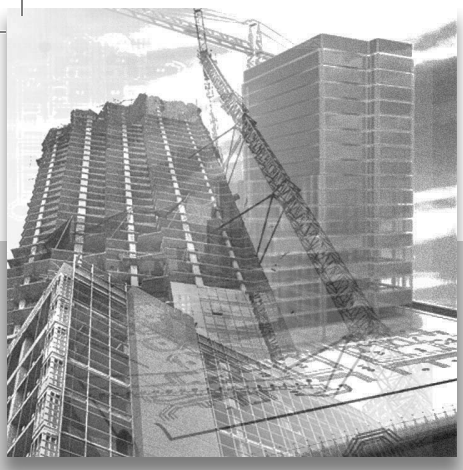


3

Construction Economics

Introduction, economic decision-making, time value of money, cash-flow diagrams, using interest tables, evaluating alternatives by equivalence, effect of taxation on comparison of alternatives, effect of inflation on cash flow, evaluation of public projects: discussion on benefit-cost ratio

3.1 INTRODUCTION

The relationship between engineering and economics is very close, and has in fact been heightened in the present scenario, wherein engineers are expected to not only create technical alternatives but also evaluate them for economic efficiency. Highlighting the economic aspect of engineering decision-making, Wellington (Wellington 1887, cited in Riggs et al. 2004) in his book *The Economic Theory of the Location of Railways* wrote, '… it would be well if engineering were less generally thought of, and even defined, as the art of constructing. In a certain important sense it is rather the art of not constructing; or, to define it rudely but not ineptly, it is the art of doing that well with one dollar which any bungler can do with two …' .

With the growing maintenance cost, especially in the infrastructure sector, it has been realized that not only the initial cost, but also the overall life-cycle cost of a project should be taken into account when evaluating options. In fact, in the case of construction projects, economics affects decision-making in many ways—from the cost of materials to purchase and scrapping of equipment, bonus and/or penalty clauses, and so on. It is, therefore, extremely important that engineers and construction managers have a working knowledge of economic principles, terminology and methods.

The effort in this chapter is directed to provide some important tools relating to different aspects of economic decision-making. In the construction industry, decisions involved include deferred payments or receipts, payments (or receipts) in installments, etc. Thus, effort is directed here to discuss the concepts of value of money and cash-flow diagrams in some detail. Further, in order to be able to compare alternatives, it is important that they are reduced to a common platform. Equivalence is one of the methods commonly used. All these concepts have been discussed at some length in the following sections. The chapter also briefly discusses the use of interest tables, relationship between concepts such as inflation and taxation, and economic decision-making in the context of construction projects.

3.2 ECONOMIC DECISION-MAKING

There are various situations—such as (a) comparison of designs or elimination of over–design; (b) designing for economy of production/maintenance/transportation; (c) economy of selection; (d) economy of perfection; (e) economy of relative size; (f) economy and location; and (g) economy and standardization and simplification—in

which an engineer has to take a decision among the competing alternatives. These are referred to as *problems of present economy*. In such situations, the decision maker may not consider the time value of money.

The three most common methods of evaluation—'out-of-pocket commitment' comparison, 'payback period', and the 'average annual rate of return' methods—in which the time value of money is not considered are discussed in the following sections.

3.2.1 Out-of-Pocket Commitment

Suppose a pre-cast concrete factory has to produce 100,000 railway sleepers per year. An economic choice has to be made between using steel formwork and wooden formwork. The life of steel formwork is estimated to be one year, while that of wooden formwork is one month. The costs of preparing one set of steel formwork and one set of wooden formwork are Rs. 400,000 and Rs. 50,000, respectively. It is further estimated that the labour costs for fixing and removing the steel and wooden formwork are Rs. 10 and Rs. 9 per sleeper, respectively. Now, this is a situation wherein we may not need to consider time value of money, and use the lowest out-of-pocket commitment criteria to choose the most economical alternative.

The out-of-pocket commitment is the total expense required for an alternative. For example, the out-of pocket commitment for steel formwork option would be the sum of total labour cost incurred for production of one lakh sleepers/year plus the cost of the steel formwork/year, that is, Rs. 100,000 $\times$ 10 + Rs. 400,000 = Rs. 1,400,000.Similarly, the out-of-pocket commitment for wooden formwork option is = Rs. 9 $\times$ 100,000 + Rs. 50,000 $\times$ 12 = Rs. 1,500,000.

Since the out-of-pocket commitment for steel formwork is lesser than that for wooden formwork, the decision would be to choose the 'steel formwork' option.

3.2.2 Payback Period

The payback period for an investment may be taken as the number of years it takes to repay the original invested capital, and serves as a very simple method for evaluation of projects and investments. Though this method does not take into account the cash flows occurring after the payback period, given the ease of computation involved, the method is widely used in practice, and it is understood that the shorter the payback period, the higher the likelihood of the project being profitable. In other words, upon comparing the two projects or alternatives, the option with the shorter payback period should be selected.

For example, let a contractor have two brands of excavators, A and B, to choose from. Both the brands are available for a down payment of four lakh rupees. Both brands can be useful for a period of four years. Brand A is estimated to give a return of Rs. 50,000 for the first year, Rs. 150,000 for the second year, and Rs. 200,000 for the third and fourth year. Brand B, on the other hand, is expected to give a return of Rs. 150,000 for all the four years. The payback period for both the brands is calculated in the following manner.

The payback period for Brand A = 4 years, as the initial investment of four lakh rupees is recovered in three years (50,000 + 150,000 + 200,000 = 400,000). The method does not consider the returns after the payback period. Hence, in this case the return of fourth year (Rs. 200,000) is simply overlooked.

For Brand B, the return is Rs. 300,000 up to the end of second year and in the third year it equals Rs. 450,000. Thus, the investment amount of Rs. 400,000 is recovered somewhere between second year and third year, which can be found out by interpolation.

$$\text{Hence, the payback period for Brand B} = 2 + 3 - 2 \times \frac{(400{,}000 - 300{,}000)}{(450{,}000 - 300{,}000)}$$

$$= 2.67 \text{ Years} = 2 \text{ Years and 8 months.}$$

Here also, as in the first case we neglect the return that is expected beyond the payback period.
Hence, it is beneficial to buy Brand B excavator as it has a lesser payback period.

Although we have taken the equal initial investment and time period for the two alternatives, the payback period can also be used when these values are not same. That is, we can use payback period for problems involving unequal initial investment and service life.

3.2.3 Average Annual Rate of Return

In this method, the alternatives are evaluated on the basis of only the average rate of return as expressed in terms of a percentage (of the original capital). Since this method does not distinguish the period at which transactions take place, or the timings of cash flow, it is not possible to account for the concept of 'time value of money' in this approach. We use the previous example (used in payback period comparison method) to illustrate the computations involved.

$$\text{The average annual return from Brand A} = \frac{(50{,}000 + 150{,}000 + 200{,}000 + 200{,}000)}{4} = \frac{600{,}000}{4} = 150{,}000$$

Here, 4 is the number of years. The above-average annual return is converted into percentage to get the average annual rate of return.

$$\text{Average annual rate of return for Brand A in \%} = \frac{150{,}000}{400{,}000} \times 100 = 37.5\%.$$

Here, 400,000 is the original invested capital.

$$\text{The average annual return from Brand B} = \frac{(150{,}000 + 150{,}000 + 150{,}000 + 150{,}000)}{4} = \frac{6000{,}000}{4} = 150{,}000$$

$$\text{The average annual rate of return for equipment B in \%} = \frac{(150{,}000)}{(400{,}000)} \times 100 = 37.5\%.$$

This is also the same as Brand A. Thus, as far as average annual rate of return is concerned, both the brands are equivalent.

However, given a choice between the two brands A and B, which one would you prefer? You guessed it right! We would go for Brand B. This is because we are getting higher returns in the initial years for Brand B when compared to Brand A. The basis for taking such decisions is inherent in the 'time value of money' concept, which is discussed in the next section.

3.3 TIME VALUE OF MONEY

Although monetary units (rupee, dollar, etc.) serve as an excellent tool to compare otherwise incomparable things, e.g., a bag of cement and tons of sand, we know that the real worth of a certain amount of money is not invariant on account of factors such as inflation, dynamic interactions between demand and supply, etc. Now, time value of money is a simple concept that accounts for variations in the value of a sum of money over time.

In most decisions, the change in the value of money needs to be accounted for, and this chapter seeks to present simple tools of analysis to help a construction engineer make logical decisions based on sound economic principles. The most important principle involved is that of 'interest', which could be looked upon as the cost of using capital (Riggs et al. 2004), or the (additional) money (in any form) paid by the borrower for the use of funds provided by the lender. According to Riggs et al. (2004), the interest represents the earning power of money, and is the premium paid to compensate a lender for the administrative cost of making a loan, the risk of non-repayment, and the loss of use of the loaned money. A borrower pays interest charges for the opportunity to do something now that otherwise would have to be delayed or would never be done.

In simple terms, interest could be simple or compound, where in the former case, the interest component does not attract any interest during the repayment period, whereas in the latter case, the interest amount itself also attracts further interest, as we know from traditional arithmetic. The concept is explained briefly in the following paragraph to illustrate some other related ideas.

Let us consider the following statement by a bank: Interest on the deposit will be payable at the rate of eight per cent compounded quarterly. Here, the period of compounding represents the interval of time at which interest will be added to the principal, and the revised principal used to calculate the interest on the next time step. Thus, in a year, for quarterly compounding of interest, four periods (of three months each) should be considered. Now, if the principal amount was Rs. 100, at the end of the first three-month period, the new principal (to calculate interest on the second quarter) should be taken as $= 100 + 100 \times \frac{0.08}{4} =$ Rs. 102.00, whereas the principal amount for the third quarter would be $102 + 102 \times \frac{0.08}{4} =$ Rs. 104.04, and so on. At the end of one year, the principal would become Rs. 108.24. In other words, it can be stated that the value of Rs. 100 changes to Rs. 108.24 under the given conditions of eight per cent nominal interest and quarterly compounding.

Now, in the above example the eight per cent is sometimes referred to as the nominal rate of interest, which is the annual interest for a one-year period with no compounding. From the above example, it is clear that Rs. 8.24 can be seen as the interest amount attracted by Rs. 100 in a one-year period under the given rate (eight per cent) and condition of quarterly compounding. This modified rate of interest is sometimes referred to as the effective rate of interest (i_{eff}), which in addition to the nominal rate of interest (i_{nom}) also depends on the period of compounding. One can work out that the effective rate of interest taking i_{nom} to be 5 per cent and 10 per cent, will be 5.09 per cent and 10.38 per cent, respectively, for quarterly compounding, and 5.12 per cent and 10.47 per cent for monthly compounding. In general, given the number of periods in a year to be taken for compounding (m) and the nominal rate of interest (i_{nom}), the effective rate of interest, i_{eff}, can be calculated as below:

$$i_{eff} = \left(\left(1 + \frac{i_{nom}}{m \times 100}\right)^{m} - 1\right) \times 100 \tag{3.1}$$

3.4 CASH-FLOW DIAGRAMS

Any organization involved in a project receives and spends different amounts of money at different points in time, and a cash-flow diagram is a visual representation of this inflow and outflow of funds. Although in practice this inflow and outflow does not necessarily follow any pattern, it is sometimes assumed that all transactions (inwards or outwards) take place either at the beginning or end of a particular period, which may be a week, a month, a quarter, or a year, simply to simplify the analysis. In other words, if it is decided that all transactions in a month will be recorded as having occurred on the last day of that month, or the first day of the next month, either approach can be followed provided the system is consistently followed.

In a cash-flow diagram (see Figure 3.1), usually time is drawn on the horizontal (X) axis in an appropriate scale, in terms of weeks, months, years, etc., whereas the Y-axis represents the amount involved in the transaction,

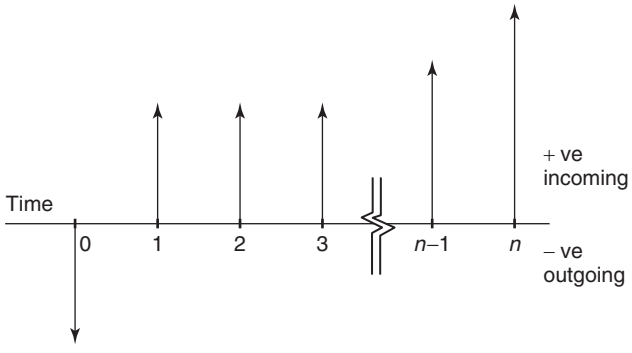


Figure 3.1 Typical cash-flow diagram

Table 3.1 Details of transactions carried out by M/s Alpha Industries for April to June

Date	Description	Amount (Rs.)
April 5	Receipt for running account bill # 9	150,000
April 10	Salary disbursement	80,000
April 16	Payment for supply of aggregates	20,000
April 21	Payment for supply of stationery for office use	5,000
May 7	Receipt for running account bill # 10	180,000
May 10	Salary disbursement	80,000
May 28	Payment for supply of cement	50,000
June 6	Receipt for running account bill # 11	250,000
June 10	Salary disbursement	80,000
June 16	Payment for supply of structural steel	100,000
June 28	Payment of rent of premises for July and August	100,000

with the receipts and disbursements being drawn on the positive and negative side, respectively, of the Y-axis. While scale is maintained for the time axis, the representation on the Y-axis is sometimes not to scale; however, effort should be made to maintain a semblance of balance. Thus, it is a practice to actually write the amount of each transaction next to the arrow. Some of the other aspects related to drawing and interpreting a cash-flow diagram are explained by way of the following example.

Example 3.1

The details of the financial transactions during the months of April, May and June for M/s Alpha Industries are given in Table 3.1. Draw a neat cash-flow diagram for the transactions using a month as a single unit, showing all transactions during a month at the end of that month.

Now, since a month is the basic unit to be used, the given transactions can be summarized as given in Table 3.2.

Table 3.2 Summary of transactions for April to June

Month	Receipts (Rs.)	Expenditures (Rs.)
April	150,000	105,000
May	180,000	130,000
June	250,000	280,000
Total	**580,000**	**515,000**

Now, this summary can be represented as a cash-flow diagram as shown in Figure 3.2 or Figure 3.3, depending on whether only the net transaction {algebraic sum of the receipt (taken positive) and the disbursement (taken negative)} for the month is to be shown or whether both the inflow and outflow are to be shown for each month. Obviously, no matter what convention is followed, it is clear that the total receipts and expenditures for the said three months are Rs. 580,000 and Rs. 515,000, respectively.

In construction economics, we come across two main types of problem: income expansion and cost reduction. Correspondingly we can distinguish the cash-flow diagrams also into revenue-dominated and cost-dominated cash-flow diagrams. In the former case, incomes or savings are emphasised, while the latter

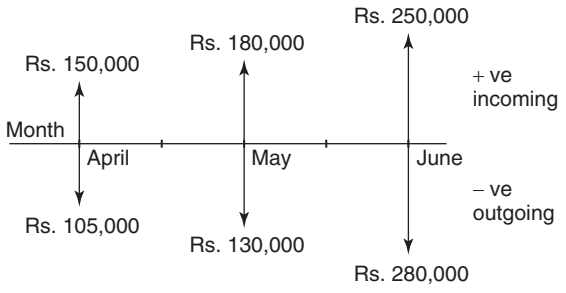


Figure 3.2 Cash-flow diagram for example problem (Alternative 1)

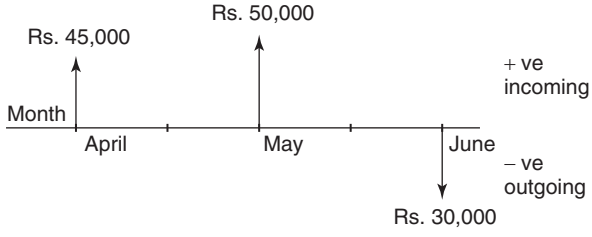


Figure 3.3 Cash-flow diagram for example problem (Alternative 2)

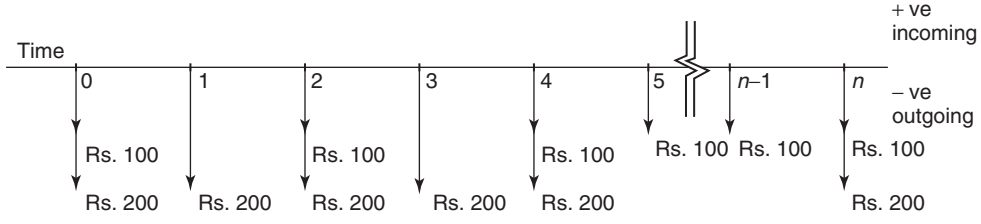


Figure 3.4 Cost-dominated cash-flow diagram

diagrams largely concentrate on periodic costs or expenditures. Illustrative representations of the diagrams are shown in Figure 3.4 and Figure 3.5.

Cash-flow diagrams are frequently utilized for finding the equivalence of money paid or received at different periods, for the purpose of comparison of different alternatives and for a host of other objectives, as we shall see in due course.

Clearly, as one party spends an amount of money, another party has to earn it, and it is inevitable that the cash-flow diagram would depend on which perspective is taken. Thus, an entry corresponding to an expenditure of Rs. 100,000 for the supply of structural steel on June 16 (refer to Example 3.1) would

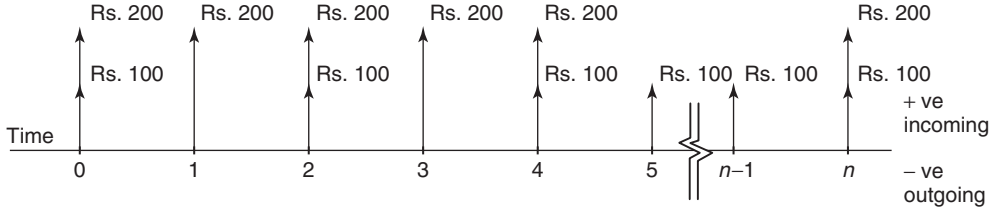


Figure 3.5 Revenue-dominated cash-flow diagram

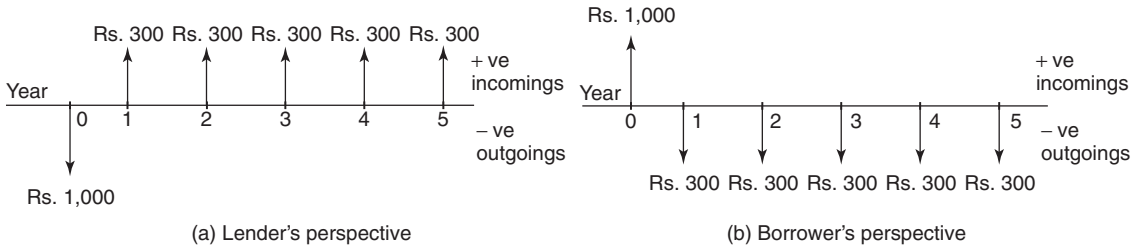


Figure 3.6 Cash-flow diagrams from lender's as well as borrower's perspectives

figure as a receipt in the cash-flow diagram drawn for the supplier of structural steel. Another illustrative example showing the difference in representation of cash-flow diagrams from different points of view is given below.

Suppose a lender has lent Rs. 1,000 to a borrower (at t = 0), and is expected to get this money back in equal instalments of Rs. 300 payable at the end of every year for a five-year period. The cash-flow diagram for this transaction from both lender's perspective and borrower's perspective is drawn in Figure 3.6. It may be noted that repayment of a Rs. 1,000 loan is amounting to Rs. 1,500, which is expected taking into account the concept of time value of money as already discussed above, and the computation of rates of interest and compounding periods under conditions such as these (payment in instalments) are discussed in subsequent sections.

3.4.1 Project Cash-Flow and Company Cash-Flow Diagrams

We have discussed the problems wherein we were given information on the cash transactions and the period of their occurrence. In practice, we have to derive such information from some conditions. Based on the concepts of cash-flow diagram discussed above, we can draw cash-flow diagrams for a construction project as well as for a construction company. In the first case it is referred to as a project cash-flow diagram, while the second one is known as company cash-flow diagram. These two are briefly explained below.

Project Cash-Flow Diagram

The project cash flow is basically a graph (pictorial representation) of receipts and disbursements versus time. The project cash flow can be prepared from different perspectives—of contractor, owner, etc. It is usual to represent time in terms of month for project cash-flow diagram. For executing a construction project, a constructor spends and receives the money at different points of time.

In order to draw a project cash-flow diagram, the following details are required.

1. The gross bill value and its time of submission
2. The measurement period. It is usual for contractors to be paid on a monthly basis. The payment can be made fortnightly or sometimes bimonthly as well. These conditions can be found under **'terms of payment'** given in the tender document
3. The certification time taken by the owner. In normal conditions, the owner takes about three to four weeks to process the bill and release the payment to the constructor or the contractor
4. The retention money deducted by the owner and the time to release the retention money
5. The mobilization advance, the plant and equipment advance, and the material advance, and the terms of their recovery

6. The details of cost incurred by the contractor for raising a particular bill value. This break-up of costs should be in terms of labour cost, materials cost, plant and equipment cost, subcontractors cost and project overheads
7. The credit period (delay between incurring a cost and the actual time at which the cost is reimbursed) enjoyed by the contractor in meeting the costs towards labour, materials, plant and equipment, and overheads

For illustrating the project cash-flow diagram, we take up an example in which a month-wise invoice is estimated and this is produced in Table 3.3. The process of preparing the estimate of monthly invoice has been explained later in the text.

Let us assume that the contractor has been awarded this contract on the following terms:

- Advance payment of Rs. 50 lakh, to be recovered in five equal instalments from the third running account bill onwards
- The total cost for the contractor to execute a particular item is 90 per cent of the quoted rate
- The total cost for a particular item consists of labour (20 per cent), material (55 per cent), plant and machinery (10 per cent), subcontractor cost (10 per cent) and project overheads (5 per cent)

Table 3.3 Monthly invoice for the example problem (Values in Rs. Lakh)

Item Description	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10
Earthwork: All soils	0.25	0.75	1.00	1.25	1.25	0.50				
Concrete works		8.00	16.00	16.00	20.00	12.00	8.00			
Formwork		4.20	8.40	8.40	10.50	6.30	4.20			
Reinforcement works		11.25	22.50	22.50	28.125	16.875	11.25			
Brickwork			3.40	5.10	6.80	6.80	6.80	3.40	1.70	
Plastering: All types				1.50	2.250	3.00	3.00	3.00	1.50	0.75
Painting: All types				0.50	0.75	1.00	1.00	1.00	0.50	0.25
Flooring: All types				11.00	16.50	22.00	22.00	22.00	11.00	5.50
Waterproofing works							6.00	6.00		
Aluminium work					3.375	3.375	3.375	3.375		
Electrical work						8.75	8.75	8.75	5.25	3.50
Sanitary and plumbing works			1.00	2.00	3.00	4.00	3.00	3.00	2.00	2.00
Road works						8.00	8.00			
Total	0.25	24.20	52.30	68.25	92.55	92.60	85.375	50.525	21.95	12.00
Cumulative	0.25	24.45	76.75	145.00	237.55	330.15	415.525	466.05	488.00	500.00

- Assume that there is no delay in payment towards labour costs and overhead costs, but a delay of one month occurs in paying to the subcontractors, material suppliers, and plant and machinery supplier
- Retention is 10 per cent of billed amount in every bill. Fifty per cent retention amount is payable after one month of practical completion, while the remaining 50 per cent is payable six months later

For the given condition, the computations for the cash inflow and the cash outflow are given in Table 3.4. The table is self-explanatory. The resulting cash inflow and outflow diagrams are shown in Figure 3.7 and Figure 3.8, respectively. The resultant cash-flow diagram is shown in Figure 3.9.

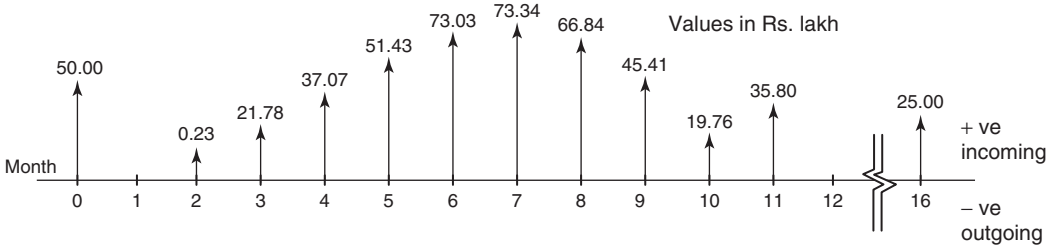


Figure 3.7 The cash-inflow diagram for the contractor for the example problem

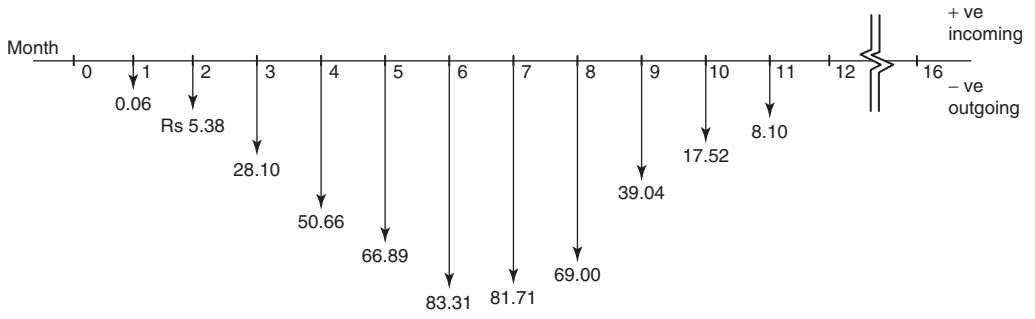


Figure 3.8 The cash-outflow diagram for the contractor for the example problem

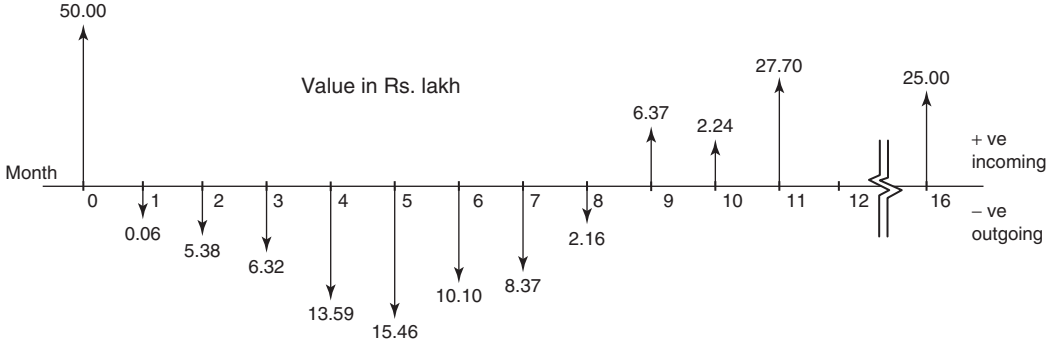


Figure 3.9 The project cash-flow (inflow–outflow) diagram for the example problem (contractor's perspective)

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Table 3.4 Computation of cash inflow and cash outflow for the example problem

	Description	t = 0	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M15	M16
1	Work done by measurement		0.25	24.20	52.30	68.25	92.55	92.60	85.375	50.525	21.95	12.00			
2	Recoveries towards advance payment of Rs 50 lakh				10.00	10.00	10.00	10.00	10.00						
3	Retention @10% of gross invoice (0.1× row 1)		0.03	2.42	5.23	6.83	9.26	9.26	8.54	5.05	2.20	1.20			
4	Payment due (1–3)		0.23	21.78	37.07	51.43	73.30	73.34	66.84	45.47	19.76	10.80			
5	Cash received (one-month delay)			0.23	21.78	37.07	51.43	73.30	73.34	66.84	45.47	19.76	10.80		
6	Release of retention												25.00		25.00
7	Total incomings	50.00	0.00	0.23	21.78	37.07	51.43	73.30	73.34	66.84	45.47	19.76	35.80	0.00	25.00
8	Cumulative incomings	**50.00**	**50.00**	**50.23**	**72.01**	**109.08**	**160.50**	**233.80**	**307.14**	**373.97**	**419.45**	**439.20**	**475.00**	**475.00**	**500.00**
	Outgoings														
9	Total cost (0.9 of 1)		0.23	21.78	47.07	61.43	83.30	83.34	76.84	45.47	19.76	10.80			
10	Cost towards labour @20% of total cost		0.05	4.36	9.41	12.29	16.66	16.67	15.37	9.09	3.95	2.16			
11	Cost towards material @55% of total cost		0.12	11.98	25.89	33.78	45.81	45.84	42.26	25.01	10.87	5.94			
12	Cost towards subcontractors @10% of total cost		0.02	2.18	4.71	6.14	8.33	8.33	7.68	4.55	1.98	1.08			

(*Continued*)

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Table 3.4 Continued

	Description	t = 0	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M15	M16
13	Cost towards plant and machinery @10% of total cost		0.02	2.18	4.71	6.14	8.33	8.33	7.68	4.55	1.98	1.08			
14	Project overheads @5% of total cost		0.01	1.09	2.35	3.07	4.16	4.17	3.84	2.27	0.99	0.54			
15	Labour payment (no delay)		0.05	4.36	9.41	12.29	16.66	16.67	15.37	9.09	3.95	2.16			
16	Material suppliers' payment (one-month delay)			0.12	11.98	25.89	33.78	45.81	45.84	42.26	25.01	10.87	5.94		
17	Subcontractors' payment (one-month delay)			0.02	2.18	4.71	6.14	8.33	8.33	7.68	4.55	1.98	1.08		
18	Plant and machinery supplier's payment (one-month delay)			0.02	2.18	4.71	6.14	8.33	8.33	7.68	4.55	1.98	1.08		
19	Overhead cost payment (no delay)		0.01	1.09	2.35	3.07	4.16	4.17	3.84	2.27	0.99	0.54			
20	Outgoings total		0.06	5.61	28.10	50.66	66.89	83.31	81.71	69.00	39.04	17.52	8.10		
21	Cumulative outgoings		**0.06**	**5.67**	**33.77**	**84.43**	**151.32**	**234.63**	**316.34**	**385.34**	**424.38**	**441.90**	**450.00**	**450.00**	**450.00**
22	Cumulative cash flow (8–21)	**50.00**	**49.94**	**44.56**	**38.23**	**24.64**	**9.18**	**−0.84**	**−9.21**	**−11.37**	**−4.94**	**−2.70**	**25.00**	**25.00**	**50.00**

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Table 3.5 Cumulative cash flow with and without mobilization advance

End of Month	Cumulative Cash Flow with Mobilization Advance	Cumulative Cash Flow without Mobilization Advance
0	50.00	0.00
1	49.94	−0.06
2	44.56	−5.45
3	38.23	−11.77
4	24.64	−15.36
5	9.18	−20.82
6	−0.84	−20.84
7	−9.21	−19.21
8	−11.37	−11.37
9	−4.94	−4.94
10	−2.70	−2.70
11	25.00	25.00
12	25.00	25.00
13	25.00	25.00
14	25.00	25.00
15	25.00	25.00
16	50.00	50.00

Assuming that the contractor is not receiving any mobilization advance, we can perform a similar analysis as shown in Table 3.5 and compute the cumulative cash flow for the total duration of the project. This is shown in column 3 of Table 3.5. In this table, column 2 summarizes the cumulative cash flow obtained for the condition wherein the contractor has received a mobilization advance of Rs. 50 lakh at time t = 0.

In the previous discussion, we have seen the impact of mobilization advance on the cash-flow position. There are many other variables that affect the cash flow of a contractor. Some of these are briefly discussed in the next section.

Factors Affecting Project Cash Flow

In addition to the mobilization advance, there are many factors that affect the project cash flow and it would be of interest to know such factors and their impact on project cash flow. These factors are—(1) the margin in a project, (2) retention, (3) extra claims, (4) distribution of margin such as front loading and back loading, (5) certification type such as over-measurement and under-measurement, (6) certification period, and (7) credit arrangement of the contractor with labour, material, and plant and equipment suppliers, and other subcontractors.

Margin The margin (profit margin or contribution) is the excess over costs. Thus, the higher the margin in a project, the better it is for the contractor's cash flow.

We illustrate this aspect with the help of a fictitious project of 10 months duration. It is also assumed that Rs. 100 (cost) is incurred every month and the margin is 10 per cent of the cost. The retention amount is five per cent of the bill value and the whole retention money is to be released six months after practical completion (i.e., 10 months). Further, it is also assumed that the owner delays the payment by one month. The cash-flow computations for this fictitious example are shown in Table 3.6.

Table 3.6 Cash-flow computations for the fictitious example

S. No.	Description	Months 1	2	3	4	5	6	7	8	9	10	11	15	16
A	Cost	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00			
B	Margin @10% of cost (B*0.1)	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00			
C	Value (A + B)	110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00	110.00			
D	Retention @5% (C*0.05)	5.50	5.50	5.50	5.50	5.50	5.50	5.50	5.50	5.50	5.50			
E	Monies due (C – D)	104.50	104.50	104.50	104.50	104.50	104.50	104.50	104.5	104.50	104.50			55.00
F	Money receipt delay by one month	0.00	104.50	104.50	104.50	104.50	104.50	104.50	104.50	104.50	104.50	104.50		55.00
G	Net cash flow (F – A)	−100.00	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	4.50	104.50		55.00

Table 3.7 Summary of net cash flow for different margins

	Variation in Net Cash Flow for Different Margin %		
End of Month	5%	10%	15%
1	−100.00	−100.00	−100.00
2	−0.25	4.50	9.25
3	−0.25	4.50	9.25
4	−0.25	4.50	9.25
5	−0.25	4.50	9.25
6	−0.25	4.50	9.25
7	−0.25	4.50	9.25
8	−0.25	4.50	9.25
9	−0.25	4.50	9.25
10	−0.25	4.50	9.25
11	99.75	104.50	109.25
12	—	—	—
13	—	—	—
14	—	—	—
15	—	—	—
16	52.50	55.00	57.50

Similar computations for cash flow can be performed assuming different margin percentages—say, five per cent and 15 per cent—keeping all the conditions same as in the case of 10 per cent margin. The results associated with different margin percentages are summarized in Table 3.7. It can be clearly seen that as the margin increases, there is betterment in the contractor's cash-flow position, keeping all other factors. same.

Retention Retention tends to reduce the margin obtained from a project. In case of very low margins, the retention can even reduce the margin to zero or less. Thus, retention affects the contractor's cash flow in a negative manner. The higher the retention, the bigger is the cash-flow problem. In order to address this issue, some contractors request the owners to get away with retention amount in lieu of bank guarantee. Thus, instead of cash retention, bank guarantee is in vogue these days. The effect of different percentage of cash retention is explained with the fictitious example discussed in the previous section. Recall that the margin was 10 per cent and retention was five per cent in the original problem. The retention percentage is changed to 0 per cent and 10 per cent from 5 per cent, and the resultant cash flow is computed. These are summarized in Table 3.8.

Extra claims Extra claims in a project may result on account of many reasons such as extra work, changes in quantity and specification and so on. In general claims take a long time to settle. Sometimes it may be settled even after the completion of project. Thus in practice the extra claims tend to worsen the cash flow position of the contractor, though after the claim is settled the contractor may be even able to achieve the original intended level of profit margin.

Table 3.8 Summary of net cash flow for different margins

End of Month	Variation in Net Cash Flow for Different Retention Conditions (Margin 10%)		
	0%	5%	10%
1	−100.00	−100.00	−100.00
2	10.00	4.50	−1.00
3	10.00	4.50	−1.00
4	10.00	4.50	−1.00
5	10.00	4.50	−1.00
6	10.00	4.50	−1.00
7	10.00	4.50	−1.00
8	10.00	4.50	−1.00
9	10.00	4.50	−1.00
10	10.00	4.50	−1.00
11	110.00	104.50	99.00
12	—	—	—
13	—	—	—
14	—	—	—
15	—	—	—
16	0.00	55.00	110.00

Distribution of Margin This aspect is covered in detail in later chapters. Keeping the overall margin amount same, the margin can be distributed across different items of a project, either in a uniform manner or it can be front- or back-loaded. In case of uniform loading, all the items of the project carry equal margin percentage, while in front-end-rate loading, the items to be executed early in the project carry a higher margin than the later items. By doing so, the cash-flow position of the contractor improves, even though the overall margin derived from the project would be the same as that obtained in case of uniform loading. In the back-end-rate loading, the items to be executed later in the project carry higher margin, while the items to be executed early in the project carry lower margin. This has a tendency to increase the negative cash flow and contractors resort to such means in a market where the inflation rate is higher than the interest rate. This way, the contractors hope to recover a substantial amount on account of price escalation.

Certification type

Over-measurement Over-measurement is the device whereby the amount of work certified in the early months of a contract is greater than the amount of work done. This is compensated for in later measurements. Thus, over-measurement has the same effect as front-end loading; it improves the cash in the early stages and reduces the capital lock-up.

Under-measurement Under-measurement is the situation whereby the amount of work certified in the early months of a contract is less than the amount actually done. This has the effect of increasing the negative cash flow for the contractor in a project.

Delay in receiving payment from client The time between interim measurement, issuing the certificate, and receiving payment is an important variable in the calculation of cash flows. Any increase in the delay in receiving this money delays all the income for the contract, with a resulting increase in the capital lock-up.

Delay in paying labour, plant hires, materials suppliers and subcontractors The time interval between receiving goods or services and paying for these is the credit the contractor receives from his suppliers.

The Company Cash-Flow Diagram

For running the projects efficiently, a company usually maintains a head office and a number of regional offices or branch offices. The expenses incurred and the revenues generated by these offices are commonly known as head office outgoings or incomings.

Some examples of head office outgoings would be rents, electricity charges, water charges, telephone and tax bills, hire charges for office equipments, payment to shareholders, taxes, etc. The examples of head office incomings would be claims made in past projects, realization of retention money not settled for past projects, etc.

A construction company executes a number of projects at any time. The company cash-flow diagram is an aggregation (sum) of all the outgoings and incomings for all the projects that the company is executing, besides the head office outgoings and incomings.

3.4.2 Using Cash-Flow Diagrams

Determining Capital Lock-Up

While estimating the effect of margin and retention money on contractor's cash-flow diagram, one can notice that the contracting company sometimes faces negative cash flow in the early stages of the project. This negative cash flow experienced in the early stages of projects represents locked-up capital that is either supplied from the contracting company's cash reserves or borrowed. If the company borrows the cash, it will have to pay interest charged to the project; if the company uses its own cash reserves, it is being deprived of the interest-earning capability of the cash and should, therefore, charge the project for this interest loss. A measure of the interest payable is obtained by calculating the area between the cash-out and the cash-in.

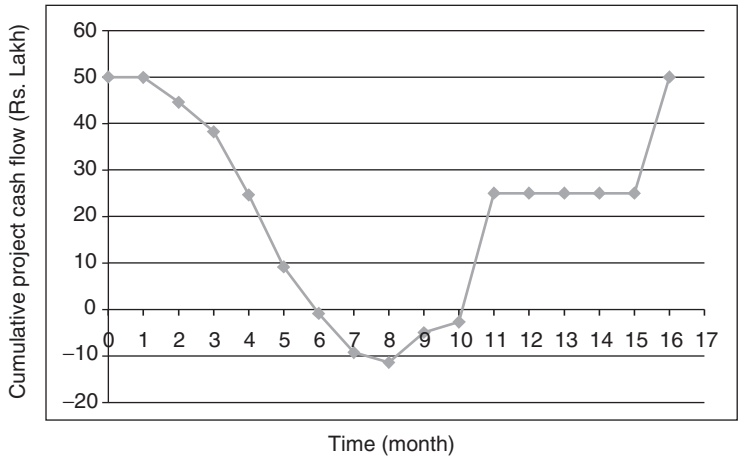


Figure 3.10 Cumulative cash-flow curve with mobilization advance

The negative cash flow indicates that the contractor has to mobilize this much fund to execute the project. The area under the 'negative cash flow' period is used to calculate the financing charges for the project by the contractor. The total area would have a unit of Rs. x months and is also known as *captim*, standing for *capital* × *time*. In order to calculate the interest charges for financing the project, we use the captim in the following manner:

$$\textit{Interest on the capital required for the project} = \frac{\textit{(Captim in Rs. Month} \times \textit{interest charges per annum \%)}}{12 \times 100} \quad (3.2)$$

For example, if the captim value is 10,000 Rs. month and the interest rate is 12 per cent per annum, the interest charges for financing the project would be $= \frac{10{,}000 \times 12}{12 \times 100} =$ Rs. 100.

The result of Table 3.5 is shown graphically in Figure 3.10 and Figure 3.11. It can be noticed that when the contractor has received the mobilization advance, there is no negative cash-flow situation in the early stages of the project—a very happy situation, indeed. In fact, there is negative cash-flow situation only for a brief period between month 6 and month 10.

Turning to Figure 3.11, which shows the cumulative cash-flow situation corresponding to the case when there is no mobilization advance received by the contractor, we find that the contractor experiences huge negative cash flow during most of the stages of the project. In fact, the positive cash-flow situation comes only after the completion of project, which is a very grim scenario.

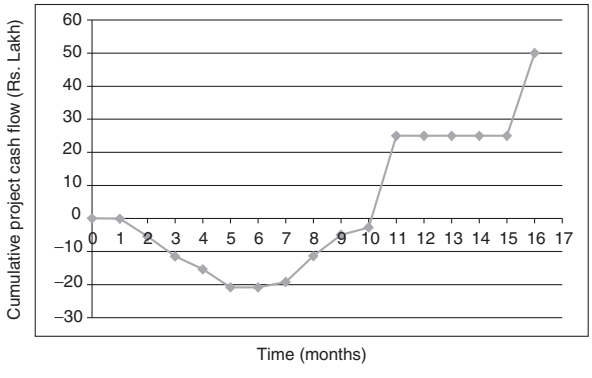


Figure 3.11 Cumulative cash-flow curve without mobilization advance

It is usual for contractors to neglect the positive cash-flow portion. This is justified because the cash available with the contractor may not fetch the same interest as the interest he has to pay for the borrowed amount. However, an exact analysis would demand the subtraction of this area from the negative cash-flow area.

Determining the Cash Requirement for a Project

With the knowledge of cumulative cash outflow and cumulative cash receipt, we can determine the maximum cash required for a project. For illustration, let us take the same problem that was used for drawing a project cash-flow diagram. The computations for month-wise cash requirement are performed in Table 3.9. The cumulative outgoings and cumulative incomings data have been taken as they are for the case in which there was no mobilization advance given to the contractor.

Month-wise requirement of cash is determined by deducting the cumulative cash outgoings for that month less the cumulative cash receipt till previous month. For example, the cash requirement for month 2 (M2) is cumulative outgoings for month 2 – cumulative cash receipt of month 1 = 5.67 − 0.00 = 5.67; for month 3 the cash requirement is 33.77 − 0.23 = 33.55; and so on. We find that the maximum cash of Rs. 104.13 (234.63 − 130.50) is needed immediately before payment is received at month 6. This is based on the assumption that incomings are received at discrete points of time while outgoings are paid continuously (refer to column 4 of Table 3.9).

The month-wise cash requirement would change if we assume that incoming and outgoing both are occurring at discrete points of time. The maximum cash required in this case would be for month 6 (M6) and is equal to 20.83 (refer to column 5 of Table 3.9).

Table 3.9 Computation of month-wise cash requirement for the project (All values in Rs. lakh)

Month	Cumulative Outgoings	Cumulative Incomings	Money Required for the Month when It is Assumed that Incoming is Received at Discrete Points of Time while Outgoings are Paid Continuously	Money Required for the Month when It is Assumed that Both Incoming and Outgoing are Occurring at Discrete Points of Time
(1)	(2)	(3)	(4)	(5)=(2) − (3)
M1	0.06	0.00	0.06	0.06
M2	5.67	0.23	5.67 − 0.00 = 5.67	5.44
M3	33.77	22.01	33.77 − 0.23 = 33.55	11.76
M4	84.43	69.08	84.43 − 69.08 = 62.43	15.35
M5	151.32	130.50	151.32 − 69.08 = 82.25	20.82
M6	234.63	213.80	**234.63 − 130.50 = 104.13**	**20.83**
M7	316.34	297.14	316.34 − 213.80 = 102.55	19.20
M8	385.34	373.97	385.34 − 297.14 = 88.21	11.37
M9	424.38	419.45	424.38 − 373.97 = 50.41	4.93
M10	441.90	439.20	441.90 − 419.45 = 22.46	2.70
M11	450.00	475.00	450.00 − 439.20 = 10.80	−25.00
M12	450.00	475.00	450 − 475 = −25.00	−25.00
M13	450.00	475.00	450 − 475 = −25.00	−25.00
M14	450.00	475.00	450 − 475 = −25.00	−25.00
M15	450.00	475.00	450 = 475 = −25.00	−25.00
M16	450.00	500.00	450 − 475 = −25.00	−25.00

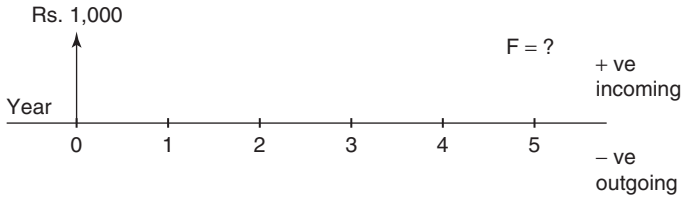


Figure 3.12 Lump of Rs. 1,000 in single instalment

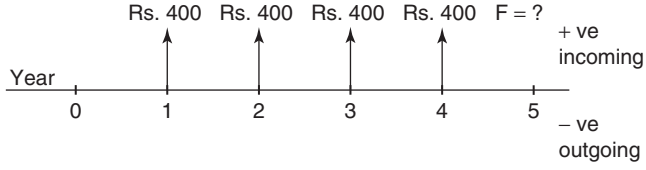


Figure 3.13 Rs. 400 every year for four years

Equivalence of Alternatives

In economic comparisons, very often there are situations wherein a comparison has to be made between alternatives, involving payments (or receipts) of different amounts of money at different points in time. Drawing cash-flow diagrams makes such comparisons easy to understand, as illustrated in the following example. The principle used is that of equivalence, which reduces different alternatives to a common baseline.

Consider making a choice between the following two alternatives of receiving payment:

1. a lump of Rs. 1,000 in a single instalment immediately, and
2. four instalments of Rs. 400 every year for four years, with payments being received at the end of years 1, 2, 3 and 4.

A cash-flow diagram representative of these options is drawn in Figure 3.12 and Figure 3.13.

Given the statement of the problem, it may be noted that Rs. 1,000 has been the inflow at $t = 0$, whereas the inflow of Rs. 400 has been drawn for four years as given (1, 2, 3, 4), and the amounts being inflows have been plotted above the X-axis. Now, the total money received in the two cases is Rs. 1,000 and Rs. 1,600 (400×4), respectively. Two things are said to be equivalent only when they have the same effect, and in this case, since money has time value, the two amounts cannot be simply compared, as they are received at different periods. The different ways of reducing alternatives to a common base are discussed later on, but for illustrative purposes, let us assume that we work out the total funds available at the end of, say, five years using an (nominal) interest rate of 10 per cent and six-monthly (half-yearly) compounding. In terms of the cash-flow diagram, the problem boils down to calculating A_1 and A_2, the amounts in the two cases, 1 and 2, as shown in the figures.

For case 1, the issue is simply calculating the compound interest on Rs. 1,000 for a period of five years, at the rate of 10 per cent compounded at six-monthly intervals. It can be verified that the total amount would be Rs. 1,628.89. For case 2, however, the amounts accruing for a principal value of Rs. 400 for periods of 4 years, 3 years, 2 years and 1 year (calculated under identical conditions) need to be added, and it will be seen that the overall sum is Rs. 2,054.22. Thus, it is clear that under the given conditions governing the time value of money, option 2 is preferable to option 1. However, if the instalment payment was reduced to Rs. 300, the total sum available in Case 2 at the end of five years would be only Rs. 1,540.67, making option 1 more preferable. It may be noted that simplistically speaking, even here the total amount received (1,200) in case 2 is higher than that in case 1 (1,000). As an extension to the problem discussed here, it can be worked out that if the instalment was Rs. 317.15, the two options would be equivalent. It should be borne in mind that the point at which the final comparisons are being made is also critical, and the calculations given would change if the numbers were worked out at another base line (say, at the end of 10 years).

Formulations for Interest Computation

It is clear from the above exercise that equivalence can be established or alternatives compared only when the applicable conditions for compounding and the rates of interest are known. The example also illustrates a possible approach to handling payments (or receipts) in instalments, etc. In this section, the more widely used formulations for interest calculations will be covered in greater detail. For the sake of convenience, three categories addressing eight commonly used interest formulations have been discussed in the following paragraphs.

1. Category A: With a single payment (SP)
   (a) Compound amount factor (SPCAF)
   (b) Present worth factor (SPPWF)
2. Category B: With an equal payment series (EPS) or several equal instalments. Equal payment series is also referred to as uniform series (US) in literature
   (a) Compound amount factor (EPSCAF)
   (b) Present worth factor (EPSPWF)
   (c) Sinking fund deposit factor (EPSSFDF)
   (d) Capital recovery factor (EPSCRF)
3. Category C: With unequal payment series
   (a) Arithmetic gradient factor (AGF)
   (b) Geometric gradient factor (GGF)

As is clear from the above classification, in category A only single lump-sum amounts are involved, while category B handles a series of uniform (equal) number of payments (or disbursements). On the other hand, category C addresses cases wherein there is a uniform increase or decrease in receipts (or disbursements).

In the following discussion, *i, n, P, F* and *A* abbreviate the interest rate per period, the number of interest periods, a present sum of money, a future sum of money, and a periodic instalment or payment, respectively. As will be seen in interest problems, 4 of these 5 parameters are always present and 3 of the 4 must be known and given as input data, and the object is to compute the missing parameter. In fact, it is convenient to discuss the problem in terms of factors, where the required information is obtained using other available information. Tables have since been developed to facilitate the calculations for different interest rates and periods, as will be illustrated subsequently. It should be noted that these tables are conventionally based upon transactions assumed to be occurring at the end of a period.

Single payment compound amount factor (SPCAF) This factor, denoted by SPCAF and read as 'single payment compound amount factor', is one by which a single payment (P) is multiplied to find its compound amount (F) at a specified time in future, as represented schematically on the cash-flow diagram. Another way

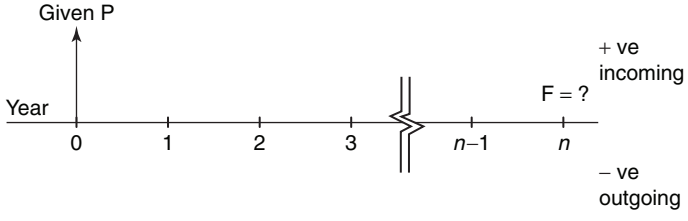


Figure 3.14 Cash-flow illustration of single payment compound amount factor

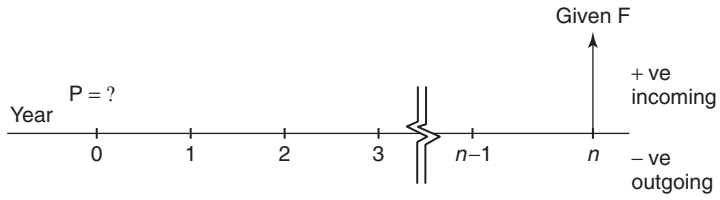


Figure 3.15 Cash-flow illustration of single payment present worth factor

of representing this factor is $(F/P, i, n)$, which is read as 'F given P at an interest rate of i for a period n'. From our understanding of compound interest, effectively

$$(F/P, i, n) = (1 + i)^n \tag{3.3}$$

Single payment present worth factor (SPPWF) When a single future payment (F) is multiplied by this factor, the present worth (P) is obtained. This factor, SPPWF, is calculated as $1/(1 + i)^n$, and also represented as $(P/F, i, n)$, which is read as 'P given F at an interest rate of i for a period n'. Hence, effectively

$$(P/F, i, n) = \frac{1}{(1 + i)^n} \tag{3.4}$$

The cash-flow diagram representation of the problem in this case is essentially the converse of the SPCAF, and is shown in Figure 3.15.

Uniform series compound amount factor (EPSCAF or USCAF) This factor converts a uniform series payment (A) to its compound amount (F), as shown in the cash-flow diagram in Figure 3.16. This factor is sometimes represented as $(F/A, i, n)$, which is read as 'F given A at an interest rate of i for a period n', and is effectively equal to

$$(F/A, i, n) = \frac{(1 + i)^n - 1}{i} \tag{3.5}$$

Uniform series present worth factor (EPSPWF or USPWF) This factor is used to determine the present worth (P) for a uniform series payment of A, as shown in Figure 3.17. This factor is represented as $(P/A, i, n)$, which is read as 'P given A at an interest rate of i for a period n'. Effectively,

$$(P/A, i, n) = \frac{(1 + i)^n - 1}{i(1 + i)^n} \tag{3.6}$$

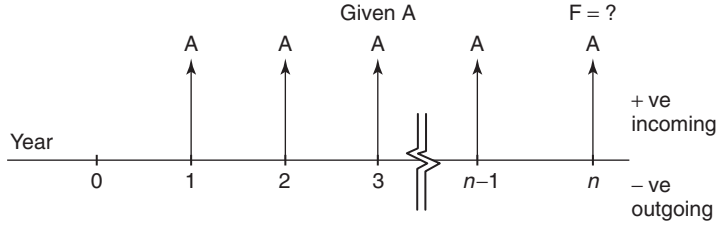


Figure 3.16 Cash-flow illustration of uniform series compound amount factor

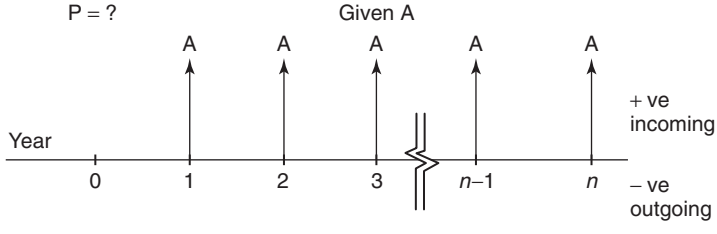


Figure 3.17 Cash-flow illustration of uniform series present worth factor

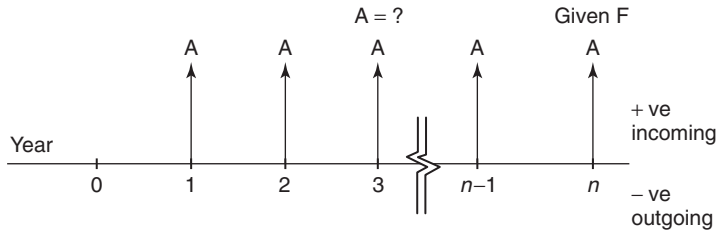


Figure 3.18 Cash-flow illustration of sinking fund deposit factor

Sinking fund deposit factor (EPSSFDF) It is the factor by which a future sum (F) is multiplied to find a uniform sum (A) that should be set regularly, such that the final value of the funds set aside is F. The sinking fund deposit factor, SFDF, is also represented as (A/F, i, n), which is read as 'A given F at an interest rate of i for a period n'. Mathematically, the EPSSFDF can be shown to be equal to

$$(A/F, i, n) = \frac{i}{(1+i)^n - 1} \tag{3.7}$$

The cash-flow diagrammatic representation of the factor is given in Figure 3.18. The formula is derived in the following manner:

$$F = A(1+i)^{n-1} + A(1+i)^{n-2} + \cdots + A(1+i) + A \tag{3.8}$$

$$\Rightarrow F = A[(1+i)^{n-1} + (1+i)^{n-2} + \cdots + (1+i) + 1] \tag{3.9}$$

Multiplying by $(1 + i)$ on both sides of Equation 3.9, we get

$$F(1+i) = A[(1+i)^n + (1+i)^{n-1} + (1+i)^{n-2} + \cdots + (1+i)^2 + (1+i)] \tag{3.10}$$

Subtracting Equation 3.9 from Equation 3.10, we get,

$$F(1 + i - 1) = A[(1+i)^n - 1]$$

$$\Rightarrow F = \frac{A[(1+i)^n - 1]}{i}$$

Thus,

$$(A/F, i, n) = \frac{i}{(1+i)^n - 1} \tag{3.11}$$

Capital recovery factor (EPSCRF) The capital recovery factor can be used to find the uniform payments of A to exactly recover a present capital sum (P) with interest. This factor is also represented as (P/A, i, n), which is read as 'P given A at an interest rate of i for a period n'. Mathematically, EPSCRF equals

$$(A/P, i, n) = \frac{i(1+i)^n}{(1+i)^n - 1} \tag{3.12}$$

and Figure 3.19 shows a schematic representation of the problem statement.

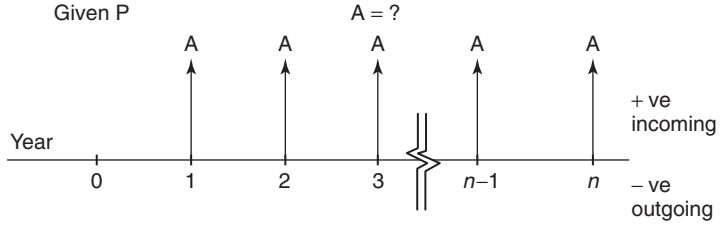


Figure 3.19 Cash-flow illustration of capital recovery factor

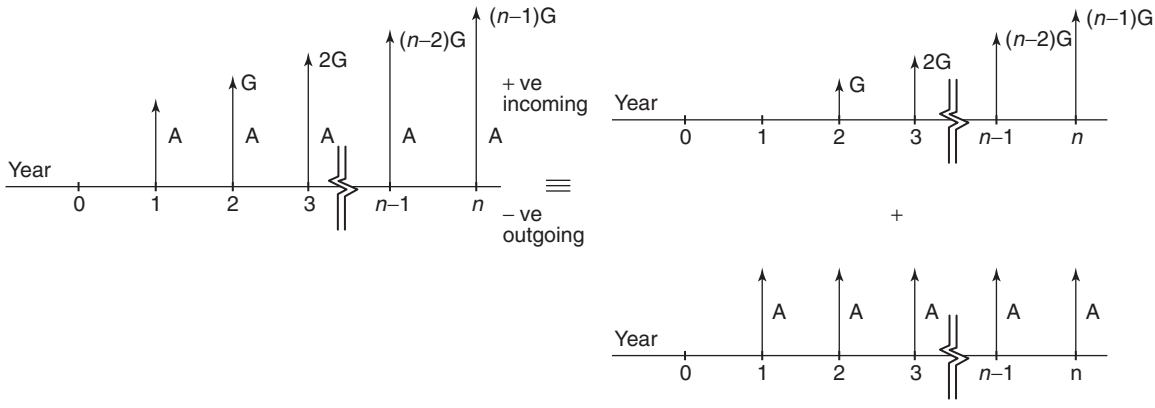


Figure 3.20 Cash-flow illustration of arithmetic gradient factor

Arithmetic gradient factor (AGF) In this case, the increase/decrease in instalments, whether it is payments or disbursements, follows an arithmetic pattern, as shown in Figure 3.20. This factor, AGF, is sometimes denoted as $(A/G, i, n)$ and read as 'A given G at an interest rate of i for a period n'. Mathematically, the factor is equal to

$$(A/G, i, n) = \left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right] \tag{3.13}$$

The formula basically converts the arithmetic increase/decrease amount into a uniform series.

Geometric gradient factor (GGF) In this case, the increase/decrease in instalments, whether it is payments or disbursements, follows a geometric pattern, as shown in Figure 3.21. This factor, GGF, is sometimes denoted as $(P/g, i, n)$ and read as 'P given g at an interest rate of i for a period n'. Mathematically, the factor is equal to

$$(P/g, i, n) = \frac{1 - (1+g)^n (1+i)^n}{(i-g)} \tag{3.14}$$

where g is equal to the rate of increase/decrease and c is the initial amount.

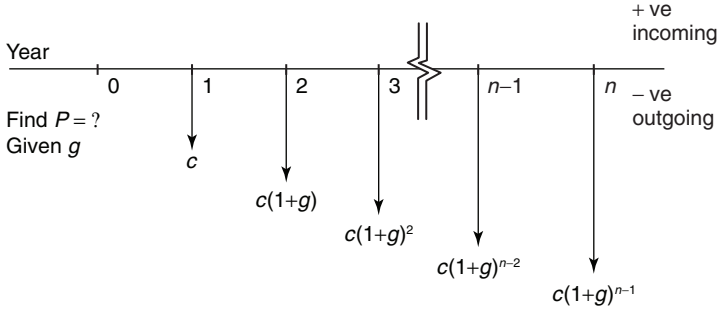


Figure 3.21 Cash-flow illustration of geometric gradient factor

Table 3.10 Summary sheet of interest formulae

1	Single payment compound amount factor	*F/P* (Find *F*, given *P*)	SPCAF = $(F/P, i, n) = (1 + \mathrm{i})^n$
2	Single-point present worth factor	*P/F* (Find *P*, given *F*)	SPPWF = $(P/F, i, n) = \frac{1}{(1+i)^n}$
3	Uniform series compound amount factor	*F/A* (Find *F*, given *A*)	USCAF = $(F/A, i, n) = \frac{(1+i)^n - 1}{i}$
4	Sinking fund deposit factor	*A/F* (Find *A*, given *F*)	SFDF = $(A/F, i, n) = \frac{i}{(1+i)^n - 1}$
5	Capital recovery factor	*A/P* (Find *A*, given *P*)	CRF = $(A/P, i, n) = \frac{i(1+i)^n}{(1+i)^n - 1}$
6	Uniform series present worth factor	*P/A* (Find *P*, given *A*)	USPWF = $(P/A, i, n) = \frac{(1+i)^n - 1}{i(1+i)^n}$
7	Arithmetic gradient conversion factor	*A/G* (Find *A*, given *G*)	AGF = $(A/G, i, n) = \left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$
8	Geometric series factor	*P/g* (Find *P*, given *g*)	GGF = $(P/g, i, n) = \frac{1 - (1+g)^n (1+i)^n}{(i-g)}$

3.5 USING INTEREST TABLES

A simplified summary of the discussion on interest formulations in the above section is presented in Table 3.10. As mentioned earlier, ready-to-use tables are available, and reproduced in the appendices to find the different interest factors at varying interest rates and time periods. Tables in Appendix 3.1 have been provided for interest rates from 0.5 per cent to 30 per cent, and period *n* up to 100. In order to find an interest factor, say compound amount factor (*F/P*, 15, 5), the user needs to look for 5 in the *n* column corresponding to interest table of 15 per cent, and then read across to the 'compound amount factor' column to find 2.01136. Similarly, other interest factors can also be found out. Illustrative examples have been included in the 'solved examples' section to show the physical significance of the different factors discussed above.

3.6 EVALUATING ALTERNATIVES BY EQUIVALENCE

Once the concept of a cash-flow diagram is well understood, the next step is to proceed to the extension of these diagrams to compare different engineering alternatives from an economic point of view. The principle involved is that of 'equivalence', wherein the alternatives are determined to be similar. It should be recalled that the concept has been briefly discussed in Section 3.4 above, and it was pointed out that equivalence exists only at a certain given rate of return, and if this rate of return is changed, the alternatives are likely to change.

Construction management often involves cost comparisons between alternatives of different engineering efficiency, namely, one with a high initial cost and low operation and maintenance costs, compared to another with a low initial cost but high operating and maintenance costs. Using the time value of money and the cash-flow diagrams for illustrative purposes, equivalence is studied to identify the better alternative, using a common basis. Some of the frequently applied methods used for the purpose in engineering economic analysis are listed below and discussed in the following paragraphs.

1. Present worth comparison
2. Future worth comparison
3. Annual cost and worth method
4. Rate of return method

3.6.1 Present Worth Comparison

In this method, the present worth (at time zero) of the cash flow in terms of equivalent single sum is determined using an interest rate, sometimes also called the discounting rate. The method is based on the following assumptions:

(a) Cash flows are known.
(b) Cash flows do not include effect of inflation. The discussion on inflation follows later in the text.
(c) The interest rate (discounting rate) is known.
(d) Comparisons are made with before-tax cash flows. The concept of tax has been discussed in later sections.
(e) Comparisons do not include intangible considerations.
(f) Comparisons do not include consideration of the availability of funds to implement alternatives.

Figure 3.22 shows three typical types of problems that are encountered in present worth analysis. The three sets of problems are—alternatives with equal lives; alternatives with unequal lives; and alternatives with infinite lives.

Type 1: Alternatives with Equal Lives

As the name suggests, in such problems the competing alternatives have equal lives. For evaluating the alternatives, the present worth of both the competing alternatives are found out. The alternative with the maximum present worth is the most economical alternative. For cost-dominated cash-flow diagrams,

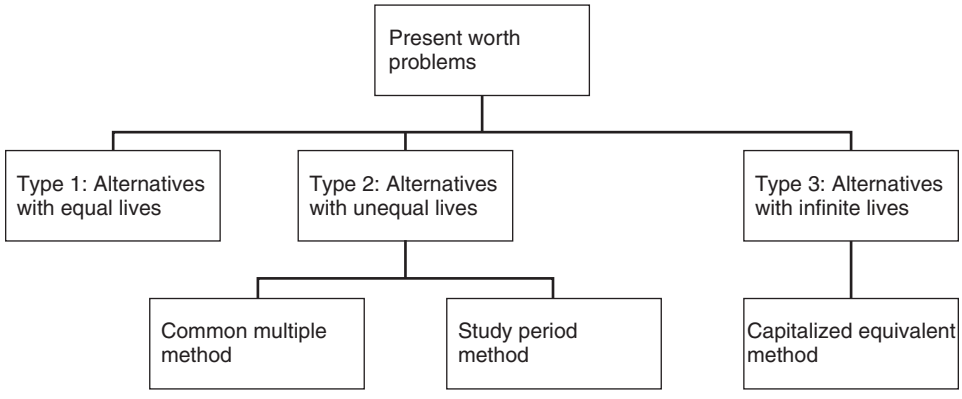


Figure 3.22 Types of problems in present worth analysis

the alternative with the lowest present cost is chosen. In case of cash-flow diagrams involving both costs and revenues, the net or difference of present worth of revenues and costs are found. This is referred to as net present worth or net present value (NPV). The method of comparison of NPV is quite popular for evaluation of alternatives. NPV is also used to calculate profitability for an investment alternative. Profitability index (PI) is the ratio of NPV and capital cost (CC) for an investment. In other words,

$$PI = \frac{NPV}{CC} \tag{3.15}$$

Type 2: Alternatives with Unequal Lives

In such problems, the alternatives do not have an equal life period of service—in other words, they are not coterminous. A relevant example would be a decision to choose between two batching plants that may have different service lives—say, 5 years and 10 years. Needless to say, a simple comparison of the two alternatives would not be accurate, as in one case there would be a need to replace the plant at the end of five years, and any cost likely to be incurred at that point in time should be appropriately accounted for in the budgeting at the outset. The common multiple method and the study period method are two approaches to discuss this class of problems.

Common multiple method In this method, a coterminous life period is chosen for the alternatives using the least common multiples of the different life periods. For example, if the alternatives have life periods of 2, 3, 4 and 6 years, they will be put in use for a period equal to the least common multiple of their life periods. In this case, it is 12 years. This means that the alternative with a two-year life period shall be replaced 6 times, the one with three years shall be replaced four times, and so on, for achieving a coterminous life period for all the alternatives. It is assumed here that the alternatives shall be replaced after their service life with same cost characteristics. This assumption stands valid if the common multiple of alternative life periods is small, and the possibility of a technically different option emerging during this time is ignored.

Study period method In this method, a study period is chosen on the basis of the length of the project or the service lives of the alternatives. An appropriate study period reflects the replacement circumstances. Thus, study period may be chosen as the shortest life period of the alternatives as a protection against technological obsolescence. In this method, we assume that all the assets will be disposed of at the end of the analysis period.

Type 3: Alternatives with Infinite Lives

Such problems are encountered in a real-life situation when the alternatives involved have long lives—for example, the appraisal of different alternatives involving construction of civil engineering structures such as dams, power projects and tunnel projects, which have a reasonably long life. A very popular approach used in such problems is the application of capitalized equivalent (CE) method, which is described in the following section.

Capitalized equivalent method This method is based on the concept of capitalized equivalent, which is the present (at time zero) worth of cash inflows and outflows. In other words, CE is a single amount determined at time zero, which at a given rate of interest will be equivalent to the net difference of receipts and disbursements if the given cash flow pattern is repeated in perpetuity (in perpetuity the period assumed is infinite). In other words, CE can be looked upon as equal to the present worth, with the added rider that the cash flow extends forever. Thus, to determine CE, the normal procedure is to first convert the actual cash flow into an equivalent cash flow of equal annual payment.

Mathematically,

$$\begin{aligned} CE &= A \times (EPSPWF)^{i}_{\infty} \\ CE &= A\,(EPSPWF)^{i}_{\infty} = A\left(\frac{P}{A}, i, n = \infty\right) = A\left[\frac{(1+i)^{\infty} - 1}{i \times (1+i)^{\infty}}\right] \\ \Rightarrow CE &= \frac{A}{i} \end{aligned} \tag{3.16}$$

CE analysis is very useful to compare long-term projects. In fact, for projects such as roads, where the cash flow may contain negative terms, only this method can be used to compare the relative merits of different types of road alignments or surfaces, etc.

3.6.2 Future Worth Comparison

In this method, the future worth of each component of cash flow is evaluated and the algebraic sum of such future worth becomes the basis for comparison. Also, there is no discounting each component of cash flow to the present, as is done in the present worth method.

Although there is no special advantage in using this method over the present worth method, it is frequently used in cases where the owner expects to sell or liquidate an investment at some future date, and wants an estimate of net worth at that point in time. In everyday life, the method is useful in situations such as planning for retirement. A comparison and evaluation using this method seems to be more meaningful as it provides some insight into future receipts also.

3.6.3 Annual Cost and Worth Comparison

This method is perhaps most widely used for comparing alternatives because it is basically simple and easy to understand and explain. Also, the computations involved are easy to carry out. In this method, all payments and disbursements are converted into an annualized cost series—annual cost is the cost pattern of each alternative converted into an equivalent uniform series of annual cost at a given interest rate. Needless to say, the alternative that yields the least cost is chosen. The method could be used to compare alternatives with equal and unequal lives. In the latter case, common multiple method and study period method could be adopted, which is similar in concept as discussed under 'present worth method' problems with unequal lives.

3.6.4 Rate of Return Method

This is another method for evaluation of different competing alternatives, especially in the area of investments. In general, rate of return may be regarded as an index of profitability, and terms such as minimum attractive rate of return (MARR), internal rate of return (IRR), incremental rate of return (IRoR) and ERR (external rate of return) are commonly encountered. The following paragraphs briefly explain some of these, through illustrative examples wherever required.

Minimum Attractive Rate of Return (MARR)

This is the minimum rate of return below which a company would not be interested in the proposed investment alternative. In other words, an investor's interest in an alternative is awakened only when the rate of return is at least equal to or greater than MARR, which itself depends on a number of factors such as conditions of the market, level of competition and cost of capital. It is interesting to note that in the context of construction projects, though the MARR values are obviously different for different companies, as they are willing to work under different rates of return, even within the same company it is always possible to have different yardsticks of MARR that could be adopted for comparison of investment alternatives. For example, in a contracting company working for construction of buildings, bridges and tunnels, the MARR may be the lowest for the building business and the largest in tunnels, on account of the level of competition faced in the different segments of the construction industry. In other words, since the competition is likely to be highest in the building sector, it will also have lower MARR, and so on.

Internal Rate of Return Method (IRR)

Internal rate of return, sometimes represented by the symbol i^* or the acronym *IRR*, is defined as the interest rate that reduces the present worth of given cash flow to zero. It represents the percentage or rate of interest

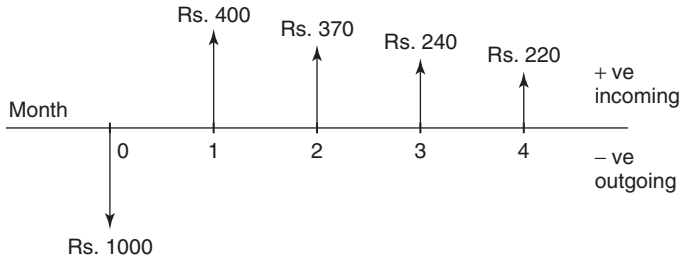


Figure 3.23 Cash-flow diagram to illustrate the IRR

earned on the unrecovered balance of an investment, at any point of time, and further, the earned recovered balance is reduced to zero at the end of the project.

An illustrative computation showing how a part of the cash flow at the end of a year goes towards payment of interest due on the outstanding (unrecovered) balance of investment, and the remainder liquidates the outstanding investment, is shown in Table 3.11, and the cash-flow diagram is shown in Figure 3.23. In this example, at the end of the proposal's life (four years), the entire investment has just been recovered, and the applicable rate of interest (10 per cent) is a special and unique rate called the *IRR*.

In principle, *IRR* can be determined by equating the net present worth of the cash flow to zero, i.e., setting the difference of the benefits and cost of the present worth to zero, as shown below:

$$(PW)_{benefits} - (PW)_{cost} = 0 \tag{3.17}$$

It may be noted that the above equation is too complex to solve directly, and usually the *IRR* is determined by a trial-and-error procedure as outlined below:

Step 1 Assume a trial rate of return (i^*).

Step 2 Counting the cost as negative and the income as positive, find the equivalent net worth of all costs and incomes.

Step 3 If the equivalent net worth is positive, then the income from the investment is worth more than the cost of investment and the actual percentage return is higher than the trial rate, and vice versa.

Step 4 Adjust the estimate of the trial rate of return and go to step 2 again until one value of i is found that results in a positive equivalent net worth, and another higher value of i is found with negative equivalent net worth.

Step 5 Solve for the applicable value of i^* by interpolation.

Table 3.11 Illustration for *IRR*

End of Year t	Cash Flow at EOY, t	Unrecovered Balance at the Beginning of Year t	Interest Earned on the Unrecovered Balance During the Year	Unrecovered Balance at the Beginning of the Year (t + 1)
0	−1,000	–	–	−1,000
1	400	−1,000	−100	−700
2	370	−700	−70	−400
3	240	−400	−40	−200
4	220	−200	−20	0

10% is the IRR

It is not possible to calculate the rate of return for the cash flows involving cost alone or revenue alone, as can be observed from Equation 3.17. Also, the *IRR* method should not be used for ranking of projects as it may give erroneous results, not in line with the results obtained from other methods of analysis such as annual cost method or present worth method. For such cases, we need to evaluate alternatives using incremental rate of return method.

Incremental Rate of Return (IRoR)

If an alternative requires a higher initial investment than the other and evaluation is of the rate of return on the increment of initial investment, the return yielded on this extra investment is called the incremental rate of return (IRoR). The incremental analysis is based on the principle that every rupee of investment is as good as the other.

The analysis makes the assumptions that sufficient funds are available to finance the alternatives with the highest investment, and that there are opportunities available to utilize the surplus funds at a rate higher than the MARR, should there be any excess funds after financing the alternative with the lower investment costs. Some of the steps involved in the incremental analysis are summarized below.

Step 1 List out all the alternatives in ascending order of their first cost or initial investment. It may be pointed out at this stage that in most cases, alternatives with the lowest investment are likely to turn out to be the 'do nothing' alternative.

Step 2 Compare the rate of return of all alternatives with the assumed MARR, and check if the rate of return is at least equal to the MARR. If not, the alternative is dropped and not considered in the further analysis.

Step 3 Prepare the cash-flow diagram on incremental basis between the alternatives being examined and the current alternative (to begin with, we have taken the alternative with the lowest initial investment).

Step 4 When an alternative that has just been examined is acceptable (rate of return is more than the MARR), it becomes the current best replacing the earlier one. The new best is examined with the next higher investment alternative.

Step 5 In case rate of return is less than MARR, the alternative under examination is ruled out and the current alternative remains the lucrative one. The current best is compared to the next higher investment.

Step 6 The above process is repeated till all the alternatives have been looked into and the best alternative is selected.

3.7 EFFECT OF TAXATION ON COMPARISON OF ALTERNATIVES

In the discussion so far, any effect that a taxation regime may have as far as comparison and evaluation of economic alternatives is considered has not been taken into account. It should, however, be borne in mind that prevailing tax laws impose taxes on companies on the basis of the annual income generated through conduct of business, and not only the rates of taxes vary from time to time, but also certain incentives and relief in taxes are provided on certain expenditures for various reasons. Companies often take advantage of such schemes and use available funds in a manner so as to optimize their interests. While it is not the intention here to discuss taxation rules at any length, the example below is included only to place in perspective the discussion on taxation and alternative comparison.

Example

Basic details of gross earnings, etc., for two companies A and B are given in Table 3.12. As can be seen from the table, apart from the gross earnings, there are 'admissible *expenses*', whose deduction from the former is allowed before working out the tax liability (the amount of taxes to be paid). The simple computations shown in the table assume a flat 40% tax rate (though this percentage is often actually a function of the level of gross or net earnings).

Table 3.12 Illustration of effect of taxation on comparison of alternatives

Sl. No.	Item Description	A	B	Remarks
1	Gross earnings	Rs. 150 lakh	Rs. 150 lakh	
2	Admissible expenses (AE)	Rs. 50 lakh	Rs. 25 lakh	
3	Pre-tax profits	Rs. 100 lakh	Rs. 125 lakh	(1) − (2)*
4	Tax payable	Rs. 40 lakh	Rs. 50 lakh	40% of (3)
5	Net profit	Rs. 60 lakh	Rs. 75 lakh	(3) − (4)

*In this example, for simplicity a 100% deduction has been made for the admissible expenses. At times, only a certain percentage of these expenses is allowed to be deducted from the gross earnings for the purpose of tax calculations. For example, in case this percentage was 50%, the pre-tax profits for A and B would be 125 and 137.5, respectively, and the subsequent figures modified accordingly.

It can be seen from Table 3.12 that for the same gross earnings, the tax payable and the net profit can be quite different depending upon the 'admissible expenses', which often include expenses on equipment purchase, research and development, etc. Depending on the prevailing regulations at any point in time, this becomes an important consideration in the comparison of different alternatives. More on taxation and their implication on selection of alternatives are given in Chapter 9.

3.8 EFFECT OF INFLATION ON CASH FLOW

Construction projects, by and large, take a number of months and during this period, the cost of labour, materials, plant and machinery may undergo an inflationary trend. Inflation in general is defined as the increase in the price level resulting in decrease in purchasing power of money.

Since general inflation results in price rise in all goods, the relative prices remain constant. Hence, it is possible to disregard escalation. In India, it is normal practice to use 12 per cent as discount rate. According to IRC:SP:61-2004, where there is a large difference between the rates of inflation and interest, the discount rate is evaluated using the following expression:

$$\textit{Modified discount rate} = \left[\left(\frac{1 + \textit{interest rate}\ (\%)}{1 + \dfrac{\textit{inflation rate}\ (\%)}{100}}\right) - 1\right] \times 100\% \qquad (3.18)$$

Considering an interest rate of 12% and an inflation rate of 8%, the modified discount rate = [{(1 + 12%) ÷ (1 + 8%) − 1}] × 100% = 3.70%

3.9 EVALUATION OF PUBLIC PROJECTS: DISCUSSION ON BENEFIT-COST RATIO

In all the preceding discussions, we focused on private projects, where the objective was to maximize the profits while adhering to the contractual requirements of the projects. The objective of public projects is to provide goods/services to the public at the minimum cost. The benefit accrued from such projects should at least recover the cost of the projects. Public projects here mean those funded by government (state or centre).

The government has a responsibility towards public welfare. Some examples of public projects are construction of dams and highways, and projects related to defence, education and public health.

Many government projects cannot be evaluated strictly in commercial terms. Profit, taxes and payoff periods take a backseat in public projects, unlike in private projects. Yet, the resources of the government, however large, are also limited, and it has to choose between different alternatives based on some criterion. The objectives could be as given below:

- To check the viability of the project economically
- To select the most viable project from a set of economically viable projects
- To rank or order the economically viable alternatives for a given situation

One of the most commonly used criteria to evaluate public projects is benefit-cost (B/C) ratio. B/C ratio is defined as the ratio of benefit to public and cost to the government. B/C ratio can be obtained using Present Worth analysis, Future Worth method, or Annual Worth method. Classification of benefits or costs is mostly arbitrary and causes confusion. When should particular project consequences be classified as benefits and not as a cost? Different classification of impacts as benefits and costs can result in significantly different benefit-cost ratio values and can even result into negative and zero values for projects with a net positive economic impact (Au 1988, Halvorsen and Ruby 1981, both cited in Lund 1992).

The numerator of all the B/C ratios is usually taken to mean the net benefit, which is the sum of all benefits minus all the disbenefits. Due care is required to determine the numerator and denominator, since some confusion may arise in a problem. However, if the definition is clearly understood, the ratio can be correctly obtained. When comparisons of several alternatives are to be made, a basis of incremental analysis should be made. B/C ratio of more than one indicates that benefit outweighs cost and, thus, the investment is justifiable. B/C ratio of 1.5 means that each rupee in cost yields Rs. 1.5 in benefits over the lifetime of the project. B/C ratio of less than one indicates that benefit accrued from the project is less than the cost required to be invested and, thus, the investment is not justifiable. B/C analysis per se does not select an alternative; however, it is a useful tool to simply distinguish between an acceptable alternative and an unacceptable one.

B/C criterion is not designed to rank or order the project. It sets only a minimum level of acceptability and does not pretend to identify the source of investment funds. Basically, it develops information useful for the purpose of decision–making, depending on the availability of investment funds, capital rationing and the special features of social merit and economic objectives.

3.9.1 Benefit/cost Criteria

Some of the benefit/cost criteria for evaluation of alternatives are:

- Minimum investment
- Maximum benefit
- Aspiration level
- Maximum advantage of benefits over cost (B−C)
- Highest B/C ratio
- Largest investment that has a benefit/cost ratio greater than 1
- Maximum incremental advantage of benefit over cost ($\Delta B - \Delta C$)
- Maximum incremental benefit/cost ratio ($\Delta B/\Delta C$)
- Largest investment that has an incremental B/C ratio greater than 1.0

Some of the limitations in the application of benefit/cost analysis relate to estimating the monetary benefits, especially in the case of intangible aspects such as recreation and the saving of human lives. Further, as in

other methods, the determination of proper interest rates and time horizon for discounting future benefits and costs has long been a source of controversy.

Example

A government is planning for a hydroelectric project that will also provide flood control, irrigation and recreation benefits. The established benefits and cost of three alternatives are given in Table 3.13. The interest rate to be used for the analysis is five per cent, and the life of each of the alternatives X, Y and Z is to be assumed as 50 years. Choose the best alternative.

Table 3.13 Data for B/C ratio computation (All values in rupees million)

Alternatives	**X**	**Y**	**Z**
Initial cost	250.00	350.00	500.00
Annual power sales	10.00	12.00	18.00
Annual flood control	2.50	3.50	5.00
Annual irrigation benefit	3.50	4.50	6.00
Annual recreation benefits	1.00	2.00	3.50
Annual operation and maintenance	2.00	2.50	3.50

Table 3.14 Solution for B/C ratio problem

Annual Cost, C, Computation	**X**	**Y**	**Z**
Annual cost equivalent = P × (A/P, 5%, 50)	250 × 0.0548 = 13.70	350 × 0.0548 = 19.18	500 × 0.0548 = 27.40
Operation and maintenance cost	2.00	2.50	3.50
Receipt on power sales	−10.00	−12.00	−18.00
Total annual cost equivalent	5.70	9.68	12.90
Annual benefit, B, computation			
Annual flood control	2.50	3.50	5.00
Annual irrigation benefit	3.50	4.50	6.00
Annual recreation benefits	1.00	2.00	3.50
Total benefit	7.00	10.00	14.50
B/C ratio	1.23	1.03	1.12

Table 3.15 Computation for incremental analysis

Annual Cost, C, Computation	**Y − X**	**Z − X**
Annual cost equivalent = P × (A/P, 5%, 50)	100 × 0.0548 = 5.48	250 × 0.0548 = 13.7
Operation and maintenance cost	0.50	1.50
Receipt on power sales	−2.00	−8.00
Total annual cost equivalent	3.98	7.20
Annual benefit, B, computations		
Annual flood control	1.00	2.50
Annual irrigation benefit	1.00	2.50
Annual recreation benefits	1.00	2.50
Total benefit	3.00	7.50
B/C ratio	3.00/3.98 = 0.75	7.50/7.20 = 1.04
Remarks	B/C < 1, Reject Y	B/C > 1, Accept Z

Solution

The computation of B/C using annual cost/worth analysis is given in Table 3.14.

B/C ratio of the three alternatives is more than 1 and, thus, all the above alternatives are economically viable. In the second stage, incremental analysis is performed to identify the best alternative. The computation is shown in Table 3.15.

REFERENCES

1. _____, IRC:SP: 61-2004, *An Approach Document on Whole Life Costing for Bridges in India*, Indian Road Congress, New Delhi.
2. Au, T., 1988, 'Profit measures and methods of economic analysis for capital project selection', *Journal of Management in Engineering*, ASCE 4(3), pp. 217–228.
3. Blank, L., and Tarquin, A., 1989, *Engineering Economy*, 3rd ed., New York: McGraw-Hill, Inc.
4. Halvorsen, R., and Ruby, M.G., 1981, *Benefit-cost analysis of air pollution control*, Lexington Books, Lexington, Mass.
5. Harris, F., and McCaffer, R., 2005, *Modern Construction Management*, 5th ed., Blackwell Publishing, India.
6. Lund, J.R., 1992, 'Benefit-cost ratios: Failures and alternatives', *Journal of Water Resources Planning and Management*, ASCE 118(1), pp. 94–100.
7. Panneerselvam, R., 2005, *Engineering Economics*, fourth print, Prentice Hall of India Private Limited, New Delhi.
8. Riggs, J.L., Bedworth, D.D., and Randhawa, S.U., 2004, *Engineering Economics*, 4th ed., Tata McGraw-Hill, New Delhi.
9. Taylor, G.A., 1980, *Managerial and Engineering Economics*, 3rd ed., D. Van Nostrand Company.
10. Thuesen, G.J., and Fabrycky, W.J., 1989, *Engineering Economy*, 7th ed., Englewood Cliffs, NJ: Prentice-Hall, Inc.
11. Van Horne, J.C., and Wachowicz J.M., 2001, *Fundamentals of Financial Management*, 11th ed., Pearson Prentice Hall, New Delhi.
12. Wellington, A.M., 1887, *The Economic Theory of the Location of Railways,* John Wiley and Sons, New York.

SOLVED EXAMPLES

Example 3.1

For the following data of a project, (a) prepare the month-wise running account bill, (b) prepare the cash inflow diagram for the contractor, and (c) prepare the cash outflow diagram for the owner.

- Value of contract: Rs. 7,625,000 (Seventy-six lakh twenty-five thousand rupees only)
- Duration: Four months
- The owner makes an advance payment of Rs. 5 lakh, which is to be recovered in four equal instalments.
- The owner also supplies materials worth Rs. 3.2 lakh, which is also to be recovered equally from each running account (RA) bill.
- The owner will recover from the payments made to the contractor two per cent of the value of the work done as income tax deducted at source, and deposit this amount with the Reserve Bank of India (RBI).

Table Q3.1.1 Construction schedule

S. No.	Item Description	Unit	Total Quantity	Rate (Rs.)	Amount (Rs.)	Quantities to be Executed in Month 1	Month 2	Month 3	Month 4
1	Earthwork in excavation	m^3	500	50	25,000	500			
2	R.C.C	m^3	1,000	4,000	4,000,000	250	500	250	
3	Brickwork	m^3	2,000	1,000	2,000,000	500	600	900	
4	Sanitary works	L.S	—	—	200,000			50%	50%
5	Electrical works	L.S	—	—	200,000			50%	50%
6	Woodwork	L.S	—	—	250,000			50%	50%
7	Finishing work	m^2	4,750	200	950,000				4,750

Contractor has prepared the construction schedule, which has been approved by the owner. The construction schedule is shown in Table Q3.1.1. Also shown are the estimated quantities that are likely to be executed during each month.

Additional conditions and assumptions:

- The cost for the contractor to execute a particular item is 90 per cent of their quoted rates.
- The total cost for a particular item consists of labour (20 per cent), material (55 per cent), plant and machinery (10 per cent), and subcontractor cost (10 per cent).
- Assume that there is no delay in payment to labour, but a delay of one month occurs in paying to the subcontractors, material suppliers, and plant and machinery supplier.
- Retention is 10 per cent of billed amount in every bill. Fifty per cent retention amount is payable after one month of practical completion, while the remaining 50 per cent is payable six months later.

Solution

The R.A. bill computation is shown in Table S3.1.1. The computation for cash inflow for the contractor is shown in Table S3.1.2.

Reconciliation

Payment to be received for work done 7,625,000

Table S3.1.1 Computation for RA bill (Value in Rs.)

S. No.	Item Description	Months 1	2	3	4
1	Earthwork in excavation	25,000			
2	R.C.C.	1,00,000	2,000,000	1,000,000	
3	Brickwork	500,000	600,000	900,000	
4	Sanitary works			100,000	100,000
5	Electrical works			100,000	100,000
6	Woodwork			125,000	125,000
7	Finishing work				950,000
	Total work progress	**1,525,000**	**2,600,000**	**2,225,000**	**1,275,000**

Table S3.1.2 Computation for cash inflow

	Item Description	0	1	2	3	4	5	6	…	10
9	Total work progress (from above)		1,525,000	26,000,00	2,225,000	1,275,000				
	Recoveries									
a	Towards advance payment	500,000	125,000	125,000	125,000	125,000				
b	Towards material advance	320,000	80,000	80,000	80,000	80,000				
c	TDS		30,500	52,000	44,500	25,500				
d	Retention @10% of gross invoice (0.1* row 9)		152,500	260,000	222,500	127,500				
e	Total deductions (a + b + c + d)		388,000	517,000	472,000	358,000				
f	Payment due (9-e)		1,289,500	2,343,000	1,975,500	1,044,500				
f	Release of retention						381,250			381,250
g	Cash received (one-month delay)	820,000		1,289,500	2,343,000	1,975,500	1,425,750			381,250

The R.A. bill will look thus:

	1st R.A. bill	2nd R.A. bill	3rd R.A. bill	4th bill
Work done by measurement	1,525,000	4,125,000 (cumulative)	6,325,000 (cumulative)	7,625,000 (cumulative)
Recoveries as per statement (TDS, mobilization advance and material advance	235,500	492,500 (cumulative)	741,500 (cumulative)	972,500 (cumulative)
Payment due	1,289,500	3,632,500 (cumulative)	5,583,500 (cumulative)	6,652,500 (cumulative)
Payment already made	Nil	1,289,500	3,632,500	5,583,500
Payment to be made now	1,289,500	2,343,000	1,951,000	1,069,000

Less TDS at 2%	(−) 152,500
Less advance payment	(−) 500,000
Less materials issued by owner	(−) 320,000
Actual payments to be received	6,652,500
Actual payments made	1,289,500 + 2,343,000 + 1,951,000 + 1,060,000 = 6,652,500 (Hence, reconciled)

The cash inflow diagram for the contractor is as shown in Figure S3.1.1.

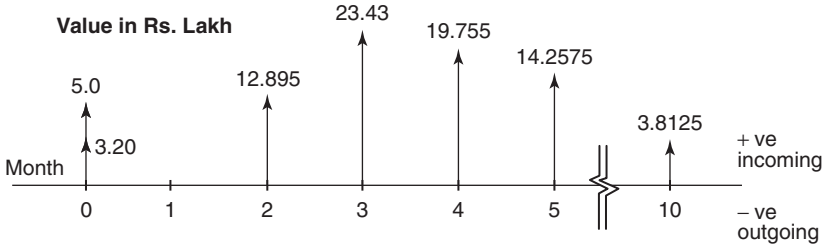


Figure S3.1.1 Cash-inflow diagram for the contractor

A contractor is also supposed to pay to his subcontractors, suppliers for the materials, and labour for the work done during a month. These factors are taken into account while preparing the cash outflows of a contractor. The computation for cash-flow diagram is performed in Table S3.1.3 using the conditions of the problem given earlier.

Table S3.1.3 Computation for cash outflows

	Item Description	1	2	3	4	5	6	…	10
9	Total work progress (from above)	1,525,000	2,600,000	2,225,000	1,275,000				
10	Total cost (0.9 of above value)	1,372,500	2,340,000	2,002,500	1,147,500				
11	Cost towards labour @20% of total cost	274,500	468,000	400,500	229,500				
12	Cost towards material @60% of total cost	823,500	1,404,000	1,201,500	688,500				
13	Cost towards subcontractors @10% of total cost	137,250	234,000	200,250	114,750				
14	Cost towards plant and machinery @10% of total cost	137,250	234,000	200,250	114,750				
15	Labour payment (no delay)	274,500	468,000	400,500	229,500				
16	Material suppliers' payment (one-month delay)		823,500	1,404,000	1,201,500	688,500			
17	Subcontractors' payment (one-month delay)		137,250	234,000	200,250	114,750			
18	Plant and machinery supplier's payment (one-month delay)		137,250	234,000	200,250	114,750			
19	Outgoings total	274,500	1,566,000	2,272,500	1,831,500	918,000			

Based on the above computation, the cash outflow diagram for the contractor is shown in Figure S3.1.2.

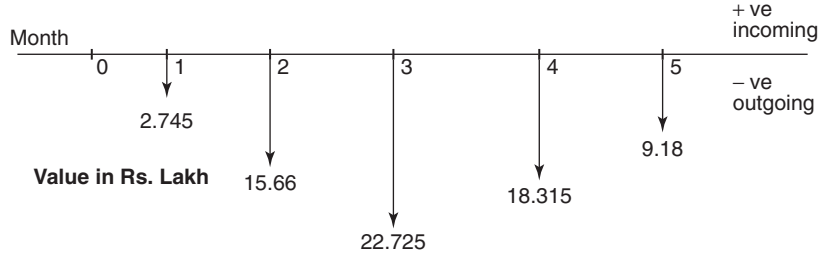


Figure S3.1.2 Cash-outflow diagram for the contractor

Example 3.2

A manufacturing company purchases materials worth Rs. 50 lakh every year. Calculate the present worth of material purchase for a five-year period, if the material price follows a geometric pattern with (a) g = −5%, (b) g = 0%, and (c) g = 5%. The interest rate can be assumed to be eight per cent.

Solution

Given c = Rs. 50 lakh. The cash-flow diagram for the given data is given in Figure S.3.2.1.

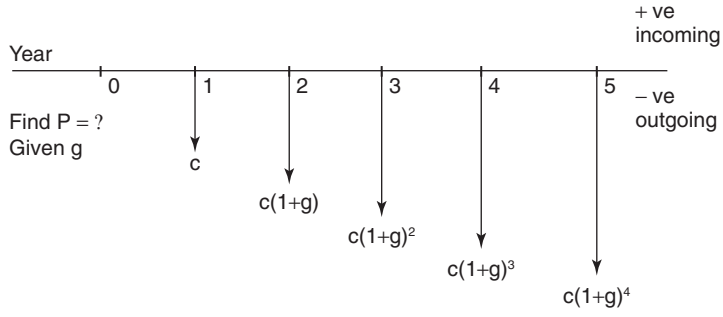


Figure S3.2.1 Cash-flow diagram

(a) The value of g = −5%

Geometric gradient factor corresponding to an interest rate of 8% = 3.641 (from interest table)
Thus, present worth of material purchase = 5,000,000 × 3.641 = Rs. 18,206,834.45

(b) The value of g = 0%

This implies there is no change in the annual cost and, thus, the cash-flow diagram would be as shown in Figure S3.2.2.

Thus, the present worth of material purchase = 5,000,000 (P/A, 8%, 5) = Rs. 28,733,000.00

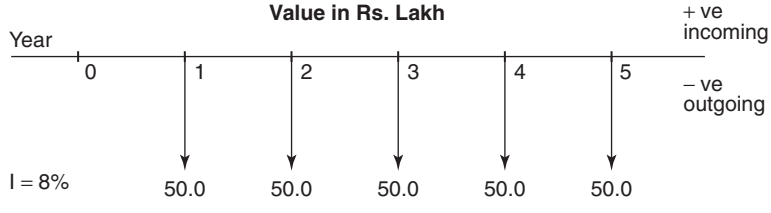


Figure S3.2.2 Cash-flow diagram (g = 0%)

(c) The value of g = 5%

Geometric gradient factor corresponding to an interest rate of 8% = 4.379 (from Equation 3.14)
Thus, present worth = 5,000,000 × 4.379 = Rs. 21,897,368.98

Example 3.3

An alternative, A, requires an initial investment of Rs. 500,000 and an annual expense of Rs. 250,000 for the next 10 years. Alternative B, on the other hand, requires an initial investment of Rs. 750,000 and an annual expense of Rs. 200,000 for the next 10 years. Which alternative would you prefer if interest rate were 10 per cent?

Solution

The cash-flow diagram corresponding to alternative A is given in Figure S3.3.1.

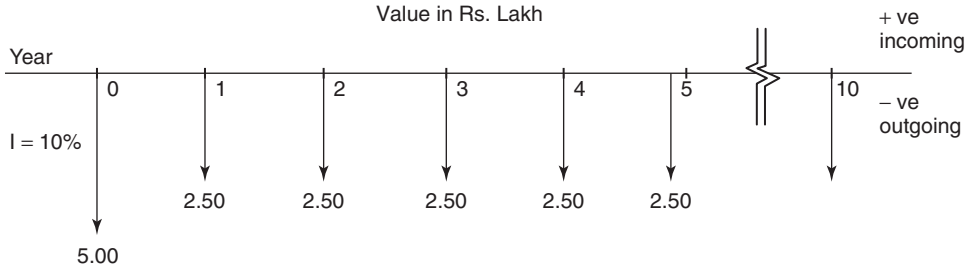


Figure S3.3.1 Cash flow diagram for Alternative A

Present cost of alternative A = 500,000 + 250,000 (P/A, 10%, 10)
= 500,000 + 250,000 (6.1446)
= Rs. 2,036,150.00

The cash-flow diagram corresponding to alternative B is given in Figure S3.3.2.

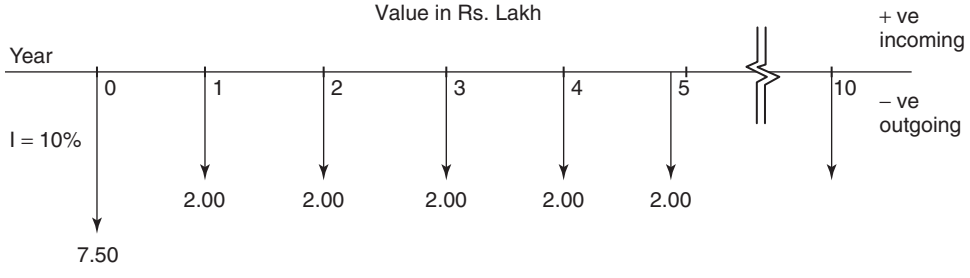


Figure S3.3.2 Cash flow diagram for Alternative B

Present cost of alternative B = 750,000 + 200,000 (P/A, 10%, 10)
= 750,000 + 200,000 (6.1446)
= Rs. 1,978,920.00

It can be noticed from the two cash-flow diagrams that the cost data alone are provided. Thus, alternative with the lowest cost at present time would be the most preferable. In this case, since the present cost for alternative B is less than that of alternative A, it is preferable to choose alternative B.

Example 3.4

What is the present equivalent value of Rs. 50,000, five years from now at 14 per cent compounded semi-annually?

Solution

The cash-flow diagram is shown in Figure S3.4.1. For an interest rate of 14 per cent compounded semi-annually, the effective interest rate can be obtained by using the formula

$$i_{eff} = \left(\left(1 + \frac{i_{nom}}{m \times 100}\right)^{m} - 1\right) \times 100$$

Here, $m = 2, i_{nom} = 14\%$

Thus,

$$i_{eff} = \left[\left(1 + \frac{14}{2 \times 100}\right)^{2} - 1\right] \times 100$$

$$= 14.49\% = 0.1449$$

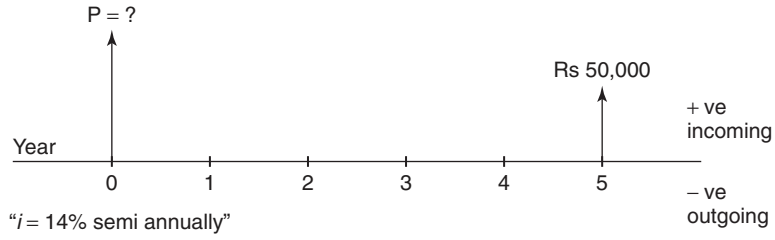


Figure S3.4.1 Cash flow diagram for Example 3.4

Thus, present equivalent $P = 50{,}000\ (P/F, i_{eff}, 5)$
The value of $(P/F, i_{eff}, 5)$ can be obtained by the formula $P = F/(1 + i_{eff})^5$
$= 50{,}000\ (.5085) = \text{Rs. } 25{,}417.46$

In order to use the interest table, one needs to interpolate for an interest rate of i = 14.49%. For this, select (P/F, 14%, 5) and (P/F, 15%, 5), and interpolate linearly for an approximate value of (P/F, 14.49%, 5).

Example 3.5

An investor has to choose between the following two investment options:

(a) Investing in a bond that earns him a rate of return of 10 per cent on an eight-year investment

(b) Depositing Rs. 7.5 lakh for first three years and earning Rs. 5 lakh every year from the end of 4th year to the end of 8th year

Which investment option is desirable?

Solution

The cash-flow diagram for investment option 'b' is shown in Figure S3.5.1.

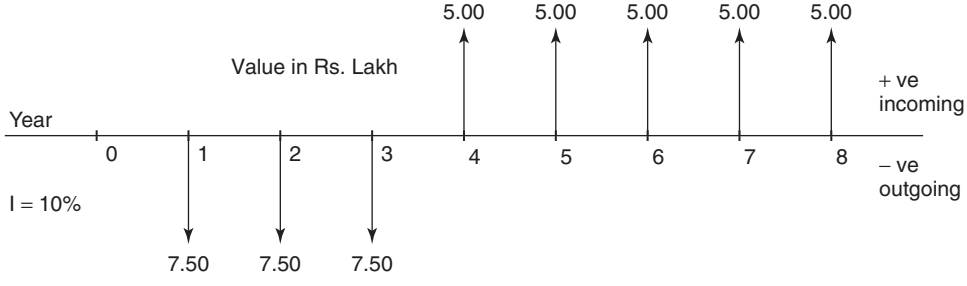


Figure S3.5.1 Cash-flow diagram for option 'b'

The net present worth of option 'b' at I = 10% (rate of return of option 'a') is found out. If it happens to be positive, it would indicate that option 'b' yields a return higher than 10%. If the net present worth were negative, it would indicate that return of option 'b' is less than 10%.

$$\text{The net present worth} = \text{total benefits} - \text{total costs}$$

$$\text{The net present worth of option 'b'} = +\ 500{,}000/(1+i)^4 + 500{,}000/(1+i)^5 + 500{,}000/(1+i)^6 + 500{,}000/(1+i)^7 + 500{,}000/(1+i)^8 - 750{,}000/(1+i)^1 - 750{,}000/(1+i)^2 - 750{,}000/(1+i)^3$$

$$= \text{Rs. } 1{,}738{,}488.45$$

Since the net present worth of option 'b' yields positive net present worth at an interest rate of 10%, this option is better than option 'a'. It may be noted that the above problem can be solved using interest tables also.

Example 3.6

Type 'A' design of a dam costs Rs. 50 crore to construct and an expense of Rs. 7.5 crore every year to operate and maintain it. Type 'B' design of the dam, on the other hand, would require Rs. 75 crore to construct and an annual expense of Rs. 5 crore to operate and maintain. Both the designs have considered 100 years as the design life of the dam. The minimum required rate of return is five per cent. Which design should be given a go-ahead? It may be noted that the above problem can be solved using interest tables also.

Solution

Both the dam options can be assumed to be permanent as the design life is 100 years (significantly large).

The cash-flow diagram corresponding to type 'A' design of dam is given in Figure S3.6.1.

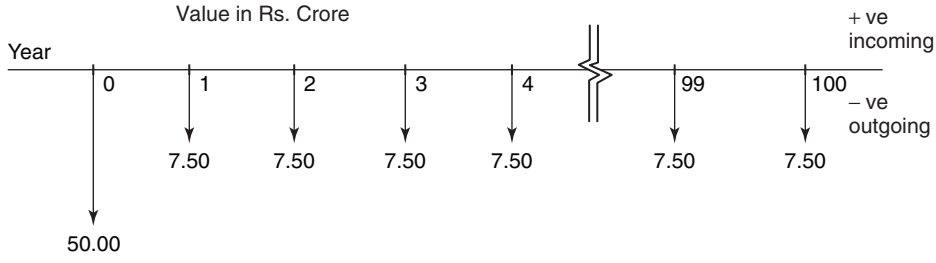


Figure S3.6.1 Cash-flow diagram for type 'A' dam design

$$\text{The net present cost for this case} = 500{,}000{,}000 + 75{,}000{,}000/0.05$$
$$= \text{Rs. } 2{,}000{,}000{,}000.00$$

The cash-flow diagram of type 'B' design of dam is given in Figure S3.6.2.

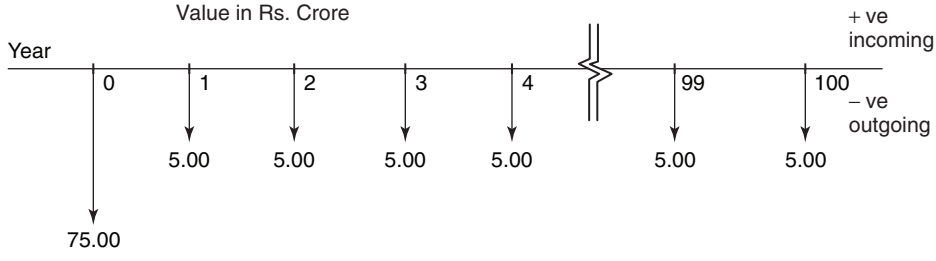


Figure S3.6.2 Cash-flow diagram for type 'B' dam design

$$\text{The net present cost for this case} = 750{,}000{,}000 + 50{,}000{,}000/0.05$$
$$= \text{Rs. } 1{,}750{,}000{,}000.00$$

The net Present cost of type 'B' design of dam is less. Hence, it is preferable to go for type 'B' design of dam (note that all the values given pertain to cost).

Example 3.7

A contractor has been awarded to do a job that requires procurement of an equipment. Two brands A and B are available to perform the job. Brand A requires an investment of Rs. 450,000, while brand B requires an investment of Rs. 725,000. The annual savings generated by brands A and B are given in Table Q3.7.1. Which brand of equipment should the contractor choose if the interest rate is eight per cent?

Table Q3.7.1

	Saving Details Year-Wise		
Option	**1**	**2**	**3**
Brand A	225,000	225,000	225,000
Brand B	300,000	300,000	300,000

Solution

The cash-flow diagram associated with brand 'A' is as shown in Figure S3.7.1.

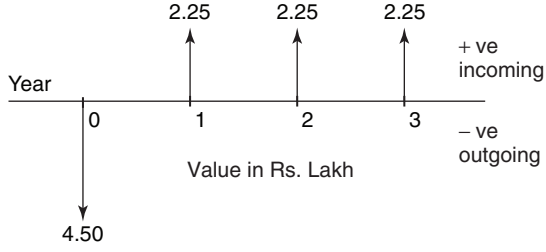


Figure S3.7.1 Cash-flow diagram for brand 'A'

The net present worth of brand 'A' is computed thus:

The net present worth = present worth of savings − present worth of cost
= 225,000 (P/A, 8%, 3) − 450,000
= 225,000 (2.5771) − 450,000
= 579,847.50 − 450,000
= Rs. 129,847.50

The cash-flow diagram associated with brand 'B' is as shown in Figure S3.7.2. The net present worth of brand 'B' is computed thus:

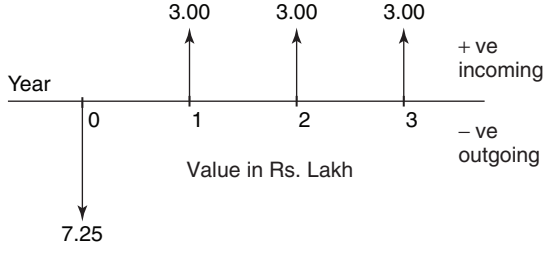


Figure S3.7.2 Cash-flow diagram for brand 'B'

The net present worth = present worth of savings − present worth of cost
= 300,000 (P/A, 8%, 3) − 725,000
= 300,000 (2.5771) − 725,000
= 773,130 − 725,0000
= Rs. 48,130.00

The net present worth associated with brand A is greater than the net present worth associated with brand B. So, select brand A.

Example 3.8

A piece of land has been purchased at Rs. 40 lakh. An investment of an additional Rs. 20 lakh has been made to construct a small shopping complex on this piece of land. It is expected to fetch an annual rental of Rs. 75,000 to the owner, while the cost towards its upkeep, tax, etc., is expected to be Rs. 30,000 annually. The owner plans to sell the entire plot with constructed facilities at an expected price of Rs. 120 lakh at the end of five years. What percent rate of return will be earned by the owner on this investment?

Solution

The cash-flow diagram for the given data is shown in Figure S3.8.1

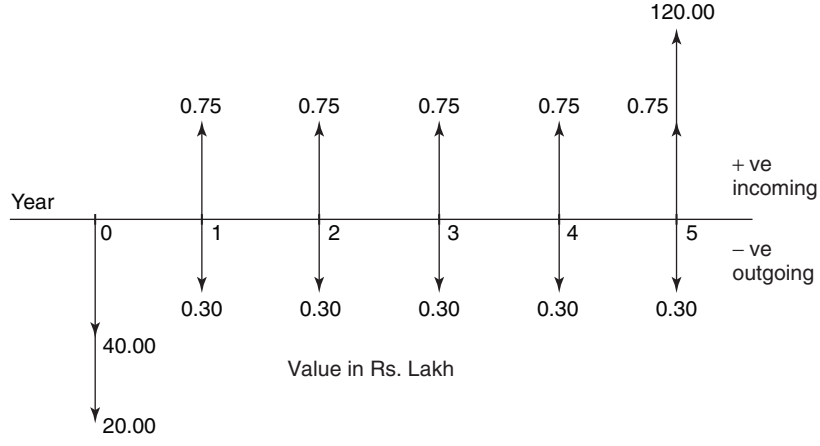


Figure S3.8.1 Cash-flow diagram

Assuming a trial rate of return of 15%,

Net present worth = −Rs. 6,000,000 − Rs. 30,000 × (P/A, 15%, 5) + Rs. 75,000 × (P/A, 15%, 5) + Rs. 12,000,000 × (P/F, 15%, 5)

Thus, net present worth = −Rs. 6,000,000 − Rs. 30,000 × 3.3522 + Rs. 75,000 × 3.3522 + Rs. 12,000,000 × 0.4972 = 117,249

The positive net present worth indicates that the rate of return is higher than 15%.

Let us assume a higher value of rate of return, say 20%.

Net present worth = −Rs. 600,000 − Rs. 30,000 × (P/A, 20%, 5) + Rs. 75,000 × (P/A, 20%, 5) + Rs. 12,000,000 × (P/F, 20%, 5)

Net present worth = −Rs. 6,000,000 − Rs. 30,000 × 2.9906 + Rs. 75,000 × 2.9906 + Rs. 12,000,000 × 0.4019 = − 1,042,623

Thus, the rate of return at which net present worth becomes zero, that is, the value of i at which exactly the same return is met, lies somewhere between 15% and 20%, which can be found out by interpolation.

$$i = 15 + \frac{(20 - 15) \times 117{,}249}{(117{,}249 - (-1{,}042{,}623)} = 15.50\%$$

Example 3.9

A supplier of prefabricated railway sleepers procures each piece of sleeper for Rs. 4,000. The demand for sleepers is 350 units, and it is estimated that a similar demand would prevail for another three years. Equipment to manufacture sleepers is available for Rs. 18 lakh. The annual operating cost for producing 350 sleepers is estimated to cost Rs. 7 lakh for year 1, with 10 per cent increase every year for years 2 and 3. If the equipment

has no salvage value at the end of three years, should the supplier continue to outsource it or should he buy the equipment and start producing the sleepers on his own? The minimum attractive rate of return is 15 per cent.

Solution

There are two options before the supplier: (1) option of outsourcing, and (2) buy the equipment and start manufacturing on his own.

The cash-flow diagram for option 1 is shown in Figure S3.9.1. The total cost to outsource = Rs. 4,000 × 350 = 1,400,000 for years 1, 2 and 3.

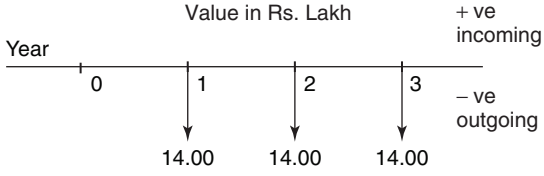


Figure S3.9.1 Cash-flow diagram for option of outsourcing

The cash-flow diagram for option 2 is shown in Figure S3.9.2.

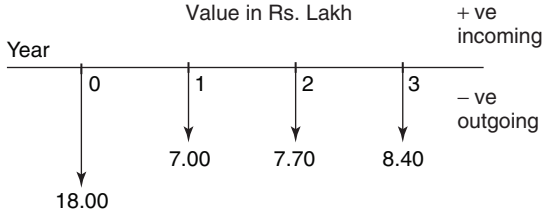


Figure S3.9.2 Cash-flow diagram for option of manufacturing on his own

As can be seen from both the cash-flow diagrams, only cost-related information is available. Thus, it is not possible to calculate the rate of return for individual options. In view of this, it is proposed that the incremental rate of return method be used for comparison of alternatives. The incremental cash-flow diagram is shown in Figure S3.9.3.

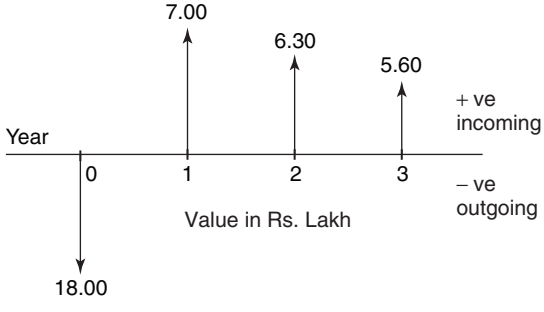


Figure S3.9.3 Cash-flow diagram on incremental basis

It can be observed that at time zero, the supplier needs to invest Rs. 1,800,000 in equipment, which will give savings of Rs. 1,400,000 − Rs. 700,000 = Rs. 700,000 in year 1; Rs. 1,400,000 − Rs. 770,000 = Rs. 630,000 in year 2; Rs. 1,400,000 − Rs. 840,000 = Rs. 560,000 in year 3.

A trial-and-error process is adopted to find out the rate of return.

Assume a trial rate of return equal to minimum attractive rate of return, that is, $i = 15\%$

The net present worth at $i = 15\%$ = −Rs. 1,800,000 + Rs. 700,000 × (P/F, 15%, 1) + Rs. 630,000 × (P/F, 15%, 2) + Rs. 560,000 × (P/F, 15%, 3)

Thus, net present worth = −Rs. 346,700

Since the net present worth at $i = 15\%$ is negative, it is not desirable to invest in purchase of equipment. In other words, it is advisable to continue with the existing system of outsourcing.

Example 3.10

Solve the problem using (a) present worth method, (b) annual worth method, and (c) internal rate of return method for the given cash-flow data. Check your results with incremental rate of return method. The minimum attractive rate of return is 10 per cent.

End of Year ⟶	**0**	**1**	**2**	**3**	**4**
Project X	−50,000	5,000	17,500	30,000	42,500
Project Y	−50,000	40,000	15,000	15,000	15,000

Solution

The cash-flow diagrams for projects X and Y are shown in Figure S3.10.1 and Figure S3.10.2, respectively.

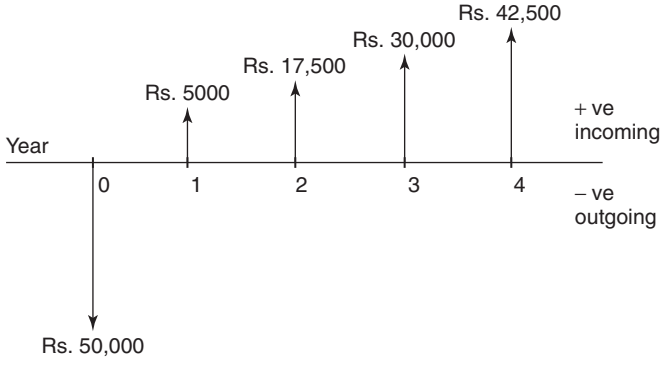


Figure S3.10.1 Cash-flow diagram for project X

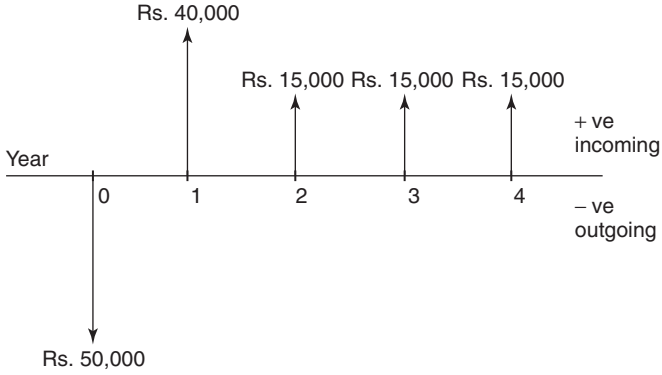


Figure S3.10.2 Cash-flow diagram for project Y

(a) Present worth method

The net present worth for project X = −50,000 + 5,000 × (P/F, 10%, 1) + 17,500 × (P/F, 10%, 2) + 30,000 × (P/F, 10%, 3) + 42,500 × (P/F, 10%, 4)

NPW = −50,000 + 5,000 × 0.9091 + 17,500 × 0.82645 + 30,000 × 0.75131 + 42500 × 0.6830
= Rs. 20,575

The net present worth for project Y = −50,000 + 40,000 × (P/F, 10%, 1) + 15,000 × (P/F, 10%, 2) + 15000 × (P/F, 10%, 3) + 15,000 × (P/F, 10%, 4)

NPW = −50,000 + 40,000 × 0.9091 + 15,000 × 0.8264 + 15,000 × 0.7513 + 15,000 × 0.6830
= Rs. 20,274

Hence, choose X since the net present worth for project X is more.

(b) Annual worth method

The equivalent annual cost for project X = −50,000 × (A/P, 10%, 4) + 5,000 × (P/F, 10%, 1) (A/P, 10%, 4) + 17,500 × (P/F, 10%, 2) (A/P, 10%, 4) + 30,000 × (P/F, 10%, 3) (A/P, 10%, 4) + 42,500 × (P/F, 10%, 4) (A/P, 10%, 4)

Equivalent annual cost = −50,000 × 0.3155 + 5,000 × 0.9091 × 0.3155 + 17,500 × 0.8264 × 0.3155 + 30,000 × 0.7513 × 0.3155 + 42,500 × 0.6830 × 0.3155

Equivalent annual worth = Rs. 6,491.09

The equivalent annual cost for project Y = −50,000 × (A/P, 10%, 4) + 40,000 × (P/F, 10%, 1) (A/P, 10%, 4) + 15,000 × (P/F, 10%, 2) (A/P, 10%, 4) + 15,000 × (P/F, 10%, 3) (A/P, 10%, 4) + 15,000 * (P/F, 10%, 4) (A/P, 10%, 4)

Equivalent annual worth = −50,000 × 0.3155 + 40,000 × 0.9091 × 0.3155 + 15,000 × 0.8264 × 0.3155 + 15000 × 0.7513 × 0.3155 + 15000 × 0.6830 × 0.3155

Equivalent annual cost = Rs. 6,396.60

Hence, choose X since equivalent annual worth for project X is higher.

(c) Internal rate of return method

For project X,

Assume i = 20%

The net present worth at i = 20% = −50,000 + 5,000 × (P/F, 20%, 1) + 17,500 × (P/F, 20%, 2) + 30,000 × (P/F, 20%, 3) + 42,500 × (P/F, 20%, 4)

NPW = −50,000 + 5,000 × 0.8333 + 17,500 × 0.6944 + 30,000 × 0.5787 + 42500 × 0.4823
= 4177.25

Assume i = 25%

The net present worth at i = 25% = −50,000 + 5,000 × (P/F, 25%, 1) + 17,500 × (P/F, 25%, 2) + 30000 × (P/F, 25%, 3) + 42,500 × (P/F, 25%, 4)

NPW = −50,000 + 5,000 × 0.8 + 17,500 × 0.64 + 30,000 × 0.512 + 42,500 × 0.4096 = −2032.00

By interpolation,

i = 23.371%

For project Y,

Assume i = 30%

The net present worth at i = 30% = −50,000 + 40,000 × (P/F, 30%, 1) + 15,000 × (P/F, 30%, 2) + 15,000 × (P/F, 30%, 3) + 15,000 × (P/F, 30%, 4)

NPW = −50,000 + 40,000 × 0.7692 + 15,000 × 0.5917 + 15,000 × 0.4552 + 15,000 × 0.3501 = 1723

Assume i = 35%

The net present worth = −50,000 + 40000 × (P/F, 35%, 1) + 15,000 × (P/F, 35%, 2) + 15,000 × (P/F, 35%, 3) + 15,000 × (P/F, 35%, 4)

NPW = −50,000 + 40,000 × 0.741 + 15,000 × 0.5509 + 15,000 × 0.4098 + 15,000 × 0.3052 = −1371.5

By interpolation,

i = 32.78%

Hence, choose project Y since it gives a higher rate of return.

(d) Incremental rate of return method

Let project X be preferred over project Y. Then, the cash-flow diagram on incremental basis would be as shown in Figure S3.10.3.

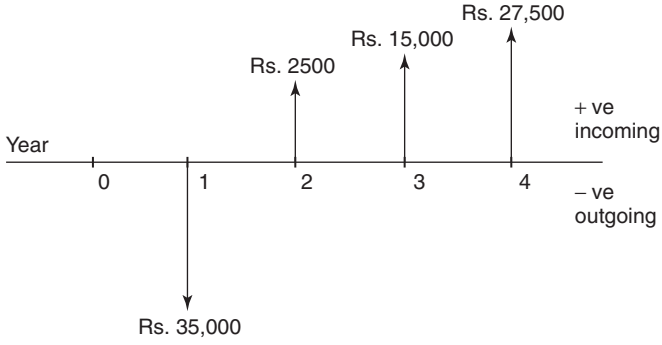


Figure S3.10.3 Incremental cash-flow diagram (project X preferred over Y

We perform trial-and-error to find the rate of return for the cash-flow diagram shown in Figure S3.10.3.

Assume i = 10%

The net present worth = 0 − 35,000/(P/F, 10%, 1) + 2,500(P/F, 10%, 2) + 15,000 (P/F, 10%, 3)^3 + 27,500 (P/F, 10%, 4) = 300.40 (check)

Assume i = 15%

The net present worth = 0 − 35,000(P/F, 15%, 1) + 2,500(P/F, 15%, 2) + 15,000 (P/F, 15%, 3) + 27,500 (P/F, 15%, 4) =− 2958.46 (check)

By interpolation, $i = 10 + \frac{(15 - 10) \times 300.4}{(300.4 - (-2958.46))} = 10.46\%$

Since 10.46% (exact value of *i*) > 10% (MARR), prefer X over Y.

Remarks

With the help of this example, it has been emphasised that the results could vary if internal rate of return method is applied for ranking the projects. In this example, we saw that although project Y has higher internal rate of return, it is not the best alternative as suggested by present worth method, annual worth method and incremental rate of return method. Thus, internal rate of return method should not be used for ranking, and we should always use incremental rate of return for the reasons described inside the text.

REVIEW QUESTIONS

1. Fill in the blanks.
   a. Rs. 50,000 now is equivalent to Rs. _____ after 10 years. at a rate of interest 10%.
   b. Rs. 50,000 is equivalent to an annual payment of Rs. _____ for a period of 10 years at an interest rate of 10%.
   c. The present value of a sum of Rs. 129,700 to be received 10 years hence at an interest rate of 10% is Rs. _____.

d. Rs. 8,137.50 every year for next 10 years at a rate of interest of 10% is equivalent to a future sum of Rs. _____ at the end of 10 years.

e. Rs. 50,000 payable at the end of 10 years is equivalent to Rs. _____ at present at an interest rate of 10%.

f. At an interest rate of _____ %, Rs. 5 lakh invested today will be worth Rs. 10 lakh in 9 years.

2. Write short notes on the payback period method.

3. A construction company is contemplating purchase of a crawler excavator costing Rs. 6,000,000. The finance officials have identified the cost and benefits of the investments, which are given below. Company proposes to use the excavator for four years. The required rate of returns is 16%. Ascertain the payback period and discounted payback period.

(Initial investment	1st year	2nd year	3rd year	4th year
Rs. 6,000,000	Rs. 1,500,000	Rs. 2,200,000	Rs. 2,800,000	Rs. 3,500,000

4. A project is estimated to cost Rs. 2.4 crore and is expected to be completed in 12 months. You may assume the following:

- 10% of the work is completed in the first two months. Take the progress to be 3% and 7% in the first and second months, respectively.
- 80% of the work is completed during the next 8 months.
- 10% of the work is completed in the last two months. Take the progress to be 8% and 2% in the eleventh and twelfth months, respectively.

The following data may also be useful:

- The contractor has to submit an earnest money deposit of 10% of the estimated cost at the time of submitting his tender in the form of demand draft. This amount is to be retained by the client as initial security deposit.
- The contractor needs to be paid a mobilization advance of Rs. 16 lakh initially, which can be recovered in four equal instalments, beginning with the third running bill.
- An amount equivalent to 5% of the gross amount due is to be retained in each bill towards (additional) security deposit.
- 50% of the total security deposit is to be returned to the contractor at the end of 6 months after the project has completed, and the remaining part after another 6 months.
- 10% recovery is to be made towards income tax. Payment to the department of income tax is to be made at the end of the project.
- The contractor submits monthly bills on the first day of the following month.
- On the client's side, it takes about 20 days to process a bill, and payment is made only on the last day of the same month in which the bill is submitted.
- Client maintains his funds with the bank and is paid an interest of 5% compounded monthly.
- Rate of interest on borrowing from the bank is 15% per annum, compounded monthly.

Based on the information provided: (a) Clearly show how much cash the client needs at different points in time as the project progresses. (b) How much money does the client need to borrow at different points of time, and what will be his total liability at the completion of the project (say, at the end of month 24)? Clearly outline all the assumptions you make in your analysis.

5. A newly opened toll bridge has a life expectancy of 25 years. Considering a rise in construction and other costs, a replacement bridge at the end of that time is expected to cost Rs. 60,000,000. The bridge is expected to have an average of 100,000 toll-paying vehicles per month for the 25 years. At the end of every month, the tolls will be deposited into an account bearing annual interest of 11 per cent compounded monthly. The operating and maintenance expenses for the toll bridge are estimated at Re 0.80 per vehicle.
   a. How much toll must be collected per vehicle in order to accumulate Rs. 60,000,000 by the end of 25 years?
   b. What should be total toll per vehicle in order to include operating and maintenance cost as well as replacement cost?
   c. What annual effective interest rate is being earned?
6. A small shopping complex can be constructed for Rs. 5,000,000. The financing requires Rs. 500,000 down (you pay) and a Rs. 4,500,000 loan at 10%, with equal annual payments. The gross income for the first year is estimated at Rs. 600,000 and is expected to increase 5% per year thereafter. The operating and maintenance costs and taxes should average about 40% of the gross income and rise at the same 5% annual rate. The resale value in 30 years is estimated at Rs. 10,000,000. Find the rate of return on this investment over the 30-year life. Solve the problem assuming that the increase follows (i) arithmetic gradient and (ii) geometric gradient.
7. Due to increasing age and downtime, the productivity of a contractor's excavator is expected to decline with each passing year. Using the given data, calculate the price in terms of rupees per m^3 that the contractor must charge to cover the cost of buying and selling the excavator.

   Cost new = Rs. 2,200,000

   Resale price at the end of year 6 = Rs. 900,000

   Contractor borrows at i = 10%

   Production decline at r = (−) 6%

   Production for year 1 = 160,000 m^3

   Assume all funds are credited at end of year
8. A contractor has a contract to construct a 3,600-metre-long tunnel in 30 months. He is trying to decide whether to do the job with his own forces or subcontract the job. You are to calculate the equivalent monthly cost under each alternative. His i = 1.75% per month. Production under both alternatives will be 120 metres per month.

   Alternative A: Buy a tunnelling machine and work with own force

   a. Cost of tunnelling machine = Rs. 20,000,000
   b. Salvage value of machine at end of month 30 = Rs. 4,000,000
   c. Cost of labour and material = Rs. 10,000/m for the first 10 months, and increasing by 0.5% per month at EOM each month thereafter (i.e., Rs. 10,050/m at EOM11, Rs. 10,100.25 at EOM 12, etc.)

   Alternative B: Subcontract the work

   Cost is Rs. 27,000 per metre of tunnel
9. A sewage treatment plant has three possible schemes of sewage carriage and treatment systems. If the life of the scheme is 20 years, which scheme should be recommended as the most economic?

Scheme	Installation Cost (Rs.)	Annual Running Cost (Rs.)
A	18,250,000	7,250,000
B	20,200,000	4, 600,000
C	24,200,000	4,000,000

   Use 15% to represent the cost of capital. If the cost of capital were 10%, would the recommendation alter?

10. A town built on a river is considering building an additional bridge across the river. Two proposals have been put forward for bridges at different sites. The costs of each proposal are summarized as follows:

Description	Bridge A	Bridge B
Initial cost of bridge	Rs. 6,500,000	Rs. 5,000,000
Initial cost of road works	Rs. 3,500,000	Rs. 3,000,000
Annual maintenance of bridge	Rs. 50,000	Rs. 90,000 for first 15 years and 110,000 thereafter
Annual maintenance of roads	Rs. 30,000	Rs. 25,000
Life of bridge	60 years	30 years
Life of roads	60 years	60 years

With the cost of capital at 9%, which proposal should be adopted? (Assessment of the proposals to be carried out by a comparison of their present worth)

11. Your firm owns a large earthmoving machine that may be renovated to increase its production output by an extra 8 m^3/hour, with no increase in operating costs. The renovation will cost Rs. 500,000. The earthmoving machine is expected to last another eight years, with zero salvage value at the end of that time. Earthmoving for this machine is currently being contracted at Rs. 14/m^3. The company is making an 18 per cent return on their invested capital. You are asked to recommend whether or not this is a good investment for the firm. Assume the equipment works 1,800 hours per year. (Hint i = 18%)

12. For the following data of a project, calculate the payment received by the contractor at different time periods. Also prepare (a) the month-wise running account bill, (b) the cash inflow diagram for the contractor, and (c) the cash outflow diagram for the owner.
    - Value of contract: Rs. 10,000,000 (one crore rupees)
    - Time period: 4 months
    - Payment schedule: First month 15% of the contract value, 2nd month 40% (cumulative), 3rd month 80% (cumulative) and 4th month 100% (cumulative)
    - The owner makes an advance payment of Rs. 5 lakh, which is to be recovered in 4 equal instalments
    - The owner also supplies materials to the value of about Rs. 3.2 lakh, which value is also to be recovered equally from each running account bill
    - In addition, the owner will recover from the payments made to the contractor 2% of the value of the work done as income tax deducted at source, and deposit this amount with the Reserve Bank of India

13. Two equipments 'A' and 'B' have the capability of satisfactorily performing a required function. Equipment 'B' has an initial cost of Rs. 160,000 and expected salvage value of Rs. 20,000 at the end of its four-year service life. Equipment 'A' costs Rs. 45,000 less initially, with an economic life one year shorter than that of 'B'; but 'A' has no salvage value, and its annual operating costs exceed those of 'B' by Rs. 12,500. When the required rate of return is 15%, state which alternative is preferred when comparison is by
    a. the common multiple method
    b. A 2 year study period (assuming the assets are needed for only 2 years).