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A Practical Equation for Elastic Modulus of Concrete

by Takafumi Noguchi, Fuminori Tomosawa, Kamran M. Nemati, Bernardino M. Chiaia, and Alessandro P. Fantilli

Many empirical equations for predicting the modulus of elasticity as a function of compressive strength can be found in the current literature. They are obtained from experiments performed on a restricted number of concrete specimens subjected to uniaxial compression. Thus, the existing equations cannot cover the entire experimental data. This is due to the fact that mechanical properties of concrete are highly dependent on the types and proportions of binders and aggregates. To introduce a new reliable formula, more than 3000 data sets, obtained by many investigators using various materials, have been collected and analyzed statistically. The compressive strengths of the considered concretes range from 40 to 160 MPa (5.8 to 23.2 ksi). As a result, a practical and universal equation, which also takes into consideration the types of coarse aggregates and mineral admixtures, is proposed.

Keywords: analysis; coarse aggregates; compressive strength; high-strength concrete; modulus of elasticity; normal-strength concrete; water-cement ratio.

INTRODUCTION

To design plain, reinforced, and prestressed concrete structures, the elastic modulus E is a fundamental parameter that needs to be defined. In fact, linear analysis of elements based on the theory of elasticity may be used to satisfy both the requirements of ultimate and serviceability limit states (ULS and SLS, respectively). This is true, for instance, in the case of prestressed concrete structures, which show uncracked cross sections up to the failure.[1] Similarly, linear elastic analysis, carried out through a suitable value of E, also permits the estimation of stresses and deflections, which need to be limited under the serviceability actions in all concrete structures.

Theoretical and experimental approaches can be applied to evaluate the elastic modulus of concretes. In the theoretical model, concretes are assumed to be a multi-phase system; thus, the modulus of elasticity is obtained as a function of the elastic behavior of its components. This is possible by modeling the concrete as a two-phase material, involving the aggregates and the hydrated cement paste (refer to Mehta and Monteiro[2] for a review), or three-phase material, if the so-called interface transition zone (ITZ) between the two phases is introduced.[3-5] Nevertheless, according to Aïtcin,[6] theoretical models can appear too complicated for a practical purpose, because the elastic modulus of concrete is a function of several parameters (that is, the elastic moduli of all the phases, the maximum aggregate diameter, and the volume of aggregate). As a consequence, such models can only be used to evaluate the effects produced by the concrete components on the modulus of elasticity.[7]

Empirical approaches, based on dynamic or static measurements,[8] are the most widely used by designers. Dynamic tests, which measure the initial tangent modulus, can be adopted when nondestructive diagnostic tests are required. On the contrary, static tests on cylindrical specimens subjected to uniaxial compression are currently used for evaluating E. From these tests, the current building codes propose more or less similar empirical formulas for the estimation of elastic modulus. Because they are directed to designers, the possible equations need to be formulated as functions of the parameters known at the design stage.[9] Thus, for both normal-strength (NSC) and high-strength (HSC) concrete, the Comité Euro-International du Béton and the Fédération Internationale de la Précontrainte (CEB-FIP) Model Code[10] and Eurocode 2[11] link the elastic modulus E to the compressive strength σ_B according to

$$E = 22{,}000\left(\frac{\sigma_B}{10}\right)^{\frac{1}{3}} \tag{1a}$$

$$E = 3191\left(\frac{\sigma_B}{1.45}\right)^{\frac{1}{3}} \tag{1b}$$

In Eq. (1a), E and σ_B are measured in MPa, whereas in Eq. (1b), E and σ_B are measured in ksi.

In the case of HSC, in the formula proposed by ACI Committee 363,[12] the elastic modulus of concrete is also function of its unit weight γ

$$E = (3321\sigma_B{}^{0.5} + 6895) \cdot (\gamma/2300)^{1.5} \tag{2a}$$

$$E = (1265\sigma_B{}^{0.5} + 1000) \cdot (\gamma/145)^{1.5} \tag{2b}$$

In Eq. (2a), E and σ_B are measured in MPa, and γ in kg/m^3, whereas in Eq. (2b), E and σ_B are measured in ksi and γ in lb/ft^3. Similarly, the Architectural Institute of Japan[13] specifies the following equation to estimate the modulus of elasticity of concrete

$$E = 21{,}000(\gamma/2300)^{1.5}(\sigma_B/20)^{1/2} \tag{3a}$$

$$E = 3046(\gamma/145)^{1.5}(\sigma_B/2.9)^{1/2} \tag{3b}$$

In Eq. (3a), E and σ_B are measured in MPa and γ in kg/m^3, whereas in Eq. (3b), E and σ_B are measured in ksi and γ in lb/ft^3.

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The effectiveness of such formulas is questionable. In fact, a simple relationship between E and σ_B can be established for normal concrete, because only a little stress is transferred at cement paste-aggregates' interface due to the high porosity of the ITZ. It cannot work in the case of HSC, for which, according to several experimental results, the modulus of elasticity is strongly dependent on the nature of coarse aggregate.[14-16] Sometimes, even different values of elastic modulus can be found in concrete having the same compressive strength, but made with different types of aggregates. Therefore, it is frequently suggested[6] to directly measure the elastic modulus of HSC rather than adopt theoretical or empirical approaches.

RESEARCH SIGNIFICANCE

Different formulas are proposed by building codes to compute the modulus of elasticity of concrete structures. Most of them based on the compressive strength are suitable for NSC. In the technical literature, similar formulas can be also found for HSC. None of them, however, are able to correctly predict the modulus of elasticity of HSC specimens made with different types of aggregates and mineral additives. Thus, by means of a statistical analysis performed on more than 3000 tests, a practical and universal equation for the evaluation of the elastic modulus E is proposed in this paper. The authors believe that such a formula can be effectively used in designing both NSC and HSC structures, because the direct measure of E through cumbersome test campaigns can be avoided.

STATISTICAL ANALYSIS OF EXPERIMENTAL DATA

Before performing any analysis, it is necessary to create a basic form for the equation of modulus of elasticity. In this study, a conventional equation is adopted in which modulus of elasticity is expressed as a function of compressive strength and unit weight. Because it is self-evident that the elastic modulus of concrete vanishes when $\sigma \to 0$ or $\gamma \to 0$, the basic formula can be expressed as a product of these two variables

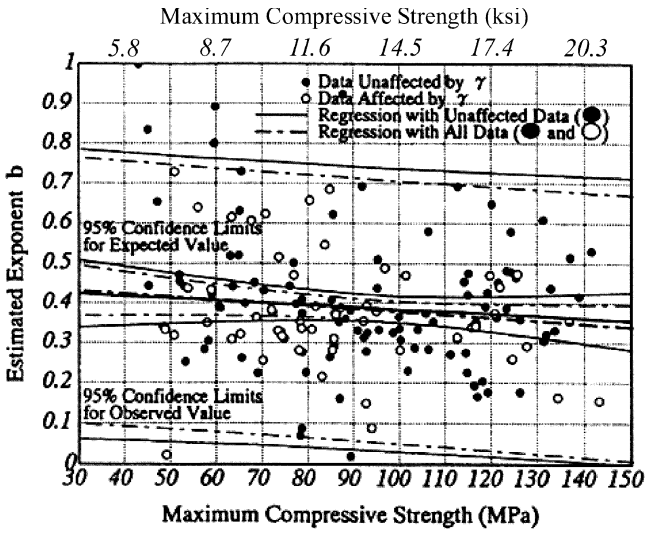

Fig. 1—Relationship between maximum compressive strength and estimated values of exponent b.

$$E = \alpha \sigma_B{}^b \gamma^c \tag{4}$$

To evaluate the values of α, b, and c, more than 3000 uniaxial compression tests on HSC of different strengths were taken into account and the results were published.[17,18] The considered parameters (compressive strength, modulus of elasticity, unit weight of concrete at the time of compression test, mechanical properties of materials for producing concrete, mixture proportioning, unit weight and air content of fresh concrete, method and temperature of curing, and age) are accurately described in a previously published report.[17]

Evaluation of exponent *b* of compressive strength

As the compressive strength increases, Eq. (2) and (3) overestimate the modulus of elasticity. Thus, it seems appropriate to reduce the value of exponent b of the compressive strength σ_B to less than 0.5 to make the estimated values more compatible with the experimental results. Possible values of exponent b have been obtained from the considered experimental data. Figure 1 shows the relationship between the maximum compressive strengths and the estimated exponent b. Similarly, Fig. 2 shows the relationship between exponent b and the ranges of compressive strengths in the available data. In both figures, exponent b tends to decrease from approximately 0.5 to approximately 0.3, as the maximum compressive strengths increase and the ranges of compressive strength widen. In other words, whereas modulus of elasticity of NSC can be predictable from the compressive strength with exponent $b \cong 0.5$, the values of $b = 0.3 \sim 0.4$ appear more appropriate in a general equation capable of estimating elastic modulus of a wide range of concretes, from normal to high strength. Consequently, $b = 1/3$ is proposed in this paper in consideration of the practical application of Eq. (4). This is in accordance with the value of b suggested by CEB-FIP Model Code[10] and Eurocode 2[11] (Eq. (1)).

Evaluation of exponent *c* of unit weight

After fixing exponent $b = 1/3$, as mentioned previously, the exponent c of the unit weight γ can be investigated. The relationship between γ and the values of elastic modulus divided by compressive strength to power of 1/3 (that is, $E/\sigma_B{}^{1/3}$) is shown in Fig. 3. From the data reported in this figure,

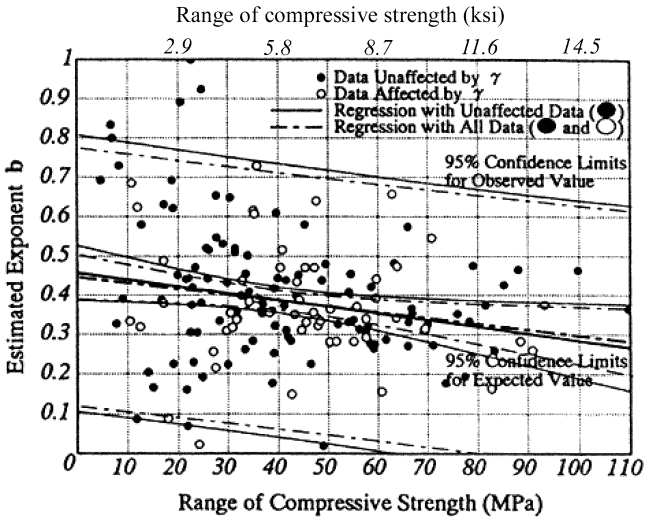

Fig. 2—Relationship between range of compressive strength and estimated values of exponent b.

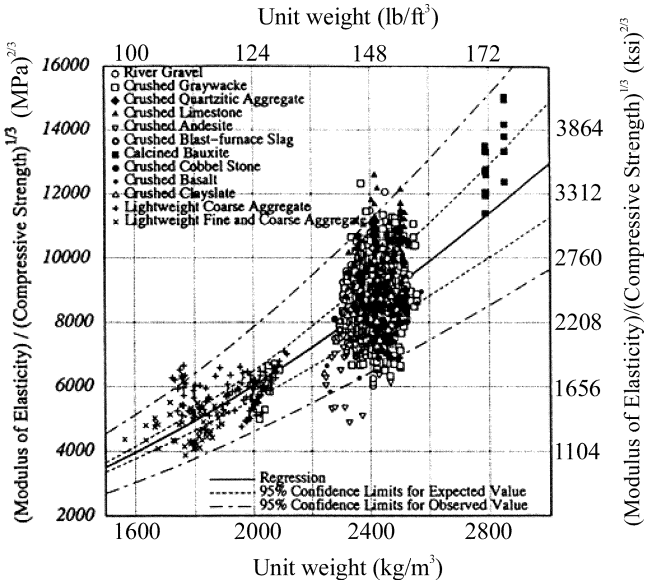

Fig. 3—Relationship between unit weight and ratio $E/\sigma_B^{1/3}$.

obtained from tests on concretes made of different type of aggregates, the following regression equation can be obtained

$$E = 3.48 \times 10^{-3}\ \sigma_B^{1/3}\gamma^{1.89} \quad (5a)$$

$$E = 0.185\sigma_B^{1/3}\gamma^{1.89} \quad (5b)$$

In Eq. (5a), E and σ_B are measured in MPa and γ in kg/m^3, whereas in Eq. (5b), E and σ_B are measured in ksi and γ in lb/ft^3.

As Fig. 3 shows by means of Eq. (5), it is possible to take into account the effect produced by the unit weight on the modulus of elasticity of concretes made with lightweight, normalweight, and heavyweight aggregates (bauxite, for example). In particular, concretes having normalweight aggregate show a scatter of $E/\sigma_B^{1/3}$ over a wide range, comprised by 6000 and 12,000 MPa$^{2/3}$ (1656 and 3312 ksi$^{2/3}$), although they gather in a relatively small unit weight range, varying from 2300 to 2500 kg/m^3 (142 to 155 lb/ft^3).

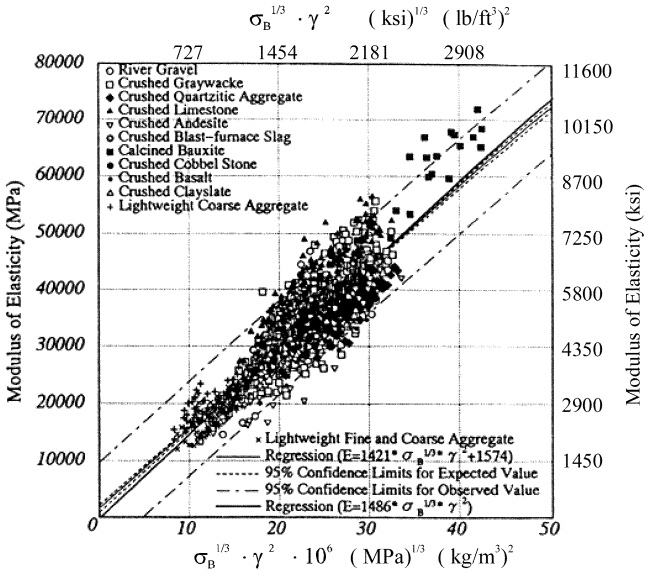

Fig. 4—Modulus of elasticity as function of $\sigma_B^{1/3}\gamma^2$.

This confirms the different effects produced by the lithological types of aggregates on modulus of elasticity,[14-16] which will be discussed in one of the following sections. Whereas c = 1.5 has been conventionally used as the exponent of unit weight (refer to Eq. (2) and (3)), c = 1.89 was obtained from the regression analysis performed on a wide range of concretes, from normal to high strength. In consideration of the utility of Eq. (4), however, c = 2 is herein proposed for the exponent of unit weight.

Evaluation of coefficient α

Because exponents b and c of Eq. (4) have been fixed at 1/3 and 2, respectively, coefficient α needs to be defined. The relationship between the modulus of elasticity E and the product of compressive strength power to 1/3 and unit weight power to 2 (that is, $\sigma_B^{1/3}\gamma^2$) is shown in Fig. 4. In the same figure, the following relationship, obtained from a regression analysis on the entire experimental data, is also reported

$$E = 1.486 \times 10^{-3}\ \sigma_B^{1/3}\gamma^2 \quad (6a)$$

$$E = 0.107\sigma_B^{1/3}\gamma^2 \quad (6b)$$

In Eq. (6a), E and σ_B are measured in MPa and γ in kg/m^3, whereas in Eq. (6b), E and σ_B are measured in ksi and γ in lb/ft^3. As shown in Fig. 4, the coefficient of determination r^2, which gives the proportion of the variance (fluctuation) of one variable that is predictable from the other variable, is approximately 0.77, and the 95% confidence interval of modulus of elasticity is within the range of ±8000 MPa (±1160 ksi). Therefore, modulus of elasticity can be effectively evaluated by Eq. (6).

EVALUATION OF CORRECTION FACTORS

Both in conventional equations (Eq. (2) and (3)) and in Eq. (4), coarse aggregates affect the values of elastic modulus through the value of its unit weight γ. Specimens made of different crushed stone, however, have revealed that unit weight is not the only factor that produces different elastic

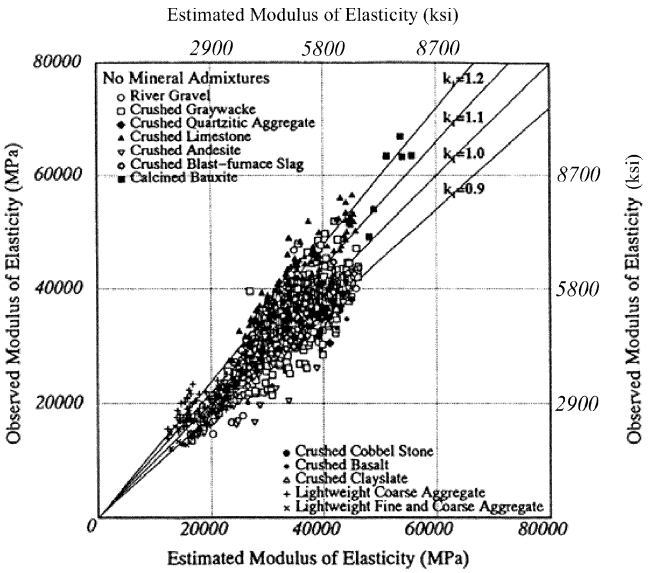

Fig. 5—Estimated modulus of elasticity versus observed modulus of elasticity.

moduli in concretes having the same compressive strength. Lithological type should also be considered as a parameter of coarse aggregate.[6] In addition, it has also been pointed out by many researchers that modulus of elasticity cannot be expected to increase with an increase in compressive strength when the concrete contains a mineral admixture, such as silica fume,[14-16] for high strength. This suggests the necessity to introduce two other corrective factors in Eq. (4) to consider the type of coarse aggregate, as well as the type and amount of mineral admixtures. In other words, Eq. (6) becomes

$$E = k_1 k_2 \cdot 1.486 \times 10^{-3}\ \sigma_B{}^{1/3}\gamma^2 \tag{7a}$$

$$E = k_1 k_2 \cdot 0.107 \sigma_B{}^{1/3}\gamma^2 \tag{7b}$$

where k_1 is the correction factor corresponding to coarse aggregates, and k_2 is the correction factor corresponding to mineral admixtures.

Evaluation of correction factor k_1 for coarse aggregate

Figure 5 shows the relationship between the values estimated by Eq. (6) and the measured values of modulus of elasticity of concretes without admixtures. According to Fig. 5, all the measured values fall in a well-defined range, whose upper and lower limits can be obtained with Eq. (7) when $k_1 = 0.9$ and $k_1 = 1.2$, respectively. In other words, for each lithological type of coarse aggregate, a suitable value of k_1 has to be introduced. The possible correction factors k_1 for each coarse aggregate is reported in Table 1. According to Table 1, the effects of coarse aggregate on modulus of elasticity can be classified into three groups. The first group, which requires no correction factor, includes river gravel and crushed graywacke. The second group, which requires correction factors greater than 1, includes crushed limestone and calcined bauxite. Finally, the third group, which requires correction factors smaller than 1, includes crushed quartzitic aggregate, crushed andesite, crushed cobble stone, crushed basalt, and crushed clayslate. In consideration of the practical use of Eq. (7), the possible values of k_1 are rearranged in Table 2.

Table 1—Correction factors for coarse aggregate

Aggregate type	k_1
River gravel	1.005
Crushed graywacke	1.002
Crushed quartzitic aggregate	0.931
Crushed limestone	1.207
Crushed andesite	0.902
Crushed basalt	0.922
Crushed clayslate	0.928
Crushed cobblestone	0.955
Blast-furnace slag	0.987
Calcined bauxite	1.163
Lightweight coarse aggregate	1.035
Lightweight fine and coarse aggregate	0.989

Table 2—Practical values of correction factor k_1

Lithological type of coarse aggregate	k_1
Crushed limestone, calcined bauxite	1.20
Crushed quartzitic aggregate, crushed andesite, crushed basalt, crushed clayslate, crushed cobblestone	0.95
Coarse aggregate, other than above	1.00

Table 3—Correction factors for concrete admixtures

	Silica fume			Granulated blast-furnace slag		Fly ash fume	Fly ash
Aggregate type	<10%	10 to 20%	20 to 30%	<30%	>30%		
River gravel	1.045	0.995	0.818	1.047	1.118	—	1.110
Crushed graywacke	0.961	0.949	0.923	0.949	0.942	0.927	—
Crushed quartzitic aggregate	0.957	0.956	—	0.942	0.961	—	—
Crushed limestone	0.968	0.913	—	—	—	—	—
Crushed andesite	—	1.072	0.959	—	—	—	—
Crushed basalt	—	—	—	—	—	—	1.087
Calcined bauxite	—	0.942	—	—	—	—	—
Lightweight coarse aggregate	1.026	—	—	—	—	—	—
Lightweight fine and coarse aggregate	1.143	—	—	—	—	—	—

Evaluation of correction factor k_2 for admixtures

Table 3 presents the average values of correction factor k_2 obtained for each lithological type of coarse aggregates as well as for each type and amount of admixtures. When fly ash is used as an admixture, the value of k_2 is generally greater than 1. Conversely, when strength-enhancing admixtures, such as silica fume, ground-granulated blast furnace slag, or fly ash fume (ultra-fine powder produced by condensation of fly ash) are added to concrete, the correction factor k_2 is usually smaller than 1. Similar to k_1, the proposed correction factors k_2 are summarized by the three groups reported in Table 4.

Practical equation for elastic modulus of concrete

Equation (7), introduced as general equations for the elastic modulus of concrete, can now be rearranged and proposed in a conventional way such as Eq. (1) through (3). In these equations, the standard moduli of elasticity can be simply obtained by substituting standard values of compressive strength and unit weight. Thus, considering 60 MPa (8.7 ksi)

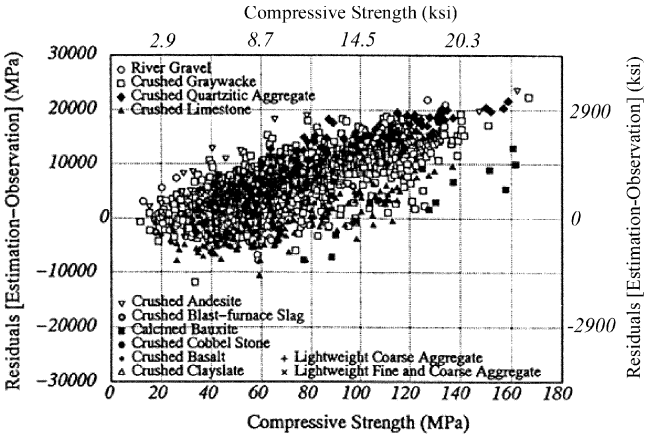

Fig. 6—Relationship between compressive strength and residuals in the case of Eq. (3).[13]

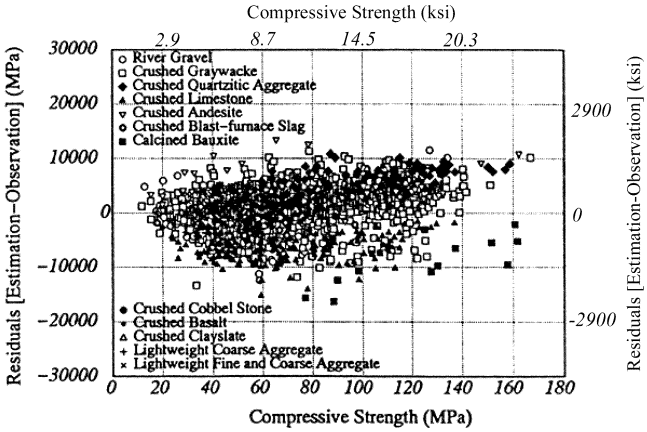

Fig. 7—Relationship between compressive strength and residuals in the case of Eq. (2).[12]

Table 4—Practical values of correction factor k_2

Type of addition	k_2
Silica fume, ground-granulated blast-furnace slag, fly ash fume	0.95
Fly ash	1.10
Addition other than above	1.00

the average compressive strength of the analyzed concretes, and using the standard unit weight of 2400 kg/m^3 (150 lb/ft^3), the following formulas are finally proposed

$$E = k_1 k_2 \cdot 3.35 \times 10^4 (\gamma/2400)^2 (\sigma_B/60)^{1/3} \quad (8a)$$

$$E = k_1 k_2 \cdot 4860 (\gamma/150)^2 (\sigma_B/8.7)^{1/3} \quad (8b)$$

In Eq. (8a), E and σ_B are measured in MPa and γ in kg/m^3, whereas in Eq. (8b), E and σ_B are measured in ksi and γ in lb/ft^3.

EXPERIMENTAL RESULTS AND PRACTICAL FORMULAS

Figures 6 to 9 show the capability of the proposed formula (Eq. (8)), as well as those adopted by code rules (Eq. (1) to (3)), to predict experimental data. Eq. (3), proposed by the Architectural Institute of Japan,[13] tends to overestimate the modulus of elasticity when compressive strengths are higher than 40 MPa (5.8 ksi), except in the cases where crushed limestone or calcined bauxite are used as coarse aggregate (Fig. 6). The residuals (that is, the difference between the estimated values and those measured experimentally) also tend to increase as the compressive strength of concrete increases.

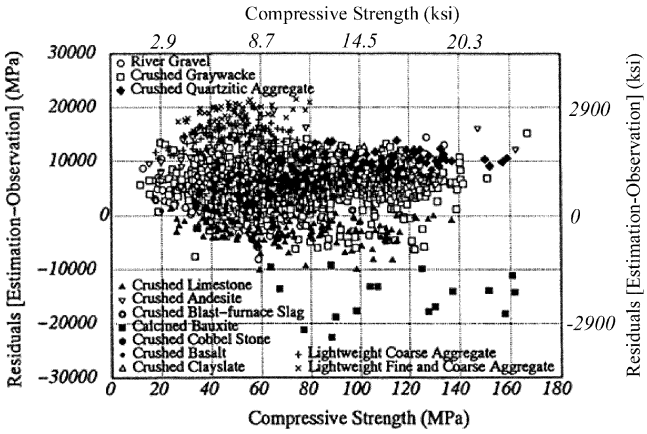

Fig. 8—Relationship between compressive strength and residuals in the case of Eq. (1).[10-11]

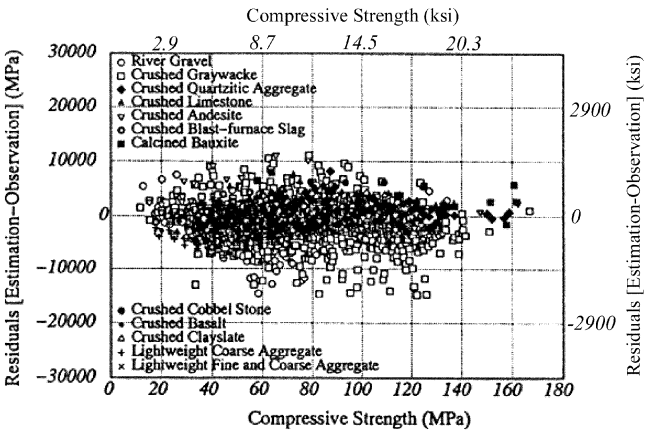

Fig. 9—Relationship between compressive strength and residuals obtained with proposed formula (Eq. (8)).

Equation (2), proposed by ACI Committee 363,[12] slightly underestimates the modulus of elasticity when crushed limestone or calcined bauxite is used as coarse aggregate, regardless of the compressive strength (Fig. 7). In the case of other aggregates, Eq. (2) tends to overestimate the moduli, though marginally, as compressive strength increases.

Equation (1), proposed by CEB-FIP Model Code[10] and Eurocode 2,[11] leads to clear differences in residuals depending on the lithological type of coarse aggregate (Fig. 8). When lightweight aggregate is used, the equation overestimates the moduli, and the value of the residuals tends to decrease as the specific gravity of coarse aggregate increases from crushed quartzitic aggregate to crashed graywacke, crushed limestone, and calcined bauxite.

The residuals obtained with Eq. (8) are shown in Fig. 9. They fall in the range of ±5000 MPa (±725 ksi) independently of σ_B, although a portion of data display residuals of approximately ±10,000 MPa (±1450 ksi). Therefore, the proposed formula (Eq. (8)) seems to be capable of estimating the modulus of elasticity of a wide range of concretes, from normal to high strength.

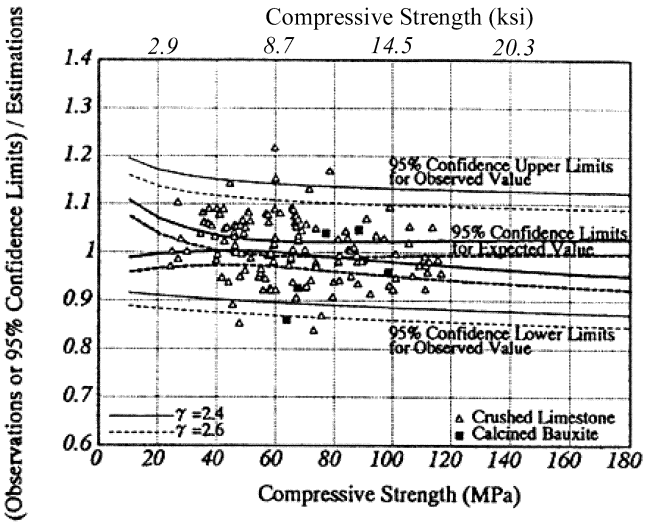

Fig. 10—Compressive strength versus confidence interval (k_1 = 1.2; k_2 = 1.0).

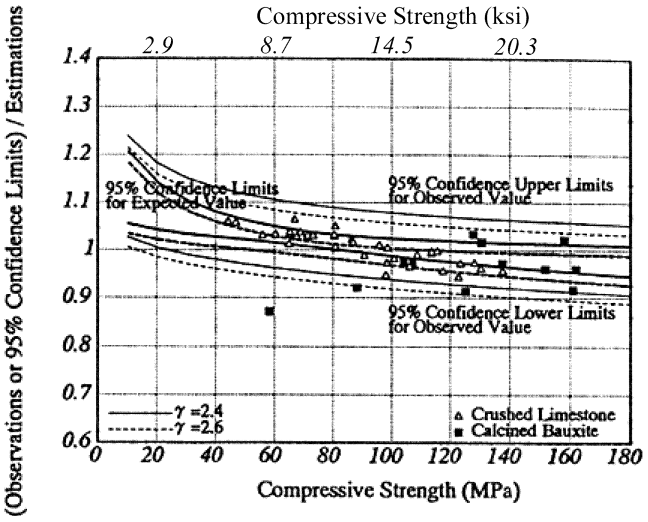

Fig. 11—Compressive strength versus confidence interval (k_1 = 1.2; k_2 = 0.95).

EVALUATION OF CONFIDENCE INTERVALS

To show the accuracy of the proposed Eq. (8), whose efficiency is enhanced by means of the correction factors k_1 and k_2, its 95% confidence intervals should be indicated. In fact, the reliability of the estimated values of E is always necessary in structural design, because it is used to determine materials and mixture proportioning for a required level of safety.

Excluding the case of using fly ash as an admixture, only five values of the product $k_1 \cdot k_2$ are possible (that is, 1.2, 1.14, 1.0, 0.95, and 0.9025). Thus, other regression analyses of Eq. (8), conducted for all the possible combinations of coarse aggregate and admixture (corresponding to the five values of $k_1 \cdot k_2$), are herein conducted to obtain 95% confidence intervals of both estimated and measured modulus of elasticity. The results are shown in Fig. 10 to 14. The curves, indicating the upper and lower limits of 95% confidence of the expected values, are within a range of approximately ±5% of the estimated values, regardless of compressive strength and unit weight. Similarly, the upper and lower limits of the measured values are included in a range of approximately ±20% of the estimated values. Consequently, the 95% confidence

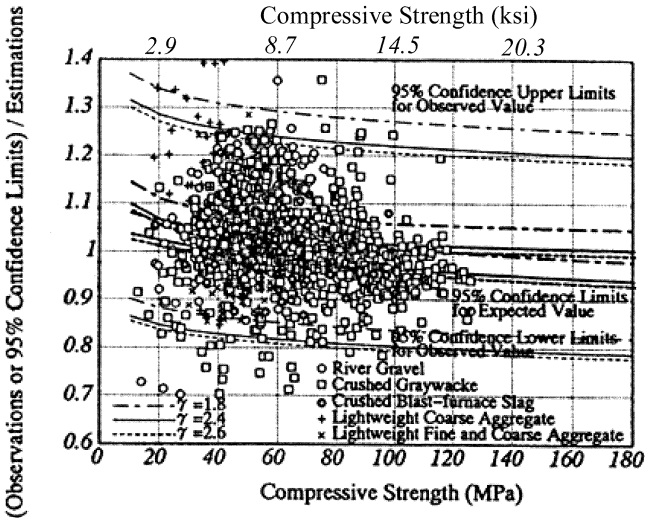

Fig. 12—Compressive strength versus confidence interval (k_1 = 1.0; k_2 = 1.0).

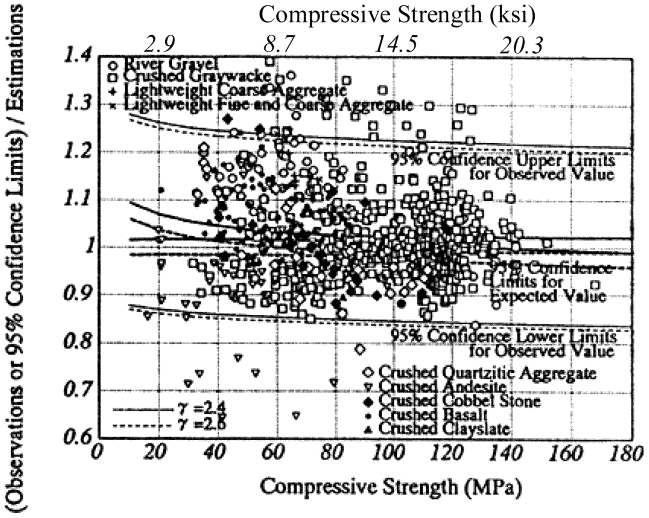

Fig. 13—Compressive strength versus confidence interval (k_1 = 0.95; k_2 = 0.95).

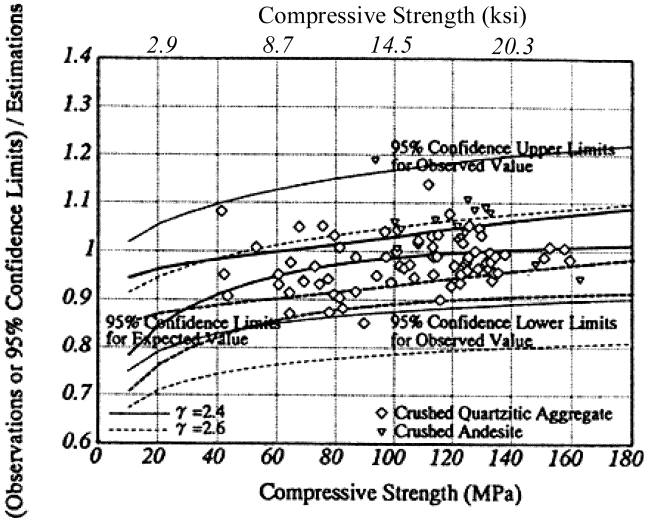

Fig. 14—Compressive strength versus confidence interval (k_1 = 0.95; k_2 = 0.95).

limits of the proposed formula (Eq. (8)), and the 95% confidence limits of measured modulus of elasticity can be respectively expressed as follows

$$E_{e95} = (1 \pm 0.05)E \quad (9)$$

$$E_{o95} = (1 \pm 0.2)E \quad (10)$$

where E_{e95} = 95% confidence limits of expected modulus of elasticity, and E_{o95} = 95% confidence limits of observed modulus of elasticity.

CONCLUSIONS

To obtain a practical and universal equation for the modulus of elasticity, multiple regression analyses have been conducted by using a large amount of data. As a result, an equation applicable to a wide range of aggregates and admixtures was introduced for different concretes, from normal to high strength. Based on the results of this investigation, the main aspects of a general formula for the elastic modulus of concrete can be summarized by the following points:

1. The modulus of elasticity of both normal-strength and high-strength concretes seems to be in direct proportion to the cube root of compressive strength, according to the European Code[10-11] rules.

2. Similarly, there is a direct proportionality between elastic modulus of concrete and its unit weight power to 2. Conversely, in the formulas proposed by Japanese[13] and American[12] Code rules, unit weight appears with an exponent $c = 1.5$.

3. In addition to compressive strength and unit weight of concrete, the modulus of elasticity needs to be expressed as a function of the lithological type of coarse aggregate and the type and amount of admixtures. For the sake of simplicity, these effects can be considered by means of two correction factors, k_1 and k_2, which are equal to 1 in the case of ordinary mixtures (refer to Tables 2 and 4).

The 95% confidence limits of the proposed equation have also been examined, and Eq. (9) and Eq. (10) are herein proposed to indicate these limits for the expected and observed values, respectively.

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