ADDIS ABABA SCIENCE AND TECHNOLOGY UNIVERSITY

MATHEMATICS DEPARTMENT

Mathematics for Natural Science(math1011)(Test I(15%))

Part I(Short answer) Write your simplified answer on the space provided(1pts for each blank space)

1. The negation of $(\forall x)(3x+1<0)$ is _____
2. All real number x and some real number y such that $x^2-y^2>0$. The open statement symbolically is _____ and the truth value is_____.
3. Let $A=\{2x:-3<x\leq 3, x\in Z\}$ and $B=\{x:0<x<6, x\in Z\}$, then
   a) List $p(A\cap B)$=_____
   b) $n(A/B)$=_____
4. Let LCM(a,45) =180 and GCF(a,45) =3, then the value of a = _____.
5. Let $z=\frac{2}{\sqrt{3}}-2i$ and $w=-\frac{2}{\sqrt{3}}+2i$, then
   a) The principal argument of zw is _____
   b) The polar form of z^4 is _____
   c) Solve $w^{1/2}$. Ans:_____

PartII(Workout): Show all the necessary steps clearly and neatly.(2+2+2 pts)

1. Identify the premises, write the argument form and check the validity of the argument: If he studies medicine, he will get a good job. If he gets a good job, he will get a good wage. He did not get a good wage. Therefore, he did not study medicine.

2. Using mathematical induction prove that $\sum_{i=1}^{n} i^3 = \frac{1}{4}n^2(n+1)^2; \ \forall n \in \mathbb{N}$.

3. For all integers a and b, if a is even and b is odd, then 4 not divide $a^2 + b^2$

   i) Write the contra positive of the statement

   ii) Prove the statement by contradiction

13. Find the value of x for which the graphs of $f(x) = 3^x$ and $g(x) = 4^{-x+2}$ intersects. [Hint: Use common logarithm] [05]

Soln $3^x = 4^{-x+2}$ gives us the point of intersection — 1pt

$\Rightarrow \log_{10} 3^x = \log_{10} 4^{-x+2}$ —

$\Rightarrow x \log_{10} 3 = (-x+2) \log_{10} 4$ } 1pt

$\Rightarrow x \log_{10} 3 + x \log_{10} 4 = 2 \log_{10} 4$

$\Rightarrow x \log_{10} 12 = \log_{10} 16$ } 3pts

$\Rightarrow x = \dfrac{\log 16}{\log 12} = \log_{12} 16$

Or

$3^x = 4^{-x+2} \Rightarrow 3^x = \dfrac{16}{4^x}$

$\Rightarrow 12^x = 16.$

$\Rightarrow \log_{12} 16 = x$

14. Suppose the polynomial function $f(x) = 2x^3 + px^2 - 3x + k$. If f is divided by $x+2$ and $x-1$, then the remainders are -20 and -2 respectively.

(i) Find the values of p and k.

(ii) Find the zeros of f.

Soln : By remainder theorem, we have

(i) $f(-2) = 20 \Rightarrow k + 4p = -10 \quad \cdots (1)$ | 1Pt [06]

$f(1) = -2 \Rightarrow k + p = -1 \quad \cdots (2)$

$\Rightarrow k = 2$ and $p = -3$ | 1Pt

Hence, $f(x) = 2x^3 - 3x^2 - 3x + 2$

(ii) By rational root theorem, we have a zero of polynomial may be one of $\pm 1, \frac{\pm 1}{\pm 2}, \pm 2$ } 2Pts

Now, $f(1) = -2 \neq 0$

$f(-1) = 0$

$\therefore x+1$ is factor of $p(x)$

— And using long division, we get that } 1Pt

$f(x) = 2x^3 - 3x^2 - 3x + 2 = (x+1)(2x^2 - 5x + 2)$

other zero of $f(x)$ will be

1Pt

$2x^2 - 5x + 2 = 0$

$x = \frac{+5 \pm \sqrt{25 - 16}}{4} = \frac{+5 \pm \sqrt{9}}{4}$

$x = \frac{+5 \pm 3}{4}$, $x = 2$ and $x = \frac{1}{2}$

1Pt Hence, the zero's of $f(x)$ are $-1, \frac{1}{2}$ and 2.

Part II *Show all the neccessary steps of all questions in this section using the answer space provided.*

11. Show that $\sqrt{2}$ is an irrational number. [03]

proof: suppose that $\sqrt{2}$ is rational number.

$\Rightarrow \sqrt{2} = a/b$ for some $a, b \in \mathbb{Z}$ and G.C.F$(a,b) = 1$

$\Rightarrow 2 = a^2/b^2 \Rightarrow a^2 = 2b^2$

$\Rightarrow a^2$ is even

$\Rightarrow a$ is even

$\Rightarrow a = 2K$, $K \in \mathbb{Z}$

$\Rightarrow 4K^2 = 2b^2$

$\Rightarrow b^2 = 2K^2$

$\Rightarrow b^2$ is even

$\Rightarrow b$ is even

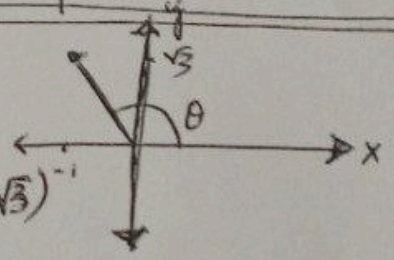
$\Rightarrow$ GCF$(a,b) \geq 2$ i.e a & b are not relatively prime or a/b is not in simplest form.

$\Rightarrow$ Which is a contradict

$\Rightarrow \sqrt{2}$ is not rational no

Hence, $\sqrt{2}$ is an irrational no.

12. Find the fourth root of $-1 + \sqrt{3}i$. [04]



Let $z = -1 + \sqrt{3}i$

$\Rightarrow r = |z| = 2$

$\Rightarrow \theta = \text{Arg}(z) = \tan^{-1}(-\sqrt{3})^{-1}$

$\therefore \theta = \frac{5\pi}{6}$

$\Rightarrow \sqrt[4]{z} = r^{1/4} e^{i\left(\frac{\theta + 2k\pi}{n}\right)}$, $n = 4$, $k = 0, 1, 2, 3$

$= \sqrt[4]{2}\, e^{i\left(\frac{5\pi/6 + 2k\pi}{4}\right)}$, $k = 0, 1, 2, 3$

$= \sqrt[4]{2}\left[\cos\left(\frac{5\pi/6 + 2k\pi}{4}\right) + i \sin\left(\frac{5\pi/6 + 2k\pi}{4}\right)\right]$

$z_0 = \sqrt[4]{2}\left(\cos(5\pi/24) + i \sin(5\pi/24)\right)$ — 0.5pt

$z_1 = \sqrt[4]{2}\left[\cos\left(\frac{17\pi}{24}\right) + i \sin\left(\frac{17\pi}{24}\right)\right]$ — 0.5pt

$z_2 = \sqrt[4]{2}\left[\cos\left(\frac{29\pi}{24}\right) + i \sin\left(\frac{29\pi}{24}\right)\right]$ — 0.5pt

$z_3 = \sqrt[4]{2}\left[\cos\left(\frac{41\pi}{24}\right) + i \sin\left(\frac{41\pi}{24}\right)\right]$ — 0.5pt

Part I Write the most simplified form of your answers only on the space provided for the following items. Each item will be awarded 2 marks for complete answers.

1. Let p and q be propositions which stands for

$$p(x) : |x| = -1 \quad q(x) : e^{2x} - 4 = 0$$

then the truth value of $(\exists x)\,(p(x) \Rightarrow q(x))$ is True (T) [02]

2. Let $A \cup B = \{a, b, d, e, x, y, z\}$ and $A \cap B = \{b, e, y\}$.

(a) If $B - A = \{x, z\}$, then $A =$ {b, e, y, a, d}

(b) If $A - B = \emptyset$, then $B =$ $A \cup B = \{a, b, d, e, x, y, z\}$

[02]

3. For propositions p, q and r, let the truth value of the compound proposition $\neg\,[\neg r \Rightarrow \neg\,(p \wedge q)]$ be True, then the truth value of

(a) $(p \wedge q) \Rightarrow (\neg q \vee p)$ is True (T)

(b) $\neg\,(p \Leftrightarrow (q \vee r))$ is False (F)

4. If $z = \sqrt{2}\,(cos\,(\frac{\pi}{24}) + isin\,(\frac{\pi}{24}))$, then the value of z^4 is $2\sqrt{3} + i$ $4[\cos(\pi/6) + i\sin(\pi/6)] = 4e^{i\pi/4}$ [02]

5. The *lub* and *glb* of the set $S = \{3.01, 3.0101, 3.010101, \cdots\}$ are $3.\overline{01}$ and 3.01 [02]

6. The inverse of the function $f(x) = \frac{x+1}{x-1}$, $x \neq 1$ is $f^{-1}(x) = \frac{x+1}{x-1}$ [02]

7. Let $f(x) = x^2 - 3x$ and $g(x) = 1 + x$, then the solution of $f(g(x)) = 0$ is {-1, 2} [02]

8. For what value of k, $x^2 + y^2 - 6y + k = 0$ represents a circle? $k \leq 9$ [02]

9. The vertex, focus and directrix of the parabola $x^2 - 8y - 6x + 1 = 0$ are (3, -1), (3, 1) and $y = -3$ respectively. [02]

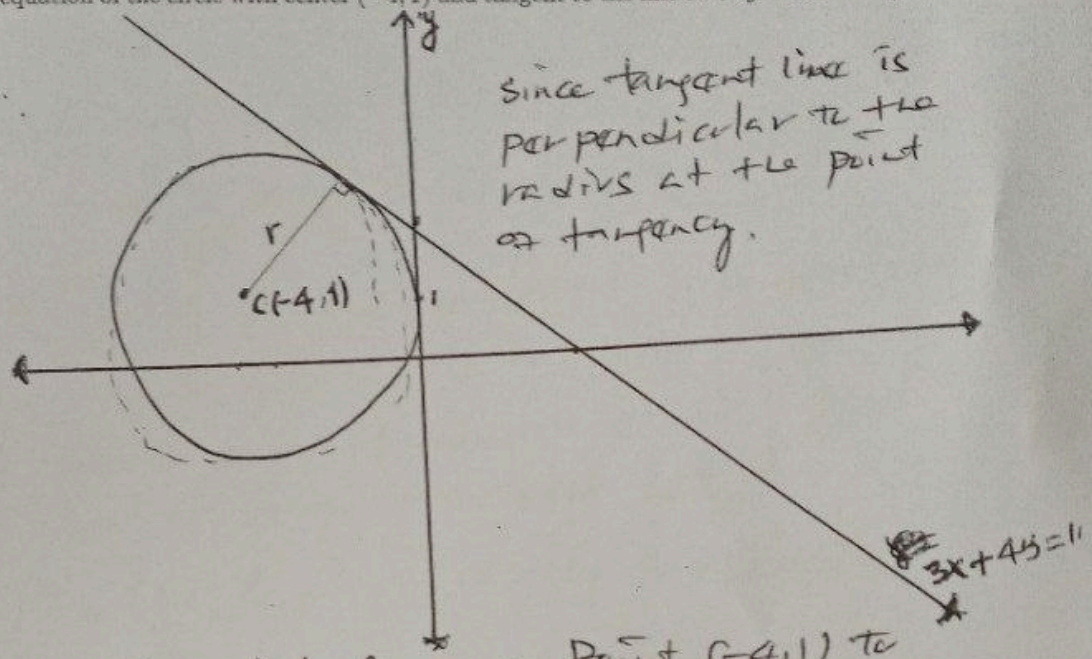
10. The eccentricity of the ellipse $\frac{(x-2)^2}{4} + \frac{(y+1)^2}{9} = 1$ is $\sqrt{5}/3$ [02]

[02]

Best Wishes!!!

15. Find the equation of the circle with center $(-4, 1)$ and tangent to the line $3x+4y = 10$. [05]

Soln :



Since tangent line is perpendicular to the radius at the point of tangency.

By distance fromulal from a Point $(-4,1)$ to the line $3x+4y-10=0$ is the radius of the circle.

3pt $\left\{ r = \dfrac{|3(-4)+4(1)-10|}{\sqrt{9+16}} = \dfrac{|-18|}{5} = 18/5 \right.$

Hence, the efuation of a circle will be

$(x-h)^2 + (y-k)^2 = r^2$ } (1pt)

1P $\left\{ \therefore (x+4)^2 + (y-1)^2 = (18/5)^2 \right.$

16. Let the length of the transverse of a hyperbola is 16 and the end points of the conjugate axis are $(5,-5)$ and $(5,3)$, then find:

(i) The equation of the hyperbola

(ii) The vertex and the foci of the hyperbola.

(iii) Sketch the graph

[07]

Soln: (i) eqn of the hyperbola is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad \text{(1Pt)}$$

where the center $C(h,k)$ = midpoint of conjugate axis

$\Rightarrow C(h,k) = (5,-1)$ — 2+P

$a = 8$ (half of length of transverse axis)

$b = 4$ (half of length of conjugate axis)

$\Rightarrow \frac{(x-5)^2}{64} - \frac{(y+1)^2}{16} = 1$ is eqn of the hyperbola.

ii) the vertices are

$V(h-a,k)$ and $V'(h+a,k)$ (1Pt)

$= V(-3,-1)$ and $V'(13,-1)$

$c = \sqrt{a^2+b^2} = \sqrt{80}$

$\Rightarrow$ the foci are $F(h-c,k) = F(5-\sqrt{80},-1)$ and $F'(h+c,k) = F'(5+\sqrt{80},-1)$ (1Pt)

