



Addis Ababa Science and Technology University
College of Natural and Social Sciences, Department of Mathematics
Applied Mathematics I (Math1014)
Worksheet I

1. Find the value of x for which $A = 3i + 2j + 9k$ and $B = i + xj + 3k$ are parallel vectors.
2. Find the value of m such that $A = mi - 3j + 3k$ and $B = 3i + 9j - 12k$ are perpendicular vectors.
3. Consider $\mathbf{u} + 2\mathbf{v} = 3j - k$ and $3\mathbf{u} - \mathbf{v} = i + j + k$. Find the values of $\mathbf{u}$ and $\mathbf{v}$.
4. Which of the following pairs of vectors are parallel?
 - (a) $(-2, 3, 1), (6, -4, -3)$
 - (b) $(10, -8, 3, 0, 27), (\frac{5}{6}, -\frac{2}{3}, \frac{3}{4}, 0, -\frac{5}{2})$
 - (c) $(12, -16), (9, -12)$
5. Normalize the vectors $(1, 2, -2, 4)$ and $(5, -4)$.
6. Suppose the vector $\overrightarrow{AB}$ has magnitude 10 units with same direction as the vector $\mathbf{v} = 4i - 3j$. Find the terminal point B if the initial point is $A(1, 2)$.
7. Let $\mathbf{u} = i + j - k$, $\mathbf{v} = i - j + k$, $\mathbf{w} = -i + j + k$. Find a unit vector in the direction of $\mathbf{u} + \mathbf{v} + \mathbf{w}$.
8. Find a vector perpendicular to both vectors $(\mathbf{u} + \mathbf{v})$ and $(\mathbf{u} - \mathbf{v})$, where $\mathbf{u} = i + j + k$ and $\mathbf{v} = i + 2j + 3k$.

9. If the sum of two vectors is a unit vector, show that the magnitude of their difference equals $\sqrt{3}$.
10. Consider $\mathbf{u}$ and $\mathbf{v}$ be vectors with $\|\mathbf{u}\| = 4$ and $\|2\mathbf{u} - \mathbf{v}\| = \|\mathbf{u} + \mathbf{v}\|$. Then find $\mathbf{u}$ and $\mathbf{v}$.
11. If $\mathbf{v} = 4i - 3j$, $\mathbf{u}$ is a unit vector and $\|\mathbf{u} + \mathbf{v}\|^2 = 27$, then find the angle, θ between the two vectors.
12. Find the value of k such that $\mathbf{u} = (k, 2k, 2k, 4)$ and $\mathbf{v} = (4, 2k, 2k, k)$ are orthogonal vectors.
13. Determine the values for $\|\mathbf{u} + \mathbf{v}\|$ and $\|\mathbf{u} - \mathbf{v}\|$ if $\|\mathbf{u}\| = 3$, $\|\mathbf{v}\| = 4$, and the angle between them is 90° .
14. If $\mathbf{u}$ is a unit vector and $(\mathbf{v} - \mathbf{u}) \cdot (\mathbf{u} + \mathbf{v}) = 80$, then find $\|\mathbf{v}\|$.
15. Let $\|\mathbf{u}\| = \sqrt{3}$, $\|\mathbf{v}\| = 1$ and the angle between $\mathbf{u}$ and $\mathbf{v}$ is $\frac{\pi}{6}$. Find the angle α between $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$.
16. Suppose that each angle between the vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ is $\frac{\pi}{3}$. If $\|\mathbf{u}\| = 4$, $\|\mathbf{v}\| = 2$ and $\|\mathbf{w}\| = 6$, then find $\|\mathbf{u} + \mathbf{v} + \mathbf{w}\|$. (*Hint: $\mathbf{u} \cdot \mathbf{v} = \|\mathbf{v}\|\|\mathbf{u}\|\cos\theta$, follow a similar approach for $\mathbf{u} \cdot \mathbf{w}$ and $\mathbf{v} \cdot \mathbf{w}$.*)
17. If the vectors $\mathbf{u}$ and $\mathbf{v}$ are given as $(2, -3, 3)$ and $(1, 2, -2)$, respectively, find the following.
 - (a) $\mathbf{u} \cdot \mathbf{v}$
 - (b) $(\mathbf{u} + \mathbf{w}) \cdot (\mathbf{v} - \mathbf{w})$
 - (c) $\|\mathbf{u} + \mathbf{v}\|^2$
 - (d) The cosine angle between $\mathbf{v}$ and $\mathbf{w}$.
 - (e) The projection of $\mathbf{u}$ along $\mathbf{v}$, and the scalar projection of $\mathbf{v}$ along $\mathbf{u}$.
 - (f) The projection of $\mathbf{u}$ along $\mathbf{v} - 2\mathbf{u}$.
 - (g) Find the directional angles and cosines of the vector $\mathbf{u}$.
18. Find the cosine angle of the triangle $\triangle ABC$ with vertices $A(1, 0, 0)$, $B(0, 1, 0)$ and $C(2, 1, -2)$.

19. Consider the vector $\mathbf{u}$ is orthogonal to the vector $\mathbf{v}$, and $\|\mathbf{u}\| = \|\mathbf{v}\| = 1$. If $\mathbf{w} = \alpha\mathbf{u} + \beta\mathbf{v}$, show that $\|\mathbf{w}\| = \sqrt{\alpha^2 + \beta^2}$, where α and β in $\mathbb{R}$.
20. If the angle between any two of the vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ in space is 60° , and $\|\mathbf{u}\| = 4$, $\mathbf{v} = 2$ and $\mathbf{w} = 4$, then find the following.
- $\mathbf{u} \cdot \mathbf{v}$
 - $\|\mathbf{u} + \mathbf{v}\|$
 - $\|\mathbf{u} + \mathbf{v} + \mathbf{w}\|$
21. Find the area of the triangle with vertices $A(1, 2, 1)$, $B(3, 2, 2)$ and $C(0, 1, 2)$.
22. Find the area of parallelogram with adjacent vertices $\overrightarrow{AB}$ and $\overrightarrow{AC}$, where $A = (1, -2, 1)$, $B = (3, 1, 4)$ and $C = (-2, 0, 3)$.
23. Find the volume and surface area of the parallelepiped with adjacent vectors.
- $\mathbf{u} = (1, 2, -1)$, $\mathbf{v} = (0, -1, 1)$ and $\mathbf{w} = (1, 2, 0)$.
 - $\mathbf{u} = (1, 2, -1)$, $\mathbf{v} = (3, 2, 0)$ and $\mathbf{w} = (3, 1, 3)$.
24. Find the perpendicular distance from the point $p(3, 0, 1)$ to the line with symmetric equation:

$$x + 2 = \frac{y-2}{3} = z.$$
25. Find the parametric and symmetric equation of the line L
- that passes through the points $P(1, 0, 4)$ and $P(3, 4, 5)$.
 - that passes through the point $P(-2, 5, -3)$ and orthogonal to the plane $2x - 3y + 4z + 7$.
 - that passes through the point $P(2, 0, -4)$ and parallel to the plane $2x + y - z = 0$.
26. Find equation of the plane
- containing the points $P_1(1, 2, 3)$, $P_2(4, 1, -1)$ and $P_3(2, 2, 0)$.
 - containing the point $P(0, 1, 2)$ and vectors $B = (-2, 4, 3)$, $C = (3, 1, -1)$.
27. Find an equation of the plane

- (a) perpendicular to the line through the points $P_1(2, 2, -4)$ and $P_2(7, -1, 3)$ and containing the point $P_3(-5, 1, 2)$.
- (b) Parallel to the plane $4x - 2y + z = 1$ and containing the point $P(2, 6, -1)$.
28. Show that the image of the point $P(3, -2, 1)$ in the plane $3x - y + z - 2 = 0$ lies on the plane $x + y + z + 4 = 0$.
29. Find the shortest distance from the point $P_0(6, 5, 9)$ to the plane determined by the points $P_1(3, -1, 2)$, $P_2(5, 2, 4)$ and $P_3(-1, -1, 3)$.
30. Determine whether the following vectors are linearly dependent or independent.
- (a) $\{(1, 1, 2), (1, 2, 1), (2, 1, 1)\}$
- (b) $\{(1, 2, 3), (2, 3, 4), (3, 4, 5)\}$
- (c) $\{t, \sin t, \sin 2t\}$
31. Determine whether or not each of the following form a basis of $\mathbb{R}^3$.
- (a) $\{(1, 1, 1), (1, 0, 1)\}$
- (b) $\{(1, 1, 1), (1, 2, 3), (2, -1, 1)\}$