

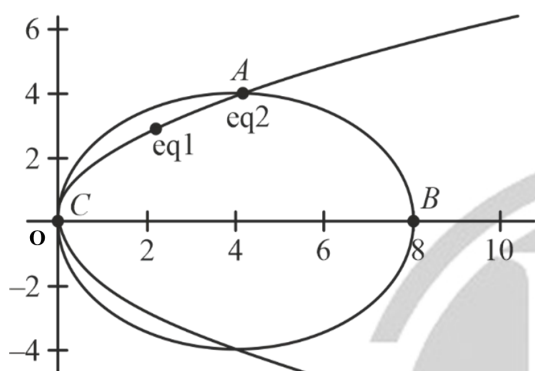
SUBJECT : MATHEMATICS

Topic Name: Area

HINTS & SOLUTIONS

1.

Sol.



The given equations are

$$x^2 + y^2 = 8x \quad \dots (1)$$

$$y^2 = 4x \quad \dots (2)$$

From (1),

$$x^2 - 8x + y^2 = 0$$

$$\Rightarrow x^2 - 2.5x + 16 + y^2 = 16$$

$$\Rightarrow (x - 4)^2 + y^2 = (4)^2 \quad \dots (3)$$

Therefore, the equation (1) is a circle with centre (4, 0) and has a radius 4.

Also, $y^2 = 4x$ is a parabola with vertex at origin and the axis along the x -axis opening in the positive direction.

To find the intersection points of the curves, we solve both the equation.

$$\therefore x^2 + 4x = 8x$$

$$x^2 - 4x = 0$$

$$x(x - 4) = 0$$

$$x = 0 \text{ and } x = 4$$

When, $x = 4$, $y = \pm 4$

But since, it is given above the x -axis.

So, $y = 4$.

$$\text{Therefore, area, } A = \int_0^4 |y_2 - y_1| dx = \int_0^4 (y_2 - y_1) dx$$

[$\therefore y_2 > y_1$]

$$= \int_0^4 \left[\sqrt{16 - (x - 4)^2} - 2\sqrt{x} \right] dx$$

$$\left[\frac{(x - 4)}{2} \sqrt{16 - (x - 4)^2} + \frac{16}{2} \sin^{-1} \left(\frac{x - 4}{4} \right) \right]_0^4 - \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4$$

$$\begin{aligned}
& \left[\frac{(x-4)}{2} \sqrt{16-(x-4)^2} + \frac{16}{2} \sin^{-1} \left(\frac{x-4}{4} \right) \right]_0^4 - \frac{4}{3} \left[x^{\frac{3}{2}} \right]_0^4 \\
&= \left[\left(\frac{(4-4)}{2} \sqrt{16-(4-4)^2} + \frac{16}{2} \sin^{-1} \left(\frac{4-4}{4} \right) \right) - \left(\frac{(0-4)}{2} \sqrt{16-(0-4)^2} + \frac{16}{2} \sin^{-1} \left(\frac{0-4}{4} \right) \right) \right] - \frac{4}{3} \left[3^{\frac{3}{2}} - 0^{\frac{3}{2}} \right] \\
&= \left[\left(\frac{(0)}{2} \sqrt{16-0} + 8 \sin^{-1}(0) \right) - \left(-2\sqrt{16-16} + 8 \sin^{-1}(-1) \right) \right] - \frac{4}{3} \left[4^{\frac{3}{2}} \right] \\
&= \left[(0+0) - (0-8 \sin^{-1}(1)) \right] - \frac{4}{3} \left[4^{\frac{3}{2}} \right] \\
&= 8 \frac{\pi}{2} - \frac{4}{3} \cdot 2^3 \\
&= 4\pi - \frac{32}{3}
\end{aligned}$$

Hence, the required area of the region is $4\pi - \frac{32}{3}$ sq. unit.

2.

Sol. The vertices of $\triangle ABC$ are $A(2, 0)$, $B(4, 5)$ and $C(6, 3)$.

Equation of line segment AB is

$$y - 0 = \left(\frac{5-0}{4-2} \right) (x-2)$$

$$\Rightarrow y = \left(\frac{5}{2} \right) (x-2) \quad \dots (1)$$

Equation of line segment BC is

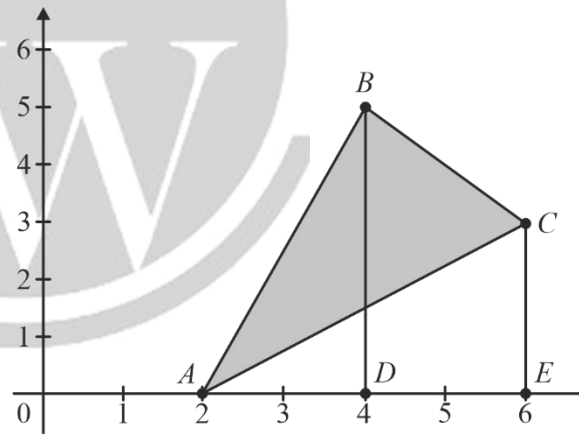
$$y - 5 = \left(\frac{3-5}{6-4} \right) (x-4)$$

$$\begin{aligned}
\Rightarrow y - 5 &= \left(\frac{-2}{2} \right) (x-4) \\
\Rightarrow y - 5 &= -x + 4 \\
\Rightarrow y &= -x + 9 \quad \dots (2)
\end{aligned}$$

Equation of line segment CA is

$$\Rightarrow y - 3 = \left(\frac{0-3}{2-6} \right) (x-6)$$

$$\begin{aligned}
\Rightarrow y - 3 &= \left(\frac{-3}{-4} \right) (x-6) \\
\Rightarrow 4(y-3) &= 3(x-6) \\
\Rightarrow 4y - 12 &= 3x - 18 \\
\Rightarrow 4y &= 3x - 6 \\
\Rightarrow y &= \frac{3}{4}(x-2) \quad \dots (3)
\end{aligned}$$



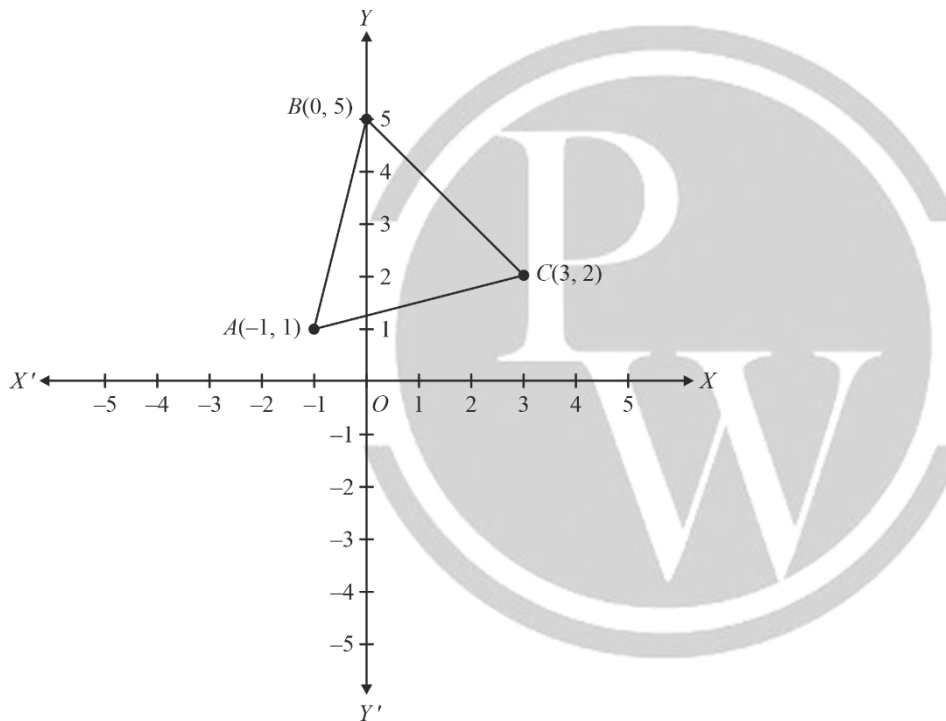
$$\text{Area } (\triangle ABC) = \text{Area } (ABDA) + \text{Area } (BDECB) - \text{Area } (AECA)$$

$$\begin{aligned}
&= \frac{5}{2} \int_2^4 (x-2) dx + \int_4^6 (-x+9) dx - \frac{3}{4} \int_2^6 (x-2) dx \\
&= \frac{5}{2} \left[\frac{x^2}{2} - 2x \right]_2^4 + \left[\frac{-x^2}{2} + 9x \right]_4^6 - \frac{3}{4} \left[\frac{x^2}{2} - 2x \right]_2^6 \\
&= \frac{5}{2} [8 - 8 - 2 + 4] + [-18 + 54 + 8 - 36] - \frac{3}{4} [18 - 12 - 2 + 4] \\
&= \frac{5}{2} [2] + [8] - \frac{3}{4} [8] = 5 + 8 - 6 \\
&= 7 \text{ sq. unit}
\end{aligned}$$

Therefore, the area of the triangle is 7 sq. unit.

3.

Sol.



Let $A(-1, 1)$, $B(0, 5)$ and $C(3, 2)$

The equation of line AB is

$$y - 1 = \frac{5 - 1}{0 - (-1)}(x + 1)$$

$$y = 4x + 5$$

The equation of line BC is

$$y - 5 = \frac{2 - 5}{3 - 0}(x - 0)$$

$$y = -x + 5$$

The equation of line CA is

$$y - 5 = \frac{1 - 2}{-1 - 3}(x - 3)$$

$$y = \frac{x}{4} + \frac{5}{4}$$

Required area = Area of $\triangle ABC$

The equation of line CA is

$$y - 2 = \frac{1-2}{-1-3}(x-3)$$

$$y = \frac{x}{4} + \frac{5}{4}$$

4.

Sol. Put $y = x$ in $x^2 + y^2 = 32$

$$\therefore x^2 + x^2 = 32$$

$$2x^2 = 32$$

$$x^2 = 16$$

$$x = 4$$

$$A = \int_0^4 y_{\text{line}} dx + \int_4^{\sqrt{32}} y_{\text{circle}} dx$$

$$A = \int_0^4 x dx + \int_4^{\sqrt{32}} (\sqrt{32 - x^2}) dx$$

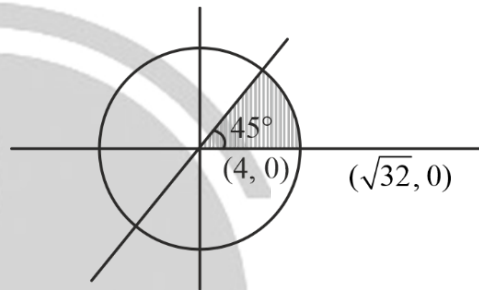
$$= \left(\frac{x^2}{2} \right)_0^4 + \int_4^{\sqrt{32}} \sqrt{(\sqrt{32})^2 - x^2} dx$$

$$= (8) + \left(\frac{x}{2} \sqrt{32 - x^2} + \frac{32}{2} \sin^{-1} \left(\frac{x}{\sqrt{32}} \right) \right)_4^{\sqrt{32}}$$

$$= (8) + \left(0 + 16 \times \frac{\pi}{2} - \left(2\sqrt{16} + 16 \sin^{-1} \left(\frac{4}{\sqrt{32}} \right) \right) \right)$$

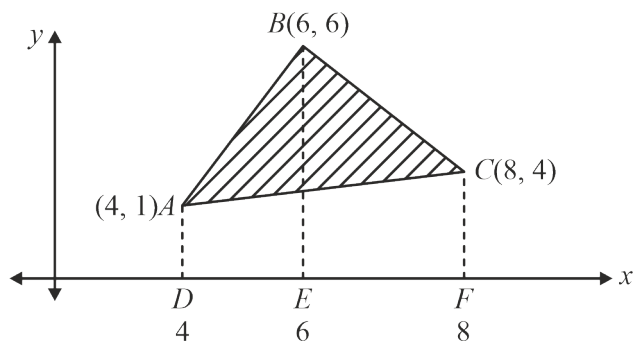
$$= 8 + 8\pi - 8 - 16 \sin^{-1} \left(\frac{1}{\sqrt{2}} \right)$$

$$= 8\pi - 16 \times \frac{\pi}{4} = 8\pi - 4\pi = 4\pi \text{ sq unit}$$



5.

Sol.



Equation of AB is $y - 1 = \frac{6-1}{6-4}(x-4)$

$$\Rightarrow 2y - 2 = 5x - 20$$

$$\Rightarrow y = \frac{5x}{2} - 9$$

Equation of BC is

$$\Rightarrow y - 6 = \frac{4-6}{8-6}(x-6)$$

$$\Rightarrow y = -x + 12$$

Equation of AC is

$$\Rightarrow y - 1 = \frac{4-1}{8-4}(x-4)$$

$$\Rightarrow 4y - 4 = 3x - 12$$

$$\Rightarrow y = \frac{3x}{4} - 2$$

Area of $\triangle ABC$ = area $ABED$ + area $BEFC$ - area $ADFC$

$$\begin{aligned} &= \int_4^6 \left(\frac{5x}{2} - 9 \right) dx + \int_6^8 (-x + 12) dx - \int_4^6 \left(\frac{3x}{4} - 2 \right) dx \\ &= \left| \left(\frac{5x^2}{4} - 9x \right) \right|_4^6 + \left| \left(\frac{-x^2}{2} + 12x \right) \right|_6^8 - \left| \left(\frac{3x^2}{8} - 2x \right) \right|_4^6 = 7 \text{ sq unit} \end{aligned}$$

OR

Parabola $4y = 3x^2$... (1)

Line $3x - 2y + 12 = 0$... (2)

From (2) $y = \frac{3x+12}{2}$

Putting this value of y is (1) we get

$$6x + 24 = 3x^2$$

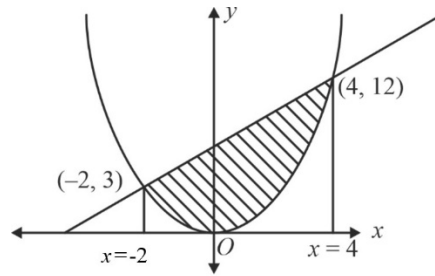
$$\Rightarrow x = 4, -2$$

when $x = 4$ then $y = 12$

$x = -2$ then $y = 3$

Required area

$$= \int_{-2}^4 (y \text{ of line}) dx - \int_{-2}^4 (y \text{ of parabola}) dx$$

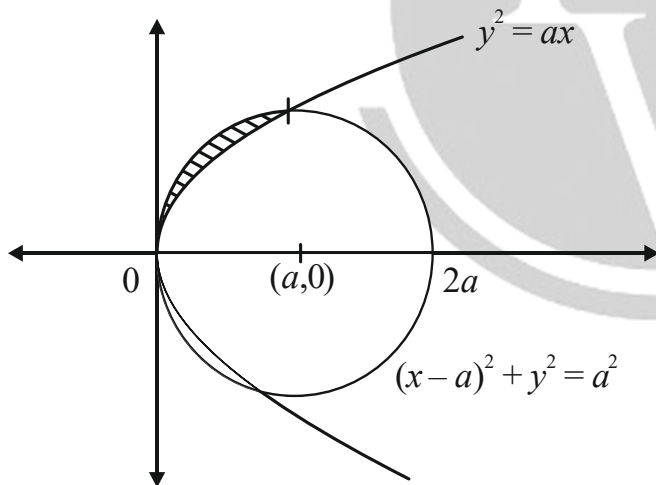


$$= \int_{-2}^4 \left(\frac{3x+12}{2} - \frac{3x^2}{4} \right) dx$$

$$= \frac{3}{4} \int_{-2}^4 (8 + 2x - x^2) dx$$

$$= \frac{3}{4} \left[8x + x^2 - \frac{x^3}{3} \right]_{-2}^4 = 27 \text{ sq. unit}$$

6.



Sol.

$$y^2 = ax, x^2 + y^2 = 2ax$$

$$\Rightarrow x^2 - ax = 0$$

$$\Rightarrow x = 0, x = a$$

Shaded area

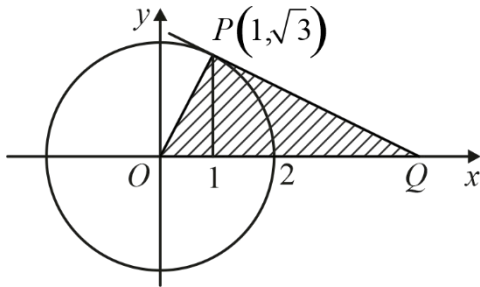
$$= \left[\int_0^a \left[\sqrt{a^2 - (x-a)^2} - \sqrt{a} \sqrt{x} \right] dx \right]$$

$$A = \left[\frac{x-a}{2} \sqrt{a^2 - (x-a)^2} + \frac{a^2}{2} \sin^{-1} \frac{x-a}{a} - \sqrt{a} \frac{2}{3} x^{3/2} \right]_0^a$$

$$= \left[-\frac{2}{3}a^2 + \frac{a^2}{2} \frac{\pi}{2} \right] = \frac{\pi a^2}{4} - \frac{2a^2}{3} \text{ sq. unit}$$

7.

Sol.



Equation of normal (OP) : $y = \sqrt{3}x$

Equation of tangent (PQ) is

$$y - \sqrt{3} = -\frac{1}{\sqrt{3}}(x-1) \text{ i.e. } y = \frac{1}{\sqrt{3}}(4-x)$$

Coordinates of $Q(4, 0)$

$$\therefore \text{Req. area} = \int_0^1 \sqrt{3}x \, dx + \int_1^4 \frac{1}{\sqrt{3}}(4-x) \, dx$$

Coordinates of $Q(4, 0)$

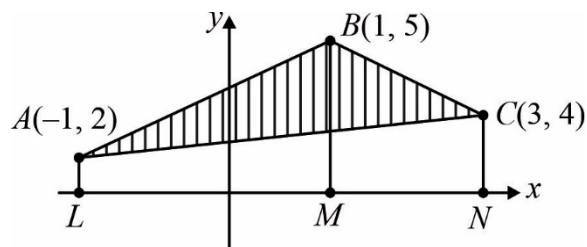
$$\begin{aligned} \therefore \text{Req. area} &= \int_0^1 \sqrt{3}x \, dx + \int_1^4 \frac{1}{\sqrt{3}}(4-x) \, dx \\ &= \left[\sqrt{3} \frac{x^2}{2} \right]_0^1 + \frac{1}{\sqrt{3}} \left[4x - \frac{x^2}{2} \right]_1^4 \\ &= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}} \left[16 - 8 - 4 + \frac{1}{2} \right] = 2\sqrt{3} \text{ sq. unit} \end{aligned}$$

8.

Sol. Let $A = (-1, 2)$

$B = (1, 5)$

$C = (3, 4)$



We have to find the area of $\triangle ABC$

Find equation of line $AB \rightarrow y - 5 = \left(\frac{2-5}{-1-1} \right) \cdot (x-1)$

$$y - 5 = \frac{3}{2}(x - 1)$$

$$2y - 10 = 3x - 3$$

$$3x - 2y + 7 = 0 \quad \dots (1)$$

$$y = \frac{3x + 7}{2}$$

$$\text{Equation of } BC \rightarrow y - 4 = \left(\frac{5-4}{1-3}\right) \cdot (x-3)$$

$$y - 4 = \frac{1}{-2}(x - 3)$$

$$2y - 8 = -x + 3$$

$$x + 2y - 11 = 0 \quad \dots (2)$$

$$y = \frac{11-x}{2}$$

$$\text{Equation of } AC \rightarrow y - 4 = \left(\frac{2-4}{-1-3}\right) \cdot (x-3)$$

$$y - 4 = \frac{1}{2}(x - 3) \Rightarrow 2y - 8 = x - 3$$

$$x - 2y + 5 = 0 \quad \dots (3)$$

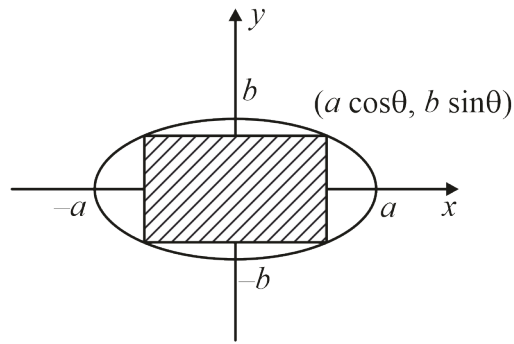
$$\Rightarrow y = \frac{x+5}{2}$$

So, required area

$$\begin{aligned} &= \int_{-1}^1 \left(\frac{3x+7}{2}\right) dx + \int_1^3 \left(\frac{11-x}{2}\right) dx - \int_{-1}^3 \left(\frac{x+5}{2}\right) dx \\ &= \frac{1}{2} \left[\frac{3x^2}{2} + 7x \right]_{-1}^1 + \frac{1}{2} \left[11x - \frac{x^2}{2} \right]_1^3 - \frac{1}{2} \left[\frac{x^2}{2} + 5x \right]_{-1}^3 \\ &= \frac{1}{2} \left[\left(\frac{3}{2} + 7 \right) - \left(\frac{3}{2} - 7 \right) \right] + \frac{1}{2} \left[\left(33 - \frac{9}{2} \right) - \left(11 - \frac{1}{2} \right) \right] - \frac{1}{2} \left[\left(\frac{9}{2} + 15 \right) - \left(\frac{1}{2} - 5 \right) \right] \\ &= \frac{1}{2} [14 + 22 - 4 - 24] = \frac{1}{2} [36 - 28] = 4 \text{ sq. unit} \end{aligned}$$

9.

Sol. Given ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



Area of rectangle

$$A = 2a \cos \theta \cdot 2b \sin \theta$$

$$A = 2ab \cdot \sin 2\theta$$

$$\therefore A_{\max} = 2ab$$

OR

Let a point (x_1, y_1)

$$3x^2 - y^2 = 8$$

$$\Rightarrow 6x - 2y \cdot y' = 0$$

$$\Rightarrow y' = \frac{3x}{y}$$

$$\therefore \text{Tangent } y - y_1 = \frac{3x_1}{y_1}(x - x_1)$$

It passing through $\left(\frac{4}{3}, 0\right)$

$$-y_1 = \frac{3x_1}{y_1} \left(\frac{4}{3} - x_1\right)$$

$$\Rightarrow -y_1^2 = 4x_1 - 3x_1^2$$

$$\Rightarrow 3x_1^2 - 8 = 3x_1^2 - 4x_1$$

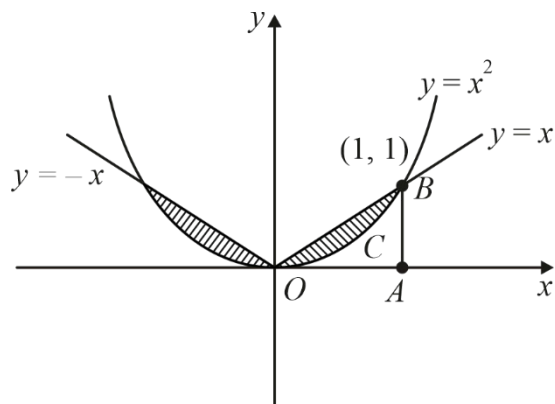
$$\therefore x_1 = 2$$

$$\text{So, } 12 - y_1^2 = 8$$

$$\Rightarrow y_1^2 = 4$$

$$\Rightarrow y_1 = \pm 2$$

10.

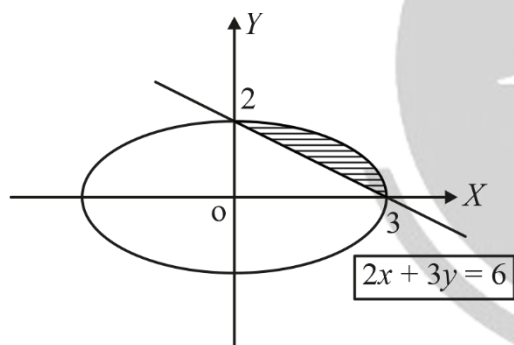
Sol.

Required area = $2[\text{area of } \triangle OAB - \text{Area of curve } OCBA]$

$$A = 2 \left[\frac{1}{2}(1)(1) - \int_0^1 x^2 dx \right]$$

$$A = 2 \left[\frac{1}{2} - \frac{1}{3} \right]$$

$$\Rightarrow A = 2 \left[\frac{1}{6} \right] = \frac{1}{3} \text{ sq. unit}$$

11.**Sol.**

Clearly points of intersection are (3, 0) and (0, 2)

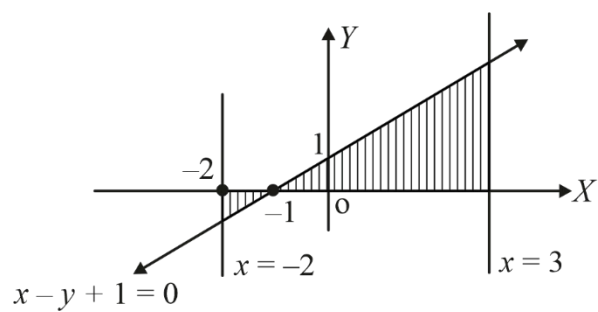
Required area

$$= \frac{2}{3} \int_0^3 \sqrt{9-x^2} dx - \frac{2}{3} \int_0^3 (3-x) dx$$

$$= \frac{2}{3} \left[\frac{x}{2} \sqrt{9-x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} + \frac{(3-x)^2}{2} \right]_0^3$$

$$= \frac{2}{3} \left[\frac{9\pi}{4} - \frac{9}{2} \right] \text{ or } \frac{3\pi}{2} - 3 \text{ sq. unit}$$

OR



Required area

$$= -\int_{-2}^{-1} (x+1)dx + \int_{-1}^3 (x+1)dx$$

$$= -\frac{(x+1)^2}{2} \Big|_{-2}^{-1} + \frac{(x+1)^2}{2} \Big|_{-1}^3$$

$$= \frac{1}{2} + 8$$

$$= \frac{17}{2} \text{ sq. unit}$$

