

APPROXIMATION

Specific outcome: work with relative and absolute error

$$\text{Relative error} = \frac{\text{Absolute error}}{\text{True measurement}}$$

- 1) Find the absolute error, the upper limit and the lower limit for 8h

Solution

$$\text{Least unit of measurement} = 1h$$

$$\text{Absolute error} = \frac{1}{2} \text{ of } 1h = 0.5h$$

$$\text{Upper limit} = 8 + 0.5 = 8.5h$$

$$\text{Lower limit} = 8 - 0.5 = 7.5h$$

- 2) Find the relative error of 5.5l

Solution

$$\text{Absolute error} = 0.05$$

$$\text{Relative error} = \frac{0.05}{5.5} = \frac{5}{550} = \frac{1}{110}$$

- 3) Find the limits between which the areas of the following shapes must lie.

- a) A square of side 3cm

- b) A right-angled triangle with hypotenuse 5m and the other sides 3m and 4m long

Solution

- a). least unit of measurement = 1cm

$$\text{Absolute error} = 0.5$$

$$\text{Upper limit is } 3 + 0.5 = 3.5\text{cm}$$

$$\text{Lower limit is } 3 - 0.5 = 2.5\text{cm}$$

$$\text{Maximum area} = (3.5 \times 3.5)\text{cm} = 12.25\text{cm}^2$$

$$\text{Minimum area} = (2.5 \times 2.5)\text{cm} = 6.25\text{cm}^2$$

Therefore, the limits are 6.25cm^2 and 12.25cm^2

b). least unit of measurement = 1m

Absolute error = 0.5

Upper limit for the height = $4 + 0.5 = 4.5\text{m}$

Lower limit for the height = $4 - 0.5 = 3.5\text{m}$

Upper limit for the base = $3 + 0.5 = 3.5\text{m}$

Lower limit for the base = $3 - 0.5 = 2.5\text{m}$

Area of a triangle $= \frac{1}{2} \times b \times h$

Therefore, maximum area $= \frac{1}{2} \times 4.5 \times 3.5 = \frac{15.75}{2} = 7.875\text{m}^2$

Minimum area $= \frac{1}{2} \times 3.5 \times 2.5 = \frac{8.75}{2} = 4.375\text{m}^2$

The limits are 4.375m^2 and 7.875m^2

1. The length and breadth of a rectangle, given to the nearest centimeter, are 15cm and 10cm respectively. Find :
 - a) The shortest possible length and shortest possible breadth of the rectangle.
 - b) The longest possible length and the longest possible breadth of the rectangle
 - c) The limits between which the area must lie.

Ans: 15cm, given to the nearest centimeter, lies between 14.5cm and 15.5cm. 10cm lies between 9.5cm and 10.5cm

- a) The shortest possible length = 14.5 and the shortest possible breadth = 9.5cm
- b) The longest possible length = 15.5cm and the longest possible breadth is 10.5cm
- c) The smallest possible Area = 14.5×9.5

$$= 137.75\text{cm}^2$$

The longest possible Area = 15.5×10.5

$$= 162.75\text{cm}^2$$

So, the area lies between 137.75cm^2 and 162.75cm^2 or $(137.75\text{cm}^2 \leq A \leq 162.75\text{cm}^2)$

2. A car is driven a distance of 30km, measured to the nearest km, in 20 minutes, measured to the nearest min. between what limits will the average speed lie?

Ans: 30km to the nearest km means.

$$29.5\text{km} \leq \text{distance} \leq 30.5\text{km}$$

20min to the nearest min means

$$19.5\text{min} \leq \text{time} \leq 20.5\text{min}$$

$$\text{Greatest average speed} = \frac{\text{greatest distance}}{\text{least time}} = \frac{30.5\text{km}}{19.5\text{min}} = \frac{1.56\text{km}}{\text{min}} = 94\text{km/h}$$

$$\text{Least possible speed} = \frac{\text{least distance}}{\text{greatest}} = \frac{29.5\text{km}}{20.5\text{min}} = 1.44\text{km/min} = 86\text{km/h}$$

Hence, the average speed = $(90 \pm 4) \text{ km/h}$

3. The true value of the length of a rectangle is 5.5m. If this is recorded as 5m, find

- A] The absolute error
- B] The relative error
- C] The percentage error

SOLUTIONS.

True value is 5.5m recorded value is 5m.

A] Absolute error =[recorded value -true value]
 =[5-5.5] m =[-0.5]= 0.5m.

B] Relative error= $\frac{\text{absolute error}}{\text{true value}} = \frac{0.5m}{5.5m} = 0.09$

C]The percentage error = relative error $\times$ 100%
 = 0.09 $\times$ 100% = 9%

4.The length of a square is given as 10cm correct to the nearest centimeter, calculate

- A) The lower bound length.
- B) The upper bound length.
- C) The smallest possible perimeter.
- D) The largest possible perimeter.
- E) The smallest possible area.
- F) The largest possible area.

Solutions

The length of a square is 10cm.

$$\text{Error} = \frac{n}{2} = \frac{1cm}{2} = 0.5cm$$

- a) The lower bound length = actual length +error
 =10cm $-$ 0.5cm
 =9.5cm

- b) The upper bound length = actual length + error

$$= 10\text{cm} + 0.5\text{cm} = 10.5\text{cm}$$

c) Perimeter of a square = $4 \times$ length

Therefore , smallest possible perimeter = $4 \times$ lower breadth length

$$= 4 \times 9.5\text{cm} = 38\text{cm}$$

d) Largest possible perimeter = $4 \times$ upper bound length

$$= 4 \times 10.5\text{cm} = 42\text{cm}$$

e) Area of a square = L^2

Therefore the smallest possible area = (lower bound length)²

$$= (9.5\text{cm})^2 = 90.25\text{cm}^2$$

f) Largest possible area = (upper bound length)²

$$= (10.5\text{cm})^2 = 110.25\text{cm}^2$$