

ADDIS ABABA UNIVERSITY
DEPARTMENT OF MATHEMATICS
Applied Mathematics I (Math 1041)
Worksheet 1 2022/23 AY

1. For a given vectors A and B
 - a) Find a real number β such that the vectors $A = (\beta, -3, 1)$ and $B = (\beta, \beta, 2)$ are perpendicular.
 - b) Find two vectors each of norm 1 that are perpendicular to the vector $A = (3, 2)$.
 - c) If U and V are perpendicular unit vectors, show that $\|U - V\| = \sqrt{2}$
 - d) Vectors A and B make an angle of $\beta = \pi/6$. If $\|A\| = \sqrt{3}$, and $\|B\| = 1$. Then calculate the angle between the vectors $A+B$ and $B-A$.
2. For a given vectors A and B
 - a) Find the cosine of the angle between the vectors $A = (4, 1, 6)$ and $B = (-3, 0, 2)$.
 - b) Find the projection of vector $U = (-7, 1, 3)$ onto vector $V = (5, 0, 1)$.
 - c) Given three vectors A, B and C such that $A+B+C = 0$; if $\|A\| = 3$, $\|B\| = 1$ and $\|C\| = 4$, then evaluate $A \cdot B + B \cdot C + A \cdot C$.
 - e) Show that $(\frac{1}{2}, \frac{1}{3}, 0)$, $(1, 1, -1)$ and $(-2, -3, 5)$ are collinear points, and find symmetric equation of the line containing them.
 - f) Let A be a vector in the first octant such that $\|A\| = 3$ and the direction cosines with respect to x-axis and y-axis are $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Then find vector A.
3. Show that the points $(-1, 1, 1)$, $(0, 2, 1)$, $(0, 0, \frac{3}{2})$ and $(13, -1, 5)$ lie on the same plane and find the equation of the plane containing them.
4. Find the equation of the line passing through the point $(-2, 5, -3)$ and perpendicular to the plane $\pi: 2x - 3y + 4z + 7 = 0$.
5. Find the equation of the plane:
 - a) that is parallel to the z-axis and contains the points $(3, -1, 5)$ and $(7, 9, 4)$.
 - b) that contains the point $(-2, 1, 4)$ a line $L: x = 2 - 3t, y = 4 + 2t, z = 3 - 5t$.
 - c) that contains the point $(4, 0, -2)$ and perpendicular to each of the planes $x - y + z = 0$ and $2x + y - 4z = 5$.
6. Let $\pi_1: 2x - 4y + 4z = 7$ and $\pi_2: 6x + 2y - 3z = 2$ be two planes
 - a) Find parametric equations of the line of intersection of the planes π_1 and π_2 .
 - b) Find the cosine of the acute angle of the intersecting planes π_1 and π_2 .
7. Find the point of intersection of the line $L: x = 2 - 3t, y = 4 + 2t, z = 3 - 5t, t \in \mathbb{R}$ and the plane $2x + 3y + 4z = -8$.
8. Let $L_1: x = 1 + 2t, y = 2 - t, z = 4 - 2t$ and $L_2: \frac{x-9}{1} = \frac{y-5}{3} = \frac{z+4}{-1}$ be two lines.
 - a) Show that L_1 and L_2 are intersecting lines.
 - b) Find the cosines of the acute angle between L_1 and L_2 at their intersection point.
 - c) Find symmetric equations for the line that is perpendicular to L_1 and L_2 and passes through their intersection point.
9. Let L be the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z+5}{7}$ and π be the plane $2x + 2y - z + 4 = 0$. Find the two points on L at a distance 3 unit from π .

10. Let $L_1: \frac{x+2}{5} = \frac{y-1}{-2} = z+4$ and $L_2: \frac{x-3}{-5} = \frac{y+4}{2} = \frac{z-3}{-1}$ be two lines. Then
- Show that L_1 and L_2 are parallel and find the distance D between L_1 and L_2 .
 - Find the equation of the plane that contains L_1 and L_2 .
11. Find the distance between the parallel planes: $z = x + 2y + 1$ and $3x + 6y - 3z = 4$.
12. Let $L_1: x = 3 - t, y = 4 + 4t, z = 1 + 2t$ and $L_2: x = s, y = 3, z = 2s$ be two lines
- Show that L_1 and L_2 are skew (neither intersecting nor parallel) lines.
 - Find two planes π_1 and π_2 such that π_1 is parallel to π_2 , π_1 contains L_1 and π_2 contains L_2 .
 - Find the distance D of the two parallel planes π_1 and π_2 (the planes you found in (b)). (Note that this distance D is the distance between the two skew lines L_1 and L_2)
13. Let $V = \{(a, b) | a, b \in \mathbb{R}\}$. Define addition in V and scalar multiplication by elements of $\mathbb{R}$ as follows: $(a, b) + (c, d) = (a + c, b + d)$ and $k(a, b) = (ka, 0)$. Show that V is not a vector space over $\mathbb{R}$ with respect to the operations defined. Which axiom(s) fail to hold?
14. Determine whether W is a subspace of V or not.
- $W = \{(x, y) \in \mathbb{R}^2 : x - 3y = 0\}, V = \mathbb{R}^2$
 - $W = \{(x, y) \in \mathbb{R}^2 : y = x^2\}, V = \mathbb{R}^2$
 - $W = \{(a, b, c) \in \mathbb{R}^3 : a - 3b + 4c = 0\}, V = \mathbb{R}^3$
 - $W = \{(x, y, z) \in \mathbb{R}^3 : z = 2y\}, V = \mathbb{R}^3$
 - $W = \{(a, b, c) \in \mathbb{R}^3 : b \text{ is an integer}\}, V = \mathbb{R}^3$
 - $W = \{(a, b, c) : a \leq b \leq c\}, V = \mathbb{R}^3$
15. Determine whether the following vectors are linearly independent or not.
- $(1, 2), (3, 2)$ in $\mathbb{R}^2$
 - $(-3, 6), (1, -2)$ in $\mathbb{R}^2$
 - $(-1, 2), (1, -2), (3, 4)$ in $\mathbb{R}^2$
 - $(1, 1, 3), (0, 2, 1)$ in $\mathbb{R}^3$
 - $(2, 1, -2), (3, 2, 2), (5, 3, 0)$ in $\mathbb{R}^3$
 - $(1, 0, 2), (0, -1, 1), (1, 3, 0)$ in $\mathbb{R}^3$
16. Let $P_n = P_n(\mathbb{R})$ be the vector space of all polynomials of degree less than $n, \forall n \in \mathbb{N}$. Determine whether the following vectors are linearly independent or not.
- $1, x + 2, 2x - 1$ in P_2
 - $x - 3, 1 - 2x$ in P_2
 - $x^2 + 1, x^2 - x, x + 1$ in P_3
 - $3 - 2x, x^2 + 2x - 1, x^2 + 2$ in P_3
 - $1, 2x - 3, x^2 + 3x - 5, 2x^2 + 2$ in P_3
17. Let V be the set of all continuous functions on $\mathbb{R}$
- Show that V is a vector space under the usual addition and scalar multiplication of functions.
 - Show that the set $W = \{f \in V : f \text{ is differentiable function}\}$ is a subspace of V .
 - Show that $2^x, 2^{2x}, 2^{3x}$ are linearly independent vectors in V
 - Show that $\sin 2x, \cos 2x, \cos x \sin x$ are linearly dependent vectors in V .
18. If possible, write $\mathbf{u}$ as a linear combination of $\mathbf{v}$ and $\mathbf{w}$.
- $\mathbf{u} = (1, -2), \mathbf{v} = (3, 1), \mathbf{w} = (0, -1)$
 - $\mathbf{u} = (-3, 2), \mathbf{v} = (-1, 3), \mathbf{w} = (2, -6)$
 - $\mathbf{u} = (0, -2, 1), \mathbf{v} = (1, -1, 2), \mathbf{w} = (0, 2, 4)$
 - $\mathbf{u} = (8, -2, 1), \mathbf{v} = (0, 1, 5), \mathbf{w} = (0, 3, -1)$
- e) $\mathbf{u} = 3x^2 + 8x - 5, \mathbf{v} = 2x^2 + 3x - 4, \mathbf{w} = x^2 - 2x - 3$