

16/20

Good!

BAHIR DAR INSTITUTE OF TECHNOLOGY

Applied Mathematics II (Math 2041) Test - Regular Program 2014 E.C

Time allowed: 1:30 Hrs

Total Marks: 20

Name: [redacted] ID: [redacted] Section: [redacted]

Part I. Choose the best answer (2 marks each)

1. Which of the following functions has its domain given by the set $\{(x, y) | x \neq 0 \text{ and } y \neq 0\}$?

- A. $f(x, y) = \frac{1}{x^2 + y^2}$ B. $f(x, y) = \frac{1}{\sqrt{x^2 - y^2}}$ C. $f(x, y) = \frac{1}{xy}$ D. A and C

2. Assuming z is an implicit function of x and y in the equation $x - yz + \cos xyz = 1$,

$\frac{\partial z}{\partial x} \big|_{(1,2,0)} =$

- A. $\frac{1}{2}$ B. 0 C. 1 D. $-\frac{1}{2}$

3. Given the function $f(x, y, z) = \ln(x^2 + y^2 + z^2)$, the direction in which the function increases most and the maximum directional derivative at point $(2, 0, 1)$ are:

- A. $(\frac{2}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}})$ and 1 B. $(\frac{4}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}})$ and $\sqrt{\frac{17}{5}}$
C. $(\frac{4}{5}, 0, \frac{1}{5})$ and $\frac{\sqrt{17}}{5}$ D. $(\frac{4}{5}, 0, \frac{2}{5})$ and $\frac{2}{\sqrt{5}}$

Part II. Fill in the blank space. (2 marks each)

1. $\lim_{(x,y) \rightarrow (0,0)} \frac{y}{x} =$ Doesn't exist

2. Given $f(x, y) = \sqrt{x^2 + y^2}$, $\sqrt{f_x^2 + f_y^2} =$ 1

3. The equation of the tangent plane to the graph of $xyz = -1$ at point $(\frac{1}{2}, -2, -1)$ is

$2x - \frac{y}{2} - z = 3$

$x = \frac{1}{2} \quad y \rightarrow 0$
 $\frac{y}{0} = 0$
 $y = 0 \quad x \rightarrow 0$
 $\frac{0}{0} = 0$

$f_x = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 + y^2}}$

$f_y = \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y = \frac{y}{\sqrt{x^2 + y^2}}$

$\sqrt{\left(\frac{x}{\sqrt{x^2 + y^2}}\right)^2 + \left(\frac{y}{\sqrt{x^2 + y^2}}\right)^2} = \sqrt{\frac{x^2 + y^2}{x^2 + y^2}} = \sqrt{1} = 1$

2. Given the function $f(x, y) = e^{xy}$, find the relative maximum value, relative minimum value or saddle points (if any).

$$f_x = e^{xy} \cdot y = ye^{xy} = 0 \quad \frac{ye^{xy}}{y} = 0/1 = 0 \quad e^{xy} = 0$$

$$f_y = e^{xy} \cdot x = xe^{xy} = 0 \quad \frac{xe^{xy}}{x} = 0/1 = 0 \quad e^{xy} = 0$$

~~$$e^{xy} = 0 \Rightarrow e^x = 0$$~~

~~$$\ln e^{xy} = \ln 0$$~~

~~$$\ln e^{xy} = -1$$~~

$$\therefore ye^{xy} = 0 \quad y = 0 \quad (0, 0) - \text{CP}$$

$$\rightarrow xe^{xy} = 0 \quad x = 0$$

$$f_{xx} = ye^{xy} \cdot y = \boxed{y^2 e^{xy}}$$

$$f_{yy} = xe^{xy} \cdot x = \boxed{x^2 e^{xy}}$$

$$f_{xy} = e^{xy} (y)'(e^{xy}) + y(e^{xy})'$$

$$e^{xy} + y \cdot xe^{xy}$$

$$e^{xy} + xy e^{xy} \quad \boxed{e^{xy} (1 + xy)}$$

$$D = f_{xx} \cdot f_{yy} - (f_{xy})^2 = \quad \text{at } (x, y) = (0, 0)$$

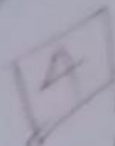
$$0 \cdot 0 - 1 = -1_{(0,0)}$$

if $D < 0$ is saddle point

but the function e^{xy} is always > 0

$\rightarrow e^{xy}$ has minimum value

$e^{(0)(0)} = 1$ is the minimum value



Part III. Workout (4 marks each)

Instructions: Show your work clearly.

1. Find the extreme value of $f(x, y) = xy$ subject to the constraint $x^2 + y^2 = 1$.

$$f_x = y \quad g_x = 2x$$

$$f_y = x \quad g_y = 2y$$

$$g(x, y) = x^2 + y^2 = 1$$

$$\begin{cases} y = \lambda 2x \\ x = \lambda 2y \\ x^2 + y^2 = 1 \end{cases}$$

$$\lambda = \frac{y}{2x} = \lambda \frac{x}{2y}$$

$$\frac{y}{2x} = \frac{x}{2y} \quad \frac{2y^2}{2} = \frac{2x^2}{2}$$

$$y^2 = x^2$$

$$x^2 + y^2 = 1 \quad \text{--- Substituted in Given Constraint}$$

$$x^2 + x^2 = 1$$

$$2x^2 = 1 \quad x^2 = \frac{1}{2} \quad x = \pm \sqrt{\frac{1}{2}}$$

$$\text{if } x = \sqrt{\frac{1}{2}} \rightarrow \left(\sqrt{\frac{1}{2}}\right)^2 + y^2 = 1 \quad y^2 = 1 - \frac{1}{2} = \frac{1}{2} \quad y = \pm \sqrt{\frac{1}{2}}$$

$$\text{if } x = -\sqrt{\frac{1}{2}} \quad \left(-\sqrt{\frac{1}{2}}\right)^2 + y^2 = 1 \quad y^2 = 1 - \frac{1}{2} = \frac{1}{2} \quad y = \pm \sqrt{\frac{1}{2}}$$

$$\pm \sqrt{\frac{1}{2}} \approx \pm \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \quad \boxed{\frac{\sqrt{2}}{2}}$$

$$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right), \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right), \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right), \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \text{ are,}$$

Critical Point

$\therefore$ We have to Evaluate the CP in $f(x, y)$

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$$f\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$$

$$f\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = -\frac{2}{4} = -\frac{1}{2}$$

$$f\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \times -\frac{\sqrt{2}}{2} = -\frac{2}{4} = -\frac{1}{2}$$

$$f\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) = -\frac{\sqrt{2}}{2} \times -\frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$$

$\therefore f(x, y) = xy$ have Relative minimum at

$$f\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \text{ and } f\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) = -\frac{1}{2}$$

and $f(x, y) = xy$ have Relative Maximum at

$$f\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \text{ and } f\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) = \frac{1}{2}$$

$$\text{Relative Minimum} = -\frac{1}{2}$$

$$\text{Relative Maximum} = \frac{1}{2}$$

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