



education

Department:
Education
PROVINCE OF KWAZULU-NATAL

KZN DEPARTMENT OF EDUCATION

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9-12)telegram channel

MATHEMATICS JUST IN TIME MATERIAL GRADE 12 ANSWERS / SOLUTIONS

TERM 1 – 2020

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EUCLIDEAN GEOMETRY

Proportionality Theorem Learner Activity Answers

QUESTION 1

$$\frac{x}{2} = \frac{x+2}{3} \quad (\text{Prop theorem, } MS \parallel QR)$$

$$3x = 2x + 4$$

$$x = 4$$

QUESTION 2

$$x = \frac{30}{8} \quad (\text{Prop theorem, } DE \parallel BC)$$

QUESTION 3

$$3.1 \quad (a) \frac{AG}{GH} = 4 \quad (b) \frac{AD}{DE} = 4$$

$$3.2 \quad DE = \frac{5}{4}$$

QUESTION 4

$$4.1 \quad PN = 12 \text{ units}$$

$$4.2 \quad MR = 24 \text{ units}$$

QUESTION 5

$$5.1 \quad AE = 15 \text{ units}$$

$$5.2 \quad AH = 8 \text{ units}$$

QUESTION 6

$$6.1 \quad CB:CA = 5:9$$

Let $CB = 5k$ and $CA = 9k$ then $BA = 4k$
 $BA = 8 \text{ units}$

$$6.2 \quad \frac{\text{Area } \triangle BDC}{\text{Area } \triangle BED} = \frac{CD}{DE} \quad (\text{shared height})$$

$$\text{But } \frac{CD}{DE} = \frac{CB}{BA} = \frac{10}{8} \quad (\text{line parallel to one side of } \triangle)$$

$$\therefore \frac{\text{Area } \triangle BDC}{\text{Area } \triangle BED} = \frac{5}{4}$$

$$6.3 \quad \text{from above } \frac{CD}{DE} = \frac{5}{4} \text{ so let } CD = 5p \text{ and } DE = 4p \text{ so } CE = 9p$$

$$\text{Now } \frac{CE}{CF} = \frac{CB}{CA} = \frac{5}{9} \quad (\text{line parallel to one side of } \triangle)$$

QUESTION 7

$$7.1 \quad \widehat{ADL} = 90^\circ \quad (\text{Angle in semi-circle})$$

$$\widehat{ACB} = 90^\circ$$

$\therefore DL \parallel CB$ (Converse corresponding angles are equal)

$$\begin{aligned} 7.2 \quad LC &= LA \text{ (radii)} \\ SD &= SL = SA \text{ (radii)} \\ \text{But } LA &= SA + SL \\ \text{Therefore } LC &= 2SD \end{aligned}$$

$$\begin{aligned} 7.3 \quad AS &= SL \text{ and } AL = LB \quad (\text{radii}) \\ \frac{SL}{AB} &= \frac{1}{4} \end{aligned}$$

$$\begin{aligned} 7.4 \quad LB &= 15 \text{ units (radius)} \\ \frac{9}{16} &= \frac{LM}{15} \quad (\text{line parallel to one side of } \Delta) \\ LM &= 8.44 \text{ units} \end{aligned}$$

QUESTION 8

$$\begin{aligned} 8.1 \quad \text{In } \Delta ABQ \\ \frac{BR}{RA} &= \frac{BT}{TQ} \quad (\text{Prop Theorem } RT \parallel AQ) \\ \frac{1}{2} &= \frac{k}{TQ} \\ \therefore TQ &= 2k \end{aligned}$$

$$\begin{aligned} 8.2.1 \quad \text{In } \Delta CRT \\ \frac{CP}{PR} &= \frac{5k}{2k} \quad (\text{Prop Theorem } RT \parallel AQ) \\ \frac{CP}{PR} &= \frac{5}{2} \end{aligned}$$

$$\begin{aligned} 8.2.2 \quad \frac{\text{Area } \Delta RCT}{\text{Area } \Delta ABC} &= \frac{\text{Area } \Delta RCT}{\text{Area } \Delta BRC} \times \frac{\text{Area } \Delta BRC}{\text{Area } \Delta ABC} \quad (\text{triangle with equal altitude}) \\ &= \frac{7}{8} \times \frac{1}{3} \\ &= \frac{7}{24} \end{aligned}$$

QUESTION 9:

$$\begin{aligned} 9.1 \quad \text{In } \Delta APQ \text{ and } \Delta ACD \\ \frac{AP}{PD} &= \frac{AQ}{QC} = \frac{1}{3} \quad (AP \parallel DC) \\ DB &= \frac{5}{9} AB \\ AD &= \frac{4}{9} AB \\ PD &= 3AP \\ DB &= \frac{3}{9} AB \text{ and } AP = \frac{1}{9} AB \\ \frac{\text{Area } \Delta QPA}{\text{Area } \Delta QPB} &= \frac{AP}{PB} = \frac{\frac{1}{9} AB}{\frac{8}{9} AB} = \frac{1}{8} \quad (\text{Same height, PQ}) \end{aligned}$$

$$\begin{aligned} 9.2 \quad \text{In } \Delta DBE \text{ and } \Delta PBQ \\ \frac{BE}{EQ} &= \frac{BD}{DP} = \frac{\frac{5}{9} AB}{\frac{3}{9} AB} = \frac{5}{3} \quad (PQ \parallel D) \end{aligned}$$

QUESTION 10

$$\begin{aligned}
 10.1 \quad \frac{\text{Area } \Delta PRA}{\text{Area } \Delta QRA} &= \frac{\frac{1}{2} \cdot 3k \cdot 2a \cdot \sin P}{\frac{1}{2} \cdot 8k \cdot 2a \cdot \sin P - \frac{1}{2} \cdot 3k \cdot 2a \cdot \sin P} \\
 &= \frac{3 \cdot a \cdot k \cdot \sin P}{8 \cdot a \cdot k \cdot \sin P - 3 \cdot a \cdot k \cdot \sin P} \\
 &= \frac{3 a k \sin P}{5 a k \sin P}
 \end{aligned}$$

$$= \frac{3}{5}$$

$$\begin{aligned}
 10.2 \quad \frac{BD}{BQ} &= \frac{AC}{CQ} \quad (\text{prop theorem } CB \parallel AR) \\
 &= \frac{1,5 k}{5k + 1,5 k} \\
 &= \frac{1,5 k}{6,5 k} \\
 &= \frac{3}{13}
 \end{aligned}$$

QUESTION 11:

$$\begin{aligned}
 11.1 \quad \frac{PQ}{RS} &= \frac{PS}{QS} = \frac{QS}{QR} = \frac{2}{1} \\
 \Delta PQS &\parallel \Delta SRQ \\
 \hat{P}SQ &= \text{alt } \hat{S}QR \\
 PS &\parallel QR
 \end{aligned}$$

$$\begin{aligned}
 11.2 \quad \frac{\text{AREA } \Delta QRS}{\text{AREA } PQRS} &= \frac{\frac{1}{2} QS \cdot QR \cdot \sin \hat{S}QR}{\frac{1}{2} QS \cdot QR \cdot \sin \hat{S}QR + \frac{1}{2} PS \cdot QS \cdot \sin \hat{P}SQ} \\
 &= \frac{\frac{1}{2}(16)(8) \sin \hat{S}QR}{\frac{1}{2}(16)(8+32) \sin \hat{S}QR} \\
 &= \frac{8}{40} \\
 &= \frac{1}{5}
 \end{aligned}$$

$$\begin{aligned}
 11.3 \quad &\text{In } \Delta POS \text{ and } \Delta ROQ \\
 (i) \quad &\hat{P}SO = \hat{O}QR \quad (PS \parallel QR) \\
 (ii) \quad &\hat{POS} = \hat{ROQ} \quad (\text{Vert. opp. } \angle \text{'s})
 \end{aligned}$$

Similarity Theorem Learner Activity Answers

1.

(a) Similar (AAA)

(b) Similar (SSS)

2. $x = 5$ $y = 9$

$$\begin{aligned} 3.1 \quad \hat{C}_1 &= \hat{A}_1 && (\text{tan. chord}) \\ A_1 &= A_2 && (\angle\text{'s subt. by = chords}) \\ \hat{C}_1 &= \hat{A}_2 \end{aligned}$$

3.2 In Δ 's BCS, DAC:

1. $\hat{C}_1 = \hat{A}_2$ (proven)
2. $\hat{B}_1 = \hat{D}$ (ext. $\angle$ cyclic quad.)
3. $\hat{S} = \hat{C}_3$ ($\angle$ sum Δ)

$\therefore \Delta BCS \parallel \Delta DAC$ (AAA)

$$\begin{aligned} 3.3 \quad \therefore \quad \frac{BC}{DA} &= \frac{BS}{CD} && (\parallel \Delta\text{'s proven}) \\ \Rightarrow BC \cdot CD &= DA \cdot BS \\ \text{but } BC &= CD \\ \therefore BC^2 &= DA \cdot BS \end{aligned}$$

$$\begin{aligned} 4.1 \quad \hat{A}_1 & \text{ (alt. } \angle\text{'s; BA//SP)} \\ \hat{R}_1 & \text{ (ext. } \angle \text{ cyclic quad.)} \end{aligned}$$

4.2 In Δ 's PTS, PSR:

1. $\hat{S}_1 = \hat{R}_1 = x$ (proven)
2. $\hat{P}$ is common
3. $\hat{T}_2 = \hat{P}\hat{S}\hat{R}$ ($\angle$ sum Δ)

$\therefore \Delta PTS \parallel \Delta PSR$ (AAA)

4.3.1 In Δ 's PQT, PRQ:

1. $\hat{Q}_1 = \hat{R}_2$ (tan. chord)
2. $\hat{P}$ is common
3. $\hat{T}_1 = \hat{P}\hat{Q}\hat{R}$ ($\angle$ sum Δ)

$\therefore \Delta PQT \parallel \Delta PRQ$ (AAA)

$$\begin{aligned} 4.3.2 \quad \frac{PQ}{PR} &= \frac{PT}{PQ} && (\parallel \Delta\text{'s proven}) \\ \therefore PQ^2 &= PR \cdot PT \end{aligned}$$

$$\begin{aligned} 4.4 \quad \frac{PS}{PR} &= \frac{PT}{PS} && (\Delta PTS \parallel \Delta PSR \text{ proven}) \\ \Rightarrow PS^2 &= PR \cdot PT \\ \text{but } PQ^2 &= PR \cdot PT \text{ (proven)} \\ \therefore PQ^2 &= PS^2 \\ \Rightarrow PQ &= PS \end{aligned}$$

5.1 In Δ 's DEC, BEA:

1. $\hat{D}_1 = \hat{B}_2$ (ext. $\angle$ cyclic quad.)
2. $\hat{E}$ is common
3. $\hat{C}_3 = \hat{B}\hat{A}\hat{E}$ ($\angle$ sum Δ)

$\therefore \Delta DEC \parallel \Delta BEA$ (AAA)

5.2 In Δ 's FAB, FCA:

1. $\hat{A}_1 = \hat{C}_1$ (ext. $\angle$ cyclic quad.)
2. $\hat{F}$ is common
3. $\hat{B}_1 = \hat{F}\hat{A}\hat{C}$ ($\angle$ sum Δ)

$\therefore \Delta FAB \parallel \Delta FCA$ (AAA)

$$5.3 \quad \therefore \frac{FA}{FC} = \frac{AB}{CA} \quad (\parallel \Delta \text{'s proven})$$

$$\Rightarrow FA \cdot CA = FC \cdot AB$$

$$6.1.1 \quad \hat{D}_2 = \hat{A}_2 + \hat{B}_1 \quad (\text{ext. } \angle \Delta)$$

$$\text{but } \hat{A}_3 = \hat{B}_1 \quad (\text{tan. chord})$$

$$\therefore \hat{D}_2 = \hat{A}_2 + \hat{A}_3$$

$$6.1.2 \quad \hat{B}_2 = \hat{D}_1 \quad (\text{tan. chord})$$

$$\hat{D}_1 = \hat{A}_2 \quad (\text{isos } \Delta)$$

$$\therefore \hat{B}_2 = \hat{A}_2$$

$$\therefore DA \parallel BC \quad (\text{alt. } \angle \text{'s } =)$$

$$6.2 \quad \frac{ED}{DB} = \frac{EA}{AC} \quad (\text{prop. int thm})$$

$$\text{but } AB = DB \quad (\text{given})$$

$$\therefore \frac{ED}{AB} = \frac{EA}{AC}$$

$$6.3 \quad EC : EA = 5 : 2 \Rightarrow EA : AC = 2 : 3$$

$$\therefore \frac{ED}{AB} = \frac{EA}{AC} \Rightarrow \frac{18}{AB} = \frac{2}{3}$$

$$\Rightarrow AB = 27$$

6.4 In Δ 's EDA, EAB

1. $\hat{A}_3 = \hat{B}_1$ (ext. $\angle$ cyclic quad.)
2. $\hat{E}$ is common
3. $\hat{D}_2 = \hat{B}\hat{A}\hat{E}$ ($\angle$ sum Δ)

$\therefore \Delta EDA \parallel \Delta EAB$ (AAA)

$$6.5 \quad \therefore \frac{EA}{EB} = \frac{ED}{EA} \quad (\parallel \Delta \text{'s proven})$$

$$\therefore EA^2 = ED \cdot EB$$

6.6 ΔEBC

$$7.1.1 \quad \hat{R}_3 = \hat{T}_2 \quad (\angle \text{'s in same segm.)}$$

$$\hat{T}_2 = \hat{T}_1 \quad (\text{given})$$

$$\hat{R}_4 = \hat{T}_1 \quad (\text{tan. chord})$$

$$\therefore \hat{R}_3 = \hat{R}_4$$

7.1.2 In Δ 's APR, MPT

1. $\hat{R}_3 = T_2$ (proven)
2. $\hat{P}_1 = \hat{P}_2$ (vert. opp.)
3. $\hat{A} = \hat{M}$ ($\angle$ sum Δ)

$\therefore \Delta APR \parallel \Delta MPT$ (AAA)

$$7.2 \quad \frac{AP}{MP} = \frac{PR}{PT}$$

$$AP \cdot PT = PR \cdot MP$$

8.1 In Δ 's JRS, JTK

1. $\hat{J}$ is common
2. $\hat{K}_1 = \hat{S}$ (ext. $\angle$ cyclic quad.)
3. $\hat{T}_1 = \hat{R}$ ($\angle$ sum Δ)

$\therefore \Delta JRS \parallel \Delta JTK$ (AAA)

$$8.2 \quad \therefore \frac{JR}{JT} = \frac{RS}{TK} \quad (\parallel \Delta \text{'s proven})$$

$$\Rightarrow JR \cdot TK = RS \cdot JT$$

$$\text{but } RS = JT \quad (\text{given})$$

$$\therefore JR \cdot TK = RS^2$$

9.1 In Δ 's CXY, XDY

1. $\hat{X}_1 = \hat{D}_1$ (tan. chord)
2. $\hat{C}_2 = \hat{X}_2$ (tan. chord)
3. $\hat{C}_1\hat{Y}\hat{X} = \hat{X}\hat{D}\hat{Y}$ ($\angle$ sum Δ)

$\therefore \Delta CXY \parallel \Delta XDY$ (AAA)

$$\therefore \frac{XY}{DY} = \frac{YC}{XY} \quad (\parallel \Delta \text{'s proven})$$

$$\Rightarrow XY^2 = DY \cdot YC$$

$$9.2 \quad \hat{A}_1 = 2\hat{X}_1 \quad (\angle \text{ at centre})$$

$$\hat{B}_1 = 2\hat{D}_2 \quad (\angle \text{ at centre})$$

$$\hat{X}_1 = 2\hat{D}_2 \quad (\text{proven})$$

$$\therefore A_1 = B_1$$

$$9.3 \quad \hat{A}\hat{C}\hat{Y} = \hat{Y}_4 \quad (\text{isos } \Delta; \text{ radii})$$

$$\therefore \hat{A}_1 = 180^\circ - 2\hat{Y}_4 \quad (\angle \text{ sum } \Delta)$$

$$\text{sim'ly } \hat{B}_1 = 180^\circ - 2\hat{Y}_2$$

$$\text{but } A_1 = B_1 \quad (\text{proven})$$

$$\therefore \hat{Y}_4 = \hat{Y}_2$$

In Δ 's CAY, YBX

1. $A_1 = B_1$ (proven)
2. $\hat{Y}_4 = \hat{Y}_2$ (proven)
3. $\hat{A}\hat{C}\hat{Y} = \hat{B}\hat{X}\hat{Y}$ ($\angle$ sum Δ)

$\therefore \Delta CAY \parallel \Delta YBX$ (AAA)

- 10.1 In Δ 's GHK, EGK
1. $\hat{K}$ is common
 2. $\hat{G}_3 = \hat{E}_1$ (tan. chord)
 3. $\hat{H}_1 = \hat{E}\hat{G}\hat{K}$ ($\angle$ sum Δ)
- $\therefore \Delta GHK \parallel \Delta EGK$ (AAA)
- $\therefore \frac{GK}{EK} = \frac{HK}{GK}$ ($\parallel \Delta$'s $\Rightarrow$)
- $\therefore GK^2 = EK.HK$
- 10.2 In Δ 's GHK, EFG
1. $\hat{H}_1 = \hat{F}$ (ext. $\angle$ cyclic quad.)
 2. $\hat{G}_3 = \hat{E}_1$ (tan. chord)
 $\hat{E}_1 = \hat{E}_2$ (given)
 $\therefore \hat{G}_3 = \hat{E}_2$
 3. $\hat{K} = \hat{G}_1$ ($\angle$ sum Δ)
- $\therefore \Delta GHK \parallel \Delta EFG$ (AAA)
- $\therefore \frac{FG}{HK} = \frac{EF}{HG}$ ($\parallel \Delta$'s $\Rightarrow$)
- $\therefore FG.HG = EF.HK$
- but $FG = GH$ (conv. $\angle$'s subt. by = chords)
- $\therefore HG^2 = EF.HK$
- $GK^2 = EK.HK$ (proven)
- $\therefore \frac{GK^2}{HG^2} = \frac{EK.HK}{EF.HK} \Rightarrow \frac{GK^2}{HG^2} = \frac{EK}{EF}$
- 11.1.1 $PR = 3\sqrt{2} \Rightarrow QR = 2\sqrt{2}$
 $PQ : QR = \sqrt{2} : 2\sqrt{2}$
 $= 1 : 2 = PK : KT$
 $\therefore TR \parallel KQ$ (conv. prop. int thm)
- 11.1.2 $\hat{K}\hat{T}\hat{S} = 90^\circ$ (rad. tan.)
 $\hat{R}_1 = 90^\circ$ ($\angle$ in semi-circle)
 $\hat{Q}_1 = 90^\circ$ (corres. $\angle$'s; $TR \parallel KQ$)
 $= \hat{K}\hat{T}\hat{S}$
 $\therefore TKQS$ cyclic (conv. ext. $\angle$ cyclic quad.)
- 11.1.3 In Δ 's QRT, KTS
1. $R_1 = \hat{K}\hat{T}\hat{S}$ (proven)
 2. $\hat{Q}_3 = \hat{K}_1$ ($\angle$'s in same segm.)
 3. $\hat{T}_2 = \hat{S}_2$ ($\angle$ sum Δ)
- $\therefore \Delta QRT \parallel \Delta KTS$ (AAA)
- 11.1.4 $\Delta RTS \parallel \Delta TPS$
- In Δ 's RTS, TPS
1. $\hat{S}$ is common
 2. $\hat{T}_1 = \hat{P}$ (tan. chord)
 3. $R_1 = \hat{K}\hat{T}\hat{S}$ (proven)
- $\therefore \Delta RTS \parallel \Delta TPS$ (AAA)

$$11.2.1 \quad \frac{ST}{PS} = \frac{RS}{ST} \quad (\parallel \Delta's \text{ proven})$$

$$ST^2 = RS \cdot PS$$

$$= \sqrt{2} \cdot 4 \cdot \sqrt{2} = 8$$

$$ST = \sqrt{8} = 2\sqrt{2}$$

$$11.2.2 \quad RT^2 = TS^2 - SR^2 \quad (\text{Pythag. thm})$$

$$= 8 - 2 = 6$$

$$RT = \sqrt{6}$$

$$\frac{KT}{QR} = \frac{ST}{RT} \quad (\Delta QRT \parallel \Delta KTS \text{ proven})$$

$$KT = \frac{ST \cdot QR}{RT}$$

$$= \frac{2\sqrt{2} \cdot 2\sqrt{2}}{\sqrt{6}} = \frac{8}{\sqrt{6}}$$

$$12.1 \quad \hat{C}_2 = 90^\circ \quad (\angle \text{ in semi-circle})$$

$$= \hat{M}_2 \quad (\text{given})$$

$$\therefore NQ \parallel CD \quad (\text{corres. } \angle's =)$$

$$12.2 \quad \hat{N} = \hat{C}_1 \quad (\text{corres. } \angle's; NQ \parallel CD)$$

$$\hat{C}_1 = \hat{A}_2 \quad (\text{tan. chord})$$

$$\therefore \hat{N} = \hat{A}_2$$

$$\therefore ANCQ \text{ cyclic} \quad (\text{conv. } \angle's \text{ in same segm.})$$

$$12.3.1 \quad \text{In } \Delta's \text{ PCD, PAC}$$

$$1. \quad \hat{P} \text{ is common}$$

$$2. \quad \hat{C}_1 = \hat{A}_2 \quad (\text{tan. chord})$$

$$3. \quad \hat{D}_1 = \hat{A}CP \quad (\angle \text{ sum } \Delta)$$

$$\therefore \Delta PCD \parallel \Delta PAC \quad (AAA)$$

$$12.3.2 \quad \frac{PC}{PA} = \frac{PD}{PC} \quad (\parallel \Delta's \text{ proven})$$

$$PC^2 = PD \cdot PA$$

$$12.4 \quad \text{In } \Delta's \text{ BCD, NBC}$$

$$1. \quad \hat{BCD} = \hat{B}_1 \quad (\text{alt. } \angle's; NQ \parallel CD)$$

$$2. \quad \hat{D}_2 = \hat{C}_4 \quad (\text{tan. chord})$$

$$3. \quad \hat{B}_2 = \hat{N} \quad (\angle \text{ sum } \Delta)$$

$$\therefore \Delta BCD \parallel \Delta NBC \quad (AAA)$$

$$\therefore \frac{BC}{NB} = \frac{CD}{BC} \quad (\parallel \Delta's =)$$

$$\Rightarrow BC^2 = CD \cdot NB$$

$$12.5 \quad PC^2 = PD \cdot PA \quad (\text{proven})$$

$$\therefore \frac{PC^2}{BC^2} = \frac{PD \cdot PA}{CD \cdot NB}$$

$$\text{but } PC = MC \quad (\text{given})$$

$$\begin{aligned}
&\therefore \frac{MC^2}{BC^2} = \frac{AP.DP}{CD.NB} \\
&\Rightarrow \frac{BC^2 - BM^2}{BC^2} = \frac{AP.DP}{CD.NB} \quad (\text{Pythag. thm}) \\
&\Rightarrow 1 - \frac{BM^2}{BC^2} = \frac{AP.DP}{CD.NB}
\end{aligned}$$

13.1 $\hat{P}_1 = x$ (tan. chord)
 $\hat{S}_2 = 90^\circ$ ($\angle$ in semi-circle)
 $\hat{E}_2 = 90^\circ$ (chord thm)
 $\therefore RS \parallel ON$ (alt. $\angle$'s =)
 $\hat{N}_1 = \hat{P}_1 = x$ (corres. $\angle$'s; $RS \parallel ON$)
 $\hat{N}_2 = \hat{N}_1 = x$ ($\triangle SNE \equiv \triangle PNE$)

13.2.1 In $\triangle$'s SEN, OPN
1. $\hat{N}_1 = \hat{N}_2$ (proven)
2. $\hat{P} = 90^\circ$ (rad. tan)
 $= \hat{E}_1$ (proven)
3. $\hat{S}_3 = \hat{O}_1$ ($\angle$ sum $\triangle$)
 $\therefore \triangle SEN \parallel \triangle OPN$ (AAA)

13.2.2 $\therefore \frac{SN}{ON} = \frac{EN}{PN}$ ($\parallel \triangle$'s proven)
 $\Rightarrow SN.PN = EN.ON$
but $SN = PN$ (tans from common pt)
 $\Rightarrow PN^2 = EN.ON$

13.3 In $\triangle$'s PEN, OEP
1. $\hat{E}_4 = \hat{E}_1 = 90^\circ$ (proven)
2. $\hat{P}_2 = \hat{S}_3$ (isos $\triangle$)
 $= \hat{O}_1$ (proven)
3. $\hat{N}_2 = \hat{P}_1$ ($\angle$ sum $\triangle$)
 $\therefore \triangle PEN \parallel \triangle OEP$ (AAA)

$\therefore \frac{PE}{OE} = \frac{EN}{PE}$ ($\parallel \triangle$'s proven)
 $\Rightarrow PE^2 = OE.EN$

13.4 $\frac{OE}{RS} = \frac{PE}{PS} = \frac{1}{2}$

13.5 $\frac{PN^2}{PE^2} = \frac{EN.ON}{OE.EN} = \frac{ON}{OE} = \frac{ON}{\frac{1}{2}RS} = \frac{2ON}{RS}$

TRIGONOMETRY

MIXED EXERCISES SOLUTIONS

1.1 $OP = \sqrt{61}$ units

1.2 $\sin \theta = \frac{5}{\sqrt{61}}$

1.3 $\frac{6}{5}$

1.4 $-\frac{6}{5}$

1.5 $\frac{2\cos^2 \theta - 1}{2\sin \theta \cos \theta}$

2.1 $\cos \alpha = -\frac{5}{35}$

2.2 $\frac{25}{169}$

2.3 $-\frac{120}{119}$

3. -1

4.1 $\frac{24}{25}$

4.2 $\frac{7}{5}$

5. $-\frac{171}{221}$

6.1 1

6.2 0

7.1 $\sin 52^\circ = \sqrt{1 - m^2}$

7.2 $\sin 38^\circ = m$

7.3 $\cos(-128^\circ) = -m$

7.4 $\cos 8^\circ = \frac{m + \sqrt{3 - 3m^2}}{2}$

7.5 $\cos 104^\circ = 2m^2 - 1$

7.6 $\frac{m - 1}{2}$

8.1 $1 - 2k^2$

$$8.2 \quad 2k\sqrt{1-k^2}$$

$$8.3 \quad \sqrt{1-k^2}$$

$$8.4 \quad \frac{\sqrt{1-k^2} + \sqrt{3}k}{2}$$

$$8.5 \quad \frac{k}{\sqrt{1-k^2}}$$

$$9. \quad -\frac{t}{8\sqrt{1-t^2}}$$

$$10.1 \quad m$$

$$10.2 \quad m$$

$$11.1 \quad \tan x$$

$$11.2 \quad \sin x$$

$$12.1 \quad 1$$

$$12.2 \quad -\frac{\sqrt{3}}{2\cos x}$$

$$14.1. \quad \alpha = 30^\circ + k.360^\circ \text{ or } \alpha = 150^\circ + k.360^\circ \text{ or } \alpha = 90^\circ + k.360^\circ \text{ or } \alpha = 270^\circ + k.360^\circ; k \in \mathbb{Z}$$

$$14.2 \quad \theta = \pm 55^\circ + k.90^\circ; k \in \mathbb{Z}$$

$$14.3 \quad y = 71,57^\circ + k.360^\circ \text{ or } y = 135^\circ + k.360^\circ; k \in \mathbb{Z}$$

$$14.4 \quad \alpha = \pm 60^\circ + k.360^\circ \text{ or } \alpha = \pm 180^\circ + k.360^\circ; k \in \mathbb{Z}$$

$$14.5 \quad x = 45^\circ - k.360^\circ; k \in \mathbb{Z} \text{ or } x = 225^\circ - k.360^\circ; k \in \mathbb{Z}$$

$$14.6 \quad x = 21^\circ + k.360^\circ; k \in \mathbb{Z} \text{ or } x = 153^\circ + k.180^\circ; k \in \mathbb{Z}$$

$$14.7 \quad x = 0^\circ + k.180^\circ; k \in \mathbb{Z} \text{ or } x = 120^\circ + k.180^\circ; k \in \mathbb{Z}$$

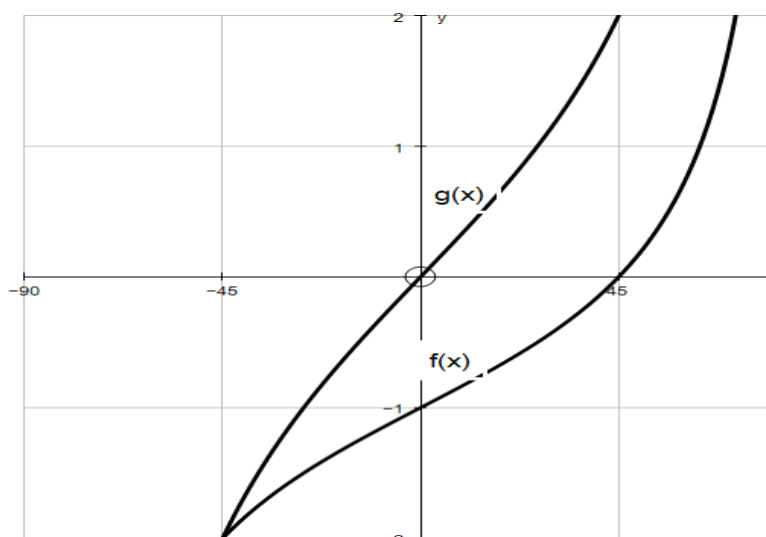
$$14.8 \quad x = -18,43^\circ; \pm 161,57^\circ \text{ or } x = \pm 45^\circ; -135^\circ; k \in \mathbb{Z}$$

$$14.9 \quad x = 0^\circ + k.360^\circ \text{ or } x = 180^\circ + k.360^\circ \text{ or } x = 30^\circ + k.360^\circ \text{ or } x = 150^\circ + k.360^\circ; k \in \mathbb{Z}$$

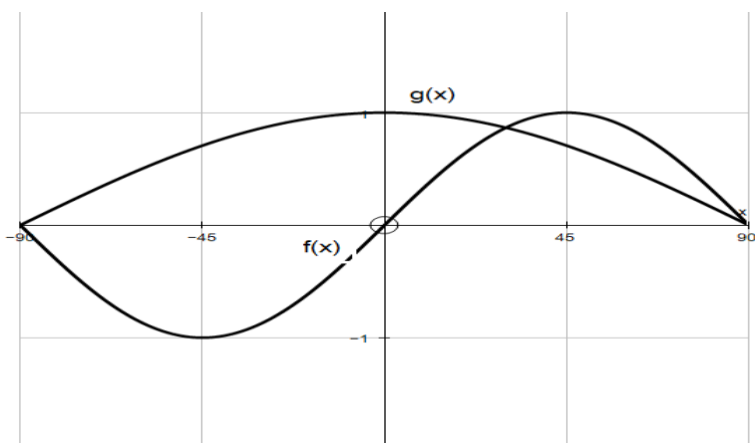
$$14.10 \quad x = 30^\circ + k.180^\circ; k \in \mathbb{Z}$$

$$14.11 \quad x = [30^\circ; -30^\circ]$$

15.



16.1



16.2 $x = 30^\circ$

16.3 $90^\circ \angle x \angle 30^\circ$

16.4 $-45^\circ \angle x \angle 45^\circ$

17.1

17.2 (i) $-150^\circ < x < 90^\circ$

(ii) $0^\circ \angle x \angle 180^\circ$

(iii) $-45^\circ \angle x \angle 45^\circ$

18.1 (i) $b = -1$

(ii) 1

(iii) $k = 2$

$$(iv) (-45^\circ; 0^\circ) \text{ and } (45^\circ; 0)$$

$$18.2 \text{ (i) } -45^\circ \angle x \angle 45^\circ$$

$$-45$$

$$19. \quad PQ = \frac{2A \tan x}{a \sin y}$$

$$20. \quad AP = \frac{10 \tan \alpha}{\cos \beta}$$

$$21. \quad h = \tan \beta \left(x \sqrt{2(1 + \cos 2x)} \right)$$

$$22. \quad HG = \frac{P \cos \beta \sin(\theta + x) \tan \alpha}{\sin \theta}$$