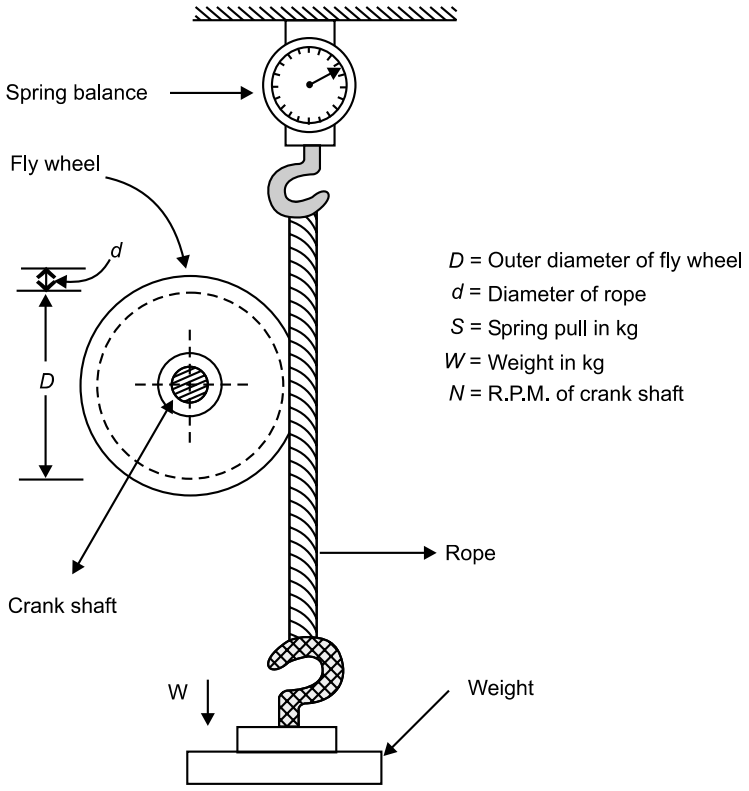


Fig. 12.18 *Schematic of rope brake dynamometer*

$$\text{Radius} = \left(\frac{D+d}{2}\right), \text{ m}$$

$$\text{Hence, torque } T = (W - S)g \cdot \left(\frac{D+d}{2}\right), \text{ N} \cdot \text{m}.$$

Power absorbed or brake power = $T \times 2\pi N$

$$\text{Brake horse power} = \left[\left\{\frac{(W-S)}{4500}\right\}\left(\frac{D+d}{2}\right)\cdot(2\pi N)\right], \text{ H.P.}$$

$$\text{Brake power in kW, BP} = \left[\left\{\frac{(W-S)\cdot g}{1000\times 60}\right\}\frac{(D+d)}{2}\cdot(2\pi N)\right], \text{ kW}$$

Heat equivalent of Brake power may be obtained by converting it from kW to kcal/min, i.e.

$$Q_a = \frac{BP \times 60}{4.18}, \text{ kcal/min}$$

(b) Heat lost in condensate, $Q_b = m_s \cdot h_{s3}$

(c) Heat lost in cooling water, $Q_c = m_w\, c_{p,w}\, (T_{w,\,\text{out}} - T_{w,\,\text{in}})$, where m_w is mass flow rate of cooling water in kg/min.

(d) Unaccounted heat lost, $Q_d = Q_1 - Q_a - Q_b - Q_c$

Other Performance Parameters: Other performance parameters of steam engine comprise of different efficiencies, steam consumption etc. For getting different efficiencies of engine the estimation of indicated power becomes necessary.

(i) ***Calculation of indicated power:*** Indicated power of steam engine refers to the power developed inside the engine. It is called indicated power because it is obtained using a mechanism called indicator diagram arrangement. Mechanism is explained along with internal combustion engines.

Indicator diagram has the actual pressure and volume variation plotted on *p-V* diagram.

For a single acting steam engine, the indicated power can be given by,

$$IP_1 = \left(\frac{p_{mep1} \cdot L \cdot A_1 \cdot N}{4500} \right), \text{ H.P.}$$

where

p_{mep1} = Mean effective pressure, i.e. $\left(\frac{A_d}{L_d} \times C \right)$, here A_d and L_d are area and length of diagram and C is indicator spring constant in N/cm^2 × cm travel.

A_1 = Piston surface area, i.e. $\left(\frac{\pi D_p^2}{4} \right)$ for bore diameter being D_p

L = Stroke length

N = Speed of rotation in r.p.m.

For a double acting engine, the indicated power shall be sum of the indicated power developed on cylinder head side and crank end side both.

Power developed on cylinder head side, $IP_1 = \left(\frac{p_{mep1} \cdot L \cdot A_1 \cdot N}{4500} \right)$, H.P.

Similarly,

Power developed on crank end side, $IP_2 = \left(\frac{p_{mep2} \cdot L \cdot A_2 \cdot N}{4500} \right)$, H.P.

Total indicated power, $\boxed{IP = IP_1 + IP_2}$

where p_{mep2} = Mean effective pressure on crank end side of piston.

A_2 = Piston surface area on crank end side i.e. (A_1 – Area of piston rod)

$= A_1 - \frac{\pi d_R^2}{4}$, where d_R is diameter of piston rod

$A_2 = \frac{\pi}{4} (D_p^2 - d_R^2)$

If piston rod area is neglected then $A_1 = A_2$ and also $p_{mep1} = p_{mep2}$

Thus in such case $IP_1 = IP_2$ and total indicated power, $IP = 2 \cdot IP_1$

Here this indicated power is expressed in horse power so may be called ‘indicated horse power’.

(ii) ***Mechanical efficiency:*** Mechanical efficiency of engine is given by the ratio of brake power to indicated power. Thus it quantifies that how effectively power developed inside cylinder can be transferred to crank shaft. Brake power i.e. power available at crank shaft is less than that developed inside cylinder due to frictional losses etc.

Mechanical efficiency, $\eta_{mech} = \dfrac{\text{Brake power}}{\text{Indicated power}}$

or, $\eta_{mech} = \dfrac{\text{Brake horse power}}{\text{Indicated horse power}}$

Also, it can be said that brake power is less than indicated power by an amount of friction power, thus

Indicated power = Brake power + Friction power

Brake power = Indicated power – Friction power

So, $\eta_{mech} = \dfrac{\text{Indicated power} - \text{Friction power}}{\text{Indicated power}}$

(iii) ***Thermal efficiency:*** Thermal efficiency is the ratio of work done to the energy supplied to the steam engine cylinder. Thermal efficiency may also be defined as the ratio of work done on brake power basis to the energy supplied to steam engine cylinder. When work done per minute is taken on IHP basis then it is called 'indicated thermal efficiency' and when work done per minute is taken on BHP basis then it is called brake thermal efficiency.'

Thus for steam flow rate of m_s, kg/min, enthalpy of steam entering being h_{s1} and enthalpy of condensate leaving to hotwell being h_{s3} the thermal efficiency may be given as,

Energy supplied to engine = $m_s\ (h_{s1} - h_{s3})$ kcal/min

Work done per minute on IHP basis = $\dfrac{\text{IHP} \times 4500}{J}$, kcal/min

Work done per minute on BHP basis = $\dfrac{\text{BHP} \times 4500}{J}$, kcal/min

Hence, $\boxed{\text{Indicated thermal efficiency, } \eta_{ith} = \dfrac{\text{IHP} \times 4500}{J \times m_s (h_{s1} - h_{s3})}}$

$\boxed{\text{Brake thermal efficiency, } \eta_{bth} = \dfrac{\text{BHP} \times 4500}{J \times m_s (h_{s1} - h_{s3})}}$

or, $\boxed{\text{Mechanical efficiency} = \dfrac{\text{Brake thermal efficiency}}{\text{Indicated thermal efficiency}}}$

(iv) ***Specific steam consumption:*** Specific steam consumption refers to steam consumption per hour per unit power. When this is defined in respect to indicated horse power then it is called specific steam consumption on IHP basis and when it is defined in respect to brake horse power then it is called specific steam consumption on BHP basis.

Specific steam consumption on IHP basis = $\dfrac{m_s \times 60}{\text{IHP}}$, kg per hour per IHP

Specific steam consumption on BHP basis = $\dfrac{m_s \times 60}{\text{BHP}}$, kg per hour per BHP

(v) ***Overall efficiency:*** This is given by the ratio of brake work (shaft work) available to the energy supplied to the boiler. Say, for m_f being mass of fuel burnt in kg/min and calorific value of fuel, being *CV*, kcal/kg. The total energy input to boiler, $Q_{\text{input}} = m_f \times CV$, kcal.

$$\text{Overall efficiency, } \eta_{\text{overall}} = \frac{\text{BHP} \times 4500}{J \times Q_{\text{input}}} = \frac{\text{BHP} \times 4500}{J \times m_f \times CV}$$

(vi) ***Rankine efficiency:*** This efficiency is given by the ratio of Rankine work available from engine to the energy supplied to steam engine.

$$\text{Rankine efficiency, } \eta_{\text{Rankine}} = \frac{(h_{s1} - h_{s2})}{(h_{s1} - h_{s3})}$$

Nomenclature is similar to that used in heat balance.

Ratio of thermal efficiency to Rankine efficiency is given as the relative efficiency.

$$\text{Relative efficiency, } \eta_{\text{Relative}} = \frac{\text{Thermal efficiency}}{\text{Rankine efficiency}}.$$

12.8 GOVERNING OF SIMPLE STEAM ENGINES

In a steam engine operating at varying loads it is desired to have some regulating mechanism so that constant speed is available at engine shaft even when the loads are varying. When load on engine is increased then the speed may get decreased and in order to have constant speed running of engine the steam supply rate should be increased. Similarly, when load on engine is decreased then the speed may get increased and in order to have constant speed of engine the steam supply rate should be decreased. This arrangement by which steam engine output is altered so as to meet fluctuating load while maintaining constant speed at engine shaft is called governing of steam engine. Generally, there are two basic principles of governing as described ahead. Governing of steam engine can be either 'throttle governing' or 'cut-off governing'.

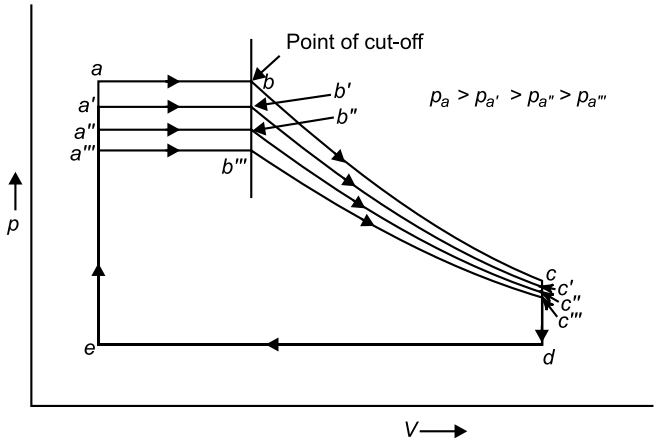


Fig. 12.19 *Effect of throttle governing shown on* p-V *diagram*

(i) ***Throttle governing:*** Throttle governing employs throttling of steam at inlet to steam engine thereby varying the output from engine. In throttle governing the cut-off point in engine is kept constant and only pressure of steam entering steam engine is altered. Throttling of steam at inlet is shown on *p-V* diagram in Fig. 12.19. In throttle governing the throttle action causes wastage of steam energy as

the available heat drop gets reduced due to it. Thermal efficiency of steam engine gets reduced due to throttling of steam at inlet to steam engine.

p-V diagram shows that at full load cycle is *abcde*. At reduced loads the throttling causes reduction in steam engine inlet pressure resulting in cycles given by *a′b′c′de*, *a″b″c″de*, *a‴b‴c‴de*. Due to throttle action the work output gets reduced as evident from modified cycles shown on *p-V* diagram. Pressure 'p_a' at full load is throttled to $p_{a'}$, $p_{a''}$, $p_{a'''}$ etc. depending upon load on engine. In throttle governing of steam engines the steam consumption is directly proportional to indicated power. Therefore, graphical pattern between steam consumption and indicated power is a straight line as shown in Fig. 12.20. Straight line variation between steam consumption and indicated power is called 'Willan's law' and straight line is called Willan's line. Willan's law is given by, $m = (S \times ip) + C$. Here '*S*' is slope of Willan's line, '*C*' is steam consumption at zero indicated power and '*ip*' is indicated power at any point.

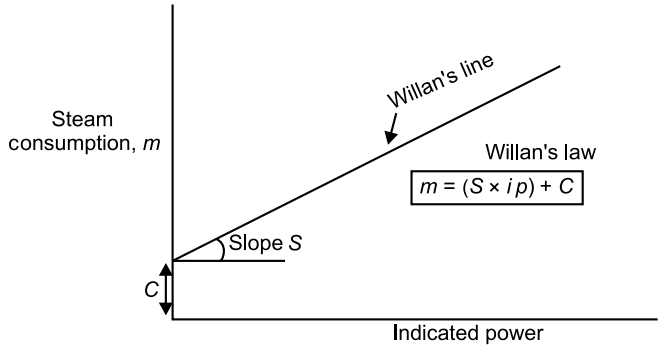


Fig. 12.20 *Steam consumption* vs. *indicated power, Willan's law*

(ii) ***Cut-off governing:*** In cut-off governing the regulation is made by varying the point of cut-off. Cut-off point is modified and thus the steam engine output is altered due to alteration in period of admission of steam. Principle behind cut-off governing is to change the quantity of steam admitted by changing the cut-off point depending on the load on engine. For full load the cut-off point is located such that the maximum amount of steam is admitted. Here the inlet pressure and back pressure do not change due to governing. Figure 12.21 shows the cut-off governing on *p-V* diagram.

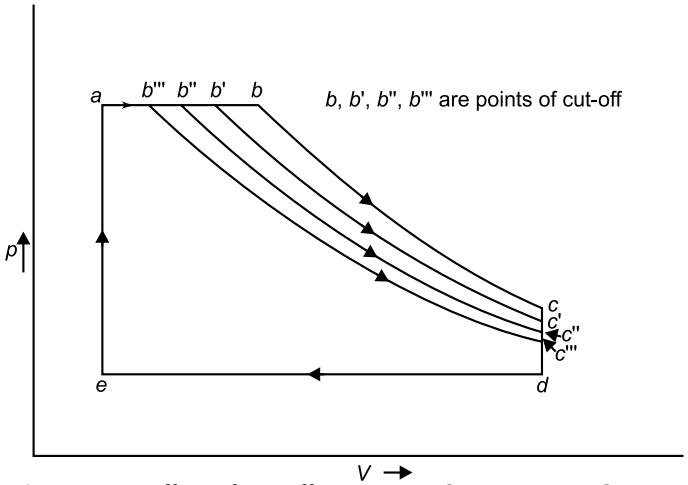


Fig. 12.21 *Effect of cut off governing shown on* p-V *diagram*

P-V diagram shows that at full load the cycle *abcde* gives the maximum output which varies upon change in cut-off point to b', b'', b''' etc. When cut off points are changed then it results in modified outputs from engine as given by area enclosed in $ab'c'de$, $ab''c''de$, $ab'''c'''de$ etc.

Cutoff governing employs special kind of valve which has provision for changing the cut-off point. This may be realized by employing Mayer's expansion valve or suitable linkages for getting the changed cut off points. Cut-off governing is successfully used in steam engine based locomotive where torque available at crank shaft should match with load in the train. Cut-off governing is efficient and economical as compared to throttle governing due to optimal steam utilization.

12.9 COMPOUND STEAM ENGINE

Performance of steam engine can be increased by increasing the pressure and temperature of steam entering the steam engine and lowering the back pressure. Since the lowering of back pressure is limited by the atmospheric pressure in case of non condensing engine and by the condenser pressure in case of condensing engines. Therefore, increasing pressure and temperature of steam at inlet to steam engine can be used for improving efficiency of steam engine. But the high pressure at inlet and low back pressure result into number of practical difficulties such as; the size of cylinder has to be of large volume and should have heavy structure in order to bear with large pressure difference, large volume after expansion and higher mechanical stresses, tendency of condensation loss and leakage loss gets increased, large temperature difference causes excessive thermal stresses, heavy structure poses the difficulty in balancing etc.

In order to overcome these difficulties instead of realizing expansion in single steam engine more than one steam engine cylinders are employed for achieving total expansion from high inlet pressure to low back pressure. Thus for any given expansion ratio (i.e. ratio of high pressure to low pressure) total expansion occurs in more than one engine cylinders successively. The arrangement in which expansion occurs in two or more engine cylinders successively is called compound steam engine.

Compounding is thus the phenomenon of realizing expansion in more than one steam engine cylinders in succession.

In case of two cylinder engines first cylinder is called high pressure cylinder (HP cylinder) and second cylinder is called low pressure cylinder (LP cylinder). In case of three cylinders the engine cylinders shall be called as high pressure (HP), intermediate pressure (IP) and low pressure (LP) cylinders.

12.10 METHODS OF COMPOUNDING

Compounding of steam engines can be done by combining more than one cylinders of steam engines such that expansion of steam occurs successively in these cylinders. There are basically two approaches of compounding, namely 'Tandem type' and 'Cross type'. Cross type compounding can be of 'Woolf type' and 'Receiver type'. Thus compound steam engines can be classified as below based on method of compounding.

(i) Tandem compound engines
(ii) Woolf compound engines } Cross type compounding.
(iii) Receiver compound engines

Arrangement in each type of compounding is described ahead.

(i) ***Tandem compound engines:*** Tandem type compounding of steam engines has the in line cylinders having pistons mounted on the same piston rod which is further having crosshead and connecting rod providing power output at crankshaft. Figure 12.22 shows schematic of tandem compound

steam engine having two in line cylinders. Steam at high pressure first enters the HP cylinder and after getting partially expanded enters the LP cylinders. Two pistons of HP and LP cylinders are mounted such that the two cylinders produce power output without any phase difference. Maximum output and minimum output from both cylinders is available at same crank angle position and output is of additive type i.e. maximum torque from compound engine.

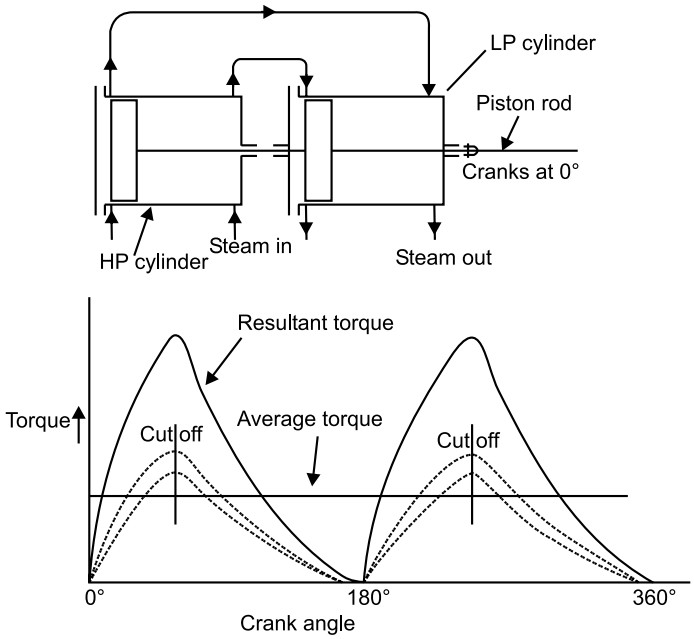


Fig. 12.22 *Tandem compound engine*

Thus there exists very large variation of torque during a cycle and for compensating this large variation heavy flywheel and large balancing weights are required.

(ii) ***Woolf compound engine:*** It is a cross type compounding having two cylinders having pistons at 180° phase difference i.e. at some position one cylinder may have piston at inner dead centre and other cylinder has piston at outer dead centre. The cylinders are arranged parallel to one another and steam leaving HP cylinder enters into LP cylinder directly thereby expansion remains continuous throughout the stroke. Since the power output from two cylinders is available with phase difference of 180° so the torque variation is very large similar to that in case of tandem compounding. Therefore, in woolf compound engine also large flywheel is required.

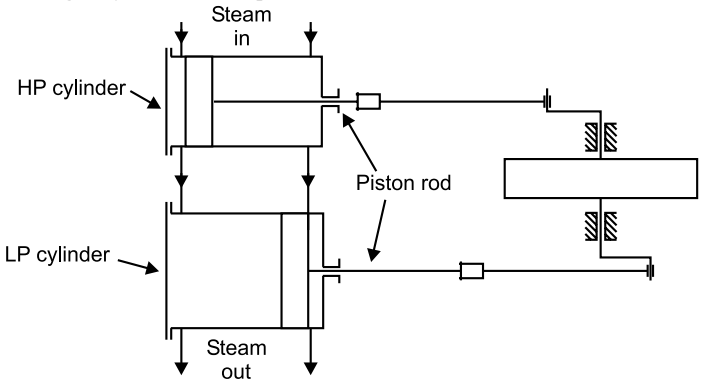


Fig. 12.23 *Woolf compound engine.*

(iii) ***Receiver compound engine:*** Receiver compound engine is also a cross compound engine having two cylinders with out of phase pistons and receiver in between. In this the cranks for two cylinders are at 90° as shown in Fig. 12.24. Here steam leaving HP cylinder passes into receiver and subsequently at correct time steam enters LP cylinder. Receiver is required because the two cylinders have out of phase pistons and exhaust from HP does not go directly into LP cylinder, so some kind of storage is a necessity. The maximum torque and minimum torques are not at the same time so the torque variation is not too much in this compounding. Hence the size of flywheel required is smaller in this arrangement. Also balancing requirement is not too much in case of receiver compounding.

Since steam is stored in reservoir so there are chances of pressure drop and heat loss taking place in receiver. These losses reduce efficiency of receiver compound engine and so require some jacketing/ steam heating of receiver to prevent heat loss.

Advantages of compounding: Compounding of steam engine offers number of advantages over the simple steam engine as given below:

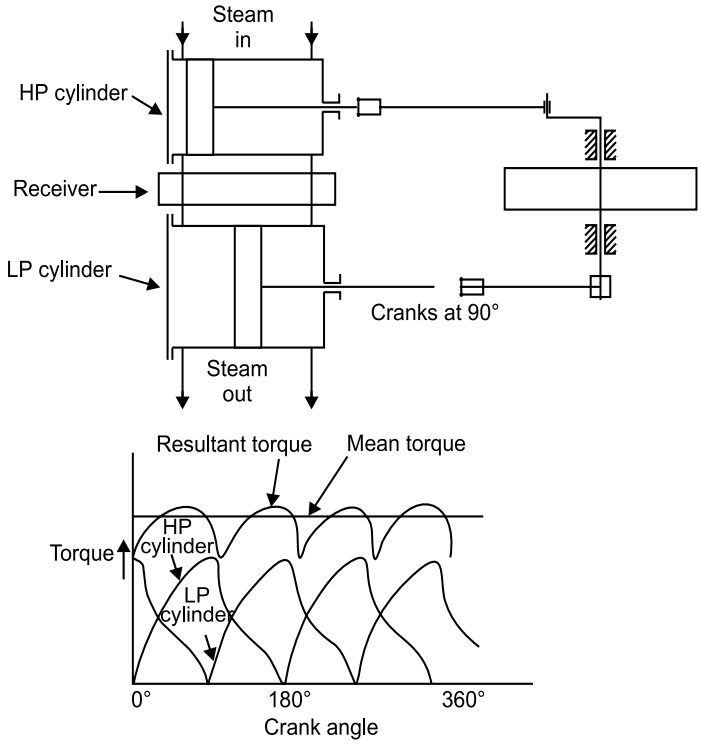


Fig. 12.24 *Receiver compound engine*

(a) Thermal stresses and phenomenon of condensation and reevaporation is reduced due to lesser temperature variation in different steam engine cylinders.
(b) Loss of work due to condensation of steam is reduced as the condensation and reevaporation is utilised in cylinders subsequent to first cylinder. Loss due to condensation is only there with last cylinder.
(c) Due to reduced pressure difference the chances of leakage are reduced.
(d) Due to expansion occurring in parts in different cylinders the reheating can be employed, if desired to control condensation of steam.

(e) In compound steam engine the HP cylinder is fed with high pressure steam so the sturdy structure requirement is there only for HP cylinder. LP cylinders need not be of very sturdy type due to low pressure steam entering it although volume of LP cylinder has to be large. Due to lighter reciprocating parts the vibrations are also minimized.
(f) Compound steam engine is capable of getting started at any crank position due to phase difference between different cylinders.
(g) Compound steam engines are better balanced and more uniform torque is available, thus requiring light flywheel.
(h) Due to multi cylinder engine the reliability of compound steam engine is better as in the event of failure of even one cylinder the reduced power will be available from other cylinder.
(i) Compound steam engines are cheaper than simple steam engine for same power output and efficiency.
(j) Steam economy is increased by 10 to 25% at rated load for non-condensing engine and by 15 to 40% for condensing engine.

Disadvantages of compound steam engine: Disadvantages with compounding are as under:

(a) Compound steam engine requires more maintenance and attention due to complexity.
(b) Lubrication requirements are excessive because of increased wear and tear in engine parts.
(c) Heat losses are more due to large surface area of compound engines.
(d) Compound steam engines are bulky and require large space.

12.11 INDICATOR DIAGRAM FOR COMPOUND STEAM ENGINE

Combined indicator diagram for the compound steam engine having HP and LP cylinder can be obtained similar to that in simple steam engine. Figure 12.25 shows the actual indicator diagram for compound steam engine. For getting the actual indicator diagram the indicator diagram arrangement is used separately and subsequently the diagrams for two cylinders are combined together.

Actually the indicator diagrams obtained from respective cylinders shall be on separate *p*-*V* scales with different spring constants, different masses of cushion steam etc. Combined indicator diagrams are obtained after getting the average indicator diagrams for HP and LP cylinders (both sides in case of double acting) being reduced to common pressure and volume scales. These individual indicator diagrams are plotted with definite clearance volumes in respective cylinders. Saturation curves shall be different for HP and LP cylinders because of different masses of cushion steam in clearance volume, pressure drop during transfer of steam from HP to LP and condensation etc. Hypothetical diagram shown has been drawn with common saturation curve as it has been drawn assuming zero clearance volume, no pressure loss and no condensation etc. In hypothetical indicator diagrams for HP and LP when combined show loss of work due to unresisted expansion at interface of HP and LP cylinders, as shown by hatched area on *p*-*V* diagram Fig. 12.25 (a). Hypothetical indicator diagram drawn on common scale is generally used for all calculations. Combined indicator diagram shown has HP cylinder having steam entering at p_1, getting expanded up to p_2 and exhausted at p_3 for entering into LP cylinder, getting expanded upto p_4 and finally exhausted at pressure p_5 from LP cylinder. Hypothetical cycle for HP cylinder is shown by '*abhog*' and for LP cylinder is given by '*gcdef*'.

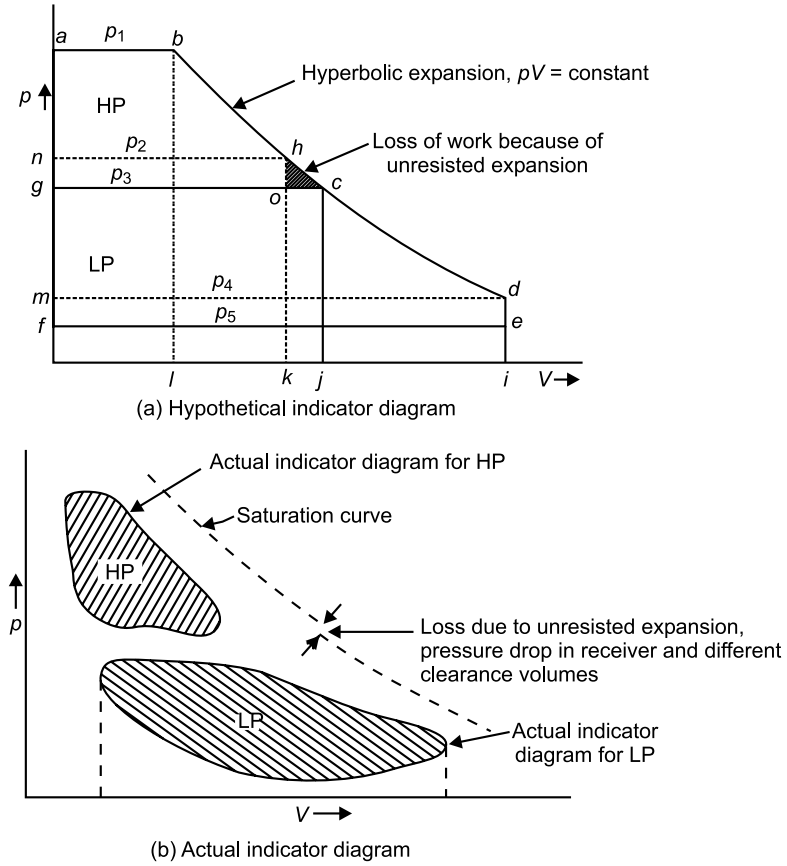


Fig. 12.25 *Indicator diagram for compound steam engine*

12.12 CALCULATIONS FOR COMPOUND STEAM ENGINES

Compound steam engine cylinders may be designed based upon the assumption of 'equal initial piston loads on all pistons' or 'equal power developed in each cylinder for getting uniform torque' or 'equal temperature drop in every cylinder for optimal use of steam.

The hypothetical work output (indicator work) for HP and LP cylinders can be given as below;

Hypothetical indicator work from HP cylinder

$$= [p_1 \, V_b \, (1 + \ln r_{HP}) - p_3 \cdot V_h]$$

Actual indicator work from HP cylinder

$$= [p_1 \, V_b \cdot (1 + \ln r_{HP}) - p_3 \cdot V_h] \cdot df_{HP}$$

where r_{HP} and df_{HP} are expansion ratio and diagram factor for HP cylinder. So, $r_{HP} = \dfrac{V_h}{V_b}$

Hypothetical indicator work from LP cylinder

$$= [p_3 \, V_c \, (1 + \ln r_{LP}) - p_5 \cdot V_d]$$

Actual indicator work from LP cylinder

$$= [p_3\ V_c\ (1 + \ln\ r_{LP}) - p_5 \cdot V_d] \cdot df_{LP}$$

where r_{LP} and df_{LP} are expansion ratio and diagram factor for LP cylinder. Here $r_{LP} = \frac{V_d}{V_c}$.

Total actual indicator work from compound steam engine

$$= [p_1\ V_b\ (\ 1 + \ln\ r_{HP}) - p_3 \cdot V_h] \cdot df_{HP} + [p_3\ V_c\ (1 + \ln\ r_{LP}) - p_5 \cdot V_d] \cdot df_{LP}$$

Expansion ratio and diagram factors for two cylinders may be same or different.

Loss of work due to unresisted expansion can be quantified by the difference between total hypothetical output and actual output from compound engine

Loss of work = Area of total hypothetical diagram – (Area of hypothetical HP diagram + Area of hypothetical LP diagram)

$$= A_{abdef} - (A_{abhog} + A_{gcdef})$$

Mean effective pressures of both cylinders may be given separately or with reference to common cylinder.

Mean effective pressure of HP cylinder

$$= \frac{\text{Area of indicator diagram for HP cylinder}}{\text{Stroke volume for HP cylinder i.e. length of diagram for HP}}$$

$$mep_{HP} = \frac{W_{HP}}{\Delta V_{HP}}$$

Similarly,

Mean effective pressure of LP cylinder

$$= \frac{\text{Area of indicator diagram for LP cylinder}}{\text{Stroke volume for LP cylinder i.e. length of diagram for LP}}$$

$$mep_{LP} = \frac{W_{LP}}{\Delta V_{LP}}$$

Mean effective pressures can be mathematically given as below,

$$\boxed{mep_{HP} = \frac{p_1 (1 + \ln\ r_{HP})}{r_{HP}} - p_3}$$

$$\boxed{mep_{LP} = \frac{p_3 (1 + \ln\ r_{LP})}{r_{LP}} - p_5}$$

In case of multi cylinder engines as in case of compound steam engines mean effective pressures when defined separately offer difficulty in comparing them. Therefore, mean effective pressure is defined in reference to LP cylinder, which means it is assumed that the stroke volume of LP cylinder is used for all cylinders and the mean effective pressure values can be directly used for comparing work output from respective cylinders. In case of compound engine having HP and LP cylinders the mean effective pressure of HP cylinder referred to LP cylinder can be given as below.

Mean effective pressure of HP referred to LP cylinder,

$$mep_{\text{HP ref LP}} = \frac{mep_{\text{HP}} \times \text{Stroke volume of HP cylinder}}{\text{Stroke volume of LP cylinder}}$$

or, $$\boxed{mep_{\text{HP ref LP}} = \frac{mep_{\text{HP}} \times \Delta V_{\text{HP}}}{\Delta V_{\text{LP}}}}$$

Mathematical conditions for different basis of compounding steam engine cylinders can be given as below.

(a) For equal initial piston loads on all pistons the loads on both HP and LP cylinder pistons can be equated. Mathematically,
Load on HP piston = Load on LP piston

$$\boxed{(p_1 - p_3) \cdot A_{\text{HP}} = (p_3 - p_5)\, A_{\text{LP}}}$$

where A_{HP} and A_{LP} are cross section areas of HP and LP piston respectively

(b) For equal power developed in each cylinder the power developed in HP and LP cylinders can be given as,
Power developed in HP cylinder = Power developed in LP cylinder
Work output in HP cylinder = Work output in LP cylinder

$$mep_{\text{HP}} \times \Delta V_{\text{HP}} = mep_{\text{LP}} \times \Delta V_{\text{LP}}$$

or

$$mep_{\text{LP}} = \frac{mep_{\text{HP}} \times \Delta V_{\text{HP}}}{\Delta V_{\text{LP}}}$$

$$\boxed{mep_{\text{LP}} = mep_{\text{HP ref. LP}}}$$

In case of actual indicator diagram of compound steam engine,
(Mean effective pressure of combined diagram) $\times\, df_{\text{combined}}$

$$= (mep_{\text{HP ref. LP}} \times df_{\text{HP}}) + (mep_{\text{LP}} \times df_{\text{LP}})$$

or
Mean effective pressure of combined diagram

$$= \frac{\left(mep_{\text{HP ref LP}} \times df_{\text{HP}}\right) + (mep_{\text{LP}} \times df_{\text{LP}})}{df_{\text{combined}}}$$

where '*df*' refers to diagram factor.

12.13 GOVERNING OF COMPOUND STEAM ENGINE

Governing of compound steam engines is done based on the principles similar to that of simple steam engine i.e. throttle governing and cut-off governing. Throttle governing is based on qualitative regulation while cut-off governing is based on quantitative regulation of steam.

(i) ***Throttle governing:*** Throttle governing of compound steam engine is done by throttling steam before being admitted to HP cylinder, thereby yielding reduced output from both HP and LP cylinders. In fact throttling reduces the heat potential of steam due to its throttling at constant heat, thereby lowering capacity of producing work by throttled steam. Figure 12.26 shows throttle governing being used in compound steam engine. Reduction in steam inlet pressure at inlet of HP cylinder without varying cut-

off point alters the discharge pressure from HP and subsequently steam with reduced enthalpy enters LP cylinder. Thus total output from engine is reduced because of less output available from both HP and LP cylinders. Throttle governing involves wastage of steam.

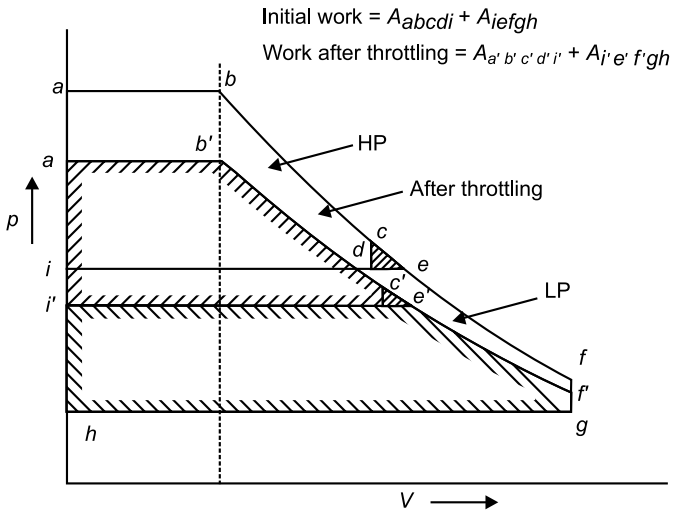


Fig. 12.26 *Throttle governing of compound steam engine*

(ii) ***Cut-off governing:*** In cut-off governing the point of cut-off of steam admission is modified in HP cylinder thereby modifying quantity of steam admitted. Here in cut-off governing the work producing capacity is not modified instead its quantity available for doing work in engine is modified. Figure 12.27 shows the cut-off governing. Mayer's expansion valve is generally used for altering the cut-off point in HP cylinder. This expansion valve is operated independently depending upon load on engine. Work output from both HP and LP cylinders is modified due to change in cut-off point. However, change in LP cylinder work is much more than change in HP cylinder work due to varying cut off point. This offers unbalance in work done by HP and LP cylinders.

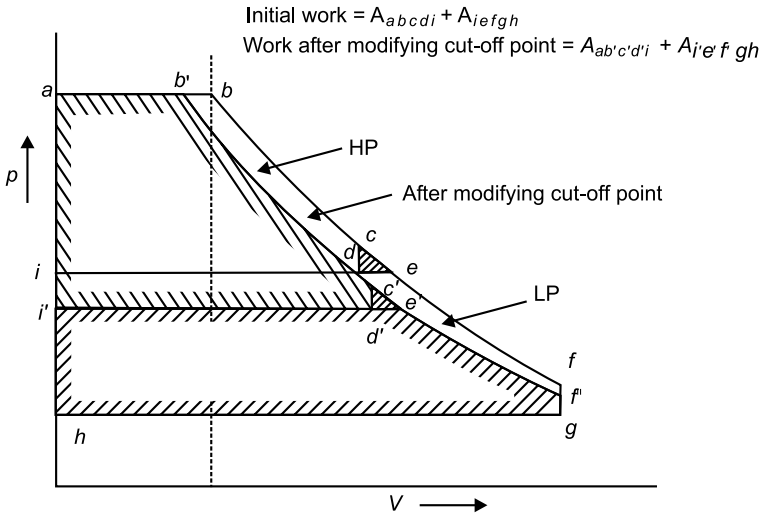


Fig. 12.27 *Cut-off governing of compound steam engine*

Therefore, in order to have balance in the work outputs from HP and LP cylinders i.e. maintaining the right proportion of works from two cylinders for getting balance, the cut off point is changed in both HP and LP cylinders. Variation of cut off point in LP and HP both is done not to change total work output but to change the proportion of work from two. Thus in some cases where unbalance in work is not problem the cut-off governing is employed for only HP cylinder while when unbalance is serious problem then cut-off governing is employed for both HP and LP cylinders.

12.14 UNIFLOW ENGINE

Uniflow engines are also called as straight flow engines. Uniflow engines have flow of steam through inlet and exit ports and passages in one direction only. Generally in a steam engine same port/passage is for inlet of steam and exit of steam from cylinder thereby causing severe problem of condensation and reevaporation. In uniflow engine steam enters at the ends of cylinder and leaves out as exhaust from centre. Thus the hot steam does not come in contact with low temperature parts and cold steam does not come in contact with hot parts of engine, similar to compound steam engine. Uniflow engines offer higher diagram factor in the absence of condensation and receiver loss. Uniflow engines of horizontal type are available in sizes up to 1200 hp while vertical type uniflow engines are available in sizes up to 7000 hp.

Figure 12.28 shows the schematic of uniflow engine in its two sectional views. It has mechanically operated valves at two ends of cylinder for admitting steam and exhaust port is located at centre of cylinder. Let us consider piston to be on inner dead centre and inlet valve gets opened. Thus live steam (high pressure and temperature steam) enters cylinder causing piston displacement towards crank end. Piston movement is caused by pushing of piston by steam till cut-off point and subsequent expansion of steam till exhaust port gets uncovered. Piston subsequently reaches to the outer dead centre when inlet valve at this end gets opened thereby causing inlet of live steam. Thus again piston moves towards inner dead centre partly due to push of live steam and partly due to its expansion. Again in this stroke exhaust shall occur as the exhaust ports get uncovered. Hence it is seen that there is straight flow of steam, eliminating problem of condensation. Here exhaust stroke is very small compared to stroke length in cylinder. Again piston reaches inner dead centre with cushion steam in clearance volume. This cushion steam gets compressed in clearance volume before inlet of live steam. Thus temperature of cushion steam gets increased before it comes in contact with live steam and condensation is regulated.

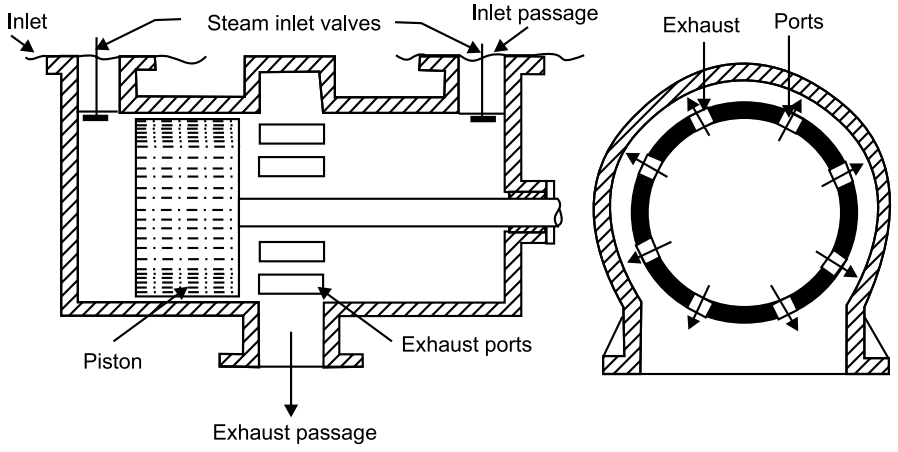


Fig. 12.28 *Sectional view of uniflow engine.*

Live steam again enters cylinder upon opening of inlet valve causing piston to move toward outer dead centre and thus engine operates.

Uniflow engine offers advantages of 'reduced condensation of steam', 'less specific steam consumption and so higher thermal efficiency', 'simple and ease of design and fabrication' and 'reduced friction loss due to elimination of D-slide or other complex valves' etc. Disadvantages of the engine are, 'large size flywheel requirement to smoothen torque fluctuations compared to compound engine', 'poor mechanical balancing of engine', 'robustness of cylinder' and 'reduced output because of early compression of steam' etc.

EXAMPLES

1. *A steam engine operates with steam being supplied at 0.2 MPa, 250°C and expanding upto 0.3 bar. Steam is finally released out at 0.05 bar. Determine the modified Rankine cycle efficiency and compare it with the efficiency of Carnot cycle operating between given limits of pressure. Neglect pump work.*

Solution:

Here states 1, 2, and 3 refer to cut-off point, state at the end of expansion and after release from engine.

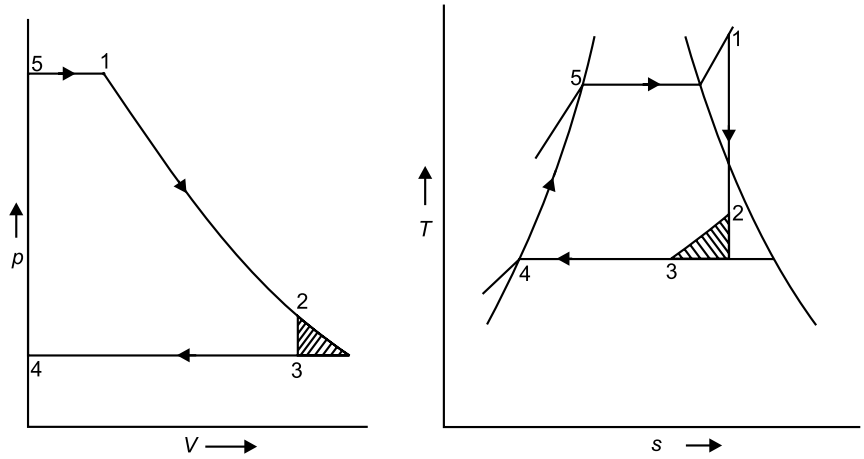


Fig. 12.29 p-V *and* T-s *representation for modified Rankine cycle*

At inlet to engine, from steam tables,

$h_1 = 2971$ kJ/kg

$s_1 = 7.7086$ kJ/kg·K

Let dryness fraction at 2 be x_2.

$s_1 = s_2$;

$s_2 = 7.7086 = s_{f \text{ at } 0.3 \text{ bar}} + x_2 \cdot s_{fg \text{ at } 0.3 \text{ bar}}$

$\Rightarrow \quad x_2 = 0.99$

$v_2 = v_{f \text{ at } 0.3 \text{ bar}} + x_2 \cdot v_{fg \text{ at } 0.3 \text{ bar}} = 5.1767 \text{ m}^3/\text{kg}$

$h_2 = 2601.97$ kJ/kg

Work output from engine cycle per kg of steam = $(h_1 - h_2) + v_2(p_2 - p_3)$

$= (2971 - 2601.97) + 5.1767\ (0.3 - 0.05) \times 10^2$

= 511.95 kJ/kg

Heat input per kg of steam = $h_1 - h_{f \text{ at } 0.05 \text{ bar}}$

= 2971 – 137.82 = 2833.18 kJ/kg

Efficiency of modified Rankine cycle = $\frac{511.95}{2833.18}$ = 0.1807

= 18.07 %

In order to find out Carnot cycle efficiency let us find out saturation temperature corresponding to 0.2 MPa and 0.05 bar.

$T_{max} = T_{sat \text{ at } 0.2 \text{ MPa}}$ = 120.23°C or 393.23 K

$T_{min} = T_{sat \text{ at } 0.05 \text{ bar}}$ = 32.88°C or 305.88 K

Carnot cycle efficiency = $1 - \frac{T_{min}}{T_{max}}$ = 0.2221 or 22.21%

Modified Rankine cycle efficiency = 18.07%,
Carnot efficiency = 22.21% ***Ans.***

2. *A steam engine has steam being supplied at 10 bar, 100°C into a cylinder of 30 cm diameter and stroke to bore ratio of 2. Steam is expanded upto 0.75 bar and then released into condenser at 0.25 bar. Determine (i) the modified Rankine efficiency and (ii) the modified stroke length considering that the same steam is expanded from inlet state to the condenser pressure. Neglect pump work.*

Solution:

States 1, 2 and 3 refer to point of cut off, end of expansion and release.

At 10 bar, 100°C

h_1 = 2676.2 kJ/kg

From steam table,

s_1 = 7.3614 kJ/kg · K

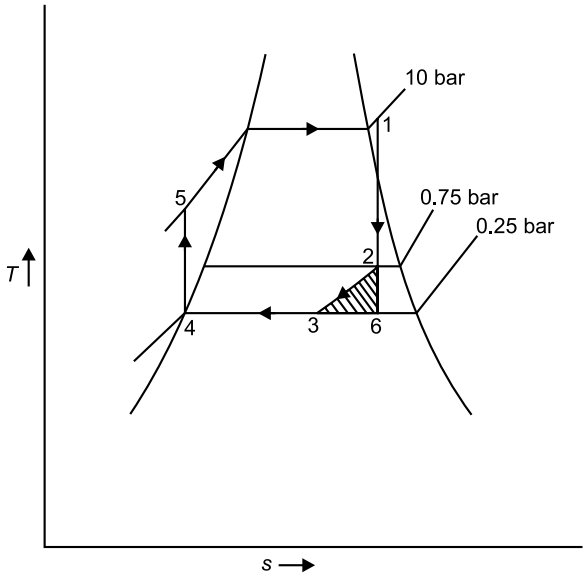


Fig. 12.30

At state 2, $s_1 = s_2$

$$s_2 = 7.3614 = s_{f \text{ at } 0.75 \text{ bar}} + x_2 \cdot s_{fg \text{ at } 0.75 \text{ bar}}$$

$$\Rightarrow \quad x_2 = 0.9848$$

$$v_2 = v_{f \text{ at } 0.75 \text{ bar}} + x_2 \cdot v_{fg \text{ at } 0.75 \text{ bar}}$$

$$v_2 = 2.1833 \text{ m}^3/\text{kg}$$

$$h_2 = h_{f \text{ at } 0.75 \text{ bar}} + x_2 \cdot h_{fg \text{ at } 0.75 \text{ bar}}$$

$$h_2 = 2628.35 \text{ kJ/kg}$$

$$h_4 = h_{f \text{ at } 0.25 \text{ bar}} = 271.93 \text{ kJ/kg}$$

Work output from steam engine per kg of steam,

$$w_{net} = (h_1 - h_2) + v_2(0.75 - 0.25) \times 10^2$$
$$= 157.015 \text{ kJ/kg}$$

Heat added, $q_{add} = (h_1 - h_4) = 2676.2 - 271.93 = 2404.27$ kJ/kg

Efficiency of modified Rankine cycle $\eta = \dfrac{w_{net}}{q_{add}} = 0.0653$ or **6.53%**

Let us now consider the expansion to occur up to condenser pressure i.e. state 6

$$s_1 = s_2 = s_6 = 7.3614 \text{ kJ/kg} \cdot \text{K}$$

$$s_6 = 7.3614 = s_{f \text{ at } 0.25 \text{ bar}} + x_6 \cdot s_{fg \text{ at } 0.25 \text{ bar}}$$

$$\Rightarrow \quad x_6 = 0.9323$$

$$h_6 = h_{f \text{ at } 0.25 \text{ bar}} + x_6 \cdot h_{fg \text{ at } 0.25 \text{ bar}} = 2459.38 \text{ kJ/kg}$$

$$v_6 = v_{f \text{ at } 0.25 \text{ bar}} + x_6 \cdot v_{fg \text{ at } 0.25 \text{ bar}} = 5.784 \text{ m}^3/\text{kg}$$

Given volume of cylinder, $V = \dfrac{\pi}{4} \cdot d^2 \cdot L = \dfrac{\pi}{4} \times (0.3)^2 \times 0.6 = 0.0424$

In a stroke the mass of steam at 2, $m = \dfrac{V}{v_2} = 0.0194$ kg

Let the modified stroke length be L' *m*.

Volume requirement at 6, $V' = m \cdot v_6 = 0.0194 \times 5.784$

$V' = 0.1122 \text{ m}^3$

or $\quad V' = \dfrac{\pi}{4} d^2 L' = 0.1122$

$\Rightarrow \quad L' = 1.5873$ m or 158.73 cm

Modified Rankine cycle efficiency = 6.53%
New stroke length = 158.73 cm

Ans.

3. *A double acting steam engine has bore of 30 cm and stroke to bore ratio of 2 with cut-off occurring at 40% of stroke. Steam enters the engine cylinder at 7.5 bar and exhausts at 0.1 bar. Engine runs at 180 rpm. Neglecting clearance volume and considering diagram factor of 0.6 determine the indicated horse power.*

Solution:

Cut off occurs at 40% of stroke, so $0.4 = \dfrac{V_1}{V_2}$

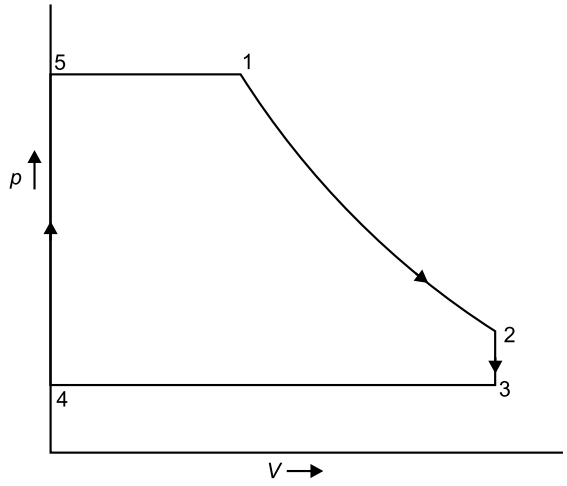


Fig. 12.31 *Indicator diagram (hypothetical)*

Expansion ratio, $r = \frac{V_2}{V_1} = \frac{1}{0.4} = 2.5$

Hypothetical mean effective pressure

$$mep = \frac{p_1}{r}(1 + \ln r) - p_3$$

$$= \left(\frac{7.5}{2.5}\right)(1 + \ln 2.5) - 0.1$$

$mep = 5.65$ bar

Actual mean effective pressure = $mep \times$ diagram factor

$= 5.65 \times 0.6$

$= 3.39$ bar

$$\text{Indicated power} = \frac{3.39 \times 0.6 \times \pi \times 0.09 \times 2 \times 180 \times 10^2}{4 \times 60},$$

(1 hp = 0.7457 kW)

= 86.26 kW or 115.67 hp

Indicated power = 115.67 hp ***Ans.***

4. *In a single acting steam engine, steam is admitted at 15 bar, 200°C and exhausts at 0.75 bar with cut-off occurring at 25% of stroke. Engine produces 150 hp at 240 rpm. The mechanical efficiency of engine is 85%, diagram factor is 0.7, brake thermal efficiency is 20% and stroke to bore ratio is 1.5. Determine the specific steam consumption and cylinder dimensions. Neglect the cross-section area of piston rod and clearance volume.*

Solution:

Neglecting clearance volume,

Expansion ratio = $\frac{1}{0.25} = 4$

$$P_1 = 15 \text{ bar}, P_3 = 0.75 \text{ bar},$$

$$mep_{\text{hypothetical}} = \frac{P_1}{r}(1 + \ln r) - P_3$$

$$= \frac{15(1+\ln 4)}{4} - 0.75$$

$$mep_{\text{hypothetical}} = 8.19 \text{ bar}$$

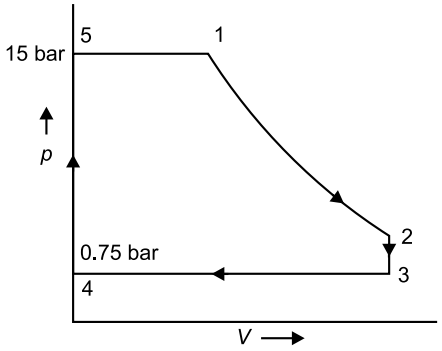


Fig. 12.32

Actual mean effective pressure = $8.19 \times 0.7 = 5.733$ bar
Indicated horse power = Brake horse power/η_{mech} = 176.47 hp
Let diameter of cylinder be 'd', Stroke = (1.5 d), {1 hp = 0.7457 kW}

$$\text{Indicated horse power} = \frac{mep \times L \times A \times N}{60 \times 0.7457}$$

$$176.47 = \frac{5.733 \times 10^2 \times 1.5d \times \pi \times d^2 \times 240}{4 \times 60 \times 0.7457}$$

$$\Rightarrow \quad d = 0.3652 \text{ m}$$

$$L = 1.5 \times 0.3652 = 0.5478 \text{ m}$$

Ans. Bore = 36.52 cm
Stroke = 54.78 cm

Heat added per kg of steam = $h_{15 \text{ bar}, 200^\circ\text{C}} - h_{f \text{ at } 0.75 \text{ bar}}$
$q_{\text{add}} = 2803.3 - 384.39 = 2418.91$ kJ/kg
Let specific steam consumption be m kg/hp. hr
So

$$\text{Brake thermal efficiency} = \frac{1 \text{hp brake output}}{m \times q_{\text{add}}}$$

$$0.2 = \frac{1 \times 0.7457 \times 3600}{m \times 2418.91}$$

$$\Rightarrow \quad m = 5.55 \text{ kg/hp. hr}$$

Specific steam consumption = 5.55 kg/hp. hr ***Ans.***

5. *A single cylinder steam engine has steam consumption of 18 kg/min and yields indicated power of 100 kW while running at 240 rpm. Engine cylinder has bore of 30 cm and stroke of 40 cm. Steam is admitted at 10 bar, 200°C and exhausted at 0.75 bar with cut-off occurring at 25% of stroke. Determine the diagram factor and indicated thermal efficiency. Neglect clearance volume.*

Solution:

p_1 = 10 bar, p_3 = 0.75 bar

Enthalpy at inlet to cylinder,

At 10 bar, 200°C, h_1 = 2875.3 kJ/kg

At 0.75 bar, h_f = 384.39 kJ/kg

Heat added per kg of steam,

$$q_{add} = h_1 - h_{f \text{ at } 0.75 \text{ bar}}$$

$$q_{add} = 2490.91 \text{ kJ/kg}$$

$$\text{Expansion ratio, } r = \frac{1}{0.25} = 4$$

$$\text{Hypothetical } mep = \frac{p_1}{r}(1 + \ln r) - p_3 = 5.215 \text{ bar}$$

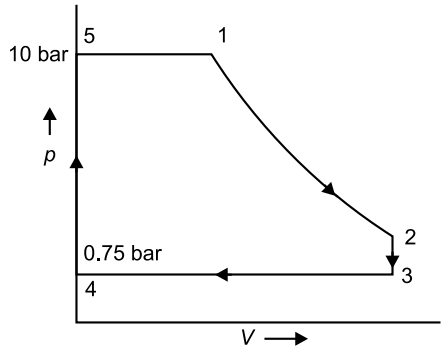


Fig. 12.33

Theoretically,

$$\text{Indicated power output} = \frac{mep \times L \times A \times N}{60}$$

$$= \frac{5.215 \times 0.40 \times \pi \times (0.3)^2 \times 240 \times 10^2}{60}$$

$$= 235.92 \text{ kW or } 316.37 \text{ hp.}$$

$$\text{Diagram factor} = \frac{\text{Actual indicated power}}{\text{Theoretical indicated power}} = \frac{100}{235.92} = 0.4238$$

$$\text{Indicated thermal efficiency} = \frac{\text{Indicated work}}{\text{Heat added}}$$

Here, steam supplied is 18 kg/min or 0.3 kg/s.

$$\text{Indicated thermal efficiency} = \frac{235.92}{m \times q_{add}} = \frac{235.92}{0.3 \times 2490.91}$$
$$= 0.3157 \text{ or } 31.57\%$$

Diagram factor = 0.4238
Indicated thermal efficiency = 31.57%

Ans.

6. *A steam engine has steam being supplied at 10 bar, 0.9 dry and exhausted at 1 bar. Cut-off occurs at 60% of stroke. Determine*

(i) the fraction of work obtained by expansion working of engine.

(ii) the efficiency of cycle

(iii) the % increase in work and efficiency when engine works as non-expansive engine. Neglect clearance volume and assume expansion to be hyperbolic.

Solution:

Let us analyze for 1 kg of steam.

At inlet to engine cylinder,

$h_1 = h_{f \text{ at } 10 \text{ bar}} + 0.9 \times h_{fg \text{ at } 10 \text{ bar}}$

$h_1 = 2576.58$ kJ/kg, $v_1 = v_{f \text{ at } 10 \text{ bar}} + 0.9 \times v_{fg \text{ at } 10 \text{ bar}}$

$v_1 = 0.1751$ m^3/kg

At 1 bar, $h_f = 417.46$ kJ/kg

Heat added per kg of steam, $q_{add} = h_1 - h_{f \text{ at } 1 \text{ bar}}$

$q_{add} = 2159.12$ kJ/kg

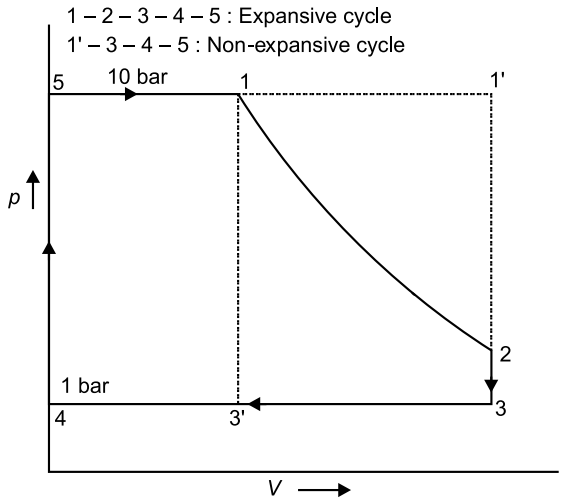


Fig. 12.34

For given cut-off volume, $v_1 = 0.6 \times v_2$

or $v_2 = 0.2918$ m^3/kg, Expansion ratio, $r = \frac{1}{0.6} = \frac{5}{3}$

Non expansive work in cycle 1-2-3-4-5 can be given by area 1-3′-4-5.

Non expansive work per kg of steam $w_{ne} = v_1 \times (10 - 1) \times 10^2 = 157.59$ kJ/kg

Expansive work in cycle 1-2-3-4-5 can be given by area 1-2-3-3′.

Expansive work per kg of steam.

$$w_e = [p_1 \, v_1 \ln r - p_3(v_2 - v_1)]$$

$$w_e = \left[10 \times 10^2 \times 0.1751 \times \ln\left(\frac{5}{3}\right) - 1 \times 10^2 \, (0.2918 - 0.1751)\right.$$

$$w_e = 77.77 \text{ kJ/kg}$$

Total work per kg of steam = $w_{ne} + w_e = 235.36$ kJ/kg

Fraction of work obtained by expansive working = $\dfrac{w_e}{w_{ne} + w_e}$

$$= \frac{77.77}{235.36} = 0.3304 \text{ or } 33.04\%$$

Thermal efficiency of cycle = $\dfrac{235.36}{q_{\text{add}}} = \dfrac{235.36}{2159.12} = 0.1090$ or 10.90%

Fraction of expansive work = 33.04% of total output
Thermal efficiency = 10.90%

Ans.

For cycle having complete non expansive work: Such modified cycle is shown by 1′-3-4-5. This refers to the situation when cut-off shall become unity. Therefore, the mass of steam admitted shall be increased as compared to previous case in which 1 kg steam is admitted up to 60% cut-off point.

Steam admitted per cycle when cut off is at 60% = 1 kg

Steam admitted per cycle when cut off becomes unity = $\dfrac{1}{0.6} = 1.66$ kg

Total work per cycle = Area 1′-3-4-5 = $(p_1 - p_3) \cdot v_1 \times 1.66 = 261.59$ kJ

$$\% \text{ increase in work} = \left(\frac{261.59 - 235.36}{235.36}\right) \times 100 = 11.14\%$$

$$\text{Modified thermal efficiency} = \frac{261.59}{1.66 \times 2159.12} = 0.0729 \text{ or } 7.29\%$$

There is reduction in thermal efficiency. % decrease in efficiency

$$= \left(\frac{10.9 - 7.29}{10.9}\right) \times 100 = 33.12\%$$

Ans. % increase in work = 11.14%
% decrease in efficiency = 33.12%

7. *A non condensing double acting steam engine produces 60 bhp when steam at 12 bar pressure is admitted and exhausted at 1 bar. Engine runs at 240 rpm and piston speed of 2 m/s. The piston rod has diameter of 4 cm. Cut off occurs at 60% of stroke. Clearance volume is 5% of stroke volume. Considering diagram factor as 0.8 and mechanical efficiency of 90% determine the bore of the cylinder.*

Solution:

$p_1 = 12$ bar, $p_3 = 1$ bar

$V_1 = 0.6\ (V_2 - V_5)$

$V_5 = 0.05\ (V_2 - V_5) \Rightarrow V_5 = \dfrac{0.05}{1.05}\ V_2$

Expansion ratio, $r = \dfrac{V_2}{V_1}$

Substituting V_1 and V_2 from above

as, $V_2 = \dfrac{1.05}{0.05}\ V_5 = 21\ V_5$

$V_1 = \dfrac{0.6\,V_5}{0.05} = 12\ V_5$

$r = 1.75$

Hypothetical mean effective pressure

$$mep = \frac{P_1V_1(1+\ln r) - P_3V_2 - (P_1 - P_3)V_5}{(V_2 - V_5)}$$

$$= \frac{12\times12\,V_5\,(1+\ln 1.75) - (1\times 21\,V_5) - (12-1)\times V_5}{(21\,V_5 - V_5)}$$

$mep = 9.63$ bar

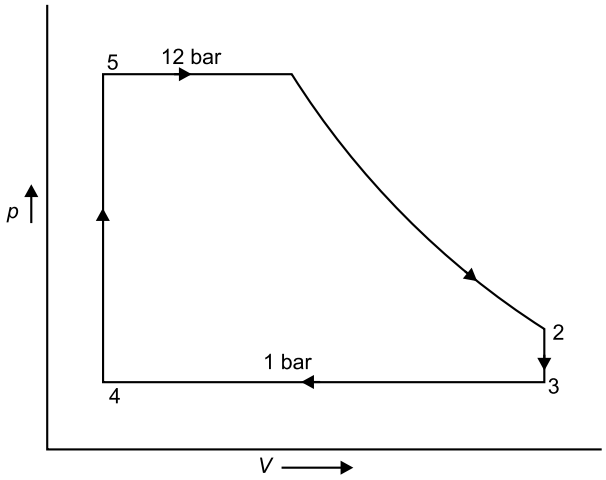


Fig. 12.35

Actual mean effective pressure mep_{actual} = Hypothetical mean effective pressure × diagram factor

= 9.63 × 0.8 = 7.704 bar ≈ 7.70 bar

Indicated power = mep_{actual} × Effective area × Piston speed

$$\frac{60\times0.7457}{0.9} = 7.70\times10^2\times A\times 2$$

$\Rightarrow$ $A = 0.03228\ m^2$

Let diameter of piston be D meter.

Since it is double acting engine so this area A calculated shall include area on both sides of piston.

$$A = 2 \cdot A_{\text{piston}} + A_{\text{piston rod}}$$

$$A = 2 \cdot \left(\frac{\pi}{4} D^2\right) + \left(\left(4 \times 10^{-2}\right)^2 \times \frac{\pi}{4}\right)$$

$$0.03228 = \left(2 \times \frac{\pi}{4} \times D^2\right) + \left(\frac{\pi}{4} \times \left(4 \times 10^{-2}\right)^2\right)$$

$$\Rightarrow \quad D = 0.1405 \text{ m or } 14.05 \text{ m}$$

Bore = 14.05 cm **Ans.**

8. *A double acting steam engine has cylinder diameter of 20 cm and stroke of 30 cm. Clearance volume is 2×10^3 cm^3. Engine uses 0.05 kg steam per stroke effectively i.e. after correction for leakage. Compression starts at 80% of stroke with pressure of 1 bar and steam is dry at the point of starting of compression. Cut-off point is at 10% of stroke and release occurs at 90% of stroke. The pressures at cut off point and point of release are 15 bar and 3 bar. Determine total mass of steam present during expansion and dryness at cut-off and release. Consider the expansion to follow PV^n = constant and find heat leaking through cylinder walls per kg of steam during expansion.*

Solution:

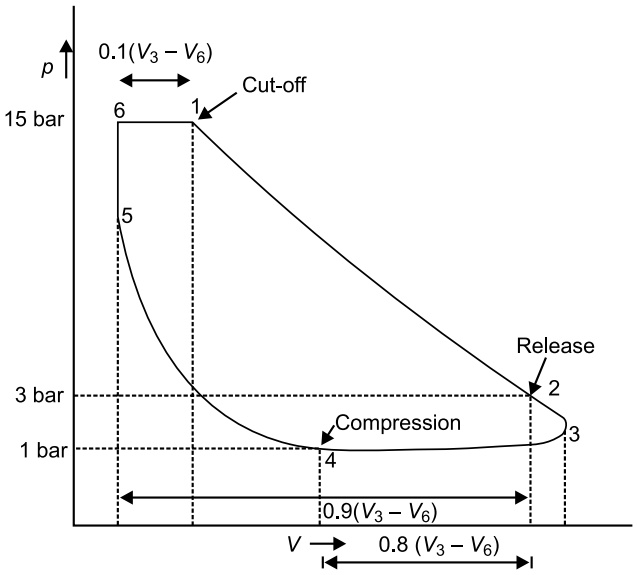


Fig. 12.36 p-V *diagram*

Clearance volume,

$$V_6 = V_5 = 2 \times 10^3 \text{ cm}^3 = 2 \times 10^{-3} \text{ m}^3$$

Stroke volume, $V_3 - V_6 = \frac{\pi}{4} \times (0.2)^2 \times 0.3 = 9.42 \times 10^{-3} \text{ m}^3$

Total volume = $V_3 = 2 \times 10^{-3} + 9.42 \times 10^{-3}$

$= 11.42 \times 10^{-3} \text{ m}^3$

At the point of beginning of compression, i.e. '4'

$$V_4 = V_3 - 0.8 \times (V_3 - V_6) = 3.884 \times 10^{-3} \text{ m}^3$$

At the start of compression, $v_4 = v_{g \text{ at 1 bar, dry saturated}} = 1.6940 \text{ m}^3/\text{kg}$

Therefore, mass of steam at state 4, i.e. cushion steam $= \dfrac{V_4}{v_4} = 2.293 \times 10^{-3}$ kg

Total mass of steam during expansion = Cushion steam + Steam used per stroke

$$= 2.293 \times 10^3 + 0.05$$
$$= 0.052293 \text{ kg}$$

Volume at cut-off point, $V_1 = V_6 + 0.1\ (V_3 - V_6)$

$$V_1 = 2.942 \times 10^{-3} \text{ m}^3$$

Dryness fraction at cut-off point, $x_1 = \dfrac{\text{Volume}}{m \cdot v_{\text{g at 15 bar}}} = \dfrac{2.49 \times 10^{-3}}{(0.052293 \times 0.13177)}$

$$x_1 = 0.4269$$

Volume at point of release, $V_2 = V_6 + 0.9\ (V_3 - V_6) = 0.010478 \text{ m}^3$

Dryness fraction at point of release, $x_2 = \dfrac{\text{Volume}, V_2}{m \cdot v_{\text{g at 3 bar}}}$

$$= \frac{0.010478}{(0.052293 \times 0.6058)}$$

$$x_2 = 0.3307$$

Let us find out index of expansion n for process 1–2.

$$P_1 V_1^n = P_2 V_2^n$$
$$15 \times (2.942 \times 10^{-3})^n = 3 \times (0.010478)^n$$
$$n = 1.267$$

$$\text{Work done in a stroke} = \left(\frac{P_1 V_1 - P_2 V_2}{n - 1} \right)$$
$$= 10^2 \times \left(\frac{15 \times 2.942 \times 10^{-3} - 3 \times 0.010478}{1.267 - 1} \right)$$
$$= 4.755 \text{ kJ}$$

$$\text{Work done per kg of steam} = \frac{4.755}{\text{Steam available per stroke}}$$
$$= \frac{4.755}{0.052293} = 90.93 \text{ kJ/kg}$$

Change in internal energy during expansion $= u_1 - u_2$

$$u_1 = u_{f \text{ at 15 bar}} + x_1 \cdot u_{fg \text{ at 15 bar}} = 1590.79 \text{ kJ/kg}$$

$$u_2 = u_{f \text{ at 3 bar}} + x_2 \cdot u_{fg \text{ at 3 bar}}$$
$$u_2 = 1216.73 \text{ kJ/kg}$$

Applying first law of thermodynamics,

$$\Delta q = \Delta u + \Delta w = (u_2 - u_1) - \Delta w$$
$$= (1216.73 - 1590.79) - 90.93$$
$$= - 464.99 \text{ kJ/kg}$$

Total mass of steam during expansion = 0.052293 kg
Dryness fraction at cut off and release = 0.4269, 0.3307
Heat leakage = 464.99 kJ/kg steam

Ans.

9. *Determine dryness fraction and missing quantity of steam in kg/hr at point of cut-off and point of release for the following data available from steam engine;*

Point of cut off: 30% of stroke

At any point on compression curve: pressure = 4 bar, indicated volume = 0.15m^3

At any point on expansion curve immediately after cut off: pressure = 12 bar

Pressure at release: 5 bar

Indicated volume at release: 0.5 m^3

Bore and stroke: 60 cm and 120 cm

Clearance volume: 10% of stroke volume

Mass of steam admitted = 1.5 kg/stroke.

Number of working strokes = 180 per minute.

Also determine the percentage re-evaporation during expansion.

Solution:

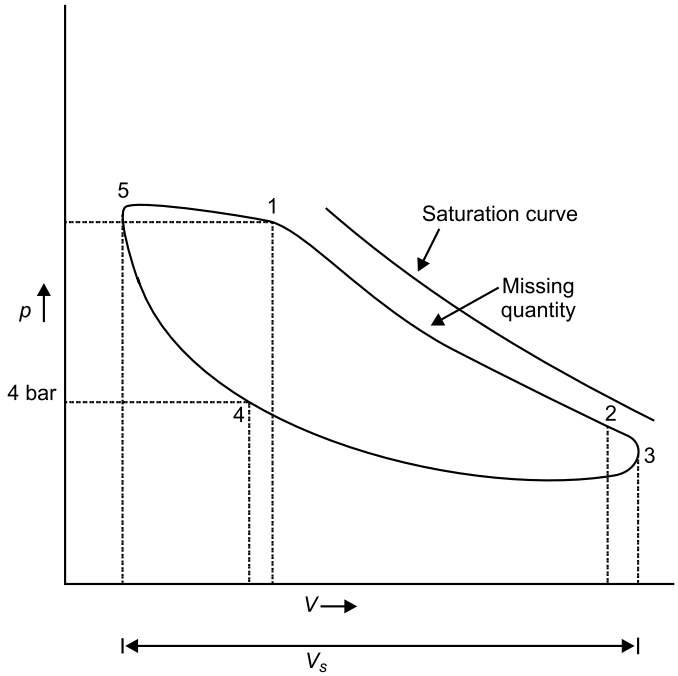


Fig. 12.37 p-V *diagram*

Let us approximate a point immediately after cut-off to be cut-off point.

V_s = Stroke volume

$$= \frac{\pi}{4} \times (0.6)^2 \times 1.2 = 0.339 \text{ m}^3$$

Clearance volume, $V_5 = 0.1 \times V_s = 0.0339 \text{ m}^3$

Total volume of cylinder, $V_3 = V_5 + V_s = 0.3729 \text{ m}^3$

V_1 = Volume at cut-off point

$V_1 = V_5 + 0.3 \times V_s$

$V_1 = 0.1356 \text{ m}^3$

Pressure at cut-off point = 12 bar

Volume at release, $V_2 = 0.5 \text{ m}^3$

At the point on compression curve, point 4

The steam present in cylinder at state 4 shall act as cushion steam.

$$\text{Mass of cushion steam} = \frac{0.15}{v_{\text{g at 4 bar}}} = \frac{0.15}{0.4625} = 0.3243 \text{ kg}$$

Total mass of steam during expansion = Cushion steam + Mass of steam admitted

$$= 0.3243 + 1.5$$

$$= 1.8243 \text{ kg}$$

At cut-off point

$$\text{Dryness fraction at cut-off point, } x_1 = \frac{V_1}{m \cdot v_{\text{g at 12 bar}}}$$

$$= \frac{0.1356}{1.8243 \times 0.16333} = 0.455$$

Missing quantity per hour = $(1.8243 - 1.8243 \times 0.455) \times 180 \times 60$

$$= 10737.83 \text{ kg}$$

At point of release

$$\text{Dryness fraction at point of release, } x_2 = \frac{V_2}{m \cdot v_{\text{g at 5 bar}}} = \frac{0.5}{1.8243 \times 0.3749}$$

$$x_2 = 0.731$$

Missing quantity per hour = $(1.8243 - 1.8243 \times 0.731) \times 180 \times 60$

$$= 5299.96 \text{ kg}$$

Percentage re-evaporation during expansion

$$= \frac{\text{Missing quantity at cut off} - \text{Missing quantity at release}}{\text{Missing quantity at cut-off}}$$

$$\text{Percentage re-evaporation} = \frac{10737.83 - 5299.96}{10737.83} = 0.5064 \text{ or } 50.64\%$$

Dryness fraction at cut-off = 0.455
Dryness fraction at release = 0.731
Missing quantity at cut-off = 10737.83 kg/hr
Missing quantity at release = 5299.96 kg/hr
Percentage re-evaporation = 50.64%

Ans.

10. *A double acting compound steam engine has two cylinders. Steam is supplied at 1.5 MPa and 0.9 dry. Each cylinder has equal initial piston loads and the exhaust from engine occurs at 40 kPa into condenser. Diagram factor referred to low pressure cylinder is 0.8 and length of stroke of both cylinders is 38 cm. Bore of cylinders of HP and LP cylinders are 20 cm and 30 cm. Expansion occur completely in HP cylinder and engine runs at 240 rpm. Neglecting clearance volume determine.*

(i) intermediate pressure, (ii) indicated power output and (iii) steam consumption in kg/hr.

Solution:

On P-V diagram, HP engine: 1267, LP engine: 23456

For equal initial piston load,

$$(P_1 - P_2) \cdot A_{HP} = (P_2 - P_4) \cdot A_{LP}$$

$$(1.5 \times 10^3 - P_2) \times \left(\frac{\pi}{4} \times (0.2)^2\right) = (P_2 - 40) \times \left(\frac{\pi}{4} \times (0.3)^2\right)$$

Solving, we get

$$P_2 = 192 \text{ kPa}$$

Ans. Intermediate pressure = 192 kPa

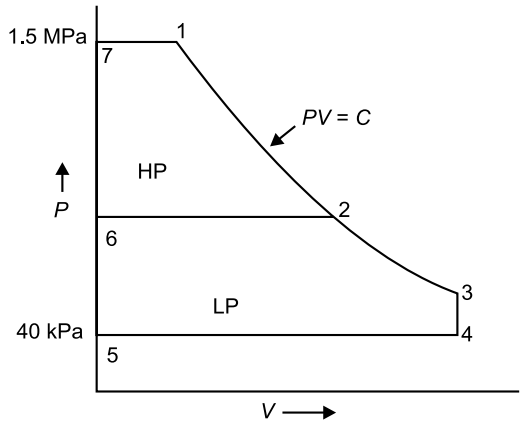


Fig. 12.38 P-V *diagram*

For expansion process 1-2-3, for hyperbolic expansion,

$$P_1 V_1 = P_2 V_2 \Rightarrow V_1 = \frac{P_2 V_2}{P_1}$$

Due to no clearance,

$$V_2 = \frac{\pi}{4} \times (0.2)^2 \times 0.38 \text{ m}^3$$

$$\Rightarrow \qquad V_1 = \frac{192 \times \frac{\pi}{4} \times (0.2)^2 \times 0.38}{1.5 \times 10^3} = 1.53 \times 10^{-3} \text{ m}^3$$

Cut-off volume in HP cylinder = 1.53×10^{-3} m^3

Volume of LP cylinder = $\frac{\pi}{4} \times (0.3)^2 \times 0.38 = 0.0268$ m^3

$$\text{Expansion ratio throughout the engine} = \frac{\text{Volume of LP cylinder}}{\text{Cut-off volume in HP cylinder}}$$

$$= \frac{0.0268}{1.53 \times 10^{-3}} = 17.52$$

Mean effective pressure for compound engine referred to LP cylinder

$$mep = \frac{P_1}{r}(1 + \ln r) - P_4$$

$$= \frac{1.5 \times 10^3}{17.52}(1 + \ln 17.52) - 40$$

$$mep = 290.76 \text{ kPa}$$

Actual mep = Hypothetical $mep \times$ diagram factor referred to LP

$= 290.76 \times 0.8$

$= 232.608$ kPa

Indicated power = $mep_{\text{actual}} \times L \times A \times N$

$$= 232.608 \times 0.38 \times \frac{\pi}{4} \times (0.3)^2 \times \frac{240}{60} \times 2$$

$= 49.98$ kW

Volume of steam admitted per hour = $V_1 \times 240 \times 2 \times 60$

$= 1.53 \times 10^{-3} \times 240 \times 2 \times 60$

$= 44.064$ m^3

Specific volume of steam being admitted, $v_1 = v_{f \text{ at1.5 MPa}} + 0.9 \times v_{fg \text{ at 1.5 MPa}}$

$v_1 = 0.1187$ m^3/kg

Steam consumption, kg/hr = $\frac{44.064}{0.1187} = 371.22$ kg/hr

Indicated power = 49.98 kW, Steam consumption = 371.22 kg/hr	***Ans.***

11. *A double acting compound steam engine has two cylinders. Steam is admitted at 1.4 MPa and is exhausted at 25 kPa. Engine runs at 240 rpm. Low pressure cylinder has bore of 60 cm and stroke of 60 cm. Diagram factor referred to LP cylinder is 0.8 while expansion is hyperbolic throughout. Expansion is complete in high pressure cylinder and the clearance volume may be*

neglected. Total expansion ratio is 8 throughout the engine. Considering equal work to be produced by each cylinder determine,

(i) the indicated power output

(ii) the diameter of high pressure cylinder considering same stroke length for both cylinders.

(iii) the intermediate pressure.

Solution:

Overall expansion ratio, $r = 8$

Hypothetical *mep* referred to LP cylinder

$$= \frac{P_1}{r}(1 + \ln r) - P_4$$

$$= \frac{1.4 \times 10^3}{8}(1 + \ln 8) - 25$$

$$= 513.9 \text{ kPa}$$

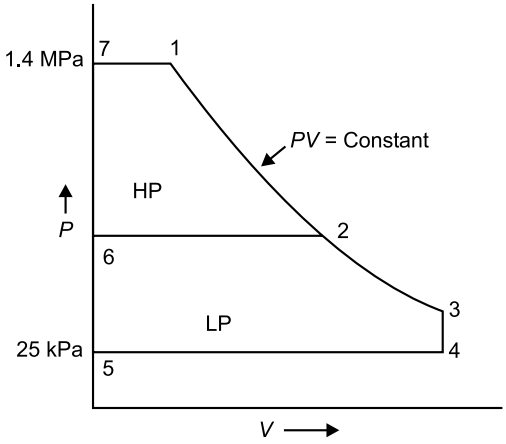


Fig. 12.39 P-V *diagram*

Actual *mep* = Hypothetical *mep* × d.f. referred to LP

$mep_{actual} = 513.9 \times 0.8 = 411.12$ kPa

Indicated power output = $mep_{actual} \times L \times A \times N$

$$= 411.12 \times 0.6 \times \frac{\pi}{4}(0.6)^2 \times \frac{240}{60} \times 2$$

$$= 557.96 \text{ kW}$$

Total area of diagram = mep_{actual} × Volume of LP cylinder

Since each HP and LP perform equal work so work done in HP cylinder

$$= \frac{\text{Total area of diagram}}{2} = \frac{411 \cdot 12 \times \pi \times (0.6)^2 \times 0.6}{4 \times 2}$$

$$= 34.87 \text{ kJ}$$

HP cylinder work = $P_1\ V_1 \ln\left(\frac{V_2}{V_1}\right) = 34.87$ kJ

Also from expansion ratio, $\frac{V_3}{V_1} = 8 \Rightarrow V_1 = \frac{V_3}{8} = \frac{\frac{\pi}{4} \times (0.6)^2 \times 0.6}{8}$

$$V_1 = 0.0212 \text{ m}^3$$

Substituting $P_1 = 1.4 \times 10^3$ kPa and $V_1 = 0.0212$ m^3, we get

$$(1.4 \times 10^3 \times 0.0212) \ln\left(\frac{V_2}{0.0212}\right) = 34.87$$

$$V_2 = 0.0686 \text{ m}^3$$

Thus, the volume of HP cylinder shall be 0.0686 m^3. Let diameter of HP cylinder be 'd' so.

$$0.0686 = \frac{\pi}{4} \times (d^2) \times 0.6$$

$$d = 0.3815 \text{ m or } 38.15 \text{ cm}$$

From hyperbolic process consideration on process 1–2.

$$P_1\ V_1 = P_2\ V_2$$

$$(1.4 \times 10^3 \times 0.0212) = P_2 \times 0.0686$$

$$\Rightarrow \quad P_2 = 432.65 \text{ kPa}$$

Indicated power = 557.96 kW
Diameter of HP cylinder = 38.15 cm
Intermediate pressure = 432.65 kPa

Ans.

12. *A double acting compound steam engine has HP and LP cylinders to produce output of 250 kW. Steam is admitted at 1.5 MPa and exhausted at 25 kPa. Overall expansion ratio referred to LP cylinder is 12. Diameter of LP cylinder is 40 cm and stroke length in both cylinders is 60 cm each. Diagram factor is 0.75. Expansion ratio in H.P. cylinder is 2.5. Neglecting clearance volume and considering expansion to be hyperbolic, determine the speed of engine in rpm and diameter of HP cylinder.*

Solution:

Hypothetical *mep* referred to LP cylinder,

$$mep_{\text{hypothetical}} = \frac{1.5 \times 10^3}{12}(1 + \ln 12) - 25$$

$$mep_{\text{hypothetical}} = 410.61 \text{ kPa}$$

$$\text{Actual } mep = 410.61 \times 0.75$$

$$mep_{\text{actual}} = 307.96 \text{ kPa}$$

$$\text{Power output} = 250 = mep_{\text{actual}} \times L \times A \times N \times 2$$

$$250 = 307.96 \times 0.6 \times \frac{\pi}{4} \times (0.4)^2 \times \frac{N}{60} \times 2$$

$$\Rightarrow \quad N = 323 \text{ rpm}$$

Speed of engine = 323 rpm ***Ans.***

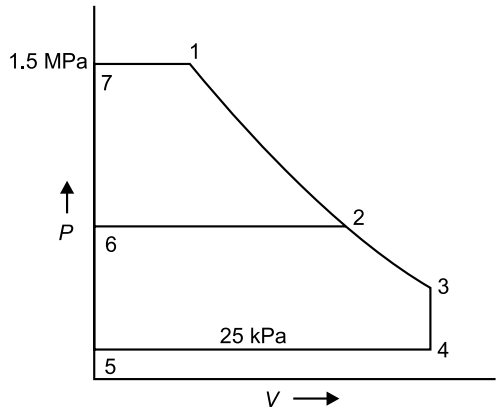


Fig. 12.40 P-V *diagram*

Volume of low-pressure cylinder = $\frac{\pi}{4} \times (0.4)^2 \times 0.6$

$= 0.0754 \text{ m}^3 = V_3 = V_4$

Cut-off volume in HP cylinder = $\frac{0.0754}{12} = 6.28 \times 10^{-3} \text{ m}^3$

Total volume in HP cylinder = $6.28 \times 10^{-3} \times 2.5 = 0.0157 \text{ m}^3$

Let diameter of HP cylinder be '*d*' meter, then

$$\frac{\pi}{4} \times d^2 \times 0.60 = 0.0157$$

$$d = 0.1825 \text{ m or } 18.25 \text{ cm}$$

Diameter of HP cylinder = 18.25 cm ***Ans.***

13. *A triple expansion steam engine has HP, IP and LP cylinder diameters of 25 cm, 40 cm and 85 cm with actual mean effective pressures of 0.5 MPa, 0.3 MPa and 0.1 MPa. Steam is supplied at 1.5 MPa and the exhaust finally occurs at 25 kPa. The cut-off in HP cylinder occurs at 60% of the stroke. Considering all the cylinders to have some stroke length determine,*

(i) the actual and hypothetical mep referred to LP cylinder,
(ii) the overall diagram factor
(iii) the contribution of each cylinder in overall output. Neglect clearance volume.

Solution:

In HP cylinder, *mep* of HP referred to LP $= 0.5 \times 10^3 \times \frac{A_{HP}}{A_{LP}}$

$$= 0.5 \times 10^3 \times \frac{\frac{\pi}{4} \times (0.25)^2}{\frac{\pi}{4} \times (0.85)^2}$$

$= 43.25 \text{ kPa}$

In IP cylinder, *mep* of IP referred to LP $= 0.3 \times 10^3 \times \frac{A_{IP}}{A_{LP}}$

$$= 0.3 \times 10^3 \times \frac{\frac{\pi}{4} \times (0.40)^2}{\frac{\pi}{4} \times (0.85)^2}$$

$$= 66.44 \text{ kPa}$$

For complete engine the overall mep referred to LP cylinder

$$= 43.25 + 66.44 + 0.1 \times 10^3$$

$$= 209.69 \text{ kPa}$$

Volume of LP cylinder $= \frac{\pi}{4} \times (0.85)^2 \times \text{L} = 0.567 \text{ L}$

Volume of HP cylinder $= \frac{\pi}{4} \times (0.25)^2 \times \text{L} = 0.049 \text{ L}$

Cut-off volume of HP cylinder = 0.6 × Volume of HP cylinder = 0.0294 L

For zero clearance volume,

$$\text{Overall expansion ratio} = \frac{\text{Volume of LP cylinder}}{\text{Cut off volume of HP cylinder}} = \frac{0.567 \text{ L}}{0.0294 \text{ L}}$$

$$r = 19.28$$

$$\text{Hypothetical m.e.p overall} = \frac{1.5 \times 10^3}{19.28} (1 + \ln 19.28) - 25 = 283.02 \text{ kPa}$$

$$\text{Overall diagram factor} = \frac{\text{Actual m.e.p.}}{\text{Hypothetical m.e.p.}}$$

$$= \frac{209.69}{283.02} = 0.741$$

Percentage of total indicated power developed in HP, IP and LP cylinder. Since stroke length for each cylinder is same so the ratio of indicated power output of each cylinder to total output shall be the ratio of m.e.p of specific cylinder referred to LP cylinder and overall mep referred to LP cylinder

$$\text{\% of HP cylinder output} = \frac{43.25}{209.69} \times 100 = 20.63\%$$

$$\text{\% of IP cylinder output} = \frac{66.44}{209.69} \times 100 = 31.68\%$$

$$\text{\% of LP cylinder output} = \frac{0.1 \times 10^3}{209.69} \times 100 = 47.69\%$$

Actual mep referred to LP = 209.69 kPa
Hypothetical mep referred to LP = 283.02 kPa
Overall diagram factor = 0.741
% of HP, IP and LP cylinder outputs = 20.63%, 31.68%, 47.69%

Ans.

14. *A double acting compound steam engine has steam supplied at 7 bar and exhausted at 0.25 bar.*

HP cylinder: diameter = 25 cm
cut-off point = 30% stroke
clearance volume = 10% of swept volume
LP cylinder: diameter = 50 cm
cut-off point = 45% stroke
clearance volume = 5% of swept volume

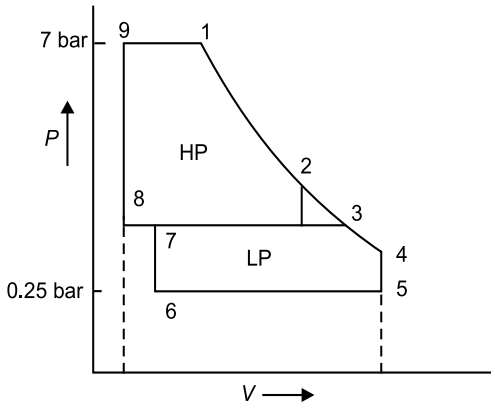


Fig. 12.41

Diagram factors of HP and LP cylinder are 0.8 and 0.7 respectively. Considering hyperbolic expansion and neglecting cushioning effect estimate mep in each cylinder and total hp developed for engine running at 100 rpm. Take account of clearance volumes.

Solution:

Let the common stroke of cylinders be 'L' m. Assuming the expansion curve to be continuous the P-V diagram is shown here.

$$\text{Volume of HP cylinder} = \frac{\pi}{4} \times (0.25)^2 \times \text{L} = 0.04908 \text{ L m}^3$$

$$\text{Clearance volume} = 0.1 \times 0.04908 \text{ L} = 0.004908 \text{ L m}^3 = V_9$$

$$\text{Total volume of HP} = 0.04908 \text{ L} + 0.004908 \text{ L} = 0.053988 \text{ L m}^3 = V_2$$

Volume at cut-off in HP cylinder

$$= (0.004908 \cdot \text{L}) + (0.3 \times 0.04908 \cdot \text{L})$$

$$= 0.019632 \text{ L m}^3 = V_1$$

For LP cylinder

$$\text{Volume of LP cylinder} = \frac{\pi}{4} \times (0.5)^2 \cdot \text{L}$$

$$= 0.1963 \text{ L m}^3$$

$$\text{Clearance volume of LP cylinder} = 0.05 \times 0.1963 \text{ L}$$

$$= 9.815 \times 10^{-3} \text{ L m}^3 = V_7$$

Total volume of LP cylinder

$= (0.1963 \text{ L} + 9.815 \times 10^{-3} \text{ L})$

$= 0.206115 \text{ L m}^3 = V_5$

Volume at cut-off of LP cylinder

$= 9.815 \times 10^{-3} \text{ L} + (0.45 \times 0.1963 \text{ L})$

$= 0.09815 \text{ L m}^3 = V_3$

Expansion ratio for HP cylinder, $r_{HP} = \dfrac{0.053988 \text{ L}}{0.019632 \text{ L}} = 2.75$

Expansion ratio for LP cylinder, $r_{LP} = \dfrac{0.206115 \text{ L}}{0.09815 \text{ L}} = 2.1$

$$P_1 V_1 = P_2 V_2 = P_3 V_3 = P_4 V_4$$

$\Rightarrow$ $P_1 V_1 = P_3 V_3$

or, $(7 \times 10^2 \times 0.019632 \text{ L}) = (P_3 \times 0.09815 \text{ L})$

$\Rightarrow$ $p_3 = 140.01 \text{ kPa}$

Actual m.e.p. for HP cylinder

$$= 0.8 \frac{\left[P_1 V_1 (1 + \ln r_{HP}) - (P_3 \cdot V_2) - \left(P_1 - P_3\right) \times V_9\right]}{\text{Stroke volume of HP}}$$

$$= 0.8 \frac{\left[\begin{array}{r} 7 \times 10^2 \times 0.019632(1 + \ln 2.75) - (140.01 \times 0.053988) - \\ \left(\left(7 \times 10^2 - 140.01\right) \times 0.004908\text{L}\right) \end{array}\right]}{0.04908 \cdot \text{L}}$$

$= 282.59 \text{ kPa}$

Actual mep of LP cylinder

$$= 0.7 \times \frac{\left[P_3 V_3 (1 + \ln r_{LP}) - (P_3 - P_5) \times V_7\right]}{\text{Stroke volume of LP}}$$

$$= 0.7 \frac{\left[\begin{array}{r} (140.01 \times 0.09815(1 + \ln 2.1)) - (25 \times 0.206115\text{L}) - \\ (140.01 - 25) \times 9.815 \times 10^{-3}\text{L} \end{array}\right]}{0.1963 \text{ L}}$$

$= 62.96 \text{ kPa}$

Actual mep of HP referred to LP cylinder $= \dfrac{282.59 \times 0.04908 \text{ L}}{0.1963 \text{ L}}$

$= 70.65 \text{ kPa}$

Total mep = 62.96 + 70.65 = 133.61 kPa

Total output = Total mep $\times L \times A \times N$

$$= \frac{133.61 \times L \times \frac{\pi}{4} \times (0.5)^2 \times 100}{60} = 43.72 \text{ L kW}$$

mep of HP referred to LP = 70.65 kPa
mep of LP = 62.96 kPa
Total output = 43.72 L kW where L is stroke length ***Ans.***

15. *During the trial of a single-cylinder double acting steam engine following is observed. Duration of trial = 15 min, Bore = 25 cm, Stroke = 30 cm, Brake diameter = 1.5 m, Net brake load = 300 N, Speed = 240 rpm, Steam pressure = 10 bar, Dryness fraction = 0.9, Cover end mean effective pressure = 0.9 bar, Crank end mean effective pressure = 0.9 bar, Steam utilized = 15 kg. Condensate temperature = 45°C. Amount of water circulating = 450 kg/hr. Rise in cooling water temperature = from 20°C to 30°C. Determine, (i) steam used per ihp. hr, (ii) mechanical efficiency, (iii) brake thermal efficiency, (iv) heat balance sheet on one hour basis.*

Solution:

From given data;

$$\text{Average value of mean effective pressure} = \frac{1}{2}\,(mep_{\text{cover end}} + mep_{\text{crank end}})$$

$$mep = 0.9 \text{ bar}$$

$$\text{Steam consumption per hour} = \frac{15}{15} \times 60 = 60 \text{ kg/hr}$$

$$\text{Indicated horse power} = \frac{mep \times L \times A \times N \times 2}{0.7457 \times 60}, \{\text{as 1 HP} = 0.7457\text{kW}\}$$

$$= \frac{0.9 \times 10^2 \times 0.30 \times \frac{\pi}{4}(0.25)^2 \times 240 \times 2}{60 \times 0.7457}$$

$$= 14.22 \text{ hp or } 10.604 \text{ kJ/s or } 38174.4 \text{ kJ/hr}$$

$$\text{Therefore, steam used per ihp. hr} = \frac{60}{14.22} = 4.22 \text{ kg/ihp. hr}$$

Steam used per ihp. hr = 4.22 kg/ihp. hr ***Ans.***

Brake power can be obtained using the dynamometer data given i.e. brake load, brake drum diameter etc.

$$\text{Brake horse power} = \frac{2\pi N \cdot T}{60 \times 0.7457 \times 10^3} = \frac{2 \times \pi \times 240 \times 300 \times 0.75}{60 \times 0.7457 \times 10^3}$$

$$\text{Brake horse power} = 7.58 \text{ hp or } 5.65 \text{ kJ/s or } 20340 \text{ kJ/hr}$$

$$\text{Mechanical efficiency} = \frac{\text{Brake horse power}}{\text{Indicated horse power}} = 0.5331 \text{ or } 53.31\%$$

$$\text{Heat available with steam/hr} = 60\,[h_{\text{at 10 bar, 0.9 dry}} - h_{\text{at 45°C}}]$$

$$= 60\,[(762.81 + 0.9 \times 2015.3) - (4.18 \times 45)]$$

$$= 143308.8 \text{ kJ/hr}$$

$$\text{Brake thermal efficiency} = \frac{\text{Heat equivalent of bhp}}{\text{Heat available with steam / sec}}$$

$$= \frac{7.58 \times 0.7457 \times 3600}{143308.8}$$

$$= 0.14199 \text{ or } 14.19\%$$

Heat carried away by circulating water in condenser

$$= 450 \times 4.18 \times [30 - 20]$$

$$= 18810 \text{ kJ/hr}$$

Heat balance sheet on one hour basis

Input	*kJ*	%	*Output*	*kJ*	*%*
Heat available with steam/hr	143308.8	100%	bhp	20340	14.19
			fhp(= ihp - bhp)	17834.4	12.44
			Heat carried away by circulating water	18810	13.13
			Unaccounted losses (by difference)	86324.4	60.24
Total	143308.8	100%	Total	143308.8	100%

16. *During trial of steam engine following observation are made; Bore:38 cm, Stroke 50 cm, Piston rod diameter: 5 cm, Speed:150 rpm, Steam consumption: 36 kg/min, Brake load = 7kN at 2 m brake diameter.*

Area of indicator diagram at cover end: 28 cm^2

Area of indicator diagram at crank end: 26cm^2

Length of indicated diagram: 7 cm

Spring scale: 15 kPa/mm

Determine brake power, indicated power, mechanical efficiency, brake specific steam consumption and indicated specific steam consumption.

Solution:

$$\text{M.E.P.}_{\text{crank}} = \frac{2600 \times 15}{70} = 557.14 \text{ kPa}$$

$$\text{M.E.P.}_{\text{cover}} = \frac{2800 \times 15}{70} = 600 \text{ kPa}$$

Indicated power at crank end, $\text{IP}_{\text{crank}} = \text{MEP}_{\text{crank}} \times L \times A_{\text{crank}} \times N$

At crank end piston area will be reduced due to piston rod,

$$\text{IP}_{\text{crank}} = 557.14 \times 50 \times 10^{-2} \times \frac{\pi}{4}\{(0.38)^2 - (0.05)^2\} \times \frac{150}{60}$$

$$\text{IP}_{\text{crank}} = 77.62 \text{ kW}$$

Indicated power at cover end, $\text{IP}_{\text{cover}} = \text{MEP}_{\text{cover}} \times L \times A_{\text{cover}} \times N$

$$\text{IP}_{\text{cover}} = 600 \times 50 \times 10^{-2} \times \frac{\pi}{4} \times (0.38)^2 \times \frac{150}{60}$$

$IP_{cover} = 85.06$ kW

Indicated power = $IP_{crank} + IP_{cover} = 162.68$ kW **Ans.**

Brake power, $BP = 2\pi NT = 2\times\pi\times\frac{150}{60}\times 7\times 1 = 109.96$ kW **Ans.**

Mechanical efficiency, $\eta_{mech} = \frac{BP}{IP} = \frac{109.96}{162.68} = 0.6759 = 67.59\%$

Indicated specific steam consumption, $ISFC = \frac{\text{Steam consumption}}{IP}$

$$= \frac{36\times 60}{162.68}$$

Indicated specific steam consumption = 13.28 kg/kWh **Ans.**

Brake specific steam consumption, $BSFC = \frac{\text{Steam consumption}}{BP}$

Brake specific steam consumption = $\frac{36\times 60}{109.96} = 19.64$ kg/kwh

17. *During trial of single cylinder, double acting steam engine of condensing type following observations were made;*

Bore: 24 cm, Stroke: 34 cm; Engine speed: 150 rpm, Piston rod diameter: 5cm, Brake load: 120kg. Spring balance reading: 100N.

Brake wheel drum diameter: 100cm;

Steam inlet state: 15 bar, 0.98 dry,

Mean effective pressure at cover end: 1.8 bar

Mean effective pressure at crank end: 1.6 bar

Cooling water flow through condenser: 42 kg/min

Rise in temperature of cooling water: 20°C

Condensate discharged from condensor: 4 kg/min

Temperature of condensate: 50°C

Determine brake power, indicated power, mechanical efficiency, brake thermal efficiency, indicated steam consumption and energy balance sheet on minute basis.

Solution:

Brake power = $2\pi N$(Brake load – Spring balance reading) × Drum radius

Brake power = $\frac{2\pi\times 150(120\times 9.81-100)\times(100/2)\times 10^{-2}}{1000\times 60}$

= 8.46 kW

Indicated power, cover end = $MEP_{cover} \times L \times A_{cover} \times N$

$$= 1.8\times 10^2\times 0.34\times\frac{\pi}{4}\times(0.24)^4\times\frac{150}{60}$$

$= 6.92$ kW

Indicated power, crank end = $\text{MEP}_{\text{crank}} \times L \times A_{\text{crank}} \times N$

$$= 1.6\times10^2 \times 0.34 \times \frac{\pi}{4} \times \{(0.24)^2 - (0.05)^2\} \times \frac{150}{60}$$

$= 5.89$ kW

Total indicated power $= 6.92 + 5.89 = 12.81$ kW

$$\text{Mechanical efficiency} = \frac{\text{Brake power}}{\text{Indicated power}} = \frac{8.46}{12.81} = 0.6604 \text{ or } 66.04\%$$

Enthalpy of steam at inlet, $h_{s,in} = h_{f \text{ at } 15 \text{ bar}} + 0.98 \cdot h_{fg}$ at 15 bar

$= 844.89 + (0.98 \times 1947.3)$

$h_{s,in} = 2753.24$ kJ/kg

Enthalpy of condensate, $h_{cond} = h_{f \text{ at } 50°C} = 209.33$ kJ/kg

Energy supplied by steam $= h_{s,in} - h_{cond}$

$= 2753.24 - 209.33 = 2543.91$ kJ/kg

Steam consumption rate $= 4$ kg/min $= 240$ kg/hr

$$\text{Brake thermal efficiency} = \frac{3600}{\frac{m_{steam}}{\text{BP}} \times (h_{s,in} - h_{cond})} = \frac{3600 \times 8.46}{240 \times 2543.91}$$

$= 0.0498$ or 4.98% **Ans.**

$$\text{Indicated steam consumption} = \frac{m_{steam}}{IP} = \frac{240}{12.81}$$

$= 18.74$ kg/kWh **Ans.**

Energy balance sheet on per minute basis.

Energy supplied per minute, $Q_{\text{in}} = 4(h_{s,in} - h_{cond})$

$= 4 \times 2543.91 = 10175.64$ kJ/min

Energy consumed by brake or available at shaft

$Q_1 = 8.46 \times 60 = 507.6$ kJ/min

Energy consumed by cooling water in condenser,

$Q_2 = m_{\text{water}} \times C_{p,\text{water}} \times \Delta T$

$= 42 \times 4.18 \times 20 = 3511.2$ kJ/min

Energy going alongwith condensate,

$Q_3 = m_{\text{steam}} \times C_p \times \Delta T$

$= 4 \times 4.18 \times 50 = 836$ kJ/min

Engery loss to surroundings, $Q_4 = Q_{in} - (Q_1 + Q_2 + Q_3)$

$= 10175.64 - (507.6 + 3511.2 + 836) = 5320.84$ kJ/min

Energy Balance sheet on per minute basis

Energy in, kJ/min	*Energy out, kJ/min*
Energy supplied = 10175.64	Energy available at shaft = 507.6 Energy consumed by cooling water = 3511.2 Energy lost with condensate = 836 Energy loss to surroundings = 5320.84
Total: 10175.64, kJ/min	Total: 10175.64 kJ/min

EXERCISE

12.1 Classify steam engines.
12.2 Give a neat sketch of simple steam engine and explain its working.
12.3 Describe with sketches any five different parts of steam engines.
12.4 Why is it necessary to provide guides for a cross-head?
12.5 Describe the function of crank.
12.6 What is the eccentric? Explain its utility?
12.7 Describe the working of D-slide valve.
12.8 Describe hypothetical and actual indicator diagrams for a simple steam engine.
12.9 What are factors responsible for actual indicator diagram differing from hypothetical diagram?
12.10 Describe significance of diagram factor.
12.11 What do you understand by missing quantity of steam?
12.12 Why the cylinder of steam engine is generally steam jacketed?
12.13 What do you understand by compound steam engines?
12.14 Describe various types of compounding in steam engines and their relative merits and demerits.
12.15 Show the hypothetical and actual P-V diagram for compound steam engine having HP and LP cylinders.
12.16 Discuss the relevance of mep referred to LP cylinder in compound steam engines.
12.17 Discuss different methods of governing of steam engines.
12.18 Explain Willan's line.
12.19 What do you understand by unresisted expansion in reference to compound steam engines?
12.20 Discuss different types of governing of compound steam engines with *p-V* diagrams.
12.21 A single cylinder double acting steam engine is supplied with steam at 5 bar and steam is exhausted at 0.34 bar. Cut-off occurs at one-third of stroke. The swept volume of cylinder is 0.05 m^3 and diagram factor is 0.85. The engine delivers brake power of 37.5 kW at the speed of 120 rpm. Determine the mechanical efficiency of engine. [85%]
12.22 In a steam engine the steam is supplied at 20 bar, 340°C and is expanded adiabatically up to 1.5 bar after which there occurs instantaneous pressure drop at constant volume up to 0.07 bar. Determine the work done per kg of steam supplied and the modified Rankine cycle efficiency. [694.72 kJ, 23.5%]
12.23 Determine the indicated power output from a double acting single cylinder steam engine having stroke of 30 cm and stroke to bore ratio of 1.3. The steam is admitted at 67.45 bar and the back

pressure is 1.3 bar with expansion ratio of 2.5. The engine runs at 210 rpm and the diagram factor is 0.8. [27 kW]

12.24 A single cylinder double acting steam engine has steam being admitted at 10 bar, 0.96 dry and exhausted at 0.5 bar. The expansion occurs up to 1.64 bar. Power output from engine is 60 hp at 210 rpm. The cylinder stroke volume is 0.021 m^3. Determine the diagram factor. [0.75]

12.25 A single cylinder double acting steam engine has bore of 25 cm and stroke to bore ratio of 1.2. What shall be the indicated power output from engine if steam is admitted at 10.35 bar and discharged at 0.34 bar. The diagram factor is 0.81 and cut off occurs at 50% of stroke. [94.27 hp]

12.26 In a double acting steam engine the steam is supplied at 6.5 bar and 0.85 dry and exhausted at 0.28 bar. The engine runs with speed of 150 rpm. The diameter of piston is 500 mm and piston rod has diameter of 64 mm. The expansion ratio at the front end and back end of piston are 10 and 8 respectively. Stroke length is 600 mm. Steam is being supplied at 6.8 kg per ihp. hr. Steam leaving engine is condensed and the feed water at 35°C is sent back. Determine thermal efficiency and total indicated horse power of engine. [19%, 159.6 hp]

12.27 A steam engine has throttle governing such that it develops 37.5 kW with steam consumption of 1000 kg/hr. The steam consumption at no load is 125 kg/hr. Determine the steam consumption for the indicated power developed being 25.8 kW. [750 kg/hr]

12.28 Determine the missing quantity of steam at cut-off, release and percentage reevaporation for the double acting steam engine having following details.

Steam pressure at cut-off = 6 bar, the cut-off point is at distance of 38 cm from zero.

Steam pressure at release = 3 bar, the point of release is at distance of 8.3 cm from zero.

Steam pressure at beginning of compression = 1.05 bar, the point of start of compression is at distance of 1.3 cm from zero.

Clearance length = 0.86 cm

Speed of engine = 150 rpm

Steam consumption = 270 kg/hr

Steam is supplied dry saturated.

Mean stroke volume = 7300 cm^3

[0.005 kg, 0.003 kg, 32.4%]

12.29 In a two cylinder double acting compound steam engine the stroke in each cylinder is 50 cm and the indicated power output is 220 kW at speed of 270 rpm. Steam is supplied at 12 bar and exhausted at 0.28 bar. The total expansion ratio through the engine is 10 and the two cylinders develop equal power. Neglecting clearance volume and assuming expansion to be complete in HP cylinder and hyperbolic throughout determine the diameters of HP and LP cylinder. [37.3 cm, 47.5 cm]

12.30 A double acting compound steam engine has HP and LP cylinder with steam being admitted at 17.25 bar and exhausted at 0.415 bar. Two cylinders have common stroke length with LP cylinder having bore of 45 cm and stroke of 40 cm and HP cylinder bore is 28.5 cm. Engine runs at 270 rpm and the total expansion ratio is 9. The diagram factor of engine referred to LP cylinder is 0.78. Considering hyperbolic expansion, zero clearance and each cylinder having equal initial piston loads determine the intermediate pressure and indicated power of engine. [5.23 bar, 364.75 hp]

12.31 A double acting compound steam engine has stroke length of 40 cm for both HP and LP cylinders. The bore of HP and LP cylinders are 30 cm and 60 cm respectively. The steam is supplied at pressure of 13.8 bar and the back pressure is 0.28 bar. The cut-off in HP cylinder occurs at $\frac{1}{3}$ rd of stroke. The engine develops brake power of 125 kW while running at 160 rpm. Considering mechanical efficiency of 0.78 determine the actual and hypothetical mean effective pressures, overall diagram factor. [2.66 bar, 3.73 bar, 0.7]

12.32 Determine the overall diagram factor and the actual mean effective pressure of a triple expansion engine having following details:

Steam admission pressure = 13.8 bar, Back pressure = 0.28 bar, Cut-off in HP = 60% of stroke

	HP	IP	LP
Bore (cm)	30	45	75
Indicator Diagram area (mm^2)	1805	1875	2000
Indicator diagram length (mm)	104	105	103
Indicator constant ($kg/mm^2/m$)	40	16	5.35

[0.76, 3.18 bar]

12.33 A compound steam engine has following details;

No. of cylinders = 2, HP and LP.

HP cylinder piston area = 480 cm^2 front and 450 cm^2 back end.

LP cylinder piston area = 1920 cm^2 front and 1890 cm^2 back end.

Mean effective pressure of HP = 2.8 bar front and 2.67 back end.

Mean effective pressure of LP = 0.77 bar front and 0.84 back end.

Stroke length = 92 cm for both HP and LP.

Steam admission = 10.5 bar, dry saturated.

Temperature of hot well = 48°C

Speed of engine = 198 rpm

Steam consumption = 1225 kg per hour

Condenser circulating water requirement = 23400 kg/hr

Circulating water temperature rise = From 11°C to 36°C

Draw up a heat balance for engine on per minute basis and also determine thermal efficiency. [Thermal efficiency = 17.7%]

13
Nozzles

13.1 INTRODUCTION

A nozzle is a flow passage of varying cross sectional area in which the velocity of fluid increases and pressure drops in the direction of flow. Thus in nozzle the fluid enters the variable cross section area duct with small velocity and high pressure and leaves it with high velocity and small pressure. During flow through nozzle the enthalpy drops and heat drop in expansion is spent in increasing the velocity of fluid. Similar to nozzle a duct with variable cross-section area will be called diffuser if the fluid gets decelerated, causing a rise in pressure along the direction of flow. Nozzles are generally used in turbines, jet engines, rockets, injectors, ejectors etc.

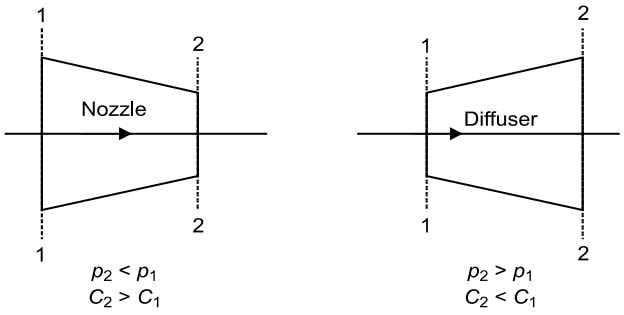


Fig. 13.1 *General arrangement in nozzle and diffuser*

Here in this chapter the one-dimensional analysis of nozzle has been considered.

Different types of nozzles, thermodynamic analysis and different phenomenon in nozzles are discussed ahead.

Momentum transfer across the control volume may be accounted for as,

[Time rate of momentum transfer across the control volume] = $m'C$

Newton's law states that resultant force F acting on the control volume equals the difference between the rates of momentum leaving and entering the control volume accompanying mass flow. Momentum equation says;

$$F = m'_2 C_2 - m'_1 C_1$$

Since at steady state, $m'_2 = m'_1$ i.e. continuity equation being satisfied

$$F = m' (C_2 - C_1)$$

The resultant force F includes forces due to pressure acting at inlet and exit, forces acting on the portion of the boundary through which there is no mass flow, and force due to gravity.

13.2 ONE DIMENSIONAL STEADY FLOW IN NOZZLES

Here one dimensional steady flow analysis in nozzle is carried out assuming the change in cross-sectional area and axis to be gradual and thermodynamic properties being uniform across planes normal to axis of duct. In general real flow through nozzle is not truly one-dimensional but this assumption offers fairly correct analysis. Flow through nozzle occurs at comparatively high velocities in many engineering applications and so exhibits changes in fluid density. Therefore, it is necessary to first look at the compressible flow preliminaries.

Compressible flow preliminaries: Let as consider compressible flow and obtain momentum equation for one dimensional steady flow.

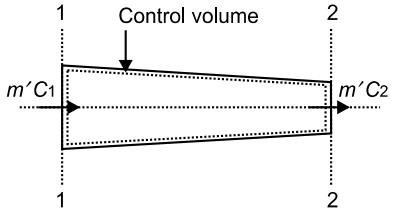


Fig. 13.2

The one dimensional steady flow through a duct is shown above. For control volume shown the principle of conservation of mass, energy and Newton's law of motion may be applied.

By Newton's law of motion, $F = m \cdot a$ where F is the resultant force acting on system of mass 'm' and acceleration 'a'.

Pressure waves and Sound waves: Let us consider a cylindrical duct having piston on one end for generating the pressure wave. Figure 13.3 shows the arrangement for producing a pressure wave moving to right with velocity 'a'. Sound wave is also a small pressure disturbance that propagates through a gas, liquid or solid with velocity 'a' that depends on the properties of medium.

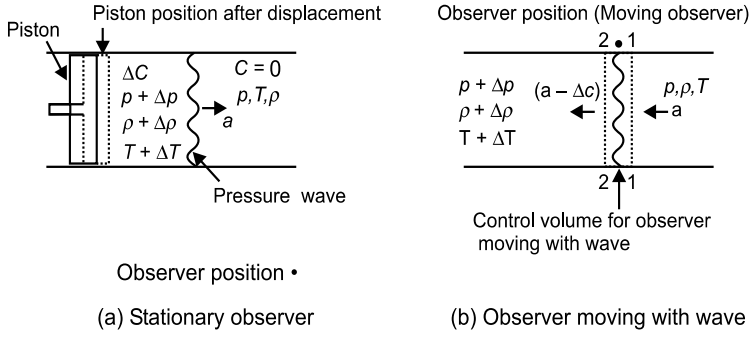


Fig. 13.3 *Propagation of pressure wave (sound wave)*

Figure 13.3 shows how the generation of pressure wave causes displacement of fluid thereby causing rise in pressure, density and temperature by Δp, $\Delta \rho$ and ΔT in respect to the region on the right of wave (undisturbed region). In the undisturbed region say pressure, density, temperature and fluid velocity be p, ρ, T and $C = 0$ respectively. Due to piston movement fluid velocity increases by ΔC and other properties also change by elemental values as shown. For analysing there are two approaches available as shown in Figs. 13.3 (a) and (b). One approach considers observer to be stationary and gas moving and second approach considers observer to be moving along with wave i.e. relative velocity of observer with respect to wave is zero.

Respective values of fluid velocity, wave propagation velocity, pressure, density and temperature are labelled on figure. For an observer at rest relative to wave (observer moving with wave) it seems as if the fluid is moving towards the stationary wave from right with velocity a, pressure p, density ρ and temperature T and moves away on left with velocity '$a - \Delta C$', pressure '$p + \Delta p$', density '$\rho + \Delta\rho$' and temperature '$T + \Delta T$'.

From conservation of mass, applying continuity equation upon control volume we get

$$m'_1 = m'_2 = m'$$

$$\rho \cdot A \cdot a = (\rho + \Delta\rho) \cdot A(a - \Delta C)$$

where A is constant cross section area of duct.

$$\rho \cdot A \cdot a = (\rho \cdot A \cdot a) - (\rho \cdot A \cdot \Delta C) + (\Delta\rho \cdot A \cdot a) - (\Delta\rho \cdot A \cdot \Delta C)$$

Upon neglecting higher order terms and rearranging we get,

$$(a \cdot \Delta\rho) - (\rho \cdot \Delta C) = 0$$

or,
$$\boxed{\Delta C = \frac{a \cdot \Delta\rho}{\rho}}$$

Applying momentum equation to the control volume;

$$(p \cdot A) - ((p + \Delta p) \cdot A) = (m' (a - \Delta C)) - (m' \cdot a)$$

$$- \Delta p \cdot A = m' \cdot (- \Delta C)$$

for mass flow rate m' we can write, $m' = \rho \cdot A \cdot a$

so, $\Delta p \cdot A = \rho \cdot A \cdot a \cdot \Delta C$

or,
$$\boxed{\Delta C = \frac{\Delta p}{\rho \cdot a}}$$

Equating two values obtained for 'ΔC' we get

$$\frac{a \cdot \Delta\rho}{\rho} = \frac{\Delta p}{\rho \cdot a}$$

$$\boxed{a = \sqrt{\frac{\Delta p}{\Delta\rho}}}$$

Thus, velocity of wave propagation comes out as the square root of the ratio of change in pressure and change in density.

In case of sound waves the magnitude of changes in pressure, density and temperature are infinitesimal and so these may also be called as infinitesimal pressure wave. It is also seen that thermodynamic process occurring across an infinitesimal pressure wave may be considered nearly isentropic. Therefore the velocity of sound can be given as square root of derivative of pressure with respect to density across the wave under isentropic conditions.

$$\boxed{a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_{s=\text{constt.}}}}$$

in terms of specific volume values; $d\rho = \dfrac{-dv}{v^2}$

so,
$$\boxed{a = \sqrt{-v^2 \left(\frac{\partial p}{\partial v}\right)_{s=\text{constt}}}}$$

Let us consider fluid to be a perfect gas following isentropic process given by pv^k = constt. Taking log of both sides and then partially differentiating we get,

$$\left(\frac{\partial p}{\partial v}\right)_s = \frac{-k \cdot p}{v}$$

Substituting in expression for sound velocity

$$\boxed{a = \sqrt{k\,p\,v}}$$

For ideal gas,

$a = \sqrt{k\,R\,T}$. In case of air, $a = \sqrt{\gamma\,R\,T}$

Using the velocity of sound and fluid velocity a non dimensional parameter called Mach number can be defined. Mach number is given by the ratio of velocity of fluid (object) to the velocity of sound. It is generally denoted by *M*.

$$\boxed{M = \frac{C}{a}}$$

Based upon Mach no. value flow can be classified as given below.

For

$M < 1$ flow is called subsonic flow.

$M = 1$ flow is called sonic flow.

$M > 1$ flow is called supersonic flow.

Nozzle flow analysis: Let us consider one dimensional steady flow in nozzles. Let us take a varying cross-section area duct such that velocity increases and pressure decreases from inlet to exit.

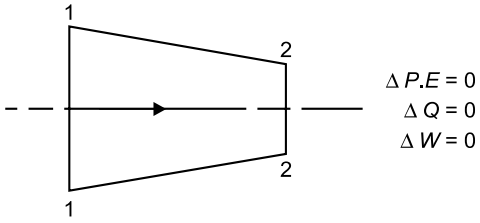


Fig. 13.4

From conservation of mass, upon applying continuity equation, it can be given that,

$\rho \cdot A \cdot C$ = constant

Taking log of both the sides,

$\ln \rho + \ln A + \ln C = \ln$ constant

Differentiating partially we get,

$$\boxed{\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dC}{C} = 0}$$

Let us now apply steady flow energy equation assuming no change in potential energy, no heat interaction and no work interaction during the flow across control volume.

S.F.E.E. yields, $dq = dh + dw + d(\text{KE}) + d(\text{PE})$

Applying assumptions,

$dh + d(\text{KE}) = 0$

It can be rewritten for section 1 and 2 as, $\left\{\text{KE} = \frac{C^2}{2}\right\}$

or $h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$

$h_{o1} = h_{o2}$

Stagnation enthalpy at section 1 = Stagnation enthalpy at section 2.

From differential form, $dh + d\left(\frac{C^2}{2}\right) = 0$

$dh + 2\,\frac{C\,dC}{2} = 0$

or, $\boxed{dh = -\,CdC}$

From first and second law combined we know,

$$\boxed{dh = Tds + vdp}$$

Using the adiabatic flow considerations, $ds = 0$, so

$$\boxed{dh = vdp = \frac{dp}{\rho}}$$

Above shows that with increase or decrease in pressure along the direction of flow the specific enthalpy also change in same way.

From thermodynamic property relations pressure can be given as function of density and entropy i.e. $p = p(\rho, s)$.

or, $$dp = \left(\frac{\partial p}{\partial \rho}\right)_s \cdot d\rho + \left(\frac{\partial p}{\partial s}\right)_\rho \cdot ds$$

For isentropic flow considerations

$$dp = \left(\frac{\partial p}{\partial \rho}\right)_s \cdot d\rho$$

We know from sound velocity $a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$ so,

$$\boxed{dp = a^2 \cdot d\rho}$$

Combining two expressions for dh we get

$-C \cdot dC = \frac{dp}{\rho}$. This shows that as pressure increases in direction of flow then velocity must decrease. Substituting from dp as obtained above, it yields,

$$-C \cdot dC = \frac{a^2 \cdot d\rho}{\rho}$$

or,

$$\boxed{-\left(\frac{dC}{C}\right) = \frac{a^2}{C^2}\left(\frac{d\rho}{\rho}\right)} \Rightarrow \boxed{\left(\frac{d\rho}{\rho}\right) = -\frac{C^2}{a^2}\left(\frac{dC}{C}\right)}$$

Substituting above in the equation available from continuity equation,

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dC}{C} = 0$$

or,

$$\boxed{\frac{dA}{A} = -\frac{dC}{C} - \frac{d\rho}{\rho}}$$

$$= -\frac{dC}{C} + \left(\frac{C^2}{a^2}\right)\frac{dC}{C}$$

or

$$\frac{dA}{A} = \frac{dC}{C}\left\{\left(\frac{C^2}{a^2}\right) - 1\right\}$$

As Mach no. $M = \frac{C}{a}$ so,

$$\boxed{\frac{dA}{A} = \frac{dC}{C}\{M^2 - 1\}}$$

Using above relation the effect of area variation upon the flow can be seen in subsonic, sonic and super sonic flow regimes.

Case 1

For subsonic flow i.e. $M < 1$

Nozzle: For positive velocity gradient i.e. velocity increases along the direction of flow as in case of nozzle $\left(\frac{dC}{C}\text{ is} + \text{ve}\right)$, it yields $\frac{dA}{A}$ as – ve. Negative area gradient means cross section area decreases along the direction of flow.

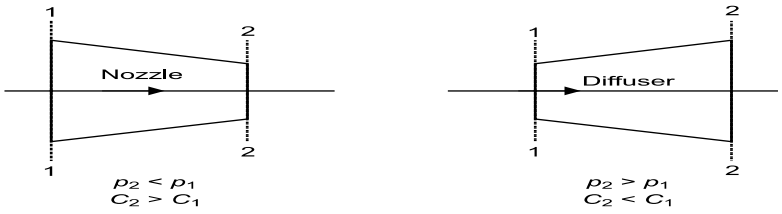
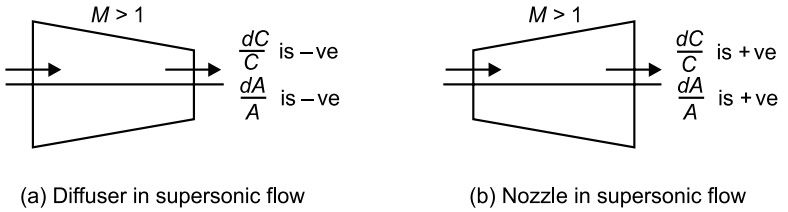


Fig. 13.5

Diffuser: For negative velocity gradient $\left(\text{i.e., } \frac{dC}{C}\text{ is} - \text{ve}\right)$ the velocity decreases along the direction of flow as in case of diffuser, it yields $\frac{dA}{A}$ as + ve. Positive area gradient means duct has diverging cross section area along the direction of flow.

Case 2 For supersonic flow i.e. M > 1

Nozzle: For positive velocity gradient i.e. $\frac{dC}{C}$ being + ve, it yields $\frac{dA}{A}$ as + ve. It means that in supersonic flow the nozzle duct shall have diverging cross-sectional area along the direction of flow.



(a) Diffuser in supersonic flow (b) Nozzle in supersonic flow

Fig. 13.6

Diffuser: For negative velocity gradient i.e. $\frac{dC}{C}$ being – ve it yields $\frac{dA}{A}$ as – ve. It means in supersonic flow the diffuser duct shall have converging cross-sectional area along the direction of flow.

From above discussion it can be concluded that

(i) Nozzle must be of convergent duct type in subsonic flow region and such nozzles are called subsonic nozzles or convergent nozzles.

(ii) Nozzle must be of divergent duct type in supersonic flow region and such nozzles are called supersonic nozzles or divergent nozzles.

(iii) For acceleration of fluid flow from subsonic to supersonic velocity the nozzle must be first of converging type till flow becomes sonic and subsequently nozzle should be of diverging type in supersonic flow. The portion of duct at which flow becomes sonic (M = 1) and dA is zero i.e. duct is constant cross-section area duct, is called throat. Thus in this type of flow from subsonic to supersonic the duct is of converging type followed by throat and a diverging duct. Such nozzles are also called convergent-divergent nozzles. Throat gives the minimum cross-section area in convergent-divergent nozzles.

Let us consider the expansion through a nozzle between sections 1 and 2. In nozzle the velocity of fluid is so high that there is no time available for heat exchange with the surroundings and the expansion may be considered adiabatic. Also the change in potential energy may be negligible if the elevation does not change from inlet to exit. Work done during flow is absent.

Application of steady flow energy equation yields,

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

Velocity at exit from nozzle:

$$\boxed{C_2 = \sqrt{2(h_1 - h_2) + C_1^2}\ \text{, m/s}}$$

For negligible velocity of fluid at inlet to nozzle, $C_1 \approx 0$

$\boxed{C_2 = \sqrt{2(h_1 - h_2)}\ \text{, m/s,}}$ where h_1 and h_2 are enthalpy in J/kg at sections 1 and 2 respectively.

Expansion of fluid on *p-v* diagram is shown below.

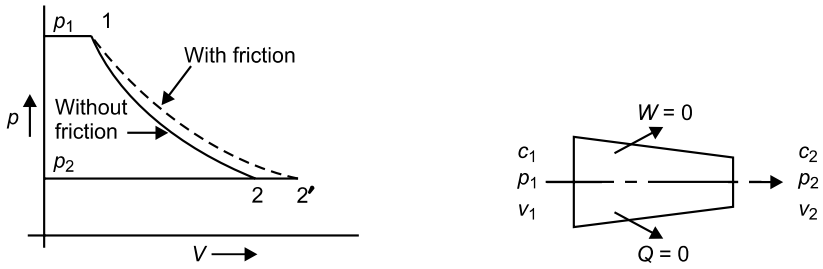


Fig. 13.7 P-V *diagram for flow through nozzle*

Expansion of gases on T-s diagram is as shown in Fig. 13.8.

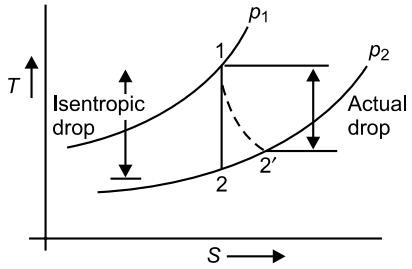


Fig. 13.8 T-s *diagram for flow through nozzle*

Expansion of steam on *T-s* and *h-s* diagram for superheated steam and wet steam is shown by 1–2 and 3–4 respectively under ideal conditions.

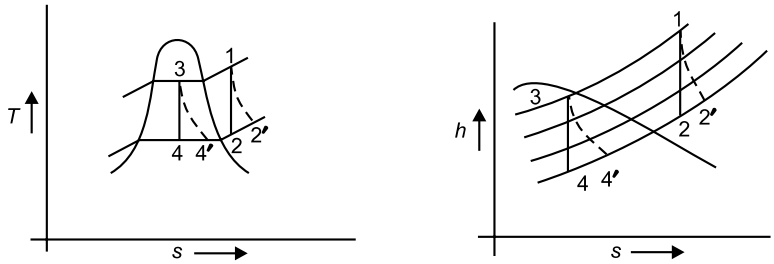


Fig. 13.9 *T-s and* h-s *representation for steam flow through nozzle*

In above representations the isentropic heat drop shown by 1–2 and 3–4 is also called 'Rankine heat drop'.

Mass flow through a nozzle can be obtained from continuity equation between sections 1 and 2.

$$m' = \frac{A_1 C_1}{v_1} = \frac{A_2 C_2}{v_2}$$

Mass flow per unit area; $\boxed{\frac{m'}{A_2} = \frac{C_2}{v_2}}$

From different from of S.F.E.E.

$$dq = dh + dw + d(\text{K.E.}) + d(\text{P.E.})$$

or, $$dh + d(\text{K.E.}) = 0$$

$du + pdv + vdp + d(\text{K.E.}) = 0$

also as $dq = du + pdv = 0$, so $d(\text{K.E.}) = -vdp$

or $$\frac{C_2^2 - C_1^2}{2} = -\int_{p_1}^{p_2} v\,dp$$

For the expansion through a nozzle being governed by process pv^n = constt.,

$$C_2^2 - C_1^2 = 2\left(\frac{n}{n-1}\right) p_1 v_1 \left(1 - \frac{p_2 v_2}{p_1 v_1}\right)$$

or,

Velocity at exit from nozzle $$\boxed{C_2 = \sqrt{2\left(\frac{n}{n-1}\right) p_1 v_1 \left(1 - \frac{p_2 v_2}{p_1 v_1}\right) + C_1^2}}$$

For negligible inlet velocity, say $C_1 \approx 0$

Velocity at exit from nozzle

$$\boxed{C_2 = \sqrt{2\left(\frac{n}{n-1}\right) p_1 v_1 \left(1 - \frac{p_2 v_2}{p_1 v_1}\right)}}$$

If the working fluid is perfect gas then $n = \gamma$ and for air $\gamma = 1.4$. However, if working fluid is steam a good approximation for n can be obtained from some polytropic considerations. For steam being dry saturated initially and process of expansion occurring in wet region the index n can be approximated as 1.135. For steam being initially superheated and expanded in superheated region the index n can be approximated as 1.3.

Looking at mathematical expression for exit velocity it could be concluded that maximum exit velocity is possible only when fluid is expanded upto zero pressure. The maximum velocity is,

$$\boxed{C_{max} = \sqrt{2\left(\frac{n}{n-1}\right) p_1 v_1}}$$

Mass flow rate,

$$m' = \frac{A_2 C_2}{v_2}$$

Mass flow rate per unit area,

$$\frac{m'}{A_2} = \frac{\sqrt{2\left(\frac{n}{n-1}\right) p_1 v_1 \left(1 - \frac{p_2 v_2}{p_1 v_1}\right)}}{v_2}$$

From expansion's governing equation, $p_1 v_1^n = p_2 v_2^n$

or, $$v_2 = \left(\frac{p_1}{p_2}\right)^{1/n} \cdot v_1$$

or,

$$\frac{m'}{A_2} = \frac{\sqrt{2\left(\frac{n}{n-1}\right)p_1v_1\left(1-\frac{p_2v_2}{p_1v_1}\right)}}{\left(\frac{p_1}{p_2}\right)^{1/n}\cdot v_1}$$

or,

$$\frac{m'}{A_2} = \left[2\left(\frac{n}{n-1}\right)\frac{p_1}{v_1}\left\{\left(\frac{p_2}{p_1}\right)^{2/n} - \left(\frac{p_2}{p_1}\right)^{\left(\frac{(n+1)}{n}\right)}\right\}\right]^{1/2}$$

This expression for mass flow rate through nozzle depends upon inlet and exit pressures, initial specific volume and index of expansion. It has been seen earlier that the mass flow per unit area is maximum at throat and nozzle should be designed for maximum discharge per unit area. Thus there will be some value of throat pressure (p_2) which offers maximum discharge per unit area. Mathematically this pressure value can be obtained by differentiating expression for mass flow per unit area and equating it to zero. This pressure at throat for maximum discharge per unit area is also called 'critical pressure' and pressure ratio with inlet pressure is called 'critical pressure ratio'.

Let pressure ratio $\frac{p_2}{p_1} = r$, then mass flow per unit area can be re-written as;

$$\frac{m'}{A_2} = \left\{2\left(\frac{n}{n-1}\right)\frac{p_1}{v_1}\left(r^{2/n} - r^{\frac{(n+1)}{n}}\right)\right\}^{1/2}$$

$$\frac{d\left(\frac{m'}{A_2}\right)}{dr} = \frac{d}{dr}\left\{2\left(\frac{n}{n-1}\right)\frac{p_1}{v_1}\left(r^{2/n} - r^{\frac{(n+1)}{n}}\right)\right\}^{1/2}$$

Here p_1, v_1 are inlet conditions and remain constant. Also n being index of expansion remains constant so differentiating and putting equal to zero.

$$\frac{2}{n}\cdot r^{\frac{(2-n)}{n}} - \left(\frac{n+1}{n}\right)r^{1/n} = 0$$

or,

$$\frac{2}{n}\cdot r^{\frac{(2-n)}{n}} = \left(\frac{n+1}{n}\right)r^{1/n}$$

$$r^{\frac{1-n}{n}} = \left(\frac{n+1}{2}\right)$$

or, Critical pressure ratio,

$$\boxed{r = \left(\frac{2}{n+1}\right)^{\frac{n}{(n-1)}}}$$

Let critical pressure at throat be given by p_c or p_t then,

$$\boxed{\frac{p_c}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{(n-1)}}} \quad \Rightarrow \quad \boxed{\frac{p_t}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{(n-1)}}}$$

Here subscript '*c*' and '*t*' refer to critical and throat respectively.

While designing a nozzle the critical pressure ratio at throat is equal to the one obtained above. Critical pressure ratio value depends only upon expansion index and so shall have constant value. Value of adiabatic expansion index and critical pressure ratio are tabulated ahead;

Table 13.1: Adiabatic expansion index and critical pressure ratio for selected fluids

Fluid	Adiabatic expansion index, n	Critical pressure ratio $\frac{p_c}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{(n-1)}}$
Wet steam	1.135 (n = 1.035 + 0.1x, where x is dryness fraction of wet steam)	0.577
Superheated steam	1.3	0.545
Air	1.4	0.528

The maximum discharge per unit area can be obtained by substituting critical pressure ratio in expression for mass flow per unit area at throat section.

$$\frac{m'}{A_t} = \sqrt{\left[2\left(\frac{n}{n-1}\right)\cdot\frac{p_1}{v_1}\left\{\left(\frac{2}{n+1}\right)^{\frac{2}{(n-1)}} - \left(\frac{2}{n+1}\right)^{\frac{(n+1)}{(n-1)}}\right\}\right]}$$

$$\frac{m'}{A_t} = \left[\left(\frac{2n}{n-1}\right)\cdot\frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{(n+1)}{(n-1)}}\cdot\left\{\left(\frac{2}{n+1}\right)^{\frac{(1-n)}{(n-1)}} - 1\right\}\right]^{1/2}$$

$$= \left[\left(\frac{2n}{n-1}\right)\cdot\frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{(n+1)}{(n-1)}}\cdot\left\{\frac{n+1}{2} - 1\right\}\right]^{1/2}$$

$$\frac{m'}{A_t} = \left[n\cdot\frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{(n+1)}{(n-1)}}\right]^{1/2}$$

Maximum discharge per unit area = $\sqrt{n\cdot\frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{(n+1)}{(n-1)}}}$

For this maximum discharge per unit area at throat the velocity at throat can be obtained for critical pressure ratio. This velocity may also be termed as 'critical velocity'.

$$C_2 = \sqrt{2\left(\frac{n}{n-1}\right)\left(p_1v_1 - p_2v_2\right)}$$

At throat

$$C_t = \sqrt{2\left(\frac{n}{n-1}\right)p_tv_t\left(\frac{p_1v_1}{p_tv_t} - 1\right)}$$

$$C_t = \sqrt{2\left(\frac{n}{n-1}\right)\cdot p_t v_t \left\{\left(\frac{p_t}{p_1}\right)^{\frac{(1-n)}{n}} - 1\right\}}$$

Substituting critical pressure ratio $\left(\frac{p_t}{p_1}\right)$

$$C_t = \sqrt{2\left(\frac{n}{n-1}\right)\cdot p_t v_t \left\{\left(\frac{n+1}{2}\right) - 1\right\}}$$

$C_t = \sqrt{n\, p_t v_t}$ Hence, critical velocity = $\sqrt{n \cdot p_t \cdot v_t}$

For perfect gas; $C_t = \sqrt{n \cdot R T_t}$

For $n = \gamma$, $C_t = \sqrt{\gamma R T_t} = a$ = Velocity of sound.

Thus it can be concluded that for maximum discharge per unit area at throat the fluid velocity (critical velocity) equals to the sonic velocity. At the throat section mach no. $M = 1$ for critical pressure ratio.

For perfect gas:

All the above equations obtained for the flow through nozzle can also be obtained for perfect gas by substituting $n = \gamma$ and $pv = RT$

Velocity at exit from nozzles

$$C_2 = \sqrt{2\left(\frac{\gamma}{\gamma-1}\right)\cdot(p_1 v_1 - p_2 v_2)}$$

or,

$$C_2 = \sqrt{2\left(\frac{\gamma}{\gamma-1}\right) R(T_1 - T_2)}$$

or,

$$C_2 = \sqrt{2\, c_p (T_1 - T_2)} \qquad \text{as } c_p = \frac{\gamma R}{\gamma - 1}$$

or,

$$C_2 = \sqrt{2\,(h_1 - h_2)}$$

Critical velocity at throat, $C_t = \sqrt{\gamma R T_t}$

Mass flow rate per unit area,

$$\frac{m'}{A_2} = \left[2\left(\frac{\gamma}{\gamma-1}\right)\cdot\frac{p_1}{v_1}\left\{\left(\frac{p_2}{p_1}\right)^{2/\gamma} - \left(\frac{p_2}{p_1}\right)^{\frac{(\gamma+1)}{\gamma}}\right\}\right]^{1/2}$$

Maximum discharge per unit area at throat for critical conditions,

$$\frac{m'}{A_t} = \sqrt{\gamma \cdot \frac{p_1}{v_1}\left(\frac{2}{\gamma+1}\right)^{\frac{(\gamma+1)}{(\gamma-1)}}}$$

Critical pressure ratio,

$$\frac{p_c}{p_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{(\gamma-1)}}$$

13.3 CHOKED FLOW

Let us consider a converging nozzle as shown in Fig. 13.10 with arrangement for varying back pressure. A valve is provided at exit of nozzle for regulating the back pressure at section 2-2. Let us denote back pressure by p_b. Expansion occurs in nozzle from pressure p_1 to p_b.

Initially when back pressure p_b is equal to p_1 there shall be no flow through the nozzle but as back pressure p_b is reduced the mass flow through nozzle increases. With the reduction in back pressure a situation comes when pressure ratio equals to critical pressure ratio (back pressure attains critical pressure value) then mass flow through nozzle is found maximum. Further reduction in back pressure beyond critical pressure value does not affect the mass flow i.e. mass flow rate does not increase beyond its' limiting value at critical pressure ratio. Thus under these situations flow is said to be choked flow or critical flow.

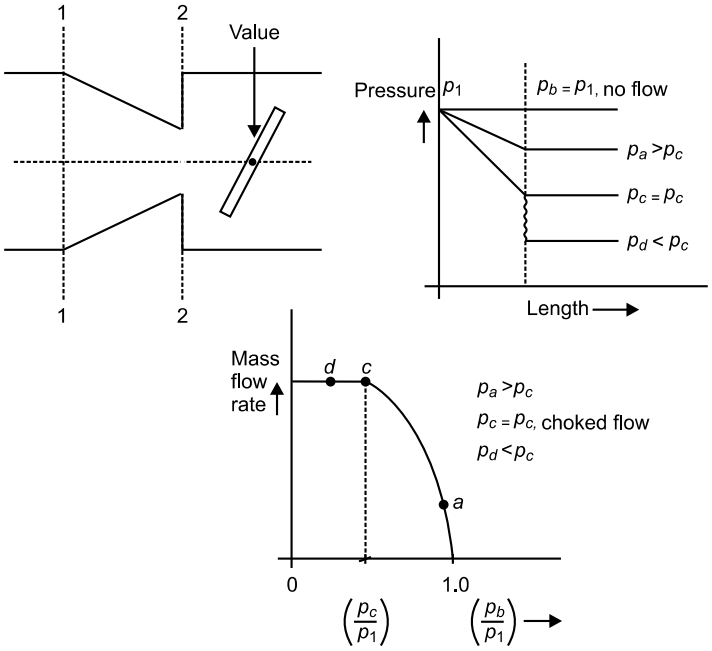


Fig. 13.10 *Flow through a convergent nozzle*

A nozzle operating with maximum mass flow rate condition is called choked flow nozzle. At the critical pressure ratio the velocity at exit is equal to the velocity of sound. If the back pressure is reduced below critical pressure then too the mass flow remains at maximum value and exit pressure remains as critical pressure and the fluid leaving nozzle at critical pressure expands violently down to the reduced back pressure value. Graphical representation of mass flow rate with pressure ratio and variation of pressure along length of nozzle explain the above phenomenon. State *a* refers to the state having back pressure more than critical pressure, state *c* refers to the state having back pressure equal to critical pressure and state *d* refers to state having back pressure less than critical pressure.

In case of convergent-divergent nozzle also the maximum mass flow through such nozzle shall be obtained when pressure ratio at throat section equals critical pressure ratio and velocity at throat equals sonic velocity. The cross-sectional area of throat decides the mass flow through nozzle for given inlet conditions.

13.4 OFF DESIGN OPERATION OF NOZZLE

Design operation of nozzle refers to the nozzle operating with pressure ratio equal to critical pressure ratio and maximum discharge rate per unit area then nozzle is said to be operating under design conditions. If the nozzle does not operate under design conditions then it is called off design operation of nozzle. Depending upon the back pressure value in reference to design value of pressure at exit of nozzle, the nozzle can be classified as under-expanding, over-expanding nozzles.

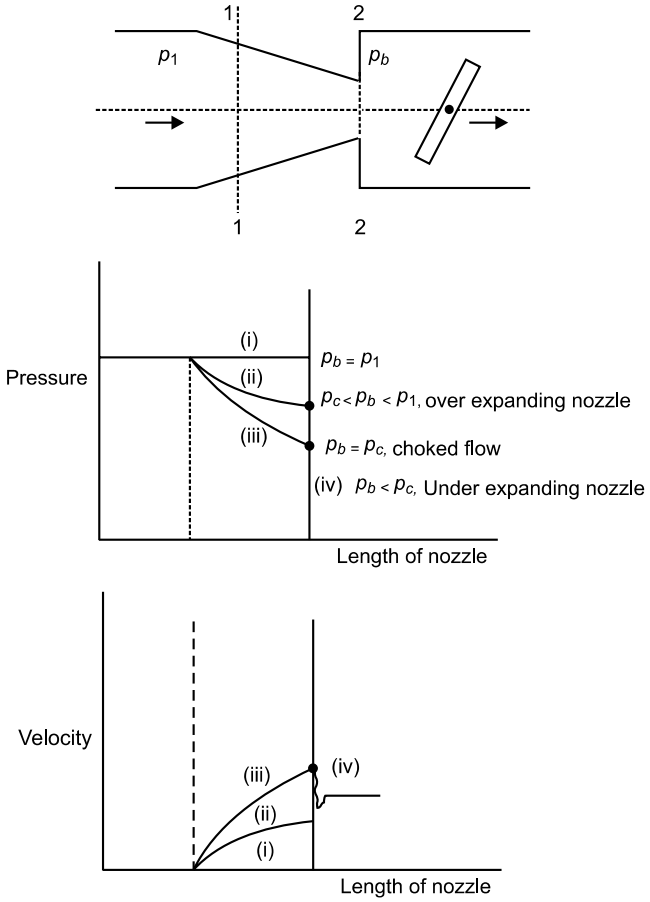


Fig. 13.11 *Off design operation of converging nozzle*

Nozzle is called under-expanding if the back pressure of nozzle is below the designed value of pressure at exit of nozzle. Nozzle is called over expanding if the back pressure of a nozzle is above designed value of pressure at exit of nozzle. Detail discussion about the off design operation of nozzle is given ahead for convergent and convergent-divergent nozzle.

Convergent nozzle: Let us look at convergent nozzle having arrangement for varying back pressure. Fluid enters the nozzle at state 1, say pressure p_1. Variation of back pressure using valve at exit of nozzle shows the pressure and velocity variation as shown in Fig. 13.11. Following significant operating states are shown here.

(i) When back pressure $p_b = p_1$, there is no flow.

(ii) When back pressure is such that back pressure is more than critical pressure i.e. $p_c < p_b < p_1$, there is flow through nozzle. Here p_c is critical pressure at exit. This operating state of nozzle having back pressure higher than critical pressure is called over expanding nozzle. In this over expanding nozzle the mass flow rate through nozzle is less than designed value.

(iii) When back pressure is such that back pressure is equal to critical pressure i.e. $p_b = p_c$. In this situation the mass flow through nozzle is maximum and nozzle is said to be choked.

(iv) When back pressure is further lowered such that back pressure is less than critical pressure i.e. $p_b < p_c$, the nozzle is said to be under expanding nozzle. In underexpanding nozzle there is no change in specific volume, velocity and mass flow rate through exit as that at choked flow state of nozzle. Since back pressure at exit is less than critical pressure while fluid leaves nozzle at critical pressure so fluid expands violently and irreversibly upto back pressure outside the nozzle.

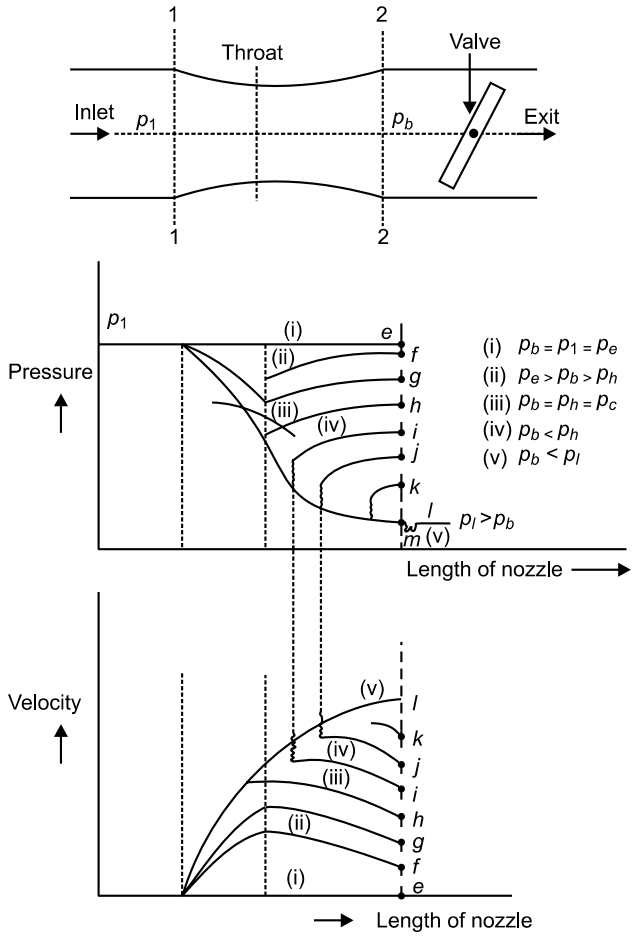


Fig. 13.12 *Off design operation of convergent-divergent nozzle.*

Convergent-divergent nozzle: Converging-diverging nozzles are generally used for accelerating flow up to supersonic velocity. Arrangement for varying back pressure is as shown in Fig. 13.12.

Different operating regime of nozzle are obtained by varying back pressure using valve at exit. Let us start with back pressure equal to inlet pressure. Following operating states are shown here;

(i) When back pressure p_b is equal to inlet pressure p_1 then there is no flow as shown by state *e* on pressure-nozzle length plot.

(ii) When back pressure is reduced and is slightly below p_1, then there is some flow through nozzle shown by state *f*. The maximum velocity and minimum pressure occurs at throat section. With further reduction in back pressure from *e* to *h*, the flow rate increases. Flow remains subsonic for back pressure between *e* to *h*. In subsonic region the diverging portion of nozzle acts as diffuser, thereby increasing pressure and decreasing velocity in the direction of flow. In this regime the convergent-divergent nozzle is also used as venturimeter for flow rate measurement. Nozzle is said to be over expanding nozzle.

(iii) With further reduction in back pressure the throat pressure decreases and the throat velocity increases. Back pressure at which throat velocity becomes sonic and throat pressure equals to critical pressure p_c is shown by state *h*. Here maximum velocity occurs at throat so the diverging portion of nozzle still acts as diffuser. Mass flow rate through nozzle has become maximum as the sonic flow conditions are obtained at throat. Thus it can be seen that flow through converging duct with subsonic velocity at inlet can never result in the velocity higher than sonic velocity and pressure less than critical pressure. This is choked flow state of nozzle.

(iv) Further lowering of back pressure less than critical pressure causes no effect on the flow in converging portion of nozzle and the pressure at throat remains equal to critical pressure and velocity at throat remains sonic. Also the flow rate through nozzle does not change. However, the nature of flow in diverging section of the duct changes as the breakage of flow occurs.

As pressure is reduced to *i* and *j* the fluid passing through the throat continues to expand and accelerate in diverging portion of nozzle. Flow velocity beyond throat is supersonic in diverging portion of nozzle. At the section downsteam of throat there occurs discontinuity in the flow due to abrupt irreversible increase in pressure accompanied by deceleration from supersonic to subsonic velocity. This discontinuity in flow is called shock and generally plane of discontinuity is normal to direction of flow so it may also be called normal shock. Flow through shock is of irreversible and steady adiabatic type. Beyond shock the fluid undergoes further isentropic deceleration as diverging section acts as a subsonic diffuser.

With further reduction in back pressure p_b the shock moves downstream till it approaches nozzle exit plane and p_b approaches the pressure given by state *k*. For the back pressure equal to pressure given by point *k* i.e. p_k the normal shock reaches at exit end of nozzle. Here flow within nozzle is isentropic, subsonic in converging portion, sonic at throat and supersonic in diverging portion. Due to shock the flow leaving nozzle becomes subsonic. These are all over expanding states of nozzle.

When back pressure is further lowered and back pressure becomes equal to exit plane pressure as at state *l* i.e. $p_b = p_l$, the isentropic expansion occurs throughout nozzle and no shock is found during flow. Fluid leaving nozzle is supersonic.

(v) Further lowering of back pressure below p_l flow remains same as for pressure upto point *j*, but the back pressure being less than design pressure causes breaking of flow at downstream of nozzle exit. An abrupt expansion of irreversible type occurs at nozzle exit.

Irrespective of reduced back pressure the pressure at nozzle exit does not go below design pressure p_l and mass flow rate and exit velocity also do not change. This operating state of nozzle is also called under expanding nozzle.

13.5 EFFECT OF FRICTION ON NOZZLE

Inspite of the inside surface of nozzle being smooth the frictional losses always prevail due to friction between fluid and nozzle surface and friction within fluid itself. Due to friction prevailing during fluid flow through nozzle the expansion process through nozzle becomes irreversible. Expansion process since occurs at quite fast rate and time available is very less for heat transfer to take place so it can be approximated as adiabatic.

Friction prevailing during flow through nozzle causes heat drop by about 10–15% and reduces the exit velocity. For the flowing fluid to be gas the T-S diagram representation is as follows:

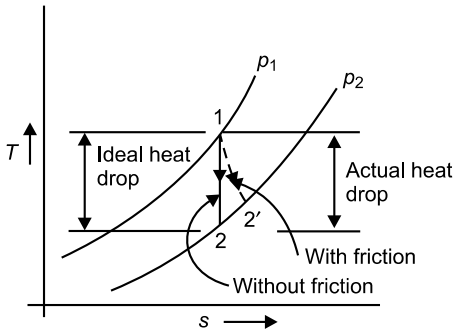


Fig. 13.13 *T-s representation for expansion through nozzle*

Non ideal operation of nozzle causes reduction in enthalpy drop. This inefficiency in nozzle can be accounted for by nozzle efficiency. Nozzle efficiency is defined as ratio of actual heat drop to ideal heat drop. Nozzle efficiency, $\eta_{\text{Nozzle}} = \dfrac{\text{Actual heat drop}}{\text{Ideal heat drop}}$

$$\eta_{\text{Nozzle}} = \frac{h_1 - h_{2'}}{h_1 - h_2}$$

$$\eta_{\text{Nozzle}} = \frac{(T_1 - T_{2'})}{(T_1 - T_2)}$$

In case of working fluid being steam the friction causes heating of steam flowing through nozzle thereby increasing dryness fraction. The volume of steam at exit also increases due to increase in dryness fraction. *T–s* and *h-s* representation of steam expanding through nozzle is given in Fig. 13.14.

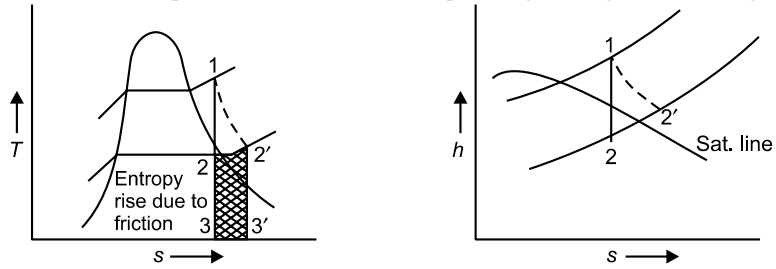


Fig. 13.14 T-s *and* h-s *representation for steam expanding through nozzle*

Due to friction the velocity at exit from nozzle gets modified by nozzle efficiency as given below.

Velocity at exit, $C_2 = \sqrt{2\,(h_1 - h_2) + C_1^2}$, for no friction

In case of nozzle with friction the enthalpy drop, $(h_1 - h_{2'})$ gives velocity at exit as,

$$C_{2'} = \sqrt{2\,(h_1 - h_{2'}) + C_1^2}$$

or, $\quad (h_1 - h_2) = \dfrac{C_2^2 - C_1^2}{2}$

and $\quad (h_1 - h_{2'}) = \dfrac{C_{2'}^2 - C_1^2}{2}$

Substituting in nozzle efficiency, $\eta_{\text{Nozzle}} = \dfrac{\left(C_{2'}^2 - C_1^2\right)}{\left(C_2^2 - C_1^2\right)}$

For negligible inlet velocity i.e. $C_1 \approx 0$

$$\text{Nozzle efficiency, } \eta_{\text{Nozzle}} = \frac{C_{2'}^2}{C_2^2}$$

Thus it could be seen that friction loss will be high with higher velocity of fluid. Generally frictional losses are found to be more in the downstream after throat in convergent-divergent nozzle because of simple fact that velocity in converging section upto throat is smaller as compared to after throat. Expansion upto throat may be considered isentropic due to small frictional losses. Apart from velocity considerations the significantly high frictional loss in diverging portion of nozzle compared to converging portion can be attributed to the contact surface area. Length of converging section upto throat is quite small compared to length of diverging portion after throat as it has subsonic acceleration which can be completed in short length. Diverging section of nozzle is designed comparatively longer than converging section so as to avoid flow separation due to adverse duct geometry (diverging type). Turbulence losses are also significant in diverging portion compared to converging portion. Due to the different factors discussed above the frictional losses are found to be more in diverging portion compared to converging portion.

Normally angle of divergence in divergent portion is kept between 10° and 25° so as to avoid flow separation. But small divergence angle causes increase in length of diverging portion therefore increasing frictional losses. Thus a compromise should be striked in selecting angle of divergence as very small angle is desirable from flow separation point of view but undesirable due to long length and larger frictional losses point of view. Length of diverging portion of nozzle can be empirically obtained as below

$$L = \sqrt{15 \cdot A_t}$$

where A_t is cross-sectional area at throat.

While designing the nozzle parameters due care should be taken for smoothness of nozzle profile, surface finish for minimum friction and ease of manufacturing etc. Thus finally, it can be concluded that nozzle efficiency depends upon nozzle material, size and shape of nozzle, angle of divergence, nature of fluid flowing and its properties etc.

Coefficient of velocity: The ‘coefficient of velocity’ or the ‘velocity coefficient’ can be given by the ratio of actual velocity at exit and the isentropic velocity at exit. Thus it measures the amount of deviation from ideal flow conditions. Mathematically,

$$\text{Coefficient of velocity} = \frac{C_{\text{actual at exit}}}{C_{\text{isentropic at exit}}}$$

Coefficient of discharge: The 'coefficient of discharge' or 'discharge coefficient' is given by the ratio of actual discharge and the discharge during isentropic flow through nozzle. Mathematically,

$$\text{Cofficient of discharge} = \frac{m_{\text{actual}}}{m_{\text{isentropic}}}$$

Here m refers to discharge rate.

13.6 SUPERSATURATION PHENOMENON IN STEAM NOZZLES

The phenomenon of supersaturation in steam nozzles is also called as supersaturated flow or metastable flow in steam nozzle. When superheated steam flows through a nozzle and expands upto the back pressure such that exit state of steam lies in wet region, then during expansion steam vapours expand isentropically and slowly get condensed up to exit state. During such expansion steam also passes across saturated steam line or saturation line having unity dryness fraction. Thus it is obvious that expansion of steam is accompanied by simultaneous state change from superheated state to wet state. At every point along expansion line there exists a mixture of vapour and liquid in equilibrium. An expansion process starting at 1 goes up to state 2 in thermal equilibrium as shown on *T-S* and *h-s* diagram.

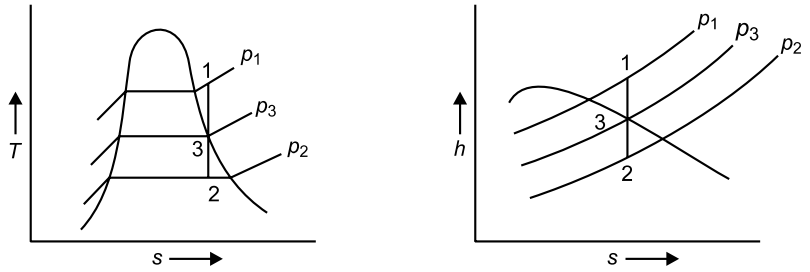


Fig. 13.15 *Expansion of steam on* T-s *and* h-s *diagram under equilibrium*

Superheated steam undergoes continuous change in state and becomes dry saturated steam at state 3 and subsequently wet steam leaving steam turbine at state 2. Some times expansion of steam occurs in metastable equilibrium or in equilibrium in which change of steam state could not maintain its pace with expanding steam. This phenomenon in which change of steam state could not occur simultaneously with expanding steam in nozzle is called phenomenon of supersaturation and flow is called supersaturated flow or metastable flow.

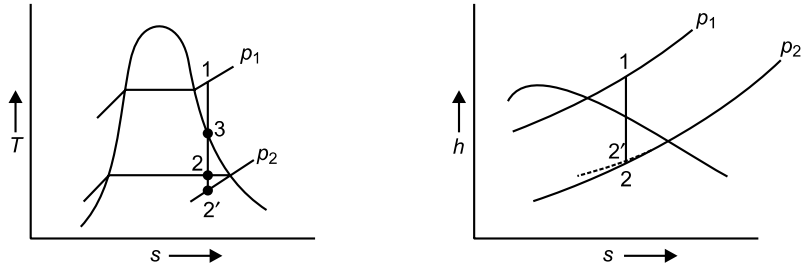


Fig. 13.16 *Metastable flow through steam nozzle*

In supersaturated flow the condensation of steam lags behind the expansion and so steam does not condense at the saturation temperature corresponding to the pressure. It may be understood as the shift in state 2 from 2 to 2′ i.e. condensation gets extended up to 2′. The dry saturated steam state which should be attained at state 3 cannot be realized at 3, but below 3 on vertical expansion line 1–2′. This

delayed phase transformation of steam causing supersaturation phenomenon may be attributed to the following.

(i) Steam flow through nozzle may be so fast that sufficient time is not available for heat transfer to take place and so the phase change lags behind the expansion. Generally time available may be of the order of 10^{-2} second for steam to flow through nozzle along with its condensation.

(ii) Also the condensation of steam may have inherent requirement of nuclei of condensation which act as initiators for condensation. These nuclei of condensation may be provided by foreign particles, solid boundary etc. In the absence of nuclei of condensation the phase change of steam may get delayed and lags behind.

These could be the factors responsible for supersaturation. Phenomenon of supersaturation or metastable equilibrium continues up to generally 94–95% dryness fraction. Beyond this the condensation of steam occurs suddenly at very fast rate irreversibly and the expansion process attains stable equilibrium. The locci of points up to which metastable equilibrium is observed is called Wilson line. Law of expansion for supersaturated flow is considered as $pv^{1.3}$ = constant.

Phenomenon of supersaturation causes increase in discharge by 2–5% because of increase in density at throat and also the heat drop gets slightly reduced thereby causing reduced velocity at exit. Supersaturation causes slight increase in dryness fraction and entropy.

Figure 13.17 shows the supersaturated flow and Wilson line.

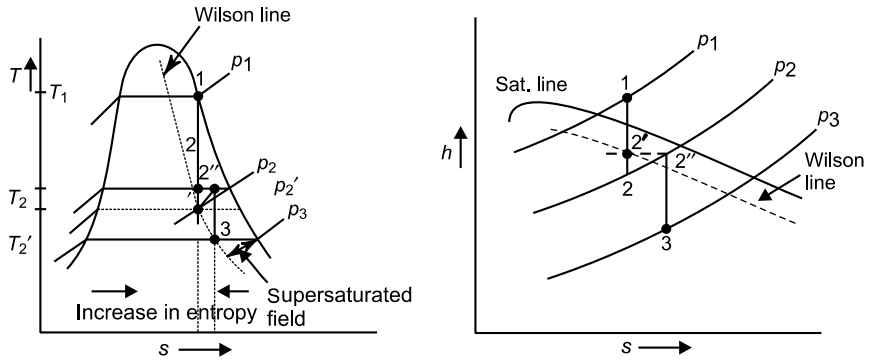


Fig. 13.17 *Supersaturated flow on* T-s *and* h-s *diagram.*

Region between saturation line and Wilson line is called supersaturated field. Here 1–2′ line shows isentropic process. In the absence of supersaturated flow expansion occurs as 1–2 while with metastable flow it gets extended up to 1–2′ as shown on *T–s* diagram. Meta stable equilibrium gets settled and stable equilibrium is attained as shown by 2′–2″ and then normal expansion in stable equilibrium continues from 2″ to 3.

Thus it is obvious that in supersaturated flow the expansion occurs as if there is no saturated steam line and state 2′ lies on the extended constant pressure line.

The temperature at 2′ is less than saturation pressure corresponding to p_2 due to excess kinetic energy of steam at the cost of sensible heat.

Metastable flow is characterized by a parameter called “degree of supersaturation” and “degree of undercooling”. ‘Degree of supersaturation’ refers to the ratio of saturation pressures corresponding to temperatures of states in stable equilibrium and metastable equilibrium (i.e. saturation pressures corresponding to 2 and 2′ states). Degree of super saturation has value more than unity. ‘Degree of

undercooling' refers to the difference of two temperatures i.e. saturation temperature at state in stable equilibrium and temperature of the state in unstable equilibrium.

13.7 STEAM INJECTOR

Steam injector refers to the device for injecting water into boiler using steam which may be available from boiler or exhaust steam from engine.

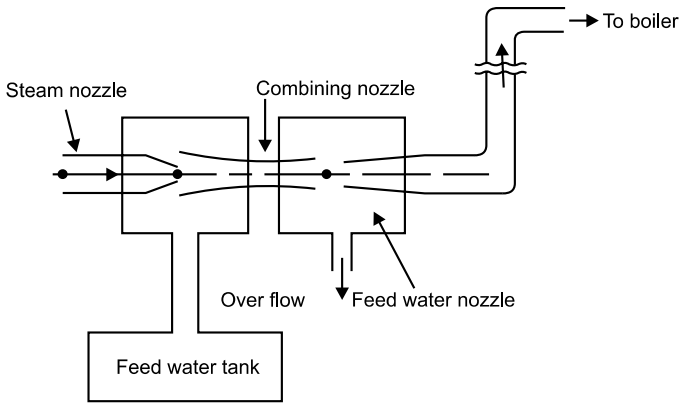


Fig. 13.18 *Steam injector*

Schematic for steam injector is shown in Fig. 13.18. Here high pressure steam enters a converging steam nozzle and leaves with high velocity and low pressure. Water gets entrained from feed tank and is carried by steam through combining nozzle. In due course steam gets condensed and the resulting mixture enters the divergent feed water nozzle where kinetic energy of water gets transformed into pressure head. Pressure head available in feed water nozzle is sufficiently above boiler pressure so that water can be fed to the boiler. Surplus water, if any gets discharged from over flow. The steam injector works on its own. It may be noted that the potential energy removed from live steam is many times more than the potential energy returned.

Potential energy removed from live steam = Boiler steam pressure × Volume of steam.

Potential energy returned = Boiler pressure × Volume of condensate and boiler feed

Difference in the two potential energy exists due to large decrease in volume as steam condenses and this difference is only used for pumping water.

EXAMPLES

1. *Dry steam at 10 bar and 100 m/s enters a nozzle and leaves it with velocity of 300 m/s at 5 bar. For 16 kg/s of steam mass flow rate determine heat drop in nozzle and final state of steam leaving nozzle assuming heat loss to surroundings as 10 kJ/kg.*

Solution:

Given, $C_1 =$ 100 m/s, $C_2 = 300$ m/s

$p_1 =$ 10 bar = 1 MPa, $p_2 = 0.5$ MPa

By steady flow energy equation between sections 1 and 2.

$$h_1 + \frac{C_1^2}{2} + q = h_2 + \frac{C_2^2}{2} + w$$

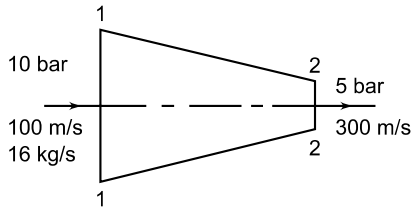


Fig. 13.19 *Nozzle*

Here there is no work so $w = 0$ and heat loss, $q = -10$ kJ/kg

$$q = (h_2 - h_1) + \left(\frac{C_2^2 - C_1^2}{2} \right)$$

From steam table, $h_1 = h_{g \text{ at } 1 \text{ MPa}} = 2778.1$ kJ/kg

$$(h_2 - h_1) = q + \left(\frac{C_1^2 - C_2^2}{2} \right)$$

$$(h_2 - h_1) = -10^4 + \left(\frac{C_1^2 - C_2^2}{2} \right)$$

$$= -10^4 + \left\{ \frac{(100)^2 - (300)^2}{2} \right\}$$

$$h_2 - h_1 = -30{,}000$$

Heat drop in nozzle = 30 kJ/kg Total heat drop = 480 kJ/s ***Ans.***

or, $h_2 = 2748.1$ kJ/kg

At 5 bar, pressure,

$h_f = 640.23$ kJ/kg, $h_{fg} = 2108.5$ kJ/kg

Let dryness fraction at exit be x_2,

$$2748.1 = 640.23 + x_2 \cdot 2108.5$$

$$\boldsymbol{x_2 = 0.99}$$

Dryness fraction at exit = 0.99 ***Ans.***

2. *Determine the mass flow rate of steam through a nozzle having isentropic flow through it. Steam enters nozzle at 10 bar, 500 °C and leaves at 6 bar. Cross-section area at exit of nozzle is 20 cm^2. Velocity of steam entering nozzle may be considered negligible. Show the process on h-s diagram also.*

Solution:

At inlet section (1),

$h_1 = 3478.5$ kJ/kg,

$s_1 = 7.7622$ kJ/kg · K

Due to isentropic expansion, $s_1 = s_2$

At exit section (2), $s_2 = 7.7622$ kJ/kg · K

Enthalpy at rate (2) can be directly looked from mollier diagram by drawing vertical line from state (1) till it intersects 6 bar line. Else from steam table it can be obtained as under, looking at steam table for 6 bar it indicates that the state (2) shall be superheated state because $s_2 > s_g$ at 6 bar.

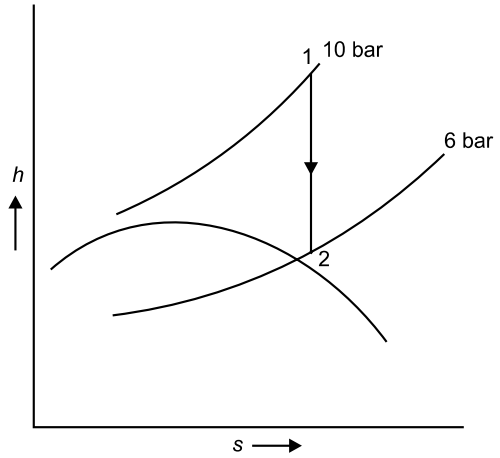


Fig. 13.20 *Flow through nozzle on* h-s *diagram*

Degree of superheat can be determined by interpolation.

Entropy 7.7622 kJ/kg · K, s_2 lies between temperature of 400°C ($s_{400°C,\ 6\ bar}$ = 7.7079 kJ/kg · K) and 500°C ($s_{500°C,\ 6\ bar}$ = 8.0021 kJ/kg · K).

$$7.7622 = 7.7079 + \frac{(8.0021 - 7.7079)}{(500 - 400)} \times (T_2 - 400)$$

$$\boldsymbol{T_2 = 418.45°C}$$

Hence enthalpy at (2), $h_2 = h_{418.45°C,\ 6\ bar}$

$$h_2 = h_{400°C,\ 6\ bar} + \frac{\left(h_{500°C,6bar} - h_{400°C,6bar}\right)}{(500 - 400)} (418.45 - 400)$$

$$= 3270.3 + \frac{(3482.8 - 3270.3)}{100} \times 18.45$$

$$h_2 = 3309.51 \text{ kJ/kg·K}$$

Neglecting inlet velocity, $C_1 \approx 0$, assuming no heat loss,

$$C_2 = \sqrt{2(h_1 - h_2)}$$

$$\boldsymbol{C_2 = 581.36 \text{ m/s}}$$

$$\text{Mass flow rate, } = \frac{A_2 \times C_2}{v_2}$$

Specific volume at exit (2)

$$v_2 = v_{400°C,\ 6\ bar} + \frac{\left(v_{500°C,6bar} - v_{400°C,6bar}\right)}{(500 - 400)} \times (418.45 - 400)$$

$$= 0.5137 + \left(\frac{0.5920 - 0.5137}{100}\right) \times 18.45$$

$$v_2 = 0.5281 \text{ kJ/kg·K}$$

Mass flow rate = $\dfrac{20\times10^{-4}\times581.36}{0.5281}$ = 2.2017

Mass flow rate = 2.202 kg/s ***Ans.***

3. *In a nozzle steam expands from 12 bar and 300°C to 6 bar with flow rate of 5 kg/s. Determine throat and exit area if exit velocity is 500 m/s and velocity at inlet to nozzle is negligible. Also find coefficient of velocity at exit. Coefficient of velocity is the ratio of actual velocity of fluid at nozzle exit to the velocity at exit considering isentropic flow through nozzle.*

Solution:

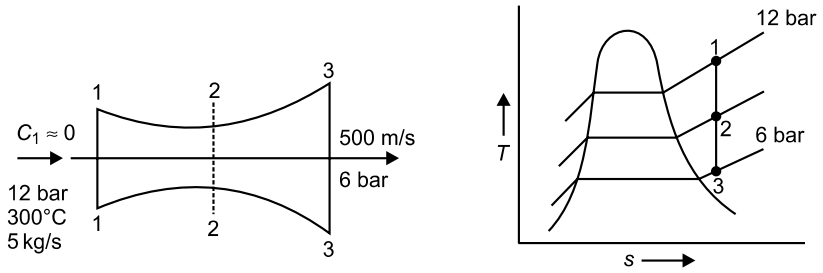


Fig. 13.21 *Nozzle and* T-s *representation of process*

Here velocity at exit is 500 m/s which is more than sonic velocity thus nozzle shall be converging-diverging nozzle as shown.

At inlet to nozzle, h_1 = 3045.8 kJ/kg, s_2 = 7.0317 kJ/kg · K

Due to isentropic flow through nozzle, $s_1 = s_2 = s_3$ = 7.0317 kJ/kg · K Pressure at throat section i.e. (2) can be estimated using critical pressure ratio, which is given as

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{2}{n+1}\right)^{\frac{n}{(n-1)}}$$

For superheated steam, $n = 1.3$

$$\left(\frac{p_2}{12}\right) = \left(\frac{2}{1+1.3}\right)^{1.3/0.3} = 0.545$$

Throat pressure, $\boldsymbol{p_2 = 6.54}$ **bar**

From steam table;

At 6.54 bar, h_2 = 2900.05 kJ/kg

T_2 = 224.48°C, v_2 = 0.3466 m^3/kg

Velocity at throat $C_2 = \sqrt{2\,(h_1 - h_2)}$

$$= \sqrt{2\times10^3(3045.8 - 2900.05)}$$

C_2 = 539.9 m/s

From continuity equation, $m_1 = m_2$ = 5 kg/s

$$m_2 = \frac{A_2\,C_2}{v_2}$$

$$5 = \frac{A_2 \times 539.9}{0.3466}$$

$$A_2 = 3.209 \times 10^{-3} \text{ m}^2$$

Cross-sectional area at throat = 3.209×10^{-3} m^2 ***Ans.***

Velocity at exit of nozzle, $C_3 = \sqrt{2(h_1 - h_3)}$

From steam tables, Temperature at exit of nozzle, $T_3 = 215.15°C$

Enthalpy at exit of nozzle, $h_3 = 2882.55$ kJ/kg

Specific volume at exit of nozzle, $v_3 = 0.3647$ m^3/kg

Ideal velocity at exit, $C_3 = \sqrt{2 \times 10^3 (3045.8 - 2882.55)}$

$C_3 = 571.40$ m/s

Actual velocity at exit, $C_{3,\text{ actual}} = 500$ m/s

Area at exit, $m_1 = m_2 = m_3 = 5 \text{ kg/s} = \frac{A_3 \times C_{3,\text{ actual}}}{v_3}$

$$A_3 = \frac{5 \times 0.3647}{500} = 3.647 \times 10^{-3} \text{ m}^2$$

Cross-sectional area at exit = 3.647×10^{-3} m^2 ***Ans.***

Coefficient of velocity = $\frac{C_{3,\text{ actual}}}{C_3} = \frac{500}{571.40} = \mathbf{0.875}$

. Coefficient of velocity = 0.875 ***Ans.***

4. *In a steam nozzle steam expands from 16 bar to 5 bar with initial temperature of 300°C and mass flow of 1 kg/s. Determine the throat and exit areas considering (i) expansion to be frictionless and, (ii) friction loss of 10% throughout the nozzle.*

Solution:

Let us assume velocity at inlet to nozzle to be negligible. From steam table, At inlet to the nozzle, enthalpy,

$h_1 = 3034.8$ kJ/kg at 16 bar, 300°C

$s_1 = 6.8844$ kJ/kg·K

$v_1 = 0.15862$ m^3/kg

(i) Considering expansion to be frictionless

Pressure at throat of nozzle, $p_2 = p_1\left(\frac{2}{n+1}\right)^{n/n-1}$, for $n = 1.3$, $p_2 = 8.73$ bar

Enthalpy at throat and exit of nozzle can be looked from steam table as,

$h_2 = 2891.39$ kJ/kg

$h_3 = 2777.0$ kJ/kg

$v_2 = 0.2559$ m^3/kg

$v_3 = 0.3882$ m^3/kg

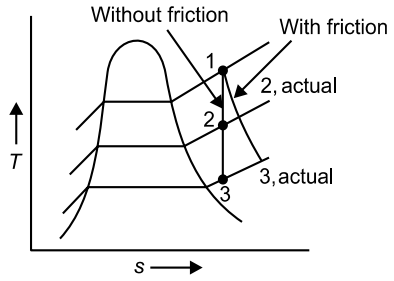


Fig. 13.22

Heat drop up to throat section = $h_1 - h_2$

Δq_{12} = 143.41 kJ/kg

Velocity at throat, $C_2 = \sqrt{2(h_1 - h_2)}$ = 535.56 m/s

Heat drop from throat to exit = $h_2 - h_3$

Δq_{23} = 114.39 kJ/kg

Velocity at exit, $C_3 = \sqrt{2(h_2 - h_3) + C_2^2}$

C_3 = 718.06 m/s

$$\text{Throat area, } A_2 = \frac{m_2 \times v_2}{C_2} = \frac{1 \times 0.2559}{535.56} = 4.78 \times 10^{-4} \text{ m}^2$$

$$\text{Exit area, } A_3 = \frac{m_3 \times v_3}{C_3} = \frac{1 \times 0.3882}{718.06} = 5.41 \times 10^{-4} \text{ m}^2$$

For frictionless expansion, ***Ans.***
Throat area = 4.78 cm^2
Exit area = 5.41 cm^2

(ii) Considering expansion to have 10% friction loss:

Due to frictional heat loss actual heat drop shall get reduced.

Actual heat drop upto throat, $\Delta q'_{12} = 0.9 \times q_{12}$

$\Delta q'_{12}$ = 129.07 kJ/kg

Thus, enthalpy at throat, $h_{2,\text{ actual}}$ = 2905.73 kJ/kg

from steam table, $v_{2,\text{ actual}}$ = 0.2598 m^3/kg corresponding to the modified state as indicated by enthalpy, $h_{2,\text{ actual}}$. This new state at throat shall have temperature of 231.35°C, which can be calculated from known enthalpy $h_{2,\text{ actual}}$ at respective pressure.

Actual velocity at throat, $C_{2,\text{ actual}} = \sqrt{2 \times 129.07 \times 10^3}$

$C_{2,\text{ actual}}$ = 508.1 m/s

Actual throat area, $A_{2,\text{ actual}} = \frac{1 \times 0.2598}{508.1}$

Actual throat area = 5.11×10^{-4} m^2 or, 5.11 cm^2 ***Ans.***

Actual heat drop from throat to exit of nozzle, $\Delta q'_{23} = 0.9 \times 114.39$

$\Delta q'_{23} = 102.95$ kJ/kg

Thus, enthalpy at exit, $h_{3,\text{ actual}} = 2905.73 - 102.95$

$= 2802.78$ kJ/kg

From steam table, corresponding to modified state as given by $h_{3,\text{ actual}}$ the specific volume can be given as, $v_{3,\text{ actual}} = 0.40023$ m^3/kg

Actual velocity at exit, $C_{3,\text{ actual}} = \sqrt{(2 \times 102.95 \times 10^3) + (508.1)^2}$

$C_{3,\text{ actual}} = 681.22$ m/s

Actual area at exit $\Rightarrow A_{3,\text{ actual}} = \dfrac{1 \times 0.40023}{681.22}$

$A_{3,\text{ actual}} = 5.875 \times 10^{-4}$ m^2

Actual area at exit, $= 5.875 \times 10^{-4}$ m^2 or 5.875 cm^2 ***Ans.***

5. *An impulse turbine of 1 MW has steam entering at 20 bar and 300°C and steam consumption of 8 kg per kW hour. Steam leaves at 0.2 bar and 10% of total heat drop is lost in overcoming friction in diverging portion of nozzle. If throat diameter of each nozzle is 1 cm then determine (i) the number of nozzles required (ii) exit diameter of each nozzle. Solve using mollier diagram.*

Solution:

In impulse turbine the impulse action of steam is used for getting shaft work. A stage of simple impulse turbine has steam nozzle and a ring of symmetrical blade. Total pressure drop in the stage thus occurs in nozzle itself.

Throat pressure $= 0.545 \times 20 = 10.9$ bar

From mollier diagram,

Heat drop up to throat, $\Delta q_{12} = h_1 - h_2$

$\Delta q_{12} = 142$ kJ/kg

$v_2 = 0.20$ m^3/kg

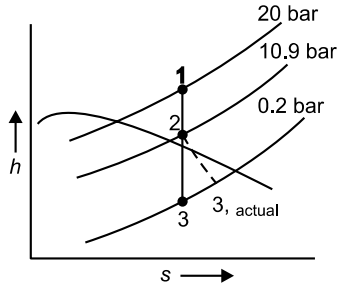


Fig. 13.23

Considering negligible inlet velocity, the velocity at throat,

$C_2 = \sqrt{2 \times 10^3 \times 142}$

$C_2 = 532.9$ m/s

$$\text{Mass flow rate through nozzle} = \frac{A_2 \cdot C_2}{v_2}$$

$$= \frac{\pi \times (10^{-2})^2 \times 532.9}{4 \times 0.20}$$

$$= 0.209 \text{ kg/s}$$

The number of nozzles can be determined by using the capacity of turbine, steam consumption and mass flow rate through a nozzle.

$$\text{Number of nozzles} = \frac{1 \times 10^3 \times 8}{3600 \times 0.209} = 10.63 \approx \textbf{11 nozzles}$$

Number of nozzles required = 11 ***Ans.***

(ii) Total heat drop in nozzle, from mollier chart, $(h_1 - h_3) = 807$ kJ/kg

Useful heat drop, $\Delta q_{13,\ \text{useful}} = 0.90 \times 807$

$= 726.3$ kJ/kg

Specific volume at exit, $v_3 = 7.2$ m^3/kg

$$\text{Velocity at exit} = \sqrt{2 \times 10^3 \times 726.3}$$

$C_3 = 1205.2$ m/s

Therefore, from continuity equation,

$$A_3 = \frac{m_3 \cdot v_3}{C_3}$$

$$= \frac{0.209 \times 7.2}{1205.2}$$

$$A_3 = 1.25 \times 10^{-3} \text{ m}^2$$

or

Area at exit = 12.5 cm^2 ***Ans.***

6. *A nozzle is supplied with steam at 0.7 mPa and 275 °C. Determine temperature and velocity at throat considering no losses.*

If diverging portion of nozzle is 6 cm long and throat diameter of 0.5 cm, determine the angle of cone in nozzle so that steam leaves nozzle at 0.1 MPa. Assume heat utilization in diverging portion to be 85%. Solve using mollier diagram.

Solution:

Pressure at throat section of nozzle

$$p_2 = 0.545 \times 0.7$$

$$= 0.38 \text{ MPa}$$

From mollier diagram,

Heat drop in nozzle up to throat,

$$h_1 - h_2 = \Delta q_{12} = 138 \text{ kJ/kg}$$

Specific volume, $v_2 = 0.58$ m^3/kg

Temperature = 203°C

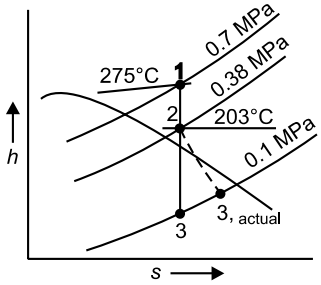


Fig. 13.24

Considering velocity at inlet to be negligible, velocity at throat,

$$C_2 = \sqrt{2 \times 138 \times 10^3}$$

$$C_2 = 525.35 \text{ m/s}$$

$$\text{Mass flow rate} = \frac{\frac{\pi}{4} \times \left(0.5 \times 10^{-2}\right)^2 \times 525.35}{0.58}$$

$$m_1 = 0.0178 \text{ kg/s} = m_2 = m_3$$

From mollier diagram, heat drop in diverging portion i.e. from throat to exit of nozzle shall be;

$$\Delta q_{23} = h_2 - h_3 = 247 \text{ kJ/kg}$$

$$\Delta q_{23,\text{ actual}} = 209.95 \text{ kJ/kg}$$

Specific volume, $v_{3,\text{ actual}} = 1.7 \text{ m}^3/\text{kg}$

Total heat drop = 138 + 209.95 = 347.95 kJ/kg

$$\text{Velocity at exit of nozzle} = \sqrt{2 \times 347.95 \times 10^3}$$

$$C_3 = 834.2 \text{ m/s}$$

$$\text{Area at exit, } A_3 = \frac{m_3 \times v_3}{C_3}$$

$$= \frac{0.0178 \times 1.7}{834.2}$$

$$= \mathbf{3.63 \times 10^{-5} \text{ m}^2}$$

$$\text{Diameter at exit} = \sqrt{\frac{3.63 \times 10^{-5} \times 4}{\pi}}$$

$$= \mathbf{6.8 \times 10^{-3} \text{ m}} \text{ or } 6.8 \text{ mm}$$

For Cone angle to be 2α,

$$\tan \alpha = \frac{6.8 - 5}{2 \times 60}, \ \alpha = 0.86°$$

$$\text{Cone angle, } 2\alpha = \mathbf{1.72°}$$

With no losses, Temperature at throat = 203°
Velocity at throat = 525.35 m/s
With losses, Cone angle = 1.72°

Ans.

7. *An impulse steam turbine generating 5000 hp requires 6 kg of steam per hp-hr at 16 bar, dry saturated. In the first stage the steam is expanded through nozzle with an efficiency of 0.90 to a pressure of 8 bar. These nozzles are placed so as to extend over approximately* $\frac{1}{3}$ *rd of the circumference with the pitch circle diameter of 600 mm and pitch of 50 mm. Considering nozzle angle as 12° to the plane of wheel and plates dividing being 3 mm thick, determine total length of nozzle arc and radial height of nozzles.*

Solution:

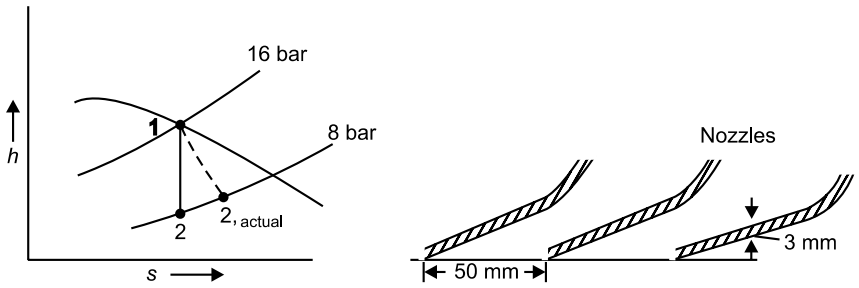


Fig. 13.25

At inlet to nozzle, from steam table, $h_1 = h_{g \text{ at } 16 \text{ bar}} = 2794$ kJ/kg

$$s_1 = s_{g \text{ at } 16 \text{ bar}} = 6.4218 \text{ kJ/kg·K}$$

At exit of nozzle,

$$s_1 = s_2 = 6.4218 \text{ kJ/kg·K}$$

Since $s_2 < s_{g \text{ at } 8 \text{ bar}}$ so the state at the end of expansion lies in wet region. Let dryness fraction at state 2 be x_2, then,

$$s_2 = s_{f \text{ at } 8 \text{ bar}} + x_2 \cdot s_{fg \text{ at } 8 \text{ bar}}$$

$$6.4218 = 2.0462 + x_2 \cdot 4.6166$$

or, $$x_2 = 0.9478$$

$$h_2 = h_{f \text{ at } 8 \text{ bar}} + x_2 \cdot h_{fg \text{ at } 8 \text{ bar}}$$

$$= 721.11 + (0.9478 \times 2048)$$

$$h_2 = 2662.2 \text{ kJ/kg}$$

Theoretically, change in enthalpy, $\Delta h_{12} = 2794 - 2662.2$

$$\Delta h_{12} = 131.8 \text{ kJ/kg}$$

$$\frac{\Delta h_{12\text{actual}}}{\Delta h_{12}} = 0.90 \text{ or } \Delta h_{12 \text{ actual}} = 0.9 \times 131.8 = 118.62 \text{ kJ/kg}$$

$$h_{2 \text{ actual}} = 2794 - 118.62 = 2675.38 \text{ kJ/kg}$$

$$h_{2, \text{ actual}} = 2675.38 = h_{f \text{ at } 8 \text{ bar}} + x_{2 \text{ actual}} \cdot h_{fg \text{ at } 8 \text{ bar}}$$

$$2675.38 = 721.11 + (x_{2\ \text{actual}} \times 2048)$$

$$x_{2\ \text{actual}} = 0.9542$$

Specific volume, $v_{2\ \text{actual}} = v_{f\ \text{at 8 bar}} + x_{2\ \text{actual}} \times v_{fg\ \text{at 8 bar}}$

$$v_{2\ \text{actual}} = 0.001115 + (0.9542 \times (0.2404 - 0.001115))$$

$$v_{2\ \text{actual}} = 0.2294\ \text{m}^3/\text{kg}$$

Neglecting the velocity at inlet to nozzle,

$$C_2 = \sqrt{2 \times \Delta h_{12\ \text{actual}}}$$

$$C_2 = 487.1\ \text{m/s}$$

By continuity equation,

$$m = \frac{A_2\ C_2}{v_{2\ \text{actual}}}$$

$$\frac{5000 \times 6}{3600} = \frac{A_2 \times 487.1}{0.2294}$$

$$A_2 = 3.925 \times 10^{-3}\ \text{m}^2$$

$$\text{Area at exit of nozzle} \approx 39.25\ \text{cm}^2$$

Approximate length of nozzle arc, $= \frac{60 \times \pi}{3} = 62.83$ cm

$$\text{Number of nozzles} = \frac{\text{Length of nozzle arc}}{\text{Pitch}}$$

$$= \frac{62.83}{5} \approx 13 \text{ nozzles.}$$

Correct length of nozzle arc $= 13 \times 5 = 65$ cm

Flow area at exit of each nozzle $= (5 \sin 12 - 0.3) \times h$

$$39.25 = (5 \sin 12 - 0.3) \times h \times 13$$

or, $h = 4.08$ cm

Length of nozzle = 65 cm
Radial height of nozzle = 4.08 cm

Ans.

8. *Air is expanded reversibly and adiabatically in a nozzle from 13 bar and 150°C to a pressure of 6 bar. The inlet velocity of the nozzle is very small and the process occurs under steady flow conditions. Calculate the exit velocity of the nozzle.* [*U.P.S.C., 1992*]

Solution:

Let inlet and exit states in nozzle be given as 1 and 2 respectively. Assuming no heat interaction and work interaction, no change in potential energy during flow across the nozzle. Steady flow energy equation gets modified as under;

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

Neglecting velocity at inlet to nozzle C_1.

$$C_2 = \sqrt{2(h_1 - h_2)}$$

$$C_2 = \sqrt{2 c_p \cdot (T_1 - T_2)}$$

Assuming adiabatic expansion in nozzle, between states 1 and 2.

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{(\gamma-1)}{\gamma}}$$

or, $$T_2 = T_1 \cdot \left(\frac{P_2}{P_1}\right)^{\frac{(\gamma-1)}{\gamma}}$$

$$= (273 + 150)\left(\frac{6}{13}\right)^{\frac{(1.4-1)}{1.4}}$$

$$T_2 = 339.16 \text{ K}$$

Substituting in expression for exit velocity, C_2,

$$C_2 = \sqrt{2 \times 1.005 \times (423 - 339.16)}$$

$$C_2 = 12.98 \text{ m/s}$$

Exit Velocity = 12.98 m/s ***Ans.***

9. *During a test on steam nozzle steam impinges a stationary flat plate which is perpendicular to the direction of flow and the force on the plate is measured. The force is found to be 350 N when dry saturated steam at 8 bar is expanded to 1 bar. Throat cross-section area is 5 cm^2 and exit area is such that the complete expansion is achieved under these conditions.*

Determine

(i) the discharge at throat.

(ii) the efficiency of nozzle assuming that all the losses occur after throat and that n = 1.13 for isentropic expansion.

Solution:

For isentropic expansion up to throat.

$$\frac{P_2}{P_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{(n-1)}}$$

$$P_2 = 8\left(\frac{2}{1.13+1}\right)^{\frac{1.13}{0.13}}$$

$$P_2 = 4.62 \text{ bar}$$

From steam table,

Enthalpy at 1, $h_1 = h_{g \text{ at } 8 \text{ bar}} = 2769.1$ kJ/kg

$s_1 = s_{g \text{ at } 8 \text{ bar}} = 6.6628$ kJ/kg·K

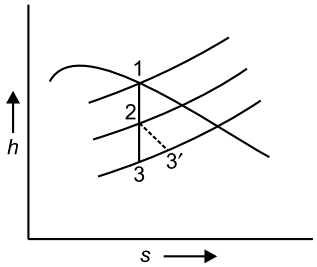


Fig. 13.26

At throat section let the dryness fraction be x_2, then

$$s_1 = s_2 = 6.6628 = s_{f \text{ at } 4.62 \text{ bar}} + s_{fg \text{ at } 4.62 \text{ bar}} \cdot x_2$$
$$6.6628 = 1.7872 + x_2 \cdot 5.0178$$
$$x_2 = 0.9717 \text{ or } 97.17\%$$
$$h_2 = h_{f \text{ at } 4.62 \text{ bar}} + x_2 \cdot h_{fg \text{ at } 4.62 \text{ bar}}$$
$$= 627.33 + (0.9717 \times 2117.77)$$
$$h_2 = 2685.17 \text{ kJ/kg}$$

Specific volume at throat, $v_2 = v_{f \text{ at } 4.62 \text{ bar}} + x_2 \cdot v_{fg \text{ at } 4.62 \text{ bar}}$
$$= 0.001089 + (0.9717 \times 0.4035)$$
$$v_2 = 0.3932 \text{ m}^3/\text{kg}$$

Enthalpy change up to throat $\Delta h_{12} = h_1 - h_2$
$$\Delta h_{12} = 83.93 \text{ kJ/kg}$$

Velocity at throat, $C_2 = \sqrt{2 \times \Delta h_{12}} = \sqrt{2 \times 83.93 \times 10^3} = 409.71$ m/s

Discharge at throat, $m_2 = \dfrac{A_2 \cdot C_2}{v_2}$

$$= \frac{5 \times 10^{-4} \times 409.71}{0.3932}$$
$$m_2 = 0.521 \text{ kg/s}$$

Discharge at throat = 0.521 kg/s **Ans.**

Force acting on plate

$$F = 350 \text{ N} = m (C_3' - C_1)$$
$$C_1 \approx 0$$

Exit actual velocity $C_3' = 671.78$ m/s

For diverging section between states 1 and 3, $s_1 = s_2 = s_3$

$$s_3 = 6.6628 = s_{f \text{ at } 1 \text{ bar}} + x_3 \cdot s_{fg \text{ at } 1 \text{ bar}}$$
$$6.6628 = 1.3026 + (x_3 \times 6.0568)$$
$$x_3 = 0.8238 \text{ or } 82.38\%$$

Enthalpy at exit

$$h_3 = h_{f \text{ at } 1 \text{ bar}} + x_3 \cdot h_{fg \text{ at } 1 \text{ bar}}$$
$$= 417.46 + (0.8238 \times 2258)$$
$$h_3 = 2277.6 \text{ kJ/kg}$$

Theoretical enthalpy drop, $\Delta h_{23} = h_2 - h_3 = 407.57$ kJ/kg

Actual enthalpy drop, $\Delta h_{23'} = \eta_N \cdot \Delta h_{23}$

Actual velocity

$$C_3' = \sqrt{2 \cdot \Delta h_{23'}}$$
$$671.78 = \sqrt{2 \times \eta_N \times 407.5 \times 10^3}$$

$\eta_N = 0.5537$ or 55.37%

Nozzle efficiency = 55.37% ***Ans.***

10. *A convergent-divergent nozzle operates with 5 kg of steam per minute being discharged at 1 bar. For the steam supplied to nozzle being at 10 bar and 200 °C and supersaturation occurring up to throat and normal afterwards, determine,*

(i) the diameter of nozzle at exit.

(ii) the maximum degree of supersaturation.

(iii) the amount of undercooling at throat.

For supersaturation take $Pv^{1.3}$ *= constant and* $\frac{P}{T^{1.3/0.3}}$ *= constant*

Solution:

Pressure at throat,

$$P_2 = P_1 \left(\frac{2}{n+1}\right)^{\frac{n}{(n-1)}}.$$

$n = 1.3$

$P_2 = 5.45$ bar

From steam tables,

Enthalpy at inlet, $h_1 = 2827.9$ kJ/kg

$s_1 = 6.6940$ kJ/kg·K

$v_1 = 0.2060$ m^3/kg

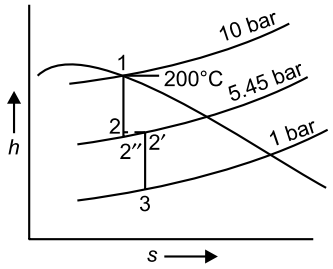


Fig. 13.27

Enthalpy drop up to throat,

$$h_1 - h_2' = \left(\frac{n}{n-1}\right) P_1 v_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}}\right]$$

$$= \left(\frac{1.3}{0.3}\right) (10 \times 10^2 \times 0.2060) \left[1 - \left(\frac{5.45}{10}\right)^{0.3/1.3}\right]$$

$h_1 - h_2' = 116.67$ kJ/kg

$h_2' = 2711.23$ kJ/kg

Let dryness fraction be x_2 at throat, so using steam tables it can be obtained as under,

$$h_2' = h_{f \text{ at } 5.45 \text{ bar}} + (x_2 \times h_{fg \text{ at } 5.45 \text{ bar}})$$

$$2711.23 = 663.36 + (x_2 \times 2098.15)$$

$$x_2 = 0.9760$$

$$s_2' = s_{f \text{ at } 5.45 \text{ bar}} + (x_2 \cdot s_{fg \text{ at } 5.45 \text{ bar}})$$

$$= (1.8936 + (0.9760 \times 4.8989) = 6.6749 \text{ kJ/kg·K}$$

Let dryness fraction at exit be x_3, so

$$s_3 = s_2' = 6.6749 \text{ kJ/kg·K}$$

$$s_3 = s_{f \text{ at } 1 \text{ bar}} + x_3 \cdot s_{fg \text{ at } 1 \text{ bar}}$$

$$6.6749 = 1.3026 + x_3 \times 6.0568$$

$$x_3 = 0.8869$$

Enthalpy at exit, $h_3 = h_{f \text{ at } 1 \text{ bar}} + x_3 \cdot h_{fg \text{ at } 1 \text{ bar}}$

$$= 417.46 + (0.8869 \times 2258)$$

$$h_3 = 2420.08 \text{ kJ/kg}$$

Specific volume at exit, $v_3 = v_{f \text{ at } 1 \text{ bar}} + x_3 \cdot v_{fg \text{ at } 1 \text{ ba}}$

$$= (0.001043 + 0.8869 \times 1.6929)$$

$$v_3 = 1.5025 \text{ m}^3/\text{kg}$$

Velocity at exit, $C_3 = \sqrt{2 \cdot \Delta h_{13}} = 903.13 \text{ m/s}$

Mass flow rate, $m_3 = \dfrac{A_3 \cdot C_3}{v_3}$

$$\frac{5}{60} = \frac{A_3 \times 903.13}{1.5025}$$

$$A_3 = 1.386 \times 10^{-4} \text{ m}^2 \text{ or } 1.386 \text{ cm}^2$$

Diameter of nozzle at exit $d_3 = 0.0133$ m or 1.33 cm

Diameter of nozzle at exit = 1.33 cm ***Ans.***

Theoretically up to throat,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}}$$

$$T_2 = 473\ (0.545)^{\frac{(1.3-1)}{1.3}} = 411.18 \text{ K} = 138.18°\text{C}$$

Saturation pressure corresponding to 411.18 K,

$P_{\text{sat, } 138.18°\text{C}} = 3.44$ bar

$$\text{Degree of supersaturation} = \frac{P_2}{P_{\text{sat},138.18°\text{C}}}$$

$$= \frac{5.45}{3.44} = 1.58$$

$$\text{Amount of undercooling} = (T_{\text{sat at } 5.45 \text{ bar}} - T_2)$$

$$= (155.12 - 138.18)$$

$$= 16.94°\text{C}$$

Degree of supersaturation = 1.58
Amount of undercooling = 16.94°C

Ans.

11. *Steam undergoes expansion from 4 bar, 180°C to a pressure of 1.5 bar. If the expansion is supersaturated and occurs with friction loss of 5 percent, determine the actual heat drop and degree of undercooling. Following equations may be used for supersaturated steam,*

$P \cdot v \times 10^2 = (h - 2614)$, *(Here P is in bar, v is specific volume in* m^3/kg, *h is enthalpy in kJ/kg)*

$\frac{P}{T^{\left(\frac{1.3}{0.3}\right)}}$ = *constant and* $P \cdot v^{1.3}$ = *constant*

Consider specific heat as 2.174 kJ/kg·K.

Solution:

At inlet for expansion, state 1, specific volume can be seen from steam table, $v_1 = 0.5088\ m^3/kg$

Enthalpy at state 1 from empirical relation,

$$4 \times 0.5088 \times 10^2 = (h_1 - 2614)$$
$$h_1 = 2817.52\ kJ/kg$$

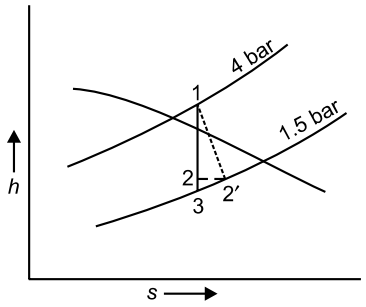


Fig. 13.28

Using governing equation given,

$$P_1\ v_1^{1.3} = P_2\ v_2^{1.3}$$
$$4 \times 10^2 \times (0.5088)^{1.3} = 1.5 \times 10^2 \times v_2^{1.3}$$
$$v_2 = 1.082\ m^3/kg$$

Enthalpy at 2,

$$P_2\ v_2 \times 10^2 = (h_2 - 2614)$$
$$(1.5 \times 1.082 \times 10^2) = (h_2 - 2614)$$

or,
$$h_2 = 2776.3\ kJ/kg$$
$$\text{Actual heat drop} = \eta_N \times (h_1 - h_2)$$
$$= 0.95 \times (2817.52 - 2776.3)$$
$$= 39.16\ kJ/kg$$

Actual heat drop = 39.16 kJ/kg ***Ans.***

Temperature at state 2,

$$\frac{P}{T^{\left(\frac{1.3}{0.3}\right)}} = \text{constant}$$

$$\frac{4\times10^2}{(453)^{1.3/0.3}} = \frac{1.5\times10^2}{(T_2)^{1.3/0.3}}$$

$$T_2 = 361.24 \text{ K}$$

Supersaturation causes rise in temperature due to reheat

$$= \frac{0.05\times(2817.52-2776.3)}{2.174}$$

$$= 0.95 \text{ K or } 0.95°\text{C}$$

$$T_2' = 361.24 + 0.95 = 362.19 \text{ K}$$

$$\text{Amount of undercooling} = T_3 - T_2'$$

$$= T_{\text{sat at 1.5 bar}} - T_2'$$

$$= ((273 + 111.37) - 362.19) = 22.18°\text{C}$$

Degree of undercooling = 22.18°C ***Ans.***

12. *Steam is supplied at 14 bar, 400 ℃ to set of 16 nozzles of impulse turbine. Pressure of steam at exit of nozzle is 10 bar and the discharge is 5 kg/s. Nozzle efficiency is 90%. Determine the cross-sectional area at exit of each nozzle. Also determine the percentage increase in discharge if steam is supplied to nozzle with velocity of 100 m/s.*

Solution:

Let inlet and exit states be 1 and 2. Therefore, enthalpy at state 1,

$$h_1 = h_{\text{at 14 bar, 400°C}}$$

From steam tables,

$$h_1 = 3257.5 \text{ kJ/kg}$$

$$s_1 = 7.3026 \text{ kJ/kg·K}$$

Since steam is superheated at inlet, so $n = 1.3$

For maximum discharge, throat pressure

$$\frac{P_{2'}}{P_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{(n-1)}}$$

$$P_2' = 14\times\left(\frac{2}{1.3+1}\right)^{\left(\frac{1.3}{0.3}\right)}$$

$$P_2' = 7.64 \text{ bar}$$

The actual pressure given at exit of nozzle is 10 bar which means that the nozzle is of convergent type.

Actual pressure at exit of nozzle, P_2. For isentropic expansion from 14 bar to 10 bar, $s_1 = s_2 = 7.3026$. State of steam at exit is superheated because s_2 is more than s_g at 10 bar (= 6.5865 kJ/kg·K). This entropy s_2 lies between superheat steam temperature of 350°C and 400°C at 10 bar.

$$s_2 = s_{\text{at 10 bar, 350°C}} + \frac{\left(s_{\text{at10bar, 400°C}} - s_{\text{at10bar,350°C}}\right)}{(400 - 350)} \times (T_2 - 350)$$

$$7.3026 = 7.3011 + \left(\frac{7.4651 - 7.3011}{50}\right) \times (T_2 - 350)$$

$$T_2 = 350.46°\text{C}$$

At 350.46°C and 10 bar from steam tables by interpolation,

$$h_2 = h_{\text{at 10 bar, 350°C}} + \left(\frac{h_{\text{at10bar, 400°C}} - h_{\text{at10bar,350°C}}}{400 - 350}\right) \times (T_2 - 350)$$

$$= 3157.7 + \left(\frac{3263.9 - 3157.7}{50}\right) (350.46 - 350)$$

$$h_2 = 3158.7 \text{ kJ/kg}$$

Specific volume at exit, $v_2 = 0.2827 \text{ m}^3/\text{kg}$

Actual enthalpy change, $\Delta h_{12} = (h_1 - h_2) \times \eta_N$

$= (3257.5 - 3158.7) \times 0.9$

$= 88.92 \text{ kJ/kg}$

Velocity at exit of nozzle, $C_2 = \sqrt{2 \cdot \Delta h_{12}}$

$= \sqrt{2 \times 10^3 \times 88.92}$

$= 421.71 \text{ m/s}$

Cross-sectional area at exit of nozzle $A_2 = \dfrac{m \times v_2}{C_2 \times N}$

$= \dfrac{5 \times 0.2827}{421.71 \times 16} \times 10^4$

$= 2.13 \text{ cm}^2$

Cross-sectional area at exit of nozzle, = 2.13 cm^2 ***Ans.***

Inlet velocity being $C_1 = 100$ m/s

Modified velocity at nozzle exit $C_2' = \sqrt{2 \cdot \Delta h_{12} + C_1^2}$

$= \sqrt{\left(2 \times 10^3 \times 88.92\right) + (100)^2}$

$C_2' = 433.41 \text{ m/s}$

Discharge with modified velocity $= \dfrac{A_2 \times C_2' \times 16}{v_2}$

$= \dfrac{16 \times 2.13 \times 10^{-4} \times 433.41}{0.2827}$

$= 5.22 \text{ kg/s}$

Percentage increase in discharge $= \left(\frac{5.22 - 5}{5}\right) \times 100$

$= 4.4\%$

% Increase in discharge = 4.4% ***Ans.***

13. *In a nozzle the steam enters at 20 bar, dry saturated and expands up to 5 bar. Considering the expansion to be frictionless throughout and steam remaining in dry state during expansion upto throat, determine the degree of supersaturation and degree of undercooling. Also determine the change of entropy, the loss due to undercooling and percentage loss if steam has to revert instantaneously to saturated state at constant enthalpy and if the further expansion takes place in thermal equilibrium.*

Solution:

At inlet to nozzle, $P_1 = 20$ bar, $T_1 = T_{\text{sat at 20 bar}}$

or, $T_1 = 212.42°C$

Pressure at throat, $P_2 = P_1 \times 0.58$

$P_2 = 11.6$ bar

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{n-1/n} \Rightarrow T_2 = T_1 \times (0.58)^{\frac{(1.3-1)}{1.3}}$$

$T_2 = 485.42 \times 0.882$

or, $T_2 = 428.14K$

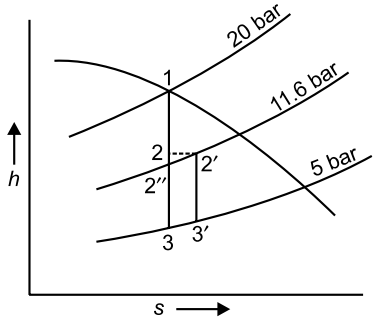


Fig. 13.29

But saturation temperature corresponding to throat pressure,

$T_{\text{sat 11.6 bar}} = 186.43°C$

Saturation pressure corresponding to 428.14K (155.14°C)

$P_{\text{sat at 155.14°C}} = 0.5452$ MPa

$P_{\text{sat at 155.14°C}} = 5.452$ bar

Degree of supersaturation $= \frac{11.6}{5.452}$

$= 2.13$

Degree of undercooling $= T_{\text{sat at 11.6 bar}} - T_2$

$= 186.43 - 155.14 = 31.29°C$

Degree of supersaturation = 2.13
Degree of undercooling = 31.29°C **Ans.**

Enthalpy at state 1, $h_1 = h_{f \text{ at } 20 \text{ bar}} = 2799.5$ kJ/kg

$v_1 = v_{g \text{ at } 20 \text{ bar}} = 0.009963$ m^3/kg

$s_1 = s_{g \text{ at } 20 \text{ bar}} = 6.3409$ kJ/kg·K

Isentropic enthalpy drop,

$$h_1 - h_2 = \left(\frac{n}{n-1}\right) \cdot P_1 v_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}}\right]$$

$$h_1 - h_2 = \left(\frac{1.3}{1.3-1}\right) \times 20 \times 10^2 \times 0.009963 \left[1 - \left(\frac{11.6}{20}\right)^{\left(\frac{1.3-1}{1.3}\right)}\right]$$

$$\Delta h_{12} = h_1 - h_2 = 10.19 \text{ kJ/kg}$$

or, $h_2 = 2789.31$ kJ/kg

In the absence of supersaturation the state at throat will be 2″.

Thus for getting enthalpy at 2″ the dryness fraction may be assumed x_2''. The entropy at inlet and throat shall be equal,

$$s_2'' = s_1$$

$$s_2'' = 6.3409$$

$$6.3409 = s_{f \text{ at } 11.6 \text{ bar}} + x_2'' \cdot s_{fg \text{ at } 11.6 \text{ bar}}$$

$$6.3409 = 2.2016 + (x_2'' \times 4.3338)$$

$$x_2'' = 0.955$$

Enthalpy, $h_2'' = h_{f \text{ at } 11.6 \text{ bar}} + x_2'' \cdot h_{fg \text{ at } 11.6 \text{ bar}}$

$= 791.73 + (0.955 \times 1991.88)$

$h_2'' = 2693.98$ kJ/kg

Heat drop with no supersaturation upto throat, $\Delta h_{12}'' = h_1 - h_2''$

$\Delta h_{12}'' = 105.52$

Loss of available heat drop $= \Delta h_{12}'' - \Delta h_{12} = 95.33$ kJ/kg

Increase in entropy, $\Delta s_{12}' = \dfrac{\Delta h_{12}'' - \Delta h_{12}}{T_{\text{sat at } 11.6 \text{ bar}}} = \dfrac{95.33}{459.43}$

$\Delta s_{12}' = 0.2075$ kJ/kg·K

Hence, entropy at 2′, $s_2' = s_1 + \Delta s_{12}'$

$s_2' = 6.5484$ kJ/kg·K

With super saturation

For isentropic expansion after throat, $s_2' = s_3'$

Let dryness fraction at exit be x_3'. So,

$$s_3' = s_{f \text{ at 5 bar}} + (x_3' \cdot s_{fg \text{ at 5 bar}})$$
$$6.5484 = 1.8607 + (x_3' \times 4.9606)$$
$$x_3' = 0.945$$
$$\text{Enthalpy at } 3',\ h_3' = h_{f \text{ at 5 bar}} + (x_3' \times h_{fg \text{ at 5 bar}})$$
$$= 640.23 + (0.945 \times 2108.5)$$
$$h_3' = 2632.76 \text{ kJ/kg}$$

Without supersaturation

Let dryness fraction be x_3 at state 3, which will be obtained in the absence of supersaturation.

$$s_1 = s_3$$
$$s_3 = 6.3409 = s_{f \text{ at 5 bar}} + (x_3 \times s_{fg \text{ at 5 bar}})$$
$$6.3409 = 1.8607 + (x_3 \times 4.9606)$$
$$x_3 = 0.903$$
$$\text{Enthalpy at 3, } h_3 = h_{\text{f at 5 bar}} + (x_3 \times h_{fg \text{ at 5 bar}})$$
$$= 640.23 + (0.903 \times 2108.5)$$
$$h_3 = 2544.21 \text{ kJ/kg}$$

Thus, loss due to undercooling, $h_3' - h_3 = 2632.76 - 2544.21$

$$\Delta h_{3'3} = 88.55 \text{ kJ/kg}$$

$$\text{Percentage loss} = \frac{\Delta h_{3'3}}{(h_1 - h_3)} \times 100$$
$$= \frac{88.55}{(2799.5 - 2544.21)} \times 100$$
$$= 34.68\%$$

Entropy change = 0.2075 kJ/kg·K
Loss due to undercooling = 88.55 kJ/kg
Percentage loss = 34.68%

Ans.

14. *A steam injector delivers 150 kg of water per minute from a water tank where the constant water level of 5 m below the axis of injector is maintained. Injector injects into the boiler having steam at 20 bar. Water level in boiler is 0.8 m above the injector. Steam for injector is taken from boiler and has dryness fraction of 0.95. Water is supplied at 25°C and velocity in delivery pipe is 20 m/s.*

Determine,

(i) the mass of water pumped per kg of steam
(ii) the diameter of throat of mixing nozzle
(iii) the diameter of the steam nozzle considering pressure at throat to be 0.7 times of supply pressure.
(iv) the temperature of water coming out of injector.

Solution:

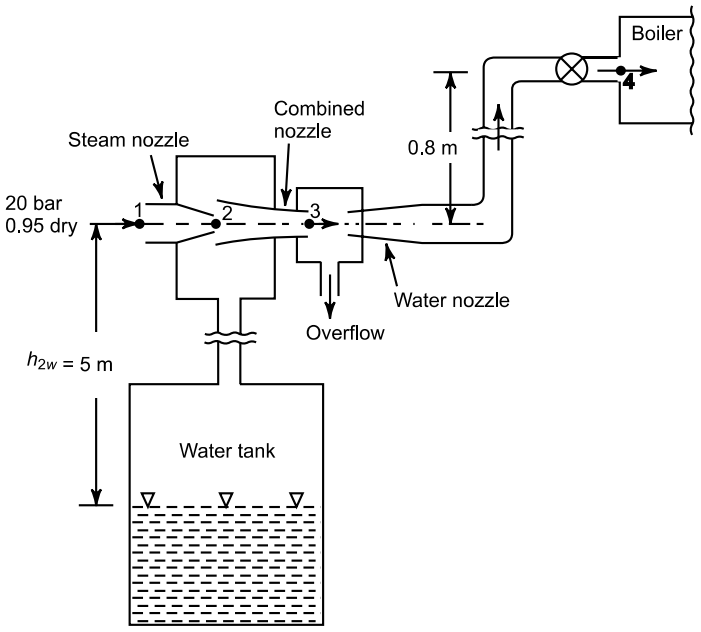


Fig. 13.30

Pressure at inlet of nozzle, $P_1 = 20$ bar,

Temperature $T_1 = T_{\text{sat at 20 bar}} = 212.42°\text{C}$

Pressure at throat, $P_2 = 0.7 \times 20 = 14$ bar

Enthalpy at state 1, $h_1 = h_{f \text{ at 20 bar}} + x_1 \cdot h_{fg \text{ at 20 bar}}$

$= (908.79) + (0.95 \times 1890.7)$

$h_1 = 2704.95$ kJ/kg

$s_1 = s_{f \text{ at 20 bar}} + x_1 \cdot s_{fg \text{ at 20 bar}}$

$= 2.4474 + (0.95 \times 3.8935)$

$s_1 = 6.1462$ kJ/kg·K

At throat $s_2 = s_1 = 6.1462$ kJ/kg·K

Let dryness fraction at throat be x_2

$s_2 = s_{f \text{ at 14 bar}} + x_2 \cdot s_{fg \text{ at 14 bar}}$

$6.1462 = 2.2842 + (x_2 \times 4.1850)$

$x_2 = 0.923$

$h_2 = h_{f \text{ at 14 bar}} + x_2 \cdot h_{fg \text{ at 14 bar}}$

$= 830.30 + (0.923 \times 1959.7)$

$h_2 = 2639.10$ kJ/kg

Specific volume $v_2 = v_{f \text{ at 14 bar}} + x_2 \cdot v_{fg \text{ at 14 bar}} = 0.13 \text{ m}^3/\text{kg}$

Velocity of steam at throat, $C_2 = \sqrt{2(h_1 - h_2)}$

$= \sqrt{2 \times (2704.95 - 2639.10) \times 10^3}$

$$C_2 = 362.9 \text{ m/s}$$

Applying Bernoulli's equation between states 3 and 4,

$$gZ_3 + \frac{P_3}{\rho} + \frac{C_3^2}{2} = gZ_4 + \frac{P_4}{\rho} + \frac{C_4^2}{2}$$

$$\frac{C_3^2}{2} + \frac{1.013 \times 10^5}{10^3} = \frac{20 \times 10^5}{10^3} + (0.8 \times 9.81) + \frac{(20)^2}{2}$$

$$C_3 = 64.91 \text{ m/s}$$

Let mass of water lifted by per kg of steam be m kg, i.e. m kg of water per kg of steam.

$$C_2 - m\sqrt{2gh_2w} = (m + 1) \times C_3$$

$$C_2 - m \times \sqrt{2 \times 9.81 \times 5} = (m + 1) \times C_3$$

$$362.9 - 9.90\, m = (m + 1) \times 64.91$$

$$m = 3.98 \text{ kg of water per kg steam}$$

Mass of water pumped per kg of steam = 3.98 kg **Ans.**

Mass of mixture passing through state 3 for given water flow,

$$= \frac{150}{60} + \frac{150}{60} \times \frac{1}{3.98}$$

$$= 2.5 + 0.628$$

$$m_3 = 3.128 \text{ kg/s}$$

Area of throat of mixing nozzle = $\frac{m_3}{\rho \times C_3} = \frac{3.128}{10^3 \times 64.91} = 0.482 \text{ cm}^2$

Diameter of throat of mixing nozzle, $d_3 = 0.783$ cm

Diameter of throat of mixing nozzle = 0.783 cm **Ans.**

Mass of steam required for given water flow rate = $\frac{150}{60} \times \frac{1}{3.98}$

$$m_{steam} = 0.628 \text{ kg/s}$$

$$m_{steam} = \frac{A_{steam} \times C_{steam}}{v_{steam}}$$

$$m_{steam} = \frac{A_2 \times C_2}{v_2}$$

$$0.628 = \frac{A_2 \times 362.9}{0.13}$$

$$A_2 = 2.25 \text{ cm}^2$$

$$d_2 = 1.69 \text{ cm}$$

Diameter of throat of steam nozzle = 1.69 cm **Ans.**

In order to determine temperature of water coming out of injector the heat balance may be written assuming no heat loss. For unit mass of steam entering injector

$x_1 \times h_{fg1} + c_{ps}\,(T_1 - T_3) = m\,c_{pw}(T_3 - T_w)$

$(0.95 \times 1890.7) + 4.18\,(212.42 - T_3) = 3.98 \times 4.18\,(T_3 - 25)$

$T_3 = 148.92°C$

Temperature of water coming out of injector = 148.92°C ***Ans.***

15. *A steam injector is used for maintaining supply of the feed water to boiler producing steam at 20 bar. The pressure of exhaust steam for operating injector is 1.5 bar and dryness fraction is 0.9. The mass of water taken from feed water tank is 5000 kg/hr and temperature is 17°C. Determine the mass of water that can be pumped per kg of steam, area of steam and water discharge orifices. Delivery pressure may be assumed to be 20% more than boiler pressure so as to overcome frictional losses and positive delivery. Neglect change in elevation.*

Solution:

At inlet to the steam nozzle pressure, $P_1 = 1.5$ bar

Pressure at throat, $P_2 = P_1 \left(\frac{2}{n+1}\right)^{\frac{n}{(n-1)}}$, here $n = 1.13$, $P_2 = 0.87$ bar

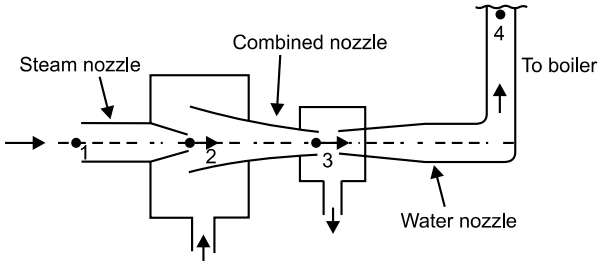


Fig. 13.31

Temperature at state 1, $T_1 = T_{\text{sat at 1.5 bar}} = 111.37°C = 384.37$ K

Enthalpy at state 1, $h_1 = h_{f \text{ at 1.5 bar}} + x_1 \cdot h_{fg \text{ at 1.5 bar}}$

$= 467.11 + (0.9 \times 2226.5)$

$h_1 = 2470.96$ kJ/kg

$s_1 = 1.4336 + (0.9 \times 5.7897)$

$s_1 = 6.6443$ kJ/kg·K

At state 2, $s_2 = s_1 = 6.6443 = s_{f \text{ at 0.87 bar}} + x_2 \cdot s_{fg \text{ at 0.87 bar}}$

$6.6443 = 1.2560 + x_2 \cdot 6.1538$

$x_2 = 0.88$

$h_2 = h_{f \text{ at 0.87 bar}} + x_2 \times h_{fg \text{ at 0.87 bar}}$

$= 400.26 + (0.88 \times 2268.71)$

$h_2 = 2396.72$ kJ/kg

Specific volume, $v_2 = v_{f \text{ at 0.87 bar}} + x_2 \cdot v_{fg \text{ at 0.87 bar}}$

$= 0.001039 + (0.88 \times 1.9649)$, $v_2 = 1.7302$ m^3/kg

Steam velocity, $C_2 = \sqrt{2(h_1 - h_2)}$

$C_2 = 385.33$ m/s

By Bernoulli's equation,

$$\frac{C_3^2}{2} = \frac{P_4}{\rho}$$

$$C_3 = \sqrt{\frac{1.2 \times 20 \times 2 \times 10^5}{1000}}$$

$C_3 = 69.28$ m/s

Let mass of water entrained per kg of steam be m kg water per kg of steam.
Neglecting momentum of entering water the momentum balance can be given as under.

$1 \times C_2 = (m + 1) \times C_3$

$385.33 = (m + 1) \times 69.28$

$m = 4.56$ kg water per kg of steam

Mass of steam supplied per second $= \dfrac{5000}{3600 \times 4.56} = 0.3046$ kg/s

Area of steam nozzle, $A_2 = \dfrac{m v_2}{C_2}$

$$= \frac{0.3046 \times 1.7302}{385.33}$$

$A_2 = 13.68$ cm^2

Total discharge from injector $= \left(\dfrac{5000}{3600} + 0.3046\right)$

$= 1.693$ kg/s

Area of discharge orifice, $= \dfrac{1.693}{69.28 \times 10^3} = 0.2444 \times 10^{-4}$ m^2

$= 0.2444$ cm^2

Mass of water pumped per kg of steam = 4.56 kg water/kg of steam
Area of steam nozzle = 13.68 cm^2
Area of discharge orifice = 0.244 cm^2

Ans.

EXERCISES

13.1 What do you understand by nozzle? Discuss different types of nozzles.

13.2 Derive the expression of velocity of fluid leaving nozzle, considering flow to be frictionless and adiabatic.

13.3 Obtain expression for discharge through nozzle.

13.4 For a convergent-divergent nozzle sketch variation of specific volume, velocity, area and pressure along the nozzle axis.

13.5 Show the following statement, "Converging duct behaves as nozzle for subsonic flows and the diverging duct shall behave as nozzle when flow is supersonic."

13.6 Prove that the maximum discharge of fluid per unit area through a nozzle shall occur when the ratio of fluid pressure at throat to the inlet pressure is $\left(\frac{2}{n+1}\right)^{\left(\frac{n}{(n-1)}\right)}$, where n is the index of adiabatic expansion.

Also obtain the expression for maximum mass flow through a convergent-divergent nozzle having isentropic expansion starting from rest.

13.7 For isentropic flow through a nozzle prove that area on p-v diagram gives the heat drop during expansion. Also show processes on T-s and h-s diagram.

13.8 Explain the significance of choked flow.

13.9 Describe the 'over expansion' and 'under expansion' in nozzles.

13.10 Give the pressure and velocity variations along the length of 'convergent nozzle' and 'convergent-divergent nozzle' when back pressure is reduced gradually from inlet pressure to the pressure below designed back pressure value.

13.11 Discuss the effects of friction on the performance of nozzles. Also show these effects on T-s and h-s diagrams.

13.12 What do you understand by supersaturation of steam flowing through nozzles? Explain the phenomenon and factors responsible for it.

13.13 Describe the physical significance of supersaturation in steam nozzle.

13.14 Describe the working of steam injector and its applications.

13.15 Write short notes on the following:

(a) Nozzle efficiency (b) Velocity coefficient

(c) Discharge coefficient (d) Critical velocity

13.16 Steam at 10 bar and 250°C is admitted in convergent-divergent nozzle with initial velocity of 75 m/s. Determine the velocity at exit considering isentropic flow through nozzle. [886.35 m/s]

13.17 Air enters into a nozzle at 4 bar and leaves at 1 bar, 350°C and 700 m/s. Determine temperature of air entering nozzle, nozzle efficiency and nozzle exit area for air flow rate of 4 kg/s. Take c_p = 1.003 kJ/kg·K. [594.27°C, 86.04%, 10.4 cm^2]

13.18 In a steam nozzle the expansion pressure ratio of 7.5 is achieved with steam entering at 300°C and leaving at 2 bar. For the steam mass flow rate of 1.2 kg/s determine the throat area considering adiabatic expansion in the absence of friction loss. [4.45 cm^2]

13.19 Steam at dry saturated state enters the nozzle at 10 bar and 90 m/s velocity. Steam leaves the nozzle at 6 bar, 435 m/s. Considering heat loss of 1.51 kcal/kg of steam flow determine dryness fraction at exit of nozzle and ratio of inlet to exit cross-section area. [0.92, 3.22]

13.20 A convergent-divergent nozzle has steam entering at 10 bar and 0.97 dry and leaving at 0.6 bar. For the steam mass flow rate of 0.2 kg/s determine the ratio of throat diameter to exit diameter considering that 10% of overall isentropic heat drop is the frictional heat generated in divergent portion of nozzle. [0.516]

13.21 A steam nozzle has steam entering at 6 bar, 300°C and expansion occurs upto steam pressure of 1.5 bar. For the mass flow rate being 5 kg/s and nozzle exit area being 6.75 cm^2 determine nozzle efficiency. [90%]

13.22 In a convergent-divergent nozzle steam enters at 7 bar, 0.96 dry and back pressure is 1 bar. Throat area of nozzle is 6.5 cm^2. In the diverging portion the friction causes reduction in total enthalpy drop by 15% compared to ideal enthalpy drop. Determine mass flow rate, velocity and condition of steam leaving nozzle assuming negligible inlet velocity. [0.684 kg/s, 736 m/s, 0.925]

13.23 Convergent-divergent nozzle operates with steam entering at 30 bar, 300°C and leaving at 5 bar, 800 m/s. Assuming negligible inlet velocity and the friction to occur in only divergent portion of nozzle with mass flow rate of 10 kg/s, determine,

(a) the efficiency for expansion in divergent portion of nozzle,

(b) the velocity of steam at throat. [90%, 530 m/s]

13.24 Steam enters a convergent-divergent nozzle at 7 bar, 773 K and isentropic expansion occurs up to pressure of 3 bar. Determine the rate of steam mass flow considering

(i) equilibrium flow

(ii) supersaturated flow with $pv^{1.3}$ = constant

Also determine degree of supersaturation and degree of undercooling. Assume inlet velocity to be negligible. [96 kg/s, 1.71, 17.5°C]

13.25 Steam expands from 20 bar, 250°C to pressure of 3 bar. Nozzle efficiency is 90% and exit area is 3.15 cm^2. Assuming supersaturated flow to be governed by $pv^{1.3}$ = constant and mean value of specific heat for supersaturated steam being 1.93 kJ/kg·K determine the mass flow rate of steam on per minute basis. [36 kg/min]

13.26 Steam at 8 bar, dry saturated enters a convergent-divergent nozzle to expand upto 1.5 bar. Determine the ratio of cross-section area at exit and throat considering flow to be isentropic and expansion index being 1.135. [1.94]

13.27 Convergent nozzle has steam entering at 4 bar, 150°C and leaving at atmospheric pressure. The supersaturated expansion occurs following the equation,

$Pv \times 10^3 = 2.308\ (h - 1943)$ where v is in m^3/kg, P is in bar, h is in kJ/kg. Determine the mass flow rate of steam for nozzle efficiency of 90% and throat area of 16.75 cm^2. [1 kg/s]

13.28 A convergent-divergent nozzle has steam entering at 10 bar, 270°C and leaving at 1.2 bar. Loss in nozzle occurs only in diverging portion because of friction and amounts to 15% of total enthalpy drop. For the cone angle being 5° in diverging portion, throat diameter being 6 mm determine the length of diverging portion of nozzle. [32 mm]

13.29 Steam at 1.5 bar, dry saturated is used in steam injector for feeding water into a boiler. Water enters into boiler at pressure being 20% more than boiler pressure of 20 bar. Water is taken from feed water tank at 20°C and rate of water supply is 1.67 kg/s. Determine

(a) the mass of water injected per kg of steam if coefficient of steam nozzle is 0.95.

(b) ratio of diameters of steam nozzle to water discharge nozzle.

(c) temperature of water entering the boiler.

Also state the assumptions made for analysis, if any.

[5.2 kg/kg of steam, 7.3, 121°C]

13.30 A steam injector lifts water from 4 m below the injector axis and feeds 150 kg per minute water into the boiler which is 1 m above the injector axis. Boiler produces steam at 15 bar, dry saturated and for steam injector the steam is taken from boiler itself. Water in the tank is at 30°C and the velocity of water being delivered into boiler is 20 m/s. Determine (a) the mass of steam required per hour for operating injector, (b) diameter of throat of mixing nozzle, (c) temperature of water leaving injector, (d) diameter of throat of steam nozzle for the maximum steam flow and nozzle efficiency of 92%. [1560 kg/hr, 8.15 mm, 124°C, 16.35 mm]

14
Steam Turbines

14.1 INTRODUCTION

Steam turbines date back to 120 B.C. when the first steam turbine was developed by Hero of Alexandria. Subsequently number of steam turbines came up but the practically successful steam turbine appeared at the end of nineteenth century when Gustaf De Laval designed a high speed turbine built on the principle of reaction turbine in 1883. Before this in 1629 G. Branca developed the first impulse turbine. Branca's impulse turbine and Hero's reaction turbine are shown in Fig. 14.1.

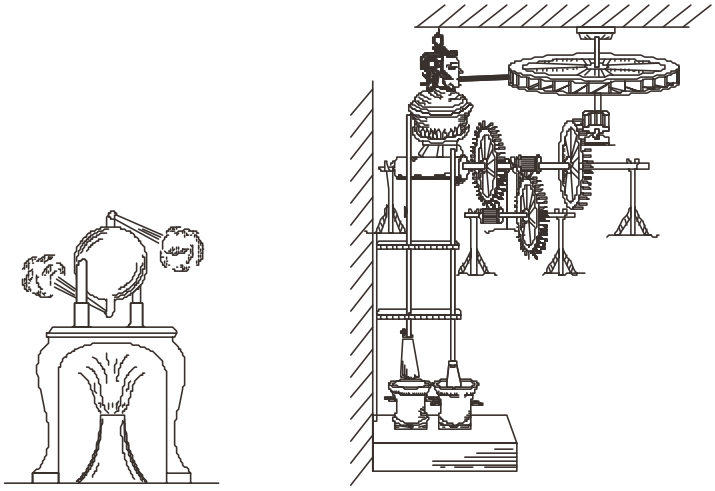


Fig. 14.1 *Hero's reaction turbine and Branca's impulse turbine*

In nineteenth century some more steam turbines were developed by Sir Charles A. Parsons and C.G. Curtis which gave a filip to the development to the modern steam turbine. Over the period of time the modern steam turbines evolved with capacity from few kilowatts to 350,000 kW and in speed from 1000 rpm to 40,000 rpm. Steam turbines offer the advantages over other prime movers in terms of simplicity, reliability and low maintenance costs. Reciprocating steam engines use pressure energy of steam while steam turbines use dynamic action of the steam. Steam turbines require less space as compared to diesel engine or steam engine and also the absence of reciprocating parts & reciprocating motion in steam turbine results in lesser vibrations and lighter foundation. In steam turbine the expanding steam does not come into contact with lubricant and so exhaust steam leaves uncontaminated.

14.2 WORKING OF STEAM TURBINE

A steam turbine has basically nozzle and ring of moving blades mounted on a shaft called rotor. The shaft motion of turbine depends solely upon the dynamic action of steam. Steam is injected through nozzles over to the ring of moving blades. Thermal energy of steam is partly converted into kinetic energy due to static pressure drop in nozzle. High velocity steam leaving nozzle enters the moving blade and the direction of steam flow gets changed from inlet to exit. This change in direction of steam flow causes change of momentum, which results in dynamic force acting as driving thrust for rotation of shaft. Steam turbine can be impulse turbine or reaction turbine based on mechanism of driving thrust creation. If the static pressure drop occurs principally in stationary nozzle with little or no static pressure drop occurring in rotor blade passage, then turbine is called an 'impulse turbine'. If the substantial static pressure drop occurs in stationary nozzle and rotor blade passage both then turbine is called 'reaction turbine'. Thus in case of impulse turbine driving thrust is available due to change in momentum because of change in velocity direction while moving across the blade from inlet to exit. In case of reaction turbine the static pressure drop in rotor blade passage causes further conversion of thermal energy into kinetic energy and hence generation of resultant reactive force. Driving thrust in reaction turbine comprises of reactive force and force associated with change in momentum due to change in direction of velocity. Impulse Turbine has basically following components which are also described in Fig. 14.2.

***Stationary nozzle*:** Impulse turbine has stationary passage comprising of one or more stationary nozzles in which steam at high static pressure and low velocity is expanded so as to increase velocity of steam at exit of nozzle. High velocity stream leaves nozzle so as to deliver steam with correct direction for smooth entry into moving blade.

Moving blades: Moving blades are fixed around the circumference of a rotor shaft with top of blades connected together for rigidity by means of a blade shroud ring. Blades actually cause change in direction of steam and so the momentum change occurs and thus impulse is generated. Steam turbine blades are made of alloy steel and manufactured by machining from bar stock. Typical blade material called nickel based super alloy may have Ni, Co, W, Cr, Al, Tn, Ti, Hf, Fe, Mo, C, Si, Mn, Cu, Za, B, S, Pb in percentage by weight of 59, 10, 10, 9, 5.5, 2.5, 1.5, 1.5, 0.25, 0.25, 0.15, 0.1, 0.1, 0.05, 0.05, 0.015, 0.008 and 0.0005, respectively.

Guide blades: Turbine rotor has guide blades prior to moving blades so as to guide steam in proper direction for smooth entry into moving blade. Guide blades are stationary guides mounted between the rotor blade rings. Guide blades perform function of reversing the direction of steam leaving the preceding moving blade row so that direction of steam entering moving blade rows is similar.

Casing: Casing refers to the outer enclosure housing nozzles and fixed blades. Casing is also referred to as shell or cylinder. Casing confines steam to flow passages and also provides the structural frame. Casings of large steam turbines are normally horizontally split for convenience of repair and inspection. For high pressures and temperatures casings are made of cast carbon steel while low pressure turbine casings are made from rolled flat steel plate. For very high temperatures stainless steel casings are good.

Shaft: Shaft or rotor or spindle refers to the rotating member upon which moving blade ring is mounted.

Ring or Wheel: Wheel refers to the ring upon which moving blades are mounted. Wheel is keyed on to the shaft. This ring is also called disc.

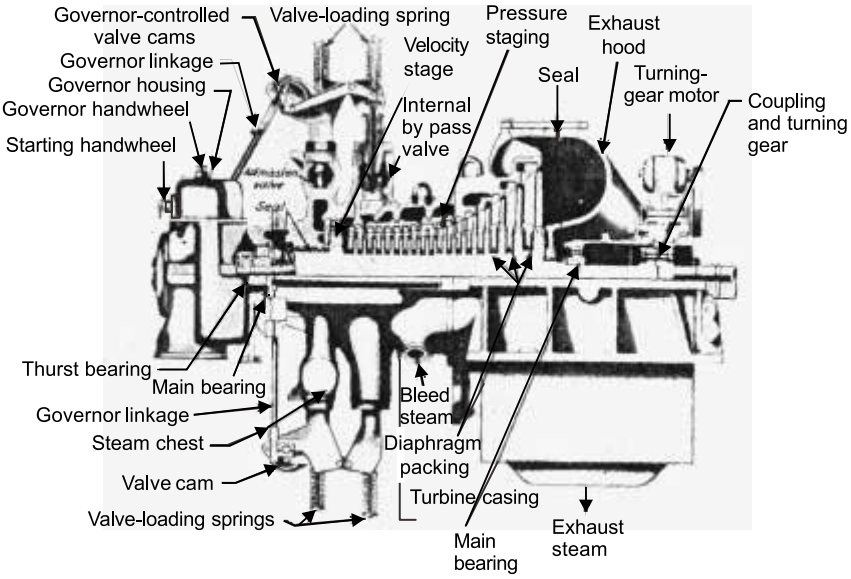


Fig. 14.2 *Impulse turbine*

Diaphragm: Diaphragm is attached to the casing containing the nozzles and performs function of confining steam flow to nozzle passage.

Packing: Packing is provided for preventing the leakage across the annular space between the diaphragm and shaft, casing and shaft. Packing is provided in the form of carbon rings or labyrinth glands. Carbon rings present an effective seal against shaft leakage in small turbines and are extensively used with labyrinth and water seals for preventing shaft leakage in larger turbines. Carbon ring consists of a ring of carbon rectangular in cross section and usually divided into four segments. Rings snugly fit into a recess in casing and are kept tight against shaft by means of garter spring. Labyrinth seals consist of series of thin strips fixed to the casing or other stationary member and arranged so as to maintain the smallest possible clearance with the shaft. The labyrinth seals have small restrictions that increase the velocity of leaking fluid only to have it dissipated in the pockets, thereby throttling the fluid. Figure 14.3 shows the straight and stepped labyrinth seals. The tips of the strips are extremely thin so that if rubbing occurs the tip gets worn away without damaging the shaft. These labyrinth seals do not prevent complete leakage of fluid. These are effective in only reducing leakage. Complete leakage prevention is done in association with other seals as carbon seals described earlier.

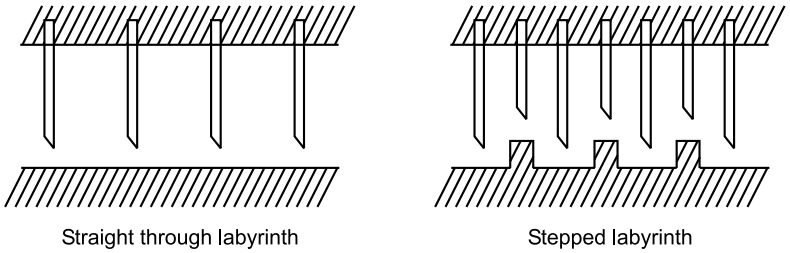


Fig. 14.3 *Labyrinth seals*

Steam chest: This is the steam supply chamber which houses steam before being supplied to nozzles.

Exhaust hood: The portion of casing which collects and delivers the exhaust steam to exhaust pipe or condenser is called exhaust hood.

Throttle valve: Throttle valve, also called stop valve is located in steam supply line of the steam turbine. Throttle valve may be manually operated or hydraulically operated for regulating steam flow during start and stop of turbine.

Governor: Governor is also provided in steam turbine for controlling the steam mass flow so as to maintain constant speed with load fluctuations. An overspeed governor with trip mechanism is also provided to shut off the supply of steam.

Bearings: Turbine has main bearings to support the shaft. Along with these thrust bearings are also provided to support the axial thrust.

Turning gear: Turning gear is generally used with large turbines and consists of a gear integral with turbine shaft driven by electric motor through necessary speed reduction. Turning gear is used so as to keep the turbine shaft rotating at about 1–20 rpm in order to avoid springing of shaft occurring due to unequal expansions and contractions during warming and cooling of turbine.

Trip mechanism: Steam turbine is provided with a trip mechanism operating through an overspeed governor to shut off supply of steam to turbine. Trip mechanism is actually safety device which gets activated upon number of other adverse operating conditions of turbine such as loss of lubricating oil pressure or condenser vacuum or excessive axial thrust etc.

14.3 CLASSIFICATION OF STEAM TURBINES

Steam turbines may be classified into different categories based on various attributes as given below.

(a) *Based on the blade flow passage:* In steam turbine thermal energy available with steam is converted into kinetic energy which in turn produces driving thrust on the shaft. Based upon the rotor blades the blade flow passage may be of (i) constant cross section area type from blade inlet to exit, or, (ii) varying cross section area type from blade inlet to exit.
Turbines having former type blading are called *impulse turbines* while later type are in *reaction turbines*. Figure 14.4 shows the impulse and reaction turbine blades.

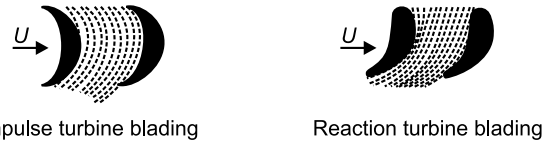


Fig. 14.4 *Impulse turbine and Reaction turbine blades*

The mechanism of impulse and reaction forces getting generated is discussed here. From Newton's second law we know,

$$F = m \cdot a$$

$$F = (m) \cdot \left(\frac{dV}{dt} \right)$$

For steady flow, F = (mass flow rate) × (change in velocity)

Similarly,Tangential force = (mass flow rate) × (change in tangential component of velocity)

The impulse force can be defined as the force because of change in tangential component of velocity of fluid which may be due to change in direction or magnitude. Figure 14.5 shows the impulse force generating because of change in velocity of fluid.

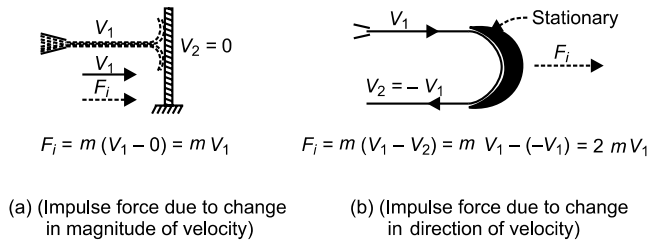


Fig. 14.5 *Impulse force*

In case of Fig. 14.5 (a) the impulse force is available due to change in magnitude of velocity and shall be given by the product of mass flow rate and change in velocity. In case (b) the impulse force is generated due to change in direction of velocity and if the blade is stationary and frictionless then there shall be no decrease in magnitude of velocity.

Reaction force is available when the tangential velocity of fluid is increased and is opposite in reference to the direction of velocity.

In the case shown in Fig 14.5 (b), the total force exerted on blade is actually a combination of impulse and reaction. Impulse force is available in the entrance half of the blade where jet impinges causing a force to right.

While in the exit half, the leaving jet exerts a reactive force on the blade which also acts to the right. Combined effect of the two forces on the impulse blade is arbitrarily called impulse force.

Reaction force available due to increase in tangential velocity of fluid can be seen in case of nozzle due to acceleration of fluid.

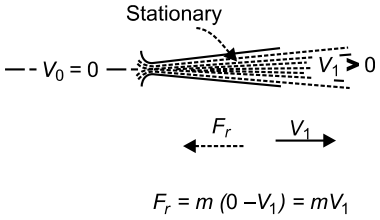


Fig. 14.6 *Reaction force*

Detailed discussion on the reaction turbines running on shaft work resulting from reaction forces is given along with "reaction turbines".

(b) *Based on the cylinder flow arrangement:* Steam turbines may be classified based upon the flow arrangement into following types.

(i) Single flow single casing turbine
(ii) Double flow single casing turbine
(iii) Cross flow compound turbine with single flow
(iv) Cross flow compound turbine with double flow
(v) Triple cross flow compound turbine with double flow

Various arrangements in the above listed turbines are shown by line diagram in Fig 14.7 Single cylinder turbines are the one which have all the stages enclosed in one cylinder while in multi cylinder

turbines the stages are accommodated in more than one cylinders, say two or three. Flow in these can be single flow, double flow, cross flow or reversed flow.

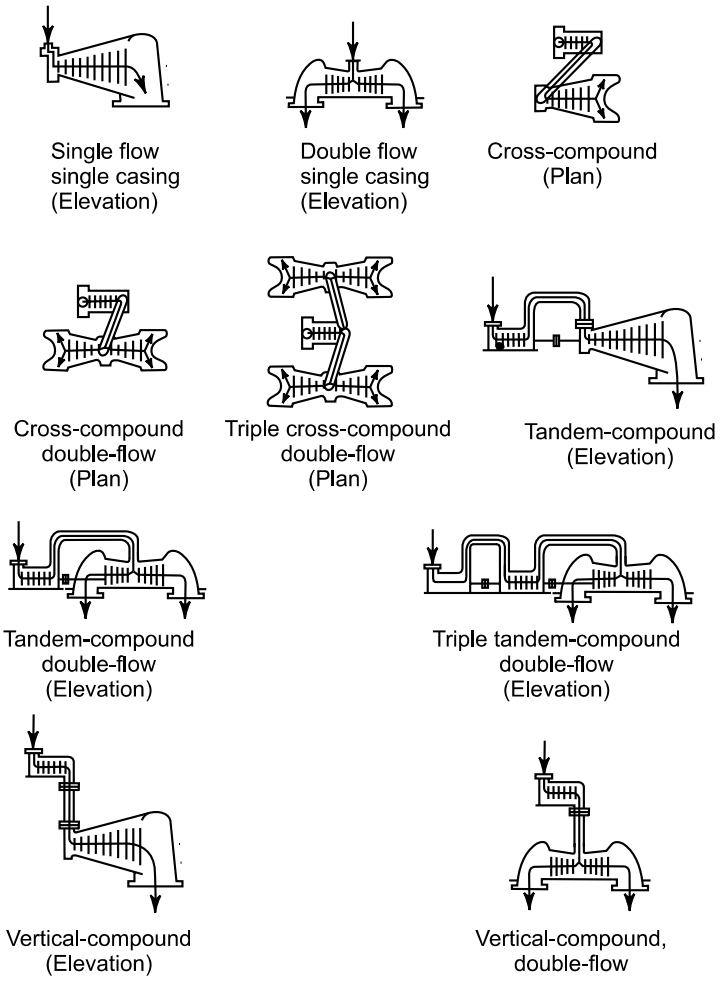


Fig. 14.7 *Classification based on flow arrangement*

(c) *Based on direction of flow:* Steam turbines can be classified based on the direction of flow by which steam flows through turbine blading. Steam turbines can be:

(i) Radial flow turbine
(ii) Tangential flow turbine
(iii) Axial flow turbine

Radial flow turbines were developed by B.F. Ljungstorm of Sweden as shown in Fig. 14.8. Radial flow turbine incorporates two shafts end to end and can be of suitably small sizes. Radial flow turbines can be started quickly and so well suited for peak load and used as stand by turbine or peak load turbines. These are also termed as Ljungstrom turbines.

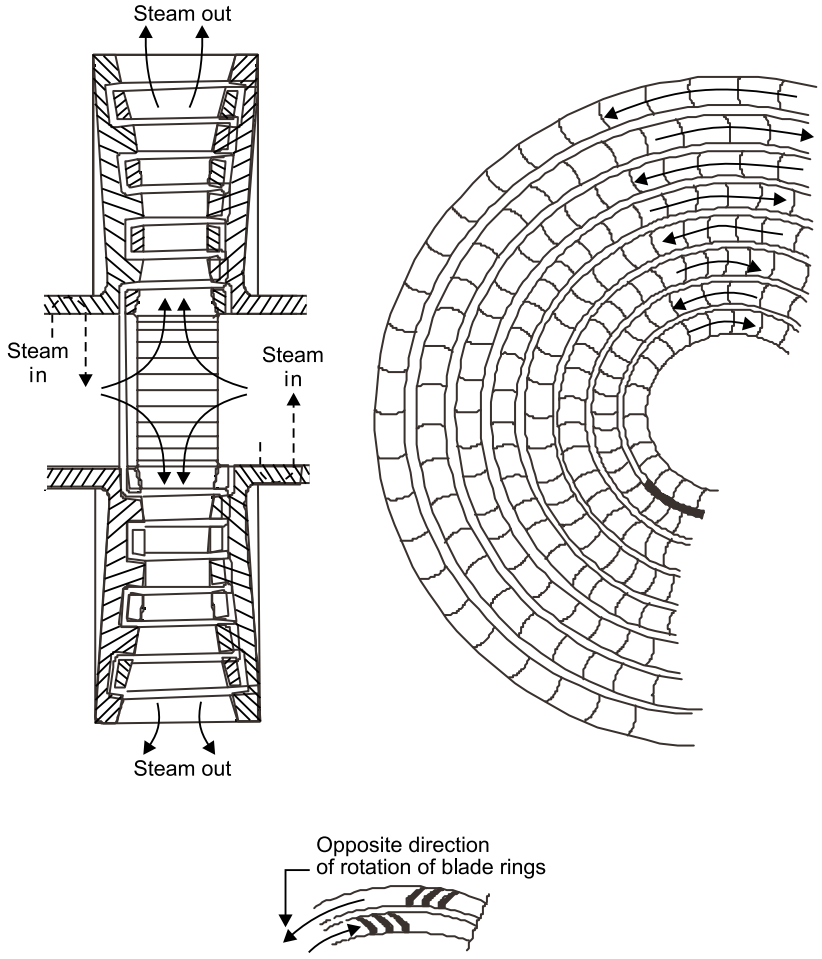


Fig. 14.8 *Radial flow turbine (Ljungstrom turbine)*

In radial flow turbines the steam is injected in middle near shaft and steam flows radially outwards through the successive moving blades placed concentrically. In radial flow turbines there are no stationary blades so pressure drop occurs in moving blade passage. Concentric moving blades rings are designed to move in opposite directions.

In tangential flow turbines the nozzle directs steam tangentially into buckets at the periphery of single wheel and steam reverses back and re-enters other bucket at its' periphery. This is repeated several times as steam follows the helical path. Tangential flow turbines are very robust but less efficient.

In axial flow turbines steam flows along the axis of turbine over blades. These axial flow turbines are well suited for large turbo generators and very commonly used presently.

(d) *Based on number of stages:* Steam turbines can also be classified based upon the number of stages in steam turbines *i.e.* depending upon the amount of heat drop. It can be:

(i) Single stage turbine
(ii) Multi stage turbine.

Single stage turbines have the expansion occurring in single stage while in multi stage turbines the expansion occurs in more than one stages of turbine. When expansion occurs in two stages it is called double stage turbine and with expansion occurring in three stages it is called triple stage turbine.

(e) *Based on the application of turbine:* Depending upon application the steam turbine can be classified as below:

(i) Condensing turbine
(ii) Non-condensing turbine
(iii) Back pressure turbine
(iv) Pass out turbine

Condensing steam turbines are those in which steam leaving turbine enters into condenser. Such type of steam turbines permit for recirculation of condensate leaving condenser. Also the pressure at the end of expansion can be lowered much below atmospheric pressure as the expanded steam is rejected into condenser where vacuum can be maintained. Condensing turbines are frequently used in thermal power plants.

Non-condensing steam turbines are those in which steam leaving turbine is rejected to atmosphere and not to condenser as in case of condensing turbine.

Back pressure turbines reject steam at a pressure much above the atmospheric pressure and steam leaving turbine with substantially high pressure can be used for some other purposes such as heating or running small condensing turbines.

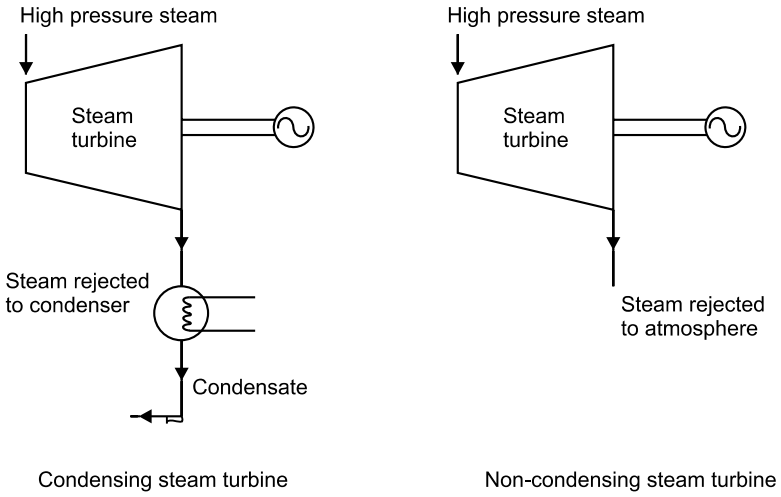


Fig. 14.9 *Condensing and non-condensing steam turbines*

Pass out turbines are those in which certain quantity of steam is continuously extracted for the purpose of heating and allowing remaining steam to pass through pressure control valve into the low pressure section of turbine. Pressure control valve and control gear is required so as to keep the speeds of turbine and pressure of steam constant irrespective of variations of power and heating loads.

(f) *Based on speed of turbine:* Steam turbines can be classified based upon the steam turbine as low speed, normal speed and high speed turbines as given below.

(i) Low speed steam turbine.
(ii) Normal speed steam turbine.
(iii) High speed steam turbine.

Low speed turbines are those steam turbines which run at speed below 3000 rpm. Normal speed steam turbines are those turbines which run at speed of about 3000 rpm while high speed steam turbines are the one which run at more than 3000 rpm.

(g) *Based on pressure in steam turbines:* Steam turbines can also be classified based upon the inlet pressure of steam turbine as follows:

(i) Low pressure steam turbine
(ii) Medium pressure steam turbine
(iii) High pressure steam turbine
(iv) Super pressure steam turbine

Low pressure steam turbines have pressure of inlet steam less than 20 kg/cm^2 while medium pressure steam turbines have steam inlet pressure between 20 kg/cm^2 to 40 kg/cm^2. High pressure steam turbines have steam inlet pressure lying between 40 kg/cm^2 to 170 kg/cm^2 while turbines having inlet steam pressure more than 170 kg/cm^2 are called super pressure steam turbines.

14.4 IMPULSE TURBINE

Schematic of impulse steam turbine is shown in Fig. 14.10. It has single-stage having a nozzle fitted in the casing followed by ring of moving blades mounted on the shaft. Variation of velocity and pressure along the axis of turbine is also shown here. Here pressure drop occurs only in the nozzle and ideally no pressure drop occurs in blades.

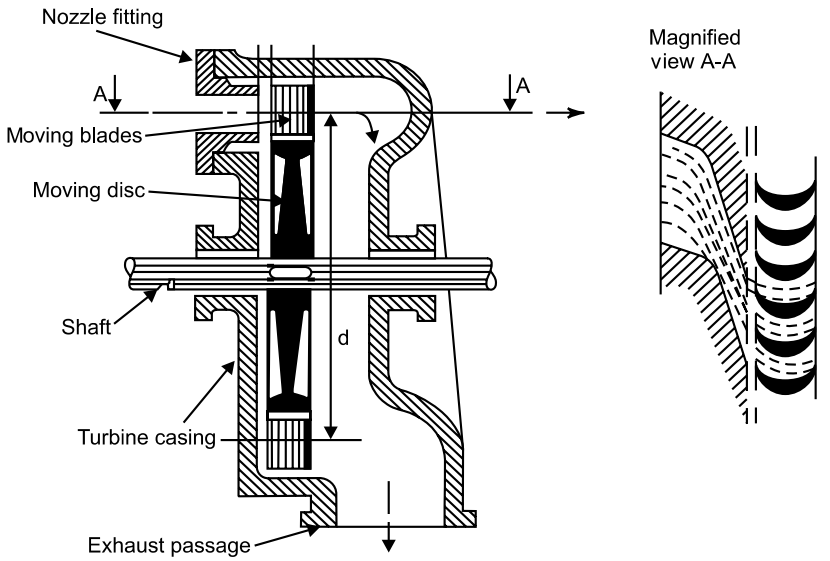


Fig. 14.10 *Schematic of simple impulse steam turbine stage*

High pressure steam from boiler enters the nozzle through pipings and leaves nozzle at predefined angle so as to smoothly flow over the moving blades. Steam velocity gets increased during its flow through nozzle due to its expansion occurring in it. During the passage of steam over the moving blades steam undergoes change in its' direction while losing the velocity and thus causing rotation of moving blade ring mounted on shaft.

Simple impulse turbine is used where small output at very high speed is required or only a small pressure drop is available. These are not suited for applications requiring conversion of large thermal energy into work. Speed of operation of turbine can be regulated by 'compounding' of impulse turbine discussed ahead. Compounding of steam turbine is required as in case of simple impulse turbine, the single stage may offer speed of the order of 30,000 rpm which can not be directly used for any engineering application and needs to be reduced. Also such a high speed shall induce large stresses in the blades. Compounding is a thermodynamic means for reducing the speed of turbine where speed reduction is realized without employing a gear box.

Compounding can be of following three types:

(i) Pressure compounded impulse turbine
(ii) Velocity compounded impulse turbine
(iii) Pressure-velocity compounded impulse turbine

Detailed discussion upon the above three types of compoundings is given below:

(i) ***Pressure compounded impulse turbine:*** Pressure compounded impulse turbine is also called as 'Rateau' turbine. Here pressure staging is done to utilize high velocity steam at acceptable shaft speed. In this the entire pressure drop is realized in parts instead of taking it in single stroke. This segmentation of pressure drop results in moderate steam velocities and thus yielding acceptable rotational speed. In case of pressure compounding there is a ring of fixed nozzles followed by ring of moving blades and subsequently there is again a ring of nozzles followed by a ring of moving blades. Thus pressure compounded impulse turbine consists of a series of simple impulse stages or De Laval turbine stages. Discharge from each moving blade row is supplied to stationary nozzle ring of the subsequent stage. In pressure compounding high pressure steam enters the first ring of stationary nozzles where part expansion occurs causing drop in pressure and increase in steam velocity. Steam subsequently enters moving blade ring where no pressure drop occurs due to symmetrical blading but velocity drops. Steam leaving moving blade ring enters the stationary nozzle ring where remaining part of expansion occurs and expanded steam subsequently enters the moving blade ring. Pressure and velocity variation in a pressure compounded impulse turbine stage are shown in Fig. 14.11 along with the schematic of such compounding.

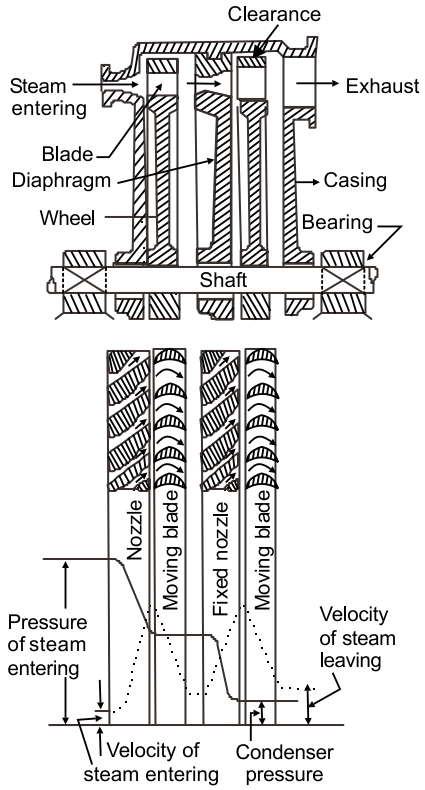


Fig. 14.11 *Pressure compounded impulse turbine stage (Rateau turbine)*

In pressure compounding as the pressure drop occurs in parts so the steam velocities are not very large and hence the turbine velocity gets reduced to low value. Turbine velocity may be further lowered if number of stages is increased. Therefore, pressure compounded impulse turbine has large number of stages which make it most expensive. This type of compounding is of most efficient type as in this ratio of blade velocity to steam velocity remains constant. Pressure compounding is more prone to leakage of steam from one section to other section at the shaft and outer casing as all pressure drop occurs in the nozzles.

(ii) ***Velocity compounded impulse turbine:*** Velocity compounded impulse turbine is called 'Curtis' turbine. Here velocity staging is employed in order to utilize the high velocity steam jet with acceptable rotational speed. In velocity compounded impulse turbine instead of absorbing all kinetic energy in a single moving blade ring it is divided into two or more moving blade rings with guide blades in between the rows. Schematic of velocity compounded impulse turbine stage with pressure and velocity distribution is shown in Fig. 14.12.

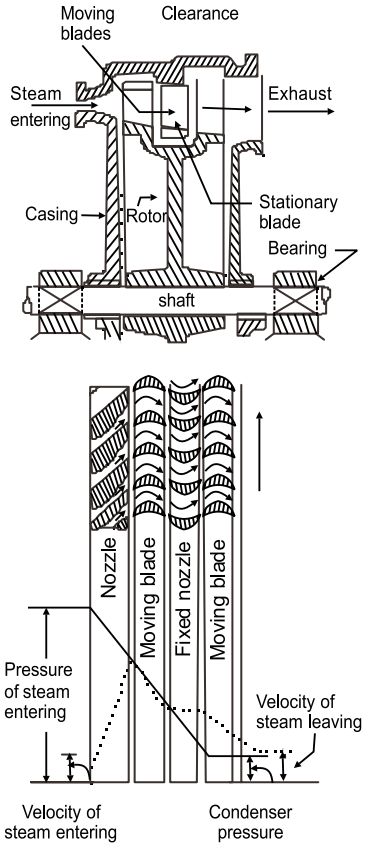


Fig. 14.12 *Velocity compounded impulse turbine or 'Curtis turbine'*

In velocity compounded impulse turbine the high velocity steam from boiler enters the first ring of stationary nozzles and undergoes the complete pressure drop as desired in a stage along with increase in velocity. Low pressure and high velocity steam leaving nozzle enters the moving blade ring where a part of velocity drop takes place while pressure drop does not occur due to symmetrical blade profile.

Steam leaves moving blade ring and enters the fixed blades which act as guide blades. Steam is smoothly guided by fixed blade ring and passes on to ring of moving blades in which remaining part of velocity drop takes place. Thus in a stage of velocity compounded impulse turbine there is stationary nozzle ring followed by moving blade ring and subsequently a fixed blade ring and moving blade ring. Here pressure drop occurs only in nozzle and the velocity drop occurs in two parts in two moving blade rings respectively. For the smooth and symmetrical impulse turbine blades used as fixed guide blades there is no drop in velocity of steam passing through fixed blade ring.

Velocity compounded impulse turbine offers advantages such as less number of stages compared to pressure compounding and so less cost. It also requires less space and is relatively more reliable and easy to start. In multi stage velocity compounded impulse turbine the first stage has large pressure drop and remaining turbine stages are subjected to lower pressure range, thus lesser number of stages. In velocity compounded impulse turbine since pressure drop occurs in nozzle itself so the rest of turbine and its' casing need not be manufactured very strong. But the efficiency is low due to large frictional losses due to large initial velocity and 'non optimum value of ratio of blade velocity to steam velocity for all blade rings'. Efficiency of velocity compounded impulse turbine goes on decreasing with increase in number of stages.

(iii) ***Pressure-velocity compounded impulse turbine:*** Pressure-velocity compounded impulse turbine is a combination of the two types of compoundings described earlier. In this, steam coming from boiler enters the stationary nozzle ring followed by moving blade ring and subsequently fixed blade ring followed by moving blade ring. Steam leaving moving blade ring enters the stationary nozzle ring followed by moving blade, fixed blade and moving blade ring respectively. Schematic of pressure-velocity compounded impulse turbine stage is shown in Fig. 14.13 along with pressure and velocity variation across the different sections.

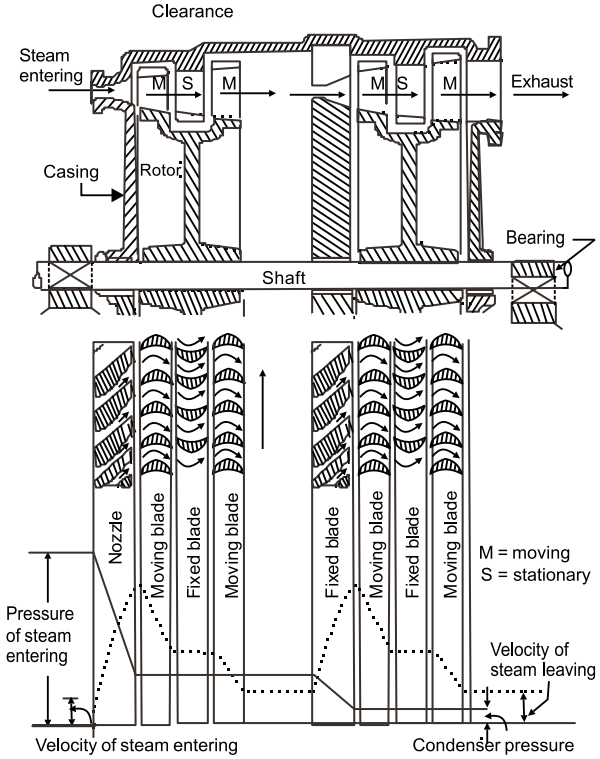


Fig. 14.13 *Pressure-velocity compounded impulse turbine*

Here both pressure drop and velocity drop are divided into different sections as shown in Fig. 14.13. Thus here one or more 'Curtis stage' (velocity compound) followed by 'Rateau stage' (pressure compound) are provided. Curtis stages reduce pressure to a moderate level with high proportion of work per stage and then the highly efficient 'Rateau stages' absorb the remaining energy available. Here it is possible to reduce over all-length of turbine and thus there is saving in initial cost which more than offsets the lower efficiency.

14.5 VELOCITY DIAGRAM AND CALCULATIONS FOR IMPULSE TURBINES

Velocity diagram gives an account of velocity of fluid entering and leaving the turbine. Velocity diagrams for single stage of simple impulse turbine and compound steam turbine are described here.

Simple impulse turbine: Single stage of simple impulse turbine is shown in Fig. 14.14. It comprises of a row of nozzle followed by moving blade row. Pressure and velocity variations along the stage in nozzle ring and moving blade ring are also shown. Subscript 0, 1 and 2 refer to nozzle inlet, nozzle exit or moving blade inlet and moving blade exit respectively.

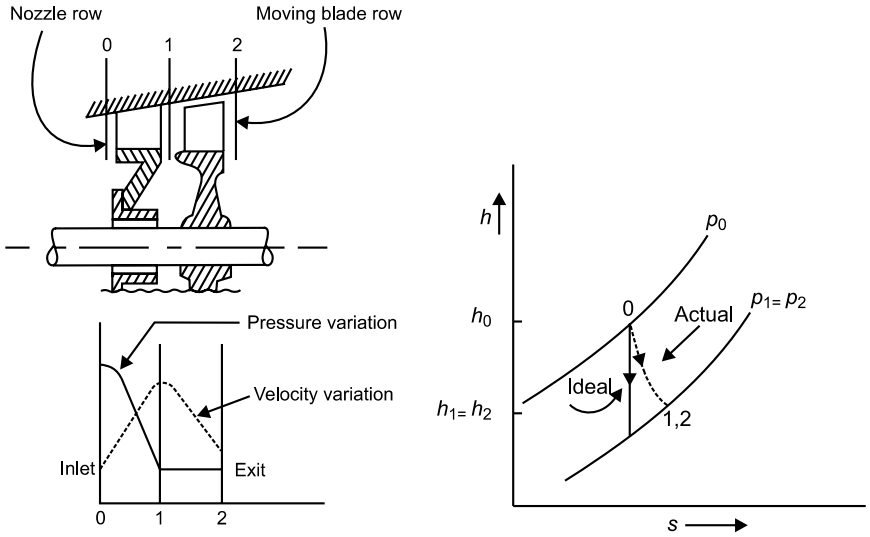


Fig. 14.14 *Single stage of simple impulse turbine*

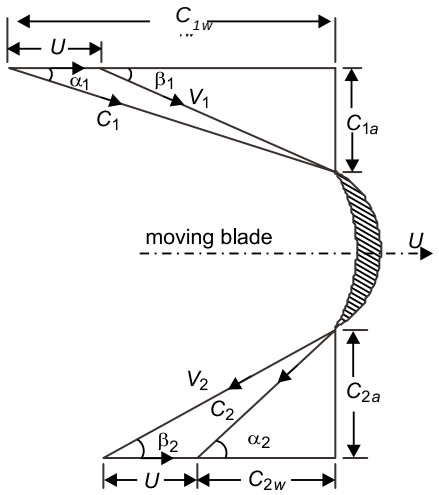
Figure 14.15 gives the inlet and outlet velocity diagrams at inlet edge and outlet edge of moving blade along with the combined inlet and outlet velocity diagram for a stage of simple impulse turbine. The notations used for denoting velocity angles and other parameters during calculations are explained as under, (SI system of units is used here).

U = Linear velocity of blade = $\frac{\pi d N}{60}$, m/s where d = mean diameter of wheel in 'm'

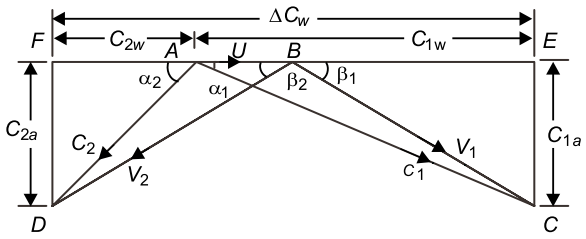
N = Speed in rpm.

C_1 = Absolute velocity of steam at inlet to moving blade or velocity of steam leaving nozzle. (Absolute velocity is the velocity of an object relative to the earth)

C_2 = Absolute velocity of steam at exit of moving bade.



Inlet and outlet velocity diagrams.



Combined inlet and outlet velocity diagrams.

Fig. 14.15

C_{1w} = Whirl velocity at inlet to moving blade or tangential component of absolute velocity at inlet to moving blade.

C_{2w} = Whirl velocity at exit of moving blade or tangential component of absolute velocity at exit of moving blade.

C_{1a} = Flow velocity at inlet to moving blade or axial component of absolute velocity at inlet to moving blade.

C_{2a} = Flow velocity at exit of moving blade or axial component of absolute velocity at exit of moving blade.

V_1 = Relative velocity of steam at inlet of moving blade (Blade velocity at inlet) (Relative velocity is the absolute velocity of one moving object compared with absolute velocity of other object.)

V_2 = Relative velocity of steam at exit of moving blade (Blade velocity at exit).

m = Mass of steam flowing over blade

ρ = Ratio of linear velocity of blade and absolute velocity at inlet of moving blade = $\frac{U}{C_1}$

k = Blade velocity coefficient (Ratio of relative velocity at exit and inlet) = $\frac{V_2}{V_1}$

α = Angle of absolute velocity with respect to the direction of blade motion.

α_1 = Angle of absolute velocity at inlet to moving blade or nozzle angle.

α_2 = Angle of absolute velocity at exit of moving blade or inlet angle of fixed blade in next stage.

β = Angle of relative velocity with respect to the direction of blade motion.

β_1 = Angle of relative velocity at inlet or inlet angle of moving blade.

β_2 = Angle of relative velocity at exit or exit angle of moving blade.

Here steam enters the nozzle and leaves so as to smoothly glide into the ring of moving blades. Steam leaves nozzle with absolute velocity C_1 and at an angle of α_1. This steam stream will be delivered to moving blade with velocity C_1 and angle α_1 but due to linear velocity of moving blade the steam stream actually glides over the moving blade with velocity V_1 and blade angle at inlet β_1. This velocity V_1 is actually the result of two velocity vectors C_1 and U. V_1 is called the relative velocity of steam at inlet of moving blade. Steam stream leaves the moving blade with velocity V_2 which is relative velocity of steam at exit of moving blade. Thus relative velocity is the actual velocity with which steam flows over the moving blade. For a perfectly smooth and frictionless blade this relative velocity should not change from inlet to exit as there is no expansion of steam in moving blade (blades are symmetrical and passage between two consecutive moving blades is of constant area type from inlet to exit). Actually there always exist some friction over the blade so the relative velocity at outlet will be smaller than the relative velocity at inlet, i.e. $V_2 < V_1$. This reduction in relative velocity is quantified by parameter called blade velocity coefficient (K). Blade velocity coefficient (K) is defined as, $K = \frac{V_2}{V_1}$.

If we look at inlet section 1, then it is obvious that for the maximum change in momentum steam should be delivered to the moving blade horizontally i.e. $\alpha_1 = 0$ and also leave horizontally i.e. $\alpha_2 = 0$ with the semi-circular shaped moving blade. This semi-circular moving blade is not possible practically as the moving blade wheel (ring) has series of blades and each blade has to receive steam from series of nozzles one after the other. This is the reason due to which nozzles are placed at some angle to the blade, say angle α_1 in this case.

Due to injection of steam at angle α_1 with velocity C_1 over the blade, steam shall have two components of velocity i.e. one tangential component and other axial component. Tangential component of velocity is parallel to the direction of rotation of blades and is also called as whirl velocity. Axial component of velocity is perpendicular to the direction of rotation of blade and is also called flow velocity. Axial component or flow velocity is responsible for maintaining flow of steam across the moving blade row. Volume flow rate of steam across the moving blade ring can be given by the product of flow velocity and effective passage area available for flow. The magnitude of flow velocity influences the size of wheel for given steam volume flow rate.

The whirl component of velocity is responsible for generation of thrust force due to change in momentum. Both whirl velocity and flow velocity being the two perpendicular components of absolute velocity depend largely upon the angle of absolute velocity i.e, α. At inlet the angle α_1 should be selected depending upon the thrust requirement and maintenance of flow across the blade row. With increase in angle α_1 the whirl velocity, $C_1 \cos \alpha_1$, decreases while the flow velocity, $C_1 \sin \alpha_1$, increases. Thus the

maximization of one leads to minimization of the other and so the compromise should be had for selecting the angle of absolute velocity. Similarly at exit of blade again there shall be whirl velocity and flow velocity components. For absolute velocity at exit being C_2 and angle of absolute velocity at exit being α_2 (this shall be the inlet angle for subsequent nozzle if more than one similar stages are there), the whirl velocity shall be $C_2 \cos \alpha_2$ and flow velocity as $C_2 \sin \alpha_2$. In the simple impulse turbine stage this whirl velocity component at exit is a kind of loss at exit. Therefore in order to minimize loss at exit this component should be minimum. For minimizing the whirl velocity at exit i.e. $C_2 \cos \alpha_2$, the angle α_2 should be made minimum. Minimum loss could be reduced to zero if angle α_2 is equal to 90°. This becomes a typical case in which the turbine discharges axially (at 90°) and such turbines are also called axial discharge turbine.

Thus it is obvious that in case of impulse turbine stage the moving blade merely deflects the steam and the change in direction of steam from inlet to exit causes change of momentum and thus thrust is generated. Moving blades being of symmetrical type offer a constant cross-section area between two consecutive blades from inlet to exit and so no expansion occurs in the moving blade. The steam expansion only occurs in the nozzle. Also in the absence of expansion across moving blade the pressure of steam remains constant from inlet to exit under ideal conditions. For symmetrical blades the inlet and exit angles of blade are same i.e. $\beta_1 = \beta_2$.

Velocity diagrams are separately drawn at inlet of moving blade and at exit of moving blade as shown in Fig. 14.15. Combined velocity diagram for the stage is also given here. Using the velocity diagram various parameters can be estimated as discussed ahead.

The driving thrust on the rotor is produced as reaction of the force estimated by the rate of change of momentum. Tangential force will be the force component producing driving thrust while axial force will be the force component producing axial thrust. Driving thrust causes rotation of rotor while axial thrust is to be absorbed by the thrust bearings provided for this purpose.

Let the mass flow rate be m, kg/s.

Tangential force $F_T = m \times$ (change of tangential component of velocity or whirl velocity)

$$= m \times (-C_2 \cos \alpha_2 - C_1 \cos \alpha_1)$$

$$F_T = -m (C_2 \cos \alpha_2 + C_1 \cos \alpha_1) = -m \cdot \Delta C_w$$

Driving thrust on rotor will be reaction of this force and will be equal and opposite.

Driving thrust, $F_D = -F_T$

$$F_D = m (C_2 \cos \alpha_2 + C_1 \cos a_1)$$

$$F_D = m \cdot \Delta C_w$$

From velocity triangles,

$$(C_1 \cos \alpha_1 + C_2 \cos \alpha_2) = (V_1 \cos \beta_1 + V_2 \cos \beta_2)$$

or, $$\Delta C_w = \Delta V_w$$

Hence $$F_D = m (C_2 \cos \alpha_2 + C_1 \cos \alpha_1)$$

$$= m \cdot (V_1 \cos \beta_1 + V_2 \cos \beta_2)$$

This driving thrust can be used for getting the rate of work done on the rotor.

Rate of work done = $W = F_D \times U$

$$W = m \cdot \Delta C_w . U$$

Work done per unit of steam mass flow, $w = U \cdot \Delta C_w$

Rate of work done will be the power produced by the turbine stage.

$$W = m \cdot U \cdot (C_2 \cos \alpha_2 + C_1 \cos \alpha_1)$$

$$W = m \cdot U \cdot (V_1 \cos \beta_1 + V_2 \cos \beta_2)$$

$$= m \cdot U \cdot V_1 \cos \beta_1 \left(1 + \frac{V_2 \cos \beta_2}{V_1 \cos \beta_1}\right)$$

$$= m \cdot U \cdot (C_1 \cos \alpha_1 - U) \left\{1 + \left(\frac{V_2}{V_1}\right) \cdot \left(\frac{\cos \beta_2}{\cos \beta_1}\right)\right\}$$

$$\boxed{W = m \cdot U \cdot (C_1 \cos \alpha_1 - U) \cdot \{1 + K \cdot C\}}$$

where K = Blade velocity coefficient = $\frac{V_2}{V_1}$

C = Ratio of cosines of blade angles = $\frac{\cos \beta_2}{\cos \beta_1}$

For perfectly smooth and symmetrical blade both K and C shall have unity value.

i.e. $K = 1$, $C = 1$

Therefore for simple impulse turbine stage having perfectly smooth and symmetrical blade, rate of work done,

$$\boxed{W = 2\ mU\ (C_1 \cos \alpha_1 - U)}$$

From velocity diagram also the rate of work done per unit of steam mass flow '*w*', can be estimated as below,

$$w = U \cdot \Delta C_w$$

Combined velocity diagram if drawn to the scale gives, $U = AB$, $\Delta C_w = EF$. Thus length *EF* and *AB* shall give $\Delta\ C_w$ and *U* respectively.

Hence, $\boxed{w = AB \times EF,}$ rate of work done per unit of steam mass flow

Rate of work done $\boxed{W = m \times AB \times EF}$. The suitable scale factors are to be used while using lengths.

The velocity diagram at inlet, outlet or combined velocity diagram can be suitably drawn to the scale and various parameters be obtained by measuring respective geometrical entity i.e. length and angle.

The work available at rotor can also be obtained using steady flow energy equation between section 1 and 2. Assuming no change in potential energy from inlet to exit across the moving blade and no heat interaction across the stage, the S.F.E.E. can be given as;

$$m\left(h_1 + \frac{C_1^2}{2}\right) = m\left(h_2 + \frac{C_2^2}{2}\right) + W$$

$$W = m\left\{(h_1 - h_2) + \left(\frac{C_1^2 - C_2^2}{2}\right)\right\}$$

In case of impulse stage the change in enthalpy from section 1 to 2 can be given by the change in kinetic energy associated with relative velocity from 1 to 2.

$$(h_1 - h_2) = \left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right)$$

or, $(h_1 - h_2) = \frac{1}{2}\left(V_2^2 - V_1^2\right)$

Substituting in W,

Rate of work done in a stage:

$$\boxed{W = \frac{m}{2}\left\{\left(V_2^2 - V_1^2\right) + \left(C_1^2 - C_2^2\right)\right\}}$$

For perfectly smooth moving blade, $V_1 = V_2$ and $(h_1 - h_2) = 0$.

Hence $\boxed{W = \frac{m}{2}\left\{\left(C_1^2 - C_2^2\right)\right\} \text{ for stage with smooth blade}}$

From velocity triangle at inlet

$$C_1^2 = V_1^2 + U^2 + 2UV_1 \cos \beta_1$$

and from velocity triangle at outlet, $C_2^2 = V_2^2 + U^2 - 2UV_2 \cos \beta_2$

Combining above two we get,

$$C_1^2 - C_2^2 = (V_1^2 - V_2^2) + 2U(V_1 \cos \beta_1 + V_2 \cos \beta_2)$$

It is already seen, $(V_1 \cos \beta_1 + V_2 \cos \beta_2) = (C_1 \cos \alpha_1 + C_2 \cos \alpha_2)$

or, $\Delta V_w = \Delta C_w$

or, $(C_1^2 - C_2^2) = (V_1^2 - V_2^2) + 2U \cdot \Delta C_w$

or, $(C_1^2 - C_2^2) + (V_2^2 - V_1^2) = 2U \cdot \Delta C_w$

Substituting in the expression for rate of work done,

$$W = \frac{m}{2}\{2U \cdot \Delta C_w\}$$

or, $\boxed{W = m \cdot U \cdot \Delta C_w}$ $\boxed{W = m \cdot U \cdot (C_1 \cos \alpha_1 + C_2 \cos \alpha_2)}$

Axial thrust: Axial component of velocity or flow velocity change causes creation of axial thrust. Axial thrust due to change in momentum because of change in flow velocity.

Axial thrust $= m \times (C_{1a} - C_{2a})$

$= m \times (C_1 \sin \alpha_1 - C_2 \sin \alpha_2)$

$\boxed{\text{Axial thrust, } F_a = m \times \Delta C_a}$

From velocity diagrams the change in axial velocity is given by length DG. Length, $DG = EC - FD$

$\boxed{\text{Axial thrust} = m \times DG = m \times (EC - FD)}$

Diagram efficiency or blading efficiency: Diagram efficiency or blading efficiency refers to the ratio of work done to the energy supplied to rotor in a stage. In case of impulse turbine this energy supplied to rotor in a stage can be given by the kinetic energy supplied at inlet of moving blade. Assuming no expansion in moving blades the energy supplied to rotor can be given by $\left(m \cdot \frac{C_1^2}{2}\right)$. Rate of work

done per unit mass of steam is also called diagram work.

Hence, diagram efficiency $= \eta_D = \dfrac{\text{Rate of work done}}{\text{Energy supplied to rotor}}$

$$\eta_D = \frac{m \cdot U \cdot \Delta C_w}{m \cdot \dfrac{C_1^2}{2}}$$

$$\eta_D = \frac{2U \cdot \Delta C_w}{C_1^2}$$

$$\boxed{\text{Diagram or blading efficiency} = \frac{2 \cdot U \cdot \Delta C_w}{C_1^2}}$$

Stage efficiency or Gross efficiency*:* Stage efficiency refers to the ratio of rate of work done and energy supplied to the stage. Energy supplied to the stage can be accounted by the change in enthalpy between section 0 and 1 i.e. inlet of nozzle to exit of nozzle. Stage efficiency is thus the output of stage divided by the available energy for the stage.

Energy supplied to stage $= m \cdot (h_0 - h_1)$

Stage efficiency, $\eta_s = \dfrac{\text{Rate of work done}}{\text{Energy supplied to stage}}$

$$= \frac{m \cdot U \cdot \Delta C_w}{m \cdot (h_0 - h_1)}$$

$$\boxed{\text{Stage efficiency} = \frac{U \cdot \Delta C_w}{(h_0 - h_1)}}$$

Nozzle efficiency: Nozzle efficiency refers to the ratio of kinetic energy available and the enthalpy change occurring across the nozzle i.e. between inlet and outlet (sections 0 and 1).

Nozzle efficiency, $\eta_N = \dfrac{m\dfrac{C_1^2}{2}}{m(h_0 - h_1)}$

$$\boxed{\text{Nozzle efficiency} = \frac{C_1^2}{2(h_0 - h_1)}}$$

Combining the stage efficiency, diagram efficiency and nozzle efficiency, it can be given that;

$$\boxed{\text{Stage efficiency} = \text{Diagram efficiency} \times \text{Nozzle efficiency}}$$

Overall efficiency: The overall efficiency of stage can be given by the ratio of work delivered at turbine shaft to the energy supplied to the stage.

Overall efficiency, $\eta_o = \dfrac{\text{Work delivered at shaft}}{\text{Energy supplied to the stage}}$

$$\boxed{\text{Overall efficiency} = \text{Nozzle efficiency} \times \text{Diagram efficiency} \times \text{Mechanical efficiency}}$$

Optimization of turbine stage: Turbine being work producing machine is designed with the aim of providing maximum work output. The diagram efficiency of turbine should be maximized as it indicates the rate of work done per unit of energy supplied to the rotor. Work output per unit time in a simple impulse turbine stage is given by, $W = m \cdot U \cdot (C_1 \cos \alpha_1 - U)(1 + KC)$.

In this expression for work the output is seen to depend upon m, C_1, U, α_1, K and C. For all other conditions fixed work output is seen to vary inversely with the angle α_1 i.e. the work output increases with decrease in nozzle exit angle, α_1. Work output shall be maximum when angle α_1 is reduced to zero but nozzle exit angle α_1 can not be reduced to zero as it shall not allow steam injection to successive blades. Angle α_1 also decides the axial or flow velocity at inlet, C_a, i.e. smaller the value of α_1 smaller will be axial velocity or flow velocity. As described earlier that the flow velocity maintains the flow across the stage so smaller flow velocity requires larger steam flow passage area for the same mass flow of steam. Small value of angle α_1 will thus yield larger blade surface area resulting into larger surface friction loss. Therefore the angle α_1 is chosen as compromise between above considerations. Generally α_1 lies between 15 and 30°.

Thus the work can not be maximized by only selecting the minimum value of angle α_1 and so requires optimization of turbine stage performance with respect to some other parameter.

The diagram efficiency should be maximized with respect to suitable parameter.

Diagram efficiency,

$$\eta_D = \frac{2U \cdot \Delta C_w}{C_1^2}$$

$$\eta_D = \frac{2U \cdot (C_1 \cos \alpha_1 - U)(1 + KC)}{C_1^2}$$

$$\eta_D = 2\left(\frac{U}{C_1}\right)\left(\cos \alpha_1 - \frac{U}{C_1}\right)(1 + KC)$$

Here $\frac{U}{C_1}$, is non-dimensional form of velocity. Let us denote by ρ

i.e. $\rho = \frac{U}{C_1}$, so, $\eta_D = 2\rho (\cos \alpha_1 - \rho)(1 + KC)$

Diagram efficiency can be optimized with respect to non-dimensional velocity denoted by ρ. This non-dimensional velocity ρ is also called as 'blade-steam velocity ratio' or 'blade speed-steam velocity ratio' or 'blade-steam speed ratio'.

Differentiating η_D with respect to ρ, we get,

$$\frac{d\eta_D}{d\rho} = 2 (\cos \alpha_1 - 2\rho)(1 + KC)$$

For a perfectly smooth and symmetrical blade, $K = 1$, $C = 1$

$$\frac{d\eta_D}{d\rho} = 4 (\cos \alpha_1 - 2\rho)$$

Equating first differential to zero;

$$\cos \alpha_1 - 2\rho = 0$$

$$\boxed{\rho = \frac{\cos \alpha_1}{2}}$$

Second order differential of η_D with respect to ρ indicates that the diagram efficiency is maximum corresponding to the blade speed-steam velocity ratio given as $\frac{\cos \alpha_1}{2}$.

Hence, maximum diagram efficiency, $\boxed{\eta_{D,\max} = \frac{\cos^2 \alpha_1 \cdot (1 + KC)}{2}}$

For perfectly smooth and symmetrical blade, maximum diagram efficiency,

$$\boxed{\eta_{D,\max} = \cos^2 \alpha_1}$$

Corresponding to $\rho = \frac{\cos \alpha_1}{2}$, the rate of work done can be given as the maximum rate of work done. Mathematically,

Maximum rate of work done can be obtained by substituting

$$\rho = \frac{U}{C_1} = \frac{\cos \alpha_1}{2} \text{ or } C_1 = \frac{2U}{\cos \alpha_1}$$

Rate of work, $W = mU \times (C_1 \cos a_1 - U)\ (1 + KC)$

$$W_{\max} = m \cdot U \cdot U\ (1 + KC)$$

$$\boxed{W_{\max} = mU^2\ (1 + KC)}$$

For perfectly smooth and symmetrical blade, $K = 1$, $C = 1$,

$$\boxed{W_{\max} = 2\ mU^2}$$

The variation of diagram efficiency can be plotted with varying blade-steam velocity ratio as given below.

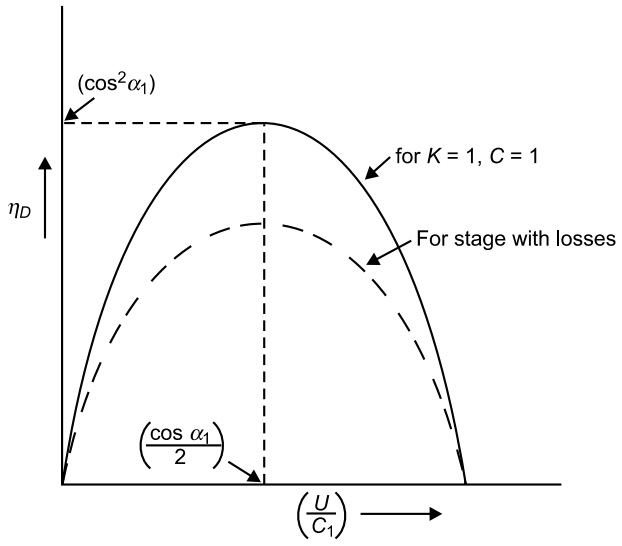


Fig. 14.16 *Diagram efficiency with blade speed-steam velocity ratio.*

14.6 IMPULSE TURBINE BLADE HEIGHT

Impulse turbine blade height is a very important parameter. Blade height is a function of total annular area required for the flow of fluid. It depends upon the mass flow rate through the section, specific volume of steam at that section,

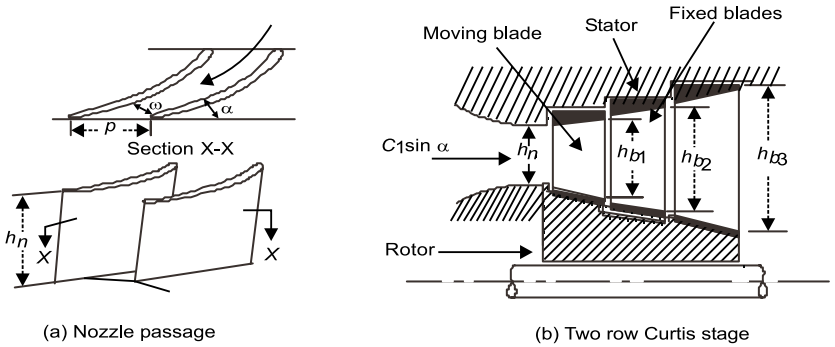


Fig. 14.17 *Nozzle and blade passage*

area of section through which steam is passing and flow velocity at the section. In estimating the net annular area available for flow, edge thickness and blade angles are taken into account. The nozzle passage and two row Curtis stage are shown in Fig 14.17.

Here estimation of both nozzle height and blade height has been described ahead.

For the nozzle passage shown the area available for flow at exit of one nozzle passage can be given as A;

$A = \omega \cdot h_n$, where ω is the width of flow passage at exit at mean nozzle height and h_n is nozzle height.

The width of flow passage at exit at mean nozzle height can be given as,

$$\omega = (p \sin \alpha - t),$$

where 'p' is nozzle pitch at mean nozzle height and 't' is edge thickness of nozzle and 'α' is nozzle angle.

Therefore, $A = (p \sin \alpha - t) \cdot h_n$

In the absence of information about edge thickness considerations above equation can be used to get nozzle height.

For full peripheral admission when nozzle diaphragm is occupied by nozzles, the number of nozzles can be obtained as,

$$n = \frac{\pi d}{p}$$

where 'd' is the diameter of mean nozzle ring and 'n' is number of nozzles.

Edge thickness in nozzle calculations can be accounted by edge thickness factor (k) denoted as,

$$k = \left(\frac{p \sin \alpha - t}{p \sin \alpha} \right)$$

Thus, $\boxed{A = k\, h_n \cdot p \sin \alpha}$, can be used to get nozzle height. Considering the total number of nozzles, the total nozzle area can be given as,

$$A_n = n \cdot A$$

$$\boxed{A_n = \pi d \cdot h_n \cdot k \sin \alpha}$$

Figure 14.17(b) shows the two stage Curtis stage having blade heights as h_{b1}, h_{b2}, h_{b3}. Theoretically blade height at entrance is equal to nozzle height at exit but in order to avoid spilling of the fluid issuing from nozzle passage the blade height is slightly increased at entrance. In case of impulse turbine blade the pressure on both sides of blade remains constant and by continuity equation the volume flow rate shall also remain same from inlet to exit.

From continuity equation for nozzle to moving blade, the volume flow rate shall be,

$$C_1 \pi d \cdot h_n \cdot k \sin \alpha = V_1 \cdot \pi d \cdot h_b \cdot k_b \cdot \sin \beta$$

where h_b is blade height, k_b is blade edge thickness factor and β is blade angle.

Generally it is assumed, $k = k_b$, hence

$$\frac{h_b}{h_n} = \frac{C_1 \sin \alpha}{V_1 \sin \beta} = \frac{C_{1a}}{V_{1a}} = 1$$

From velocity diagram at blade inlet it can be seen that, $C_1 \sin \alpha = C_{1a} = V_1 \sin \beta = V_{1a}$.

Hence $h_b = h_n$, ideally but as described earlier the height at blade entrance is slightly increased, which is also called 'blade step-up' or 'overlap'. Similarly for two row Curtis stage considered here the blade height can be related as,

$$h_{b1} = \frac{h_n \cdot C_{1a}}{V_{2a}}$$

$$h_{b2} = \frac{h_n \cdot C_{1a}}{V_{3a}}$$

$$h_{b3} = \frac{h_n \cdot C_{1a}}{V_{4a}}$$

Step up or overlap is arbitrarily taken and varies from 0.2 cm in high pressure stages to 2 cm in low pressure stages of large turbines. Here in this calculation no consideration is made for increase in specific volume of fluid in blade passage due to reheat although it shall be there but may be neglected on account of being small.

Maximum height of blade is restricted by the stresses getting generated due to bending and centrifugal forces.

Alternatively, the blade height can be estimated by the volume flow rate considerations. Moving blade height at exit can be given as below;

$$mv = \left(\frac{l}{p_b} \right) (p_b \sin \beta_2 - t_b)\, l h_{b1} \cdot V_2$$

where, l is the length of arc covered by nozzle and p_b is the distance between two consecutive blades, t_b is edge thickness of blade as shown in Fig 14.18. Here $\left(\frac{l}{p_b} \right)$ gives the number of exit passages in nozzles through which steam arrives over the moving blade.

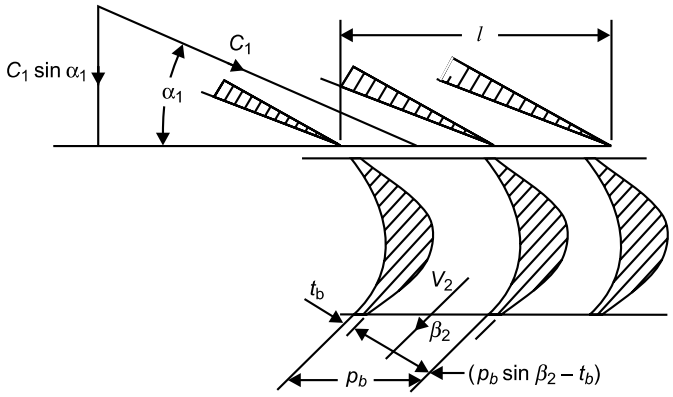


Fig. 14.18 *Blade height description*

Steam turbines are called full admission turbines when the nozzle coverage of blades is complete i.e. in such case $l = \pi d$, where 'd' is mean blade diameter of the rotor.

14.7 CALCULATIONS FOR COMPOUNDED IMPULSE TURBINE

Velocity diagram for simple impulse turbine is described earlier. Similar to the velocity diagram described, the velocity diagram for the compounded impulse turbine stage can be drawn. Velocity diagram for velocity compounded impulse turbine stages is shown in Fig. 14.19.

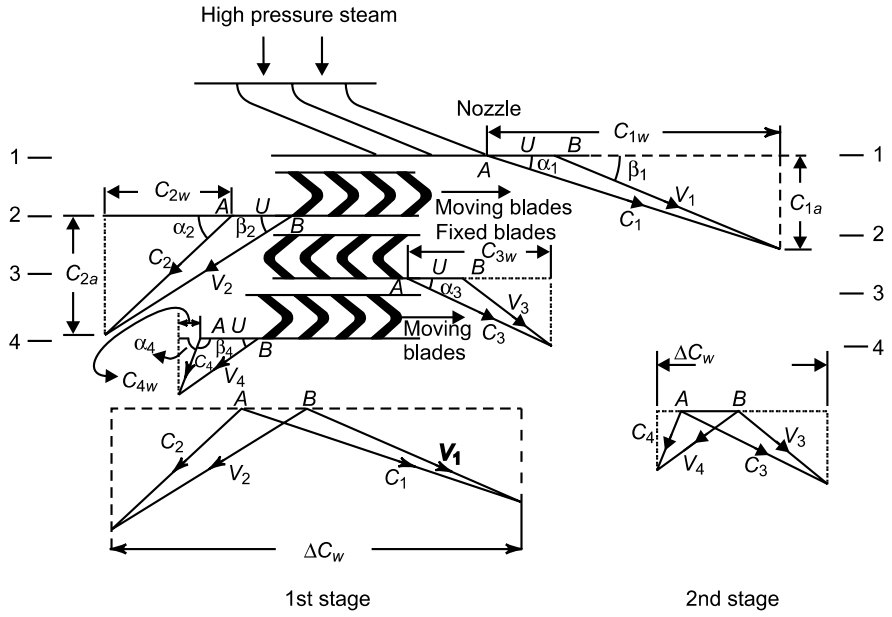


Fig. 14.19 *Stage of velocity compounded impulse turbine and velocity diagram for the stage.*

Here high pressure steam first enters the nozzle and steam leaving nozzle enters the row of moving blades. Steam leaving moving blades enters the fixed blade row. Fixed blades just act as guide

blades and are fixed to casing. Steam is guided by fixed blade to smoothly enter into next row of moving blades. Thus nozzle, moving blade row, fixed blade row and moving blade row together constitute stage of velocity compounded impulse turbine. Let us consider section at exit of nozzle as 1 – 1, at exit of first moving blade row as 2 – 2, at exit of fixed blade row as 3 – 3 and at exit of second row of moving blades as 4 – 4.

Absolute velocity, relative velocity and different angles are shown in velocity diagrams at sections 1 – 1, 2 – 2, 3 – 3, and 4 – 4.

Steam enters first row of moving blades at section 1 – 1 with absolute velocity C_1 and leaves moving blade with velocity C_2 at angle α_2. Steam passes through fixed blades and leaves it with absolute velocity C_3 at angle α_3. Steam finally leaves second row of moving blades with absolute velocity C_4 at angle α_4. Let us assume that the moving blades and fixed blades are symmetrical and the blades are perfectly smooth i.e. no friction loss occurs on moving blades and fixed blades.

Mathematically the velocity and angles can be related as follows for the above assumptions.

Constant relative velocity across moving blades due to smooth blades surface;

$$V_1 = V_2$$

and

$$V_3 = V_4$$

Across the fixed blade also there shall be no friction loss resulting in;

$$C_2 = C_3$$

Due to symmetry of moving blades;

$$\beta_1 = \beta_2 \ \& \ \beta_3 = \beta_4$$

Due to symmetry of fixed blades; $\alpha_2 = \alpha_3$

Let us also assume that the stream finally leaving the stage goes out with minimum energy for yielding maximum efficiency. Stream should leave the stage axially for above assumption, i.e. $\alpha_4 = 90°$. In this case when discharge from last blade row is axial then the velocity of whirl at exit is zero. Such turbines are also called axial discharge turbine.

The total work available from turbine stage shall be algebraic summation of work available from first row of moving blades and second row of moving blades. Diagram efficiency shall be the ratio of total work and energy supplied to the stage.

Thus, Total work from stage = Work from first row of moving blades
+ Work from second row of moving blades

Work from first row of moving blades can be written as already described for simple impulse stage.

$$W_I = 2 \cdot m \cdot U \cdot (C_1 \cos \alpha_1 - U)$$

Similarly, work from second row of moving blades,

$$W_{II} = 2 \cdot m \cdot U \, (C_3 \cdot \cos \alpha_3 - U)$$

or

$$W_{II} = 2 \cdot m \cdot U \, (C_2 \cdot \cos \alpha_2 - U)$$

From velocity diagram, $C_2 \cdot \cos \alpha_2 = V_2 \cdot \cos \beta_2 - U$

Substituting,

$$W_{II} = 2 \cdot m \cdot U\,(V_2 \cos \beta_2 - 2U)$$

$$W_{II} = 2 \cdot m \cdot U\,(V_1 \cos \beta_1 - 2U)$$

from velocity diagram,

$$V_1 \cos \beta_1 = C_1 \cos \alpha_1 - U$$

Substituting,

$$W_{II} = 2m \cdot U\,(C_1 \cos \alpha_1 - 3U)$$

Total work from stage, $W_T = W_I + W_{II}$

$$W_T = 2m \cdot U \cdot (C_1 \cos \alpha_1 - U) + 2m \cdot U\,(C_1 \cos \alpha_1 - 3U)$$

$$\boxed{W_T = 2m \cdot U \cdot (2C_1 \cos \alpha_1 - 4U)}$$

Diagram efficiency, $\eta_d = \dfrac{\text{Total work}}{\text{Energy supplied}}$

$$= \frac{\{2m \cdot U \cdot (2C_1 \cdot \cos \alpha_1 - 4U)\}}{\left(\dfrac{m \cdot C_1^2}{2}\right)}$$

$$\eta_d = \frac{4U}{C_1^2}\,(2C_1 \cdot \cos \alpha_1 - 4U)$$

Non dimensional form of velocity $\dfrac{U}{C_1}$, $= \rho$, i.e. blade speed to steam velocity ratio

or, $\boxed{\eta_d = 4 \cdot \rho \cdot (2\cos \alpha_1 - 4\rho)}$

Differentiating diagram efficiency with respect to ρ and applying conditions to get the optimum value of ρ for maximum diagram efficiency;

$$\frac{d\eta_d}{d\rho} = \frac{d}{d\rho}\,(8\rho \cos \alpha_1 - 16\rho^2)$$

Substituting

$$\frac{d\eta_d}{d\rho} = 0$$

or, $$\rho_{opt} = \frac{\cos \alpha_1}{4} \text{ or } 4U = C_1 \cos \alpha_1$$

Maximum efficiency, $\eta_{d,\max} = \cos^2 \alpha_1$

For the optimum value of ρ_{opt} the work may be estimated as;

Work from Ist row, $W_I = 6m \cdot U^2$

Work from IInd row, $W_{II} = 2m \cdot U^2$

Total work from velocity compounded turbine stage, $W_T = W_I + W_{II}$

$$W_T = 8mU^2$$

Here it is obvious that for optimum value of ρ i.e. ρ_{opt} the work available from second row is $\frac{1}{4}$th of the total work from stage. It can be extrapolated from here that if there are more than two rows of moving blades in a velocity compounded turbine stage i.e. three, four or more number of rows, then the optimum blade speed to steam velocity ratio ρ can be generalised for 'n' number of moving blade rows.

$$\rho_{opt} = \frac{\cos \alpha_1}{2n} \text{ i.e. for three rows, } \rho_{opt} = \frac{\cos \alpha_1}{6}$$

Similarly, work from the last row of moving blades shall be,

$$W_{\text{last row}} = \frac{1}{2^n} \text{ of total work.}$$

Thus from a three row stage, work from last row shall be $\frac{1}{8}$th of the total work and from a four row stage, work from last row shall be $\frac{1}{16}$th of the total work. Here it is obvious that as the number of rows increases the work from last row goes on giving diminishing work. Therefore, with increase in number of rows the work obtained from last row decreases, so the number of rows is generally limited to two. If we look at diagram efficiency then it shows that the efficiency also diminishes with increase in number of stages. Graphical pattern between the diagram efficiency versus blade speed to steam velocity ratio ρ is shown in Fig. 14.20.

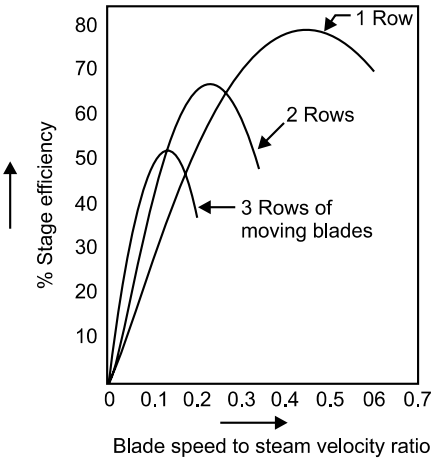


Fig. 14.20 *Diagram efficiency vs. blade speed to steam velocity ratio for velocity compounded impulse turbine*

14.8 REACTION TURBINES

In a reaction turbine the pressure drop occurs in both stationary and moving rows contrary to the impulse turbine where the total pressure drop occurs in stationary nozzles alone. The difference in blading of reaction and impulse has already been described earlier. In reaction turbine the passage

between two consecutive blades is of converging type as compared to impulse turbine blading which has constant cross-sectional area passage between two consecutive blades.

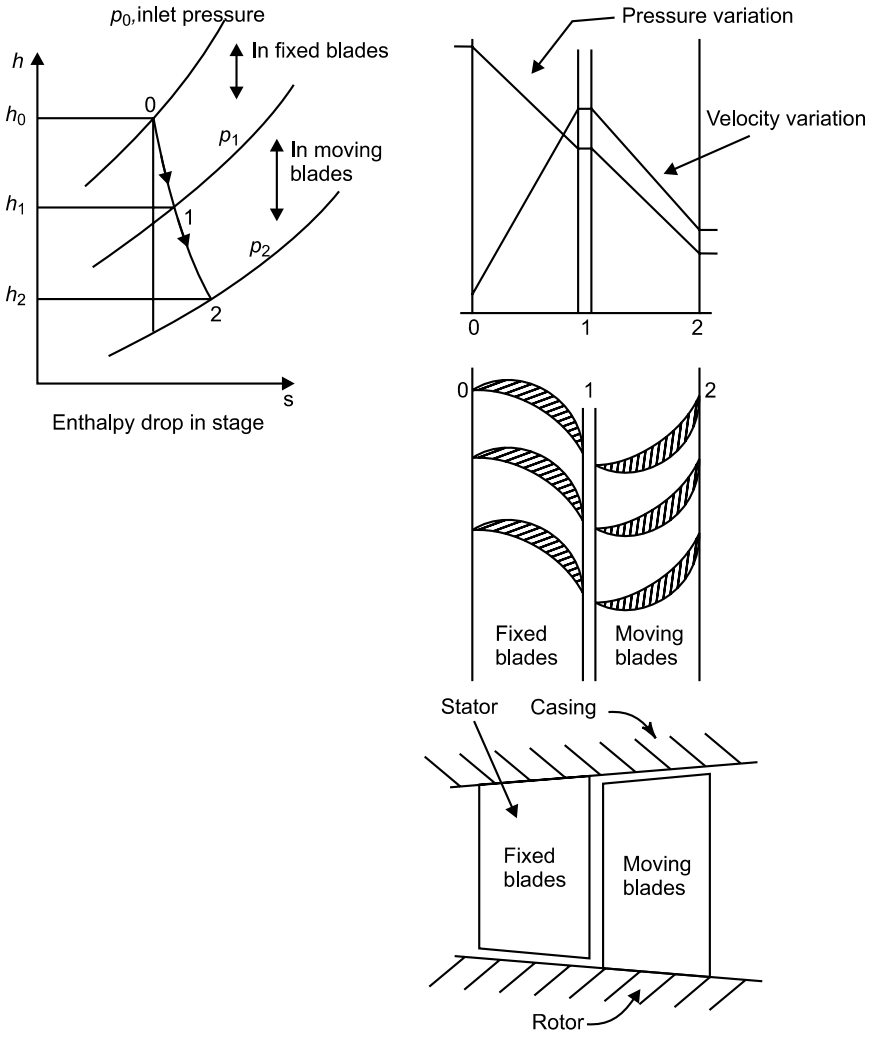


Fig. 14.21 *Reaction turbine stage*

Figure 14.21 shows the schematic of a reaction turbine stage having fixed blades followed by moving blades row. Due to the varying cross section area for steam flow the pressure drop occurs in both stationary (fixed) blades row and moving blades row. The velocity increases in stationary blades which act as nozzles. Thus the passage formed in the stationary blades in reaction turbine are of nozzle type although they do not have conventional nozzle shape. Steam stream leaving stationary blades impinges upon the moving blades. This impinging stream exerts a force to the right as evident from the velocity diagrams of reaction blading. Velocity diagram of reaction turbine is similar in principle to the velocity diagram in impulse turbine.

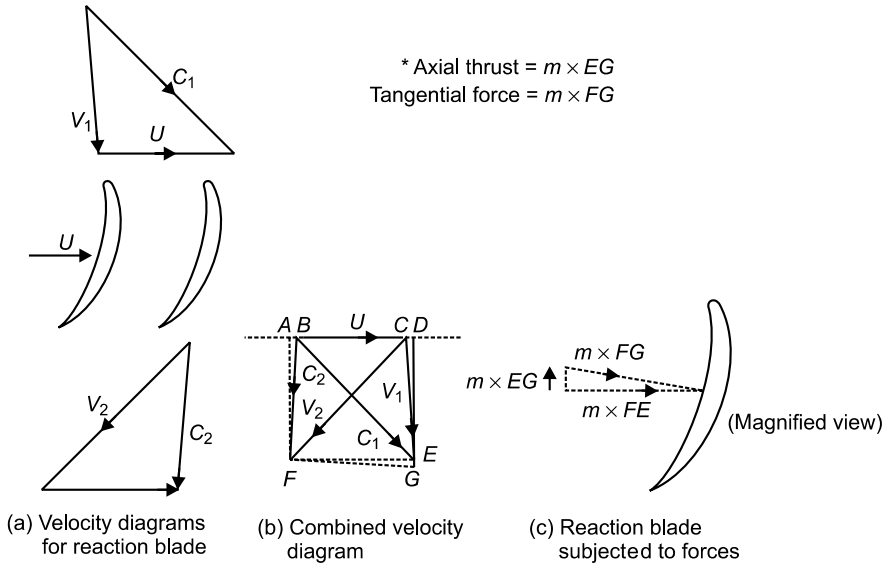


Fig. 14.22 *Velocity and force diagram of reaction blading*

Steam entering moving blades is subjected to pass through converging area passage along with change in direction. Thus there is increase in velocity ($V_2 > V_1$) from inlet to exit in moving blade which results in a reaction force. Change in direction of velocity is accompanied by change in momentum thus an impulse force. It shows that the rotation of shaft is caused by the combination of impulse and reaction forces. The magnitude of impulse force depends upon the pressure drop in fixed blades. It may be noted that due to shaft rotation being caused by combination of impulse and reaction forces these reaction turbines are also termed as impulse-reaction turbine. These are also called full admission turbines as the steam enters through fixed blade row over complete annulus. The enthalpy drop over the reaction turbine stage shows that heat drop occurs in both fixed blades and moving blades rows. If the total enthalpy drop in stage is equally divided between the stationary and moving blades then the stage is called 50% reaction stage. A mathematical parameter called 'degree of reaction' is used to quantify the proportion of enthalpy drops occurring in stationary and moving blades. The 'degree of reaction' is defined as the ratio of enthalpy drop in moving blades row (rotor blades) to the total enthalpy drop in the stage. Mathematically it can be given as,

$$\text{Degree of reaction} = \frac{\text{Enthalpy drop in moving blades (rotor blades)}}{\text{Total enthalpy drop in the stage}}$$

Degree of reaction, $$\varepsilon = \frac{h_1 - h_2}{h_0 - h_2}$$

In case of 50% degree of reaction, $\varepsilon = 0.5$ and $h_1 - h_2 = \frac{1}{2}(h_0 - h_2)$

or, $$(h_0 - h_1) = (h_1 - h_2) = \frac{1}{2}(h_0 - h_2)$$

Such turbines having 50% degree of reaction are called 'Parson's turbine'. Parson's turbine has symmetrical blades for moving and stationary blades i.e. inlet angles of stationary and moving blades are equal and also the exit angles of stationary and moving blades are equal. Term symmetrical blading in reaction turbine refers to the 50 per cent reaction stage.

A typical degree of reaction value of zero refers to the condition when there is no enthalpy drop in moving blades. This refers to the impulse turbine stage where no heat drop occurs in moving blades.

In reaction turbines also the compounding is done similar to that in impulse turbine for reducing the speed of rotation. The compounding in reaction turbine is shown in Fig. 14.23. where pressure drop gradually occurs in the stationary and moving blade rows. In reaction turbine blading there is possibility of steam leakage across the tip of blade and casing as there is large clearance here. This leakage of pressure from the tip clearance is more vulnerable in high pressure region as there occurs large pressure drop in high pressure region for given enthalpy drop. Due to this fact the initial stages in high pressure region are taken to be of impulse type and subsequent stages are reaction stages. Above fact is generally interpreted as "the inefficient reaction blading in high pressure stage". Generally, the modern turbines have first stage as velocity compounded (Curtis stage) followed by stages of pressure compounded type (Rateau stages) or reaction stages.

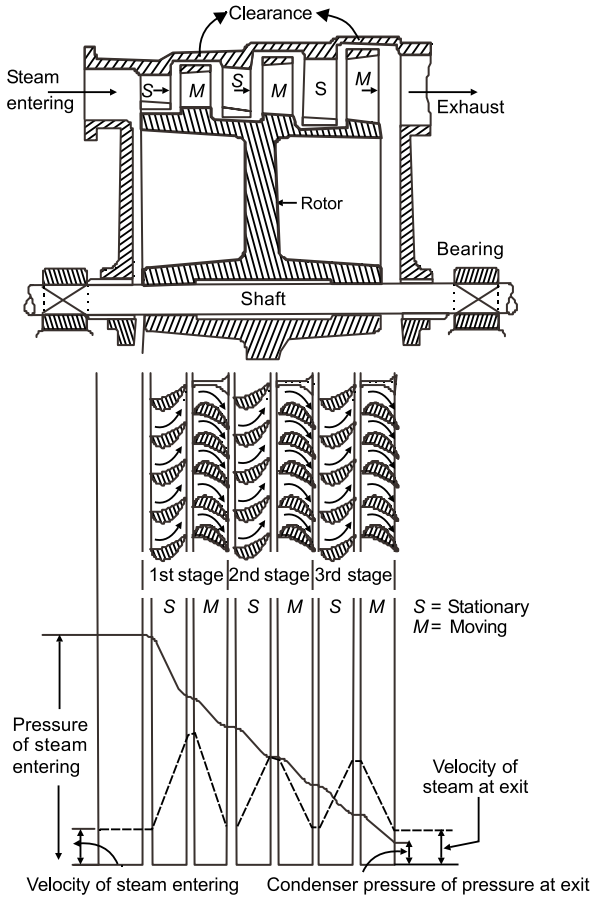


Fig. 14.23 *Reaction turbine having compounding*

Velocity diagram for 50% reaction turbine: 50% degree of reaction turbine stage has symmetrical stationary and moving blades. The combined velocity diagram for a moving blade having section 1 – 1 as inlet section and section 2 – 2 as exit section is shown in Fig. 14.24.

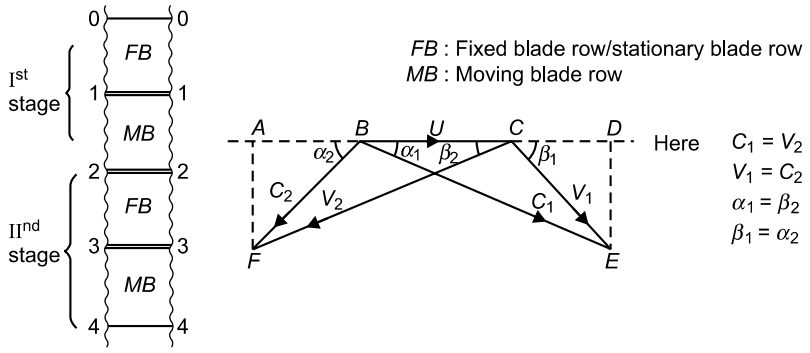


Fig. 14.24 *Combined velocity diagram for a moving blade of 50% reaction turbine.*

From earlier discussions it is clear that 50% degree of reaction turbine has equal enthalpy drops occuring in stationary blade row and moving blade rows. Steady flow energy equation (S.F.E.E.) when applied over turbine stage yields the following assuming no heat interaction, no change in potential energy.

Let us assume that turbine has more than one symmetrical stage such that absolute velocity of steam leaving moving blade row equals to the velocity of steam entering fixed blade. Thus, from given velocity diagram steam enters fixed blade with velocity C_2 while leaves it with velocity C_1. In case of moving blades the relative velocities are to be considered i.e. velocity increasing from V_1 to V_2 from inlet to exit.

For stationary blade row the S.F.E.E. gives,

$$h_0 - h_1 = \frac{C_2^2 - C_1^2}{2}$$

For moving blade row the S.F.E.E. gives, $h_1 - h_2 = \dfrac{V_2^2 - V_1^2}{2}$

Total enthalpy drop in stage, $h_0 - h_2 = (h_0 - h_1) + (h_1 - h_2)$

and for 50% degree of reaction as $C_1 = V_2$ and $V_1 = C_2$

so,

$$h_0 - h_1 = h_1 - h_2$$

$$h_0 = 2h_1 - h_2$$

or, $$(h_0 - h_2) = 2\,(h_1 - h_2)$$

i.e. $$\frac{h_1 - h_2}{h_0 - h_2} = \frac{1}{2}$$

Hence it proves that symmetrical blading results in 50% degree of reaction turbine.

Rate of work done from reaction stage can be estimated similar to that of impulse stage. Work done = $m \cdot U \cdot \Delta C_w$

where $\Delta C_w = C_{w2} + C_{w1} = (C_1 \cos \alpha_1 + C_2 \cos \alpha_2)$

For symmetrical blading, $C_2 = V_1$, $\alpha_2 = \beta_1$, so, $\Delta C_w = C_1 \cos \alpha_1 + V_1 \cos \beta_1$

or, $$\Delta C_w = C_1 \cos \alpha_1 + (C_1 \cos \alpha_1 - U)$$

$$\Delta C_w = 2C_1, \cos \alpha_1 - U$$

Therefore, work done, $$\boxed{W = m \cdot U \cdot (2C_1 \cos \alpha_1 - U)}$$

Axial thrust in turbine can be quantified by the sum of thrust due to change in axial velocity and due to pressure difference across the moving blade row. Mathematically the axial thrust can be given as,

$F_a = m \cdot (C_{1a} - C_{2a}) + (p_1A_1 - p_2A_2)$, where A_1 and A_2 are cross-sectional areas available as flow passage at inlet and exit of moving blade row and p_1 & p_2 are pressures of steam at inlet and exit sections of moving blade rows.

In case of 50% reaction turbine $C_{1a} = C_{2a}$, therefore axial thrust is only due to pressure difference across moving blade row.

Diagram efficiency of reaction stage can be estimated by knowing the energy input to the moving blades and taking ratio of work done to energy input.

Energy input to moving blades, $E_{in} = m \cdot \frac{C_1^2}{2} + m \cdot \left(\frac{V_2^2 - V_1^2}{2} \right)$

$$E_{in} = \frac{m}{2} \cdot (C_1^2 + V_2^2 - V_1^2)$$

$$= \frac{m}{2} (C_1^2 + C_1^2 - V_1^2), \text{ as } C_1 = V_2$$

$$E_{in} = m (C_1^2 - \frac{V_1^2}{2})$$

From velocity diagram, $V_1^2 = C_1^2 + U^2 - 2C_1U \cos \alpha_1$

Substituting value of V_1 in energy input expression,

$$E_{in} = m \cdot \left\{ C_1^2 - \frac{\left(C_1^2 + U^2 - 2C_1U\cos\alpha_1\right)}{2} \right\}$$

$$E_{in} = m \cdot \left\{ \frac{C_1^2 + 2UC_1\cos\alpha_1 - U^2}{2} \right\}$$

Thus, diagram efficiency, $\eta_d = \frac{W}{E_{in}}$

$$= \frac{m \cdot U \cdot (2C_1\cos\alpha_1 - U)}{m \cdot \frac{\{C_1^2 + 2UC_1\cos\alpha_1 - U^2\}}{2}}$$

$$\eta_d = \frac{2U \cdot (2C_1\cos\alpha_1 - U)}{\left(C_1^2 + 2UC_1\cos\alpha_1 - U^2\right)}$$

Substituting $\rho = \frac{U}{C_1}$, $\boxed{\eta_d = \frac{2\rho \cdot (2\cos\alpha_1 - \rho)}{\left(1 + 2\rho\cos\alpha_1 - \rho^2\right)}}$

The maximum value of diagram efficiency and the optimum value of blade speed to steam velocity ratio, ρ can be estimated by differentiating with respect to ρ and equating to zero.

$$\frac{d\eta_d}{d\rho} = 0$$

or, $2 \cos \alpha_1 - 2\rho = 0$

or, $\rho = \cos \alpha_1$

Substituting 'ρ' for getting maximum value of diagram efficiency, $\eta_{d,max}$

$$\eta_{d,\max} = \frac{2\cos\alpha_1 \cdot (2\cos\alpha_1 - \cos\alpha_1)}{(1 + 2\cos^2\alpha_1 - \cos^2\alpha_1)}$$

or, $$\boxed{\eta_{d,\max} = \frac{2\cos^2\alpha_1}{1 + \cos^2\alpha_1}}$$

Work output for the condition of maximum diagram efficiency can be obtained by substituting $\rho = \cos\alpha_1$

or, $$\frac{U}{C_1} = \cos\alpha_1$$

Substituting in expression for work, or, $U = C_1 \cdot \cos\alpha_1$

For $\eta_{d,\max}$ work; $W = m \cdot C_1 \cos\alpha_1 (2C_1 \cos\alpha_1 - C_1 \cos\alpha_1)$

$$W = m \cdot C_1^2 \cos^2\alpha_1$$

$\boxed{W = m \cdot U^2}$ for maximum diagram efficiency.

Optimum blade speed to steam velocity ratio yields the condition $U = C_1\cos\alpha_1$ which shall modify the velocity diagram for such optimum reaction stage as shown in Fig. 14.25.

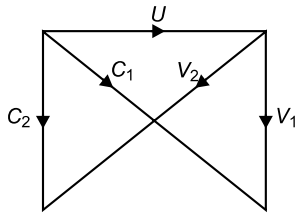


Fig. 14.25 *Velocity diagram for optimum reaction turbine stage having* $\rho = \cos\alpha_1$

Height of blade in reaction turbine can be estimated from the volume flow rate as given below. Reaction turbine stage is full admission stage.

Taking number of blades as '*N*', thickness of blade as '*t*', the mass flow rate as '*m*', average specific volume as '*v*' it can be written that,

$$m \cdot v = (\pi d - Nt) \cdot l \cdot V_2 \sin\beta_2$$

Height of blade, $$l = \frac{m \cdot v}{(\pi \cdot d - N \cdot t) \cdot V_2 \cdot \sin\beta_2}$$

It is observed that the minimum height for reaction turbine blade should not be less than 5 per cent of mean blade ring diameter and the maximum height should not exceed beyond 20 per cent of mean blade ring diameter. If the blade is of tapered type then the maximum height may go up to 30 per cent of mean blade ring diameter. The lower and upper limits of blade height are to be maintained while deciding blade height as shorter blades have increased energy losses while longer blades may have excessive stresses in them. Generally in reaction stages of turbine the blade height is allowed to increase gradually so as to give increasing annular area for expanding fluid.

14.9 LOSSES IN STEAM TURBINES

Steam turbine being work producing device running at quite high speed has number of losses occurring in it. These losses when put together result into substantial loss of energy. Therefore, while selecting a turbine due attention should be paid to the losses in turbine. Some of the losses occur within turbine stages while some are external to stage. These losses are described ahead.

1. Losses in nozzles: Steam turbine nozzle is designed for isentropic expansion so as to result in increase in velocity from inlet to exit. Practically in a nozzle the steam leaving nozzle may not have velocity equal to the designed velocity value. This deviation in operating state of nozzle may occur because of non-isentropic expansion. The reasons for non-isentropic expansion may be friction losses between the steam and nozzle wall, viscous friction resistance to flow in the steam particles, boundary layer formation and separation, heat loss during flow etc. Mathematically, this shift from isentropic expansion to non-isentropic expansion is quantified using the parameter called 'nozzle efficiency'. Nozzle efficiency as described earlier is defined by the ratio of 'actual enthalpy drop' to the 'isentropic enthalpy drop' between inlet and exit of nozzle.

2. Losses in moving blades: In steam turbine stage steam is supposed to glide smoothly over the moving blades after leaving nozzles or fixed blades. In actual turbine stage during flow of steam over moving blade, there may be number of factors causing loss of energy as given under:

(i) Blade friction may incur frictional loss which is taken into account by the blade friction factor. This friction factor largely depends upon the Reynolds number, although it is earlier defined as the ratio of relative velocity leaving blades to the relative velocity of steam entering blades. This loss may be termed as "passage loss".

(ii) "Boundary layer separation" may occur due to sharp deflection of fluid within the blade passage. Deflection results in centrifugal force which causes compression near concave surface and the rarefaction near the convex surface of blade, thus resulting in separation of boundary layer.

(iii) Loss of energy may be due to turbulence at outlet of preceding row of nozzles due to finite thickness of nozzle exit edge. There is mixing of steam jet leaving nozzles and entering moving blade. Due to this transition of flow from nozzle passage to blade passage there is formation of eddies and turbulence gets set in. This turbulence is generally in the form of trailing vortices which keep on disappearing at high velocities. These cause the reduction of kinetic energy delivered to blades and are called "wake losses". Wake losses are visible at the trailing edge of fixed blades too due to thickness of trailing edge.

(iv) Loss of energy is also there due to breakage of flow which occurs upon the impingement of steam upon the leading edge of moving blade. This is also termed as 'impingement loss'. These losses are less if the flow is laminar as compared to the turbulent flow.

(v) Loss of energy also occurs during passage of steam from one stage to other i.e. rows. This loss is also termed as 'carry over loss'. This carry over loss is minimum if spacing between consecutive rows is kept small.

The different losses as described above are accounted by taking the profile loss coefficient (k_p), incidence loss coefficient (k_i) and carry over loss coefficient (k_c). Profile loss coefficient takes into account the losses due to turbulence, friction, fluid deflection in blade passage, curvature of blade and actual exit angle being different from blade exit angle. Incidence loss coefficient takes care of losses due to turbulence introduced by angle of incidence. The carry over loss coefficient takes care of losses due to kinetic energy loss during transition of flow between the rows. Therefore, the actual relative velocity leaving blade shall be, $\boxed{V_2 = k_p \cdot k_i \cdot k_c \cdot V_1}$

3. Disc friction loss: This is a kind of loss of energy visible whenever any object say disc is rotated in air or other medium. The disc would cut the atmosphere and impart motion to surrounding air. There shall always exist relative motion between solid wall of object and the air or surrounding fluid. Due to this relative motion surrounding medium always exerts a resistance to motion of moving object. This may result in loss of energy due to friction which may be felt by the increase in enthalpy of surrounding fluid. In case of steam turbines too the rotor is completely surrounded by the steam which offers resistance to the rotor motion. The loss of energy of rotor may go into the steam enveloping it. This loss of energy is termed as 'disc friction loss'. The disc friction loss may cause heating of steam surrounding the rotor i.e. a portion of kinetic energy is transferred from the rotor disc to steam by heating of steam. Disc friction loss is substantial in case of impulse stages as compared to reaction stages where it is very small and can be neglected.

4. Windage loss: Windage loss occurs when the rotor blades come in contact with near stationary fluid (steam). In case of partial admission turbines i.e. generally impulse turbines there is churning of steam in the region having no active steam in steam turbine. When moving rotor blades come in contact with inactive steam then there is transfer of energy from blade to steam. This loss of energy from rotor to fluid is termed as 'windage loss'.

In case of full admission turbine the region inside turbine having inactive steam is negligible and so the windage loss is nearly negligible. To minimize windage loss the turbine should be filled with moving steam (active steam). Windage losses are very small in case of low pressure stages. Reaction turbines have negligible windage losses as they are full admission turbines.

5. Loss due to leakage: Steam leakage may occur across the turbine shaft and between stages. Leakage of steam may result in availability of less work from stage as steam is not fully utilized for producing work. Leakage occurs during flow from one stage to other or from one row to other through the clearance space between diaphragm and shaft. Leakage also occurs across the blade tip. Leakage across diaphragm occurs in both impulse turbine and reaction turbine stages. Leakage across tip is not prominent in case of impulse turbine as the pressure difference is very small. Tip leakage is prominent in reaction turbine stages.

Due to this diaphragm and tip leakage effective mass flow rate for doing work gets reduced and is consequently a loss of energy. Leakage is accompanied by the increase in entropy and so the decrease in availability of work due to throttling of steam which is an irreversible process. Leakage loss can be minimized by reducing the clearances as much as possible after providing for expansion of turbine parts so that the metal-to-metal rubbing is avoided. Different seals such as labyrinths, carbon rings, water, steam or air seals are used to prevent this leakage through clearance. Also in order to reduce leakage loss the drum type construction is preferred to diaphragm and wheel type construction in reaction turbines.

Another type of leakage may be of balance-piston leakage which refers to leakage between balance piston and casing. Here fluid leaks out in high pressure region of turbine and atmospheric air bleeds into casing in low pressure sub atmospheric region (condensor side). Generally this is not a total loss as the leakage out and leakage in are not varying too much. This kind of loss can also be prevented by employing labyrinth packing. At low pressure/subsonic region (condenser side) of turbine the labyrinth packing is fed with low pressure steam so that steam leaks in instead of air in case of unavoidable leakage.

6. Losses in bearings: Turbine bearings are critical parts to support high speed rotation of shaft. Generally, a loss to the tune of 1% of turbine output occurs in bearings. Although this loss depends upon bearing load, oil viscosity, speed of shaft, bearing surface area and film thickness etc.

7. Losses at inlet and exit: Loss at inlet of steam turbine occurs at regulating valves at entry. At these valves which may be stop valve or governor valves the throttling loss generally occurs causing lowering of entering steam pressure.

At the exit of steam turbine the steam becomes wet and the fluid now is mixture of water droplet and steam. Due to wetness of steam at exit end the water particles being heavier cause loss of kinetic energy. These water particles, if in excessive amount may also endanger the turbine blade. The kinetic energy of fluid at exit of steam turbine is a total loss and theoretically the fluid leaving turbine must have the lowest possible energy in it. This loss of energy may be of the order of 10 – 13 per cent.

8. Losses due to radiations: Radiation losses also occur in steam turbines, although they are very small compared to other losses and may be neglected. In case of steam turbines the high temperature steam is limited to small part of casing so losses are small. But the radiation losses are quite significant in gas turbines. In order to prevent radiation losses the pipings, turbine casing etc. carrying hot fluid should be well insulated.

14.10 REHEAT FACTOR

In case of steam turbine the expansion process must occur isentropically under ideal conditions. But in actual practice due to large number of losses such as friction, leakage, entry and exit losses etc. the expansion inside does not remain isentropic. These factors are actually resulting into the situation having incomplete conversion of available energy into the shaftwork. Thus, the actual steam leaving turbine will be leaving with higher enthalpy than the enthalpy of steam at exit under isentropic expansion. Figure. 14.26 shows the ideal and actual expansion process in case of single stage and multi stage steam turbine. Single stage expansion shows that ideally steam enters at state 1 and leaves at 2.

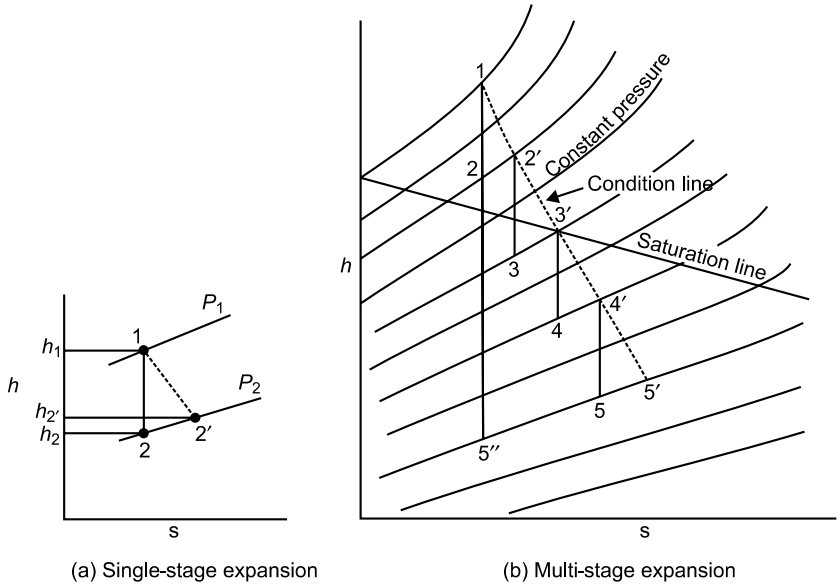


Fig. 14.26 *Ideal and actual expansion in steam turbine showing reheat effect on* h–*s diagram*

While actually steam enters at state 1 and leaves at state 2′ i.e. enthalpy at state 2′ is higher than at 2. Comparison of expansion process 1–2 and 1–2′ shows that due to various losses the total available energy $(h_1 - h_2)$ is not completely converted into shaft work. Portion of available energy given by $\{(h_1 - h_2) - (h_1 - h_{2'})\}$ remains with expanding fluid i.e. steam itself. On *h*–*s* diagram the shift in state after expansion from 2 to 2′ due to non-ideal operation is understood as reheating of steam at constant pressure. For the sake of simplicity this reheating from 2 to 2′ is considered to be of constant pressure

type, whereas actually it may not be so. If the state of steam after expansion lies in wet region then it can be seen that non-isentropic expansion causes increase in dryness of steam at exit i.e. dryness fraction at 2, (x_2) will be less than that at 2′($x_{2'}$), $x_2 < x_{2'}$.

Reheating of steam is accompanied by increase in entropy i.e. $s_{2'} > s_2$. Figure 14.27 shows the ideal and actual expansion in steam turbine on *T*–*s* diagram. For single stage expansion the *T*– *s* diagram shows the amount of heat lost/dissipated due to the friction etc. and subsequently recovered by steam expanding in steam turbine in the form of reheating by the area 22′ *BA*.

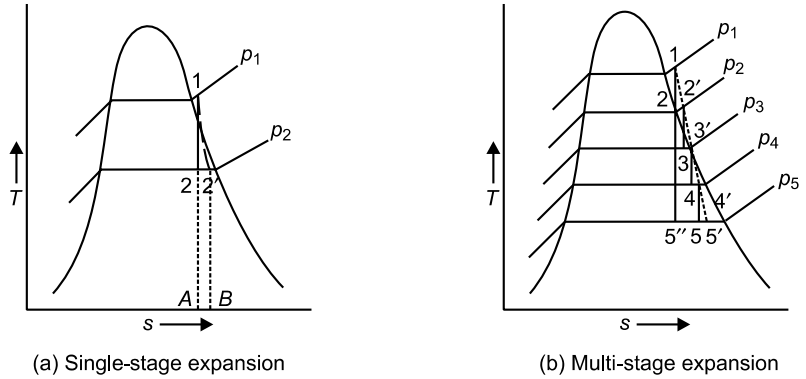


Fig. 14.27 *Ideal and actual expansion in steam turbine on T-S diagram*

In case of multi stage expansion the *h* – *s* diagram shown in Fig. 14.26 indicates the four stage expansion starting from state 1. In the first stage theoretically the ideal expansion is given by the line 1–2 while actually it is 1–2′. Second stage expansion shall begin from state 2′ and isentropic and non-isentropic expansion processes are shown by 2′–3 and 2′–3′ respectively. Actual state after expansion in second stage is 3′ and the subsequent stage expansion begins from this point. Expansion in third stage results in theoretical and actual states as 4 and 4′ respectively. In the last stage starting from 4′ the ideal and actual states after expansion are at 5 and 5′ respectively.

Thus for multi stage expansion the actual states at the end of each stage are 2′, 3′, 4′ and 5′. For any stage, say first stage the isentropic and non-isentropic enthalpy drop may be related to stage efficiency as below,

$$\text{Stage efficiency, } \eta_s \text{ for Ist stage} = \frac{(h_1 - h_{2'})}{(h_1 - h_2)}$$

Here the locus of actual state points 1–2′ – 3′ – 4′ – 5′ is also called the ‘condition’ line as it approximates actual state of steam during its exit through turbine stages. If we look at the enthalpy drops occurring in ideal and actual conditions then it is seen that the sum of isentropic enthalpy drops in stages i.e. ($\Delta h_{12} + \Delta h_{2'3} + \Delta h_{3'4} + \Delta h_{4'-5}$) shall be different from the actual enthalpy drop ($\Delta h_{12'} + \Delta h_{2'3'} + \Delta h_{3'4'} + \Delta h_{4'5'}$).

This sum of isentropic enthalpy drop (heat drop) is generally termed as cumulative heat drop (Δh_c). Total enthalpy drop for isentropic expansion can be given by $\Delta h_{1\text{-}5'}$ and this is also termed as ‘Rankine heat drop’ ($\Delta h_{\text{Rankine}}$).

Let us look actual and ideal expansion in a stage, say second stage where expansion begins at 2′ and should go up to 3 while it actually ends at 3′. The shift in state from 3 to 3′ indicates reheating of steam during expansion. This phenomenon is called reheating of steam during expansion in steam turbine and is quantified by ‘Reheat Factor’. ‘Reheat factor’ is defined by the ratio of ‘cumulative heat

drop' to 'Rankine heat drop' i.e. cumulative heat drop (Δh_c) to direct isentropic heat drop ($\Delta h_{\text{Rankine}}$).

$$\text{Reheat factor} = \frac{\text{Cumulative heat drop}}{\text{Rankine heat drop}} = \frac{\Delta h_{12} + \Delta h_{2'3} + \Delta h_{3'4} + \Delta h_{4'5} + \cdots}{\Delta h_{1-5''}}$$

$$\text{RF} = \frac{\Delta h_{12} + \Delta h_{2'3} + \Delta h_{3'4} + \Delta h_{4'5}}{\Delta h_{1-5''}} = \frac{\Delta h_c}{\Delta h_{\text{Rankine}}}$$

Reheat factor value is always greater than unity because cumulative heat drop shall always be more than Rankine heat drop. This may be seen from *h–s* diagram which has diverging constant pressure lines because of which the cumulative heat drop becomes more than Rankine heat drop. Reheat factor value increases with increase in number of stages for a given pressure range. If the stage efficiency is reduced then also the value of reheat factor increases for given pressure range and number of stages. From designer's point of view the reheat factor must have value of unity under ideal conditions. But, since unity reheat factor value can not be attained so efforts must be made to reduce the reheat factor value to make it close to unity. Unity reheat factor indicates that actual expansion is similar to ideal expansion and 'cumulative heat drop' equals to 'Rankine heat drop.'

Reheat factor may be related to internal efficiency of turbine as detailed below. Internal efficiency is given by the ratio of 'sum of actual outputs of stages' and 'the available energy in turbine'. Internal efficiency may also be given by the ratio of internal work done of turbine to the Rankine heat drop.

Mathematically, internal efficiency, $\eta_{\text{int}} = \dfrac{\Delta h_{\text{actual}}}{\Delta h_{\text{Rankine}}}$

$$\eta_{\text{int}} = \frac{\Delta h_{12'} + \Delta h_{2'3'} + \Delta h_{3'4'} + \Delta h_{3'5'}}{\Delta h_{15''}}$$

or, $$\eta_{\text{int}} = \frac{\Delta h_{15'}}{\Delta h_{15''}}$$

If the stage efficiency for turbine stages is given by η_S and stage efficiency remains same for all stages, then stage efficiency can be given as,

$$\eta_S = \frac{\Delta h_{12'}}{\Delta h_{12}} = \frac{\Delta h_{2'3'}}{\Delta h_{2'3}} = \frac{\Delta h_{3'4'}}{\Delta h_{3'4}} = \frac{\Delta h_{4'5'}}{\Delta h_{4'5}}$$

Also, $\Delta h_{15'} = \Delta h_{12'} + \Delta h_{2'3'} + \Delta h_{3'4'} + \Delta h_{4'5'}$

Let us assume the available energy in each stage to be equal as, Δh_{stage}

$$\Delta h_{12} = \Delta h_{2'3} = \Delta h_{3'4} = \Delta h_{4'5} = \Delta h_{\text{stage}}$$

or,

$$\Delta h_{12'} = \eta_S \cdot \Delta h_{\text{stage}} = \Delta h_{2'3'} = \Delta h_{3'4'} = \Delta h_{4'5'}$$

Substituting in internal turbine efficiency, for 'n' number of stages

$$\Delta h_{15'} = \eta_S \cdot \Delta h_{\text{stage}} \times n$$

$$\eta_{\text{int}} = \frac{\eta_S \cdot \Delta h_{stage} \times n}{\Delta h_{15''}} = \frac{\eta_S \cdot \Delta h_{stage} \times n}{\Delta h_{\text{Rankine}}}$$

From definition of reheat factor,

$$\text{RF} = \frac{\Delta h_c}{\Delta h_{\text{Rankine}}}$$

$$\Delta h_{\text{Rankine}} = \frac{\Delta h_c}{\text{RF}}$$

Substituting $\Delta h_{\text{Rankine}}$ in η_{int},

$$\eta_{\text{int}} = \frac{\eta_S \cdot \Delta h_{\text{stage}} \times n \times \text{RF}}{\Delta h_c}$$

$$\boxed{\eta_{\text{int}} = \frac{\eta_S \cdot \left(n \cdot \Delta h_{\text{stage}}\right) \cdot \text{RF}}{\Delta h_c}}$$

For single stage turbine,

$$\boxed{\eta_{\text{int}} = \frac{\Delta h_{12'} \cdot \text{RF}}{\Delta h_{12}}}$$

For the four stage turbine as considered here,

$$\boxed{\eta_{\text{int}} = \frac{(\Delta h_{15'}) \cdot \text{RF}}{(\Delta h_{15''})}}$$

Thus it is obvious that the internal efficiency of a turbine having multiple stages is always more than stage efficiency as Reheat factor is more than unity.

14.11 STEAM TURBINE CONTROL

In actual operation of steam turbine the speed and output of turbine are to be constantly monitored and controlled as rarely any turbine operates at constant load. This operation of turbine at off-design conditions also deteriorates performance of turbine. Therefore, for safe and optimum operation of turbine it has certain control and monitoring instruments. Number and size of these control and monitoring instruments depends upon size of turbine and its' application. Some of typical controls and instruments used are described below for monitoring pressure, temperature, speed, noise and vibration etc.

(i) Pressure measurement is done for oil pressure to bearing, water/steam pressure to gland seals, steam pressure at stop valves, steam chest, inlet and exit of each stage in turbine. Pressure gauges are mounted at suitable locations for each one of above.

(ii) Temperature measurement are done at almost all the locations of pressure measurement.

(iii) Speed measurement is required to know speed of turbine in rpm during starting and shutdown of turbine. Turbine speed is obtained from the generator frequency recorder.

(iv) Eccentricity of shaft is also to be monitored for which an eccentricity recorder is provided to indicate and record eccentricity of shaft at high pressure end of turbine. Eccentricity recorder has a detector coil at low pressure end of shaft, a transfer switch on turning gear, and a power unit and recorder mounted on control panel. Eccentricity of shaft causes variation in air space between shaft and coil detector and therefore change in current flowing through coil, which is being sensed appropriately.

(v) Vibrations at main bearings of turbine and generator are being monitored using vibration amplitude recorder. This type of recorder has a detector at each bearing and detector voltage is proportional to the bearing vibrations. Voltage variation is sent for suitable signal transformation and vibration amplitude is recorded.

(vi) Due to excessive temperature the turbine casing may undergo axial expansion for which an expansion indicator is put. Expansion is monitored with respect to some selected mark on

foundation. Steady expansion of casing is permitted while unsteady expansion may be harmful.

(vii) Noise of turbine operation is being measured using noise meter as any unusual sound level may indicate possibility of failure.

(viii) Flow rate of steam at all crucial points in steam turbine is being measured employing flow meters and signals monitored at control panel.

(ix) Control panel also has wattmeters, voltmeters, ammeters for respective measurements. Control panel also has the digital control of various valves, governor control etc.

Now-a-days the control panel of turbines has all digital controls and monitoring employing suitable microprocessors.

14.12 GOVERNING OF STEAM TURBINES

Governing of steam turbine is required for controlling and regulating the output of a turbine according to variable demands in service. This regulation of turbine is done by maintaining accurate and positive control of speed, pressure and flow by employing governors. Governing of turbine maintains the speed of turbine at constant level irrespective of load. Governing of steam turbines can be of following types depending upon the mechanism of regulation.

(i) Throttle governing

(ii) Nozzle control governing

(iii) By-pass governing

(iv) Combined governing i.e., Combination of above as 'throttle and nozzle control governing' or 'throttle and by-pass governing'.

(i) *Throttle Governing:* Throttle governing of steam turbine bases upon the throttling of steam up to suitable pressure and regulate the mass flow of steam through the turbine so as to control the output of turbine. Here in throttle governing, aim is to alter the mass flow rate as for example by reducing it, the available energy gets reduced and hence lowering of output.

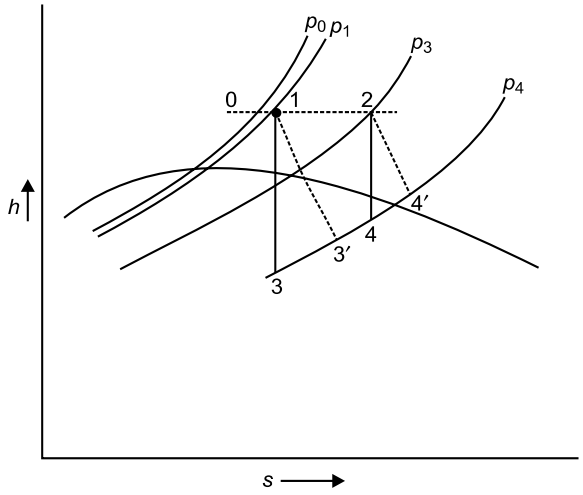


Fig. 14.28 *Throttle governing on h-s diagram*

In throttle governing the steam entering is regulated by opening and closing of valve. As the valve is closed, the throttling or constant enthalpy process occurs across the valve with an increase in entropy and corresponding decrease in availability of energy per unit mass flow of steam. Also due to throttling

the state of steam entering turbine stage gets modified and the modified expansion line for each load is obtained. It may be noted that even when the governor valve is full open the pressure drop does occur and thus it can be said that throttling is evident at all loads on turbine. Representation of throttle governing on *h-s* diagram shows that the steam is available at state '0' at p_0 pressure in the main steam line. At the inlet from main steam line when the governor valve is full open the throttling results in modified state 1 from where expansion occurs following path 1–3 under insentropic expansion and non-isentropic expansion occurs following path 1–3′. When mass flow rate is reduced for reduced load on turbine then this partial closing of valve causes throttling as shown by 1–2. Now as a result of this throttling from 1 to 2 the modified expansion paths are 2–4 and 2–4′ for isentropic and non-isentropic expansion in turbine. Thus it shows that as a result of throttling the available energy gets reduced from $\Delta h_{1\text{-}3}$ to $\Delta h_{2\text{-}4}$ and $\Delta h_{2\text{-}4} < \Delta h_{1\text{-}3}$. Hence this lowering of available energy causes reduced output from turbine.

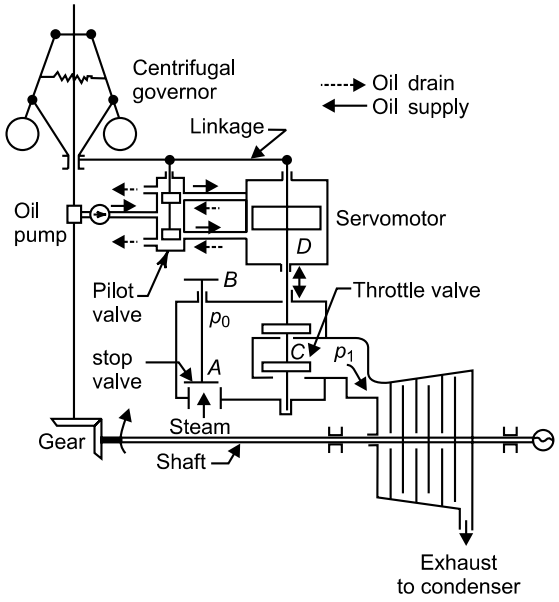


Fig. 14.29 *Schematic of simple throttle governing*

Schematic of simple throttle governing in steam turbines is shown in Fig. 14.29. Here a centrifgal governor is used to sense the change in speed of shaft. The relay system has a pilot valve and servomotor. The displacement of servomotor piston either upward or downward decides the opening of throttle valve '*C*'. Servomotor piston is actuated by the high pressure oil entering from pilot valve to upper or lower half of servomotor piston *D*. Under normal operation the servomotor piston occupies middle position and pilot valves keep the inlet and exit ports in closed position. When oil enters the upper half of servomotor then servomotor piston lowers down and the throttle valve starts closing causing reduction of steam flow rate and so the output till the speed is maintained to normal running speed. Simultaneously, the oil from lower half of servomotor gets drained out through pilot valve port. When oil under pressure enters lower half of servomotor then servomotor piston gets lifted up causing lift of throttle valve.

For throttle governing of steam turbine the steam consumption rate may be plotted with load resulting into characteristic line called Willans line as shown in Fig. 14.30.

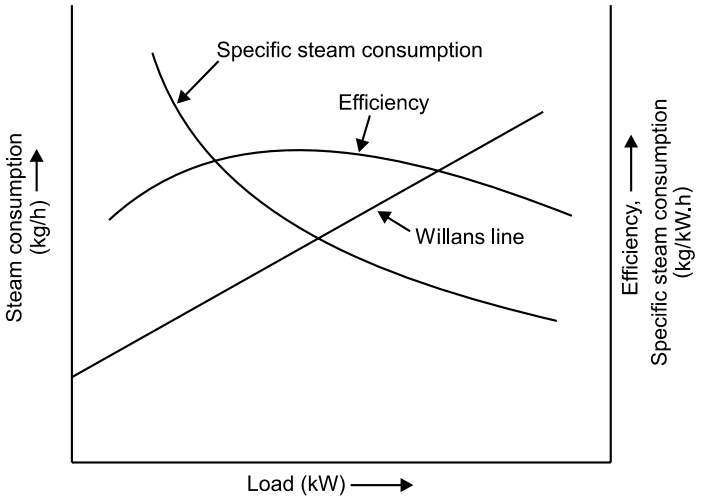


Fig. 14.30 *Characteristics of throttle governing*

During throttle governing the Willan's line is straight line making an intercept on *y*-axis. Mathematically, it can be given as,

$$M = K \cdot L + M_0$$

where M is steam consumption in kg/h at any load L.

M_0 is steam consumption in kg/h at no load i.e. $L = 0$

L is any load on turbine in kW.

K is the constant and gives slope of Willan's line.

Here it shows that even at no load the steam consumption shall be M_0 (kg/h) which is graphically given by intercept on *y*-axis.

Above equation can also be written as, $\frac{M}{L} = K + \frac{M_0}{L}$

where $\frac{M}{L}$ is the specific steam consumption at any load, kg/kW·h

and $\frac{M_0}{L}$ is the specific steam consumption at no load, kg/kW·h.

Throttle governing offers following disadvantages due to throttle action at inlet:

(a) Throttling increases initial superheat at inlet and the greatest variations in steam velocity occur in the later stages.

(b) The wetness of steam gets reduced in later stages due to throttling. Due to this reduced wetness there occurs reduction in stage efficiency at part load operation of turbine.

(ii) *Nozzle Control Governing:* Nozzle control governing is the one in which steam flowing through nozzles is regulated by valves. Nozzle control governing is generally employed at first stage of turbine due to practical limitations. The nozzle areas in remaining stages remain constant. If some how the nozzle governing is provided for all nozzles in each and every stage then an ideal condition of turbine flow passage areas conforming to mass flow rate at all loads shall exist. Under such ideal conditions the pressure, velocities and nozzle and blade efficiencies would be constant with load. For such ideal condition the Willan's line would be straight line as indicated for throttle governing of turbine.

In nozzle governing the nozzles of turbines are grouped in two, three or more groups upto six or eight groups. When nozzle governing is employed then the pressure and temperature of steam entering first stage nozzles are independent of load. Figure 14.31 shows the schematic of nozzle control governing.

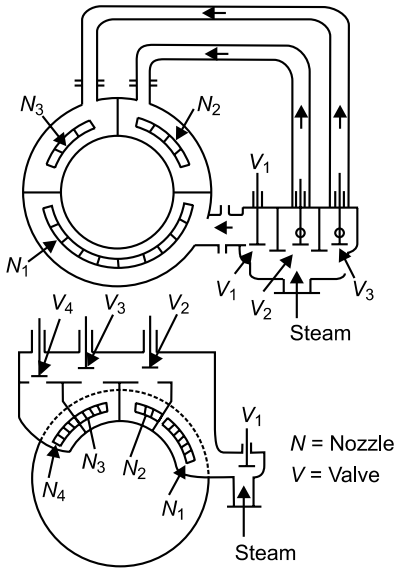


Fig. 14.31 *Schematic of nozzle control governing*

As the valves are being regulated for actuating nozzle control governing so there occurs some throttling of steam at each valve. However, the amount of throttling is considerably lesser and the decrease in availability of energy to turbine is not too much. In order to avoid this occurrence of throttling very large number (infinite) of nozzle and governing valves may be put.

(iii) *By Pass Governing:* In case of by-pass governing arrangement is made for by-passing surplus quantity of steam without allowing total steam quantity to contribute in turbine output when load reduces. Arrangement of by pass governing is shown in Fig. 14.32.

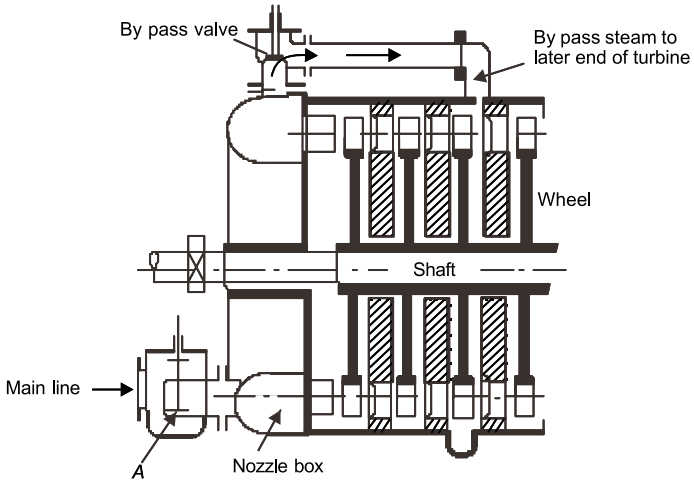


Fig. 14.32 *Arrangement in by pass governing*

Diagram shows that steam from main line enters the main valve which is controlled by speed governor. Steam from main valve enters the nozzle box or steam chest. By pass valve is also provided on the nozzle box. By pass valve is connected to a passage which delivers steam being by passed to later end of turbine. By pass valve is actuated when load varies, thus allowing only part of steam entering main valve to contribute in power output. By pass valve is controlled by speed governor for all loads within its' range. In this kind of governing depending upon turbine and its' application there may be more than one by pass valves.

(iv) *Combined governing:* Some times when the governing requirements are not met by any one kind of arrangements of governing i.e. throttle, nozzle control and by pass governing, then the combination of two governing mechanisms may be employed. These popular combinations are 'throttle and nozzle control combined governing' and 'throttle and by pass combined governing'.

14.13 DIFFERENCE BETWEEN THROTTLE GOVERNING AND NOZZLE CONTROL GOVERNING

Differences between throttle governing and nozzle control governing are as given;

Throttle governing	*Nozzle control governing*
(a) This governing is realized by throttling of steam at inlet to turbine so there occur severe throttling losses.	(a) This governing is realized by nozzle control so the degree of throttling losses in valves are negligible.
(b) It has smaller losses due to partial admission of steam.	(b) It has large partial admission losses.
(c) This governing is generally employed in both impulse turbines and reaction turbines.	(c) This governing is employed in impulse turbine and reaction turbines having initial impulse stages.

14.14 DIFFERENCE BETWEEN IMPULSE AND REACTION TURBINES

Differences between the impulse turbine and reaction turbines are as under.

Impulse turbines	*Reaction turbines*
(a) Impulse turbine has profile type blades and has constant area between two consecutive blades.	(a) Reaction turbine has airfoil type blades and has converging area between two consecutive blades.
(b) Impulse turbine stage has pressure drop occurring only in nozzles. No pressure drop occurs in moving blade row.	(b) Reaction turbine stage has pressure drop occurring in both fixed as well as moving blades.
(c) Impulse turbines have incomplete admission of steam or steam being admitted at selected positions around the motor.	(c) Reaction turbines have complete admission of steam or steam being admitted all around the rotor through fixed blade ring.
(d) Impulse turbine diaphragm has nozzles mounted on it. Velocity of	(d) Reaction turbine has fixed blade ring attached to casing to serve

Cont.

steam is quite large.	as nozzle and guide blades for entering steam. Velocity of steam is comparatively smaller.
(e) Impulse turbine has lesser efficiency and much power can not be developed from it.	(e) Reaction turbine has higher efficiency and is capable of producing large power output.
(f) These occupy less space for same power output.	(f) These occupy large space for same power output.
(g) These are not very costly as the manufacturing of impulse turbine blades is much simpler.	(g) Reaction turbines are costly as their blades are very difficult to be manufactured.
(h) Impulse turbines are suitable for small power requirement.	(h) Reaction turbines are suited for medium and higher power requirements.

EXAMPLES

1. *Single stage impulse turbine has equal blade angles and nozzle angle of 15°. Determine the maximum possible blade efficiency if the blade velocity coefficient is 0.85. Determine the blade speed to steam velocity ratio if the actual blade efficiency is 90% of maximum blade efficiency.*

Solution:

Given, $\alpha_1 = 15°$, $\beta_1 = \beta_2$, $K = 0.85$

Maximum blade efficiency, $$\eta_{b,\max} = \frac{\cos^2 \alpha_1}{2}\left(1 + K \cdot \frac{\cos \beta_1}{\cos \beta_2}\right)$$

$$= \frac{\cos^2 15°}{2} (1 + 0.85)$$

$$= 0.863 \text{ or } 86.3\%$$

Maximum blade efficiency = 86.3% ***Ans.***

Actual blade efficiency = 0.90×0.863

$$= 0.7767$$

Blade efficiency for single stage impulse turbine can be given as,

$$\eta_b = 2(1 + K)\ (\rho \cos \alpha_1 - \rho^2)$$

$$0.7767 = 2(1 + 0.85)\ (\rho \cos 15 - \rho^2)$$

$$0.2099 = (\rho \times 0.966 - \rho^2)$$

or,

$$\rho^2 - 0.966\rho + 0.21 = 0$$

$$\rho = 0.653 \text{ or } 0.347$$

Blade speed to steam velocity ratio = 0.653 or 0.347 ***Ans.***

2. A *single stage of simple impulse turbine produces 120 kW at blade speed of 150 m/s when steam mass flow rate is 3 kg/s. Steam enters moving blade at 350 m/s and leaves the stage axially. Considering velocity coefficient of 0.9 and smooth steam entry without shock into blades, determine the nozzle angle and blade angles. Solve using velocity diagram.*

Solution:

Given: $U = 150$ m/s, $C_1 = 350$ m/s, $\frac{V_2}{V_1} = 0.9$, $\alpha_2 = 90°$

For impulse turbine single stage,

$$\text{Output} = 120 \times 10^3 = 150 \times \Delta C_w \times 3$$

$$\Delta C_w = 266.67 \text{ m/s.}$$

Let scale be 1 cm = 50 m/s for drawing velocity diagram using given information.

$V_1 = 255$ m/s $\Rightarrow BD = 5.1$ cm

$C_1 = 350$ m $\Rightarrow AD = 7$ cm

$V_2 = 229.5$ m/s $\Rightarrow BE = 4.59 \approx 4.6$ cm

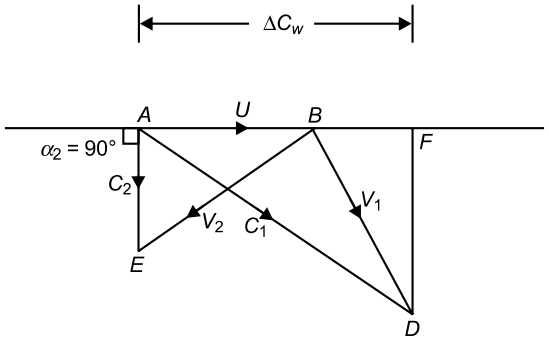


Fig. 14.33

Steps:

(i) Draw $AB = 3$ cm $= \left(\frac{150}{50}\right)$ corresponding to U.

(ii) Draw perpendicular from A as $\alpha_2 = 90°$ for axial discharge from turbine. Mark point F along AB corresponding to ΔC_w (= 266.67 m/s), $AF = 5.33$ cm

(iii) Draw perpendicular from F along FD. Open compass for C_1 (= 350 m/s) i.e. 7 cm and mark point D keeping needle at A. Join AD and DB corresponding to C_1 and V_1 respectively. Measure V_1 (= 5.1 cm or 255 m/s) by length BD. Estimate $V_2 = V_1 \times 0.9 = 229.5$ m/s.

(iv) Open compass for V_2 (= 229.5 m/s) i.e. 4.59 cm and mark point E keeping needle on B. Join points E and A, E, B to show C_2 and V_2 respectively. This completes velocity diagram.

Measure, $\angle BAD = \alpha_1 = 39°$

$\angle FBD = \beta_1 = 61.5°$

$\angle ABE = \beta_2 = 47°$

Nozzle angle = 39°
Blade angles at inlet and exit = 61.5° and 47°

Ans.

3. *In a single stage simple impulse turbine the steam flows at rate of 5 kg/s. It has rotor of 1.2 m diameter running at 3000 rpm. Nozzle angle is 18°, blade speed ratio is 0.4, velocity coefficient is 0.9, outlet angle of blade is 3° less than inlet angle. Determine blade angles and power developed.*

Solution:

Given: $\rho = \dfrac{U}{C_1} = 0.4,\ \dfrac{V_2}{V_1} = 0.9,\ \alpha_1 = 18°,\ \beta_2 = \beta_1 - 3$

$$U = \frac{2 \times \pi \times 3000 \times 1.2}{60 \times 2} = 188.5 \text{ m/s} \Rightarrow C_1 = 471.25 \text{ m/s}$$

Using these parameters the velocity diagram is drawn.

Scale 1 cm = 50 m/s.

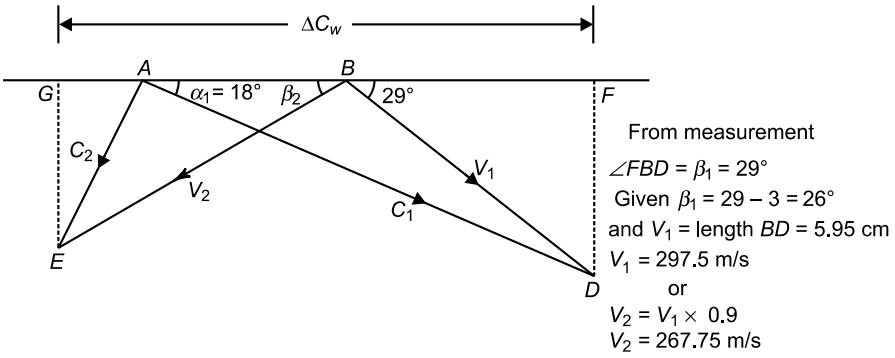


Fig. 14.34

From velocity diagram, $\beta_1 = 29°$, $\beta_2 = 26°$, $\Delta C_w = 495$ m/s (= length FG)

Power developed = $m \cdot U \cdot \Delta C_w$

$= 5 \times 188.5 \times 495 = 466537.5$ J/s

$= 466.54$ kW

Blade angles at inlet and exit = 29° and 26°
Power developed = 466.54 kW

Ans.

4. *In a single stage impulse turbine the isentropic enthalpy drop of 200 kJ/kg occurs in the nozzle having efficiency of 96% and nozzle angle of 15°. The blade velocity coefficient is 0.96 and ratio of blade speed to steam velocity is 0.5. The steam mass flow rate is 20 kg/s and velocity of steam entering is 50 m/s. Determine*

(a) the blade angles at inlet and outlet if the steam enters blades smoothly and leaves axially.
(b) the blade efficiency
(c) the power developed in kW
(d) the axial thrust.

Solve using velocity diagram.

Solution:

Let the state at inlet to nozzle, leaving nozzle and leaving blade be denoted using subscripts 0, 1 and 2.

Using nozzle efficiency the actual enthalpy drop may be calculated as.

Actual enthalpy drop in nozzle, $\Delta h_N = 200 \times 0.96 = 192$ kJ/kg

Velocity of steam leaving nozzle, $C_1 = \sqrt{2 \cdot \Delta h_N + (C_0)^2}$

$$C_1 = \sqrt{2 \times 192 \times 10^3 + (50)^2}$$

$$C_1 = 621.69 \text{ m/s}$$

Given: $\frac{U}{C_1} = 0.5$, $\frac{V_2}{V_1} = 0.96$, m = 20 kg/s, $\alpha_1 = 15°$

or, $U = 0.5 \times 621.69 = 310.845$ m/s

Let us draw the velocity diagram across blade i.e. states 1 and 2. Let us use scale as 1 cm = 100 m/s.

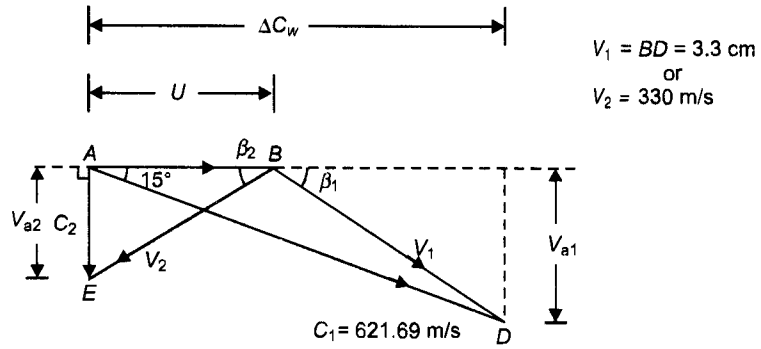


Fig. 14.35

From ΔABD, $AD = C_1 = 621.69$ m/s, $AB = U = 310.845$ m/s

Length BD gives V_1, from measurement it is seen to be 3.3 cm.

hence, $V_1 = 330$ m/s, so $V_2 = 0.96 \times 330$

$V_2 = 316.8$

From velocity diagram, $\beta_1 = 28.5°$, $\beta_2 = 12°$, $\Delta C_w = 595$ m/s,

$V_{a1} = 170$ m/s

$V_{a2} = 75$ m/s

Blade angle at inlet = 28.5°
Blade angle at exit = 12° ***Ans.***

$$\text{Blade efficiency} = \frac{mU \cdot \Delta C_w}{m\left(\frac{C_1^2}{2}\right)} = \frac{2 \times 310.845 \times 595}{(621.69)^2} = 0.9571$$

or $= 95.71\%$

Blade efficiency = 95.71% ***Ans.***

Power developed = $U \cdot \Delta C_w \times m = 20 \times 310.845 \times 595$

$= 369905.55$ J/s

or $= 369.91$ kJ/s

Power developed = 369.91 kW ***Ans.***

Axial thrust, $F_a = m \cdot (V_{a1} - V_{a2})$
$= 20 \times (170 - 75)$
$= 1900$ N

Axial thrust = 1900 N ***Ans.***

5. *A De Laval turbine runs with steam supplied at 16 bar, 250°C.*
The nozzle efficiency is 90%, blade velocity coefficient is 0.98, mechanical efficiency is 95%, nozzle angle is 15°, symmetrical blades with 30° angle, mean diameter of wheel is 80 cm, back pressure is 0.15 bar. Determine

(a) the speed of rotation,
(b) the steam consumption per-bhp-hr,
(c) the stage efficiency,
(d) the percentage energy loss at exit.

Solution:

From steam table at inlet to turbine nozzle, $h_0 = 2919.2$ kJ/kg
$s_0 = 6.6732$ kJ/kg·K

At exit i.e. back pressure 0.15 bar, $s_1 = s_0 = 6.6732$ kJ/kg·K

$$s_1 = s_{\text{f at 0.15 bar}} + x_1 \cdot s_{\text{fg at 0.15 bar}}$$
$$6.6732 = 0.7549 + x_1 \times 7.2536$$
$$x_1 = 0.8159$$

$$\text{Enthalpy } h_1 = h_{\text{f at 0.15 bar}} + x_1 \times h_{\text{fg at 0.15 bar}}$$
$$= (225.94 + (0.8159 \times 2373.1))$$
$$h_1 = 2162.15 \text{ kJ/kg}$$

Isentropic heat drop $= h_0 - h_1$
$= (2919.2 - 2162.15)$
$= 757.05$ kJ/kg

Velocity of steam $C_1 = \sqrt{2 \times 757.05 \times 0.90 \times 10^3}$
$C_1 = 1167.34$ m/s

Nozzle angle $\alpha_1 = 15°$, Blade angle $\beta_1 = \beta_2 = 30°$

Blade velocity coefficient, $\frac{V_2}{V_1} = 0.98$

The velocity diagram is drawn using above information. Taking scale 1 cm = 100 m/s.

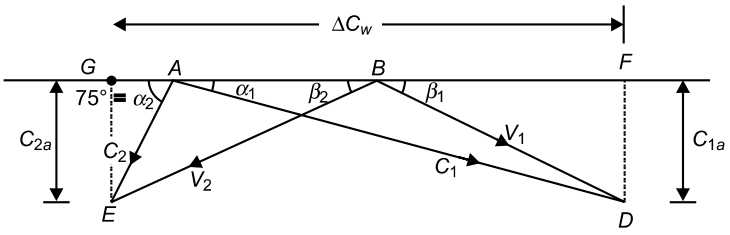


Fig. 14.36

From velocity diagram, V_1 = 630 m/s, given by length *BD*,

Length *AB* gives U = 570 m/s

Using blade velocity coefficient, $V_2 = 0.98 \times V_1$ = 617.4 m/s

ΔC_w = Given by length *GF* = 1080 m/s

Length *AE* gives C_2 velocity,

C_2 = 320 m/s

$$U = 2\pi N \times r \Rightarrow 570 = 2 \times \frac{22}{7} \times N \times \frac{0.80}{2}$$

N = 226.7 rps

N = 13602 rpm

Speed of rotation = 13602 rpm ***Ans.***

Work done per kg of steam, $W = U \cdot \Delta C_w$

$= 570 \times 1080 = 615600$ W/kg

$= 615.6$ kW/kg

1hp = 0.7457 kW

hp per kg of steam per second = 826.86 hp

Using mechanical efficiency, bhp can be obtained as $= 0.95 \times 826.86$

= 785.52 hp

$$\text{Steam consumption per bhp-hr} = \frac{3600}{785.52} = 4.58 \text{ kg}$$

Steam consumption per bhp-hr = 4.58 kg ***Ans.***

$$\text{Stage efficiency} = \frac{\text{Work done}}{\text{Energy supplied to stage}}$$

$$= \frac{615.6}{(757.05)}$$

$$= 0.8132$$

Stage efficiency = 81.32% ***Ans.***

$$\text{Loss at turbine exit per kg of steam} = \frac{C_2^2}{2}$$

= 51200 J/kg

$$\text{Percentage energy loss at exit} = \frac{51200}{\left(757.05 \times 10^3\right)} \times 100$$

$$= 6.76\%$$

Percentage energy loss at exit = 6.76% ***Ans.***

6. *Following data refer to a De Laval steam turbine having equiangular blades;*

Steam entering nozzle = 100 m/s

Nozzle efficiency = *0.90*
Blade speed = *200 m/s*
Blade velocity coefficient = *0.85*
Rate of steam mass flow = *3 kg/s*

Absolute velocity of steam at exit from stage = 90 m/s

Angle of absolute velocity of steam at exit from stage with tangent of wheel = 75°

Determine,

(a) the blade angles,

(b) the nozzle angle,

(c) the absolute velocity of steam at inlet,

(d) the axial thrust,

(e) the hp developed.

Solution:

Given, $U = 200$ m/s, $C_2 = 90$ m/s, $a_2 = 75°$, $\beta_1 = \beta_2$, $\frac{V_2}{V_1} = 0.85$,

Velocity diagram can be drawn considering scale of 1 cm = 50 m/s as below;

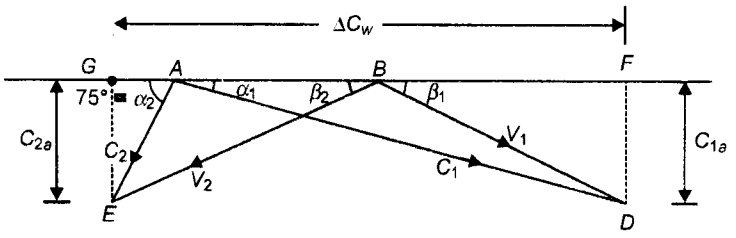


Fig. 14.37

From velocity diagram,

Length *AD* gives $C_1 = 470$ m/s, $C_{1a} = 105$ m/s, $C_{2a} = 87.5$ m/s,

$\Delta C_w = 485$ m/s

$V_2 = 240$ m/s from length *BE*. Angle $\angle ABE = \beta_2 = 21°$ so, $\beta_1 = 21°$

$V_1 = \frac{240}{0.85} = 282.35$ m/s. Nozzle angle = $\angle BAD = \alpha_1 = 11.5°$

Blade angles = 21°
Nozzle angle = 11.5°

Ans.

Absolute velocity of steam at inlet to blades = 470 m/s

The axial thrust = $m\,(C_{1a} - C_{2a})$

$= 3(105 - 87.5)$

$= 52.5$ N

Axial thrust = 52.5 N

Ans.

$$\text{Horse power developed} = \frac{m \cdot U \cdot \Delta C_w}{745.7}$$

$$= \frac{3 \times 200 \times 485}{745.7}$$

$$= 390.23 \text{ hp}$$

HP developed = 390.23 hp **Ans.**

7. *In a simple impulse steam turbine stage steam enters the nozzle at 15 bar, dry saturated with velocity of 150 m/s. Nozzle angle is 20° and steam leaves nozzle at 8 bar and enters into smooth blades. Considering nozzle velocity coefficient of 0.90 and blades to be equiangular determine the following for maximum diagram efficiency.*

(a) the blade angles,
(b) the blading efficiency,
(c) the stage efficiency.

Solution:

From steam tables, enthalpy at inlet to nozzle, $h_0 = h_{\text{g at 15 bar,}} = 2792.2$ kJ/kg

entropy, $s_0 = s_{\text{g at 15 bar}} = 6.4448$ kJ/kg.K

Let dryness fraction at exit of nozzle be x_1 at 8 bar.

For isentropic expansion, $s_1 = s_0 = 6.4448$ kJ/kg·K

$$s_1 = 6.4448 = s_{\text{f at 8 bar}} + x_1 \times s_{\text{fg at 8 bar}}$$

$$6.4448 = 2.0462 + (x_1 \times 4.6166)$$

$$x_1 = 0.9528$$

$$h_1 = h_{\text{f at 8 bar}} + x_1 \times h_{\text{fg at 8 bar}}$$

$$= 721.11 + (0.9528 \times 2048)$$

$$h_1 = 2672.44 \text{ kJ/kg}$$

$$\text{Isentropic heat drop in nozzle} = (h_0 - h_1) = 119.76 \text{ kJ/kg}$$

$$\text{Velocity at exit of nozzle } C_1 = \sqrt{2(h_0 - h_1) + C_0^2}$$

$$= \sqrt{2\left(119.76 \times 10^3 \times 0.9\right) + (150)^2}$$

$$C_1 = 487.92 \text{ m/s}$$

For maximum diagram efficiency the blade velocity can be obtained by,

$$\rho = \frac{\cos \alpha_1}{2} \Rightarrow \frac{U}{C_1} = \frac{\cos \alpha_1}{2}$$

$$U = 487.92 \times \frac{\cos 15}{2} \Rightarrow U = 229.25 \text{ m/s}$$

Velocity diagram is drawn considering $C_1 = 487.92$ m/s, $U = 229.25$ m/s $\alpha_1 = 20°$, $\beta_1 = \beta_2$.
Scale 1 cm = 50 m/s.

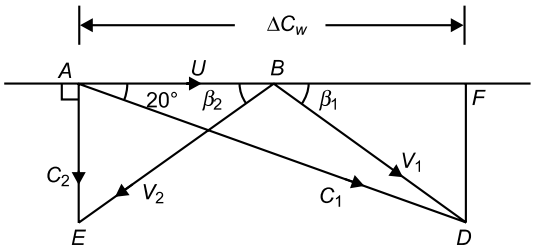


Fig. 14.38

From velocity diagram, length BD = 5.7 cm i.e. V_1 = 285 m/s

Since blades are smooth so $V_1 = V_2$

C_2 = 150 m/s (= length AE)

ΔC_w = 455 m/s (= length AF)

V_2 = 285 m/s

$\beta_1 = 36° = \beta_2$

Blade angle at inlet = 36°
Blade angle at exit = 36° ***Ans.***

Work done per kg of steam = $U \cdot \Delta C_w = (229.25 \times 455) = 104.31 \times 10^3$ J/kg

Kinetic energy supplied per kg of steam = $\frac{C_1^2}{2} = \frac{(487.92)^2}{2}$

$= 119.03 \times 10^3$ J/kg

$$\text{Blading efficiency} = \frac{104.31 \times 10^3}{119.03 \times 10^3} = 0.8763$$

$$\text{Stage efficiency} = \frac{\text{Work done per kg of steam}}{\text{Energy supplied per kg of steam to stage}}$$

$$= \frac{104.31 \times 10^3}{\left\{119.76 \times 10^3 + \frac{(150)^2}{2}\right\}} = 0.7962$$

Blading efficiency = 87.63%
Stage efficiency = 79.62% ***Ans.***

8. *Single row impulse turbine operates between 10 bar and 5 bar with expansion efficiency of 95%. 10 kg of dry saturated steam per second enters into nozzle and leaves nozzle at angle of 20° to the axis of rotation of blades. The blade velocity coefficient is 0.90, blade speed is 200 m/s and internal losses due to disc friction and windage losses is 0.5 kJ/kg of steam. Consider that there is no axial thrust on the blades.*

Determine,

(a) Blade angles,

(b) Stage efficiency,

(c) Stage output in hp and prepare heat balance sheet. Also reason out the error if any by solving using velocity diagram.

Solution:

From steam tables:

Enthalpy of steam entering, $h_0 = h_{g \text{ at } 10 \text{ bar}} = 2778.1$ kJ/kg

Entropy. $s_0 = s_{g \text{ at } 10 \text{ bar}} = 6.5865$ kJ/kg·K

For isentropic expansion $s_1 = s_0 = 6.5865$ kJ/kg·K

Let dryness fraction at exit of nozzle be x_1 at pressure of 5 bar.

$$s_1 = s_{f \text{ at } 5 \text{ bar}} + x_1 \times s_{fg \text{ at } 5 \text{ bar}}$$

$$6.5865 = 1.8607 + (x_1 \times 4.9606)$$

$$x_1 = 0.9527$$

Enthalpy at exit of nozzle, $h_1 = h_{f \text{ at } 5 \text{ bar}} + x_1, h_{fg \text{ at } 5 \text{ bar}}$

$$= 640.23 + (x_1 \times 2108.5)$$

$$h_1 = 2648.99 \text{ kJ/kg}$$

Isentropic heat drop $= (h_0 - h_1) = 129.11$ kJ/kg

Actual heat drop $= 0.95 \times (h_0 - h_1) = 122.65$ kJ/kg

Velocity of steam leaving nozzle considering negligible inlet velocity,

$$C_1 = \sqrt{2(h_0 - h_1) \times 0.95}$$

$$= \sqrt{2 \times 122.65 \times 10^3}$$

$$C_1 = 495.28 \text{ m/s}$$

Given: $U = 200$ m/s, $\alpha_1 = 20°$, $\frac{V_2}{V_1} = 0.90$

$C_{1a} = C_{2a}$ (As there is no axial thrust)

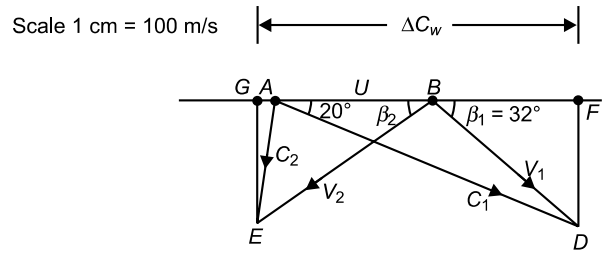


Fig. 14.39

From velocity diagram $V_1 = 320$ m/s, $V_2 = 0.90 \times V_1 = 288$ m/s,

$C_2 = 180$ m/s, $\Delta C_w = 485$ m/s

angle $\angle FBD = \beta_1 = 32°$

angle $\angle ABE = \beta_2 = 39°$

Work done per kg of steam $= U \cdot \Delta C_w$

$= (200 \times 485)$

$= 97000$ J/kg, Actual work $= (97 - 0.5) = 96.5$ kJ/kg

$$\text{Stage efficiency} = \frac{\text{Work done per kg of steam}}{\text{Energy input per kg of steam to stage}}$$

$$= \frac{\left(97000 \times 10^{-3} - 0.5\right)}{129.11} = 0.7474 \text{ or } 74.74\%$$

$$\text{Output in hp} = \frac{(97 - 0.5) \times 10}{0.7457} = 1294.09 \text{ hp}$$

$$\text{Energy lost in nozzle} = (h_0 - h_1) \times 0.05 = 6.46 \text{ kJ/kg}$$

$$\text{Energy lost in blade friction} = \left(\frac{V_1^2 - V_2^2}{2}\right) = \left(\frac{(320)^2 - (288)^2}{2}\right)$$

$$= 9.73 \text{ kJ/kg}$$

$$\text{Exit energy loss} = \frac{C_2^2}{2} = \frac{(180)^2}{2} = 16.2 \text{ kJ/kg}$$

Heat balance sheet in kJ/kg of steam

Input, kJ/kg	*%*	*Expenditure, kJ/kg*	*%*
Heat supplied = 129.11	100%	(i) Useful work = 96.5	74.63
		(ii) Nozzle friction loss = 6.4	4.95
		(iii) Blade friction loss = 9.7	7.50
		(iv) Windage & disc friction loss = 0.5	0.38
		(v) Exit energy loss = 16.2	12.55
129.11		129.3 kJ/kg, Error = 0.19 kJ/kg	

Error of 0.19 kJ/kg is due to cumulative error creeping in from construction and rounding off measurements from velocity diagram.

9. *Following data refer to the velocity compounded impulse turbine with two rows of moving blades:*

Steam supply: 60 bar, 500°C
Speed of rotation: 3600 rpm
Mean radius of blades: 60 cm
Steam supply: 6 kg/s
Blade speed to steam velocity ratio: 0.2
Nozzle efficiency: 90%
Nozzle angle: 20°
Exit angles: 25°, 25° and 40° for first moving, fixed and second moving blades respectively
Blade velocity coefficient: 0.85 for all blades
Disc friction and Windage loss: 20 hp.

Determine,

(a) the stage pressure
(b) the blading efficiency
(c) the power output
(d) the state of steam leaving stage.

Solution:

Blade speed, $U = r\omega = \dfrac{60 \times 10^{-2} \times 2 \times \pi \times 3600}{60} = 226.2$ m/s

$\rho = 0.2 = \dfrac{U}{C_1}$ so $C_1 = 1131$ m/s

Actual heat drop in stage, $\Delta h'_{01} = \dfrac{C_1^2}{2} = \dfrac{(1131)^2}{2} = 639.58$ kJ/kg

Theoretical heat drop $\Delta h_{01} = \left(\dfrac{\Delta h'_{01}}{0.9}\right) = 710.64$ kJ/kg

Enthalpy at state 1, $h_1 = h_0 - \Delta h_{01}$

Enthalpy at inlet to nozzle, $h_0 = h_{\text{at 60 bar, 500°C}}$

$h_0 = 3422.2$ kJ/kg

Entropy $s_0 = 6.8803$ kJ/kg·K

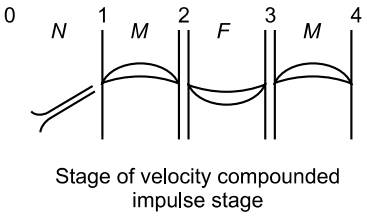


Fig. 14.40

hence, $h_1 = 2711.56$ kJ/kg, $s_1 = s_0 = 6.8803$ kJ/kg·K

From mollies diagram, $P_1 = 3.8$ bar corresponding to h_1 vertically below 60 bar and 500°C state.

Stage pressure = 3.8 bar ***Ans.***

Velocity diagram is drawn using scale of 1 cm = 100 m/s and following,

$U = 226.2$ m/s, $C_1 = 1131$ m/s, $\alpha_1 = 20°$, $\beta_2 = 25°$, $\alpha_3 = 25°$, $\beta_4 = 40°$

$$\frac{V_2}{V_1} = \frac{V_4}{V_3} = 0.85$$

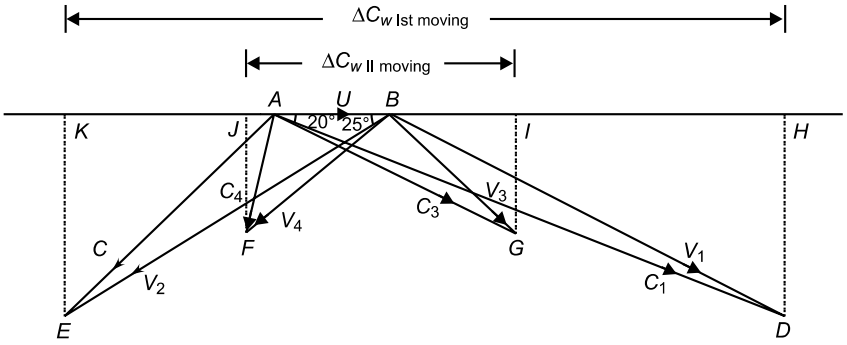


Fig. 14.41

From diagram, $V_1 = 935$ m/s, $V_2 = 794.75$ m/s, $C_2 = 610$ m/s

$$\frac{C_3}{C_2} = 0.85, \Rightarrow C_3 = 0.85 \times C_2 = 518.5 \text{ m/s}$$

$$V_3 = 335 \text{ m/s} \Rightarrow V_4 = 284.75 \text{ m/s}$$

Velocity diagram is drawn in following steps:

(i) Draw AB corresponding to U (= 226.2 m/s)

(ii) Draw AD at angle 20° corresponding to C_1 (= 1131 m/s)

(iii) Join D with B and measure BD (= 9.35 cm) or V_1 = 935 m/s, Estimate $V_2 = 0.85 \times V_1$

(iv) Draw BE corresponding to V_2 = (794.75 m/s) at angle β_2 (= 25°)

(v) Join E with A and measure AE (= 6.1 cm) or C_2 = 610 m/s. For fixed blade $C_3 = 0.85 \times C_2$, Estimate C_3.

(vi) Draw AG at angle α_3 (= 25°) corresponding to C_3 (= 518.5 m/s)

(vii) Join G with B and measure BG (= 3.35 cm) or V_3 = 335 m/s. Estimate $V_4 = 0.85 \times V_3$ for IInd moving blade.

(viii) Draw BF at angle 40° corresponding to V_4 (= 284.75 m/s)

(ix) Join F with A and measure AF (= 2.8 cm) or C_4 = 280 m/s.

Length KH (= 15.6 cm) or $\Delta C_{w \text{ Ist moving}}$ = 1560 m/s

Length JI (= 5.45 cm) or $\Delta C_{w \text{ II moving}}$ = 545 m/s

Total Work done = $(U \cdot \Delta C_{w \text{ Ist moving}}) + (U \cdot \Delta C_{w \text{ II moving}})$ per kg of steam

$$= 226.2 (1560 + 545)$$

$$= 476.15 \text{ kJ/kg}$$

$$\text{Blading efficiency} = \frac{476.15}{639.58} = 0.7444 \text{ or } 74.44\%$$

$$\text{Net power output} = \left(\frac{476.15 \times 6}{0.7457}\right) - 20$$

$$= 3811.16 \text{ hp}$$

$$\text{Energy loss at exit per kg} = \left(\frac{C_4^2}{2}\right) = \frac{(280)^2}{2} = 39.2 \text{ kJ/kg}$$

$$\text{Heat equivalent of net work} = \frac{3811.16 \times 0.7457}{6} = 473.66 \text{ kJ/kg}$$

$$\text{Hence } \Delta h = 473.66 + 39.2 = 512.86 \text{ kJ/kg}$$

Corresponding to enthalpy drop of 512.86 kJ/kg from h_0 the final condition of steam can be seen from mollier diagram. The state is at 225° (superheated steam at 3.8 bar (approximately)

Blading efficiency = 74.44%
Net output = 3811.16 hp
State of steam leaving stage = 225°C

Ans.

10. *Determine blade speed, blade and nozzle angles and work done per kg of steam, ratio of height required for moving blades rows and fixed blade row with respect to nozzle height. Increase in*

volume of steam during expansion in velocity compounded impulse turbine having two moving blade rows may be neglected. Further data required for analysis is given as under,

Velocity of steam leaving nozzle = 500 m/s

Angle at exit of second moving blade = 30° from plane of rotation of blades

Velocity coefficient = 0.90 for each blade.

There is no axial thrust on any of moving blade row and the discharge from stage is axial. The axial velocity in first moving blade is twice that of second moving blade. Solve using velocity diagram.

Solution:

Let us draw the velocity diagram for the stages in backward manner starting from exit of second moving blade and coming gradually to inlet of Ist moving Blade.

Given, $C_1 = 500$ m/s, $\beta_4 = 30°$, $\frac{V_2}{V_1} = \frac{V_4}{V_3} = \frac{C_3}{C_2} = 0.90$

$\alpha_4 = 90°$, $C_{2a} = C_{1a}$ and $C_{2a} = 2 \cdot C_{3a}$

$C_{3a} = C_{4a}$

Steps:

(i) Start with any length *AB* designating *U*, say *AB* = 2 cm

(ii) Draw line *BG* at angle β_4 (= 30°) such that *G* lies vertically below *A*, there is axial discharge from stage. Measure *BG* = 2.7 cm which gives V_4. Estimate V_3, as $V_3 = \frac{V_4}{0.9} = 3$ cm

(iii) Draw a line *GF* parallel to *AB* as $C_{3a} = C_{4a}$. Using compass for 3 cm width mark state *F* with compass needle at *B*. Join *B* and *F*. Measure *AF* (= 4.8 cm) which gives C_3.

Estimate $C_2 = \frac{C_3}{0.9} = 5.33$ cm

(iv) Draw a line *ED* parallel to *AB* as $C_{2a} = 2 \cdot C_{3a}$ i.e. 3.5 cm below *AB* and using compass for 5.33 cm width mark state *E* with compass needle at *A*. Join *E* and *A* to show C_2. Join *E* and *B* which shows V_2. Measure *BE* (= 7 cm) which gives V_2. Estimate $V_1 = \frac{V_2}{0.9} =$ 7.78 cm.

(v) Using compass open for 7.78 cm mark point *D* with compass needle at *B*. Join line *BD* and *AD* which show V_1 and C_1 respectively. Measure length *AD* = 9.6 cm. Thus C_1 of 500 m/s velocity is shown on length of 9.6 cm. Scale comes out, 9.6 cm = 500 m/s, 1cm = 52.08 m/s

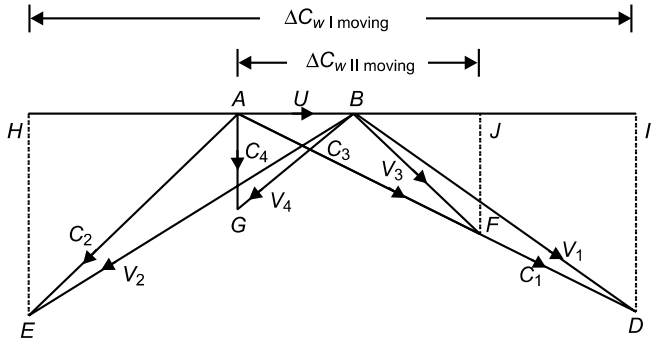


Fig. 14.42

Hence using scale, 1 cm = 52.08 m/s.

So AB = 2 cm $\Rightarrow U = 2 \times 52.08$ m/s, U = 104.16 m/s

$\Delta C_{w\text{I moving}} = HI$ = 13.05 cm = 679.64 m/s

$\Delta C_{w\text{II moving}}$ = 4.55 cm or 236.96 m/s

$AG = C_4 = C_{3a}$ = 1.75 cm = 91.14 m/s

$HE = C_{2a} = C_{1a}$ = 3.5 cm = 182.28 m/s

Angles $\angle BAD = \alpha_1 = 21°$

$\angle IBD = \beta_1 = 26°$

$\angle ABE = \beta_2 = 28.5°$

$\angle HAE = \alpha_2 = 39°$

$\angle BAH = \alpha_3 = 20°$

$\angle ABF = \beta_3 = 33°$

Blade speed =	104.16 m/s
Ist moving blade inlet and exit angle =	26°, 28.5°
Fixed blade inlet and exit angle =	39°, 20°
IInd moving blade inlet angle =	33°
Nozzle angle =	21°

Ans.

Total work done per kg of steam = $U \cdot \Delta C_{w\text{I moving}} + U \cdot \Delta C_{w\text{II moving}}$

= 104.16 (679.64 + 236.96)

= 95473.056 J/kg ≈ 95.47 kJ/kg

Using continuity equation, $\dfrac{\text{Area} \times \text{Flow velocity or axial velocity}}{\text{Specific volume}}$ =constant,

Let height of nozzle first moving, fixed blade and second moving blades be l_n, l_{m1}, l_f, l_{m2} respectively

$$l_n \times C_{a1} = l_{m1} \times C_{a2}$$

$$\frac{l_{m1}}{l_n} = \frac{C_{a1}}{C_{a2}} = \frac{182.28}{182.28} = 1$$

Ratio of height of Ist moving blade to nozzle = 1

$$\text{Ratio of height of fixed blade to nozzle} = \frac{l_f}{l_n} = \frac{C_{a2}}{C_{a2}} = 1$$

Ratio of height of IInd moving blade to nozzle,

$$\frac{l_{m2}}{l_n} = \frac{C_{a1}}{C_{a3}} = \frac{182.28}{91.14} = 2$$

Ratio of height of Ist moving blade to nozzle =	1
Ratio of height of fixed blade to nozzle =	1
Ratio of height of IInd moving blade to nozzle =	2

Ans.

11. *In a Parson's reaction turbine the rotor of 1m diameter runs at 3000 rpm. Determine the isentropic enthalpy drop in the stage considering stage efficiency of 0.80 , ρ = 0.7, blade outlet angle = 20°.*

Solution:

Given, $\rho = \dfrac{U}{C_1} = 0.7$, $\eta_{stage} = 0.80$, $\beta_2 = 20°$

$$U = r\omega = \frac{2 \times \pi \times 3000 \times 1}{60 \times 2} = 157.08 \text{ m/s}$$

or, $C_1 = 224.4$ m/s

Drawing velocity diagram considering given information.

As it is Parson's reaction turbine so $\alpha_1 = \beta_2$ and $\beta_1 = \alpha_2$

Scale 1 cm = 50m/s

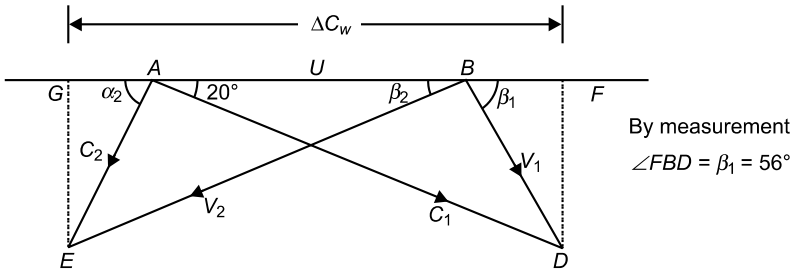


Fig. 14.43

Steps:

(i) Draw *AB* corresponding to U = 157.08 m/s.

(ii) Draw *AD* at angle $\alpha_1 = \beta_2 = 20°$ corresponding to C_1 (= 224.4 m/s).

(iii) Join point *D* with *B* which gives V_1 (= 95 m/s). Measure angle $\angle FBD = \beta_1 = 56°$

(iv) Draw C_2 along *AE* at angle $\alpha_2 = \beta_1 = 56°$. Also draw *EB* at angle 20°. It intersects at *E*. Lines *AE* and *BE* give C_2 and V_2 respectively.

(v) Length *FG* gives ΔC_w (= 260 m/s) of 5.2 cm.

$$\text{Work output (per kg of steam)} = U \cdot \Delta C_w = 157.8 \times 260 = 40840.8 \text{ J/kg}$$

$$\text{State efficiency } \eta_{stage} = 0.80 = \frac{40840.8}{\Delta h_{isentropic}}$$

$$\Delta h_{isentropic} = 51051 \text{ J/kg or } 51.05 \text{ kJ/kg}$$

Enthalpy drop in stage = 51.05 kJ/kg ***Ans.***

12. *A Parson's reaction turbine has mean diameter of blades as 1.6 m and rotor moving at 1500 rpm. The inlet and outlet angles are 80° and 20° respectively. Turbine receives steam at 12 bar, 200°C and has isentropic heat drop of 26 kJ/kg. 5% of steam supplied is lost through leakage. Determine the following considering horse power developed in stage to be 600 hp.*

(a) the stage efficiency

(b) the blade height.

Solution:

Given: 50% reaction turbine with $\beta_1 = 80°$ & $\beta_2 = 20°$

$N = 1500$ rpm, $d = 1.6$ m $\Rightarrow \alpha_2 = 80°$ and $\alpha_1 = 20°$

$$U = r\omega = \frac{1.6 \times 2 \times \pi \times 1500}{2 \times 60} = 125.66 \text{ m/s}$$

Velocity diagram for turbine stage is drawn as below using above data.

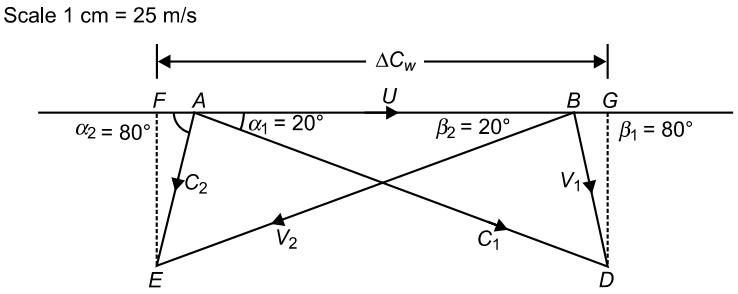


Fig. 14.44

From velocity diagram, Length $AD = C_1 = 168.75$ m/s

Length $FG = \Delta C_w = 170$ m/s

Length $FE = C_{2a} = 52.5$ m/s $= C_{1a}$

Work done $= U \cdot \Delta Cw = 125.66 \times 170 = 21362.2$ J/kg or 21.36 kJ/kg per kg of steam.

Since 5% of steam gets leaked so only 95% of steam supplied will be used for production of work

Net work per kg of steam $= 0.95 \times 21.36 = 20.29$ kJ/kg

$$\text{Stage efficiency} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{20.29}{26} = 0.7804 \text{ or } 78.04\%$$

Let mass of steam supplied be m kg per second to produce 600 hp.

$0.7457 \times 600 = m \times 20.29$

$m = 22.05$ kg/s

Assuming heat drop per stage to be small so that the specific volume remains nearly constant throughout in the stage.

Specific volume at inlet from steam table, $v_1 = v_{\text{at 12 bar, 200°c}}$

$v_1 = 0.16930$ m^3/kg

Let height of blades be 'h'.

Volume of steam flow rate $= \pi \times d \times h \times C_{1a}$

$22.05 \times 0.16930 = \pi \times 1.2 \times h \times 52.5$

$h = 0.01886$ m or 1.89 cm

Stage efficiency = 78.04%
Height of blade = 1.89 cm

Ans.

13. *In a reaction turbine 6 kg/s steam is admitted at 15 bar dry saturated in the first stage. Turbine has eight pairs on mean diameter of 50 cm and run at 3000 rpm with mean blade speed to steam velocity*

ratio of 0.8. There occurs tip leakage of steam at all rows amounting to 10% of total and efficiency of working steam is 85%. Considering blade outlet angles for both fixed and moving blades to be 20°, determine the following analytically.

(i) the output from turbine in hp.
(ii) the pressure of steam leaving turbine,
(iii) the mean blade height.

Solution:

Let the degree of reaction be 50% for the reaction turbine.

Mean blade speed $$U = \frac{\pi \times 50 \times 10^{-2} \times 2 \times 3000}{2 \times 60} = 78.54 \text{ m/s}$$

Steam velocity, $$C_1 = \frac{U}{\rho} = \frac{78.54}{0.8} = 98.18 \text{ m/s}$$

Angles, $\alpha_1 = 20°$, $\beta_2 = 20°$

$$\Delta C_w = 2C_1 \cos \alpha_1 - U$$
$$= \{(2 \times 98.18 \times \cos 20) - 78.54)\}$$
$$\Delta C_w = 105.98 \text{ m/s}$$

Actual mass of steam flowing through rows, $m_{\text{actual}} = 6 \times 0.90 = 5.4$ kg/s

$$\text{Power output} = U \cdot \Delta C_w \times m_{\text{actual}} \times \text{no. of pairs}$$
$$= \frac{78.54 \times 105.98 \times 5.4 \times 8 \times 10^{-3}}{0.7454}$$
$$= 482.4 \text{ hp}$$

Output from turbine = 482.4 hp ***Ans.***

Theoretical heat drop in turbine can be obtained using efficiency of working steam,

$$\Delta h = \frac{482.4 \times 0.7454}{6 \times 0.85} = 70.51 \text{ kJ/kg}$$

At inlet to steam turbine the enthalpy may be seen from steam table as,

$$h_1 = h_{\text{at 15 bar, dry saturated}} = h_{\text{g at 15 bar}} = 2792.2 \text{ kJ/kg}$$
$$s_1 = s_{\text{g at 15 bar}} = 6.448 \text{ kJ/kg·K}, \; v_1 = 0.13177 \text{ m}^3\text{/kg}$$

Hence, enthalpy at exit of turbine

$$h_{\text{exit}} = h_1 - \Delta h$$
$$= 2792.2 - 70.51$$
$$h_{\text{exit}} = 2721.69 \text{ kJ/kg}$$

Pressure at exit of turbine can be approximated from the mollier diagram corresponding to h_{exit} and s_{exit} (= s_1) vertically below state 1. Pressure at exit = 12 bar (approx.)

Pressure at exit = 12 bar ***Ans.***

Dryness fraction at exit of turbine, is obtained using entropy values,

$$s_1 = s_{\text{exit}}$$

$$= s_{f\ at}\ 12\ bar + x_{exit} \times s_{fg\ at\ 12\ bar}$$

$$6.448 = 2.2166 + x_{exit} \times 4.3067$$

$$x_{exit} = 0.98$$

$$v_{exit} = v_{f\ at\ 12\ bar} + x_{exit},\ v_{fg\ at\ 12\ bar}$$

$$= 0.001139 + 0.98 \times (0.16333 - 0.001139)$$

$$v_{exit} = 0.16009\ m^3/kg$$

$$\text{Mean specific volume} = \frac{v_1 + v_{exit}}{2} = \frac{(0.13177 + 0.16009)}{2}$$

$$= 0.14593\ m^3/kg$$

$$\text{Steam mass flow rate} = 6 \times 0.92 \times 0.14593 = \frac{\pi \times 50 \times h \times C_{1a}}{100}$$

$$\text{Axial component of velocity } C_{1a} = C_1 \sin \alpha_1$$

$$= 98.18 \sin 20$$

$$C_{1a} = 33.58\ m/s$$

This axial component of velocity, C_{1a} is approximated as average flow velocity

Substituting C_{1a}, h is estimated as under

$$h = \frac{6 \times 0.92 \times 0.14593 \times 100}{\pi \times 50 \times 33.58} = 0.0153\ m$$

Mean blade height = 1.53 cm ***Ans.***

14. *A reaction turbine has mean blade speed of 180 m/s, blade speed to steam velocity ratio of 0.8, outlet angles of fixed and moving blades as 20° and 30°, specific volume at outlet of fixed blade as 0.5 m^3 and at moving blade outlet as 0.6 m^3. Areas at exit of fixed blade and moving blades are same, Consider the efficiency of blades as 90% when considered as nozzles and $K^2 = 0.88$ where K is blade velocity coefficient. Determine*

(i) the degree of reaction,

(ii) adiabatic heat drop,

(iii) the overall stage efficiency.

Solution:

Given: $\alpha_1 = 20°$, $\beta_2 = 30°$, $\rho = 0.8$, $U = 180$ m/s.

$$\text{Steam velocity } C_1 = \frac{\text{Mean blade speed}}{\text{Blade speed to steam velocity ratio}}$$

$$= \frac{180}{0.8}$$

$$C_1 = 225\ m/s$$

$$\text{Axial velocity component, at inlet } C_{1a} = C_1 \sin \alpha_1$$

$$= 225 \sin 20$$

$$C_{1a} = 76.95\ m/s$$

From continuity equation, the steam mass flow rate at exit of fixed blades and moving blades shall be same, so

$$\frac{C_{1a}}{v_{\text{at exit of fixed blades}}} = \frac{C_{2a}}{v_{\text{at exit of moving blades}}}$$

$$\frac{76.95}{0.5} = \frac{C_{2a}}{0.6}$$

$$C_{2a} = 92.34 \text{ m/s}$$

From geometry of velocity diagrams,

$$C_{2a} = V_2 \sin \beta_2$$

$$V_2 = \frac{C_{2a}}{\sin \beta_2} = \frac{92.34}{\sin 30}$$

$$V_2 = 184.68 \text{ m/s}$$

$$V_1^2 = C_1^2 + U^2 - 2C_1 U \cos \alpha_1$$
$$= (225)^2 + (180)^2 - (2 \times 225 \times 180 \times \cos 20)$$
$$= 6309.9$$

$$\text{Heat drop in fixed blades} = \left\{\frac{C_1^2 - K^2 C_2^2}{2 \times \eta_N}\right\}$$

$$\text{Heat drop in moving blades} = \left\{\frac{V_2^2 - K^2 V_1^2}{2 \times \eta_N}\right\}$$

Here nozzle efficiency is given as 0.9 for both fixed and moving blades.

$$C_2^2 = V_2^2 + U^2 - 2UV_2 \cos \beta_2$$
$$= (184.68)^2 + (180)^2 - (2 \times 180 \times 184.68 \times \cos 30)$$
$$C_2^2 = 8929.18$$
$$C_2 = 94.49 \text{ m/s}$$

$$\text{Heat drop in fixed blades} = \left\{\frac{(225)^2 - (0.88 \times 8929.18)}{2 \times 0.9}\right\}$$
$$= 23.76 \text{ kJ/kg}$$

$$\text{Heat drop in moving blades} = \left\{\frac{(184.68)^2 - (0.88 \times 6309.9)}{2 \times 0.9}\right\}$$
$$= 15.86 \text{ kJ/kg}$$

$$\text{Therefore, degree of reaction} = \frac{\text{Heat drop in moving blades}}{\text{Total heat drop}}$$
$$= \frac{15.86}{(23.76 + 15.86)}$$
$$= 0.4003 \text{ or } 40.03\%$$

Degree of reaction = 40.03% ***Ans.***

Adiabatic heat drop in one pair of blades = (23.76 + 15.86)
= 39.62 kJ/kg.

Heat drop in a pair of blades = 39.62 kJ/kg **Ans.**

Work done = $U \cdot \Delta C_w$

$= U \cdot (V_2 \cos \beta_2 + C_1 \cos \alpha_1 - U)$

$= 180 \times (184.68 \cos 30 + 225 \cos 20 - 180)$

$= 34446.31$ J/kg $= 34.45$ kJ/kg

Stage efficiency $= \dfrac{34.45}{39.62} = 0.8695$ or 86.95%

Stage efficiency = 86.95% **Ans.**

15. *A multi stage steam turbine has steam entering at 20 bar, 300°C and leaving at 0.05 bar and 0.95 dry. Determine reheat factor, condition of steam at exit from each of five stages considering efficiency ratio of 0.555 and all stages doing equal work.*

Solution:

At inlet to turbine, the enthalpy $h_1 = h_{\text{at 20 bar, 300°C}}$

$h_1 = 3023.5$ kJ/kg

$s_1 = 6.7664$ kJ/kg·K

Enthalpy at exit, $h_6 = h_{\text{f at 0.05 bar}} + x \times h_{\text{fg at 0.05 bar}}$

$= 137.82 + (0.95 \times 2423.7)$

$h_6 = 2440.34$ kJ/kg

$h_1 - h_6 = 583.16$ kJ/kg = Actual heat drop

Given: Stage efficiency or efficiency ratio = 0.55

Since all the stages do equal work, so the useful enthalpy drop in each of stage shall be equal.

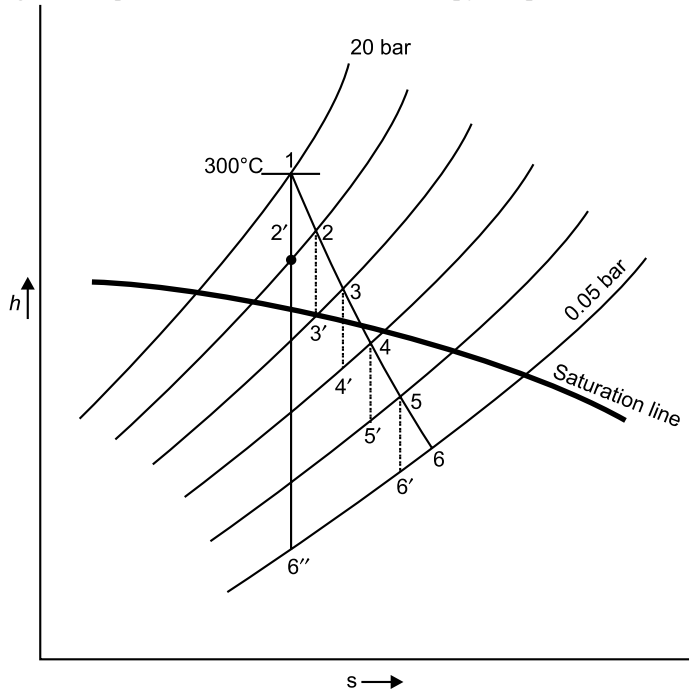


Fig. 14.45

$$\text{Enthalpy drop in each stage} = \left(\frac{h_1 - h_6}{5}\right) = 116.63 \text{ kJ/kg}$$

$$\text{For first stage, } h_1 - h_2 = 116.63 \text{ kJ/kg}$$

$$\text{Stage efficiency} = \frac{h_1 - h_2}{h_1 - h_{2'}}$$

$$h_1 - h_{2'} = \frac{116.63}{0.555}$$

$$= 210.05 \text{ kJ/kg}$$

$$\text{For second stage, } h_2 - h_3 = 116.63 \text{ kJ/kg}$$

$$\text{Stage efficiency} = 0.555 = \frac{h_2 - h_3}{h_2 - h_{3'}}$$

$$h_2 - h_{3'} = \frac{116.63}{0.555} = 210.05 \text{ kJ/kg}$$

Similarly, for third stage $h_3 - h_{4'} = 210.05$ kJ/kg

for fourth stage, $h_4 - h_{5'} = 210.05$ kJ/kg

for fifth stage, $h_5 - h_{6'} = 210.05$ kJ/kg

After isentropic heat drop up to state 6″, $s_1 = s_{6''} = 6.7664$ kJ/kg·K

$$s_{6''} = s_{\text{f at 0.05 bar}} + x_{6''} \cdot s_{\text{fg at 0.05 bar}}$$

$$x_{6''} = \left(\frac{6.7664 - 0.4764}{7.9187}\right)$$

$$x_{6''} = 0.7943$$

$$\text{Enthalpy, } h_{6''} = h_{\text{f at 0.05 bar}} + x_{6''} \cdot h_{\text{fg at 0.05 bar}}$$

$$= 137.82 + (0.7943 \times 2423.7)$$

$$h_{6''} = 2062.96 \text{ kJ/kg}$$

$$\text{Reheat factor, RF} = \frac{\text{Cumulative heat drop}\left(\begin{array}{l}=(h_1 - h_{2'}) + (h_2 - h_{3'}) \\ +(h_3 - h_{4'}) + (h_4 - h_{5'}) + (h_5 - h_{6'})\end{array}\right)}{\text{Adialatic heat drop}\,(= h_1 - h_{6''})}$$

$$= \frac{5 \times 210.05}{(3023.5 - 2062.96)}$$

RF = 1.09

Reheat factor = 1.09 ***Ans.***

Plotting the actual heat drop shown by 1-6 on Mollier diagram state of steam leaving each stage can be given as under.

No. of stage	1	2	3	4	5
Pressure at exit of stage, bar	6.9	2.6	0.8	0.2	0.05
Condition of steam	235°C	160°C	dry sat.	0.975 dryness factor	0.95 dryness factor

Ans.

16. *In a five stage turbine the steam is supplied at 35 bar, 450°C and leaves at 0.07 bar, 0.89 dry. Determine the following for equal work between the stages:*

(i) the state of steam at inlet to each stage,
(ii) the stage efficiency or efficiency ratio for all stages,
(iii) the reheat factor,
(iv) the overall turbine efficiency.

Solve using Mollier diagram.

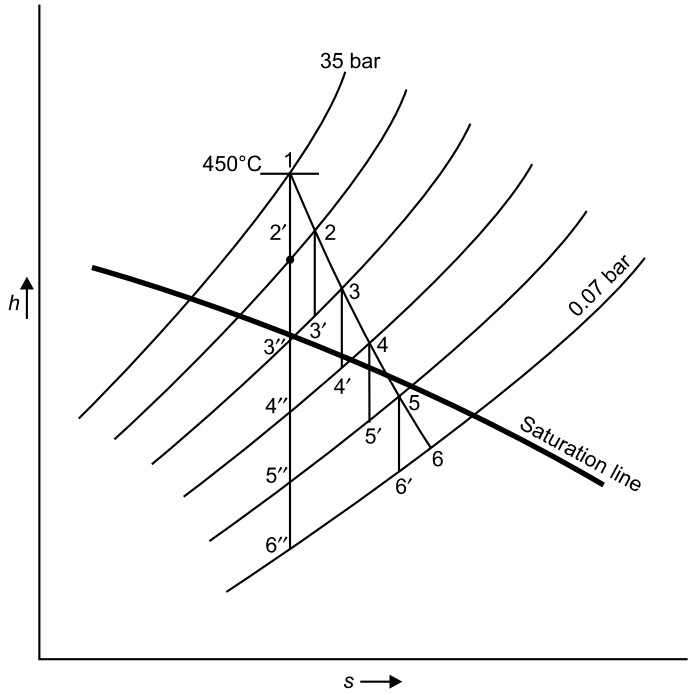


Fig. 14.46

Solution:

From mollier diagram,

$h_1 - h_6 = 1028$ kJ/kg

Considering equal work the enthalpy drop in each stage $= \dfrac{h_1 - h_6}{5}$

$= 205.6$ kJ/kg

Plotting the condition line as straight line between 1 and 6, the following states are obtained at inlet of each stage

No. of stage	1	2	3	4	5
Pressure, at inlet bar	35	14.5	5.5	1.7	0.35
State of steam	450°C	342°C	235°C	125°C	0.95

The enthalpy drops as evident from mollier diagrams,

$h_1 - h_{2'} = 264$ kJ/kg, $h_2 - h_{3'} = 246$ kJ/kg, $h_3 - h_{4'} = 240$ kJ/kg,

$h_4 - h_{5'} = 240$ kJ/kg, $h_5 - h_{6'} = 235$ kJ/kg
and $h_1 - h_2 = h_2 - h_3 = h_3 - h_4 = h_4 - h_5 = h_5 - h_6 = 205.6$ kJ/kg

Stage efficiency for different stages, $\eta_{\text{stage,I}} = \dfrac{h_1 - h_2}{h_1 - h_{2'}} = \dfrac{205.6}{264} = 0.7788$

$$\eta_{\text{stage,II}} = \frac{h_2 - h_3}{h_2 - h_{3'}} = \frac{205.6}{246} = 0.8358$$

$$\eta_{\text{stage,III}} = \frac{h_3 - h_4}{h_3 - h_{4'}} = \frac{205.6}{240} = 0.8567$$

$$\eta_{\text{stage,IV}} = \frac{h_4 - h_5}{h_4 - h_{5'}} = \frac{205.6}{240} = 0.8567$$

$$\eta_{\text{stage,V}} = \frac{h_5 - h_6}{h_5 - h_{6'}} = \frac{205.6}{235} = 0.8749$$

Stage efficiencies = 77.88%, 83.58%, 85.67%, 85.67%, 87.49% ***Ans.***

From mollier chart. $h_1 - h_{6''} = 1162$ kJ/kg

$$\text{Reheat factor , RF} = \frac{(h_1 - h_{2'}) + (h_2 - h_{3'}) + (h_3 - h_{4'}) + (h_4 - h_{5'}) + (h_5 - h_{6'})}{(h_1 - h_{6''})}$$

$$= \frac{(264 + 246 + 240 + 240 + 235)}{1162}$$

$$\text{RF} = 1.05$$

Ans. Reheat factor = 1.05

$$\text{Overall efficiency, } \eta = \frac{(h_1 - h_6)}{(h_1 - h_{6''})} = \frac{1028}{1162} = 0.8847$$

Overall efficiency = 88.47% ***Ans.***

17. *One stage of an impulse turbine consists of a converging nozzle ring and one ring of moving blades. The nozzles are inclined at 22° to the blades whose tip angles are both 35°. If the velocity of steam at exit from the nozzle is 660 m/s, find the blade speed so that the steam passes without shock. Find diagram efficiency neglecting losses if the blades are run at this speed.*

[U.P.S.C., 1992]

Solution:

Diagram efficiency of impulse turbine, when losses are neglected, shall be equal to the maximum diagram efficiency obtained for optimum blade speed-steam velocity ratio (ρ).

$$\eta_{\text{diag, max}} = \frac{\cos^2 \alpha_1}{2} (1 + K \cdot C)$$

Considering turbine to have smooth and symmetrical blades i.e. $K = 1, C = 1$ and Blade angles, $\beta_1 = \beta_2$

Nozzle angle = $\alpha_1 = 22°$

$$\eta_{\text{diag, max}} = \cos^2 \alpha_1 = 0.8597 \text{ or } 85.97\%$$

$$\text{Also } \rho_{\text{optimum}} = \frac{\cos \alpha_1}{2} = \frac{U}{C_1}$$

Velocity of steam leaving nozzle is C_1 and given as 660 m/s.

$$\frac{\cos 22}{2} = \frac{U}{660}$$

$$U = 305.97 \text{ m/s}$$

Blade speed = 305.97 m/s Diagram efficiency = 85.97%	***Ans.***

18. *A simple impulse turbine has a mean blade speed of 200 m/s. The nozzles are inclined at 20° to the plane of rotation of the blades. The steam velocity from nozzles is 600 m/s. The turbine uses 3500 kg/hr of steam. The absolute velocity at exit is along the axis of the turbine. Determine (i) the inlet and exit angles of the blades, (ii) the power output of the turbine, (iii) the diagram efficiency, and (iv) the end thrust (per kg steam per second) and its' direction.*

[U.P.S.C., 1998]

Solution:

Given, U = 200 m/s, α_1 = 20°, C_1 = 600 m/s, α_2 = 90°, m = 3500 kg/hr

scale: 1 cm = 50 m/s

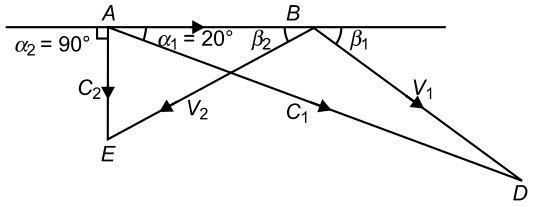


Fig. 14.47

From geometry, $C_1 \sin \alpha_1 = 600 \sin 20 = 205.2$ m/s

$$\tan \beta_1 = \frac{C_1 \sin \alpha_1}{C_1 \cos \alpha_1 - U}$$

$$\beta_1 = \tan^{-1}\left(\frac{205.2}{(600.\cos 20 - 200)}\right)$$

$$\beta_1 = 29.42°$$

For symmetrical blade, $\beta_1 = \beta_2 = 29.42°$

Blade inlet angle = Blade exit angle = 29.42°	***Ans.***

$$\text{Power output} = m \cdot U \cdot \Delta C_w$$

$$= \frac{3500 \times 200 \times 600 \cdot \cos 20}{1000 \times 60 \times 60}$$

$$= 109.63 \text{ kW}$$

Power output = 109.63 kW	***Ans.***

$$\text{Diagram efficiency} = \frac{(m \cdot U \cdot \Delta C_w)}{\left(m \cdot \frac{C_1^2}{2}\right)}$$

$$= \frac{2U \cdot C_1 \cos\alpha_1}{C_1^2}$$

$$= \frac{2U \cdot \cos\alpha_1}{C_1}$$

$$= \frac{2 \times 200 \times \cos 20}{600}$$

$$= 0.6265$$

Diagram efficiency = 62.65% ***Ans.***

From velocity diagram, $V_2 \cos \beta_2 = U$, $V_2 = \frac{U}{\cos\beta_2} = \frac{200}{\cos 29.42} = 229.61$ m/s

End thrust = $(C_2 - C_1 \sin \alpha_1)$

= $(V_2 \sin \beta_2 - C_1 \sin \alpha_1)$

= (229.61 (sin 29.42) – 600 sin 20)

= – 92.43 N/kg·s

End thrust = 92.43 N/kgs in reverse direction ***Ans.***

EXERCISE

14.1 Describe the principle of steam turbine operation.

14.2 Classify the steam turbines and differentiate between steam turbine and steam engines.

14.3 Differentiate between impulse and reaction turbines.

14.4 What do you understand by compounding of steam turbines? Describe different types of compounding of steam turbines.

14.5 Describe the velocity diagram for single stage impulse turbine. Also obtain the expressions for force, work done, diagram efficiency, gross stage efficiency and axial thrust.

14.6 Obtain the condition for maximum blade efficiency in single stage impulse turbine. Also show how this efficiency varies with blade speed to steam velocity ratio.

14.7 Compare different types of compounding of steam turbines with one another.

14.8 Sketch the velocity diagram of a two stage velocity compounded turbine.

14.9 Obtain the optimum blade speed to steam velocity ratio for two stage velocity compounded impulse turbine. Also plot the variation of diagram efficiency with blade speed to steam velocity ratio.

14.10 What do you understand by axial discharge turbine?

14.11 Discuss the significance of "whirl component of velocity" and "flow component or axial component of velocity" in steam turbine.

14.12 Describe the working of reaction turbine. Explain, why pure reaction turbine is not used in practice.

14.13 How does reaction turbine blades differ from impulse turbine.

14.14 Discuss different losses in steam turbines.

14.15 Sketch the velocity diagram for a single stage reaction turbine having 50% degree of reaction.

14.16 Obtain the condition for maximum diagram efficiency of a reaction turbine in terms of absolute angle at inlet to moving blade.

14.17 What is meant by reheat factor? Also discuss its' relevance with the help of *T–s* and *h–s* diagrams

14.18 Describe the governing in steam turbines. Give different methods for governing in steam turbines.

14.19 Differentiate amongst different methods of governing of steam turbine.

14.20 Give the reason for following statement;

"Velocity compounded impulse stages are generally placed at the high pressure inlet end of steam turbine".

14.21 Show that the reheat factor in steam turbine having steam entering dry and saturated at absolute temperature T_1, and expanding up to the absolute temperature T_2 with stage efficiency η_s, shall be

$$\text{given as, RF} = \frac{T_1 + T_2}{2T_2 + \eta_s(T_1 - T_2)}$$

The number of stages may be considered infinite and the condition line is straight line.

14.22 A steam turbine stage is supplied with steam at 50 bar, 350°C and leaves turbine at pressure of 5 bar. Determine steam supply rate if the isentropic efficiency is 82% and power output is 15224 kW [37.8 kg/s]

14.23 A De Laval steam turbine has steam leaving nozzle with velocity of 1200 m/s at angle of 20° and mean blade speed of 400 m/s. Considering blades to be symmetrical, blade velocity coefficient of 0.8 and steam flowing at rate of 1000 kg/hr, determine the following using velocity diagram,

(a) the blade angle
(b) the power developed
(c) the relative velocity of steam entering blades
(d) the blade efficiency.

[30°, 30°, 145.5 kW, 830 m/s, 72.8%]

14.24 Determine the turbine output from a single stage impulse turbine having smooth blades, steam flow rate of 20 kg/s, steam velocity of 600 m/s, blade speed of 250 m/s, nozzle angle of 20° and blade outlet angle of 25°. Also calculate axial thrust. [3275 kW, 800N]

14.25 Determine nozzle angle, blade angles at inlet and exit for a single stage impulse steam turbine developing 132 kW, speed of 3340 rpm, mean rotor diameter of 1 m, steam flow rate of 2 kg/s, blade velocity coefficient of 0.9, steam leaving nozzle at 400 m/s and steam discharges axially from turbine. Solve using velocity diagram. [21°, 36°, 32°]

14.26 De-Laval turbine has nozzle angle of 20°, symmetrical blades and blade velocity coefficient of 0.85. Determine maximum blade efficiency. Also find out the blade speed to steam velocity ratio if the actual blade efficiency is 92% of maximum blade efficiency. [81.6%, 0.6 or 0.34]

14.27 What shall be the nozzle angle for a De-Laval turbine offering maximum possible efficiency of 90%? [18.44°]

14.28 A stage of simple impulse turbine has 10 kg/s steam entering the rotor having mean diameter of 105 cm, running at 50 r.p.s.. Nozzle angle is 18°, blade speed to steam velocity ratio is 0.42, blade velocity coefficient is 0.84 and the exit angle of blade 3° less than inlet angle. Draw the velocity diagram and obtain (a) the axial thrust on blades, (b) the tangential thrust on blades, (c) the resultant thrust on blades (d) the power developed. [250 N, 3900 N, 3908 N, 641.5 kW]

14.29 A singe stage impulse steam turbine has mean diameter of rotor as 1 m and runs at 3000 rpm. Steam leaves nozzle at 20° with velocity of 300 m/s. Determine the power developed if the axial thrust on

blades is 98 N, blades are symmetrical and there occurs 19% frictional loss of kinetic energy of relative velocity at inlet to blades. [181.2 kW]

14.30 Single row impulse turbine has blade speed of 175 m/s and steam leaves nozzle at 400 m/s. Steam flows at the rate of 163.2 kg/min and leaves turbine axially. Considering blade velocity coefficient of 0.9 determine nozzle angle, blade angles at inlet and exit, axial thrust, energy loss at exit, energy loss in blades and diagram efficiency if turbine produces 180 kW output.

[19°, 32.6°, 36.3°, 4.35 N, 8.27 kJ/kg, 5.53 kJ/kg. 41.36%]

14.31 Following data refer to a stage of velocity compounded impulse steam turbine;

Nozzle angle: 16°

Angle at exit of first moving blade: 20°

Angle at exit of fixed blade: 25°

Angle at exit of second moving blade: 30°.

Velocity of steam leaving nozzle: 800 m/s

Blade speed: 150 m/s

Blade velocity coefficient for first moving, fixed and second moving blades: 0.8, 0.85 and 0.85.

Steam flow rate: 5 kg/s

Determine, (a) diagram efficiency, (b) the energy carried at exit, (c) the axial thrust on each moving blade ring, (d) the power developed. [66.2%, 3.79 kJ/kg, 20.9 N, 27 N, 1059 kW]

14.32 A two stage velocity compound impulse steam turbine has mean blade speed of 150 m/s, steam leaves nozzle with 675 m/s velocity at angle of 20°. The exit angles of first row moving blade, fixed blade and second row moving blade are 25°, 25° and 30° respectively. For the blade velocity coefficient of 0.9 for all blades and flow rate of4.5 kg/s determine,

(a) the power output, (b) the diagram efficiency.

[80.7 kW, 78.5%]

14.33 A simple impulse turbine has nozzles at angle of 20° to the direction of motion of moving blades and steam leaves nozzle at 375 m/s, blade velocity coefficient is 0.85, steam flow rate is 10 kg/s and blade speed is 165 m/s. Determine,

(a) Inlet and outlet angles for blades such that there is no axial thrust.

(b) The power output.

[34.4°, 41.6°, 532 kW]

14.34 A Parson's reaction turbine has blade speed of 78.5 m/s, stage efficiency of 80%, blade speed to steam velocity ratio of 0.7 and blade outlet angle of 20°. Estimate the isentropic enthalpy drop in the stage. [12.32 kJ/kg]

14.35 In a 50% reaction turbine the inlet and exit angles are 80° and 20° respectively. Blade speed is 113 m/s and steam is admitted in turbine at 12 bar, 200°C and leaves after adiabatic enthalpy drop of 17.5 kJ/kg with only 95% of heat drop being utilized for producing work of 620 hp. Calculate the stage efficiency and blade speed. [78.5%, 2.29 cm]

14.36 A reaction turbine has mean blade speed of 220 m/s and blade speed to steam velocity ratio of 0.7. Determine the angle at inlet of blade and work done per kg of steam flow if angle at exit is 20°.

Also determine the percentage increase in diagram efficiency if turbine is run for optimum blade speed to steam velocity ratio.

[55°, 81.5 kJ/kg, 93.8%]

14.37 A reaction turbine has eight pairs with mean blade speed of 68 m/s and average blade speed to steam velocity ratio of 0.8. Steam at 15 bar, dry saturated is supplied at the rate of 300 kg/min.

Angle at exit of fixed and moving blade is 20°. Determine (a) the horse power, (b) the average blade height. [312.6 hp, 1.35 cm]

14.38 In a four stage velocity compounded impulse turbine, steam is supplied at 35 bar, 420°C and leaves at 0.07 bar. Obtain the initial pressure, quality at inlet of each stage and internal efficiency of turbine if the average efficiency is 70%. Also find reheat factor.

[11.3 bar, 304°C, 2.9 bar, 189°C, 0.55 bar, 0.99, 74.6% 1.07]

14.39 A reaction turbine stage has steam leaving fixed blade at 3 bar, 0.94 dry and velocity of 143 m/s. Ratio of axial velocity of flow to blade velocity is 0.7 at entry and 0.75 at exit from moving blades. Angles at exit of fixed and moving blades are same. The height of blade is 1.8 cm and steam flow is 150 kg/min. For blade velocity of 70 m/s determine the degree of reaction.

[58.2%]

14.40 A three stage turbine has steam entering at 30 bar, 350°C and leaves first, second and third stages at 7 bar, 1 bar and 0.1 bar respectively. For equal stage efficiency of 70% determine (a) the final condition of steam, (b) the reheat factor, (c) the overall thermal efficiency.

[0.94, 1.05, 24.5%]

14.41 A four stage turbine has steam supplied at 20 bar, 350°C and leaves at 0.05 bar. Considering overall efficiency to be 80% and equal work from the stages and straight condition line determine, (a) the steam pressure at exit of each stage, (b) the each stage efficiency, (c) the reheat factor.

[9 bar, 2.4 bar, 0.5 bar, 0.05 bar, 72.5%, 76%, 78.4%, 82.5%, 1.04]

15
Steam Condenser

15.1 INTRODUCTION

Condenser is one of the essential components of steam power plants as it facilitates condensation of steam at given conditions with minimum expenditure of energy and minimum loss of heat and finally gives condensate which can be recirculated by feed pump to boiler for steam generation. Condenser generally operates at pressure less than atmospheric pressure. In the steam power plant the use of condenser permits expansion in steam turbine even upto less than atmospheric pressure and subsequently condensing steam to yield condensate for recirculation thus improving plant efficiency and output. Expansion in steam turbine/engine can not be extended to pressures less than atmospheric in the absence of condenser.

"Condenser can be defined as device used for condensation of steam at constant pressure; generally pressure is less than atmospheric pressure". Condenser is thus a closed vessel which is generally maintained at vacuum and cold fluid is circulated for picking heat from steam to cause its condensation. Use of Condenser offers advantages such as hotter feed water for being sent to boiler', 'removal of air and non condensable dissolved gases from feed water', 'recovery of condensate reduces treated water requirement', 'expansion upto subatmospheric conditions and capital cost is reduced by recycling of feed water' etc. Increase in expansion work due to use of condenser is shown in Fig. 15.1 on *p-V* diagram.

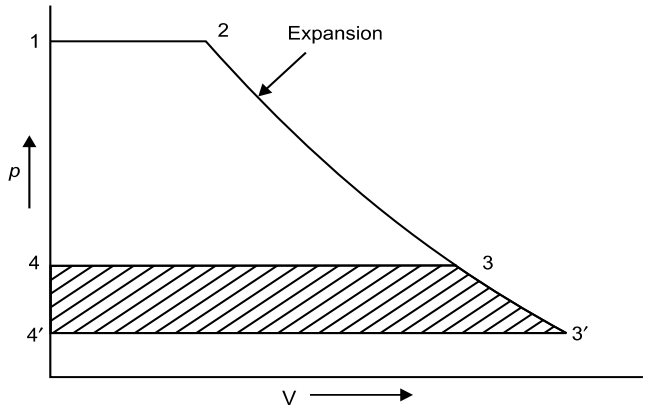


Fig. 15.1 p-V *diagram showing how condenser increases work output in steam engine*

Steam power plant employing condenser and the condensing plant are shown in Fig. 15.2.

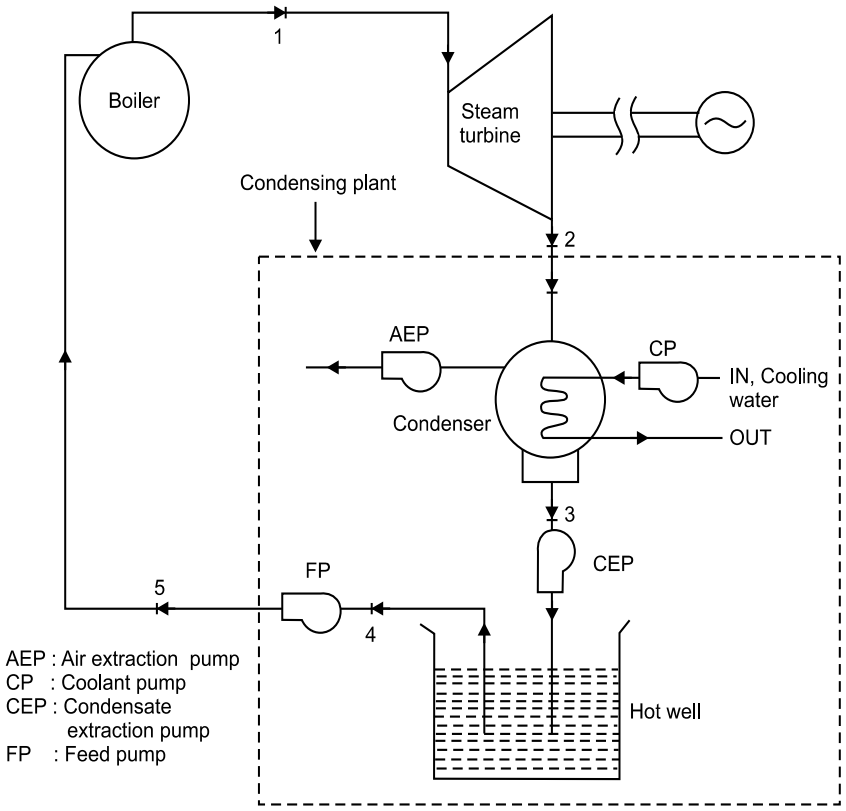


Fig. 15.2 *Schematic for steam power plant having condensing plant*

Discharge from steam turbine passes into condenser where it is condensed using cooling water being circulated employing coolant pump. Condensate being at pressure less than atmospheric pressure is to be sucked out using condensate extraction pump. Condensate is extracted and sent to hot well from where it is pumped to boiler using feed pump. Dissolved gases and air etc. if any are extracted out from condenser using air extraction pump. This air or vapour may be present because of air leaking into vacuum system and air coming with steam. Cooling water for supply to condenser is taken either from some river or from cooling tower. Cooling water requirement may be up to 100 kg water per kg of steam or even more depending upon the type of condenser and its capacity. Cooling tower cools the hot cooling water leaving condenser to get cooled by evaporation of water and heat exchange with air.

Water evaporated or lost in cooling tower is compensated by the make up treated water available from feed water treatment plant.

15.2 CLASSIFICATION OF CONDENSER

Condenser can be broadly classified on the basis of type of heat exchange i.e. direct or indirect contact condensers.

(i) Direct contact type or Mixing type or Jet condenser
(ii) Indirect contact type or Non-mixing type or Surface condenser
(iii) Evaporative condenser

Jet condensers have direct contact between steam and cooling fluid thereby causing contamination of condensate. Surface condensers have indirect heat exchange through metal interface and the two

fluids do not come in direct contact to each other. Evaporative condensers use evaporation of water for heat extraction and is well suited for dry weather so that evaporation is not difficult. Due to direct contact of two fluids the circulating water requirement is much less in jet condenser as compared to other types of condensers. Space requirement and size of condenser etc. are also less with jet condensers. Surface condenser is advantageous over direct contact type condensers because any type of cooling fluid can be used in it and also there is no scope of contamination etc. Different types of condensers are discussed ahead.

(i) ***Jet condenser:*** In jet condenser the steam to be condensed and cooling water are intimately mixed by breaking up of water in the form of spray and allowing small sized water particles to fall down through the body of steam. The water may also be discharged out through suitably shaped nozzles into body of steam. Thus it is desired to atomize water into small sized particles so that increased surface area is available for heat exchange between hot and cold fluid. Number of arrangements for flow of steam and water are available such as; counter flow type having steam entering from bottom and flowing upwards while water enters from top and falls downwards with air pump connected on top where air is colder etc. Jet condenser may be further classified based on relative movement of two fluids, and based on arrangement used for removal of condensate.

Based on relative moment of two fluids jet condenser can be,

(a) Counter flow jet condenser
(b) Parallel flow jet condenser

Based on arrangement for removal of condensate jet condenser can be,

(a) Low level jet condenser
(b) High level jet condenser
(c) Ejector condenser

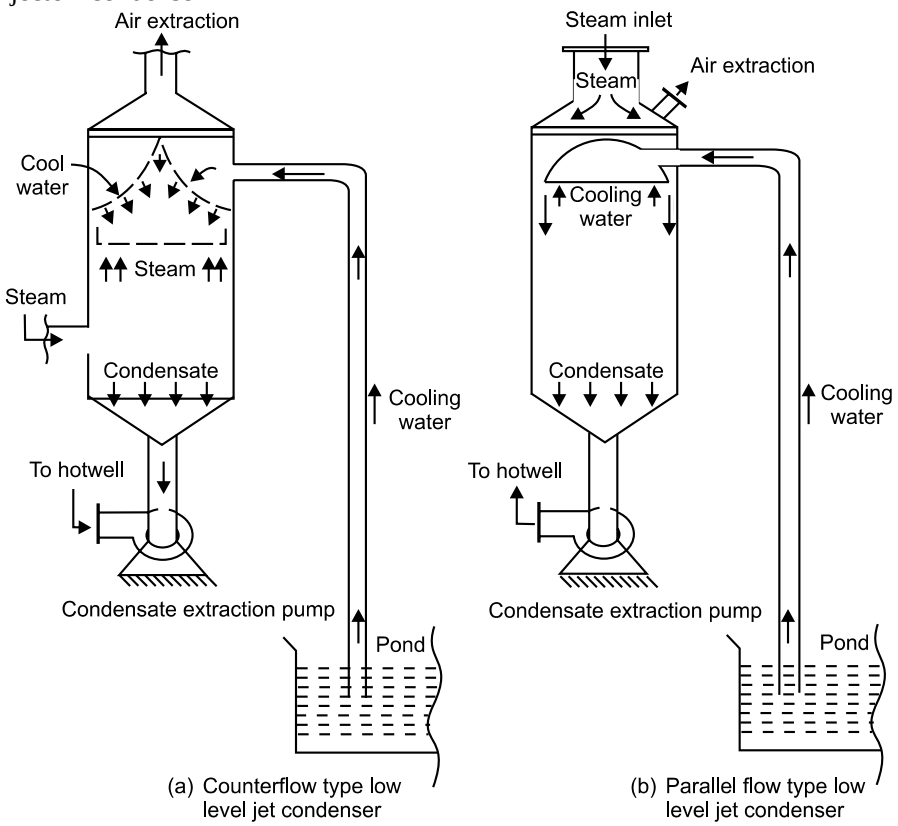


Fig. 15.3 *Schematic of low level jet condenser*

(a) *Low level jet condenser:* Low level jet condenser is the one which is placed at low level such that vacuum inside condenser draws cooling water into condenser from river/pond/cooling tower. Difference between atmospheric pressure (at which cooling water is available) and condenser pressure causes flow of cooling water from cooling water reservoir to condenser i.e. ($p_{atm} - p_{cond}$). Flow of steam and cooling water could be parallel flow or counter flow type. Counter flow type and parallel flow type low level jet condensers are shown in Fig. 15.3. There is provision for extraction of air and dissolved gases from top of condenser by using air extraction pump. Condensate extraction pump is used for taking out condensate from condenser and sending it to hot well.

Cooling water supplied to jet condenser has generally a large percentage of dissolved air which gets liberated due to atomization of water, vacuum and heating of water and is extracted out. Low level jet condenser suffers from inherent drawback that in the event of failure of condensate extraction pump condenser shall be flooded with cooling water.

(b) *High level jet condenser:* High level jet condenser is the one which is placed at a height more than that of water and water is to be injected into condenser using a pump and the condensate will flow out of condenser because of gravity. Here no condensate extraction pump is required, instead pump is required for pumping water upto condenser inlet. High level jet condenser is also called as 'barometric condenser'. High level jet condenser is placed at suitable height depending upon efficient drainage and capacity of sump (hot well) into which tail pipe of condenser discharges out. Mathematically, it could be said that jet condenser placed above hotwell by 10.36 m shall be high level jet condenser or barometric condenser. High level jet condenser may also be of counterflow type or parallel flow type depending upon the direction of flow of steam and cooling water. Figure 15.4 shows counterflow high level jet condenser.

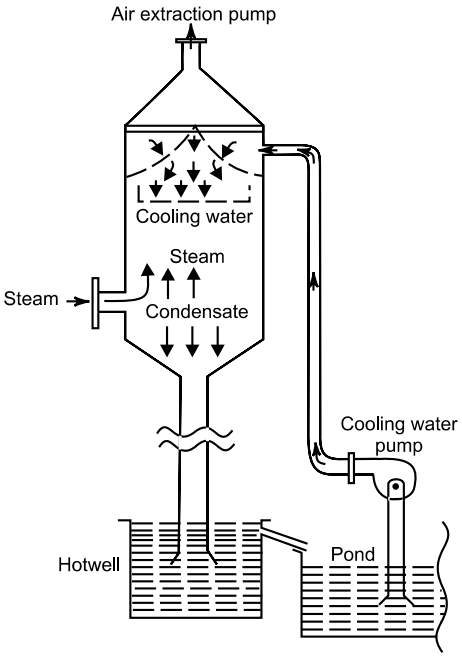


Fig. 15.4 *Schematic of high level condenser*

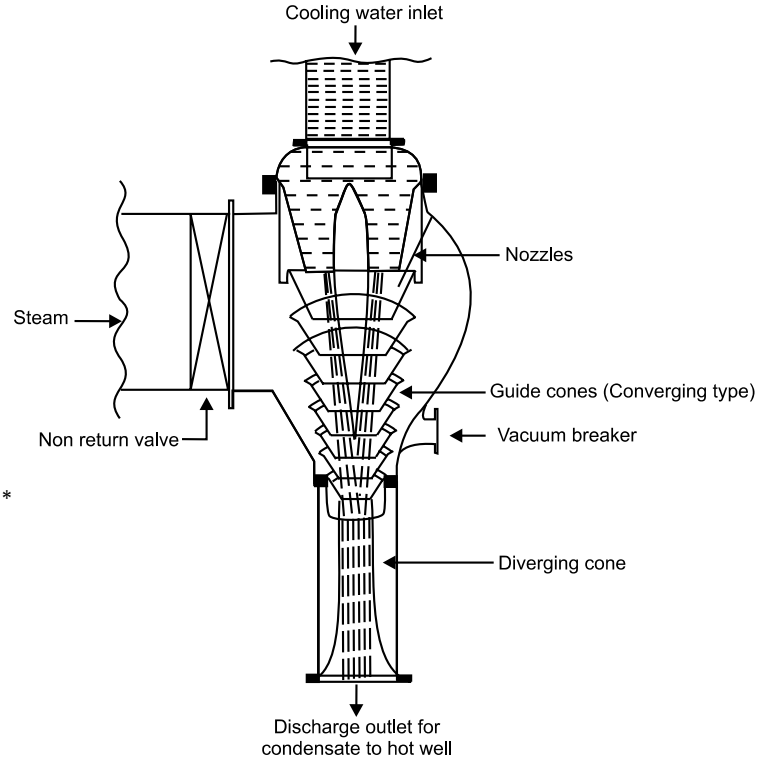


Fig. 15.5 *Ejector condenser*

High level jet condenser do not pose problem of flooding of condenser in the event of failure of pump as it is in case of low level jet condensers. But high level jet condensers are costlier than low level jet condenser. Also there is loss of vacuum between turbine and condenser in case of high level jet condenser.

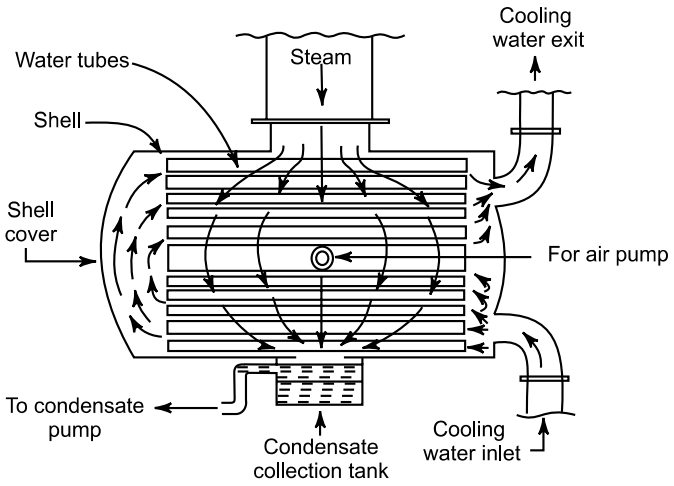
Ejector condenser: Ejector condenser has water jet discharging through the series of guide cones which guide steam on to the surface of water jet. Discharge of water through these convergent nozzles causes partial vacuum due to conversion of potential energy into kinetic energy. Subsequently water jet enters the diffuser nozzle where kinetic energy is converted into the pressure head and water is discharged against the vacuum pull. Ejector condensers are well suited for moderate vacuum only.

Steam is injected in condenser with non return valve in between and is condensed by the mixing with cooling water. Condensation of steam further increases vacuum.

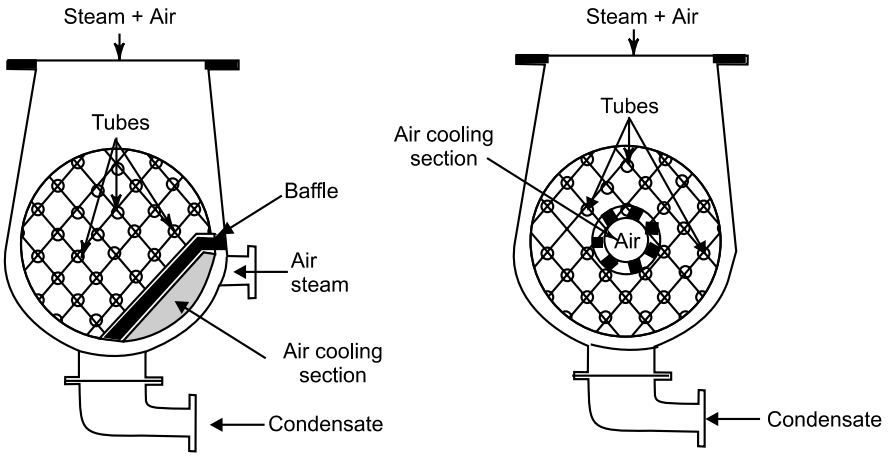
Ejector condenser do not require air pump because of air entraining effect of water jet itself. Here condensing jet has number of nozzles arranged concentrically and have their axis inclined at such an angle that water jet assumes the form of inverted cone. Around the water jet the guide cones are arranged with increasing area from bottom to top. Water will be colder in upper part of condensing cone as compared to lower down. The temperature difference between steam and water at top will be greater than at lower end and so the condensation is greatest at top and gradually diminishes to zero at bottom. In case of failure of cooling water supply water may be sucked from hot well to go into steam pipe, but this is prevented by non-return valve in steam supply line.

(ii) ***Non mixing type or surface condensers****:* Surface condensers are the most common types of condenser and offer great advantage in terms of no contamination of feed water. In these condensers the steam to be condensed and cooling fluid (water) do not come in contact with one another, instead

the heat transfer occurs between two fluids through surface in between. Generally, cooling water flows through the pipes/tubes and steam surrounds them. These condensers are preferred in the locations where large quantity of poor quality cooling fluid (impure water) is available and condensate is to be recirculated. Surface condensers can be classified based on number of passes of condenser i.e. single pass or multipass. Number of times the cooling water crosses any transverse section is called a pass. Surface condensers may be of 'down flow type' or 'central flow type' depending on the type of flow of condensate and tube arrangement. Typical surface condenser having two passes, down flow type and central flow type arrangement are shown in Fig. 15.6.



(a) Two pass surface condenser



(b) Down flow condenser (c) Central flow surface condenser

Fig. 15.6 *Surface condenser*

Two pass surface condenser has cooling water entering from one end and coming out after twice traversing through the tubes (generally, brass) containing water and surrounded by steam to be condensed. Condensate gets collected at bottom and is subsequently sucked by condensate extraction pump. Steam is admitted from the top. Cooling water may be picked directly from river/pond/cooling

tower. For extraction of air the provision is made for air pump. Thus, this type of condenser has three pumps i.e. one for circulating cooling water, second for condensate extraction and third for air extraction. In surface condenser the space occupied by tubes in shell is about 10% of shell volume. Steam is not passed through the tubes because at this steam pressure the specific volume of steam is large requiring large number of tubes.

Down flow condenser has steam and air entering from top and flowing downwards across the bundle of tubes having cooling water flowing through them. Air is extracted from bottom and before being handled by air pump it is flown through air cooler so as to reduce the temperature of air. Low temperature of air enhances the air handling capacity of pump. With the flow of steam down and simultaneous heat exchange the condensate is taken out by condensate extraction pump.

Central flow condenser has air cooling section in the centre of condenser. Steam enters from top and passes over the tube banks of similar type as in case of down flow condenser. As air is being sucked from centre so the flow of steam is radially inwards towards the centre. During this flow steam passes over tubes. Condensate is collected from bottom. In this type of condenser there is better contact between steam and tubes because of radial flow of steam in whole of condenser, thus arrangement is better as compared to down flow condenser.

In different designs of condenser it is always attempted to have maximum heat transfer between two fluids. Also air extraction should be done effectively.

Thus designer of condenser should keep following things in consideration for making a better design surface condenser.

(i) Steam should be uniformly distributed over cooling water tubes. i.e. cooling surface.
(ii) Distribution of steam should be such that there is minimum pressure loss.
(iii) Number of tubes should be minimum. Water must be flown inside tubes and steam should surround them.
(iv) Tubes should be cleaned from inside and outside both. Although on external surface the steam surrounding tubes prevents deposition. For internal cleaning of tubes mechanical or chemical means of cleaning be used at frequent intervals.
(v) Leakage of air into condenser (due to vacuum) should be prevented as it reduces the work output. Also this reduces the heat transfer rate. Even if there is leakage of air, arrangement should be made for quick and effective removal of air with minimum work input.
(vi) Air should be cooled to maximum extent inside condenser before being thrown out as this shall cause condensation if possible within condenser and thus reduce loss of condensate. Also the cool air shall enhance air handling capacity of pump.
(vii) Rate of circulation of cooling water should be such that the range of temperature variation in cooling water lies near the optimum temperature range. Generally, the cooling water temperature rise is limited to 10°C for having maximum heat exchange between two fluids.
(viii) Material of tubes is generally taken as brass. Tube material should be such as to offer maximum heat transfer rate i.e. high thermal conductivity. Generally, surface condensers are bulky and require large space.
(ix) Cost of surface condenser should be kept low. Capital cost, running and maintenance cost should be maintained as low as possible. Generally, these costs are high in case of surface condenser as compared to other types of condensers.

(iii) ***Evaporative condenser:*** Evaporative condensers are generally used where the availability of water is very poor. Figure 15.7 shows the schematic of such type of condenser where water falls from top through the nozzles over the condenser coil. Water picks up heat from the steam flowing through condenser coil and gets warmed up. This water is recirculated by circulation pump. Air flow inside

condenser is maintained by using exhaust fan. This flow of air across condenser coil may be natural or forced to enhance the cooling rate.

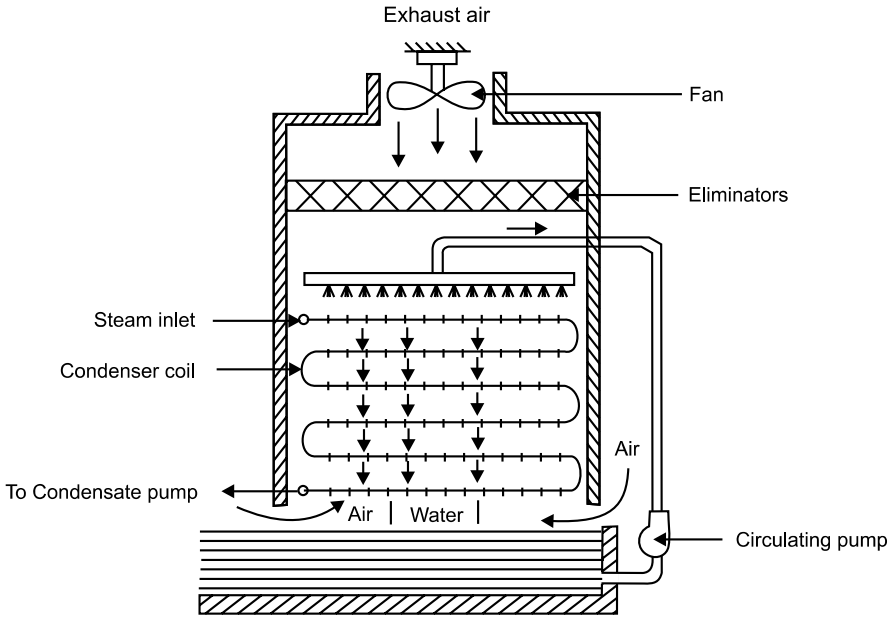


Fig. 15.7 *Evaporative condenser*

Water gets evaporated and evaporated vapours are taken by air leaving condenser. Heat required for evaporation is extracted finally from the steam flowing inside tubes and thus causing its phase transformation. For preventing the exit of water vapours with air going out the separator/eliminator is put on the top before the final exit by which water vapour are recovered upto certain extent. Evaporative condensers are named so because the technique of evaporation is used for realizing the cooling. Amount of water to be sprinkled on condenser tubes should be just sufficient to maintain tube surface in thoroughly wet state. In case of air being humid the vapourizing capacity of wet air gets reduced compared to dry air and so the performance of evaporative condenser deteriorates when humidity in atmosphere is high.

Evaporative condenser is advantageous over the surface condenser as the vacuum maintained in evaporative condenser is not very high and the water requirement is small. These condensers are generally used in small capacity power plants where shortage of water supply is there.

15.3 AIR LEAKAGE

Generally, inside the condenser pressure less than atmospheric pressure is maintained, thereby increasing the chances of air leakage into condenser. Leakage of air occurs due to leaking joints, packings, glands etc. along with air in dissolved form coming with feed water. This leakage of air accounts up to 0.005% and 0.5% of steam condensed in case of jet type condenser and surface condensers respectively. Thus, leakage of air is practically always there in the condensers. Air leakage causes the reduction in work done per kg of steam as it increases the back pressure. Also the quantity of water required for condensation of steam is increased due to lowering of partial pressure of steam due to pressure of air. At low pressure the latent heat of steam to be released is more than at higher pressure. Air (having lower conductivity) when present between water and steam hampers the heat exchange and also takes away a portion of

heat. Because of this reason also the more quantity of cooling water is required. Hence, leakage of air reduces the condenser efficiency and auxiliary devices such as reciprocating pump, rotary pump, steam ejector or air pumps etc. are required. Also the presence of air increases corrosive action as the corrosion depends largely upon the oxygen content.

Since air leakage in condenser is quite damaging to the performance of condenser so air leakage should be detected and subsequent extraction of air being done. Air leakage is detected by isolating condenser from the rest of plant after the steady states are attained in it i.e. pressure and temperature become steady. After isolation of condenser from plant by stopping the steam and cooling fluid pump, if the vacuum gauge and thermometer readings change then it shows that their is air leakage. For identifying location of leak points, the soap bubble test is carried out in carried out in which the bubble formation occurs at leak point if soapy water is put on that after filling condenser with air. Thus, for soap bubble test condenser needs to be emptied and filled with high pressure air. This test has drawback that condenser is to be made non functional and running of plant suffers till the test is performed.

For locating the sources of air leakage during the operational state of condenser peppermint oil test may be used. In this peppermint oil is applied at suspected sources of leak point and in case of leakage at the joint peppermint oil fumes enter the condenser and will come out with the air. Odour of peppermint could be felt in the air leaving condenser. Leak joints can also be detected by passing candle flame over the probable joint. In case of leakage the flame gets distorted.

15.4 CONDENSER PERFORMANCE MEASUREMENT

The vacuum inside the condenser can be quantified by looking at barometer reading which gives atmospheric pressure and vacuum gauge reading and taking their difference to get absolute pressure inside condenser.

Thus, Absolute pressure (in cm) in condenser = (Barometric head in cm of Hg) – (Vacuum pressure in cm of Hg)

Generally, this barometric head depends upon the atmospheric conditions and so the absolute pressure also keeps on changing depending upon it. In order to take care for these variations a pressure head called as corrected vacuum in condenser is being defined. This corrected vacuum pressure is defined in reference to 76 cm of mercury which is the standard barometric head as below,

Corrected vacuum pressure (in cm of Hg) = 76 – Absolute pressure in condenser (in cm of Hg).

Therefore, the corrected vacuum pressure is used in cases where barometric head differs from 76 cm of mercury.

By the Dalton's law of partial pressures, the absolute pressure inside condenser is the sum of partial pressures of steam and air inside it. The partial pressure of steam shall be equal to the saturation pressure corresponding to entering steam temperature. This partial pressure of steam could be seen from steam table. Mathematically, absolute pressure in condenser (p_c), as per Dalton's law;

$$p_c = p_a + p_s$$

where p_a is partial pressure of air and p_s is partial pressure of steam.

Theoretically the vacuum in condenser can be given as, $p_{v,\,th} = p_b - p_s$ where $p_{v,\,th}$ is theoretical vacuum is condenser and p_b is barometric pressure. It could be understood that the leakage of air into condenser shall disturb the vacuum inside the condenser and actually due to this air leaked into condenser the condenser pressure is always greater than the theoretical condenser pressure.

In the absence of air leakage and with air leakage their is loss in performance of condenser and so we need to quantify this effect. 'Vacuum efficiency' of condenser is such parameter which is defined by the ratio of actual vacuum to theoretical vacuum inside condenser. Actually vacuum in condenser in the

presence of air leakage can be given by;

$$p_{v,\,\text{act}} = p_b - (p_a + p_s)$$

$$\text{Thus, vacuum efficiency of condenser} = \frac{\text{Actual vacuum in condenser } \left(p_{v,\,\text{act}}\right)}{\text{Theoretical vacuum in condenser, } \left(p_{v,\,\text{th}}\right)}$$

$$\text{Vacuum efficiency} = \frac{p_b - (p_a + p_s)}{(p_b - p_s)}$$

Here, partial pressure of air (p_a) shall be zero in the absence of air leakage and vacuum efficiency shall be 100%. If we look at mathematical expression of efficiency, it is obvious that vacuum efficiency increases with reduction in partial pressure of air.

Also the vacuum efficiency shall increase with decrease in barometric pressure for constant exit steam pressure and condenser actual pressure. In case of less cooling water the condenser pressure increases and reduces the vacuum efficiency of condenser for other pressures remaining same. Designer always wishes to have condenser with highest vacuum efficiency i.e. close to 100%.

'Condenser efficiency' is another condenser performance parameter. It is given by the ratio of actual rise in cooling water temperature to the maximum possible temperature rise. Condenser facilitates heat exchange between two fluids and under ideal conditions the steam should only reject latent heat to cooling water so as to yield condensate at saturated liquid condition. Thus, there should be no undercooling of condensate in ideal condenser. Therefore, ideal condenser may be defined as condenser in which steam rejects only latent heat to cooling fluid and condensate is available without any undercooling. Ideal condenser requires minimum quantity of cooling water and shows maximum gain in cooling water temperature so as to condense the steam. Mathematically,

$$\text{Condenser efficiency} = \frac{\text{Actual rise in cooling water temperature}}{\text{Maximum possible temperature rise}}$$

Here, the maximum possible temperature rise = {(Saturated temperature corresponding to condenser pressure) – (Cooling water inlet temperature)}

15.5 COOLING TOWER

Cooling tower is similar to evaporative condenser where water used for cooling is being cooled effectively. Water used for cooling becomes hotter after extracting heat from condenser steam and needs to be cooled down if it is to be recycled. Cooling towers are preferably used where the water supply is limited and cooling water has to be recirculated without being thrown out.

Cooling tower is such an arrangement made of wood or metal structure having baffles inside to facilitate better heat exchange between hot water falling down and atmospheric air blowing across it. Generally, hot water is admitted from top and is broken into small size (atomized) while falling down. Air enters tower at bottom and flows upward either due to natural draught or forced draught as the case may be. Air picks up heat by intimate contact with hot water particles and leaves cooling tower from exit passage at top. Cooled water falls down and is collected in a tank at bottom of cooling tower. The heat transfer from hot water to air occurs due to evaporative cooling of water and convective heating of air both. The effectiveness of cooling tower diminishes in humid weather conditions due to reduced capacity of air. Dry air shall offer better cooling effectiveness as compared to moist air. During cooling there occurs some loss of water as it is carried away by air. This water loss may be from 1 to 4% due to evaporation and drift losses.

Typical values for a 500 MW steam power plant indicate that this plant has exhaust of steam at the rate of 3.6 kg/kWh at full load, requiring 18×10^7 kg per hour of cooling water and to cool this cooling water air requirement is about 30×10^6 kg per hour in cooling tower.

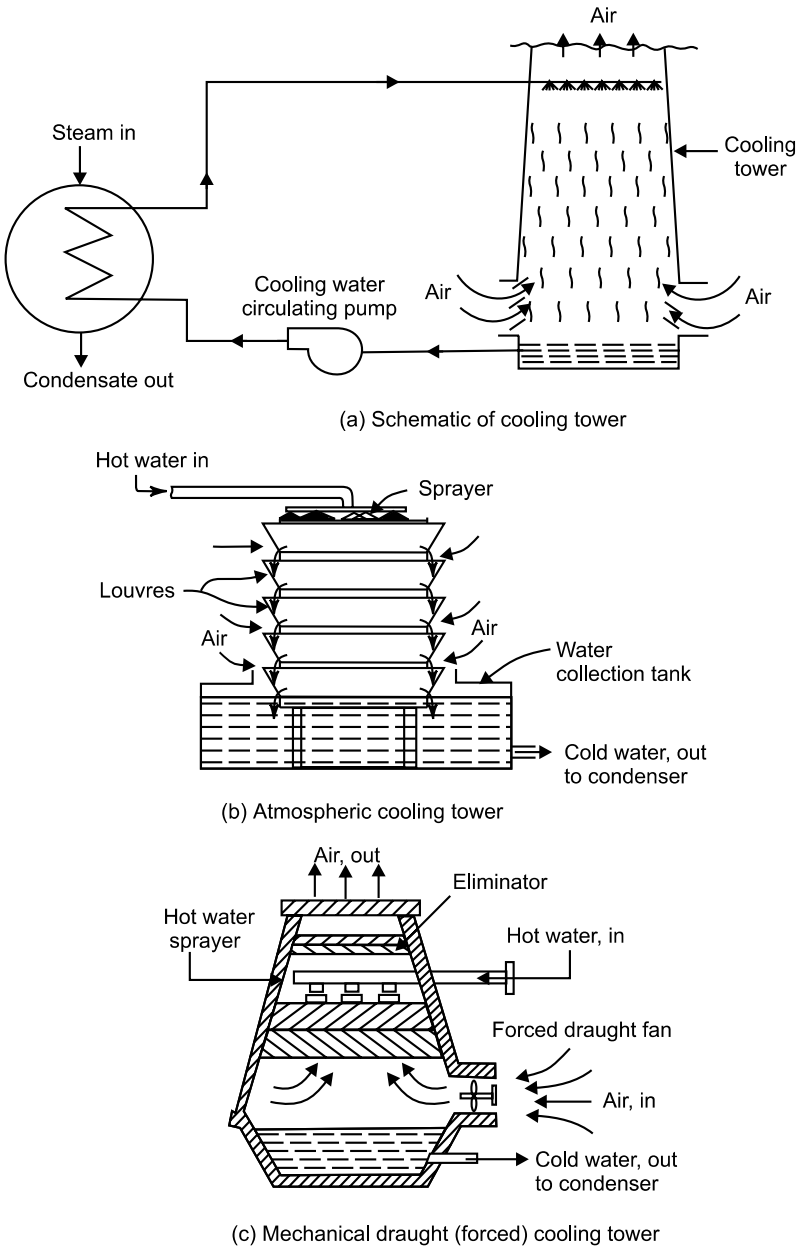


Fig. 15.8 *Cooling tower*

Figure 15.8 shows schematic of different cooling towers. The performance of cooling tower depends largely upon the duration of contact between water particle and air, surface area of contact between water particle and air, humidity of air and relative velocity of air and water flow etc.

EXAMPLES

1. *In a condenser the vacuum of 71 cm of Hg is maintained with barometer reading of 76 cm in Hg. Temperature in condenser is 35°C while hot well is at temperature of 30°C. The cooling water is circulated at the rate of 800 kg/min and condensate is available at 25 kg/min. The temperature of cooling water at inlet and outlet are 15°C and 25°C. Determine the mass of air in kg/m³ of condenser volume, dryness fraction of steam entering condenser and vacuum efficiency. Take mercury density as 0.0135951 kg/cm³, g = 9.81 m/s².*

Solution:

Absolute pressure in the condenser

$$p_t = (76 - 71) \times 10^{-2} \times 0.0135951 \times 10^6 \times 9.81 = 6668.396 \text{ N/m}^2$$
$$= 6.67 \text{ kPa}$$

Partial pressure of steam in condenser = Saturation pressure of steam corresponding to 35°C (from steam table)

$$p_s = 5.62 \text{ kPa}$$

Partial pressure of air, $p_a = p_t - p_s = 6.67 - 5.63 = 1.04$ kPa

Mass of air per m³ of condenser volume can be obtained from gas equation,

$$m_a = \frac{p_a \cdot V}{RT} = \frac{1.04 \times 1}{(273 + 35) \times 0.287} = 0.012 \text{ kg/m}^3$$

Let the enthalpy of steam entering condenser be h_s so by heat balance,

$$m_w \times C_{p_w} \times (T_{w_o} - T_{w_i}) = m_s \cdot (h_s - C_{p_w} \cdot T_c)$$

m_w= 800 kg/min, m_s = 25 kg/min, T_{w_o} = 25°C, T_{w_i} = 15°C, T_c = 30°C

$$800 \times 4.18 (25 - 15) = 25 (h_s - 4.18 \times 30)$$
$$h_s = 1463 \text{ kJ/kg}$$

Let dryness fraction of steam entering be x.

$$h_s = 1463 = h_{f \text{ at } 35°C} + x \cdot h_{fg \text{ at } 35°C}$$
$$1463 = 146.68 + x \cdot 2418.6$$
$$\Rightarrow \quad x = 0.5442$$

$$\text{Vacuum efficiency} = \frac{(76 - 5) \times 0.0135951 \times 10^4 \times 9.81}{\left(\left(76 \times 0.0135951 \times 10^4 \times 9.81\right) - 5.63 \times 10^3\right)}$$
$$= 0.9891 \text{ or } 98.91\%$$

Mass of air in kg/m³ of condenser volume = 0.012 kg/m³, dryness fraction of steam entering = 0.5442, Vacuum efficiency = 98.91%

Ans.

2. *A condenser has vacuum of 70 cm of Hg when barometer reading is 76 cm. Condenser has temperature of 30°C. Air leaks into condenser at the rate of 1 kg air per 2500 kg steam. Calculate (i) the capacity of air pump per kg of steam for removal of air from steam entering condenser, and (ii) the mass of water vapour accompanying this air.*

Solution:

Absolute pressure in condenser = (76 – 70) cm of Hg

$= 6 \times 0.0135951 \times 10^4 \times 9.81 = 8002.67$ Pa

$p_t = 8.003$ kPa

Partial pressure of steam, p_s = Saturation pressure corresponding to 30°C from steam table

$p_s = 4.246$ kPa

Partial pressure of air, p_a = Total pressure in condenser – Partial pressure of steam

$p_a = p_t - p_s = 3.757$ kPa

Mass of air accompanying per kg steam due to leakage = $\frac{1}{2500} = 0.0004$ kg

Using gas equation, $p_a \cdot V = mRT$

Volume of air per kg of steam = $\frac{mRT}{p_a} = \frac{0.0004 \times 0.287 \times (273 + 30)}{3.757}$

$= 9.26 \times 10^{-3}$ m^3/kg

Capacity of air pump = 9.26×10^{-3} m^3 per kg steam ***Ans.***

Volume of water vapour accompanying air shall be equal to the volume of air.

So volume of water vapour accompanying air = 9.26×10^{-3} m^3/kg

Specific volume of dry steam at condenser temperature of 30°C = $v_{g \text{ at } 30°C} = 32.89$ m^3/kg

Mass of water vapour accompanying air = $\frac{9.26 \times 10^{-3}}{32.89}$

$= 2.82 \times 10^{-4}$ kg/kg of steam

Mass of water vapour accompanying air = 2.82×10^{-4} kg/kg of steam ***Ans.***

3. *During the trial on a condenser it is seen to have vacuum of 67 cm of Hg while barometer reading is 75 cm of Hg. The mean condenser temperature is 40°C and temperature of hot well is 35°C. Circulating water flows at 1000 kg/min for giving condensate at the rate of 50 kg/min. Temperature of cooling water at inlet and exit are 10°C and 25°C. Determine, (i) the vacuum corrected to standard barometer reading of 76 cm. (ii) the vacuum efficiency of condenser, (iii) the undercooling of condensate, (iv) the condenser efficiency, (v) the state of steam entering condenser, (vi) the mass of air per m^3 of condenser volume, and (vii) the mass of air per kg of uncondensed steam.*

Solution:

Vacuum corrected to standard barometer reading of 76 cm

$= 76 - (75 - 67) = 68$ cm Hg

Corrected vacuum = 68 cm Hg ***Ans.***

Absolute pressure in condenser = 75 – 67 = 8 cm of Hg or ($8 \times 0.0135951 \times 10^4 \times 9.81 = 10.67$ kPa)

or, $p_t = 10.67$ kPa

Partial pressure of steam, p_s = Saturation pressure at 40°C from steam table

p_s = 7.384 kPa

Partial pressure of air, $p_a = p_t - p_s$ = 3.286 kPa

$$\text{Vacuum efficiency} = \frac{\text{Actual vacuum}}{\text{Theoretical vacuum}} = \frac{(75-8)\times 0.0135951\times 10^4 \times 9.81}{\left(75\times 0.0145951\times 10^4 \times 9.81\right) - 7.384\times 10^3}$$

= 0.9645 or 96.45%

Ans. Vacuum efficiency = 96.45%

Undercooling of condensate = 40 – 35 = 5°C

Ans. Undercooling = 5°C

$$\text{Condenser efficiency} = \frac{\left(T_{w\,\text{out}} - T_{w\,\text{in}}\right)}{\left(T_{\text{sat corresponding to } p_t} - T_{w\,\text{in}}\right)}$$

Saturation temperature corresponding to absolute pressure in condenser, 40.01 kPa.

$T_{\text{sat corresponding to } p_t}$ = 46.9°C, from steam table.

$T_{w\ \text{out}}$ = 25°C, $T_{w\ \text{in}}$ = 10°C

$$\text{Condenser efficiency} = \frac{25-10}{46.9-10} = 0.4065 \text{ or } 40.65\%$$

Let us consider enthalpy of steam entering be h kJ/kg and dryness fraction be x. By applying heat balance,

$$\begin{aligned} m_s \cdot (h - c_{p_w} \cdot T_c) &= m_w \cdot c_{p_w} \cdot (T_{w_o} - T_{w_i}) \\ m_s &= 50 \text{ kg/min}, \ m_w = 1000 \text{ kg/min} \\ 50(h - 4.18 \times 40) &= 1000 \times 4.18\ (25 - 10) \\ h &= 1421.2 \text{ kJ/kg} = h_{f \text{ at } 40°C} + x \cdot h_{fg \text{ at } 40°C} \\ 1421.2 &= 167.57 + x \cdot 2406.7 \\ x &= 0.5209 \end{aligned}$$

Dryness fraction of steam entering = 0.5209	***Ans.***

Mass of air per m^3 of condenser volume

$$m = \frac{p_a \cdot V}{RT_c} = \frac{3.286 \times 1}{0.287 \times (273 + 40)} = \mathbf{0.0366\ kg/m^3}$$

Volume of per kg of uncondensed steam = $v_{g \text{ at } 40°C}$ = 19.52 m^3/kg

Mass of air in one kg of uncondensed steam = Mass of air in 19.52 m^3 volume

$$= \frac{3.286 \times 19.52}{0.287 \times (40 + 273)} = 0.714 \text{ kg}$$

Mass of air/m^3 of condenser volume = 0.0366 kg/m^3 Mass of air in per kg of uncondensed steam = 0.714 kg	***Ans.***

4. *In a surface condenser operating with steam turbine the vacuum near inlet of air pump is 69 cm of Hg when barometer reading is 76 cm of Hg. Temperature at inlet of vacuum pump is 30°C. Air leakage occurs at the rate of 60 kg/hr. Determine,*

(i) the capacity of air pump in m^3/hr and mass of vapour extracted with air in kg/hr.

(ii) the dimension of reciprocating air pump cylinder if it runs at 240 rpm and L/D ratio is 1.5.

Solution:

Absolute pressure at inlet to air pump = (76 – 69) = 7 cm Hg, or

$= 7 \times 0.0135951 \times 10^4 \times 9.81$

$p_t = 9.336$ kPa

Partial pressure of vapour at 30°C = Saturation pressure at 30°C

$p_s = 4.246$ kPa

Partial pressure of air, $p_a = p_t - p_s = 5.09$ kPa

Volume of 60 kg air at pressure of 89.112 kPa

$$V = \frac{60 \times 0.287 \times (273 + 30)}{5.09}$$

$= 1025.1$ m^3/hr

Capacity of air pump in m^3/hr = 1025.1 m^3/hr ***Ans.***

Volume handled, m^3/hr $= \frac{\pi}{4} D^2 \times L \times N \times 60 = 1025.1$

$$1025.1 = \frac{\pi}{4} \times 1.5\, D^3 \times 240 \times 60$$

$D = 0.3924$ m or 39.24 cm

$L = 58.86$ cm

Mass of water vapour going with air $= \frac{V}{V_{g \text{ at } 30°C}} = \frac{1025.1}{32.89} = 31.17$ kg/hr

Bore = 39.24 cm, Stroke = 58.86 cm

Mass of water vapour extracted with air = 31.17 kg/hr ***Ans.***

5. *A steam condenser is supplied with 1000 kg/min steam in 0.9 dry state. The pressure at suction of air extraction pump on condenser is 70 cm of Hg and barometer reads 77 cm of Hg. Temperature in suction pipe is 30°C and air leaks at the rate of 5×10^{-4} kg per kg of steam. Cooling water temperature gets increased by 15°C. Determine the mass handled by dry air extractor and cooling water circulation rate in kg/min.*

Solution:

Absolute pressure in condenser = (77 – 70) = 7 cm of Hg or $(7 \times 0.0135951 \times 10^4 \times 9.81)$

$p_t = 9.34$ kPa

Partial pressure of steam, p_s = Saturation pressure corresponding to 30°C = 4.246 kPa

Partial pressure of air, $p_a = p_t - p_s = 5.094$ kPa

Rate of air extraction per minute $= 5 \times 10^{-4} \times 1000 = 0.5$ kg/min

Volume of air extracted per minute $= \frac{mRT}{p_a} = \frac{0.5 \times 0.287 \times (273 + 30)}{5.094}$

$= 8.54$ m^3/min

Specific volume corresponding to 30°C, v_g = 32.89 m^3/kg
Volume of air extracted = Volume of mixture sucked per minute = 8.54 m^3/min

Mass of steam extracted in mixture handled per minute = $\frac{8.54}{32.89}$ = 0.2596 kg/min

Therefore, mass handled by air extraction pump = 0.5 + 0.2596 = 0.7596 kg/min

Mass handled by air pump = 0.7596 kg/min ***Ans.***

Enthalpy of steam entering condenser, $h = h_{f \text{ at } 30°C} + 0.9\ (h_{fg \text{ at } 30°C})$
$= 125.79 + (0.9 \times 2430.5)$
$= 2313.24$ kJ/kg

Mass flow rate of circulating water can be obtained by energy balance on condenser,

$$m_w \times Cp_w \times (\Delta T_w) = m_s \times (h - Cp_w \times T_c)$$
$$m_w \times 4.18 \times 15 = 1000 \times (2313.24 - 4.18 \times 303)$$
$$m_w = 16693.78 \text{ kg/min}$$

Water circulation rate = 16693.78 kg/min ***Ans.***

6. *In a surface condenser vacuum of 70 cm Hg is maintained when the barometric pressure is 76 cm Hg. Steam enters 0.85 dry into condenser at the rate of 300 kg/min. Temperature of condensate is 30°C and the rise in circulating water temperature is 20°C. For sending water through condenser and piping a pressure head of 5 m is required. For surface condenser determine,*

(i) the flow surface area required when water flows at 50 m/min,
(ii) the cooling surface area required when heat transfer rate is 15 × 10^5 kJ/m^2 · hr °C.
(iii) the total head required to be developed by pump

Solution:

Absolute pressure in condenser = 76 – 70 = 6 cm of Hg, or, $(6 \times 0.0135951 \times 10^4 \times 9.81)$ Pa

$$p_t = 8.002 \text{ kPa}$$

Partial pressure of steam, p_s = Saturation pressure corresponding to 30°C
$= 4.246$ kPa

Applying heat balance on condenser,

$$m_w \times 4.18 \times 20 = (h_{f \text{ at } 30°C} + 0.85 \cdot h_{fg \text{ at } 30°C} - 4.18 \times 30) \times 300$$
$$m_w = 7415 \text{ kg/min}$$

Volume flow of water = $\frac{7415}{1000}$ m^3/min = 7.415 m^3/min

$$\text{Flow surface area requirement} = \frac{\text{Velocity flow rate of water}}{\text{Velocity of water flow}} = \frac{7.415}{50}$$
$$= 0.1483 \text{ m}^2$$

Flow surface area required = 0.1483 m^2 ***Ans.***

$$\text{Cooling surface area required} = \frac{\text{Total heat given by steam}}{\text{Heat transfer rate}}$$
$$= \frac{300 \times \left(\begin{matrix} h_{f \text{ at } 30°C} + 0.85 \\ \times h_{fg at 30°C} - 4.18 \times 30 \end{matrix} \right) \times 60}{15 \times 10^5}$$

$$= 0.413 \times 60$$
$$= 24.79 \text{ m}^2$$

Cooling surface area required = 24.79 m^2 ***Ans.***

$$\text{Velocity head present} = \frac{1}{2} \times \left(\frac{50}{60}\right)^2 \times \frac{1}{9.81} = 0.0354 \text{ m}$$

Total head required = Pressure head + Velocity head
$$= 5 + 0.0354 = 5.0354 \text{ m}$$

Head required = 5.0354 m ***Ans.***

7. *A jet condenser has steam entering at 350 kg/min when vacuum of 680 mm is maintained in it and the barometer reads 760 mm. Air mass going into condenser is 0.05% of steam mass entering. Water at 20°C enters the condenser to condense the steam such that temperature of condensate is 30°C. Volume of water required is 0.02 m^3 per kg steam. The volume of air dissolved in the water injected may be considered as 5% of volume of water at atmospheric pressure. Determine the volume handling capacity of air pump for removing air and condensate when pump has volumetric efficiency of 90%.*

Solution:

Absolute pressure in condenser = (76 – 68) = 8 cm of Hg

or, $= (8 \times 0.0135951 \times 9.81 \times 10^4 \times 10^{-3})$

$p_t = 10.67$ kPa

Partial pressure of steam,

p_s = Saturation pressure corresponding to 30°C

$p_s = 4.246$ kPa

Partial pressure of air, $p_a = p_t - p_s = 10.67 - 4.246 = 6.424$ kPa

Volume of cooling water required per minute = $350 \times 0.02 = 7 \text{m}^3/\text{min}$

Mass of air going into condenser with steam per minute = $\frac{350 \times 0.05}{100}$

$= 0.175$ kg/min

Volume of air entering per minute with cooling water = $\frac{7 \times 5}{100}$

$= 0.35 \text{ m}^3/\text{min}$

Mass of air with cooling water, using $pV = mRT$, $m = \frac{pV}{RT}$

here p = atmospheric pressure = 101.3 kPa, $V = 0.35 \text{ m}^3$, $R = 0.287$ kJ/kg·K

$T = 273 + 20 = 293$ K

Mass of air with cooling water = $\frac{101.3 \times 0.35}{0.287 \times 293} = 0.422$ kg/min

Thus, total mass of air inside condenser per minute

= (Mass of air with steam + Mass of air with cooling water)

$= 0.175 + 0.422 = 0.597$ kg/min

Volume of air corresponding to 0.597 kg/min

$$= \frac{0.597 \times 0.287 \times (273 + 30)}{6.424} = 8.08 \text{ m}^3/\text{min}$$

Volume of steam condensed (condensate) = $350 \times v_{f \text{ at } 30°C}$

$$= 350 \times 0.001004 = 0.3514 \text{ m}^3/\text{min}$$

Total capacity of air pump (wet air pump) = Volume of condensate/min + Volume of air/min + Volume of cooling water/min.

$$= 0.3514 + 8.08 + 7$$

$$= 15.4314 \text{ m}^3/\text{min}$$

Actual capacity of air pump = $\frac{15.4314}{0.9} = 17.15 \text{ m}^3/\text{min}$

Capacity of air pump = 17.15 m^3/min **Ans.**

8. *A barometric jet condenser has steam entering at the rate of 20 kg/min and 12 kg cooling water per kg of steam is supplied into it for condensation. After condensation the cooling water and condensate leave at 40°C. Cooling water enters the condenser at 20°C. Vacuum of 650 mm Hg is maintained inside condenser while barometer read 760 mm Hg. An air pump is put for extracting air from condensate at the rate of 2m^3 of wet mixture per minute. At the suction side of air pump a vacuum of 660 mm Hg and temperature of 35°C is observed.*

Determine,

(i) the dryness fraction of steam entering condenser,

(ii) the mass of air entering condenser per minute,

(iii) the effective pressure head at tail of barometric condenser.

Solution:

Absolute pressure in condenser, $p_t = (760 - 650) \times 10^{-1} = 11$ cm Hg

or,

$$p_t = 11 \times 10^4 \times 0.0135951 \times 9.81 \times 10^{-3}, \text{ kPa}$$

$$p_t = 14.67 \text{ kPa}$$

Partial pressure of steam, p_s = Saturation pressure corresponding to 40°C

$$= 7.384 \text{ kPa}$$

Partial pressure of air, $p_a = p_t - p_s = 7.286$ kPa

Cooling water required = $12 \times 20 = 240$ kg/min

Let the dryness fraction of steam entering condenser be x and enthalpy h.

Heat balance on condenser yields;

$$m_s \cdot h + m_w \cdot c_{pw} \cdot T_{wi} = (m_s + m_w)\, c_{pw} \cdot T_{wo}$$

$$20 \cdot h + (240 \times 4.18 \times 20) = (20 + 240) \times 4.18 \times 40$$

$$h = 1170.4 \text{ kJ/kg} = h_{f \text{ at } 40°C} + x \cdot h_{fg \text{ at } 40°C}$$

$$1170.4 = 167.57 + (x \cdot 2406.7)$$

$$\Rightarrow \quad x = 0.4167$$

Dryness fraction of steam entering = 0.4167 **Ans.**

Absolute pressure at suction of air pump = $(760 - 660) \times 10^{-1}$ = 10 cm Hg or
$(10 \times 0.0135951 \times 10^4 \times 9.81 \times 10^{-3}$ = 13.34 kPa)
Partial pressure of steam at suction of air pump = Saturation pressure at 35°C.

$$p'_s = 5.628 \text{ kPa}$$

Partial pressure of air, $p'_a = 13.34 - 5.628 = 7.712$ kPa
Now, at suction of pump volume of air will be equal to the volume of mixture.

$$\text{Volume of mixture} = \text{Volume of air} = 2 \text{ m}^3$$

$$\text{Mass of air entering} = \frac{p'_a V}{RT'_a} = \frac{7.712 \times 2}{0.287 \times (273 + 35)}$$

$$= 0.1745 \text{ kg/min}$$

Effective pressure head tail of barometric condenser

= Head corresponding to (Barometric pressure – Absolute pressure in condenser)

$$= \frac{(101.3 - 14.67) \times 10^3}{9.81 \times 0.0135951 \times 10^6} = \mathbf{0.649\ m}$$

Mass of air entering = 0.1745 kg/min Head = 0.649 m

Ans.

9. *A steam condenser has steam entering at 35°C and condensate being removed at 34°C. Condenser has two pumps one for extracting air and other for extraction of condensate. Air is removed at temperature of 33°C. The air leaks into condenser at the rate of 3 kg/hr. Consider the pressure inside condenser to remain uniform and neglect change in pressure due to air at steam inlet. Determine the volume of air handled by air pump in kg/hr and also determine the volume to be handled if a combined air and condensate pump is being used.*

Solution:

Partial pressure of steam at 35°C = Saturation pressure corresponding to 35°C.

$$p_s = 5.628 \text{ kPa}$$

If the pressure of air at inlet is neglected then the total pressure in condenser,

$$p_t = p_s = 5.628 \text{ kPa}.$$

At the suction of air pump, partial pressure of steam

= Saturation pressure corresponding to 33°C

$$p_{s'} = 5.075 \text{ kPa}$$

Partial pressure of air, $p_{a'} = p_t - p_{s'} = 5.628 - 5.075 = 0.553$ kPa

$$\text{Volume of air handled by air pump} = \frac{mRT_{a'}}{p_{a'}} = \frac{3 \times 0.287 \times (273 + 33)}{0.553}$$

$$= 467.43 \text{ m}^3/\text{hr}$$

In case the air and condensate mixture is to be handled by same pump then,
Partial pressure of steam = Saturation pressure corresponding to 34°C

$$p_{s''} = 5.352 \text{ kPa}$$

Partial pressure of air, $p_{a''} = p_t - p_{s''} = 5.628 - 5.352 = 0.276$ kPa

Volume of air = Volume of mixture handled by pump = $\dfrac{3 \times 0.287 \times (273 + 34)}{0.276}$

$= 957.71 \text{ m}^3/\text{hr}$

Volume of air handled = 467.43 m^3/hr
Volume of mixture handled = 957.71 m^3/hr

Ans.

10. *In a surface condenser the vacuum at inlet is seen to be 72 cm Hg and at outlet it is 73 cm Hg. The barometer reading is 76 cm and the dryness fraction of steam at inlet is 0.92. Cooling water entering the condenser is at 20°C. Considering no air in the condenser and the temperature rise in cooling water to be maximum determine.*

(i) the minimum amount of undercooling.

(ii) the amount of cooling water required per kg of steam.

Solution:

Inlet pressure in condenser = (76 – 72) cm Hg = 4 cm Hg.

$= 4 \times 10^4 \times 0.0135951 \times 9.81 \times 10^{-3}$

$= 5.335$ kPa

Outlet pressure in condenser = (76 – 73) cm Hg = 3 cm Hg

$= 3 \times 10^4 \times 0.0135951 \times 9.81 \times 10^{-3}$

$= 4.001$ kPa

Since there is no air in condenser so the 5.335 kPa and 4.001 kPa will be the pressure of steam. Saturation temperature corresponding to above pressures give temperature at inlet and outlet respectively.

Saturation temperature at inlet =33.87°C, (from steam table at 5.335 kPa)

Saturation temperature at outlet =28.96°C, (from steam table at 4.001 kPa)

Thus, steam will leave at maximum temperature of 28.96°C

The minimum amount of undercooling = 33.87 – 28.96 = 4.91°C

For maximum temperature rise of cooling water the temperature of cooling water outlet will be equal to the temperature of steam at inlet of 33.87°C.

Therefore, the maximum rise in cooling water temperature

$= 33.87 - 20 = 13.87°\text{C}$

Enthalpy of steam entering, $h = h_{f \text{ at } 33.87°\text{C}} + 0.92 \times h_{fg \text{ at } 33.87°\text{C}}$

$h = 141.97 + (0.92 \times 2421.33) = 2369.59$ kJ/kg

Let mass of cooling water required be m kg per kg steam.

Heat balance on condenser yields,

$m \times 4.18 \times 13.87 = 1 \times (2369.59 - 4.18 \times 28.96)$

$\Rightarrow \quad m = 38.78$ kg water per kg of steam

Undercooling = 4.91°C
Cooling water requirement = 38.78 kg/kg steam

Ans.

EXERCISE

15.1 What do you understand by condenser? Discuss its significance.
15.2 How does condenser improve performance of steam power plant?
15.3 Discuss different types of condenser briefly.
15.4 Differentiate between surface condenser and jet condenser.
15.5 Give a sketch of barometric jet condenser and explain its working.
15.6 Discuss the effect of air leakage upon the performance of condenser.
15.7 How the air leaking into condenser is extracted out? Explain.
15.8 Describe the factors affecting the efficiency of condensing plant.
15.9 Discuss the relevance of Dalton's law of partial pressures in condenser calculations.
15.10 What do you understand by cooling towers? Explain their utility.
15.11 Determine the vacuum efficiency of a surface condenser having vacuum of 715 mm of Hg and temperature of 32°C. The barometer reading is 765 mm of Hg. [98%]
15.12 A surface condenser having vacuum of 715 mm Hg and temperature of 32°C has cooling water circulated at 800 kg/min. The cooling water entering condenser becomes warmer by 14°C. The condensate is available from condenser at 25 kg/min. The hot well temperature is 30°C. Barometer reading is 765 mm of Hg. Determine the mass of air in kg/m^3 of condenser volume and dryness fraction of steam entering. [0.022 kg/m^3, 0.84]
15.13 A surface condenser has vacuum of 71 cm Hg and mean temperature of 35°C. The barometer reading is 76.5 cm Hg. The hot well temperature is 28°C. Steam enters condenser at 2000 kg/hr and requires cooling water at 8°C at the rate of 1000 kg/min. Cooling water leaves condenser at 24°C. Determine
(*i*) the vacuum efficiency of condenser,
(*ii*) the undercooling in condenser
(*iii*) corrected vacuum in reference to standard barometer reading, (iv) the condenser efficiency. [0.982, 7°C, 70.5 cm Hg, 0.505]
15.14 In a surface condenser steam enters at 40°C and dryness fraction of 0.85. Air leaks into it at 0.25 kg/min. An air pump is provided upon the condenser for extracting out air. Temperature at suction of air pump is 32°C while condensate temperature is 35°C. Determine.
(*i*) the reading of vacuum gauge
(*ii*) the volume handling capacity of air pump in m^3/hr
(*iii*) the loss of condensate in kg/hr.
[705 mm Hg, 500 m^3/hr, 16.9 kg/hr]
15.15 A steam turbine discharges steam into a surface condenser having vacuum of 700 mm Hg. The barometer reading is 760 mm Hg. Leakage into condenser is seen to be 1.4 kg/min. The air pump is employed for extracting out air leaking in. Temperature at the inlet of air pump is 20°C. The air pump is of reciprocating type running at 300 rpm and has L : D ratio of 2 : 1. Determine,
(*i*) the capacity of air pump is m^3/hr
(*ii*) the dimensions of air pump
(*iii*) the mass of vapour going out with air in air pump, kg/hr.
[1250 m^3/hr, bore: 35.36 cm, stroke: 70.72 cm, 21.5 kg/hr]
15.16 A surface condenser handles condensate at 70.15 cm Hg when barometer reads 76 cm Hg. Steam entering at 2360 kg/hr requires cooling water at 6.81×10^2 kg/hr, 10°C. Cooling water leaves condenser at 27.8°C while condenser has mean temperature of 37°C. Air leaks into condenser at 0.3 kg/min. Determine,
(*i*) the mass of vapour going out with air per hour
(*ii*) the state of steam entering.
[119 kg/hr, 0.89]

15.17 A steam condenses working at 71.5 cm Hg vacuum and temperature of 32.28°C. Condenser has steam entering at 13750 kg/hr. The leakage of air into condenser is 0.4×10^{-3} kg/kg steam entering. Considering volumetric capacity of air pump as 80% determine the capacity of air pump, handling mixture of air and water vapour. [10.3 m^3/min]

15.18 A jet condenser has pressure at inlet as 0.07 bar. It is supplied steam at 4000 kg/hr. Water is supplied for condensation at 16×10^4 kg/hr. The volume of air dissolved in water at 1 bar, 15°C is 0.05 of that water.

Air leaks in with steam at 0.05 kg/min. Temperature at suction of air pump is 30°C and volumetric efficiency is 80%. Determine the capacity of air pump in m^3/min for extracting out air.

[519 kg/hr]

15.19 A surface condenser has steam entering at 0.09 bar, 0.88 dry at the rate of 16000 kg/hr. The air leakage into condenser is 8 kg/hr. Temperature at condensate and air extraction pipe is 36°C. For the average heat transfer rate of 133760 kJ/m^2·hr, determine the surface required. [254 m^2]

15.20 A surface condenser has vacuum of 581 mm of Hg and steam enters into it at 57.4°C, 3100 kg/hr. The barometer reads 726 mm of Hg. The temperature at suction of air pump is 50°C. The cooling water is supplied at 5.6×10^4 kg/hr, 15°C and leaves at 43.6°C. Determine,

(*i*) the mass of air entering condenser per kg of steam

(*ii*) the vacuum efficiency.

[0.136 kg/kg steam, 0.981]

16

Reciprocating and Rotary Compressor

16.1 INTRODUCTION

Compressors are work absorbing devices which are used for increasing pressure of fluid at the expense of work done on fluid.

The compressors used for compressing air are called air compressors. Compressors are invariably used for all applications requiring high pressure air. Some of popular applications of compressor are, for driving pneumatic tools and air operated equipments, spray painting, compressed air engine, supercharging in internal combustion engines, material handling (for transfer of material), surface cleaning, refrigeration and air conditioning, chemical industry etc. Compressors are supplied with low pressure air (or any fluid) at inlet which comes out as high pressure air (or any fluid) at outlet, Fig. 16.1. Work required for increasing pressure of air is available from the prime mover driving the compressor. Generally, electric motor, internal combustion engine or steam engine, turbine etc. are used as prime movers. Compressors are similar to fans and blowers but differ in terms of pressure ratios. Fan is said to have pressure ratio up to 1.1 and blowers have pressure ratio between 1.1 and 4 while compressors have pressure ratios more than 4.

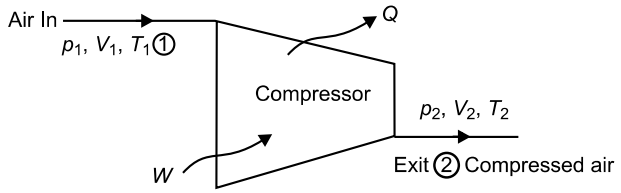


Fig. 16.1 *Compressor*

Compressors can be classified in the following different ways.

(a) ***Based on principle of operation:*** Based on the principle of operation compressors can be classified as,

(i) Positive displacement compressors

(ii) Non-positive displacement compressors

In positive displacement compressors the compression is realized by displacement of solid boundary and preventing fluid by solid boundary from flowing back in the direction of pressure gradient. Due to solid wall displacement these are capable of providing quite large pressure ratios. Positive displacement compressors can be further classified based on the type of mechanism used for compression. These can be

(i) Reciprocating type positive displacement compressors
(ii) Rotary type positive displacement compressors

Reciprocating compressors generally, employ piston-cylinder arrangement where displacement of piston in cylinder causes rise in pressure. Reciprocating compressors are capable of giving large pressure ratios but the mass handling capacity is limited or small. Reciprocating compressors may also be single acting compressor or double acting compressor. Single acting compressor has one delivery stroke per revolution while in double acting there are two delivery strokes per revolution of crank shaft. Rotary compressors employing positive displacement have a rotary part whose boundary causes positive displacement of fluid and thereby compression. Rotary compressors of this type are available in the names as given below;

(i) Roots blower
(ii) Vaned type compressors

Rotary compressors of above type are capable of running at higher speed and can handle large mass flow rate than reciprocating compressors of positive displacement type.

Non-positive displacement compressors, also called as steady flow compressors use dynamic action of solid boundary for realizing pressure rise. Here fluid is not contained in definite volume and subsequent volume reduction does not occur as in case of positive displacement compressors. Non-positive displacement compressor may be of 'axial flow type' or 'centrifugal type' depending upon type of flow in compressor.

(b) ***Based on number of stages:*** Compressors may also be classified on the basis of number of stages. Generally, the number of stages depend upon the maximum delivery pressure. Compressors can be single stage or multistage. Normally maximum compression ratio of 5 is realized in single stage compressors. For compression ratio more than 5 the multi-stage compressors are used.

Typical values of maximum delivery pressures generally available from different types of compressor are,

(i) Single stage compressor, for delivery pressure up to 5 bar
(ii) Two stage compressor, for delivery pressure between 5 and 35 bar
(iii) Three stage compressor, for delivery pressure between 35 and 85 bar
(iv) Four stage compressor, for delivery pressure more than 85 bar

(c) ***Based on capacity of compressors:*** Compressors can also be classified depending upon the capacity of compressor or air delivered per unit time. Typical values of capacity for different compressors are given as;

(i) Low capacity compressors, having air delivery capacity of 0.15 m^3/s or less
(ii) Medium capacity compressors, having air delivery capacity between 0.15 and 5 m^3/s.
(iii) High capacity compressors, having air delivery capacity more than 5 m^3/s.

(d) ***Based on highest pressure developed:*** Depending upon the maximum pressure available from compressor they can be classified as low pressure, medium pressure, high pressure and super high pressure compressors. Typical values of maximum pressure developed for different compressors are as under;

(i) Low pressure compressor, having maximum pressure up to 1 bar
(ii) Medium pressure compressor, having maximum pressure from 1 to 8 bar
(iii) High pressure compressor, having maximum pressure from 8 to 10 bar
(iv) Super high pressure compressor, having maximum pressure more than 10 bar.

16.2 RECIPROCATING COMPRESSORS

Reciprocating compressor has piston cylinder arrangement as shown in Fig. 16.2.

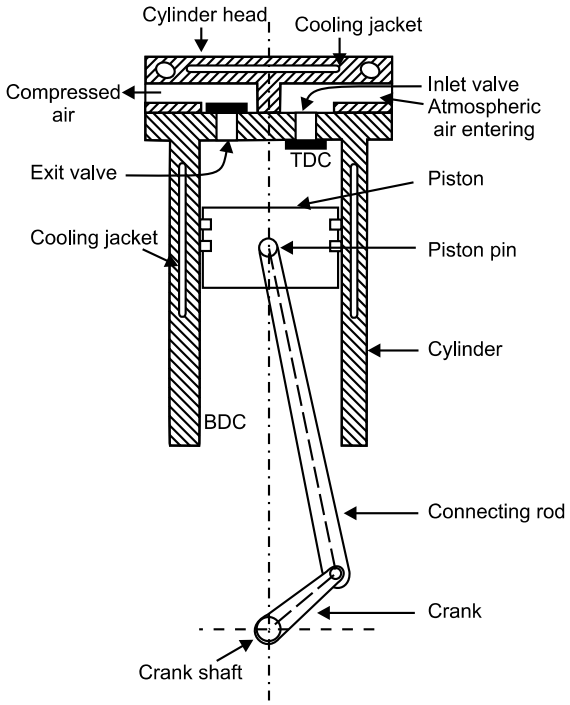


Fig. 16.2 *Line diagram of reciprocating compressor*

Reciprocating compressor has piston, cylinder, inlet valve, exit valve, connecting rod, crank, piston pin, crank pin and crank shaft. Inlet valve and exit valves may be of spring loaded type which get opened and closed due to pressure differential across them. Let us consider piston to be at top dead centre (TDC) and move towards bottom dead centre (BDC). Due to this piston movement from TDC to BDC suction pressure is created causing opening of inlet valve. With this opening of inlet valve and suction pressure the atmospheric air enters the cylinder.

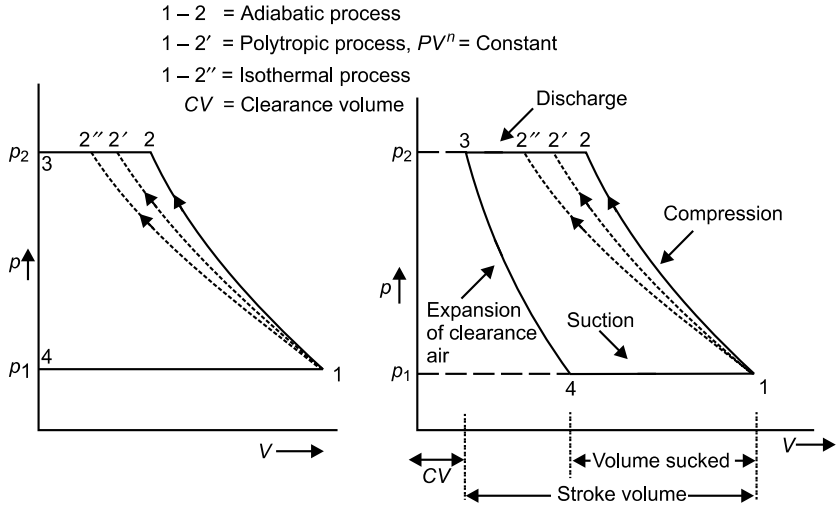
Air gets into cylinder during this stroke and is subsequently compressed in next stroke with both inlet valve and exit valve closed. Both inlet valve and exit valves are of plate type and spring loaded so as to operate automatically as and when sufficient pressure difference is available to cause deflection in spring of valve plates to open them. After piston reaching BDC it reverses its motion and compresses the air inducted in previous stroke. Compression is continued till the pressure of air inside becomes sufficient to cause deflection in exit valve. At the moment when exit valve plate gets lifted the exhaust of compressed air takes place. This piston again reaches TDC from where downward piston movement is again accompanied by suction. This is how reciprocating compressor keeps on working as flow device. In order to counter for the heating of piston-cylinder arrangement during compression the provision of cooling the cylinder is there in the form of cooling jackets in the body. Reciprocating compressor described above has suction, compression and discharge as three prominent processes getting completed in two strokes of piston or one revolution of crank shaft.

16.3 THERMODYNAMIC ANALYSIS

Compression of air in compressor may be carried out following number of thermodynamic processes such as isothermal compression, polytropic compression or adiabatic compression. Figure 16.3 shows the thermodynamic cycle involved in compression. Theoretical cycle is shown neglecting clearance volume but in actual cycle clearance volume can not be negligible. Clearance volume is necessary in order to prevent collision of piston with cylinder head, accommodating valve mechanism etc. Compression process is shown by process 1–2, 1–2′, 1–2″ following adiabatic, polytropic and isothermal processes.

On *p-V* diagram process 4–1 shows the suction process followed by compression during 1–2 and discharge through compressor is shown by process 2–3.

Air enters compressor at pressure p_1 and is compressed up to p_2. Compression work requirement can be estimated from the area below the each compression process. Area on *p–V* diagram shows that work requirement shall be minimum with isothermal process 1–2″. Work requirement is maximum with process 1–2 i.e. adiabatic process. As a designer one shall be interested in a compressor having minimum compression work requirement. Therefore, ideally compression should occur isothermally for minimum work input. In practice it is not possible to have isothermal compression because constancy of temperature



(a) Compression cycle without clearance (b) Compression cycle with clearance

Fig. 16.3 *Compression cycle on* p-V *diagram*

during compression can not be realized. Generally, compressors run at substantially high speed while isothermal compression requires compressor to run at very slow speed so that heat evolved during compression is dissipated out and temperature remains constant. Actually due to high speed running of compressor the compression process may be assumed to be near adiabatic or polytropic process following law of compression as $pV^n = C$ with value of '*n*' varying between 1.25 and 1.35 for air. Compression process following three processes is also shown on *T-s* diagram in Fig. 16.4. It is thus obvious that actual compression process should be compared with isothermal compression process. A mathematical parameter called isothermal efficiency is defined for quantifying the degree of deviation of actual compression process from ideal compression process. Isothermal efficiency is defined by the ratio of isothermal work and actual indicated work in reciprocating compressor.

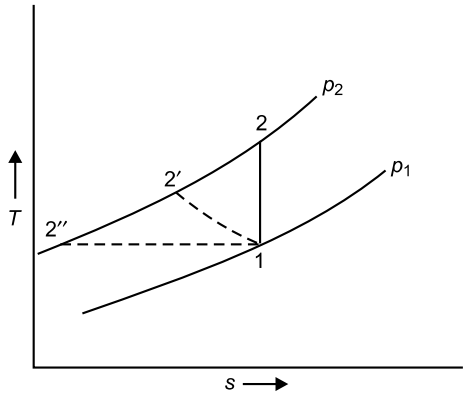


Fig. 16.4 *Compression process on* T-s *diagram*

$$\text{Isothermal efficiency} = \frac{\text{Isothermal work}}{\text{Actual indicated work}}$$

Practically, compression process is attempted to be closed to isothermal process by air/water cooling, spraying cold water during compression process. In case of multi-stage compression process the compression in different stages is accompanied by intercooling in between the stages.

Mathematically, for the compression work following polytropic process, $pV^n = C$. Assuming negligible clearance volume the cycle work done,

W_c = Area on p-V diagram

$$= \left[p_2V_2 + \left(\frac{p_2V_2 - p_1V_1}{n-1} \right) \right] - p_1V_1$$

$$= \left(\frac{n}{n-1} \right) [p_2V_2 - p_1V_1]$$

$$= \left(\frac{n}{n-1} \right) (p_1V_1) \left[\frac{p_2V_2}{p_1V_1} - 1 \right]$$

$$W_c = \left(\frac{n}{n-1} \right) (p_1V_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{(n-1)}{n}} - 1 \right]$$

$$\boxed{W_c = \left(\frac{n}{n-1} \right) (mRT_1) \left[\left(\frac{p_2}{p_1} \right)^{\frac{(n-1)}{n}} - 1 \right]}$$

or,

$$\boxed{W_c = \left(\frac{n}{n-1} \right) mR\ (T_2 - T_1)}$$

In case of compressor having isothermal compression process, $n = 1$, i.e. $p_1V_1 = p_1V_2$

$$W_{c,\ \text{iso}} = p_2V_2 + p_1V_1 \ln r - p_1V_1$$

$$W_{c,\ \text{iso}} = p_1V_1 \ln r, \text{ where } r = \frac{V_1}{V_2}$$

In case of compressor having adiabatic compression process, $n = \gamma$

$$\boxed{W_{c,\ \text{adiabatic}} = \left(\frac{\gamma}{\gamma - 1}\right) mR\ (T_2 - T_1)}$$

or,

$$W_{c,\ \text{adiabatic}} = mC_p\ (T_2 - T_1)$$

$$\boxed{W_{c,\ \text{adiabatic}} = m\ (h_2 - h_1)}$$

Hence, isothermal efficiency, $$\boxed{\eta_{\text{iso}} = \frac{p_1V_1 \ln r}{\left(\frac{n}{n-1}\right)(p_1V_1)\left[\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right]}}$$

The isothermal efficiency of a compressor should be close to 100% which means that actual compression should occur following a process close to isothermal process. For this the mechanism be derived to maintain constant temperature during compression process. Different arrangements which can be used are:

(i) Faster heat dissipation from inside of compressor to outside by use of fins over cylinder. Fins facilitate quick heat transfer from air being compressed to atmosphere so that temperature rise during compression can be minimized.

(ii) Water jacket may be provided around compressor cylinder so that heat can be picked by cooling water circulating through water jacket. Cooling water circulation around compressor regulates rise in temperature to great extent.

(iii) The water may also be injected at the end of compression process in order to cool the air being compressed. This water injection near the end of compression process requires special arrangement in compressor and also the air gets mixed with water and needs to be separated out before being used. Water injection also contaminates the lubricant film on inner surface of cylinder and may initiate corrosion etc. The water injection is not popularly used.

(iv) In case of multistage compression in different compressors operating serially, the air leaving one compressor may be cooled up to ambient state or somewhat high temperature before being injected into subsequent compressor. This cooling of fluid being compressed between two consecutive compressors is called intercooling and is frequently used in case of multistage compressors.

Considering clearance volume: With clearance volume the cycle is represented on Fig. 16.3 (b). The work done for compression of air polytropically can be given by the area enclosed in cycle 1–2–3–4. Clearance volume in compressors varies from 1.5% to 35% depending upon type of compressor.

$$W_{c,\ \text{with CV}} = \text{Area1234}$$

$$= \left(\frac{n}{n-1}\right)(p_1V_1)\left[\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right] - \left(\frac{n}{n-1}\right)(p_4V_4)\left[\left(\frac{p_3}{p_4}\right)^{\frac{(n-1)}{n}} - 1\right]$$

Here $p_1 = p_4,\ p_2 = p_3$

$$W_{c,\text{ with }CV} = \left(\frac{n}{n-1}\right)(p_1V_1)\left[\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right] - \left(\frac{n}{n-1}\right)(p_1V_4)\cdot\left[\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right]$$

$$\boxed{W_{c,\text{ with }CV} = \left(\frac{n}{n-1}\right)p_1\cdot\left[\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right]\cdot(V_1 - V_4)}$$

In the cylinder of reciprocating compressor ($V_1 - V_4$) shall be the actual volume of air delivered per cycle. $V_d = V_1 - V_4$. This ($V_1 - V_4$) is actually the volume of air inhaled in the cycle and delivered subsequently.

$$\boxed{W_{c,\text{ with }CV} = \left(\frac{n}{n-1}\right)p_1V_d\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]}$$

If air is considered to behave as perfect gas then pressure, temperature, volume and mass can be inter related using perfect gas equation. The mass at state 1 may be given as m_1, mass at state 2 shall be m_1, but at state 3 after delivery mass reduces to m_2 and at state 4 it shall be m_2.

So, at state 1, $p_1V_1 = m_1RT_1$
at state 2, $p_2V_2 = m_1RT_2$
at state 3, $p_3V_3 = m_2RT_3$ or, $p_2V_3 = m_2RT_3$
at state 4, $p_4V_4 = m_2RT_4$, or $p_1V_4 = m_2RT_4$

Ideally there shall be no change in temperature during suction and delivery i.e., $T_4 = T_1$ and $T_2 = T_3$. From earlier equation,

$$W_{c,\text{ with }CV} = \left(\frac{n}{n-1}\right)p_1(V_1 - V_4)\left[\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right]$$

Temperature and pressure can be related as,

$$\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} = \frac{T_2}{T_1}$$

and $$\left(\frac{p_4}{p_3}\right)^{\frac{(n-1)}{n}} = \frac{T_4}{T_3} \Rightarrow \left(\frac{p_1}{p_2}\right)^{\frac{(n-1)}{n}} = \frac{T_4}{T_3}$$

Substituting,

$$W_{c,\text{ with }CV} = \left(\frac{n}{n-1}\right)(m_1RT_1 - m_2RT_4)\left[\frac{T_2}{T_1} - 1\right]$$

Substituting for constancy of temperature during suction and delivery.

$$W_{c,\text{ with }CV} = \left(\frac{n}{n-1}\right)(m_1RT_1 - m_2RT_1)\left[\frac{T_2 - T_1}{T_1}\right]$$

or,

$$\boxed{W_{c,\text{ with }CV} = \left(\frac{n}{n-1}\right)(m_1 - m_2)\,R(T_2 - T_1)}$$

Thus, $(m_1 - m_2)$ denotes the mass of air sucked or delivered. For unit mass of air delivered the work done per kg of air can be given as,

$$W_{c,\text{ with } CV} = \left(\frac{n}{n-1}\right) R(T_2 - T_1), \text{ per kg of air}$$

Thus from above expressions it is obvious that the clearance volume reduces the effective swept volume i.e. the mass of air handled but the work done per kg of air delivered remains unaffected.

From the cycle work estimated as above the theoretical power required for running compressor shall be as given ahead.

For single acting compressor running with N rpm, power input required, assuming clearance volume.

$$\text{Power required} = \left[\left(\frac{n}{n-1}\right) p_1(V_1 - V_4)\left\{\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right\}\right] \times N$$

$$\text{for double acting compressor, power} = \left[\left(\frac{n}{n-1}\right) p_1(V_1 - V_4)\left\{\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right\}\right] \times 2N$$

Volumetric efficiency: Volumetric efficiency of compressor is the measure of the deviation from volume handling capacity of compressor. Mathematically, the volumetric efficiency is given by the ratio of actual volume of air sucked and swept volume of cylinder. Ideally the volume of air sucked should be equal to the swept volume of cylinder, but it is not so in actual case. Practically the volumetric efficiency lies between 60 and 90%.

Volumetric efficiency can be overall volumetric efficiency and absolute volumetric efficiency as given below:

$$\text{Overall volumetric efficiency} = \frac{\text{Volume of free air sucked into cylinder}}{\text{Swept volume of LP cylinder}}$$

or

$$\text{(Volumetric efficiency referred to free air conditions)} = \frac{\text{(Mass of air delivered per unit time)}}{\left(\begin{array}{l}\text{Mass of air corresponding to swept}\\ \text{volume of LP cylinder per unit time}\\ \text{for free air conditions}\end{array}\right)}$$

Here free air condition refers to the standard conditions. Free air condition may be taken as 1 atm or 1.01325 bar and 15°C or 288 K. Consideration for free air is necessary as otherwise the different compressors can not be compared using volumetric efficiency because specific volume or density of air varies with altitude. It may be seen that a compressor at datum level (sea level) shall deliver large mass than the same compressor at high altitude.

This concept is used for giving the capacity of compressor in terms of 'free air delivery' (FAD). "Free air delivery is the volume of air delivered being reduced to free air conditions." In case of air the free air delivery can be obtained using perfect gas equation as,

$$\frac{p_a \cdot V_a}{T_a} = \frac{p_1 (V_1 - V_4)}{T_1} = \frac{p_2 (V_2 - V_3)}{T_2}$$

where subscript a or p_a, V_a, T_a denote properties at free air conditions

or, $$V_a = \frac{p_1 \cdot T_a \cdot (V_1 - V_4)}{p_a \cdot T_1} = \text{FAD per cycle.}$$

This volume V_a gives 'free air delivered' per cycle by the compressor.

Absolute volumetric efficiency can be defined, using NTP conditions in place of free air conditions.

$$\text{Absolute volumetric efficiency} = \frac{\text{Volume of air sucked into cylinder at NTP}}{\text{Swept volume of LP cylinder}}$$

$$= \frac{(\text{Mass of air delivered per unit time})}{\left(\begin{matrix}\text{Mass of air corresponding to swept volume} \\ \text{of LP cylinder per unit time estimated at NTP}\end{matrix}\right)}$$

Thus, volumetric efficiency referred to free air conditions.

$$\eta_{\text{vol.}} = \frac{\text{Volume of air sucked referred to free air conditions (FAD)}}{\text{Swept volume}}$$

$$= \frac{V_a}{(V_1 - V_3)}$$

$$= \frac{p_1 T_a (V_1 - V_4)}{p_a T_1 (V_1 - V_3)}$$

$$\eta_{\text{vol.}} = \left(\frac{p_1 T_a}{p_a T_1}\right) \cdot \left\{\frac{(V_s + V_c) - V_4}{V_s}\right\}.$$

Here, V_s is swept volume, $V_s = V_1 - V_3$

and V_c is clearance volume, $V_c = V_3$

$$\eta_{\text{vol.}} = \left(\frac{p_1 T_a}{p_a T_1}\right) \left\{1 + \left(\frac{V_c}{V_s}\right) - \left(\frac{V_4}{V_s}\right)\right\}$$

Here,

$$\frac{V_4}{V_s} = \frac{V_4}{V_c} \cdot \frac{V_c}{V_s}$$

$$= \left(\frac{V_4}{V_3} \cdot \frac{V_c}{V_s}\right)$$

Let the ratio of clearance volume to swept volume be given by C.i.e. $\frac{V_c}{V_s} = C$.

$$\eta_{\text{vol.}} = \left(\frac{p_1 T_a}{p_a T_1}\right) \cdot \left\{1 + C - C \cdot \left(\frac{V_4}{V_3}\right)\right\}$$

or, $$\boxed{\eta_{\text{vol.}} = \left(\frac{p_1 T_a}{p_a T_1}\right) \cdot \left\{1 + C - C \cdot \left(\frac{p_2}{p_1}\right)^{1/n}\right\}}$$

Volumetric efficiency depends on ambient pressure and temperature, suction pressure and temperature, ratio of clearance to swept volume, and pressure limits. Volumetric efficiency increases with decrease in pressure ratio in compressor.

16.4 ACTUAL INDICATOR DIAGRAM

Theoretical indicator diagram of reciprocating compressor as shown in earlier discussion refers to the ideal state of operation of compressor. The practical limitations, when considered in the indicator diagram yield actual indicator diagram as shown in Fig. 16.5.

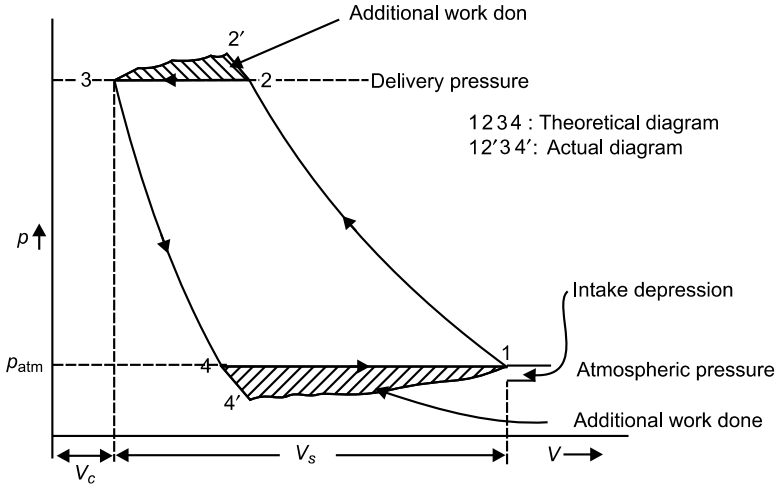


Fig. 16.5 *Actual indicator diagram*

Actual *p-V* diagram varies from theoretical *p-V* diagram due to following: Compressor has mechanical types of valves so the instantaneous opening and closing of valves can never be achieved. Also during suction and discharge there occurs throttling due to reduction in area of flow across inlet and exit valve. 1234 shows theoretical indicator diagram and actual indicator diagram is shown by 12′34′ on *p-V* diagram. Compression process 1–2 ends at state 2. At state 2 exit valve should open instantaneously which does not occur and also due to restricted opening there shall be throttling causing drop in pressure. Due to time lag in opening of exit valve compression process is continued up to 2′. Thus, additional work is done during delivery from compressor as shown by hatched area 22′3.

After delivery stroke the inlet valve should theoretically open at 4 but does not open at this point instead is opened fully at 4′. Shift from state 4 to 4′ is there due to inertia in opening of valve throttling, gradual opening, and friction losses etc. Thus it is seen that during suction there occurs intake depression as shown in actual indicator diagram. Work required as shown in actual indicator diagram is more than theoretical diagram. In order to have compressor close to ideal compressor with minimum losses it shall be desired to have actual indicator diagram close to theoretical diagram, which requires less inertia and efficient operation of valves. Friction losses in pipings and across valves should be minimized.

16.5 MULTISTAGE COMPRESSION

Multistage compression refers to the compression process completed in more than one stage i.e. a part of compression occurs in one cylinder and subsequently compressed air is sent to subsequent cylinders for further compression. In case it is desired to increase the compression ratio of compressor then multi-stage compression becomes inevitable. If we look at the expression for volumetric efficiency then it shows that the volumetric efficiency decreases with increase in pressure ratio. This aspect can also be explained using *p*-*V* representation shown in Fig. 16.6.

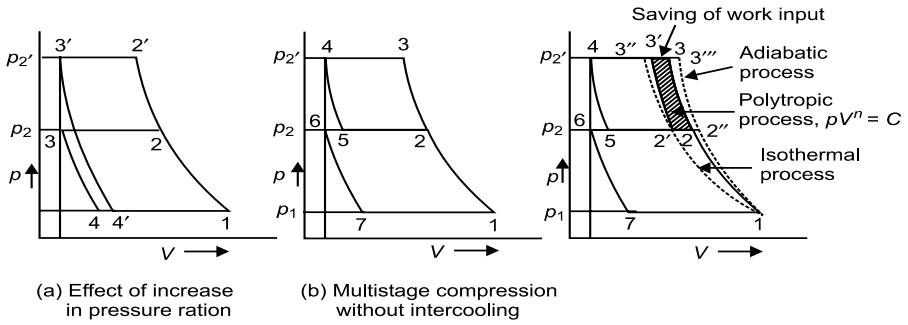


Fig. 16.6 *Multistage compression*

Figure 16.6 a, shows that by increasing pressure ratio i.e. increasing delivery pressure the volume of air being sucked goes on reducing as evident from cycles 1234 and 12′3′4′. Let us increase pressure from p_2 to $p_{2'}$ and this shall cause the suction process to get modified from 4–1 to 4′–1. Thus volume sucked reduces from $(V_1 - V_4)$ to $(V_1 - V_{4'})$ with increased pressure ratio from $\left(\frac{p_2}{p_1}\right)$ to $\left(\frac{p_2'}{p_1}\right)$, thereby reducing the free air delivery while swept volume remains same.

Therefore, the volumetric efficiency reduces with increasing pressure ratio in compressor with single stage compression. Also for getting the same amount of free air delivery the size of cylinder is to be increased with increasing pressure ratio. The increase in pressure ratio also requires sturdy structure from mechanical strength point of view for withstanding large pressure difference.

The solution to number of difficulties discussed above lies in using the multistage compression where compression occurs in parts in different cylinders one after the other. Figure 16.6 b, shows the multistage compression occurring in two stages. Here first stage of compression occurs in cycle 12671 and after first stage compression partly compressed air enters second stage of compression and occurs in cycle 2345. In case of multistage compression the compression in first stage occurs at low temperature and subsequent compression in following stages occurs at higher temperature. The compression work requirement depends largely upon the average temperature during compression. Higher average temperature during compression has larger work requirement compared to low temperature so it is always desired to keep the low average temperature during compression.

Apart from the cooling during compression the temperature of air at inlet to compressor can be reduced so as to reduce compression work. In multistage compression the partly compressed air leaving first stage is cooled up to ambient air temperature in intercooler and then sent to subsequent cylinder (stage) for compression. Thus, intercoolers when put between the stages reduce the compression work and compression is called intercooled compression. Intercooling is called perfect when temperature at

inlet to subsequent stages of compression is reduced to ambient temperature. Figure 16.6 c, shows multistage (two stage) intercooled compression. Intercooling between two stages causes temperature drop from 2 to 2′ i.e. discharge from first stage (at 2) is cooled up to the ambient temperature state (at 2′) which lies on isothermal compression process 1–2′–3″. In the absence of intercooling the discharge from first stage shall enter at 2. Final discharge from second stage occurs at 3′ in case of intercooled compression compared to discharge at 3 in case of non-intercooled compression. Thus, intercooling offers reduced work requirement by the amount shown by area 22′3′3 on *p-V* diagram. If the inter-cooling is not perfect then the inlet state to second/subsequent stage shall not lie on the isothermal compression process line and this state shall lie between actual discharge state from first stage and isothermal compression line.

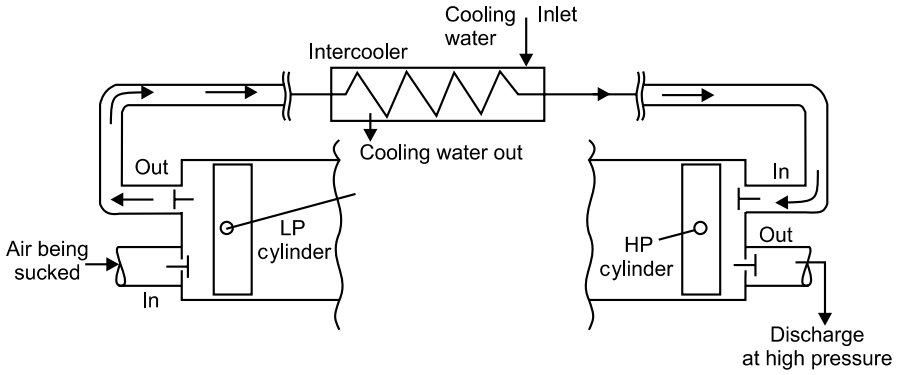


Fig. 16.7 *Schematic for two-stage compression (Multistage compressor)*

Figure 16.7 shows the schematic of multi stage compressor (double stage) with intercooler between stages. *T–s* representation is shown in Fig. 16.8. The total work requirement for running this shall be algebraic summation of work required for low pressure (LP) and high pressure (HP) stages. The size of HP cylinder is smaller than LP cylinder as HP cylinder handles high pressure air having smaller specific volume.

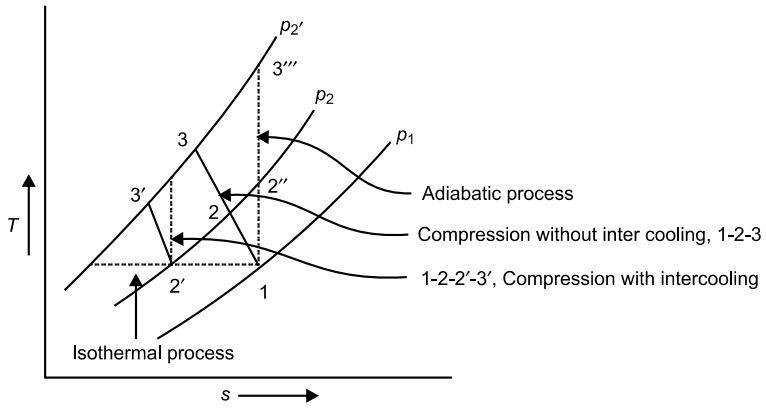


Fig. 16.8 T-s *representation of multistage compression*

Mathematical analysis of multistage compressor is done with following assumptions:

(i) Compression in all the stages is done following same index of compression and there is no pressure drop in suction and delivery pressures in each stage. Suction and delivery pressure remains constant in the stages.

(ii) There is perfect intercooling between compression stages.
(iii) Mass handled in different stages is same i.e. mass of air in LP and HP stages are same.
(iv) Air behaves as perfect gas during compression.

From combined *p-V* diagram the compressor work requirement can be given as,

Work requirement in LP cylinder, $W_{LP} = \left(\frac{n}{n-1}\right) p_1 V_1 \left\{\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right\}$

Work requirement in HP cylinder, $W_{HP} = \left(\frac{n}{n-1}\right) p_2 V_2 \left\{\left(\frac{p_{2'}}{p_2}\right)^{\frac{(n-1)}{n}} - 1\right\}$

For perfect intercooling, $p_1 V_1 = p_2 V_{2'}$ and

$$W_{HP} = \left(\frac{n}{n-1}\right) p_2 V_{2'} \left\{\left(\frac{p_{2'}}{p_2}\right)^{\frac{(n-1)}{n}} - 1\right\}$$

Therefore, total work requirement, $W_c = W_{LP} + W_{HP}$, for perfect intercooling

$$W_c = \left(\frac{n}{n-1}\right)\left[p_1 V_1 \left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} + p_2 V_{2'} \left\{\left(\frac{p_{2'}}{p_2}\right)^{\frac{n-1}{n}} - 1\right\}\right]$$

$$= \left(\frac{n}{n-1}\right)\left[p_1 V_1 \left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} + p_1 V_1 \left\{\left(\frac{p_{2'}}{p_2}\right)^{\frac{n-1}{n}} - 1\right\}\right]$$

$$\boxed{W_c = \left(\frac{n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_{2'}}{p_2}\right)^{\frac{n-1}{n}} - 2\right]}$$

Power required can be given in HP as below, considering speed to be *N* rpm.

$$\text{Power} = \frac{W_c \times N}{75 \times 60}, \text{ HP}$$

If we look at compressor work then it shows that with the initial and final pressures p_1 and $p_{2'}$ remaining same the intermediate pressure p_2 may have value floating between p_1 and $p_{2'}$ and change the work requirement W_c. Thus, the compressor work can be optimized with respect to intermediate pressure p_2. Mathematically, it can be differentiated with respect to p_2.

$$\frac{dW_c}{dp_2} = \frac{d}{dp_2}\left[\left(\frac{n}{n-1}\right) p_1 V_1 \left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_{2'}}{p_2}\right)^{\frac{n-1}{n}} - 2\right\}\right]$$

$$\frac{dW_c}{dp_2} = \left(\frac{n}{n-1}\right) p_1 V_1 \cdot \frac{d}{dp_2}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_{2'}}{p_2}\right)^{\frac{n-1}{n}} - 2\right\}$$

$$\frac{dW_c}{dp_2} = \left(\frac{n}{n-1}\right) p_1 V_1 \left\{\left(\frac{n-1}{n}\right) \cdot p_1^{\frac{1-n}{n}} \cdot p_2^{\frac{-1}{n}} - \left(\frac{n-1}{n}\right) \cdot p_{2'}^{\frac{n-1}{n}} \; p_2^{\frac{1-2n}{n}}\right\}$$

Equating, $\frac{dW_c}{dp_2} = 0$ yields.

$$p_1^{\frac{1-n}{n}} \cdot p_2^{\frac{-1}{n}} = p_{2'}^{\frac{n-1}{n}} \cdot p_2^{\frac{1-2n}{n}}$$

or $$p_2^{\frac{-2+2n}{n}} = p_{2'}^{\frac{n-1}{n}} \cdot p_1^{\frac{n-1}{n}}$$

or $$p_2^{2\left(\frac{n-1}{n}\right)} = (p_1 \cdot p_{2'})^{\left(\frac{n-1}{n}\right)} \text{ or, } \quad p_2^2 = p_1 \cdot p_{2'}, \quad p_2 = \sqrt{p_1 \cdot p_{2'}}$$

or $$\boxed{\frac{p_2}{p_1} = \frac{p_{2'}}{p_2}} \text{ or } \boxed{\frac{p_2}{p_1} = \left(\frac{p_{2'}}{p_1}\right)^{1/2}}$$

Pressure ratio in Ist stage = Pressure ratio in IInd stage

Thus, it is established that the compressor work requirement shall be minimum when the pressure ratio in each stage is equal.

In case of multiple stages, say i number of stages, for the delivery and suction pressures of p_{i+1} and p_1 the optimum stage pressure ratio shall be,

Optimum stage pressure ratio = $\left(\frac{p_{i+1}}{p_1}\right)^{1/i}$ for pressures at stages being p_1, p_2, p_3, p_4, p_5, …, p_{i-1}, p_i, p_{i+1}

Minimum work required in two stage compressor can be given by

$$W_{c,\,\min} = \left(\frac{n}{n-1}\right) p_1 V_1 \cdot 2 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

For i number of stages, minimum work,

$$W_{c,\,\min} = \left(\frac{n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} + \left(\frac{p_4}{p_3}\right)^{\frac{n-1}{n}} + \cdots + \left(\frac{p_{i+1}}{p_i}\right)^{\frac{n-1}{n}} - i\right]$$

$$W_{c,\,\min} = i \cdot \left(\frac{n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

or,

$$W_{c,\,\min} = i \cdot \left(\frac{n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_{i+1}}{p_i}\right)^{\frac{n-1}{n \cdot i}} - 1\right]$$

It also shows that for optimum pressure ratio the work required in different stages remains same for the assumptions made for present analysis. Due to pressure ratio being equal in all stages the temperature ratios and maximum temperature in each stage remains same for perfect intercooling.

Cylinder dimensions: In case of multistage compressor the dimension of cylinders can be estimated basing upon the fact that the mass flow rate of air across the stages remains same. For perfect intercooling the temperature of air at suction of each stage shall be same.

If the actual volume sucked during suction stroke is V_1, V_2, V_3 …, for different stages then by perfect gas law, $p_1V_1 = RT_1$, $p_2V_2 = RT_2$, $p_3V_3 = RT_3$

For perfect intercooling ($T_1 = T_2 = T_3 = \ldots$) so

$$p_1V_1 = RT_1,\ p_2V_2 = RT_1,\ p_3V_3 = RT_1,$$

or $$p_1V_1 = p_2V_2 = p_3V_3 = \ldots$$

If the volumetric efficiency of respective stages in η_{V_1}, η_{V_2}, η_{V_3}, …

Then theoretical volume of cylinder 1, $V_{1,\,th} = \dfrac{V_1}{\eta_{V_1}}$; $V_1 = \eta_{V_1} \cdot V_{1,\,th}$

cylinder 2, $$V_{2,\,th} = \frac{V_2}{\eta_{V_2}};\ V_2 = \eta_{V2} \cdot V_{2,\,th}$$

cylinder 3, $$V_{3,\,th} = \frac{V_3}{\eta_{V_3}};\ V_3 = \eta_{V3} \cdot V_{3,\,th}$$

Substituting,

$$p_1 \cdot \eta_{V1} \cdot V_{1,\,th} = p_2 \cdot \eta_{V2} \cdot V_{2,\,th} = p_3 \cdot \eta_{V3} \cdot V_{3,\,th} = \ldots$$

Theoretical volumes of cylinder can be given using geometrical dimensions of cylinder as diameters D_1, D_2, D_3, … and stroke lengths L_1, L_2, L_3, ….

or $$V_{1,\,th} = \frac{\pi}{4} D_1^2 \cdot L_1$$

$$V_{2,\,th} = \frac{\pi}{4} D_2^2 \cdot L_2$$

$$V_{3,\,th} = \frac{\pi}{4} D_3^2 \cdot L_3$$

or,

$$p_1 \cdot \eta_{Vi} \cdot \frac{\pi}{4} D_1^2 L_1 = p_2 \cdot \eta_{V2} \cdot \frac{\pi}{4} D_2^2 L_2 = p_3 \cdot \eta_{V3} \cdot \frac{\pi}{4} D_3^2 \cdot L_3 = \ldots$$

$$p_1\ \eta_{V1} \cdot D_1^2 L_1 = p_2 \cdot \eta_{V2} \cdot D_2^2 \cdot L_2 = p_3 \cdot \eta_{V3} \cdot D_3^2 \cdot L_3 = \ldots$$

If the volumetric efficiency is same for all cylinders, i.e. $\eta_{V1} = \eta_{V2} = \eta_{V3} = \ldots$ and stroke for all cylinder is same i.e. $L_1 = L_2 = L_3 = \ldots$

Then, $$D_1^2\, p_1 = D_2^2\, p_2 = D_3^2\, p_3 = \ldots$$

These generic relations may be used for getting the ratio of diameters of cylinders of multistage compression.

Energy balance: Energy balance may be applied on the different components constituting multistage compression.

For LP stage the steady flow energy equation can be written as below:

$$m \cdot h_1 + W_{LP} = mh_2 + Q_{LP}$$
$$Q_{LP} = W_{LP} - m\,(h_2 - h_1)$$
$$Q_{LP} = W_{LP} - mC_p\,(T_2 - T_1)$$

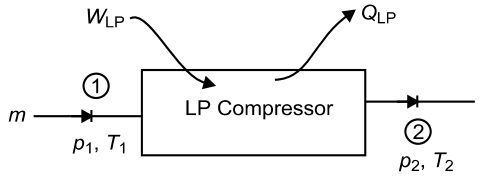


Fig. 16.9

For intercooling (Fig. 16.10) between LP and HP stage steady flow energy equation shall be;

$$mh_2 = mh_{2'} + Q_{Int}$$

or,

$$Q_{Int} = m\,(h_2 - h_{2'})$$
$$Q_{Int} = mC_p\,(T_2 - T_{2'})$$

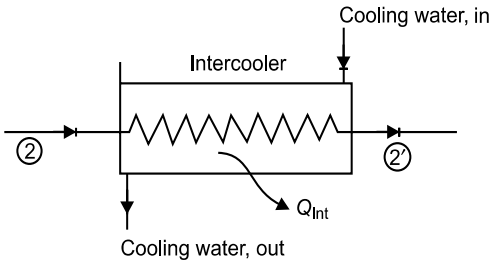


Fig. 16.10

For HP stage (Fig. 16.11) the steady flow energy equation yields.

$$mh_{2'} + W_{HP} = mh_{3'} + Q_{HP}$$
$$Q_{HP} = W_{HP} + m\,(h_{2'} - h_{3'})$$
$$Q_{HP} = W_{HP} + mC_p\,(T_{2'} - T_{3'}) = W_{HP} - mC_p\,(T_3' - T_2)$$

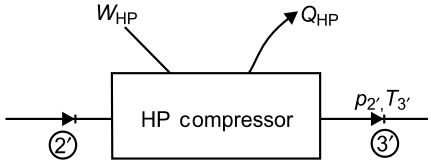


Fig. 16.11

In case of perfect intercooling and optimum pressure ratio, $T_{2'} = T_1$ and $T_2 = T_{3'}$
Hence for these conditions,

$$Q_{LP} = W_{LP} - mC_p\,(T_2 - T_1)$$

$$Q_{\text{Int}} = mC_p\,(T_2 - T_1)$$
$$Q_{\text{HP}} = W_{\text{HP}} - mC_p\,(T_2 - T_1)$$

Total heat rejected during compression shall be the sum of heat rejected during compression and heat extracted in intercooler for perfect intercooling.

Heat rejected during compression for polytropic process $= \left(\frac{\gamma - n}{\gamma - 1}\right) \times$ Work

16.6 CONTROL OF RECIPROCATING COMPRESSORS

Output from the compressors can be controlled by different measures which regulate the compressor output. In practical applications the compressors are fitted with air receiver to store the high pressure air and supply as and when required. Therefore, the compressors are run only for the duration required to maintain the limiting pressure inside receiver. When the pressure inside receiver starts dropping down then the compressor again starts supplying compressed air till the level is restored. Different ways for this control are based on throttle, clearance, blow off control and speed control.

(i) Throttle control has the regulation of opening/closing of inlet valve so that the quantity of air entering can be varied. With partial opening of inlet valve throttling occurs at valve and quantity of air entering is reduced while the pressure ratio gets increased.

(ii) Clearance control is the arrangement in which the clearance volume is increased when pressure ratio exceeds the limit. For this cylinder has clearance openings which are closed by spring loaded valves. Whenever pressure exceeds, then the clearance openings get opened and the increased clearance volume reduces maximum pressure.

(iii) Blow-off control has spring loaded safety valve or by pass valve for blowing out excess air in receiver. After release of excess air automatically the valve gets closed on its own.

(iv) Speed of compressor can also be controlled by regulating the prime mover, thereby regulating compressor output. Thus, compressor is run on variable speed for its' control.

16.7 RECIPROCATING AIR MOTOR

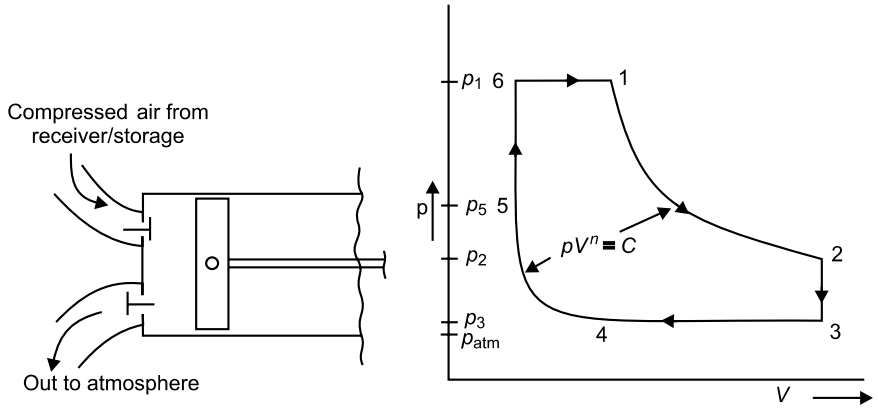
Air motors are actually prime movers run using the compressed air. It is used extensively in the applications where electric motor/IC engine/Gas turbine etc. can not be used due to fire hazard, specially in coal/oil mining applications. Apart from use of air motors in coal/oil mines these are also used for running pneumatic tools in workshops and manufacturing/assembly lines. Air motors have a reciprocating piston-cylinder arrangement where compressed air is admitted in cylinder with inlet valve open for limited period in suction stroke and this causes piston movement to yield shaft work. Thus air motors are reverse of air compressors. Schematic for air motor and p–V diagram for cycle used is shown in Fig. 16.12.

Compressed air enters the cylinder up to state 1 and expands up to state 2. Expansion process is generally polytropic and yields expansion work. Expansion is terminated even before the atmospheric pressure is reached because in later part of expansion the positive work is less than negative work. The exhaust valve opens causing drop in pressure from 2 to 3. Exhaust pressure is slightly more or nearly equal to atmospheric pressure. The exhaust of air occurs at constant pressure up to state 4 where the closing of exhaust valve occurs. The inlet valve is opened at state 5 causing sudden rise in pressure upto state 6 after which piston displacement begins and is continued with compressed air inlet up to state 1 after which expansion occurs with inlet valve closed. Here point of cut-off is at state 1 which can be

suitably varied depending upon output required from air motor. Here, total work available is the result of non-expansive work during 6–1 and expansion work during 1–2.

Work available from air motor can be quantified by the area enclosed on *p-V* diagram.

$$W_{\text{motor}} = p_1 (V_1 - V_6) + \left(\frac{p_1V_1 - p_2V_2}{n-1} \right) - (V_3 - V_4)\, p_3 - \left(\frac{p_5V_5 - p_4V_4}{n-1} \right)$$



(a) Schematic for air motor (b) *p-V* diagram for air motor cycle

Fig. 16.12 *Reciprocating air motor*

Air motor suffers from the drawback that in case of expansion the air temperature may even go below the dew point temperature and moisture may condense or even get frozen also inside the cylinder. This phenomenon may be prevented by increasing the temperature of air entering cylinder so that temperature after expansion does not go near the dew point temperature. Some times air is heated before being supplied to air motors in order to avoid condensation.

16.8 ROTARY COMPRESSORS

Rotary compressors are those compressors in which rotating action is used for compression of fluid. Rotary air compressors have capability of running at high speeds up to 40,000 rpm and can be directly coupled to any prime mover such as electric motor, turbine etc. due to compact design, no balancing problem and less no. of sliding parts. Comparative study of rotary compressor with reciprocating compressor shows that rotary compressors can be used for delivering large quantity of air but the maximum pressure at delivery is less compared to reciprocating compressors. Generally, rotary compressors can yield delivery pressure up to 10 bar and free air delivery of 3000 m^3/min. Rotary compressors are less bulky, and offer uniform discharge compared to reciprocating compressor even in the absence of big size receiver. Lubrication requirement and wear and tear is less due to rotary motion of parts in rotary compressors compared to reciprocating compressors.

Rotary compressors may work on the principle of positive displacement and dynamic action both. Rotary compressors having positive displacement may be of following types:

(i) Roots blower

(ii) Screw type or Helical type compressor

(iii) Vane type compressor

Rotary compressors employing dynamic action may be of centrifugal type or axial type depending upon the direction of flow. These centrifugal type or axial compressors may also be termed as non-positive displacement type steady flow compressors.

(i) *Roots blower:* Roots blower is a positive displacement type rotary compressor. It has two rotors having two or three lobes having epicycloid and hypocycloid or involute profiles such that they remain in proper contact. Figure 16.13 shows two lobe rotors in a roots blower. To prevent wear and tear two rotors have clearance in between. Out of two rotors one is driven by prime mover while other one is driven by first rotor. When two rotors rotate then their typical geometry divides the region inside casing into two regions i.e. high pressure region and low pressure region. Although there occurs slight leakage across the mating parts which can only be minimised not eliminated completely.

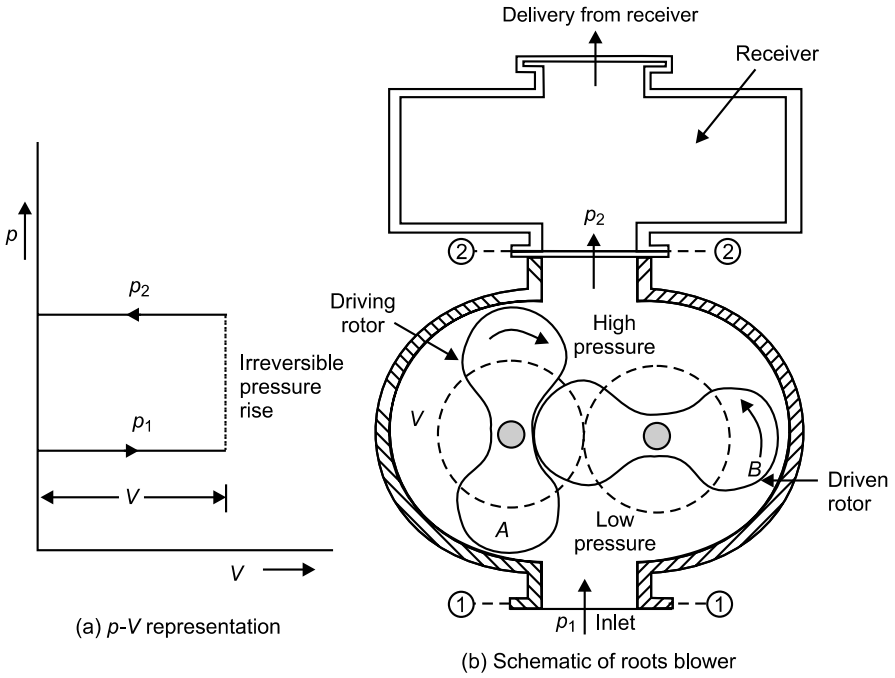


Fig. 16.13 *Roots blower*

Figure 16.13 b shows the general arrangement in roots blower. It has inlet at section 1–1 and exit at 2–2. Air at atmospheric pressure enters the casing and is trapped between rotor *A* and the casing. When the rotor rotate then air trapped in volume space *V* is displaced towards high pressure region due to rotation of rotor. Exit end is connected to receiver in which air is gradually transferred and the pressure inside receiver increases due to cumulative effect of air being transferred from atmospheric pressure region to receiver region. In one revolution this positive displacement of air trapped between rotor and casing from inlet end to receiver end shall occur four times in case of two lobe rotor as shown. While in case of three lobes rotor this transfer shall occur six times. Every time when *V* volume of air is displaced without being compressed to the receiver side high pressure region, then the high pressure air rushes back from receiver and mixes irreversibly with this air until the pressure gets equalized. Thus, gradually air pressure builds up and say this pressure becomes p_2. For inlet air pressure being p_1, the work done per cycle,

$$W = (p_2 - p_1) \times V$$

In one revolution the total work required shall depend on number of lobes in rotor and in case of two lobes rotor,

$$W_{\text{revolution}} = (p_2 - p_1) \times (4\ V)$$

Let volume thus compressed in one revolution, be $V_c = 4V$

$$W_{\text{revolution}} = (p_2 - p_1) \cdot V_c = p_1 \left(\frac{p_2}{p_1} - 1 \right) V_c$$

The efficiency of compression can be defined by taking the ratio of work required in compressing the air isentropically and actual work done. This efficiency is also called *roots efficiency*.

$$\text{Roots efficiency} = \frac{\text{Work required in reversible adiabatic (isentropic) compression}}{\text{Actual work required in compression}}$$

For compressing volume V_c from pressure p_1 to p_2 the work required in reversible adiabatic (isentropic) compression shall be;

$$W_{\text{isen}} = \left(\frac{\gamma}{\gamma - 1} \right) p_1\ V_c \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right\}$$

$$\eta_{\text{roots}} = \frac{\left(\frac{\gamma}{\gamma - 1} \right) p_1 V_c \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right\}}{W_{\text{actual}} (= W_{\text{revolution}})}$$

$$\eta_{\text{roots}} = \frac{\left(\frac{\gamma}{\gamma - 1} \right) p_1 V_c \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right\}}{p_1 \left(\frac{p_2}{p_1} - 1 \right) V_c}$$

Let pressure ratio be given by, $r = \frac{p_2}{p_1}$

so,

$$\eta_{\text{roots}} = \left(\frac{\gamma}{\gamma - 1} \right) \frac{\left\{ r^{\frac{\gamma - 1}{\gamma}} - 1 \right\}}{\{r - 1\}}$$

The expression for roots blower efficiency shows that the efficiency depends upon pressure ratio and the increase in pressure ratio yields decrease in roots efficiency. At low pressure ratios the roots efficiency is quite high. Reduced efficiency with high pressure ratios is attributed to the mechanism involved in pressure rise. With high pressure ratio there occurs the reversal of flow during positive displacement of air from casing to receiver. This reversal causes the loss of energy thereby lowering efficiency due to increased actual work requirement.

Roots blower are available in the capacity ranging from 0.15 m^3/min to 1500 m^3/min and the pressure ratios are of the order of 1 to 3.5. These are generally used for scavenging, supercharging of IC engines etc. Generally the roots efficiency is around 50–60% but high efficiency values up to 80% can also be achieved with low pressure ratios.

(ii) *Screw type or Helical type compressor:* Screw type compressor is very much similar to roots blower.

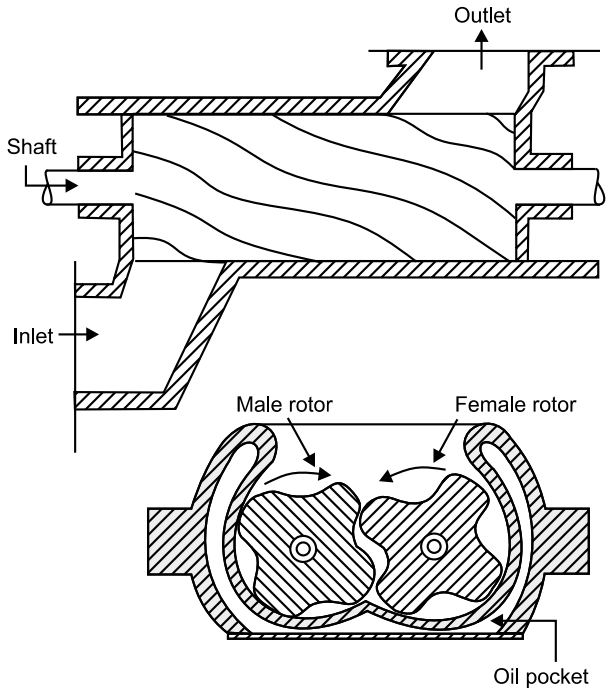


Fig. 16.14 *Line sketch of screw compressor*

These may have two spiral lobed rotors, out of which one may be called male rotor having 3–4 lobes and other female rotor having 4–6 lobes which intermesh with small clearance. Meshing is such that lobes jutting out of male rotor get placed in matching hollow portion in female rotors. Initially, before this intermeshing the hollows remain filled with gaseous fluid at inlet port. As rotation begins the surface in contact move parallel to the axis of rotors toward the outlet end gradually compressing the fluid till the trapped volume reaches up to outlet port for getting discharged out at designed pressure. Since the number of lobes are different so the rotors operate at different speed.

The material of casing may be cast iron or cast steel while rotors may be of steel and generally internally cooled by circulation of lubrication oil. Surface of lobes are smooth and the shaft is sealed by carbon rings at oil pressure. Two rotors are brought into synchronization by the screw gears. Thrust upon rotors is taken care of by oil lubricated thrust bearings.

These compressors are capable of handling gas flows ranging from 200 to 20000 m^3/h under discharge pressures of 3 bar gauge in single stage and up to 13 bar gauge in two stages. Even with increase in number of stages pressures up to 100 bar absolute have been obtained with stage pressure ratio of 2.

Mechanical efficiency of these compressors is quite high and their isothermal efficiencies are even more than vane blowers and may be compared with centrifugal and axial compressors. But these are very noisy, sensitive to dust and fragile due to small clearances.

(iii) *Vane type compressor:* Schematic of vane type compressor is shown in Fig. 16.15. It has cylindrical casing having an eccentrically mounted rotor inside it. The rotor has number of slots in it with rectangular vanes of spring loaded type mounted in slots. These vanes are generally non metallic and made of fibre or carbon composites or any other wear resistant material. These vanes remain in continuous contact with casing such that leakage across the vane-casing interface is minimum or absent. It has one end as inlet end and other as the delivery end connected to receiver. Upon rotation the eccentric rotor has the vanes having differential projection out of rotor depending upon their position. Air is trapped between each set of two consecutive blades in front of inlet passage and is positively displaced to the delivery end after compressing the volume V_1 initially to V_2, V_3 and V_4. When compressed volume comes in front of delivery passage and further rotation results in the situation when partly compressed air is forced to enter the receiver as their is no other way out. This cumulative transfer of partly compressed air in receiver causes irreversible compression resulting in gradual pressure rise. The *p-V* representation shown in Fig. 16.15 (b) indicates that the total pressure rise is due to the combined effect of reversible pressure rise inside casing and irreversible pressure rise inside receiver. Generally, the contribution of reversible pressure rise and irreversible pressure rise is in proportion of 50 : 50.

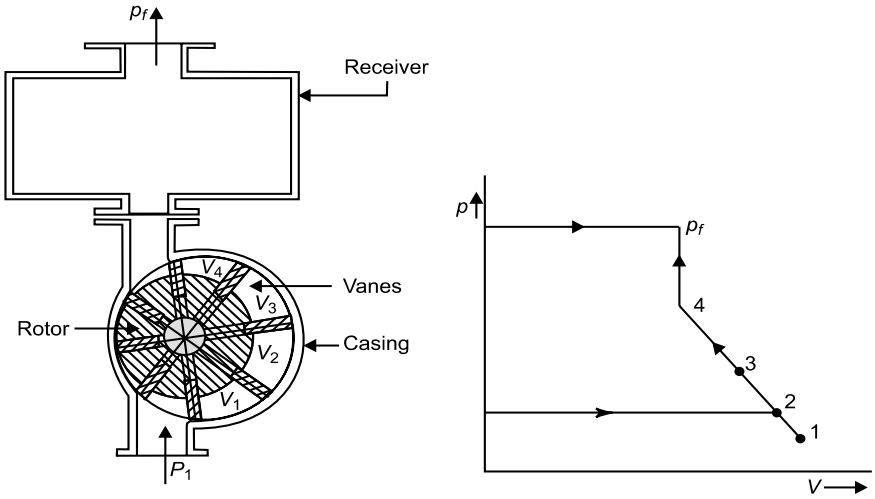


Fig. 16.15 *Vane type compressor*

Vane compressors are available for capacity up to 150 m^3/min and pressure ratios up to 8 and efficiency up to 75%. For higher pressure ratios the efficiency of vane compressors is more than that of roots blower but the vane compressors have maximum speed up to 2500 rpm as compared to 7500 rpm in case of roots blower. Vane compressors have large power requirement as compared to roots blower

due to large number of vanes on rotor. However, with small number of vanes the power required by vaned compressor may be comparable to roots blower. Vane compressors require large maintenance due to wear of vanes and other moving parts.

Work required per revolution for vane compressor with n number of vanes shall be partly in reversible and partly in irreversible process.

Let the inlet air be at pressure p_1 and volume trapped is V_1. This volume is partly compressed up to volume V_4 and pressure rises from to p_1 to p_2, p_3 and p_4. Thus, the irreversible compression occurs for volume V_4 and pressure rising from p_4 to p_f.

$$\text{Total work required, } W = n\left(\frac{\gamma}{\gamma-1}\right)\cdot p_1 V_1\left\{\left(\frac{p_4}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\} + n\,(p_f - p_4)\cdot V_4$$

16.9 CENTRIFUGAL COMPRESSORS

Centrifugal compressor is a radial flow machine compressing the fluid due to the dynamic action of impeller. Centrifugal compressors have impeller mounted on driving shaft, diffuser and volute casing as shown in Fig. 16.16. Centrifugal compressors have air inlet at the centre of impeller. The portion of impeller in front of inlet passage is called impeller eye.

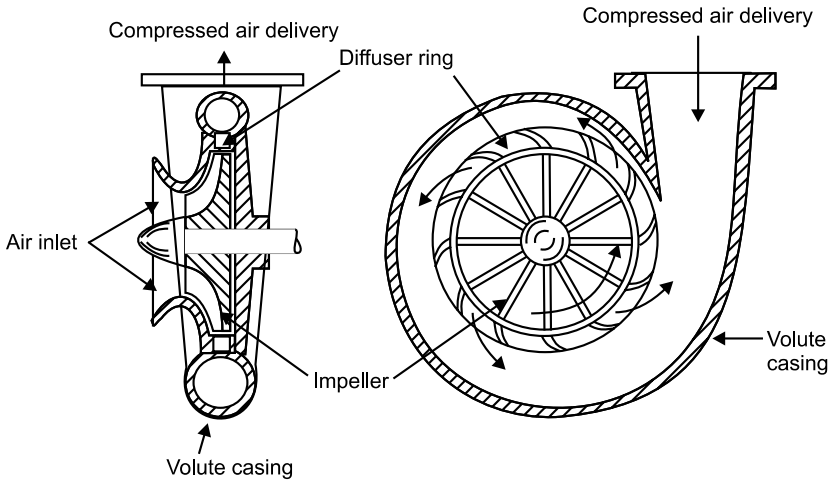


Fig. 16.16 *Centrifugal compressor*

Impeller is a type of disc having radial blades mounted upon it. Compressor casing has a diffuser ring surrounding impeller and the air enters the impeller eye and leaves from impeller tip to enter diffuser ring. Volute casing surrounds the diffuser ring. Volute casing has cross section area increasing gradually up to the exit of compressor. These impellers of centrifugal compressors may also be of double sided type such that air can enter from two sides (both) of impeller. Thus double sided impeller shall have double impeller eye compared to single impeller eye as shown in Fig. 16.17.

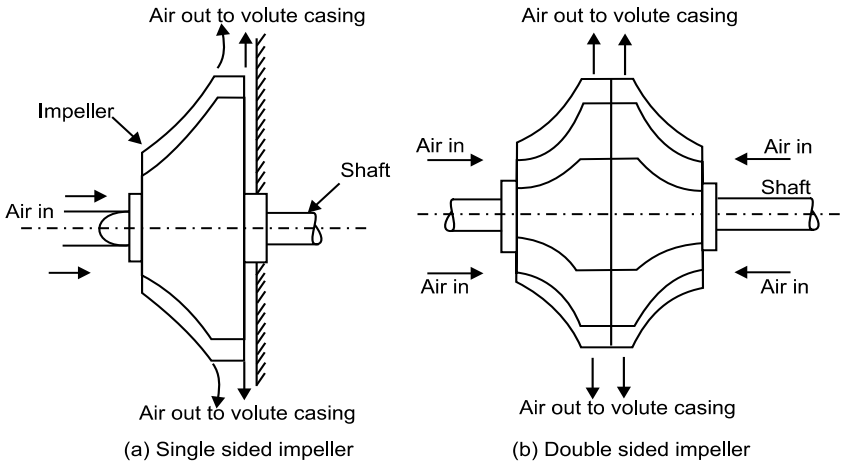


Fig. 16.17 *Single sided impeller and double sided impeller*

Air enters the impeller eye axially and flows radially outwards after having entered compressor. Radial flow of air inside compressor is due to impeller (blades) rotating about its axis. These impeller blades impart momentum to the air entering, thereby rising its pressure and temperature. Subsequently the high pressure fluid leaving impeller enters the diffuser ring where the velocity of air is lowered with further increase in pressure of air. Thus in diffuser ring the kinetic energy of air is transformed into pressure head. High pressure air leaving diffuser is carried by volute casing to the exit of compressor. Due to increased cross section area of volute casing some velocity is further reduced causing rise in its pressure, although this is very small. Total pressure rise in compressor may be due to 'impeller action' and 'diffuser action' both. Generally, about half of total pressure rise is available in impeller and remaining half in diffuser. Pressure and velocity variation in centrifugal compressor is shown in Fig. 16.18.

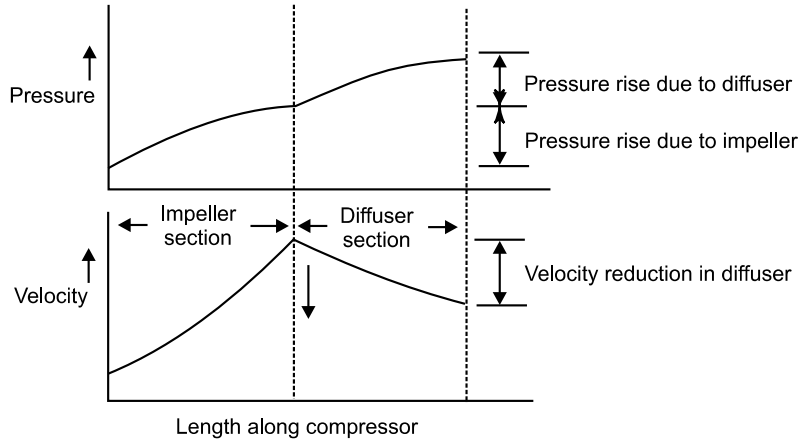


Fig. 16.18 *Pressure and velocity variation in compressors*

Centrifugal compressors are used in aircrafts, blowers, superchargers, etc. where large quantity of air is to be supplied at smaller pressure ratios. Generally, pressure ratio up to 4 is achieved in single

stage centrifugal compressors while in multistage compressors the pressure ratio up to 12 can be achieved. These compressors run at speed of 20,000–30,000 rpm.

Thermodynamic analysis: In case of centrifugal compressors the fluid is being compressed following reversible adiabatic process ideally. In actual case the frictionless adiabatic compression is not possible due the frictional losses between fluid and casing. Irreversibilities also creep into compression process due to the entry and exit losses such as throttling, shock at inlet and exit and eddy formation etc. Therefore, the actual compression occurs following polytropic compression process. Figure 16.19 shows the *T–s* representation for ideal and actual compression process.

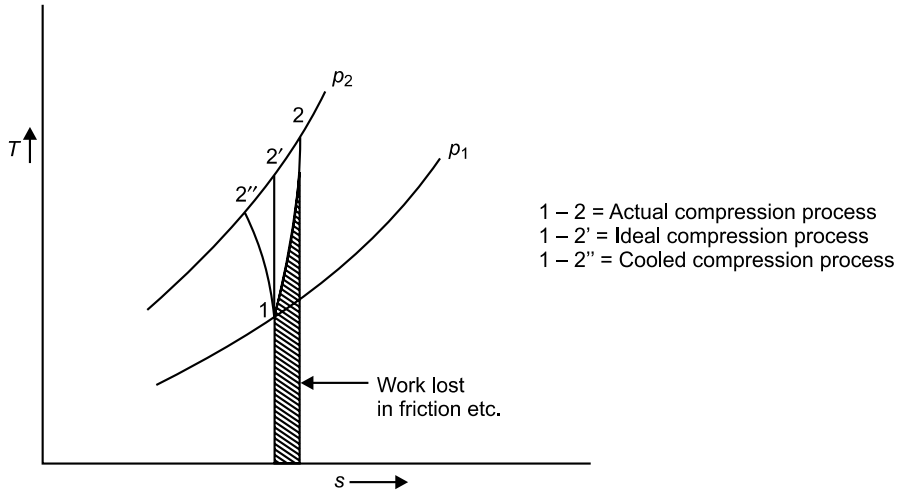


Fig. 16.19 T-s *representation for compression in centrifugal compressor*

Compression process may also be of cooled type and this cooling during compression modifies the process to 1–2″. Actual compression process may have the index of compression value (generally, n = 1.5 to 1.7) more than that of adiabatic process (for air, γ = 1.4). But the cooling reduces the index value even less than the adiabatic index.

In centrifugal compressor the fluid moves with significant velocity during compression i.e. it has considerable kinetic energy which can not be neglected. For correct thermodynamic analysis the kinetic energy of fluid should be considered. The stagnation properties are evolved for considering the effect of fluid velocity. These considerations are not required in analysis of reciprocating compressors because the velocities are quite small and so negligible. Stagnation property refers to thermodynamic property of fluid upon being decelerated to zero velocity isentropically as described in earlier chapter of gas power cycles.

Velocity diagram for centrifugal compressor: Figure 16.20 shows the velocity diagram for impeller blade of centrifugal compressor. Air enters the blade at 1 and leaves at 2. The air velocity at inlet and exit corresponding to the blade angles are shown on velocity diagrams for axial entry i.e. $\alpha_1 = 90°$.

Tangential force acting on blade can be estimated by change in whirl velocity value;

$$= m\,(C_{w2} - C_{w1}); \text{ for axial entry } C_{w1} = 0$$

$F_T = m\,C_{w2}$. This tangential force shall be responsible for rotation of shaft. The work done on impeller can be estimated using tangential force and distance travelled along the direction of force. This work is also called Euler's work.

$$\boxed{W = m \cdot C_{w2} \times U_2}$$

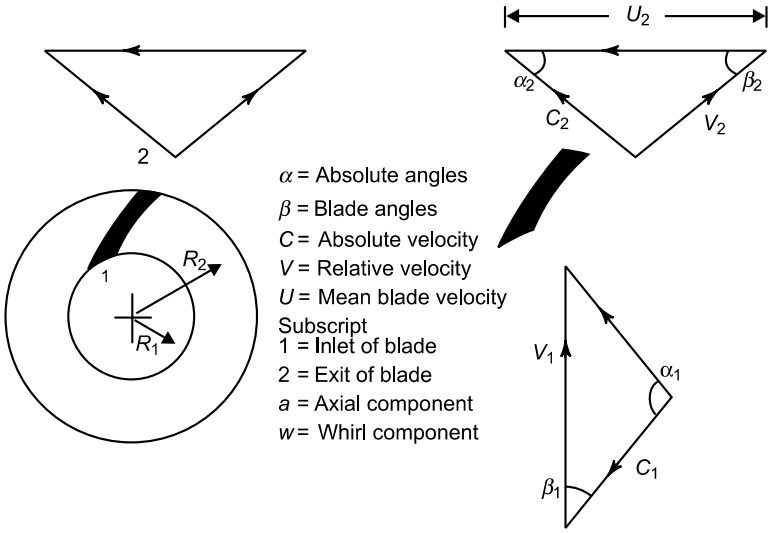
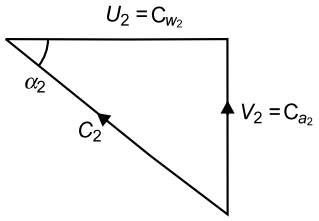


Fig. 16.20 *Velocity diagrams for centrifugal compressor*

For radial blades on impeller the whirl velocity C_{w2} shall coincide with U_2 i.e. velocity diagram at exit of impeller blade;



and, $\boxed{W = mU_2^2}$

This work required can also be estimated in terms of thermodynamic properties. By steady flow energy equation between sections 1 and 2 for no heat interaction in case of adiabatic compression process for no change in potential energy.

$$h_1 + \frac{C_1^2}{2} + W = h_2 + \frac{C_2^2}{2}$$

$$W = m\left(h_2 + \frac{C_2^2}{2}\right) - m\left(h_1 + \frac{C_1^2}{2}\right)$$

$$W = mC_p\left(T_2 + \frac{C_2^2}{2C_p}\right) - mC_p\left(T_1 + \frac{C_1^2}{2C_p}\right)$$

From definition of stagnation temperature,

$$\left(T_2 + \frac{C_2^2}{2C_p}\right) = T_{o2} \text{ and } \left(T_1 + \frac{C_1^2}{2C_p}\right) = T_{01}$$

or $$\boxed{W = mC_p\,(T_{02} - T_{01})}$$

Thus, impeller work can be given by the product of specific heat and change in stagnation temperature from inlet to exit.

$$W = mC_p\,T_{o1}\left[\frac{T_{o2}}{T_{o1}} - 1\right]$$

$$W = mC_p\,T_{o1}\left[\frac{T_2\left(\frac{p_{02}}{p_2}\right)^{\frac{\gamma-1}{\gamma}}}{T_1\left(\frac{p_{o1}}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}\right]$$

$$W = m\,C_p\,T_{o1}\left[\left(\frac{p_{o2}}{p_{o1}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

Here, $\left(\frac{p_{02}}{p_{01}}\right)$ is ratio of stagnation pressure across impeller. For radial flow blades, substituting for W,

$$mU_2^2 = mC_pT_{01}\left[\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

or, $$\left(\frac{p_{02}}{p_{01}}\right) = \left[\frac{U_2^2}{C_pT_{01}} + 1\right]^{\frac{\gamma}{\gamma-1}}$$

Thus, it shows that stagnation pressure ratio depends on the square of blade tip speed. Here it has been assumed that in case of radial blades blade tip speed is equal to the whirl velocity component, which is not valid condition in reference to actual operation.

Actually, due to turbulence and secondary flow effects the $C_{w2} \neq U_2$ and the $C_{w2} < U_2$. Difference between U_2 and C_{w2} i.e. $(U_2 - C_{w2})$ is called the slip. This slip is quantified by the parameter called slip factor. Slip factor is the ratio of actual whirl velocity component to the ideal whirl velocity component.

$$\text{Slip factor} = \frac{C_{w_2}}{U_2} = \frac{\text{Actual whirl velocity}\,(C_{w2})}{\text{Ideal whirl velocity}\,(U_2)}$$

Losses in centrifugal compressor: Centrifugal compressor has losses at inlet. Every compressor offers its optimum performance at design operation. Any deviation from design condition increases the different losses occuring in it. Due to friction, turbulence, eddy formation etc. there are flow losses inside the compressor. These losses are proportional to the square of flow velocity. Compressibility of fluid at high Mach nos. also offers losses. Losses in diffuser, volute casing are also there due to friction. Friction and secondary flow losses also cause slip and actual whirl component is less than theoretical value.

16.10 AXIAL FLOW COMPRESSORS

Axial flow compressors have the fixed blades and moving blades mounted along the axis of compressor. Air enters axially and leaves axially. It has primarily two components i.e. rotor and casing. The rotor has

blades mounted on it constituting moving blade ring. Blades are also mounted on the inner side of casing thereby constituting stages as fixed blade ring followed by moving blade ring followed by fixed blade ring, moving blade ring and so on.

Due to the reduction in volume the volume space for compressed air may be gradually reduced. Gradual reduction in volume can be done by flaring the rotor while keeping stator diameter uniform or by flaring the stator while keeping rotor diameter constant as shown in fig. 16.21. The pressure of fluid entering the axial flow compressor increases upon passing through the fixed and moving blades. This flow of fluid over moving blades is accompanied by enthalpy rise while the fixed blades merely deflect the fluid so as to facilitate smooth entry into moving blades. Absolute velocity of air increases along axis of rotor due to work input from the prime mover. Relative velocity of air decreases during its flow through rotor. Blades have aerofoil section so as to have minimum losses due to turbulence, boundary layer formation and separation, eddy formation etc.

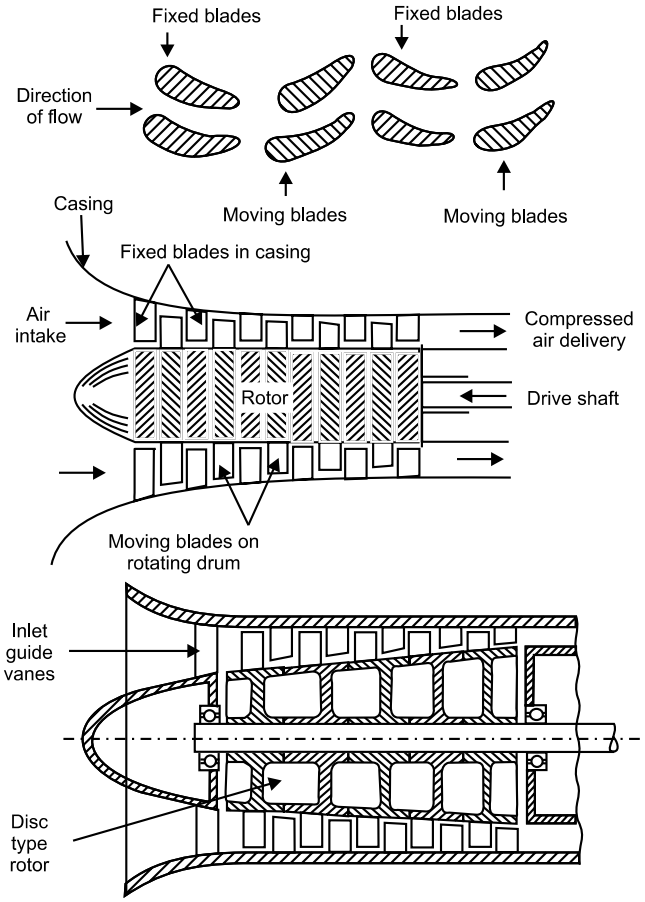


Fig. 16.21 *Axial flow compressor*

16.11 SURGING AND CHOKING

Let us look at the variation of theoretical pressure ratio with mass flow rate in case of a centrifugal compressor running at constant speed, as shown in Fig. 16.22.

Let the compressor be run with completely closed delivery valve thus the mass flow rate is zero and pressure ratio available is shown by point *a*. This operating state of compressor shall not have any amount of air entering into it instead the air trapped inside the impeller and casing shall be subjected to impeller action (compression) causing its pressure rise upto the state *a*. Now upon opening of delivery valve the flow of air into compressor and its discharge begins. With this gradual opening of delivery valve the pressure ratio increases, say from state *a* to *b* and *c* along with increase in mass flow rate. The pressure ratio reaches to some maximum value at point *c*. Further opening of delivery valve shows that the mass flow rate increases but the pressure ratio decreases after point *c* i.e. beyond the point having maximum pressure ratio. The decrease in pressure ratio is due to the dominance of pressure loss due to friction etc. over the pressure rise in diffuser. After point *c* pressure losses are more than the pressure rise thereby, showing net decrease in pressure ratio as evident from points *d* and *e* on the delivery pressure vs mass flow rate characteristic curve.

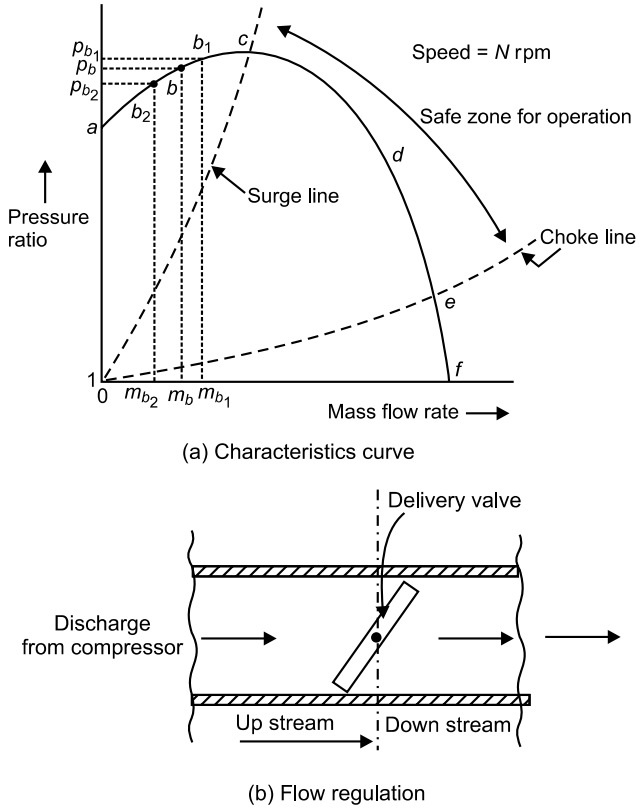


Fig. 16.22 *Surging and choking*

Theoretically, the characteristic curve may be extrapolated and the maximum discharge (mass flow rate) is evident from point *f* at which the pressure ratio becomes unity and efficiency is zero. Maximum efficiency is available at the point (*c*) corresponding to the maximum pressure ratio. At the state *f* all work input to compressor is used in overcoming frictional resistance. In actual case the state 'e' shall indicate the maximum mass flow rate practically possible. This maximum mass flow state is known as choked mass flow. State of operation of compressor at point *e* is called 'choking' of compressor. Choking gives the maximum mass flow from compressor and the pressure ratio corresponding to this is less than maximum pressure ratio.

Let us open delivery valve for increase in mass flow rate from m_b to m_{b1}. With this opening of valve the delivery pressure increases from p_b to p_{b1}. At point b_1 the pressure on upstream and down stream do not rise simultaneously instead the pressure at upstream becomes p_{b1} while in downstream it remains p_b for some time, due to inertia till pressure is not equalized. Since $p_b < p_{b1}$ so there is no problem in terms of flow reversal. Similarly as the delivery valve is gradually opened up to state *c* there is no problem of flow reversal as upstream pressure is always more than downstream pressure up to *c*. Upon partly closing the delivery valve, say from *b* to b_2 the mass flow rate decreases from m_b to m_{b2}. This is accompanied by decrease in pressure at both upstream and downstream. Pressure at upstream gets changed (lowered) from p_b to p_{b2} ($p_{b2} < p_b$) instantaneously while the downstream pressure remains equal to p_b for some time due to inertia. Pressure gets equalized to p_{b2} after some time lag. During this time lag the reversal of flow (stoppage of discharge from compressor) occurs from downstream to upstream as in transient period downstream pressure p_b is more than upstream pressure p_{b2}. Reversed flow stabilizes to normal flow after the pressure gets equalized. This reversal of flow occurs during part closing of valve in the operating range from *a* to *c*. Reversal of flow causes pulsations to the compressor and the noise, vibration and, jerks are felt during this transition period. This phenomenon is called 'surging'. Similar, study of part closing and opening of valve when carried out in between *c* and *e* shows that no such reversed flow is observed. This changed behaviour is seen as even in case of closing of valve the mass flow rate shall decrease but the pressure ratio increases and in case of opening of valve the mass flow rate increases although pressure ratio decreases. Region lying between *c* and *e* gives the safe operating zone where surging phenomenon is not observed and also it is within the limits of choking of compressor. Compressor should be therefore run between *c* and *e* for its' smooth running. The excessive surging may even lead to failure (fracture) of rotating parts in compressor and is therefore completely avoided. For every compressor the safe zone for its operation is obtained and compressors are run in that region.

Surging generally occurs in diffuser passages where the flow is slightly obstructed and gets retarded due to friction near vanes. Surging tends to increase with increasing number of diffuser vanes as with large no. of vanes it becomes difficult to divide air equally in all the passages. To avoid this the number of diffuser vanes is kept less than the impeller blades so that each of diffuser vane passage is provided with flow from more than one impeller blade channel. Thus, the condition of flow remain nearly same for all diffuser passages and surging due to flow reversal does not occur.

16.12 STALLING

Stalling refers to the instability in the flow occurring due to flow separation from blade surfaces. Stalling may occur due to the non-uniform flow passing through the channels/passage between consecutive blades/vanes. In case of centrifugal compressor the flow output from impeller passes into the diffuser vanes and subsequently to exit end. In case of breakdown of flow in any one channel in diffuser, which may be due to reduced angle of incidence/increased angle of incidence, the flow pattern gets disturbed. For example the Fig. 16.23 shows the reduced angle of incidence in channel *B* of diffuser section in centrifugal compressor. Since the diffuser vanes are supposed to handle the similar amount of total flow therefore, inspite of reduced flow through the channel *B* (due to reduced angle of incidence) the mass flow rate does not change. Due to reduced flow through *B* the remaining channels have to carry additional flow in order to compensate for flow reduction in one of channels. These different angles of incidence into different channels result into non-uniform mass of air being handled by these channels. Such as in present case channel *A* has higher angle of incidence and channel *C* has reduced angle of incidence due to reduced flow through channel *B*. It results into flow separation due to increase and decrease in angle of incidence. Flow separation disturbs the flow pattern and results into excessive

vibrations. This stall phenomenon passes from one channel to the other gradually. Stall may rotate in direction opposite to that of impeller rotation and is termed as rotating stall.

This phenomenon of flow separation from blade surface, called as stalling also occurs in axial flow compressors, in the manner similar to that of centrifugal compressor. Stalling results in reduction of delivery pressure at compressor exit which may eventually lead to reversal of flow or surging. Stalling also leads to reduction in stage efficiency along with vibrations in compressor which may even rise upto the level causing failure of compressor parts when frequency matches with natural frequency.

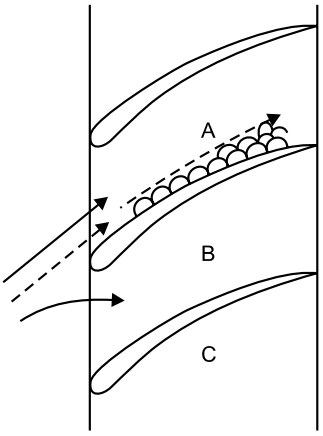


Fig.16.23 *Stall propagation*

16.13 CENTRIFUGAL COMPRESSOR CHARACTERISTICS

Compressor characteristics are the set of curves obtained between delivery pressure and temperature plotted against mass flow for different values of rotational speed. Characteristics curves are machine specific i.e. every compressor shall have its own unique characteristic curves. These characteristic curves are plotted using non-dimensional parameters. Non-dimensional parameters used as the performance indicator of compressor. depend upon entry pressure, temperature, physical properties of fluid being compressed, mass flow rate and rotational speed etc. In order to obtain characteristic curve excessively large number of experimental observations are needed which can not be presented concisely. Use of non-dimensional parameters helps in combining number of parameters in order to get dimensionless groups which can be used for getting compressor characteristic curves. The non-dimensional groups thus used are, $\frac{p_{02}}{p_{01}}, \frac{T_{02}}{T_{01}}, \frac{m\sqrt{RT_{01}}}{D^2 p_{01}}, \frac{ND}{\sqrt{RT_{01}}}$ where p, T, m, R, D, N are pressure, temperature, mass flow rate, gas constant, characteristics linear dimension and rotational speed respectively. Here for a particular compressor running with definite fluid the R and D shall be fixed and non-dimensional groups can be modified as, $\frac{p_{02}}{p_{01}}, \frac{T_{02}}{T_{01}}, \frac{m\sqrt{T_{01}}}{p_{01}}, \frac{N}{\sqrt{T_{01}}}$.

Different non-dimensional groups given here are termed as described below inspite of the fact that some of these are no more dimensionless due to omission of R and D terms.

$$\frac{p_{02}}{p_{01}} = \text{Dimensionless pressure}$$

$\frac{T_{02}}{T_{01}}$ = Dimensionless temperature

$\frac{m\sqrt{T_{01}}}{p_{01}}$ = Dimensionless mass flow

$\frac{N}{\sqrt{T_{01}}}$ = Dimensionless rotational speed

Characteristic curves for centrifugal compressor are obtained by plotting; (i) "dimensionless pressure" $\left(\frac{p_{02}}{p_{01}}\right)$ with "dimensionless mass flow $\left(\frac{m\sqrt{T_{01}}}{p_{01}}\right)$ relative to design value" for different "dimensionless rotational speeds $\left(\frac{N}{\sqrt{T_{01}}}\right)$ relative to design value" and, (ii) "isentropic efficiency" with "dimensionless mass flow $\left(\frac{m\sqrt{T_{01}}}{p_{01}}\right)$ relative to design value" for different "dimensionless rotational speeds $\left(\frac{N}{\sqrt{T_{01}}}\right)$ relative to design value."

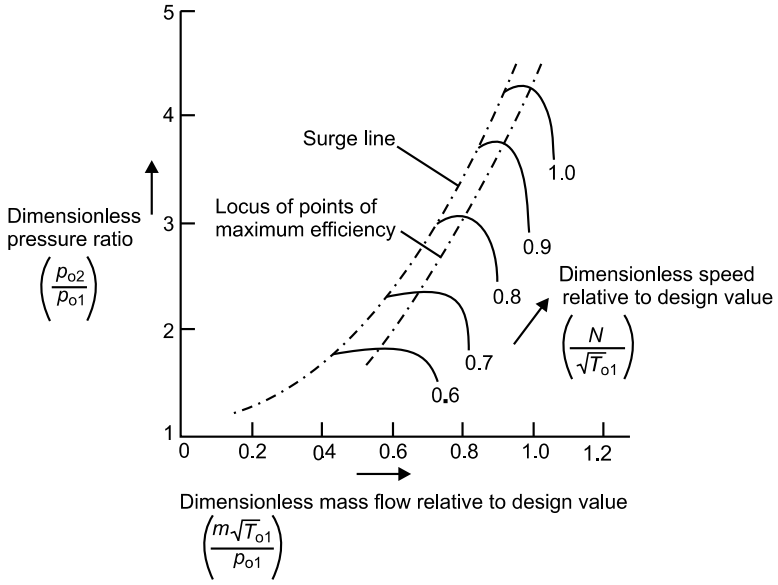


Fig. 16.24 *Centrifugal compressor characteristic curve*

Figures 16.24 and 16.25 shows the compressor characteristic curves. First curve is plot between pressure ratio and mass flow rate. Similar type of plot has been referred while discussing surging and choking. Here only the portion between surge line and choke line has been shown for different dimensionless rotational speeds relative to design value of 0.6, 0.7, 0.8, 0.9, 1.0. The locus of points having maximum efficiency is also shown on characteristic curve. This curve identifies the two extreme operating states between which the compressor runs.

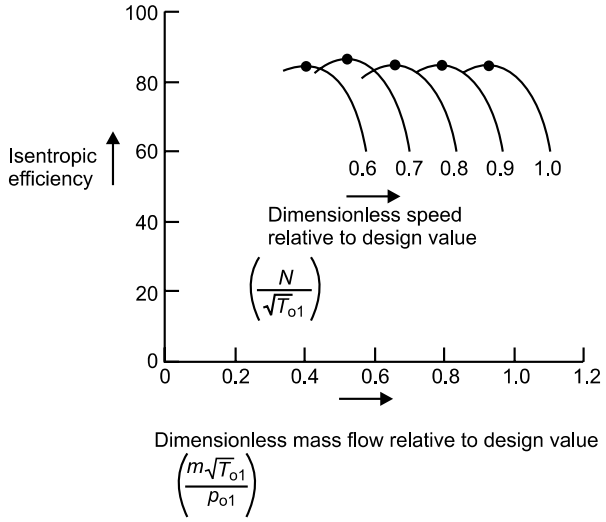


Fig. 16.25 *Isentropic efficiency of a centrifugal compressor (characteristic curve)*

Similar to the plot between dimensionless pressure and dimensionless mass flow rate the plot between dimensionless temperature and dimensionless mass flow rate can be obtained. Using these patterns the plot between isentropic efficiency and dimensionless mass flow rate for different dimensionless rotational speeds can be obtained as shown in Fig. 16.25. These characteristic curves give the operating range of compressor and are also used for matching of compressors with other devices such as turbine etc. operating together. Characteristics of compressor and turbine operating together in any plant should be matching else inefficiencies are observed. Looking at the centrifugal compressor characteristic curves it can be concluded that, "*for a given pressure ratio the increase in speed shows increase in flow rate and simultaneous reduction in efficiency," "for any particular speed of compressor the decrease in mass flow rate is accompanied by increase in pressure ratio" and "at all speeds the compressor operation is limited by surging on one end and choking on other end*" etc. Figure 16.26 shows a typical characteristic curve for some centrifugal compressor.

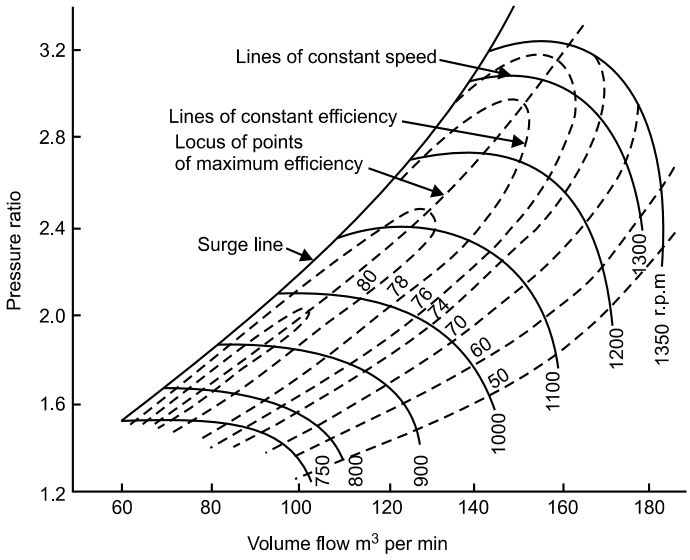


Fig. 16.26 *Typical centrifugal compressor characteristic curve*

16.14 AXIAL FLOW COMPRESSOR CHARACTERISTICS

Characteristic curves for axial flow compressor can be obtained similarly as it has been in case of centrifugal compressor. Characteristic curves for axial flow compressors are also plotted on non-dimensional basis showing the variation of "dimensionless pressure ratio" and "isentropic efficiency" with respect to the "dimensionless mass flow rate relative to design value" for varying dimensionless rotation speeds. Apart from above curves the plot between "*loading coefficient*" and "*flow coefficient*" may also be required. The "*loading coefficient* refers to the ratio of actual stagnation enthalpy rise in a stage to the energy (enthalpy) equivalent of peripheral speed of compressor rotor. "*Flow coefficient*" also termed as "*compressor-velocity ratio*" is the ratio of axial velocity to peripheral speed of blades. Flow coefficient is used for regulating the stall phenomenon in compressors. Figures 16.27 (a) and (b) show the characteristic curves for axial flow compressors at different speeds.

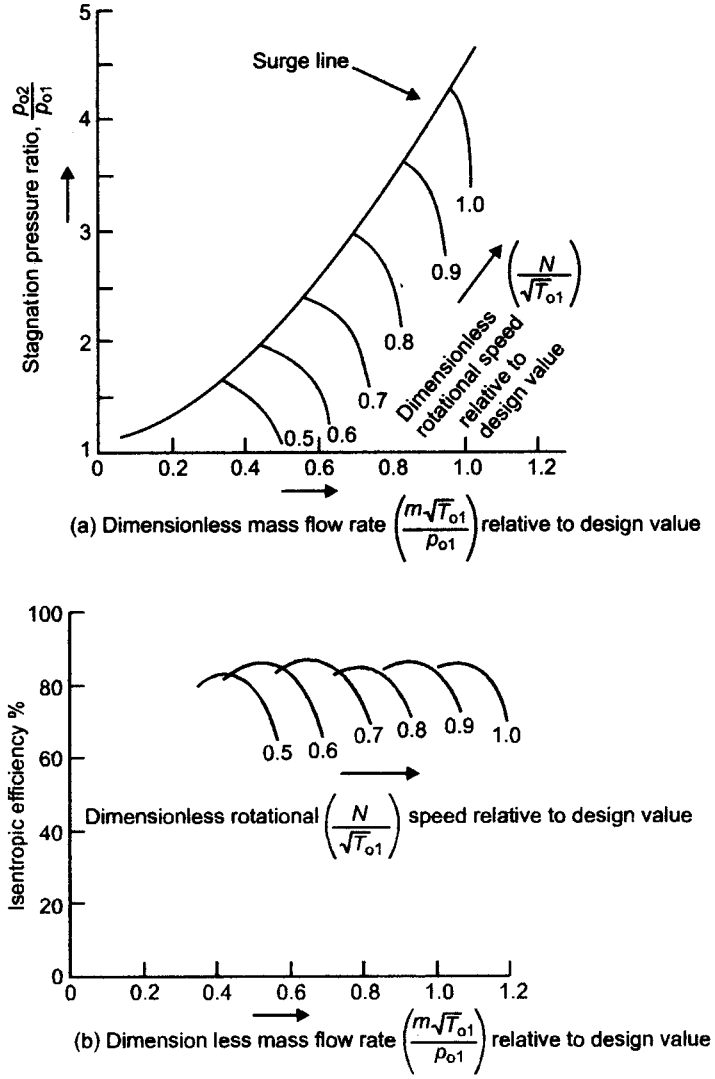


Fig. 16.27 Axial flow compressor characteristic curves

Here it may be noted that since axial flow compressor comprises of number of stages so each stage shall also have its' own characteristic curve, although stage characteristic curves are similar to over all characteristic curve but with small pressure ratio. Characteristic curves show that the flow rate does not change much with increase in pressure ratio near design conditions. The efficiency is seen to decrease with decrease in flow rate and pressure ratios. Characteristic curves for axial flow compressor can also be shown as in Fig. 16.28.

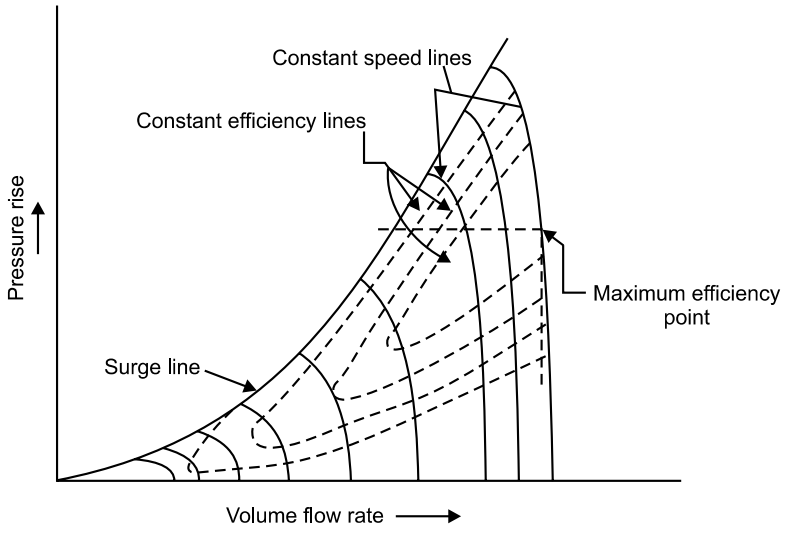


Fig. 16.28 *Performance characteristic of axial flow compressor*

It is evident from characteristic curves that when compressor is run at less than design speed then rise in pressure and temperature is less than design value and also the choking decides the limiting operating point for compressor.

16.15 COMPARATIVE STUDY OF COMPRESSORS

Comparative study of different types of compressors is presented below:

(i) Comparison of reciprocating compressors with rotary compressors

Reciprocating compressors	*Rotary compressors*
1. These are suitable for low discharge rate at higher delivery pressures. Speed of rotation of reciprocating compressors is quite small due to higher inertia of reciprocating parts.	1. These are suitable for high discharge rate at smaller delivery pressures. Speed of rotation of rotary compressors is quite high up to 40,000 rpm.
2. Reciprocating compressor has intermittent delivery of high pressure air thereby requiring receiver for maintaining continuous delivery.	2. Rotary compressor has continuous delivery of high pressure air therefore no receiver is required.
3. Reciprocating compressors have balancing problems.	3. Rotary compressors have better balancing as compared to reciprocating compressors.

(*Contd.*)

Reciprocating compressors	*Rotary compressors*
4. Quality of air delivered from reciprocating compressors is poor due to its contamination with lubricant.	4. Air delivered is comparatively cleaner.
5. Lubrication requirements are severe in these compressors.	5. Lubricating requirements are less compared to reciprocating compressors.
6. Reciprocating compressors can not be directly coupled to high speed prime movers such as turbines etc.	6. Rotary compressors can be directly coupled to high speed prime movers.
7. Reciprocating compressors are desi-gned to achieve compression close to isothermal compression by cooling etc. compression process with index more	7. Rotary compressors have isentropic process ideally but actually the compression occurs following polytropic compression with index more than adiabatic index.

(ii) Comparison between the centrifugal compressor and axial flow compressor is given as under.

Centrifugal Compressor	*Axial flow compressor*
(i) Centrifugal compressors generally have stage pressure ratio of aro-und 5 and have radial flow inside with axial entry of fluid. Maxi-mum pressure up to 400 bar can be achieved by multistaging, although multistaging is slightly difficult.	(i) Axial flow compressors generally have stage pressure around 1.5 and have axial inlet and axial exit with flow inside compressor being parallel to the compressor axis. Maximum pressure up to 10 bar is possible to be achieved through multistaging which is very convenient in axial compressors.
(ii) Centrifugal compressors have good part load efficiency and wide operating range between surging and choking limits.	(ii) Axial flow compressors have poor part load efficiency and narrow operating range between surging and choking limits.
(iii) Centrifugal compressors have small starting torque requirement.	(iii) Axial flow compressors have large starting torque requirement.
(iv) Due to large frontal area these are well suited for land applications.	(iv) Due to small frontal area axial flow compressors are well suited for aviation applications.
(v) These have simple construction and are comparatively cheap.	(v) These have complex construction and are costly.
(vi) These have poor isentropic efficiency (≈ 70%). Power required per kg of air flow rate is more.	(vi) These have better isentropic efficiency (≈ 85 to 90%). Power required per kg of air flow rate is less in axial flow compressors.
(vii) Efficiency vs. speed characteristics for centrifugal compressor is flat. It has higher efficiency in large speed range.	(vii) Efficiency vs. speed characteristics is slightly bell shaped. It means higher efficiency in narrow range of speed.
(viii) Centrifugal compressor's performance does not alter with any kind of deposition on blade surface.	(viii) Axial compressor's performance deteriorates with any kind of deposition on blade surface.

EXAMPLES

1. *A reciprocating air compressor has cylinder with 24 cm bore and 36 cm stroke. Compressor admits air at 1 bar, 17°C and compresses it up to 6 bar. Compressor runs at 120 rpm. Considering compressor to be single acting and single stage determine mean effective pressure and the horse power required to run compressor when it compresses following the isothermal process and polytropic process with index of 1.3. Also find isothermal efficiency when compression is of polytropic and adiabatic type.*

Solution:

$$\text{Compression ratio} = \frac{P_2}{P_1} = 6 = r$$

From cylinder dimensions the stroke volume

$$= \frac{\pi}{4} \times (0.24)^2 \times (0.36)$$

$$= 0.01628 \text{ m}^3$$

Volume of air compressed per minute = 0.01628×120

$$= 1.954 \text{ m}^3/\text{min}$$

Let us neglect clearance volume.

Work done in isothermal process

$$W_{iso} = P_1 V_1 \ln r$$

Mean effective pressure in isothermal process

$$mep_{iso} = \frac{p_1 V_1 \ln r}{V_1} = P_1 \ln r$$

$$= 1 \times 10^2 \times \ln 6 = 179.18 \text{ kPa}$$

Work done in polytropic process with index $n = 1.3$, *i.e.* $PV^{1.3} = C$

$$W_{poly} = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

Mean effective pressure in polytropic process,

$$mep_{poly} = \frac{\left(\frac{n}{n-1} \right) P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]}{V_1}$$

$$mep_{poly} = \left(\frac{n}{n-1} \right) P_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \left(\frac{1.3}{1.3-1} \right) \times 1 \times 10^2 \left[(6)^{\frac{1.3-1}{1.3}} - 1 \right]$$

$$mep_{poly} = 221.89 \text{ kPa}$$

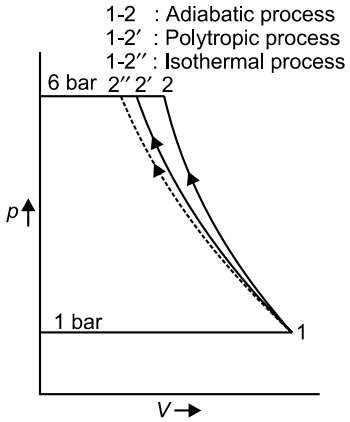


Fig. 16.29 p-V *representation*

Work done in adiabatic process, $W_{\text{adibatic}} = \frac{\gamma}{\gamma - 1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]$

Mean effective pressure in adiabatic process, $mep_{\text{adiabatic}} = \frac{W_{\text{adiabatic}}}{V_1}$

$$mep_{\text{adiabatic}} = \frac{\gamma}{\gamma - 1} P_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]$$

$$= \left(\frac{1.4}{1.4 - 1} \right) \times 1 \times 10^2 \left[(6)^{\frac{1.4 - 1}{1.4}} - 1 \right]$$

$$mep_{\text{adiabatic}} = 233.98 \text{ kPa}$$

Horse power required for isothermal process,

$$\text{HP}_{\text{iso}} = \frac{mep_{\text{iso}} \times \text{Volume per minute}}{0.7457 \times 60}$$

(As 1 hp = 0.7457 kW)

$$= \frac{179.18 \times 1.954}{0.7457 \times 60}$$

$$\text{HP}_{\text{iso}} = 7.825 \text{ hp}$$

Horse power required for polytropic process,

$$\text{HP}_{\text{poly}} = \frac{mep_{\text{poly}} \times \text{Volume per minute}}{0.7457 \times 60}$$

$$= \frac{221.89 \times 1.954}{0.7457 \times 60}$$

$$\text{HP}_{\text{poly}} = 9.69 \text{ hp}$$

Horse power required for adiabatic process,

$$\text{HP}_{\text{adiabatic}} = \frac{mep_{\text{adiabatic}} \times \text{Volume per minute}}{0.7457 \times 60}$$

$$= \frac{233.98 \times 1.954}{0.7457 \times 60}$$

$$= 10.22 \text{ hp}$$

$$\text{Isothermal efficiency} = \frac{\text{Isothermal process power required}}{\text{Actual power required}}$$

$$\text{Isothermal efficiency}_{\text{poly}} = \frac{HP_{\text{iso}}}{HP_{\text{poly}}} = \frac{7.825}{9.69} = 0.8075 \text{ or } 80.75\%$$

$$\text{Isothermal efficiency}_{\text{adiabatic}} = \frac{HP_{\text{adiabatic}}}{HP_{\text{poly}}} = \frac{7.825}{10.22} = 0.7657 \text{ or } 76.57\%$$

mep: 179.18 kPa for isothermal, 221.89 kPa for polytropic process
HP required: 7.825 HP for isothermal, 9.69 HP for polytropic
Isothermal efficiency: 80.75% for polytropic process, 76.57% for adiabatic process **Ans.**

2. *A single stage single acting reciprocating air compressor has air entering at 1 bar, 20°C and compression occurs following polytropic process with index 1.2 upto the delivery pressure of 12 bar. The compressor runs at the speed of 240 rpm and has L/D ratio of 1.8. The compressor has mechanical efficiency of 0.88. Determine the isothermal efficiency and cylinder dimensions. Also find out the rating of drive required to run the compressor which admits 1 m^3 of air perminute.*

Solution:

Using perfect gas equation the mass of air delivered per minute can be obtained as,

$$m = \frac{P_1 V_1}{RT_1}$$

$$= \frac{\left(1 \times 10^2 \times 1\right)}{(0.287 \times 293)}$$

$$= 1.189 \text{ kg/min}$$

Compression process follows $PV^{1.2}$ = constt.

Temperature at the end of compression;

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$T_2 = 293 \left(\frac{12}{1}\right)^{\frac{1.2-1}{1.2}}$$

$$T_2 = 443.33 \text{ K}$$

Work required during compression process, $W = \left(\frac{n}{n-1}\right) \times mR\ (T_2 - T_1)$

$$= \left(\frac{1.2}{1.2-1}\right) \times 1.189 \times 0.287\ (443.33 - 293)$$

$$W = 307.79 \text{ kJ/min} = \frac{307.79}{60 \times 0.7457} \text{ hp}$$

$$W = 6.88 \text{ hp}$$

Capacity of drive required to run compressor = $\frac{6.88}{0.88}$ = 7.82 hp

Isothermal work required for same compression,

$$W_{iso} = m\,RT_1 \ln\left(\frac{P_2}{P_1}\right)$$

$$= (1.189 \times 0.287 \times 293) \cdot \ln\left(\frac{12}{1}\right)$$

$$W_{iso} = 248.45 \text{ kJ/min}$$

$$\text{Isothermal efficiency} = \frac{\text{Isothermal work}}{\text{Actual work}} = \frac{248.45}{307.79} = 0.8072$$

$$\text{Volume of air entering per cycle} = \frac{1}{240} = 4.167 \times 10^{-3} \text{ m}^3\text{/cycle}$$

$$\text{Volume of cylinder} = 4.167 \times 10^{-3} = \frac{\pi}{4} D^2 L$$

$$= 4.167 \times 10^{-3} = \frac{\pi}{4} \times D^2 \times 1.8D$$

Bore, $D = 0.1434$ m or 14.34 cm

Stroke length $L = 1.8 \times D = 1.8 \times 14.34 = 25.812$ cm

Isothermal efficiency = 80.72 %
Cylinder dimension, D = 14.34 cm
L = 25.812 cm
Rating of drive = 7.82 hp

Ans.

3. *A reciprocating compressor of single stage, double acting type delivers 20 m³/min when measured at free air condition of 1 bar, 27°C. The compressor has compression ratio of 7 and the conditions at the end of suction are 0.97 bar, 35°C. Compressor runs at 240 rpm with clearance volume of 5% of swept volume. The L/D ratio is 1.2. Determine the volumetric efficiency and dimensions of cylinder and isothermal efficiency taking the index of compression and expansion as 1.25. Also show the cycle on P-V diagram.*

Solution:

$P_1 = 0.97$ bar, $T_1 = 273 + 35 = 308$ K, $\frac{P_2}{P_1} = 7$, $N = 240$ rpm

$$V_3 = 0.05\ V_s,\ V_s = V_1 - V_3,\ V_1 = 1.05\ V_s$$

For free air condition of 1 bar, 27°C the mass of air delivered,

$$m = \frac{PV}{RT} = \frac{1\times10^2\times20}{0.287\times300}$$

$$m = 23.23 \text{ kg/min}$$

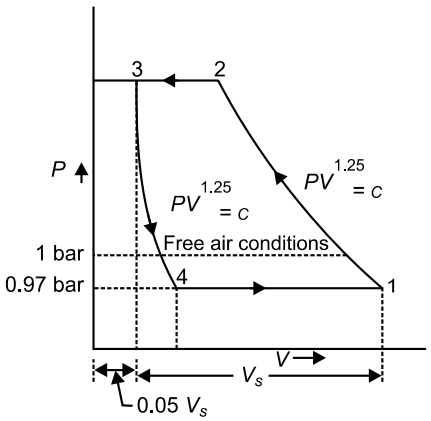


Fig. 16.30 P-V *representation*

For compression process 1–2, $PV^{1.25}$ = Constant

$$T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{1.25-1}{1.25}}$$

$$= 308 \times (7)^{0.2}$$

$$T_2 = 454.54 \text{ K}$$

For expansion process, 3–4, $PV^{1.25}$ = Constant

$$V_4 = V_3 \times \left(\frac{P_3}{P_4}\right)^{\frac{1}{1.25}}$$

$$V_4 = 0.05\ V_s\ (7)^{1/1.25}$$

$$V_4 = 0.273\ V_s$$

$$V_1 - V_4 = 1.05\ V_s - 0.237\ V_s = 0.813\ V_s$$

Volume of air corresponding to the suction conditions can be obtained using the volume of free air delivered

$$= \frac{0.97\times300}{1\times308} \times (V_1 - V_4) = \frac{0.97\times300}{1\times308} \times 0.813\ V_s$$

$$= 0.768\ V_s$$

Volumetric efficiency = $\frac{0.768\,V_s}{V_s}$ = 0.768 or 76.8%

Volumetric efficiency = 76.8% ***Ans.***

In order to find out the dimension of cylinder the volume of air sucked in one cycle. Volume of air sucked in a cycle = $\frac{20}{2 \times 240}$ = 0.0208 m^3/cycle.

Volume per cycle = Swept volume

$$0.0208 = \frac{\pi}{4} D^2 \cdot L = \frac{\pi}{4} D^2 \cdot (1.2\ D)$$

$$\Rightarrow \quad D = 0.2805 \text{ m or } 28.05 \text{ cm}$$

$$L = 33.66 \text{ cm}$$

Bore =	28.05 cm
Stroke =	33.66 cm

Ans.

Work required in reciprocating compressor

$$W = \frac{n}{n-1} mR\ (T_2 - T_1)$$

$$W = \left(\frac{1.25}{1.25-1}\right) \times 23.23 \times 0.287\ (454.54 - 308)$$

$$= 4884.92 \text{ kJ/min}$$

$$= \frac{4884.92}{60 \times 0.7457}, \text{ {as 1 hp = 0.7457 kW]}}$$

$$W = 109.18 \text{ hp}$$

Work required when compression is isothermal, $W_{iso} = m\ RT_1 \ln\left(\frac{P_2}{P_1}\right)$

$$= 23.23 \times 0.287 \times 308 \ln (7)$$

$$= 3995.81 \text{ kJ/min}$$

$$= \frac{3995.81}{60 \times 0.7457}$$

$$W_{iso} = 89.31 \text{ hp}$$

Isothermal efficiency = $\frac{W_{iso}}{W} = \frac{89.31}{109.81}$ = 0.8133 or 81.33%

Isothermal efficiency = 81.33% ***Ans.***

4. *A reciprocating compressor of single stage and double acting type is running at 200 rpm with mechanical efficiency of 85%. Air flows into compressor at the rate of 5 m^3/min measured at atmospheric condition of 1.02 bar, 27°C. Compressor has compressed air leaving at 8 bar with compression following polytropic process with index of 1.3. Compressor has clearance volume of 5% of stroke volume. During suction of air from atmosphere into compressor its temperature rises by 10°C. There occurs pressure loss of 0.03 bar during suction and pressure loss of 0.05 bar during discharge passage through valves. Determine the dimensions of cylinder, volumetric efficiency and power input required to drive the compressor if stroke to bore ratio is 1.5.*

Solution:

Considering the losses at suction and discharge, the actual pressure at suction and delivery shall be as under.

Atmospheric pressure, P_a = 1.02 bar, T_a = 273 + 27 = 300 K, V_a = 5 m^3/min

Pressure at suction, P_1 = 1.02 – 0.03 = 0.99 bar

T_1 = 300 + 10 = 310 K

Pressure at delivery, P_2 = 8 + 0.05 = 8.05 bar

Volume corresponding to suction condition of P_1, T_1,

$$V_1 = \frac{P_a \cdot T_1 \cdot V_a}{P_1 \cdot T_a} = \frac{1.02 \times 310 \times 5}{0.99 \times 300} = 5.32 \text{ m}^3/\text{min}$$

Therefore, work required for compression,

$$W = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \left(\frac{1.3}{1.3-1} \right) \times \frac{0.99 \times 10^2 \times 5.32}{60} \left[\left(\frac{8.05}{0.99} \right)^{\frac{0.3}{1.3}} - 1 \right]$$

= 23.66 kW or 31.73 hp

$$\text{Power input required} = \frac{31.73}{0.85} = 37.33 \text{ hp}$$

Power input = 37.33 hp **Ans.**

Volumetric efficiency, $\eta_{vol} = \frac{P_1 T_a}{P_a T_1} \left[1 + C - C \left(\frac{P_2}{P_1} \right)^{\frac{1}{n}} \right]$

Here, C = 0.05, so $\eta_{vol} = \frac{0.99 \times 300}{1.02 \times 310} \left[1 + 0.05 - 0.05 \left(\frac{8.05}{0.99} \right)^{\frac{1}{1.3}} \right]$

= 0.7508 or 75.08%

$$\text{Stroke volume per cycle} = \frac{5}{2 \times 200} = 0.0125 \text{ m}^3/\text{cycle}$$

$$\text{Actual stroke volume taking care of volumetric efficiency} = \frac{0.0125}{0.7508}$$

= 0.0167 m^3/cycle

$$\text{Stroke volume} = \frac{\pi}{4} D^2 L = 0.0167$$

$$\frac{\pi}{4} D^2 \times 1.5D = 0.0167$$

D = 0.2420 m or 24.20 cm

Stroke L = 1.5 D = 36.3 cm

Cylinder dimensions = bore of 24.20 cm, stroke of 36.3 cm. ***Ans.***
Volumetric efficiency = 75.08%

5. *In a reciprocating air compressor the air is compressed at the rate of 4 m^3/min at 1 bar, 27°C up to the pressure of 8 bar following index of compression as 1.2. The compression occurs in two stages with intercooling at optimum intercooler pressure and perfect intercooling. Compare the work input required if the same compression occurs in single stage. Also compare the work input if same compression occurs in two stages with imperfect intercooling up to 30°C at the optimum intercooling pressure. Consider C_p = 1.0032 kJ/kg · K and R = 0.287 kJ/kg · K.*

Solution:

Mass of air compressed per minute;

$$m = \frac{1\times10^2\times4}{0.287\times300} = 4.65 \text{ kg/min}$$

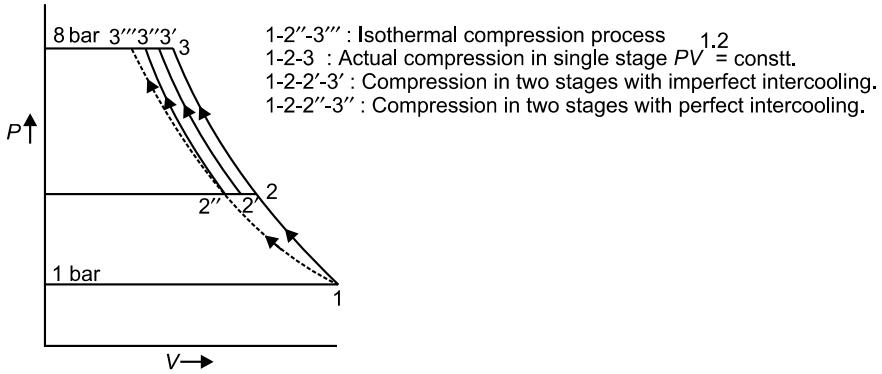


Fig. 16.31 *P-V diagram*

Different types of compression are shown on *P–V* diagram

For single stage compression from 1 bar to 8 bar, process 1–2–3, $PV^{1.2}$ = Constt.

$$\text{Work input, } W_I = \frac{n}{n-1} P_1V_1 \left[\left(\frac{P_3}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \left(\frac{1.2}{1.2-1}\right) \times 1 \times 10^2 \times 4 \left[\left(\frac{8}{1}\right)^{\frac{1.2-1}{1.2}} - 1\right]$$

$$W_I = 994.113 \text{ kJ/min or } \left(\frac{994.113}{60\times0.7457}\right) = 22.22 \text{ hp}$$

Optimum intercooling pressure = $\sqrt{8\times1}$ = 2.83 bar

For two stage compression with perfect intercooling;

$$\text{Work input, } W_{II,\ I/C} = 2 \times \frac{n}{n-1} P_1V_1 \left[\left(\frac{P_3}{P_1}\right)^{\frac{n-1}{2n}} - 1\right]$$

$$= 2 \times \left(\frac{1.2}{1.2-1}\right) \times 1 \times 10^2 \times 4 \times \left[\left(\frac{8}{1}\right)^{\frac{1.2-1}{2.4}} - 1\right]$$

$W_{II,\ I/C}$ = 908.19 kJ/min or 20.29 hp

When there is two stage compression with imperfect intercooling:

Intercooler pressure, P_2 = 2.83 bar, $T_{2'}$ = 273 + 30 = 303 K

Volume of air at inlet of HP cylinder, $V_{2'} = \frac{P_1 V_1}{T_1} \times \frac{T_{2'}}{P_2}$

$$V_{2'} = \frac{1 \times 4 \times 303}{300 \times 2.83} = 1.43 \text{ m}^3/\text{min}$$

Hence, work required for two stage compression,

$$W'_{II,I/C} = W'_{HP} + W'_{LP}$$

$$= \left[\frac{n}{n-1} P_1 V_1 \left\{\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right\}\right] + \left[\frac{n}{n-1} P_2 V_2' \left\{\left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} - 1\right\}\right]$$

$$W'_{\text{II,I/C}} = \left[\left(\frac{1.2}{1.2-1}\right) \times 1 \times 10^2 \times 4 \left\{(2.83)^{\frac{(1.2-1)}{1.2}} - 1\right\}\right]$$

$$+ \left[\left(\frac{1.2}{1.2-1}\right) \times 2.83 \times 10^2 \times 1.43 \times \left\{\left(\frac{8}{2.83}\right)^{\frac{1.2-1}{1.2}} - 1\right\}\right]$$

= [454.3] + [459.2] = 913.5 kJ/min or 20.42 hp

% saving in work when compression occur with perfect intercooling as compared to single stage compression

$$= 100 \times \left(\frac{W_I - W_{II,I/C}}{W_I}\right)$$

= 8.69% ***Ans.***

% excess work to be done when two stage compression occurs with imperfect intercooling as compared to two stage compression with perfect intercooling:

$$= 100 \times \left(\frac{W'_{II,I/C} - W_{II,I/C}}{W'_{II,\ I/C}}\right)$$

= 0.636% ***Ans.***

6. *A reciprocating air compressor has four stage compression with 2 m³/min of air being delivered at 150 bar when initial pressure and temperature are 1 bar, 27°C. Compression occur polytropically following polytropic index of 1.25 in four stages with perfect intercooling between stages. For the optimum intercooling conditions determine the intermediate pressures and the work required for driving compressor.*

Solution:

Here there is four stage compression with perfect intercooling at optimum intercooling conditions.

So optimum stage pressure ratio = $(150)^{1/4}$ = 3.499 ≈ 3.5

Intermediate pressure shall be as follows:

Between Ist and IInd stage = 3.5 bar

Between IInd and IIIrd stage = 12.25 bar

Between IIIrd and IVth stage = 42.87 bar

Intermediate pressure: 3.5 bar, 12.25 bar, 42.87 bar ***Ans.***

Since it is perfect intercooling so temperature at inlet of each stage will be 300 k.

So temperature at the end of fourth stage, $T = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$

$= 300 \times (3.5)^{\left(\frac{1.25-1}{1.25}\right)}$

$T = 385.42$ K

Mass of air, kg/min, $m = \frac{PV}{RT} = \frac{150 \times 10^2 \times 2}{0.287 \times 385.42} = 271.21$ kg/min

Work required for driving compressor,

$$W = \left(\frac{n}{n-1}\right) m\, RT_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right] \times 4$$

$$= \left(\frac{1.25}{1.25-1}\right) \times 271.21 \times 0.287 \times 300 \left[(3.5)^{\frac{(1.25-1)}{1.25}} - 1\right] \times 4$$

$$= 132978.04 \text{ kJ/min or } 2972.11 \text{ hp } \left(= \frac{132978.04}{60 \times 0.7457}\right)$$

. Work input = 2972.11 hp ***Ans.***

7. *A two stage reciprocating compressor has air entering at 1 bar, 17°C into LP compressor and leaving HP stage at 16 bar. An intercooler working at 4 bar pressure is provided between the HP and LP stages. The compression process follows the process given by $PV^{1.3}$ = constant. The bore diameters of HP and LP cylinder are 6 cm and 12 cm respectively while stroke lengths are equal. For perfect intercooling determine the work done in compressing per unit mass of air. Also state whether the intercooler pressure will rise, fall or no change if the volumetric efficiency is taken as 0.90 for LP cylinder. Neglect clearance volumes.*

Solution:

For perfect intercooling the amount of work required shall be;

$$W = \frac{n \times m}{n-1} RT_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} + \left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} - 2\right]$$

$$= \frac{1.3 \times 1}{1.3-1} \times 0.287 \times (273 + 17) \left[(4)^{\frac{(1.3-1)}{1.3}} + \left(\frac{16}{4}\right)^{\frac{1.3-1}{1.3}} - 2\right]$$

= 271.94 kJ/kg

. Work = 271.94 kJ/kg ***Ans.***

The ratio of volumes handled by HP and LP stages. Using the perfect intercooling.

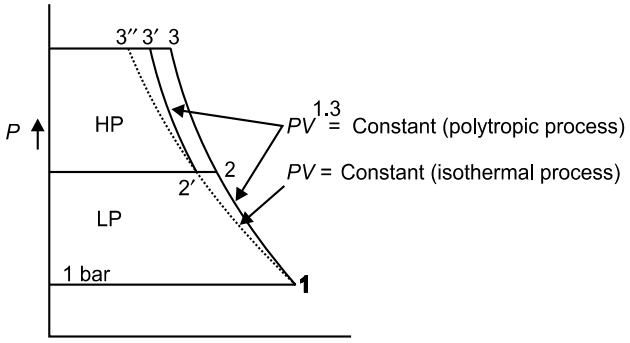


Fig. 16.32 *Two stage compression on* P-V *diagram*

State at inlet of LP cylinder = $P_1\ V_1$

State at inlet of HP cylinder = $P_2\ V_{2'}$

Since state 2′ lies on isothermal compression process line so, $P_1\ V_1 = P_2\ V_{2'}$ or,

$$\frac{V_1}{V_{2'}} = \frac{P_2}{P_1} = \left(\frac{4}{1}\right) = 4$$

$$\frac{\text{Volume at inlet to LP}}{\text{Volume at inlet to HP}} = 4$$

From given cylinder dimensions, for given bore diameters and common stroke length,

$$\text{Ratio of effective cylinder volumes} = \frac{\text{Effective volume of LP cylinder}}{\text{Effective volume of HP cylinder}}$$

$$= \frac{0.90 \times \frac{\pi}{4} D_{\text{LP}}^2 \cdot L}{\frac{\pi}{4} D_{\text{HP}}^2 \cdot L}$$

$$= \frac{0.9 \times (0.12)^2}{(0.06)^2} = 3.6$$

Theoretically, the volume ratio is 4 while considering volumetric efficiency the ratio of effective cylinder volumes comes out to be 3.6 which is less than the theoretical volume ratio. Therefore, it can be concluded that less amount of air is given into HP cylinder than its' capacity. Thus, the HP cylinder would inhale volume equal to its' capacity and since it has larger capacity than volume available so the pressure of intercooler shall drop.

8. *In a two stage reciprocating air compressor running at 200 rpm the air is admitted at 1 bar, 17°C and discharged at 25 bar. At low pressure stage suction conditions the rate of air flow is 4 kg/minute. The low pressure cylinder and high pressure cylinders have clearance volumes of 4% and 5% of respective cylinder stroke volumes. The index for compression and expansion processes in two stages are same*

as 1.25. Considering an optimum and perfect intercooling in between two stages determine the power required, isothermal efficiency, free air delivered, heat transferred in each cylinder and the cylinder volumes.

Solution:

For the optimum intercooling the pressure ratio in each stage = $\sqrt{\frac{25}{1}} = 5$

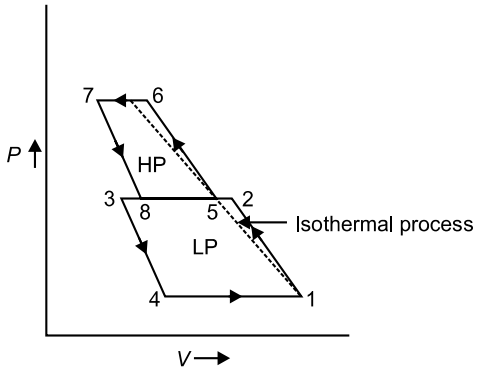


Fig. 16.33 P-V *representation*

$$\frac{P_2}{P_1} = \frac{P_6}{P_5} = 5$$

Perfect intercooling indicates, $T_1 = T_5 = 273 + 17 = 290$ K

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 400.12 \text{ K}$$

$$T_6 = T_5 \left(\frac{P_6}{P_5}\right)^{\frac{n-1}{n}} = 400.12 \text{ K}$$

Actual compression work requirement,

$$W = W_{HP} + W_{LP}$$

$$= 2\left(\frac{n}{n-1}\right) m\, RT_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= 2 \times \left(\frac{1.25}{1.25-1}\right) \times 4 \times 0.287 \times 290 \left[(5)^{\frac{(1.25-1)}{1.25}} - 1\right]$$

$$W = 1264.19 \text{ kJ/min or } 28.25 \text{ hp}$$

Work requirement if the process is isothermal compression,

$$W_{iso} = m\, RT_1 \ln\left(\frac{P_6}{P_1}\right) = 4 \times 0.287 \times 290 \ln (25)$$

$$W_{iso} = 1071.63 \text{ kJ/min}$$

Isothermal efficiency = $\frac{W_{iso}}{W}$ = 0.8477 or 84.77%

Free air delivered = $\frac{mRT_1}{P_1} = \frac{4 \times 0.287 \times 290}{1 \times 10^2}$ = 3.33 m^3/min

Heat transferred in HP cylinder = Heat transferred in LP cylinder = Q
(Due to optimum and perfect intercooling)

$$Q = \left(\frac{W}{2}\right) - m\, C_p\, (T_2 - T_1)$$

$$= \left(\frac{1264.19}{2}\right) - 4 \times 1.0032 \times (400.12 - 290)$$

$$Q = 190.21 \text{ kJ/min}$$

Volumetric efficiency, $\eta_{\text{vol}} = \frac{P_1 T_a}{P_a \cdot T_1}\left[1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right]$

Here the ambient conditions and suction conditions are same so expression gets modified as,

$$\eta_{\text{vol}} = 1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

Volumetric efficiency of HP,

$$\eta_{\text{vol, HP}} = 1 + C_{\text{HP}} - C_{\text{HP}}\left(\frac{P_6}{P_5}\right)^{\frac{1}{n}}$$

$$C_{\text{HP}} = 0.04$$

$$= 1 + 0.04 - 0.04\ (5)^{1/1.25}$$

$$\eta_{\text{vol, HP}} = 0.895 \text{ or } 89.5\%$$

Volumetric efficiency of LP,

$$\eta_{\text{vol, LP}} = 1 + C_{\text{LP}} - C_{\text{LP}}\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

$$C_{\text{LP}} = 0.05$$

$$= 1 + 0.05 - 0.05\ (5)^{1/1.25} = 0.8688 \text{ or } 86.88\%$$

Stroke volume of HP cylinder = $\frac{\text{Free air delivery}}{\text{Pressure ratio} \times \text{speed} \times \eta_{\text{vol, HP}}}$

$$V_{s,\text{ HP}} = \frac{3.33}{5 \times 200 \times 0.895} = 3.721 \times 10^{-3} \text{ m}^3$$

Clearance volume, $V_{c,\text{ HP}} = 0.05 \times 3.721 \times 10^{-3} = 1.861 \times 10^{-4}$ m^3
Total HP cylinder volume, $V_{\text{HP}} = V_{s,\text{ HP}} + V_{c,\text{ HP}} = 3.907 \times 10^{-3}$ m^3 ;
$V_{c,\text{ HP}}$ = Clearance volume of HP

Stroke volume of LP cylinder = $\frac{\text{Free air delivery}}{\text{Speed} \times \eta_{\text{vol, LP}}} = \frac{3.33}{200 \times 0.8688}$

$$V_{s,\ LP} = 0.01916\ m^3$$

Clearance volume, $V_{c,\ LP} = 0.04 \times V_{s,\ LP} = 7.664 \times 10^{-4}\ m^3$

Total LP cylinder volume, $V_{LP} = V_{s,\ LP} + V_{c,\ LP} = 0.019926\ m^3$

Power required = 28.25 hp,
Isothermal efficiency = 84.77%
Free air delivered = 3.33 m^3/min,
Heat transfer in HP cylinder = 190.21 kJ/min
Heat transferred in LP cylinder = 190.21 kJ/min,
HP cylinder volume = $3.907 \times 10^{-3}\ m^3$
LP cylinder volume = 0.019926 m^3

Ans.

9. *A two stage double acting reciprocating air compressor running at 200 rpm has air entering at 1 bar, 25°C. The low pressure stage discharges air at optimum intercooling pressure into intercooler after which it enters at 2.9 bar, 25°C into high pressure stage. Compressed air leaves HP stage at 9 bar. The LP cylinder and HP cylinder have same stroke lengths and equal clearance volumes of 5% of respective cylinder swept volumes. Bore of LP cylinder is 30 cm and stroke is 40 cm. Index of compression for both stages may be taken as 1.2. Determine,*

(i) the heat rejected in intercooler,
(ii) the bore of HP cylinder,
(iii) the hp required to drive the HP cylinder.

Solution:

Optimum intercooling pressure = $\sqrt{9}$ = 3 bar
LP stage pressure ratio = HP stage pressure ratio = 3
From the given dimensions of LP cylinder, the volume of LP cylinder, in m^3/min

$$V_{LP} = \frac{\pi}{4} \times (0.30)^2 \times (0.40) \times 200 \times 2$$

$$V_{LP} = 11.31\ m^3/\text{min}$$

Volumetric efficiency of LP compressor, here ambient and suction conditions are same,

$$\eta_{vol,\ LP} = 1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} = 1 + 0.05 - \left(0.05\left(\frac{3}{1}\right)^{\frac{1}{1.2}}\right)$$

$$\eta_{vol,\ LP} = 0.9251 \text{ or } 92.51\%$$

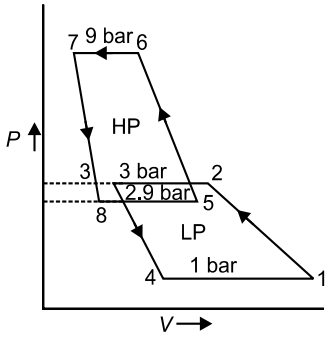


Fig. 16.34 P-V *diagram*

Volume of air inhaled in LP stage = $V_{LP} \times \eta_{vol,\ LP}$

$= 11.31 \times 0.9251$

$= 10.46\ m^3/min$

Mass of air per minute, $m = \dfrac{P_1 V_1}{RT_1}$

$= \dfrac{1 \times 10^2 \times 10.46}{0.287 \times 298} = 12.23$ kg/min

Temperature after compression in LP stage,

$$T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$= 298 \times \left(\frac{3}{1}\right)^{\frac{1.2-1}{1.2}}$$

$$T_2 = 357.88\ K$$

Volume of air going into HP cylinder

$$V_5 = \frac{mRT_5}{P_5}$$

After intercooling, $T_5 = 298$ K, $P_5 = 2.9$ bar,

$$V_5 = \frac{12.23 \times 0.287 \times 298}{2.9 \times 10^2}$$

$$V_5 = 3.61\ m^3/min$$

Since the clearance volume fraction and pressure ratio for both HP and LP stages are same so the volumetric efficiency of HP stage referred to LP stage suction condition shall be same

$$\eta_{vol,\ HP} = \eta_{vol,\ LP} = 0.9251$$

Hence, the volume of HP cylinder/min = $\dfrac{V_5}{\eta_{vol,HP}} = \dfrac{3.61}{0.9251} = 3.902\ m^3/min$

Let bore of HP cylinder be D_{HP},

$$3.902 = \frac{\pi}{4} \times (D_{HP})^2 \times 0.40 \times 2 \times 200$$

$$D_{HP} = 0.1762\ m \text{ or } 17.62\ cm$$

Heat rejected in intercooler, $Q = mC_p\ (T_2 - T_5)$

$= 12.23 \times 1.0032 \times (357.88 - 298)$

$= 734.68$ kJ/min

In HP stage, $\dfrac{T_6}{T_5} = \left(\dfrac{P_6}{P_5}\right)^{\frac{n-1}{n}} \Rightarrow T_6 = 298 \times \left(\dfrac{9}{2.9}\right)^{\frac{1.2-1}{1.2}}$

$$T_6 = 359.91\ K$$

Work input required for HP stage, $W_{HP} = \left(\frac{n}{n-1}\right) mR\,(T_6 - T_5)$

$$= \left(\frac{1.2}{1.2-1}\right) \times 12.23 \times 0.287 \cdot (359.9 - 298)$$

$W_{HP} = 1303.62$ kJ/min

or $W_{HP} = 29.14$ hp

Heat rejected in intercooler = 734.68 kJ/min
Bore of HP cylinder = 17.62 cm
Horse power required to drive HP stage = 29.14 hp

Ans.

10. *During an experiment on reciprocating air compressor the following observations are being taken;*

Barometer reading = 75.6 cm Hg, Manometer reading across orifice = 13 cm Hg.
Atmospheric temperature = 25°C. Diameter of orifice = 15 mm.
Coefficient of discharge across the orifice = 0.65
Take density of Hg = 0.0135951 kg/cm^3
Determine the volume of free air handled by compressor in m^3/min.

Solution:

Cross-sectional area of orifice, $A = \frac{\pi}{4} \times (15 \times 10^{-3})^2 = 1.77 \times 10^{-4}$ m^2

Atmospheric pressure $= 75.6 \times 0.0135951 \times 9.81 \times 10^4 \times 10^{-3}$
$= 100.83$ kPa

Specific volume of air per kg at atmospheric conditions,

$$v = \frac{RT}{P} = \frac{1 \times 0.287 \times 298}{100.83} = 0.848 \text{ m}^3/\text{kg}$$

Density of air $= \frac{1}{v} = 1.18$ kg/m^3

Pressure difference across orifice $= 13 \times 0.0135951 \times 9.81 \times 10^4 \times 10^{-3}$
$= 17.34$ kPa

Height of air column for pressure difference across orifice.

$\rho_a \times h_a \times g = 17.34 \times 10^3$

Put, $\rho_a = 1.18$ kg/m$^3 \Rightarrow h_a = 1497.95$ m

Free air delivery $= C_d \cdot A \cdot \sqrt{2gh_a}$
$= 0.65 \times 1.77 \times 10^{-4} \sqrt{2 \times 9.81 \times 1497.95}$
$= 0.01972$ m^3/s or 1.183 m^3/min

. | Free air delivery = 1.183 m^3/min | ***Ans.***

11. *During a trial on single acting single stage compression the following observations are made;*

Dimensions of cylinder: 10 cm bore and 8 cm stroke.
Speed of rotation: 500 rpm.

Barometer reading: 76 cm Hg

Atmospheric temperature: 27°C

Delivery air temperature = 130°C

Free air delivery = 15 m^3/hr

Spring balance of dynamometer type (electric motor) reading: 10 kg
Radius of arm of spring balance: 30 cm

Take mechanical efficiency = 0.90.

Determine the volumetric efficiency, shaft output per m^3 of free air per minute.

Solution:

Free air delivery = 15 m^3/hr = 0.25 m^3/min

Volume of cylinder = $\frac{\pi}{4} \times (0.10)^2 \times (0.08) = 6.28 \times 10^{-4}$ m^3

Volumetric efficiency = $\frac{(15/60)}{\left(6.28 \times 10^{-4} \times 500\right)}$ = 0.7962 or 79.62% *Ans.*

Shaft output = $\frac{2\pi NT}{60}$

Shaft output = $\frac{2 \times \pi \times 500 \times 10 \times 9.81 \times 0.30 \times 10^{-3}}{60}$

= 15.41 kJ/s or 20.66 hp

Shaft output per m^3 of free air per minute = $\frac{20.66}{0.25}$

= 82.64 hp per m^3 of free air per minute. ***Ans.***

12. *Determine the minimum number of stages required in an air compressor which admits air at 1 bar, 27°C and delivers at 180 bar. The maximum discharge temperature at any stage is limited to 150°C. Consider the index for polytropic compression as 1.25 and perfect and optimum intercooling in between the stages. Neglect the effect of clearance.*

Solution:

Let there be '*i*' number of stages. So the overall pressure ratio considering inlet state as P_a and T_a and delivery state pressure as P_i.

$$\frac{P_i}{P_a} = \frac{P_1}{P_a} \times \frac{P_2}{P_1} \times \frac{P_3}{P_2} \times \cdots \times \frac{P_i}{P_{i-1}}$$

When perfect and optimum intercooling is considered then pressure ratio in each stage will be same.

$$\frac{P_1}{P_a} = \frac{P_2}{P_1} = \frac{P_3}{P_2} = \ldots = \frac{P_i}{P_{i-1}} = r$$

$\frac{P_i}{P_a} = (r)^i$, for any stage, say second stage,

$T_1 = 273 + 27 = 300$ K

and $T_2 = 273 + 150 = 423$ K

and $\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}}$, $\frac{P_i}{P_a} = (r)^i = \left(\frac{T_2}{T_1}\right)^{\frac{in}{n-1}}$

$$\left(\frac{180}{1}\right) = \left(\frac{423}{300}\right)^{\frac{i \times 1.25}{1.25-1}}, \text{ Taking log for solving,}$$

$$\ln 180 = \left(\frac{1.25i}{0.25}\right) \ln\left(\frac{423}{300}\right)$$

Upon solving, $i = 3.022 \approx$ say 3 stages

3 stages ***Ans.***

13. *In a triple stage reciprocating compressor of single acting type the air enters at 1 bar, 27°C. The compressor has low pressure cylinder with bore of 30 cm and stroke of 20 cm. Clearance volume of LP cylinder is 4% of the swept volume. The final discharge from compressor takes place at 20 bar. The expansion and compression index may be taken uniformly as 1.25 for all the stages. The intercooling between the stages may be considered to be at optimum intercooling pressure and perfect intercooling. Determine, the interstage pressures, effective swept volume of low pressure cylinder, temperature and volume of air delivered in each stroke and the work done per kg of air.*

Solution:

Here $P_1 = 1$ bar, $T_1 = 300$ K, $C = 0.04$, $P_{10} = 20$ bar, $n = 1.25$, See Fig. 16.35

For optimum and perfect intercooling,

$$\frac{P_2}{P_1} = \frac{P_6}{P_2} = \frac{P_{10}}{P_6} = \left(\frac{20}{1}\right)^{\frac{1}{3}} = 2.714$$

$P_2 = 2.714$ bar, $T_5 = T_1 = 300$ K

$P_6 = 7.366$ bar $T_9 = T_1 = 300$ K

Volumetric efficiency of LP stage,

$$\eta_{\text{vol, LP}} = 1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{1.25}}$$

$$= 1 + 0.04 - 0.04\ (2.714)^{1/1.25}$$

$$= 0.9511 \text{ or } 95.11\%$$

LP swept volume, $V_1 - V_3 = \frac{\pi}{4} D^2 L = \frac{\pi}{4} \times (0.30)^2 \times 0.20 = 0.01414$ m^3

Effective swept volume of LP cylinder, $V_1 - V_4 = \eta_{\text{vol, LP}} \times (V_1 - V_3)$

$$V_1 - V_4 = 0.9511 \times 0.01414 = 0.01345 \text{ m}^3$$

Temperature of air delivered, $T_{10} = T_9 \times \left(\frac{P_{10}}{P_6}\right)^{\frac{1.25-1}{1.25}} = 300 \times (2.714)^{\frac{(1.25-1)}{1.25}}$

$$= 366.31 \text{ K}$$

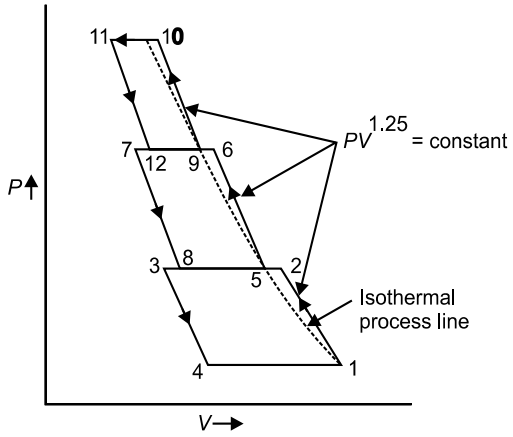


Fig. 16.35 P-V *representation*

For the compression process of air as perfect gas;

$$\frac{P_1 \times (V_1 - V_4)}{T_1} = \frac{P_{10} \times (V_{10} - V_{11})}{T_{10}}$$

$$V_{10} - V_{11} = \frac{(V_1 - V_4) \times T_{10} \times P_1}{T_1 \times P_{10}}$$

$$= \frac{0.01345 \times 366.31 \times 1 \times 10^2}{300 \times 20 \times 10^2}$$

Volume of air delivered = $V_{10} - V_{11} = 8.2115 \times 10^{-4}$ m^3

Total Work done per kg air,

$$W = 3 \times \left\{ \frac{n}{n-1} RT_1 \left(\left(\frac{P_2}{P_1} \right)^{\frac{1.25-1}{1.25}} - 1 \right) \right\}$$

$$= 3 \times \left\{ \left(\frac{1.25}{1.25-1} \right) \times 0.287 \times 300 \times \left((2.714)^{\frac{(1.25-1)}{1.25}} - 1 \right) \right\}$$

$$= 285.44 \text{ kJ/kg of air}$$

Intermediate pressure = 2.714 bar, 7.366 bar
Effective swept volume of LP cylinder = 0.01345 m^3
Temperature of air delivered = 366.31 K
Volume of air delivered = 8.2115×10^{-4} m^3
Work done = 285.44 kJ/kg of air

Ans.

14. *A two stage reciprocating air compressor has air being admitted at 1 bar, 27°C and delivered at 30 bar, 150°C with interstage pressure of 6 bar and intercooling up to 35°C. Compressor delivers at the rate of 2 kg/s. Clearance volumes of LP and HP cylinders are 5% and 7% of stroke volume respectively. The index of compression and expansion are same throughout. Determine the swept volume of both cylinders in m^3/min, amount of cooling required in intercooler and total power required. Also estimate the amount of cooling required in each cylinder.*

Solution:

Given: $P_1 = 1$ bar, $T_1 = 300$ K, $P_2 = 6$ bar, $P_6 = 30$ bar,
$T_6 = 273 + 150 = 423$ K, $T_5 = 273 + 35 = 308$ K, $C_{LP} = 0.05$, $C_{HP} = 0.07$, $m = 2$ kg/s

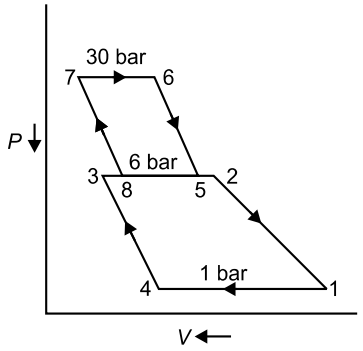


Fig. 16.36 P-V *representation*

For process 5–6, $P_2 = P_5$

$$\frac{P_6}{P_5} = \left(\frac{T_6}{T_5}\right)^{\frac{n}{n-1}}$$

$$\Rightarrow \quad \left(\frac{30}{6}\right) = \left(\frac{423}{308}\right)^{\frac{n}{n-1}}$$

Taking log of both sides,

$$\Rightarrow \quad \ln (5) = \frac{n}{n-1} \ln (1.3734)$$

Upon solving we get,

$$n = 1.245$$

Volumetric efficiency of LP cylinder,

$$\eta_{\text{vol, LP}} = 1 + C_{LP} - C_{LP}\left(\frac{P_2}{P_1}\right)^{\frac{1}{1.245}}$$

$$= 1 + 0.05 - 0.05\left(\frac{6}{1}\right)^{\frac{1}{1.245}}$$

$$= 0.8391 \text{ or } 83.91\%$$

Volumetric efficiency of HP cylinder,

$$\eta_{\text{vol, HP}} = 1 + C_{HP} - C_{HP}\left(\frac{P_6}{P_5}\right)^{\frac{1}{1.245}}$$

$$= 1 + 0.07 - 0.07\left(\frac{30}{6}\right)^{\frac{1}{1.245}}$$

$$\eta_{\text{vol, HP}} = 0.815 \text{ or } 81.50\%$$

For suction of LP cylinder

$$P_1 \cdot (V_1 - V_4) = mRT_1$$

$$(V_1 - V_4) = \frac{2 \times 0.287 \times 300}{1 \times 10^2} = 1.722 \text{ m}^3\text{/s or } 103.32 \text{ m}^3\text{/min}$$

$$\eta_{\text{vol, LP}} = \frac{V_1 - V_4}{V_1 - V_3} = 0.8391 \Rightarrow (V_1 - V_3)$$

$$= \frac{103.32}{0.8391} = 123.13 \text{ m}^3\text{/min}$$

Swept volume of LP cylinder = 123.13 m^3/min **Ans.**

For HP cylinder, $P_2 \times (V_5 - V_8) = m\,RT_5$

$$\Rightarrow \quad (V_5 - V_8) = \frac{mRT_5}{P_2}$$

$$= \frac{2 \times 0.287 \times 308}{6 \times 10^2} = 0.2946 \text{ m}^3\text{/s or } 17.676 \text{ m}^3\text{/min}$$

$$\eta_{\text{vol, HP}} = \frac{V_5 - V_8}{V_6 - V_7} \Rightarrow (V_6 - V_7) = \frac{17.676}{0.815} = 21.69 \text{ m}^3\text{/min}$$

Swept volume of HP cylinder = 21.69 m^3/min **Ans.**

For compression in LP stage,

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$= 300 \times \left(\frac{6}{1}\right)^{\frac{1.245-1}{1.245}}$$

$$T_2 = 426.83 \text{ K}$$

Cooling required in intercooler, $Q_{I/C} = m \times C_p \times (T_2 - T_5)$

$$= 2 \times 1.0032 \times (426.83 - 308)$$

$$Q_{I/C} = 238.42 \text{ kJ/s}$$

Heat picked in intercooler = 238.42 kW **Ans.**

Work input required = $W_{\text{LP}} + W_{\text{HP}}$

$$= \frac{n}{n-1}\, m\, RT_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right] + \frac{n}{n-1}\, m\, RT_5 \left[\left(\frac{P_6}{P_5}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= \frac{n}{n-1}\, mR \left[T_1\left\{\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right\} + T_5\left\{\left(\frac{P_6}{P_5}\right)^{\frac{n-1}{n}} - 1\right\}\right]$$

$$= \left(\frac{1.245}{1.245-1}\right) \times 2 \times 0.287 \left[300\left\{\left(\frac{6}{1}\right)^{\frac{1.245-1}{1.245}} - 1\right\} + 308\left\{\left(\frac{30}{6}\right)^{\frac{1.245-1}{1.245}} - 1\right\}\right]$$

$$= 704.71 \text{ kJ/s}$$

Total work required = 704.71 kW ***Ans.***

Heat transferred in LP cylinder = Amount of cooling required in LP cylinder

$$Q_{LP} = m\left(\frac{\gamma - n}{n-1}\right) \times C_v \times (T_2 - T_1)$$

$$= 2 \times \left(\frac{1.4 - 1.245}{1.245 - 1}\right) \times 0.72 \times (426.83 - 300)$$

$$= 115.55 \text{ kJ/s}$$

. Amount of cooling required in LP cylinder = 115.55 kW ***Ans.***

Heat transferred in HP cylinder = Amount of cooling required in HP cylinder

$$Q_{HP} = m\left(\frac{\gamma - n}{n-1}\right) \times C_v \times (T_6 - T_5)$$

$$= 2 \times \left(\frac{1.4 - 1.245}{1.245 - 1}\right) \times 0.72 \times (423 - 308)$$

$$= 104.77 \text{ kJ/s}$$

Amount of cooling required in HP cylinder = 104.77 kW ***Ans.***

15. *A roots blower handles free air of 0.5 m^3/s at 1 bar and 27°C and delivers air at pressure of 2 bar. Determine indicated power required to drive compressor and isentropic efficiency.*

Solution:

Indicated power required = $(P_2 - P_1) \times V_1$

$$W_{roots} = (2 - 1) \times 10^2 \times 0.5 = 50 \text{ kJ/s}$$

$$W_{roots} = 50 \text{ kW or } 67.05 \text{ hp}$$

Indicated power when isentropic compression occurs,

$$W_{isentropic} = \frac{\gamma}{\gamma - 1} \times P_1 V_1 \left\{\left(\frac{P_2}{P_1}\right)^{\frac{(\gamma-1)}{\gamma}} - 1\right\}$$

$$= \left(\frac{1.4}{1.4-1}\right) \times 1 \times 10^2 \times 0.5 \left\{\left(\frac{2}{1}\right)^{\frac{1.4-1}{1.4}} - 1\right\}$$

$$= 38.33 \text{ kW or } 51.4 \text{ hp}$$

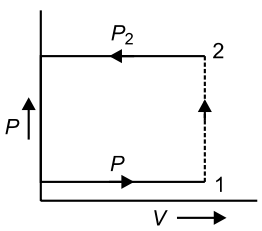


Fig. 16.37

Isentropic efficiency of roots blower = $\frac{W_{\text{isentropic}}}{W_{\text{roots}}} = \frac{38.33}{50}$ = 0.7666 or 76.66%

Indicated power of roots blower = 67.05 hp, Isentropic efficiency = 76.66%

Ans.

16. *A vaned compressor handles free air of 0.6 m^3/s at 1 bar and compresses up to 2.3 bar. There occurs 30% reduction in volume before the back flow occurs. Determine the indicated power required and isentropic efficiency.*

Solution:

Here on *P–V* diagram the state 2 indicates the point at which delivery occurs. While 2′ is the point up to which air is compressed inside. $V_2 = 0.7 \times V_1$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = \left(\frac{V_1}{0.7V_1}\right)^{1.4}$$

$$\Rightarrow \qquad P_2 = 1 \times \left(\frac{1}{0.7}\right)^{1.4} = 1.65 \text{ bar}$$

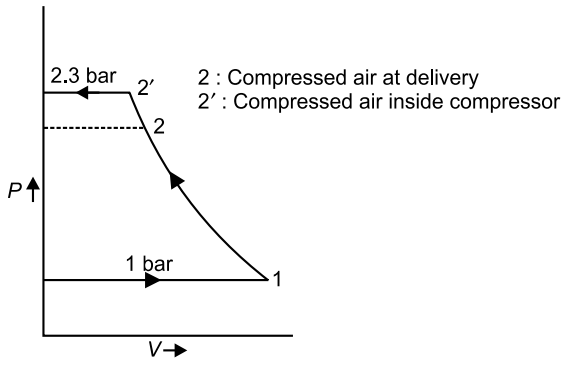


Fig. 16.38 *P-V diagram*

Indicated power required for vaned compressor

$$W_{\text{vane}} = \left(\frac{\gamma}{\gamma - 1}\right) P_1 V_1 \left\{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\} + (P_{2'} - P_2) \times V_2$$

$$= \left(\frac{1.4}{1.4-1}\right) \times 1 \times 10^2 \times 0.6 \left\{(1.65)^{\frac{(1.4-1)}{1.4}} - 1\right\} + (2.3 - 1.65) \times 10^2 \times (0.7 \times 0.6)$$

= 59.60 kJ/s or 79.925 hp

Power requirement when compression occurs isentropically,

$$W_{\text{isentropic}} = \frac{\gamma}{\gamma - 1} P_1 V_1 \left\{\left(\frac{P_{2'}}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\}$$

$$= \left(\frac{1.4}{1.4-1}\right) \times 1 \times 10^2 \times 0.6 \left\{\left(\frac{2.3}{1}\right)^{\frac{1.4-1}{1.4}} - 1\right\}$$

= 56.42 kJ/s or 75.66 hp

Isentropic efficiency of Vane compressor = $\frac{W_{\text{isentropic}}}{W_{\text{vane}}} = \frac{75.66}{79.925}$

= 0.9466 or 94.66%

Indicated power required = 79.925 hp, isentropic efficiency = 94.66% ***Ans.***

17. *A centrifugal compressor delivers free air of 18 kg/min. Air is sucked at static states of 1 bar, 27°C with inlet velocity of 50 m/s. The total head pressure ratio is 4 and isentropic efficiency of compressor is 0.75. The mechanical efficiency of motor attached to it is 0.90. Determine total head temperature of air at exit of compressor and brake power required to drive compressor.*

Solution:

Stagnation temperature at inlet, $T_{01} = T_1 + \frac{V_1^2}{2C_p}$

$$T_{01} = 300 + \frac{(50)^2}{2 \times 1.0032 \times 10^3} = 301.25 \text{ K}$$

Isentropic efficiency of compressor,

$$\eta_{\text{isen}} = \frac{T_{02'} - T_{01}}{T_{02} - T_{01}}$$

For process 1–2,

$$\frac{T_{02'}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\Rightarrow \quad T_{02'} = T_{01} \times (4)^{\frac{(1.4-1)}{1.4}}$$

$$T_{02'} = 301.25 \times (4)^{0.4/1.4} = 447.66 \text{ K}$$

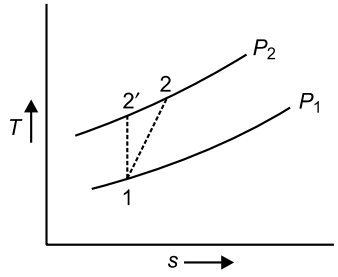


Fig. 16.39

Substituting temperature values in expression of isentropic efficiency,

$$T_{02} = \frac{(T_{02'} - T_{01})}{\eta_{isen}} + T_{01}$$

$$= \frac{(447.66 - 301.25)}{0.75} + 301.25$$

$$T_{02} = 496.46 \text{ K}$$

Total head temperature at exit = 496.46 K **Ans.**

Brake power of drive required $= \dfrac{m \times C_p \times (T_{02} - T_{01})}{\eta_{mech}}$

$$= \frac{18 \times 1.0032 \times (496.46 - 301.25)}{60 \times 0.9}$$

= 65.28 kW or 87.54 hp

Brake power required = 87.54 hp **Ans.**

18. *A double-acting single cylinder reciprocating air compressor has a piston displacement of 0.015 m^3 per revolution, operates at 500 r.p.m. and has a 5% clearance. The air is received at 1 bar and delivered at 6 bar. The compression and expansion are polytropic with n = 1.3. Determine,*

(i) the volumetric efficiency

(ii) the power required

(iii) the heat transferred and its direction, during compression if inlet temperature of air is 20°C.

[*U.P.S.C., 1998*]

Solution:

V = 0.015 m^3 per revolution, N = 500 r.p.m., C = 5% or 0.05, n = 1.3, P_1 = 1 bar, P_2 = 6 bar, T_1 = 20°C

Volumetric efficiency, $\eta_{vol} = 1 + C - C\left(\dfrac{P_2}{P_1}\right)^{\frac{1}{n}}$

$$= 1 + 0.05 - 0.05\left(\frac{6}{1}\right)^{\frac{1}{1.3}}$$

= 0.8516 or 85.16%

Ans. Volumetric efficiency = 85.16%

$$\text{Power required} = \frac{n}{n-1} P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

Here swept volume in cylinder = $V_s = V \times 2N$

$$= 0.015 \times 2 \times 500$$

V_s = 15 m^3/min

Actual air inhaled = V_1 = 15 × 0.85

V_1 = 12.75 m^3/min

Mass of air entering, $m = \dfrac{P_1V_1}{RT_1} = \dfrac{1\times10^2\times12.75}{0.287\times293} = 15.16$ kg/min.

$$\text{Power required} = \frac{(1.3)}{(1.3-1)}\times1\times10^2\times12.75\left[\left(\frac{6}{1}\right)^{\frac{1.3-1}{1.3}}-1\right]$$

$$= 2829.21 \text{ kJ/min}$$

Ans. Power required = 2829.21 kJ/min

Heat transferred during compression, (this is heat rejected), for a polytropic process

$$Q = m\,C_v\cdot\left(\frac{\gamma-n}{n-1}\right)(T_2 - T_1)$$

During compression process

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$T_2 = 298\times\left(\frac{6}{1}\right)^{\frac{1.3-1}{1.3}}$$

$$T_2 = 450.59 \text{ K}$$

Substituting in heat transferred Q,

$$Q = 15.16\times0.718\times\left(\frac{1.4-1.3}{1.3-1}\right)(450.59 - 293)$$

$$Q = 571.78 \text{ kJ/min}$$

Heat rejected during compression = 571.78 kJ/min ***Ans.***

EXERCISE

16.1 Classify the compressors.

16.2 Discuss the applications of compressed air to highlight the significance of compressors.

16.3 Describe the working of single stage reciprocating compressor.

16.4 Discuss the indicator diagram for reciprocating compressor. Also describe the factors responsible for deviation of hypothetical indicator diagram to actual indicator diagram.

16.5 Obtain the volumetric efficiency of single stage reciprocating compressor with clearance volume and without clearance volume.

16.6 Discuss the effects of clearance upon the performance of reciprocating compressor.

16.7 Define isothermal efficiency. Also discuss its significance.

16.8 What do you understand by multistage compression? What are its' merits over single stage compression?

16.9 Show that the volumetric efficiency with respect to free air delivery is given by,

$$\eta_{\text{vol.}} = \frac{P_1T_a}{P_a\cdot T_1}\left[1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}\right]$$

where all the terms of expression have their usual meanings.

16.10 Discuss the significance of intercooling upon the performance of multi-stage compression.

16.11 What is the optimum pressure ratio for perfect intercooling in between two stages of compression? The inlet and outlet pressures may be taken as P_1 and P_3.

16.12 Discuss the control of reciprocating air compressor.

16.13 Discuss the working of positive displacement rotary compressors.

16.14 Describe the working of centrifugal compressors.

16.15 What do you understand by surging and choking phenomenon?

16.16 Explain the stalling and its effect on the compressor performance.

16.17 Describe the characteristics of centrifugal compressor.

16.18 Compare the axial flow compressor with centrifugal compressors.

16.19 Show that the heat rejected in each stage of a reciprocating compressor with perfect intercooling is given by,

$$Q = \left[C_p + C_v \left(\frac{\gamma - n}{n - 1} \right) \right] (T_2 - T_1)$$

16.20 Write short notes on the following:

(*i*) Free air delivery

(*ii*) Volumetric efficiency

(*iii*) Axial flow compressors

(*iv*) Air flow rate measurement in reciprocating compressors.

16.21 A single stage single cylinder reciprocating compressor has 60 m^3/hr air entering at 1.013 bar, 15°C and air leaves at 7 bar. Compression follows polytropic process with index of 1.35. Considering negligible clearance determine mass of air delivered per minute, delivery temperature, indicated power and isothermal efficiency. [1.225 kg/min, 202.37°C, 4.23 kW, 77.1%]

16.22 A reciprocating compressor of single stage and double acting type has free air delivered at 14 m^3/min measured at 1.013 bar, 288 K. Pressure and temperature at suction are 0.95 bar and 305 K. The cylinder has clearance volume of 5% of swept volume. The air is delivered at pressure of 7 bar and expansion and compression follow the common index of 1.3. Determine the indicated power required and volumetric efficiency with respect to free air delivery. [63.55 kW, 72.4%]

16.23 A single stage double acting reciprocating compressor delivers 14 m^3/min measured at suction states of 1 bar and 20°C. Compressor runs at 300 rpm and air is delivered after compression with compression ratio of 7. Compressor has clearance volume of 5% of swept volume and compression follows polytropic process with index 1.3. Determine the swept volume of cylinder and indicated power in hp. [0.028 m^3, 76.86 hp]

16.24 A single stage single acting reciprocating air compressor handles 0.5 m^3/min of free air measured at 1 bar. Compressor delivers air at 6.5 bar while running at 450 rpm. The volumetric efficiency is 0.75, isothermal efficiency is 0.76 and mechanical efficiency is 0.80. Determine indicated mean effective pressure and power required to drive the compressor. [0.185 MPa, 3.44 hp]

16.25 A single stage single acting reciprocating air compressor compresses air by a ratio of 7. The polytropic index of both compression and expansion is 1.35. The clearance volume is 6.2% of cylinder volume. For volumetric efficiency of 0.8 and stroke to bore ratio of 1.3 determine the dimensions of cylinder. [14.67 cm and 19.08 cm]

16.26 A single stage single acting reciprocating air compressor runs with air entering at 1 bar and leaving at 7 bar following $PV^{1.3}$ = constant. Free air delivery is 5.6 m^3/minute and mean piston speed is 150 m/min. Take stroke to bore ratio of 1.3 and clearance volume to be $\frac{1}{15}$th of swept

volume per stroke. The suction pressure and temperature are equal to atmospheric air pressure and temperature. Determine volumetric efficiency, speed of rotation, stroke and bore. Take mean piston speed = 2 × stroke × rpm. [76.88%, 164 rpm, 45.7 cm, 35.1 cm]

16.27 A reciprocating compressor of single acting type has air entering at 1.013 bar, 15°C and leaving at 8 bar. Compressor is driven by electric motor of 30.84 hp and the mechanical efficiency is 0.87. The clearance volume is 7% of swept volume and the bore is equal to stroke. The compression and expansion follow $PV^{1.3}$ = constant. Determine (*i*) free air delivered in m^3/min, (*ii*) volumetric efficiency, and (*iii*) cylinder dimensions. [4.47 m^3/min, 72.68%, $L = D$ = 29.7 cm]

16.28 A reciprocating compressor has two stages with inlet air going into LP stage at 1 bar, 16°C and at the rate of 12 m^3/min. Air is finally delivered at 7 bar and there is perfect intercooling at optimum pressure between the stages. The index for compression is 1.25 and compressor runs at 600 rpm. Neglecting clearance volume determine intermediate pressure, total volume of each cylinder and total work required. [2.645 bar, 0.02 m^3, 0.0075 m^3, 57.6 hp]

16.29 A two stage reciprocating air compressor delivers 4.2 kg of free air per min at 1.01325 bar and 15°C. The suction conditions are 0.95 bar, 22°C. Compressor delivers air at 13 bar. Compression throughout occurs following $PV^{1.25} = C$. There is optimum and perfect intercooling between the two stages. Mechanical efficiency is 0.75. Neglecting clearance volume determine

(*i*) the heat transfer in intercooler per second.

(*ii*) the capacity of electric motor.

(*iii*) the % saving in work if two stage intercooling is compared with single stage compressor between same limits. [7.6 kJ/s, 44.65 hp, 13%]

16.30 A single acting reciprocating air compressor has two stages with the optimum and perfect intercooling in between. Compressor has air sucked at 1 bar and at the rate of 2.4 m^3/min when measured at 1.013 bar, 288 K. Compressor delivers air at 70 bar. Temperature at the end of suction stroke is 32°C. The compression and expansion follows polytropic process $PV^{1.25} = C$ uniformly. The clearance volume is 3% of swept volume in each HP and LP cylinder. Compressor runs at 750 rpm. If the mechanical efficiency is 0.85 then determine the power of drive required, swept volumes of each cylinder, % saving in power as compared to single stage compression within limits. [35.8 hp, 3963 cm^3, 473 cm^3, 20.89%]

17

Introduction to Internal Combustion Engines

17.1 INTRODUCTION

History of internal combustion engines dates back to year 1680 when Christian Huygens developed an engine using gun powder for explosion inside a cylindrical tube. Subsequently, in year 1860 a non compression engine utilising coal gas was developed by 'Lenoir' in France. Engine was called Lenoir engine and it was operationally similar to steam engine. In 1866 'Otto-Langen free piston engine' was developed in Germany and it had thermal efficiency more than 'Lenoir engine.' In 1876 four stroke engine based on Otto cycle was developed by Nikolous Otto in Germany which revolutionised the developments of internal combustion engines and are even used till date. The compression ignition engine technology and engines based on it evolved in year 1892. Such engines are credited to Rudolf Diesel another German engineer and are so named as Diesel engine. Two stroke engines came up in 1881 in Scotland and were first developed by Dugald Clerke. 'Brayton engine' based on Brayton cycle and 'Atkinson engine' based on Atkinson cycle were developed in year 1873 and 1885 respectively. Apart from conventional two stroke and four stroke engines described earlier the 'Wankel engine' which was rotary engine and the 'Stirling engine' which was external combustion engine came up in years 1957 and 1938 respectively. In 1923 an engine called as free piston engine was developed by Pateras Pescara in France. Free piston engine is actually a combination of reciprocating piston-cylinder engine and turbine. "Engine refers to a device which transforms one form of energy into the other form". "Heat engine is a modified form of engine used for transforming chemical energy of fuel into thermal energy and subsequently for producing work". Based on the mechanism used for adding thermal energy they can be classified into the following:

(*a*) External combustion engine

(*b*) Internal combustion engine.

External combustion engine have combustion occuring outside engine and adding heat to the working fluid used in the engine. Thus, in external combustion engines heat released during combustion is indirectly utilized by the working fluid in external combustion engine. Internal combustion engines have combustion occuring in engine itself and heat released during combustion is directly utilized for getting shaft work. Internal combustion engines have numerous advantages over external combustion engines such as lower weight to power output ratio, simplicity, smaller initial cost, and higher efficiency etc. Due to indirect heat transfer the external combustion engines permit for the use of any cheaper fuel such as coal, wood, oil etc. for combustion.

Internal combustion engines are exhaustively used in automobiles, gas turbine etc. and external combustion engines are used in steam turbine, steam engine, nuclear power plant etc. Internal combustion

engines are efficient than external combustion engines and also are smaller than external combustion engine of similar capacity.

17.2 CLASSIFICATION OF IC ENGINES

Internal combustion engines can be classified on the following basis.

(*a*) *Based on number of strokes :* Number of strokes involved in a cycle of IC engine can be two strokes or four strokes. Such engine can be;
  (*i*) Two stroke engines
  (*ii*) Four stroke engines

(*b*) *Based on thermodynamic cycle :* Depending upon thermodynamic cycle used in the internal combustion engines these can be classified as:
  (*i*) Engines based on Otto cycle ('Spark-Ignition engine')
  (*ii*) Engines based on Diesel or Dual cycle ('Compression-Ignition engine')

(*c*) *Based on mechanism of ignition:* Internal combustion engines have combustion as the basic process. Combustion process may be initiated using externally assisted ignition (spark ignition) or it may get initiated on its' own due to excessive compression (compression ignition). Such engines are called:
  (*i*) Spark ignition engines
  (*ii*) Compression ignition engines.
  The spark ignition engines may have "magneto ignition system" or "battery ignition system" for creating necessary electric potential for producing spark.

(*d*) *Based on type of fuel used:* IC engines may be classified depending upon the type of fuel being used. These can be:
  (*i*) Petrol engines (petrol being used as fuel)
  (*ii*) Gas engines (gaseous fuel being used)
  (*iii*) Diesel engines (diesel being used as fuel)
  (*iv*) Multi-fuel engines (more than one fuel being used)

(*e*) *Based on fuel admission:* IC engines can be of different types depending upon arrangement used for fuel admission:
  (*i*) Carburettor type engines (use carburettor fuel metering)
  (*ii*) Injection type engines (use fuel injector and injection system)

(*f*) *Based on type of cooling:* IC engines have inherent requirement of continuous cooling of engine. Based on type of cooling these can be classified as:
  (*i*) Air cooled engines (Generally used in small sized engines)
  (*ii*) Water cooled engines (Generally used in large sized engines)

(*g*) *Based on type of motion:* IC engines may have reciprocating motion of piston or it may also have rotary motion. Such engines can be:
  (*i*) Reciprocating engines
  (*ii*) Rotary engines
  Reciprocating engines may have different cylinder arrangements such as:
  (*i*) Opposed cylinder engines
  (*ii*) Inclined cylinder engines
  (*iii*) V-shaped cylinder arrangement.
  Rotary engines may be further classified as single rotor engines or multirotor engines i.e.
  (*i*) Single rotor engine
  (*ii*) Multi rotor engine

17.3 IC ENGINE TERMINOLOGY

Some of the basic components and generally used terms in internal combustion engines are given as under.

1. Cylinder: It is a cylindrical block having cylindrical space inside for piston to make reciprocating motion. Upper portion of cylinder which covers it from the top is called cylinder head. This is manufactured by casting process and materials used are cast iron or alloy steel.

2. Piston and Piston rings: Piston is a cylindrical part which reciprocates inside the cylinder and is used for doing work and getting work. Piston has piston rings tightly fitted in groove around piston and provide a tight seal so as to prevent leakage across piston and cylinder wall during piston's reciprocating motion. Pistons are manufactured by casting or forging process. Pistons are made of cast iron, aluminum alloy. Piston rings are made of silicon, cast iron, steel alloy by casting process.

3. Combustion space: It is the space available between the cylinder head and top of piston when piston is at farthest position from crankshaft (TDC).

4. Intake manifold: It is the passage/duct connecting intake system to the inlet valve upon cylinder. Through intake manifold the air/air-fuel mixture goes into cylinder.

5. Exhaust manifold: It is the passage/duct connecting exhaust system to the exhaust valve upon cylinder. Through exhaust manifold burnt gases go out of cylinder.

6. Valves: Engine has both intake and exhaust type of valves which are operated by valve operating mechanism comprising of cam, camshaft, follower, valve rod, rocker arm, valve spring etc. Valves are generally of spring loaded type and made out of special alloy steels by forging process.

7. Spark plug: It is the external ignitor used for initiating combustion process. Spark plug is activated by electrical energy fed by electrical system with engine. It delivers spark with suitable energy to initiate combustion at appropriate time for suitable duration.

8. Bearing: Bearings are required to support crank shaft. Bearings are made of white metal leaded bronze.

9. Connecting rod: It is the member connecting piston and crankshaft. It has generally I section and is made of steel by forging process.

10. Crank: It is the rigid member connecting the crankshaft and connecting rod. Crank is mounted on crankshaft. Crank transfers motion from connecting rod to crankshaft as it is linked to connecting rod through crank pin.

11. Crankshaft: It is the shaft at which useful positive work is available from the piston-cylinder arrangement. Reciprocating motion of piston gets converted into rotary motion of crankshaft. Crankshaft are manufactured by forging process from alloy steel.

12. Crankcase: Crankcase actually acts like a sump housing crank, crankshaft, connecting rod and is attached to cylinder. These are made of aluminium alloy, steel, cast iron etc. by casting process.

13. Gudgeon pin: It is the pin joining small end of the connecting rod and piston. This is made of steel by forging process.

14. Cams and Camshafts: Cams are mounted upon camshaft for opening and closing the valves at righ timings and for correct duration. Camshaft gets motion from crankshaft through timing gears.

15. Carburettor: Carburettor is device to prepare the air fuel mixture in right proportion and supply at right time.

16. Bore: It is nominal inner diameter of the cylinder.

17. Piston area: It is the area of a circle of diameter equal to bore.

18. Stroke: It is the nominal distance travelled by the piston between two extreme positions in the cylinder.

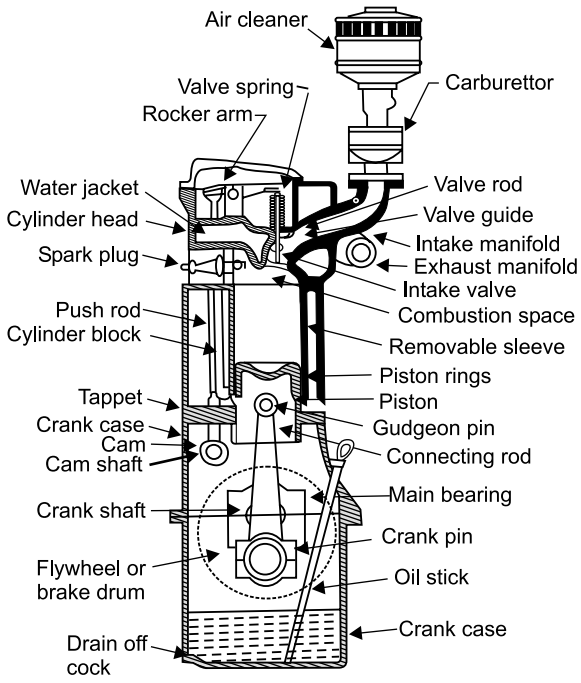


Fig. 17.1 *Internal combustion engine*

19. Dead centre: It refers to the extreme end positions inside the cylinder at which piston reverses it's motion. Thus, there are two dead centres in cylinder, called as 'top dead centre' or 'inner dead centre' and 'bottom dead centre' or 'outer dead centre'.

Top dead centre (TDC) is the farthest position of piston from crankshaft. It is also called inner dead centre (IDC).

Bottom dead centre (BDC) refers to the closed position of piston from crankshaft. It is also called outer dead center (ODC).

20. Swept volume : It is the volume swept by piston while travelling from one dead centre to the other. It may also be called stroke volume or displacement volume.

Mathematically, Swept volume = Piston area × Stroke

21. Clearance volume: It is the volume space above the piston inside cylinder, when piston is at top dead centre. It is provided for cushioning considerations and depends, largely upon compression ratio.

22. Compression ratio: It is the ratio of the total cylinder volume when piston is at BDC to the clearance volume.

$$\text{Compression ratio} = \frac{\text{Swept volume} + \text{Clearance volume}}{\text{Clearance volume}}$$

17.4 4-STROKE SI ENGINE

Spark ignition (SI) engines employ external ignition system for initiating the combustion process. Spark plug is the most commonly used ignition method used in spark ignition engines. Let us first understand

working of 4-stroke SI engine. Figure 17.3 shows the different strokes and processes associated with them.

State	*Valve position*		*Action*	*Work*	*Heat*
	Inlet	*Exhaust*			
1	Just opened	Just closed			
1–2	Open	Closed	Air-fuel intake	Air to piston	None
2	Just closed	Closed			
2–3	Closed	Closed	Compression	Piston to air	None
3	Closed	Closed	Ignition		
3–4	Closed	Closed	Combustion		Input to gas
4	Closed	Closed	Max. *T*, *P*		
4–5	Closed	Closed	Expansion	Gas to piston	None
5	Closed	Just opened			
5–6	Closed	Open	Exhaust blowdown	None	Gas to atmosphere
6	Closed	Open	Pressure drop to atmosphere	None	Gas to atmosphere
6–1	Closed	Open	Exhaust	Piston to gas	Gas to atmosphere

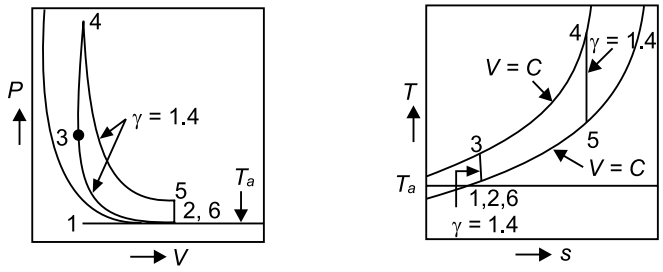
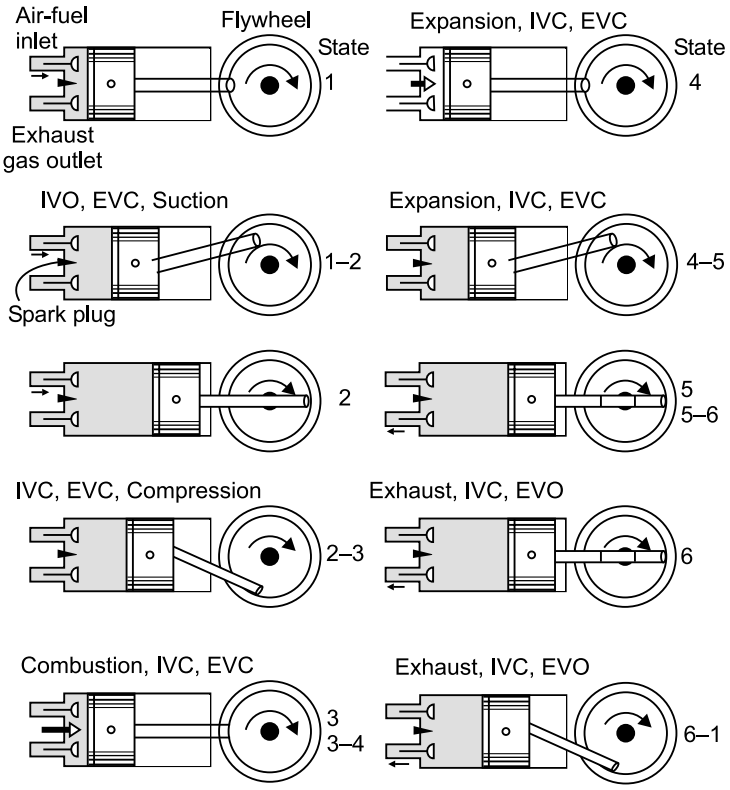


Fig. 17.2 *P-V and T-s diagram for SI engine cycle (Otto cycle)*

Let us start with piston at TDC, state 1. As piston moves for TDC to BDC, the inlet vlave gets opened and fresh air-fuel mixture prepared in carburettor enters the cylinder. This supply of air-fuel mixture into cylinder is called suction process or suction stroke during which inlet valve is open while exhaust valve remains closed. After the piston reaches BDC, it reverses its motion and moves towards TDC. During this piston travel both inlet and exhaust valves remain closed. Thus, the air-fuel mixture inside cylinder gets compressed till piston reaches TDC. This is the second stroke and called compression stroke or compression process (2–3).



IVC : Inlet valve closed, IVO: Inlet valve open, EVC: Exhaust valve closed, EVO: Exhaust valve open.

Fig. 17.3 *Working of 4-stroke S.I. engine*

Now highly compressed air-fuel mixture is available inside the cylinder and ready for combustion. With piston at TDC, the spark plug is activated and it releases spark for igniting air-fuel mixture. Spark plug used in the SI engines gets activated by suitable mechanism in the engine and provides suitable amount of energy in the form of spark for initiating combustion process. This burning of mixture is accompanied by sudden increase in pressure and temperature while piston passes the TDC position. Here constant volume heat addition takes place and state 4 is attained. Due to this release of fuel energy, the combustion products try to expand and piston moves from 4–5 i.e. TDC to BDC. During this travel the inlet and exhaust valves remain closed. This stroke is called expansion stroke or power stroke or expansion process. This is the stroke accompanied by positive work available at shaft. Now while piston is at BDC the exhaust valve gets opened and combustion products are exhausted out during 5–6. Cylinder is further emptied and made ready for being recharged while piston travels from BDC to TDC, 6–1 pushing out burnt gases. This is called exhaust stroke.

Here we have seen that out of suction, compression, expansion and exhaust strokes only expansion stroke is accompanied by the production of positive work, rest three strokes are work absorbing strokes. Work requirement for the three strokes is met from the work available during expansion stroke. For

storing the excess energy and releasing it when required, a flywheel is mounted over the crankshsaft. Cycle gets completed in two revolutions of crankshaft. Examples of four stroke engines are petrol engines used in cars etc.

17.5 2-STROKE SI ENGINE

2-stroke SI engine is a modified form of 4-stroke engine where all the four processes required for completion of one cycle of SI engine get completed in two strokes. Thus, obviously in each stroke two processes get completed. Figure 17.4 shows the line diagram of 2-stroke SI engine.

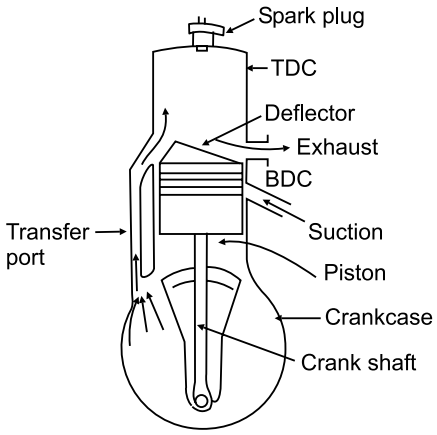


Fig. 17.4 *2-stroke SI engine*

General arrangement shows that here there are no valves as in case of 4-stroke engines, instead it has exhaust and suction ports. Piston has a projection on its top, which acts like deflector. Mixture of air-fuel goes into crank case first and then gets transferred to top of piston at appropriate time. Let us start piston movement from TDC to BDC. When piston reverses its motion from BDC to TDC then the suction port gets uncovered and fresh mixture enters and goes into crank case. With piston moving from TDC to BDC and during covered position of suction port the mixture gets transferred to the top of piston through transfer port. Upon reversal during piston travelling from BDC to TDC, the air fuel mixture on top of piston gets compressed and subsequently gets ignited by spark from spark plug. The combustion of fuel-air mixture results in release of excessive energy which forces piston to move from TDC to BDC. Simultaneously as piston uncovers exhaust port the burnt gases go out through exhaust port. Again when piston reaches BDC it reverses its motion and during travel from BDC to TDC the suction takes place as explained above at the bottom of piston while compression of fuel air mixture takes place on top of piston. Thus, suction and compression, both processes get completed during travel of piston from BDC to TDC. Expansion and exhaust processes occur during travel of piston from TDC to BDC along with transfer of fresh fuel air mixture from crankcase to top of piston.

Here all four processes occur during two strokes and one revolution of crank shaft. Thermodynamic cycle followed by 2-stroke SI engine is Otto cycle. Scooter engines are generally two stroke engines. 2-stroke SI engines are used for smaller applications.

17.6 4-STROKE CI ENGINE

Compression ignition (CI) engines operate generally on “Diesel”/“Dual” cycle. In these engines the combustion is realized due to excessive compression and are so called compression ignition engines. Here air alone is sucked inside the cylinder during suction stoke and compressed. Degree of compression is much more than that of spark ignition (SI) engines. After compression of air the fuel is injected into the high pressure and high temperature compressed air. Due to high temperature of air the combustion of fuel gets set on its’ own. Self ignition of fuel takes place due to temperature of air-fuel mixture being higher than self ignition temperature of fuel. Thus in CI engines, larger amount of compression causes high temperature, therefore unassisted combustion.

Schematic of 4-stroke CI engine is quite similar to that of 4-stroke SI engine with the only major difference that spark plug is replaced by fuel injector for injecting fuel at high pressure into compressed air. 4-Stroke CI engine works with following four processes getting completed in separate strokes. General arrangement in CI engine is similar to that of SI engine with spark plug replaced by fuel injector.

Stroke 1: Piston travels from TDC to BDC and air is sucked.

Stroke 2: Piston travels from BDC to TDC, while air is compressed with inlet and exit passages closed.

Stroke 3: Piston reaches TDC and air gets compressed. Fuel injector injects fuel into compressed air for certain duration. Ignition of fuel also takes place simultaneously as air temperature is much higher than self ignition temperature of fuel. Burning of fuel results in release of fuel chemical energy, which forces piston to travel from TDC to BDC. Contrary to SI engine where heat addition gets completed near instantaneously, in CI engines fuel injection and thus heat addition is spread in certain stroke travel of piston i.e. heat addition takes place at constant pressure during which piston travels certain stroke length as decided by cut-off ratio. This is expansion process and piston comes down to BDC with both inlet and exit valves closed.

Stroke 4: After expansion piston reverses its motion upon reaching BDC and travels up to TDC with exit passage open. During this piston travel burnt gases are expelled out of cylinder i.e. exhaust stroke.

Completion of above four stroke requires two revolutions of crankshaft.

17.7 2-STROKE CI ENGINE

Fig. 17.5 shows the general arrangement in 2-stroke CI engine. Here the structure of CI engine is very much similar to that of SI engine with the major difference that spark plug is replaced by fuel injector, structure is made more sturdy to withstand high compression ratio.

Working of 2-stroke compression ignition engine is shown in (*a*), (*b*), (*c*), (*d*) explaining the suction, compression, expansion and exhaust processes. During piston travel from BDC to TDC air enters crankcase. When piston reaches TDC and reverses its motion to BDC air in crankcase gets partly compressed and is transferred from crankcase to top of piston through transfer port. Upon reversal of piston motion from BDC to TDC the compression of air occurs by the top side of piston while on the bottom side of piston air again enters into crankcase. Upon piston reaching TDC fuel is injected into compressed air which is at high temperature and pressure. As fuel is injected into compressed air the fuel ignition gets set on its own due to temperature being more than self ignition temperature of fuel, i.e. compression ignition. Fuel injection is continued for some duration along with its ignition which causes release of excessive fuel energy. This energy release causes piston to go back from TDC to BDC, i.e. the expansion process as shown in (*c*). As piston reaches BDC it simultaneously forces air in crank case to get transferred to cylinder space and forces burnt gases out of cylinder i.e. exhaust process.

Here also cycle gets completed in single revolution of crankshaft i.e two processes occurring simultaneously in each stroke.

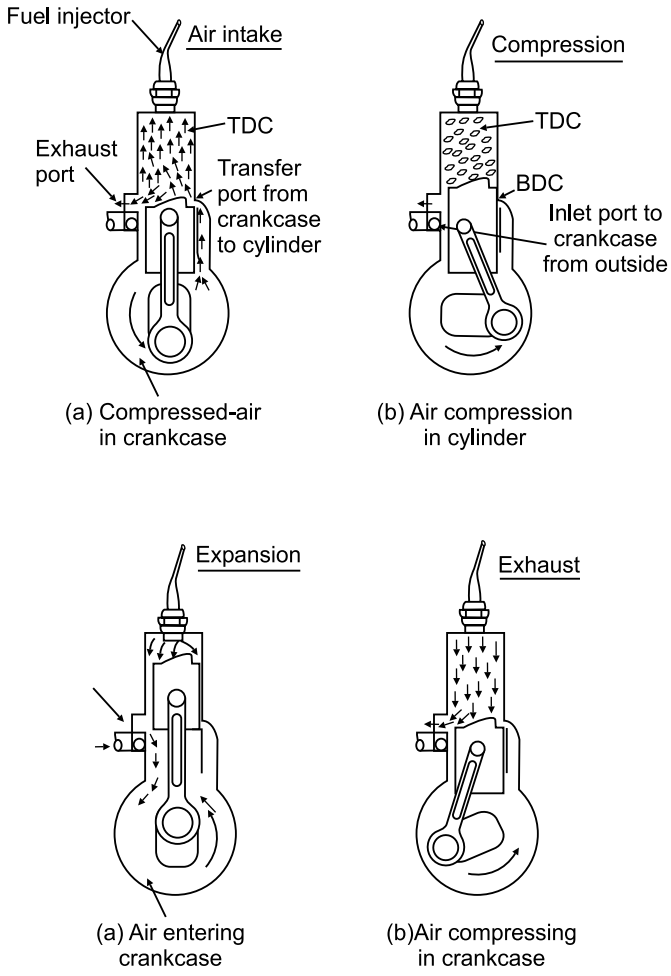


Fig. 17.5 *2-stroke CI engine*

17.8 THERMODYNAMIC CYCLES IN IC ENGINES

Thermodynamic cycles used in IC engines are Otto cycle, Diesel cycle and Dual cycle and are described in Chapter 9 under title of 'Air standard cycles' However, the comparison of these cycles with one another for different reference conditions is presented here.

(*a*) *Comparison of Otto, Diesel and Dual cycles for same compression ratio and heat input:*

Figure 17.6 shows the three cycles on *P-V* and *T-s* diagram for same compression ratio and heat input.

For same heat input, *T-s* diagram shows;

Area *bcef* = Area *bc" e" f* = Area *bb' c'e'f*

(Otto) (Diesel) (Dual)

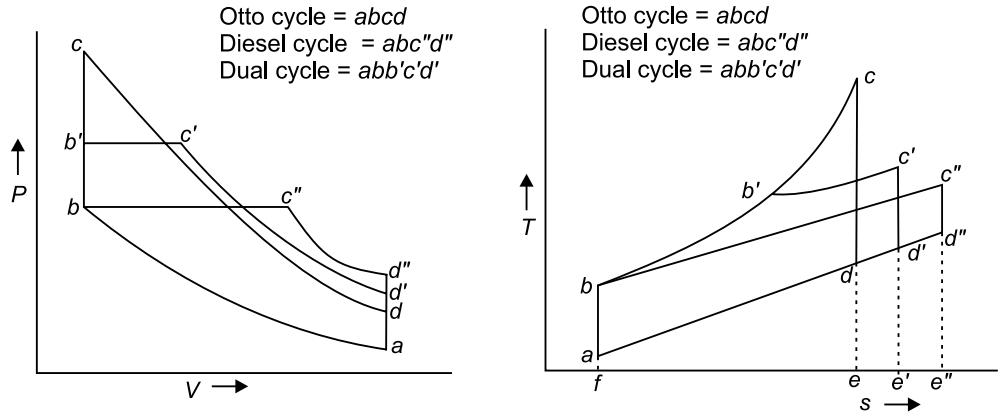


Fig. 17.6 *Comparison of Otto Diesel and Dual cycles on P-V and T-s diagram for same compression ratio and heat input. Comparison can be made by looking at these cycle representations.*

Heat rejected can be given as;

For Otto = Area *a d e f*

For Diesel = Area *a d"e" f*

For Dual = Area *a d' e' f*

A look at areas on *T-s* diagram shows

Area *a d e f* < Area *a d'e'f* < Area *a d" e" f.*

Since heat input is same therefore based on the heat rejected one can say that as heat rejected for Otto cycle is smallest so this has highest efficiency. It is followed by Dual cycle and smallest efficiency is for Diesel cycle.

(*b*) *Comparison of Otto, Diesel and Dual cycles for same maximum pressure and same heat input:*

For carrying out comparison the *P-V* and *T-s* representations for three cycles are shown in Fig. 17.7.

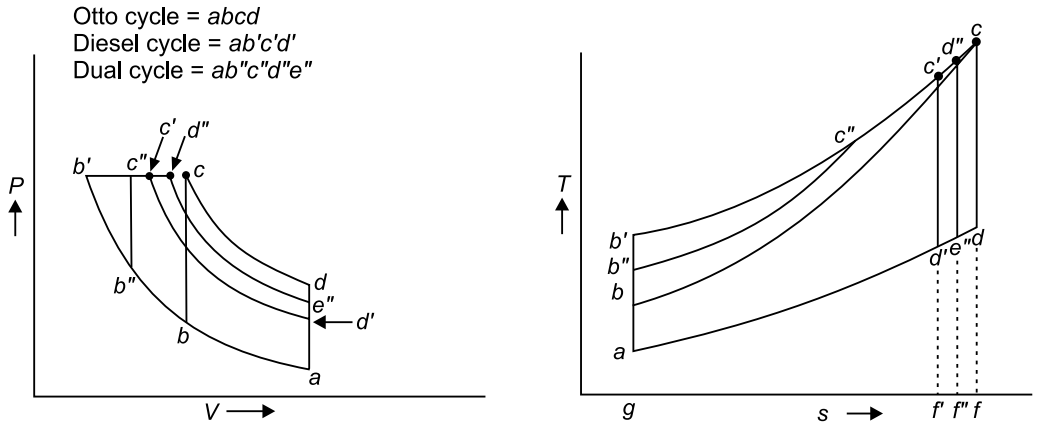


Fig. 17.7 *Comparison of Otto, diesel and Dual cycles on P-V and T-s diagram for same maximum pressure and same heat input.*

Here heat input is same so,

Area $bcfg$ = Area $b'\,c'\,f'\,g$ = Area $b''\,c''\,d''\,f''\,g$.

(Otto) (Diesel) (Dual)

Heat rejected shall be given by;

Area $adfg$ for Otto cycle

Area $ad'\,f'\,g$ for Diesel cycle

Area $ae''\,f''\,g$ for Dual cycle

Area $adfg$ > Area $ae''\,f''\,g$ > Area $ad'\,f'\,g \Rightarrow W_{\text{Diesel}} > W_{\text{Dual}} > W_{\text{Otto}}$

Here W refers to net work in respective cycle.

Hence, it can be seen that heat rejected is highest in Otto cycle followed by Dual cycle and Diesel cycle. Thus, efficiency shall be maximum for Diesel cycle due to largest work available followed by Dual cycle and minimum for Otto cycle at same maximum pressure and same heat input.

17.9 INDICATOR DIAGRAM AND POWER MEASUREMENT

Internal combustion engines have combustion taking place inside and power is available at crankshaft. The shaft work available is less than the total energy released inside the cylinder due to frictional and other losses.

For performance evaluation of internal combustion engine one is interested in following different powers.

(a) **Indicated power:** It refers to the power available inside the cylinder i.e. the power provided to piston.

Mathematically:

Indicated power = (Energy in fuel) – (Energy loss in exhaust, coolant, radiation etc.)

It is measured from the indicator diagram which is obtained using indicator mechanism.

(b) **Brake power:** It refers to the power available at crankshaft i.e. it is the useful shaft work.

Mathematically:

Brake power = (Indicated power) – (Energy loss in friction, pumping and unaccounted losses etc.)

Brake power is usually measured by absorption or transmission type dynamometers. It can be given as:

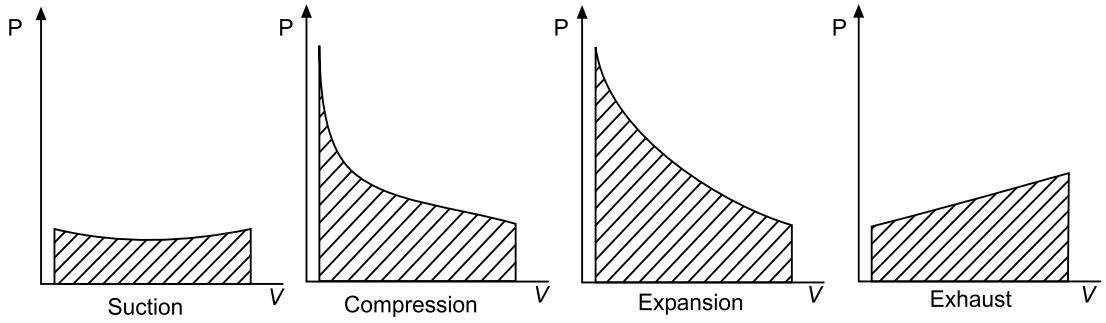
$$\text{Brake power} = \frac{2\pi\, NT}{60} \text{ Watt}$$

where {N is speed of rotation of shaft in rpm, T is torque, N · m.}

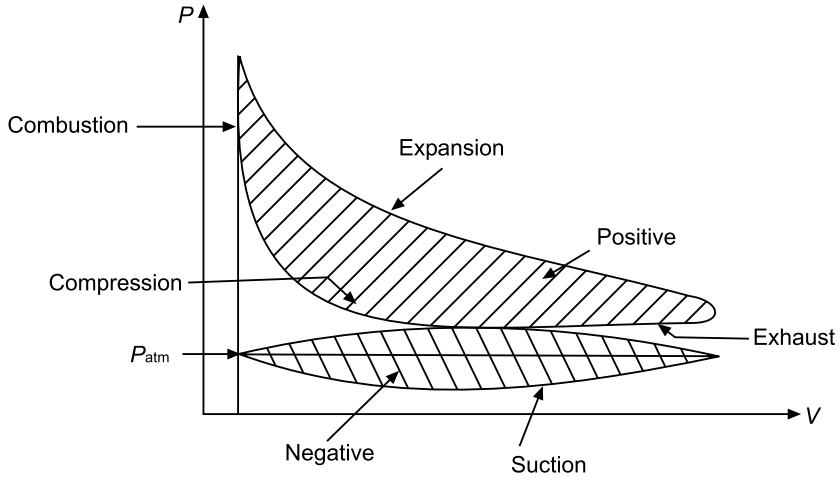
(c) **Friction power:** It refers to the power lost due to friction and other reasons. It is quantified by the difference between indicated power and brake power.

Friction power = Indicated power – Brake power

Indicator diagram: Indicator diagram is the graphical description of pressure and volume variations occurring inside cylinder. An indicator diagram for a four-stroke internal combustion engine is shown in Fig. 17.8 along with the four different processes.



(*a*) Four processes in 4-stroke IC engine (Otto cycle)



(b) Indicator diagram of 4-stroke IC engine (Otto cycle).

Fig 17.8 *Indicator diagram and processes in IC engine*

For getting the indicator diagram, "indicator diagram mechanism" is put over the IC engine and real variation of pressure and volume states obtained.

Indicator mechanism shown in Fig. 17.9, has number of links '*g*' and chord for transferring linear displacement of piston rod into the rotation of drum '*f*'. A stylus '*d*' is operated over the drum through a parallel motion link '*e*'. Displacement of '*e*' is caused along y-axis by an indicator piston rod '*h*', which is connected to indicator piston '*b*' put in indicator cylinder '*a*' as shown. Indicator piston is resisted by an indicator spring '*c*'. In between indicator cylinder and engine cylinder a cock is provided to isolate it after indicator diagram is obtained. Indicator mechanism has thus drum '*f*' rotating about its axis and stylus '*d*' operated by pressure variation inside cylinder along y-axis. Thus, displacement along volume axis is made available by rotation of drum and along pressure axis it is made available by vertical reciprocatory movement of stylus depending upon pressure inside engine cylinder.

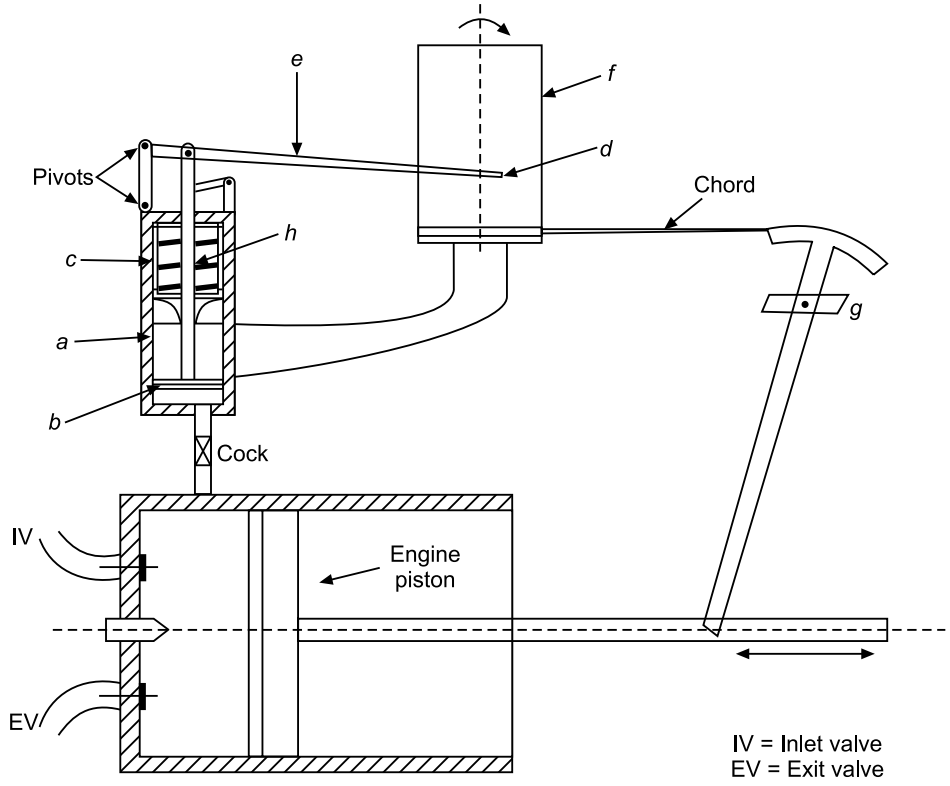


Fig. 17.9 *Indicator diagram mechanism*

For getting indicated power we need mean effective pressure (average pressure) and volume displaced.

From the indicator diagram obtained the mean effective pressure can be obtained, knowing area of diagram, length of diagram and indicator spring constant. Area of diagram can be estimated using planimeter and length of diagram measured.

Mean effective pressure, $mep = \dfrac{A_d}{L_d} \times C$

where A_d is area enclosed in diagram

L_d is length of diagram

C is indicator spring constant given as N/cm^2 × cm travel.

Using mean effective pressure the indicated power can be obtained by product of pressure and volume displaced.

Indicated power for 2-stroke engine, $= \dfrac{mep \times A \times L \times N \times n}{60}$ Watt

where mep = mean effective pressure in Pa

A = cross-sectional area of cylinder. $A = \dfrac{\pi}{4} D^2$, where D is bore.

L = length of stroke.

N = rpm

n = no. of cylinders.

In case of 4-stroke engines, 2 revolutions are required for completing one cycle, therefore effective rpm shall be $N/2$.

$$\text{Indicated power for 4-stroke engine} = \frac{mep \times A \times L \times (N/2) \times n}{60} \text{ Watt}$$

Based on indicated power and brake power, the mechanical efficiency of engine can be obtained as below.

Mechanical efficiency of engine

$$\eta_{\text{mech}} = \frac{\text{Brake power}}{\text{Indicated power}}$$

Mechanical efficiency is thus indicator of how efficiently indicated power is converted into brake power.

17.10 COMBUSTION IN SI ENGINE

Combustion is the process of oxidation of fuel resulting into the release of energy equivalent to calorific value of fuel. Energy released in combustion is in the form of heat. Combustion process in spark ignition engine has requirement of the '*mixture of fuel and air in right proportion,*' '*mechanism for initiation of combustion process*' and '*stabilization and propagation of flame for complete burning.*'

For complete combustion of every fuel there is chemically correct fuel-air ratio also called stoichiometric fuel-air ratio. This fuel air ratio may be rich or lean depending upon the proportion of fuel and air present in mixture. In SI engine this fuel air ratio generally varies between 1 : 7 to 1 : 30 with lean mixture at 1 : 30 and rich mixture at 1 : 7. Stoichiometric fuel-air ratio is around 1 : 14 to 1 : 15 for hydrocarbon fuel. The extreme values of fuel-air ratio permissible in SI engine on both rich and lean ends put limits as '*lower ignition limit*' and '*upper ignition limit*'. Varying fuel-air ratio is required in SI engine due to varying loads on engine between no load to full load on engine. The ratio of actual fuel-air ratio to stoichiometic fuel-air ratio is given by 'equivalence ratio' or 'relative fuel-air ratio'. Appropriate fuel-air ratio is maintained in SI engines through 'carburettor' (the fuel metering system).

Combustion in SI engine may be described to be occurring in following significant phase:

(*i*) After compression of fuel-air mixture in cylinder the high temperature spark is delivered by spark plug in the compressed fuel-air mixture. Temperature at the tip of spark plug electrode may go even more than 10,000ºC at the time of release of spark. Sparkles released have sufficiently high temperature to initiate the combustion of fuel. For complete combustion of fuel mere initiation of combustion does not serve the purpose instead a sustainable combustion process is required. After setting up of combustion a sustainable flame front or flame nuclei is needed so that it proceeds across the combustion space to ensure complete combustion. Thus, this phase in which spark is first released followed by setting up of sustainable flame front is called "preparation phase" and may consume around 10º of crank angle rotation. Crank angle rotation consumed in "preparation phase" depends upon the speed of engine, constructional feature of cylinder, piston, location of spark plug, strength of spark, characteristics of fuel, fuel-air ratio etc. Designer always wishes to complete combustion in smallest possible time as, theoretically Otto cycle requires combustion to occur instantaneously. Therefore, this first phase termed as preparation phase should be as small as possible. Figure 17.10 shows different phases of combustion.

Preparation phase is shown to occur from '*a*' to '*b*' with small or negligible pressure rise as initially rate of burning is very small.

(*ii*) After sustainable combustion flame is set up then the flame nuclei get scattered due to excessive turbulence in combustion space causing pressure to rise from '*b*' to '*c*'. This phase of combustion

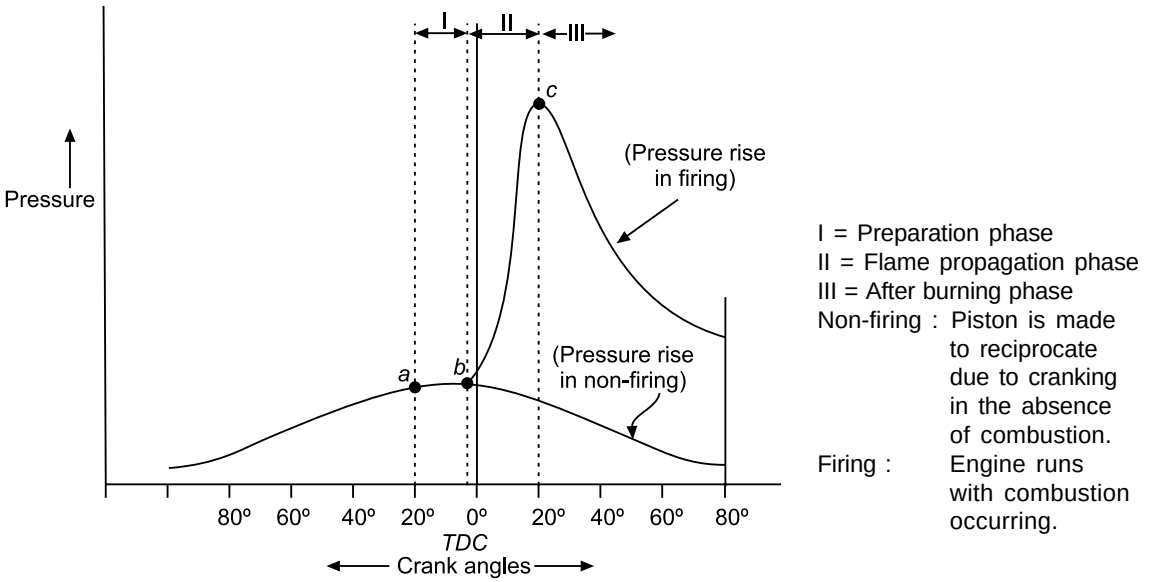


Fig. 17.10 *Phases of combustion in SI engine*

depends upon the turbulence inside cylinder, strength of compbustion nuclei, fuel-air ratio, strength of spark, cylinder geometry, fuel properties etc. This phase of combustion is called as "flame propagation phase" and is accompanied by the excessive pressure rise. Flame propagation phase should also be as small as possible.

(*iii*) After the maximum amount of fuel-air mixture is burnt, the residual gets burnt after the piston has moved across the TDC. This last phase is termed as "after burning phase" and occurs during the expansion stroke.

Hence, it can be summarised that the complete combustion in SI engine occurs in three distinct zones i.e. preparation phase, flame propagation phase and after burning phase. In order to complete combustion in smallest possible time the flame propagation phase and preparation phase should be shortened. It is seen that out of total distance travelled in combustion space in first phase i.e. preparation phase about 10% of combustion space length is covered in about 20–30% of total time for combustion. Flame propagation phase is spread in about 80% of combustion space length and is covered in 60–70% of total time of combustion. 'After burning' occurs in less than 10% of combustion space in less than 10% of total combustion time as evident from Fig. 17.11. Combustion may also sometimes occur abnormally. "Abnormal combustion" is said to occur when combustion begins inside the cylinder on its' own before the stipulated time for it. This abnormal combustion may be due to preignition (i.e. ignition of fuel even before spark plug ignites it) and results in uncontrolled pressure rise. Abnormal combustion is also termed as detonation or knocking and can be felt by jerky operation of engine, excessive noise, reduced power output etc.

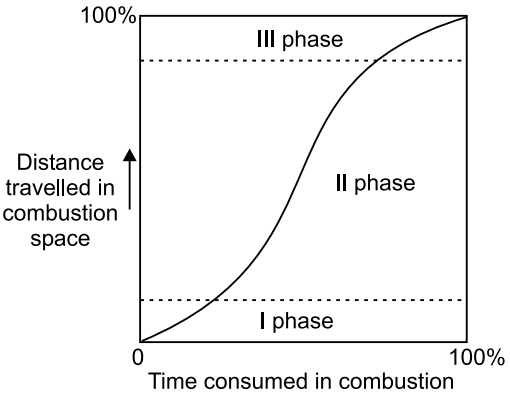


Fig. 17.11 *Duration of combustion phases*

17.11 COMBUSTION IN CI ENGINES

Combustion in CI engines differ from SI engine due to the basic fact that CI engine combustion is unassisted combustion occurring on its' own. In CI engine the fuel is injected into combustion space after the compression of air is completed. Due to excessively high temperature and pressure of air the fuel when injected in atomised form gets burnt on its' own and burning of fuel is continued till the fuel is injected. Theoretically this injection of fuel and its' burning should occur simultaneously upto the cut-off point, but this does not occur in actual CI engine. Different significant phases of combustion are explained as under.

(*i*) Injection of fuel in atomized form is initiated into the combustion space containing compressed air. Fuel upon injection does not get burnt immediately instead some time is required for preparation before start of combustion. Fuel droplet injected into high temperature air first gets transformed into vapour (gaseous form) and then gets enveloped around by suitable amount of oxygen present so as to form combustible mixture. Subsequently, if temperature inside is greater than self ignition temperature at respective pressure then ignition gets set. Thus, the delay in start of ignition may be said to occur due to 'physical delay' i.e. time consumed in transformation from liquid droplet into gaseous form, and 'chemical delay' i.e. time consumed in preparation for setting up of chemical reaction (combustion). The duration of ignition delay depends upon fuel characteristic, compression ratio (i.e. pressure and temperature after compression), fuel injection, ambient air temperature, speed of engine, and geometry of combustion chamber etc. Ignition delay is inevitable stage and in order to accommodate it, the fuel injection is advanced by about 20º before TDC. Ignition delay is shown by *a* – *b* in Fig. 17.12, showing pressure rise during combustion. Fuel injection begins at '*a*' and ignition begins at '*b*'. Theoretically, this ignition delay should be as small as possible.

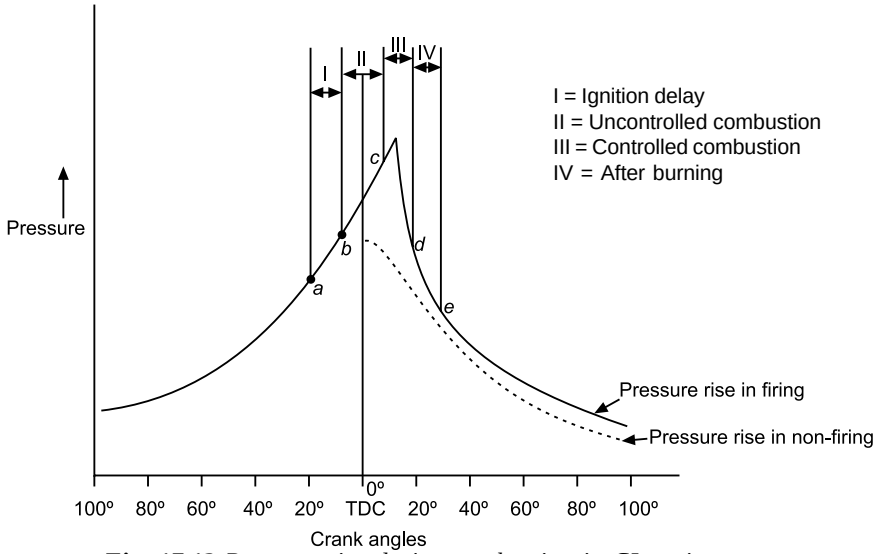


Fig. 17.12 *Pressure rise during combustion in CI engines*

(*ii*) During the ignition delay period also the injection of fuel is continued as it has begun at point '*a*' and shall continue upto the point of cut-off. For the duration in which preparation for ignition is made, the continuous fuel injection results in accumulation of fuel in combustion space. The moment when ignition just begins, if the sustainable flame front is established then this accumulated fuel also gets burnt rapidly. This burning of accumulated fuel occurs in such a manner that combustion process becomes uncontrolled resulting into steep pressure rise as shown from '*b*' to '*c*'. The uncontrolled burning

continues till the collected fuel gets burnt. During this 'uncontrolled combustion' phase if the pressure rise is very abrupt then combustion is termed as 'abnormal combustion' and may even lead to damage of engine parts in extreme conditions.

Thus, it is obvious that 'uncontrolled combustion' depends upon the 'ignition delay' period as during ignition delay itself the accumulation of unburnt fuel occurs and its' burning results in steep pressure rise. Hence in order to have minimum uncontrolled combustion the ignition delay should be as small as possible. During this uncontrolled combustion phase about one-third of total fuel heat is released.

(*iii*) After the 'uncontrolled combustion' is over then the rate of burning matches with rate of fuel injection and the combustion is termed as 'controlled combustion'. Controlled combustion is shown between '*c*' to '*d*' and during this phase maximum of heat gets evolved in controlled manner. In controlled combustion phase rate of combustion can be directly regulated by the rate of fuel injection i.e. through fuel injector. Controlled combustion phase has smooth pressure variation and maximum temperature is attained during this period. It is seen that about two-third of total fuel heat is released during this phase.

(*iv*) After controlled combustion, the residual if any gets burnt and the combustion is termed as 'after burning'. This after burning may be there due to fuel particles residing in remote position in combustion space where flame front could not reach. 'After burning' is spread over 60 – 70º of crank angle rotation and occurs even during expansion stroke.

Thus, it is seen that the complete combustion in CI engines may be comprising of four distinct phases i.e. 'ignition delay' followed by 'uncontrolled combustion,' 'controlled combustion' and 'after burning'. Combustion generally becomes abnormal combustion in CI engines when the ignition delay is too large resulting into large uncontrolled combustion and zig-zag pressure rise. Abnormal combustion in CI engines may also be termed as 'knocking' in engines and can be felt by excessive vibrations, excessive noise, excessive heat release, pitting of cylinder head and piston head etc. In order to control the knocking some additives are put in CI engine fuel so as to reduce its' self ignition temperature and accelerate ignition process. Also, the combustion chambers are properly designed so as to have reduced physical and chemical delay.

17.12 IC ENGINE FUELS

Fuels used in SI engines and CI engines are different as the nature of combustion process is different in the two engines. Normally hydrocarbon fuels are used for both the applications and should possess desirable properties such as high calorific value, suitable combustion characteristics, ease of handling, environment friendly etc. In SI engines the fuel used is generally 'gasoline' also called as petrol and is mixture of different hydrocarbons. Gasoline is available from refining process and its' exact composition depends upon the degree of refining and source etc. Gasoline should have capability of mixing rapidly with air, suitable volatility, resistance to abnormal combustion etc.

SI engine fuels are characterised in respect to its' resistance to abnormal combustion by its' rating in terms of "Octane number". Octane number of fuel is determined by comparing the combustion performance of actual fuel with that of reference fuel. Octane number is defined as the percentage by volume of iso-octane in a mixture of iso-octane (C_8H_{18}) and *n*-heptane (C_7H_{16}). Iso octane is low boiling point fuel having very high resistance to abnormal combustion and is arbitrarily assigned octane number of 100. '*n*-heptane' is seen to have very poor resistance to abnormal combustion and is so assigned octane number of 0. These 'iso-octane' and '*n*-heptane' are also called as 'primary reference fuels'. In order to determine octane number of any fuel its combustion characteristics are matched with those of a variable mixture of 'iso-octane' and '*n*-heptane'. The composition of iso-octane fraction and *n*-heptane fraction which yields the similar combustion characteristics is used to know the octane number. Say, for example in a particular test the mixture of 80% by volume of iso-octane and 20% by volume of *n*-

heptane offers the combustion similar to that of test fuel. Then the test fuel shall have octane number of 80. Octane number generally lies between 0 and 100 but can be extended beyond 100 by using certain additives. 'Tetra ethyl lead' is the most popular additive used in SI engines which further increases resistance to abnormal combustion i.e. even more than that of iso-octane.

In CI engines the Diesel fuel is generally used which is also available during pertroleum refining but is impure as compared to SI engine fuel. CI engine fuels are also characterized by a rating given in terms of 'Cetane number' which also indicates the resistance of fuel to knocking. Cetane number for any fuel is given by percentage by volume of cetane (C_{16} H_{34}) in a mixture of cetane and α-methyl napthalene ($C_{10}H_7CH_3$) which offers the combustion characteristics similar to that of test fuel.

Cetane is arbitrarily assigned cetane number of 100 as it offers maximum knock resistance and α-methyl napthalene is assigned cetane number of 0 as it has mimimum knock resistance. Cetane rating is also obtained by the test similar to that for octane rating.

17.13 MORSE TEST

In IC engines there are three powers namely 'indicated power' which is developed inside cylinder, 'brake power' which is available at crank shaft and 'friction power' which is lost in overcoming friction and other losses. Brake power can be measured by using dynamometer at the crank shaft of engine. Friction power can be experimentally determined by number of methods namely 'Willan's line method', 'Motoring test' and by the numerical difference between indicated power and brake power.

Morse test is experimental method for estimating the friction power of the multi cylinder engine. Here engine is first run upto certain speed of rotation. Subsequently, one by one each cylinder of the engine is isolated from contributing to shaft work either by switching off electric supply to spark plug or by stopping fuel supply. When one cylinder is not producing power i.e. due to its isolation then also piston reciprocates in this cylinder similar to other cylinders thus the frictional losses remain constant for the shaft running at same speed. When second cylinder is cut then also frictional losses remain same but the brake power and indicated power get reduced accordingly. This process of cutting off each cylinder one by one is carried out till last cylinder and the brake power is measured by dynamometer attached to crank shaft which is maintained to run at same speed of rotation when cylinders are cut off one by one.

Mathematically, if there are *n* number of cylinders in a multicylinder engine. Then initially when all cylinders are working.

Indicated power from *n* cylinders, $(ip)_n = (bp)_n + (fp)_n$; here *ip*, *bp* and *fp* refer to indicated power, brake power and friction power respectively.

When one cylinder is cut then,

$$(ip)_{n-1} = (bp)_{n-1} + (fp)_n$$

When second cylinder is cut then,

$$(ip)_{n-2} = (bp)_{n-2} + (fp)_n$$

From above the power produced from first cylinder being cut off can be obtained as:

$$(ip)_{n\text{th cylinder}} = (bp)_n - (bp)_{n-1}$$

Total indicated power from engine: $(ip)_n = \sum (ip)_{n\text{th cylinder}}$

Thus, after knowing indicated power of engine the friction power can be determined as under, because brake power is also known through dynamometer.

$$(fp)_n = (ip)_n - (bp)_n$$

Friction power estimated using Morse test gives fairly correct estimate. Although, there may be slight deviation because when each cylinder is cut then there is substantial increase in engine vibrations and the friction losses may change due to change in back pressure etc.

17.14 COMPARATIVE STUDY OF IC ENGINES

Comparative study of IC engines with other prime movers is described here.

(a) Difference between SI and CI engine

SI engine	*CI engine*
1. It works on Otto cycle	2. It works on Diesel/Dual cycle
2. Compression ratio is from 5 to 15.	2. Compression ratio is from 12 to 30.
3. Petrol (Gasoline) is used as fuel.	3. Diesel is used as fuel.
4. Fuel should have high self ignition temperature.	4. Fuel should have low self ignition temperature.
5. A mixture of air and fuel is sucked inside the cylinder during suction process.	5. Air alone is sucked during suction process.
6. Carburettor is required for preparing air-fuel mixture.	6. Carburettor is not required.
7. Fuel goes into cylinder alongwith air during suction.	7. Fuel is injected after compression. For injection of fuel a fuel pump and injector is required
8. Spark plug is required to initiate combustion.	8. Ignition of fuel takes place on its' own due to high temperature of air inside cylinder.
9. Combustion takes place isochorically.	9. Combustion takes place isobarically.
10. Due to low compression ratio the engine structure is light.	10. To withstand high Compression ratio the engine structure is sturdy.
11. Engine speed is higher.	11. Engine speed is lower.
12. Efficiency is low.	12. Efficiency is high.
13. Generally used in 2-wheeler automobiles and smaller and lighter engines.	13. Generally used in 4 wheelers and bigger engines.

(b) Comparison of 2-stroke engine with 4-stroke engine

2-stroke engine	*4-stroke engine*
1. One cycle is completed in one revolution of crankshaft.	1. One cycle is completed in two revolution of crankshaft.
2. Power stroke is available in each revolution of crankshaft.	2. Power stroke is available in alternate revolution of crankshaft.
3. Thermal efficiency is low.	3. Thermal efficiency is high.
4. Volumetric efficiency is less due to less time available for suction.	4. Volumetric efficiency is high due to large time available for suction.
5. Lighter flywheel is required due to more uniform torque available.	5. Heavier flywheel is required due to less uniform torque available.
6. Engine is compact and light in weight.	6. Engine is heavy.
7. Cooling requirements are less.	7. Cooling requirements are high.
8. Valves are not required.	8. Complex valve operating mechanism is required.
9. Cost is low.	9. Cost is high.
10. Used in light vehicles and small engines.	10. Used in heavy duty vehicles and bigger engines.

(c) Comparison of IC engines with gas turbine

IC engines	*Gas turbine*
1. IC engine have good part load efficiency. These have internal combustion.	1. Gas turbines have poor part load efficiency. These may have external combustion or internal combustion both as per requirement.
2. IC engines are well suited for smaller applications such as automobiles. IC engines can be used for small capacity power plants.	2. Gas turbines are not well suited for smaller applications, specially automobiles. These are well suited for aviation applications and land applications in power plants etc. as prime movers. It has small specific weight and small size capability which make these suitable for aeroplanes.
3. Balancing is poor due to reciprocating parts. Mechanical efficiency is also poor.	3. Balancing is good due to absence of reciprocating parts. Mechanical efficiency is good due to rotary parts and no sliding members
4. Maintenance requirement is high.	4. Poor maintenance requirement.
5. Type of fuels which can be used is limited in number.	5. Large number of different types of fuels can be used.
6. Operation is noisy and large vibrations are present.	6. Operation is silent and smooth.
7. No modification is possible for improving efficiency and output in existing IC engines.	7. Modifications such as reheating, regeneration and intercooling etc. are possible in order to improve efficiency and output.

(d) Comparison of gas turbine with steam turbines

Gas turbine	*Steam turbines*
1. These are generally internal combustion type, but may be of external combustion type too.	1. These are of external combustion type.
2. High quality fuel is being used.	2. Any type of poor or good quality fuel can be utilized.
3. Thermal efficiency is better.	3. Thermal efficiency is poor than gas turbines.
4. It does not require boiler, feed water supply, condensing plant etc. Number of components involved is small.	4. It requires boiler, feed water supply, feed pump, condensing plant etc. instead of compressor, combustion chamber as in gas turbines. Number of components involved is large.
5. These can be started quickly.	5. These take time to start because of time required in generating steam.
6. Maintenance cost is small.	6. Maintenance cost is high.
7. These have smaller specific weight and size i.e. small weight per kW output and so these can be used in aviation applications.	7. These have large specific weight and size and are exclusively suited for land applications.
8. Initial cost and running cost are small.	8. Initial cost and running cost are large in steam turbines.

(e) Comparison between Diesel engine, Gas turbine and Steam turbine based plants: (Parametric form)

Parameter	*Diesel engine*	*Steam turbine plant*	*Gas turbine*
1. Power generation capacity	10 kW to 10 MW	1 MW to 1000 MW	100 kW to 100 MW
2. Efficiency	0.35 to 0.40	0.30 to 0.40	0.20 to 0.40
3. Fuel	High grade fuel, liquid fuel of diesel grade	Medium to low grade fuel, may be gaseous, liquid or solid fuel.	High to medium grade, may be liquid or gas fuel.
4. Size and weight	Medium	Large	Small
5. Capital cost	$US 110–180 US$ per kW	$US 140–1200 per kW	$US 80–300 per kW
6. Lubrication requirement	Frequent oil and filter changes	Negligible	Oil topping up.
7. Maintenance period	Weekly and monthly	Daily	Monthly and annual
8. Specific installation needs	Very heavy foundations, vibration isolation, enclosures for making its operation quiet etc.	Coal handling facility, Ash handling facility, enough supplies of water, exhaust emission control etc.	Intake and exhaust mufflers etc.
9. Plant life	10–12 years	25 to 35 years	15 to 30 years

EXAMPLES

1. *A four cylinder diesel engine of 4-stroke type has stroke to bore ratio as 1.2 and the cylinder diameter is 12 cm. Estimate indicated power of the engine using the indicator diagram arrangement. Indicator card shows the diagram having area of 30 cm^2 and length as half of stroke. Indicator spring constant is 20 × 10^3 kN/m^2 and engine is running at 2000 rpm. Also find out mechanical efficiency of engine if 10% of power is lost in friction and other losses.*

Solution:

From stroke to bore ratio i.e. $\frac{L}{D}$ = 1.2 and cylinder diameter = bore, i.e. D = 12 cm, so L = 14.4 cm

Area of indicator diagram = 30 cm^2 = 30 × 10^{-4} m^2

$$\text{Length of indicator diagram} = \frac{1}{2} \times \text{stroke}$$
$$= 7.2 \text{ cm or } 7.2 \times 10^{-2} \text{ m}$$

$$\text{Mean effective pressure} = \frac{30\times10^{-4}\times20\times10^{3}\times10^{3}}{7.2\times10^{-2}}$$
$$\text{m.e.p.} = 8333.333 \times 10^2 \text{ N/m}^2$$

$$\text{Cross-sectional area of piston} = \frac{\pi}{4}D^2 = 0.01131 \text{ m}^2$$

For one cylinder indicated power

$$= \frac{8333.333\times10^{2}\times0.01131\times14.4\times10^{-2}\times2000}{2\times60}$$

$$= 22619.9 \text{ W}$$

For four cylinders total indicated power = 90479.6 W

Frictional power loss = 0.10 × 90479.6 = 9047.96 W

Brake power available = Indicated power – Frictional power

= (90479.6 – 9047.96)

= 81431.64 W

$$\text{Therefore, mechanical efficiency of engine} = \frac{\text{brake power}}{\text{indicated power}}$$

$$= \frac{81431.64}{90479.6}$$

$$= 0.90$$

Indicated power = **90479.6 W**
Mechanical efficiency = **90%** **Ans.**

2. *Determine the power required to drive a double acting reciprocating pump having indicator diagram with area 40 cm^2 and length 8 cm. Bore and stroke of the pump are 15 cm and 20 cm. The pump motor runs at 100 rpm. Indicator spring constant is 1.5 × 10^8 Pa per m.*

Solution:

Reciprocating Pump is work absorbing machine having its mechanism similar to the piston-cylinder arrangement in IC engine.

$$\text{Mean effective pressure} = \frac{40\times10^{-4}\times1.5\times10^{8}}{8\times10^{-2}}$$

m.e.p. = 7.5 × 10^6 Pa

Indicated power = (Piston area × Stroke × mep × Speed × No. of cylinder)

(It is double acting so let us assume total power to be double of that in single acting)

$$= \left(\frac{\pi}{4}\times(0.15)^{2}\times0.20\times7.5\times10^{6}\times\frac{100}{60}\times1\times2\right)$$

$$= 88357.29 \text{ W}$$

Power required to drive = **88.36 kW** **Ans.**

3. *An engine with 90% mechanical efficiency has rating of 38 kW brake power. Estimate its indicated power and frictional power loss. Also determine the mechanical efficiency at quarter load assuming frictional power to remain same.*

Solution:

$$\text{Indicated power} = \frac{\text{Brake power}}{\text{Mechanical efficiency}} = \frac{38}{0.9}$$

$$= 42.22 \text{ kW}$$

$$\text{Frictional power loss} = \text{Indicated power} - \text{Brake power}$$
$$= 42.22 - 38$$
$$= 4.22 \text{ kW}$$

$$\text{Brake power at quarter load} = 0.25 \times 38$$
$$= 9.5 \text{ kW}$$

$$\text{Mechanical efficiency} = \frac{\text{Brake power}}{\text{Indicated power}} = \frac{9.5}{9.5 + 4.22}$$
$$= 0.6924 \text{ or } 69.24\%$$

Indicated power = **42.22 kW**
Frictional power loss = **4.22 kW**
Mechanical efficiency = **69.24%** **Ans.**

4. *An eight cylinder diesel engine of two stroke type has specific fuel consumption of 0.25 kg/kWh. The brake mean effective pressure of each cylinder is 1.5 MPa and engine run at 100 rpm. The bore and stroke of cylinder are 85 cm and 220 cm respectively. Considering the calorific value of diesel as 43 MJ/kg determine the brake power of engine, fuel consumption in kg/hr and brake thermal efficiency of engine.*

Solution:

$$\text{Brake power of engine} = P_{b\,\text{mep}} \times L \times A \times N$$
$$= \frac{1.5 \times 10^3 \times (2.2) \times \left(\frac{\pi}{4} \times (0.85)^2\right) \times 100}{60}$$
$$= 3120.97 \text{ kW or } 31.21 \text{ MW}$$

Brake power = **31.21 MW** **Ans.**

$$\text{The fuel consumption in kg/hr} = 0.25 \times 3120.97 = 780.24 \text{ kg/hr}$$

In order to find out brake thermal efficiency the heat input from fuel per second is required.

$$\text{Heat from fuel, kJ/s} = \frac{780.24 \times 43 \times 10^3}{3600} = 9319.53 \text{ kJ/s}$$

$$\text{Energy to brake power} = 3120.97 \text{ kW}$$

$$\text{Brake thermal efficiency} = \frac{3120.97}{9319.53} = 0.33488 \text{ or } 33.49\%$$

Fuel consumption = 780.24 kg/hr, Brake thermal efficiency = **33.49%** **Ans.**

5. *A four cylinder, four stroke diesel engine has brake mean effective pressure of 6 bar at full load speed of 600 rpm and specific fuel consumption of 0.25 kg/kWh. The cylinder has bore of 20 cm and stroke length of 30 cm. The air fuel ratio is measured as 26 from the exhaust gas analysis. The ambient conditions are 1 bar, 27°C. Assuming the calorific value of fuel as 43 MJ/kg determine the brake thermal efficiency and the volumetric efficiency. Also find out brake power.*

Solution:

$$\text{Brake thermal efficiency} = \frac{3600}{(\text{brake specific fuel consumption} \times \text{Calorific value})}$$

$$= \frac{3600}{(0.25 \times 43 \times 10^3)} = 0.3349 \text{ or } 33.49\%$$

Brake thermal efficiency = **33.49%** **Ans.**

$$\text{Brake power} = \frac{4 \times 6 \times 10^2 \times 0.30 \times \left(\frac{\pi}{4} \times (0.20)^2\right) \times 600}{60} = 226.19 \text{ kW}$$

Brake power = **226.19 kW** **Ans.**

$$\text{Brake specific fuel consumption} = \frac{m_f}{\text{brake power}}$$

$$0.25 = \frac{m_f}{226.19} \Rightarrow m_f = 56.55 \text{ kg/hr}$$

Air consumption from given air fuel ratio = 26 × 56.55 = 1470.3 kg/hr
or

$$m_a = 24.505 \text{ kg/min}$$

Using perfect gas equation,

$$P \cdot V_a = m_a RT$$
$$1 \times 10^2 \times V_a = 24.505 \times 0.287 \times 300$$
$$V_a = 21.098 \text{ m}^3/\text{min}$$

$$\text{Swept volume, } V_s = \frac{\pi}{4} D^2 L = \frac{\pi}{4} \times (0.30)^2 \times 0.4 = 0.02827 \text{ m}^3$$

$$\text{Volumetric efficiency, } \eta_{\text{vol.}} = \frac{V_a}{V_s \times \frac{N}{2} \times \text{number or cylinders}}$$

$$= \frac{21.09 \times 2}{0.02827 \times 600 \times 4}$$
$$= 0.6217 \text{ or } 62.17\%$$

Volumetric efficiency = **62.17%** **Ans.**

6. *A two stroke two cylinder engine runs with speed of 3000 rpm and fuel consumption of 5 litres/hr. The fuel has specific gravity of 0.7 and air-fuel ratio is 19. The piston speed is 500 m/min and indicated mean effective pressure is 6 bar. The ambient conditions are 1.013 bar, 15°C. The volumetric efficiency is 0.7 and mechanical efficiency is 0.8. Determine brake power output considering R for gas = 0.287 kJ/kg · K*

(Take piston speed, m/min = 2 *LN* where *L* is stroke (*m*) and *N* is rpm)

Solution:

Let the bore of cylinder be 'D' meter.

Using piston speed, $500 = 2 \times L \times 3000$

$\Rightarrow$ $L = 0.0833$ m

$$\text{Volumetric efficiency, } \eta_{\text{vol.}} = \frac{\text{Air sucked (m}^3\text{/min)}}{\text{Swept volume} \times N \times \text{number of cylinders}}$$

$$0.7 = \frac{\text{Air sucked}}{\left(\frac{\pi}{4} D^2 \cdot L\right) \times 3000 \times 2}$$

$\Rightarrow$ Air sucked $= 274.78\ D^2$ m^3/min

Air requirement, kg/min = A/F ratio × Fuel consumption per minute

$$= 19 \times \frac{5 \times 0.7 \times 10^3 \times 10^{-3}}{60}$$

$m_a = 1.11$ kg/min

Using perfect gas equation, $PV_a = m_a RT$

$$V_a = \frac{1.11 \times 0.287 \times 288}{1.013 \times 10^2}$$

$V_a = 0.906$ m^3/min

Ideally, Air sucked $= V_a$

$274.78\ D^2 = 0.906$

$\Rightarrow$ $D = 0.0574$ m or 57.4 mm

Indicated power $= P_{\text{imep}} \times L \times A \times N \times$ no. of cylinders

$$= 6 \times 10^2 \times 0.0833 \times \frac{\pi}{4} \times \frac{(0.0574)^2 \times 3000 \times 2}{60}$$

$= 12.93$ kW

Brake power = Indicated power × Mechanical efficiency

$= 12.93 \times 0.8 = 10.34$ kW

Brake power = **10.34 kW** **Ans.**

7. *During trial of four stroke single cylinder engine the load on dynamometer is found 20 kg at radius of 50 cm. The speed of rotation is 3000 rpm. The bore and stroke are 20 cm and 30 respectively. Fuel is supplied at the rate of 0.15 kg/min. The calorific value of fuel may be taken as 43 MJ/kg. After some time the fuel supply is cut and the engine is rotated with motor which required 5 kW to maintain the same speed of rotation of engine. Determine the brake power, indicated power, mechanical efficiency, brake thermal efficiency, indicated thermal efficiency, brake mean effective pressure, indicated mean effective pressure.*

Solution:

After switching off fuel supply the capacity of motor required to run engine will be the friction power required at this speed of engine.

Friction power = 5 kW

$$\text{Brake power} = 2\pi NT = \frac{2\times\pi\times3000\times20\times9.81\times0.5\times10^{-3}}{60}$$

$$= 30.82 \text{ kW}$$

$$\text{Indicated power} = \text{Brake power} + \text{Friction power} = 35.82 \text{ kW}$$

$$\text{Mechanical efficiency} = \frac{\text{Brake power}}{\text{Indicated power}} = \frac{30.82}{35.82} = 0.8604 \text{ or } 86.04\%$$

$$\text{Brake specific fuel consumption} = \frac{\text{Specific fuel consumption}}{\text{Brake power}}$$

$$= \frac{0.15\times60}{30.82} = 0.292 \text{ kg/kW hr}$$

$$\text{Brake thermal efficiency} = \frac{3600}{\text{Brake specific fuel consumption}\times\text{calorific value}}$$

$$= \frac{3600}{0.292\times43\times10^3} = 0.2867 \text{ or } 28.67\%$$

$$\text{Also, Mechanical efficiency} = \frac{\text{Brake thermal efficiency}}{\text{Indicated thermal efficiency}}$$

$$\Rightarrow \quad \text{Indicated thermal efficiency} = \frac{0.2867}{0.8604} = 0.3332 \text{ or } 33.32\%$$

$$\text{Indicated power} = P_{\text{imep}} \times L \times A \times N$$

$$35.82 = P_{\text{imep}} \times 0.30 \times \frac{\pi}{4} \times (0.20)^2 \times \frac{3000}{60}$$

$$\Rightarrow \quad P_{\text{imep}} = 76.01 \text{ kPa}$$

$$\text{Also, Mechanical efficiency} = \frac{P_{bmep}}{P_{imep}}$$

$$\text{Brake mean effective pressure} = P_{imep} \times \text{Mechanical efficiency}$$

$$P_{imep} = 76.01 \times 0.8604 = 65.39 \text{ kPa}$$

Brake power = **30.82 kW,** Indicated power = **35.82 kW**
Mechanical efficiency = **86.04%.** Brake thermal efficiency = **28.67%**
Indicated thermal efficiency = **33.32%,** Brake mean effective pressure = **65.39 kPa,**
Indicated mean effective pressure = **76.01 kPa** **Ans.**

8. *A four stroke four cylinder diesel engine running at 300 rpm produces 250 kW of brake power. The cylinder dimensions are 30 cm bore and 25 cm stroke. Fuel consumption rate is 1 kg/min while air fuel ratio is 10. The average indicated mean effective pressure is 0.8 MPa. Determine indicated power, mechanical efficiency, brake thermal efficiency and volumetric efficiency of engine. The calorific value of fuel is 43 MJ/kg. The ambient conditions are 1.013 bar, 27ºC.*

Solution:

Given, D = 0.3 m, L = 0.25 m, N = 300 rpm, m_f = 1 kg/min, F/A = 20,
P_{imep} = 0.8 MPa, Brake power = 250 kW

$$\text{Indicated power kW} = P_{imep} \times L \times A \times \frac{N \times 4}{60} = 0.8 \times 0.25 \times \frac{\pi}{4} \times (0.3)^2 \times \frac{300}{60} \times 4 \times 10^3$$

$$= 282.74 \text{ kW}$$

$$\text{Mechanical efficiency} = \frac{\text{Brake power}}{\text{Indicated power}} = \frac{250}{282.74} = 0.8842 \text{ or } 88.42\%$$

$$\text{Brake specific fuel consumption} = \frac{1 \times 60}{250} = 0.24 \text{ kg/kW hr}$$

$$\text{Brake thermal efficiency} = \frac{3600}{bsfc \times CV} = \frac{3600}{0.24 \times 43000} = 0.3488 \text{ or } 34.88\%$$

$$\text{Swept volume, } V_s = \frac{\pi}{4} D^2 L = \frac{\pi}{4} \times (0.30)^2 \times 0.25 = 0.01767 \text{ m}^3$$

Mass of air corresponding to above swept volume, using Perfect gas equation

$$PV_s = m_a RT$$

$$m_a = \frac{1.013 \times 10^2 \times 0.017}{0.287 \times 300}$$

$$= 0.02 \text{ kg}$$

$$\text{Volumetric efficiency , } \eta_{vol} = \frac{\text{Mass of air taken/minute}}{\text{Mass corresponding to swept volume per minute}}$$

Mass of air taken per minute = 1 × 10 = kg/min

Mass corresponding to swept volume per minute = $0.02 \times 4 \times \frac{300}{2}$ = 12 kg/min

$$\text{Volumetric efficiency} = \frac{10}{12} = 0.8333 \text{ or } 83.33\%$$

Indicated power = 282.74 kW, Mechanical efficiency = 88.42%

Brake thermal efficiency = **34.88%,** Volumetric efficiency = **83.33%** **Ans.**

9. *During an experiment on four stroke single cylinder engine the indicator diagram obtained has average height of 1 cm while indicator constant is 25 kN/m^2 per mm. The engine run at 300 rpm and the swept volume is 1.5 × 10^4 cm^3. The effective brake load upon dynamometer is 60 kg while the effective brake drum radius is 50 cm. The fuel consumption is 0.12 kg/min and the calorific value of fuel oil is 42 MJ/kg. The engine is cooled by circulating water around it at the rate of 6 kg/min.*

The cooling water enters at 35º C and leaves at 70ºC. Exhaust gases leaving have energy of 30 kJ/s with them. Take specific heat of water as 4.18 kJ/kg K. Determine indicated power output, brake power output and mechanical efficiency. Also draw the overall energy balance in kJ/s.

Solution:

Indicated mean effective pressure = 10 × 25 = 250 kPa

$$\text{Indicated power} = P_{imep} \times L \times A \times \frac{N}{2} \quad \left\{\text{here } N = \frac{N}{2} \text{ as it is four stroke}\right\}$$

$$= 250 \times 1.5 \times 10^4 \times \frac{300}{2\times 60} \times 10^{-6} = 9.375 \text{ kW}$$

$$\text{Brake power} = 2\pi NT = 2 \times \pi \times \frac{300}{2\times 60} \times (60 \times 9.81 \times 0.50) \times 10^{-3} = 4.62 \text{ kW}$$

$$\text{Mechanical efficiency} = \frac{\text{Brake power}}{\text{Indicated power}} = \frac{4.62}{9.375}$$

$$= 0.4928 \text{ or } 49.28\%$$

Indicated power = **9.375 kW,**
Brake power = **4.62 kW,**
Mechanical efficiency = **49.28%** **Ans.**

$$\text{Energy liberated from fuel} = \frac{42\times 10^3 \times 0.12}{60} = 84 \text{ kJ/s}$$

Energy available as brake power = 4.62 kW

$$\text{Energy to coolant} = \frac{6}{60} \times 4.18 \times (70 - 35) = 14.63 \text{ kW}$$

Energy carried by exhaust gases = 30 kJ/s
Unaccounted energy loss = 84 – 4.62 – 14.63 – 30 = 34.75 kW
Energy Balance on per second basis

Energy input, kJ/s	*Energy consumed, kJ/s*	*in %*
Heat from fuel = 84 kJ/s, 100%	Energy consumed as brake power = 4.62 kW	5.5%
	Energy carried by coolant = 14.63 kW	17.42%
	Energy carried by exhaust gases = 30 kW	35.71%
	Unaccounted losses = 34.75 kW	41.37%

10. *During 15 minutes trial of an internal combustion engine of 2-stroke single cylinder type the total 4 kg fuel is consumed while the engine is run at 1500 rpm. Engine is cooled employing water being circulated at 15 kg/min with its inlet and exit temperatures as 27ºC and 50ºC. The total air consumed is 150 kg and the exhaust temperature is 400ºC. The atmospheric temperature is 27ºC. The mean specific heat of exhaust gases may be taken as 1.25 kJ/kg K. The mechanical efficiency is 0.9. Determine, the brake power, brake specific fuel consumption and indicated thermal efficiency. Also draw energy balance on per minute basis. Brake torque is 300 Nm and the fuel calorific value is 42 MJ/kg.*

Solution:

$$\text{Brake power} = 2\pi NT = \frac{2\times \pi \times 1500 \times 300}{60} = 47.124 \text{ kW}$$

Brake power = **47.124 kW** **Ans.**

$$\text{Brake specific fuel consumption} = \frac{4\times 60}{15\times 47.124} = 0.339 \text{ kg/kW hr.}$$

Brake specific fuel consumption = **0.339 kg/kW hr** **Ans.**

$$\text{Indicated power} = \frac{\text{Brake power}}{\text{Mechanical efficiency}} = \frac{47.124}{0.9} = 52.36 \text{ kW}$$

$$\text{Indicated thermal efficiency} = \frac{52.36}{\left(\frac{4\times 42\times 10^3}{15\times 60}\right)} = 0.2805 \text{ or } 28.05\%$$

Indicated thermal efficiency = **28.05%** **Ans.**

$$\text{Heat available from fuel, kJ/min} = \frac{4}{15} \times 42 \times 10^3 = 11200 \text{ kJ/min}$$

Energy consumed as brake power = 47.124 × 60 = 2827.44 kJ/min

Energy carried by cooling water = 15 × 4.18 × (50 – 27) = 1442.1 kJ/min

$$\text{Energy carried by exhaust gases} = \left(\frac{150+4}{15}\right) \times 1.25 \times (400 - 27)$$

= 4786.83 kJ/min

Unaccounted energy loss = 11200 – (2827.44 + 1442.1 + 4786.83)

= 2143.63 kJ/min. Heat balance on per minute basis

Heat input, kJ/min	*%*	*Heat consumed, kJ/min*	*%*
Heat from fuel = 11200	100%	1. Brake power = 2827.44	25.24%
		2. Cooling water = 1442.1	12.88%
		3. Exhaust gases = 4786.83	42.74%
		4. Unaccounted loss = 2143.63 (by difference)	19.14%
Total = 11200 kJ/min	100%	Total = 11200 kJ/min	100%

11. *During Morse Test experiment on a six cylinder petrol engine the brake power output was found 50 kW when all cylinders run at full load. When one by one each cylinder is cut and load is reduced to bring engine back to original speed, the measured brake power outputs are as under. Determine the indicated power of engine and mechanical efficiency of engine.*

No. of cylinders cut	*Brake power, kW*
1	40.1
2	39.5
3	39.1
4	39.6
5	39.8
6	40

Solution:

Indicated power of Ist cylinder = 50 – 40.1 = 9.9 kW
Indicated power of IInd cylinder = 50 – 39.5 = 10.5 kW
Indicated power of IIIrd cylinder = 50 – 39.1 = 10.9 kW
Indicated power of IVth cylinder = 50 – 39.6 = 10.4 kW
Indicated power of Vth cylinder = 50 – 39.8 = 10.2 kW
Indicated power of VIth cylinder = 50 – 40 = 10 kW
Total indicated power = 61.9 kW

$$\text{Mechanical efficiency} = \frac{\text{Brake power}}{\text{Indicated power}} = \frac{50}{61.9} = 0.8077 \text{ or } \mathbf{80.77\%}$$

Indicated power = **61.9 kW**
Mechanical efficiency = **80.77%** **Ans.**

12. *During trial of a four cylinder four stroke petrol engine running at full load it has speed of 1500 rpm and brake load of 250 N when all cylinders are working. After some time each cylinder is cut one by one and then again brought back to same speed of engine. The brake readings are measured as 175 N, 180 N, 182 N and 170 N. The brake drum radius is 50 cm. The fuel consumption rate is 0.189 kg/min with the fuel whose calorific value is 43 MJ/kg and* A/F *ratio of 12. Exhaust gas temperature is found to be 600ºC. The cooling water flows at 18 kg/min and enters at 27ºC and leaves at 50ºC. The atmospheric air temperature is 27ºC. Take specific heat of exhaust gas as 1.02 kJ/kg K. Determine the brake power output of engine, its indicated power and mechanical efficiency. Also draw a heat balance on per minute basis.*

Solution:

$$\text{Brake power output of engine} = \frac{2\pi NT}{60} = \frac{2\pi \times 1500 \times 250 \times 0.50 \times 10^{-3}}{60} = 19.63 \text{ kW}$$

$$\text{Brake power when cylinder 1 is cut} = \frac{2 \times \pi \times 1500 \times 175 \times 0.5 \times 10^{-3}}{60} = 13.74 \text{ kW}$$

Indicated power of first cylinder = 19.63 – 13.74 = 5.89 kW

$$\text{Brake power when cylinder 2 is cut} = \frac{2 \times \pi \times 1500 \times 180 \times 0.5 \times 10^{-3}}{60} = 14.14 \text{ kW}$$

Indicated power of second cylinder = 19.63 – 14.14 = 5.49 kW

$$\text{Brake power when cylinder 3 is cut} = \frac{2 \times \pi \times 1500 \times 182 \times 0.5 \times 10^{-3}}{60} = 14.29 \text{ kW}$$

Indicated power of third cylinder = 19.63 – 14.29 = 5.34 kW

$$\text{Brake power when cylinder 4 is cut} = \frac{2 \times \pi \times 1500 \times 170 \times 0.5 \times 10^{-3}}{60} = 13.35 \text{ kW}$$

Indicated power of fourth cylinder = 19.63 – 13.35 = 6.28 kW
Total indicated power output = 5.89 + 5.49 + 5.34 + 6.28 = 23 kW

$$\text{Engine's mechanical efficiency} = \frac{\text{Brake power}}{\text{Indicated power}} = \frac{19.63}{23} = 0.8535 \text{ or } 85.35\%$$

B.P. = 19.63 kW, I.P. = 23 kW, η_{mech} = **83.35%** **Ans.**

Heat liberated by fuel, kJ/min = $0.189 \times 43 \times 10^3 = 8127$ kJ/min

Heat carried by exhaust gases = $(12 + 1) \times 0.189 \times 1.02 \times (600 - 27) = 1436.02$ kJ/min

Heat carried by cooling water = $18 \times 4.18 \times (50 - 27) = 1730.52$ kJ/min

Energy to brake power = $19.63 \times 60 = 1177.8$ kJ/min

Unaccounted losses = $8127 - (1436.02 + 1730.52 + 1177.8) = 3782.66$ kJ/min

Heat balance sheet on per minute basis

Energy in, kJ/min	*%*	*Energy out, kJ/min*	*%*
Heat released by fuel = 8127	100%	(a) Brake power = 1177.8 (b) Exhaust gases = 1436.02 (c) Cooling water = 1730.52 (d) Unaccounted = 3782.66 (by difference)	14.50 17.67 21.29 40.54
Total = 8127, kJ/min	100%	Total = 8127 kJ/min	100%

13. *During the trial of a single acting oil engine, cylinder diameter is 20 cm, stroke 28 cm, working on two stroke cycle and firing every cycle, the following observations were made:*

Duration of trial :1 hour
Total fuel used :4.22 kg
Calorific value :44670 kJ/kg
Proportion of hydrogen in fuel : 15%
Total number of revolutions : 21000
Mean effective pressure : 2.74 bar
Net brake load applied to a drum of 100 cm diameter : 600 N
Total mass of cooling water circulated : 495 kg
Total mass of cooling water : inlet 13ºC, outlet 38ºC
Air used : 135 kg
Temperature of air in test room : 20ºC
Temperature of exhaust gases : 370ºC
Assume $C_{p,\ gases}$ *= 1.005 kJ/kg K,*
$C_{p,\ steam}$ *at atmospheric pressure = 2.093 kJ/kg K*
Calculate thermal efficiency and draw up the heat balance. *[U.P.S.C. 1997]*

Solution:

Brake power = $2\pi NT$

$$N = \frac{21,000}{60} \text{ rpm} = 350 \text{ rpm}$$

$$T = 600 \times 0.5 = 300 \text{ N} \cdot \text{m}$$

$$\text{Brake power} = \frac{2\pi \times 350 \times 300}{60 \times 1000} = 10.996 \text{ kW}$$

$$\text{Indicated power} = \frac{\text{mep} \times L \cdot A \cdot N}{60 \times 1000} \text{ kW}$$

$$= \frac{2.74 \times 10^5 \times 0.28 \times \pi \times (0.2)^2 \times 300}{60 \times 1000 \times 4} = 14.06 \text{ kW}$$

(A) $$\text{Heat added} = \frac{4.22}{3600} \times 44670 = 52.36 \text{ kJ/s or } 3141.79 \text{ kJ/min}$$

$$\text{Thermal efficiency, } \eta = \frac{\text{Indicated power}}{\text{Heat added}}$$

$$= \frac{14.06}{\frac{4.22}{3600} \times 44670}$$

$$= 0.2685 \text{ or } 26.85\%$$

(B) Heat equivalent of brake power = 10.996 × 60 = 659.76 kJ/min

(C) Heat lost to cooling water = $m_w \times C_{pw} \times \Delta T$

$$= \frac{495}{60} \times 4.18 \times (38 - 13)$$

$$= 862.13 \text{ kJ/min}$$

Heat carried by exhaust gases = Heat carried by steam in exhaust gases + Heat carried by fuel gases (dry gases) in exhaust gases

Mass of exhaust gases = mass of air + mass of fuel

$$= \frac{(135 + 4.22)}{60} \text{ kg/min}$$

$$= 2.32 \text{ kg/min}$$

$$\text{Mass of steam in exhaust gases} = 9 \times \left(0.15 \times \frac{4.22}{60}\right)$$

$$= 0.095 \text{ kg/min}$$

Mass of dry exhaust gases = 2.32 – 0.095

$$= 2.225 \text{ kg/min}$$

(D) Heat carried by steam in exhaust = 0.095 × {4.18 (100 –20) + 2256.9 + 2.093 × (370 – 100)}

$$= 299.86 \text{ kJ/min}$$

(E) Heat carried by fuel gases (dry gases) in exhaust gases

$$= 2.225 \times 1.005 (370 - 20)$$

$$= 782.64 \text{ kJ/min}$$

(F) Unaccounted loss = A – B – C – D – E

$$= (3141.79) - (659.76) - (862.13) - (299.86) - (782.64)$$

$$= 537.4 \text{ kJ/min}$$

Heat balance sheet (on per minute basis)

Input		*Expenditure*	
Heat, kJ/min	*%, Percentage*	*Heat, kJ/min*	*%, Percentage*
(A) Heat added by fuel = 3141.79	100	(B) Heat equivalent to brake power = 659.76	21%
		(C) Heat lost to cooling water = 862.13	27.44%
		(D) Heat carried by steam in exhaust gases = 299.86	9.54%
		(E) Heat carried by dry fuel gases in exhaust gases = 782.64	24.92%
		(F) Unaccounted losses = 537.4	17.10%

Ans.

EXERCISE

17.1 Give classification of internal combustion engines.

17.2 Give a sketch of simple one cylinder internal combustion engine and label important parts.

17.3 What is meant by spark ignition and compression ignition engines?

17.4 Explain the working of 4-stroke SI engine.

17.5 Describe the working of 4-stroke CI engine.

17.6 Explain working of 2-stroke CI engine.

17.7 Explain working of 2-stroke SI engine.

17.8 What is meant by indicator diagram? Describe the arrangement for getting it?

17.9 Explain the power measurement using indicator diagram arrangement.

17.10 Explain the combustion in SI engines.

17.11 Explain the combustion in CI engines.

17.12 Describe some IC engine fuels.

17.13 What is More test? How is it carried out?

17.14 Compare SI engines with CI engines.

17.15 Compare 2 stroke SI engine with 4-stroke SI engines.

17.16 Write short notes on the following:

Brake power, Indicated power, Brake specific fuel consumption, Indicated specific fuel consumption, Brake mean effective pressure, Indicated mean effective pressure, Mechanical efficiency. Brake thermal efficiency, Indicated thermal efficiency.

17.17 A single cylinder four stroke CI engine runs at 200 rpm and the cylinder has bore and stroke of 10 cm and 12.5 cm respectively. The indicator diagram taken has length of 10 cm and area of 20 cm^2. The indicator constant is 30 kPa per mm. Considering mechanical efficiency of 0.8 determine indicated mean effective pressure and indicated power. [600 kPa, 5.89 kW]

17.18 Determine the dimensions of cylinder, brake thermal efficiency and indicated mean effective pressure for the single cylinder four stroke diesel engine producing 100 kW of brake power. Engine runs at speed of 400 rpm, brake mean effective pressure of 850 kPa, brake specific fuel consumption of 0.335 kg/kW hr. The mechanical efficiency is 0.8 and stroke to bore ratio is 1.25.

The calorific value of fuel may be taken as 43.5 MJ/kg. [D = 33 cm, L = 41.25 cm, 24.7%, 10.62 bar]

17.19 Determine thermal efficiency and mean effective pressure of thermodynamic cycle used by a 4-stroke petrol engine. Details of cycle are as follows.

Compression ratio = 7

Initial state = 100 kPa and 90°C

Swept volume = $0.1\,m^3$

Heat added to cycle at constant volume = 100 kJ/cycle.

Consider air as working fluid. [54%, 530 kPa]

17.20 A 4-stroke four cylinder petrol engine has bore and stroke as 0.15 m and 0.17 m respectively. Indicator diagram arrangement when put on a cylinder yields indicator diagram having area of 25 cm^2 and length of diagram as 6 cm. The speed of engine is 2500 rpm. Determine the indicated power and frictional power if mechanical efficiency is 88%. Take indicator spring constant as 20×10^3 kN/m^3.

[208.6 kW, 25 kW]

17.21 Determine the cylinder diameter and stroke length for a 8 cylinder, single acting, 2-stroke diesel engine for which details of engine and indicator diagram are as under;

Indicated power = 16266.7 kW

Speed = 150 rpm

L : D ratio = 1.5

Area of indicator diagram = $550\,mm^2$

Length of diagram = 60 mm

Indicator spring constant = 14.7×10^7 Pa per m [80 cm, 120 cm]

17.22 Determine the capacity of motor required for running a single-cylinder, double acting reciprocating water pump at 50 rpm. Indicator diagram arrangement gives an indicator diagram with area of 37.5 cm^2 and length 7.5 cm. Indicator spring constant is 14.7×10^7 Pa per m. Pump cylinder diameter and stroke are 15 cm and 20 cm Mechanical efficiency of arrangement is 90%. [33.66 kW]

17.23 During a test on single cylinder four stroke diesel engine having bore of 15 cm, stroke of 18 cm gives indicator diagram of 10 cm length and area of 9 cm^2. Engine has speed of 480 rpm, fuel consumption of 0.05 kg/min and cooling water flow rate of 250 kg/hr. Cooling water enters at 27°C and leaves at 67°C. The calorific value of fuel is 44000 kJ/kg. Brake torque is 200 Nm. Determine the indicator constant, brake specific fuel consumption and brake thermal efficiency. Take mechanical efficiency as 87.8%.

[0.298 kg.kW hr, 27.4%]

17.24 A two cylinder four stroke engine runs at 240 rpm developing a torque of 5.16 kN · m. The bore and stroke of cylinder are 30 cm and 58.5 cm respectively.

Engine runs with gaseous fuel having calorific value of 16.8 MJ/m^3. The gas and air mixture is supplied in proporation of 1 : 7 by volume.

The volumetric efficiency is 0.85. Determine

(*i*) the brake power

(*ii*) the mean piston speed in m/s

(*iii*) the brake mean effective pressure

(*iv*) the brake thermal efficiency. [129.68 kg, 4.68 m/s, 4.89 bar, 27.37%]

17.25 A four cylinder petrol engine has output of 52 kW at 200 rpm. During the Morse test being performed each cylinder is cut one by one. The brake load readings are 35.47 kg, 36.08 kg, 34.25 kg and 34.66 kg when one by one all cylinders are cut and the engine is brought back to original running speed. The brake drum radius may be taken as 50 cm. The specific fuel consumption at this speed is 364 gm/kW hr. The calorific value of fuel may be taken as 42 MJ/kg. Determine the mechanical efficiency, brake thermal efficiency, and indicated thermal efficiency of engine. [81.61%, 23.54%, 28.87%]

17.26 In the testing of two stroke engine the following observations are made.

Bore and stroke = 20 cm, 25 cm

Brake drum radius = 60 cm

Speed = 360 rpm
Brake load = 500 N
Fuel consumption = 0.06 kg/min
Calorific value of fuel = 42000 kJ/kg
Cooling water flow rate = 350 kg/hr
Cooling water temperature rise = 30°C
Area of indicator diagram = 2.8 cm^2
Length of indicator diagram = 10 cm
Spring constant = 100 kPa/min

Determine indicated power, brake power and thermal efficiency. [13.19 kW, 11.31 kW, 26.93%]

17.27 A four stroke engine has single cylinder with bore of 18 cm and stroke of 34 cm. Engine has indicated mean effective pressure of 555 kPa at 390 rpm. At this the brake load is 510 N at brake radius of 56.5 cm. Cooling water is flown at the rate of 270 kg/hr with its inlet and exit temperatures being 18ºC and 58ºC. The fuel has calorific value of 43000 kJ/kg and the fuel is supplied at 3.6 kg/ hr. The exhaust gases carry 38% of fuel energy. Determine mechanical efficiency and brake thermal efficiency. Also prepare a heat balance sheet on per second basis. [75.54%, 27.36%]

17.28 Trial on a four stroke single cylinder engine results in following observations:

Bore = 16.5 cm, Stroke = 30.5 cm
Speed = 300 rpm
Brake load = 360 N
Brake diameter = 0.9 m
Gas consumption = 4.95 m^3/hr
Calorific value of gaseous fuel = 18850 kJ/m^3
Cooling water supply = 204 kg/hr
Cooling water temperature rise = 30ºC
Indicated mean effective pressure = 455 kPa

Determine

(*i*) the mechanical efficiency.

(*ii*) the indicated thermal efficiency

(*iii*) the brake thermal efficiency

Draw the energy balance on per second basis. [68.86%, 28.51%, 19.63%]

18

Introduction to Refrigeration and Airconditioning

18.1 INTRODUCTION

Refrigeration refers to the use of mechanical devices or heat activated devices for producing and maintaining the temperature in a region less than that of surroundings. American Society of Refrigerating Engineers defines refrigeration as "the science of providing and maintaining temperature below that of the surrounding atmosphere". Refrigerating a region in reference to the human comfort is called air conditioning. Thus, air conditioning refers to maintaining a space for the human comfort i.e. temperature, humidity and ventilation are the parameters to be controlled. When refrigeration provides temperature below – 150ºC then this is known as cryogenics. Equipment used for removing the heat continuously for maintaining low temperature in a space is called 'refrigerator' and cycles on which it operates are called 'refrigeration cycles'. The working fluids used for carrying away heat are called 'refrigerants' which are used in both refrigeration and air conditioning equipments. Equipment used for air conditioning are called airconditioners. The air conditioning of space may also be the requirement of various goods, equipments and processes etc. When air conditioning is done for human comfort then it is called comfort air conditioning while when it is done for industrial purposes then it is called 'industrial air conditioning'. Comfort air conditioning can be further categorized as summer air conditioning and winter air conditioning. Summer air conditioning is used in summer season and generally requires cooling and dehumidification. Winter air conditioning used in winter generally requires heating and humidification.

Comfort air conditioning is extensively used in residential buildings, hospitals, offices, working spaces, vehicles, trains, aeroplanes etc.

Industrial air conditioning is used for spot cooling/heating, environmental laboratories, printing industry, textile industry, precision parts manufacturing, photographic product handling, computer rooms, control rooms of power plants etc.

Refrigeration is extensively used for increasing the storage life of perishable items specially food products, vegetables, fruits, milk, beverages, chilling of water, ice formation etc. along with industrial applications in chemical manufacturing, petroleum refinery, petrochemical plants, paper and plup industry etc.

History of refrigeration system dates back to 1834 when Jacob Perkins had got first patent on mechanical type refrigeration system using 'ether' as refrigerant. First successful commercial refrigeration system was produced in 1857 by James Harrison and D.E. Siebe using ethyl ether as refrigerant.

Refrigeration can be achieved by using refrigeration cycle which may be open cycle type or closed cycle type. In open cycle the refrigerant passes through the system once for refrigerating the space and is thrown away as it gets contaminated. In closed cycle the refrigerant flows inside tubes while

refrigerating the space and does not get contaminated. Thus, in closed cycle a definite mass of refrigerant can be used for quite long time while it is not so in case of open cycle where for every run fresh refrigerant is to be picked up.

Different methods for refrigeration are described below:

(*i*) *Refrigeration by evaporation:* Refrigeration effect can be realized by evaporation of liquid. Evaporation of liquid requires latent heat of evaporation which is provided by the surroundings of region where liquid is being evaporated. This causes the cooling of the region supplying latent heat for evaporation. Examples of this type of refrigeration are:
- Cooling of water kept in earthen pot. Earthen pot has porosity thereby allowing evaporation of water and cooling down the inside remaining water.
- Cooling of water kept in desert bag (made of thick canvas i.e. tight woven fabric). Here some quantity of water always keeps on leaking out across the cloth surface. This water which appears on the surface of bag gets evaporated after absorbing heat from atmospheric air and water inside the bag, thus cooling the water kept in bag.
- Sweating from the human skin is also example of such refrigeration which keeps the human being cool.
- Evaporative condensers also use this concept for cooling.

(*ii*) *Refrigeration by ice:* Ice is being popularly used for refrigeration. In this kind of refrigeration ice is kept in between the walls of an insulated cabinet. Apart from above there can be some other arrangements too such as ice may be kept in a tray upon the insulated box having flow of air across ice blocks to the refrigerated space.

Temperature inside refrigerated space is controlled by flow rate of air and the temperature upto 5ºC can be attained like this.

(*iii*) *Refrigeration by expansion of air:* Generally, the expansion of air is accompanied by the cooling air due to reduction in its temperature. Subsequently cool air is passed into the region where refrigeration is to be done. Generally the adiabatic expansion of air is used in air refrigeration system described ahead.

(*iv*) *Refrigeration by throttling process:* Throttling of certain gases show the reduction in temperature of gas after throttling. Effect of throttling is evident by Joule-Thomson coefficient which shows whether the throttling is accompanied by cooling or heating of gases. Throttling is constant enthalpy process for which Joule-Thomson coefficient (μ) is given as:

$$\mu = \left(\frac{\partial p}{\partial T}\right)_h \text{; for } \mu > 0 \text{ the cooling is seen.}$$

(*v*) *Refrigeration by dry ice:* Dry ice is oftenly used in packaging of frozen foods for maintaining them at low temperature. Carbon dioxide in solid phase is termed as dry ice. It has typical property of getting transformed directly from solid phase to vapour phase at its sublimation temperature of –78ºC at atmospheric pressure. Dry ice blocks in suitable sizes are stacked around the frozen food for keeping them in frozen state.

(*vi*) *Refrigeration using liquid gases:* Liquid gases such as liquid nitrogen, liquid carbon dioxide are used for refrigeration in refrigerated cargo vehicles. Liquid gases are expanded thereby causing its transformation from liquid to gas accompanied by lowering of temperature of refrigerated space.

Liquid gases are sprayed inside the cooling area and it gets transformed into vapour which is exhausted out from vehicle. Nitrogen and carbon dioxide being cheaply available are generally used as liquid refrigerant. It facilitates attainable temperature up to –30ºC.

(*vii*) V*apour Refrigeration system:* Vapour refrigeration system based on vapour compression refrigeration system and vapour absorption refrigeration system are discussed ahead.

18.2 PERFORMANCE PARAMETER

Refrigerator's performance is given by coefficient of performance (COP), COP for a refrigerator shown in block diagram, Fig. 18.1 can be given as under. COP is the ratio of refrigeration effect and net work done upon refrigerator. COP is dimensionless quantity so the two quantities should have same units.

$$\text{COP of refrigeration} = \frac{\text{Desired effect}}{\text{Net work done}}$$

Here the desired effect is the continuous removal of heat at the rate of Q_2 for maintaining body at lower temperature T_2. Net work done for getting this refrigeration effect is W.

$$\text{COP} = \frac{Q_2}{W} = \frac{Q_2}{(Q_1 - Q_2)}$$

COP of refrigerator may have any magnitude i.e. less than unity or greater than unity.

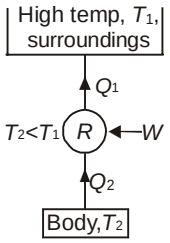


Fig. 18.1 *Refrigerator*

Heat pump is the device which runs exactly similar to the refrigerator but its purpose is to maintain the body at temperature higher than that of surroundings. Therefore, heat pump supplies heat at constant rate to the body to be maintained at higher temperature. Performance of heat pump is also given by the parameter defined exactly like COP but instead of calling it COP, it is called as energy performance ratio. Different name of EPR is assigned to differentiate it from COP of refrigerator.

Thus, COP of heat pump = Energy performance ratio (EPR)

$$= \frac{Q_1}{W}$$

$$\text{EPR} = \frac{Q_1}{Q_1 - Q_2}$$

or, $$\boxed{\text{EPR} = \text{COP} + 1}$$

EPR shall always have its magnitude greater than unity.

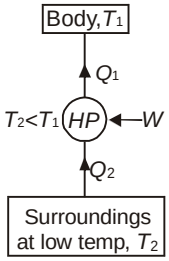


Fig. 18.2 *Heat pump*

18.3 UNIT OF REFRIGERATION

Refrigeration effect is the amount of heat extracted by refrigerator from the refrigerated space. The higher capacity of refrigerator means higher shall be the refrigeration effect. This refrigeration effect is defined by the unit of refrigeration called 'Ton' of refrigeration. 'Ton' as unit of refrigeration has been defined based on formation of ice as described ahead.

"One 'Ton' of refrigeration can be defined by the amount of heat being removed from one ton of water at 0°C to form one ton of ice at 0°C within 24 hours." Thus, a Ton of refrigeration shall quantify the latent heat required to be removed for solidification of water at 0°C.

Latent heat of solidification from water to ice at 0°C = 334.5 kJ/kg

1 Ton of refrigeration = mass of water × latent heat at 0°C from water to ice (in SI units)

$$= \frac{(1000 \times 334.5)}{24} \text{ kJ/hr}$$

$$= \frac{334.5 \times 10^3}{24 \times 3600} = 3.87 \text{ kJ/sec}$$

Generally, 1 Ton is taken as 3.5 kJ/sec. This deviation from 3.87 to 3.5 kJ/sec is there because Ton was originally defined in fps units and the approximations during conversion yielded numerical value of 3.5 kJ/sec which is now universally accepted.

$$\text{1 Ton of refrigeration (in MKS units)} = \frac{1000 \times 80}{24 \times 60} = 55.4 \text{ kcal/min}$$

$$= 50 \text{ kcal/min}$$

In MKS units this is commonly taken as 50 kcal/min.

Therefore,

$$1 \text{ Ton} = 3.5 \text{ kJ/sec}$$

and

$$1 \text{ Ton} = 50 \text{ kcal/min}$$

18.4 CARNOT REFRIGERATION CYCLE

Carnot cycle being the most efficient and ideal cycle can also be used for getting the refrigeration effect upon its reversal. Reversed carnot cycle is shown in Fig. 18.3.

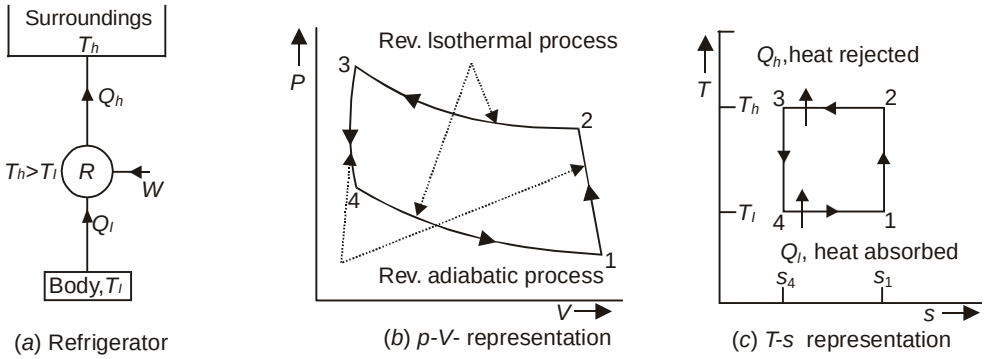


Fig. 18.3 *Reversed Carnot cycle for refrigeration*

Here, the refrigerated space/body is to be maintained at low temperature T_l for which heat Q_l should be removed at constant rate and rejected to surroundings at high temperature, T_h. Amount of heat rejected to surroundings is Q_h while the net work done upon refrigerator is W.

Different processes in reversed Carnot cycle are,

1–2 Reversible adiabatic compression

2–3 Reversible isothermal heat rejection, Q_h at temperature, T_h

3–4 Reversible adiabatic expansion

4–1 Reversible isothermal heat absorption, Q_l at temperature, T_l

Refrigeration effect is seen during process 4–1.

For cyclic process; Net work $W = Q_h - Q_l$

$$W = T_h (S_1 - S_4) - T_l (S_1 - S_4)$$

$$W = (S_1 - S_4) (T_h - T_l)$$

For reversed Carnot cycle

$$\text{COP} = \frac{Q_l}{W} = \frac{T_l (S_1 - S_4)}{(T_h - T_l)(S_1 - S_4)} = \left\{\frac{T_l}{T_h - T_l}\right\}$$

The coefficient of performance of refrigerator depends upon the two temperature values i.e. low temperature T_l and high temperature T_h. For COP value to be high the low temperature T_l should be high while higher temperature T_h should be small. With increasing low temperature value the COP goes on increasing, while with decreasing high temperature T_h value the COP goes on increasing. Practically, the lower temperature requirement is decided by the user while higher temperature is generally fixed by the atmospheric temperature value. Thus, it could be said that for certain low temperature to be maintained, COP of refrigerator shall be more during cold days as compared to hot days.

$$\text{COP}_{\text{cold days}} > \text{COP}_{\text{hot days}} \text{ because } T_{h,\text{ cold days}} < T_{h,\text{ hot days}}$$

Similar to the carnot power cycles, reversed carnot cycle for refrigeration is not feasible in practice. Therefore, number of modified cycles are practically used for refrigeration as discussed ahead. But the reversed carnot cycle provides the basis for comparison and provides the benchmark for achieving maximum COP.

18.5 AIR REFRIGERATION CYCLES

Refrigeration cycles using air as refrigerant are termed as air refrigeration cycles. Air refrigeration cycles offer distinct advantage of refrigerant being available in abundance and free of cost. Bell-Coleman cycle is one such cycle running on air as refrigerant. Bell-Coleman cycle is modified form of reversed carnot cycle in order to make it feasible cycle. Here isothermal heat addition and release are replaced by isobaric processes, Bell-Coleman cycle based refrigerators are the ancient refrigeration systems used in cargo ships.

Figure 18.4 shows the schematic of refrigeration system based on this cycle. It has a compressor, heat exchanger, expander and refrigeration unit.

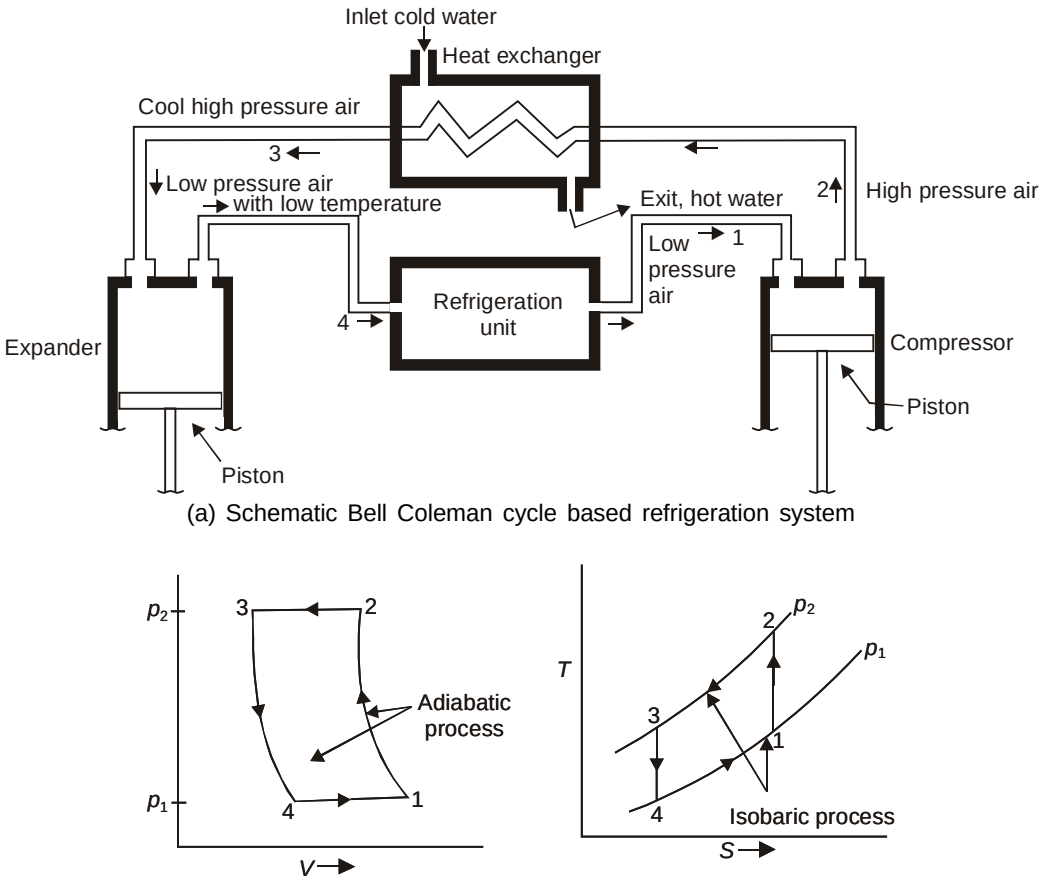


Fig. 18.4 *Bell Coleman cycle*

Bell-Coleman cycle is actually the reversed Brayton cycle.

Different processes in Bell-Coleman cycle are as under.

1–2: Adiabatic compression of air causing rise in its pressure and temperature inside compressor. Ideally, this compression process is considered isentropic whereas actually it may not be so.

2–3: Isobaric heat rejection causing cooling of high pressure and high temperature air coming from compressor making it low temperature and high pressure air inside heat exchanger. Water may be used as cooling fluid inside heat exchanger.

3–4: Adiabatic expansion inside expander causing further cooling of high pressure low temperature air entering expander. This very cool air enters the refrigeration unit subsequently. Ideally expansion can be considered to be of isentropic type whereas actually it may not be isentropic.

4–1: Low temperature cool air enters refrigeration space and picks up heat from space thereby showing refrigeration effect. Subsequently low temperature air gets heated up and leaves the space after showing desired refrigeration to enter in to compressor. Thus, process 4–1 is the actual process which is giving the desired refrigeration effect.

Thermodynamically, the refrigeration effect, net work supplied and COP estimated are as under.

Heat picked up from refrigerated space:

$$Q_{\text{absorbed}} = mC_p\,(T_1 - T_4)$$

Net work done upon system, W = (Heat rejected – heat absorbed), as for a cycle.

$$\Sigma W_{\text{cycle}} = \Sigma Q_{\text{cycle}}$$

heat rejected,

$$Q_{\text{rejected}} = mC_p\,(T_2 - T_3)$$

Hence,
$$W = mC_p\,(T_2 - T_3) - mC_p\,(T_1 - T_4)$$

$$\text{COP} = \frac{\text{Desired effect}}{\text{Net work}} = \frac{mC_p(T_1 - T_4)}{mC_p(T_2 - T_3) - mC_p(T_1 - T_4)}$$

Theoretical COP for ideal cycle, $\boxed{\text{COP} = \dfrac{1}{\left(\dfrac{T_2 - T_3}{T_1 - T_4}\right) - 1} = \text{COP}_{\text{ideal}}}$

Above expression for COP is correct till the different processes are of isobaric or isentropic type. In actual practice the expansion and compression processes are of polytropic type and given by pV^n = constant i.e. both expansion and compression are assumed to follow same index. Actual cycle shall have heat absorption and heat rejection occurring with some pressure loss thereby deviating from ideal isobaric process.

Work required in compressor, $W_c = \dfrac{n}{n-1}\,[p_2\,V_2 - p_1 V_1] \times m$

$$W_c = \left(\frac{n}{n-1}\right) R[T_2 - T_1] \times m$$

Work available from expander, $W_e = \dfrac{n}{n-1}\,[p_2\,V_3 - p_1\,V_4] \times m$

$$W_c = \left(\frac{n}{n-1}\right) R[T_3 - T_4] \times m$$

Net work,
$$W_{\text{actual}} = W_c - W_e$$

$$W_{\text{actual}} = \left(\frac{n}{n-1}\right) \cdot m \cdot R\,[(T_2 - T_1) - (T_3 - T_4)]$$

as
$$C_p = \frac{\gamma R}{(\gamma - 1)}, \text{ so}$$

$$W_{\text{actual}} = \left(\frac{n}{n-1}\right) \cdot m \cdot C_p \cdot \left(\frac{\gamma - 1}{\gamma}\right)[(T_2 - T_1) - (T_3 - T_4)]$$

Therefore,
$$\text{COP}_{\text{actual}} = \frac{mC_p(T_1 - T_4)}{mC_p\left(\dfrac{n}{n-1}\right)\left(\dfrac{\gamma - 1}{\gamma}\right)[(T_2 - T_1) - (T_3 - T_4)]}$$

$$\text{COP}_{\text{actual}} = \frac{1}{\left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right)\left[\left(\frac{T_2-T_3}{T_1-T_4}\right)-1\right]}$$

For isentropic process, $\gamma = n$ and $\text{COP}_{\text{ideal}} = \text{COP}_{\text{actual}}$

Using isentropic considerations, $\text{COP}_{\text{ideal}} = \frac{1}{\left[\left(\frac{T_2-T_3}{T_1-T_4}\right)-1\right]}$

$$\text{COP}_{\text{ideal}} = \frac{1}{\left[\left(\frac{T_2}{T_1}\right)\left(\frac{1-\frac{T_3}{T_2}}{1-\frac{T_4}{T_1}}\right)-1\right]}$$

For isentropic process between states 1 and 2, 3 and 4.

$$\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1} \text{ and } \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{T_3}{T_4}\right)$$

Upon equating we get,

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$

or,
$$\frac{T_4}{T_1} = \frac{T_3}{T_2} \text{ or } \frac{T_1-T_4}{T_2-T_3} = \frac{T_1}{T_2}$$

Therefore, upon substitution,

$$\text{COP}_{\text{ideal}} = \frac{1}{\left(\frac{T_2}{T_1}-1\right)}$$

$$\boxed{\text{COP}_{\text{ideal}} = \left(\frac{T_1}{T_2-T_1}\right)}$$

$$\boxed{\text{COP}_{\text{actual}} = \frac{T_1}{\left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right)(T_2-T_1)}}$$

Comparing the COP of ideal Bell Coleman cycle with reversed Carnot cycle COP we find that COP of Bell Coleman cycle will be less than that of COP of reversed Carnot cycle.

$$\text{COP}_{\text{Bell Coleman}} < \text{COP}_{\text{Carnot}}$$

Carnot cycle operating between the temperature limits of Bell Coleman cycle shall have COP given by, $COP_{carnot} = \dfrac{T_1}{T_3 - T_1}$

Since $T_2 > T_3$ so, $COP_{Bell\ Coleman} < COP_{Carnot}$.

Constant pressure heat transfer results in operating range which is greater than minimum range given by temperature of those parts of the surroundings with which heat is exchanged.

Air refrigeration systems offer advantages as these run on air as refrigerant which is cheap, non-inflammable and non toxic. Also, air refrigeration systems are light weight per ton of refrigeration compared to other refrigeration systems and is used in aircraft applications. Leakage of air refrigerant may not be dangerous problem.

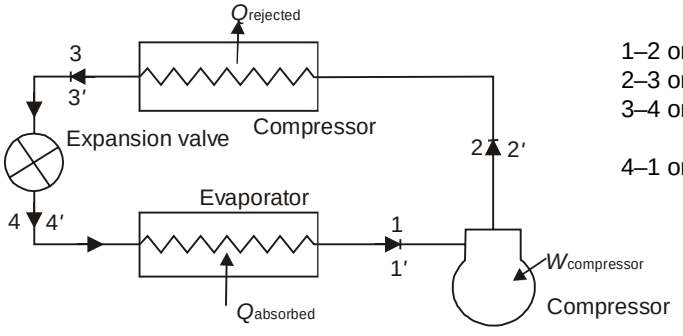
Air refrigeration systems are disadvantageous due to lower COP compared to other refrigeration systems. There is significant possibility of freezing of moisture accompanied by atmospheric air due to low temperature in refrigeration space. Air may also pick up moisture while passing through refrigeration space. Quantity of air refrigerant required is more compared to other refrigerants for similar refrigeration capacity. This is because of fact that air only carries heat in the form of sensible heat and so its large quantity is required compared to other refrigerants.

18.6 VAPOUR COMPRESSION CYCLES

Vapour compression cycle based refrigeration systems are extensively used in refrigeration systems. These cycles are used for most of small domestic and large industrial applications.

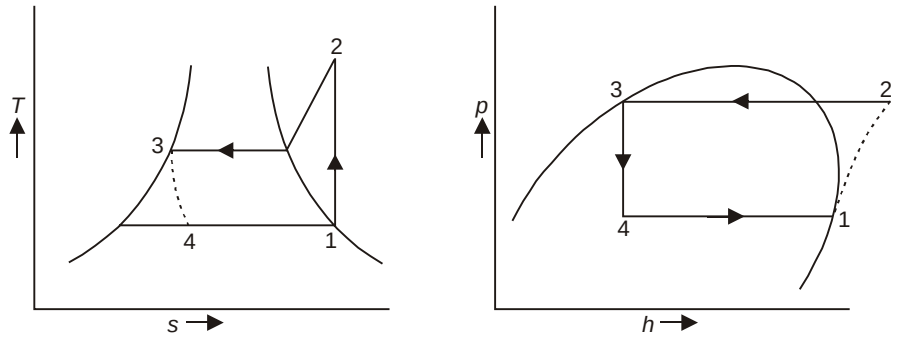
The vapour compression cycle has the refrigerant being circulated in closed circuit through compressor, condenser, throttle valve or expansion valve and evaporator as shown in Fig. 18.5. Refrigerant (in gas/vapour) is compressed isentropically in compressor from state 1 to 2. High pressure and high temperature refrigerant enters the condenser at state 2 where its condensation occurs and refrigerant is available in liquid form at state 3. Refrigerant in the form of saturated liquid at high pressure is passed through expansion valve where isenthalpic expansion occurs. Refrigerant leaving expansion valve at state 4 is in the form of low pressure wet mixture of liquid and vapour. Low pressure liquid-vapour mixture is passed through evaporator section in which refrigerant picks up heat from surroundings thereby showing refrigeration effect. As a result of this heat absorption the liquid-vapour mixture refrigerant gets transformed into dry gaseous refrigerant in case of *dry compression* as shown in Fig. 18.5 (*b*). Compression of dry and saturated or superheated refrigerant is called dry compression.

Compression of dry refrigerant yields superheated state of refrigerant as shown by state 2. (Fig. 18.5 (*b*))

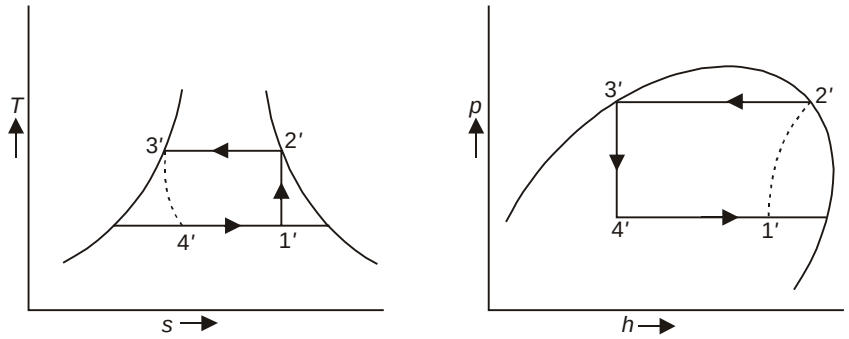


(*a*) Schematic for vapour compression cycle

1–2 or 1′–2′ : - Isentropic compression
2–3 or 2′–3′ : - Isobaric heat rejection
3–4 or 3′–4′ : - Isenthalpic expansion process or throttling process
4–1 or 4′–1′ : - Isobaric heat absorption



(*b*) *T-s* and *p-h* diagram for dry compression



(*c*) *T-s* and *p-h* diagram for wet compression

Fig. 18.5 *Vapour compression cycle*

Wet compression: It is also possible that the refrigerant is in wet state i.e. liquid-vapour mixture at inlet of compressor, state 1'. Compression of wet mixture gets transformed into dry refrigerant (gaseous form) as shown by state 2'. Dry refrigerant at high pressure and high temperature is passed through condenser where refrigerant gets condensed and condensate is available in saturated liquid form at high pressure. Subsequently refrigerant is throttled from high pressure to low pressure inside expansion valve from state 3' to 4'. Low pressure refrigerant in wet state is passed through evaporator from state 4' to 1' where it picks up heat from surroundings and some of its liquid fraction gets transformed into vapour but it does not become dry (gas) refrigerant at inlet to compressor. This wet refrigerant is compressed inside compressor.

In case of dry compression the compressor efficiency is found to be more than that of wet compression due to higher volumetric efficiency with dry refrigerant and also chances of damage by liquid refrigerant are absent. During dry compression the temperature of compressed refrigerant becomes more than condensation temperature. Due to this high temperature after compression the compressor becomes very hot therefore requiring cooling. Coolant used in condenser may also be used for cooling compressor as its temperature is lower than compressor temperature. This cooling of compressor reduces compression work.

T-s and *p-h* diagram for vapour compression cycle shows that the refrigeration capacity can be increased by subcooling the condensate before it enters the expansion valve and also by increasing the degree of expansion in expansion valve.

Here in second option the condensation temperature is fixed and evaporator temperature is lowered which increases expansion ratio and compression ratio. This increase in compression ratio shows poor

volumetric efficiency in single stage dry compression. The temperature after compression also gets increased with increased compression ratio which may be harmful to refrigerant properties. To regulate the excessive rise in refrigerant temperature the intercooling during compression with multistage compression is used. This intercooled compression may be devised using refrigerant itself as intercooling medium.

Thermodynamic analysis: For simple vapour compression cycle shown in Fig. 18.6 the COP, refrigeration effect and work input can be estimated based on following assumptions:

(*i*) All process of refrigeration cycle are internally reversible except the expansion through valve which is throttling process and is irreversible.

(*ii*) Compressor and expansion valve have no heat interaction with surroundings during their operation i.e. they operate adiabatically.

(*iii*) Refrigerant leaving condenser is saturated liquid.

(*iv*) Refrigerant entering compressor is saturated vapour in case of 'dry compression' and liquid-vapour mixture in case of 'wet compression'.

(*v*) Changes in kinetic energy and potential energy are negligible.

For mass flow rate of refrigerant being given by 'm' kg/s the refrigeration capacity or refrigeration effect. $Q_{\text{absorbed}} = m(h_1 - h_4)$, in dry compression

$$Q_{\text{absorbed}} = m(h_{1'} - h_{4'}) \text{ in wet compression}$$

Net work = Work input, $W_{\text{compressor}} = m(h_2 - h_1)$, in dry compression

$$W_{\text{compressor}} = m(h_{2'} - h_{1'}), \text{ in wet compression.}$$

Therefore,
$$\text{COP} = \frac{m(h_1 - h_4)}{m(h_2 - h_1)} = \left\{\frac{(h_1 - h_4)}{(h_2 - h_1)}\right\}, \text{ in dry compression}$$

$$\text{COP} = \frac{m(h_{1'} - h_{4'})}{m(h_{2'} - h_{1'})} = \left\{\frac{(h_{1'} - h_{4'})}{(h_{2'} - h_{1'})}\right\}, \text{ in wet compression.}$$

Some calculations in vapour compression cycle: Let us take three different cases of vapour compression cycle similar to the dry and wet compression.

Case 1: *Compression yields dry and saturated vapour at the end of compression.*

Vapour compression cycle in which the compression process produces vapour dry and saturated, is shown on *T-s* diagram in Fig. 18.6.

Different processes are,

1–2 : Isentropic compression

2–3 : Heat rejection in condenser

3–4 : Isenthalpic expansion in expansion valve (throttling)

4–1 : Heat absorption in evaporator or refrigerated space.

Work done in the cycle can be given by the area enclosed 1–2–3–5–1 on *T-s* diagram. The work required for compression process.

The refrigeration effect in evaporator can be given by area enclosed 1–4–9–10–1 on *T–s* diagram.

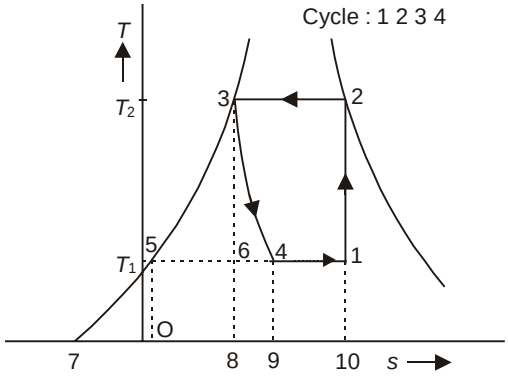


Fig. 18.6

Compressor work, W = Area 1 – 2 – 3 – 5 – 1
= (Area 2 – 3 – 5 – 7 – 10 – 2) – (Area 1 – 5 – 7 – 10 – 1)
= (Enthalpy at 2) – (Enthalpy at 1)
$W = h_2 - h_1$

Here, $h_2 = h_{g \text{ at } T_2}$

Refrigeration effect = Area 1 – 4 – 9 – 10 – 1 = (Area 1 – 5 – 7 – 10 – 1) – (Area 4 – 5 – 7 – 9 – 4)
= (Enthalpy at 1) – (Enthalpy at 4)
Refrigeration effect $= h_1 - h_4$
Enthalpy at 3 = Enthalpy at 4
For throttling process 3 – 4, $h_3 = h_4$
(Area 3 – 7 – 8 – 3) = (Area 4 – 5 – 7 – 9 – 4)

or,

(Area 3 – 5 – 6 – 3) = (Area 6 – 5 – 7 – 8 – 6)
= (Area 6 – 5 – 7 – 8 – 6) – (Area 4 – 6 – 8 – 9 – 4)

or,

(Area 3 – 5 – 6 – 3) = (Area 4 – 6 – 8 – 9 – 4)

(Area 3 – 5 – 7 – 8 – 3) – (Area 5 – 7 – 0 – 5) – (Area 6 – 5 – 0 – 8 – 6) = (Area 4 – 6 – 8 – 9 – 4)

(Enthalpy at 3) – (Enthalpy at 5) – $(T_1 \times \Delta s_{56}) = (T_1 \times \Delta s_{46})$

$\Rightarrow$ (Area 3 – 5 – 6 – 3) = $\{(h_{f \text{ at } T_2} - h_{f \text{ at } T_1}) - T_1 \times (s_{f \text{ at } T_2} - s_{f \text{ at } T_1})\}$

Here enthalpy at 3 and enthalpy at 5 are enthalpies for saturated liquid states at temperature T_2 and T_1 respectively.

Enthalpy at 3 = $h_{f \text{ at } T_2} = h_3$

Enthalpy at 5 = $h_{f \text{ at } T_1} = h_5$

State 6 lies vertically below 3 so, $s_3 = s_6$ and,

Entropy at 5 = $s_{f \text{ at } T_1} = s_5$

Entropy at 3 = $s_{f \text{ at } T_2} = s_3 = s_6$

$$\Delta s_{56} = s_6 - s_5 = (s_3 - s_5) = (s_{f \text{ at } T_2} - s_{f \text{ at } T_1})$$

So, for known h_3, h_5, Δs_{56} the entropy change Δs_{46} can be obtained as under;

$$\Delta s_{46} = \frac{1}{T_1}(h_{f \text{ at } T_2} - h_{f \text{ at } T_1}) - \Delta s_{56}$$

Therefore, for getting work,

W = Area 1 – 2 – 3 – 5 –1
= (Area 3 – 5 – 6 – 3) + Area (2 – 3 – 6 – 1 – 2)
$= \{(h_{f \text{ at } T_2} - h_{f \text{ at } T_1}) - T_1 \times (s_{f \text{ at } T_2} - s_{f \text{ at } T_1})\} + \{(T_2 - T_1) \times \Delta s_{23}\}$

$W = \{(h_{f \text{ at } T_2} - h_{f \text{ at } T_1}) - T_1 \times (s_{f \text{ at } T_2} - s_{f \text{ at } T_1})\} + \{(T_2 - T_1) \times (s_2 - s_3)\}$

$W = \{(h_{f \text{ at } T_2} - h_{f \text{ at } T_1}) - T_1 \times (s_{f \text{ at } T_2} - s_{f \text{ at } T_1})\} + \{(T_2 - T_1) \times (s_{g \text{ at } T_2} - s_{f \text{ at } T_2})\}$

The refrigeration effect = (Area 1 – 4 – 9 – 10 – 1)
$= T_1 \times \Delta s_{14} = T_1 \times (\Delta s_{16} - \Delta s_{46})$

here, $\Delta s_{16} = s_1 - s_6 = s_{g \text{ at } T_2} - s_{f \text{ at } T_2}$

Substituting Δs_{16} and Δs_{46} values in expression for refrigeration effect.

$$\text{Refrigeration effect} = T_1 \{(s_{g \text{ at } T_2} - s_{f \text{ at } T_2}) - (\frac{1}{T_1} (h_{f \text{ at } T_2} - h_{f \text{ at } T_1}) - \Delta s_{56})\}$$

$$= T_1 \{(s_{g \text{ at } T_2} - s_{f \text{ at } T_2}) - \frac{1}{T_1} (h_{f \text{ at } T_2} - s_{f \text{ at } T_1}) + (s_{f \text{ at } T_2} - s_{f \text{ at } T_1})\}$$

$$= T_1 \{(s_{g \text{ at } T_2} - s_{f \text{ at } T_1}) - \frac{1}{T_1} (h_{f \text{ at } T_2} - h_{f \text{ at } T_1})\}$$

$$\text{Here, COP of cycle} = \frac{\text{Refrigeration effect}}{\text{Work done}}$$

$$\text{COP} = \frac{T_1\left\{(s_{g \text{ at } T_2} - s_{f \text{ at } T_1}) - (h_{f \text{ at } T_2} - h_{f \text{ at } T_1})(1/T_1)\right\}}{\left\{(h_{f \text{ at } T_2} - h_{f \text{ at } T_1}) - T_1(s_{f \text{ at } T_2} - s_{f \text{ at } T_1}) + (T_2 - T_1)\cdot(s_{g \text{ at } T_2} - s_{f \text{ at } T_2})\right\}}$$

Case 2 : *Compression yields wet vapour at the end of compression*

Figure 18.7 gives *T-s* representation of the vapour compression cycle having compression process yielding wet vapour after compression i.e. at state 2. Let wetness after compression be x_2.

Work done = Area 1 – 2 – 3 – 5 – 1

= (Area 3 – 5 – 6 – 3) + (Area 2 – 3 – 6 – 1 –2)

From earlier analysis,

Area 3 – 5 – 6 – 3 = Area (4 – 6 – 8 – 9 – 4)

$$\text{Area } 3 - 5 - 6 - 3 = \{(h_{f \text{ at } T_2} - h_{f \text{ at } T_1}) - T_1 (s_{f \text{ at } T_2} - s_{f \text{ at } T_1})\}$$

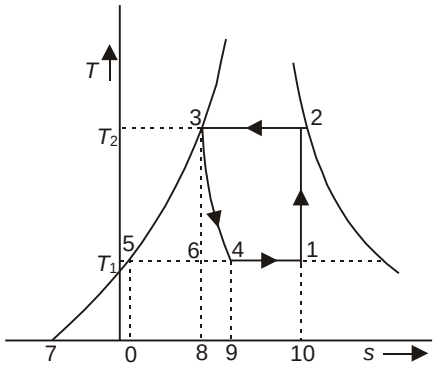


Fig. 18.7

Work = {Area 3 – 5 – 6 – 3} + {Area 2 – 3 – 6 – 1 – 2}

$$\text{Work} = \{(h_{f \text{ at } T_2} - h_{f \text{ at } T_1}) - T_1 (s_{f \text{ at } T_2} - s_{f \text{ at } T_1})\} + \left\{(T_2 - T_1) \times \frac{x_2 \cdot h_{fg \text{ at } T_2}}{T_2}\right\}$$

Refrigeration effect = Area 1 – 4 – 9 – 10 – 1

$$= \{\text{Area } 1-6-8-10-1\} - \{\text{Area } 4-6-8-9-4\}$$
$$= \{\text{Area } 1-6-8-10-1\} - \{\text{Area } 3-5-6-3\}$$
$$= \{T_1 \cdot (x_2 \cdot s_{fg \text{ at } T_2})\} - \{(h_{f \text{ at } T_2} - h_{f \text{ at } T_1}) - T_1 (s_{f \text{ at } T_2} - s_{f \text{ at } T_1})\}$$

$$\text{Refrigeration effect} = \{T_1 \cdot ((x_2 \cdot s_{fg \text{ at } T_2} + s_{f \text{ at } T_2}) - s_{f \text{ at } T_1})\} - \{h_{f \text{ at } T_2} - h_{f \text{ at } T_1}\}$$

COP may be obtained by taking the ratio of refrigeration effect and work.

Case 3: *Compression yields superheated vapour after compression*

Vapour compression cycle having the compression process yielding superheated vapour at the end of compression is shown on *T-s* diagram in Fig. 18.8.

Cycle is shown as 1–2–3–4–1.

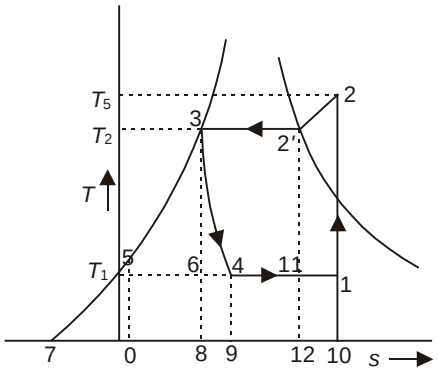


Fig. 18.8

In this cycle additional work shown by area 2 – 2′ – 11 – 1 – 2 is done and also the additional refrigeration effect shown by area 1 – 11 – 12 – 10 – 1 is available.

Additional work W' = Area 2 – 2′ – 11 – 1 – 2

= (Area 2 – 2′ – 12 – 10 – 2) – (Area 1 – 11 – 12 – 10 – 1)

$$W' = C_p (T_5 - T_2) - \left(T_1 C_p \cdot \ln\left(\frac{T_5}{T_2}\right)\right)$$

Additional refrigeration effect = (Area 1 – 11 – 12 – 10 – 1)

$$= T_1 C_p \ln\left(\frac{T_5}{T_1}\right)$$

Therefore, COP of system could be obtained by ratio of total refrigeration effect and total work done.

$$\text{COP} = \frac{\text{Refrigeration effect (= Area } 1-4-9-10-1)}{\text{Work done (= Area } 2-2'-3-5-1-2)}$$

This ratio of mass fraction of gas (vapour) to liquid i.e. $\left(\frac{m_9}{m_7}\right)$ can be estimated by energy balance on flash chamber. State 3 can be determined by the energy balance on mixing box.

Thermodynamic analysis

Thermodynamic analysis of multistage vapour compression cycle can also be carried out for the dry compression considered here based on the assumptions as under

(*i*) All processes of refrigeration cycle are internally reversible except the processes in expansion valve, flash chamber and mixing box.

(*ii*) The processes in expansion valve, flash chamber, mixing box and compressors operate adiabatically.

(*iii*) Refrigerant leaving condenser is saturated liquid and the refrigerant entering low pressure compressor is saturated vapour.

(*iv*) There are no changes in kinetic and potential energies in different processes.

For mass flow rate of m kg/s and dryness fraction of refrigerant at inlet of flash chamber being x, the refrigeration effect, net work and COP can be given as under.

Refrigeration capacity or refrigeration effect,

$$Q_{\text{absorbed}} = (m - x \cdot m) \cdot (h_1 - h_8) \text{ kJ/s}$$

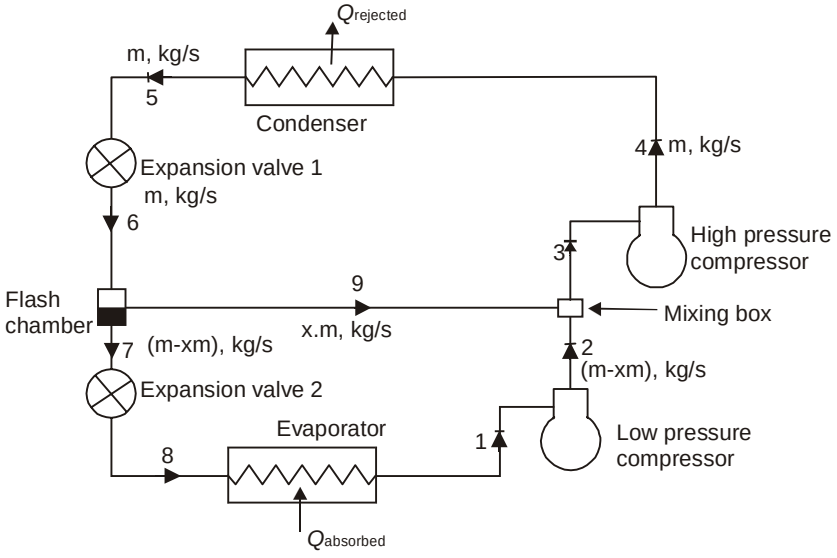
Net work or Work input, $W_{\text{compressor}} = \{(m - x \cdot m)\ (h_2 - h_1) + m\ (h_4 - h_3)\}$kJ/s

$$\text{COP} = \frac{(m - xm)(h_1 - h_8)}{\{(m - xm)(h_2 - h_1) + m(h_4 - h_3)\}}$$

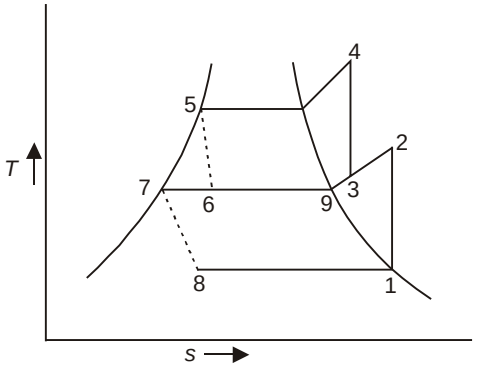
$$= \frac{(1 - x)(h_1 - h_8)}{\{(1 - x)(h_2 - h_1) + (h_4 - h_3)\}}$$

18.7 MULTISTAGE VAPOUR COMPRESSION CYCLE

Intercooled compression using flash chamber is shown in Fig. 18.9. Here compression for total compression ratio occurs in two stages i.e. LP compressor and HP compressor.



(*a*) Two stage intercooled vapour compression cycle with flash chamber



1–2, 3–4	Reversible adiabatic compression
5–6, 7–8	Irreversible throttling process
6–7, 6–9	In flash chamber
4–5	Isobaric condensation
9–3–2	Mixing of refrigerant from LP compressor with cool vapour from flash chamber.

(*b*) T-s representation of two stage intercooled vapour compression cycle

Fig. 18.9 *Multistage vapour compression cycle*

Refrigerant is partly compressed in LP stage from 1 to 2. Compressed refrigerant leaving compressor is cooled by mixing it with the cool vapour leaving flash chamber at state 9 and finally yielding low pressure refrigerant at low temperature, state 3 (Temperature at 3 < Temperature at 2). This reduced temperature of refrigerant from 2 to 3 results in smaller compressor work requirement. Subsequently, refrigerant is compressed in high pressure compressor upto state 4. High pressure refrigerant is passed through condenser where it gets condensed into saturated liquid form of refrigerant and passed through expansion valve (expansion valve 1) from state 5 to state 6. Expansion is carried out till the refrigerant's pressure gets reduced to interstage pressure and sent to flash chamber. Liquid vapour mixture entering flash chamber is separated such that liquid fraction of refrigerant passes into next expansion valve (expansion valve 2) at state 7 and the gaseous (vapour) fraction of refrigerant leaves flash chamber at state 9 for intercooling refrigerant in mixing box between state 2 and 3. Fraction of liquid refrigerant entering expansion valve is expanded from state 7 to 8. As a result of expansion the refrigerant's pressure gets reduced and low pressure liquid refrigerant is passed through evaporator from state 8 to 1 and its phase transformation occurs from liquid to vapour. Inside evaporator this phase transformation from liquid to vapour (gas) occurs by absorbing latent heat of vaporization from the refrigerated space thereby showing cooling affect.

18.8 ABSORPTION REFRIGERATION CYCLE

In vapour compression cycle it is seen that the power input is made for compressing refrigerant from low pressure to high pressure. For constant refrigerating capacity the COP of refrigeration system can be improved by reducing the work input to cycle. Here in order to reduce work input to cycle the compressor is being replaced by pump if the refrigerant is in liquid form instead of being in gas (vapour) form. Since the average specific volume of liquid solution is much less than that of refrigerant in vapour form so considerably less amount of work is required for increasing its pressure in pump.

The absorption refrigeration cycle is based on this premise of employing pump for raising pressure of refrigerant in order to take advantage of less work requirement. For realizing it entirely different arrangement is employed as shown in Fig. 18.10. Such arrangement is called absorption refrigeration system. Here aqua-ammonia absorption refrigeration system has been discussed. Ammonia works as refrigerant and water acts as carrier for refrigerant in this cycle. Ammonia vapour leaving evaporator enter the absorber where ammonia vapours are released through perforated pipe into water for getting

absorbed in water. This dissolving of NH_3 in water result in strong aqua-ammonia solution. Mixing of NH_3 in water is an exothermic process where heat is released similar to condensation process of pure substance. Due to heat release in absorber some arrangement should be made so that heat could be extracted out for maintaining its constant temperature. This mixing of ammonia into water is more when temperature of water mixing is low. Therefore, it is always tried to maintain absorber at low temperature and for this cooling water may be circulated in absorber. This strong aqua-ammonia solution prepared in absorber is pumped employing a pump for being sent to generator at high pressure i.e. condenser pressure. Generator is used for separating out NH_3 vapour. For driving out ammonia from strong-aqua-ammonia solution heat is added to generator. The separation of NH_3 from strong solution is an endothermic process similar to evaporation process. Generator may have arrangement for heat addition employing burners. Ammonia vapours leaving generator at high pressure are sent to condenser at state 2 where the condensation of ammonia takes place yielding high pressure ammonia in liquid form at state 3. High pressure ammonia liquid is passed through expansion valve from state 3 to 4. Weak solution of aqua-ammonia left inside generator is sent to absorber at state *d*. Thus, it is seen that the combination of absorber, pump and generator helps in getting high pressure refrigerant (ammonia) without using compressor.

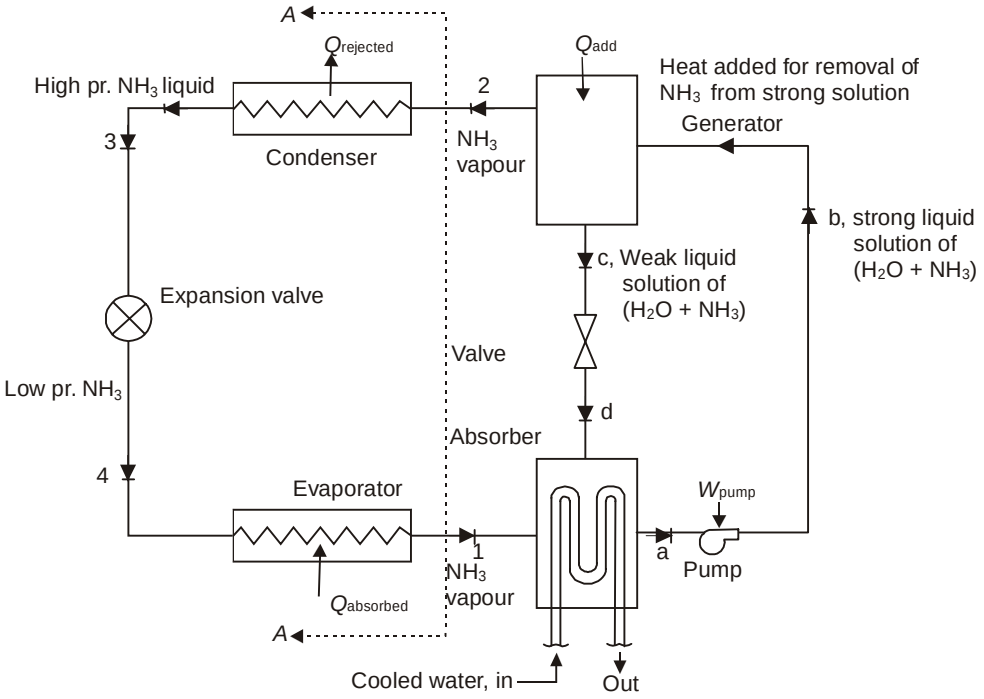


Fig 18.10 *Ammonia-water absorption system*

Low pressure ammonia leaving expansion valve may be pure liquid or liquid-vapour mixture for being sent to evaporator at state 4. Ammonia leaving evaporator after picking up heat from refrigerating space gets transformed into vapour at state 1. Ammonia vapours enter into absorber for getting mixed with weak aqua-ammonia solution to result in strong aqua-ammonia solution at state *a*. It is obvious from the arrangement that work input to absorption cycle gets considerably reduced but the cost incurred in replacing compressor by absorber, pump and generator and heat source etc. is quite large and nullifies the advantage of small work input. Here in absorption system the arrangement of components

between stages 2, 3, 4 and 1 is similar to that of vapour compression system i.e. condenser followed by expansion valve and evaporator as shown on left of section *A–A* on schematic of ammonia water absorption system (Fig. 18.10).

Performance of aqua-ammonia absorption depends largely upon the characteristics of binary mixtures i.e. water and ammonia in present case. The generic phase equilibrium diagram for ammonia-water mixture is shown in Fig. 18.11.

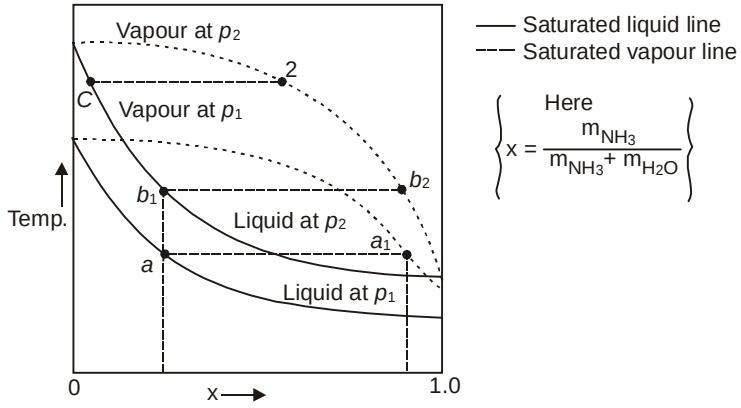


Fig. 18.11 *Ammonia-water mixture equilibrium diagram*

Phase equilibrium diagram has temperature on *y* axis and mass fraction of ammonia, *x* on x-axis. The saturated liquid states and saturated vapour states are shown on saturated liquid line and saturated vapour lines given by firm lines and dotted lines respectively. Saturation curves are shown for two pressures p_1 and p_2 between which vapour absorption cycle operates. Here when $x = 0$ then mixture shall be complete water and the temperature shall be saturation temperature of water at respective pressure. When $x = 1$ then mixture shall be complete ammonia and temperature is saturation temperature of ammonia at respective pressure. Phase equilibrium diagram shows that the saturated liquid and saturated vapour can exist together in equilibrium as evident from states *a* and a_1 respectively at any temperature. It also shows that at any temperature the concentration of ammonia is more in vapour than liquid i.e. $(x_{a_1} > x_a)$. For the absorption cycle shown in Fig. 18.11 the saturated liquid at state *a* leaving absorber at pressure p_1 enters the pump for being pumped upto pressure p_2. For causing evaporation of NH_3 the mixture is heated in generator upto state *b*. Further heat addition in generator causes evaporation of liquid to vapour yielding state b_2. Transformation from b_1 to b_2 results in formation of ammonia rich vapour thereby reducing concentration of ammonia in liquid. If the pressure and temperature are not changed then evaporation gets stopped as lower concentration liquid is not at its boiling point. With further heat addition the evaporation of mixture continues and the mixture's temperature is raised showing new liquid and vapour states as *C* and 2. Liquid at *C* is weak aqua-ammonia mixture and is passed through valve back to absorber. NH_3 vapour (gas) leaves at state 2 from generator to the condenser for being condensed.

18.9 MODIFIED ABSORPTION REFRIGERATION CYCLE

In the absorption cycle discussed earlier it is seen that the strong ammonia solution preparation requires low temperature in absorber while vaporization of NH_3 from strong aqua-ammonia solution needs heating up. Thus, weak solution leaving generator shall be at higher temperature while strong solution leaving absorber shall be at lower temperature. It may be noted that it will be desirable to send strong solution

into generator at high temperature so that less external heat addition is required. Also, it is desirable to send weak solution into absorber at lower possible temperature so that less external cooling is needed for effective absorption. Using above two facts the absorption cycle can be modified by employing a heat exchanger between generator and absorber as shown in Fig. 18.12.

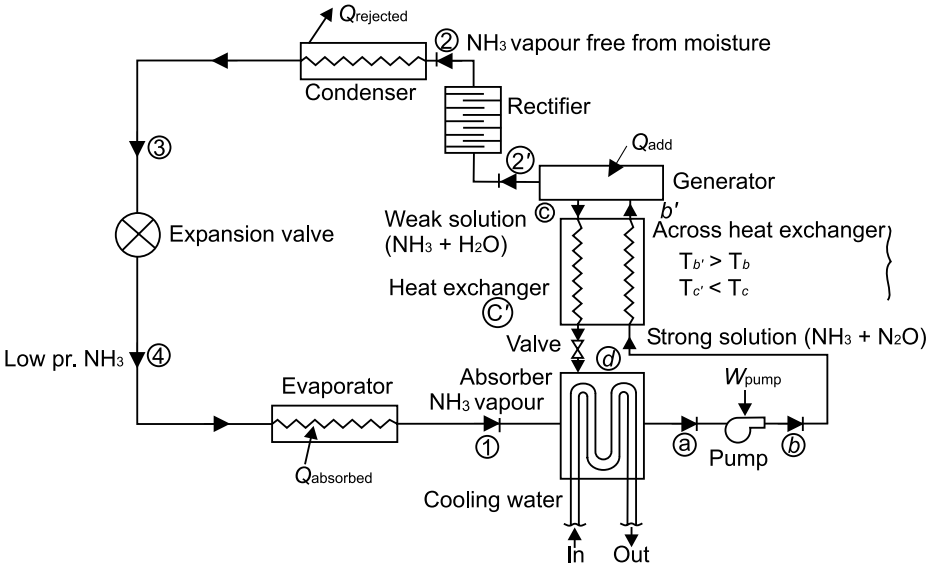


Fig. 18.12 *Modified ammonia-water absorption system*

Ammonia vapour leaving generator are outcome of evaporation from aqua-ammonia mixture due to its heating and there is possibility of water vapour accompanying the ammonia vapour leaving generator. This moisture in refrigerant may undergo freezing (probably at expansion valve or evaporator) and choke the refrigerant flow circuit which is not desirable. Therefore, for preventing moisture accompanying refrigerant from entering into circuit the device called rectifier is put after generator where traces of water are separated out and retained. Rectifier is shown in schematic of modified absorption cycle, Fig. 18.12. Thus, modified absorption cycle employing heat exchanger and rectifier permits to take advantage of heat of weak solution and make refrigerant free of moisture content.

Such type of absorption cycle can also be run using 'lithium-bromide' as absorbent and 'water' as refrigerant on the principle similar to 'water-ammonia system' discussed here. This lithium bromide and water system may not offer very low temperature refrigeration and so this cycle may be cascaded with another cycle using refrigerant having good low temperature characteristic, such as ammonia. This combination is termed as cascade refrigeration system.

18.10 HEAT PUMP SYSTEMS

Heat pump refers to the device which is used for maintaining a region or body at temperature more than that of surroundings. Heat pumps have their working similar to refrigerator with only difference of objective i.e. heat pump maintains temperature more than surroundings contrary to refrigerator which maintains temperature less than surroundings temperature. Heat pump systems can be based upon vapour compression cycle, absorption cycle etc. and are extensively used for space heating applications in domestic and industrial premises.

Reversed Carnot cycle is the ideal thermodynamic cycle for heat pumps similar to the refrigerators. Heat pump system based on vapour compression cycle is described here in Fig. 18.13.

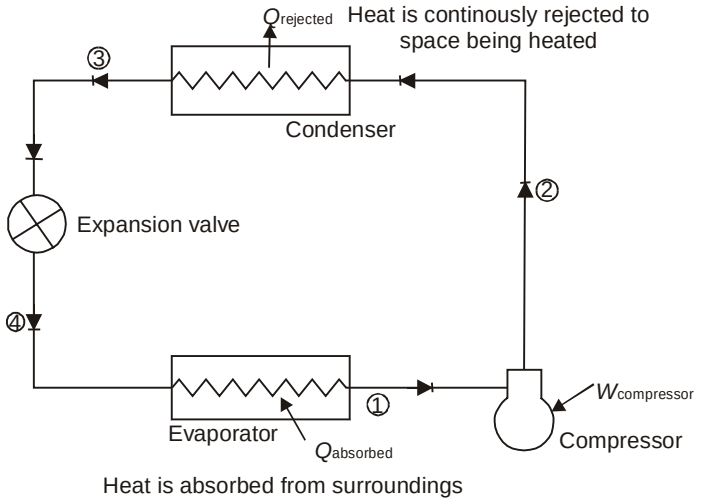


Fig. 18.13 *Heat pump system based on vapour compression cycle*

Here compressor compresses the refrigerant from low pressure state 1 to high pressure state 2. High pressure gaseous refrigerant is sent to condenser where the condensation of refrigerant occurs rejecting heat equal to the latent heat of condensation to the space being heated up. Subsequently refrigerant in saturated liquid form passes into expansion valve from state 3 to 4. Low pressure liquid refrigerant leaving expansion valve subsequently enters the evaporator where its evaporation occurs up to state 1. Evaporation of refrigerant occurs due to heat being provided by surroundings, water in rivers/ ponds, wells or solar collectors etc. Hence the performance of heat pump can be given by:

$$\text{COP of heat pump} = \frac{\text{Desired effect (heating of space)}}{\text{Net work}}$$

$$= \frac{m(h_2 - h_3)}{m(h_2 - h_1)}$$

$$\text{COP of heat pump} = \left\{\frac{(h_2 - h_3)}{(h_2 - h_1)}\right\}$$

Here, heat pump works between the low temperature surroundings or other heat sources giving heat of $Q_{absorbed}$ and the region/body acting as high temperature body where heat $Q_{rejected}$ is rejected to region/body. These heat pumps are not very attractive options when used for only direct heating but they become very attractive when used with some other application.

18.11 REFRIGERANTS

Refrigerant is the working fluid used in refrigeration/air conditioning equipments having capability of carrying heat/rejecting heat in the form of sensible heat or latent heat. Refrigerants carrying heat/rejecting heat in the form of latent heat are more effective and for this refrigerant should possess suitable properties to get transformed from liquid to gas and vice-versa. During selection of refrigerant its chemical, physical and other general properties are being looked into along with refrigeration cycle requirements and application. Refrigerants can be classified into two main categories as 'Primary refrigerants' and 'Secondary refrigerants'. The *primary refrigerants* are those refrigerants which are directly involved in

refrigeration system while *secondary refrigerants* are first cooled by primary refrigerants and then used for imparting refrigeration. The primary refrigerants are used in vapour compression systems while secondary refrigerants are liquids used for transporting low-temperature heat energy from one place to another as done by 'brine', antifreeze agents etc. Commonly used refrigerants are air, ammonia, carbon-dioxide, sulphur-dioxide, fluorinated hydrocarbons and Freons etc. All refrigerants are assigned with internationally acceptable number such as *R*–12, *R*–717 etc.

(*i*) *Primary refrigerants:* Primary refrigerants can be further classified into following categories depending upon their characteristics.

(*a*) *Halocarbon compounds:* Halocarbon group includes refrigerants containing one or more of three halogens such as chlorine, fluorine and bromine. These refrigerants are traded in market under the brand names of Freon, Genetron, Isotron and Arctron. Halocarbons, specially chlorine containing halocarbons (chloro fluoro carbons, CFCs) were commonly used from about 1940 to 1990s in most of vapour compression systems. But due to disastrous effect of chlorine in refrigerant upon the earth's protective ozone layer the efforts are being made to replace the use of CFCs by the class of refrigerants having hydrogen in place of chlorine. Such new ecofriendly refrigerants are called hydrofluoro carbons (HFCs) for example *R*–134a (CF_3 CH_2F) can replace *R*-12 (CCl_2F_2).

Table 18.1 *Halocarbon refrigerants*

Name of refrigerant	*Chemical name*	*Chemical formula*
R–11	Trichloro mono fluoro-methane	CCl_3F
R–12	Dichloro difluoro-methane	CCl_2F_2
R–13	Monochloro trifluoro-methane	$CClF_3$
R–21	Dichloro monofluoro-methane	$CHCl_2F$
R–22	Monochloro difluoro-methane	$CHClF_2$
R–30	Methylene-chloride	CH_2Cl_2
R–40	Methly chloride	CH_3Cl
R–100	Ethyl chloride	CH_3CH_2Cl
R–113	Trichloro tetrafluoro-ethane	$C_2F_3Cl_3$
R–114	Dichloro tetrafluoro-ethane	$CClF_2CClF_2$
R–152	Difluoro-ethane	$CH_3CH_3F_2$

The numbering of halocarbon group refrigerants is based on;

- first digit on right gives number of fluorine atoms.
- second digit from right is one more than the number of hydrogen atoms.
- third digit from right is one less than the number of carbon atoms. Third digit is not given when it is zero.

Some halocarbon refrigerants are as given in Table 18.1.

(*b*) *Inorganic compounds:* Refrigerants used in olden days were inorganic compounds. Some of such refrgerants are still used in different applications such as ice plants, steam refrigeration, aeroplanes and ship refrigeration etc. due to their inherent thermodynamic and physical properties. These inorganic compounds are listed in Table 18.2.

Table 18.2 *Inorganic compound refrigerants*

Name of refrigerant	*Chemical name*	*Chemical formula*
R–717	Ammonia	NH_3
R–718	Water	H_2O
R–729	Air	
R–744	Carbondioxide	CO_2
R–764	Sulfurdioxide	SO_2

(*c*) *Hydrocarbons*: Some of hydrocarbons as given in Table 18.3, are also used as refrigerants particularly in petroleum and petrochemical industry.

Table 18.3 *Hydrocarbon refrigerants*

Name of refrigerant	*Chemical name*	*Chemical formula*
R–50	Methane	CH_4
R–170	Ethane	C_2H_6
R–290	Propane	C_3H_8

(*d*) *Azeotropes:* Azeotropes are those mixture of different refrigerants which do not separate into their components by distillation or with the change in pressure and temperature. Azeotrope refrigerants behave differently from their constituents such as azeotrope evaporates and condenses as single substance having properties that are different from those of either constituents.

Examples of azeotrope are; *R*–502 is azeotrope which is mixture of 48.8%, *R*–22 and 51.2% of *R*–115. *R*–500 is azeotrope which is mixture of 73.8% *R*–12 and 26.2% of *R*–152.

(*e*) *Unsaturated organic compounds:* These are refrigerants from hydrocarbon group and have ethylene or propylene as the base. Table 18.4 gives-such refrigerants.

Table 18.4 *Unsaturated organic compounds*

Name of refrigerant	*Chemical name*	*Chemical formula*
R–1120	Trichloro ethylene	$C_2H_4Cl_3$
R–1130	Dichloro ethylene	$C_2H_4Cl_2$
R–1150	Ethylene	C_2H_4
R–1270	Propylene	C_3H_6

(*ii*) *Secondary refrigerants:* Secondary refrigerants are indirect refrigerants that carry heat from the substance being cooled to the evaporator. Secondary refrigerants undergo change in their temperature by absorbing heat and rejecting it at evaporator without any phase change. Examples of secondary refrigerants are water, brine and antifreezes etc. Antifreezing agents may be solution of water and ethylene glycol, propylene glycol or calcium chloride, with freezing temperature well below 0ºC.

Secondary refrigerants offer advantages of ease of handling and control, ease of maintaining temperature of large buildings by controlling flow of secondary refrigerants such as brine etc. Secondary refrigerants also eliminate requirement of long primary refrigerant lines.

18.12 DESIRED PROPERTIES OF REFRIGERANTS

A refrigerant being used in refrigeration should have following thermodynamic properties, physical properties and working properties.

(*i*) Boiling temperature of refrigerant should be quite low at atmospheric conditions for effective refrigeration. For refrigerants having higher boiling temperatures at atmospheric conditions the compressor is run at higher vacuum.

(*ii*) For an ideal refrigerant the freezing temperature of refrigerant should be quite low so as to prevent its freezing at evaporator temperature. Freezing point temperature should be less than evaporator temperature. For example, refrigerant *R*–22 has freezing point of – 160ºC and normally most of refrigerants have freezing point below – 30ºC.

(*iii*) Critical temperature of the ideal refrigerant should be higher than the condenser temperature for the ease of condensation.

(*iv*) Refrigerant should have large latent heat at evaporator temperature as this shall increase the refrigerating capacity per kg of refrigerant.

(*v*) Refrigerant should have small specific volume at inlet to compressor as this reduces compressor size for same refrigeration capacity.

(*vi*) Specific heat of refrigerant in liquid form should be small and it should be large for refrigerant in vapour form, since these increase the refrigerating capacity per kg of refrigerant.

(*vii*) Thermal conductivity of refrigerant should be high.

(*viii*) Viscosity of refrigerants should be small for the ease of better heat transfer and small pumping work requirement.

(*ix*) Refrigerant should be chemically inert and non toxic.

(*x*) Refrigerant should be non-flammable, non explosive and do not have any harmful effect upon coming in contact with material stored in refrigeration space.

(*xi*) Refrigerant may have pleasant distinct odour so as to know about its leakage.

(*xii*) Refrigerant should be readily available at lesser price.

Choice of refrigerant: Selection of suitable refrigerant depends upon the number of parameters as there can not be single refrigerant well suited for all kinds of refrigeration systems. Thermodynamic properties, physical properties and other properties are considered in reference to the refrigeration system during choice of suitable refrigerant. Apart from these properties, due consideration should be given to the working pressure and temperature range and pressure ratio, space restrictions, corrosiveness and inflammability, oil miscibility etc. before selecting refrigerant with suitable compressor and other equipments of refrigeration system.

18.13 PSYCHROMETRY

Psychrometry or Psychrometrics refers to the study of system involving dry air and water. Properties of mixture of air and water are called psychrometric properties. Psychrometry becomes very important from air conditioning point of view as in air conditioning the human comfort conditions defined in terms of temperature, humidity and air circulation/ventilation are being considered. Different psychrometric properties, charts and processes generally used are defined ahead.

(*i*) ***Dry air:*** Atmospheric air having 79% nitrogen and 21% oxygen by volume is considered dry air. Its, molecular weight is taken as 29.

(*ii*) ***Moist air:*** Moist air is the mixture of dry air and water vapour in which dry air is treated as if it were pure component. Quantity of water vapour present in the mixture depends upon the temperature of air and it may vary from zero in dry air to the maximum quantity when mixture is

saturated of water vapour (called saturation capacity of air). Moist air is assumed to behave as ideal gas for the purpose of analysis. Mixture pressure is the sum of partial pressures of dry air and water vapour. When the partial pressure of water vapour corresponds to the saturation pressure of water at mixture temperature then mixture is said to be saturated. Saturated air is the mixture of dry air and saturated water vapour. When the temperature of mixture of air and vapour is above the saturation temperature of water vapour then the vapour is called superheated vapour.

(*iii*) ***Humidity ratio or specific humidity:*** This is given by the ratio of the mass of water vapour to the mass of dry air. It can also be defined as mass of water vapour present in per kg of dry air.

It is given in terms of grams per kg of dry air. Mathematically, humidity ratio or specific humidity,

$$\omega = \frac{m_v}{m_a}$$

where m_v and m_a are mass of water vapour and dry air respectively.

Humidity ratio can also be given in terms of partial pressures and molecular weights.

$$m_v = \frac{M_v \cdot p_v V}{\bar{R}T}$$

$$m_a = \frac{M_a \cdot p_a V}{\bar{R}T}$$

where p_v and p_a are partial pressures of water vapour and dry air respectively, M_v and M_a are molecular weights of water vapour and dry air respectively. For total pressure of mixture being p, the Dalton's law of partial pressure says,

$$p = p_a + p_v$$

or,
$$p_a = p - p_v$$

Hence, upon substitution,

$$\omega = \frac{M_v \cdot p_v}{M_a \cdot p_a} = \frac{M_v \cdot p_v}{M_a \cdot (p - p_v)}$$

or,
$$\boxed{\omega = 0.622\left(\frac{p_v}{p - p_v}\right)}$$

Humidity ratio is measured using an instrument called 'hygrometer' in which moist air sample is processed till the moisture present in it gets absorbed and moisture absorbed can be quantified.

(*iv*) ***Absolute humidity:*** Absolute humidity refers to the weight of water vapour present in unit volume of air.

(*v*) ***Relative humidity:*** Relative humidity gives an account of moisture content in an actual mixture as compared to the mixture in saturated state at same temperature and pressure. It can be given by the ratio of actual mass of water vapour in given volume to the mass of water vapour if the air is saturated at the same temperature and pressure.

Mathematically, relative humidity (ϕ) can be given as:

$$\phi = \frac{(\text{Actual mass of water vapour in given volume of mixture})}{\begin{pmatrix}\text{Maximum mass of water vapour in same volume of mixture being staurated at} \\ \text{same temperature and pressure}\end{pmatrix}}$$

$\phi = \dfrac{m_v}{m_{v,\,\text{sat}}}$, where m_v and $m_{v,\,\text{sat}}$ are the actual and maximum mass of water vapour in same volume of mixture as defined earlier.

$$\phi = \frac{\left(\dfrac{M_v \cdot p_v \cdot V}{\bar{R}T}\right)}{\left(\dfrac{M_v \cdot p_{v,\,\text{sat}} \cdot V}{\bar{R}T}\right)}$$

or,

$\phi = \dfrac{p_v}{p_{v,\,\text{sat}}}$, where p_v and $p_{v,\,\text{sat}}$ are the partial pressures of 'actual mass of water vapour' and 'maximum mass of water vapour in same volume of mixture as defined earlier'.

(*vi*) ***Enthalpy of moist air or Mixture enthalpy:*** The enthalpy of moist air can be estimated by addition of the enthalpy contribution of each constituent component i.e water vapour and dry air at the state of mixture. Thus, enthalpy of mixture (moist air) of dry air and water can be given as,

$$H = H_a + H_v = m_a\, h_a + m_v\, h_{v,} \text{ or, } \frac{H}{m_a} = h_a + \left(\frac{m_v}{m_a}\right) h_v$$

(Enthalpy of moist air per unit mass of dry air) = (Enthalpy of unit mass of dry air)
+ (Enthalpy of water vapour associated with unit mass of dry air)

Here H and h refer to total enthalpy and specific enthalpy values respectively. Subscripts a and v refer to dry air and water vapour respectively.

(*vii*) ***Dry bulb temperature*:** Dry bulb temperature refers to the temperature of air measured with ordinary thermometer having its bulb open.

(*viii*) ***Wet bulb temperature:*** Wet bulb temperature refers to the temperature measurement with the thermometer having its bulb covered with wet cloth and exposed to air stream whose temperature is being measured. For getting wet bulb temperature thermometer bulb is covered with water wet cloth/wick moistened with water and then temperature is measured. Difference between dry bulb temperature and wet bulb temperature is called wet bulb depression. Wet bulb depression is zero in case of saturated air as the dry bulb temperature and wet bulb temperatures are equal.

(*ix*) ***Dew point temperature*:** It is the temperature at which condensation of water vapour present in moist air just begins. It could be understood from different examples that if the moist air temperature is reduced then water vapour gets condensed. Examples for such condensation is formation of dew on grass, condensation of water vapour on exterior surface of steel tumbler having cold water inside, condensation of water vapour on glass window-panes during winter season etc.
Dew point temperature shall be equal to the saturation temperature corresponding to partial pressure of water vapour in moist air. Difference between dew point temperature and dry bulb temperature is called dew point depression.

(*x*) ***Humid specific volume:*** It refers to the volume of mixture per kg of dry air in mixture and is given by m^3/kg of dry air.

(*xi*) ***Psychrometric charts:*** Psychrometric chart gives the graphical representation of different important properties of moist air. These charts are readily available for different mixture pressure. Fig. 18.14 shows psychrometric chart. Abscissa (x-axis) of chart gives the dry bulb temperature (°C) and the ordinate (y-axis) has humidity ratio (ω) in kg or gram of water vapour per kg of dry air.

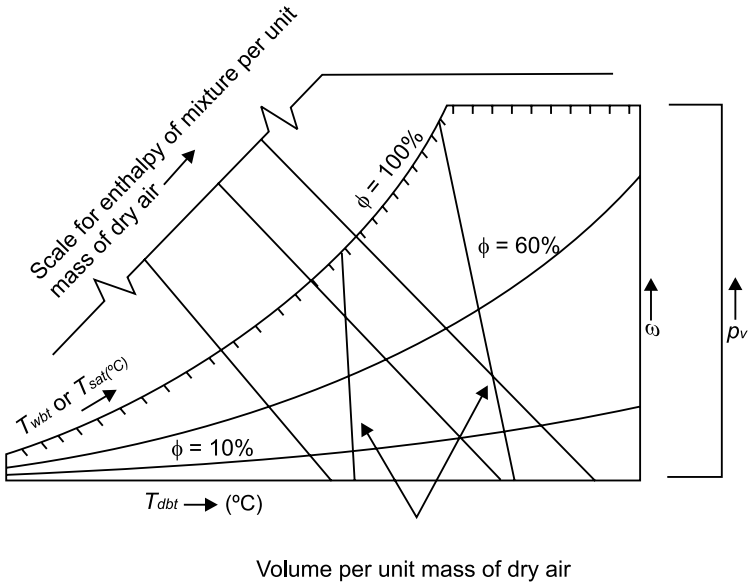
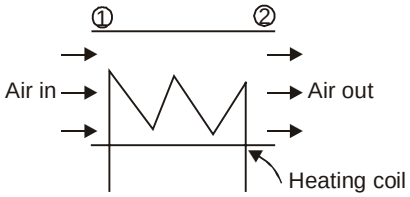


Fig. 18.14 *Psychrometric chart*

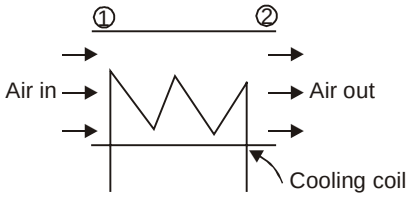
Since humidity ratio can be directly related to partial pressure of water vapour so vapour pressure (p_v) can also be shown on ordinate. Constant relative humidity (ϕ) curves are also shown on psychrometric chart for different ϕ values such as ϕ = 10%, ..., 60%, ..., 100%. Mixture enthalpy per unit mass of dry air (kJ/kg of dry air) is also available on psychrometric chart. Dew point temperature for the moist air can be known by following the line of constant ω (or constant p_v) up to saturation line, ϕ = 100% as the dew point refers to the state where mixture becomes saturated when cooled at constant vapour pressure. Wet bulb temperature is also available on psychrometric chart where constant wet bulb temperature is also available on psychrometric chart where constant wet bulb temperature lines run from upper left to the lower right of chart. These constant wet bulb temperature lines approximate to the lines of constant mixture enthalpy per unit mass of dry air. Pscychrometric chart also has lines representing volume per unit mass of dry air (m^3/kg). These specific volume lines can be approximated as state giving volume of dry air or water vapour per unit mass of dry air since each component of mixture i.e. air and water vapour occupy the same volume.

(*xii*) ***Psychrometric processes*:** Different psychrometric processes commonly encountered in air conditioning are as follows.

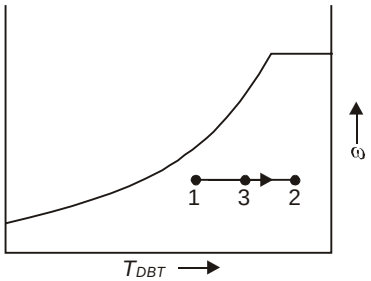
(*a*) *Sensible heating or sensible cooling:* Sensible heating or cooling refers to the heating or cooling without phase change i.e. heating or cooling of air without increase or decrease of moisture content. For sensible heating the air is passed over heating coils (electrical resistance type or steam type) while for sensible cooling the air is passed over cooling coils (such as evaporator coil of refrigeration cycle), as shown in Fig. 18.15.



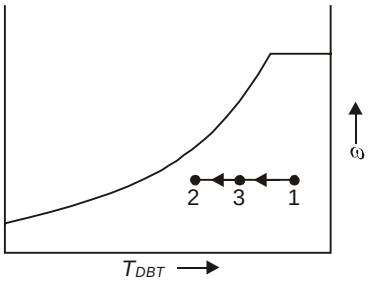
(*a*) Sensible heating (without by-pass)



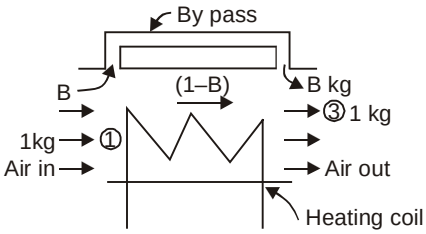
(*b*) Sensible cooling (without by-pass)



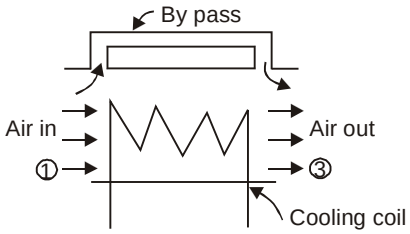
(*c*) Sensible heating on psychrometric chart



(*d*) Sensible cooling on psychrometric chart



(*e*) Sensible heating with by-pass



(*f*) Sensible cooling with by-pass

Fig. 18.15 *Sensible heating and sensible cooling*

This heating and cooling may be for complete amount of air being passed over the coil as shown in Fig. 18.15 *a* and *b*, or the portion of air flowing can be by passed through by-pass passage thus, only a fraction is passed over coil. Later arrangement is called sensible heating or sensible cooling with by pass as shown in Fig. 18.15 *e* and *f*. Sensible heating and sensible cooling without by pass is shown on psychrometric charts Fig. 18.15 *c* and *d*, respectively between states 1 and 2. Thus, for inlet air temperature being T_1, the heating/cooling causes change in temperature up to T_2. When some amount of air is by passed then the temperature of air coming out shall be different from T_2. In case of sensible heating with by pass, temperature of air coming out is less than temperature of air wihout by pass i.e. $T_3 < T_2$. Similarly in case of sensible cooling with by pass temperature of air leaving is more than temperature of air without by pass i.e. $T_3 > T_2$.

Heat added can be quantified by the amount of heat gained by dry air and water vapour between dry bulb temperature T_1 and T_2.

$$Q_{\text{sensible heating}} = C_{pa}\,(T_2 - T_1) + \omega \cdot C_{pv}\,(T_2 > T_1)$$
$$= (Cp_a + \omega\, Cp_v)\,(T_2 - T_1)$$
$$Q_{\text{sensible heating}} = C_{pm} \cdot (T_2 - T_1),\ \text{Hence } C_{pm} \text{ is mean specific heat.}$$

In case of sensible heating with by pass, if the amount of air by passed is B kg per kg of air then heat balance can be given as below. Here $(1 - B)$ kg of air passes over the coil and later on mixes with by passed air at exit.

$B \cdot (C_{pa} + \omega\, C_{pv})\, T_1 + (1 - B)\, (C_{pa} + \omega\, C_{pv})\, T_2 = 1 \times (C_{pa} + \omega\, C_{pv}) \cdot T_3$

or,

$$\text{By pass factor, } B = \frac{T_2 - T_3}{T_2 - T_1} = \frac{\text{length}_{2\text{-}3} \text{ on psychrometric chart}}{\text{length}_{2\text{-}1} \text{ on psychrometric chart}}$$

This amount of air by passed per kg of air flowing is also called as By-pass factor of the coil. Factor $(1 - B)$ is called contact factor.

(*b*) *Cooling with dehumidification*: This process refers to the reducing air temperature along with removal of water vapour from air. Removal of moisture content can be done by reducing the temperature of air below dew point temperature so as to allow condensation of water vapour. Arrangement for cooling with humidification is shown in Fig. 18.16 where moist air entering is first passed over cooling coil having cool refrigerant inside. Water vapour present in moist air gets condensed and saturated moist air leaves dehumidification section at state 2.

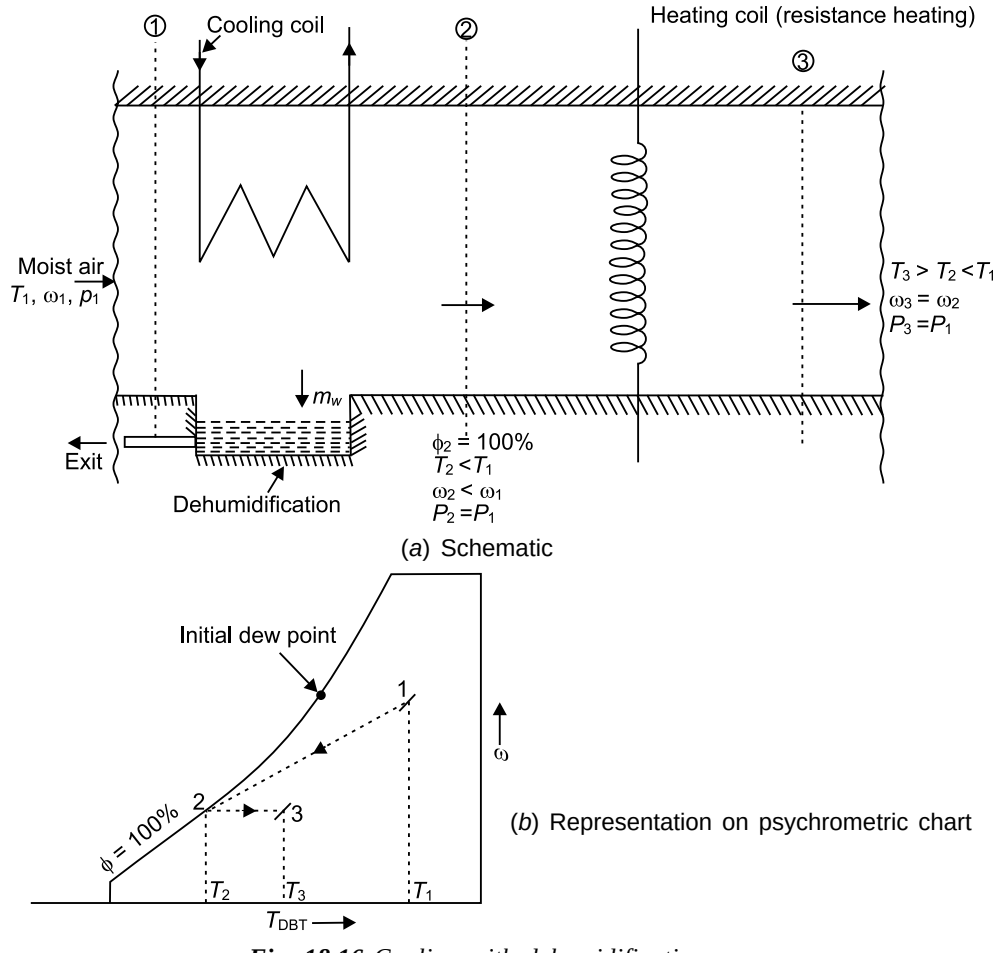


Fig. 18.16 *Cooling with dehumidification*

Condensate also gets cooled up to temperature T_2 before leaving dehumidification section. Moist air at state (2) may be excessively cooled so that it can be subsequently heated by passing it over heating coil. Thus, at state (3) the available air shall be at least temperature and in dehumidified form. States (1), (2) and (3) are also shown on psychrometric chart.

Mass balance shows,

Dry air at section (1) and (2) $m_{a1} = m_{a2} = m_a$

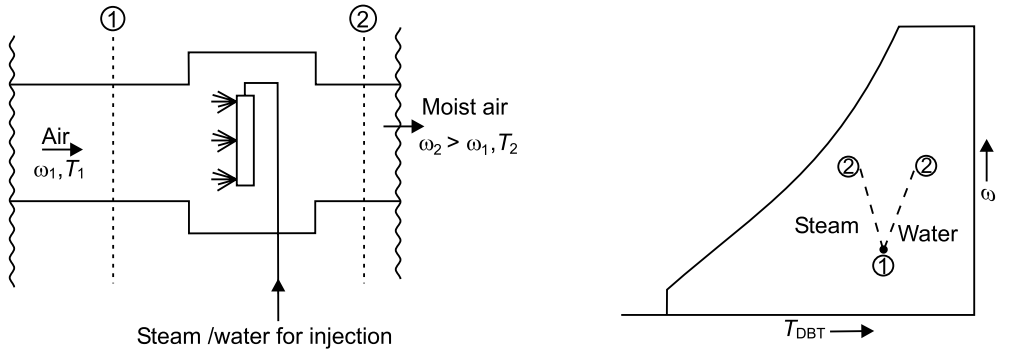
Vapour content at section (1) and (2); $m_{v1} = m_w + m_{v2}$, or $m_w = m_{v1} - m_{v2}$,

It can also be given: $m_{v1} = \omega_1 \cdot m_a$; $m_{v2} = \omega_2 \cdot m_a$

Mass of water condensed per unit mass of dry air; $\omega_1 - \omega_2 = \dfrac{m_w}{m_a}$

Between section (2) and (3), $\omega_2 = \omega_3$.

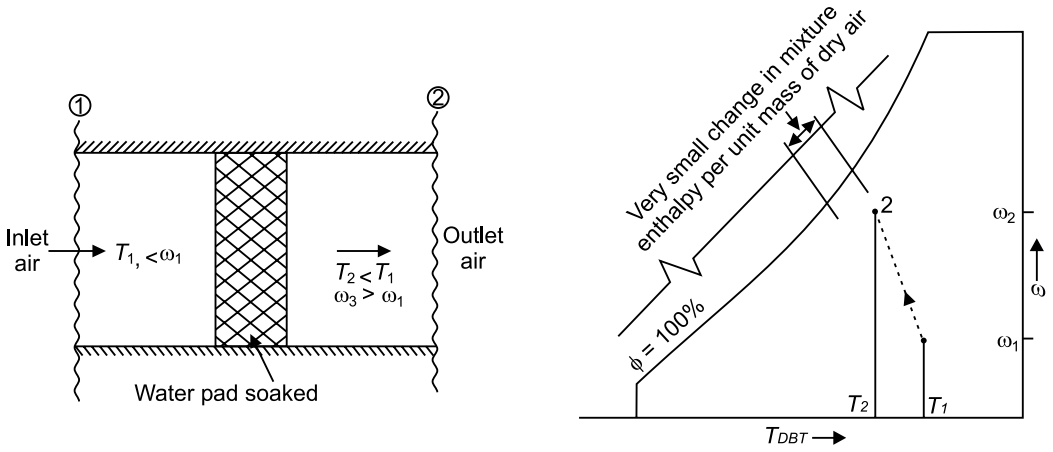
(*c*) *Humidification*: This is the process of adding moisture to the air. Humidification may be required during air conditioning when air being circulated in occupied space may have little moisture in it. Available means for increasing humidity are to inject steam or spray liquid water into air. Normally the steam being injected shall be at temperature more than that of air and humidification shall be accompanied by increase in humidity ratio and dry-bulb temperature. When liquid water is sprayed then the moist air shall leave humidification section with lower temperature than at inlet and increased humidity ratio. Figure 18.17 shows schematic for humidification process along with representation on psychrometric chart for humidification using steam and liquid water spray both.



(*a*) Schematic for humidifier (*b*) Psychrometric chart representation for humidification

Fig. 18.17 *Humidification*

(*d*) *Evaporator cooling:* This type of cooling is needed in hot and dry climates. Evaporative cooling has arrangement for spray of spraying liquid water into air or passing air through a pad soaked with water. Due to less humidity of air it shall evaporate some amount of water in its contact and thus reduce its temperature because of heat extracted for evaporation of water. This air leaving evaporative cooler shall have temperature less than inlet air temperature and also due to moisture being picked up the humidity ratio gets increased. Arrangement for such type of cooling and its representation on psychrometric chart is shown in Fig. 18.18.



(*a*) Evaporative cooler (*b*) Representation of evaporative cooling on psychrometric chart

Fig. 18.18 *Evaporative cooling*

(*e*) *Adiabatic mixing of moist air stream:* Adiabatic mixing of moist air streams is the one in which there is no heat interaction during mixing.

Obviously there is no work interaction during mixing. This process of mixing of moist air streams is used very frequently in air conditioning systems. Figure 18.19 shows adiabatic mixing of two streams entering at (1) and (2) and leaving at (3).

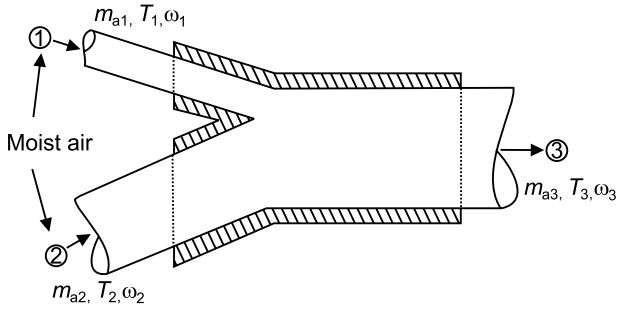


Fig. 18.19 *Adiabatic mixing*

If dry air entering be m_{a1} & m_{a2} and water vapour flow rate be m_{v1} & m_{v2} with streams (1) and (2) respectively at steady state then mass balances and energy balances can be given as under:

$$\text{For dry air, } m_{a1} + m_{a2} = m_{a3}$$

$$\text{For water vapour, } m_{v1} + m_{v2} = m_{v3}$$

Since $m_{v1} = \omega_1 \cdot m_{a1}$, $m_{v2} = \omega_2 \cdot m_{a2}$, $m_{v3} = \omega_3 \cdot m_{a3}$, then substitution yields;

$$\omega_1 \cdot m_{a1} + \omega_2 \cdot m_{a2} = \omega_3 \cdot m_{a3}$$

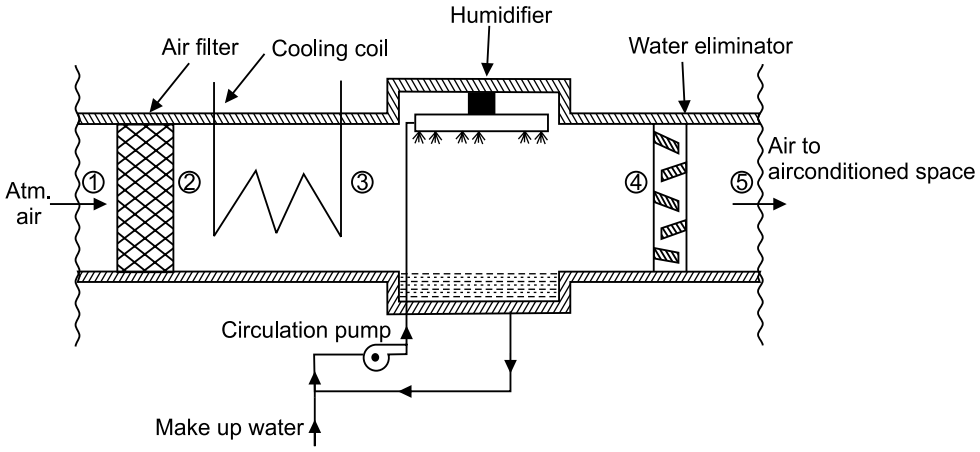
or,
$$\omega_1 \cdot m_{a1} + \omega_2 \cdot m_{a2} = \omega_3 (m_{a1} + m_{a2})$$

Energy balance yields; following as $Q = 0$, $W = 0$.

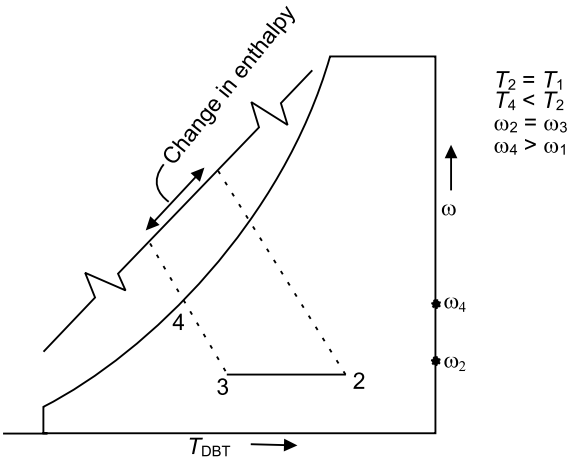
$$m_{a1} \cdot (h_{a1} + \omega_1 h_{g1}) + m_{a2} \cdot (h_{a2} + \omega_2 h_{g2}) = m_{a3} \cdot (h_{a3} + \omega_3 h_{g3})$$

18.14 AIR CONDITIONING SYSTEMS

Air conditioning systems require basic arrangement for getting refrigeration effect through cooling coil followed by subsequent humidification/dehumidification and heating etc. in order to provide air conditioned space with air at desired temperature and humidity. Air conditioning systems require different arrangements depending upon the atmospheric air condition and comfort condition requirement. Such as summer air conditioning systems and winter air conditioning systems are different. These systems have different arrangement if outdoor conditions are hot and humid, hot and dry etc. Summer air conditioning system for hot and dry outdoor condition is given in Fig. 18.20. Here the comfort conditions may require delivery of air to air-conditioned space at about 25ºC DBT and 60% relative humidity where the outdoor conditions may be up to 40–44º C DBT and 20% relative humidity in Indian conditions. Generic arrangement has air blower which blows air across the air filter between (1) and (2). Air coming out from filter passes over cooling coils and is subsequently sent for humidification between states (3) and (4). Large size water particles carried by air are retained by water eliminator. Air finally coming out at state (5) is sent to air conditioned space. Here psychrometric representation is made considering negligible change in humidity in water eliminator.



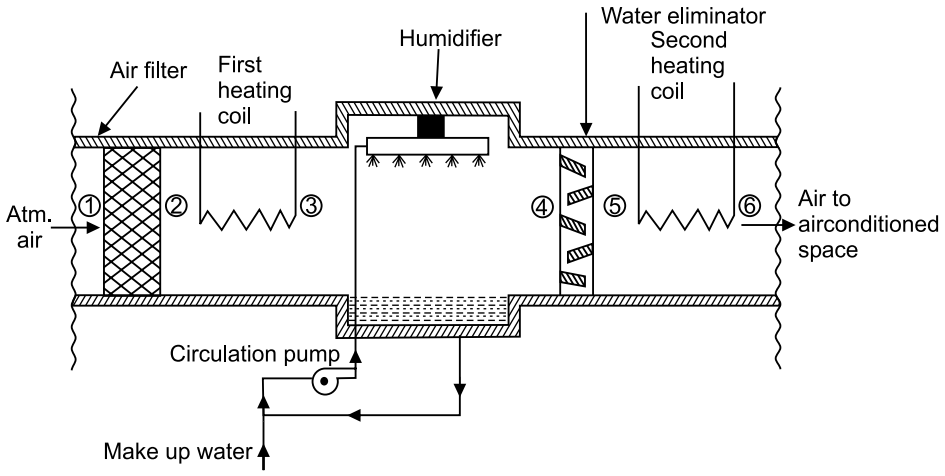
(*a*) Schematic summer airconditioning



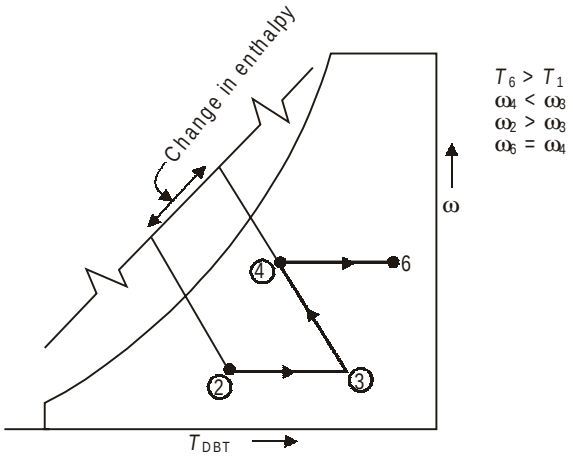
(*b*) Representation on psychrometric chart

Fig. 18.20 *Summer air conditioning*

Winter air conditioning has slightly different arrangement as compared to summer air conditioning. Figure 18.21 shows the schematic for winter air-conditioning along with representation on psychrometric chart.



(*a*) Schematic for winter airconditioning



(*b*) Representation on psychrometric chart

Fig. 18.21 *Winter air conditioning*

Let us assume air from atmosphere to be available at 10ºC and 80% relative humidity while comfort conditions are 25ºC DBT and 60% relative humidity.

Here atmospheric air is blown into air filter and it comes out at (2). Air is passed across heating coil for being heated. Hot air at (3) passes through humidifier section where its humidity increases as shown on psychrometric chart by state (4). It is further passed through water eliminator to retain large sized water particles. Air may again be passed through heating coil section so as to compensate for temperature reduction in humidifier section and attain desired temperature.

18.15 COMPARISON OF VARIOUS REFRIGERATION METHODS

Merits and demerits of Vapour compression system over Air-refrigeration system.

Vapour compression system when compared with air refrigeration system possess following merits and demerits.

Merits

- Thermodynamic cycle involved in vapour compression system is very close to Carnot cycle hence its COP is quite high of the order of 3–4 while for air refrigeration COP is generally less than unity.
- Refrigerant required is smaller in vapour compression system because the heat is carried away by latent heat of vapour and so the amount of liquid refrigerant circulated per ton is less. Also size of evaporator is small.
- Operating cost of vapour compression system is very less as compared to air refrigeration system on ground.
- Refrigerated space temperature can be easily regulated by controlling the throttle valve.

Demerits

- Initial investment is too large as compared to air refrigeration system.
- There are chances of leakage of refrigerant as large pressure differential exists throughout the cycle.
- Air leakage into refrigerant offers reduced performance of vapour compression system and so it should be prevented.

Merits and demerits of Vapour absorption system over Vapour compression system.

The merits and demerits of vapour absorption system over vapour compression system are listed as under:

Merits

- Vapour absorption system has no moving component throughout cycle, so it works quietly and has reduced maintenance requirement.
- Lesser work input is required for pump as compared to compression in vapour compression system.
- Vapour absorption system may be run with some installation rejecting waste heat, which may be used to run generator, thus energy conservation.
- Vapour absorption system is well suited for large refrigeration capacity applications such as even above 1000 tons, which is difficult in vapour compression systems.
- Vapour absorption systems offer smaller variation with load whereas COP of vapour compression system varies largely with load. At smaller loads the COP of absorption system is more than that of vapour absorption system.

Demerits

- Size of absorption system is large and is bulky to handle.
- If heating source (such as electric heating) is costly then the vapour absorption system becomes costly than vapour compression system.
- At very high loads the COP of vapour compression system is more than that of vapour absorption system.

EXAMPLES

1. *A refrigerator operating on reversed Carnot cycle extracts 500 kJ/min heat from a refrigerated space being maintained at –16ºC and rejects heat to the atmosphere at 27ºC. Determine the work input required to run the refrigerator.*

Solution:

For refrigerator working on reversed Carnot cycle.

$$\frac{Q_1}{(-16+273)} = \frac{Q_2}{(27+273)}$$

$$\frac{Q_1}{Q_2} = \frac{257}{300}$$

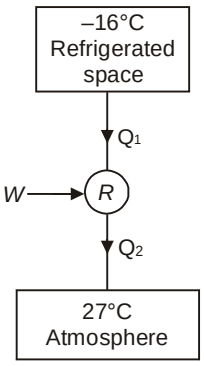


Fig. 18.22

Also it is given that $Q_1 = 500$ kJ/min

$Q_2 = 583.66$ kJ/min

Work input required, $W = Q_2 - Q_1$

$= 83.66$ kJ/min

Work input = **83.66 kJ/min** **Ans.**

2. *A refrigerator operating between –5ºC and 27°C has the capacity of 800 tons. Determine the mass rate of ice formation considering water being supplied at atmospheric temperature of 27ºC. Also find the minimum hp required. Consider the refrigerator to work upon reversed Carnot cycle. Latent heat for ice formation from water may be taken as 335 kJ/kg.*

Solution:

Refrigeration capacity = 800 × 3.5 = 2800 kJ/s = Heat extraction rate

Let the ice formation rate be '*m*' kg/s.

Heat to be removed from per kg of water for its transformation into ice.

$= 4.18 \times (27 - 0) + 335$ kJ/kg

$= 447.86$ kJ/kg

$$\text{Ice formation rate} = \frac{\text{Refrigeration capacity}}{\text{Heat removed for getting per kg of ice}}$$

$$= \frac{2800}{447.86} = 6.25 \text{ kg/s}$$

Mass rate of ice formation = **6.25 kg/s** **Ans.**

$$\text{COP of refrigerator,} = \frac{T_1}{T_2 - T_1} = \frac{\text{Refrigeration capacity}}{\text{Work done}}$$

Here $T_1 = 273 - 5 = 268$ K, $T_2 = 27 + 273 = 300$ K

$$\text{COP} = \frac{268}{300 - 268} = \frac{2800}{W}$$

$$W = 334.33 \text{ kJ/s}$$

$$= \frac{334.33}{0.7457} \text{ hp} \qquad \{\text{as 1 hp} = 0.7457 \text{ kW}\}$$

$$= 448.34 \text{ hp}$$

HP required = **448.34 hp** **Ans.**

3. *3 hp per ton of refrigeration is required to maintain temperature of –27ºC in refrigerator working on Carnot cycle. Determine COP of cycle and temperature of surroundings to which heat is rejected by refrigerator.*

Solution:

$$\text{COP} = \frac{T_1}{T_2 - T_1} = \frac{246}{T_2 - 246}$$

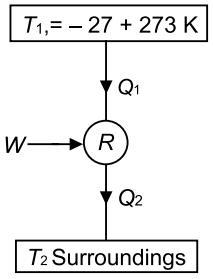


Fig. 18.23

Also,
$$\text{COP} = \frac{\text{Refrigeration effect}}{\text{Work input}}$$

$$= \frac{1 \times 3.5}{3 \times 0.7457}$$

$$\text{COP} = 1.56$$

COP = **1.56** **Ans.**

Equating COP,

$$1.56 = \frac{246}{T_2 \times 246}$$

$$T_2 = 403.69 \text{ K}$$

Temperature of surroundings = **403.69 K** **Ans.**

4. *An air refrigeration cycle has pressure ratio of 8, temperature of air at beginning of compression is –30ºC and air is cooled up to 27ºC after compression. Compression and expansion processes have isentropic efficiency of 85% and 90% respectively. Determine the refrigeration capacity and coefficient of performance. Consider* C_p *= 1.005 kJ/kg ·K,* γ *= 1.4 and air flow rate of 1 kg/s.*

Solution:

Given:

$T_1 = –30$ºC, $T_3 = 27$ºC or 300 K

During process 1–2′,

$$\frac{p_2}{p_1} = \left(\frac{T_{2'}}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$T_1 = –30 + 273 = 240 \text{ K}$$

$$\frac{T_{2'}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{2'} = 240 \times (8)^{\frac{(1.4-1)}{1.4}}$$

Theoretical temperature after compression $T_{2'} = 434.75$ K

For compression process,

$$\eta_{\text{isen,compr.}} = 0.85 = \frac{T_{2'} - T_1}{T_2 - T_1}$$

$$(T_2 - 240) = \frac{(434.75 - 240)}{0.85}$$

$$T_2 = 469.12 \text{ K}$$

For expansion process, 3–4', $\dfrac{T_{4'}}{T_3} = \left(\dfrac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$

$$T_{4'} = 300 \times \left(\frac{1}{8}\right)^{\frac{1.4-1}{1.4}}$$

$$T_{4'} = 165.61 \text{ K}$$

$$\eta_{\text{isen, expansion}} = 0.90 = \frac{T_3 - T_4}{T_3 - T_{4'}} = \frac{300 - T_4}{300 - 165.61}$$

or, $T_4 = 179.05$ K

Work during compression, $W_c = 1 \times C_p \times (T_2 - T_1)$

$= 1 \times 1.005\ (469.12 - 240)$

$= 230.27$ kJ/s

Work during expansion $W_T = 1 \times C_p \times (T_3 - T_4)$

$W_T = 1 \times 1.005 \times (300 - 179.05)$

$= 121.55$ kJ/s

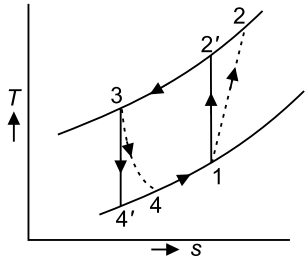


Fig. 18.24

Refrigeration effect is realized during process, 4–1. So refrigeration effect shall be,

$$Q_{\text{ref}} = 1 \times C_p \times (T_1 - T_4)$$

$$= 1 \times 1.005\ (240 - 179.05)$$

$$Q_{\text{ref}} = 61.25 \text{ kJ/s or } 17.5 \text{ ton}$$

Net work required = $W_C - W_T = 230.27 - 121.55 = 108.72$ kJ/s

$$\text{COP} = \frac{Q_{\text{ref}}}{W_C - W_T} = \frac{61.25}{108.72} = 0.56$$

Refrigeration capacity = **17.5** ton or **61.25 kJ/s**
COP = **0.56**

Ans.

5. *A refrigeration system has air leaving refrigerated space at 7°C and 1 bar and subsequently air is compressed isentropically to 5 bar. After being compressed air is cooled up to 27°C at this pressure and then expanded isentropically up to 1 bar and discharged into refrigerated space. Determine COP of system.*

Solution:

Given

T_1 = 7ºC or 280 K

T_3 = 27ºC or 300 K

P_1 = 1 bar, P_2 = 5 bar

For isentropic compression process:

$$\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1}$$

$$T_2 = 280 \times (5)^{\frac{(1.4-1)}{1.4}}$$

$$T_2 = 443.47 \text{ K}$$

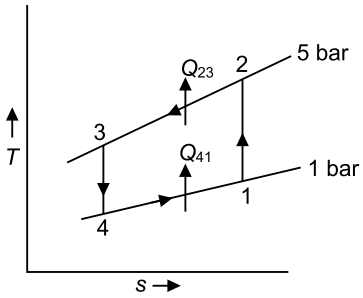


Fig. 18.25

For isentropic expansion process:

$$\left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{T_3}{T_4}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\left(\frac{T_3}{T_4}\right) = (5)^{\frac{(1.4-1)}{1.4}}$$

$$T_4 = 189.42 \text{ K}$$

Heat rejected during process 2–3, $Q_{23} = m \times C_p \times (T_2 - T_3)$

$= m \times 1.005 \times (443.47 - 300)$

$Q_{23} = 144.19$ m

Refrigeration process, heat picked during process 4–1, $Q_{41} = m \times C_p \times (T_1 - T_4)$

$= m \times 1.005 \times (280 - 189.42)$

$Q_{41} = 91.03$ m

Net work $= Q_{23} - Q_{41}$

$= (144.19 - 91.03)$ m

$W = 53.16$ m kJ

$$\text{COP} = \frac{\text{Refrigeration effect}}{\text{Net work}} = \frac{91.03\text{m}}{53.16\text{m}} = 1.71$$

COP = **1.71** **Ans.**

6. *In a refrigerator working on Bell Coleman cycle the air enters compressor at 1 bar and –10ºC and gets compressed up to 5.5 bar. Compressed air is cooled to 27ºC at same pressure before being sent to expander for expansion up to 1 bar and then passes through refrigerated space. Determine refrigeration capacity, hp required to run compressor and COP of system if air flow rate is 0.8 kg/s.*

Solution:

Given:

T_1 = –10ºC or 263 K, P_1 = 1 bar, T_3 = 27ºC or 300 K, P_2 = 5.5 bar, m = 0.8 kg/s.

For process 1–2,

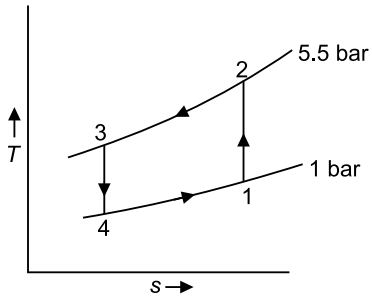


Fig. 18.26

$$\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1}$$

$$T_2 = 263 \times (5.5)^{\frac{(1.4-1)}{1.4}}$$

$$T_2 = 428.04 \text{ K}$$

For process 3–4,

$$\left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_3}{T_4}$$

$$T_4 = T_3 \times \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = T_3 \times \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_4 = 184.33 \text{ K}$$

$$\text{Refrigeration capacity} = mC_p (T_1 - T_4)$$

$$= 0.8 \times 1.005 (263 - 184.33)$$

$$= 63.25 \text{ kJ/s or } 18.07 \text{ ton}$$

Refrigeration capacity = **18.07 ton** **Ans.**

$$\text{Work required to run the compressor} = \frac{m \times n}{n-1} (p_2 v_2 - p_1 v_1)$$

$$= \frac{m \times n}{n-1} R(T_2 - T_1)$$

$$= \frac{0.8 \times 1.4}{(0.4)} \times 0.287 (428.04 - 263)$$

$$= 132.63 \text{ kJ/s}$$

or

$$\text{HP required to run compressor} = \frac{132.63}{0.7457} = 177.86 \text{ hp}$$

HP required to run compressor = **177.86 hp** **Ans.**

$$\text{Net work input} = mC_p \{(T_2 - T_3) - (T_1 - T_4)\}$$

$$= 0.8 \times 1.005 \{(428.04 - 300) - (263 - 184.33)\}$$

$$W = 39.69 \text{ kJ/s}$$

$$\text{COP} = \frac{\text{Refrigeration capacity}}{\text{Work}} = \frac{63.25}{39.69} = 1.59$$

COP = **1.59** **Ans.**

7. *Air refrigeration system for an aircraft has two cooling turbines, one intercooler and a fan. First cooling turbine is supplied with ram air at 1.2 bar, 15ºC and delivers it after expansion to intercooler at 0.9 bar for cooling the air coming from compressor at 4 bar. The cooling air from intercooler is sucked by fan and discharged to atmosphere and the cooled air from intercooler is expanded upto 1 bar in second turbine and discharged into passenger cabin. The air is exhausted from cabin at 25ºC and 10 ton of refrigeration is required in cabin. Index of compression and expansion are 1.45 and 1.30 respectively. Determine the air mass flow rate in cabin, ram air mass flow rate, and COP of system if compressed air is cooled to 50ºC in intercooler and temperature rise in intercooler for ram air is limited to 30ºC.*

Solution:

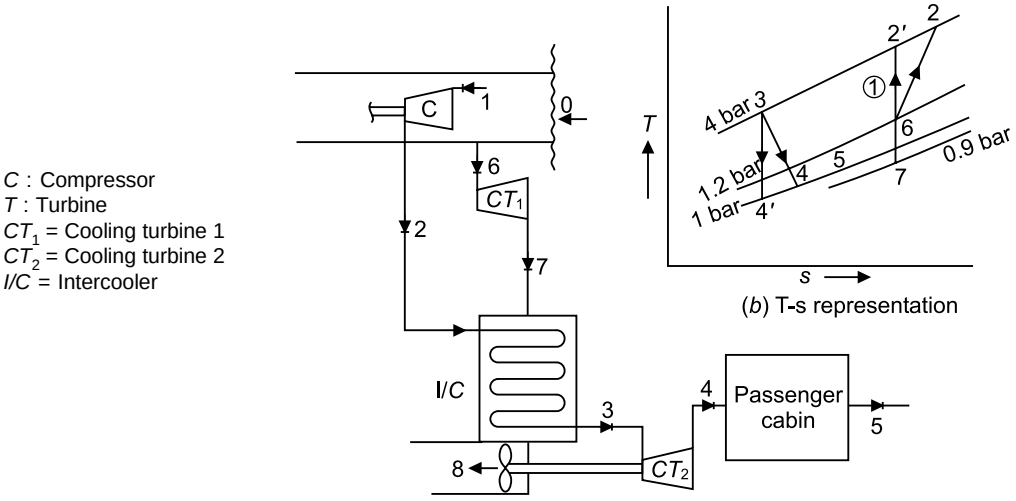


Fig. 18.27 *Schematic air refrigeration cycle*

Given:

$P_1 = P_6 = 1.2$ bar, $T_1 = T_6 = 15$ºC = 288 K, $P_7 = 0.9$ bar

$P_3 = P_2 = 4$ bar, $P_4 = 1$ bar, $T_5 = 25$ºC, $T_3 =$ 50ºC or 323 K, $T_8 = 30$ºC

For process 1–2, $n = 1.45$;

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

$$T_2 = 288 \left(\frac{4}{1.2}\right)^{\frac{1.45-1}{1.45}}$$

$$T_2 = 418.47 \text{ K}$$

For process 3–4, $n' = 1.3$,

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\left(\frac{n'-1}{n'}\right)}$$

$$T_4 = 323 \left(\frac{1}{4}\right)^{\frac{1.3-1}{1.3}}$$

$$T_4 = 234.57 \text{ K}$$

Refrigeration effect in passenger cabin with m kg/s mass flow rate of air.

$$10 \times 3.5 = m \times C_P (T_5 - T_4)$$

$$10 \times 3.5 = m \times 1.005 (298 - 234.57)$$

$$m = 0.55 \text{ kg/s}$$

Air mass flow rate in cabin = **0.55 kg/s** **Ans.**

Let the mass flow rate of air through intercooler be m_1 kg/s then the energy balance upon intercooler yields,

$$m_1 \times C_P \times (T_8 - T_7) = m \times C_P (T_2 - T_3)$$

During process 6 – 7, $\frac{T_7}{T_6} = \left(\frac{P_7}{P_6}\right)^{\frac{1.3-1}{1.3}}$

$$T_7 = 288 \left(\frac{0.9}{1.2}\right)^{\frac{1.3-1}{1.3}} = 269.5 \text{ K}$$

Substituting T_2, T_3, T_7, T_8 and m in energy balance on intercooler,

$$m_1 = 1.005 \times (303 - 269.5) = 0.55 \times 1.005 \times (418.47 - 323)$$

$$m_1 = 1.57 \text{ kg/s}$$

Total ram air mass flow rate $= m + m_1 = 0.55 + 1.57$

$= 2.12$ kg/s

Ram air mass flow rate = **2.12 kg/s** **Ans.**

Work input to compressor $= m \times C_P (T_2 - T_1)$

$= 0.55 \times 1.005 (418.47 - 288)$

$= 72.12$ kJ/s

$$\text{COP} = \frac{\text{Refrigeration effect}}{\text{Work input}}$$

$$= \frac{10 \times 3.5}{72.12}$$

$$\text{COP} = 0.485$$

COP = **0.485** **Ans.**

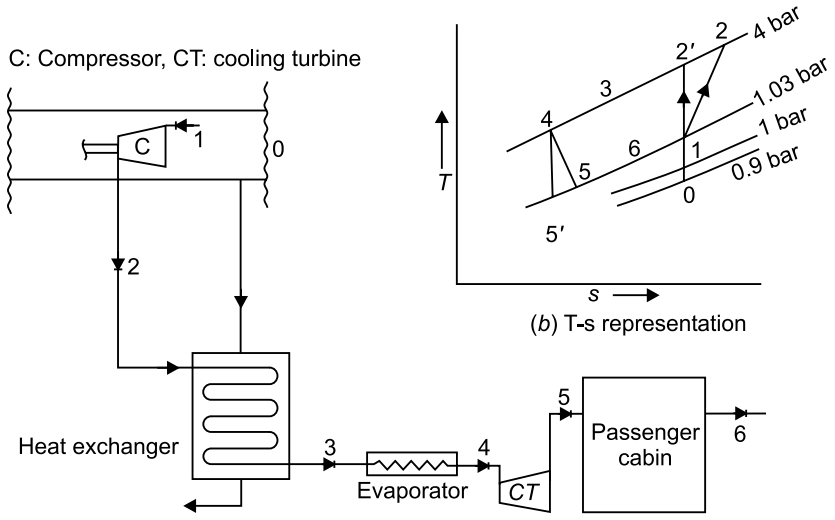
8. *An air cooling system for aeroplane has refrigeration capacity of 15 tons. Atmospheric air is available at 0.9 bar, 3ºC and due to ramming action pressure rises up to 1 bar. The ram air leaves main compressor at 4 bar and subsequently 66% of heat of compressed air is removed in air cooled heat exchanger and then passed through an evaporator for further cooling. Temperature of air gets lowered by 10ºC in evaporator and this cool air is then expanded in cooling turbine before being supplied to passenger cabin maintained at 1.03 bar. Air leaves cabin at 25ºC. Considering isentropic efficiency of compressor and turbine as 0.9 and 0.8 determine, (a) HP required to run the system, (b) COP of system.*

Solution:

Given:

P_0 = 0.9 bar, T_0 = 3ºC or 276 K, $P_2 = P_3 = P_4$ = 4 bar, P_5 = 1.03 bar.

$T_3 - T_4$ = 10, T_6 = 25ºC or 298 K



Schematic or air refrigeration system

Fig. 18.28

Considering index of compression and expansion as 1.4.

During ramming action, process 0–1,

$$\frac{T_1}{T_0} = \left(\frac{P_1}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_1 = 276 \times \left(\frac{1}{0.9}\right)^{\frac{1.4-1}{1.4}}$$

$$T_1 = 284.43 \text{ K}$$

During compression process 1–2′

$$\frac{T_{2'}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{2'} = 284.43 \left(\frac{4}{1}\right)^{\frac{1.4-1}{1.4}}$$

$$T_{2'} = 422.66 \text{ K}$$

$$\eta_{\text{isen, } c} = 0.9 = \frac{T_{2'} - T_1}{T_2 - T_1} \text{ or, } T_2 = 438.02 \text{ K}$$

In heat exchanger 66% of heat loss shall result in temperature at exit from heat exchanger to be, $T_3 = 0.34 \times 438.02 = 148.93$ K. Subsequently for 10ºC temperature drop in evaporator,

$$T_4 = 148.93 - 10$$
$$T_4 = 138.93 \text{ K}$$

Expansion in cooling turbine during process 4–5′,

$$\frac{T_{5'}}{T_4} = \left(\frac{P_5}{P_4}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{5'} = 138.93 \left(\frac{1.03}{4}\right)^{\frac{1.4-1}{1.4}}$$

$$T_{5'} = 94.29 \text{ K}$$

$$\eta_{\text{isen, } T} = \frac{T_4 - T_5}{T_4 - T_{5'}}$$

$$0.8 = \frac{138.93 - T_5}{138.93 - 94.29}$$

$$T_5 = 103.22 \text{ K}$$

Let the mass flow rate of air through cabin be m kg/s. Using refrigeration capacity heat balance yields.

$$15 \times 3.5 = m \times C_P (T_6 - T_5)$$

$$m = \frac{15 \times 3.5}{1.005 \times (298 - 103.22)}$$

$$m = 0.268 \text{ kg/s}$$

$$\text{Work input to compressor} = 0.268 \times 1.005 (T_2 - T_1)$$
$$= 41.37 \text{ kJ/s or } 55.48 \text{ hp}$$

$$\text{COP} = \frac{\text{Refrigeration effect}}{\text{Work input}}$$

$$= \frac{15 \times 3.5}{41.37}$$

$$\text{COP} = 1.27$$

COP = **1.27**
HP required = **55.48 hp** **Ans.**

9. *A refrigerator working on simple vapour compression cycle operates between the temperature of 25ºC and – 15ºC with NH_3, refrigerant. Ammonia is found to be dry after compression and no under cooling of liquid refrigerant occurs in cycle. Using following data obtain COP of system.*

Properties of NH_3

Temperature	*Enthalpy, kJ/kg*		*Entropy, kJ/kg · K*	
	h_f	h_g	s_f	s_g
– 15ºC	– 54.51	1303.74	– 0.2132	5.0536
25ºC	99.94	1317.95	0.3386	4.4809

Solution:

T_1 = –15ºC or 258 K, T_2 = 25ºC or 298 K

$h_2 = h_{g \text{ at } 25ºC} = 1317.95$ kJ/kg

$s_2 = s_{g \text{ at } 25ºC} = 4.4809$ kJ/kg·K

$h_3 = h_{f \text{ at } 25ºC} = 99.94$ kJ/kg

$s_3 = s_{f \text{ at } 25ºC} = 0.3386$ kJ/kg·K

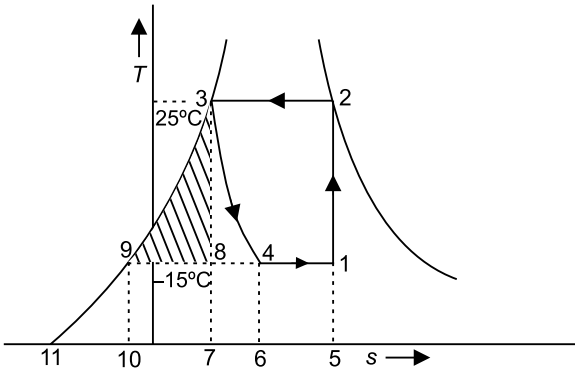


Fig. 18.29 *T-s representation*

Here work done, W = Area 1– 2 – 3 – 9 – 1

Refrigeration effect = Area 1– 5 – 6 – 4 – 1

Area 3 – 8 – 9 = (Area 3 – 11 – 7) – (Area 9 – 11 – 10) – (Area 9 – 8 – 7 – 10)

$= (h_3) - (h_9) - T_1 \times (s_3 - s_9)$

here $h_9 = h_{f \text{ at } -15ºC} = -54.51$ kJ/kg,

$s_9 = h_{f \text{ at } -15ºC} = -0.2132$ kJ/kg · K

Substituting

Area 3 – 8 – 9 = (99.94 – (– 54.51) – 258 (0.3386 + 0.2132)

= 12.09 kJ/kg

During throttling process between 3 and 4.

$h_3 = h_4$

(Area 3 – 11 – 7 – 3) = (Area 4 – 9 – 11 – 6 – 4)

(Area 3 – 8 – 9) + (Area 8 – 9 – 11 – 7 – 8)

= (Area 4 – 6 – 7 – 8 – 4) + (Area 8 – 9 – 11 – 7 – 8)

or (Area 3 – 8 – 9) = (Area 4 – 6 – 7 – 8 – 4)

$$\text{(Area 4 – 6 – 7 – 8 – 4)} = 12.09 \text{ kJ/kg}$$

$$\text{Also, (Area 4 – 6 – 7 – 8 – 4)} = T_1 \times (s_4 - s_8)$$

$$12.09 = 258 \times (s_4 - s_8)$$

$$s_4 - s_8 = 0.0469 \text{ kJ/kg} \cdot \text{K}$$

Also, $s_3 = s_8 = 0.3386 \text{ kJ/kg} \cdot \text{K}$

therefore, $s_4 = 0.3855 \text{ kJ/kg} \cdot \text{K}$

Also $s_1 = s_2 = 4.4809 \text{ kJ/kg}$

$$\text{Refrigeration effect} = \text{Area } (1 - 5 - 6 - 4 - 1) = T_1 \times (s_1 - s_4)$$

$$= 258 \times (4.4809 - 0.3855)$$

$$= 1056.61 \text{ kJ/kg}$$

$$\text{Work done} = \text{Area } 1 - 2 - 3 - 9 - 1$$

$$= (\text{Area } 3 - 8 - 9) + ((T_2 - T_1) \times (s_1 - s_8))$$

$$= 12.09 + ((298 - 258) \times (4.4809 - 0.3386))$$

$$= 177.782 \text{ kJ/kg}$$

$$\text{COP} = \frac{\text{Refrigeration effect}}{\text{Work done}} = \frac{1056.61}{177.782} = 5.94$$

$$\boxed{\text{COP} = \mathbf{5.94}} \quad \textbf{Ans.}$$

10. *A vapour compression cycle works on Freon-12 refrigerant with condensation temperature of 40°C and evaporator temperature of – 20°C. Refrigeration effect of 2.86 ton is desired from the cycle. The compressor runs with 1200 rpm and has clearance volume of 2%. Considering compression index of 1.13 and following data determine.*

(a) the COP

(b) the piston displacement in the reciprocating compressor used for compression.

Properties of Freon - 12

Temperature °C	*Saturation pressure, bar*	*Specific volume, $v_g m^3/kg$*	*Enthalpy, kJ/kg*		*Entropy, kJ/kg·K*		*specific heat kJ/kg·K*	
			h_f	h_g	s_f	s_g	C_{pf}	C_{pg}
– 20	1.51	0.1088	17.8	178.61	0.0730	0.7082	–	0.605
40	9.61	–	74.53	203.05	0.2716	0.682	0.976	0.747

Solution:

Given $T_1 = – 20°C = 253$ K. During expansion (throttling) between 3 and 4,

$h_3 = h_4$, $h_3 = h_{f \text{ at } 40°C} = 74.53 \text{ kJ/kg} = h_4$

Process 1–2 is adiabatic compression so

$$s_1 = s_2$$

and $$s_1 = s_{g \text{ at } -20°C} = 0.7082 \text{ kJ/kg} \cdot \text{K}$$

$$0.7082 = s_{g \text{ at } 40°C} + C_{pg \text{ at } 40°C} \cdot \ln\left(\frac{T_2}{313}\right)$$

$$0.7082 = 0.682 + 0.747 \ln\left(\frac{T_2}{313}\right)$$

Temperature after compression $T_2 = 324.17$ K

Enthalpy after compression, $h_2 = h_{g \text{ at } 40^\circ C} + C_{pg \text{ at } 40^\circ C} \cdot (324.17 - 313)$

$= 203.05 + (0.747 \times (324.17 - 313))$

$h_2 = 211.39$ kJ/kg

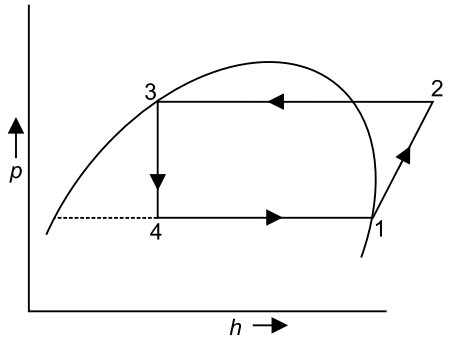


Fig. 18.30

Compression work required, per kg $= h_2 - h_1$

$= 211.39 - h_{g \text{ at } -20^\circ C}$

$= 211.39 - 178.61$

$W_c = 32.78$ kJ/kg

Refrigeration effect during cycle, per kg $= h_1 - h_4$

$= 178.61 - 74.53$

$= 104.08$ kJ/kg

Mass flow rate of refrigerant, $m = \dfrac{2.86 \times 3.5}{104.08}$

$m = 0.096$ kg/s

$$\text{COP} = \frac{104.08}{32.78} = 3.175$$

Volumetric efficiency of reciprocating compressor, given $C = 0.02$

$$\eta_{vol} = 1 + C - C\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

$$= 1 + 0.02 - 0.02\left(\frac{9.61}{1.51}\right)^{\frac{1}{1.13}}$$

$\eta_{vol} = 0.9171$

Let piston displacement by V, m^3.

$$\text{Mass flow rate, } 0.096 = \frac{V \times 0.9171 \times 1200}{60 \times v_{g\ \text{at}-20^\circ\text{C}}}$$

$$0.096 = \frac{V \times 0.9171 \times 1200}{60 \times 0.1088}$$

$$V = 569.45 \text{ cm}^3$$

COP = **3.175**
Piston displacement = **569.45 cm³** **Ans.**

11. *A vapour compression refrigeration cycle operates between the condensation temperature of 20°C and evaporator temperature of –10°C with carbon dioxide as refrigerant. Temperature after isentropic compression is 40°C and condensate leaves at 10°C before being passed through expansion valve. Determine COP of system and mass flow rate of CO_2 required to get refrigeration effect of 2 kW. Properties of CO_2 are given as under:*

Properties of CO_2

Temperature °C	*Saturation Pressure, bar*	*Specific Volume, v_g m^3/kg*	*Enthalpy, kJ/kg*		*Entropy, kJ/kg·K*		*Specific heat kJ/kg·K*	
			h_f	h_g	s_f	s_g	C_{pf}	C_{pg}
20	*57.27*	*–*	*144.11*	*299.62*	*0.523*	*1.0527*	*2.889*	*2.135*
– 10	*26.49*	*0.014*	*60.78*	*322.28*	*0.2381*	*1.2324*	*–*	*–*

Solution:

Processes of vapour compression cycle are shown on *T-s* diagram

1 – 2 : Isentropic compression process

2 – 3 – 4: Condensation process

4 – 5: Isenthalpic expansion process

5 – 1: Refrigeration process in evaporator

$$h_1 = h_{g\ \text{at}-10^\circ\text{C}} = 322.28 \text{ kJ/kg}$$

$$h_2 = h_{g\ \text{at}\ 20^\circ\text{C}} + C_{pg\ \text{at}\ 20^\circ\text{C}} \times (40 - 20)$$

$$= 299.62 + (2.135 \times 20)$$

$$h_2 = 342.32 \text{ kJ/kg}$$

$$\text{Entropy at state 2, } s_2 = s_{g\ \text{at}+20^\circ\text{C}} + C_{pg\ \text{at}\ 20^\circ\text{C}} \times \ln\left(\frac{273+40}{273+20}\right)$$

$$= 1.0527 + 2.135 \times \ln\left(\frac{313}{293}\right)$$

$$s_2 = 1.1937 \text{ kJ/kg·K}$$

Entropy during isentropic process, $s_1 = s_2$

$$1.1937 = s_{f\ \text{at}-10^\circ\text{C}} + x_1 \times s_{fg\ \text{at}-10^\circ\text{C}}$$

$$1.1937 = 0.2381 + x_1 \times (1.2324 - 0.2381)$$

$$x_1 = 0.961$$

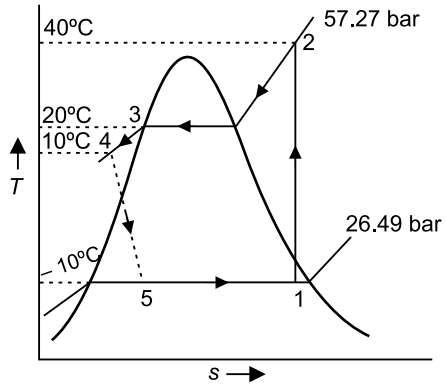


Fig. 18.31 *T-s representation*

Enthalpy at state 1, $h_1 = h_{f \text{ at } -10^\circ C} + x_1 \times h_{fg \text{ at } -10^\circ C}$

$= 60.78 + (0.961 \times (322.28 - 60.78))$

$h_1 = 312.08$ kJ/kg

$h_3 = h_{f \text{ at } 20^\circ C} = 144.11$ kJ/kg

Since undercooling occurs up to 10°C, so, $h_4 = h_3 - C_{pf} \cdot \Delta T$

$= 144.11 - (2.889 \times (20 - 10))$

$h_4 = 115.22$ kJ/kg

Also, $h_4 = h_5 = 115.22$ kJ/kg

Refrigeration effect per kg of refrigerant = $(h_1 - h_5) = (312.08 - 115.22)$

$= 196.86$ kJ/kg

Let refrigerant flow rate be m kg/s.

Refrigeration effect $\Rightarrow 2 = m \times 196.86$

$m = 0.1016$ kg/s

Compressor work, $w_c = h_2 - h_1 = 342.32 - 312.08$

$w_c = 30.24$ kJ/kg

$$\text{COP} = \frac{\text{Refrigeration effect per kg}}{\text{Compressor work per kg}} = \frac{196.86}{30.24} = 6.51$$

COP = **6.51**, Mass flow rate = **0.1016 kg/s** **Ans.**

12. *Determine partial pressure of vapour and relative humidity in the atmospheric air having specific humidity of 16 gm/kg of air and 25°C DBT.*

Solution:

Here pressure of atmospheric air may be taken as 1.013 bar

$$\text{Specific humidity, } \omega = 0.622 \left(\frac{p_v}{p - p_v} \right)$$

$$0.016 = 0.622 \left(\frac{p_v}{1.013 - p_v} \right)$$

or, p_v = Partial pressure of vapour = 0.0254 bar

$$\text{Relative humidity } \phi = \left(\frac{p_v}{p_{v,\text{sat}}} \right)$$

From psychrometric properties of air table $p_{v, \text{ sat at } 25^\circ C} = 0.03098$ bar

$$\phi = \frac{0.0254}{0.03098}$$

$= 0.8198$ or 81.98%

Partial pressure of vapour = **0.0254**
Relative humidity = **81.98%** **Ans.**

13. *For the atmospheric air at room temperature of 30ºC and relative humidity of 60% determine partial pressure of air, humidity ratio, dew point temperature, density and enthalpy of air.*

Solution:

At 30ºC from steam table, saturation pressure, $p_{v,\,sat} = 0.0425$ bar

Partial pressure of vapour = Relative humidity × $p_{v,\,sat}$

$= 0.6 \times 0.0425 = 0.0255$ bar

Partial pressure of air = Total pressure of mixture – Partial pressure of vapour

$= 1.013 - 0.0255$

$= 0.9875$ bar

Partial pressure of air = **0.9875 bar** **Ans.**

$$\text{Humidity ratio, } \omega = 0.622 \frac{p_v}{p - p_v}$$

$$= 0.622 \left(\frac{0.0255}{1.013 - 0.0255} \right)$$

$\omega = 0.01606$ kg/kg of dry air.

Humidity ratio = **0.01606 kg/kg of dry air** **Ans.**

Dew point temperature may be seen from the steam table. The saturation temperature corresponding to the partial pressure of vapour is 0.0255 bar. Dew point temperature can be approximated as 21.4ºC by interpolation.

Dew point temperatrure = **21.4ºC** **Ans.**

Density of mixture = Density of air (ρ_a) + Density of vapour (ρ_v)

$= \rho_a + \rho_v$

$= \rho_a + \omega \cdot \rho_a$

$= \rho_a (1 + \omega)$

$$= \frac{P_a}{RT_a} (1 + 0.01606)$$

$$= \frac{1.013 \times 10^2 \times 1.01606}{0.287 \times 303}$$

$= 1.1835$ kg/m^3

Density = **1.1835 kg/m^3** **Ans.**

Enthalpy of mixture, $h = C_p \cdot T + \omega (h_{g \text{ at } 30^\circ C} + 1.860 (30 - 21.4))$

$= (1.005 \times 30) + (0.01606 \times (2540.1 + 1.86 \times 8.6))$

$= 71.2$ kJ/kg of dry air

Enthalpy of mixture = **71.2 kJ/kg** of dry air **Ans.**

14. *Determine the mass of water added and heat transferred for conditioning atmospheric air at 15ºC and 80% relative humidity to temperature of 25ºC and relative humidity of 50%. Final volume of conditioned air is 0.8 m^3/s.*

Solution:

Initial state at 15ºC and 80% relative humidity is shown by point 1 and final state at 25ºC and 50% relative humidity is shown by point 2 on psychrometric chart.

ω_1 = 0.0086 kg/kg of air and h_1 = 37 kJ/kg

ω_2 = 0.01 kg/kg of air and h_2 = 50 kJ/kg

v_2 = 0.854 m^3/kg

Mass of water added between states 1 and 2 = $\omega_2 - \omega_1$

= 0.01 – 0.0086

= 0.0014 kg/kg of air

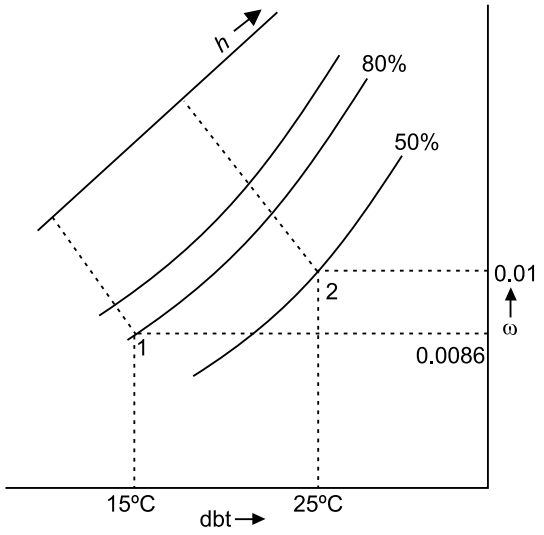


Fig. 18.32 *Psychrometric chart*

$$\text{Mass flow rate of air} = \frac{0.8}{v_2} = \frac{0.8}{0.854} = 0.9368 \text{ kg/s}$$

$$\text{Total mass of water added} = 0.9368 \times (\omega_2 - \omega_1) = 0.9368 \times 0.0014$$

$$= 0.001312 \text{ kg/s}$$

$$\text{Heat transferred} = m_a (h_2 - h_1)$$

$$= 0.9368 \times (50 - 37) = 12.18 \text{ kJ/s}$$

Mass of water added = **0.001312 kg/s,** Heat transferred = **12.18 kW** **Ans.**

15. *Two streams of moist air, one having flow rate of 3 kg/s at 30ºC and 30% relative humidity, other having flow rate of 2 kg/s at 35ºC and 85% relative humidity get mixed adiabatically. Determine specific humidity and partial pressure of water vapour after mixing. Take* $C_{p,\ stream}$ *= 1.86 kJ/kg · K*

Solution:

Let temperature after mixing be *T*ºC. For getting final enthalpy after adiabatic mixing the enthalpy of two streams are required.

For moist air stream at 30ºC and 30% relative humidity.

$$\phi_1 = \frac{p_{v_1}}{p_{v_{\text{sat at } 30^\circ\text{C}}}}$$

$p_{v \text{ sat at } 30^\circ\text{C}} = 0.04246$ bar, so, $p_{v1} = 0.3 \times 0.04246 = 0.01274$ bar

Corresponding to vapour pressure of 0.01274 bar the dew point temperature shall be 10.5ºC.

$$\text{Specific humidity, } \omega_1 = \frac{0.622 \times p_{v1}}{(p - p_{v1})} = \frac{0.622 \times 0.01274}{(1.013 - 0.01274)}$$

$\omega_1 = 0.007922$ kg/kg of air

At dew point temperature of 10.5ºC, enthalpy, $h_{g \text{ at } 10.5^\circ\text{C}} = 2520.7$ kJ/kg.

$$h_1 = C_{p, \text{ air}} \times T_1 + \omega_1 \{h_{g \text{ at } 10.5^\circ\text{C}} - C_{p, \text{ stream}} \times (T_1 - T_{\text{dp1}})\}$$
$$= (1.005 \times 30) + 0.007922 \{2520.7 - 1.86 \times (30 - 10.5)\}$$
$$h_1 = 49.83 \text{ kJ/kg of dry air}$$

For second moist air stream at 35ºC and 85% relative humidity.

$$\phi_2 = 0.85 = \frac{p_{v_2}}{p_{v_{\text{sat at } 35^\circ\text{C}}}} = \frac{p_{v_2}}{0.005628}$$

or, $p_{v2} = 0.004784$ bar

$$\text{Specific humidity } \omega_2 = \frac{0.622 \times p_{v_2}}{(p - p_{v_2})} = \frac{0.622 \times 0.004784}{(1.013 - 0.004784)}$$

$\omega_2 = 0.002951$ kg/kg of air

Corresponding to vapour pressure of 0.004784 bar the dew point temperature is 32ºC.

So, enthalpy of second stream,

$$h_2 = C_{p, \text{ air}} \cdot T_2 + \omega_2 \{h_{g \text{ at } 32^\circ\text{C}} + C_{p, \text{ stream}} \times (T_2 - T_{\text{dp2}})\}$$
$$= 1.005 \times 35 + 0.002951 \{2559.9 + 1.86 \times (35 - 32)\}$$
$$h_2 = 42.75 \text{ kJ/kg of dry air}$$

Enthalpy of mixture after adiabatic mixing

$$= \frac{1}{(m_1 + m_2)} \times \left\{\frac{h_1 \times m_1}{(1 + \omega_1)} + \frac{h_2 \times m_2}{(1 + \omega_2)}\right\}$$

$$= \frac{1}{5} \times \left\{\left(\frac{49.83 \times 3}{1.007922}\right) + \left(\frac{42.75 \times 2}{1.002951}\right)\right\}$$

$= 46.71$ kJ/kg of moist air

$$\text{Mass of vapour per kg of moist air} = \frac{1}{5} \times \left\{\frac{\omega_1 \times m_1}{(1 + \omega_1)} + \frac{\omega_2 \times m_2}{(1 + \omega_2)}\right\}$$

$$= \frac{1}{5} \times \left\{\left(\frac{0.007922 \times 3}{1.007922}\right) + \left(\frac{0.002951 \times 2}{1.002951}\right)\right\}$$

$= 0.005893$ kg per kg of moist air

$$\text{Specific humidity of mixture} = \frac{0.005893}{1 - 0.005893}$$

$$= 0.00593 \text{ kg/kg of dry air}$$

$$\omega = \frac{0.622 \times p_v}{p - p_v}$$

$$0.00593 = \frac{0.622 \times p_v}{1.013 - p_v}$$

Partial pressure of water vapour = 0.00957 bar

Specific humidity of mixture = **0.00593 kg/kg** dry air
Partial pressure of water vapour in mixture = **0.00957 bar** **Ans.**

16. *Calculate the amount of heat added to moist air entering heating coil at 15ºC dbt and 80% relative humidity at a rate of 3m³/s. Air leaves the heating coil at 30ºC and atmospheric pressure is 1.013 bar.*

Solution:

The type of heating involved is sensible heating. Locating state 1 on psychrometric chart corresponding to 15ºC dbt and 80% relative humidity the other property values shall be,

h_1 = 36.4 kJ/kg, ω_1 = 0.0086 kg/kg of air, v_1 = 0.825 m³/kg

Final state 2 has, h_2 = 52 kJ/kg

$$\text{Mass of air} = \frac{3}{0.825} = 3.64 \text{ kg/s}$$

$$\text{Amount of heat added} = 3.64 \times (52 - 36.4)$$
$$= 56.78 \text{ kJ/s.}$$

Heat added = **56.78 kJ/s** **Ans.**

EXERCISE

18.1 Define refrigeration and compare it with air conditioning.
18.2 Explain reversed Carnot cycle and its limitations.
18.3 Define unit of refrigeration.
18.4 Explain working of a Carnot refrigerator and state why it cannot be used in practice.
18.5 Describe air-refrigeration cycles.
18.6 Explain simple air refrigeration cycle being used in aeroplanes.
18.7 What do you understand by vapour compression cycle? Explain the processes on *T-s* and *p-h* diagrams.
18.8 Compare Carnot refrigeration cycle with vapour compression cycle.
18.9 What is meant by wet compression and dry compression in vapour compression cycles? Explain.
18.10 What is there in multistage vapour compression cycle? List its advantages over simple vapour compression cycle.
18.11 Describe the vapour absorption cycle for ammonia-water system and its working.
18.12 Explain the modified absorption-refrigeration cycle and its working.
18.13 Describe heat pump system based on vapour compression cycle.
18.14 Classify refrigerants and list desired properties of refrigerants.
18.15 Explain sensible heating and sensible cooling on psychrometric chart.
18.16 Write short notes on relative humidity, specific humidity, dry bulb temperature, dew point temperature, wet bulb temperature.
18.17 Discuss the working of summer air-conditioning system.
18.18 Describe working of winter air-conditioning system.

18.19 Show that the moisture content 'ω' of the atmospheric air is given as,

$$\omega = \frac{0.622 \times p_v}{p_a}$$

where p_v and p_a are partial pressure of vapour and air respectively.

18.20 Determine power required to run the compressor and heat rejected by compressor per second from a refrigerator working on reversed Carnot cycle between temperatures of 266 K and 300 K. Refrigerator produces 0.139 kg/s ice at –5ºC from water at 287 K. Also determine COP of refrigerator.
[9.76 hp, 63.19 kJ/s, 7.82]

18.21 A refrigerator working between –5ºC and 20ºC has capacity of 600 tons. Determine the minimum hp required for machine. [266 hp]

18.22 An air refrigeration cycle has pressure ratio of 4 and the air enters compressor at 15ºC and expander at 30ºC. Consider compression and expansion index to be 1.25 and 1.35 respectively. Determine COP of refrigeration cycle. [1.7]

18.23 Show that the ratio of maximum COP of Bell-Coleman cycle and COP of Carnot cycle is given by:

$$\frac{\text{COP}_{\text{Bell–Colemen}}}{\text{COP}_{\text{carnot}}} = \left(\frac{T_1 - T_2}{T_2}\right)^2$$

where T_1 and T_2 are the temperature of surroundings and refrigerated space. Assume that the rise in temperature of air passing through heat exchanger (cooler) is same in both cases and is constant.

18.24 A Bell-coleman refrigeration cycle of 75.8 kJ/s refrigeration capacity runs between pressures of 1 bar and 5 bar. Refrigerated space is maintained at 0ºC and ambient temperature is 300 K. For isentropic efficiency of compression and expansion being 0.85 and 0.90 respectively. Determine COP of system.
[0.815]

18.25 An air refrigeration system working on reversed Brayton cycle has refrigeration capacity of 10 ton and pressure ratio of 3. Pressure in refrigerated space is 1.4 bar, temperature of air at inlet to compressor is 253 K and air is cooled to a temperature of 323 K in cooler. Considering C_p = 1.07 kJ/kg·K determine COP of system, air circulation per minute, theoretical piston displacement and power required.
[2.43, 114 kg/min, 59.13 m^3/min, 13 kW]

18.26 An air refrigeration cycle works between 1 bar and 12 bar to produce refrigeration capacity of 25 tons when air leaves cooler at 25ºC and air leaves refrigerated space at 0ºC. Considering compression and expansion processes to have index of 1.35 determine.
(*i*) the COP of system
(*ii*) the HP required
(*iii*) the mass of air circulated per minute
(*iv*) the piston displacements of compressor and expanders.
[1.94, 116 hp, 133 kg/min, 35.5 m^3/min, 20.35 m^3/min]

18.27 Air cooled aeroplane system has pressure of air getting increased from atmospheric pressure of 0.9 bar to 1 bar due to ram action. Compressed air leaves compressor at 3.5 bar and temperature of compressed air leaving heat exchanger is 355 K and subsequently leaving the evaporator is 348 K. High pressure cool air leaving evaporator enters the cooling turbine and subsequently to passenger cabin at 1.03 bar. Considering isentropic efficiencies of compression and expansion to be 0.8 and 0.75 determine hp required for desired cooling and COP of system when stale air leaves passenger cabin at 298 K. Ambient air temperature is 296 K. [596 hp, 0.16]

18.28 The passenger cabin of an aircraft is maintained at 298 K when ambient air is at 0.35 bar and –15ºC. Compressor in aircraft has pressure ratio of 3 and aircraft is moving at 1000 kmph. Compressed air leaving compressor is passed through a heat exchanger and cooled there upto ambient air temperature. Subsequently this cool air passes through cooling turbine so as to leave it at 1.06 bar. Considering cooling load to be 200 ton determine.
(*i*) the stagnation temperature and pressure

(*ii*) the COP of system
(*iii*) the mass flow rate of air
Take C_{pair} = 1 kJ/kg·K, γ = 1.4 [296.5 K, 0.57 bar, 0.31, 2 kg/s]

18.29 Calculate COP of vapour compression system running on ammonia (NH_3) as refrigerant between 25ºC and – 10ºC such that the refrigerant is dry saturated at the end of compression and there is no undercooling. Properties of NH_3 are;

Temperature, ºC	*Enthalpy, kJ/kg* h_f	h_{fg}	*Entropy, kJ/kg·K* s_f
25	100.32	1228.92	0.3469
–10	– 33.52	1345.96	– 0.1379

[7.16]

18.30 A vapour compression cycle running on NH_3 had refrigeration capacity of 1990.5 ton while operating between temperature limits of 298 K and 258 K. The vapour is dry saturated at the end of compression and actual COP of cycle is only 60% of theoretical COP, then determine power required to drive compressor.
Properties of NH_3

Temperature, ºC	*Enthalpy, kJ/kg* h_f	h_g	*Entropy, kJ/kg·K* s_f	s_g
25ºC	99.94	1318	0.0347	4.4809
–15ºC	–54.51	1303.7	– 0.2132	5.0536

[43.8 hp]

18.31 Determine the COP of vapour compression system running with CO_2 and operating between temperature of 25ºC and – 5ºC such that carbon dioxide entering compressor is 0.6.
Properties of CO_2

Temperature, ºC	*Enthalpy, kJ/kg* h_f	h_g	*Entropy, kJ/kg·K* s_f	s_g
25	81.09	202.31	0.2508	0.6287
– 5	– 7.52	236.59	– 0.0418	0.8402

[3.3]

18.32 A water cooler working on Freon– 12 as refrigerant operates between 299 K and 275 K such that the vapour leaves evaporator dry saturated. Considering volumetric efficiency of compressor as 80%, mechanical efficiency of compressor as 85%, 20% of useful cooling getting lost and cooler cooling water from 299 K to 279 K at the rate of 100 kg/hr determine.
(*i*) the hp required in compressor
(*ii*) the volumetric capacity of compressor
Properties of Freon-12

Temperature, ºC	*Sp. volume* v_g, m^3/kg	*Enthalpy, kJ/kg* h_f	h_g	*Entropy, kg/kg·K* s_f	s_g	*Specific heat kg/kg·K* C_{pf}	C_{pg}
26	0.027	443.12	583.95	4.2665	4.737	60.9948	0.6729
2	0.053	419.88	573.54	4.1867	4.745	60.9405	0.618

[0.442 hp, 0.085 m^3/min.]

19

Jet Propulsion and Rocket Engines

19.1 INTRODUCTION

World War-II witnessed the use of a new gas turbine engine type aircraft being developed independently in Germany and in England. These engines were called as jet engines. In these aircraft thrust generated by propulsion system caused movement of aircraft in air. Underlying principle for the creation of thrust was Newton's third law of motion. In jet propulsion system working fluid is accelerated by the system and reaction to this acceleration produces force on the system. Amount of thrust created is directly proportional to the mass flow rate through engine and exit velocity of gas. Jet engines have atmospheric air being inducted into compressor for being compressed and subsequently used for burning fuel in combustor. High temperature, high pressure burnt gases are then expanded in gas turbine and hot exhaust gases are finally passed through a nozzle for creation of thrust. Rocket engines also work on similar principle but unlike jet engine these rocket engines carry oxygen with them for combustion.

History of jet propulsion dates back to 100 B.C. when Hero of Alexandria first developed engine called 'aeolipile' which was basically reaction turbine working on steam. In 1500 A.D. Leonardo Vinci used hot air coming out from chimney to drive rotor called chimney jack. Steam jet propelled wagon was developed in 1687 A.D. in Dutch. Major breakthrough was achieved when the gas turbine was patented in the name of John Barbar in 1791 A.D. This evolution of gas turbine paved way for development of gas based engines for use in gas turbine power plants and aircraft etc. By the year 1903 A.D. number of gas turbine based plants came up and gas turbine was found to be successful.

19.2 PRINCIPLE OF JET PROPULSION

Jet propulsion refers to the imparting of forward motion to the object as a reaction to exit of high velocity gas/liquid stream from the rear end of object. Jet propulsion devices are popularly used in high-speed, high altitude aircraft/missile/spacecrafts etc. Simple example of jet propulsion is forward motion of an inflated balloon when air is suddenly released from it.

Jet propulsion is based on the principle of Newton's second law and third law of motion. In a jet propulsion engine the objective is to get the propelling thrust for the engine and for getting it the momentum change occurs in fluid stream such that the reaction to the impulse created by momentum change gives propelling thrust. Thus, the change of momentum of the fluid stream flowing across engine and the reaction to the impulse due to momentum change are responsible for jet propulsion. For realizing momentum change the high temperature and pressure gas stream is expanded through a nozzle so that

the gas stream comes out in atmosphere with significantly high velocity, thus giving change of momentum. This momentum change yields impulse force whose reaction produces propelling thrust. Figure. 19.1 gives block diagram for jet propulsion engine.

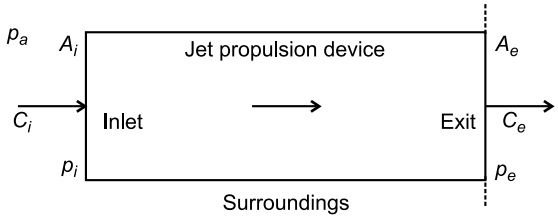


Fig. 19.1 *Principle of jet propulsion engine*

Let us consider jet propulsion device as shown in Fig. 19.1. Let the inlet and exit of device be indicated by subscript '*i*' and '*e*'. Cross-sectional area, velocity of gas, pressure of gas at inlet and exit are given by A_i , C_i , p_i and A_e, C_e, p_e respectively. Let us analyze for air entering device and flowing through device such that velocity changes from inlet 'C_i' to exit 'C_e'.

Atmospheric air enters device at atmospheric pressure p_a, velocity C_i. Consider mass of air entering into engine be at the rate of m_a kg/s. Mass of gas leaving engine will be at the rate of $(m_a + m_f)$ kg/s where m_f is mass flow rate of fuel required for running. Expansion occurring in engine shall be complete when the pressure at exit p_e equals to atmospheric pressure, p_a. In actual case this expansion may not be complete.

For simplicity the mass flowing through device may be assumed constant *i.e.*, fuel added is negligible, then the thrust T may be estimated by the rate of change of momentum. This is called as momentum thrust.

$$T_m = m_a (C_e - C_i)$$

considering mass of fuel added.

$$F_m = \{(m_a + m_f) \cdot C_e - m_a \cdot C_i\}$$

In case of incomplete expansion the exhaust gas may leave device with pressure p_e which is greater than p_a. This incomplete expansion would cause additional pressure thrust exerted over the device exit area A_e equal to $A_e \cdot (p_e - p_a)$. This total thrust will have two components *i.e.*, momentum thrust and pressure thrust.

Total thrust, $T =$ Momentum thrust, T_m + Pressure thrust, T_p

$$T = m_a (C_e - C_i) + A_e (p_e - p_i)$$

Above mathematical equation shows that the total thrust may thus be maximized by increasing momentum thrust and pressure thrust both or any one of these. Key parameters responsible for increase in total thrust could be high velocity of jet leaving (C_e), high mass flow rate (m_a) high pressure difference at exit and higher cross sectional area at exit. But in view of the interdependency of these parameters the required thrust may be said to be available either by high velocity jet of small mass flow or a low velocity jet of high mass flow.

While considering the fuel mass addition the total thrust may be given as,

$$T = \left\{(m_a + m_f) \cdot C_e - m_a \cdot C_i\right\} + A_e (p_e - p_i)$$

Basic arrangement required in a jet propulsion engine comprises of a compressor for compressing air, followed by combustor (combustion chamber) for generating high pressure and high temperature

gases followed by a turbine for generating shaft work to drive compressor and a nozzle at end to finally exhaust the gas stream. Figure 19.2 shows the schematic of turbojet engine. The jet leaving engine has quite high velocity and the reaction of this jet propels the engine (device) in direction opposite to that of jet.

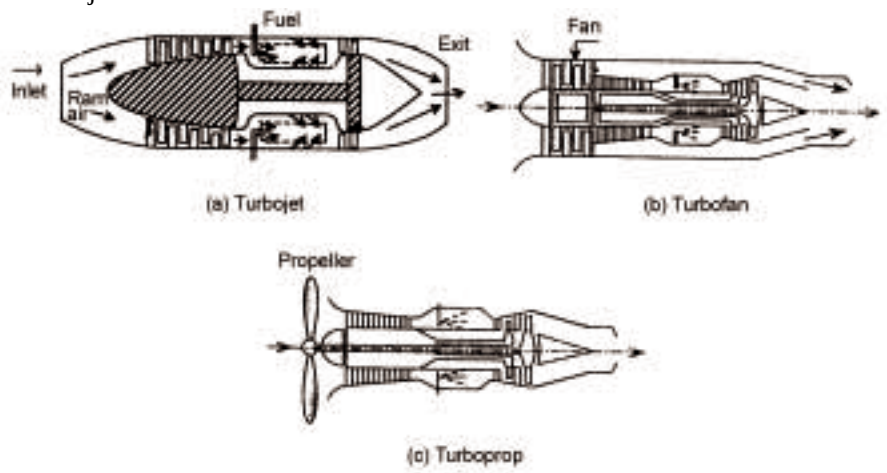


Fig. 19.2 *Turbojet, Turbofan and Turboprop engines*

Total thrust, $T = m_a\,(C_e - C_i) + A_e\,(p_e - p_a)$; (neglecting fuel addition)

$T = \left\{(m_a + m_f)\,C_e - m_a \cdot C_i\right\} + A_e\,(p_e - p_a)$; (considering fuel addition)

19.3 CLASSIFICATION OF JET PROPULSION ENGINES

Jet propulsion engines can be broadly classified based on type of suction. In case of jet engine sucking atmospheric air it is called air-breathing jet engines or atmospheric jet engines. Jet engines may be fed with oxygen carried separately in engine and such engines do not induct atmospheric air. These engines which carry their own oxidizer for combustion of fuel are called non-air breathing jet engines or rocket engines. Thus, jet propulsion engines can be;

(*i*) Air breathing jet engines or atmospheric jet engines

(*ii*) Non-air breathing jet engines or rocket engines

(*i*) Air breathing jet engines or atmospheric jet engines can be further classified depending on the type of air flow as,

(*a*) Continuous air flow engines or steady combustion type engines.

(*b*) Intermittent air flow engines or intermittent combustion type engines.

Continuous air flow type engines can be further classified as,

(*a*.1) Turbojet engines

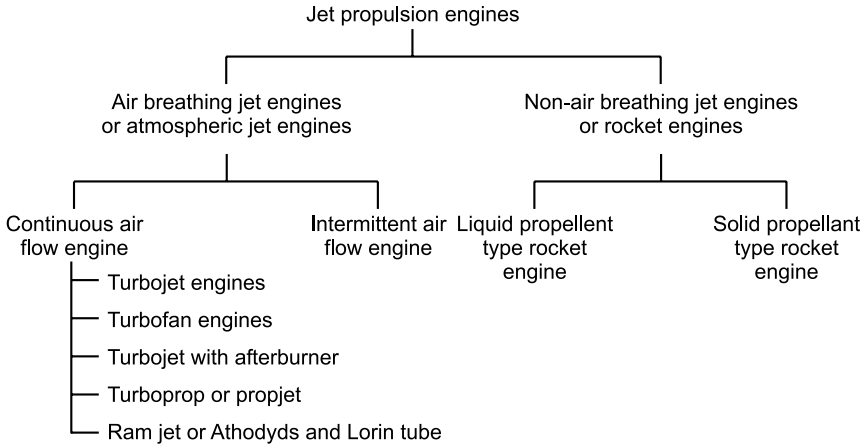
(*a*.2) Turbofan engines

(*a*.3) Turbojet with afterburner

(*a*.4) Turboprop or propjet

(*a*.5) Ramjet or Athodyds and Lorin tube

(*ii*) Non-air breathing jet engine or rocket engine can be classified based on the type of propelling fluid used. It could be liquid propellant type rocket engine and solid propellant type rocket engine.



19.4 PERFORMANCE OF JET PROPULSION ENGINES

Performance of jet propulsion engine can be judged by the performance parameters given below. Let us consider turbojet engine as shown in Fig. 19.1, moving with velocity through air as C_a. Thus the velocity of air entering engine would be C_a (approach velocity) if the atmospheric air is at rest.

Thrust obtained per kg of air (N/kg of air/s) flowing can be given as :

$$T = \left[\left\{\left(1 + \frac{m_f}{m_a}\right)\cdot C_e - C_a\right\} + \frac{A_e}{m_a}(p_e - p_a)\right]$$

(*i*) **Thrust power (TP):** Thrust power indicates the actual power available for propulsion. It refers to the work done per unit time by the engine. This thrust power can be expressed by the product of thrust and velocity with which engine moves (flight velocity).

$$TP = T \times C_a$$

$$= \left[\left\{\left(1 + \frac{m_f}{m_a}\right)\cdot C_e - C_a\right\} + \frac{A_e}{m_a}(p_e - p_a)\right]\cdot C_a$$

Neglecting pressure thrust it could be given as:

$$TP = \left[\left(1 + \frac{m_f}{m_a}\right)\cdot C_e - C_a\right]\cdot C_a$$

Neglecting mass of fuel added;

$$TP = (C_e - C_a)\cdot C_a, \qquad \text{W/kg of air}$$

(*ii*) **Propulsive power (PP):** Propulsive power indicates the total energy available for propulsion. It can be estimated by the difference between the rate of kinetic energy entering with air and leaving with jet of exit gases. Mathematically;

$$PP = \text{K.E.}_{\text{leaving}} - \text{K.E.}_{\text{entering}} = \Delta\text{KE}$$

$$\Rightarrow \qquad \Delta\text{KE} = \frac{1}{2}\left(1 + \frac{m_f}{m_a}\right).C_e^2 - \frac{C_a^{\,2}}{2}; \qquad \text{W/kg of air}$$

$$\Rightarrow \qquad PP = \frac{1}{2}\left\{\left(1+\frac{m_f}{m_a}\right)\cdot C_e^2 - C_a^2\right\}; \quad \text{W/kg of air}$$

Neglecting fuel mass addition, propulsive power can be given by,

$$PP = \frac{C_e^2 - C_a^2}{2}; \qquad \text{W/kg of air}$$

(*iii*) **Propulsive efficiency (η_{Prop}):** Propulsive efficiency is measure of effectiveness by which propulsive power is transformed into thrust power *i.e.*, how efficiently propelling duct can propel the engine.

Mathematically, it can be given by ratio of thrust power (TP) to propulsive power (PP). Propulsive efficiency is also called Froude efficiency.

$$\eta_{prop} = \frac{TP}{PP} = \frac{\left[\left(1+\frac{m_f}{m_a}\right)C_e - C_a\right]C_a}{\frac{1}{2}\left[\left(1+\frac{m_f}{m_a}\right)\cdot C_e^2 - C_a^2\right]}$$

Neglecting fuel mass addition, propulsive efficiency can be given as,

$$\eta_{prop} = \frac{2\,(C_e - C_a)\cdot C_a}{(C_e^2 - C_a^2)}$$

$$= \frac{2C_a}{(C_e + C_a)} = \frac{2}{\left\{1+\left(\frac{C_e}{C_a}\right)\right\}}$$

The above expression indicates that propulsive efficiency would be unity when exit jet velocity equals to approach velocity, but this is not desired as it decreases thrust available.

(*iv*) **Thermal efficiency (η_{th}):** Thermal efficiency can be given by the ratio of kinetic energy available and total heat supplied.

$$\eta_{th} = \frac{\text{Kinetic energy available}}{\text{Heat supplied}}$$

$$= \frac{\left\{\frac{1}{2}\left((m_a + m_f)\,C_e^2 - m_a\,C_a^2\right)\right\}}{m_f \times \text{Calorific value of fuel}}$$

or,
$$\eta_{th} = \frac{\left(\left(1+\frac{m_f}{m_a}\right)C_e^2 - C_a^2\right)}{2\left(\frac{m_f}{m_a}\right)\times \text{Calorific value of fuel}},$$

$$= \frac{(1+FA)\,C_e^2 - C_a^2}{2 \times FA \times \text{Calorific value of fuel}}$$

where *FA* is fuel-air ratio.

(*v*) **Overall efficiency ($\eta_{overall}$):** Overall efficiency can be given by the ratio of useful workdone to the energy supplied.

$$\eta_{overall} = \frac{\left[\left(1 + \frac{m_f}{m_a}\right) . C_e - C_a\right] C_a}{\left(\frac{m_f}{m_a}\right) \times \text{Calorific value of fuel}}$$

$$= \eta_{th} \times \eta_{prop}$$

(*vi*) **Jet efficiency (η_{jet}):** Jet efficiency is given by the ratio of final kinetic energy in jet to the total of isentropic heat drop in jet pipe and carry over from turbine.

Comparative performance of turbojet and other propulsion engines in respect to the propulsive efficiency and jet velocity is shown in Fig. 19.3. Figure 19.4 shows comparison of specific fuel consumption with jet plane speed for these different devices.

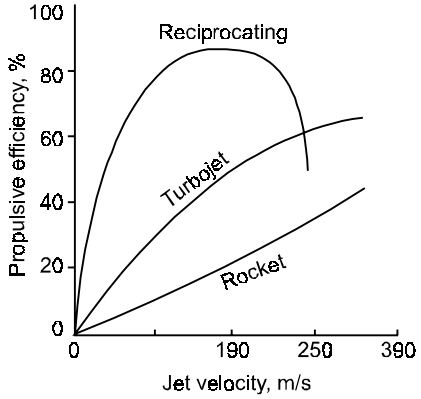


Fig. 19.3 *Comparison of propulsive efficiency*

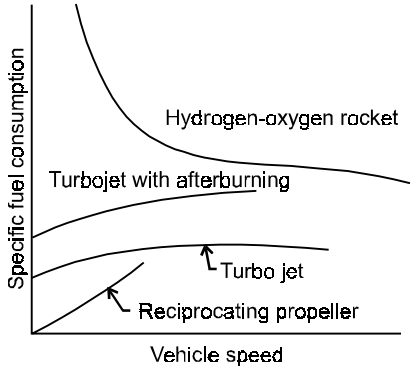


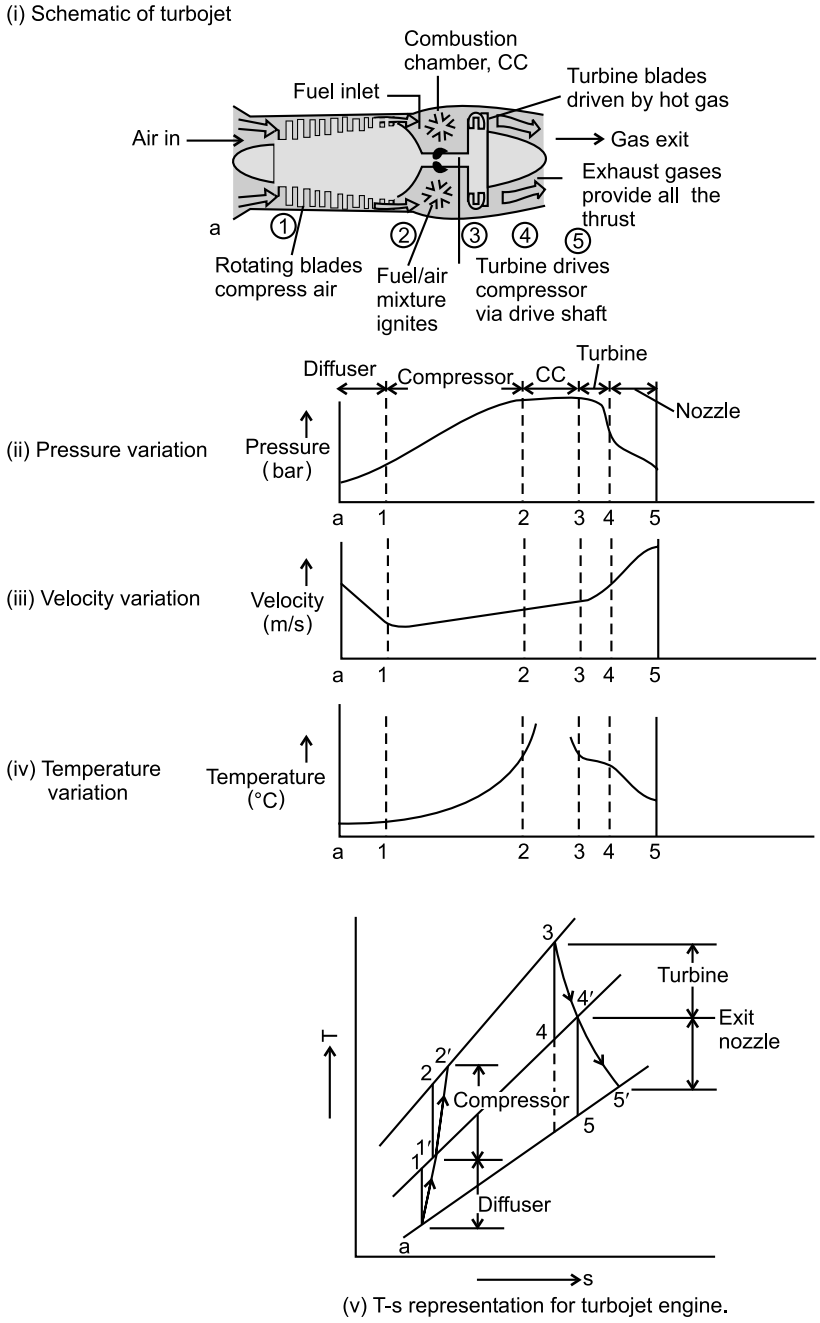
Fig. 19.4 *Comparison of specific fuel consumption*

19.5 TURBOJET ENGINE

Fig. 19.5 shows the schematic of turbojet engine. It has a diffuser section at inlet for realizing some compression of air passing through this section. Due to this air reaching compressor section has pressure more than ambient pressure. This action of partly compressing air by passing it through diffuser section is called "ramming action" or "ram effect". Subsequently compressor section compresses air which is fed to combustion chamber and fuel is added to it for causing combustion. Combustion products available at high pressure and temperature are then passed through turbine and expanded there. Thus, turbine yields positive work which is used for driving compressor.

Expanding gases leaving turbine are passed through exit nozzle where it is further expanded and results in high velocity jet at exit. This high velocity jet leaving nozzle is responsible for getting desired thrust for propulsion. Pressure variation along the length of turbojet is shown in Fig. 19.5. Velocity and temperature variations are also shown there in. Different salient states shown are plotted on temperature-entropy (*T*–*s*) diagram as given in Fig. 19.5. Ambient air at state '*a*' enters the diffuser section and ram effect is seen from '*a*' to '1' resulting into small pressure rise. Desired compression occurs between '1' to '2'. Subsequent combustion occurs in combustor section resulting into combustion products at state '3'. Expansion occurs in turbine from '3' to '4' and subsequently expanded from '4' to '5'.

Between states '*a*' and '1' diffusion of air entering occurs isentropically with velocity varying from C_a to zero for diffuser section having 100% efficiency. Actually diffuser efficiency may be 0.9–0.95. T–s diagram depicts both theoretical states and actual states. Actual states have been shown as 1′, 2′, 4′ and 5′, while theoretical states are 1, 2, 3, 4 and 5.



(v) T-s representation for turbojet engine.

Fig. 19.5 *Turbojet engine*

Considering the states shown on *T–s* diagram to be static states the mathematical equations for different sections can be given as under.

In diffuser section ($a - 1$) the energy equation yields;

$$\frac{C_a^2}{2} + h_a + q_{a-1} = \frac{C_1^2}{2} + h_1 + w_{a-1}$$

For ideal diffuser, $C_1 = 0$, $q_{a-1} = 0$ and $w_{a-1} = 0$, which means after ram compression air gets decelerated to $C_1 = 0$, diffuser section is perfectly insulated and no work interaction occurs in this section.

$$h_1 = h_a + \frac{C_a^2}{2}$$

or,
$$T_1 = T_a + \frac{C_a^2}{2C_p}$$

Deviation from ideal diffusion process to actual can be quantified through diffuser efficiency.

$$\eta_{\text{diffuser}} = \frac{h_1 - h_a}{h_{1'} - h_a} = \frac{T_1 - T_a}{T_{1'} - T_a}$$

$$\Rightarrow \quad h_{1'} = h_a + \frac{C_a^2}{2\eta_{\text{diffuser}}} \Rightarrow T_{1'} = T_a + \frac{C_a^2}{2\eta_{\text{diffusor}}}$$

Pressure rise in diffuser can be given by pressure ratio $\left(\frac{p_1}{p_a}\right)$.

$$\frac{p_1}{p_a} = \left[\frac{C_a^2}{2C_p \cdot T_a}\eta_{\text{diffuser}} + 1\right]^{\frac{\gamma}{(\gamma-1)}};$$

here substituting, $C_p = \frac{\gamma R}{(\gamma - 1)}$ and

velocity of sound = $\sqrt{\gamma \cdot RT}$,

or,
$$\frac{p_1}{p_a} = \left[1 + \eta_{\text{diffuser}}\left(\frac{\gamma - 1}{2}\right) M_a^2\right]^{\frac{\gamma}{(\gamma-1)}}$$; Here M_a is Mach no. at '*a*'.

Similar to diffuser section the energy equation when applied to compressor section between states 1 and 2 yields, (neglecting change in potential energy)

$$h_1 + \frac{C_1^2}{2} + q_{1-2} = h_2 + \frac{C_2^2}{2} + (-w_{\text{comp}})$$

or,
$$w_{\text{comp}} = \left(h_2 + \frac{C_2^2}{2}\right) - \left(h_1 + \frac{C_1^2}{2}\right)$$

substituting stagnation property,

$$h_{02} = h_2 + \frac{C_2^2}{2}, \quad h_{01} = h_1 + \frac{C_1^2}{2}$$

$$w_{comp} = h_{02} - h_{01}$$

Neglecting change in kinetic energy and potential energy the work required by compressor shall be,

$$w_{comp} = h_2 - h_1 = C_p (T_2 - T_1)$$

$$w_{comp} = h_2 - h_1 = C_p (T_2 - T_1)$$

Actual state at the end of compressor lies at state 2′ instead of 2. This shift from 2 to 2′ can be quantified by compressor efficiency (η_{comp}) as given below:

$$\eta_{comp} = \frac{(h_2 - h_1)}{(h_{2'} - h_1)} = \frac{\text{Ideal compressor work}}{\text{Actual compressor work}}$$

Heat addition in combustion chamber occurs between states 2 and 3. Neglecting mass of fuel added, the heat supplied can be estimated as;

$$q_{2-3} = h_3 - h_2 = C_p (T_3 - T_2)$$

Actual heat supplied can be given as:

$$Q_{2-3} = \left\{(m_a + m_f)\cdot h_3 - (m_a \cdot h_{2'})\right\}$$

After combustion the hot gases enter turbine for expansion between 3 and 4. Energy equation can be written for turbine, as

$$h_3 + \frac{C_3^2}{2} + q_{3-4} = h_4 + \frac{C_4^2}{2} + w_{turbine}$$

Considering expansion in turbine to be occurring adiabatically; $q_{3-4} = 0$

or,
$$w_{turbine} = \left(h_3 + \frac{C_3^2}{2}\right) - \left(h_4 - \frac{C_4^2}{2}\right)$$

In terms of stagnation properties;

$$w_{turbine} = h_{03} - h_{04}$$

Neglecting change in kinetic energy across turbine,

$$w_{turbine} = h_3 - h_4$$

$$w_{turbine} = C_p T_3 - C_p T_4$$

Actual turbine work, $w'_{turbine} = (h_3 - h_{4'})$

Turbine efficiency, $\eta_{turbine} = \frac{w'_{turbine}}{w_{turbine}} = \frac{(h_3 - h_{4'})}{(h_3 - h_4)} = \frac{(T_3 - T_{4'})}{(T_3 - T_4)}$

At the exit of turbine gases pass through exit nozzle from 4 to 5. Energy equation can be given as,

$$h_4 + \frac{C_4^2}{2} = h_5 + \frac{C_5^2}{2}$$

for actual expansion in exit nozzle;

$$h_{4'} + \frac{C_{4'}^2}{2} = h_{5'} + \frac{C_{5'}^2}{2}$$

Nozzle efficiency, $\eta_{nozzle} = \frac{h_{4'} - h_{5'}}{h_{4'} - h_5} = \frac{T_{4'} - T_{5'}}{T_{4'} - T_5}$

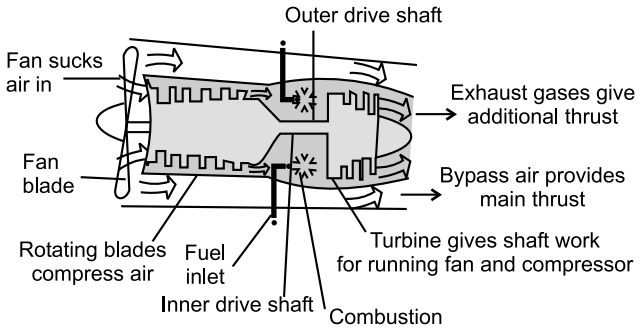
Considering above *T*-*s* diagram thermal efficiency can be mathematically given as

$$\eta_{th} = \frac{(h_3 - h_{5'}) - (h_{2'} - h_a)}{(h_3 - h_{2'})}$$

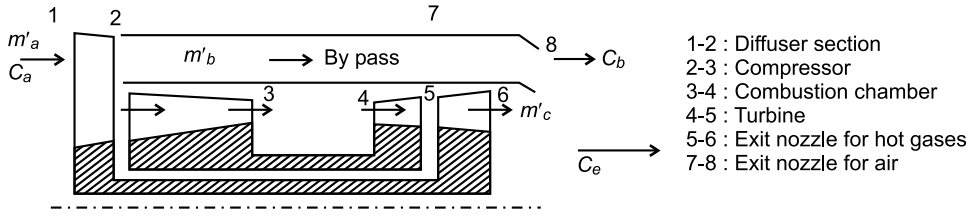
$$\eta_{th} = \frac{(T_3 - T_{5'}) - (T_{2'} - T_a)}{(T_3 - T_{2'})}$$

19.6 TURBOFAN ENGINE

Turbofan engine is the modified turbojet engine in which additional thrust is realised by putting fan at the entry of the engine casing. Fan blades propel by pass air around engine core between inner and outer engine casing. This air does not participate in combustion but provides additional thrust while leaving through exit nozzle. Figure. 19.6 shows the schematic of turbofan engine. The propeller fan put at inlet to engine sucks air and it passes through by pass passage as shown up to the exit nozzle end. Thus there are two streams of air flowing, one air stream gets rammed, compressed, burnt, expanded in turbine and finally passes through exit nozzle and other air stream passes through passage between outer and inner casings from inlet to nozzle exit. Total thrust created will be due to two jet streams one due to cold air or fan air and other due to burnt gases leaving turbine.



(a) Turbofan engine arrangement



(b) Line diagram showing different sections (not to scale)

Fig. 19.6 *Turbofan engine*

Arrangement in turbofan engine indicates that the proportion of air flowing through bypass and engine play significant role, this is mathematically defined as bypass ratio. Bypass ratio is given by the ratio of flow through bypass duct and flow through compressor. Generally bypass ratio has value in the range 0.3 to 0.8. Thus, for bypass mass flow rate being m'_b and mass flow rate through compressor being m'_c the bypass factor (B) shall be

$$B = \frac{m'_b}{m'_c} \text{ and}$$

total air entering turbofan, $m'_a = m'_b + m'_c$

or $$m'_a = m'_c\,(1 + B) \Rightarrow m'_c = \frac{m'_a}{B + 1}$$

Let the velocity of bypass duct air leaving jet be given by C_b and velocity of gas leaving jet be C_e, then the thrust can be given as;

$$\text{Thrust} = (m'_c \cdot C_e + m'_b \cdot C_b) - m'_a \cdot C_a$$

Turbo fan engine's performance primarily depends on the fan pressure ratio, bypass ratio, overall pressure ratio, turbine inlet temperature, cruising speed and altitude. Amongst these thermodynamic parameters. For optimum performance of turbofan engine the plot of specific fuel consumption for the varying bypass ratio is shown in Fig. 19.7.

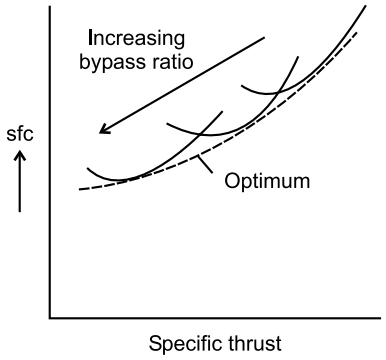


Fig. 19.7 *Specific fuel consumption vs. specific thrust for turbofan engine*

19.7 TURBOPROP ENGINE

Turboprop (Turbo-propeller) engine, also called turboshaft engine or propjet engine differs slightly from turbofan engine. It uses thrust to turn a propeller. It consists of a compressor for compressing the inlet air, combustion chamber and turbine followed by exit nozzle. A part of turbine output is used to drive the compressor and remaining for driving propeller. Thus some stages of turbine give shaft work for driving compressor and some stages produce shaft work for driving propeller as shown in Fig. 19.8. It can also be understood as if the gases expand through main turbine which drives compressor and also expands through power turbine which drive propeller through suitable reduction gear box. Turboprop engines are used in small passenger planes, cargo planes etc.

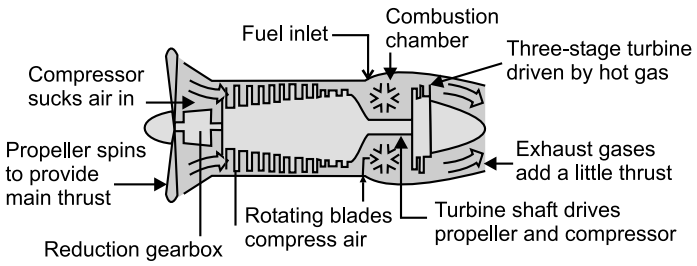


Fig. 19.8 *Turboprop engine*

19.8 TURBOJET ENGINE WITH AFTERBURNER

Turbojet engine with afterburner is similar to turbojet engine with afterburner in tail pipe. Figure 19.9 shows schematic for turbojet engine with afterburner where exhaust leaving turbine is passed through afterburner in which combustion causes rise in exhaust gas temperature. Thus exhaust gas have high potential to produce thrust for propulsion.

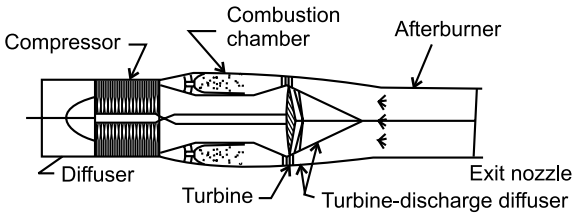


Fig. 19.9 *Turbojet engine with afterburner*

19.9 RAMJET ENGINE

Ramjet engine is the simplest of jet engines having no moving parts. Ramjet is a typically shaped duct open at both ends with air being compressed merely due to forward motion of engine. Fuel is subsequently added for combustion and thus high pressure, high temperature gases exit from exhaust nozzle. High pressure air is continuously available as engines keeps on moving forward. These ramjets are extensively used for propulsion in number of high speed aircrafts.

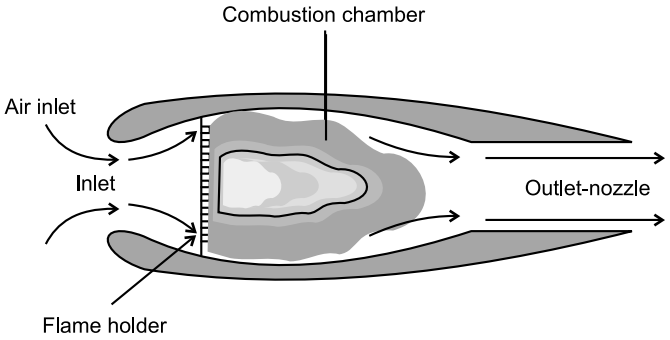
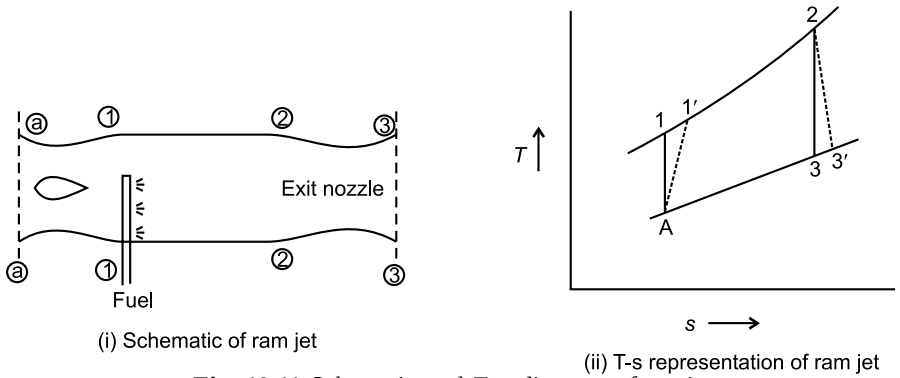


Fig. 19.10 *Ramjet engine*

Ramjet is also called Athodyd Lorin tube based on the name of its inventor Rene Lorin, a French engineer. First ramjet engine was developed in 1913 and it had steady flow through inlet diffuser, combustion chamber and outlet nozzle. These can not operate under stationary condition as sole compression of air occurs due to ram compression. Ram pressure ratio is relatively small initially as forward speed is slow but increases as the speed increases. Ramjet is boosted up to substantially high speed using turbojet or rocket engine for getting desired thrust by ramjet. Ramjet are boosted up to speed of 300 km/hr.

The efficiency of ramjet largely depends upon the design of diffuser section.



(i) Schematic of ram jet

(ii) T-s representation of ram jet

Fig. 19.11 *Schematic and T–s diagram of ramjet*

Advantages of ramjet engine are:

(*i*) It has no moving parts and hence ramjet are better balanced.
(*ii*) It yields greater thrust per unit mass as compared to any other propulsion engine at supersonic speed.
(*iii*) It is much simpler in construction and light in weight.
(*iv*) It yields much greater thrust per unit frontal area at supersonic speeds. Best performance can be had at 1700 km/hr to 2200 km/hr speed range.
(*v*) Variety of fuels can be used in ramjet.
(*vi*) These are ideal propulsion device for aircraft missiles.

Disadvantages of ramjet engine are:

(*i*) Forward motion is very much necessary to realize ram compression.
(*ii*) Ram pressure ratio increases gradually.
(*iii*) Ramjets are unable to work at low flight speeds.

Ram compression results in decrease in velocity and increase of pressure of air passing through this diffuser section. At the end of diffuser section a pressure barrier is created and after this point fuel is injected through nozzles and ignited using spark plug. Combustion results in expansion of gases which is restrained due to pressure barrier on one side and so move out through exit nozzle with high velocity.

19.10 PULSE JET ENGINE

Pulse jet engine was developed by German scientist Paul Schmidt in 1930. It is quite similar to ramjet engine except the difference that pulse jet employs a non-return type mechanical valve of V-type for preventing flow of hot gases through diffuser. Pulse jet engine has diffuser section in which ram compression occurs and after diffuser section a grid of non-return valves is put for maintaining intermittent flow of compressed air. In combustion section fuel is atomized during injection and burnt using spark plug or igniters. Combustion of air and fuel results in combustion products having high pressure and temperature. Due to high pressure of combustion product non-return valve remains closed and causes flow through tail pipe so as to produce thrust. When combustion gases move out from combustion chamber then the pressure lowers down in this side as compared to pressure available in diffuser section. Due to this pressure differential the flow of compressed air again occurs into the combustion chamber which again burns and flows out from exit nozzle. Thus there occur the processes of suction, combustion and exhaust one after the other. Pulse jet engine tube shown in Fig. 19.12 has temperature varying continuously from inlet to exit. This pulse jet engine has advantages of being cheap compared to turbojet engine and produces static thrust. Pulse jet engine produces thrust more than drag at lower speed compared to ramjet. Pulse jet engines have disadvantages of noise, maintenance in view of mechanical operation of valves, high fuel consumption rate and vibration etc.

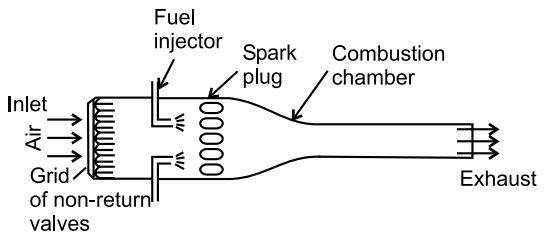


Fig. 19.12 *Pulse jet engine*

19.11 PRINCIPLE OF ROCKET PROPULSION

Rocket engines are non-air breathing engines and carry their own oxidiser for burning of fuel. Rocket propulsion is realized by the thrust produced by combustion products leaving exit nozzle. It has injection system for fuel and oxidizer followed by combustion chamber and exit nozzle as shown in Fig. 19.13.

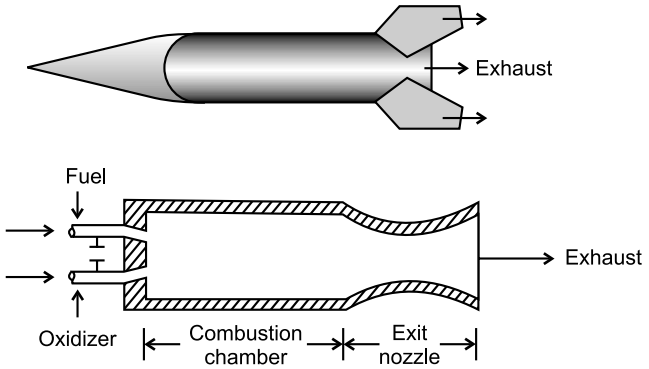


Fig. 19.13 *Layout for rocket engine*

In rocket engines the combustion products get discharged from the exit nozzle with supersonic velocity and thus have very high kinetic energy. Rocket gets desired thrust by the reaction available from the nozzle stream. Thrust is available due to change of momentum and pressure with which jet comes out.

Net thrust available; $$T = m'_p C_e + A_e (p_e - p_a)$$

where m'_p is mass flow rate of propellant, jet exit velocity C_e, area of exit nozzle A_e, pressure of exit jet p_e and atmospheric pressure is p_a.

Above expression shows that for maximizing thrust exit velocity should be maximized, pressure difference at exit $(p_e - p_a)$ should be maximized. Thus rocket would get maximum thrust when atmospheric pressure is not there *i.e.*, $p_a = 0$, which means maximum thrust would be available in vacuum.

Thrust could also be given in terms of rocket performance parameter called effective jet velocity.

$$T = m'_p C_e + A_e (p_e - p_a) = m'_p \cdot C_{ej}$$

Here C_{ej} is effective jet velocity which could be given as,

$$C_{ej} = C_e + \frac{A_e}{m'_p} (p_e - p_a)$$

Specific impulse can be given as

$$I_{sp} = \frac{T}{m'_p} = C_{ej}$$

i.e., Specific impulse is the thrust produced per unit mass flow rate of propellant.

Thrust power in case of rocket engine can be given as;

$$TP = T \cdot C_a = m'_p \cdot C_{ej} \cdot C_a$$

here C_a is the velocity with which rocket moves forward.

Propulsive efficiency: For rocket engine

$$\eta_{prop} = \frac{TP}{TP + \text{loss of kinetic energy}}$$

$$= \frac{m'_p \cdot C_{ej} \cdot C_a}{(m'_p C_{ej} \cdot C_a) + (m'_p (C_{ej} - C_a)^2/2)}$$

$$\eta_{prop} = \frac{2\,(C_a/C_{ej})}{1 + (C_a/C_{ej})^2}$$

19.12 ROCKET ENGINE

Rocket engines as described earlier can be classified based on type of propellant carried by them. These could be solid propellant and liquid propellant resulting into,

(*i*) Solid propellant rocket engine
(*ii*) Liquid propellant rocket engine.

Since Rocket engines carry propellant fuel and oxidizer along with, therefore, the characteristics of propellant have to be such as to give best performance with minimum oxidizer requirement. Desired properties of propellant for rocket engine are,

(*i*) propellant should have large heating value.
(*ii*) propellant should have high density so that storage space required is small.
(*iii*) propellant should be capable of having smooth ignition.
(*iv*) propellant should have stability and ease of handling and storage.
(*v*) propellant should be non-toxic and non-corrosive.
(*vi*) propellant should be environment friendly.

Applications of rocket engines have proved boon for our civilization as satellites in space which are serving air communication and other requirements are rocket engines. Rocket engines are also extensively used in spacecrafts, missiles, jet assisted air planes, pilotless aircraft, etc.

19.13 SOLID PROPELLANT ROCKET ENGINES

These rocket engines use solid propellants which burns using oxidizer present within it. There are two types of solid propellant rockets depending upon type of burning *i.e.*, restricted burning and unrestricted burning. Figure 19.14 shows the different arrangements in solid propellant rocket engines. Solid propellant has composition such that all essential requirements for combustion are met. Specific requirements of solid propellant are,

(*i*) Propellant should have sufficient compressive and impact strength at low temperature.
(*ii*) Propellant should give uniform burning.
(*iii*) Propellant should give high specific impulse.

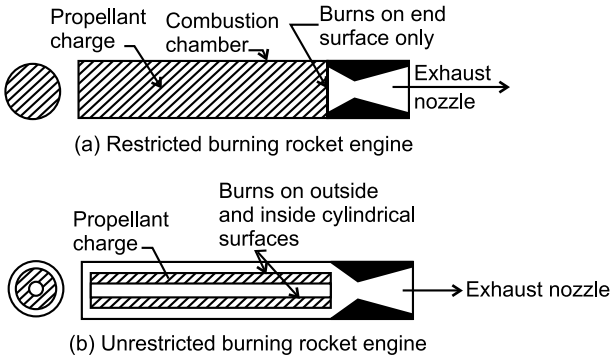


Fig. 19.14 *Solid propellant rocket engines*

Arrangement shows that it has seamless sturdy steel tube closed at one end. At the open end exit nozzle is provided and this may be single nozzle type or multi nozzle type exit. Propellant is filled in tube while in liquid state and gets solidified gradually upon cooling and fits tube completely. Solid propellant filled inside tube is burnt gradually so as to give combustion products for producing desired thrust. Restricted burning rocket engines have the propellant burnt only on its exposed surface towards exit nozzle. Gradually complete propellant is burnt and combustion products move out through exit nozzle. Thus restricted burning nozzles are very similar to cigarette whose burning begins at one end and goes till end. Restricted burning rockets are suited for small thrust applications for longer duration.

Unrestricted burning in rocket engine occurs such that all exposed surfaces of solid propellant are burnt simultaneously inside tube. Since burning of whole solid fuel is initiated so it gets burnt quickly and produces large thrust.

19.14 LIQUID PROPELLANT ROCKET ENGINES

Figure 19.15 shows the schematic of liquid propellant rocket engines. These contain liquid propellants stored in containers outside combustion chamber. Liquid propellant fuel is being fed using either pressure feed or pump feed to transfer fuel from storage tank to combustion chamber. Pressure feed system has pump driven by small turbine. This feed system is relatively cheap and simple compared to other feed systems.

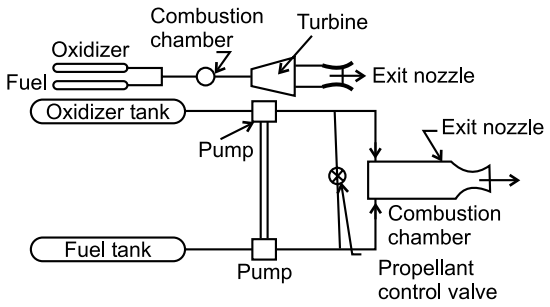


Fig. 19.15 *Liquid propellant rocket engines*

There could be mono-propellant or bi-propellant type rocket engines. Mono-propellant systems are heavier in view of additional quantity of fuel required. Schematic shows two liquid bi-propellant rocket system with rocket motor and propellant system etc. Rocket motor has propellant injector for injecting liquid fuel and oxidizer, ignition systems, combustion chamber and exit nozzle as shown. Liquid propellant is heated up by circulating it around the combustion chamber and nozzle walls for cooling it before being injected for combustion. Cooling is done so as to maintain temperature of walls low, so that excessive thermal stresses are not developed. Liquid propellants are selected based on specific impulse, availability, cost, ease of handling and density etc. There are number of oxidizer and liquid propellants available, namely, liquid oxygen + liquid hydrogen, liquid oxygen + ethyl alcohol, liquid oxygen + high quality gasoline, nitric acid + aniline etc.

EXAMPLES

1. *A jet propulsion engine moves with speed of 200 m/s at an altitude where total pressure and total temperature at inlet to compressor are 0.5 bar and 272 K. Isentropic efficiency of compressor and turbine are 0.84 and 0.82 respectively.*

Total pressure and temperature of gases at inlet to turbine are 3 bar and 1000 K and static back pressure of propulsion nozzle is 0.4 bar and nozzle efficiency based on total

pressure drop is 92%. Determine (i) power required to run compressor per kg of air, (ii) the air-fuel ratio, (iii) the total pressure of gas leaving turbine, (iv) thrust per unit air flow rate, (v) Plot the processes on h–s diagram.

Take $C_{p_{gas}} = 1.13$ kJ/kg.K, $\gamma_{gas} = 1.33$, $C_{p_{air}} = 1.005$ kJ/kg.K, $\gamma_{air} = 1.4$ and Calorific value = 41.84 MJ/kg of fuel.

Solution:

We know, $$\frac{T_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_{02} = 272\left(\frac{3}{0.5}\right)^{\left(\frac{1.4-1}{1.4}\right)}$$

$$T_{02} = 453.83 \text{ K}$$

Considering compressor efficiency,

$$\eta_{compr} = 0.84 = \frac{(T_{02} - T_{01})}{(T_{02'} - T_{01})} \Rightarrow T_{02'} = 488.46 \text{ K}$$

$$W_{compr} = C_{p_{air}}\,(T_{02'} - T_{01})$$

$$= 1.005\,(488.46 - 272) = 217.54 \text{ kW/kg}$$

Power required for compressor = 217.54 kW/kg ***Ans.***

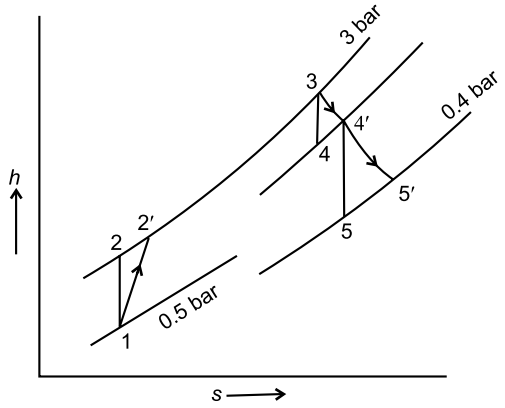


Fig. 19.16

In combustion chamber,

$$(m'_a + m'_f)\,C_{p_{gas}} \cdot T_{03} - m'_a\,C_{p_{air}} \cdot T_{02'} = m'_f \times \text{C.V}$$

or, $$\frac{m'_a}{m'_f} = \left\{\frac{\text{CV} - C_{p_{gas}} \cdot T_{03}}{C_{p_{gas}} \cdot T_{03} - C_{p_{air}} T_{02'}}\right\}$$

$$= \frac{(41840 - 1.13 \times 1000)}{(1.13 \times 1000 - 1.005 \times 488.46)}$$

$$\frac{m'_a}{m'_f} = 63.69$$

Air fuel ratio : 63.69 ***Ans.***

In this arrangement the power developed in turbine is used for driving compressor so,

$$C_{p_{\text{air}}}\,(T_{02'} - T_{01}) = C_{p_{\text{gas}}}\,(T_{03} - T_{04'})\left(1 + \frac{m'_f}{m'_a}\right)$$

$$\Rightarrow \quad 1.005\,(488.46 - 272) = 1.13\,(1000 - T_{04'})\left(1 + \frac{1}{63.69}\right)$$

$$\Rightarrow \quad T_{04'} = 810.46 \text{ K}$$

From turbine isentropic efficiency, $\eta_{\text{turb}} = \dfrac{(T_{03} - T_{04'})}{(T_{03} - T_{04})}$

$$\text{or} \quad 0.82 = \left(\frac{T_{03} - 810.46}{T_{03} - T_{04}}\right)$$

$$\Rightarrow \quad T_{04} = 768.85 \text{ K}$$

For isentropic expansion,

$$\frac{p_{04}}{p_{03}} = \left(\frac{T_{04}}{T_{03}}\right)^{(\gamma_{gas}/\gamma_{\text{gas}} - 1)}$$

Pressure of gas leaving turbine,

$$\Rightarrow \quad p_{04} = 3\left(\frac{768.85}{1000}\right)^{\left(\frac{1.33}{0.33}\right)} = 1.04 \text{ bar} \quad \textbf{\textit{Ans.}}$$

Considering T_{04} and p_{04} as static values,

$$T_s = T_{4'}\left(\frac{p_5}{p_4}\right)^{\frac{(\gamma - 1)}{\gamma}} = 810.46\left(\frac{0.4}{1.04}\right)^{\frac{0.33}{1.33}} = 639.4 \text{ K}$$

For exit nozzle, $\eta_{\text{nozzle}} = \dfrac{(T_{4'} - T_{5'})}{(T_{4'} - T_5)}$

$$0.92 = \frac{(810.46 - T_{5'})}{(810.46 - 639.4)}$$

$$\Rightarrow \quad T_{5'} = 653.08 \text{ K}$$

Exit jet velocity, $C_{5'} = \sqrt{2\,(h_{4'} - h_{5'})}$

$$= \sqrt{2\,C_{p_{\text{gas}}}(T_{4'} - T_{5'})}$$

$$= \sqrt{2 \times 1.13 \times (810.46 - 653.08) \times 1000}$$

$$C_{5'} = 596.4 \text{ m/s} = C_e$$

Thrust per kg of air per second

$$T = \left(1 + \frac{m'_f}{m'_a}\right) C_e - C_a$$

$$= \left(1 + \frac{1}{63.69}\right) \times 596.4 - 200$$

$$\text{Thrust} = 405.73 \text{ N/kg/s} \quad \textbf{\textit{Ans.}}$$

2. *A jet propulsion engine has compressor with pressure ratio 4 and compressed air enters into combustion chamber where combustion occurs so as to yield temperature of 500°C at turbine inlet. Actual temperature at inlet to combustion chamber is 10% more than that of isentropic compressor temperature rise. Exhaust from turbine is expanded up to atmospheric pressure of 1 bar. The ambient temperature is 285 K. Determine, (i) power required to drive compressor, (ii) air fuel ratio if calorific value of fuel is 43100 kJ/kg, (iii) static thrust developed per kg of air per second.*

Solution:

$$T_1 = 285 \text{ K}, p_1 = 1 \text{ bar}, T_3 = 773 \text{ K}$$

$$\frac{p_2}{p_1} = 4 \Rightarrow p_2 = 4 \text{ bar},$$

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}}$$

$$= 285 \times (4)^{0.286}$$

$$T_2 = 423.7 \text{ K}$$

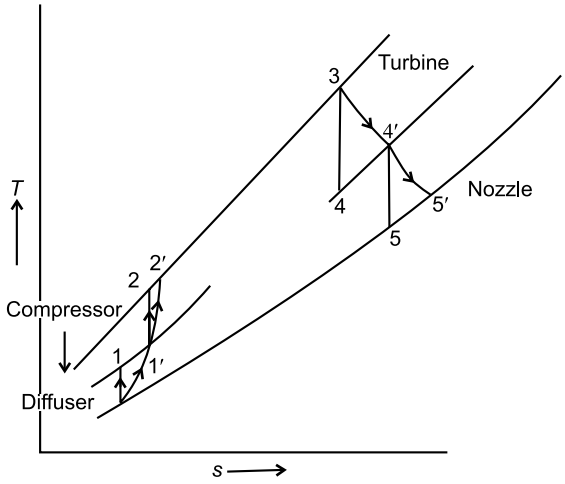


Fig. 19.17

Actual temperature rise, $(T_{2'} - T_1) = 1.1\,(T_2 - T_1)$

$\Rightarrow$ $T_{2'} = T_1 + 1.1\,(T_2 - T_1)$

$= 285 + 1.1\,(423.7 - 285)$

$T_{2'} = 437.57 \text{ K}$

Work required in compressor, $W_{\text{compr}} = C_{p_{air}}\,(T_{2'} - T_1)$

$= 1.005\,(437.57 - 285)$

$= 153.3$ kJ/kg of air.

Power required to drive compressor = 153.3 kW/kg of air ***Ans.***

Heat added in combustion chamber,

$$q_{\text{add}} = C_{p_{\text{air}}}\,(T_3 - T_{2'})$$

$$= 1.005\,(773 - 437.57)$$

$$q_{add} = 337.11 \text{ kJ/kg of air}$$

for air-fuel mass ratio being $\frac{m'_a}{m'_f}$;

$$\frac{m'_a}{m'_f} \times q_{add} = \text{C.V.}$$

$$\left(\frac{m'_a}{m'_f}\right) = \frac{43100}{337.11} = 127.85 \text{ kg of air / kg of fuel}$$

$$\text{Air-fuel ratio} = 127.85 \quad \textbf{\textit{Ans.}}$$

For isentropic expansion, $\frac{T_5}{T_3} = \left(\frac{p_1}{p_2}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}$

$$\Rightarrow \quad T_5 = T_3\left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = 773\left(\frac{1}{4}\right)^{0.286}$$

$$T_5 = 519.9 \text{ K}$$

$$\text{Enthalpy drop in nozzle} = (h_3 - h_5) - (h_{2'} - h_1)$$

$$= C_p\,(T_3 - T_5) - C_p\,(T_{2'} - T_1)$$

$$= 1.005\left\{(773 - 519.9) - (437.57 - 285)\right\}$$

$$= 101.03 \text{ kJ/kg of air}$$

Velocity of exit gas from nozzle, $C_e = \sqrt{2 \times 101.03 \times 1000}$

$$C_e = 449.5 \text{ m/s} \quad \textbf{\textit{Ans.}}$$

Thrust, $T = \left(1 + \frac{m'_f}{m'_a}\right) C_e - C_a$

for negligible velocity at inlet, *i.e.*, $C_a = 0$

$$T = \left(1 + \frac{1}{127.85}\right) 449.5 - 0$$

$$\textit{Thrust} = 453.02 \text{ N/kg of air/s} \quad \textbf{\textit{Ans.}}$$

3. *Following data refer to double spool turbofan engine in which low pressure turbine drives fan and high pressure turbine drives compressor.*

Ambient state : 1 bar, 15°C
Fan pressure ratio : 1.5
Overall pressure ratio : 28
By pass ratio, $\left(\frac{m'_p}{m'_c}\right)$ *: 5.5*
Mass flow rate of air : 200 kg/s.
Stagnation temperature at turbine inlet : 1300°C
Mechanical efficiency $\eta_{mech.} = 96\%$

Polytropic efficiency of fan and compressor = $\eta_{poly,\ fan} = \eta_{poly,\ compr.} = 90\%$

Polytropic efficiency of turbine, $\eta_{poly,\ turb} = 95\%$

Pressure loss in combustion chamber = 1.5 bar

Nozzle efficiency, $\eta_{nozzle} = 95\%$

Lower calorific value of fuel : 43100 kJ/kg

$$C_{p_{air}} = 1.005\ kJ/kg{\cdot}K$$

$$C_{p_{gas}} = 1.14\ kJ/kg{\cdot}K$$

$$\gamma_{gas} = 1.33$$

Determine air-fuel ratio, specific fuel consumption and specific thrust.

Solution:

Using the polytropic efficiency for compression and expansion the index of compression and expansion can be obtained.

For compression: $\left(\frac{n_e - 1}{n_e}\right) = \frac{1}{\eta_{poly,\ compr}}\left(\frac{\gamma - 1}{\gamma}\right)$

$$= \frac{1}{0.87}\left(\frac{1.4 - 1}{1.4}\right) = 0.328$$

For expansion: $\left(\frac{n_T - 1}{n_T}\right) = \eta_{poly,\ turb}\left(\frac{\gamma_{gas} - 1}{\gamma_{gas}}\right) = 0.90\left(\frac{0.33}{1.33}\right)$

$$= 0.223$$

In fan, $\frac{T_{02'}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\left(\frac{n_c - 1}{n_c}\right)}$; Take, $p_{01} = p_a$ and $T_{01} = T_a$

$$T_{02'} = 288\left(\frac{1.5}{1}\right)^{0.328}$$

$$T_{02'} = 328.96\ \text{K}$$

In compressor $\frac{T_{03'}}{T_{02'}} = \left(\frac{p_{03}}{p_{02}}\right)^{\left(\frac{n_T - 1}{n_T}\right)}$

$$T_{03'} = 328.96\left(\frac{28}{1.5}\right)^{0.223} = 631.82\ \text{K}$$

For flow of bypass atmospheric air through nozzle the nozzle may be checked for whether it is in choked state or not.

Critical pressure ratio for by pass air nozzle, $\frac{p_c}{p_{02}} = \left(\frac{2}{\gamma + 1}\right)^{\gamma/(\gamma - 1)}$

$$\frac{p_c}{p_{02}} = \left(\frac{2}{1.4 + 1}\right)^{\frac{1.4}{(1.4 - 1)}} \Rightarrow \frac{p_c}{p_{02}} = 0.528$$

$$\Rightarrow \qquad \frac{p_{02}}{p_c} = 1.89$$

Actual pressure ratio for this nozzle, $\frac{p_{02}}{p_a} = \frac{1.5}{1} = 1.5$

Since critical pressure ratio (1.89) is more than actual expansion pressure ratio (1.5) so it means that by pass air nozzle is not choked and p_8 may be taken equal to p_a.

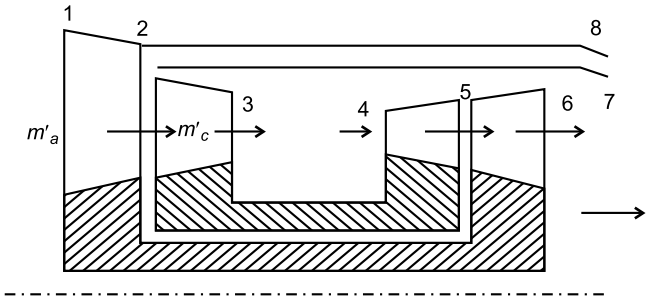


Fig. 19.18 *Twin spool turbofan engine.*

Using nozzle efficiency

$$T_{02'} - T_{8'} = \eta_{\text{nozzle}}\ T_{02'}\left[1 - \left(\frac{p_a}{p_{02}}\right)^{\frac{(\gamma-1)}{\gamma}}\right]$$

$$= 0.95 \times 328.96\left[1 - \left(\frac{1}{1.5}\right)^{1/3.5}\right]$$

$$T_{02'} - T_{8'} = 34.22 \text{ K}$$

Velocity at exit of nozzle,

$$C_8 = \sqrt{\left[2 \times 1.005 \times 10^3 \times 34.22\right]} = 26.2 \text{ m/s}$$

Mass flow rate of bypass air, $M'_b = \frac{m'_a\ B}{(B+1)} = \frac{200 \times 5.5}{(6.5)} = 169.23$ kg/s

Mass flow rate of hot gases, $m_{c'} = 200 - 169.23 = 30.77$ kg/s

Thrust available due to by pass air,

$$T_b = m'_b \times C_8$$
$$= 169.23 \times 262.26$$
$$T_b = 44.382 \text{ kN}$$

Applying energy balance on combustion chamber we get,

$$\left(1 + \frac{m'_f}{m'_c}\right) C_{p_{\text{gas}}}\ T_{04} = C_{p_{\text{air}}} \cdot T_{03'} + \frac{m'_f}{m'_c} \times LCV$$

$$\left(1 + \frac{m'_f}{m'_c}\right) \times 1.14 \times 1573 = (1.005 \times 631.82) + \left(\frac{m'_f}{m'_c}\right) \times 43100$$

$$\frac{m'_f}{m'_c} = 0.028$$

For HP turbine driving the HP compressor the work output from HP turbine may be equated to work required in HP compressor.

$$C_{p_{air}} (T_{03'} - T_{02'}) = \eta_{mech} \left(1 + \frac{m'_f}{m'_c}\right) \times C_{p_{gas}} (T_{04} - T_{05'})$$

$$1.005 (631.82 - 328.96) = 0.96 (1 + 0.028) \times 1.14 \times (1573 - T_{05'})$$

$$T_{05'} = 1302.45 \text{ K}$$

LP turbine is used for driving fan so, output of LP turbine goes for driving fan.

$$C_{p_{gas}} \cdot \eta_{mech} (T_{05'} - T_{06'}) = (1 + B) C_{p_{air}} (T_{02'} - T_{01})$$

$$\Rightarrow 1.14 \times 0.96 (1302.45 - T_{06'}) = (1 + 55) \times 1.005 (328.96 - 288)$$

$$T_{06¢} = 1057.96 \text{ K}$$

Pressure at inlet to HP turbine, $p_{04} = p_{03} - \Delta p_{loss} = 28 - 1.5 = 26.5$ bar

Pressure at exit of LP turbine, (p_{06}) may be estimated as under,

$$p_{05} = p_{04} \left(\frac{T_{05'}}{T_{04}}\right)^{\left(\frac{n_T}{n_T - 1}\right)} = 26.5 \left(\frac{1302.45}{1573}\right)^{1/0.223} = 11.37 \text{ bar}$$

and

$$p_{06} = p_{05} \left(\frac{T_{06'}}{T_{05'}}\right)^{\left(\frac{n_T}{n_T - 1}\right)} = 11.37 \left(\frac{1057.96}{1302.45}\right)^{1/0.223} = 4.476 \text{ bar}$$

Critical expansion pressure ratio of nozzle handling hot gases,

$$= \left(\frac{\gamma_{gas} + 1}{2}\right)^{\frac{\gamma_{gas}}{(\gamma_{gas} - 1)}} = \left(\frac{2.33}{2}\right)^{\frac{1.33}{0.33}}$$

Critical pressure ratio = 1.85

Since critical pressure ratio is less than expansion ratio in nozzle for hot gas so nozzle will be in choked state.

$$\frac{p_{06}}{p_a} > \text{Critical pressure ratio}$$

so,

$$\frac{p_{06}}{p_7} = 1.85$$

For exit nozzle $\Rightarrow$

$$p_7 = 2.42 \text{ bar}$$

$$(T_{06'} - T_{7'}) = \eta_{nozzle} T_{06'} \left[1 - \left(\frac{p_7}{p_{06}}\right)^{\left(\frac{\gamma_{gas} - 1}{\gamma_{gas}}\right)}\right]$$

$$= 0.95 \times 1057.96 \left[1 - \left(\frac{1}{1.85}\right)^{\left(\frac{0.33}{1.33}\right)}\right]$$

$$(T_{06'} - T_{7'}) = 142.28 \text{ K}$$

Velocity of hot gases at exit of nozzle,

$$C_7 = \sqrt{2 C_{p_{gas}} \times (T_{06'} - T_{7'})}$$

$$C_7 = \sqrt{2 \times 1.14 \times 1000 \times 142.28}$$

$$C_7 = 569.55 \text{ m/s}$$

$$\text{Thrust due to hot gases} = m_{c'} \times C_7$$

$$= 30.77 \times 569.55 = 17.525 \text{ kN}$$

$$\text{Total thrust} = \text{thrust due to bypass air + thrust due to hot gases}$$

$$= 44.382 + 17.525 = 61.91 \text{ kN}$$

$$\text{Specific thrust} = \frac{61.91}{200} = 0.309 \text{ kN/kg/s} \quad \textbf{\textit{Ans.}}$$

$$\text{Specific fuel consumption} = \frac{\left(\frac{m'_f}{m'_c}\right) \times m'_c \times 3600}{\text{Total thrust}}$$

$$= \frac{0.028 \times 30.77 \times 3600}{61.90 \times 10^3} = 0.05 \text{ kg/h.N} \quad \textbf{\textit{Ans.}}$$

4. *A turbojet plane flies with speed of 1000 kmph and inducts air at the rate of 50 kg/s. Air fuel ratio is 52 and the expansion of gases occurs up to ambient pressure. Lower calorific value of fuel is 43100 kJ/kg. For maximum thrust the flight to jet velocity ratio is 0.5. Determine (i) jet velocity, (ii) maximum thrust, (iii) specific thrust, (iv) thrust power, (v) propulsive, thermal and overall efficiencies, (vi) Specific fuel consumption.*

Solution:

Velocity of turbojet plane = 1000 kmph = 277.78 m/s

For maximum thrust power, Velocity of jet at exit, $C_e = \frac{277.78}{0.5}$

$$C_e = 555.56 \text{ m/s}$$

Jet velocity = 555.56 m/s *****Ans.*****

Air will be admitted in plane with the velocity with which plane moves forward.

Thrust, $$T = (m'_a + m'_f)\, C_e - m'_a \cdot C_a$$

$$= \left\{\left(\left(50 + \frac{50}{52}\right) \times 555.56\right) - (50 \times 277.78)\right\}$$

$$T = 14423.19 \simeq 14.42 \text{ kN} \quad \textbf{\textit{Ans.}}$$

$$\text{Specific thrust} = \frac{14423.19}{50} = 288.46 \text{ N/kg/s} \quad \textbf{\textit{Ans.}}$$

$$\text{Thrust power} = 14.42 \times 277.78 = 4005.58 \text{ kW} \quad \textbf{\textit{Ans.}}$$

$$\text{Propulsive efficiency } \eta_{\text{prop}} = \frac{2}{1 + \left(\frac{1}{0.5}\right)} = 0.6666 \approx 66.66\% \quad \textbf{\textit{Ans.}}$$

$$\text{Thermal efficiency, } \eta_{\text{th}} = \frac{(1 + FA)\, C_e^2 - C_a^2}{2 \times FA \times \text{Calorific value}}$$

$$= \frac{\left(1+\frac{1}{52}\right)(555.56)^2 - (277.78)^2}{2 \times \frac{1}{52} \times 43100 \times 10^3}$$

$$= 0.1432 \approx 14.32\% \quad \textbf{\textit{Ans.}}$$

Overall efficiency $\eta_{overall} = \eta_{prop} \times \eta_{th}$

$$= 0.1432 \times 0.6666$$

$$= 0.0954 = 9.54\% \text{ Ans.}$$

Specific fuel consumption $= \dfrac{m'_f \times 3600}{\text{Thrust}}$

$$= \frac{\left(\frac{50}{52}\right) \times 3600}{14423.19} = 0.2399 \text{ kg/h N} \quad \textbf{\textit{Ans.}}$$

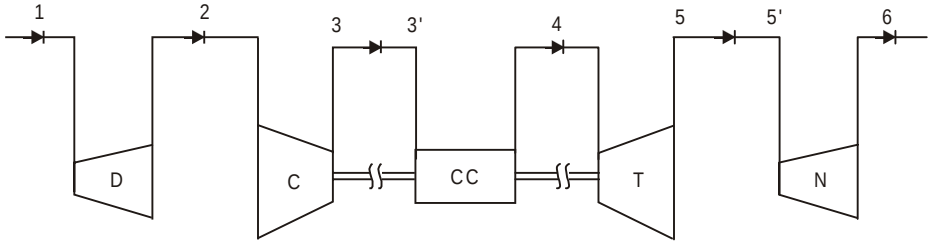
5. *A turboject engine has air entering at 2.2 bar, – 53°C and 260 m/s. Turbojet has compressor pressure ratio of 12, turbine inlet temperature of 1000°C, pressure at exit of nozzle as 2.2 bar, isentropic processes in diffuser & nozzle and isentropic efficiecies of compressor and turbine as 85% and 88% respectively. Neglecting pressure drop in combustion chamber determine the pressure and temperature at all salient points and the velocity at exit of nozzle. Also consider air as working fluid throughout.*

Solution:

$p_1 = 2.2$ bar

$T_1 = 220$ K or -53°C

$C_1 = 260$ m/s



D : Diffuser, CC : Combustion chamber, C : Compressor
T : Turbine, N : Nozzle, 3′ and 5′ are actual states.

Fig. 19.19

Stagnation properties at inlet state can be estimated for knowing the states at point 2 which is inlet of compressor. In diffuser section kinetic energy will be getting transformed to modify pressure and temperature at inlet to diffuser leading to negligible kinetic energy at inlet to compressor.

$$T_2 = T_1 + \frac{C_1^2}{2C_p} = 220 + \frac{(260)^2}{(2 \times 1.005 \times 10^3)} = 253.63\text{K}$$

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)} = 2.2\left(\frac{253.63}{220}\right)^{3.5} = 3.62 \text{ bar}$$

$$p_3 = p_2 \times 12 = 3.62 \times 12 = 43.44 \text{ bar} = p_4$$

$$T_3 = T_2\left(\frac{p_3}{p_2}\right)^{\frac{(\gamma-1)}{\gamma}} = 253.63(12)^{\frac{(1.4-1)}{1}} = 516.23\text{K}$$

Compressor isentropic efficiency, $0.85 = \dfrac{T_3 - T_2}{T_{3'} - T_2}$

$$0.85 = \frac{516.23 - 253.63}{T_{3'} - 253.63} \Rightarrow T_{3'} = 562.57\text{K}$$

In case of turbojet engine, under ideal condition the work requirement of compressor should be equal, $W_{comp} = W_{trubine}$

$$C_p(T_{3'} - T_2) = C_p(T_4 - T_{5'})$$

Given turbine inlet temperature, $T_4 = 1273$K

$$(562.57 - 253.63) = (1273 - T_{5'})$$

$$\Rightarrow \quad T_{5'} = 964.06 \text{ K}$$

Turbine iseutropic efficiency, $0.88 = \dfrac{(T_4 - T_{5'})}{(T_4 - T_5)}$

$$0.88 = \frac{(1273 - 964.06)}{(1273 - T_5)} \Rightarrow T_5 = 921.93 \text{ K}$$

Corresponding to this temperature T_5, pressure p_5 can be determined

$$p_5 = p_4\left(\frac{T_5}{T_4}\right)^{\gamma/(\gamma-1)} = 43.44\left(\frac{921.93}{1273}\right)^{3.5}$$

$$p_5 = 14.042 \text{ bar}$$

$$T_2 = 260 + \frac{(200)^2}{2\times1005}$$

$$T_2 = 279.9 \text{ K}$$

Considering diffuser efficiency,

$$0.9 = \frac{T_2 - T_1}{T_{2'} - T_1}$$

$$\Rightarrow T_{2'} = 282.11 \text{ K}$$

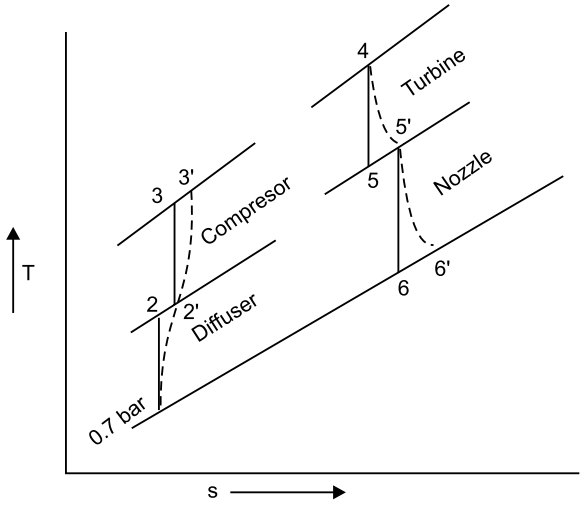


Fig. 19.20

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{(\gamma-1)}{\gamma}} = 282.11(8)^{0.286}$$

Considering compressor efficiency, $0.8 = \dfrac{T_3 - T_{2'}}{T_{3'} - T_{2'}}$

$$0.8 = \frac{511.33 - 282.11}{(T_{3'} - 282.11)}$$

$$T_{3'} = 568.635 \text{ K}$$

Turbine inlet temperature, $T_4 = 1000\text{K}$

Considering energy balance on combustion chamber;

$$m_f \times \text{CV} + m_3 \times C_p \times T_{3'} = (m_3 + m_f) \times C_p \times T_4$$

$$\text{CV} + \left(\frac{m_3}{m_f}\right) C_p T_{3'} = \left(1 + \frac{m_3}{m_f}\right) C_p \cdot T_4$$

$$\left(\frac{m_3}{m_f}\right)(C_p T_{3'} - C_p T_4) = CpT_4 - \text{CV}$$

Air–Fuel ratio, $\left(\dfrac{m_3}{m_f}\right) = \dfrac{(C_p T_4 - \text{CV})}{(C_p T_{3'} - C_p T_4)}$

Temperature at exit of nozzle for given exit pressure of 2.2 bars

$$p_6 = 2.2 \text{ bar; } T_6 = T_5\left(\frac{p_6}{p_5}\right)^{\frac{(\gamma-1)}{\gamma}}$$

$$T_6 = 921.93\left(\frac{2.2}{14.042}\right)^{\frac{(1.4-1)}{1.4}}$$

$$T_6 = 542.83 \text{ K}$$

Velocity at exit of nozzle,

$$\frac{C_6^2}{2C_p} + T_6 = T_5$$

$$C_6 = \sqrt{2(T_5 - T_6)C_p}$$

$$C_6 = 872.92 \text{ m/s}$$

Velocity of exit of nozzle = 872.92 m/s. ***Ans.***

6. *A jet engine plane is moving at 200 m/s and admits air at 0.7 bar, 260 K in diffuser section having inlet cross section area of 1000 cm^2 from where it enters compressor with negligible velocity. Compressor pressure ratio is 8 and turbine inlet temperature is 1000°C. Combustion gases expand in turbine and subsequently leave nozzle at 0.7 bar. Efficiencies of nozzle section, diffuser section, compressor and turbine are 0.9, 0.9, 0.8 and 0.8 respectively. Fuel used in combustion chamber has calorific value of 45000 kJ/kg of fuel and pressure drop in combustion chamber is 0.15 bar. Determine the fuel air ratio, specific thrust and total thrust available. Neglect change in specific heat of air when transformed into combustion products and treat it as air. Take C_p = 1.005 kJ/kg·K.*

Solution: Temperature at inlet to compressor

$$T_2 = T_1 + \frac{C_1^2}{2C_p}$$

$$\frac{m_3}{m_f} = \frac{(45000 - (1000 \times 1.005)}{((1.005 \times 1000) - (1.005 \times 568.635))}$$

Air-Fuel ratio = 101.48 : 1

Turbine inlet pressure, $p_4 = p_3 - 0.15 = 7.248 - 0.15 = 7.098$ bar

We know that in jet engines normally the turbine output should be just sufficient to run compressor; so $W_{\text{compressor}} = W_{\text{turbine}}$

$$C_p(T_{3'} - T_{2'}) = C_p(T_4 - T_{5'})$$

$$(568.635 - 282.11) = (1000 - T_{5'})$$

$$\Rightarrow \quad T_{5'} = 713.475\text{K}$$

Considering turbine efficiency

$$0.8 = \frac{T_4 - T_{5'}}{T_4 - T_5} = \frac{1000 - 713.475}{1000 - T_5}$$

$$\Rightarrow \quad T_5 = 641.84\text{K}$$

Considering adiabatic expansion

$$p_5 = p_4\left(\frac{T_5}{T_4}\right)^{\frac{\gamma}{(\gamma-1)}} = 7.098\left(\frac{641.84}{1000}\right)^{3.5} = 1.5036 \text{ bar}$$

In nozzle exit pressure is given, so, $p_6 = 0.7$ bar

$$T_6 = T_{5'}\left(\frac{p_6}{p_5}\right)^{\frac{(\gamma-1)}{\gamma}} = 713.475\left(\frac{0.7}{1.5036}\right)^{0.286} = 573.35\text{K}$$

Considering nozzle efficiency; $0.9 = \dfrac{T_{5'} - T_{6'}}{T_{5'} - T_6} = \dfrac{(713.475 - T_{6'})}{(713.475 - 573.35)}$

$$T_{6'} = 587.36 \text{ K}$$

Velocity at exit of nozzle, $C_6 = \sqrt{2C_p(T_{5'} - T_{6'})}$

$= \sqrt{2\times1005(713.475 - 587.36)}$

$C_6 = 503.48$ m/s

Volume flow rate of air $= 200 \times 1000 \times 10^{-4} = 20$ m^3/s

$$\text{Mass flow rate} = \frac{p \cdot v}{RT} = \frac{0.7\times10^2\times20}{0.287\times260} = 18.76 \text{ kg/s}$$

$$\text{Specific thrust} = \left(1 + \frac{m_f}{m_3}\right)\times C_6$$

Specific thrust = 508.44 N/kg of air/s ***Ans.***

Total thurst = 18.76 × 508.44 = 9538.33N ***Ans.***

7. *An aeroplane flying at speed of 250 m/s, running on jet engine inducts air at 0.3 bar and –20°C from atmosphere, compressor pressure ratio is 6 and the turbine inlet temperature is 700 °C. Velocity at exit of turbine is 180 m/s, turbine and compressor efficiencies are 0.85 each and jet engine efficiency is 90%. Thurst power produced is 800kW, and calorific value of fuel is 43000 kJ/kg. Considering following properties for air and gas determine overall efficiency, rate of air consumption, power developed by turbine, and exit area of jet.*

For air, $C_{p,air} = 1.005$ kJ/kg·K, $\gamma_{air} = 1.4$, $R = 0.287$ kJ/kg·K

For gas, $C_{p,gas} = 1.087$ kJ/kg·K, $\gamma_{gas} = 1.33$

Solution: Let mass flow rate of air and fuel be m_{air} & m_{fuel} kg/s

Writing down energy balance on combustion chamber;

$(m_{fuel} \times CV) + (m_{air}\cdot C_{pair}\cdot T_{2'}) = (m_{fuel} + m_{air})\cdot C_{pgas}.T_3$

In diffuser

$$T_1 = T_0 + \frac{C_0^2}{2C_{pair}} = 253 + \frac{(250)^2}{2\times1005} = 284.09\text{K}$$

$$p_1 = p_0\left(\frac{T_1}{T_0}\right)^{\gamma/(\gamma-1)} = 0.31 \text{ bar}$$

Let, $\left(\frac{m_{fuel}}{m_{air}} = FA\right)$

Thus energy balance on combustion chamber;

$FA \times CV = (1 + FA)C_{p,gas}\cdot T_3 - C_{p,air}\, T_{2'}$

In compressor, $T_2 = T_1(6)^{0.286} = 474.25$K

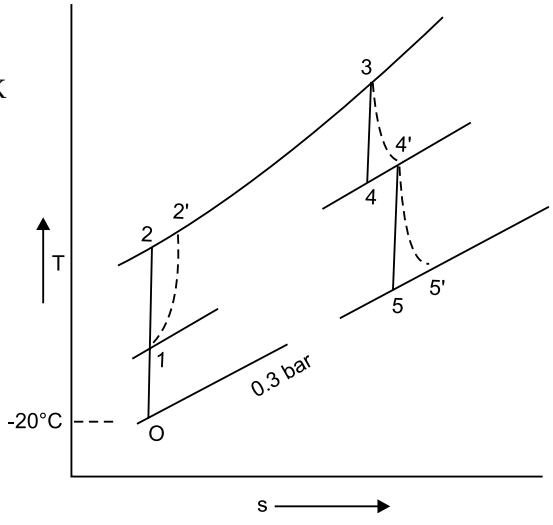


Fig. 19.21

$$T_{2'} = T_1 + \frac{(T_2 - T_1)}{\eta_{\text{compr.}}}$$

$$= \frac{284.09 + (474.25 - 284.09)}{0.85}$$

$$T_{2'} = 507.81\text{K},\ T_3 = 973\ \text{K}$$

$p_2 = p_1 \times$ pressure ratio $= 0.31 \times 6 = 1.86$ bar $= p_3$

Substituting in energy balance on combustion chamber;

$$FA \times 43000 = \{(1 + FA) \times 1.005 \times 973\} - (1.087 \times 507.81)$$

$$FA = 0.01013 \text{ or Air – fuel ratio} = \mathbf{98.67 : 1} \text{ Ans.}$$

Ideally, $W_{\text{compr}} = W_{\text{turb}}$

$$C_{\text{p,air}} = (T_{2'} - T_1) = c_{\text{pgas}}\,(1 + FA) \times (T_3 - T_{4'})$$

$$1.005(507.81 - 284.09) = 1.087(1 + 0.01013)\ (973 - T_{4'})$$

$$T_{4'} = 768.23\ \text{K}$$

Isentropic expansion in turbine yields,

$$\eta_{\text{turb}} = \frac{T_3 - T_{4'}}{T_3 - T_4}$$

$$\Rightarrow \qquad 0.85 = \frac{(973 - 768.23)}{(973 - T_4)} \Rightarrow T_4 = 732.09\ \text{K}$$

$$p_4 = p_3 \times \left(\frac{T_4}{T_3}\right)^{\frac{\gamma_{\text{gas}}}{(\gamma_{\text{gas}} - 1)}}$$

$$p_4 = 1.86\left(\frac{732.09}{973}\right)^{\frac{1.33}{(1.33-1)}} \quad p_4 = 0.5909\ \text{bar}$$

In exit nozzle jet, $T_5 = T_{4'}\left(\frac{p_5}{p_4}\right)^{\left(\frac{\gamma_{\text{gas}} - 1}{\gamma_{\text{gas}}}\right)}$

$$T_5 = 768.23\left(\frac{0.3}{0.5909}\right)^{\frac{(1.33-1)}{1.33}}$$

$$T_5 = 649.3\text{K}$$

Jet engine efficiency $= 0.9 = \dfrac{(C_5^2/2)}{C_{p\ \text{gas}}(T_{4'} - T_5) + (C_{4'}^2/2)}$

Given $C_{4'} = 180$ m/s $\Rightarrow 0.9 = \dfrac{(C_5^2/2)}{\{1087(786.23-649.3)+(180^2/2)\}}$

Nozzle exit velocity. $C_5 = 511.72$ m/s.

$$\text{Overall efficiency} = \frac{\{(1+FA)C_5 - C_0\}\cdot C_0}{\{FA\times CV\}}$$

$$= \frac{\{(1+0.01013)\times 511.72 - 250\}\times 250}{(0.01013\times 43000)\times 10^3}$$

$$= 0.1532 \text{ or } 15.32\%$$

It is given that engine develops thrust power of 800 kW.

$$\text{Hence, Thrust power} = 800 = \frac{\{(1+FA)C_5 - C_0\}, m_{air}\times C_0}{1000}$$

$$\Rightarrow \quad m_{air} = 11.99 \text{ kg/s}$$

Rate of air consumption = 11.99 kg/s Ans.

Power produced by turbine $= m_{air}\,(1 + FA)C_{p\,gas}(T_3 - T_{4'})$

$= 11.99(1 + 0.01013) \times 1.087(973 - 768.23)$

Power produced by turbine = 2695.83 kW ***Ans.***

The actual state at exit of nozzle jet may not be result of isentropic expansion in nozzle, let actual state at exit of nozzle be 5′,

$$C_{p,gas}(T_{4'} - T_{5'}) = \frac{C_5^2 - C_{4'}^2}{2}$$

$$1.087 \times 10^3(768.23 - T_{5'}) = \frac{(511.72)^2 - (180)^2}{2}$$

$$T_{5'} = 662.68\text{K}$$

Density of exhaust gases, $\rho_{5'} = \dfrac{p_{5'}}{RT_{5'}} = \dfrac{0.3\times 10^2}{0.287\times 662.68}$

$= 0.1577\ m^3/\text{kg}$

Let cross-sectional area of jet at exit be A_j m^2.

$$A_j \times C_5 \times \rho_{5'} = m_{air}(1 + FA)$$

$$A_j = \frac{11.99(1+0.01013)}{511.72\times 0.1577} = 0.15\text{m}^2$$

Jet exit area = 0.15m^2 ***Ans.***

8. *A jet aeroplane flies at speed of 900 km/h. The density of air at this altitude is 0.15 kg/m^3 and drag on plane is 6800kW. Considering jet plane to have 2 jets and engine working on turbo-prop system with propulsive efficiency of 56% determine the absolute velocity of jet and diameter of jet.*

Solution:

$$\text{Velocity of aeroplane} = \frac{900\times1000}{3600} = 250 \text{ m/s}$$

$$\text{Propulsive efficiency} = \frac{2C_a}{(C_e + C_a)}$$

Here C_a = 250 m/s, C_e = Relative velocity of jet leaving

$$0.56 = \frac{2\times250}{(C_e + 250)}$$

$$C_e = 642.86 \text{ m/s}$$

$$\text{Absolute velocity of jet} = C_e - C_a = 642.86 - 250 = 392.86 \text{ m/s}$$

$$\text{Drag on plane or thrust} = m'(C_e - C_a)$$

$$.6800 = m'(642.86 - 250)$$

$$m' = 17.31 \text{ kg/s}$$

$$\text{Volume flow rate} = \frac{17.31}{0.15} = 115.4 \text{ m}^3\text{/s}$$

Let jet diameter of jet be d_j;

$$\text{Volume flow rate} = 115.4 = 2\times\frac{\pi}{4}d_j^2\times C_e$$

$$\Rightarrow \quad d_j = 0.338 \text{ m}$$

Jet diameter = 33.8 cm ***Ans.***

9. *An aeroplane is flying at speed of 900 km/hr. It is run by turbojet engine. The plane has wing area of 2 × 10^5 cm^2, drag coefficient is 0.018 and jet velocity is 550 m/s. Considering C_p = 1.005 kJ/kg. K for air and density ratio between STP and flight altitude being 0.4 determine specific thrust.*

Solution:

$$\text{Density at STP} = \frac{1.01325\times10^2}{0.287\times288} = 1.226 \text{ kg/m}^3$$

$$\text{Density of air at altitude} = 0.4 \times 1.226 = 0.488 \text{ kg/m}^3$$

$$\text{Thrust on Aeroplane} = c_d \times A \times \rho \times \frac{C_a^2}{2}$$

$$= 0.018\times(2\times10^5\times10^{-4})\times0.488\times\frac{(900\times10^3/3600)^2}{2}$$

$$= 5490 \text{ N or } 5.49 \text{ kN}$$

Jet velocity, $C_e = 550$ m/s

Thurst can also be given as, $= m'(C_e - C_a)$

$$5490 = m'\left(550 - \left(\frac{900\times10^3}{3600}\right)\right)$$

$$m' = 18.3 \text{ kg/s}$$

$$\text{Specific thrust} = \frac{5490}{18.3} = 300 \text{ N/kg of air} \quad \textbf{\textit{Ans.}}$$

10. *An air craft running with speed of 900 km/hr uses turbo– jet engine taking air at rate of 55 kg/s. The air-fuel ratio is 85 and combustion efficiency is 0.96. Fuel has lower calorific value of 43000 kJ/kg. The turbo jet engine nozzle has isentropic enthalpy change of 220 kJ/kg and velocity coefficient is 0.95. Calculate propulsive power, thrust power, propulsive efficiency, overall efficiency, fuel flow rate and thrust neglecting fuel mass addition.*

Solution: Given $C_a = 900$ km/hr $= \dfrac{900\times1000}{3600} = 250$ m/s

$$\text{Jet velocity} = \text{Velocity coeff} \times \sqrt{2\times\text{Enthalpy change in nozzle}}$$

$$= 0.95\times\sqrt{2\times220\times10^3}$$

$$C_e = 615.67 \text{ m/s}$$

$$\text{Specific thrust per kg of air} = 1 \times (615.67 - 250) = 400.67 \text{ N/kg air}$$

$$\text{For given Air-Fuel ratio, fuel flowrate} = \left(\frac{1}{85}\right)\times3600\times55$$

$$\text{Fuel flow rate} = 2329.4 \text{ kg/hr}$$

$$\text{Specific fuel consumption} = \frac{2329.4}{400.67\times55} = 0.1057 \text{ kg/N·hr}$$

$$\text{Thrust power} = m'(C_e - C_a) \times C_a = 55 \times 250 \times (615.67 - 250)$$

$$= 5027962.5 \text{ W or } 5027.96 \text{ kW}$$

$$\text{Propulsive power} = \frac{m'(C_e^2 - C_a^2)}{2} = \frac{55\times\left\{(615.67)^2 - (250)^2\right\}}{2}$$

$$= 8705112.59 \text{ W or } 8705.11 \text{ kW} \quad \textbf{\textit{Ans.}}$$

$$\text{Propulsive efficiency} = \frac{\text{Thrust power}}{\text{Propulsive power}} = \frac{5027.96}{8705.11} = 0.5776 \text{ or } 57.76\% \quad \textbf{\textit{Ans.}}$$

$$\text{Overall efficiency} = \frac{\text{Thrust power}}{\text{Heat supplied}} = \frac{5027.96}{55\times\left(\frac{1}{85}\right)\times43000\times0.96}$$

$$= 0.1882 \text{ or } 18.82\% \quad \textbf{\textit{Ans.}}$$

EXERCISE

19.1 Describe the principle of jet propulsion.

19.2 Classify the jet propulsion engines.

19.3 Define and give mathematical expressions for thrust power, propulsive power and propulsive efficiency.

19.4 Compare turbojet engine with other jet propulsion engines.

19.5 Compare between solid propellent rocket engine and liquid propellant rocket engines.

19.6 Discuss the working of turboprop, ramjet and pulse jet engines.

19.7 Determine specific fuel consumption and propulsive efficiency for the following turbojet engine.

Forward speed of engine : 862.5 km/hr

Ambient pressure and temperature : 0.265 bar, –49.8°C

Compressor pressure ratio : 10

Stagnation temperature at turbine inlet : 1400 K

Lower calorific value of fuel : 4.31×10^5 kJ/kg

Isentropic efficiency of intake : 0.93

Isentropic efficiency of compressor : 0.87

Isentropic efficiency of turbine : 0.90

Isentropic efficiency of exit nozzle : 0.95

Mechanical efficiency : 0.98

Combustion efficiency : 0.98

Combustion pressure loss : 4% of compressor exit pressure

C_{p_a} = 1.005 kJ/kg·k, C_{p_g} = 1.148 kJ/kg·K, γ = 1.33

[0.13 kg/h·N, 0.57]

19.8 Considering a jet propulsion unit to have isentropic compression and expansion and heat supply at constant pressure show that thrust developed per kg of air per second for negligible velocity of approach can be given by; $[2C_p \cdot T_a (t-1) (r_p^{\frac{(\gamma-1)}{\gamma}} - 1)]$

Here t is the ratio of absolute temperature after combustion and before combustion, r_p is compression ratio and T_a is absolute atmospheric temperature.

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Multiple Answer Type Questions

1. Thermodynamic system refers to a portion of
(a) Universe (b) Surroundings
(c) Control surface (d) None of these

2. Units of temperature, mass, time, length and energy in SI system of units are given by
(a) Celsius, kilogram, second, metre and watt
(b) Kelvin, kilogram, second, metre and joule
(c) Kelvin, gram, hour, metre and joule
(d) Kelvin, kilogram, second, centimetre, joule

3. Which of the following set has all intensive properties?
(a) Pressure, volume, energy, heat, specific volume
(b) Pressure, volume, energy, heat density
(c) Pressure, volume, temperature, density, specific volume
(d) Pressure, volume, heat, density, specific volume

4. Which of the following sets has all open systems?
(a) Window air conditioner, scooter engine, thermometer
(b) Boiler, gas turbine, compressor
(c) Pump, thermosflask, refrigerator
(d) None of these

5. Locii of the series of state change during heat addition to a gas shall be,
(a) Process (b) Path
(c) State change (d) All of the above

6. In engineering thermodynamics the macroscopic approach is followed and it may also be called as:
(a) Statistical thermodynamics (b) Classical thermodynamics
(c) Microscopic study (d) None of these

7. Time, length, mass and temperature are:
(a) Primary dimensions (b) Secondary dimensions
(c) Both of above (d) None of these

8. Difference between the pressure of a fluid and the pressure of atmosphere is called as:
(a) Absolute pressure (b) Barometric pressure
(c) Gauge pressure (d) None of these

9. Which of the following sets has all properties as point functions?
(a) Pressure, temperature, heat
(b) Entropy, enthalpy, work
(c) Temperature, enthalpy, internal energy
(d) Heat, work, enthalpy

10. Which of the following is not property of thermodynamic system?
(a) Temperature (b) Work
(c) Pressure (d) Internal energy

11. At a given temperature and pressure all molecules of gas occupy the same volume. This is called
(a) Charle's law (b) Boyle's law
(c) Avagadro's law (d) Regnault's law

12. At constant pressure the volume of gas is directly proportional to its temperature. This refers to
(a) Charle's law (b) Boyle's law
(c) Dalton's law (d) None of these

13. If temperature in degree Celsius is 37° then the equivalent temperature in Fahrenheit will be:
(a) 98.6 (b) 99
(c) 98.3 (d) 98

14. The characteristic gas constant R and universal gas constant $\overline{R}$ can be related through molecular weight M as under,
(a) $R = \overline{R} \times M$ (b) $\overline{R} = R \times M$
(c) $\overline{R} = R + M$ (d) $R = \overline{R} + M$

15. 'Work' done upon the system and 'heat' added to the system shall have sign convention as shown respectively.
(a) (–) and (–) (b) (–) and (+)
(c) (+) and (+) (d) (+) and (–)

16. In a constant volume gas thermometer the thermometric property used is
(a) length (b) volume (c) mass (d) resistance

17. A car's tyre become hotter due to their exposure to sunlight. Considering air in tyre as system determine sign of heat and work interactions.
(a) (+) and (+) (b) (–) and (+)
(c) (–) and (–) (d) (+) and (–)

18. Specific heats at constant pressure (C_p) and at constant volume (C_v) can be related by
(a) $C_p = C_v$ (b) $C_p < C_v$
(c) $C_p > C_v$ (d) $C_p + C_v = 0$

19. In reference to thermodynamic equilibrium it is required to have,
(a) Mechanical equilibrium
(b) Chemical equilibrium
(c) Thermal equilibrium
(d) Mechanical, chemical and thermal equilibrium

20. The real gas follows the modified form of gas equation involving compressibility factor Z and may be given by:
(a) $P^{-} = Z\overline{R}T$ (b) $P^{-} = \dfrac{\overline{R}T}{Z}$
(c) $P^{-} = \overline{R}T$ (d) None of these

21. If ideal gas equation $p^{-} = \overline{R}T$ is used to model a real gas then modeling will becoming more accurate when

(a) Pressure decreases, temperature decreases and molar mass decreases
(b) Pressure decreases, temperature increases and molar mass decreases
(c) Pressure increases, temperature increases and molar mass increases
(d) Pressure increases, temperature decreases and molar mass increases.

22. Compressibility factor (correction factor for real gas) is function of following properties of real gas
(a) V and ρ (b) U and s
(c) P and T (d) T and U

23. In the van der Waals equation for real gas the constant b refers to
(a) Co-volume
(b) Mutual attraction amongst molecules
(c) Both
(d) None of these

24. Which of the following represent the real gas law?
(a) $nRT = (P + a/v^2)\ (v - b)$ (b) $nRT = (P - a/v^2)\ (v + b)$
(c) $nRT = (P - a/v^2)\ (v + nb)$ (d) $nRT = (P + n^2 \cdot a/v^2)\ (v - nb)$

25. Gas constants for hydrogen and carbon dioxide shall be:
(a) 4.124 and 0.189 (b) 0.287 and 2.077
(c) 0.260 and 0.287 (d) 0.461 and 0.260

26. Steam point and ice point temperatures in Fahrenheit scale can be given as:
(a) 491.67 and 671.67 (b) 212 and 32
(c) 373.15 and 273.15 (d) 100 and 0

27. Triple point of water temperature in Rankine scale can be given by:
(a) 491.69 (b) 0.01 (c) 273.16 (d) 32.02

28. Solid, liquids and gases have how many values for specific heat of respective phases.
(a) 1, 1 and 1 (b) 2, 2 and 2
(c) 1, 1 and 2 (d) 0, 1 and 2

29. A polytropic process $pV^n = C$ having the value of index 'n' as infinity refers to
(a) Isobaric process (b) Isochoric process
(c) Isothermal process (d) None of these

30. For an ideal gas the hyperbolic process will be similar to:
(a) Isobaric process (b) Isochoric process
(c) Isothermal process (d) Isentropic process

31. As per the Zeroth law of thermodynamics if the two bodies A and B are in thermal equilibrium with a third body C separately then the two bodies A and B shall also be in thermal equilibrium with each other. Using this law as basis of temperature measurement of the body A the thermometer shall be refering to
(a) Body A (b) Body B
(c) Body C (d) None of these

32. Which of the following temperature and volume relation is correct for an adiabatic process between state 1 and 2?

(a) $\frac{T_1}{T_2} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$ (b) $\left(\frac{T_2}{T_1}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$

(c) $\left(\frac{T_1}{T_2}\right)^{\gamma - 1} = \left(\frac{V_1}{V_2}\right)$ (d) $\left(\frac{T_2}{T_1}\right)^{\gamma - 1} = \left(\frac{V_1}{V_2}\right)$

33. For a hyperbolic process between the states 1 and 2, the heat interaction can be given as,

(a) $q = C_v \left(\frac{\gamma - n}{1 - n} \right) \times (T_2 - T_1)$ (b) $q = 0$

(c) $q = C_p (T_2 - T_1)$ (d) $q = C_v (T_2 - T_1) + RT_1 \ln \left(\frac{V_2}{V_1} \right)$

34. The specific heat for polytropic process can be given as,

(a) $C_n = C_v \left(\frac{\gamma - n}{1 - n} \right)$ (b) $C_n = C_v \left(\frac{n - \gamma}{1 - n} \right)$

(c) $C_n = C_v \left(\frac{1 - n}{\gamma - n} \right)$ (d) $C_n = C_v \left(\frac{n - 1}{\gamma - n} \right)$

35. In case of free expansion between the states 1 and 2 following is correct, considering no heat interaction.

(a) $U_1 = U_2$ (b) $W_{1-2} = 0$

(c) $Q_{1-2} = 0$ (d) All of these

36. The work interaction between two states 1 and 2, if given by $W_{1-2} = \int_1^2 P \times dV$ refers to,

(a) Flow work (b) Non-flow work

(c) Steady flow work (d) None of these

37. Enthalpy remains constant during which of the following process?

(a) Throttling process (b) Free expansion

(c) Isothermal process (d) None of these

38. Amongst the following which cannot be reversible process

(a) Isothermal (b) Throttling

(c) Adiabatic process (d) None of these

39. For a perfect gas the internal energy is the function of,

(a) Pressure only (b) Temperature only

(c) Volume only (d) None of these

40. Specific heat at constant pressure can be related to R and γ as under,

(a) $\frac{\gamma R}{\gamma - 1}$ (b) $\frac{R}{\gamma - 1}$

(c) $\gamma \cdot R$ (d) None of these

41. The work done by $2m^3$ of air at $5N/m^2$ while getting expanded freely to 10 m^3 shall be,

(a) 40 J (b) Zero

(c) 10 J (d) 50 J

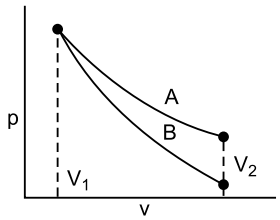
42. For the expansion of air occurring between volume state V_1 and V_2 the adiabatic process and isothermal process lines shall be as under,

(a) A : adiabatic, B : isothermal

(b) A : isothermal, B : adiabatic

(c) Any one may refer to any of process

(d) None of these



43. Change in internal energy in a reversible process occurring in closed system will be equal to heat transferred if process occurs at constant
(a) Pressure (b) Volume (c) Temperature (d) Enthalpy

44. Flow work between the state 1 and 2 is given by

(a) $\int_1^2 v dp$ (b) $\int_1^2 p dv$

(c) $\int_1^2 v dp + \int_1^2 p \cdot dv$ (d) None of these

45. Steady flow energy equation when applied to boiler shall yield,
(a) $\Delta q = h_2 - h_1$ (b) $\Delta q = \Delta w$
(c) $\Delta q = 0$ (d) None of these

46. Law of energy conservation is also stated by,
(a) Zeroth law of thermodynamics (b) First law of thermodynamics
(c) Second law of thermodynamics (d) Third law of thermodynamics

47. Which of the following is/are correct?
(a) $h = u + pv$ (b) $dq = du + d\omega$

(c) $\Delta h = \int_1^2 C_p \, dT$ (d) All of these

48. Heat interaction in polytropic process can be given by,

(a) $\left(\frac{\gamma - n}{\gamma - 1}\right) \cdot \Delta w$ (b) $\left(\frac{\gamma - n}{n - 1}\right) \cdot \Delta w$

(c) $\left(\frac{\gamma - n}{n + \gamma}\right) \cdot \Delta w$ (d) $\left(\frac{\gamma}{\gamma - n}\right) \cdot \Delta w$

49. If a Carnot engine rejects heat at temperature of 400 K and accepts at 750 K. What shall be heat absorbed, if heat rejected is 1000 kJ,
(a) 946 kJ (b) 800 kJ (c) 1875 kJ (d) 750 kJ

50. A perpetual motion machine of second kind refers to a heat engine having 100% efficiency. This heat engine will not be possible as it violates
(a) Zeroth law of thermodynamics
(b) First law of thermodynamics
(c) Second law of thermodynamics
(d) Third law of thermodynamics

51. Efficiency of Carnot engine depends upon the highest and lowest
(a) Temperatures (b) Pressures
(c) Volumes (d) None of these

52. Efficiency of Carnot engine having the limiting temperatures T_{max} and T_{min}, shall be minimum when,
(a) $T_{max} > T_{min}$ (b) $T_{max} = T_{min}$
(c) $T_{min} = 0$ (d) None of these

53. Which of the following is correct in reference to first law of thermodynamics?
(a) No difference between heat and work
(b) No indication of direction of process
(c) Law of energy conservation
(d) All of these

54. Which amongst the following is not necessary requirement for a device to be heat engine?
(a) Cyclic processes
(b) Operation between high temperature source and low temperature sink
(c) Reversible processes
(d) Converting heat into work

55. A heat engine has efficiency of 60% and operates between temperatures of T_1 and 300 K. What shall be the temperature T_1?
(a) 120 K (b) 750 K
(c) 120 K or 750 K (d) None of these

56. Joule's law states that the internal energy of an ideal gas is function of only
(a) Temperature (b) Volume (c) Pressure (d) None of these

57. Coefficient of performance for heat pump may have value:
(a) Slightly more than 0 (b) Equal to 1
(c) Greater than 1 (d) All of these

58. Difference between the COP of heat pump and COP of refrigerator is
(a) 0 (b) 1 (c) > 1 (d) < 1

59. For reversible heat engine operating between source and sink having temperatures T_1 and T_2 and heat interactions being Q_1 and Q_2 respectively thermodynamic temperature scale states
(a) $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$ (b) $\frac{Q_1}{Q_2} = \frac{T_2}{T_1}$
(c) $\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 1$ (d) None of these

60. In case of adiabatic process which of the following may not be true.
(a) $\Delta s = 0$ (b) $\Delta Q = 0$
(c) $\Delta W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$ (d) None of these

61. For a reversible adiabatic process the following is/are true.
(a) $\Delta s = 0$ (b) $\Delta Q = 0$
(c) $\Delta W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$ (d) All of these

62. Carnot cycle has following processes:
(a) Two adiabatics and two isothermal processes
(b) Two reversible adiabatics and two reversible isothermal processes
(c) Two isentropic processes and two isothermal processes
(d) None of these

63. C.O.P. of a reversible heat pump is 1.2. If it is reversed to run as reversible heat engine then its efficiency shall be:
(a) 0.833 (b) 0.2 (c) 1.2 (d) None of these

64. For a reversible engine cycle the Clausius inequality says,
(a) $\oint \frac{dQ}{T} > 0$ (b) $\oint \frac{dQ}{T} < 0$
(c) $\oint \frac{dQ}{T} = 0$ (d) None of these

65. For an irreversible process in isolated system the entropy change can be given as,
(a) $dS_{\text{isolated}} > 0$ (b) $dS_{\text{isolated}} < 0$
(c) $dS_{\text{isolated}} = 0$ (d) None of these

66. For a reversible process in isolated system the entropy change can be given as,
(a) $dS_{\text{isolated}} > 0$ (b) $dS_{\text{isolated}} < 0$
(c) $dS_{\text{isolated}} = 0$ (d) None of these

67. The entropy generated and the total entropy change for a process to be impossible shall be,
(a) S_{gen} or $\Delta S_{\text{total}} = 0$ (b) S_{gen} or $\Delta S_{\text{total}} > 0$
(c) S_{gen} or $\Delta S_{\text{total}} < 0$ (d) None of these

68. The entropy change in polytropic process can be given by,
(a) $C_v \ln \left(\frac{T_2}{T_1}\right) \cdot \left(\frac{n-\gamma}{n-1}\right)$ (b) $C_v \ln \left(\frac{T_2}{T_1}\right) \cdot \left(\frac{n-1}{n-\gamma}\right)$
(c) $C_v \ln \frac{T_2}{T_1}$ (d) None of these

69. Entropy change for an adiabatic process can be given by,
(a) $\int_1^2 \frac{C_p(T)dT}{T} - R \ln \left(\frac{P_2}{P_1}\right)$ (b) $C_v \ln \left(\frac{T_2}{T_1}\right) - R \ln \left(\frac{v_2}{v_1}\right)$
(c) $C_p \ln \frac{T_2}{T_1}$ (d) $C_v \ln \frac{T_2}{T_1}$

70. Which of the following relation(s) is/are correct?
(a) $Tds = du + p \cdot dv$ (b) $dh = T \cdot ds + v \cdot dp$
(c) $Tds = C_p \cdot dT - v\, dp$ (d) All of these

71. On *T-s* diagram the slope of constant pressure line (m_p) and the slope of constant volume line (m_v) can be related as,
(a) $m_p = m_v$ (b) $m_v > m_p$
(c) $m_p > m_v$ (d) $m_p \cdot m_v = 1$

72. Exergy can not be less than zero as the maximum work interaction can not be
(a) Less than zero (b) Greater than zero
(c) Zero (d) None of these

73. Law of degradation of energy says that the unavailable energy is gradually increasing due to
(a) Increase in reversible processes
(b) Increase in irreversible processes
(c) Increase in unavailable energy
(d) None of these

74. Which of the following is/are correct? I: Irreversibility, A: Availability $W_{c,\ max}$: Maximum work, T_0: Dead state temperature, ΔS_c: Combined system entropy change.
(a) $I = T_0 \cdot \Delta S_c$ (b) $W_c = A - I$
(c) $A = W_{c,\ max}$ (d) All of these

75. The efficiency of reversible and irreversible heat engine cycles can be related as,
(a) $\eta_{rev} = \eta_{irrev}$ (b) $\eta_{rev} < \eta_{irrev}$
(c) $\eta_{rev} > \eta_{irrev}$ (d) None of these

76. Which of the following is/are correct?
(a) Entropy is used as a convenient coordinate in thermodynamics.
(b) Entropy is used to decide whether a process is reversible, irreversible or impossible.
(c) Entropy is used to determine the heat transfer in reversible process
(d) All of these.

77. Equation $\Delta G = \Delta H - T \cdot \Delta S$ tells us that an exothermic reaction will be associated with which of the following:
(i) – ve ΔH (ii) + ve ΔH
(iii) More disordered, +ve Δs (iv) Spontaneous reaction
(v) More ordered – ve Δs
Possible answers
(a) (i) and (v) (b) (ii) and (v)
(c) (ii) and (iii) (d) (i), (iii) and (iv)

78. Availability of a closed system at state 1 and dead states referred with subscript 0 is given by
(a) $(E_1 - E_0) + p_0(V_1 - V_0) - T_0(S_1 - S_0)$
(b) $(E_1 - E_2) + p_0(V_1 - V_2) - T_0(S_1 - S_2)$
(c) $(h_1 - h_0) + g(Z_1 - Z_0) - T_0(S_1 - S_0) + \dfrac{V_1^2}{2}$
(d) None of these

79. The availability of steadily flowing fluid, also called stream availability is given by; (the current state is 1 and dead state is given by 0).
(a) $(E_1 - E_0) + p_0(V_1 - V_0) - T_0(S_1 - S_0)$
(b) $(h_1 - h_2) + g(Z_1 - Z_2) - T_0(S_1 - S_2) + \left(\dfrac{V_1^2 - V_2^2}{2}\right)$
(c) $(h_1 - h_0) + g(Z_1 - Z_0) - T_0(S_1 - S_0) + \dfrac{V_1^2}{2}$
(d) None of these

80. A characteristic function is one from which all properties of a substance can be obtained by differentiation alone, so that no arbitrary functions requiring supplementary data for their evaluation are required. Characteristic function is/are out of the following:
(a) $f(u,\ v,\ s) = 0$ (b) $f(h,\ p,\ s) = 0$
(c) $f(g,\ p,\ t) = 0$ (d) All of these

81. Which of the following is/are incorrect Maxwell equations?

(a) $\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v$ (b) $\left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p$

(c) $\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T$ (d) None of these

82. The equation relating the following three measurable properties is called:

(i) The slope of saturation pressure-temperature line

(ii) The latent heat

(iii) The change in volume during phase transformation

(a) Joule's equation

(b) Clapeyron equation

(c) Maxwell relation

(d) None of these

83. Joule Thomoson coefficient has its value changing on two sides of inversion line as under,

(a) + ve to the right and – ve on the left of inversion line

(b) – ve to the right and + ve on the left of inversion line

(c) Zero on the right and zero on the left of inversion line

(d) None of these

84. Critical point pressure and temperature for water are,

(a) 22.12 MPa and 374.15°C (b) 0.23 MPa and –268°C

(c) 18.2 MPa and 899°C (d) None of these

85. On *T-s* diagram the state change during transformation of 0°C water to 0°C steam at atmospheric pressure due to heat addition can be shown by

(a) Slant line (b) Vertical line

(c) Horizontal line (d) None of these

86. On *T-s* diagram the state change during transformation of 0°C water to 100°C water at atmospheric pressure due to heat addition can be shown by

(a) Line with positive slope (b) Vertical line

(c) Horizontal line (d) None of these

87. Slope of constant pressure line on enthalpy-entropy diagram of steam with increasing pressure.

(a) Increases (b) Decreases

(c) Remains unaltered (d) None of these

88. The value of dryness fraction at critical point for water-steam phase transformation may be,

(a) 0 (b) 1 (c) Either 0 or 1 (d) All of these

89. During the dryness fraction measurement using throttling calorimeter the wet state of steam is throttled so as to lie in

(a) Superheated state (b) Wet state

(c) Subcooled state (d) None of these

90. The expression for air standard efficiency of Diesel cycle is

(a) $\eta = 1 - \frac{1}{r^{\gamma-1}}$ (b) $\eta = 1 - \frac{1}{\gamma}\left\{\frac{\rho^{\gamma}-1}{r^{\gamma-1}(\rho-1)}\right\}$

(c) $\eta = 1 - \frac{1}{r^{\gamma-1}}\left\{\frac{\alpha\cdot\rho^{\gamma}-1}{(\alpha-1)+\alpha\cdot\gamma(\rho-1)}\right\}$ (d) None of these

Here r, ρ, α are compression ratio, cut-off ratio and pressure ratio.

91. The mean effective pressure for Diesel cycle is,

(a) mep = $\dfrac{r\left(r^{\gamma-1}-1\right)\cdot p_1\cdot(\alpha-1)}{(r-1)(\gamma-1)}$ (b) mep = $\dfrac{p_1\cdot r^{\gamma}\left[\gamma\cdot(\rho-1)-r^{1-\gamma}\cdot\left(\rho^{\gamma}-1\right)\right]}{(\gamma-1)(r-1)}$

(c) Both of these (d) None of these

Here r, ρ, p_1 and α are compression ratio, cut-off ratio pressure at beginning of compression and pressure ratio.

92. Thermodynamic processes constituting a Rankine cycle are isobaric heat addition, adiabatic expansion, isobaric heat release and ...,

(a) Adiabatic pumping (b) Isothermal pumping
(c) Polytropic pumping (d) None of these

93. Rankine cycle efficiency can be improved by

(a) Reduced heat addition in boiler (b) Increasing expansion work
(c) Reduced feed pump work (d) All of these

94. For the same compression ratio and same heat input the Otto, Diesel and Dual cycle efficiencies can be given as,

(a) $\eta_{\text{Otto}} > \eta_{\text{Dual}} > \eta_{\text{Diesel}}$ (b) $\eta_{\text{Otto}} > \eta_{\text{Diesel}} > \eta_{\text{Dual}}$
(c) $\eta_{\text{Diesel}} > \eta_{\text{Dual}} > \eta_{\text{Otto}}$ (d) None of these

95. Carnot vapour power cycle comprises of following processes.

(a) Two reversible isothermals and two reversible adiabatic processes
(b) Two isothermals and two adiabatic processes
(c) Two isothermals and two isentropic processes
(d) Two isobarics and two adiabatic processes

96. The steam is superheated in boiler at

(a) Isothermal process (b) Isobaric process
(c) Isochoric process (d) None of these

97. Throttling process can be shown on Mollier diagram by,

(a) Vertical line (b) Horizontal line
(c) Inclined line (d) None of these

98. The heat required for vaporization of water to steam decreases with

(a) Increasing steam generation pressure
(b) Decreasing steam generation pressure
(c) Decreasing steam generation temperature
(d) None of these

99. Adding heat to wet steam causes increase in its dryness fraction while the temperature and pressure remain constant in

(a) Subcooled region (b) Superheated region
(c) Wet region (d) None of these

100. Latent heat of vaporization of water at critical point is

(a) Zero (b) 539 kcal/kg
(c) 1 kcal/kg (d) None of these

101. The combustion analysis in which the fuel is separated into arbitrary constituents such as moisture, volatile matter, fixed carbon and ash etc. is called as

(a) Ultimate analysis (b) Proximate analysis
(c) Stoichiometric analysis (d) None of these

102. A combustion has all the oxygen supplied with air to the reactants being used and no free oxygen appears in products. Such air supplied is called as
(a) Stoichiometric air (b) Excess air
(c) Dry air (d) None of these

103. Composition of dry air on mass basis is
(a) 0.232 kg O_2 and 0.768 kg N_2 (b) 0.21 kg O_2 and 0.79 kg N_2
(c) 1 kg O_2 and 3.31 kg N_2 (d) None of these

104. Enthalpy of combustion of any fuel containing hydrogen in it and the H_2O present in liquid form in products is termed as,
(a) Higher heating valve (b) Lower heating valve
(c) Enthalpy of hydration (d) None of these

105. When the temperature of a pure substance approaches zero on Kelvin scale then the entropy of substance approaches zero. This is stated by
(a) First law of thermodynamics (b) Second law of thermodynamics
(c) Third law of thermodynamics (d) None of these

106. The efficiency of Carnot cycle may be equal to which of the following cycle running between same temperature limits,
(a) Rankine cycle (b) Stirling cycle
(c) Brayton cycle (d) Otto cycle

107. Which of the following is not vapour power cycle?
(a) Rankine cycle (b) Carnot vapour power cycle
(c) Otto cycle (d) None of these

108. The amount of energy added by heat transfer to the cycle to produce unit of net work output is called
(a) Heat rate (b) Work ratio
(c) Back work ratio (d) Thermal efficiency

109. Difference between temperature of feed water outlet and saturation temperature of steam entering the heater is called
(a) Pinch point (b) Terminal temperature difference
(c) LMTD (d) None of these

110. Deaerator is feed water heater of which type
(a) Closed type feed water heater (b) Surface type feed water heater
(c) Open type feed water heater (d) None of these

111. Effectiveness by second law of thermodynamics can be given by
(a) $\dfrac{\text{Availability output}}{\text{Availability in}}$ (b) $\dfrac{\eta_{\text{thermal}}}{\eta_{\text{thermal, rev}}}$
(c) $\dfrac{\text{Availability used}}{\text{Availability supplied}}$ (d) All of these

112. Which of the following is incorrect?
(a) Availability follows the law of conservation
(b) Availability is function of states of matter under consideration and environment
(c) Availability always depends upon pressure
(d) Availability increases with temperature drop at low temperatures

113. For a reversible isothermal and isochoric process the Helmholtz free energy,
(a) Increases (b) Decreases
(c) Remains constant (d) Nothing can be said

114. Clausius-Clapeyron equation given by $\left(\frac{dp}{p}\right)_{sat} = h_{fg} \cdot \left(\frac{dT}{RT^2}\right)_{sat}$ is valid for

(a) Low pressure phase transformation of liquid-vapour type
(b) High pressure phase transformation of liquid-vapour type
(c) High pressure phase transformation of solid-vapour type
(d) None of these

115. Expression given by $\left\{\frac{1}{v}\cdot\left(\frac{\partial v}{\partial T}\right)_P\right\}$ refers to

(a) Isothermal compressibility (b) Coefficient of volume expansion
(c) Isothermal bulk modulus (d) None of these

116. Mayer's relation is given by; (Here β and α are volume expansivity and isothermal compressibility).

(a) $C_p - C_v = \frac{v \cdot T \cdot \beta^2}{\alpha}$ (b) $C_p - C_v = \frac{v \cdot T}{\alpha}$

(c) $C_p - C_v = v \cdot T \cdot \beta^2$ (d) None of these

117. Fuel-air mixture will be called lean when the equivalence ratio has its value
(a) < 1 (b) > 1 (c) 1 (d) None of these

118. Which of the following is/are type of coal?
(a) Peat (b) Lignite
(c) Bituminous coal (d) All of these

119. Natural gas has highest fraction of which of the following in it,
(a) CH_4 (b) C_2H_4 (c) C_2H_6 (d) CO

120. Minimum air required per kg of fuel for complete combustion can be given by, (considering mass fraction being C, H, O and S of respective constituent),

(a) $\frac{1}{23}\left[\frac{8}{3}C + 8\left(H - \frac{O}{8}\right) + S\right]$ (b) $\frac{1}{21}\left[\frac{8}{3}C + 8\left(H - \frac{O}{8}\right) + S\right]$

(c) $\frac{1}{23}\left[8C + 8\left(H - \frac{O}{8}\right) + S\right]$ (d) None of these

121. Equipment used for carrying out volumetric analysis of dry products of combustion is called,
(a) Bomb calorimeter (b) Orsat analyzer
(c) Gas calorimeter (d) None of these

122. Lancashire boiler has how many number of fire tubes in it?
(a) Three (b) Four (c) Two (d) One

123. Cornish boiler has how many flue tubes in it?
(a) One (b) Two (c) Three (d) None

124. Which of the following is/are super critical boiler?
(a) Cochran boiler (b) Locomotive boiler
(c) Benson boiler (d) Lancashire boiler

125. Which of the following is/are fire tube boiler(s)?
(a) Velox boiler (b) Nestler boiler
(c) Locomotive boilers (d) All of these

126. Water level indicator has how many vertical tubes?
(a) One (b) Two (c) Three (d) Four

127. Fusible plug has plug made of copper and the body is made of
(a) Gun metal (b) Aluminium (c) Iron (d) Bronze

128. Feed pump in boilers may be of
(a) reciprocating pump type (b) centrifugal pump type
(c) injector type (d) All of these

129. The maximum discharge through a natural draught chimney can be given as function of m (mass of air supplied per kg of fuel), T_a (atmospheric temperature) and T_g (hot gas temperature).
(a) $T_g = \frac{T_a \cdot 2(m+1)}{m}$ (b) $T_g = \frac{T_a \cdot 2m}{(m+1)}$
(c) $T_g = \frac{T_a \cdot (m+1)}{2m}$ (d) None of these

130. The power requirement in forced draught and induced draught shall be related as,
(a) $P_{\text{induced}} = P_{\text{forced}}$ (b) $P_{\text{induced}} < P_{\text{forced}}$
(c) $P_{\text{induced}} > P_{\text{forced}}$ (d) None of these

131. State which of the following is/are incorrect?
(a) Combustion is better in forced draught
(b) Fan size of induced draught is larger than forced draught fan
(c) Maintenance in F.D. fan is easy as compared to ID fan
(d) None of these

132. Steam jet draught is not of following type
(a) Natural draught (b) Induced draught
(c) Forced draught (d) Artificial draught

133. Boiler efficiency is given by the the ratio of heat used in steam generation and
(a) Heat recovered using artificial draught
(b) Heat lost due to use of natural draught
(c) Heat available due to fuel burning
(d) None of these

134. Mean temperature of heat addition gets increased and shows increase in cycle thermal efficiency. This cycle is called,
(a) Regenerative cycle (b) Reheat cycle
(c) Carnot cycle (d) None of these

135. At a given compression ratio the Otto cycle efficiency for the working fluids having $\gamma = 1.2$, 1.4 and 1.67 can be related as,
(a) $\eta_{\gamma = 1.2} > \eta_{\gamma = 1.4} > \eta_{\gamma = 1.67}$ (b) $\eta_{\gamma = 1.2} < \eta_{\gamma = 1.4} < \eta_{\gamma = 1.67}$
(c) $\eta_{\gamma = 1.2} = \eta_{\gamma = 1.4} = \eta_{\gamma = 1.67}$ (d) None of these

136. Intercooling in between the compression process of gas turbine cycle causes
(a) Reduction in expansion work (b) Increase in compression work
(c) Increase in expansion work (d) Reduction in compression work

137. Stagnation temperature can be related to the static temperature and dynamic temperature as under
(a) Stagnation temperature = Static temperature – Dynamic temperature
(b) Stagnation temperature = Static temperature + Dynamic temperature
(c) Stagnation temperature = Dynamic temperature – Static temperature
(d) None of these

138. In case of multistage compressor inspite of constant stage efficiency the isentropic compression efficiency goes on reducing with increasing pressure ratio due to
(a) Preheating effect
(b) Increase in temperature due to friction
(c) Friction in a stage results in more work requirement in next stage
(d) All of above

139. Modified index of compression (n) accounting for non-isentropic compression can be given as function of compressor's polytropic efficiency, $\eta_{\text{poly,c}}$ as

(a) $\dfrac{n-1}{n} = \left(\dfrac{\gamma-1}{\gamma \cdot \eta_{\text{poly,c}}}\right)$ (b) $\dfrac{n-1}{n} = \left(\dfrac{\gamma-1}{\gamma}\right) \eta_{\text{poly,c}}$

(c) $\dfrac{n-1}{n} = \left(\dfrac{\gamma \cdot \eta_{\text{poly,c}}}{\gamma-1}\right)$ (d) None of these

140. Modified index of expansion (n), accounting for non-isentropic expansion can be given as function of turbine's polytropic efficiency, $\eta_{\text{poly,T}}$.

(a) $\dfrac{n-1}{n} = \left(\dfrac{\gamma-1}{\gamma \cdot \eta_{\text{poly,T}}}\right)$ (b) $\dfrac{n-1}{n} = \left(\dfrac{\eta_{\text{poly,T}} \cdot (\gamma-1)}{\gamma}\right)$

(c) $\dfrac{n-1}{n} = \left(\dfrac{\eta_{\text{poy,T}} \cdot \gamma}{(\gamma-1)}\right)$ (d) None of these

141. Which of the following cycles may have highest efficiency under ideal conditions?
(a) Ericsson cycle (b) Stirling cycle
(c) Carnot cycle (d) All of these

142. In a steam engine eccentric the ratio of 'throw of eccentric' and 'radius of eccentric' is equal to
(a) 1 (b) 2 (c) 0.5 (d) None of these

143. Distance by which valve uncovers the steam port when piston is at dead centre is called
(a) Lead (b) Stroke
(c) Clearance (d) None of these

144. Expansion in steam engine cylinder is following type of thermodynamic process
(a) Adiabatic process (b) Isobaric process
(c) Isochoric process (d) Hyperbolic process

145. Mean effective pressure for steam engine without clearance volume can be given as function of inlet pressure p_1, back pressure p_b, expansion ratio 'r' as under:

(a) $\dfrac{p_1(1-\ln r)}{r} - p_b$ (b) $\dfrac{p_1(1+\ln r)}{r} + p_b$

(c) $\dfrac{p_1(1+\ln r)}{r} - p_b$ (d) None of these

146. Actual indicator diagram differs from the hypothetical indicator diagram in steam engine due to
(a) Pressure drop because of friction
(b) Non instantaneous operation of inlet and exit valves
(c) Expansion not occurring hyperbolically in actual engine
(d) All of these

147. Diagram factor in steam engines has its value
(a) Less than unity (b) Greater than unity
(c) Equal to unity (d) None of these

148. Condensation can be prevented in steam engines by
(a) Supplying superheated steam
(b) Providing steam jacket around cylinder wall
(c) Increasing speed of engine for certain output
(d) All of these

149. If all the steam states during expansion are found to be dry and saturated then the expansion curve is called
(a) Saturation curve (b) Willan's line
(c) Missing curve (d) None of these

150. A special kind of valve employed for cut-off governing of steam engine is called
(a) D-slide valve (b) Piston valve
(c) Meyer's expansion valve (d) None of these

151. Compounding of steam engine offers the advantages of
(a) Possibility of increased pressure and temperature of steam at inlet
(b) Reduced thermal stresses
(c) Reduced condensation losses
(d) All of these

152. Two pistons of HP and LP cylinders are mounted in a compound steam engine with zero phase difference. This type of engine is called
(a) Tandem compound engine (b) Woolf compound engine
(c) Receiver compound engine (d) None of these

153. The mean effective pressure of HP referred to LP cylinder can be given by
(a) $\frac{\text{mep}_{\text{HP}} \times \Delta V_{\text{LP}}}{\Delta V_{\text{HP}}}$ (b) $\frac{mep_{\text{HP}} \times \Delta V_{\text{HP}}}{\Delta V_{\text{LP}}}$
(c) $\frac{\text{mep}_{\text{LP}} \times \Delta V_{\text{HP}}}{\Delta V_{\text{LP}}}$ (d) None of these

154. For equal power developed in each cylinder of compound steam engine having LP and HP cylinders it can be said that
(a) $\text{mep}_{\text{LP}} = \text{mep}_{\text{HP referred to LP}}$ (b) $\text{mep}_{\text{LP}} \times \text{mep}_{\text{HP}} = 1$
(c) $\text{mep}_{\text{LP}} \times \Delta V_{\text{HP}} = \text{mep}_{\text{HP}} \times \Delta V_{\text{LP}}$ (d) None of these

155. The variation of cross-sectional area (A), velocity of gas flowing (C), and Mach number (M) during flow through a variable cross section area duct can be related as,
(a) $\frac{dA}{A} = \frac{dC}{C}(1 - M^2)$ (b) $\frac{dA}{A} = \frac{dC}{C}(M^2 - 1)$
(c) $\frac{dC}{C} = \frac{dA}{A}(M^2 - 1)$ (d) None of these

156. For a subsonic flow the increase in velocity from inlet to exit may be obtained from a duct of,
(a) Diverging cross-sectional area type
(b) Diverging-converging cross-sectional area type
(c) Converging cross sectional area type
(d) None of these

157. For supersonic flow the increase in velocity from inlet to exit may be obtained from a duct of,
(a) Diverging cross-sectional area type
(b) Converging-diverging cross-sectional area type
(c) Converging cross-sectional area type
(d) None of these

158. The critical pressure ratio for maximum discharge through nozzle can be given as,

(a) $\left(\frac{1}{n+1}\right)^{\frac{n}{n-1}}$ (b) $\left(\frac{n+1}{2}\right)^{\frac{n}{n-1}}$

(c) $\left(\frac{2}{n+1}\right)^{\frac{n-1}{n}}$ (d) $\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$

159. The critical pressure ratio for air flowing through nozzle shall be,
(a) 0.528 (b) 0.545 (c) 0.577 (d) none of these

160. The maximum discharge per unit area at throat for critical conditions for perfect gas flowing through nozzle can be given by,

(a) $\sqrt{\gamma \cdot \frac{p_1}{v_1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma-1}{\gamma+1}}}$ (b) $\sqrt{\frac{\gamma \cdot p_1}{v_1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$

(c) $\sqrt{\gamma \cdot p_1 v_1 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$ (d) None of these

161. When the back pressure of nozzle is below the designed value of pressure at exit of nozzle then the nozzle is called,
(a) Inder-expanding nozzle (b) Over-expanding nozzle
(c) Designed operation nozzle (d) None of these

162. When the back pressure of a nozzle is equal to critical pressure then such operating state of nozzle is called,
(a) Under-expanding nozzle (b) Over-expanding nozzle
(c) Choked operation of nozzle (d) None of these

163. If the velocity at inlet to nozzle is C_1 and the velocity at exit of nozzle during isentropic expansion and non isentropic expansion are C_2 and $C_{2'}$ respectively, then the nozzle efficiency shall be;

(a) $\frac{C_2^2 - C_1^2}{C_{2'}^2 - C_1^2}$ (b) $\frac{C_{2'}^2}{C_2^2}$ (c) $\frac{C_1^2 - C_{2'}^2}{C_2^2 - C_1^2}$ (d) $\frac{C_{2'}^2 - C_1^2}{C_2^2 - C_1^2}$

164. Phenomenon of supersaturation, when occurring in nozzle causes,
(a) Increase in discharge (b) Reduced velocity at exit
(c) Increase in dryness fraction and entropy (d) All of these

165. The ratio of saturation pressure corresponding to the temperatures of steam states in stable equilibrium and metastable equilibrium during steam flow through nozzle is called
(a) Degree of supersaturation (b) Degree of undercooling
(c) Critical pressure ratio (d) None of these

166. The difference between the saturation temperature corresponding to the steam states in stable equilibrium and metastable equilibrium during steam flow through nozzle is called
(a) Degree of supersaturation (b) Degree of undercooling
(c) Critical temperature difference (d) None of these

167. In a steam turbine the substantial static pressure drop occurs in stationary nozzle and rotor blade passage, the type of turbine may be
(a) Impulse turbine (b) Reaction turbine
(c) Branca's turbine (d) None of these

168. In a steam turbine if the leakage along the mating surface of shaft and casing is reduced by using the principle of steam throttling then the type of sealing arrangement may be
(a) Carbon rings (b) Packings
(c) Labyrinth glands (d) None of these

169. Radial flow turbines are also called as
(a) Rateau turbine (b) Ljungstrom turbine
(c) Curtis turbine (d) None of these

170. A steam turbine rejects steam at a pressure much above the atmospheric pressure and this steam is being used for some purposes such as heating etc. This could be:
(a) Back pressure turbine (b) Passout turbine
(c) Mixed pressure turbine (d) None of these

171. Which of the following is not a part of impulse steam turbine?
(a) Nozzle (b) Symmetrical moving blades ring
(c) Aerofoil type blades (d) None of these

172. A steam turbine stage has nozzle followed by moving blade ring followed by fixed blade ring and lastly moving blade ring. This could be
(a) Rateau turbine
(b) Curtis turbine
(c) Pressure compounded impulse turbine
(d) None of these

173. The ratio of blade velocity to steam velocity remains constant in which type of compounding of impulse steam turbines
(a) Pressure compounding (b) Velocity compounding
(c) Pressure-Velocity compounding (d) None of these

174. The efficiency of a compound impulse turbine goes on decreasing with increase in the number of stages. This could be,
(a) Pressure compounding (b) Velocity compounding
(c) Pressure-Velocity compounding (d) None of these

175. In case of an impulse turbine the blade velocity coefficient shall have its value for actual case as,
(a) < 1 (b) > 1 (c) $= 1$ (d) None of these

176. The component of velocity which is responsible for maintaining the flow of steam across the moving blade row is called,
(a) Relative velocity (b) Absolute velocity
(c) Tangential component of velocity (d) Axial component of velocity

177. In an impulse turbine the absolute angle at inlet should be selected depending upon
(a) Thrust requirement (b) Maintaining flow across the blade row
(c) Both (a) and (b) (d) None of these

178. The ratio of work done to the energy supplied to rotor in a turbine stage is called,
(a) Blading efficiency (b) Stage efficiency
(c) Nozzle efficiency (d) None of these

179. The ratio of rate of work done and energy supplied to the stage in a turbine is called
(a) Blading efficiency (b) Stage efficiency
(c) Nozzle efficiency (d) None of these

180. Stage efficiency, nozzle efficiency and diagram efficiency can be related as,

(a) $\eta_{stage} = \dfrac{\eta_{diagram}}{\eta_{nozzle}}$ (b) $\eta_{stage} = \eta_{diagram} \times \eta_{nozzle}$

(c) $\eta_{nozzle} = \eta_{stage} \times \eta_{diagram}$ (d) None of these

181. The diagram efficiency is highest for a simple impulse turbine stage having smooth and symmetrical blade when blade—steam speed ratio can be given as,

(a) $\cos\ \alpha_1$ (b) $\dfrac{\cos\alpha_1}{4}$ (c) $\dfrac{\cos\alpha_1}{2}$ (d) none of these

182. Steam turbines having the complete nozzle coverage of blades are called,
(a) Full admission turbine (b) Partial admission turbine
(c) Reaction turbines (d) None of these

183. The optimum value of blade to steam speed ratio can be generalised for '*n*' number of moving blade rows in impulse turbine as,

(a) $\dfrac{\cos\alpha_1}{2^n}$ (b) $\dfrac{\cos\alpha_1}{2 \cdot n}$ (c) $\cos\ \alpha_1$ (d) None of these

184. If the degree of reaction is 0.75 then the enthalpy changes occurring in rotor and stator blades in a stage can be related as,

(a) $\Delta h_{stator} = \dfrac{1}{3} \cdot\ \Delta h_{rotor}$ (b) $3 \cdot \Delta h_{stator} = 4 \cdot \Delta h_{rotor}$

(c) $\Delta h_{rotor} = \dfrac{1}{3} \cdot\ \Delta h_{stator}$ (d) None of these

185. Which of the following may be reaction turbine?
(a) Curtis turbine (b) Rateau turbine
(c) Parson's turbine (d) None of these

186. Reheat factor for steam turbine stages has its value,
(a) > 1 (b) < 1 (c) = 1 (d) None of these

187. The ratio of sum of actual output of stages and the available energy in turbine is called
(a) Reheat factor (b) Stage efficiency
(c) Internal efficiency (d) None of these

188. Use of condenser in a steam power plant offers advantages such as
(a) It permits the recirculation of condensate
(b) It allows for lower back pressure
(c) It increases the turbine output
(d) All of these

189. Condenser generally operates at
(a) Sub-atmospheric pressure (b) Atmospheric pressure
(c) Above atmospheric pressure (d) None of these

190. Which of the following can not be achieved by using condenser?
(a) Removal of air and other non-condensable dissolved gases
(b) Reduced load on water treatment plant
(c) Availability of hotter feed water
(d) None of these

191. The condenser in which there is direct contact between the steam and cooling fluid could be
(a) Surface condenser (b) Jet condenser
(c) Evaporative condenser (d) None of these

192. A condenser is placed at low level such that the vacuum inside condenser draws cooling water into condenser from river. Such condenser may be
(a) Low level jet condenser (b) High level jet condenser
(c) Surface condenser (d) None of these

193. A condenser is placed at a height more than that of water and water is to be injected into condenser using a pump and condensate flows out of condenser due to gravity. This condenser may be,
(a) Low level jet condenser (b) High level condenser
(c) Surface condenser (d) None of these

194. Ejector condenser does not require air pump because of
(a) No air present
(b) Air entraining effect of water jet itself
(c) Air being separated and released separately
(d) None of these

195. In ejector condenser in the event of failure of cooling water supply water may be sucked from hot well to go into steam pipe. But this is prevented due to
(a) Non return valve in steam line (b) High pressure
(c) Low pressure (d) None of these

196. While designing the condenser it is desired to have,
(a) Maximum heat transfer between two fluids
(b) Effective air extraction
(c) No air leakage
(d) All of these

197. A surface condenser should preferably have,
(a) Uniform distribution of steam over cooling water tubes
(b) Minimum pressure loss of steam
(c) Minimum number of tubes
(d) All of these

198. Leakage of air into condenser causes,
(a) Reduction in work done per kg of steam due to increase in back pressure
(b) Increased heat exchange between water and steam
(c) Increased efficiency of auxiliary devices
(d) None of these

199. Air leakage into condenser could be identified by
(a) Drop in thermometer reading, after isolation of condenser from plant
(b) Soap bubble test
(c) Peppermint oil test (d) All of above

200. Vacuum efficiency of condenser can be given by,

(a) $\dfrac{\text{Theoretical vacuum}}{\text{Actual vacuum}}$ (b) $\dfrac{\text{Actual vacuum}}{\text{Theoretical vacuum}}$

(c) $\dfrac{\text{Condenser pressure}}{\text{Ideal condenser pressure}}$ (d) None of these

201. Condenser efficiency is given by,

(a) $\dfrac{\text{Actual cooling water temperature rise}}{\text{Maximum possible temperature rise}}$

(b) $\dfrac{\text{Actual vacuum}}{\text{Theoretical vacuum}}$

(c) $\dfrac{\text{Actual cooling water required}}{\text{Minimum cooling water requirement}}$

(d) none of these

202. Effectiveness of cooling tower diminishes due to,

(a) Humid weather conditions (b) Drift losses

(c) Poor atomization of hot water (d) All of these

203. The performance of cooling tower depends upon,

(a) Duration of contact between water and air

(b) Area of contact between water and air

(c) Relative velocity of air and water flow

(d) All of these

204. For delivery pressure more than 85 bar the number of stages desired in compressor may be

(a) 1 (b) 2 (c) 3 (d) 4

205. The compression work requirement is minimum in case of the compression following process of,

(a) Adiabatic type (b) Isochoric type

(c) Isothermal type (d) Hyperbolic type

206. The ratio of volume of free air sucked into cylinder and the swept volume of LP cylinder is called,

(a) Volumetric efficiency (b) Diagram factor

(c) Compression efficiency (d) None of these

207. Volumetric efficiency referred to free air conditions in case of reciprocating air compressor can be given by,

(a) $\left(\dfrac{P_1 \cdot T_a}{P_a \cdot T_1}\right)\left\{1 + C - C\left(\dfrac{P_2}{P_1}\right)^{1/n}\right\}$ (b) $\left(\dfrac{P_a \cdot T_1}{P_1 \cdot T_a}\right)\left\{1 + C - C\left(\dfrac{P_2}{P_1}\right)^{1/n}\right\}$

(c) $\left(\dfrac{P_1 \cdot T_a}{P_a \cdot T_1}\right)\left\{1 - C + C\left(\dfrac{P_2}{P_1}\right)^{1/n}\right\}$ (d) None of these

208. What will happen to the volumetric efficiency with increasing pressure ratio in case of single stage compression in compressors?

(a) Decrease (b) Increase

(c) Remains unaffected (d) None of these

209. In order to get the same free air delivery from a compressor how should the size of cylinder be changed with increasing pressure ratio?
(a) Increased (b) Reduced (c) No change (d) None of these

210. For the multistage compression with perfect intercooling the work requirement shall be minimum when,
(a) Pressure ratio in each stage are equal
(b) Pressure ratio in first stage is greater than second stage
(c) Pressure ratio in second stage is double of pressure ratio in first stage
(d) None of these

211. Which of the following is not a positive displacement type compressor?
(a) Roots blower (b) Screw type compressor
(c) Vane type compressor (d) None of these

212. In vane type compressor the contribution of reversible pressure rise and irrversible pressure rise is generally in the proportion of
(a) 3 : 1 (b) 1 : 3 (c) 1 : 1 (d) None of these

213. Slip factor for a centrifugal compressor may be given by the ratio of
(a) Actual whirl velocity and ideal whirl velocity
(b) Actual tangential velocity and ideal absolute velocity
(c) Relative velocity and absolute velocity
(d) None of these

214. For the centrifugal compressor having radial flow blades the stagnation pressure may be given by

(a) $\left[1+\frac{C_p T_{01}}{U_2^2}\right]^{\frac{\gamma}{(\gamma-1)}}$ (b) $\left[1+\frac{C_p T_{01}}{U_2^2}\right]^{\frac{\gamma-1}{\gamma}}$

(c) $\left[1+\frac{U_2^2}{C_p T_{01}}\right]^{\frac{\gamma}{(\gamma-1)}}$ (d) None of these

215. The index of compression value in case of air compression process of cooled type in centrifugal compressor may
(a) Be equal to adiabatic index (b) Be even less than adiabatic index
(c) Be more than from adiabatic index (d) None of these

216. What shall happen to surging if the number of diffuser vanes is increased in a centrifugal compressor?
(a) Increases (b) Decreases
(c) Remains unchanged (d) None of these

217. In order to prevent surging due to flow reversal the number of diffuser vanes (n_d) and the number of impeller blades (n_i) shall be as under in case of centrifugal compressor,
(a) $n_d = n_i$ (b) $n_i < n_d$ (c) $n_i > n_d$ (d) None of these

218. Which of the following occurs due to stalling?
(a) Reduced stage efficiency (b) Increased vibrations
(c) Reduced delivery pressure (d) All of these

219. Which of the following refers to centrifugal compressor characteristics?
(a) For a given pressure ratio the increase in speed shows increase in flow rate and simultaneous reduction in efficiency.
(b) For a particular speed of compressor the decrease in mass flow rate is accompanied by increase in pressure ratio.

(c) At all speeds the compressor operation is limited by surging and choking at the two ends.
(d) All of these

220. Which of the following refers to axial flow compressor characteristics?
(a) Flow rate changes too much with increase in pressure ratio at design conditions
(b) Efficiency decreases with decreasing flow rate
(c) Efficiency increases with decreasing pressure ratio
(d) All of these

221. Isentropic efficiencies for the centrifugal compressor and the axial flow compressor can be related as,
(a) $\eta_{\text{isen, cent}} = \eta_{\text{isen, axial}}$ (b) $\eta_{\text{isen, cent}} > \eta_{\text{isen, axial}}$
(c) $\eta_{\text{isen, cent}} < \eta_{\text{isen, axial}}$ (d) None of these

222. Starting torque requirements of centrifugal compressor and axial flow compressor can be related as,
(a) $T_{\text{centrifugal}} > T_{\text{axial}}$ (b) $T_{\text{centrifugal}} < T_{\text{axial}}$
(c) $T_{\text{centrifugal}} = T_{\text{axial}}$ (d) None of these

223. Morse test is used for measurement of,
(a) Torque (b) Specific fuel consumption
(c) Brake power (d) Friction power

224. Which of the following are phases of combustion in CI engine?
(a) Ignition delay (b) Uncontrolled combustion
(c) Controlled combustion (d) All of these

225. One ton of refrigeration is equivalent to
(a) 3.87 kJ/s (b) 3 kJ/s
(c) 56 kcal/min (d) None of these

226. For a definite low temperature to be maintained, the COP of refrigerator can be related for cold and hot days as,
(a) $\text{COP}_{\text{hot days}} > \text{COP}_{\text{cold days}}$ (b) $\text{COP}_{\text{hot days}} < \text{COP}_{\text{cold days}}$
(c) $\text{COP}_{\text{hot days}} = \text{COP}_{\text{cold days}}$ (d) None of these

227. Bell-Coleman cycle based refrigeration system has
(a) Isobaric heat addition (b) Isothermal heat release
(c) Isochoric heat addition (d) Isothermal heat addition

228. COP of Bell-Coleman cycle and reversed Carnot cycle can be related as
(a) $\text{COP}_{\text{Bell-Coleman}} > \text{COP}_{\text{Carnot}}$ (b) $\text{COP}_{\text{Bell-Coleman}} < \text{COP}_{\text{Carnot}}$
(c) $\text{COP}_{\text{Bell-Coleman}} = \text{COP}_{\text{Carnot}}$ (d) None of these

229. For the compression of refrigerant beginning with wet state of refrigerant in compressor, the compression process is called,
(a) Dry compression (b) Wet compression
(c) Adiabatic compression (d) None of these

230. Throttling of certain gases may be used for getting the refrigeration. The value of Joule-Thomson coefficient for such throttling processes shall be,
(a) $\mu = 1$ (b) $\mu < 0$ (c) $\mu > 0$ (d) $\mu = 0$

231. Energy performance ratio (EPR) and Coefficient of performance (COP) can be related by,
(a) $EPR = \text{COP} + 1$ (b) $EPR + \text{COP} = 1$
(c) $EPR = \text{COP}$ (d) None of these

232. The absorption refrigeration cycle employs following devices for raising pressure of refrigerant,
(a) Compressor (b) Pump (c) Generator (d) Absorber

233. The coolant used in ammonia-LiBr, water absorption refrigeration system is
(a) Ammonia only (b) Water only
(c) Ammonia-LiBr (d) Ammonia-water

234. In ammonia-water absorption system the separation of ammonia from strong aqua-ammonia solution requires,
(a) Heat addition to it (b) Heat extraction from it
(c) No energy interaction (d) None of these

235. Absorber used in ammonia-water absorption system should be kept at low temperature because it,
(a) Helps in generating high pressure ammonia
(b) Facilitates better mixing of ammonia into water
(c) Drives out ammonia quickly
(d) None of these

236. The low pressure ammonia leaving expansion valve in ammonia-water absorption system is
(a) Pure vapour (b) Pure liquid
(c) Liquid-vapour mixture (d) Pure liquid or liquid-vapour mixture

237. Device used in absorption system for preventing the moisture accompanying refrigerant from entering into the circuit of absorption cycle is called,
(a) Rectifier (b) Evaporator (c) Generator (d) Absorber

238. Which of the following are secondary refrigerants?
(a) Brine (b) Water
(c) Anti-freeze agents (d) All of these

239. For increasing the refrigerating capacity the refrigerant should have small value of specific heat in liquid form and its specific heat value in vapour form should be,
(a) Small (b) Large
(c) Nothing can be said (d) None of these

240. Which of the following is/are desired properties of refrigerants?
(a) High viscosity (b) Low thermal conductivity
(c) Large specific volume (d) None of these

241. Humidity ratio can be given in terms of partial pressures of dry air (p_a) and water vapour (p_v) as,
(a) $0.622\left(\frac{p_a}{p_v}\right)$ (b) $0.622\left(\frac{p_v}{p_a}\right)$
(c) $0.622\left(\frac{p_v}{p_v - p_a}\right)$ (d) None of these

242. Wet bulb depression in case of saturated air shall be,
(a) 0 (b) $\frac{wbt}{dbt}$ (c) wbt (d) dbt

243. The saturation temperature corresponding to the partial pressure of water vapour present in moist air can be given as,
(a) Dry bulb temperature (b) Wet bulb temperature
(c) Dew point temperature (d) None of these

244. If air is passed over the cooling coils then this process is termed as
(a) Sensible cooling (b) Cooling with humidification
(c) Cooling with dehumidification (d) None of these

245. If in a process the humidity ratio and dry bulb temperature increase then the process may be,
(a) Sensible heating (b) Dehumidification
(c) Humidification (d) None of these

246. What processes could be there in summer air conditioning?
(a) Cooling (b) Humidification
(c) Dehumidification (d) All of these

247. The generally used comfort conditions required in air conditioned space in India are
(a) 30°C *DBT* and 60% relative humidity
(b) 15°C *DBT* and 50% relative humidity
(c) 25°C *DBT* and 60% relative humidity
(d) 25°C *DBT* and 50% relative humidity

248. Which of the following is demerit with vapour compression system over air refrigeration system
(a) Large initial investment (b) Small refrigerant requirement
(c) Smaller operating cost (d) All of these

249. Which of the following is advantage with vapour absorption system over the vapour compression system?
(a) No moving component (b) Less work input requirement
(c) Utilization of waste energy (d) All of these

250. Which of the following gives demerit with vapour absorption system over vapour compression system?
(a) Large size (b) Requirement of heating source
(c) Poor COP at high loads (d) All of these

ANSWERS

1. (a)	**2.** (b)	**3.** (c)	**4.** (b)	**5.** (b)	**6.** (b)
7. (a)	**8.** (c)	**9.** (c)	**10.** (b)	**11.** (c)	**12.** (a)
13. (a)	**14.** (b)	**15.** (b)	**16.** (a)	**17.** (a)	**18.** (c)
19. (d)	**20.** (a)	**21.** (b)	**22.** (c)	**23.** (a)	**24.** (d)
25. (a)	**26.** (b)	**27.** (a)	**28.** (c)	**29.** (b)	**30.** (c)
31. (c)	**32.** (b)	**33.** (d)	**34.** (a)	**35.** (d)	**36.** (b)
37. (a)	**38.** (b)	**39.** (b)	**40.** (a)	**41.** (b)	**42.** (b)
43. (b)	**44.** (a)	**45.** (a)	**46.** (b)	**47.** (d)	**48.** (a)
49. (c)	**50.** (c)	**51.** (a)	**52.** (b)	**53.** (d)	**54.** (c)
55. (c)	**56.** (a)	**57.** (d)	**58.** (b)	**59.** (a)	**60.** (a)
61. (d)	**62.** (b)	**63.** (a)	**64.** (c)	**65.** (a)	**66.** (c)
67. (c)	**68.** (a)	**69.** (a)	**70.** (d)	**71.** (b)	**72.** (a)
73. (b)	**74.** (d)	**75.** (c)	**76.** (d)	**77.** (d)	**78.** (a)
79. (c)	**80.** (d)	**81.** (d)	**82.** (b)	**83.** (a)	**84.** (a)
85. (c)	**86.** (a)	**87.** (a)	**88.** (d)	**89.** (a)	**90.** (b)
91. (b)	**92.** (a)	**93.** (d)	**94.** (a)	**95.** (a)	**96.** (b)
97. (b)	**98.** (a)	**99.** (c)	**100.** (a)	**101.** (b)	**102.** (a)

103. (a) **104.** (b) **105.** (c) **106.** (b) **107.** (c) **108.** (a)
109. (b) **110.** (c) **111.** (d) **112.** (a) **113.** (c) **114.** (a)
115. (b) **116.** (a) **117.** (a) **118.** (d) **119.** (a) **120.** (a)
121. (b) **122.** (c) **123.** (a) **124.** (c) **125.** (d) **126.** (b)
127. (a) **128.** (d) **129.** (a) **130.** (c) **131.** (d) **132.** (a)
133. (c) **134.** (a) **135.** (b) **136.** (d) **137.** (b) **138.** (d)
139. (a) **140.** (b) **141.** (d) **142.** (b) **143.** (a) **144.** (d)
145. (c) **146.** (d) **147.** (a) **148.** (d) **149.** (a) **150.** (c)
151. (d) **152.** (a) **153.** (b) **154.** (a) **155.** (b) **156.** (c)
157. (a) **158.** (d) **159.** (a) **160.** (b) **161.** (a) **162.** (c)
163. (d) **164.** (d) **165.** (a) **166.** (b) **167.** (b) **168.** (c)
169. (b) **170.** (a) **171.** (c) **172.** (b) **173.** (a) **174.** (b)
175. (a) **176.** (d) **177.** (c) **178.** (a) **179.** (b) **180.** (b)
181. (c) **182.** (a) **183.** (b) **184.** (a) **185.** (c) **186.** (a)
187. (c) **188.** (d) **189.** (a) **190.** (d) **191.** (b) **192.** (a)
193. (b) **194.** (b) **195.** (a) **196.** (d) **197.** (d) **198.** (a)
199. (d) **200.** (b) **201.** (a) **202.** (d) **203.** (d) **204.** (d)
205. (c) **206.** (a) **207.** (a) **208.** (a) **209.** (a) **210.** (a)
211. (d) **212.** (c) **213.** (a) **214.** (c) **215.** (b) **216.** (a)
217. (c) **218.** (d) **219.** (d) **220.** (b) **221.** (c) **222.** (b)
223. (d) **224.** (d) **225.** (a) **226.** (b) **227.** (a) **228.** (b)
229. (b) **230.** (c) **231.** (a) **232.** (b) **233.** (b) **234.** (a)
235. (b) **236.** (d) **237.** (a) **238.** (d) **239.** (b) **240.** (d)
241. (b) **242.** (a) **243.** (c) **244.** (a) **245.** (c) **246.** (d)
247. (c) **248.** (a) **249.** (d) **250.** (d)

APPENDIX

Table 1 Ideal-gas specific heats of various common gases at 300 K

Gas	*Formula*	*Gas constant R kJ/(kg.K)*	c_p *kJ/(kg.K)*	c_v *kJ/(kg.K)*	γ
Air	–	0.2870	1.005	0.718	1.400
Argon	Ar	0.2081	0.5203	0.3122	1.667
Butane	C_4H_{10}	0.1433	1.7164	1.5734	1.091
Carbon dioxide	CO_2	0.1889	0.846	0.657	1.289
Carbon monoxide	CO	0.2968	1.040	0.744	1.400
Ethane	C_2H_6	0.2765	1.7662	1.4897	1.186
Ethylene	C_2H_4	0.2964	1.5482	1.2518	1.237
Helium	He	2.0769	5.1926	3.1156	1.667
Hydrogen	H_2	4.1240	14.307	10.183	1.405
Methane	CH_4	0.5182	2.2537	1.7354	1.299
Neon	Ne	0.4119	1.0299	0.6179	1.667
Nitrogen	N_2	0.2968	1.039	0.743	1.400
Octane	C_8H_{18}	0.0729	1.7113	1.6385	1.044
Oxygen	O_2	0.2598	0.918	0.658	1.395
Propane	C_3H_8	0.1885	1.6794	1.4909	1.126
Steam	H_2O	0.4615	1.8723	1.4108	1.327

Table 2 Saturated Steam (temperature) table

		Specific volume m^3/kg		*Internal energy kJ/kg*			*Enthalpy kJ/kg*			*Entropy kJ/(kg.K)*		
Temp. °C T	*Sat. press.* kPa P_{sat}	*Sat. liquid* v_f	*Sat. vapour* v_g	*Sat. liquid* u_f	*Evap.* u_{fg}	*Sat. vapour* u_g	*Sat. liquid* h_f	*Evap.* h_{fg}	*Sat. vapour* h_g	*Sat. liquid* s_f	*Evap.* s_{fg}	*Sat. vapour* s_g
0.01	0.6113	0.001000	206.14	0.0	2375.3	2375.3	0.01	2501.4	2501.4	0.000	9.1562	9.1562
5	0.8721	0.001000	147.12	20.97	2361.3	2382.3	20.98	2489.6	2510.6	0.0761	8.9496	9.0257
10	1.2276	0.001000	106.38	42.00	2347.2	2389.2	42.01	2477.7	2519.8	0.1510	8.7498	8.9008
15	1.7051	0.001001	77.93	62.99	2333.1	2396.1	62.99	2465.9	2528.9	0.2245	8.5569	8.7814
20	2.339	0.001002	57.79	83.95	2319.0	2402.9	83.96	2454.1	2538.1	0.2966	8.3706	8.6672
25	3.169	0.001003	43.36	104.88	2409.8	2309.8	104.89	2442.3	2547.2	0.3674	8.1905	8.5580
30	4.246	0.001004	32.89	125.78	2290.8	2416.6	125.79	2430.5	2556.3	0.4369	8.0164	8.4533
35	5.628	0.001006	25.22	146.67	2276.7	2423.4	146.68	2418.6	2565.3	0.5053	7.8478	8.3531
40	7.384	0.001008	19.52	167.56	2262.6	2430.1	167.57	2406.7	2574.3	0.5725	7.6845	8.2570
45	9.593	0.001010	15.26	188.44	2248.4	2436.8	188.45	2394.8	2583.2	0.6387	7.5261	8.1648
50	12.349	0.001012	12.03	209.32	2234.2	2443.5	209.33	2382.7	2592.1	0.7038	7.3725	8.0763
55	15.758	0.001015	9.568	230.21	2219.9	2450.1	230.23	2370.7	2600.9	0.7679	7.2234	7.9913
60	19.940	0.001017	7.671	251.11	2205.5	2456.6	251.13	2358.5	2609.6	0.8312	7.0784	7.9096
65	25.03	0.001020	6.197	272.02	2191.1	2463.1	272.06	2346.2	2618.3	0.8935	6.9375	7.8310
70	31.19	0.001023	5.042	292.95	2176.6	2469.6	292.98	2333.8	2626.8	0.9549	6.8004	7.7553
75	38.58	0.001026	4.131	313.90	2162.0	2475.9	313.93	2321.4	2635.3	1.0155	6.6669	7.6824
80	47.39	0.001029	3.407	334.86	2147.4	2482.2	334.91	2308.8	2643.7	1.0753	6.5369	7.6122
85	57.83	0.001033	2.828	355.84	2132.6	2488.4	355.90	2296.0	2651.9	1.1343	6.4102	7.5445
90	70.14	0.001036	2.361	376.85	2117.7	2494.5	376.92	2283.2	2660.1	1.1925	6.2866	7.4791
95	84.55	0.001040	1.982	397.88	2102.7	2500.6	397.96	2270.2	2668.1	1.2500	6.1659	7.4159

Table 2 Saturated Steam (temperature) table

Temp. °C T	Sat. press. kPa P_{sat}	Specific volume m^3/kg		Internal energy kJ/kg			Enthalpy kJ/kg			Entropy kJ/(kg.K)		
		Sat. liquid v_f	Sat. vapour v_g	Sat. liquid u_f	Evap. u_{fg}	Sat. vapour u_g	Sat. liquid h_f	Evap. h_{fg}	Sat. vapour h_g	Sat. liquid s_f	Evap. s_{fg}	Sat. vapour s_g
100	0.10135	0.001044	1.6729	418.94	2087.6	2506.5	419.04	2257.0	2676.1	1.3069	6.0480	7.3549
105	0.12082	0.001048	1.4194	440.02	2072.3	2512.4	440.15	2243.7	2683.8	1.3630	5.9328	7.2958
110	0.14327	0.001052	1.2102	461.14	2057.0	2518.1	461.30	2230.2	2691.5	1.4185	5.8202	7.2387
115	0.16906	0.001056	1.0366	482.30	2041.4	2523.7	482.48	2216.5	2699.0	1.4734	5.7100	7.1833
120	0.19853	0.001060	0.8919	503.50	2025.8	2529.3	503.71	2202.6	2706.3	1.5276	5.6020	7.1296
125	0.2321	0.001065	0.7706	524.74	2009.9	2534.6	524.99	2188.5	2713.5	1.5813	5.4962	7.0775
130	0.2701	0.001070	0.6685	546.02	1993.9	2539.9	546.31	2174.2	2720.5	1.6344	5.3925	7.0269
135	0.3130	0.001075	0.5822	567.35	1977.7	2545.0	567.69	2159.6	2727.3	1.6870	5.2907	6.9777
140	0.3613	0.001080	0.5089	588.74	1961.3	2550.0	589.13	2144.7	2733.9	1.7391	5.1908	6.9299
145	0.4154	0.001085	0.4463	610.18	1944.7	2554.9	610.63	2129.6	2740.3	1.7907	5.0926	6.8833
150	0.4758	0.001091	0.3928	631.68	1927.9	2559.5	632.20	2114.3	2746.5	1.8418	4.9960	6.8379
155	0.5431	0.001096	0.3468	653.24	1910.8	2564.1	653.84	2098.6	2752.4	1.8925	4.9010	6.7935
160	0.6178	0.001102	0.3071	674.87	1893.5	2568.4	675.55	2082.6	2758.1	1.9427	4.8075	6.7502
165	0.7005	0.001108	0.2727	696.56	1876.0	2572.5	697.34	2066.2	2763.5	1.9925	4.7153	6.7078
170	0.7917	0.001114	0.2428	718.33	1858.1	2576.5	719.21	2049.5	2768.7	2.0419	4.6244	6.6663
175	0.8920	0.001121	0.2168	740.17	1840.0	2580.2	741.17	2032.4	2773.6	2.0909	4.5347	6.6256
180	1.0021	0.001127	0.19405	762.09	1821.6	2583.7	763.22	2015.0	2778.2	2.1396	4.4461	6.5857
185	1.1227	0.001134	0.17409	784.10	1802.9	2587.0	785.37	1997.1	2782.4	2.1879	4.3586	6.5465
190	1.2544	0.001141	0.15654	806.19	1783.8	2590.0	807.62	1978.8	2786.4	2.2359	4.2720	6.5079
195	1.3978	0.001149	0.14105	828.37	1764.4	2592.8	829.98	1960.0	2790.0	2.2835	4.1863	6.4698
200	1.5538	0.001157	0.12736	850.65	1744.7	2595.3	852.45	1940.7	2793.2	2.3309	4.1014	6.4323
205	1.7230	0.001164	0.11521	873.04	1724.5	2597.5	875.04	1921.0	2796.0	2.3780	4.0172	6.3952
210	1.9062	0.001173	0.10441	895.53	1703.9	2599.5	897.76	1900.7	2798.5	2.4248	3.9337	6.3585
215	2.104	0.001181	0.09479	918.14	1682.9	2601.1	920.62	1879.9	2800.5	2.4714	3.8507	6.3221
220	2.318	0.001190	0.08619	940.87	1661.5	2602.4	943.62	1858.5	2802.1	2.5178	3.7683	6.2861
225	2.548	0.001199	0.07849	963.73	1639.6	2603.3	966.78	1836.5	2803.3	2.5639	3.6863	6.2503
230	2.795	0.001209	0.07158	986.74	1617.2	2603.9	990.12	1813.8	2804.0	2.6099	3.6047	6.2146
235	3.060	0.001219	0.06537	1009.89	1594.2	2604.1	1013.62	1790.5	2804.2	2.6558	3.5233	6.1791
240	3.344	0.001229	0.05976	1033.21	1570.8	2604.0	1037.32	1766.5	2803.8	2.7015	3.4422	6.1437
245	3.648	0.001240	0.05471	1056.71	1546.7	2603.4	1061.23	1741.7	2803.0	2.7472	3.3612	6.1083
250	3.973	0.001251	0.05013	1080.39	1522.0	2602.4	1085.36	1716.2	2801.5	2.7927	3.2802	6.0730
255	4.319	0.001263	0.04598	1104.28	1596.7	2600.9	1109.73	1689.8	2799.5	2.8383	3.1992	6.0375
260	4.688	0.001276	0.04221	1128.39	1470.6	2599.0	1134.37	1662.5	2796.9	2.8838	3.1181	6.0019
265	5.081	0.001289	0.03877	1152.74	1443.9	2596.6	1159.28	1634.4	2793.6	2.9294	3.0368	5.9662
270	5.499	0.001302	0.03564	1177.36	1416.3	2593.7	1184.51	1605.2	2789.7	2.9751	2.9551	5.9301
275	5.942	0.001317	0.03279	1202.25	1387.9	2590.2	1210.07	1574.9	2785.0	3.0208	2.8730	5.8938
280	6.412	0.001332	0.03017	1227.46	1358.7	2586.1	1235.99	1543.6	2779.6	3.0668	2.7903	5.8571
285	6.909	0.001348	0.02777	1253.00	1328.4	2581.4	1262.31	1511.0	2773.3	3.1130	2.7070	5.8199
290	7.436	0.001366	0.02557	1278.92	1297.1	2576.0	1289.07	1477.1	2766.2	3.1594	2.6227	5.7821
295	7.993	0.001384	0.02354	1305.2	1264.7	2569.9	1316.3	1441.8	2758.1	3.2062	2.5375	5.7437
300	8.581	0.001404	0.02167	1332.0	1231.0	2563.0	1344.0	1404.9	2749.0	3.2534	2.4511	5.7045
305	9.202	0.001425	0.019948	1359.3	1195.9	2555.2	1372.4	1366.4	2738.7	3.3010	2.3633	5.6643
310	9.856	0.001447	0.018350	1387.1	1159.4	2546.4	1401.3	1326.0	2727.3	3.3493	2.2737	5.6230

Table 2 Saturated Steam (temperature) table

Temp. °C T	Sat. press. MPa P_{sat}	Specific volume m^3/kg		Internal energy kJ/kg			Enthalpy kJ/kg			Entropy $kJ/(kg.K)$		
		Sat. liquid v_f	Sat. vapour v_g	Sat. liquid u_f	Evap. u_{fg}	Sat. vapour u_g	Sat. liquid h_f	Evap. h_{fg}	Sat. vapour h_g	Sat. liquid s_f	Evap. s_{fg}	Sat. vapour s_g
315	10.547	0.001472	0.016867	1415.5	1121.1	2536.6	1431.0	1283.5	2714.5	3.3982	2.1821	5.5804
320	11.274	0.001499	0.015488	1444.6	1080.9	2525.5	1461.5	1238.6	2700.1	3.4480	2.0882	5.5362
330	12.845	0.001561	0.012996	1505.3	993.7	2498.9	1525.3	1140.6	2665.9	3.5507	1.8909	5.4417
340	14.586	0.001638	0.010797	1570.3	894.3	2464.6	1594.2	1027.9	2622.0	3.6594	1.6763	5.3357
350	16.513	0.001740	0.008813	1641.9	776.6	2418.4	1670.6	893.4	2563.9	3.7777	1.4335	5.2112
360	18.651	0.001893	0.006945	1725.2	626.3	2351.5	1760.5	720.3	2481.0	3.9147	1.1379	5.0526
370	21.03	0.002213	0.004925	1844.0	384.5	2228.5	1890.5	441.6	2332.1	4.1106	0.6865	4.7971
374.14	22.09	0.003155	0.003155	2029.6	0	2029.6	2099.3	0	2099.3	4.4298	0	4.4298

Table 3 Saturated steam (pressure) table

Press. kPa P	Sat. Temp. °C T_{sat}	Specific volume m^3/kg		Internal energy kJ/kg			Enthalpy kJ/kg			Entropy $kJ/(kg.K)$		
		Sat. liquid v_f	Sat. vapour v_g	Sat. liquid u_f	Evap. u_{fg}	Sat. vapour u_g	Sat. liquid h_f	Evap. h_{fg}	Sat. vapour h_g	Sat. liquid s_f	Evap. s_{fg}	Sat. vapour s_g
0.6113	0.01	0.001000	206.14	0.00	2375.3	2375.3	0.01	2501.3	2501.4	0.0000	9.1562	9.1562
1.0	6.98	0.001000	129.21	29.30	2355.7	2385.0	29.30	2484.9	2514.2	0.1059	8.8697	8.9756
1.5	13.03	0.001001	87.98	54.71	2338.6	2393.3	54.71	2470.6	2525.3	0.1957	8.6322	8.8279
2.0	17.50	0.001001	67.00	73.48	2326.0	2399.5	73.48	2460.0	2533.5	0.2607	8.4629	8.7237
2.5	21.08	0.001002	54.25	88.48	2315.9	2404.4	88.49	2451.6	2540.0	0.3120	8.3311	8.6432
3.0	24.08	0.001003	45.67	101.04	2307.5	2408.5	101.05	2444.5	2545.5	0.3545	8.2231	8.5776
4.0	28.96	0.001004	34.80	121.45	2293.7	2415.2	121.46	2432.9	2554.4	0.4226	8.0520	8.4746
5.0	32.88	0.001005	28.19	137.81	2282.7	2420.5	137.82	2423.7	2561.5	0.4764	7.9187	8.3951
7.5	40.29	0.001008	19.24	168.78	2261.7	2430.5	168.79	2406.0	2576.8	0.5764	7.6750	8.2515
10	45.81	0.001010	14.67	191.82	2246.1	2437.9	191.83	2392.8	2584.7	0.6493	7.5009	8.1502
15	53.97	0.001014	10.02	225.92	2222.8	2448.7	225.94	2373.1	2599.1	0.7549	7.2536	8.0085
20	60.06	0.001017	7.649	251.38	2205.4	2456.7	251.40	2358.3	2609.7	0.8320	7.0766	7.9085
25	64.97	0.001020	6.204	271.90	2191.2	2463.1	271.93	2346.3	2618.2	0.8931	6.9383	7.8314
30	69.10	0.001022	5.229	289.20	2179.2	2468.4	289.23	2336.1	2625.3	0.9439	6.8247	7.7686
40	75.87	0.001027	3.993	317.53	2159.5	2477.0	317.58	2319.2	2636.8	1.0259	6.6441	7.6700
50	81.33	0.001030	3.240	340.44	2143.4	2483.9	340.49	2305.4	2645.9	1.0910	6.5029	7.5939
75	91.78	0.001037	2.217	384.31	2112.4	2496.7	384.39	2278.6	2663.0	1.2130	6.2434	7.4564

Table 3 Saturated steam (pressure) table

Press. MPa P	Sat. Temp. °C T_{sat}	Specific volume m^3/kg		Internal energy kJ/kg			Enthalpy kJ/kg			Entropy $kJ/(kg.K)$		
		Sat. liquid v_f	Sat. vapour v_g	Sat. liquid u_f	Evap. u_{fg}	Sat. vapour u_g	Sat. liquid h_f	Evap. h_{fg}	Sat. vapour h_g	Sat. liquid s_f	Evap. s_{fg}	Sat. vapour s_g
0.100	99.63	0.001043	1.6940	417.36	2088.7	2506.1	417.46	2258.0	2675.5	1.3026	6.0568	7.3594
0.125	105.99	0.001048	1.3749	444.19	2069.3	2513.5	444.32	2241.0	2685.4	1.3740	5.9104	7.2844
0.150	111.37	0.001053	1.1593	466.94	2052.7	2519.7	467.11	2226.5	2693.6	1.4336	5.7897	7.2233
0.175	116.06	0.001057	1.0036	486.80	2038.1	2524.9	486.99	2213.6	2700.6	1.4849	5.6868	7.1717
0.200	120.23	0.001061	0.8857	504.49	2025.0	2529.5	504.70	2201.9	2706.7	1.5301	5.5970	7.1271
0.225	124.00	0.001064	0.7933	520.47	2013.1	2533.6	520.72	2191.3	2712.1	1.5706	5.5173	7.0878
0.250	127.44	0.001067	0.7187	535.10	2002.1	2537.2	535.37	2181.5	2716.9	1.6072	5.4455	7.0527
0.275	130.60	0.001070	0.6573	548.59	1991.9	2540.5	548.89	2172.3	2721.3	1.6408	5.3801	7.0209
0.300	133.55	0.001073	0.6058	561.15	1982.4	2543.6	561.47	2163.8	2725.3	1.6718	5.3201	6.9919
0.325	136.30	0.001076	0.5620	572.90	1973.5	2546.4	573.25	2155.8	2729.0	1.7006	5.2646	6.9652
0.350	138.88	0.001079	0.5243	583.95	1965.0	2548.9	584.33	2148.1	2732.4	1.7275	5.2130	6.9405
0.375	141.32	0.001081	0.4914	594.40	1956.9	2551.3	594.81	2140.8	2735.6	1.7528	5.1647	6.9175
0.40	143.63	0.001084	0.4625	604.31	1949.3	2553.6	604.74	2133.8	2738.6	1.7766	5.1193	6.8959
0.45	147.93	0.001088	0.4140	622.77	1934.9	2557.6	623.25	2120.7	2743.9	1.8207	5.0359	6.8565
0.50	151.86	0.001093	0.3749	639.68	1921.6	2561.2	640.23	2108.5	2748.7	1.8607	4.9606	6.8213
0.55	155.48	0.001097	0.3427	655.32	1909.2	2564.5	665.93	2097.0	2753.0	1.8973	4.8920	6.7893
0.60	158.85	0.001101	0.3157	669.90	1897.5	2567.4	670.56	2086.3	2756.8	1.9312	4.8288	6.7600
0.65	162.01	0.001104	0.2927	683.56	1886.5	2570.1	684.28	2076.0	2760.3	1.9627	4.7703	6.7331
0.70	164.97	0.001008	0.2729	696.44	1876.1	2572.5	697.22	2066.3	2763.5	1.9922	4.7158	6.7080
0.75	167.78	0.001112	0.2556	708.64	1866.1	2574.7	709.47	2057.0	2766.4	2.0200	4.6647	6.6847
0.80	170.43	0.001115	0.2404	720.22	1856.6	2576.8	721.11	2048.0	2769.1	2.0462	4.6166	6.6628
0.85	172.96	0.001118	0.2270	731.27	1847.4	2578.7	732.22	2039.4	2771.6	2.0710	4.5711	6.6421
0.90	175.38	0.001121	0.2150	741.83	1838.6	2580.5	742.83	2031.0	2773.9	2.0946	4.5280	6.6226
0.95	177.69	0.001124	0.2042	751.95	1830.2	2582.1	753.02	2023.1	2776.1	2.1172	4.4869	6.6041
1.00	179.91	0.001127	0.19444	761.68	1822.0	2583.6	762.81	2015.3	2778.1	2.1387	4.4478	6.5865
1.10	184.09	0.001133	017753	780.09	1806.3	2586.4	781.34	2000.4	2781.7	2.1792	4.3744	6.5536
1.20	187.99	0.001139	0.16333	797.29	1791.5	2588.8	798.65	1986.2	2784.8	2.2166	4.3067	6.5233
1.30	191.64	0.001144	0.15125	813.44	1777.5	2591.0	814.93	1972.7	2787.6	2.2515	4.2438	6.4953
1.40	195.07	0.001149	0.14084	828.70	1764.1	2592.8	830.30	1959.7	2790.0	2.2842	4.1850	6.4693
1.50	198.32	0.001154	0.13177	843.16	1751.3	2594.5	844.89	1947.3	2792.2	2.3150	4.1298	6.4448
1.75	205.76	0.001166	0.11349	876.46	1721.4	2597.8	878.50	1917.9	2796.4	2.3851	4.0044	6.3896
2.00	212.42	0.001177	0.09963	906.44	1693.8	2600.3	908.79	1890.7	2799.5	2.4474	3.8935	6.3409
2.25	218.45	0.001187	0.08875	933.83	1668.2	2602.0	936.49	1865.2	2801.7	2.5035	3.7937	6.2972
2.5	223.99	0.001197	0.07998	959.11	1644.0	2603.1	962.11	1841.0	2803.1	2.5547	3.7028	6.2575
3.0	233.90	0.001217	0.06668	1004.78	1599.3	2604.1	1008.42	1795.7	2804.2	2.6457	3.5412	6.1869
3.5	242.60	0.001235	0.05707	1045.43	1558.3	2603.7	1049.75	1753.7	2803.4	2.7253	3.4000	6.1253
4	250.40	0.001252	0.04978	1082.31	1520.0	2602.3	1087.31	1741.1	2801.4	2.7964	3.2737	6.0701
5	263.99	0.001286	0.03944	1147.81	1449.3	2597.1	1154.23	1640.1	2794.3	2.9202	3.0532	5.9734
6	275.64	0.001319	0.03244	1205.44	1384.3	2589.7	1213.35	1571.0	2784.3	3.0267	2.8625	5.8892
7	285.88	0.001351	0.02737	1257.55	1323.0	2580.5	1267.00	1505.1	2772.1	3.1211	2.6922	5.8133
8	295.06	0.001384	0.02352	1305.57	1264.2	2569.8	1316.64	1441.3	2758.0	3.2068	2.5364	5.7432
9	303.40	0.001418	0.02048	1350.51	1207.3	2557.8	1363.26	1378.9	2742.1	3.2858	2.3915	5.6722
10	311.06	0.001452	0.018026	1393.04	1151.4	2544.4	1407.56	1317.1	2724.7	3.3596	2.2544	5.6141

Contd.

Table 3 Saturated steam (pressure) table

Press. *MPa* *P*	*Sat. Temp.* *°C* T_{sat}	*Specific volume* m^3/kg *Sat. liquid* v_f	*Sat. vapour* v_g	*Internal energy* *kJ/kg* *Sat. liquid* u_f	*Evap.* u_{fg}	*Sat. vapour* u_g	*Enthalpy* *kJ/kg* *Sat. liquid* h_f	*Evap.* h_{fg}	*Sat. vapour* h_g	*Entropy* *kJ/(kg.K)* *Sat. liquid* s_f	*Evap.* s_{fg}	*Sat. vapour* s_g
11	318.15	0.001489	0.015987	1433.7	1096.0	2529.8	1450.1	1255.5	2705.6	3.4295	2.1233	5.5527
12	324.75	0.001527	0.014263	1473.0	1040.7	2513.7	1491.3	1193.3	2684.9	3.4962	1.9962	5.4924
13	330.93	0.001567	0.012780	1511.1	985.0	2496.1	1531.5	1130.7	2662.2	3.5606	1.8718	5.4323
14	336.75	0.001611	0.011485	1548.6	928.2	2476.8	1571.1	1066.5	2637.6	3.6232	1.7485	5.3717
15	342.24	0.001658	0.010337	1585.6	869.8	2455.5	1610.5	1000.0	2610.5	3.6848	1.6249	5.3098
16	347.44	0.001711	0.009306	1622.7	809.0	2431.7	1650.1	930.6	2580.6	3.7461	1.4994	5.2455
17	352.37	0.001770	0.008364	1660.2	744.8	2405.0	1690.3	856.9	2547.2	3.8079	1.3698	5.1777
18	357.06	0.001840	0.007489	1698.9	675.4	2374.3	1732.0	777.1	2509.1	3.8715	1.2329	5.1044
19	361.54	0.001924	0.006657	1739.9	598.1	2338.1	1776.5	688.0	2464.5	3.9388	1.0839	5.0228
20	365.81	0.002036	0.005834	1785.6	507.5	2293.0	1826.3	583.4	2409.7	4.0139	0.9130	4.9269
21	369.89	0.002207	0.004952	1842.1	388.5	2230.6	1888.4	446.2	2334.6	4.1075	0.6938	4.8013
22	373.80	0.002742	0.003568	1961.9	125.2	2087.1	2022.2	143.4	2165.6	4.3110	0.2216	4.5327
22.09	374.12	0.003155	0.003155	2029.6	0	2029.6	2099.3	0	2099.3	4.4298	0	4.4298

Table 4 Superheated steam table

T *°C*	*v* m^3/kg	*u* *kJ/kg*	*h* *kJ/kg*	*s* *kJ/(kg·K)*	*v* m^3/kg	*u* *kJ/kg*	*h* *kJ/kg*	*s* *kJ/(kg·K)*	*v* m^3/kg	*u* *kJ/kg*	*h* *kJ/kg*	*s* *kJ/(kg·K)*
	P = 0.01 MPa (45.81 °C)				*P = 0.05 MPa (81.33 °C)*				*P = 0.10 MPa (99.63 °C)*			
Sat.	14.674	2437.9	2584.7	8.1502	3.240	2483.9	2645.9	7.5939	1.6940	2506.1	2675.5	7.3594
50	14.869	2483.9	2592.6	8.1749								
100	17.196	2515.5	2687.5	8.4479	3.418	2511.6	2682.5	7.6947	1.6958	2506.7	2676.2	7.3614
150	19.512	2587.9	2783.0	8.6882	3.889	2585.6	2780.1	7.9401	1.9364	2582.8	2776.4	7.6134
200	21.825	2661.3	2879.5	8.9038	4.356	2659.9	2877.7	8.1580	2.172	2658.1	2875.3	7.8343
250	24.136	2736.0	2977.3	9.1002	4.820	2735.0	2976.0	8.3556	2.406	2733.7	2974.3	8.0333
300	26.445	2812.1	3075.5	9.2813	5.284	2811.3	3075.5	8.5373	2.639	2810.4	3074.3	8.2158
400	31.063	2968.9	3279.6	9.6077	6.209	2968.5	3278.9	8.8642	3.103	2967.9	3278.2	8.5435
500	35.679	3132.3	3489.1	9.8978	7.134	3132.0	3488.7	9.1546	3.565	3131.6	3488.1	8.8342
600	40.295	3302.5	3705.4	10.1608	8.057	3302.2	3705.1	9.4178	4.028	3301.9	3704.4	9.0976
700	44.911	3479.6	3928.7	10.4028	8.981	3479.4	3928.5	9.6599	4.490	3479.2	3928.2	9.3398
800	49.526	3663.8	4159.0	10.6281	9.904	3663.6	4158.0	9.8852	4.952	3663.5	4158.6	9.5652
900	54.141	3855.0	4396.4	10.8396	10.828	3854.9	4396.3	10.0967	5.414	3854.8	4396.1	9.7767
1000	58.757	4053.0	4640.6	11.0393	11.751	4052.9	4640.5	10.2964	5.875	4052.8	4640.3	9.9764
1100	63.372	4257.5	4891.2	11.2287	12.674	4257.4	4891.1	10.4859	6.337	4257.3	4891.0	10.1659
1200	67.987	4467.9	5147.8	11.4091	13.597	4467.8	5147.7	10.6662	6.799	4467.7	5147.6	10.3463
1300	72.602	4683.7	5409.7	11.5811	14.521	4683.6	5409.6	10.8382	7.260	4683.5	5409.5	10.5183

Table 4 Superheated steam table

T °C	*v* m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)
	P = 0.20 MPa (120.23 °C)				*P = 0.30 MPa (133.55 °C)*				*P = 0.40 MPa (143.63 °C)*			
Sat.	0.8857	2529.5	2706.7	7.1272	0.6058	2543.6	2725.3	6.9919	0.4625	2553.6	2738.6	6.8959
150	0.9596	2576.9	2768.8	7.2795	0.6339	2570.8	2761.0	7.0778	0.4708	2564.5	2752.8	6.9299
200	1.0803	2654.4	2870.5	7.5066	0.7163	2650.7	2865.6	7.3115	0.5342	2646.8	2860.5	7.1706
250	1.1988	2731.2	2971.0	7.7086	0.7964	2728.7	2967.6	7.5166	0.5951	2726.1	2964.2	7.3789
300	1.3162	2808.6	3071.8	7.8926	0.8753	2806.7	3069.3	7.7022	0.6548	2804.8	3066.8	7.5662
400	1.5493	2966.7	3276.6	8.2218	1.0315	2965.6	3275.0	8.0330	0.7726	2964.4	3273.4	7.8985
500	1.7814	3130.8	3487.1	8.5133	1.1867	3130.0	3486.0	8.3251	0.8893	3129.2	3484.9	8.1913
600	2.013	3301.4	3704.0	8.7770	1.3414	3300.8	3703.2	8.5892	1.0055	3300.2	3702.4	8.4558
700	2.244	3478.8	3927.6	9.0194	1.4957	3478.4	3927.1	8.8319	1.1215	3477.9	3926.5	8.6987
800	2.475	3663.1	4158.2	9.2449	1.6499	3662.9	4157.8	9.0576	1.2312	3662.4	4157.3	8.9244
900	2.705	3854.5	4395.8	9.4566	1.8041	3854.2	4395.4	9.2692	1.3529	3853.9	4395.1	9.1362
1000	2.937	4052.5	4640.0	9.5663	1.9581	4052.3	4639.7	9.4690	1.4685	4052.0	4639.4	9.3360
1100	3.168	4257.0	4890.7	9.8458	2.1121	4256.8	4890.4	9.6585	1.5840	4256.5	4890.2	9.5256
1200	3.399	4467.5	5147.5	10.0262	2.2661	4467.2	5147.1	9.8389	1.6996	4467.0	5146.8	9.7060
1300	3.630	4683.2	5409.3	10.1982	2.4201	4683.0	5409.0	10.0110	1.8151	4682.8	5408.8	9.8780

Table 4 Superheated steam table

T °C	*v* m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)
	P = 0.50 MPa (151.86 °C)				*P = 0.60 MPa (158.85 °C)*				*P = 0.80 MPa (170.43 °C)*			
Sat.	0.3749	2561.2	2748.7	6.8213	0.3157	2567.4	2756.8	6.7600	0.2404	2576.8	2769.1	6.6628
200	0.4249	2642.9	2855.4	7.0592	0.3520	2638.9	2850.1	6.9665	0.2608	2630.6	2839.3	6.8158
250	0.4744	2723.5	2960.7	7.2709	0.3938	2720.9	2957.2	7.1816	0.2931	2715.5	2950.0	7.0384
300	0.5226	2802.9	3064.2	7.4599	0.4344	2801.0	3061.6	7.3724	0.3241	2797.2	3056.5	7.2328
350	0.5701	2882.6	3167.7	7.6329	0.4742	2881.2	3165.7	7.5464	0.3544	2878.2	3161.7	7.4089
400	0.6173	2963.2	327l.9	7.7938	0.5137	2962.1	3270.3	7.7079	0.3843	2959.7	3267.1	7.5716
500	0.7109	3128.4	3483.9	8.0873	0.5920	3127.6	3482.8	8.0021	0.4433	3126.0	3480.6	7.8673
600	0.8041	3299.6	3701.7	7.3522	0.6697	3299.1	3700.9	8.2674	0.5018	3297.9	3699.4	8.1333
700	0.8969	3477.5	3925.9	8.5952	0.7472	3477.0	3925.3	8.5107	0.5601	3476.2	3924.2	8.3770
800	0.9896	3662.1	4156.9	8.8211	0.8215	3661.8	4156.5	8.7367	0.6181	3661.1	4155.6	8.6033
900	1.0822	3853.6	4394.7	9.0329	0.9017	3853.4	4394.4	8.9486	0.6761	3852.8	4393.7	8.8153
1000	1.1747	4051.8	4639.1	9.2328	0.9788	4051.5	4638.8	9.1485	0.7340	4051.0	4638.2	9.0153
1100	1.2672	4256.3	4889.9	9.4224	1.0559	4256.1	4889.6	9.3381	0.7919	4255.6	4889.1	9.2050
1200	1.3596	4466.8	5146.6	9.6029	1.1330	4466.5	5146.3	9.5185	0.8497	4466.1	5145.9	9.3855
1300	1.4521	4682.5	5408.6	9.7749	1.2101	4682.3	5408.3	9.6906	0.9076	4681.8	5407.9	9.5575

Table 4 Superheated steam table

T ºC	*v* m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)
	P = 1.00 MPa (179.91 ºC)				*P = 1.20 MPa (187.99 ºC)*				*P = 1.40 MPa (195.70 ºC)*			
Sat.	0.19444	2583.6	2778.1	6.5865	0.16333	2588.8	2784.8	6.5233	0.14084	2592.8	2790.0	6.4693
200	0.2060	2621.9	2827.9	6.6940	0.16930	2612.8	2815.9	6.5898	0.14302	2603.1	2803.3	6.4975
250	0.2327	2709.9	2942.6	6.9247	0.19234	2704.2	2935.0	6.8294	0.16350	2698.3	2927.2	6.7467
300	0.2579	2793.2	3051.2	7.1229	0.2138	2789.2	3045.8	7.0317	0.18228	2785.2	3040.4	6.9534
350	0.2825	2875.2	3157.7	7.3011	0.2345	2872.2	3153.6	7.2121	0.2003	2869.2	3149.5	7.1360
400	0.3066	2957.3	3263.9	7.4651	0.2548	2954.9	3260.7	7.3774	0.2178	2952.5	3257.5	7.3026
500	0.3541	3124.4	3478.5	7.7622	0.2946	3122.8	3476.3	7.6759	0.2521	3121.1	3474.1	7.6027
600	0.4011	3296.8	3697.9	8.0290	0.3339	3295.6	3696.3	7.9435	0.2860	3294.4	3694.8	7.8710
700	0.4478	3475.3	3923.1	8.2731	0.3729	3474.4	3922.0	8.1881	0.3195	3473.6	3920.8	8.1160
800	0.4943	3660.4	4154.7	8.4996	0.4118	3659.7	4153.8	8.4148	0.3528	3659.0	4153.0	8.3431
900	0.5407	3852.2	4392.9	8.7118	0.4505	3851.6	4392.2	8.6272	0.3861	3851.1	4391.5	8.5556
1000	0.5871	4050.5	4637.6	8.9119	0.4892	4050.0	4637.0	8.8274	0.4192	4049.5	4636.4	8.7559
1100	0.6335	4255.1	4888.6	9.1017	0.5278	4254.6	4888.0	9.0172	0.4524	4254.1	4887.5	8.9457
1200	0.6798	4465.6	5145.4	9.2822	0.5665	4465.1	5144.9	9.1977	0.4855	4464.7	5144.4	9.1262
1300	0.7261	4681.3	5407.4	9.4543	0.6051	4680.9	5407.0	9.3698	0.5186	4680.4	5406.5	9.2984

Table 4 Superheated steam table

T ºC	*v* m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)
	P = 1.60 MPa (201.41 ºC)				*P = 1.80 MPa (207.15 ºC)*				*p = 2.00 MPa (212.42 ºC)*			
Sat	0.12380	2596.0	2794.0	6.4218	0.11042	2598.4	2797.1	6.3794	0.09963	2600.3	2799.5	6.3409
225	0.13287	2644.7	2857.3	6.5518	0.11673	2636.6	2846.7	6.4808	0.10377	2628.3	2835.8	6.4147
250	0.14184	2692.3	2919.2	6.6732	0.12497	2686.0	2911.0	6.6066	0.11144	2679.6	2902.5	6.5453
300	0.15862	2781.1	3034.8	6.8844	0.14021	2776.9	3029.2	6.8226	0.12547	2772.6	3023.5	6.7664
350	0.17456	2866.1	3145.4	7.0694	0.15457	2863.0	3141.2	7.0100	0.13857	2859.8	3137.0	6.9563
400	0.19005	2950.1	3254.2	7.2374	0.16847	2947.7	3250.9	7.1794	0.15120	2945.2	3247.6	7.1271
500	0.2203	3119.5	3472.0	7.5390	0.19550	3117.9	3469.8	7.4825	0.17568	3116.2	3467.6	7.4317
600	0.2500	3293.3	3693.2	7.8080	0.2220	3292.1	3691.7	7.7523	0.19960	3290.9	3690.1	7.7024
700	0.2794	3472.7	3919.7	8.0535	0.2482	3471.8	3971.8	7.9983	0.2232	3470.9	3917.4	7.9487
800	0.3086	3658.3	4152.1	8.2808	0.2742	3657.6	4151.2	8.2258	0.2467	3657.0	4150.3	8.1765
900	0.3377	3850.5	4390.8	8.4935	0.3001	3849.9	4390.1	8.4386	0.2700	3849.3	4389.4	8.3895
1000	0.3668	4049.0	4635.8	8.6938	0.3260	4048.5	4635.2	8.6391	0.2933	4048.0	4634.6	8.5901
1100	0.3958	4253.7	4887.0	8.8837	0.3518	4253.2	4886.4	8.8290	0.3166	4252.7	4885.9	8.7800
1200	0.4248	4464.2	5143.9	9.0643	0.3776	4463.7	5143.4	9.0096	0.3398	4463.3	5142.9	8.9607
1300	0.4538	4679.9	5406.0	9.2364	0.4034	4679.5	5405.6	9.1818	0.3631	4679.0	5405.1	9.1329

Table 4 Superheated steam table

T °C	*v* m³/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m³/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m³/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)
	P = 2.50 MPa (223.99 °C)				*P = 3.00 MPa (233.90 °C)*				*P = 3.50 MPa (242.60 °C)*			
Sat.	0.07998	2603.1	2803.1	6.2575	0.06668	2604.1	2804.2	6.1869	0.05707	2603.7	2803.4	6.1253
225	0.08027	2605.6	2806.3	6.2639								
250	0.08700	2662.6	2880.1	6.4085	0.07058	2644.0	2855.8	6.2872	0.05872	2623.7	2829.2	6.1749
300	0.09890	2761.6	3008.8	6.6438	0.08114	2750.1	2993.5	6.5390	0.06842	2738.0	2977.5	6.4461
350	0.10976	2851.9	3126.3	6.8403	0.09053	2843.7	3115.3	6.7428	0.07678	2835.3	3104.0	6.6579
400	0.12010	2939.1	3239.3	7.0148	0.09936	2932.8	3230.9	6.9212	0.08453	2926.4	3222.3	6.8405
450	0.13014	3025.5	3350.8	7.1746	0.10787	3020.4	3344.0	7.0834	0.09196	3015.3	3337.2	7.0052
500	0.13993	3112.1	3462.1	7.3234	0.11619	3108.0	3456.5	7.2338	0.09918	3103.0	3450.9	7.1572
600	0.15930	3288.0	3686.3	7.5960	0.13243	3285.0	3682.3	7.5085	0.11324	3282.1	3678.4	7.4339
700	0.17832	3468.7	3914.5	7.8435	0.14838	3466.5	3911.7	7.7571	0.12699	3464.3	3908.8	7.6837
800	0.19716	3655.3	4148.2	8.0720	0.16414	3653.5	4145.9	7.9862	0.14056	3651.8	4143.7	7.9134
900	0.21590	3847.9	4387.6	8.2853	0.17980	3846.5	4385.9	8.1999	0.15402	3845.0	4384.1	8.1276
1000	0.2346	4046.7	4633.1	8.4861	0.19541	4045.4	4631.6	8.4009	0.16743	4044.1	4630.1	8.3288
1100	0.2532	4251.5	4884.6	8.6762	0.21098	4250.3	4883.3	8.5912	0.18080	4249.2	4881.9	8.5192
1200	0.2718	4462.1	5141.7	8.8569	0.22652	4460.9	5140.5	8.7720	0.19415	4459.8	5139.3	8.7000
1300	0.2905	4677.8	5404.0	9.0291	0.24206	4676.6	5402.8	8.9442	0.20749	4675.5	5401.7	8.8723

Table 4 Superheated steam table

T °C	*v* m³/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m³/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m³/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)
	P = 4.0 MPa (250.40 °C)				*P = 4.5 MPa (257.49 °C)*				*P = 5.0 MPa (263.99 °C)*			
Sat.	0.04978	2602.3	2801.4	6.0701	0.04406	2600.1	2798.3	6.0198	0.03944	2597.1	2794.3	5.9734
275	0.05457	2667.9	2886.2	6.2285	0.04730	2650.3	2863.2	6.1401	0.04141	2631.3	2838.3	6.0544
300	0.05884	2725.3	2960.7	6.3615	0.05135	2712.0	2943.1	6.2828	0.04532	2698.0	2924.5	6.2084
350	0.06645	2826.7	3092.5	6.5821	0.05840	2817.8	3080.6	6.5131	0.05194	2808.7	3068.4	6.4493
400	0.07341	2919.9	3213.6	6.7690	0.06475	2913.3	3204.7	6.7047	0.05781	2906.6	3195.7	6.6459
450	0.08002	3010.2	3330.3	6.9363	0.07074	3005.0	3323.3	6.8746	0.06330	2999.7	3316.2	6.8186
500	0.08643	3099.5	3445.3	7.0901	0.07651	3095.3	3439.6	7.0301	0.06857	3091.0	3433.8	6.9759
600	0.09885	3279.1	3674.4	7.3688	0.08765	3276.0	3670.5	7.3110	0.07869	3273.0	3666.5	7.2589
700	0.11095	3462.1	3905.9	7.6198	0.09847	3459.9	3903.0	7.5631	0.08849	3457.6	3900.1	7.5122
800	0.12287	3650.0	4141.5	7.8502	0.10911	3648.3	4139.3	7.7942	0.09811	3646.6	4137.1	7.7440
900	0.13469	3843.6	4382.3	8.0647	0.11965	3842.2	4380.6	8.0091	0.10762	3840.7	4378.8	7.9593
1000	0.14645	4042.9	4628.7	8.2662	0.13013	4041.6	4627.2	8.2108	0.11707	4040.4	4625.7	8.1612
1100	0.15817	4248.0	4880.6	8.4567	0.14056	4246.8	4879.3	8.4015	0.12648	4245.6	4878.0	8.3520
1200	0.16987	4458.6	5138.1	8.6376	0.15098	4457.5	5136.9	8.5825	0.13587	4456.3	5135.7	8.5331
1300	0.18156	4674.3	5400.5	8.8100	0.16139	4673.1	5399.4	8.7549	0.14526	4672.0	5398.2	8.7055

Table 4 Superheated steam table

T ºC	*v* m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)
	P = 6.0 MPa (275.64 ºC)				*P = 7.0 MPa (285.88 ºC)*				*P = 8.0 MPa (295.06 ºC)*			
Sat.	0.03244	2589.7	2784.3	5.8892	0.02737	2580.5	2772.1	5.8133	0.02352	2569.0	2758.0	5.7432
300	0.03616	2667.2	2884.2	6.0674	0.02947	2632.2	2838.4	5.9305	0.02426	2590.9	2785.0	5.7906
350	0.04223	2789.6	3043.0	6.3335	0.03524	2769.4	3016.0	6.2283	0.02995	2747.7	2987.3	6.1301
400	0.04739	2892.9	3177.2	6.5408	0.03993	2878.6	3158.1	6.4478	0.03432	2863.8	3138.3	6.3634
450	0.05214	2988.9	3301.8	6.7193	0.04416	2978.0	3287.1	6.6327	0.03817	2966.7	3272.0	6.5551
500	0.05665	3082.2	3422.2	6.8803	0.04814	3073.4	3410.3	6.7975	0.04175	3064.3	3398.3	6.7240
550	0.06101	3174.6	3540.6	7.0288	0.05195	3167.2	3530.9	6.9486	0.04516	3159.8	3521.0	6.8778
600	0.06525	3266.9	3658.4	7.1677	0.05565	3260.7	3650.3	7.0894	0.04845	3254.4	3642.0	7.0206
700	0.07352	3453.1	3894.2	7.4234	0.06283	3448.5	3888.3	7.3476	0.05481	3443.9	3882.4	7.2812
800	0.08160	3643.1	4132.7	7.6566	0.06981	3639.5	4128.2	7.5822	0.06097	3636.0	4123.8	7.5173
900	0.08958	3837.8	4375.3	7.8727	0.07669	3835.0	4371.8	7.7991	0.06702	3832.1	4368.3	7.7351
1000	0.09749	4037.8	4622.7	8.0751	0.08350	4035.3	4619.8	8.0020	0.07301	4032.8	4616.9	7.9384
1100	0.10536	4243.3	4875.4	8.2661	0.09027	4240.9	4872.8	8.1933	0.07896	4238.6	4870.3	8.1300
1200	0.11321	4454.0	5133.3	8.4474	0.09703	4451.7	5130.9	8.3747	0.08489	4449.5	5128.5	8.3115
1300	0.12106	4669.6	5396.0	8.6199	0.10377	4667.3	5393.7	8.5475	0.09080	4665.0	5391.5	8.4842

Table 4 Superheated steam table

T ºC	*v* m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m^3/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)
	P = 9.0 MPa (303.40 ºC)				*P = 10.0 MPa (311.06 ºC)*				*P = 12.5 MPa (327.89 ºC)*			
Sat.	0.02048	2557.8	2742.1	5.6772	0.018026	2544.4	2724.7	5.6141	0.013495	2505.1	2673.8	5.4624
325	0.02327	2646.6	2856.0	5.8712	0.019861	2610.4	2809.1	5.7568				
350	0.02580	2724.4	2956.6	6.0361	0.02242	2699.2	2923.4	5.9443	0.016126	2624.6	2826.2	5.7118
400	0.02993	2848.4	3117.8	6.2854	0.02641	2832.4	3096.5	6.2120	0.02000	2789.3	3039.3	6.0417
450	0.03350	2955.2	3256.6	6.4844	0.02975	2943.4	3240.9	6.4190	0.02299	2912.5	3199.8	6.2719
500	0.03677	3055.2	3386.1	6.6576	0.03279	3045.8	3373.7	6.5966	0.02560	3021.7	3341.8	6.4618
550	0.03987	3152.2	3511.0	6.8142	0.03564	3144.1	3500.9	6.7561	0.02801	3125.0	3475.2	6.6290
600	0.04285	3248.1	3633.7	6.9589	0.03837	3241.7	3625.3	6.9029	0.03029	3225.4	3604.0	6.7810
650	0.04574	3343.6	3755.3	7.0943	0.04101	3338.2	3748.2	7.0398	0.03248	3324.4	3730.4	6.9218
700	0.04857	3439.3	3876.5	7.2221	0.04358	3434.7	3870.5	7.1687	0.03460	3422.9	3855.3	7.0536
800	0.05409	3632.5	4119.3	7.4596	0.04859	3628.9	4114.8	7.4077	0.03869	3620.0	4103.6	7.2965
900	0.05950	3829.2	4364.8	7.6783	0.05349	3826.3	4361.2	7.6272	0.04267	3819.1	4352.5	7.5182
1000	0.06485	4030.3	4614.0	7.8821	0.05832	4027.8	4611.0	7.8315	0.04658	4021.6	4603.8	7.7237
1100	0.07016	4236.3	4867.7	8.0740	0.06312	4234.0	4865.1	8.0237	0.05045	4228.2	4858.8	7.9165
1200	0.07544	4447.2	5126.2	8.2556	0.06789	4444.9	5123.8	8.2055	0.05430	4439.3	5118.0	8.0937
1300	0.08072	4662.7	5389.2	8.4284	0.07265	4460.5	5387.0	8.3783	0.05813	4654.8	5381.4	8.2717

Table 4 Superheated steam table

T °C	*v* m³/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m³/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m³/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)
	P = 15.0 MPa (342.24 °C)				*P = 17.5 MPa (354.75 °C)*				*P = 20.0 MPa (365.81 °C)*			
Sat.	0.010337	2455.5	2610.5	5.3098	0.007920	2390.2	2528.8	5.1419	0.005834	2293.0	2409.7	4.9269
350	0.011470	2520.4	2692.4	5.4421								
400	0.015649	2740.7	2975.5	5.8811	0.012447	2685.0	2902.9	5.7213	0.009942	2619.3	2818.1	5.5540
450	0.018445	2879.5	3156.2	6.1404	0.015174	2844.2	3109.7	6.0184	0.012695	2806.2	3060.1	5.9017
500	0.02080	2996.6	3308.6	6.3443	0.017358	2970.3	3274.1	6.2383	0.014768	2942.9	3238.2	6.1401
550	0.02293	3104.7	3448.6	6.5199	0.019288	3083.9	3421.4	6.4230	0.016555	3062.4	3393.5	6.3348
600	0.02491	3208.6	3582.3	6.6776	0.02106	3191.5	3560.1	6.5866	0.018178	3174.0	3537.6	6.5048
650	0.02680	3310.3	3712.3	6.8224	0.02274	3296.0	3693.9	6.7357	0.019693	3281.4	3675.3	6.6582
700	0.02861	3410.9	3840.1	6.9572	0.02434	3324.6	398.7	6.8736	0.02113	3386.4	3809.0	6.7993
800	0.03210	3610.9	4092.4	7.2040	0.02738	3601.8	4081.1	7.1244	0.02385	3592.7	4069.7	7.0544
900	0.03546	3811.9	4343.8	7.4279	0.03031	3804.7	4335.1	7.3507	0.02645	3797.5	4326.4	7.2830
1000	0.03875	4015.4	4596.6	7.6348	0.03316	4009.3	4589.5	7.5589	0.02897	4003.1	4582.5	7.4925
1100	0.04200	4222.6	4852.6	7.8283	0.03597	4216.9	4846.4	7.7531	0.03145	4211.3	4840.2	7.6874
1200	0.04523	4433.8	5112.3	8.0108	0.03876	4428.3	5106.6	7.9360	0.03391	4422.8	5101.0	7.8707
1300	0.04845	4649.1	5376.0	8.1840	0.04154	4643.5	5370.5	8.1093	0.03636	4638.0	5365.1	8.0442

Table 4 Superheated steam table

T °C	*v* m³/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m³/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)	*v* m³/kg	*u* kJ/kg	*h* kJ/kg	*s* kJ/(kg·K)
	P = 25.0 MPa				*P = 30.0 MPa*				*P = 35.0 MPa*			
375	0.0019731	1798.7	1848.0	4.0320	0.0017892	1737.8	1791.5	3.9305	0.0017003	1702.9	1762.4	3.8722
400	0.006004	2430.1	2580.2	5.1418	0.002790	2067.4	2151.1	4.4728	0.002100	1914.1	1987.6	4.2126
425	0.007881	2609.2	2806.3	5.4723	0.005303	2455.1	2614.2	5.1504	0.003428	2253.4	2373.4	4.7747
450	0.009162	2720.7	2949.7	5.6744	0.006735	2619.3	2821.4	5.4424	0.004961	2498.7	2672.4	5.1962
500	0.011123	2884.3	3162.4	5.9592	0.008678	2820.7	3081.1	5.7905	0.006927	2751.9	2994.4	5.6282
550	0.012724	3017.5	3335.6	6.1765	0.010168	2970.3	3275.4	6.0342	0.008345	2921.0	3213.0	5.9026
600	0.014137	3137.9	3491.4	6.3602	0.011446	3100.5	3443.9	6.2331	0.009527	3062.0	3395.5	6.1179
650	0.015433	3251.6	3637.4	6.5229	0.012596	3221.0	3598.9	6.4058	0.010575	3189.8	3559.9	6.3010
700	0.016646	3361.3	3777.5	6.6707	0.013661	3335.8	3745.6	6.5606	0.011533	3309.8	3713.5	6.4631
800	0.018912	3574.3	4047.1	6.9345	0.015623	3555.5	4024.2	6.8332	0.013278	3536.7	4001.5	6.7450
900	0.021045	3783.0	4309.1	7.1680	0.017448	3768.5	4291.9	7.0718	0.014883	3754.0	4274.9	6.9386
1000	0.02310	3990.9	4568.5	7.3802	0.019196	3978.8	4554.7	7.2867	0.016410	3966.7	4541.1	7.2064
1100	0.02512	4200.2	4828.2	7.5765	0.020903	4189.2	4816.3	7.4845	0.017895	4178.3	4804.6	7.4037
1200	0.02711	4412.0	5089.9	7.7605	0.022589	4401.3	5079.0	7.6692	0.019360	4390.7	5068.3	7.5910
1300	0.02910	4626.9	5354.4	7.9342	0.024266	4616.0	5344.0	7.8432	0.020815	4605.1	5333.6	7.7653

Table 4 Superheated steam table

T	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*
°C	*m³/kg*	*kJ/kg*	*kJ/kg*	*kJ/(kg·K)*	*m³/kg*	*kJ/kg*	*kJ/kg*	*kJ/(kg·K)*	*m³/kg*	*kJ/kg*	*kJ/kg*	*kJ/(kg·K)*
	P = 40.0 MPa				*P = 50.0 MPa*				*P = 60.0 MPa*			
375	0.0016407	1677.1	1742.8	3.8290	0.0015594	1638.6	1716.6	3.7639	0.0015028	1609.4	1699.5	3.7141
400	0.0019077	1854.6	1930.9	4.1135	0.0017309	1788.1	1874.6	4.0031	0.0016335	1745.4	1843.4	3.9318
425	0.002532	2096.9	2198.1	4.5029	0.002007	1959.7	2060.0	4.2734	0.0018165	1892.7	2001.7	4.1626
450	0.003693	2365.1	2512.8	4.9459	0.002486	2159.6	2284.0	4.5884	0.002085	2053.9	2179.0	4.4121
500	0.005622	2678.4	2903.3	5.4700	0.003892	2525.5	2720.1	5.1726	0.002956	2390.6	2567.9	4.9321
550	0.006984	2869.7	3149.1	5.7785	0.005118	2763.6	3019.5	5.5485	0.003956	2658.8	2896.2	5.3441
600	0.008094	3022.6	3346.4	6.0144	0.006112	2942.0	3247.6	5.8178	0.004834	2861.1	3151.2	5.6452
650	0.009063	3158.0	3520.6	6.2054	0.006966	3093.5	3441.8	6.0342	0.005595	3028.8	3364.5	5.8829
700	0.009941	3283.6	3681.2	6.3750	0.007727	3230.5	3616.8	6.2189	0.006272	3177.2	3553.5	6.0824
800	0.011523	3517.8	3978.7	6.6662	0.009076	3479.8	3933.6	6.5290	0.007459	3441.5	3889.1	6.4109
900	0.012962	3739.4	4257.9	6.9150	0.010283	3710.3	4224.4	6.7882	0.008508	3681.0	4191.5	6.6805
1000	0.014324	3954.6	4527.6	7.1356	0.011411	3930.5	4501.1	7.0146	0.009480	3906.4	4475.2	6.9121
1100	0.015642	4167.4	4793.1	7.3364	0.012496	4145.7	4770.5	7.2184	0.010409	4124.1	4748.6	7.1195
1200	0.016940	4380.1	5057.7	7.5224	0.013561	4359.1	5037.2	7.4058	0.011317	4338.2	5017.2	7.3083
1300	0.018229	4594.3	5323.5	7.6969	0.014616	4572.8	5303.6	7.5808	0.012215	4551.4	5284.3	7.4837

Table 5 Compressed liquid water table

T	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*
°C	*m³/kg*	*kJ/kg*	*kJ/kg*	*kJ/(kg·K)*	*m³/kg*	*kJ/kg*	*kJ/kg*	*kJ/(kg·K)*	*m³/kg*	*kJ/kg*	*kJ/kg*	*kJ/(kg·K)*
	P = 5 MPa (263.99 °C)				*P = 10 MPa (311.06 °C)*				*P = 15 MPa (342.24 °C)*			
Sat.	0.0012859	1147.8	1154.2	2.9202	0.0014524	1393.0	1407.6	3.3596	0.0016581	l585.6	1610.5	3.6848
0	0.0009977	0.04	5.04	0.0001	0.0009952	0.09	10.04	0.0002	0.0009928	0.15	15.05	0.0004
20	0.0009995	83.65	83.65	0.2956	0.0009972	83.36	93.33	0.2945	0.0009950	83.06	97.99	0.2934
40	0.0010056	166.95	171.97	0.5705	0.0010034	166.35	176.38	0.5686	0.0010013	165.76	180.78	0.5666
60	0.0010149	250.23	255.30	0.8285	0.0010127	249.36	259.49	0.8258	0.0010105	248.51	263.67	0.8232
80	0.0010268	333.72	338.85	1.0720	0.0010245	332.59	342.83	1.0688	0.0010222	331.48	346.81	1.0656
100	0.0010410	417.52	422.72	1.3030	0.0010385	416.12	426.50	1.2992	0.0010361	414.74	430.28	1.2955
120	0.0010576	501.80	507.09	1.5233	0.0010549	500.08	510.64	1.5189	0.0010522	498.40	514.19	1.5145
140	0.0010768	586.76	592.15	1.7343	0.0010737	584.68	595.42	1.7292	0.0010707	582.66	598.72	1.7242
160	0.0010988	672.62	678.12	1.9375	0.0010953	670.13	681.08	1.9317	0.0010918	667.71	684.09	1.9260
180	0.0011240	759.63	765.25	2.1341	0.0011199	756.65	767.84	2.1275	0.0011159	753.76	770.50	2.1210
200	0.0011530	848.1	853.9	2.3255	0.0011480	844.5	856.0	2.3178	0.0011433	841.0	858.2	2.3104
220	0.0011866	938.4	944.4	2.5128	0.0011805	934.1	945.9	2.5039	0.0011748	929.9	947.5	2.4953
240	0.0012264	1031.4	1037.5	2.6979	0.0012187	1026.0	1038.1	2.6872	0.0012114	1020.8	1039.0	2.6771
260	0.0012749	1127.9	1134.3	2.8830	0.0012645	1121.1	1133.7	2.8699	0.0012550	1114.6	1133.4	2.8576
280					0.0013216	1220.9	1234.1	3.0548	0.0013084	1212.5	1232.1	3.0393
300					0.0013972	1328.4	1342.3	3.2469	0.0013770	1316.6	1337.3	3.2260
320									0.0014724	1431.1	1453.2	3.4247
340									0.0016311	1567.5	1591.9	3.6546

Table 5 Compressed liquid water table

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/(kg·K)	v m³/kg	u kJ/kg	h kJ/kg	s kJ/(kg·K)	v m³/kg	u kJ/kg	h kJ/kg	s kJ/(kg·K)
	P = 20.0 MPa (365.81 °C)				P = 30 MPa				P = 50 MPa			
Sat.	0.002036	1785.6	1826.3	4.0139								
0	0.0009904	0.19	20.01	0.0004	0.0009856	0.25	29.82	0.0001	0.0009766	0.20	49.03	0.0014
20	0.0009928	82.77	102.62	0.2923	0.0009886	82.17	111.84	0.2899	0.0009804	81.00	130.02	0.2848
40	0.0009992	165.17	185.16	0.5646	0.0009951	164.04	193.89	0.5607	0.0009872	161.86	211.21	0.5527
60	0.0010084	247.68	267.85	0.8206	0.0010042	246.06	276.19	0.8154	0.0009962	242.98	292.79	0.8052
80	0.0010199	330.40	350.80	1.0624	0.0010156	328.30	358.77	1.0561	0.0010073	324.34	374.70	1.0440
100	0.0010337	413.39	434.06	1.2917	0.0010290	410.78	441.66	1.2844	0.0010201	405.88	456.89	1.2703
120	0.0010496	496.76	517.76	1.5102	0.0010445	493.59	524.93	1.5018	0.0010348	487.65	539.39	1.4857
140	0.0010678	580.69	602.04	1.7193	0.0010621	576.88	608.75	1.7098	0.0010515	569.77	622.35	1.6915
160	0.0010885	665.35	687.12	1.9204	0.0010821	660.82	693.28	1.9096	0.0010703	652.41	705.92	1.8891
180	0.0011120	750.95	773.20	2.1147	0.0011047	745.59	778.73	2.1024	0.0010912	735.69	790.25	2.0794
200	0.0011388	837.7	860.5	2.3031	0.0011302	831.4	865.3	2.2893	0.0011146	819.7	875.5	2.2634
220	0.0011695	925.9	949.3	2.4870	0.0011590	918.3	953.1	2.4711	0.0011408	904.7	961.7	2.4419
240	0.0012046	1016.0	1040.0	2.6674	0.0011920	1006.9	1042.6	2.6490	0.0011702	990.7	1049.2	2.6158
260	0.0012462	1108.6	1133.5	2.8459	0.0012303	1097.4	1134.3	2.8243	0.0012034	1078.1	1138.2	2.7860
280	0.0012965	1204.7	1230.6	3.0248	0.0012755	1190.7	1229.0	2.9986	0.0012415	1167.2	1229.3	2.9537
300	0.0013596	1306.1	1333.3	3.2071	0.0013304	1287.9	1327.8	3.1741	0.0012860	1258.7	1323.0	3.1200
320	0.0014437	1415.7	1444.6	3.3979	0.0013997	1390.7	1432.7	3.3539	0.0013388	1353.3	1420.2	3.2868
340	0.0015684	1539.7	1571.0	3.6075	0.0014920	1501.7	1546.5	3.5426	0.0014032	1452.0	1522.1	3.4557
360	0.0018226	1702.8	1739.3	3.8772	0.0016265	1626.6	1675.4	3.7494	0.0014838	1556.0	1630.2	3.6291
380					0.0018691	1781.4	1837.5	4.0012	0.0015884	1667.2	1746.6	3.8101

Table 6 Saturated ice-steam (temperature) table

	Sat.	Specific volume m³/kg		Internal energy kJ/kg			Enthalpy kJ/kg			Entropy kJ/(kg.K)		
Temp. °C	Press. kPa	Sat. ice	Sat. vapour	Sat. ice	Subl.	Sat. vapour	Sat. ice	Subl.	Sat. vapour	Sat. ice	Subl.	Sat. vapour
T	T_{sat}	$v_i \times 10^3$	v_g	u_i	u_{ig}	u_g	h_i	h_{ig}	h_g	s_i	s_{ig}	s_g
0.01	0.6113	1.0908	206.1	–333.40	2708.7	2375.3	–333.40	2834.8	2501.4	–1.221	10.378	9.156
0	0.6108	1.0908	206.3	–333.43	2708.8	2375.3	–333.43	2834.8	2501.3	–1.221	10.378	9.157
–2	0.5176	1.0904	241.7	–337.62	2710.2	2372.6	–337.62	2835.3	2497.7	–1.237	10.456	9.219
–4	0.4375	1.0901	283.8	–341.78	2711.6	2369.8	–341.78	2835.7	2494.0	–1.253	10.536	9.283
–6	0.3689	1.0898	334.2	–345.91	2712.9	2367.0	–345.91	2836.2	2490.3	–1.268	10.616	9.348
–8	0.3102	1.0894	394.4	–350.02	2714.2	2364.2	–350.02	2836.6	2486.6	–1.284	10.698	9.414
–10	0.2602	1.0891	466.7	–354.09	2715.5	2361.4	–354.09	2837.0	2482.9	–1.299	10.781	9.481
–l2	0.2176	1.0888	553.7	–358.14	2716.8	2358.7	–358.14	2837.3	2479.2	–1.315	10.865	9.550
–14	0.1815	1.0884	658.8	–362.15	2718.0	2355.9	–362.15	2837.6	2475.5	–1.331	10.950	9.619
–16	0.1510	1.0881	786.0	–366.14	2719.2	2353.1	–366.14	2837.9	2471.8	–1.346	11.036	9.690
–18	0.l252	1.0878	940.5	–370.10	2720.4	2350.3	–370.10	2838.2	2468.1	–1.362	11.123	9.762
–20	0.1035	1.0874	1128.6	–374.03	2721.6	2347.5	–374.03	2838.4	2464.3	–1.377	11.212	9.835
–22	0.0853	1.0871	1358.4	–377.93	2722.7	2344.7	–377.93	2838.6	2460.6	–1.393	11.302	9.909
–24	0.0701	1.0868	1640.1	–381.80	2723.7	2342.0	–381.80	2838.7	2456.9	–1.408	11.394	9.985
–26	0.0574	1.0864	1986.4	–385.64	2724.8	2339.2	–385.64	2838.9	2453.2	–1.424	11.486	10.062

Contd.

		Specific volume m^3/kg		Internal energy kJ/kg			Enthalpy kJ/kg			Entropy $kJ/(kg.K)$		
Temp. °C	Sat. Press. kPa	Sat. ice	Sat. vapour	Sat. ice	Subl.	Sat. vapour	Sat. ice	Subl.	Sat. vapour	Sat. ice	Subl.	Sat. vapour
T	T_{sat}	$v_i \times 10^3$	v_g	u_i	u_{ig}	u_g	h_i	h_{ig}	h_g	s_i	s_{ig}	s_g
–28	0.0469	1.0861	2413.7	–389.45	2725.8	2336.4	–389.45	2839.0	2449.5	–1.439	11.580	10.141
–30	0.0381	1.0858	2943	–393.23	2726.8	2333.6	–393.23	2839.0	2445.8	–1.455	11.676	10.221
–32	0.0309	1.0854	3600	–396.98	2727.8	2330.8	–396.98	2839.1	2442.1	–1.471	11.773	10.303
–34	0.0250	1.0851	4419	–400.71	2728.7	2328.0	–400.71	2839.1	2438.4	–1.486	11.872	10.386
–36	0.0201	1.0848	5444	–404.40	2729.6	2325.2	–404.40	2839.1	2434.7	–1.501	11.972	10.470
–38	0.0161	1.0844	6731	–408.06	2730.5	2322.4	–408.06	2839.0	2430.9	–1.517	12.073	10.556
–40	0.0129	1.0841	8354	–411.70	2731.3	2319.6	–411.70	2839.9	2427.2	–1.532	12.176	10.644

Table 7 Critical Point Data For Some Substances

Substance	*Formula*	*Molecular Weight*	*Temperature* T_c, K	*Pressure* p_c, MPa	*Volume* $m^3/kmol$
Ammonia	NH_3	17.03	405.5	11.28	.0724
Argon	Ar	39.948	151	4.86	.0749
Bromine	Br_2	159.808	584	10.34	.1355
Carbon Dioxide	CO_2	44.01	304.2	7.39	.0943
Carbon Monoxide	CO	28.001	133	3.50	0.930
Chlorine	Cl_2	70.906	417	7.71	.1242
Deuterium	D_2	4.00	38.4	1.66	—
Helium	He	4.003	5.3	0.23	0.578
Helium[3]	He	3.00	3.3	0.12	—
Hydrogen	H_2	2.016	33.3	1.30	.0649
Krypton	Kr	83.80	209.4	5.50	.0924
Neon	Ne	20.183	44.5	2.73	.0417
Nitrogen	N_2	28.013	126.2	3.39	.0899
Nitrous Oxide	N_2O	44.013	309.7	7.27	.0961
Oxygen	O_2	31.999	154.8	5.08	.0780
Sulfur Dioxide	SO_2	64.063	430.7	7.88	.1217
Water	H_2O	18.015	647.3	22.09	.0568
Xenon	Xe	131.30	289.8	5.88	.1186
Benzene	X_2H_6	78.115	562	4.92	.2603
n-Butane	C_4H_{10}	58.124	425.5	3.80	.2547
Carbon Tetrachloride	CCl_4	153.82	556.4	4.56	.2759
Chloroform	$CHCl_2$	119.38	536.6	5.47	.2403
Dichlorodifluoromethane	CCl_2F_2	120.91	384.7	4.01	.2179
Dichlorofluoromethane	$CHCl_2F$	102.92	451.7	5.17	.1973
Ethane	C_2H_6	30.070	305.5	4.88	.1480
Ethyl Alcohol	C_2H_5OH	46.07	516	6.38	.1673
Ethylene	C_2H_{14}	28.054	282.4	5.12	.1242
n-Hexane	C_6H_4	86.178	507.9	3.03	.3677
Methane	CH_4	16.043	191.1	4.64	.993
Methyl Alcohol	CH_3OH	32.042	513.2	7.95	.1180
Methyl Chloride	CH_3Cl	50.488	416.3	6.68	.1430
Propane	C_3H_8	44.097	370	4.26	.1998
Propene	C_3H_6	42.081	365	4.62	.1810
Propyne	C_3H_4	40.065	401	5.35	—
Trichlorofluoromethane	CCl_3F	137.37	471.2	4.38	.2478

Table 8 Saturated Ammonia Table

Temp. °C	Abs. Press. kpa P	Specific volume m^3/kg			Enthalpy kJ/kg			Entropy kJ/(kg.K)		
		Sat. Liquid v_f	Evap. v_{fg}	Sat. Vapor v_g	Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapor h_g	Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapor s_g
–50	40.88	0.001 424	2.6239	2.6254	–44.3	1416.7	1372.4	–0.1942	6.3502	6.1561
–48	45.96	0.001 429	2.3518	2.3533	–35.5	1411.3	1375.8	–0.1547	6.2696	6.1149
–46	51.55	0.001 434	2.1126	2.1124	–26.6	1405.8	1379.2	–0.1156	6.1902	6.0746
–44	57.69	0.001 439	1.9018	1.9032	–17.8	1400.3	1382.5	–0.0768	6.1120	6.0352
–42	64.42	0.001 444	1.7155	1.7170	–8.9	1394.7	1385.8	–0.0382	6.0349	5.9967
–40	71.77	0.001 449	1.5506	1.5521	0.0	1389.0	1389.0	0.0000	5.9589	5.9589
–38	79.80	0.001 454	1.4043	1.4058	8.9	1383.3	1392.2	0.0380	5.8840	5.9220
–36	88.54	0.001 460	1.2742	1.2757	17.8	1377.6	1395.4	0.0757	5.8101	5.8858
–34	98.05	0.001 465	1.1582	1.1597	26.8	1371.8	1398.5	0.1132	5.7372	5.8504
–32	108.37	0.001 470	1.0547	1.0562	35.7	1365.9	1401.6	0.1504	5.6652	5.8156
–30	119.55	0.001 476	0.9621	0.9635	44.7	1360.0	1404.6	0.1873	5.5942	5.7815
–28	131.64	0.001 481	0.8790	0.8805	53.6	1354.0	1407.6	0.2240	5.5241	5.7481
–26	144.70	0.001 487	0.8044	0.8059	62.6	1347.9	1410.5	0.2605	5.4548	5.7153
– 24	158.78	0.001 492	0.7373	0.7388	71.6	1341.8	1413.4	0.2967	5.3864	5.6831
–22	173.93	0.001 498	0.6768	0.6783	80.7	1335.6	1416.2	0.3327	5.3188	5.6515
–20	190.22	0.001 504	0.6222	0.6237	89.7	1329.3	1419.0	0.3684	5.2520	5.6205
–18	207.71	0.001 510	0.5728	0.5743	98.8	1322.9	1421.7	0.4040	5.1860	5.5900
–16	226.45	0.001 515	0.5280	0.5296	107.8	1316.5	1424.4	0.4393	5.1207	5.5600
–14	246.51	0.001 521	0.4874	0.4889	116.9	1310.0	1427.0	0.4744	5.0561	5.5305
–12	267.95	0.001 528	0.4505	0.4520	126.0	1303.5	1429.5	0.5093	4.9922	5.5015
–10	290.85	0.001 534	0.4169	0.4185	135.2	1296.8	1432.0	0.5440	4.9290	5.4730
–8	315.25	0.001 540	0.3863	0.3878	144.3	1290.1	1434.4	0.5785	4.8664	5.4449
–6	341.25	0.001 546	0.3583	0.3599	153.5	1283.3	1436.8	0.6128	4.8045	5.4173
–4	368.90	0.001 553	0.3328	0.3343	162.7	1276.4	1439.1	0.6469	4.7432	5.3901
–2	398.27	0.001 559	0.3094	0.3109	171.9	1269.4	1441.3	0.6808	4.6825	5.3633
0	429.44	0.001 566	0.2879	0.2895	181.1	1262.4	1443.5	0.7145	4.6223	5.3369
2	462.49	0.001 573	0.2683	0.2698	190.4	1255.2	1445.6	0.7481	4.5627	5.3108
4	497.49	0.001 580	0.2502	0.2517	199.6	1248.0	1447.6	0.7815	4.5037	5.2852
6	534.51	0.001 587	0.2335	0.2351	208.9	1240.6	1449.6	0.8148	4.4451	5.2599
8	573.64	0.001 594	0.2182	0.2198	218.3	1233.2	1451.5	0.8479	4.3871	5.2350
10	614.95	0.001 601	0.2040	0.2056	227.6	1225.7	1453.3	0.8808	4.3295	5.2104
12	658.52	0.001 608	0.1910	0.1926	237.0	1218.1	1455.1	0.9136	4.2725	5.1861
14	407.44	0.001 616	0.1789	0.1805	246.4	1210.4	1456.8	0.9463	4.2159	5.1621
16	752.79	0.001 623	0.1677	0.1693	255.9	1202.6	1458.5	0.9788	4.1597	5.1385
18	803.66	0.001 631	0.1574	0.1590	265.4	1194.7	1460.0	1.0112	4.1039	5.1151
20	857.12	0.001 639	0.1477	0.1494	274.9	1186.7	1461.5	1.0434	4.0486	5.0920
22	913.27	0.001 647	0.1388	0.1405	284.4	1178.5	1462.9	1.0755	3.9937	5.0692
24	972.19	0.001 655	0.1305	0.1322	294.0	1170.3	1464.3	1.1075	3.9392	5.0467
26	1033.97	0.001 663	0.1228	0.1245	303.6	1162.0	1465.6	1.1394	3.8850	5.0244
28	1098.71	0.001 671	0.1156	0.1173	313.6	1153.6	1466.8	1.1711	3.8312	5.0023
30	1166.49	0.001 680	1.1089	0.1106	322.9	1145.0	1467.9	1.2028	3.7777	4.9805
32	1237.41	0.001 689	0.1027	0.1044	332.6	1136.4	1469.0	1.2343	3.7246	4.9589
34	1311.55	0.001 698	0.0969	0.0986	342.3	1127.6	1469.9	1.2656	3.6718	4.9374
36	1389.03	0.001 707	0.0914	0.0931	352.1	1118.7	1470.8	1.2969	3.6192	4.9161
38	1469.92	0.001 716	0.0863	0.0880	361.9	1109.7	1471.5	1.3281	3.5669	4.8950
40	1554.33	0.001 726	0.0815	0.0833	371.7	1100.5	1472.2	1.3591	3.5148	4.8740
42	1442.35	0.001 735	0.0771	0.0788	381.6	1091.2	1472.8	1.3901	3.4630	4.8530
44	1734.09	0.001 745	0.0728	0.0746	391.5	1081.7	1473.2	1.4209	3.4112	4.8322
46	1829.65	0.001 756	0.0689	0.0707	401.5	1072.0	1473.5	1.4518	3.3595	4.8113
48	1929.13	0.001 766	0.0652	0.0669	411.5	1062.2	1473.7	1.4826	3.3079	4.7905
50	2032.62	0.001 777	0.0617	0.0635	421.7	1052.0	1473.7	1.5135	3.2561	4.7696

Table 9 Superheated Ammonia Table

Abs. Press. kPa		*Temperature, °C*											
		–20	–10	0	10	20	30	40	50	60	70	80	100
	v	2.4474	–2.5481	2.6482	2.7479	2.8473	2.9464	3.0453	3.1441	3.2427	3.3413	3.4397	
50	*h*	1435.8	1457.0	1478.1	1499.2	1520.4	1541.7	1563.0	1584.5	1606.1	1627.8	1649.7	
	s	6.3256	6.4077	6.4865	6.5625	6.6360	6.7073	6.7766	6.8441	6.9099	6.9743	7.0372	
	v	1.6233	1.6915	1.7591	1.8263	1.8932	1.9597	2.0261	2.0923	2.1584	2.2244	2.2903	
75	*h*	1433.0	1454.7	1476.1	1497.5	1518.9	1540.3	1561.8	1583.4	1605.1	1626.9	1648.9	
	s	6.1190	6.2028	6.2828	6.3597	6.4339	6.5058	6.5756	6.6434	6.7096	6.7742	6.8373	
	v	1.2110	1.2631	1.3145	1.3654	1.4160	1.4664	1.5165	1.5664	1.6163	1.6659	1.7155	1.8145
100	*h*	1430.1	1452.2	1474.1	1495.7	1517.3	1538.9	1560.5	1582.2	1604.1	1626.0	1648.0	1692.6
	s	5.9695	6.0552	6.1366	6.2144	6.2894	6.3618	6.4321	6.5003	6.5668	6.6316	6.6950	6.8l77
	v	0.9635	1.0059	1.0476	1.0889	1.1297	1.1703	1.2107	1.2509	1.2909	1.3309	1.3707	1.4501
125	*h*	1427.2	1449.8	1472.0	1493.9	1515.7	1537.5	1559.3	1581.1	1603.0	1625.0	1647.2	1691.8
	s	5.8512	5.9389	6.0217	6.1006	6.1763	6.2494	6.3201	6.3887	6.4555	6.5206	6.5842	6.7072
	v	0.7984	0.8344	0.8697	0.9045	0.9388	0.9729	1.0068	1.0405	1.0740	1.1074	1.1408	1.2072
150	*h*	1424.1	1447.3	1469.8	1492.1	1514.1	1536.1	1558.0	1580.0	1602.0	1624.1	1646.3	1691.1
	s	5.7526	5.8424	5.9266	6.0066	6.0831	6.1568	6.2280	6.2970	6.3641	6.4295	6.4933	6.6167
	v		0.6199	0.6471	0.6738	0.7001	0.7261	0.7519	0.7774	0.8029	0.8282	0.8533	0.9035
200	*h*		1442.0	1465.5	1488.4	1510.9	1533.2	1555.5	1577.7	1599.9	1622.2	1644.6	1689.6
	s		5.6863	5.7737	5.8559	5.9342	6.0091	6.0813	6.1512	6.2189	6.2849	6.3491	6.4732
	v		0.4910	0.5135	0.5354	0.5568	0.5780	0.5989	0.6196	0.6401	0.6605	0.6809	0.7212
250	*h*		1436.6	1461.0	1484.5	1507.6	1530.3	1552.9	1575.4	1597.8	1620.3	1642.8	1688.2
	s		5.5609	5.6517	5.7365	5.8165	5.8928	5.9661	6.0368	6.1052	6.1717	6.2365	6.3613
	v			0.4243	0.4430	0.4613	0.4792	0.4968	0.5143	0.5316	0.5488	0.5658	0.5997
	h			1456.3	1480.6	1504.2	1527.4	1550.3	1573.0	1595.7	1618.4	1641.1	1686.7
300	*s*			5.5493	5.6366	5.7186	5.7963	5.8707	5.9423	6.0114	6.0785	6.1437	6.2693
	v			0.3605	0.3770	0.3929	0.4086	0.4239	0.4391	0.4541	0.4689	0.4837	0.5129
350	*h*			1451.5	1476.5	1500.7	1524.4	1547.6	1570.7	1593.6	1616.5	1639.3	1685.2
	s			5.4600	5.5502	5.6342	5.7135	5.7890	5.8615	5.9314	5.9990	6.0647	6.1910
	v			0.3125	0.3274	0.3417	0.3556	0.3692	0.3826	0.3959	0.4090	0.4220	0.4478
400	*h*			1446.5	1472.4	1497.2	1521.3	1544.9	1568.3	1591.5	1614.5	1637.6	1683.7
	s			5.3803	5.4735	5.5597	5.6405	5.7173	5.7907	5.8613	5.9296	5.9957	6.1228
	v			0.2752	0.2887	0.3017	0.3143	0.3266	0.3387	0.3506	0.3624	0.3740	0.3971
450	*h*			1441.3	1468.1	1493.6	1518.2	1542.2	1565.9	1589.3	1612.6	1635.8	1682.2
	s			5.3078	5.4042	5.4926	5.5752	5.6532	5.7275	5.7989	5.8678	5.9345	6.0623
		20	30	40	50	60	70	80	100	120	140	160	180
	v	0.2698	0.2813	0.2926	0.3036	0.3144	0.3251	0.3357	0.3565	0.3771	0.3975		
500	*h*	1489.9	1515.0	1539.5	1563.4	1587.1	1610.6	1634.0	1680.7	1727.5	1774.7		
	s	5.4314	5.5157	5.5950	5.6704	5.7425	5.8120	5.8793	6.0079	6.1301	6.2472		
	v	0.2217	0.2317	0.2414	0.2508	0.2600	0.2691	0.2781	0.2957	0.3130	0.3302		
600	*h*	1482.4	1508.6	1533.8	1558.5	1582.7	1606.6	1630.4	1677.7	1724.9	1772.4		

	s	5.3222	5.4102	5.4923	5.5697	5.6436	5.7144	5.7826	5.9129	6.0363	6.1541		
	v	0.1874	0.1963	0.2048	0.2131	0.2212	0.2291	0.2369	0.2522	0.2672	0.2821		
700	*h*	1474.5	1501.9	1528.1	1553.4	1578.2	1602.6	1626.8	1674.6	1722.4	1770.2		
	s	5.2259	5.3179	5.4029	5.4826	5.5582	5.6303	5.6997	5.8316	5.9562	6.0749		
	v	0.1615	0.1696	0.1773	0.1848	0.1920	0.1991	0.2060	0.2196	0.2329	0.2459	0.2589	
800	*h*	1466.3	1495.0	1522.2	1548.3	1573.7	1598.6	1623.1	1671.6	1719.8	1768.0	1816.4	
	s	5.1387	5.2351	5.3232	5.4053	5.4827	5.5562	5.6268	5.7603	5.8861	6.0057	6.1202	
	v		0.1488	0.1559	0.1627	0.1693	0.1757	0.1820	0.1942	0.2061	0.2178	0.2294	
900	*h*		1488.0	1516.2	1543.0	1569.1	1594.4	1619.4	1668.5	1717.1	1765.7	1814.4	
	s		5.1593	5.2508	5.3354	5.4147	5.4897	5.5614	5.6968	5.8237	5.9442	6.0594	
	v		0.1321	0.1388	0.1450	0.1511	0.1570	0.1627	0.1739	0.1847	0.1954	0.2058	0.2162
1000	*h*		1480.6	1510.0	1537.7	1564.4	1590.3	1615.6	1665.4	1714.5	1763.4	1812.4	1861.7
	s		5.0889	5.1840	5.2713	5.3525	5.4292	5.5021	5.6392	5.7674	5.8888	6.0047	6.1159
	v			0.1129	0.1185	0.1238	0.1289	0.1338	0.1434	0.1526	0.1616	0.1705	0.1792
1200	*h*			1497.1	1526.6	1554.7	1581.7	1608.0	1659.2	1709.2	1758.9	1808.5	1858.2
	s			5.0629	5.1560	5.2416	5.3215	5.3970	5.5379	5.6687	5.7919	5.9091	6.0214
	v			0.0944	0.0995	0.1042	0.1088	0.1132	0.1216	0.1297	0.1376	0.1452	0.1528
1400	*h*			1483.4	1515.1	1544.7	1573.0	1600.2	1652.8	1703.9	1754.3	1804.5	1854.7
	s			4.9534	5.0530	5.1434	5.2270	5.3053	5.4501	5.5836	5.7087	5.8273	5.9406
	v				0.0851	0.0895	0.0937	0.0977	0.1053	0.1125	0.1195	0.1263	0.1330
1600	*h*				1502.9	1534.4	1564.0	1592.3	1646.4	1698.5	1749.7	1800.5	1851.2
	s				4.9584	5.0543	5.1419	5.2232	5.3722	5.5084	5.6355	5.7555	5.8699
	v				0.0739	0.0781	0.0820	0.0856	0.0926	0.0992	0.1055	0.1116	0.1177
1800	*h*				1490.0	1523.5	1554.6	1584.1	1639.8	1693.1	1745.1	1796.5	1847.7
	s				4.8693	4.9715	5.0635	5.14825	3018	5.4409	5.5699	5.6914	5.8069
	v				0.0648	0.0688	0.0725	0.0760	0.0824	0.0885	0.0943	0.0999	0.1054
2000	*h*				1476.1	1512.0	1544.9	1575.6	1633.2	1687.6	1740.4	1792.4	1844.1
	s				4.7834	4.8930	4.9902	5.0786	5.2371	5.3793	5.5104	5.6333	5.7499

Table 10 Saturated Freon - 12 Table

		Specific volume m^3/kg			*Enthalpy* kJ/kg			*Entropy* $kJ/(kg.K)$		
Temp. °C	*Abs. Press.* MPa P	*Sat. Liquid* v_f	*Evap.* v_{fg}	*Sat. Vapor* v_g	*Sat. Liquid* h_f	*Evap.* h_{fg}	*Sat. Vapor* h_g	*Sat. Liquid* s_f	*Evap.* s_{fg}	*Sat. Vapor* s_g
–90	0.0028	0.000 608	4.414 937	4.415 545	–43.243	189.618	146.375	–0.2084	1.0352	0.8268
–85	0.0042	0.000 612	3.036 704	3.037 316	–38.968	187.608	148.640	–0.1854	0.9970	0.8116
–80	0.0062	0.000 617	2.137 728	2.138 345	–34.688	185.612	150.924	–0.1630	0.9609	0.7979
–75	0.0088	0.000 622	1.537 030	1.537 651	–30.401	183.625	153.224	–0.1411	0.9266	0.7855
–70	0.0123	0.000 627	1.126 654	1.127 280	–26.103	181.640	155.536	–0.1197	0.8940	0.7744
–65	0.0168	0.000 632	0.840 534	0.841 166	–21.793	179.651	157.857	–0.0987	0.8630	0.7643
–60	0.0226	0.000 637	0.637 274	0.637 910	–17.469	177.653	160.184	–0.0782	0.8334	0.7552
–55	0.0300	0.000 642	0.490 358	0.491 000	–13.129	175.641	162.512	–0.0581	0.8051	0.7470
–50	0.0391	0.000 648	0.382 457	0.383 105	–8.772	173.611	164.840	–0.0384	0.7779	0.7396
–45	0.0504	0.000 654	0.302 029	0.302 682	–4.396	171.558	167.163	–0.0190	0.7519	0.7329
–40	0.0642	0.000 659	0.241 251	0.241 910	–0.000	169.479	169.479	–0.0000	0.7269	0.7269
–35	0.0807	0.000 666	0.194 732	0.195 398	4.416	167.368	171.784	0.0187	0.7027	0.7214
–30	0.1004	0.000 672	0.158 703	0.159 375	8.854	165.222	174.076	0.0371	0.6795	0.7165
–25	0.1237	0.000 679	0.130 487	0.131 166	13.315	163.037	176.352	0.0552	0.6570	0.7121
–20	0.1509	0.000 685	0.108 162	0.108 847	17.800	160.810	178.610	0.0730	0.6352	0.7082
–15	0.1826	0.000 693	0.090 326	0.091 018	22.312	158.534	180.846	0.0906	0.6141	0.7046
–10	0.2191	0.000 700	0.075 946	0.076 646	26.851	156.207	183.058	0.1079	0.5936	0.7014
–5	0.2610	0.000 708	0.064 255	0.064 963	31.420	153.823	185.243	0.1250	0.5736	0.6986
0	0.3086	0.000 716	0.054 673	0.055 389	36.022	151.376	187.397	0.1418	0.5542	0.6960
5	0.3626	0.000 724	0.046 761	0.047 485	40.659	148.859	189.518	0.1585	0.5351	0.6937
10	0.4233	0.000 733	0.040 180	0.040 914	45.337	146.265	191.602	0.1750	0.5165	0.6916
15	0.4914	0.000 743	0.034 671	0.035 413	50.058	143.586	193.644	0.1914	0.4983	0.6897
20	0.5673	0.000 752	0.030 028	0.030 780	54.828	140.812	195.641	0.2076	0.4803	0.6879
25	0.6516	0.000 763	0.026 091	0.026 854	59.653	137.933	197.586	0.2237	0.4626	0.6863
30	0.7449	0.000 774	0.022 734	0.023 508	64.539	134.936	199.475	0.2397	0.4451	0.6848
35	0.8477	0.000 786	0.019 855	0.020 641	69.494	131.805	201.299	0.2557	0.4277	0.6834
40	0.9607	0.000 798	0.017 373	0.018 171	74.527	128.525	203.051	0.2716	0.4104	0.6820
45	1.0843	0.000 811	0.015 220	0.016 032	79.647	125.074	204.722	0.2875	0.3931	0.6806
50	1.2193	0.000 826	0.013 344	0.014 170	84.868	121.430	206.298	0.3034	0.3758	0.6792
55	1.3663	0.000 841	0.011 701	0.012 542	90.201	117.565	207.766	0.3194	0.3582	0.6777
60	1.5259	0.000 858	0.010 253	0.011 111	95.665	113.443	209.109	0.3355	0.3405	0.6760
65	1.6988	0.000 877	0.008 971	0.009 847	101.279	109.024	210.303	0.3518	0.3224	0.6742
70	1.8858	0.000 897	0.007 828	0.008 725	107.067	104.255	211.321	0.3683	0.3038	0.6721
75	2.0874	0.000 920	0.006 802	0.007 723	113.058	99.068	212.126	0.3851	0.2845	0.6697
80	2.3046	0.000 946	0.005 875	0.006 821	119.291	93.373	212.665	0.4023	0.2644	0.6667
85	2.5380	0.000 976	0.005 029	0.006 005	125.818	87.047	212.865	0.4201	0.2430	0.6631
90	2.7885	0.001 012	0.004 246	0.005 258	132.708	79.907	212.614	0.4385	0.2200	0.6585
95	3.0569	0.001 056	0.003 508	0.004 563	140.068	71.658	211.726	0.4579	0.1946	0.6526
100	3.3440	0.001 113	0.002 790	0.003 903	148.076	61.768	209.843	0.4788	0.1655	0.6444
105	3.6509	0.001 197	0.002 045	0.003 242	157.085	49.014	206.099	0.5023	0.1296	0.6319
110	3.9784	0.001 364	0.001 098	0.002 462	168.059	28.425	196.484	0.5322	0.0742	0.6064
112	4.1155	0.001 792	0.000 005	0.001 797	174.920	0.151	175.071	0.5651	0.0004	0.5655

Table 11 Superheated Freon - 12 Table

Temp. °C	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg K*
	0.05 *MPa*			0. 10 *MPa*			0.15 *MPa*		
–20.0	0.341 857	181.042	0.7912	0.167 701	179.861	0.7401			
–10.0	0.356 227	186.757	0.8133	0.175 222	185.707	0.7628	0.114 716	184.619	0.7318
0.0	0.370 508	102.567	0.8350	0.182 647	191.628	0.7849	0.119 866	190.660	0.7543
10.0	0.384 716	198.471	0.8562	0.189 994	197.628	0.8064	0.124 932	196.762	0.7763
20.0	0.398 863	204.469	0.8770	0.197 277	203.707	0.8275	0.129 930	202.927	0.7977
30.0	0.412 959	210.557	0.8974	0.204 506	209.866	0.8482	0.134 873	209.160	0.8186
40.0	0.427 012	216.733	0.9175	0.211 691	216.104	0.8684	0.139 768	215.463	0.8390
50.0	0.441 030	222.997	0.9372	0.218 839	222.421	0.8883	0.144 625	221.835	0.8591
60.0	0.455 017	229.344	0.9565	0.225 955	228.815	0.9078	0.149 450	228.277	0.8787
70.0	0.468 978	235.774	0.9755	0.233 044	235.285	0.9269	0.154 247	234.789	0.8980
80.0	0.482 917	242.282	0.9942	0.240 111	241.829	0.9457	0.159 020	241.371	0.9169
90.0	0.496 838	248.868	1.0126	0.247 159	248.446	0.9642	0.163 774	248.020	0.9354
	0.20 *MPa*			0.25 *MPa*			0.30 *MPa*		
0.0	0.088 608	189.669	0.7320	0.069 752	188.644	0.7139	0.057 150	187.583	0.6984
10.0	0.092 550	195.878	0.7543	0.073 024	194.969	0.7366	0.059 984	194.034	0.7216
20.0	0.096 418	202.135	0.7760	0.076 218	201.322	0.7587	0.062 734	200.490	0.7440
30.0	0.100 228	208.446	0.7972	0.079 350	207.715	0.7801	0.065 418	206.969	0.7658
40.0	0.103 989	214.814	0.8178	0.082 431	214.153	0.8010	0.068 049	213.480	0.7869
50.0	0.107 710	221.243	0.8381	0.085 470	220.642	0.8214	0.070 635	220.030	0.8075
60.0	0.111 397	227.735	0.8578	0.088 474	227.185	0.8413	0.073 185	226.627	0.8276
70.0	0.115 055	234.291	0.8772	0.091 449	233.785	0.8608	0.075 705	233.273	0.8473
80.0	0.118 690	240.910	0.8962	0.094 398	240.443	0.8800	0.078 200	239.971	0.8665
90.0	0.122 304	247.593	0.9149	0.097 327	247.160	0.8987	0.080 673	246.723	0.8853
100.0	0.125 901	254.339	0.9332	0.100 238	253.936	0.9171	0.083 127	253.530	0.9038
110.0	0.129 483	261.147	0.9512	0.103 114	260.770	0.9352	0.085 566	260.391	0.9220
	0.40 *MPa*			0.50 *MPa*			0.60 *MPa*		
120.0	0.045 836	198.762	0.7199	0.035 646	196.935	0.6999			
130.0	0.047 971	205.428	0.7423	0.037 464	203.814	0.7230	0.030 422	202.116	0.7063
140.0	0.050 046	212.095	0.7639	0.039 214	210.656	0.7452	0.031 966	209.154	0.7291
150.0	0.052 072	218.779	0.7849	0.040 911	217.484	0.7667	0.033 450	216.141	0.7511
160.0	0.054 059	225.488	0.8054	0.042 565	224.315	0.7875	0.034 887	223.104	0.7723
170.0	0.056 014	232.230	0.8253	0.044 184	231.161	0.8077	0.036 285	230.062	0.7929
180.0	0.057 941	239.012	0.8448	0.045 774	238.031	0.8275	0.037 653	237.027	0.8129
190.0	0.059 846	245.837	0.8638	0.047 340	244.932	0.8467	0.038 995	244.009	0.8324
200.0	0.061 731	252.707	0.8825	0.048 886	251.869	0.8656	0.040 316	251.016	0.8514
210.0	0.063 600	259.624	0.9008	0.050 415	258.845	0.8840	0.041 619	258.053	0.8700
220.0	0.065 455	266.590	0.9187	0.051 929	265.862	0.9021	0.042 907	265.124	0.8882
230.0	0.067 298	273.605	0.9364	0.053 430	772.923	0.9198	0.044 181	272.231	0.9061

	0.70 *MPa*			0.80 *MPa*			0.90 *MPa*		
40.0	0.026 761	207.580	0.7148	0.022 830	205.924	0.7016	0.019 744	204.170	0.6982
50.0	0.028 100	214.745	0.7373	0.024 068	213.290	0.7248	0.020 912	211.765	0.7131
60.0	0.029 387	221.854	0.7590	0.025 247	220.558	0.7469	0.022 012	219.212	0.7358
70.0	0.030 632	228.931	0.7799	0.026 380	277.766	0.7682	0.023 062	226.564	0.7575
80.0	0.031 843	235.997	0.8002	0.027 477	234.941	0.7888	0.024 072	233.856	0.7785
90.0	0.033 027	243.066	0.8199	0.028 545	242.101	0.8088	0.025 051	241.113	0.7987
100.0	0.034 189	250.146	0.8392	0.029 588	249.260	0.8283	0.026 005	248.355	0.8184
110.0	0.035 332	257.247	0.8579	0.030 612	256.428	0.8472	0.026 937	255.593	0.8376
120.0	0.036 458	264.374	0.8763	0.031 619	263.613	0.8657	0.027 851	262.839	0.862
130.0	0.037 572	271.531	0.8943	0.032 612	270.820	0.8838	0.028 751	270.100	0.8745
140.0	0.038 673	278.720	0.9119	0.033 592	278.055	0.9016	0.029 639	277.381	0.8923
150.0	0.039 764	285.946	0.9292	0.034 563	285.320	0.9189	0.030 515	284.687	0.9098
	1.00 *MPa*			1.20 *MPa*			1.40 *MPa*		
50.0	0.018 366	210.162	0.7021	0.014 483	206.661	0.6812			
60.0	0.019 410	217.810	0.7254	0.015 463	214.805	0.7060	0.012 579	211.457	0.6876
70.0	0.020 397	225.319	0.7476	0.016 368	222.687	0.7293	0.013 448	219.822	0.7123
80.0	0.021 341	232.739	0.7689	0.017 221	230.398	0.7514	0.014 247	227.891	0.7355
90.0	0.022 251	240.101	0.7895	0.018 032	237.995	0.7727	0.014 997	235.766	0.7575
100.0	0.023 133	247.430	0.8094	0.018 812	245.518	0.7931	0.015 710	243.512	0.7785
110.0	0.023 993	254.743	0.8287	0.019 567	252.993	0.8129	0.016 393	251.170	0.7988
120.0	0.024 835	262.053	0.8475	0.020 301	260.441	0.8320	0.017 053	258.770	0.8183
130.0	0.025 661	269.369	0.8659	0.021 018	267.875	0.8507	0.017 695	266.334	0.8373
140.0	0.026 474	276.699	0.8839	0.021 721	275.307	0.8689	0.018 321	273.877	0.8558
150.0	0.027 275	284.047	0.9015	0.022 412	282.745	0.8867	0.018 934	281.411	0.8738
160.0	0.028 068	291.419	0.9187	0.023 093	290.195	0.9041	0.019 535	288.946	0.8914
	1.60 *MPa*			1.80 *MPa*			2.00 *MPa*		
70.0	0.011 208	216.650	0.6959	0.009 406	213.049	0.6794			
80.0	0.011 984	225.177	0.7204	0.010 187	222.198	0.7057	0.008 704	218.859	0.6909
90.0	0.012 698	233.390	0.7433	0.010 884	230.835	0.7298	0.009 406	228.056	0.7166
100.0	0.013 366	241.397	0.7651	0.011 526	239.155	0.7524	0.010 035	236.760	0.7402
110.0	0.014 000	249.264	0.7859	0.012 126	247.264	0.7739	0.010 615	245.154	0.7624
l20.0	0.014 608	257.035	0.8059	0.012 697	255.228	0.7944	0.011 159	253.341	0.7835
130.0	0.015 195	264.742	0.8253	0.013 244	263.094	0.8141	0.011 676	261.384	0.8037
140.0	0.015 765	272.406	0.8440	0.013 772	270.891	0.8332	0.012 172	269.327	0.8232
150.0	0.016 320	280.044	0.8623	0.014 284	278.642	0.8518	0.012 651	277.201	0.8420
160.0	0.016 864	287.669	0.8801	0.014 784	286.364	0.8698	0.013 116	285.027	0.8603
170.0	0.017 398	295.290	0.8975	0.015 272	294.069	0.8874	0.013 570	292.822	0.8781
180.0	0.017 923	302.914	0.9145	0.015 752	301.767	0.9046	0.014 013	300.598	0.8955

Table 11 Superheated Freon - 12 Table

Temp. ^oC	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg K*
	2.50 *MPa*			3.00 *MPa*			3.50 *MPa*		
90.0	0.006 595	219.562	0.6823						
100.0	0.007 264	229.852	0.7103	0.005 231	220.529	0.6770			
110.0	0.007 837	239.271	0.7352	0.005 886	232.068	0.7075	0.004 324	222.121	0.6750
120.0	0.008 351	248.192	0.7582	0.006 419	242.208	0.7336	0.004 959	234.875	0.7078
130.0	0.008 827	256.794	0.7798	0.006 887	251.632	0.7573	0.005 456	245.661	0.7349
140.0	0.009 273	265.180	0.8003	0.007 313	260.620	0.7793	0.005 884	255.524	0.7591
150.0	0.009 697	273.414	0.8200	0.007 709	269.319	0.8001	0.006 270	264.846	0.7814
160.0	0.010 104	281.540	0.8390	0.008 083	277.817	0.8200	0.006 626	273.817	0.8023
170.0	0.010 497	289.589	0.8574	0.008 439	286.171	0.8391	0.006 961	282.545	0.8222
180.0	0.010 879	297.583	0.8752	0.008 782	294.422	0.8575	0.007 279	291.100	0.8413
190.0	0.011 250	305.540	0.8926	0.009 114	302.597	0.8753	0.007 584	299.528	0.8597
200.0	0.011 614	313.472	0.9095	0.009 436	310.718	0.8927	0.007 878	307.864	0.8775
	4.00 *MPa*								
120.0	0.003 736	224.863	0.6771						
130.0	0.004 325	238.443	0.7111						
140.0	0.004 781	249.703	0.7386						
150.0	0.005 172	259.904	0.7630						
160.0	0.005 522	269.492	0.7854						
170.0	0.005 845	278.684	0.8063						
180.0	0.006 147	287.602	0.8262						
190.0	0.006 434	296.326	0.8453						
200.0	0.006 708	304.906	0.8636						
210.0	0.006 972	313.380	0.8813						
220.0	0.007 228	321.774	0.8985						
230.0	0.007 477	330.108	0.9152						

Table 12 Enthalpies of Formation. Gibbs Function of Formation and Absolute Entropy at 25°C and 1 atm Pressure

Substance	*M*	*Enthalpy of formation (kJ/kgmol)*	*Gibbs function of formation (kJ/kgmol)*	*Absolute Entropy (kJ/kgmol.K)*
Acetylene $C_2H_2(g)$	26.038	226 866	209 290	200.98
Ammonia $NH_3(g)$	17.032	–45 926	–16 390	192.72
Benzene $C_6H_6(g)$	78.108	82 976	129 732	269.38
Butane $C_4H_{10}(g)$	58.124	–126 223	–17 164	310.32
Carbon C graphite	12.011	0	0	5.69
Carbon Dioxide $CO_2(g)$	44.01	–393 757	–344 631	213.83
Carbon Monoxide CO(g)	28.001	–110 596	–137 242	197.68
Ethane $C_2H_6(g)$	30.07	–84 718	–32 905	229.64
Ethene $C_2H_4(g)$	28.054	52 315	681 59	219.58
Hydrazine $N_2H_4(g)$	32.048	95 410	159 260	238.77
Methane $CH_4(g)$	16.043	–74 917	–50 844	186.27
Octane $C_8H_{18}(g)$	114.23	–208 581	16 536	467.04
Octane $C_8H_{18}(l)$	114.23	–250 102	6 614	361.03
Propane $C_3H_8(g)$	44.097	–103 909	–23 502	270.09
Water $H_2O(g)$	18.015	–241 971	–228 729	188.85
Water $H_2O(l)$	18.015	–286 010	–237 327	69.98
Dodecane $C_{12}H_{26}(g)$	170.328	–290 971		
Dodecane $C_{12}H_{26}(l)$	170.328	–394 199		

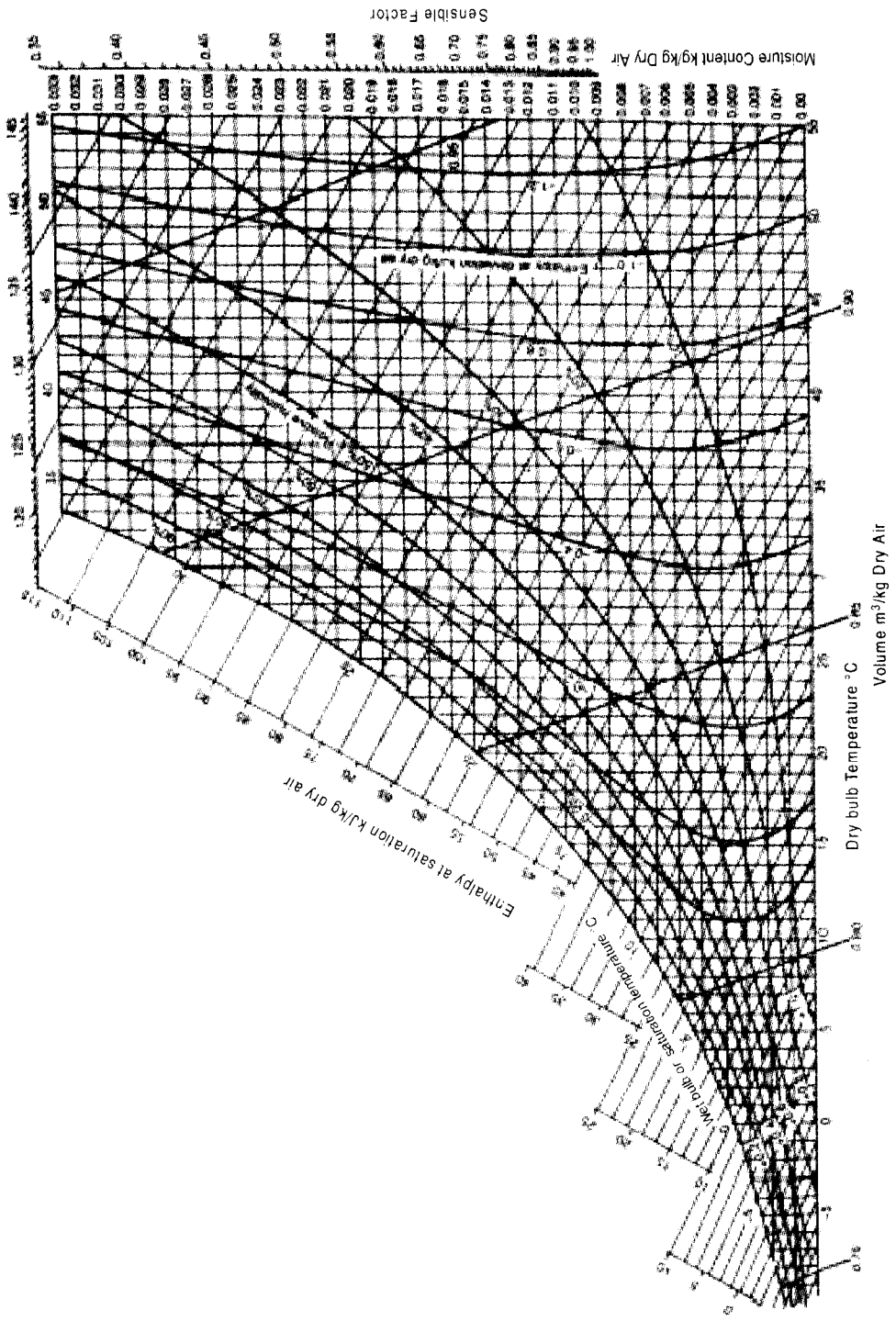


Chart 1 : Psychrometric Chart

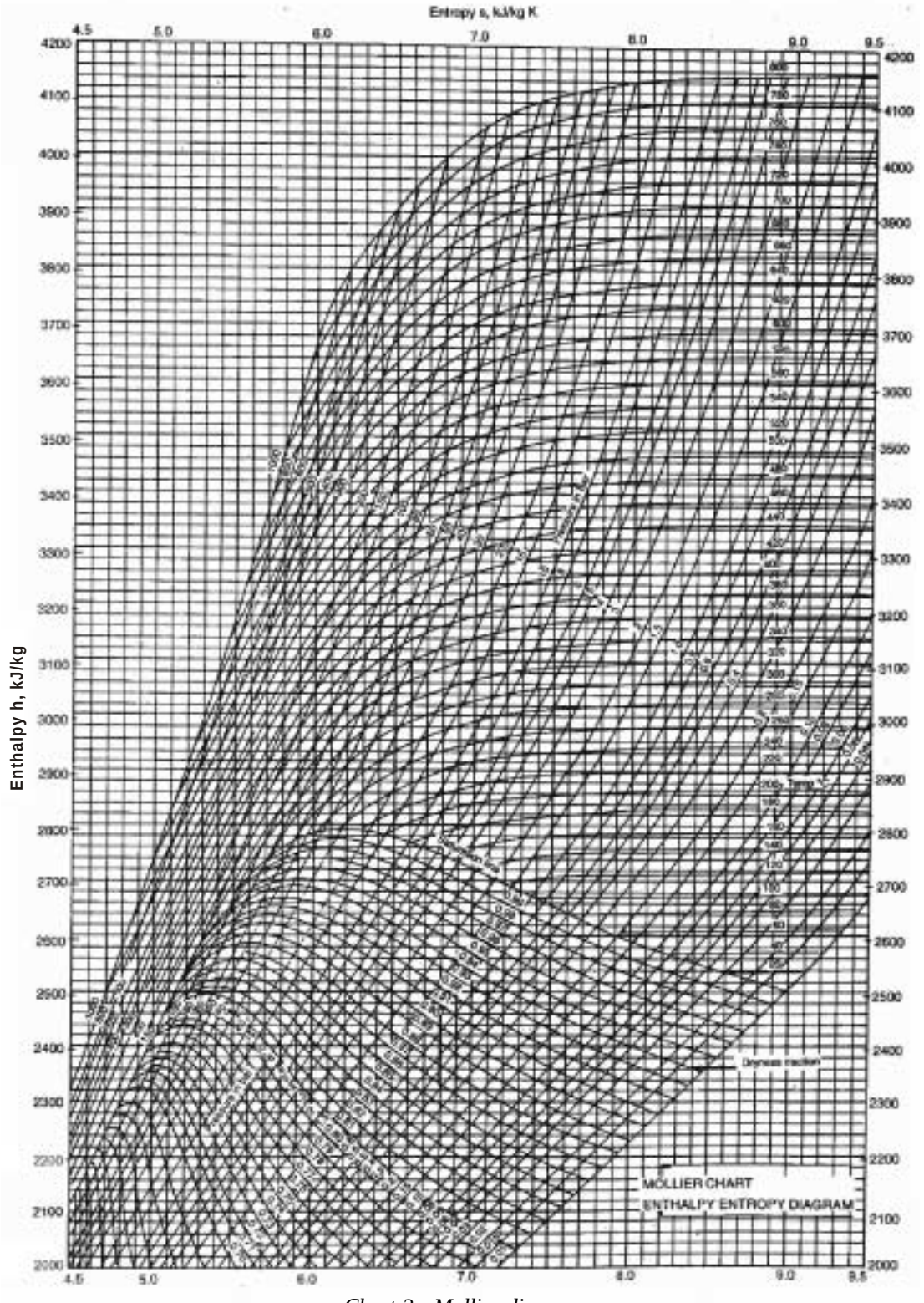


Chart 2 : Mollier diagram

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