UNIT 4: ELECTROSTATICS

Electric charge:- is the property of some elementary particles which causes them to exert forces on one another.

Electrostatics:- is the study of electric charges at rest, their electric field and potentials.

Properties of electric charges

If an object

*gains electrons, it will be negatively charged.

*loses electrons, it will be positively charged.

Processes of charging:-

1.*Charging by rubbing (friction)*:-

When two neutral bodies are rubbed with each other, there will be a transfer of electrons from one body to the other.

When a rubber rod is rubbed with fur, the **rubber rod** *gains electros* while the **fur** *loses electrons*.

When a glass rod is rubbed with silk cloth, the **glass rod** *loses electrons* while the **silk cloth** *gains electrons*.

By rubbing (friction), bodies are oppositely charged.

2.Charging by contact(touching)

When a charged body is brought into contact with a neutral body, charges of the same sign will be transferred from the charging body to the neutral body. This process is known as *charging by conduction*.

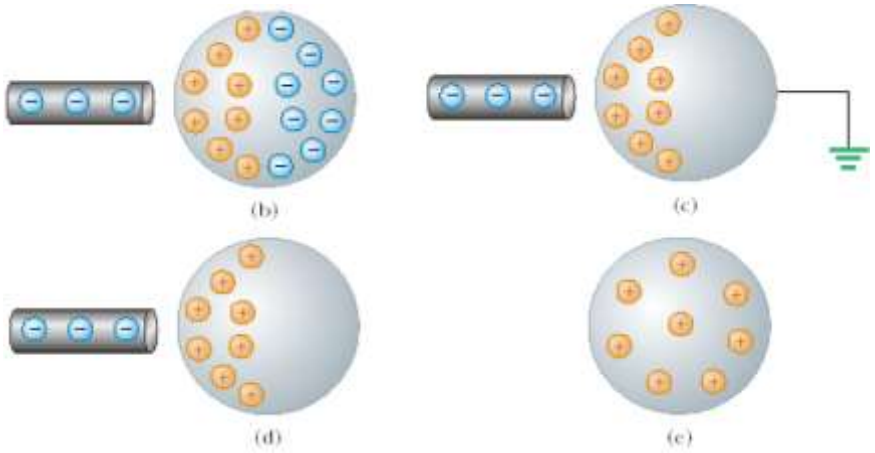
By conduction, bodies are similarly charged.

3.Charging by induction

Charging an object by induction requires no contact with the object inducing the charge. This is in contrast to charging an object by *rubbing* and *conduction,* which does require contact between the two objects.

By induction, bodies are oppositely charged.

Consider a neutral conducting sphere insulated from ground, as shown in the figure. When a negatively charged rubber rod is brought near the sphere, the region of the sphere nearest the rod obtains an excess of positive charge while the region farthest from the rod obtains an equal excess of negative charge. If a conducting wire is connected from the sphere to ground, some of the electrons in the conductor move out of the sphere through the ground wire and into the Earth. If the wire to ground is then removed, the conducting sphere contains an excess of *induced* positive charge. This induced positive charge remains on the ungrounded sphere

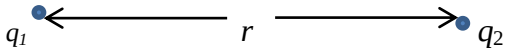


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(a) A neutral metallic sphere, with equal numbers of positive and negative charges.
(b) The charge on the neutral sphere is redistributed when a charged rubber rod is placed near the sphere.
(c) When the sphere is grounded, some of its electrons leave through the ground wire.
(d) When the ground connection is removed, the sphere has excess positive charge that is non-uniformly distributed.
(e) When the rod is removed, the excess positive charge becomes uniformly distributed over the surface of the sphere.

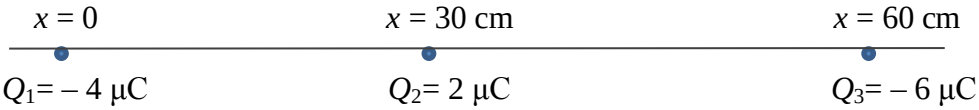
Coulombs Law:- " The electrostatic force between two charged bodies is directly proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between them."

$F = \frac{Kq_1q_2}{r^2}$ Where $K = 9\times10^9$ N m^2/C^2 is a proportionality constant.



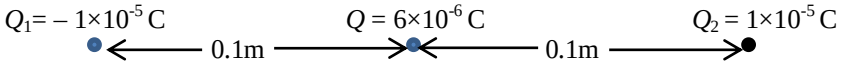
Example

1. A 6 μC charge is located 30 cm from an 8 μC charge. What is the electrostatic force exerted on each other? **Ans**. F= 4.8 N repulsive
2. What force does a 4 μC charge exert on a – 2 μC charge when they are 2 cm apart? **Ans**. F= 180 N attractive.
3. Find the resultant force on Q_2 by the two fixed charges Q_1 and Q_3. **Ans**. F= 0.4 N toward right



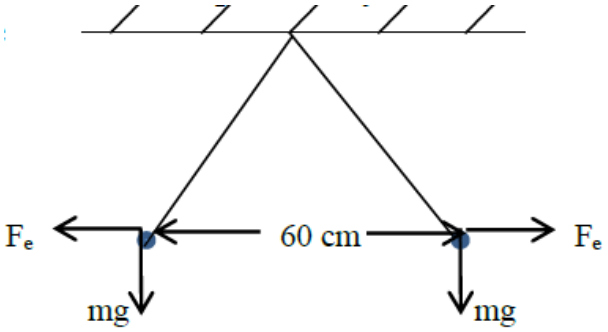
Exercise

1. Two equal point charges exert a repulsive force of 0.9 N on each other when they are 40 cm apart. Calculate the charge on each. **Ans**. $Q = \pm 4$ μC.
2. What is the resultant force on Q by Q_1 and Q_2? **Ans**. F_R= 108 N toward left

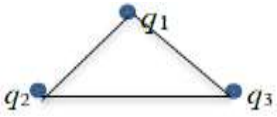


3. Two charges, 6 μC and – 5 μC, exert a 3 N force on each other. How far are they apart? **Ans**. r = 0.3 m
4. If the force between two spheres 4cm apart is 3.6×10^{-18} N, how many excess electrons are on each sphere? **Ans**. n = 5000 electrons
5. Two point charges, Q_1= 1 μC and Q_2 = 4 μC, are separated by 3 m. At what point between these charges will the resultant force on a charge q be zero? **Ans**. x = 1 m from Q_1
6. Two balls, each 0.3 g, are suspended from a 50 cm long string. The balls are separated by 60 cm and stay in equilibrium due to a repulsive force. If the charges on each are equal, calculate the charge on each ball.

Ans. $Q = \pm 3\times10^{-7}$ C



7. Two charges, Q_1 = 0.4 μC and Q_3 = –0.3 μC, are placed at x = –30 *cm* and y = –30 *cm* respectively. What is the magnitude of the resultant force on charge Q_2 = 0.1 μC placed at the origin due to Q_1 and Q_3? **Ans**. $F_R = 5\times10^{-3}$ N 37^0 below +ve x- axis
8. Three point charges, each +10 μC, are placed at the vertices of an equilateral triangle 30 cm on a side. Calculate the force exerted on q_3 by q_1 and q_2. **Ans**. $F_R = 10\sqrt{3}$ N



9. Two point charges q_1 and q_2 that are 90 cm apart exert a force of 0.05 N on each other. If q_1+q_2= 4.5 μC, what are q_1 and q_2? **Ans**. 3 μC and 1.5 μC

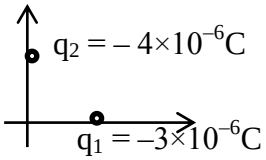
10. Two small charged spheres repel each other with a force of 0.09 N when they are 10 cm apart. What will be the repulsive force if they are moved until they are 30 cm apart? **Ans**. F= 0.01 N
11. The electric force between two charges is 4×10^{-3} N when they are 3m apart. For what separation would the electric force between the two charges be 1.6×10^{-2} N? **Ans**. r = 1.5 m
12. If a shower of protons from outer space deposits equal charges ***q*** on the Earth (mass M) as well as on the moon (mass m) until electric repulsion equals the gravitational attraction, the charge ***q*** is equal to

A) GmM B) $\frac{GmM}{k}$ C) $\sqrt{\frac{mM}{k}}$ D) $\sqrt{\frac{GmM}{k}}$

13. Two particles, with charges q_1 and q_2, are on the x axis, with q_1 at x= **a** and q_2 at x= –2**a**. For the net force on a third charged particle, at x = 0, to be zero, q_1 and q_2 must be related by

A**)** $q_2= 4q_1$ B) $q_2= 2q_1$ C) $q_2= -2q_1$ D) $q_2= -4q_1$

14. Charges q_1 and q_2 are located at positions (3m, 0) on the x axis and (0, 3m) on the y axis respectively as shown in the following figure.



What should be the x and y components of the net electric force on a 1μC charge placed at the origin?

A) $F_x = 4\times10^{-3}$N and $F_y = 3\times10^{-3}$N C) $F_x = -4\times10^{-3}$N and $F_y = -3\times10^{-3}$N
B) $F_x = 3\times10^{-3}$N and $F_y = 4\times10^{-3}$N D) $F_x = -3\times10^{-3}$N and $F_y = -4\times10^{-3}$N

Questions from national exam

1. Two point charges $q_1= 4\times10^{-7}$ C and $q_2 = 6\times10^{-7}$ C are 3 m apart. What is the magnitude of the electric force between the two charges?

A**)** 2.4×10^{-4} N B) 7.2×10^{-4} N C) 2.4×10^{-5} N D) 7.2×10^{-5} N

2. The force between two charges is 1.5 N. What would be the force if the distance between the charges is increased to five times its original value?

A**)** 0.06 N B) 0.6 N C) 0.03 N D) 0.3 N

3. If the coulombs force between two like charges Q_1 and Q_2 separated by a distance r is equal to F_o. What would be the new force of repulsion if the magnitude of the charges Q_1 is doubled and that of Q_2 is halved? (Provided the separation remains the same)

A) $\frac{F_0}{3}$ B) $\frac{F_0}{2}$ C) F_o D) $2F_o$

4. Coulomb's law for the force between electric charges most closely resembles

A) the law conservation of energy. C) Newton's law of gravitation.
B) Newton's second law of motion. D) the law of conservation of mass.

5. Two negative point charges are 2 m apart and repel each other with a force of 2 N. When the distance between the charges is doubled, the force between them is

A) one fourth as great. C) twice as great.
B) one half as great. D) four times as great.

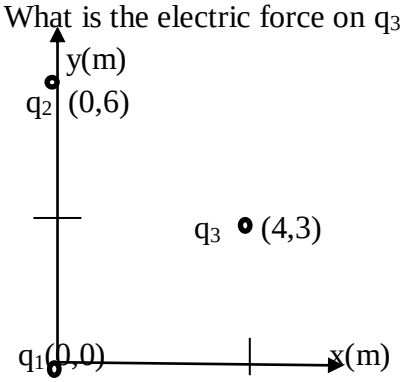
6. What is the magnitude of the electric force between two protons separated by 2×10^{-15} m? ($q_{pro}= 1.6\times10^{-19}$ C)

A) 67.5 N B) 115.0 N C**)** 57.6 N D) 83.4 N

7. The distance between two point charges is doubled. What will happen to the force of interaction between the charges?

A) The force will be doubled.
B) The force will be reduced by half.
C) The force will be reduced by a factor of 1/4
D) The force will not be affected.

8. A charge of $q_1 = -10\times10^{-9}$C, $q_2 = 10\times10^{-9}$C and $q_3 = -2.5\times10^{-8}$C are placed on x-y plane as shown in the figure below. What is the electric force on q_3 by q_1 and q_2?



A**)** 1.08×10^{-7}N in +ve y-direction

B) 9×10^{-7}N in –ve y-direction

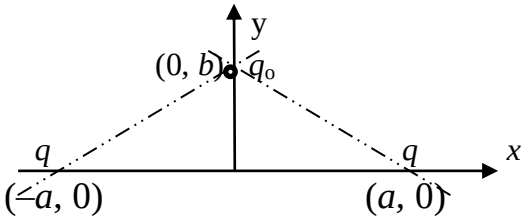
C) 1.42×10^{-7}N in +ve x-direction

D) 1.42×10^{-7}N in –ve x-direction

9. Suppose that three point charges are placed along a straight line and the electrostatic force exerted on the middle charge is zero. What can we conclude about the charges?

(A) The sighns of the three charges should be the same.
(B) The charges at the ends must have the same sign.
(C) The middle charge should be placed at the center between the two charges.
(D) The charge at the middle and at the left end should have same sign.

10. Two like charges of value q are fixed at $x = a$ and $x = -a$ as shown in the diagram below.



If a test charge q_o is released at y = b, which path will it follow?

A) It stays at y = b
B) along y-axis
C) along a line $y = \frac{b}{a}x + b$
D) along a line $y = -\frac{b}{a}x + b$

Electric Field

Electric field:- is a region around a charged object where another charged object experiences electric force.

Electric lines of force (Electric field lines):- are imaginary lines drawn to visualize electric field.

Electric lines of force

*originate from positive charge and terminate at negative charge.

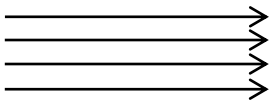
*never cross each other.

*are closely spaced in a region where the field is strong.

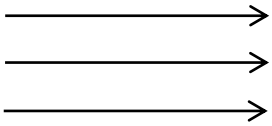
*are widely spaced in a region where the field is weak.

*are straight, parallel and uniformly spaced if the field is uniform.

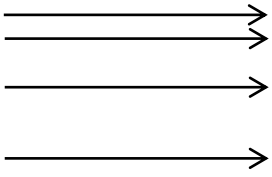
*are not uniformly spaced if the field is non-uniform.



strong and uniform field

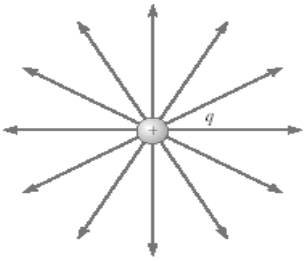


weak and uniform field

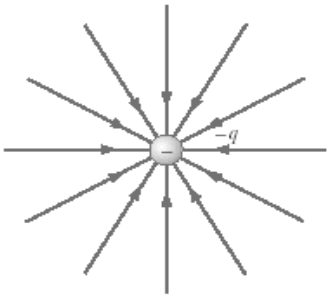


non-uniform field

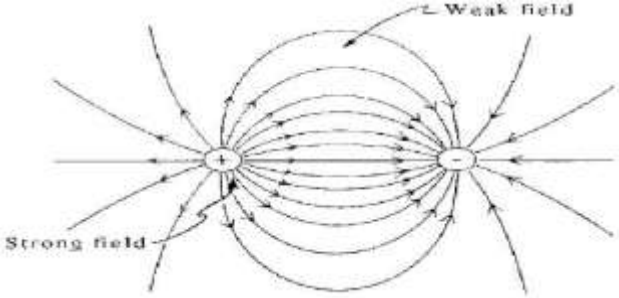
Electric field lines



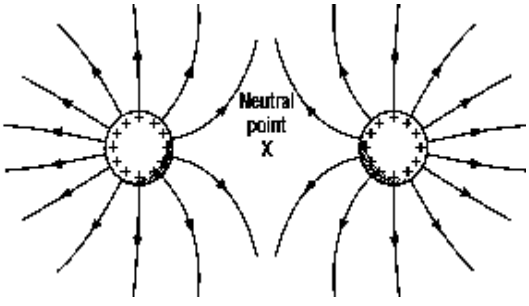
around positive point charge



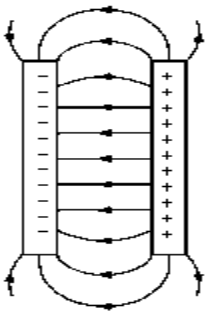
around negative point charge



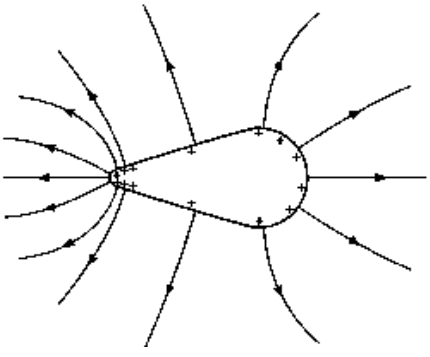
around positive and negative charge



near two positive point charges



between two oppositely charged parallel plates



around a positively charged object, rounded at one end and sharp at the other end

Electric field strength (intensity) at a point in space is defined as the ratio of electric force on a positive test charge placed at that point to the magnitude of the test charge.

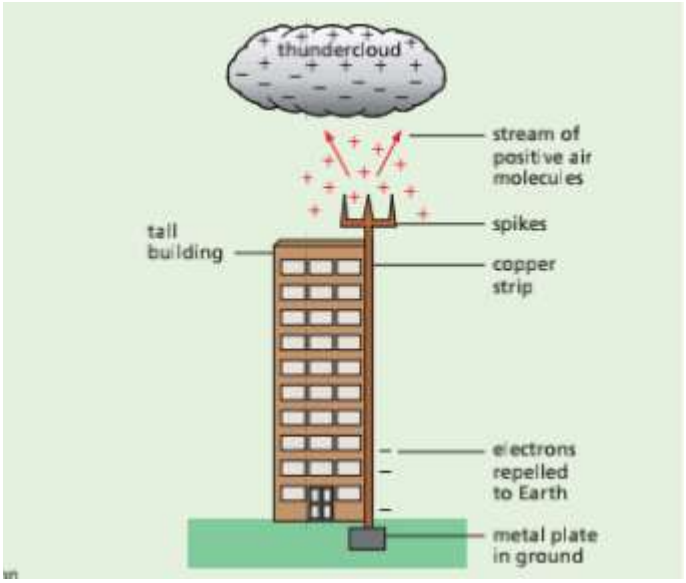
$\vec{E} = \frac{\vec{F}}{q^+}$ or E= $\frac{KQ}{r^2}$ The magnitude of an electric field at a distance r from a point charge Q

Dangers Of Static Electricity

Lightning

A tall building is protected by a lightning conductor consisting of a thick copper strip fixed on the outside of the building connecting metal spikes at the top to the metal plate in the ground as in figure below.

Thunderclouds carry charges; a negatively charged cloud passing overhead repels electrons from the spikes to the Earth. The points of the spikes are left with a large positive charge (charge concentrates on sharp points) which removes electrons from the nearby air molecules, so charging them positively and causing them to be repelled from the spike. This effect, called **action at points**, results in an 'electric wind' of positive air molecules streaming from the thundercloud in a lightning flash. If a flash occurs it is now less violent and the conductor gives it an easy path to ground.



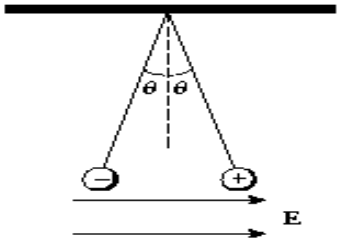
Example

1. What is the field strength at a point 0.1 m from a point charge of 5 μC? **Ans**. E = 4.5×10^6 N/C outward
2. Compute the electric field intensity at 30 cm from a charge of –5 μC. **Ans**. E=5×10^5N/C toward the charge
3. Two point charges, +5 μC and +6 μC, are placed 60 cm apart. Calculate the electric field intensity at the midpoint between the charges. **Ans**. $E_{mid} = 10^5$ N/C toward +5 μC
4. An object, having a charge of 24 μC, is placed in a uniform electric field of 600 N/C directed vertically upward. What is the mass of this object if it floats in the electric field? **Ans**. $m = 1.44\times10^{-3}$ kg

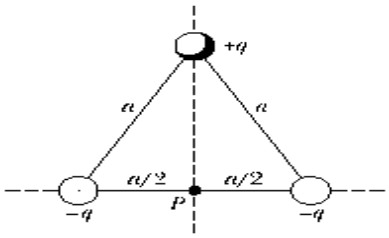
Exercise

1. Two point charges, $q_1 = 4\times10^{-7}$ C and $q_2 = -4\times10^{-7}$ C, are held 6 cm apart. What is electric field at a point midway between the two charges? **Ans**. $E_{mid} = 8\times10^6$ N/C toward q_2
2. Two point charges, $q_1 = 2\times10^{-11}$ C and $q_2 = 3\times10^{-11}$ C, are 20 cm apart. What is the electric field at the midpoint between the charges? **Ans**. $E_{mid} = 9$ N/C toward q_1

3. A particle has a mass of 5×10^{-2} kg and a charge of 4 μC. When the particle is released at a place where there is a uniform electric field, it experiences an acceleration of 6×10^{3}m/s^2. What is the magnitude of the electric field? **Ans**. E= 7.5×10^{7} N/C
4. If an electric force on a 5 μC charge in a uniform electric field is 2×10^{-2} N, what would be the force on a 9 μC charge placed at the same point? **Ans**. F = 3.6×10^{-2} N
5. An object having a net charge of 24 μC is placed in a uniform electric field of 500 N/C. What is mass of this object if it floats in the electric field? **Ans**. $m = 1.2\times10^{-3}$ kg
6. An electron, $m = 9\times10^{-31}$ kg and $q = 1.6\times10^{-19}$ C, is released from rest in a region where there is a uniform field of 900 N/C.
   a) What is acceleration of the electron? **Ans**. a = 1.6×10^{14} m/s^2
   b) How long will it take to move a distance of 3.2 m? **Ans**. $t = 2\times10^{-7}$ sec
7. A point charge Q_1 is at x = 0 and Q_2 is at x = 8 cm. What is the relationship between these charges if the resultant electric field strength is zero at x = 16 cm?
   A) $Q_1 = 2Q_2$ B) $Q_1 = -0.25Q_2$ C) $Q_2 = 0.25Q_1$ D**)** $Q_1 = -4Q_2$
8. Two small spheres, each 0.6g, are suspended by light strings 10cm in length as in the figure. If the spheres have charges of -5μC and 5μC, determine the electric field that enables the spheres to be in equilibrium at 37^0 with the vertical. **Ans**. E= 900 N/C



9. Three charges of equal magnitude q are placed at the corners of an equilateral triangle of side length a. Find the magnitude of the electric field at point P, midway between the negative charges. **Ans**. E= $\frac{4kq}{3a^2}$



Questions from national exam

1. Two charges q_1 and q_2 are placed 50 cm apart. If $q_2 = -4q_1$, at what point along the line passing through the charges is the electric field zero?
   A) 50cm from q_2 B) 50cm form q_1 C) 100cm from q_2 D**)** B and C
2. An electron ($m = 9\times10^{-31}$ kg and $q_e = 1.6\times10^{-19}$ C is released from rest in a uniform electric field of 9×10^{3} N/C. How far does the electron move in 2×10^{-8} sec?
   A) 0.16 m B**)** 0.32 m C) 1.6 m D) 3.2 m
3. A particle of mass 2×10^{-4} kg and charge 8×10^{-6} C is released in a region where there is a constant electric field of 250 N/C. What is the acceleration of the particle due to the electric field?
   A) 5 m/s^2 B**)** 10 m/s^2 C) 15 m/s^2 D) 20 m/s^2

4. Electric field at a point is given by $(9i - 16j) \times 10^4$ N/C. The force acting on a point charge $q = -20$ μC placed at the given field point is

   A) $(-4.5i + 8j)$ N B) $(4.5i - 8j)$ N C) $(1.8i - 3.2j)$ N D) $(-1.8i + 3.2j)$ N

5. Two point charges, Q_1 and Q_2 having positive sign, are placed on the *x*-axis. They are separated by a distance **a** and $Q_1 = 4Q_2$. At what distance on the *x*- axis between the charges and measured from Q_1 will the electric field vanish?

   A) 2**a** B) $\frac{2}{3}$**a** C) $\frac{1}{3}$**a** D) A and C

6. An object having a net charge of 24 μC is placed in a uniform electric field of 610 N/C directed vertically upward. What is the mass of the object if it floats in this electric field?

   A) 14.640 kg B) 1.464 g C) 14.460 g D) 14.460 kg

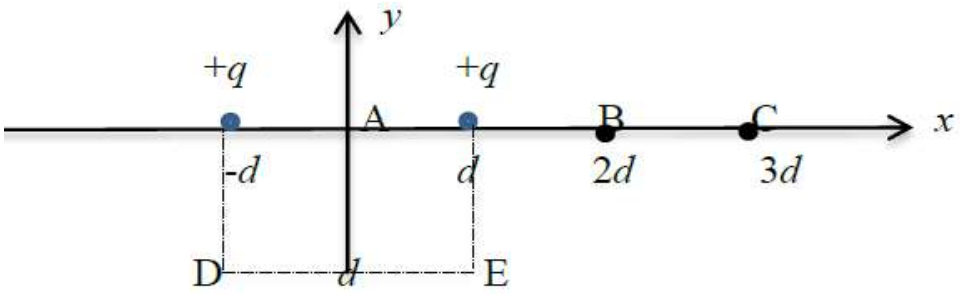
7. One of the following statements is NOT characteristic of electric field.

   A) An electric field is the region around a charged object where another charged object will experience a force.
   B) Lines of electric field have a direction which moves away from negative and toward positive.
   C) The direction of electric field is the direction of force acting on a small positive test charge.
   D) Lines of electric flux do not cross each other.

8. An electron enters a region of uniform electric field of 5×10^3 N/C. What is the force on the electron?

   A) 8×10^{-22} N B) 5.3×10^{-22} N C) 8×10^{-16} N D) 1.88×10^{22} N

9. Two positive charges of magnitude *q* are each a distance *d* from the origin A of a coordinate system as shown below.



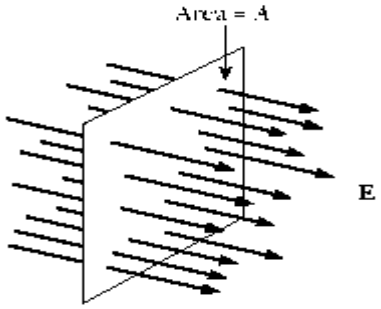
   The magnitude of the electric field is minimum at point

   A) A B) B C) C D) D

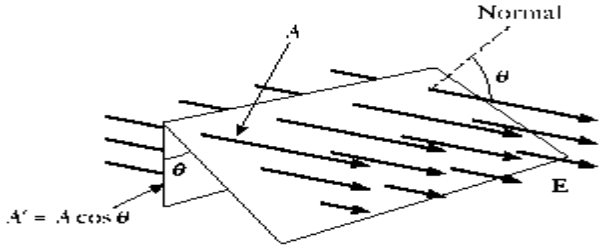
Gauss' Law

The number of electric lines of force crossing a given area perpendicularly is known as **electric flux** (Φ). The electric flux through a surface is

$\Phi = \vec{A} \bullet \vec{E}$



$\Phi = A\,E$



$\Phi = AE\cos\theta$

Gauss' Law: The electric flux Φ_E through any closed surface is equal to the net charge inside the surface, Q_{inside}, divided by ε_0:

$$\Phi_E = \frac{Q_{inside}}{\varepsilon_0}$$ Where $\varepsilon_0 = 8.85\times10^{-12}\ C^2/N\ m^2$ permittivity of free space.

Note that:- ***flux is independent of the shape of that surface***.

Equating the two we get

$$E = \frac{Q}{A\varepsilon_0}$$

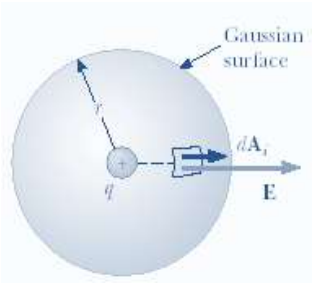
The surface area of imaginary spherical surface, called Gaussian surface is

$$A = 4\pi r^2$$

The electric field at a distance *r* from an isolated point charge *Q* is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$ or

$$E = \frac{KQ}{r^2}$$ Where $K = \frac{1}{4\pi\varepsilon_0}$ proportionality constant.



Example

1. Consider a uniform electric field oriented in the *x*-direction.
   a) Find the electric flux through each surface of a cube with edges *L* oriented as shown in figure below.

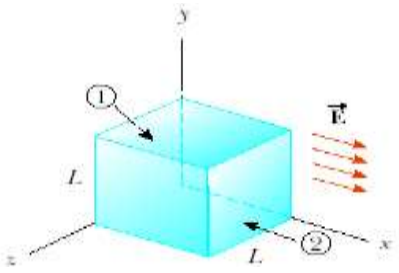
Solution

a) The normal vector to the *xy* -plane points in the negative *z*-direction; back of the cube. This, in turn, is perpendicular to $\vec{E}$, so $\theta = 90°$.
(The opposite side works similarly; front side of the cube)

$$\Phi_{xy} = EA\cos90^0 = 0$$

The normal vector to the *xz*-plane points in the negative *y*-direction. This, in turn, is perpendicular to $\vec{E}$, so again $\theta = 90°$. (The opposite side works similarly.)

$$\Phi_{__} = EA\cos 90^0 = 0$$ top and bottom



The normal vector to surface ① (the *yz*-plane) points in the negative *x*-direction. This is antiparallel to $\vec{E}$, so $\theta = 180^0$. (The opposite side works similarly.)

$$\Phi_{yz} = EA\cos180^0 = -EL^2$$ surface ①

Surface ② has normal vector pointing in the positive *x*-direction, so $\theta = 0°$.

$$\Phi_{yz} = EA\cos0^0 = EL^2$$ surface ②

b) The net flux is

$$\Phi_{net} = 0 + 0 + 0 + 0 - E\,L^2 + EL^2 = 0$$

Notice that there is no charge in the box. The net electric flux is always zero for closed surfaces that don't contain net charge.

2. Suppose the constant electric field in Example 1 points in the positive *y*-direction instead. Calculate the flux through the *xz*-plane and the surface parallel to it. What is the net electric flux through the surface of the cube? **Ans**. $\Phi_{xz} = -EL^2$ (bottom), $\Phi_{xz} = +EL^2$ (top). The net flux is sill zero.

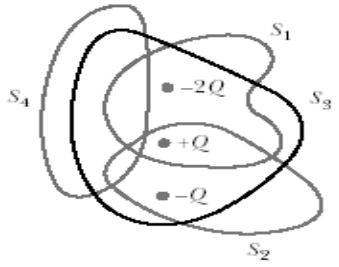
3. An electric field of intensity 3.5×10^3 N/C is along positive *x*-axis. Calculate the flux through a rectangular plane of 0.35 m wide and 0.8 m long if:

   a) the plane is parallel to the *yz* plane. **Ans**. $\Phi = 980\ \text{N}m^2/\text{C}$

   b) the plane is parallel to the *xy* plane. **Ans**. $\Phi = 0$

   c) its normal makes an angle of 60^0 with the *x*-axis. **Ans**. $\Phi = 490\ \text{N}m^2/\text{C}$

4. Find the electric flux through each surface if $Q = 2\ n\text{C}$



   Ans. $\Phi_1 = -226\ \text{N}m^2/\text{C}$

   $\Phi_2 = 0$

   $\Phi_3 = -452\ \text{N}m^2/\text{C}$

   $\Phi_4 = 0$

5. A spherical balloon contains a positively charged particle at its center. As the balloon is inflated to a greater volume while the charged particle remains at the center, the electric flux at the surface of the balloon

   A) increase C) remains the same

   B) decrease D) impossible to determine

6. Find the electric field at a distance *r* from a line of positive charge of infinite length and constant charge per unit length λ. **Ans**. $\mathrm{E} = \dfrac{1}{2\pi\varepsilon_0}\dfrac{\lambda}{r}$

7. Which one of the following is true about an electric flux of a point charge $+Q$?

   A) The electric flux at a distance *d* is half the flux at a distance 2*d* from $+Q$.

   B) The electric flux at a distance *d* has the same magnitude as the flux at a distance 2*d* from $+Q$.

   C) The electric flux at a distance *d* is twice the flux at a distance 2*d* from $+Q$.

   D) The electric flux does not depend on the magnitude of $+Q$.

CONDUCTORS IN ELECTROSTATIC EQUILIBRIUM

A good electric conductor like copper, though electrically neutral, contains electrons that aren't bound to any atom and are free to move about within the material. When no net motion of charge occurs within a conductor, the conductor is said to be in **electrostatic equilibrium**.

An isolated conductor (one that is insulated from ground) has the following properties:

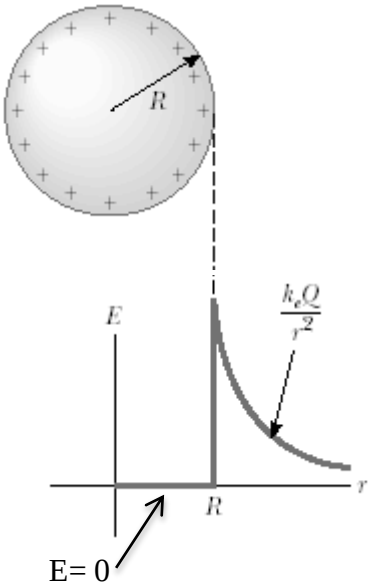
1. The electric field is zero everywhere inside the conducting material.
2. Any excess charge on an *isolated conductor* resides entirely on its surface.
3. The electric field just outside a charged conductor is perpendicular to the conductor's surface.
4. On an irregularly shaped conductor, the charge accumulates at sharp points, where the radius of curvature of the surface is smallest.

** Why is it safe to stay inside an automobile during a lightning storm?*

Many people believe that staying inside a car is safe because of the insulating characteristics of the rubbers in the car, but in fact this isn't true. Lightning can travel through centimeters of rubber. The safety of remaining in the car is due to the fact that charges on the metal shell of the car will reside on the outer surface of the car, as noted in property 2 discussed earlier. As a result, an occupant in the automobile touching the inner surfaces is not in danger.

For a conducting sphere of radius R:

* the net charge *Q* of the conductor is uniformly distributed over the surface of the conductor



E= $\frac{kQ}{r^2}$ for $r \geq R$

E= 0 for $r < R$

For a conducting spherical shell of outer radius R_2 and inner radius R_1

i) If a charge +*Q* is placed on the spherical conducting shell and a particle with a charge −*q* at the center of the cavity, the net charge on

* the inner surface of the spherical conducting shell is +*q*

*the outer surface the spherical conducting shell is +*Q* –*q*

ii) If a charge +*Q* is placed on the spherical conducting shell and a particle with a charge +*q* at the center of the cavity, the net charge on

* the inner surface of the spherical conducting shell is –*q*

*the outer surface the spherical conducting shell is +*Q* +*q*

iii) If a charge –*Q* is placed on the spherical conducting shell and a particle with a charge +*q* at the center of the cavity, the net charge on

*the inner surface of the spherical conducting shell is –*q*

*the outer surface the spherical conducting shell is –*Q* +*q*

iv) If a charge –*Q* is placed on the spherical conducting shell and a particle with a charge –*q* at the center of the cavity, the net charge on

* the inner surface of the spherical conducting shell is +*q*

*the outer surface the spherical conducting shell is –*Q* –*q*

Example

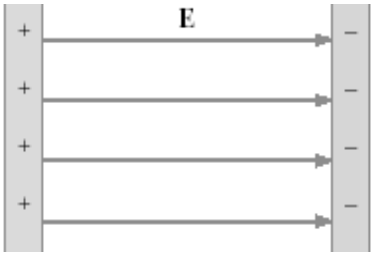
1. A 10C charge is placed on a *spherical conducting shell*. If a particle with a charge −3C is placed at the center of the cavity, what is the net charge on
   a) the inner surface of the shell? **Ans**. +3C. b) the outer surface of the shell? **Ans**. +7C
2. Positive charge Q is placed on a *conducting spherical shell* with inner radius R_1 and outer radius R_2. A point charge +q is placed at the center of the cavity. What is the magnitude of the electric field at a point *outside the shell*, a distance *r* from the center? **Ans**. E= $\frac{1}{4\pi\varepsilon_0}\frac{(q+Q)}{r^2}$

* The electric field at the surface of the conductor is perpendicular to that surface.

Electric field between two oppositely charged parallel plates is

$$E = \frac{V}{d} = \frac{Q}{C}.\frac{1}{d} = \frac{Q}{\varepsilon_o \frac{A}{d}.d} = \frac{Q}{A}.\frac{1}{\varepsilon_o} \quad \Rightarrow \quad E = \frac{\sigma}{\varepsilon_0}$$

Where $\sigma = \frac{Q}{A}$ charge density on each plate

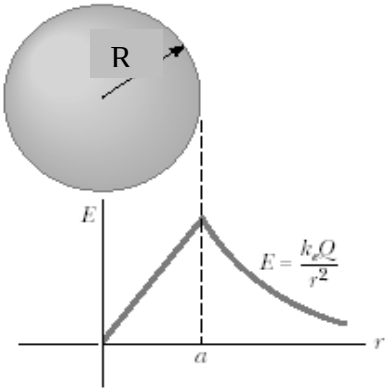


Example

1. A solid conducting sphere of radius 30cm has a net charge of 5μC. Calculate the electric field intensity at
   a) the center of the sphere. **Ans**. E = 0
   b) 10cm from the center. **Ans**. E = 0
   c) 30cm from the center. **Ans**. E = 5×10^5 N/C outward
   d) 40cm from the center. **Ans**. E = 2.8×10^5 N/C outward
2. How many electrons must be added to a spherical conductor of radius 0.1 m to produce an electric field of 3.6×10^{-3} N/C just above the surface?
   A) 5×10^3 B) 5×10^4 C) 2.5×10^3 D) 2.5×10^4
3. Consider a uniformly charged conducting spherical shell of inner radius R_1 and outer radius R_2 carrying a total charge of negative charge q. A positive point charge of magnitude q is placed at the center of the spherical shell. What are the electric fields at $r < R_1$, $R_1 < r < R_2$ and $r > R_2$?

   A) $\frac{q}{4\pi\varepsilon_0 r^2}$, 0, $\frac{q}{4\pi\varepsilon_0\left(R_2^2 - R_1^2\right)}$
   B) $\frac{q}{4\pi\varepsilon_0 R_2^2}$, 0, $\frac{q}{4\pi\varepsilon_0 r^2}$
   C) $\frac{q}{4\pi\varepsilon_0 r^2}$, 0, 0
   D) $\frac{q}{4\pi\varepsilon_0\left(R_2^2 - R_1^2\right)}$, 0, 0

For an insulating sphere of radius R: the charge is uniformly distributed throughout its volume



$E = \frac{kQ}{r^2}$ for $r \geq R$

$E = \frac{kQ}{R^3} r$ for $r < R$

Exercise

1. Given a uniformly charged insulating sphere of charge density ρ, what is the electric field at a distance *r* from the center inside the sphere?

A) $\frac{\rho r}{3\varepsilon_0}$ B) $\frac{2\rho r}{3\varepsilon_0}$ C) $\frac{\rho r}{2\varepsilon_0}$ D) $\frac{3\rho r}{2\varepsilon_0}$

2. Consider a uniformly charged insulating spherical shell of inner radius R_1 and outer radius R_2. What are the electric fields at $r>R_2$ and $r < R_1$ respectively?

A) $\frac{q}{4\pi\varepsilon_0 r^2}$, 0 B) $\frac{q}{4\pi\varepsilon_0 R_2^2}$, 0 C) $\frac{q}{4\pi\varepsilon_0 R_1^2}$, 0 D) $\frac{q}{4\pi\varepsilon_0\left(R_2^2 - R_1^2\right)}$, 0

3. Consider a long cylindrical charge distribution of radius R with a uniform charge density ρ. What is the magnitude of electric field at distance *r* from the axis where $r < R$?

A) $\frac{\rho r}{3\varepsilon_0}$ B) $\frac{\rho r}{2\varepsilon_0}$ C) $\frac{\rho r}{7\varepsilon_0}$ D) $\frac{\rho r}{5\varepsilon_0}$

Electric dipole

Two point charges of equal magnitude with opposite sign, separated by a distance ℓ is known as an **electric dipole**.

ℓ is the dipole axis

The torque of the couple is

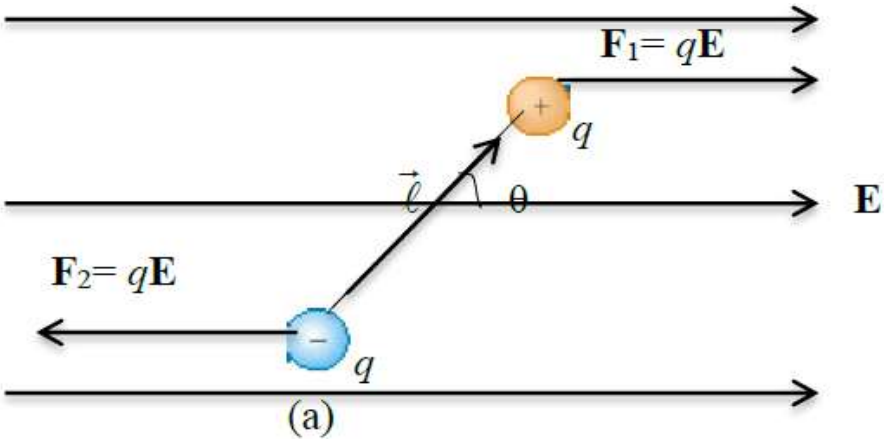
$\vec{\tau} = \vec{\ell} \times \vec{F}$

$= \vec{\ell} \times (q\vec{E})$

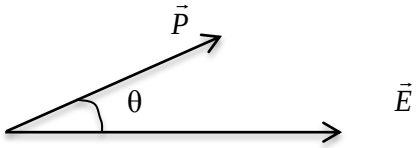
$= q\vec{\ell} \times \vec{E}$

$= \vec{P} \times \vec{E}$

$\tau = PE \sin\theta$



Where $\vec{P} = q\vec{\ell}$ is the electric dipole moment which points toward the positive charge.



The effect of the torque is to tend to rotate the dipole to a position in which the dipole moment is parallel to the electric field $\vec{E}$, as in figure (b).



Which one of the following statements is correct?

A) The electric field strength due to two point charges of equal magnitude and the same sign is zero only at one point on the line joining the two charges.

B) If a point charge of mass *m* is released from rest in a uniform electric field, it will move along a line of force with uniform speed.

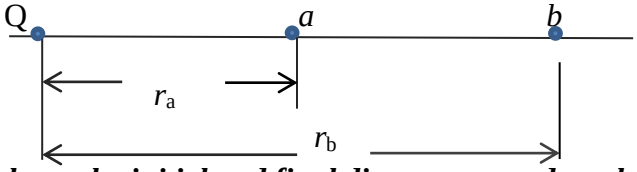
C) The variation of the strength of the electric field due to an electric dipole is inversely proportional with the square of the distance from its center of mass.

D) An electric dipole moment ***P*** in a uniform external electric field ***E*** is in stable equilibrium if ***P*** and ***E*** are in opposite directions.

Work done by electric force

Consider a single stationary charge Q which is a source of an electric field in space and a test charge *q* moving in the field. The work done by the field as *q* moves from *a* to *b* is,

$$W = kQq\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$



The work done on q by the field of Q depends only on the initial and final distances r_a and r_b whatever the path followed between the points.

*When the work done by a force depends only on the initial and final positions, the force is said to be a **conservative force.**

$$W_{ab} = kQq\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$
$$= \frac{kQq}{r_a} - \frac{kQq}{r_b}$$
$$= U_a - U_b = -(U_b - U_a)$$
$$= -\Delta U$$

Where $U_a = \frac{kQq}{r_a}$ and $U_b = \frac{kQq}{r_b}$ are the electric potential energies of *q* at r_a and r_b from Q, respectively.

Electric potential energy:- is the energy required to bring a charge *q* from infinity to the point in an electric field at which the potential energy is being specified.

- The electric potential energy of a charge *q* at a distance *r* from charge Q is given by

$$U = \frac{kQq}{r}$$

The amount of work done by electrostatic force to change the separation between two charges form r_a to infinity is

$$W = kQq\left(\frac{1}{r_a} - \frac{1}{\infty}\right)$$

$$W = \frac{kQq}{r_a}$$

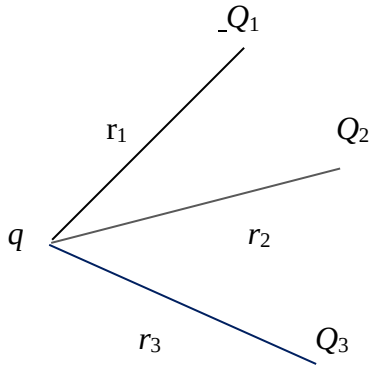
Electric potential energy of

i) a charge q due to a system of charges is given by

$$U = \frac{kQ_1q}{r_1} + \frac{kQ_2q}{r_2} + \frac{kQ_3q}{r_3} + \ldots$$

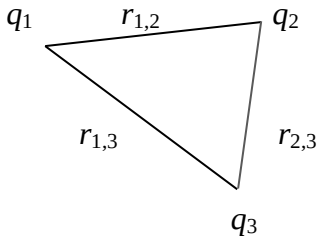
$$= kq\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3} + \ldots\right)$$

$$= k\,q\sum\frac{Q_i}{r_i}$$



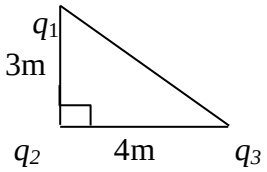
ii) charges placed at the corners of a triangle is given by

$$U = \frac{kq_1q_2}{r_{1,2}} + \frac{kq_2q_3}{r_{2,3}} + \frac{kq_1q_3}{r_{1,3}}$$

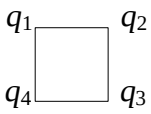


Example

1. What is the potential energy of
   a) a 0.1 μC charge at a distance of 10 cm from a stationary charge of 6 μC? **Ans**. U= 0.054 J
   b) a charge q= – 1 μC at a distance of 20 cm from a stationary charge of 4 μC? **Ans**. U= – 0.18 J
2. Three point charges, q_1= 1μC, q_2 = – 2μC and q_3= 3μC, are placed at the corners of a triangle shown blow. Calculate the potential energy of the three charges.
   Ans. U= -1.41×10^{-2}J



3. Three point charges of 3×10^{-7} C are placed at the corners of an equilateral triangle of side 1 m. What is potential energy of the system of charges? **Ans**. U= 2.43×10^{-3} J
4. Four point charges, q_1, q_2, q_3 and q_4 are placed at the corners of a square side ***a***. Show that the potential energy of the charges is given by: $U = \frac{k}{\mathbf{a}}\left[q_1q_2 + \frac{q_1q_3}{\sqrt{2}} + q_1q_4 + q_2q_3 + \frac{q_2q_4}{\sqrt{2}} + q_3q_4\right]$



Questions from national exam

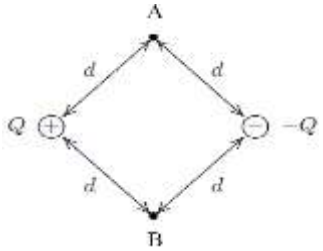
1. Two positive point charges are held in place, 1m apart. They are then moved so that their electric potential energy doubles. What is the new separation?

   A) $\sqrt{2}$ m B) $\frac{\sqrt{2}}{2}$ m C) $\frac{1}{2}$ m D) $\frac{1}{4}$ m

2. Total potential energy of three equal charges of charge q placed at the three corners of equilateral triangle of side ℓ is given by

   A) $\frac{kq^2}{\ell}$ B) $\frac{2kq^2}{\ell}$ C) $\frac{3kq^2}{\ell}$ D) $\frac{3kq^2}{2\ell}$

3. By what factor does the electric potential energy change if the distance between two point charges is tripled?
   A**)** 1/3 B) 3 C) 1/9 D) 9
4. What is the potential energy of two equal positive point charges 1 μC each held 1 m apart?
   A) 1J B) 1eV C**)** 9×10^{-3}J D) zero
5. What is electric potential energy of a system of four point charges, each with charge *q* located at the corners of square of side *a*?
   A) $\frac{\sqrt{2}kq}{a}\left(2\sqrt{2}+1\right)$ B) $\frac{\sqrt{2}kq^2}{a}\left(2\sqrt{2}-1\right)$ C**)** $\frac{\sqrt{2}kq^2}{a}\left(2\sqrt{2}+1\right)$ D) $\frac{\sqrt{2}kq}{a}\left(2\sqrt{2}-1\right)$
6. Point A and B are each a distance *d* from each of two equal and opposite charges as shown below.



What will be the work equired to move a negative *q* from A to B?
   A**)** Zero B) $\frac{qQ}{4\pi\varepsilon_0 d^2}$ C) $\frac{qQ}{4\pi\varepsilon_0 d}$ D) $\frac{qQ}{4\pi\varepsilon_0\left(4d^2\right)}$

Electric potential

Electric potential (V):- **electric potential at an arbitrary point in an electric field equals the work required per unit charge to bring a positive test charge from infinity to that point.**

$V=\frac{U}{q_0}$ Its SI unit is Joule/coloumb= Volt

Electric potential due to a point charge V(r) at any point a distance *r* from the point charge *Q* is:

$$V(r)=\frac{U(r)}{q_0}=\frac{kQq_0/r}{q_0}=\frac{kQ}{r}$$

Example

1. What is an electric potential due to
   a) a 4 μC charge at a distance of 30 cm? **Ans**. $V=1.2\times10^5$ V
   b) a –6 μC at a distance of 90 cm? **Ans**. $V=-6\times10^4$ V
2. If the electric potential at a distance of 3 m from a point charge is 3000 V, what is the potential at a distance of 1.5 m from the same charge? **Ans**. V= 6000 V
3. At a distance *r* from a point charge *Q*, the electric potential is 1.8×10^4 V and the magnitude of the electric field is 4.5×10^3 N/C. Determine the values of *r* and *Q*. **Ans**. *r* = 4 m, *Q* = 8 μC

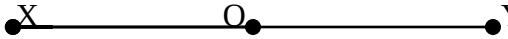
Exercise

1. If an electric potential at a point 4 m from a charge is 250 V, what is the potential at a point 2 m from the same charge? **Ans**. V= 500 V
2. At a certain distance from a point charge, the electric field is 500 N/C and the potential is 3000 V. What is
   a) the distance from the charge? **Ans**. *r* = 6 m.
   b) the magnitude of the charge? **Ans**. *Q* = 2 μC

Questions from national exam

1. An electric potential of point X is V_x = 15V and that of point Y is V_Y = 5V as shown in figure below. Which of the following is true about an electron placed at mid-point O between points X and Y?

The electron will



A) move toward point Y
C**)** start moving toward point X.
B) stay at rest at point O
D) vibrate between X and Y.

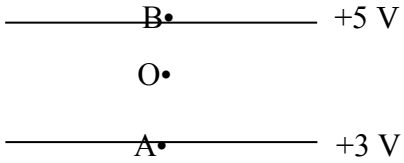
2. If the electric potential at a point 2 m from a positive charge is 12 V, what force would be exerted on a charge of 5 *n*C placed at this point?

A) 6×10^{-8} N B**)** 3×10^{-8} N C) 6×10^{-9} N D) 3×10^{-9} N

3. What is the potential (V) at a distance of 10m from a charge of +5C?

A) -4.45×10^{9}V B) –0.5V C**)** $+4.45\times10^{9}$V D) +0.5V

4. Two parallel plates are kept at 3 V and 5 V as shown in the diagram below. If an electron is released at O, then wich way will the electron move?



(A) towards B (C) remains at O.
(B) towards A (D) parallel to the plates.

Potential due to a system of point charges

The electric potential due to point charges Q_1, Q_2,, . . ., Q_n at distances r_1, r_2, , . . . , r_n respectively is given by

$$V = \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2} + \ldots + \frac{kQ_n}{r_n}$$

$$= k\sum_{i=1}^{r=n}\frac{Q_i}{r_i}$$

Example

1. Two charges, q_1 and $q_2 = 4q_1$, are 1 m apart. If the electric potential at a point midway between the charges is 4.5×10^5 V, what is the electric field at this point? **Ans**. E= 5.4×10^5 N/C.
2. Two charges, q_1 and $q_2 = 2q_1$, are 2 m apart. If the electric potential at a point midway between the charges is 6×10^5 V, what electric force would be exerted on a 5 μC charge placed at this point? **Ans**. F= 1N
3. A point charge $+q$ is placed at $x = a$ and another point charge $+q$ is placed at $x = -a$. Show that the potential

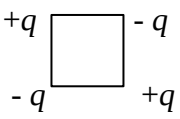
a) at any point along the y-axis is given by: $V = k\frac{2q}{\sqrt{a^2+y^2}}$. b) at the origin is, $V = \frac{2kq}{a}$

4. Consider four point charges, each $+q$, placed at the corners of a square of side a.

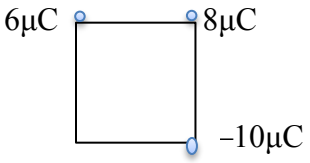
a) Calculate the potential at the center of the square. **Ans**. $V = \frac{4\sqrt{2}\,kq}{a}$

b) Repeat part (a) if the charges at the second and fourth corners are replaced by $-q$ each.

Ans. V= 0



5. Three point charges 6 μC, 8 μC and –10 μC are placed at the three corners of a square of side 10 cm. Calculate the potential at the fourth (vacant) corner.



Ans. $V = 3.6\times10^{5}\left(\sqrt{2}-1\right)V$

6. Two positive charges of magnitude 2.4×10^{-9}C and 1.6×10^{-9}C are separated in air by 2 cm. How much electrostatic potential is produced by the two charges at midway between them?

A**)** 3.6 kV B) 0.72 kV C) 2.16 kV D) 1.44 kV

7. Two equal and opposite charges (– 4 μC and + 4 μC) are located along x-axis at – 2 *cm* and + 2 *cm* respectively. How much would be the electric potential at the origin?

A) 3.6 MV B) 1.8 MV C) Zero D) 4.5 MV

Potential due to a conducting sphere

Consider a conducting sphere of radius R, with total charge *Q* distributed uniformly over the surface of the sphere.

E= 0, for $r < R$ (inside the sphere)

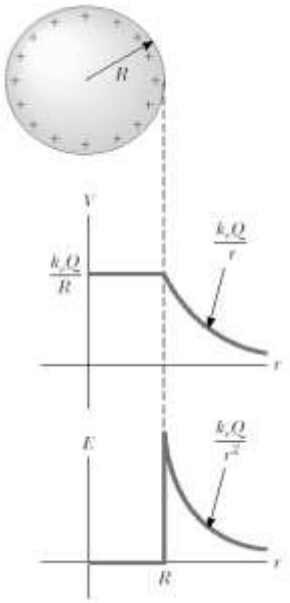
E= $\frac{kQ}{r^2}$ for $r \geq R$ (outside the sphere)

The potential V at a radial distance relative to a point at infinity is

V= $\frac{kQ}{r}$ for $r > R$ (outside the sphere)

V= $\frac{kQ}{R}$ for $r = R$

V= $\frac{kQ}{R}$ for $r < R$ (inside the sphere)



Example

1. A total charge of 4.*8 nC* is distributed uniformly over the surface of sphere with radius R= 0.2 m. What is the electric potential

a) on the surface of the sphere? **Ans**. V= 216 V.

b) inside the sphere 0.1 m from the center? **Ans**. V= 216 V.

c) at the center of the sphere? **Ans**. V= 216 V

d) outside the sphere 1 m from the center? **Ans**. V = 43.2 V

e) outside the sphere 100 m from the center? **Ans**. V= 0.432 V

2. A total electric charge of 3.5×10^{-9} C is distributed uniformly over the surface of metal sphere with radius of 0.2 m. What is the value of electric potential at a point 0.1 m from the center of the sphere?

A**)** 157.5 V B) 787.5 V C) 315 V D) 3150 V

3. A hollow metal sphere of radius 10 cm is charged to a potential of 100 V. What is the potential at the center of the sphere?

A) Zero
B) 100V
C) The same as that at a point 50cm from the surface.
D) The same as that at a point 10cm from the surface.

4. If a hollow metal sphere placed on an insulating stand is charged positively, then what is the electric potential inside the sphere?

A) Zero.
B) The same as on the surface.
C) Greater than the potential on the surface.
D) Less than the potential on the surface.

Potential difference

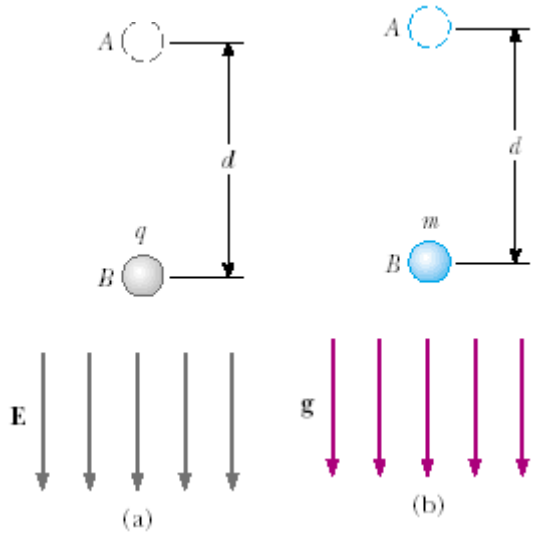
The *potential difference* $\Delta V = V_B - V_A$ *between two points B and A in an electric field is* defined as *the change in potential energy of the system when a test charge is moved between the points divided by the test charge* q_0*:*

$$\Delta V = \frac{\Delta U}{q_0}$$

$\Delta V = V_B - V_A = -Ed$

This equation works only for a uniform electric field.

The negative sign indicates that the electric potential at point *B* is lower than at point A; that is, $V_B < V_A$.
Electric field lines always point in the direction of decreasing electric potential, as shown in Figure (a).



(a) When the electric field **E** is directed downward, point *B* is at a lower electric potential than point *A*. When a positive test charge moves from point *A* to point *B*, the charge –field system loses electric potential energy. (b) When an object of mass *m* moves downward in the direction of the gravitational field **g,** the object – field system loses gravitational potential energy.

A system consisting of a positive charge and an electric field loses electric potential energy when the charge moves in the direction of the field. This means that an electric field does work on a positive charge when the charge moves in the direction of the electric field. (This is analogous to the work done by the gravitational field on a falling object, as shown in figure (b) above.

A system consisting of a negative charge and an electric field gains electric potential energy when the charge moves in the direction of the field.

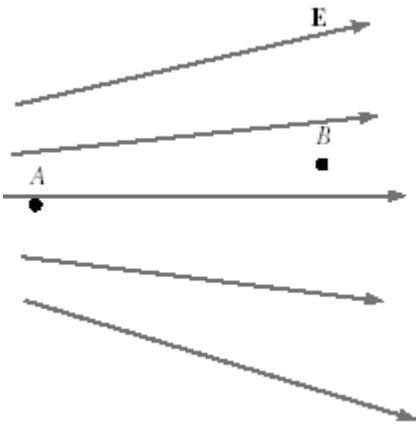
For non-uniform electric field, the potential will change everywhere like the field.

$E = -\frac{\Delta V}{\Delta s}$ for an infinitesimal distance Δs

From the above equation, the unit of E is Volt/meter. 1V/m= 1N/C

Positive charges *move from* ***high potential*** *to* ***low potential***.

Negative charges *move from* ***low potential*** *to* ***high potential***.



Example

1. Two large parallel metal plates, separated by 0.1 m, carry equal and opposite charges. If the potential difference between them is 500 V, calculate:
   a) magnitude of the electric field between them. **Ans**. E = 5000 V/m.
   b) work done by the field on a 2 *n*C charge as it moves from higher potential to lower potential. **Ans**. $W=10^{-6}$ J
2. The potential difference between points A and B is 2.5×10^{4} V. A particle of mass 4×10^{-6} kg and charge 5 μC is released from rest at point A. What is the speed of the particle when it arrives at point B? **Ans**. V= 250 m/s
3. How much work is required to move a 2 μC charge from a point where the potential is – 4 V to 12 V? **Ans**. W= 3.2×10^{-5} J

Exercise

1. Two parallel plates separated by 5 cm are maintained at a potential difference of 180 V. An electron ($m_e= 9\times10^{-31}$ kg) is released from the positive plate. Calculate
   a) its velocity when it strikes the positive plate. **Ans**. V= 8×10^{6} m/s.
   b) its kinetic energy at the instant it strikes positive plate. **Ans**. K.E= 2.88×10^{-17} J
2. A small sphere of mass 0.2 g and charge 4 *n*C hangs by thread between two vertical parallel plates 5 cm apart. What potential difference between the plates will cause the thread to make an angle of 45^{0} with the vertical? **Ans**. V = 25000 V
3. Two point charges, q_1= 40 *n*C and q_2 = –30 *n*C, are 10 cm apart. Point A is midway between them; point B is 8 cm from q_1 and 6 cm from q_2. Find
   a) the potential at point A. **Ans**. $V_A= 1.8\times10^{3}$ V
   b) the potential at point B. **Ans**. V_B= 0
   c) the potential difference between points A and B. **Ans**. $V_A - V_B = 1.8\times10^{3}$ V.
   d) the work required to transfer a charge of 25 *n*C from point B to A. **Ans**. W = 4.5×10^{-5} J.

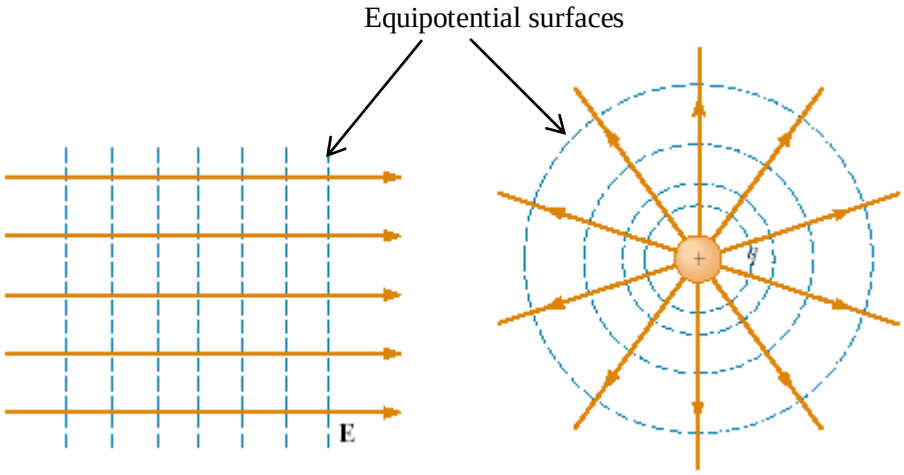
Questions from national exam

1. If a charge of q= 5 μC is moved from a point with 25 V potential to a point with 125 V potential, what is the increase in potential energy?
   A**)** 5×10^{-4} J B) 2×10^{-5} J C) 1.25×10^{-4} J D) 6.25×10^{-4} J
2. Two parallel plates 2.5 cm apart carry equal and opposite charges. If the potential difference between the plates is 500 V, what electric force would be experienced on a particle of charge 3.5×10^{-9} C placed in the region between the plates?
   A) 3.5×10^{-5} N B**)** 7×10^{-5} N C) 3.5×10^{5} N D) 7×10^{5} N
3. A charge of 20μC is put at the origin of x-y coordinate. The potential difference between points $(a,0)$ and $\left(\frac{a}{\sqrt{2}},\frac{a}{\sqrt{2}}\right)$ is
   A) $\frac{a}{\sqrt{2}}$ B) a C**)** zero D) $2a$
4. An ion accelerated through a potential difference of 115 V experiences an increase in kinetic energy of 7.37×10^{-17} J. What is the charge on the ion?
   A**)** 6.41×10^{-19} C B) 2.32×10^{-15} C C) 4.51×10^{-15} C D) 2×10^{-21} C
5. Moving 2.5×10^{-6} C of charge from point A to point B in an electric field requires 6.3×10^{-4} J of work. The potential difference between points A and B is approximately: **Ans**. V_{AB} = 252 V
   A) 4.0×10^{-3} V B) 2.5×10^{-2} V C) 1.6×10^{-19} V D) 1.0×10^{-14} V
6. In a certain region of space, a uniform electric field is directed in the positive x direction. If a particle with negative charge is carried from x = 20 cm to x = 60 cm, what can we conclude about the electric potential and the electric potential energy of the charge-field system?
   A) The electric potential at x = 20 cm is larger than that at x = 60 cm.
   B) The electric potential at x = 20 cm is the same as that at x = 60 cm.
   C) The electric potential energy decreases.
   D) The electric potential energy increases.
7. A positive charge of 1×10^{-4}C is on a negatively charged plate of a parallel plate capacitor. If the potential on the positive plate is +10,000V and on the negative plate is –10,000V, how much work is required to move the charge from the negative plate to the positive plate?
   A) 2J B) 1J C) Zero D) 2×10^{4}J

Equipotential Surfaces

Equipotential surface:- is one on which the potential has the same value at all points on the surface. The potential energy of a charged body is the same at all points of a given equipotential surface and no electrical work is done to move a charged body over the surface.

Equipotential surfaces are at right angles to the direction of the field at that point.



A circle of radius *r* is drawn with center as charge +*q*. A charge *q*, is taken from one point to another diametrically opposite along the circumference of the circle. What is the value of the work done?

A) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\times 2r$ B) $\frac{1}{4\pi\varepsilon_0}\frac{qq^+r}{r}\times 2r$ C) $\frac{1}{4\pi\varepsilon_0}\frac{qq^+}{r^2}\times 2r$ D) Zero

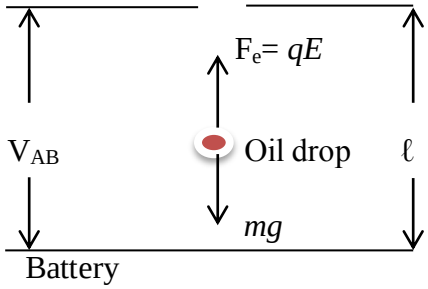
The Millikan oil drop experiment

In the Millikan oil drop experiment, the forces on the drop are its weight *mg* and upward electric force $F_e = qE$. By adjusting the electric field *E*, $F_e = qE$ can be made equal to *mg*, so that the drop remains at rest.

$qE = mg$

$$q = \frac{mg}{E} = \frac{\rho Vg}{V_{AB}/\ell}$$

$$q = \frac{4}{3}\frac{\pi\rho g\ell r^3}{V_{AB}} \quad \ldots\ldots\ldots (1)$$



Where ρ is density of the oil drop.

V is volume of the oil drop.

V_{AB} is potential difference between A and B.

r is radius of the drop.

Millikan could not measure *r* directly, so by cutting off the electric field and measuring terminal velocity V_{ter} of the drop as it falls a distance ℓ, the viscous force on the sphere is given by Stoke's law

$f = 6\pi\eta V_{ter} r \quad \ldots\ldots (2)$ where η is coefficient of viscosity of a fluid

At the terminal velocity, weight equals the viscous force.

$f = mg \quad \ldots\ldots\ldots (3)$

Equating (2) and (3) we get

$mg = 6\pi\eta V_{ter} r$

$$\frac{4}{3}\pi r^3 \rho g = 6\pi\eta V_{ter} r$$

$$r = 3\sqrt{\frac{\eta V_{ter}}{2\rho g}} \quad \ldots\ldots\ldots (4)$$

Substituting eq(4) into eq (1) we get

$$q = \frac{4\pi\rho g \ell (27)}{3V_{AB}}\left(\sqrt{\frac{\eta V_{ter}}{2\rho g}}\right)^3$$

$$q = \frac{18\pi\ell}{V_{AB}}\sqrt{\frac{\eta^3 V_{ter}^3}{2\rho g}}$$

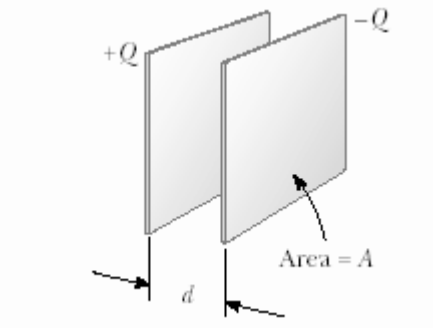
After recording measurements on thousands of droplets, Millikan had a charge equal to some integer multiple of the elementary charge *e:*

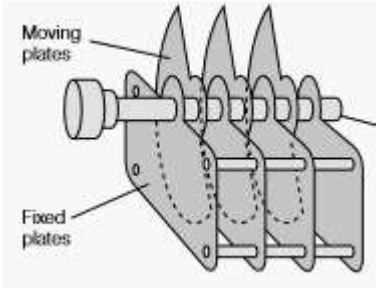
$q = n$ e $\quad n = 0, -1, -2, -3 \ldots$

Where $e = 1.60\times10^{-19}$ C. Millikan's experiment yields conclusive evidence that charge is *quantized*. For this work, he was awarded the Nobel Prize in Physics in 1923.

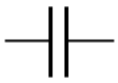
Capacitors

Capacitor:- is a device used to store electric charge. Capacitors usually consist of two parallel metal plates separated by an insulating material (dielectric).





A symbol for fixed capacitor





A symbol for variable capacitor

Uses of capacitors

- for tuning radio and TV.
- for smoothening a rectified current delivered by an ac power supply.
- to eliminate sparking when a circuit containing coil (inductor) is suddenly opened.
- to eliminate the sparking of points in ignition system of car engines when they are opened and closed.
- to increase the efficiency of alternating current transmission.
- to release energy almost instantaneously for flash lamp used in photography. The flash lamp requires a short burst of electric energy with in milliseconds what a battery can generally provide.
- use as a filter; unwanted noise is blocked from entering loudspeaker.
- used to sense the presence of a person.
- used in indicator light on a motor vehicle when a driver signals his/her intension to turn left or right.

Capacitance:- *is the property of a capacitor that describes its ability to store electric charge*. *Capacitance* of a capacitor is defined as *the ratio of the magnitude of the charge on either conductor* to *the magnitude of the potential difference between the conductors.*

$$C=\frac{Q}{V}$$ Its SI unit is Farad. 1F= 1C/V

Example

1. When a potential difference of 300 V is applied, 6 μC charge is stored on a capacitor. What is capacitance of the capacitor? **Ans**. C= 2×10^{-8} F
2. a)What is the capacitance of a capacitor which can store 18 *n*C of charge when the potential difference across it is 6 V? **Ans**. C= 3 *n*F
   b)How much charge will be stored if the potential difference is increased to 20 V? **Ans**. Q = *60 nC*
3. When the voltage applied to a capacitor increases from 80 V to 110 V, the charge on the capacitor increases by 9×10^{-5} C. What is the capacitance of the capacitor?

A) 4 μF B**)** 3 μF C) 2 μF D) 5 μC

Parallel plate capacitor

The capacitance of a device depends on the geometric arrangement of the conductors. If the plates are separated by a very short distance, the field between the plates is then uniform and the charge on the plates is uniformly distributed over the plates.

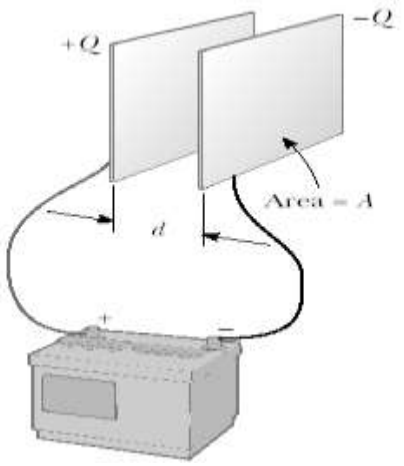
The capacitance of a parallel-plate capacitor plates separated by air is

$$C=\frac{Q}{V}$$

$$From \quad \sigma=\frac{Q}{A}, \qquad Q=\sigma A \quad and \quad V=Ed$$

$$C=\frac{\sigma A}{Ed} \quad but\ E=\frac{\sigma}{\varepsilon_o} \quad therefore \quad C=\frac{\sigma A}{\frac{\sigma}{\varepsilon_o}d}$$

$$Which\ gives\ C=\frac{\varepsilon_o A}{d}$$



Where ε_0= 8.85×10^{-12}F/m is permittivity of free space
(the degree to which the medium resists the flow of electric charge)

Since ε_0, ***A*** *and* ***d*** *are constants, for a given capacitor* ***the capacitance is a constant independent of the charge on the plates or the Potential difference.***

Example

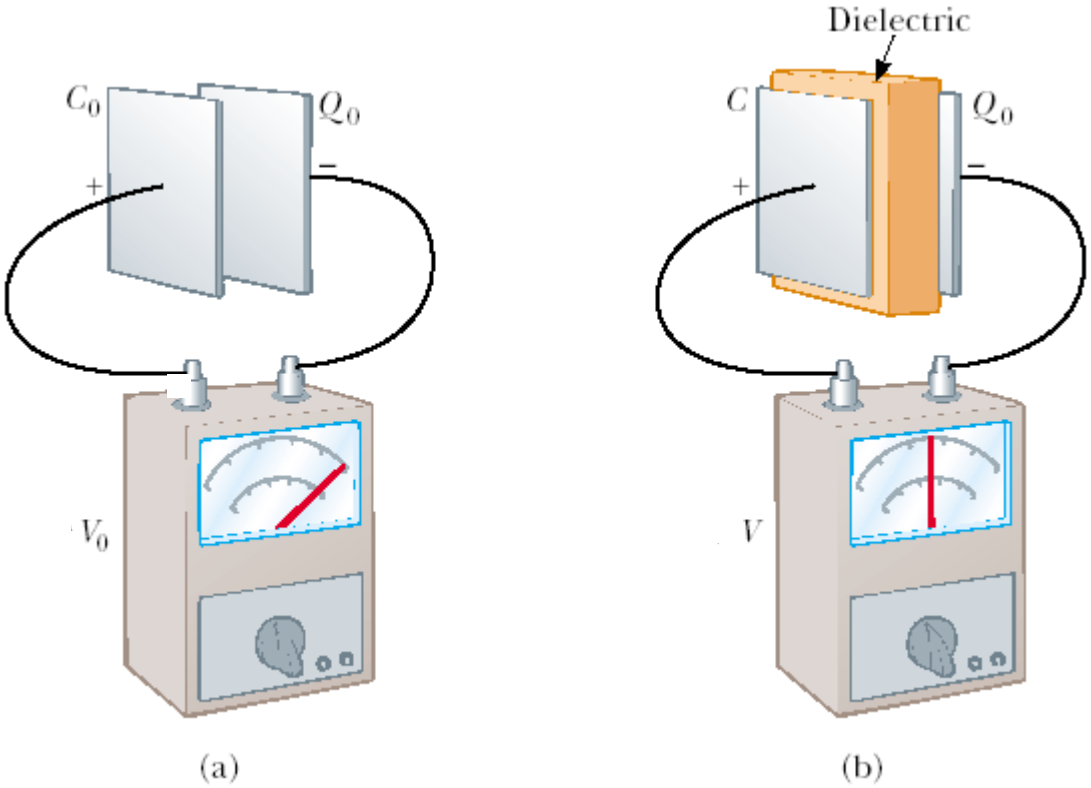
1. A parallel-plate capacitor with air between the plates has an area $A = 2\times 10^{-4}\ m^2$ and a plate separation d =1 mm. Find its capacitance. Ans. C=1.77 pF
2. A capacitor is charged to 80 pC when the potential difference between the plates is 40 V.
   a) What is the capacitance? **Ans**. C= 2 pF
   b) The charge is now increased to 100 pC for the same capacitor. What is the capacitance now? **Ans**.
   c) What is the new potential difference? **Ans**. V= 50 V
3. A 1 cm^2 parallel plate capacitor in vacuum are 2 mm apart. What is
   a) the capacitance? **Ans**. $C = 4.425\times10^{-13}\ F$
   b) the charge on the plates if potential difference is 100 V? **Ans**. $Q = 4.425\times10^{-11}\ C$
   c) the electric field between the plates? **Ans**. $E = 5\times10^{4}\ V/m$
4. What will be the capacitance in question 3 above if the Potential difference is doubled?
5. If the separation between the plates of a parallel plate capacitor is reduced to one-third of its original value, by what factor will the capacitance change? **Ans**. $C_2 = 3C_1$

Questions from national exam

1. A battery is used to charge a parallel capacitor to 3 V. The battery is then disconnected so that the charge remains on the plates. The plates are then pulled apart until the distance between them is doubled. What is the new potential across the plates?
   A) 1.5 V B) 3 V C**)** 6 V D) 9 V
2. What is the capacitance of a parallel plate capacitor whose plates are separated by air, if the area of each plate is 1 cm^2 and the plates are separated by a distance of 1 mm?
   A) $26.55\times10^{-13}\ F$ B) $14.25\times10^{-13}\ F$ C**)** $8.85\times10^{-13}\ F$ D) $6.43\times10^{-13}\ F$
3. When a potential difference of 150 V is applied to the plates of a parallel plate capacitor, the plates carry a surface charge density of 30 nC/cm^2. What is the spacing between the plates?
   A) 2.24 μm B) 5.32 μm C) 12.24 μm D**)** 4.42 μm
4. A small object of mass *m* carries a charge *q* and is suspended by a thread between the vertical plates of a parallel plate capacitor. The plate separation is *d*. If the thread makes an angle θ with the vertical, what is the potential difference between the plates?
   A**)** $\frac{mgd\tan\theta}{q}$ B) $\frac{md\tan\theta}{q}$ C) $\frac{mgd\sin\theta}{q}$ D) $\frac{md\sin\theta}{q}$
5. A parallel plate capacitor has a capacitance of 10 μF. If the distance between the plates is doubled, keeping all other quantities constant, what is the new capacitance of the capacitor?
   A) 0.2 μF B**)** 5 μF C) 10 μF D) 20 μF
6. The capacitance of a parallel plate capacitor is 10 μF. If the distance between the plates is halved, keeping all other quantities constant, what is the new capacitance of the capacitor?
   A) 0.2 μF B) 5 μF C) 10 μF D**)** 20 μF

Dielectrics

Practically the space between the plates of a parallel plate capacitor is filled with non-conducting materials such as glass, wood, paper, plastic, etc. Such insulators are called ***dielectric material*** (**dielectrics**).



A charged capacitor (a) before and (b) after insertion of a dielectric between the plates. The charge on the plates remains unchanged, but the potential difference decreases from v_0 to $v = v_0/k$. Thus, the capacitance increases from C_0 to kC_0.

The uses of dielectric

- Increases the capacitance of a capacitor and allows much more charge to be stored at the same p.d .
- Solve the mechanical problem of maintaining two metal plates at extremely small separation without actual contact.
- Makes the plates withstand much higher voltage. It increases the voltage at which the breakdown will take place, that is, the value of the voltage beyond which the space between the plates begin to conduct.

The capacitance of a parallel plate with a dielectric is given by:

$C = \frac{k\varepsilon_0 A}{d}$ Where k is the dielectric constant

$C = k\,C_0$ C_0 is capacitance without dielectric

$V = \frac{V_0}{k}$ C is capacitance with dielectric

V_0 is p.d between the plates without dielectric

V is p.d between the plates with dielectric

For any given plate separation, there is a maximum electric field that can be produced in the dielectric before it breaks down and begins to conduct. This maximum electric field is called the **dielectric strength**, and for air its value is about 3×10^6 V/m.

Approximate dielectric constants and dielectric strengths of various materials at room temperature

Material	Dielectric Constant	Dielectric Strength (10^6V/m)
Air (dry)	1.00059	3
Fused quartz	3.78	8
Nylon	3.4	14
Paper	3.7	16
Pyrex glass	5.6	14

The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown.

Example

1. An air filled, 10 cm by 10 cm, parallel plate capacitor is charged to a potential difference of 200 V. If the plates are separated by 5 mm, what is
   a) the capacitance? **Ans**. C_0= 17.7 pF
   b) the charge on each plate? Q = 3.54 *n*C
   c) the electric field between the plates? **Ans**. E= 4×10^4 V/m
   d) the surface charge density? **Ans**. σ = 354 *n*C/m^2
2. If a dielectric with k= 5 was inserted between the plates of the capacitor in question 1 and then charged to a potential difference of 200 V, calculate
   a) the capacitance. **Ans**. C= 88.5 *p*F
   b) the charge on each plate. **Ans**. Q= 17.7 *n*C
   c) the electric field between the plates. E= 4×10^4 V/m
   d) the surface charge density. **Ans**. σ = 1770 *n*C/m^2

Exercise

1. A parallel-plate capacitor has plates of dimensions 2.0 cm by 3.0 cm separated by a 1.0 mm thickness of paper with k = 3.7.
   a) Find its capacitance. **Ans**. C= 20 pF
   b) What is the maximum charge that can be placed on the capacitor? the dielectric strength of paper is 16×10^6 V/m. **Ans**. Q_{max}= 0.32 μC
2. A parallel plate air capacitor has A= 2000 cm^2 and spacing 1 cm. When a sheet of dielectric with k = 3 is inserted between the plates the potential difference decreases from 3000 V to 1000 V. Calculate
   a) the original capacitance. **Ans**. C_0 = 177 pF
   b) the charge on each plate. **Ans**. *Q* = 531 *n*C
   c) the capacitance after the dielectric is inserted. **Ans**. C= 531 pF
   d) the electric field after the dielectric is inserted. **Ans**. E=10^5 V/m

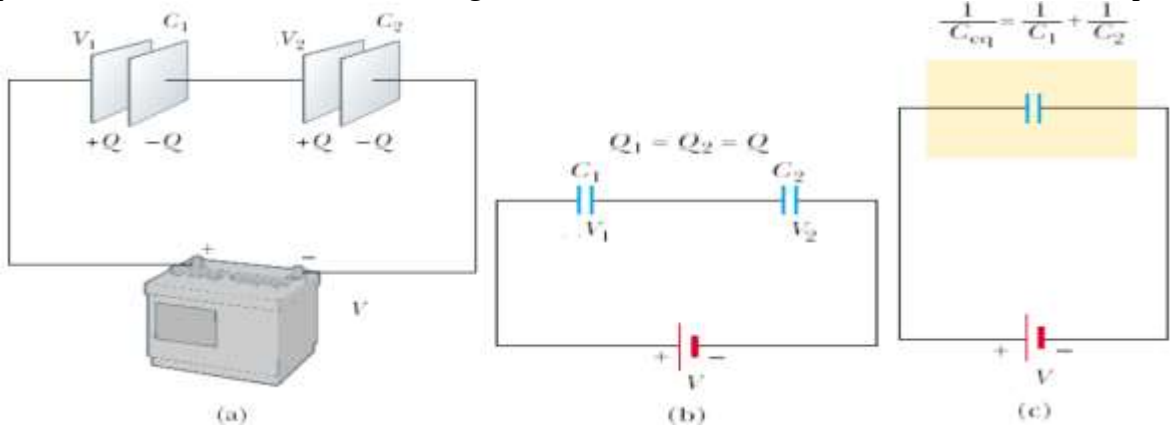
Questions from national exam

1. A parallel plate air capacitor has a capacitance of 4×10^{-8}F. What would be the value of the capacitance if the region between the plates is filled with a material having a dielectric constant of 8? **Ans**.C'= 3.2×10^{-7}F
   A) 5×10^{-7} F B) 5×10^{7} F C) 3.2×10^{-9} F D) 3.2×10^{9} F
2. A Parallel plate air capacitor is charged to a potential difference of 75 V. The capacitor is then removed from the charging source and the region between the plates is filled with a material having a dielectric constant of 3. What is the new potential difference across the capacitor?
   A) 20 V B**)** 25 V C) 30 V D) 35 V
3. An air-filled parallel plate capacitor has a capacitance of 1 pF. The plate separation is then doubled and a wax dielectric is inserted, completely filling the space between the plates. As a result, the capacitance becomes 2 pF. What is the dielectric constant of the wax?
   A) 2.0 B**)** 4.0 C) 6.0 D) 8.0
4. A sheet of polythelene (*k* = 2.3) and 0.25 *mm* thick is to be used in a capacitor by sandwiching it between two sheets of aluminum foil. What area must the sheets have if the capacitor is to have a capacitance of 0.5 μF?
   A) 12 m^2 B) 0.12 m^2 C) 0.6 m^2 D) 6.14 m^2

Capacitor networks

i) Capacitors in series

Capacitors connected as shown in the figure below are known as a *series combination* of capacitors.



(a) A series combination of two capacitors. The charges on the two capacitors are the same. (b) The circuit diagram for the series combination. (c) The equivalent capacitance.

The total potential difference across any number of capacitors connected in series is the sum of the potential differences across the individual capacitors. That is

$$V = V_1 + V_2$$

The charges on capacitors connected in series are the same.

Where $V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$ and $V = \frac{Q}{C_{eff}}$

$$\frac{Q}{C_{eff}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad \text{OR} \qquad C_{eff} = \frac{C_1 C_2}{C_1 + C_2}$$

When N capacitors are connected in series, the relationship for the equivalent capacitance is

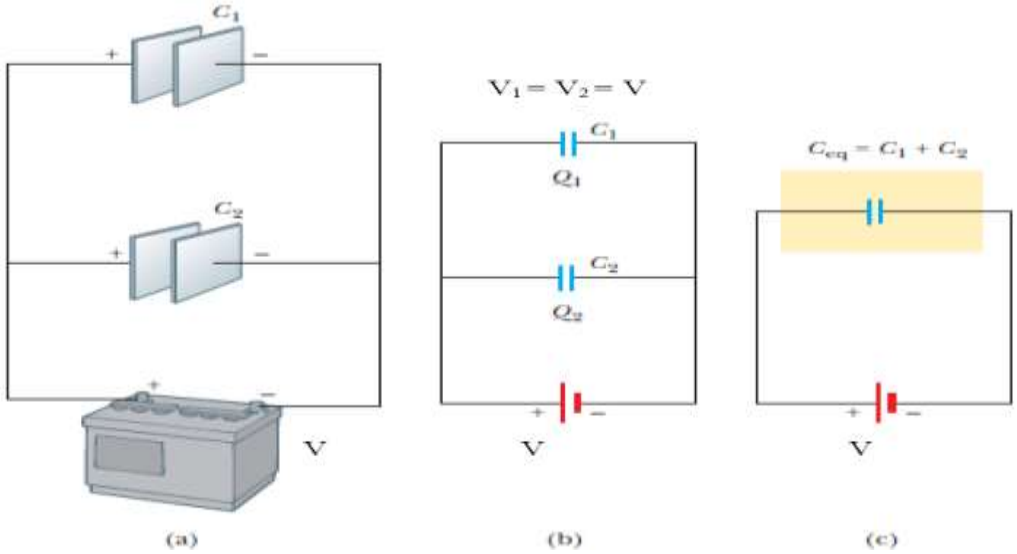
$$\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_N}$$

If there are *n* identical resistors in series, show that the equivalent capacitance is given by

$$C_{eq} = \frac{C}{n}$$

ii) Capacitors in parallel

Two capacitors connected as shown in the figure below are known as a *parallel combination* of capacitors.



(a) A parallel combination of two capacitors in an electric circuit in which the potential difference across the battery terminals is *V*. (b) The circuit diagram f(the parallel combination. (c) The equivalent capacitance.

The total charge on capacitors connected in parallel is the sum of the charges on the individual capacitors.

$$Q = Q_1 + Q_2$$

The individual potential differences across capacitors connected in parallel are the same and are equal to the potential difference applied across the combination.

Because $V_1 = V_2 = V$,

$Q = Q_1 + Q_2$ becomes

$C_{eq} V = C_1 V + C_2 V$

$C_{eq} = C_1 + C_2$

If we extend this treatment for N capacitors connected in parallel, we find the equivalent capacitance to be

$$C_{eq} = C_1 + C_2 + C_3 + \ldots + C_N$$

Thus, the equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitances.

If there are *n* identical capacitors in parallel, show that the equivalent capacitance is given by

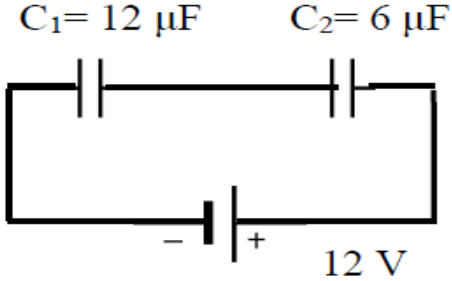
$C_{eq} = nC$

Example

1. Calculate

a) the equivalent capacitance.
b) the total charge.
c) the charge on each capacitor.
d) the potential difference across each capacitor.

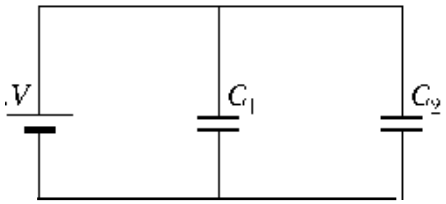
i)



Ans. a) $C_{eq} = 4\ \mu F$ c) $Q_1 = Q_2 = Q_{Tot} = 48\ \mu C$

b) $Q_{Tot} = 48\ \mu C$ d) $V_1 = 4V$, $V_2 = 8\ V$

ii)

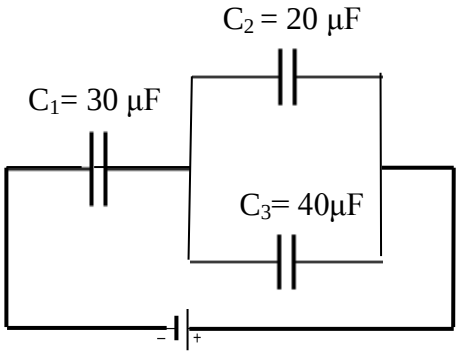


If C_1= 20 μF, C_2 = 40 μF and V= 3 V

Ans. a) C_{eq}= 60 μF c) Q_1= 60 μC, Q_2= 120 μC

b) Q_{Tot} = 180 μC d) V_1= 3 V, V_2= 3 V

iii)

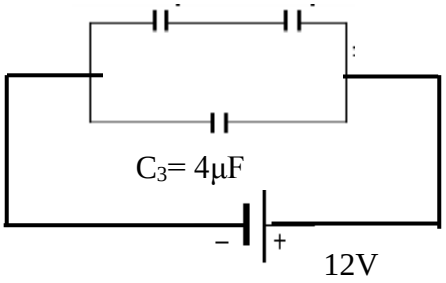


Ans. a) C_{eq}= 20 μF c) V_1= 8 V, V_2= 4 V & V_3= 4 V

b) Q_{Tot} = 240 μC d) Q_1 = 240 μC, Q_2 = 80 μC

Q_3 = 160 μC

iv)

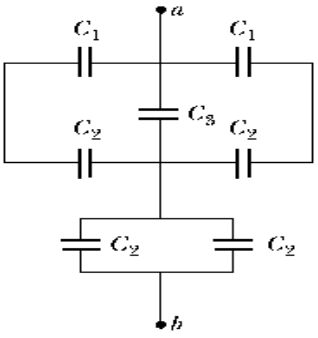


Ans. a) C_{eq}= 6 μF d) V_1= 8 V, V_2 = 4V, V_3= 12 V

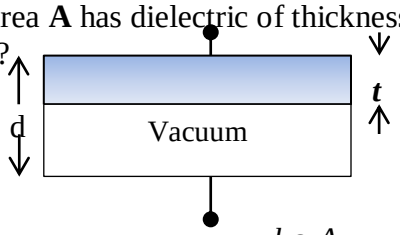
b) Q_{Tot} = 72 μC

c) Q_1= 24 μC, Q_2= 24 μC, Q_3= 48 μC & $Q_{1,2}$ = 24 μC

2. Two capacitors, C_1= 3 μF and C_2 = 6 μF, are connected in series across a 24 V battery. What is
   a) the equivalent capacitance? **Ans**. C_{eq}= 2 μF. c) the charge on each capacitor? **Ans**. Q_1=Q_2=Q_{Tot}= 48 μC
   b) the total charge? **Ans**. Q_{Tot} = 48 μC, d) the p.d across each capacitor? **Ans**. V_1= 16 V, V_2= 8 V
3. If the capacitors in question 2 above are connected in parallel across a 24 V battery, what is
   a) the equivalent capacitance? **Ans**. C_{eq}= 9 μF. c)the charge on each capacitor? **Ans**. Q_1 =72 μC, Q_2= 144 μC
   b) the total charge? **Ans**. Q_{Tot} = 216 μC. d) the p.d across each capacitor? **Ans**. V_1 = V_2= 24 V
4. Prove that in series combination, the equivalent capacitance of two capacitors is always smaller than the smaller capacitance of the two.
5. Find the equivalent capacitance between points *a* and *b* for the group of capacitors connected in figure below if C_1= 9 μF, C_2 = 6 μF and C_3 = 4.8 μF. **Ans**. C_{eq}= 6 μF



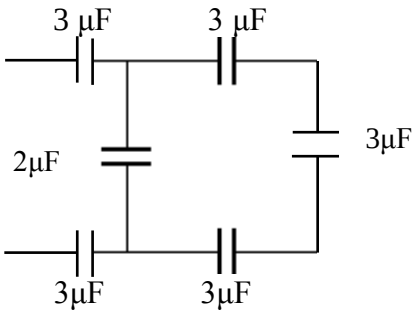
6. A parallel plate capacitor with a plate area **A** has dielectric of thickness ***t*** between its plates. If the plats separation is ***d***, what is the capacitance?



Ans. $C_{eq} = \dfrac{k\varepsilon_0 A}{k(d-t)+t}$

7. Consider question 6 above. Check if the equation $C_{eq} = \dfrac{k\varepsilon_0 A}{k(d-t)+t}$ gives the expression for capacitor in vacuum and with full dielectric. Hint: For a capacitor in vacuum t =0 and with a full dielectric $t = d$.

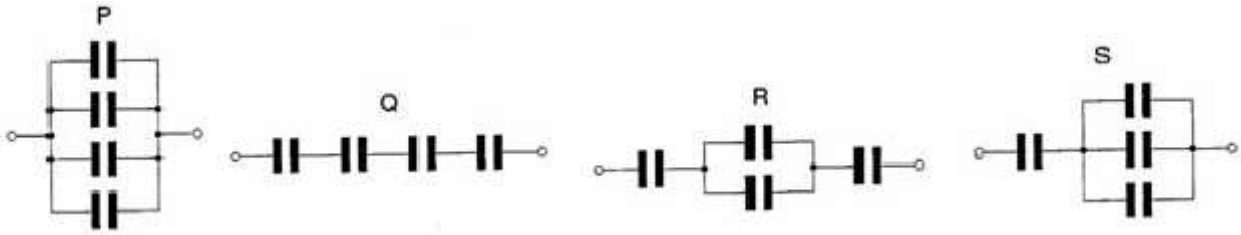
8. What is the equivalent capacitance for the circuit below?



Ans. $C_{eq} = 1\mu F$

Exercise

1. Four identical capacitors are connected in four different combinations, P, Q, R and S as shown. Organize these circuits into order of increasing effective capacitance.
   Ans. $C_Q = 0.25C$, $C_R = 0.4C$, $C_S = 0.75C$ and $C_P = 4C$

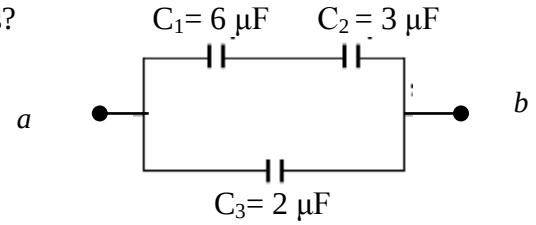


2. Three capacitors, each has area *A* and plate separation d, are connected
   a) in parallel. What must be the spacing of a single capacitor with area A if its capacitance equals that of the parallel combination? **Ans**. $d' = 1/3d$.
   b) in series. What must be the spacing of a single capacitor with area A if its capacitance equals that of the series combination? **Ans**. $d' = 3d$.
3. The potential difference applied across a series combination of 3 μF and 6 μF capacitors is 600 V. What is the p.d across each capacitor? **Ans**. $V_{4\mu F} = 400$ V and $V_{6\mu F} = 200$ V
4. Two capacitors when connected in parallel give an equivalent capacitance of 9 pF and give an equivalent capacitance of 2 pF when connected in series. What is the capacitance of each capacitor?
   Ans. $C_1 = 3$ pF, $C_2 = 6$ pF
5. The equivalent capacitance of three identical capacitors connected in parallel is 9 μF. What will be their equivalent capacitance when they are connected in series? **Ans**. $C_{eq} = 1$ μF
6. Two capacitors, $C_1 = 6$ μF and $C_2 = 3$ μF, are connected in series. If the p.d across this combination is 18 V, what is the p.d across C_1 and C_2? **Ans**. $V_1 = 6$ V, $V_2 = 12$ V
7. Two capacitors each with a capacitance of 2 μF are connected in parallel. If this combination is connected in series with a 4 μF capacitor, what is the equivalent capacitance? **Ans**. $C_{eq} = 2$ μF
8. Seven capacitors each of capacitance 1F are connected in parallel, and their combination is connected in series to a 3 F capacitor. What is the equivalent capacitance of the combination? **Ans**. $C_{eq} = 2.1$ F

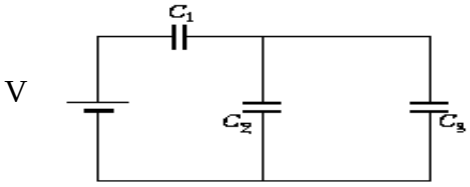
9. Three capacitors with capacitance of 0.2 μF, 0.3 μF and 0.5 μF are connected in parallel. If this combination is connected across a potential difference of 100 V, what is the total charge on the capacitors? **Ans**. $Q_{Tot} = 100$ μC

10. Three capacitors C_1, C_2 and C_3 are connected as shown in the figure below. If the p.d across points *a* and *b* is 15 V, what is the total charge on the capacitors?

Ans. $Q_{Tot} = 60$ μC



11. Three capacitors are connected to a battery as shown in the figure below. Their capacitances are $C_1 = 3C$, $C_2 = C$, and $C_3 = 5C$.



a) What is the equivalent capacitance of this set of capacitors? **Ans**. $C_{eq} = 2C$

b) State the ranking of the capacitors according to the charge they store, from largest to smallest.

Ans. $Q_1 = Q_{Tot} = 2CV$, $Q_3 = \frac{5}{3}CV$ and $Q_2 = \frac{1}{3}CV$

c) Rank the capacitors according to the potential differences across them, from largest to smallest.

Ans. $V_1 = \frac{2}{3}V$, $V_2 = V_3 = \frac{1}{3}V$

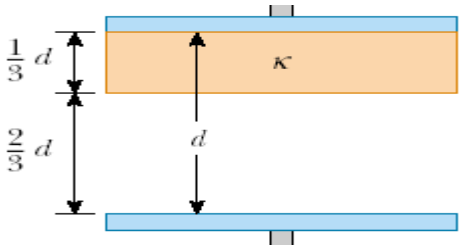
12. When two capacitors are connected in parallel, the equivalent capacitance is 4 μF. If the same capacitors are connected in series, the equivalent capacitance is one-fourth the capacitance of one of the two capacitors. What are the two capacitors? **Ans**. 1 μF and 3 μF

13. Two capacitors, C_1 and C_2, when connected in parallel give an equivalent capacitance of C_p and an equivalent capacitance of C_s when connected in series. Show that the capacitance C_1 and C_2 are:

$$C_1 = \frac{C_p - \sqrt{C_p^2 - 4C_sC_p}}{2} \text{ and } C_2 = \frac{C_p + \sqrt{C_p^2 - 4C_sC_p}}{2} \quad \text{OR}$$

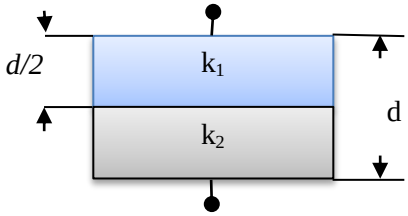
$$C_1 = \frac{C_p + \sqrt{C_p^2 - 4C_sC_p}}{2} \text{ and } C_2 = \frac{C_p - \sqrt{C_p^2 - 4C_sC_p}}{2}$$

14. A parallel-plate capacitor with a plate separation *d* has a capacitance C_0 in the absence of a dielectric. What is the capacitance when a slab of dielectric material of dielectric constant k and thickness $\frac{1}{3}d$ is inserted between the plates? **Ans**. $C = \left(\frac{3k}{2k+1}\right)C_0$

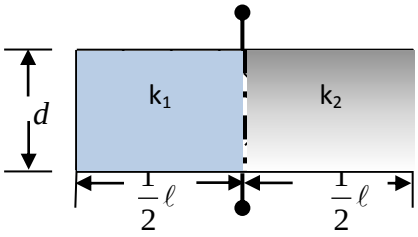


15. A parallel plate capacitor, filled with two dielectrics of dielectric constants k_1 and k_2, each occupy half of the volume as in the figure below. Show that the capacitance is given by:

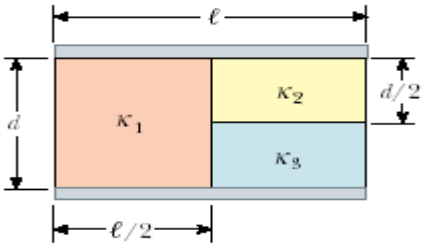
a) $C = \frac{2\varepsilon_0 A}{d}\left(\frac{k_1 k_2}{k_1 + k_2}\right)$



b) $C = \frac{\varepsilon_0 A}{2d}(k_1 + k_2)$

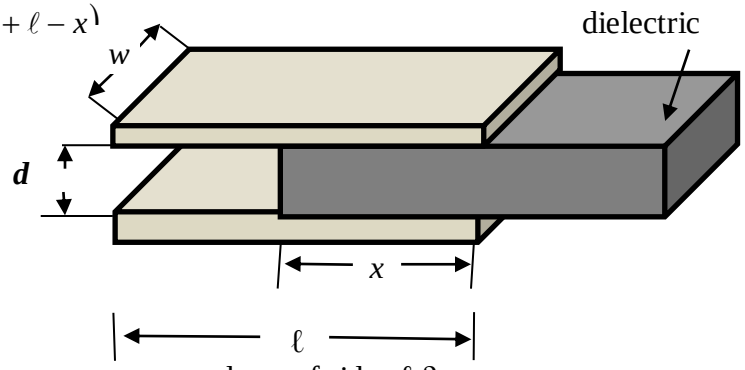


16. A parallel-plate capacitor is constructed by filling the space between two square plates with blocks of three dielectric materials, as in the figure below. (a) Find an expression for the capacitance of the device in terms of the plate area *A*, *d*, k_1, k_2 and k_3. **Ans**. $C = \frac{\varepsilon_0 A}{d}\left(\frac{k_1}{2} + \frac{k_2 k_3}{k_2 + k_3}\right)$



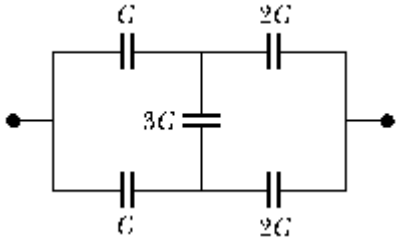
17. A dielectric slab, with dielectric constant **k**, is partially inserted a distance ***x*** between the plates of a parallel – plate capacitor of plate separation ***d*** and lateral dimensions ***w*** & ℓ , as shown in the figure.

a) Show that the capacitance is: $C = \frac{\varepsilon_0 w}{d}(kx + \ell - x)$



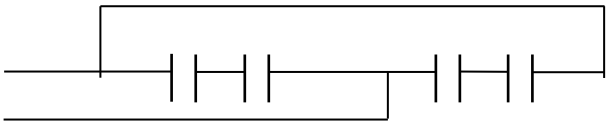
b) What is the capacitance if the parallel plates were square plates of sides ℓ ?

18. Show that the equivalent capacitance of the combination shown in figure below is: $C_{eq} = \frac{4C}{3}$

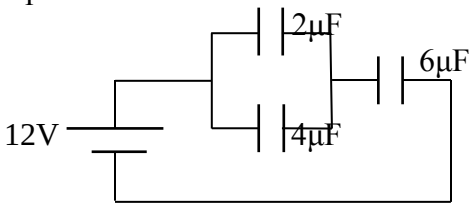


Questions from national exam

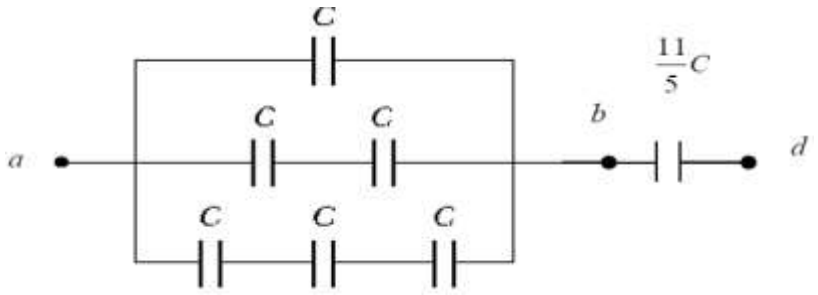
1. A group of identical capacitors is connected first in series and then in parallel. The combined capacitance in parallel is 16 times larger than for the series combination. How many capacitors are in the group?
   A**)** 4 B) 6 C) 8 D) 10
2. The potential difference applied across a series combination of a 4μF and a 12μF capacitors is 240 V. What is the potential difference across the 4μF capacitor?
   A**)** 180 V B) 150 V C) 120 V D) 60 V
3. The effective capacitance of two capacitors, when connected in parallel is 32μF and when connected in series is 6μF. What are values of the two capacitors in μF?
   A) 16, 16 B) 20, 12 C) 5, 27 D**)** 8, 24
4. In the figure below, capacitor has a capacitance of 2 μF. What is the equivalent capacitance of the arrangement?
   A**)** 2 μF C) 1 μF
   B) ½ μF D) 1.5 μF



5. What is the voltage in V across the 2 μF capacitor?
   A) 1 C**)** 6
   B) 2 D) 8



6. Three capacitors of capacitances 8 μF, 12 μF and 24 μF are connected (a) in series and then (b) in parallel. What is the ratio of the equivalent capacitance in case (a) to that in case (b)?
   A**)** 1:11 B) 11:1 C) 1:1 D) 1:3
7. Consider the network of identical capacitors, each having capacitance C between terminals *a* and *b* and capacitor of capacitance $\frac{11}{5}$C between *b* and *d* shown in figure below. What is the effective capacitance between terminal *a* and *d*?
   A) $\frac{121}{30}C$ C) $\frac{11}{6}C$
   B**)** 1C D) 3C

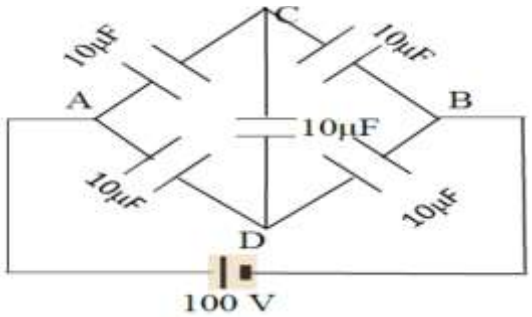


8. A parallel plate capacitor with a plate separation *d* has a capacitance *C* in the absence of a dielectric. What is the capacitance when a slab of dielectric of dielectric constant *k* and thickness 1/3*d is* inserted between the plates?
   A**)** $\frac{3k}{2k+1}C$ B) $\frac{2k+1}{3k}C$ C) $\frac{3k}{2k-1}C$ D) $\frac{2k-1}{3k}C$

9. Five capacitors 10 μF capacity each is connected to A, D, and C potential of 100 V as shown in the next figure. What is the equivalent capacitance between A and B

A**)** 10 μF C) 30 μF

B) 20 μF D) 60 μF



10. What is the capacitance of a capacitor that should be connected with 2 other capacitors having capacitances of 4 μF and 8 μF all in series to produce an effective capacitance of $\frac{24}{11}$ μF?

A**)** 12 μF B) 10 μF C) 8 μF D) 4 μF

Energy stored in a capacitor

The process of charging a capacitor is equivalent to transferring charge from lower potential plate to the plate at higher potential. The charging process therefore requires doing work and adding energy to the system. The final charge and the final potential difference are related by

$Q = CV$

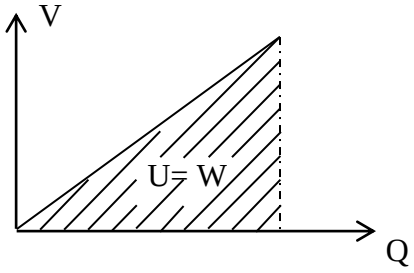
Since potential difference increases proportionally with the charge, the average p.d V_{av} during the charging process is

$$V_{av} = \frac{1}{2}\frac{Q}{C}$$

The total work is

$$W = V_{av}Q$$

$$W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$



This is equal to the shaded area

A charged capacitor is the electrical analogy of a stretched spring, whose elastic potential energy $U = \frac{1}{2}kx^2$

***Q* is analogous to the elongation *x* and 1/C is analogous to the force constant *k*.**

The energy supplied to a capacitor in the charging process is stored by the capacitor and is released when it discharges.

The capacitance of a parallel plate capacitor in vacuum is

$$C = \frac{\varepsilon_0 A}{d} \qquad \ldots\ldots\ldots\ldots .(1)$$

The electric field in the space between the plates is

$$E = \frac{V}{d} \quad \text{OR} \quad V = Ed \ldots\ldots\ldots .(2)$$

Substituting eq (1) and eq (2) in $U = \frac{1}{2}CV^2$ we get

$$U = \frac{1}{2}\varepsilon_0 E^2 Ad$$

The volume of the space between the plates of the capacitors is

$V = Ad$

The energy density U_e is defined as energy per unit volume

$$U_e = \frac{1}{2}\frac{\varepsilon_0 E^2 Ad}{Ad} \qquad \text{OR} \qquad U_e = \frac{1}{2}\varepsilon_0 E^2$$

The energy density in any electric field is proportional to the square of the magnitude of the electric field at a given point.

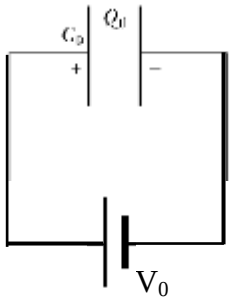
If the space between the plates is filled by a dielectric with dielectric constant k, the stored energy and energy density will take the form

$$U = \frac{1}{2}\varepsilon_0 kE^2 Ad \quad \text{and} \quad U_e = \frac{1}{2}\varepsilon_0 kE^2$$

Example

1. A parallel plate air capacitor has an area of 2 cm^2 and separation 5 *mm*. If the capacitor is charged to 100 V, calculate
   a) the stored energy. **Ans**. U= 1.77×10^{-9} J. b) the energy density. **Ans**. U_e= 1.77×10^{-3} J/m^3.
2. If the space between the plates in question 1 were filled with a dielectric of k = 5 and charged to 100 V, how would the result be altered? **Ans**. a) U= 8.85×10^{-9} J. b) U_e= 8.85×10^{-3} J/m^3
3. A parallel-plate capacitor is charged with a battery to a charge Q_0, as shown in figure below. The battery is then removed, and a slab of material that has a dielectric constant **k** is inserted between the plates. Show that the energy stored before and after the dielectric is inserted are, respectively

$$U_0 = \frac{1}{2}\frac{Q_0^2}{C_0}, \qquad U = \frac{U_0}{k}$$



Exercise

1. a) A parallel plate of area 2000 cm^2 and plat separation 1 cm is charged to 2000 V. Calculate energy stored and energy density. **Ans**. U= 3.54×10^{-4} J and U_e= 0.177 J/m^3
   b) If the capacitor in (a) is disconnected from the charging battery and a dielectric of k= 5 is inserted in the space between the plates, calculate U and U_e. **Ans**. U= 7.08×10^{-5} J, U_e= 0.0354 J/m^3
2. A parallel plate of area *A* and separation *d* is charged to potential difference V_1. The charging battery is then disconnected and the plates are pulled apart until their separation is doubled. Show that
   a) the new p.d is $V_2 = 2V_1$.
   c) the final stored energy is $U_f = \frac{\varepsilon_0 V_1^2 A}{d}$
   b) the initial stored energy is: $U_i = \frac{1}{2}\frac{\varepsilon_0 V_1^2 A}{d}$. d) the work required to pull apart the plates. $W = \frac{1}{2}\frac{\varepsilon_0 V_1^2 A}{d}$
3. A parallel plate air capacitor has a volume of 2 m^3 and the electric field between the plates is 200 N/C. What is
   a) the stored energy? **Ans**. U= 354 *n*J. b)the energy density? **Ans**. U_e= 177 *n*J/m^3
4. A parallel-plate capacitor is charged and then disconnected from the battery. By what fraction does the stored energy change (increase or decrease) when the plate separation is doubled? **Ans**. it increases twice.
5. Two capacitors, C_1 = 25μF and C_2 =5μF, are connected in parallel and charged with a 100V power supply.
   a) Draw a circuit diagram and calculate the total energy stored in the two capacitors. **Ans**. U= 0.15 J
   b) What potential difference would be required across the same two capacitors connected in series in order that the combination stores the same amount of energy as in (a)?
   Draw a circuit diagram of this circuit. Ans. V'= 268 V

6. Two identical parallel-plate capacitors, each with capacitance *C*, are charged to potential difference *V* and connected in parallel. Then the plate separation in one of the capacitors is doubled. Find
   a) the total energy of the system of two capacitors *before* the plate separation is doubled. **Ans**. $U_0 = CV^2$
   b) the potential difference across each capacitor *after* the plate separation is doubled. **Ans**. $V'_1 = 2V$, $V'_2 = V$
   c) the total energy of the system *after* the plate separation is doubled. **Ans**. $U = \frac{3}{2}CV^2 = \frac{3}{2}U_0$
7. A certain capacitor is charged to a potential V. By what percent should you increase V if you wish to increase its stored energy by:
   a) 21%? **Ans**. 10% . b) 44%? **Ans**. 20%. c) 69%? **Ans**. 30%. d) 96%? **Ans**. 40%.

Questions from national exam

1. A 4 μF capacitor and a 6 μF capacitor are connected in series across a potential difference of 500 V. What is the energy stored in the 4 μF capacitor?
   A) 0.12 J B**)** 0.18 J C) 0.24 J D) 1.2 J
2. If the energy stored in a 12 μF capacitor is 1.5×10^{-2} J, what is the potential difference across the capacitor?
   A) 20 V B) 30 V C) 40 V D**)** 50 V
3. A 10 μF capacitor is charged to a potential difference of 20 V. What is the energy stored in the capacitor?
   A) 1×10^{-4} J B) 2×10^{-4} J C**)** 2×10^{-3} J D) 4×10^{-3} J
4. Two capacitors, 1 μF and 3 μF, are connected in parallel across a 300 V potential difference. What is the energy stored in joules in the capacitors?
   A) 0.27 B**)** 0.18 C) 0.32 D) 0.4
5. A certain capacitor is charged to a potential V. If you wish to increase its stored energy by 10%, then by what percentage should you increase V?
   A**)** 4.88% B) 5.92% C) 6.98% D) 5.13%
6. Which one of the following changes to parallel plate capacitor would not increase energy stored in the capacitor at a fixed voltage across the plates?
   A) increasing the area of the plates. C) decreasing the charge on the plates.
   B) increasing the dielectric constant. D) increasing the distance between the plates.
7. A capacitor made of parallel plates is charged by connecting it to a battery. After fully charged if the plates are disconnected from the battery and then pulled apart a little, what will happen to the total stored energy, electric field, and the potential difference between the plates?
   A) The electric field between the plates increases.
   B) The potential difference between the plates decreases.
   C) The total energy stored in the capacitor increases.
   D) The electric field, the potential difference and the stored energy remain the same.
8. A 12 μF capacitor can be fully charged when it is connected to a potential difference of 9 V. How much electrical energy is used to charge the capacitor?
   (A) 1.0×10^{-3} J (B) 54×10^{-4} J (C) 9.72×10^{-4} J (D**)** 4.86×10^{-4} J

Charging and discharging a capacitor

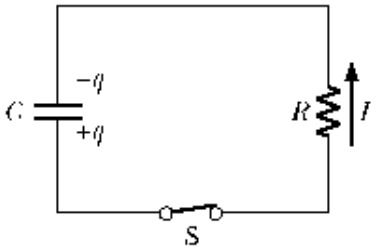
i) Discharging a capacitor

Consider a capacitor of capacitance C initially charged to a voltage V_0 and connected to resistor R. Initially the capacitor has charge Q_0. The capacitor then discharges through a resistor R and a current I= V/R flows through the resistor.

*The voltage across the capacitor after time *t* is given by

$$V=V_0 e^{-t/RC}$$

The voltage across the capacitor decreases exponentially with time.



*The charge Q on the capacitor at any time t is

$$Q= Q_0 e^{-t/RC}$$

The charge is also decreasing exponentially like the voltage.

*The current through the resistor at any time t is

$$I= I_0 e^{-t/RC}$$

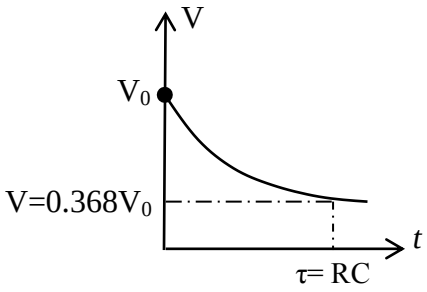
The current is also decreasing exponentially with time.

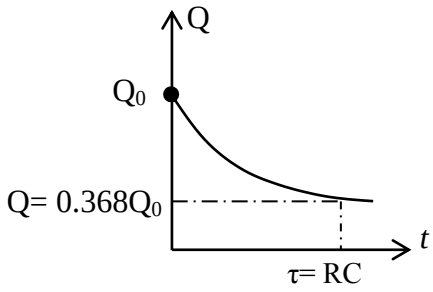
Where V_0 is voltage at t = 0, which is initially maximum.
Q_0 is charge at t = 0, which is initially maximum.
I_0 is current at t = 0, which is initially maximum.

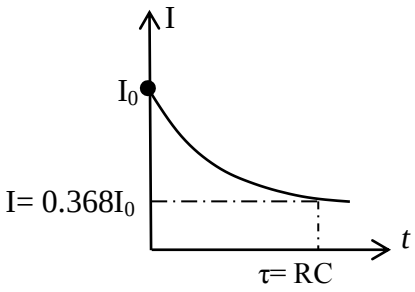
Note that, the exponent is dimensionless.

The product **RC** *is called* ***time constant (τ)***

when $t= RC= \tau$: $V= \frac{V_0}{e} = 0.368V_0$, $Q = \frac{Q_0}{e} = 0.368Q_0$, $I= \frac{I_0}{e} = 0.368I_0$







Example

1. In a discharging circuit, C= 4 μF, V_0 = 12 V and R = 10 MΩ.
   a) Calculate: τ, I_0 and Q_0. **Ans**. τ = 40 sec, I_0 = 1.2 μA and Q_0 = 48 μC
   b) Calculate: I, Q and V at
      i) t = 1 sec. **Ans**. I =1.17 μA, Q = 46.8 μC, V = 11.7 V
      ii) t =20 sec. **Ans**. I = 0.728 μA , Q = 29.12 μC, V = 7.28 V
      iii) t = 40 sec. **Ans**. I = 0.442 μA, Q = 17.66 μC, V = 4.42 V
      iv) t = 80 sec. **Ans**. I = 0.163 μA, Q = 6.5 μC, V= 1.63 V
      v) t = 120 sec. **Ans**. I = 0.06 μA, Q = 2.4 μC, V = 0.6 V
      vi) t = 160 sec. **Ans**. I = 0.022 μA, Q = 0.88 μC, V = 0.22 V

c) Calculate: the time at which

i) Q=90%Q_0, I= 90%I_0, V= 90%V_0. **Ans**. t = 4.2 sec.

ii) Q= 80%Q_0, I= 80%I_0, V=80%V_0. **Ans**. t = 8.9 sec

iii) Q= 70%Q_0, I=70%I_0, V= 70%V_0. **Ans**. t = 14.3 sec.

iv) Q= 60%Q_0, I= 60%I_0, V= 60%V_0. **Ans**.t = 20.4 sec.

v) Q= 50%Q_0, I= 50%I_0, V=50%V_0. **Ans**. t = 27.7 sec.

d) What do you conclude from (b) and (c) above?

2. A capacitor is connected across 10kΩ resistor. After 8sec the current has decreased to 36.8% of its initial value. What is the capacitance? **Ans**. C = 800 μF
3. How long will it take for a current to decrease from 120 μA to 60 μA? (C=500μF, R=100kΩ). **Ans**. t=34.7 s
4. A capacitor of capacitance *C* is being discharged through a resistor of resistance *R*.

a) After how many time constants is the charge on the capacitor decreases to one-fourth of its initial value? **Ans**. t =1.386τ

b) Show that the stored energy in the discharging capacitor decreases to U= $U_0e^{-2t/RC}$ after time *t*.

c) After how many time constants is this stored energy one- fourth its initial value? **Ans**. t = 0.693τ

Exercise

1. A 50μF capacitor is discharged through a 10kΩ resistor. How long will it take for the p.d across the capacitor to fall to 40% of its initial value? **Ans**. t= 0.46 sec
2. The initial potential difference across a capacitor is 100V. If the Potential difference is decreased to 25 V after 10 sec. Calculate

a) the time constant. **Ans**. τ = 7.2 sec c) p.d at three time constant. **Ans**. V = 5 V

b) p.d at twice time constant. **Ans**. V = 13.5V d) p.d at four time constant. **Ans**. 1.83V

3. A fully charged capacitor stores energy U_0. How much energy remains when its charge has decreased to half its original value? **Ans**. U= 1/4U_0
4. How long will it take for the charge to decrease to 13.54% of its initial value?(13.54%=0.368 × 0.368%) **Ans**. t = 2RC

ii) Charging a capacitor

Consider a circuit that consists resistance of resistor **R**, uncharged capacitor of capacitance **C**, a battery with p.d $\mathbf{V_0}$ and a switch **S**. When the switch is closed, current starts to pass through **R** and charges the capacitor. As the capacitor charges, V_c increases and V_R decreases.

Initially

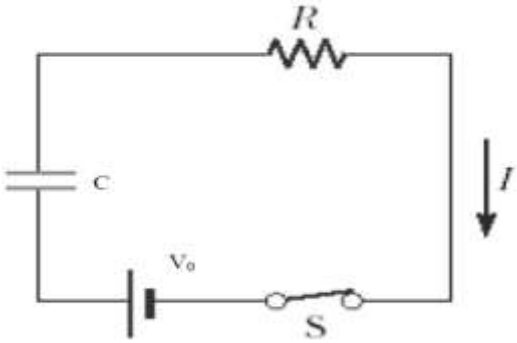
- the charge on the capacitor is zero. $Q_0 = 0$
- the p.d across the capacitor is zero. $V_{0C} = 0$
- the current through the resistor $I_0 = \frac{V_0}{R}$ is maximum. As time passes, the charge on the capacitor increases, reaching a maximum value.

Note that during charging, charges do not jump across the capacitor plates because the gap between the plates represents an open circuit. Instead, charge is transferred between each plate and its connecting wires.

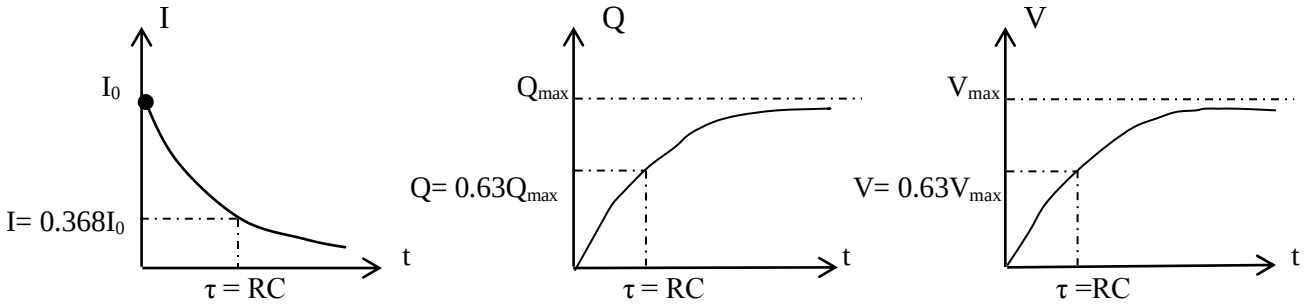
$Q_{max} = CV_0$ after which the battery cannot charge the capacitor.

*Current I is maximum at t= 0 and approaches zero as t becomes larger.

- The variation in I with time is given by
  $I= I_0e^{-t/RC}$
- The dependence of Q on time is
  $Q = Q_{max}(1- e^{-t/RC})$
- The p.d across the capacitor after time t is
  $V=V_0(1- e^{-t/RC})$ Where $V_0= \frac{Q_{max}}{C}$



Like the charge, the p.d V across the capacitor increases as time elapses.
When t = RC, $I= 0.368I_0$, $Q= 0.63Q_{max}$ and $V= 0.63V_{max}$



When τ is *small*, the *capacitor charges quickly*;
τ is *large*, the *capacitor takes more time to charge*.

Example

1. C=2.2 μF, R= 10 MΩ, V_{max}= 50 V.
   a) Calculate: τ, Q_{max} and I_0. **Ans**. τ= 22 sec, Q_{max}= 110 μC, I_0= 5 μA
   b) Calculate: the time taken for the capacitor to charge to 25% of Q_{max}. **Ans**. t= 6.34 sec
   c) Calculate: I, Q and V
      i) at t = 1 sec. **Ans**. I= 4.8 μA, Q = 4.4 μC, V= 2 V.
      ii) at t =11 sec. **Ans**. I=3.03 μA, Q =43.28 μC, V=19.65V
      iii) at t = 22 sec. **Ans**. I= 1.84 μA, Q =69.53 μC, V= 31.6 V
      iv) at t = 44 sec. **Ans**. I= 0.677 μA, Q =95.15 μC, V=43.25 V
      v) at t = 66 sec. **Ans**. I= 0.25 μA, Q = 104.5 μC, V= 47.5 V
      vi) at t = 88 sec. **Ans**. I = 0.09 μA, Q = 108.02 μC, V= 49.1 V
   d) Calculate the time when
      i) $Q= 10\%Q_{max}$ & $V= 10\%V_{max}$ **Ans**. t = 2.3 sec , $I = 10\%I_0$. **Ans**. t = 50.7 sec
      ii) $Q= 20\%Q_{max}$ & $V= 20V_{max}$. **Ans**. t = 4.9 sec , $I = 20\%I_0$. **Ans**. t= 35.4 sec
      iii) $Q= 30\%Q_{max}$ & $V= 30V_{max}$. **Ans**. t = 7.8 sec, $I = 30\%I_0$. **Ans**. t = 26.5 sec
      iv) $Q= 40\%Q_{max}$ & $V= 40\%V_{max}$. **Ans**. t = 11.24 sec, $I = 40\%I_0$. **Ans**. t = 20.2 sec
      v) $Q= 50\%Q_{max}$ & $V= 50\%V_{max}$. **Ans**. t = 15.25 sec, $I = 50\%I_0$. **Ans**. t = 15.25 sec
      vi) $Q = 60\%Q_{max}$ & $V= 60\%V_{max}$. **Ans**. t = 20.2 sec, $I = 60\%I_0$. **Ans**. t = 11.24 sec
   e) What do you conclude from (c) and (d) above?
2. A 100 V power supply is used to charge a 2 μF capacitor which is connected in series with a 10 MΩ resistor. Calculate
   a) the time constant, Q_{max} & I_0. **Ans**. τ = 20 sec, Q_{max} = 200 μC, I_0 = 10 μA.
   b) the time at which
      i) Q reaches $1/4Q_{max}$. **Ans**. t = 5.75 sec
      ii) Q reaches $1/2Q_{max}$. **Ans**. t = 13.86 sec.
      iii) Q reaches $3/4Q_{max}$. **Ans**. t = 27.7 sec.

3. A capacitor in an *RC* circuit is charged to 60% of its maximum value in 0.9 s. What is the time constant of the circuit? **Ans**. $\tau = 0.098$ sec
4. Consider a series RC circuit for which R =1.0 MΩ, C = 5.0 μF and ε = 30.0 V. What is the time constant of the circuit and the maximum charge on the capacitor after which the switch is closed respectively?

A**)** 5 s, 150 μC B) 10 s, 160 μC C) 7 s, 155 μC D) 9 s, 135 μC

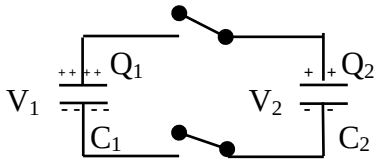
Joining two charged capacitors

If two capacitors, C_1 and C_2 with $V_1 > V_2$, are charged separately then disconnected from the charging batteries and

i) +ve plate of one capacitor connected to +ve plate of the other and –ve plate with –ve plate of the other, charge will move until the potential difference across the two capacitors is the same. At equilibrium the p.d across the two capacitors must be the same. The potential across C_1 must therefore decrease, which means that the charge stored on C_1 must decrease. At the same time the p.d across C_2 must increase, which means that the charge on C_2 must increase. The mobile charges in metallic conductors are electrons. Until the system comes to equilibrium, electrons will move from the +ve plate of C_2 to the +ve plate of C_1. An equal number of electors will move from the –ve plate of C_1 to the –ve plate of C_2.

At equilibrium, $V_1 = V_2 = V$

$$Q_1 + Q_2 = Q_{1f} + Q_{2f}$$
$$= C_1V + C_2V$$
$$Q_1 + Q_2 = (C_1 + C_2)V$$
$$V = \frac{Q_1 + Q_2}{C_1 + C_2}$$

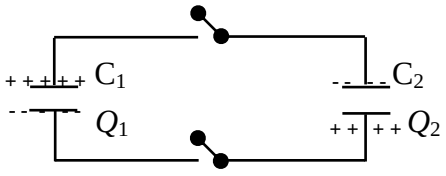


Where Q_1 is initial charge on C_1
Q_2 is initial charge on C_2
Q_{1f} is final charge on C_1
Q_{2f} is final charge on C_2.

ii) +ve plate of one capacitor is joined to –ve plate of the other as in the figure below, electrons will move from the –ve plate of C_2, to the +ve plate of C_1. An equal number of electrons will move from –ve plate of C_1 to +ve plate of C_2. After connection these charges partially cancel and the potential difference across each are equal.

$$Q_{1f} + Q_{2f} = |Q_2 - Q_1|$$
$$C_1V + C_2V = |Q_2 - Q_1|$$

$$V = \frac{|Q_2 - Q_1|}{C_1 + C_2}$$



Example

1. Two capacitors, 4 μF and 6 μF, are individually charged across a 50 V and 100 V batteries respectively. After being disconnected from the batteries, they are connected together with +ve plate of one with +ve plate of the other, –ve plate to –ve plate of the other. Calculate:
   a) the p.d across each capacitor. **Ans**. 80 V
   b) the final charge on C_1 and C_2. **Ans**. Q_{1f} = 320 μC & Q_{2f} = 480 μC
2. Consider the capacitors in question 1. If the –ve plate of one were connected to the +ve plate of the other, calculate
   a) the p.d across each capacitor. **Ans**. 40 V
   b) the final charge on C_1 and C_2. **Ans**. Q_{1f} = 160 μC & Q_{2f} = 240 μC

3. Two different capacitors of capacitances C_1 = 2 μF and C_2 = 3 μF are connected in series across a 120 V supply line. The charged capacitors are disconnected from the line and from each other, and reconnected with terminals of like sign together. What will be the final charges on C_1 and C_2, respectively?

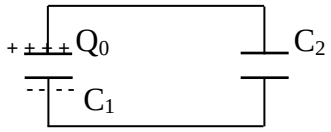
A) 57.6 μC and 86.4 μC
B**)** 115.2 μC and 172.8 μC
C) 144 μC across each
D) 288 μC across each

Connecting charged capacitor with uncharged capacitor

If capacitor C_1 in the figure below is initially charged by a source of V_0 then disconnected from the charging battery and connected with uncharged capacitor C_2, electrons will move from the upper plate of C_2 to the +ve plate of C_1. An equal number of electrons will move from the –ve plate of C_1 to the lower plate of C_2.

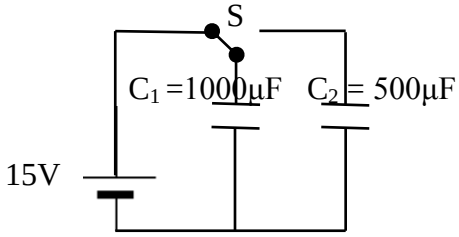
The initial charge on capacitor C_1 *is distributed over* C_1 *and* C_2.

$$Q_0 = Q_{1f} + Q_{2f}$$
$$= C_1V + C_2V$$
$$Q_0 = (C_1 + C_2)V$$
$$V = \frac{Q_0}{C_1 + C_2}$$ The final p.d across each capacitor



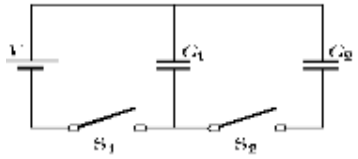
Example

1. If C_1 = 8 μF is charged to 120 V and disconnected from the charging battery, then connected to an uncharged 4 μF capacitor, what is
   a) the p.d across each capacitor? **Ans**. V= 80 V
   b) the final charge on each capacitor? **Ans**. Q_{1f} = 640 μC, Q_{2f} = 320 μC
2. A 1000 μF capacitor is charged from a 15 V battery. Then the switch is thrown to connect it to an uncharged 500 μF capacitor. Calculate
   a)the initial charge on C_1. **Ans**. Q_0 = 15 *m*C
   b) the p.d across each capacitor after C_1 is connected to C_2.
   Ans. V= 10 V
   c) the final charge on C_1 and C_2.
   Ans. Q_{1f} = 10 *m*C, Q_{2f} = 5 *m*C



3. Consider the circuit shown in figure below, where C_1 =6 μF, C_2 = 3 μF, and V = 20 V. Capacitor C_1 is first charged by the closing of switch S_1. Switch S_1 is then opened, and the charged capacitor is connected to the uncharged capacitor by the closing of S_2. Calculate

   a) the initial charge acquired by C_1. **Ans**. Q_{1i} =120 μC
   b) the final charge on each capacitor . **Ans**. Q_{1f} = 80 μC, Q_{2f} = 40μC



Questions from national exam

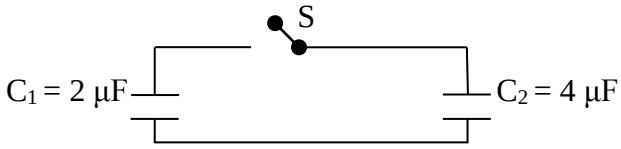
1. A 0.008 μF capacitor is charged to a p.d of 50 V. After disconnecting from the charging battery, the capacitor is connected in parallel with another uncharged capacitor. If the p.d drops to 32 V, what is the capacitance of the second capacitor?

   A**)** 4.5×10^{-9} F B) 4×10^{-7} F C) 4.5×10^{9} F D) 4×10^{7} F

2. A capacitor C_1 is charged to a p.d $V_0 = 6$ V, using a battery. The charging battery is then removed and the capacitor is connected as shown in the figure below to uncharged capacitor C_2. What is the common p.d in volts?

   A) 3 C) 4

   B) 1 D**)** 2



3. When switch S is thrown to the left in the figure shown below, the plates of the capacitor C_1 acquire a p.d V_0. C_2 and C_3 are initially uncharged. The switch is now thrown to the right. What is the final charge Q_1 on C_1?

   A) $\left(\frac{C_1C_2 + C_2C_3}{C_1C_2 + C_1C_3 + C_2C_3}\right)C_1V_0$

   B) $\left(\frac{C_1C_2 + C_1C_3}{C_1C_2 + C_1C_3 + C_2C_3}\right)C_3V_0$

   C) $\left(\frac{C_1C_2 + C_1C_3}{C_1C_2 + C_1C_3 + C_2C_3}\right)C_2V_0$

   D**)** $\left(\frac{C_1C_2 + C_1C_3}{C_1C_2 + C_1C_3 + C_2C_3}\right)C_1V_0$

