Jimma University

College of Natural Sciences

Department of Mathematics

Math-1011 (Mathematics for Social sciences) Worksheet I

Chapter I: Propositional Logic and Set Theory

1. Let p: 387 is a composite number.

   a. q: 123 is a prime number.

   Write each of the following in words, and determine the truth value of each.

   $a. q \vee \neg p \qquad b. \neg q \wedge p \qquad c. \neg p \Rightarrow \neg q \qquad d. \neg q \Leftrightarrow \neg p$

2. Suppose $p, q, and\ r$ are propositions. Use truth table to show that each of the following pairs of propositions is logically equivalent.

   a. $a.\ (p \vee q) \Rightarrow r\ and\ (p \Rightarrow q)\ \wedge (q \Rightarrow r)$

   $b.\ p \Rightarrow (q \vee r)\ and\ (\neg r)\ \Rightarrow (p \Rightarrow q)$

   $c.\ p \Rightarrow (q \vee r)\ and\ ((\neg r) \wedge p)\ \Rightarrow q$

3. Let $P(x)$: x is an integer, $Q(x)$: x is a rational number,

   $R(x)$: x is an irrational number and $S(x)$: x is a real number.

   Express each of the following in symbolic form.

   (a) x is an irrational number if x is a real number

   (b) x is not an integer or x is not a rational number.

   (c) x is a real sufficient condition for x is an irrational number.

   (d) x is an irrational number only if x is a real number.

4. Let $\cup$ = the set of real numbers. Determine the truth value of the following.

   (a) $(\forall x)(x^2 + x + 1 > 0)$

   (b) $(\exists x)(x^3 + x = 0)$

   (c) $(\forall x)(\forall y)\ [x^2 - y^2 = (x + y)(x - y)]$

   (d) $(\forall x)(\exists y)\ (xy = 1)$

   (e) $(\exists x)(\forall y)(y < x)$

   (f) $(\exists y)(\exists x)\ (y < x)$

(g) $(\exists y)\ (\forall x)\ (xy = x)$

(h) $(\forall x)\ (\forall y)\ (x < y \vee x > y)$

(i) $(\forall x)\ (\exists y)\ (x + y = 0)$

(j) $(\forall x)\ (\forall y)\ (\exists z)\ (x + y = z)$

5. **With and without using truth table** show that the following pairs of sentences are equivalent:

(a) $p \Rightarrow (q \vee r);\ \neg r \Rightarrow (p \Rightarrow q)$

(b) $\neg[p \vee (\neg p \wedge q)];\ \neg p \wedge \neg q$

(c) $(p \Rightarrow q) \wedge (p \Rightarrow r);\ p \Rightarrow (q \wedge r)$

6. **Use both methods (truth table & formal proof)** to show that the following argument forms are valid.

a) $\neg p \wedge q, r \Rightarrow p,\ \neg r \Rightarrow s,\ s \Rightarrow t \mid -t$

b) $\neg p \Rightarrow r,\ p \Rightarrow q,\ \neg q,\ r \Rightarrow s \mid -s \vee t$

c) $(u \vee \neg q) \Rightarrow (\neg s \vee t),\ u,\ \neg s \Rightarrow l,\ (t \vee \neg r) \Rightarrow p \mid -p$

d) $q \Rightarrow r,\ \neg s \Rightarrow (t \Rightarrow p),\ s \vee (q \vee t),\ \neg s \mid -r \vee p$

7. Let A and B be two finite sets such that $n(A) = 20,\ n(B) = 28$ and $n(A \cup B) = 36$, find $n(A \cap B)$.

8. If $n(A - B) = 18,\ n(A \cup B) = 70$ and $n(A \cap B) = 25$, , then find $n(B)$.

9. In a group of 60 people, 27 like coffee and 42 like tea and each person likes at least one of the two drinks. How many like both coffee and tea?

10. In a competition, a school awarded medals in different categories. 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 persons and only 4 persons got medals in all the three categories, how many received medals in exactly two of these categories?

11. If $A, B, and\ C$ are any three sets, then show that,

(a) $(B - A) \cup (C - A) = (B \cup C) - A,$

(b) $(A - B) - (B - C) = A - B$

(c) $P(A) \subseteq P(B) \Leftrightarrow A \subseteq B$

(d) $A\Delta(B\Delta C) = (A\Delta B)\Delta C$

12. Determine whether each of the following arguments is valid or not:

(a) If the moon is made of bananas, then there are monkeys on the moon. The moon is not made of bananas. Therefore, there are no monkeys on the moon.

(b) A sufficient condition for this year to be 2020 is that last year was 2019. This year is 2020. Therefore last year was 2019.

Chapter 2: FUNCTIONS

1. Find x and y if: (i) (4x + 3, y) = (3x + 5, – 2) (ii) (x – y, x + y) = (6, 10)

2. If $x \neq 0$ and $3+10x-5=(a+1)x-2$, find the value of a.

3. Solve each of the following problems.

a) $4x-7(2-x)=3x+2$ b) $8(2+3z)+1=z-10(z+1)$

c) $2x(6x-1)+21=8x-x(3-12x)$ d) $\frac{3w-1}{5}+1=\frac{7w+2}{15}$

e) $2(3-\frac{x}{4})=\frac{2x+5}{3}-\frac{1}{3}$ f) $\frac{8-4z}{3z-2}=2-\frac{10z}{3z-2}$

g) $\frac{4w-1}{w-2}+\frac{8w}{w^2-6w+8}=\frac{4w+3}{w-4}$ h) $\frac{3x-1}{5}-\frac{5x+1}{6}=\frac{x+1}{8}-3$

i) $\frac{4}{x-3}-\frac{3}{x-2}=\frac{1}{x-4}$ j) $(6x-5)(x-2)-(3x-1)(2x-3)=4$

k) $\frac{6}{x^2-3x}=\frac{12}{x}+\frac{1}{x-3}$

4. Solve each of the following inequalities and give the solution in both inequality and interval notations.

a) $4(z+2)-1>5-7(4-z)$ b) $\frac{1}{2}(3+4t)\le 6(\frac{1}{3}-\frac{1}{2}t)-\frac{1}{4}(2+10t)$

c) $\frac{x-4}{2}<\frac{7x}{2}-(3x+2)$ d) $(x-1)(x-2)\ge(x-3)(x+1)$

e) $4(x^2+1)+8(3x-4)>4x^2$ f) $\frac{4x+3}{2x-5}>0$

h) $\frac{5x-2}{3}\ge\frac{x+3}{4}$ I) $-\frac{x+5}{2}\le\frac{12+3x}{4}$

5. Solve the quadratic equations :

a) $x^2+2x-63=0$ b) $2x^2+7x-9=0$ c) $7-5x^2+2x=0$

d) $(6-x)(2x-5)+30=0$ e) $(3x-2)-(x-3)^2+11=0$

f) $\frac{x+3}{x-3}+\frac{x-6}{x+6}=\frac{11}{5}$ g) $\frac{5}{x-1}+\frac{5}{x+1}=\frac{6}{x-2}$

6. Solve the quadratic inequalities :

$a)\ 4x - x^2 > 12$ $b)\ x^2 + 6 > -2x$ $c)\ -x^2 - 4x - 3 \geq 0$

$d)\ x(x-2) \leq 2x - 4$ $e)\ x^2 + 2x - 3 \geq 0$ $f)\ -x^2 - 2x + 8 > 0$

$g)\ 2x^2 + 3x + 4 > 0$ $h)\ x^2 - 8x + 12 \geq 0$ $i)\ -2(x^2 - 1) + 2(1 - x) - 5 \leq 0$

$j)\ \dfrac{x^2 + 4x + 4}{2x^2 - x - 1} > 0$ $k)\ \dfrac{4}{1+x} + \dfrac{2}{1-x} < 1$ $l)\ \dfrac{1 - 2x - 3x^2}{3x - x^2 - 5} > 0$

7. If α *and* β are roots of $x^2 - x + 3 = 0$, what is the value of $\alpha^4 + \beta^4$?

8. If the roots of a quadratic equation $x^2 - bx + c = 0$ are two consecutive integers, find the value of the discriminant.

9. Let P and Q be two sets and n (P × Q) = 6. If three elements of p × Q are (3, 6); (4, 8); (5, 8), then find P × Q.

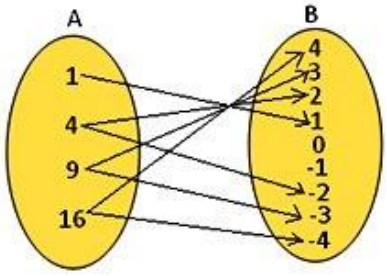
10. If A = {1, 2, 3} and B = {4, 5, 6}, state which of the following is a relation from A to B.

(a) R_1 = {(1, 4); (2, 5); (6, 3)} (b) R_2 = {(2, 5); (3, 6)}

(c) R_3 = {(6, 3); (5, 2); (4, 1)} (d) R_4 = {(1, 5); (1, 6); (2, 4); (2, 6), (3, 4), (3, 5)}

11. Let A = {3, 4, 5, 6} B = {1, 2, 3, 4, 5, 6} Let R = {(a, b): a ∈ A, b ∈ B and a < b}.Write R in the roster form and find its domain and range.

12. Adjoining figure shows relation between A and B. Write relation in



(i) Set builder form. (ii) Roster form. (iii) Find domain and range of R.

13. Write the domain and range of the relation R given by

$$R = \{(x, y) : y = x + \frac{6}{x}, \text{where } x, y \in \mathbb{N} \text{ and } x < 6\}$$

13. Is the following relation a function? Justify your answer

(i) $R_1 = \{(2, 3), (12, 0), (2, 7), (-4, 6)\}$

ii) $R=\{(x,y)/\ y=3x^2+4x+1, x\in\mathbb{R}\}$ *iii*) $R=\{(x,y)/\ y^2=10-3x, x\in\mathbb{R}\}$

14. Let $A=\{1,\ 2,\ 3,\ 4\}$, $B=\{0,3,\ 6,\ 8,12,15\}$ and $f: A\to B$ defined *by* $f(x)=x^2-1,\ x\in A$,

Then a) write the function in roster form b) find domain, codomain and range of f.

15. Find the domain and range of each of the following functions.

a) $f(x)=\dfrac{x}{x^2+3x+2}$ *b*) $f(x)=|x|+x$ c) $\dfrac{|x-3|}{x-3}$ *d*) $\sqrt{9-x^2}$ *e*) $f(x)=\begin{cases}3x+4, & -1\le x<2\\ 1+x, & 2\le x\le 5\end{cases}$

16. If $f(x)=x^3-\dfrac{1}{x^3}$, find $f(x)+f(\dfrac{1}{x})$

17 Let $f(x)=4x^2+3x+2$ *and* $g(x)=2x^2-5x-6$, find

a) $(f+g)(x)$ *b*) $(f-g)(x)$ *c*) $(fg)(x)$ *d*) $(\dfrac{f}{g})(x)$ *e*) $(fog)(x)$ *f*) $(gof)(x)$

18. Find $g(x)$ *if* *a*) $f(x)=2x-5$ and $(fog)(x)=2x-1$

b) $f(x)=x^2+7$ and $(fog)(x)=x^2-6x+16$ *c*) $f(x)=3x+2$ *and* $(fog)(x)=3x^2+12x+17$

19. Let $f(x)=4x+1$ and $g(x)=3x+k$, find the value of *k* for which $(fog)(x)=(gof)(x)$.

20. Given the function $f:\{(1,3),(2,5),(3,1)\}\to\mathbb{N}$ defined by $f=\{(1,3),(2,5),(3,1)\}$

a) Is f one –to-one? b) Is f onto? c) Is f bijective?

21. Which of the following functions are injective, surjective, and bijective on their respective domains?

a) $f(x)=x^3-2x+1$ *b*) $f(x)=\dfrac{x+1}{x-1}$ *c*) $f(x)=\begin{cases}x^2, & x\le 0\\ x+1, & x>0\end{cases}$ *d*) $f(x)=\dfrac{1}{x^2+1}$

22. Find $f^{-1}(x)$ if a) $f(x)=7x-6$ *b*) $f(x)=\dfrac{4-x}{3x}$ *c*) $f(x)=\dfrac{5x+3}{1-2x}$ *d*) $f(x)=\dfrac{2x}{1+x}$

23. Find the value of a if -4 is a root of the function $f(x)=x^4+ax^3-19x^2-46x+120$.

24. If the remainder is 10 when the polynomial $p(x)=x^4-3x^2+kx-2$ is divided by $(x-2)$. Find the value of k.

25. What number should be added to x^2+5 so that the resulting polynomial leaves the remainder 3 when divided by x + 3?

26. If the polynomials ax^3+4x^2+3x-4 and x^3-4x+a leave the same remainder when divided by $(x-3)$, find the value of a.

27. If $x=-1$ and $x=2i$ are the roots of a polynomial of third degree with integral coefficients, write the formula for the polynomial.

28. State the domain and sketch the graph of each of the following rational functions:

$a)\ f(x)=\frac{1}{x}$ $b)\ f(x)=\frac{1}{x+3}-5$ $c)\ f(x)=\frac{5}{x-1}$ $d)\ f(x)=3x+\frac{2}{x+1}$

$e)\ f(x)=\frac{2x}{x+7}$ $f)\ h(x)=\frac{x+7}{x^2-6x+8}$ $g)\ f(x)=\frac{x^2-2x+1}{x^2-1}$

29. a) Write the general polynomial $p(x)$ whose only zeros are 1, 2 and 3, with multiplicity 2, 3 and 1 respectively. What is its degree?

b) Find $p(x)$ described in part (a) if $p(0)=5$.

30. If $2+3i$ is a root of $p(x)=2x^3-5x^2+14x+39,$ find the remaining zeros of p(x).

31. Determine the rational zeros of the polynomials,

$a)\ f(x)=12x^4+8x^3-11x^2-8x-1$ $b)\ f(x)=x^3-4x^2-7x+10$

$c)\ p(x)=2x^3-5x^2-28x+15$ $d)\ p(x)=6x^3+x^2-4x+1$

32. Solve each of the following equations

$a)\ 4^{2x-1}=8^{x+3}$ $b)\ 5^{x^2+8}-125^{2x}=0$ $c)4^x.2^{x^2}=256$ $d)\ 3^x-2=12$ $e)\ e^{2x}-2e^x=15$

33. 15, 000$ is invested in an account that yields 5% interest per year. After how many years will the account be worth 91, 221.04$ if the interest is compounded yearly?

34. Starting at the age of 40, an average man loses 5% of his hair every year. At what age should an average man expect to have half his hair left?

35. A bacteria culture starts with 10 00 bacteria and the number doubles every 40 minutes. (a) Find a formula for the number of bacteria at time t. (b) Find the number of bacteria after one hour. (c) After how many minutes will there be 50 000 bacteria?

36. Let $f(x)=3^x$. Show that $\frac{f(x+2)-f(2)}{2}=4(3^x)$.

37. Find the solution set of $(\log_3 x)^2+36(\log_x 3)^2=13$.

38. If $\log 2=x, \log 3=y$ *and* $\log 5=z$,find $\log(120)^{1/2}$.

39. Given that $\log 2 = x,\ \log 3 = y$ and $\log 7 = z,$ express the following expressions in terms of x, y, and z.

a. $\log \frac{28}{3}$

b. $\log 10.5$

c. $\log \frac{6000}{3}$

40. Prove the following statements.

a. $\log_{\sqrt{b}} x = 2 \log_b x$

b. $(\log_{b^4} x^2 = \log_b \sqrt{x}$

41. Find the inverse of each of the following functions.

a. $f(x) = \log_2 (x - 3) - 5$

b. $f(x) = 3 \log_3 (x + 3) + 1$

c. $f(x) = -\ln(1 - 2x) + 1$

42. Solve the following equations for x, if $0 \le x < 2\pi$.

a) $\sqrt{3}\sin x - 2\sin x \cos x = 0$ *b*) $\sin^2 x - \sin x - 2 = 0$ *c*) $\cos^2 x - \sin x \cos x = 0$

43. Let $\cos\beta = \frac{12}{13}$, *find* *a*) $\sin\beta$ *b*) $\tan\beta$ *c*) $\cos(\frac{\pi}{2} - \beta)$

44. Use trigonometric definitions and identities to prove the statements below.

a) $(1+\cos\beta)(1-\cos\beta) = \sin^2\beta$ *b*) $(\sin x - \cos x)(\sec x + \csc x) = \tan x - \cot x$

45. The angle of elevation of the top of the building at a distance of 50 m from its foot on a horizontal plane is found to be 60 degree. Find the height of the building.