

SUBJECT : MATHEMATICS

Topic Name: AOD

HINTS & SOLUTIONS

1.

Sol. Let r be the radius and C the circumference of the circle.

$$\text{Then } C = 2\pi r$$

$$\text{It is given that } \frac{dr}{dt} = 0.5 \text{ cm/s}$$

$$\text{Now, } C = 2\pi r$$

Differentiating both sides w.r.t t , we get,

$$\Rightarrow \frac{dC}{dt} = 2\pi \cdot \frac{dr}{dt}$$

$$\Rightarrow \frac{dC}{dt} = 2\pi \cdot 0.5$$

$$\Rightarrow \frac{dC}{dt} = \pi \text{ cm/s}$$

Therefore, the rate in increase of the circumference is π cm/s.

2.

Sol. Given that the volume of the right circular cylindrical box $V = 125\pi \text{ cm}^3$.

Let the radius of the cylinder be h and the height be equal to h .

$$\text{Volume, } V = \pi r^2 h$$

$$\Rightarrow 125\pi = \pi r^2 h$$

$$\Rightarrow r^2 h = 125$$

$$\Rightarrow h = \frac{125}{r^2}$$

$$\text{Surface area of the box, } S = \pi r h + \pi r^2$$

$$S = \pi r \left(\frac{125}{r^2} \right) + \pi r^2$$

Differentiating S w.r.t r to find the point of minima,

$$\Rightarrow \frac{dS}{dr} = \frac{-250\pi}{r^2} + 2\pi r$$

therefore, for the point of minima,

$$\Rightarrow \frac{dS}{dr} = 0$$

$$\Rightarrow \frac{-250\pi}{r^2} + 2\pi r = 0$$

$$\Rightarrow 2\pi r = \frac{250\pi}{r^2}$$

$$\Rightarrow r^3 = 125$$

$$\Rightarrow r = 5 \text{ cm}$$

[Only positive value will be considered, as length can't have negative value]

$$\text{Now, } h = \frac{125}{r^2} = \frac{125}{25} = 5 \text{ cm}$$

The dimension of the cylindrical box is radius, $r = 5$ cm and height, $h = 5$ cm

3.

Sol. Equation of the curve is

$$y = \frac{(x-7)}{(x-2)(x-3)}$$

put $y = 0$ in the above equation we get $x = 7$

$$\frac{dy}{dx} = \frac{(x-2)(x-3) - (x-7)(2x-5)}{(x-2)^2 \cdot (x-3)^2}$$

The slope of the tangent at point $(7, 0)$ is

$$m_i = \left. \frac{dy}{dx} \right|_{(7,0)} = \frac{20}{400} = \frac{1}{20}$$

$$(y-0) = \frac{1}{20}(x-7) \Rightarrow x - 20y - 7 = 0$$

$$m_i \cdot m_n = -1$$

$$\Rightarrow m_i = \frac{-1}{1/20} = -20$$

Equation of the normal is

$$(y-0) = -20(x-7) \Rightarrow 20x + y - 140 = 0$$

4.

Sol. Let R be the radius

H be the height

V be the volume

S be the total surface area

$$V = \pi R^2 H$$

$$S = \pi R^2 + 2\pi RH$$

$$\Rightarrow H = \frac{S - \pi R^2}{2\pi R}$$

Substituting value of H in V

$$V = \frac{1}{2}(SR - \pi R^3)$$

$$\frac{dV}{dR} = \frac{1}{2}(S - 3\pi R^2)$$

$$\frac{dV}{dR} = 0$$

$$\Rightarrow \frac{1}{2}(S - 3\pi R^2) = 0$$

$$R = \sqrt{\frac{S}{3\pi}}$$

$$\frac{d^2V}{dR^2} = \frac{1}{2}(0 - 6\pi R)$$

$$= -3\pi R$$

$$V \text{ is greatest when } R = \sqrt{\frac{S}{3\pi}}$$

$$H = \frac{S - \pi \times \frac{S}{3\pi}}{2\pi \sqrt{\frac{S}{3\pi}}}$$

$$H = \frac{\frac{2S}{3}}{2\sqrt{\frac{\pi S}{3}}}$$

$$H = \sqrt{\frac{S}{3\pi}}$$

5.

Sol. $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$

$$\text{Marginal Cost } (C_M) = \frac{d}{dx} C(x) = 0.005 \times 3x^2 - 0.02 \times 2x + 30$$

$$\because x = 3$$

$$C_M = 0.005 \times 3 \times 9 - 0.02 \times 2 \times 3 + 30$$

$$= 0.135 - 0.12 + 30$$

$$= 30.135 - 0.12$$

$$= 30.015$$

6.

Sol. $\because P(x_1, y_1) = (2, y_1)$ lies on $16x^2 + 9y^2 = 145$

$$16(2)^2 + 9y_1^2 = 145$$

$$9y_1^2 = 145 - 64$$

$$9y_1^2 = 81$$

$$y_1^2 = 9$$

$$y_1 = \pm 3$$

$$\text{But } y_1 > 0 \quad y_1 = 3$$

$$\therefore P = (2, 3)$$

$$16x^2 + 9y^2 = 145 \quad \dots(i)$$

$$32x + 18y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-32x}{18y} = \frac{-16x}{9y}$$

$$\text{Slope of tangent} = m_{(2,3)} = \frac{-16 \times 2}{9 \times 3} = \frac{-32}{27}$$

$$\text{Slope of normal} = m_{(2,3)} = \frac{27}{32}$$

Equation of tangent is,

$$(y - 3) = \frac{-32}{27}(x - 2)$$

$$27y - 81 = -32x + 64$$

$$32x + 27y - 145 = 0$$

Equation of normal is,

$$(y - 3) = \frac{27}{32}(x - 2)$$

$$32y - 96 = 27x - 54$$

$$27x - 32y - 54 + 96 = 0$$

$$27x - 32y + 42 = 0$$

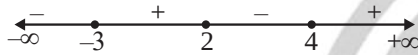
7.

Sol. $f'(x) = x^3 - 3x^2 - 10x + 24$

$$f'(x) = (x + 3)(x - 2)(x - 4)$$

$f(x)$ is strictly increasing

If $f'(x) > 0$



$$\therefore x \in (-3, 2) \cup (4, \infty)$$

$f(x)$ is strictly decreasing is $f'(x) < 0$

$$\therefore x \in (-\infty, -3) \cup (2, 4)$$

8.

Sol. Let the length, width and height of the open tank be x , x and y units respectively. Then, its volume is x^2y and the total surface area is $x^2 + 4xy$.

It is given that the tank can hold a given quantity of water. This means that its volume is constant. Let it be V . then, $V = x^2y$

The cost of the material will be least if the total surface area is least. Let S denote the total surface area.

$$\text{Then, } S = x^2 + 4xy$$

We have to minimize S subject to the condition that the volume V is constant.

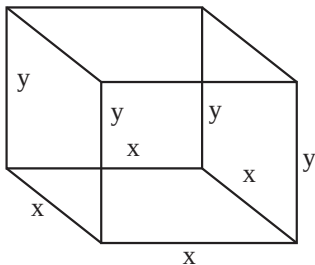
Now,

$$S = x^2 + 4xy$$

$$\Rightarrow S = x^2 + \frac{4V}{x}$$

$$\Rightarrow \frac{dS}{dx} = 2x - \frac{4V}{x^2} \text{ and } \frac{d^2S}{dx^2} = 2 + \frac{8V}{x^3}$$

The critical numbers of S are given by $\frac{dS}{dx} = 0$



$$\Rightarrow 2x - \frac{4V}{x^2} = 0$$

$$\Rightarrow x = 2y$$

$$\text{Clearly, } \frac{d^2S}{dx^2} = 2 + \frac{8V}{x^3} > 0 \text{ for all } x.$$

Hence, S is minimum when $x = 2y$ i.e. the depth (height) of the tank is half of its width.

Comment: Base is directly proportional to height.

9.

Sol. $f(x) = x^3 - 3x$

(i) $f(x)$ being a polynomial is continuous on $[-\sqrt{3}, 0]$

(ii) $f(-\sqrt{3}) = f(0) = 0$

(iii) $f'(x) = 3x^2 - 3$ and this exist uniquely on $[-\sqrt{3}, 0]$

$\therefore f(x)$ is derivable on $(-\sqrt{3}, 0)$

$\therefore f(x)$ satisfies all condition of Roll's theorem

$\therefore$ There exist at least one $c \in (-\sqrt{3}, 0)$ where $f'(c) = 0$

$$\Rightarrow 3c^2 - 3 = 0$$

$$\Rightarrow c = \pm 1 \Rightarrow c = -1$$

10.

Sol. Assumed volume of cube = V

Given that, $\frac{dV}{dt} = 9 \text{ cm}^3/\text{sec}$

$$\frac{dV}{dt} = ?$$

$$l = 10 \text{ cm}$$

$$\frac{dV}{dt} = \frac{d}{dt}(l)^3 = 9 \Rightarrow 3l^2 \frac{dl}{dt} = 9$$

$$\frac{dl}{dt} = \frac{3}{l^2}. \quad \dots(1)$$

Now $\frac{dA}{dt} = \frac{d}{dt}(6l^2) = 12l \frac{dl}{dt} = 12l \times \frac{3}{l^2}$ (form (1))

$$= \frac{36}{l} = \frac{36}{10} = 3.6 \text{ cm}^2/\text{sec}$$

11.

Sol. $f(x) = x^3 - 3x^2 + 6x - 100$

$$f'(x) = 3x^2 - 6x + 6$$

$$f'(x) = 3(x^2 - 2x + 2)$$

$$f'(x) = 3[(x-1)^2 + 1]$$

$$f'(x) > 0 \text{ for all } x \in R$$

So, $f'(x)$ is increasing on R .

12.

Sol. If each side of square base is x and height is h then volume

$$V = x^2 h \Rightarrow h = \frac{V}{x^2}$$

S is surface area then

$$S = 4hx + 2x^2 = 4\left(\frac{V}{x^2}\right)x + 2x^2$$

$$\Rightarrow S = \frac{4V}{x} + 2x^2$$

Diff. w.r.t x then we get,

$$\frac{dS}{dx} = -\frac{4V}{x^2} + 4x \text{ and } \frac{d^2S}{dx^2} = +\frac{8V}{x^3} + 4$$

$$\text{Now } \frac{dS}{dx} = 0 \Rightarrow 4x = \frac{4V}{x^2}$$

$$\Rightarrow x^3 = V \Rightarrow x = V^{1/3}$$

$$\text{at } x = V^{1/3}, \frac{d^2S}{dx^2} > 0$$

$$\Rightarrow S \text{ is minimum when } x = V^{1/3}$$

$$\text{and } h = \frac{V}{x^2} = \frac{V}{V^{2/3}} = V^{1/3} \Rightarrow x = h$$

$$\Rightarrow x = h \text{ means it is a cube}$$

$$13. \quad \frac{dx}{dt} = -3\sin t + 3\cos^2 t \sin t = -3\sin t(1 - \cos^2 t) = -3\sin^3 t$$

$$\frac{dy}{dt} = 3\cos t - 3\sin^2 t \cos t = 3\cos t(1 - \sin^2 t) = 3\cos^3 t$$

$$\text{Slope of normal} = -\frac{dx}{dy} = \frac{\sin^3 t}{\cos^3 t}$$

Eqn. of normal is

$$y - (3\sin t - \sin^3 t) = \frac{\sin^3 t}{\cos^3 t} [x - (3\cos t - \cos^3 t)]$$

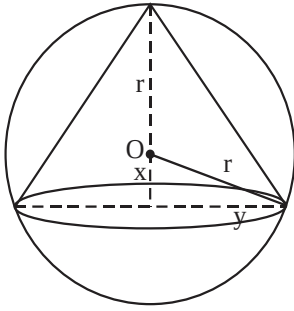
$$\Rightarrow y\cos^3 t - x\sin^3 t = 3\sin t \cos t (\cos^2 t - \sin^2 t)$$

$$\Rightarrow y\cos^3 t - x\sin^3 t = \frac{3}{4} \sin 4t$$

$$\Rightarrow 4(y\cos^3 t - x\sin^3 t) = 3\sin 4t$$

14.

Sol.



Let radius of cone be y and the altitude be $r + x$

$$\therefore x^2 + y^2 = r^2 \quad \dots(i)$$

$$\text{Volume } V = \frac{1}{3}\pi y^2(r+x)$$

$$= \frac{1}{3}\pi(r^2 - x^2)(r+x)$$

$$\frac{dV}{dx} = \frac{\pi}{3}[(r^2 - x^2)1 + (r+x)(-2x)] = \frac{\pi}{3}(r+x)(r-3x)$$

$$\frac{dV}{dx} = 0 \Rightarrow x = \frac{r}{3}$$

$$\therefore \text{Altitude} = r + \frac{r}{3} = \frac{4r}{3}$$

$$\text{and } \frac{d^2V}{dx^2} = \frac{\pi}{3}[(r+x)(-3) + (r-3x)] = \frac{\pi}{3}[-2r-6x] < 0$$

$$\therefore \text{Max. Volume} = \frac{\pi}{3}\left(r^2 - \frac{r^2}{9}\right)\left(r + \frac{r}{3}\right) = \frac{8}{27}\left(\frac{4}{3}\pi r^3\right)$$

$$= \frac{8}{27}(\text{Volume of sphere})$$

15.

Sol. $f(x) = \sin 3x - \cos 3x, 0 < x < \pi$

$$f'(x) = 3\cos 3x + 3\sin 3x$$

$$f'(x) = 0 \Rightarrow \tan 3x = -1$$

$$\Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{4}, n \in \mathbb{Z}$$

$$x = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}$$

$$\text{Intervals are: } \left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{7\pi}{12}\right), \left(\frac{7\pi}{12}, \frac{11\pi}{12}\right), \left(\frac{11\pi}{12}, \pi\right)$$

$$f(x) \text{ is strictly increasing in } \left(0, \frac{\pi}{4}\right) \cup \left(\frac{7\pi}{12}, \frac{11\pi}{12}\right)$$

$$\text{and strictly decreasing in } \left(\frac{\pi}{4}, \frac{7\pi}{12}\right) \cup \left(\frac{11\pi}{12}, \pi\right)$$

16.

Sol. Let x be the side of an equilateral triangle

$$\therefore \frac{dx}{dt} = 2 \text{ cm/s}.$$

$$\text{Area (A)} = \frac{\sqrt{3}x^2}{4}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2} x \frac{dx}{dt}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\sqrt{3}}{2} \cdot (20) = 20\sqrt{3} \text{ cm}^2/\text{s}$$

17.

Sol. $f(x) = \sin x - \cos x$, $0 < x < 2\pi$

$$f'(x) = 0 \Rightarrow \cos x + \sin x = 0 \text{ or } \tan x = -1,$$

$$\therefore x = 3\pi/4, \frac{5\pi}{4}$$

$$f''(3\pi/4) = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \text{ is -ve so, } x = 3\pi/4 \text{ is Local Maxima}$$

$$\text{and } f''(5\pi/4) = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \text{ i.e +ve so, } x = 5\pi/4 \text{ is Local Minima}$$

$$\text{Local Maxima value} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$$

$$\text{Local Minima value} = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\sqrt{2}$$

18.

Sol. $y = [x(x-2)]^2$

we know, for increasing function we have

$$f'(x) \geq 0$$

$$\therefore f'(x) = 2[x(x-2)] \left[\frac{d}{dx} x(x-2) \right]$$

$$\text{Or, } f'(x) = 2[x(x-2)] \frac{d}{dx} (x^2 - 2x)$$

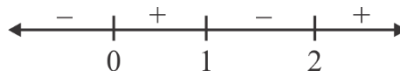
$$= 2x(x-2)(2x-2)$$

$$= 4x(x-2)(x-1)$$

$$\text{For } f'(x) \geq 0$$

$$\text{i.e., } 4x(x-1)(x-2) \geq 0$$

the values of x are :



$$x \in [0, 1] \cup [2, \infty)$$

OR

The slope of the tangent at $(\sqrt{2} a, b)$ to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\frac{2x}{a^2} - \frac{2yy'}{b^2} = 0$$

$$\Rightarrow y' = \left. \frac{b^2 x}{a^2 y} \right|_{(\sqrt{2}a, b)} = \frac{b^2 \sqrt{2}a}{a^2 b} = \frac{b\sqrt{2}}{a}$$

The equation of the tangent:

$$y - b = \frac{b\sqrt{2}}{a}(a - \sqrt{2}a) \text{ (using point-slope form: } y - y_1 = m(x - x_1)\text{)}$$

$$b\sqrt{2}x - ay - ab = 0$$

Normal:

$$\text{The slope of the normal} = \frac{-1}{dy/dx}$$

$$= \frac{-1}{\frac{b\sqrt{2}}{a}} = -\frac{a}{b\sqrt{2}}$$

Equation of Normal:

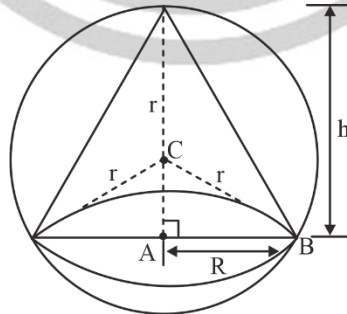
$$y - b = \frac{-a}{b\sqrt{2}}(x - \sqrt{2}a)$$

$$yb\sqrt{2} - b^2\sqrt{2} = -ax + \sqrt{2}a^2$$

$$\text{or } ax + b\sqrt{2}y - \sqrt{2}(a^2 + b^2) = 0$$

19.

Sol. Let R and h be the radius and height of the cone r be the radius of sphere.



$$\text{To show } h = \frac{4r}{3}$$

$$\text{and Maximum Volume of Sphere} = \frac{8}{27} \text{ Volume of Sphere}$$

$$\text{In } \triangle ABX, AC = h - r$$

$$\therefore (h - r)^2 + R^2 = r^2 \text{ \{Pythagoras Theorem\}}$$

$$\Rightarrow R^2 = r^2 - (h - r)^2$$

$$\text{Volume of cone: } V = \frac{1}{3} \pi R^2 h$$

$$\text{Or, } V = \frac{1}{3} \pi (r^2 - (h - r)^2) h$$

$$V = \frac{1}{3}\pi[r^2 - h^2 - r^2 + 2rh]h$$

$$V = \frac{1}{3}\pi[2h^2r - h^3]$$

For maxima or minima, $\frac{dV}{dh} = 0$

$$\text{Now, } \frac{dV}{dh} = \frac{1}{3}\pi[4hr - 3h^2]$$

Putting, $\frac{dV}{dh} = 0$ We get $4hr = 3h^2$

$$\Rightarrow h = \frac{4r}{3}$$

$$\frac{d^2V}{dh^2} = \frac{1}{3}\pi[4r - 6h]$$

Putting $h = \frac{4r}{3}$

$$\frac{d^2V}{dh^2} = \frac{1}{3}\pi\left[4r - \frac{6 \cdot 4r}{3}\right] = -\frac{1}{3}\pi[4r]$$

Which is less than zero, therefore

$$h = \frac{4r}{3} \text{ is a maxima}$$

And the volume of the cone at $h = \frac{4r}{3}$ will be maximum,

$$\begin{aligned} V &= \frac{1}{3}\pi R^2 h \\ &= \frac{1}{3}\pi[r^2 - (h-r)^2]h \\ &= \frac{1}{3}\pi\left[r^2 - \left(\frac{4r}{3} - r\right)^2\right]\left[\frac{4r}{3}\right] \\ &= \frac{1}{3}\pi\left[\frac{8r^2}{9}\right]\left[\frac{4r}{3}\right] \\ &= \frac{8}{27}\left(\frac{4\pi r^3}{3}\right) \\ &= \frac{8}{27} \text{ (Volume of the sphere)} \end{aligned}$$

20.

Sol. $R(x) = 3x^2 + 36x + 5$

$$MR = \frac{dR}{dx} = 6x + 36$$

when $x = 5$

$$MR = 30 + 36 = 66$$

21.

Sol. Given: $y = x^2 - 5x + 6$

$$\Rightarrow \frac{dy}{dx} = 2x - 5$$

Slope at (2, 0) is $m_1 = -1$

Slope at (3, 0) is $m_2 = 1$

Angle between the tangents is given by

$$\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{2}{0} \right)$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

22.

Sol. Given: $y = x^3 + 6x^2 + 6$

$$\Rightarrow f'(x) = \frac{dy}{dx} = 3x^2 + 12x = 3x(x + 4)$$

Taking $f'(x) = 0$, we get, $x = 0$ & $x = -4$

Now, $f'(x) > 0$ for $x < -4$ and $x > 0$.

And, $f'(x) < 0$ for $-4 < x < 0$.

Hence, $f(x)$ increases on the interval

$(-\infty, -4) \cup (0, \infty)$

23.

Sol. Let $f(x) = x - x^2$

Therefore, $f'(x) = 1 - 2x$

Taking $f'(x) = 0$, we get, $x = \frac{1}{2}$

Now, $f''(x) = -2$

At $x = \frac{1}{2}$, $f''(x) < 0$

Therefore, $f(x)$ is maximum at $x = \frac{1}{2}$.

24.

Sol. Given curve is $x^2 - xy + y^2 = 27$

$$\Rightarrow 2x - x \frac{dy}{dx} - y + 2y \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} (2y - x) = y - 2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - 2x}{2y - x}$$

Slope of a line parallel to x-axis is 0.

$$\Rightarrow \frac{y - 2x}{2y - x} = 0$$

$$\Rightarrow y = 2x$$

Using this in (i), we get

$$x^2 - 2x^2 + 4x^2 = 27$$

$$\text{Therefore, } x^2 = 9$$

$$\text{Thus, } x = \pm 3$$

$$\text{So, } y = \pm 6$$

Hence, the points are (3, 6) and (-3, -6)

25.

Sol. Length of the wire = 20 cm

Therefore, Length of wire = 2Radius + Arc length i.e. $2r + l = 20$ cm

$$\text{Therefore, } l = 2(10 - r) = \frac{1}{2}\pi r^2$$

$$\text{Area of sector (A)} = \frac{\theta}{360} \times \pi r^2$$

$$\text{Also } \theta = \frac{1}{r}$$

$$\Rightarrow A = \frac{1}{360} \times \pi r = \frac{20 - 2r}{360} \times \pi r = \frac{10\pi r - \pi r^2}{180}$$

$$\Rightarrow \frac{dA}{dr} = \frac{10\pi - 2\pi r}{180} = \frac{5\pi - \pi r}{90}$$

Taking $\frac{dA}{dr} = 0$, we get, $r = 5$

$$\text{Therefore, } l = 20 - 10 = 10$$

$$\text{Length of an arc} = \frac{\theta}{360} \times 2\pi r$$

$$\Rightarrow 10 = \frac{\theta}{360} \times 10\pi$$

$$\Rightarrow \theta = \frac{360}{\pi}$$

$$\Rightarrow A = \frac{360}{\pi \times 360} \times \pi \times 5^2$$

$$\Rightarrow A = 25 \text{ sq cm}$$

26.

Sol. Let $f(x) = x - \sin x$

Therefore, $f'(x) = 1 - \cos x$ Now, $f(x)$ decreases when $f'(x) < 0$

$$\text{i.e., } -1 \leq \cos x \leq 1$$

Thus, there is no such value of x

27.

Sol. Let $y^2 = 4ax$ (i)

$$\Rightarrow 2y \frac{dy}{dx} = 4a$$

$$\Rightarrow m_1 = \frac{dy}{dx} = \frac{2a}{y}$$

And, $ay = 2x^2$ (ii)

$$\Rightarrow a \frac{dy}{dx} = 4x$$

$$\Rightarrow m_2 = \frac{dy}{dx} = \frac{4x}{a}$$

Angle between the curves is given by

$$\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right| = \left| \frac{\frac{4x}{a} - \frac{2a}{y}}{1 + \frac{4x}{a} \left(\frac{2a}{y} \right)} \right| = \left| \frac{\frac{4x}{a} - \frac{2a}{y}}{1 + \frac{8x}{y}} \right|$$

$$\Rightarrow \tan \theta \Big|_{(a, 2a)} = \left| \frac{\frac{4a}{a} - \frac{2a}{2a}}{1 + \frac{8a}{2a}} \right| = \left| \frac{4-1}{1+4} \right| = \frac{3}{5}$$

28.

Sol. The curve is $3y = 6x - 5x^3$

$$\Rightarrow \frac{dy}{dx} = 2 - 5x^2$$

$$\Rightarrow \frac{dy}{dx} \Big|_{\left(1, \frac{1}{3}\right)} = -3$$

So, the slope of normal will be $\frac{1}{3}$.

Equation of normal at $\left(1, \frac{1}{3}\right)$ is given by

$$y - \frac{1}{3} = \frac{1}{3}(x - 1)$$

$$\Rightarrow x - 3y = 0$$

The point (3, 1) satisfies this equation.

Hence, the normal passes through (3, 1).

29.

Sol. Volume of half cylinder (V) = $\frac{1}{2} \pi r^2 h$

30.

Sol. Total surface area of the half cylinder (S) = $\pi r h + \pi r^2 + 2rh$

31.

Sol. $S = \pi r h + \pi r^2 + 2rh$

$$= rh(\pi + 2) + \pi r^2$$

$$= \pi r^2 + \frac{2V(\pi + 2)}{\pi r}$$

32.

Sol. $S = \pi r^2 + \frac{2V(\pi + 2)}{\pi r}$

$$\Rightarrow \frac{dS}{dr} = 2\pi r - \frac{2V(\pi + 2)}{\pi r^2}$$

Taking $\frac{dS}{dr} = 0$, we get

$$\pi^2 r^3 = V(\pi + 2)$$

33.

The total surface area is minimum when $\pi^2 r^3 = V(\pi + 2)$

Since, $V = \frac{1}{2} \pi r^2 h$

$$\Rightarrow \frac{\pi^2 r^3}{(\pi + 2)} = \frac{1}{2} \pi r^2 h$$

$$\Rightarrow \frac{\pi r}{(\pi + 2)} = \frac{1}{2} h$$

$$\Rightarrow \frac{h}{2r} = \frac{\pi}{\pi + 2}$$

34.

Given, $f(x) = x^2 e^{-x}$

Now, differentiating both sides with respect to x , we get,

$$\Rightarrow f'(x) = 2xe^{-x} - x^2 e^{-x}$$

$$\Rightarrow f'(x) = xe^{-x} (2 - x)$$

For, the function to be increasing,

$$f'(x) > 0$$

$$\Rightarrow xe^{-x} (2 - x) > 0$$

$$\Rightarrow x(2 - x) > 0$$

$$\Rightarrow x(x - 2) < 0$$

[Since, e^{-x} can never be zero]

Using, the method of intervals, we get,



Therefore, $x \in (0, 2)$.

35.

Given: $y^2 (2 - x) = x^3$

$$\Rightarrow (2 - x)2y \frac{dy}{dx} + y^2 (-1) = 3x^2$$

$$\Rightarrow (2 - x)2y \frac{dy}{dx} = 3x^2 + y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2 + y^2}{2y(2 - x)}$$

$$\Rightarrow \left. \frac{dy}{dx} \right|_{(1,1)} = 2$$