



FOR LOGO 0902822746



ΦΗΓΟ ACADEMY

O f f i c i a l T e l e g r a m C h a n n e l

Part II: Write True if the statement is correct and False if it is incorrect on the space provided.

(Each blank space worth 2pts)

False 1) The eigenvalues of a matrix $A = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 3 & 7 \\ 0 & 0 & 8 \end{bmatrix}$ are 4 and 8 only.

True 2) Let π_1 and π_2 be two planes with normal vectors $\vec{n}_1$ and $\vec{n}_2$ respectively. If $\vec{n}_1$ is perpendicular to $\vec{n}_2$, then π_1 is perpendicular to π_2 .

True 3) $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$.

False 4) The vectors $(0, 1, -1)$ and $(2, 6, 3)$ are linearly dependent.

False 5) If f is differentiable function, then $\frac{d}{dx} f(\sqrt{x}) = \frac{f'(x)}{2\sqrt{x}}$ for all $x > 0$.

True 6) Let f be a continuous function such that $f(-2) = 3$ and $f(4) = 1$. The equation $f(x) = 2$ has a solution (or root) in $[-2, 4]$.

Part III: Attempt each of the following questions by showing all necessary steps on the space provided. If necessary, you can use the back sides of the pages.

- 1) (a) Find an equation of the plane that contains the point $(1, -5, 2)$ and parallel to the plane $\pi: 3x - 7y + 4z = 5$.

(3pts)

$$(x_0, y_0, z_0) = (1, -5, 2)$$

$$\vec{n} = (a, b, c) = (3, -7, 4)$$

equation of plane

$$\pi: a(x - x_0) + b(y - y_0) + c(z - z_0)$$

$$\Rightarrow 3(x - 1) + (-7)(y - (-5)) + 4(z - 2)$$

$$\Rightarrow 3x - 3 - 7y - 35 + 4z - 8$$

$$\Rightarrow 3x - 7y + 4z - 46 = 0$$

$$\Rightarrow 3x - 7y + 4z = 46$$

- (b) Find a parametric equation of the line that contains the point $(4, 0, -2)$ and is perpendicular to

the plane $\pi: x - 2y + 4z = 0$.

(4pts)

~~$$p = (4, 0, -2)$$

$$\pi: x - 2y + 4z = 0$$

$$\vec{n} = (a, b, c) = (1, -2, 4)$$

$$\pi = a(x - x_0) + b(y - y_0) + c(z - z_0)$$

$$1(x - 4) + (-2)(y - 0) + 4(z - (-2)) = 0$$

$$x - 4 - 2y + 4z + 8 = 0$$

$$x - 2y + 4z + 4 = 0$$~~

Sorry mister, I wrote the answer at the back of this page.

2) Using the method of Cramer's rule solve the system $\begin{cases} 12x + 3y = 15 \\ 2x - 3y = 13 \end{cases}$ (4pts)

$$D = \begin{vmatrix} 12 & 3 \\ 2 & -3 \end{vmatrix} = (12)(-3) - (3)(2) = -36 - 6 = -42$$

$$D_x = \begin{vmatrix} 15 & 3 \\ 13 & -3 \end{vmatrix} = (15)(-3) - (3)(13) = -45 - 39 = -84$$

$$D_y = \begin{vmatrix} 12 & 15 \\ 2 & 13 \end{vmatrix} = (12)(13) - (15)(2) = 156 - 30 = 126$$

according to Cramer's rule

$$x = \frac{D_x}{D} = \frac{-84}{-42} = 2$$

$$y = \frac{D_y}{D} = \frac{126}{-42} = -3$$

The solution set is $\{(2, -3)\}$.

3) Let $f(x) = 2x^3 - 6x^2 - 18x + 2$.

(a) Find all critical numbers of f if exist.

(2pts)

metric equation of
the point (4,0,2)
perpendicular to the

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to find the critical number
we need to get $f'(x) = 0$
 $f(x) = 2x^3 - 6x^2 - 18x + 2$
 $f'(x) = 2 \cdot 3(x^{3-1}) - 2 \cdot 6(x^{2-1}) - 18x^{1-1} + 0$
 $\Rightarrow 6x^2 - 12x - 18$
 $\Rightarrow 6(x^2 - 2x - 3)$
 $\Rightarrow 6(x+1)(x-3) = 0 \Rightarrow x+1=0, x-3=0$
 $\Rightarrow x = -1 \text{ or } x = 3$ are critical numbers.

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- (b) Find the (i) intervals in which the function f is increasing,
(ii) intervals in which the function f is decreasing, and
(iii) local extreme points of f .

(5pts)

	-1	3
$x+1$	-	+
$x-3$	-	+
$(x+1)(x-3)$	-	+

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- (b) (i) $(-\infty, -1) \cup (3, \infty)$ the function f is increasing
(ii) $(-1, 3)$ the function f is decreasing
iii) at $x = -1$ it is changed from "+" to "-"
So $x = -1 \Rightarrow f(-1) = 2(-1)^3 - 6(-1)^2 - 18(-1) + 2$
 $\Rightarrow -2 - 6 + 18 + 2$
 $\Rightarrow 12$
 $(-1, 12)$ Local maximum
at $x = 3$ it is changed from "-" to "+"
So $x = 3 \Rightarrow f(3) = 2(3)^3 - 6(3)^2 - 18(3) + 2$
 $\Rightarrow 54 - 54 - 54 + 2$
 $\Rightarrow -52$
 $(3, -52)$ Local minimum

- (c) Find the (i) intervals in which the function f is concave upward,
(ii) intervals in which the function f is concave downward, and

(4pts)

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(iii) inflection point of the graph of f .

to find Concave upward, Concave downward and Inflection point, first we have to find $f''(x)$

$$f(x) = 2x^3 - 6x^2 - 18x + 12$$

$$f'(x) = 3 \cdot 2x^{3-1} - 6 \cdot 2x^{2-1} - 18x^{1-1} \\ = 6x^2 - 12x - 18$$

$$f''(x) = 2 \cdot 6x^{2-1} - 12x^{1-1} \\ = 12x - 12$$

$$f''(x) > 0 \quad \text{concave upward}$$

$$f''(x) < 0 \quad \text{concave downward}$$

$$12x - 12 = 0$$

$$12(x-1) = 0 \Rightarrow x=1$$

$$\begin{array}{c} \text{+} \\ \text{---} \\ x-1 \text{ ---} 0 \text{ ---} \text{+++} \\ 12(x-1) \text{ ---} \text{---} \text{+++} \end{array}$$

$$i = (1, \infty) \text{ concave upward}$$

$$ii = (-\infty, 1) \text{ concave downward}$$

iii: $f''(x) = 0$ or $f''(x)$ doesn't exist indicates inflection at $x=1$

$$x=1$$

$$f(x) = -20$$

$(1, -20)$ is inflection point.

b) Find the parametric equation of the line contains the point $(4, 0, -2)$ and is perpendicular to the plane $\pi: x - 2y + 4z = 0$

$Q = (x, y, z) = (4, 0, -2)$

$U = (a, b, c) = (1, -2, 4)$

$P = (x_0, y_0, z_0) = (4, 0, -2)$

$(x, y, z) = (x_0, y_0, z_0) + t(a, b, c)$

$(x, y, z) = (4, 0, -2) + t(1, -2, 4)$

$$\left. \begin{aligned} x &= 4 + t \\ y &= -2t \\ z &= -2 + 4t \end{aligned} \right\} \text{ is the parametric equation of the line}$$