UNIT 8: PROPERTIES OF BULK MATTER

Elastic behavior

Elasticity;- is *the property of a body to retain its original shape or size after the removal of a deforming force*.

Bodies which can recover their original condition on the removal of the deforming forces are said to be **elastic**.

Bodies which do not recover their original conditions are said to be **plastic**.

Elastic limit:- is a point beyond which all materials are permanently deformed.

Elastic Properties of Solids

Except for our discussion about springs in earlier unit, we have assumed that objects remain rigid when external forces act on them. In reality, all objects are deformable. That is, it is possible to change the shape or the size (or both) of an object by applying external forces. As these changes take place, however, internal forces in the object resist the deformation.

We shall discuss the deformation of solids in terms of the concepts of *stress* and *strain*.

Stress:-is the *external force* acting on an object *per unit cross-sectional area*.

The result of a stress is *strain*.

Strain:- is *the measure of the degree of deformation*. It is found that, **for small stresses**, **strain is proportional to stress**; the constant of proportionality depends on the material being deformed and on the nature of the deformation. We call this proportionality constant the **elastic modulus**.

The elastic modulus is therefore defined as the ratio of the stress to the resulting strain:

I) Young's Modulus: Elasticity in Length

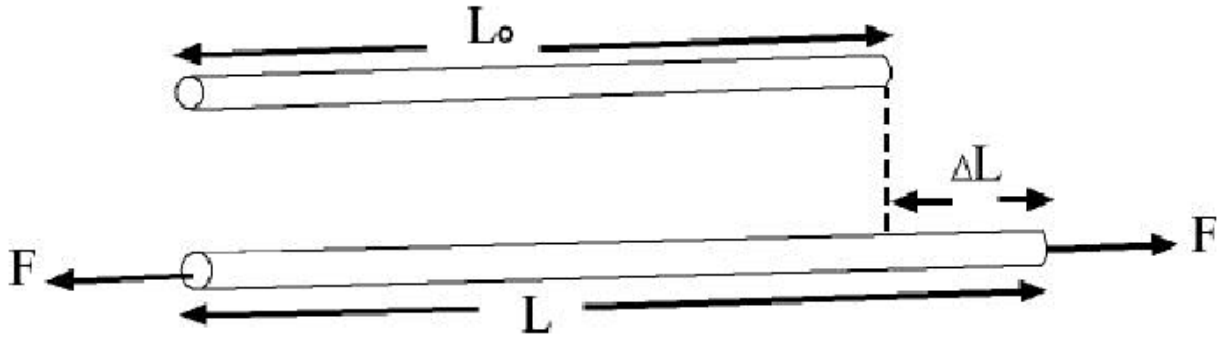
Young's modulus (Y):- measures the *resistance of a solid to a change in its length*.

Consider a long rod of cross-sectional area A and initial length L_0, as in figure below. When the rod is subjected to a tensile stress , internal forces in the rod resist distortion (stretching), but the rod reaches an equilibrium situation in which its final length L is greater than initial lengthL_0at which the external force is exactly balanced by internal forces. In such a situation, the rod is said to be stressed.

Tensile stress:- is *the ratio of the magnitude of the external force F* to the *cross-sectional area A*.

$$Tensile\,stress = \frac{External\,Force}{Cross-sectional\,area}$$

$$= \frac{F}{A}$$ Its SI unit is $N/m^2 = Pa$



Tensile strain:- is the ratio of the change in length ΔL to the original length L_0.

$$Tensile\,strain = \frac{Change\,in\,length}{Original\,length}$$

$$= \frac{\Delta L}{L_0}$$

For relatively small stresses, the bar will return to its initial length when the force is removed. The **elastic limit** of a substance is defined as the maximum stress that can be applied to the substance before it becomes permanently deformed and does not return to its initial length. It is possible to exceed the elastic limit of a substance by applying a sufficiently large stress. Initially, a stress-versus-strain curve is a straight line. As the stress increases, however, the curve is no longer a straight line. *When the stress exceeds the elastic limit, the object is permanently distorted and does not return to its original shape after the stress is removed. As the stress is increased even further, the material ultimately breaks.*

$$Young's\,Modulus = \frac{Tensile\ \ stress}{Tensile\ \ strain}$$

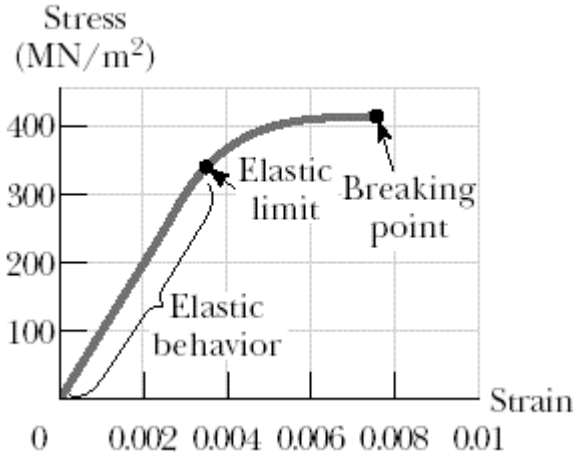
$$Y = \frac{F/A}{\Delta L/L_0}$$

$$Y = \frac{FL_0}{A\Delta L}$$

Show that the force constant $k = \frac{YA}{L_0}$

Its SI unit is N/m

Strain energy = ½ F ΔL



Example

1. A wire, 1m long and cross-sectional area $4\times10^{-6}m^2$, stretches by 0.5mm when a load of 200N hangs from it. Calculate
   a) stress. **Ans**. $5\times10^{7}N/m^2$.
   b) strain. **Ans**. 5×10^{-4} d) strain energy. **Ans**. $E = 5\times10^{-2}J$
   c) the force constant. **Ans**. $k = 4\times10^{5}N/m$
2. A wire of length 10cm and cross-sectional area $0.2cm^2$ stretches by 4mm when a load of 500N is applied. Calculate
   a) stress. **Ans**. $2.5\times10^{7}N/m^2$
   b) strain **Ans**. 4×10^{-4}
   c) the force constant. **Ans**. $k = 1.25\times10^{5}N/m$
   d) strain energy. **Ans**. E = 1J
3. A vertical steel beam in a building supports a load of $6\times10^{4}N$.If the length of the beam is 4m and its cross-sectional area is $8\times10^{-3}m^2$. What is the distance the beam is compressed along its length? **Ans**. $\Delta L = 1.5\times10^{-4}m$

Questions from national exam

1. A steel wire stretches by a certain amount when a load is hung from it. If another wire made of the same material with twice the cross-sectional area but of the same length supports the same load, then it will stretch by
   A) the same amount. C) twice as much.
   B) half as much. D) one quarter as much.
2. A steel has Young's Modulus of 2×10^{11}N/m^2 and a strain of 0.15 at the breaking point. The stress (N/m^2), at the breaking point for steel is therefore
   A)1.33×10^{11} B) 1.33×10^{12} C) 7.5×10^{-3} D**)** 3×10^{10}
3. When stretched beyond its elastic limit, a metal such as steel
   A) becomes elastic. C) obeys Hook's law.
   B) has no energy. D) becomes plastic.
4. Young's Modulus is the
   A) ratio of stress to strain. C) ratio of strain to stress.
   B) force per unit area. D) force per unit elongation.
5. When a load of 500kg is hanging from a steel wire of length 3m and cross sectional area 0.2 cm^2, the wire stretches beyond its no-load length. If the Young's Modulus for the wire made of steel is Y = 2×10^{11}Pa, what is the change in length of the wire beyond its no-load length?
   A**)** 3.75 mm B) 2.75 mm C) 1.75 mm D) 0.75 mm
6. Which one of the following laws best describes the behavior of elastic materials?
   A) Gay Lussac's law. C) Newton's second law.
   B) Hook's law. D) Newton's law of gravitation.

7. The definition of Young's Modulus is given by:
   A) force per unit area C) ratio of stress to strain
   B) ratio of strain to stress D) force per unit elongation

8. Which one of the following statements is correct?
   A) Elastic materials retain their shape after deformation.
   B) Plastic materials can recover the original size or shape after being distorted.
   C) Plastic materials change their shape by pressure.
   D) Elastic materials cannot recover the original size or shape after being distorted.

II)**Shear Modulus: Elasticity of Shape**

Shear modulus (S):- is a type of deformation that occurs when an object is subjected to a force parallel to one of its faces while the opposite face is held fixed by another force. The stress in this case is called a **shear stress**. If the object is originally a rectangular block, a shear stress results in a shape whose cross section is a parallelogram. Shear modulus is the measure of the resistance to motion of the planes within a solid parallel to each other.

$$Shear\ \ Stress = \frac{Tangentia\ \ Force}{Area\ \ of\ \ surface\ \ being\ \ sheared}$$

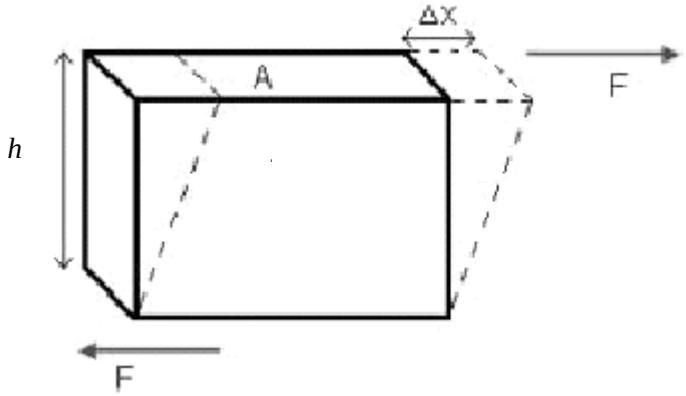
Shear stress = F/A Its SI unit is N/m^2 = Pa

The **shear strain** is defined as the ratio $\Delta x/h$ *where* Δx is the horizontal distance that the sheared face moves. *h* is the height of the object.

$$Shear\ \ Modulus = \frac{Shear\ \ stress}{Shear\ \ strain}$$

$$S = \frac{F/A}{\Delta x/h}$$

$$S = \frac{Fh}{A\Delta x}$$



Since Δx is usually very small, the ratio $\Delta x/h$ is approximately equal to shear angle ϕ (in radian),

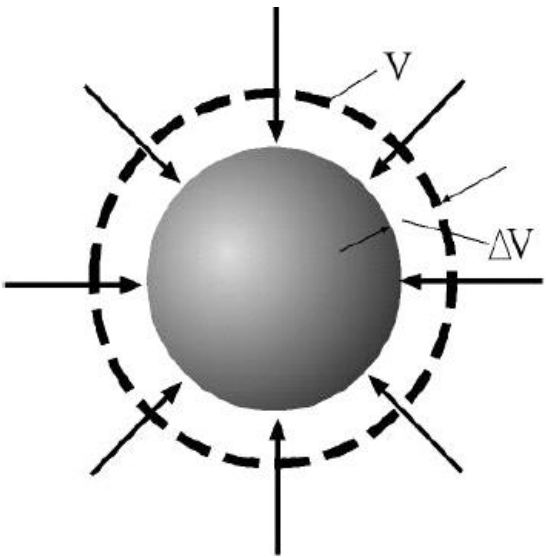
$S = \frac{Fh}{A\Delta x}$ becomes $S = \frac{F}{A\phi}$

III) Bulk Modulus: Volume Elasticity

Bulk modulus (B):- characterizes the response of an object to changes in a force of uniform magnitude applied perpendicularly over the entire surface of the object, as shown in figure below. An object subject to this type of deformation undergoes a change in volume but no change in shape.

The **volume stress** is defined as the ratio of the magnitude of the total force *F* exerted on a surface to the area face *A* of the surface. The quantity $P = F/A$ is pressure.

If the pressure on an object of original volume Vis increased by an amount ΔP, then the increase in pressure causes a volume change ΔV.



$Volume\ stress = \Delta P$

$$Volume\ strain = -\frac{\Delta V}{V}$$

$$Bulk\ Modulus = \frac{Volume\ stress}{Volume\ strain}$$

$$B = -\frac{\Delta P}{\Delta V/V} \quad \text{OR} \quad B = -\frac{V\Delta P}{\Delta V}$$

The –ve sign is used so as to cancel the –ve numerical value of ΔV and make B a +Ve number.

Typical Values for Elastic Moduli

Substance	Young' Modulus (N/m^2)	Shear Modulus (N/m^2)	Bulk Modulus (N/m^2)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5\text{-}7.8\times10^{10}$	$2.6\text{-}6.2\times10^{10}$	$5.0\text{-}5.5\times10^{10}$
Water	-	-	0.21×10^{10}
Mercury	-	-	2.8×10^{10}

Example

1. A box-shaped piece of gelatin has a top area of $15cm^2$ and a height of 3cm. A shearing force of 0.5N applied to the upper surface displaces it by 4mm realtive to the bottom surface. What is
   a) the shear stress? **Ans**. Sheat stress= 333Pa. c) the Shear Modulus? **Ans**. S = 2504Pa
   b) the shear strain?**Ans**. Shear strain = 0.133.
2. Compute the volume contraction of 100mℓ of water when subjected to a pressure of 1.5MPa. **Ans**. $\Delta V = -0.071 m\ell$
3. Determine the fractional change in volume as the pressure of the atmosphere, 10^5Pa, around a metal block is reduced to zero by placing the block in a vacuum. The bulck modulus for the metal is 125GPa. **Ans**. $\Delta V/V = 8\times10^{-7}$
4. Compute the volume change of a metal cube, 40mm on each edge, when subjected to a pressure of 20MPa. B_{metal} = 125GPa. **Ans**. $\Delta V = -10.24 mm^3$
5. Two parallel and opposite forces, each 4000N, are applied tangentialy to the upper and lower faces of a cubical metal block 25cm on a side. If the shear modulus for the metal is 80GPa, find
   a) the shear angle. **Ans**. $\phi = 8\times10^{-7}$rad.
   b) the displacement of the upper surface. **Ans**. $\Delta x = 2\times10^{-7}$m.
5. A solid lead sphere of volume $0.5m^3$, dropped in the ocean, sinks to a depth of 2.0×10^3m, where the pressure increases by 2.0×10^7Pa. Lead has a bulk modulus of 4.2×10^{10}Pa. What is the change in volume of the sphere? **Ans**. $\Delta V = -2.4\times10^{-4}m^3$
   The negative sign indicates a *decrease* in volume.

Fluid statics

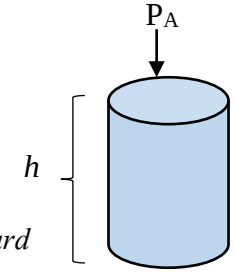
The hydrostatic (gauge) pressure in a liquid at any point in a closed container depends on:
* the height *h* above that point
*the density of the liquid and
*acceleration due to gravity

$$P = \rho gh$$

The absolute pressure in a liquid at a point in an open container is

$$P = P_A + \rho gh$$

Where P_A is atmospheric pressure.



Densities of Some Common Substances at Standard Temperature (0^0) and Pressure (Atmospheric)

Substance	ρ (kg/m^3)	Substance	ρ (kg/m^3)
Glycerin	1.26×10^3	Platinum	21.4×10^3
Seawater	1.03×10^3	Gold	19.3×10^3
Fresh water	1.00×10^3	Mercury	13.6×10^3
Benzene	0.879×10^3	Lead	11.3×10^3
Ethyl alcohol	0.806×10^3	Silver	10.5×10^3
Oxygen gas	1.43	Copper	8.92×10^3
Air	1.29	Iron	7.86×10^3
Helium gas	1.79×10^{-1}	Aluminum	2.70×10^3
Hydrogen gas	8.99×10^{-2}	Ice	0.917×10^3

Example

1. An open tank is filled with water. What is the absolute pressure at a point 0.8 m below the upper surface? Take $P_A = 10^5$Pa **Ans**. $P = 1.08 \times 10^5$Pa
2. At what depth in a lake is the absolute pressure equal to 3 times atmospheric pressure?
   Ans. h=20m

Questions from national exam

1. The pressure at the bottom of a cylindrical tube filled with water was measured to be 5000Pa. If the water in the tube were replaced with ethyl alcohol, what would be the new pressure at the bottom of the tube? ($\rho_{alchol} = 0.8$g/cm^3)
   A**)** 4000Pa B) 4800Pa C) 5000Pa D) 6250Pa
2. A very large open tank is full of oil whose density is 800kg/m^3. If the absolute pressure at the bottom of the tank is 1.81×10^5Pa, then what is the depth of oil? $P_A = 1.01 \times 10^5$Pa
   A) 100.0m B) 75.0m C**)** 10.0m D) 9.5m
3. A container resting on the floor of an elevator contains an incompressible fluid of density ρ. When the elevator has an upward acceleration of magnitude *a* then what will be the pressure difference between two points in the fluid separated by a vertical distance Δh?
   A) ρ*a*Δ*h* B**)** ρ(g+*a*)Δ*h* C) ρgΔ*h* D) ρg*a*Δ*h*
4. Which statement is correct about the pressure under a fluid in a container?
   A) It depends on the volume of the fluid. C) It depends on the shape of the container.
   B) It depends on the density of the fluid. D) It is independent of the depth of the fluid.

5. Pressure at the bottom of a liquid column is independent of :

A) shape of the container
B) the density of the fluid
C) the depth of the point
D) the acceleration due to gravity

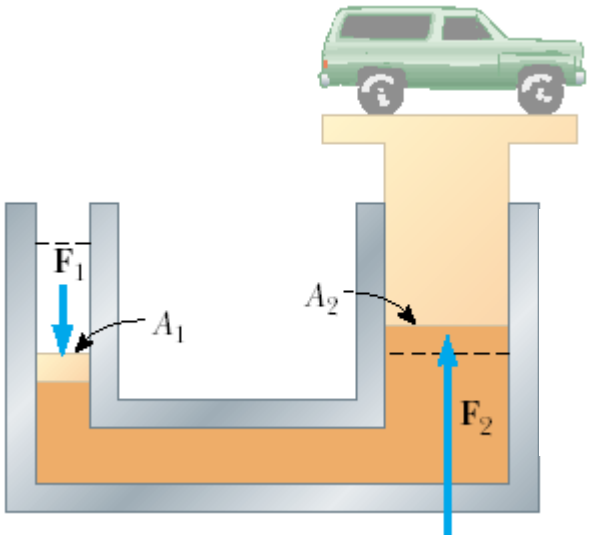
Pascal's Law

Pascal's law states "Pressure applied to an enclosed fluid is transmitted equally and undiminished to every portion of the fluid and to the walls of the container."
An important application of Pascal's law is the hydraulic press illustrated in figure bellow.

A force of magnitude F_1 is applied to a small piston of surface area face A_1. The pressure is transmitted through an incompressible liquid to a larger piston of surface area A_2. Because the pressure must be the same on both sides,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Therefore, the force F_2 is greater than the force F_1 by a factor $\frac{A_2}{A_1}$

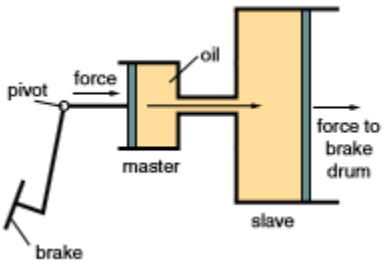


Car lift

By designing an appropriate areas A_1 and A_2, large output force can be applied by means of a small input force. Hydraulic brakes, car lifts, hydraulic jacks, and forklifts all make use of this principle.

Hydraulic brake

For hydraulic brakes of vehicles, the liquid used is oil and connects the *master piston* to the *slave piston*.



Example

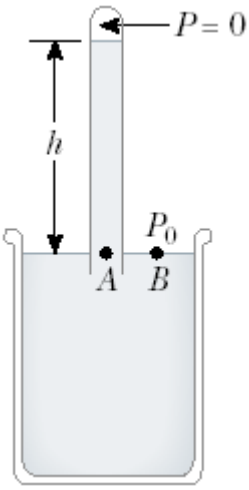
1. In a hydraulic press the smaller piston has a cross-sectional area $5cm^2$ and the larger piston has cross-sectional area $200cm^2$. If a force of 250N is applied to the smaller piston, find the force on the larger piston. **Ans**. $F_{large} = 10^4$N
2. Consider a hydraulic press with a smaller piston having a diameter $d = 2cm$ and larger piston with a diameter $D = 200cm$. What force must be applied on the smaller piston in order to support a 5000N object on the larger piston? **Ans**. $F_{smaller} = 0.5$N
3. In a car lift used in a service station, the smaller piston has a circular cross section and a radius of 5cm. The larger piston has a radius of 15cm. What force must be applied on the smaller piston to lift a car weighing 4,500N on the larger piston? **Ans**. $F_{smaller} = 500$N

4. The cross-sectional areas of the small and large pistons of a hydraulic lift are $4.0cm^2$ and $0.02m^2$, respectively. What is the magnitude of the force F that must be applied to the small piston to lift a weight W = 15 000N placed on the large piston?

A**)** 3.0×10^2N B) 1.5×10^4N C) 7.5×10^5N D) 3×10^6N

Pressure Measurements

An instrument used to measure atmospheric pressure is the common barometer invented by Evangelista Torricelli (1608-1647). A long tube closed at one end is filled with mercury and then inverted into a dish of mercury as in figure below. The closed end of the tube is nearly a vacuum, so the pressure at the top of the mercury column can be taken as zero. The pressure at point *A*, due to the column of mercury, must equal the pressure at point *B*, due to the atmosphere. ,



Therefore, it follows that

$$P_0 = \rho_m g h$$

Where ρ_m is the density of the mercury and

h is the height of the mercury column.

Let us determine the height of a mercury column for one atmosphere of pressure, P_0 = 1atm = 1.013×10^5Pa

$$h = \frac{P_0}{\rho_m g} = \frac{1.013\times10^5}{13.6\times10^3\times9.8} = 0.76m$$

Based on this calculation, one atmosphere of pressure is defined to be the pressure equivalent of a column of mercury that is exactly 0.76m.

Archimedes' Principle

Have you ever tried to push a beach ball (*a large light ball for playing with on a beach*) under water as in figure below? This is extremely difficult to do because of the large upward force exerted by The water on the ball.

The upward force exerted by a fluid on any immersed object is called a**buoyant force**.

Archimedes' principle states

"***When a body is totally or partially immersed in a fluid, the upthrust (buoyant) force on the body is equal to the weight of displaced fluid.***"



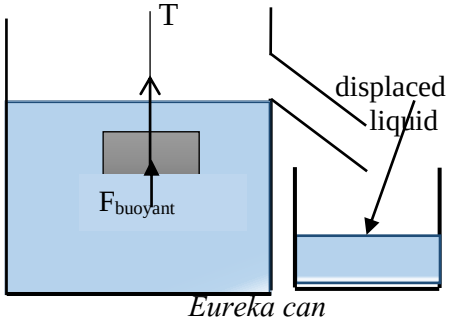
The magnitude of the *upthrust force* is equal to the *apparent loss in weight of the body* which is also equal to *weight of the displaced liquid*. When a body is totally immersed in a liquid, it displaces a liquid equal to its own volume.

Buoyant force = Weight of displaced liquid = Apparent loss in weight of the body

$$F_B = W_{dl} = W_{\text{in air}} - W_{\text{in liquid}}$$

Volume of submerged part of the body = Volume of displaced liquid

$$V_{sp} = V_{d\ell}$$

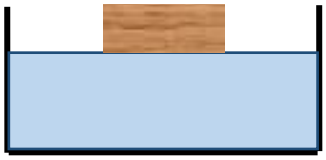


Principle of floatation

If the buoyant F_B is

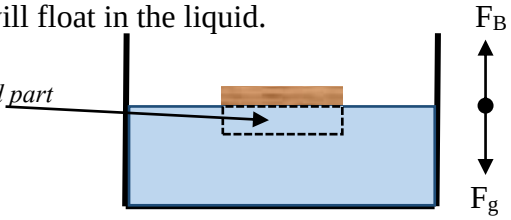
i) greater than weight of the submerged body, the body will rise to the surface and floats.

Density of the liquid is greater than density of the submerged body.



ii) equal to the weight of the submerged body, the body will float in the liquid.

$$W_{obj} = W_{d\ell}$$
$$m_{obj}\, g = m_{d\ell}\, g$$
$$m_{obj} = m_{d\ell}$$
$$\rho_{obj}\, V_{obj} = \rho_{liq}\, V_{d\ell}$$

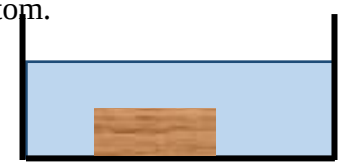


$$\frac{\rho_{obj}}{\rho_{\ell iq}} = \frac{V_{d\ell}}{V_{obj}} \quad \text{OR} \quad \frac{\rho_{obj}}{\rho_{\ell iq}} = \frac{V_{sp}}{V_{obj}}$$

This equation tells us that **the fraction of the volume of a floating object that is below the fluid surface (volume submerged part) is equal to the ratio of the density of the object to that of the fluid**.

iii) smaller than weight of the submerged body, it will sink to the bottom.

Density of the liquid is smaller than density of the submerged body.



Example

1. When a solid block of weight 9.6N, suspended from a string, is totally immersed in water, its apparent weight is 6.6N. Calculate
   a) the buoyant force. **Ans**. F_B = 3N
   b) the apparent loss in weight. **Ans**. 3N.
   c) weight of displaced water. **Ans**. W_{dw}=3N.
   d) volume of displaced water. **Ans**. $V_{dw} = 3\times10^{-4}m^3$
   e) volume of block. **Ans**. $V_{dw} = V_{block} = 3\times10^{-4}m^3$
   f) density of the block. **Ans**. $\rho_{block} = 3200kg/m^3$
2. When a2 kg object, suspended from a string, is totally immersed in water the tension in the string is 15N. What is density of the object? **Ans**. $\rho_{obj} = 4\times10^3 kg/m^3$

3. A block of wood floats in water with three-fourth of its volume immersed. What is density of the wood? **Ans**. $\rho_{wood} = 750kg/m^3$
4. A block of wood ($\rho_{wood} = 600kg/m^3$) having a volume of $0.125m^3$ floats on water. What mass should be placed on the block in order for its top surface to be at the same level with the surface of the water? **Ans**. $m = 50kg$
5. An iceberg ($\rho_{ice} = 890kg/m^3$) is floating in seawater. What fraction of the iceberg lies below the water level? **Ans**. $V_{sp}/V_{ice} = 89\%$

Exercise

1. The weight of a 2kg object, when totally immersed in water is 15N. If the weight of the same object when totally immersed in another liquid is 18N, what is the density of the other liquid? **Ans**. $\rho_{other} = 400kg/m^3$
2. A cubical block of wood having a cross-sectional area $100cm^2$ floats in water. What is mass of the block if one-fifth of the block is immersed in the water? **Ans**. $m_{block} = 0.2kg$
3. A 4 kg wood floats in water with 0.75 of its volume immersed when a 2kg object is placed on the top of the wood. What is the density of the wood? **Ans**. $\rho_{wood} = 500kg/m^3$
4. A cubical block of wood ($\rho_{wood} = 0.75g/cm^3$) floats in water. What fraction of the block is
   a) submerged in water? **Ans**. $v_{sp}/v_{cube} = ¾$. b) above the surface of the water? **Ans**. ¼
5. A cubical block of wood 20*cm* on a side and having a density of $0.65g/cm^3$ floats in water. What is the distance from the top of the cube to the water level? **Ans**. $h = 7cm$
6. A cube of wood, $\rho_{wood} = 0.6\ g/cm^3$ and volume 64 cm^3, floats in water.
   a) What is the distance from the top of the cube to the water level? **Ans**. $h = 1.6cm$
   b) What percent of the cube is submerged? **Ans**. $V_{sp} = 60\%$
7. A piece of wood is 0.5m long, 0.2m wide and 0.02m thick. If its density is $600kg/m^3$,
   a) what percent of the block is submerged? **Ans**. $V_{sp} = 60\%$
   b) what is the distance from the top of the block to the water level? **Ans**. $h = 0.008m$

Questions from national exam

1. A certain block with a mass of 0.5kg and density of $8\times10^3kg/m^3$ is suspended from a string. What is the tension in the string if the block is completely immersed in water?
   A**)** 4.375N B) 5.245N C) 6N D) 6.275N
2. The weight of a block having a density of $5\times10^3kg/m^3$ is 1.5N in air. When the block is completely immersed in a certain liquid, its weight is found to be 1.2N. What is density of the liquid?
   A) $10^2\ kg/m^3$ B**)** $10^3kg/m^3$ C) $10^4kg/m^3$ D) $10^5kg/m^3$
3. A table-tennis ball has a diameter of 3.80*cm* and average density of $0.084g/cm^3$. What force is required to hold it completely submerged under water? ($g = 9.8m/s^2$)
   A) 0.521N B**)** 0.258N C) 0.300N D) 0.510N
4. A table-tennis ball which is spherical in shape has a diameter of 6*cm* and density $0.400g/cm^3$. What force is required to hold it completely submerged under water?
   A**)** 0.68N downward C) 1.13N downward
   B) 0.68N upward D) 1.13N upward

Surface tension

Surface tension:- is the property of a liquid's surface that acts like a stretched elastic skin. It is caused by the forces of attraction between the particles of liquid and other substances with which it comes into contact.

*Some insects can walk on the surface of water, their feet making depression in the surface but not penetrating it.

*A liquid flowing slowly from the tip of a medicine dropper emerges not as a continuous stream but as a succession of drops.

All these are due to surface tension.



Surface tension(*f*):- is the force acting across the surface of a liquid per unit length perpendicular to an imaginary line drawn onto surface.

$$f = F/\ell$$

There are two ways to reduce surface tension.

i) warming the water *ii*) adding detergent.

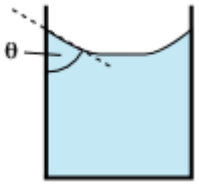
Pressure differences across a surface film

The surface of a liquid tends to take a curved shape. This observation tells us that there must be a pressure difference between the sides of the surface to cause the curvature.

Pressure difference = surface tension×$(1/R_1 + 1/R_2)$

$$\Delta P = f\,(1/R_1 + 1/R_2)$$

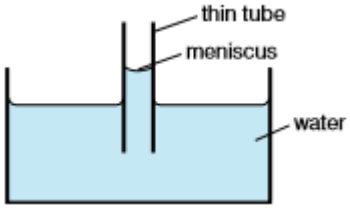
Contact angle:- is the angle formed between the tangent plane to the surface of a liquid and the tangent plane to the surface of a solid at the point of contact.



Capillarity:-is a phenomenon caused by surface tension and resulting in the elevation or depression of the surface of a liquid in contact with solid.

Capillary action:- is the movement of a liquid along the surface of a solid by attraction of molecules of the liquid to surface of the solid.

The curved liquid surface in the tube is called a **meniscus**.



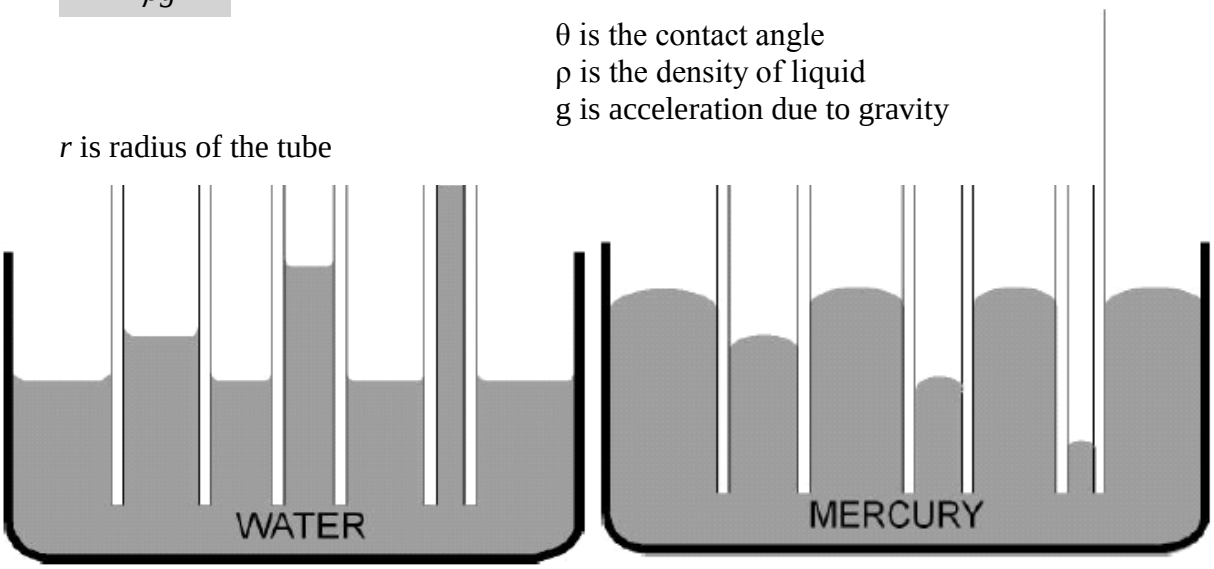
Surface tension pulls the liquid column up until there is sufficient mass of liquid for gravitational force to overcome the intermolecular forces. Narrow tube draws a liquid column higher than the wide tube.

The height of a liquid column as a result of capillary action is

$h = \frac{2f\cos\theta}{\rho g r}$ Where f is the liquid-air surface tension

θ is the contact angle
ρ is the density of liquid
g is acceleration due to gravity

r is radius of the tube

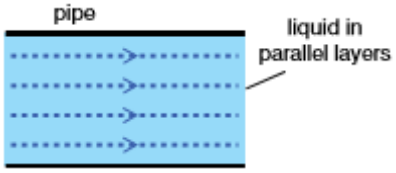


Capillary action of water and that of mercury

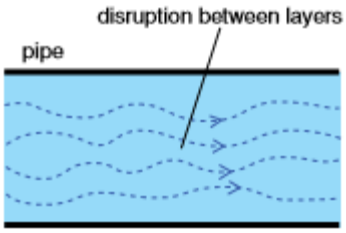
Example Water rises in a tube of inner diameter 1 mm to a height of 4cm above the surrounding liquid and the angle of contact is zero. What is the surface tension? **Ans**. f= 0.1N/m

Fluid Dynamics

Streamline or laminar flow:- is a flow of fluid with constant speed in the same direction at all times and the flow appears to be smooth and regular.



Turbulent flow:- is a flow of fluid where in the speed of the fluid change rapidly in magnitude and direction. The motion of a fluid becomes turbulent if its speed increases beyond a certain value. Below this speed, the flow is streamlined.



Factors affecting laminar flow of air

i) density of air
ii) compressibility (how much its volume can be reduced)
iii) temperature (for air, viscosity increases with the increase of temperature)
iv) viscosity (the extent to which a fluid resist a tendency to flow)

The density and compressibility of air are important factors at high speeds. As a result, other properties of air then change.

Trains, racing cars, boats and airplanes are streamlined by making their surfaces smooth and point at their fronts (noses).

Viscosity

Viscosity:- is the extent to which a fluid resist a tendency to flow.

Thick syrups and honey are examples of viscous fluids. Viscous fluids flow quite slowly.

When water flow through a pipe, the water close to the walls of the pipe scarcely moves. This is due to friction between the water and the walls of the pipe. The water near the center of the pipe moves most quickly as indicated by the length of the vectors.

The force required to drive the layers is

$$F = \frac{\eta A V}{\ell}$$

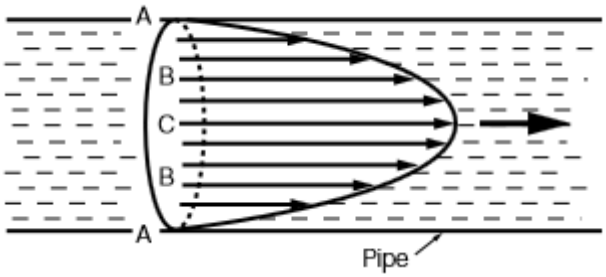
Where V is speed

ℓ is the distance from the wall

A is cross-sectional area

V/ℓ is velocity gradient

η is viscosity



Viscosity of

- liquids decreases with the increase of temperature.
- gases increases with the increase of temperature.

Typical values of viscosity

Temp.(^{0}C)	Castor oil (poise)	Water (centipoise)	Air (μ poise)
0	53	1.792	171
20	9.86	1.005	181
40	2.31	0.656	190
60	0.80	0.469	200
80	0.30	0.357	209
100	0.17	0.284	218

Reynolds number

A dimensionless number used in fluid dynamics to determine the type of flow of a liquid through a pipe is given by

$$R_e = \frac{\rho V d}{\eta}$$

Where ρ is density of the liquid

V is average forward velocity

d is the diameter of the pipe

η is viscosity

A variety of experiments have shown that when the *Reynolds number is* **less than about 2000, the flow is laminar**, whereas **above about 3000 the flow is turbulent**. In the transition region **between2000 and 3000** the **flow is unstable** and may **change from one type to the other**.

Equation of continuity

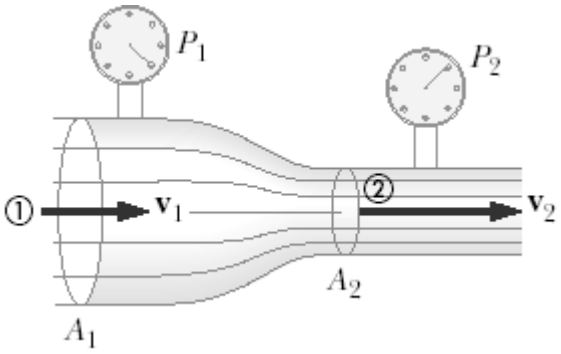
For an incompressible fluid in steady flow, between two fixed cross-sections of A_1 and A_2, the mass flowing in is $\rho A_1 V_1 \Delta t$. Similarly the mass that flows out across A_2 in the same time is $\rho A_2 V_2 \Delta t$.

$\rho A_1 V_1 \Delta t = \rho A_2 V_2 \Delta t$

$A_1 V_1 = A_2 V_2$ Equation of continuity

OR $Q_1 = Q_2$

Where Q_1 and Q_2 are the discharge (flow) rate at point 1 and point 2 respectively.



What is the SI unit of discharge rate?

Example

1. The speed of water flowing through a horizontal pipe is 4m/s at r_1 = 5mm. What is the speed at r_2 = 10mm? **Ans**. V_2 = 1m/s
2. Water flows through a 2cm diameter hose with a speed of 0.5m/s. What is the speed of the water through a 0.25cm diameter? **Ans**. V_2 = 32m/s
3. Water in a horizontal pipe with diameter d_1 = 0.2m, flows at a speed of 2m/s. If the pipe has a constriction of diameter d_2 = 0.1m, what is the speed in the constriction? **Ans**. V_2 = 8m/s
4. The radius of aorta is about 0.01m and the blood passing through it has a speed of about 0.3m/s. A typical capillary has a radius of about 4×10^{-6}m, and the blood flows through it at a speed of about 5×10^{-4}m/s. Estimate the number of capillaries in the body. **Ans**. n = 3.75×10^{9}

Questions from national exam

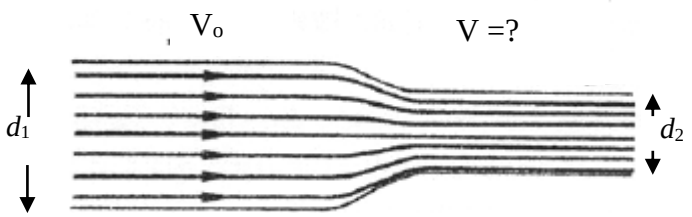
1.Water flows through a pipe with a diameter of 8cm at a speed of 12cm/s. If the pipe is connected to another pipe with a diameter of 4 cm, what is the speed of the water in the smaller pipe?

A)24cm/s B**)** 48cm/s C) 16cm/s D) 32cm/s

2. A fluid flows through the pipe shown below. It enters through the pipe of diameter of d_1 with a velocity V_0. At what velocity the fluid leaves the pipe if the diameter of the narrow pipe is $d_2 = \frac{1}{6} d_1$?

A) $40V_0$ C**)** $36V_0$

B) $30V_0$ D) $24V_0$



3. A garden hose has an inside diameter of 16cm. The hose can fill a 10 lit bucket in 20 seconds. If the diameter of the nozzele is reduced to 8cm,

A)water exits with a speed twice the speed inside the hose.

B) a 10 litter bucket can be filled in 10seconds.

C) a 20 lit bucket can be filled in 40 seconds.

D) water exits with a speed half the speed inside the hose.

4. Water flows through a pipe whose cross-sectional area is $50cm^2$ into another pipe whose cross-sectional area is $10cm^2$. Both pipes are horizontal. If the velocity of the water in the larger pipe is 12m/s, the velocity in the small pipe is,

   A) 0.24m/s B) 1.2m/s C**)** 60m/s D) 30m/s

5. Water flows through a fire hose of diameter 6.35cm at a rate of $0.012m^3/s$. The fire hose ends in a nozzele of inner diameter 2.20cm. What is the speed with which the water exits the nozzele?

   A) 23.4m/s B**)** 31.6m/s C) 63.2m/s D) 46.8m/s

6. A water pipe has a diameter of 5cm in the inlet side and 1.5cm in outlet side. Water flows at $0.5m^3$ per minute at the inlet point. What is the speed of the water at the outlet point?

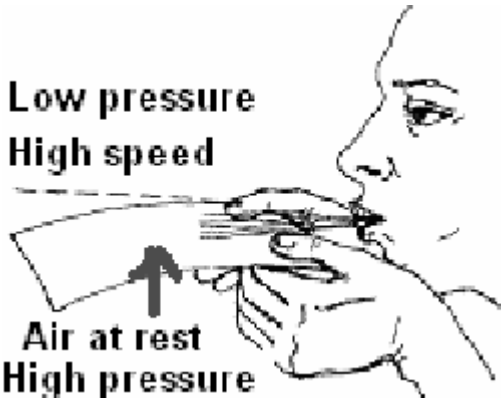
   Ans.$V_2 = 2831m/s$

   A) 26.67m/s B) 13.33m/s C) 74.1m/s D) 148.1m/s

Bernoulli's Principle

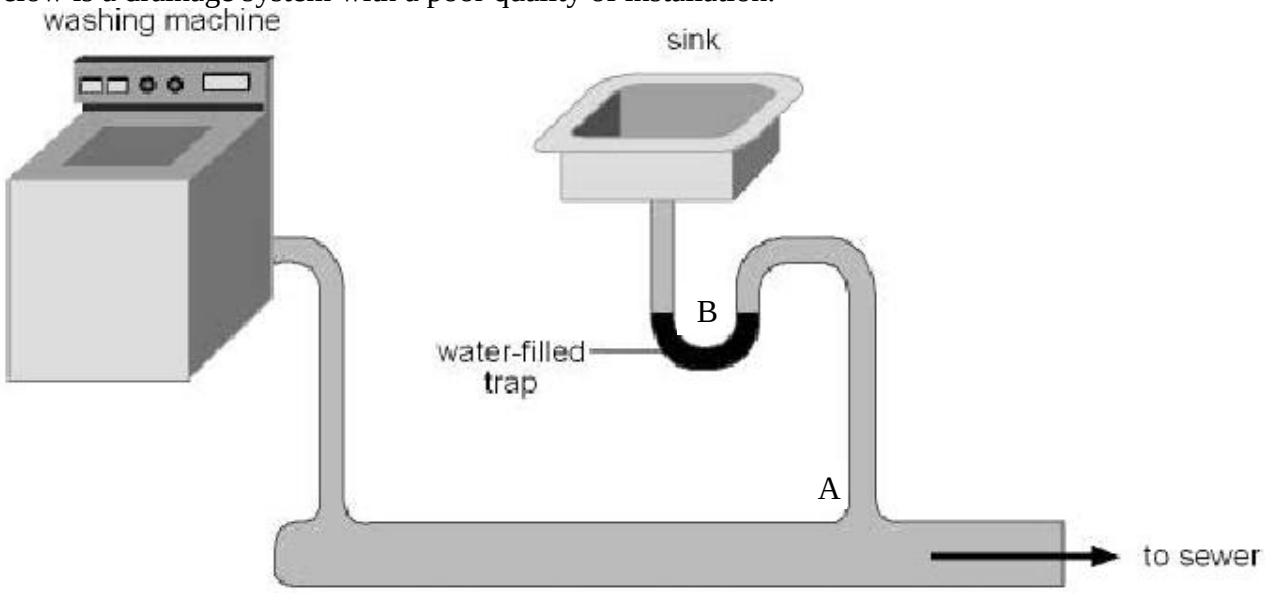
Bernoulli's principle states that "*as the speed of a fluid increases, its pressure decreases*".
The stream of air over the top of the paper is passing at a greater speed while the air is at rest below it. So the stream of air over the top of the paper is at a lower pressure than the air below the paper.

The difference in pressure results in a difference in force above and below the paper.
So there is a resultant force acting up on the paper.



When walking on a highway, you may have felt the effect of a large truck passing you at high speed. You experience a frightening feeling that you were being pulled towards the truck as it passed you. This fast moving air exerts less pressure on you than slow moving air on the other side. Thus, there is a net force pushing you toward the truck.

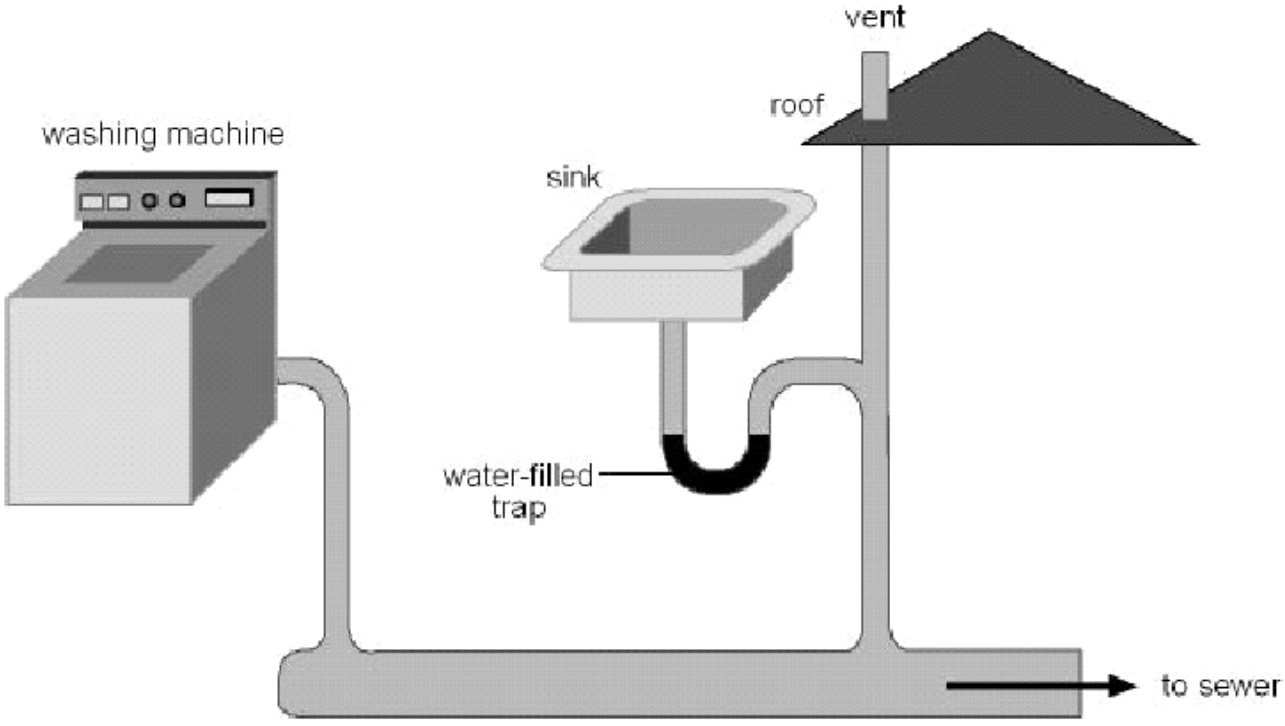
Below is a drainage system with a poor quality of installation.



Here we see the flow of water from the outlet pipe of a washing machine placed in a kitchen. From the outlet pipe the water flows through the sewer pipe to the sewer outside. Close to the washing machine is a sink where dishes are washed. The used water from the sink also flows through a system of pipes to the sewer. Why this drainage system is of poor quality, and how it can be improved?

The U- shaped section of the pipe, below the sink, traps water. The trapped water serves as a barrier to prevent sewer smells and gas from leaking into the house. When the water from the washing machine rushes through the sewer pipe, the high-speed flow of used water causes the pressure at point A to drop. The pressure at point B in the sink however remains at the higher atmospheric pressure. As a result of this pressure difference, the water is pushed out of the trap and into the sewer line, leaving no protection against sewer gas.

A correctly designed system is vented to the outside of the house.

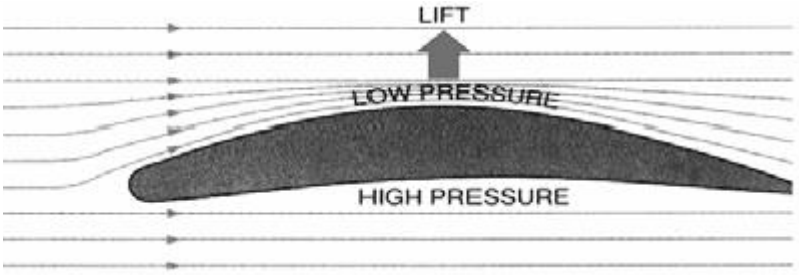


The vent ensures that the pressure at A remains the same as that at B. That is at atmospheric pressure. So, the purpose of the vent is to prevent the trap from being emptied, and to provide an external escape route for the sewer gas.

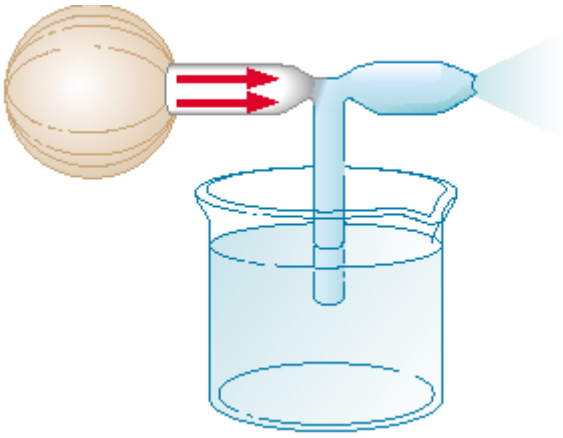
A rapidly moving air, a tornado or hurricane, causes the external pressure above the roof of a house, to fall below atmospheric pressure. However, the stationary air inside the house remains at normal atmospheric pressure. The pressure difference between the inside and outside of the roof, results in an upward force on the roof, which can be large enough to lift the roof off the house.

Applications of Fluid Dynamics

The lift on an aircraft's wing can be explained, in part, by the Bernoulli's effect. Airplane wings are designed so that the air speed above the wing is greater than that below the wing. As a result, the air pressure above the wing is less than the pressure below, and there is a resultant force acting up on the wing due to this difference in pressure, so the wing lifts.



A number of devices operate by means of the pressure differences that result from differences in a fluid's speed. For example, a stream of air passing over one end of an open tube, the other end of which is immersed in a liquid, reduces the pressure above the tube, as illustrated in the figure below.



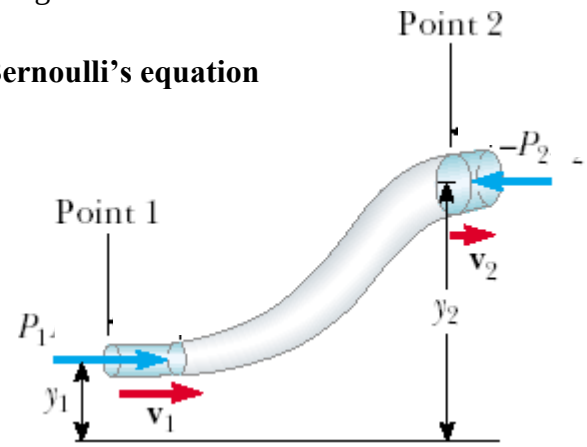
This reduction in pressure causes the liquid to rise into the air stream. The liquid is then dispersed into a fine spray of droplets. You might recognize that this principle is used in perfume bottles and paint sprayers.

Bernoulli's Equation

As a fluid moves through a region where its speed and/or elevation above the Earth's surface changes, the pressure in the fluid varies with these changes.

Bernoulli's equation

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g y$$



Where P_1 is pressure of the fluid at point 1
P_2 is pressure of the fluid at point 2
V_1 is speed of the fluid at point 1
V_2 is speed of the fluid at point 2

This is applied to an ideal fluid.

OR $$P_1 - P_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right) + \rho g\left(y_2 - y_1\right)$$

If $y_1 = y_2$, the above equation becomes

$$P_1 - P_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right)$$

This expression shows that the **pressure of a fluid decreases** as the **speed of the fluid increases**.

Example

1. The cross-sectional area of a water pipe entering the basement is $4\times10^{-4}m^2$. The pressure at this point is $3\times10^5 N/m^2$and the speed of the water is 2m/s. Its cross-sectional area at the second floor 8m above is $2\times10^{-4}m^2$. Calculate the speed and pressure at the second floor.
   Ans. V_2=4m/s and $P_2 = 2.14\times10^5$ N/m²
2. The cross-sectional area of a pipe at point A is twice that at point B, which is 10m above point A. The speed of water at point A is 5m/s, while the pressure at this point is $1.5\times10^5 N/m^2$. Calculate the speed and pressure at point B. **Ans**. V_B = 10m/s, $P_B = 1.25\times10^4 N/m^2$
3. Water flows through a horizontal pipe between two points having cross-sectional area A_1= $8cm^2$ and A_2 = $2cm^2$. If the speed at A_1 is 4m/s, what is the pressure difference between the two points? **Ans**. $P_1 - P_2 = 1.2\times10^5 N/m^2$
4. Water is flowing through a horizontal pipe with a speed of 15 m/s at a point where the pressure is 1.5×10^5Pa and cross-sectional area of $4cm^2$. What is the pressure at a point where the cross-sectional area of the pipe is $12cm^2$? **Ans**. $P = 2.5\times10^5$Pa
5. A tornado or hurricane often lifts the roof of a house. Use Bernoulli's principle to explain how this occurs.

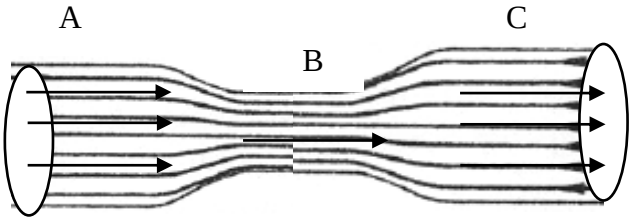
Questions from national exam

1. A horizontal pipe 10cm in diameter has a smooth reduction to a pipe 5cm in diameter. If the pressure of the water in the larger pipe is 1.5×10^5Pa and the pressure in the smaller pipe is 3×10^4Pa, what is the speed of the water through the larger pipe? **Ans**. V =4 m/s

   A) 4cm/s B) 8cm/s C) 16cm/s D) 20cm/s

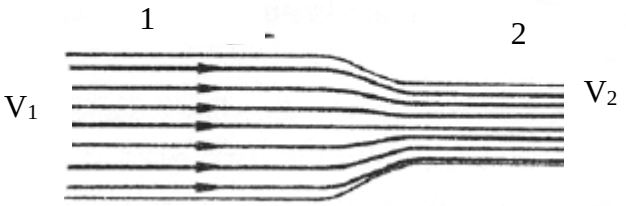
2. An incompressible fluid is flowing in a tube constricted at the center as shown below. Which statement is correct?

   A) Pressure is maximum at B.
   B) Fluid flow faster at B.
   C) Volume flux is maximum at A.
   D) Volume flux is minimum at C.



3. A fluid flows through a horizontal pipe as shown in the figure below. If the velocity of the flow at 2 is twice at 1, how does the pressure P change between the two points?

   A) $P_1 < P_2$
   B) $P_1 = P_2$
   C) $P_1 > P_2$
   D) $P_1 - P_2 >$ change in kinetic energy density.



4. The cross-sectional area of a water pipe entering the basement is $4\times10^{-4}m^2$. The pipe carries water into the basement at a velocity of 6m/s. The pressure at the basement is 3×10^5Pa. What are the speed and pressure when it reaches the second floor, which is 8m above the basement if the cross-sectional area of a water pipe is half of its value at the basement? $V=12m/s, P=1.66\times10^5 N/m^2$

   A) 12m/s, $1.12\times10^5 N/m^2$ C) 12m/s, $2.12\times10^5 N/m^2$
   B) 24m/s, $1.12\times10^5 N/m^2$ D) 18m/s, $2.12\times10^5 N/m^2$

5. Through a pipe 15cm in diameter, water is pumped from the river outside of Addis Ababa to one of the sub-cities of Addis Ababa. The river is at an elevation of 564m and the sub-city is at an elevation of 2096m. What is the minimum pressure at which the water must be pumped to arrive at the sub-city?

   A) 15.32MPa B) 15.01MPa C) 14.99MPa D) 16.03MPa

6. A horizontal pipe 10cm in diameter has a smooth reduction to a pipe 5cm in diameter. If the pressure of the water in the larger pipe is 80kPa and the pressure in the smaller pipe is 60kPa, at what rate does the water flow through the smaller pipe?

   A) 1.63m/s B**)** 6.53m/s C) 13.6m/s D) 42.67m/s

7. An incompressible fluid flows through a horizontal pipe whose diameter varies along its length. The sum of the kinetic energy per unit volume and pressure of the fluid at different sections of the pipe would_____.

   A) depend on the length of the pipe.
   B) increases as the pipe diameter increases.
   C) increases as the pipe diameter decreases.
   D) remains the same as the pipe diameter changes.

Torricelli's Law

Consider an enclosed tank containing a liquid with density ρ. Note that the diameter of the tank is much greater than the hole, $A_2 >> A_1$, implies that the fluid is approximately at rest at the top of the tank. That is $V_2 = 0$

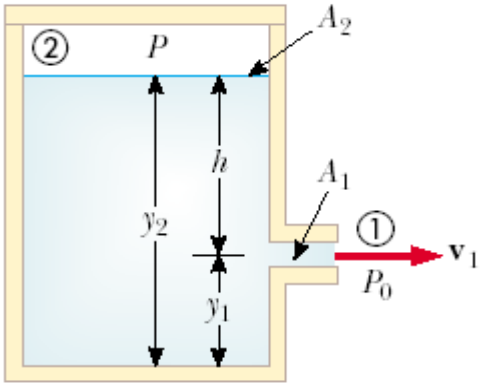
Thus, Bernoulli's equation

$P_1 + ½\,\rho V_1^2 + \rho g y_1 = P_2 + ½\,\rho V_2^2 + \rho g y_2$ becomes

$P_0 + ½\,\rho V_1^2 + \rho g y_1 = P + \rho g y_2$

$$V_1 = \sqrt{\frac{2(P - P_0)}{\rho} + 2g(y_2 - y_1)}$$

But $y_2 - y_1 = h$; thus, this expression reduces to



$$V_1 = \sqrt{\frac{2(P - P_0)}{\rho} + 2gh}$$

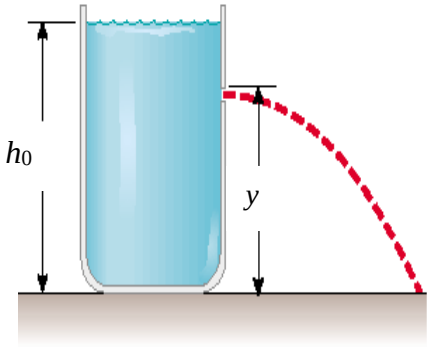
If the tank is open to the atmosphere, $P = P_0$ and

$$V_1 = \sqrt{2gh}\,.$$

Example

1. A large storage tank is filled to a height h_0. The tank is punctured at a height y above the bottom of the tank as in figure below. Find an expression for how far from the tank the exiting stream lands.

Ans. $R = 2\sqrt{y(h_0 - y)}$



2. If the water in question 1 above is to shoot as far as possible horizontally, ***estimate*** how far from:
   a) the bottom of the container should the hole be punched? **Ans.** $y = ½\,h_0$
   Hint: Calculate R for $y = 0, 0.1h_0, 0.2h_0, 0.3h_0, 0.4h_0, 0.5h_0, 0.6h_0, 0.7h_0, 0.8h_0, 0.9h_0$ and h_0.
   b) the side of the container will the water land? **Ans.** $R = h_0$

Questions from national exam

1. A large storage tank is filled to a height h_0. If the tank is punctured at a height h from the bottom of the tank, how far from the tank will the stream land?

   A) $2\sqrt{h(h_0 + h)}$ B) $\sqrt{h(h_0 - h)}$ C) $\sqrt{(h(h_0 + h))}$ D) $2\sqrt{h(h_0 - h)}$

2. An open tank containing a liquid of density ρ has a small hole in its side at a distance y_1 from the bottom. What is the speed at which the liquid leaves the hole when the liquid level is a distance h above the hole?

   A) $\sqrt{2gh}$ B) $\sqrt{2g\,y_1}$ C) $\sqrt{gh}$ D) $\sqrt{g\,y_1}$

Thermal expansion of solids

Thermal expansion is a consequence of the change in the *average* separation between the atoms in an object.

i) Linear expansion of solids

When the length is very large compared to other dimensions, the expansion is considered as linear.

The *change in length* ($\Delta\ell$) is *directly proportional* to *the product of the original length* (ℓ_0) and *the change in temperature* (ΔT).

$\Delta \ell = \alpha\ \ell_0 \Delta T$

$\ell - \ell_0 = \alpha\ \ell_0 (T - T_0)$

$\ell = \ell_0 (1 + \alpha \Delta T)$

$\ell = \ell_0 [1 + \alpha (T - T_0)]$

Where $\Delta\ell$ is change in length

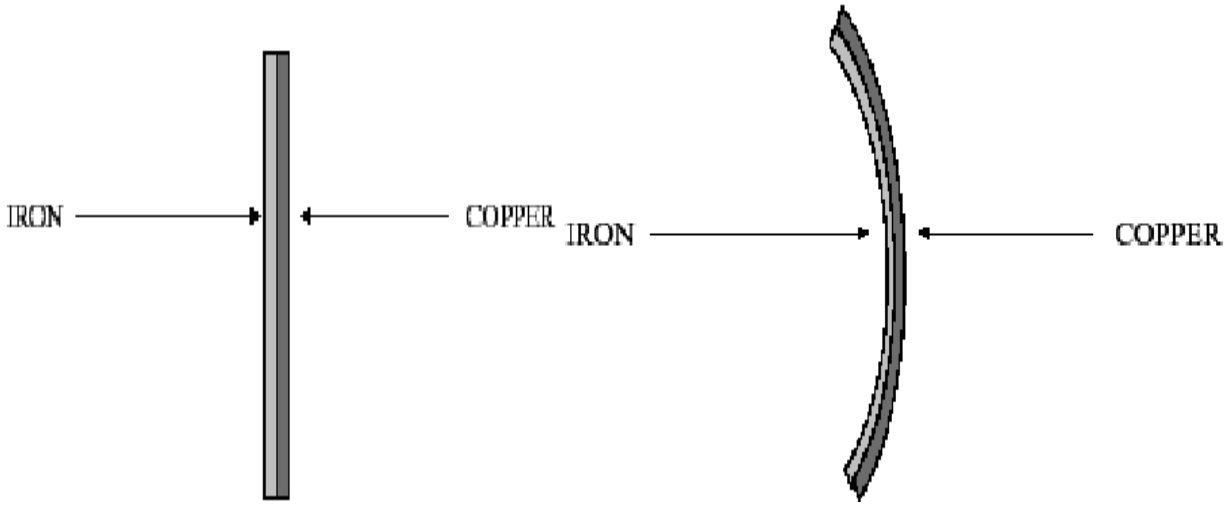
ℓ_0 is original length

ℓ is final length

α is coefficient of linear expansion

Bimetallic Strip

A *bimetallic strip* consists of two thin strips of dissimilar metals welded together. When the temperature of the strip increases, the two metals expand by different amounts because the two metals have different coefficients of expansion and the strip bends, as shown in the figure below. The copper expands more than the iron does because copper has a greater coefficient of expansion than iron. The combination will bend into an arc with the copper being on the outside of the arc.

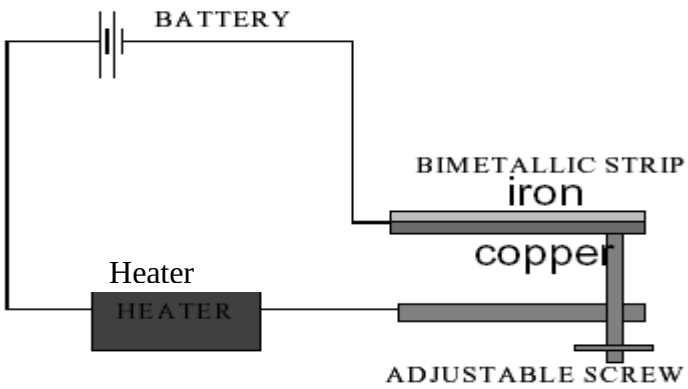


A bimetallic strip is used in a thermostat to break or make electrical contact.

What happens to the circuit when the bimetallic strip is heated up?

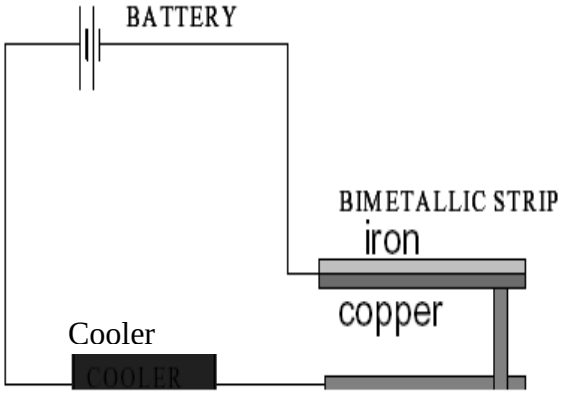
The copper expands more than the iron when it is heated causing the bimetallic strip to bend upwards.

When the bimetallic strip bends upwards, the circuit is broken and the heater is therefore switched off.



Could this thermostat arrangement be used in a cooling application?

As the room cools, it will cause the bimetallic strip to contract. The copper, having the higher coefficient of linear expansion, would contract more than the iron causing the strip to bend downwards, and effectively keeping the circuit closed. The circuit will never be broken in a cooling application. The bimetallic strip will have to be turned around so that the copper is at the top. With the copper at the top, when the room cools the bimetallic strip will bend upwards thereby breaking the circuit. As the room warms up again, the strip will return to normal and close the circuit, causing the cooler to work again.



ii) <u>Areal expansion of solids</u>

Consider a rectangular sheet of metal whose thickness is negligible.

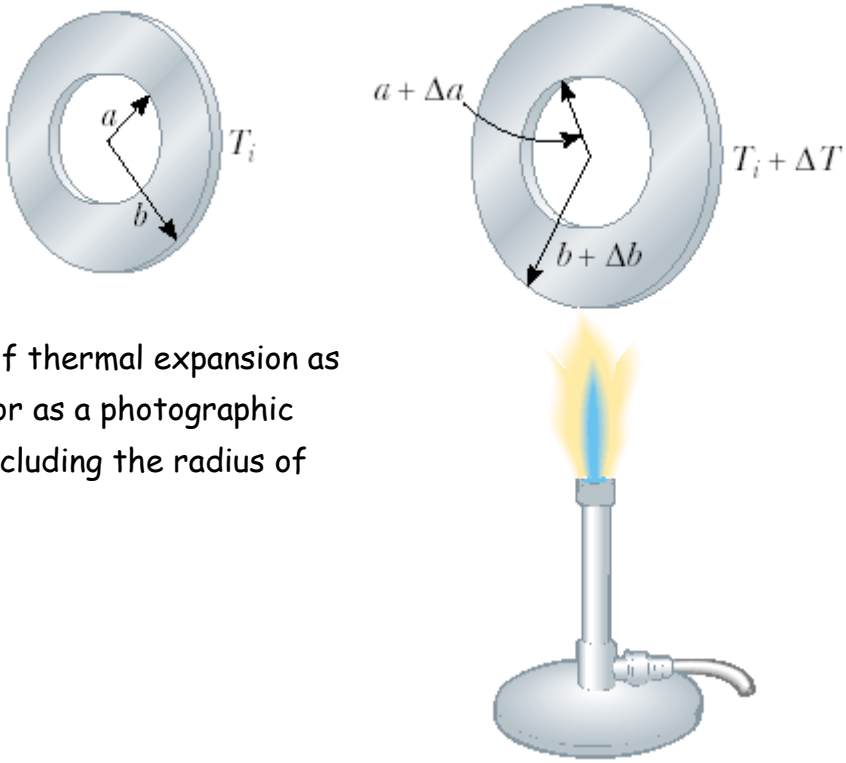
The *change in area (ΔA)* is *directly proportional* to *the product of original area (A_0)* and the *change in temperature (ΔT).*

$\Delta A = \beta A_0 \Delta T$ Where $\beta = 2\alpha$ is coefficient of areal expansion

$A - A_0 = \beta A_0 (T - T_0)$ A is final area

$A = A_0[1 + \beta(T - T_0)]$

When an object's temperature is raised, every linear dimension increases in size. This includes any holes in the material, which expand in the same way as if the hole were filled with the material, as shown in the figure below.



It may be helpful to think of thermal expansion as an effective magnification or as a photographic enlargement of an object including the radius of the hole.

iii) Volume expansion of solids

When a rectangular block of metal is heated, its length, width and height increase resulting in an increase in volume.

The *change in volume (ΔV)* is *directly proportional* to *the product of original volume (V_0)* and the *change in temperature (ΔT).*

$\Delta V = \gamma V_0 \Delta T$ Where $\gamma = 3\alpha$ is coefficient of cubical expansion

$V - V_0 = \gamma V_0(T - T_0)$ V is the final volume

$V = V_0[1+\gamma(T - T_0)]$

Average Coefficients of Expansion for Some Materials Near Room Temperature

Material	Coefficient of Linear Expansion (^{0}C)	Material	Coefficient of Linear Expansion ^{0}C)
Aluminum	24×10^{-6}	Ethyl alcohol	1.12×10^{-4}
Brass and bronze	19×10^{-6}	Benzene	1.24×10^{-4}
Copper	17×10^{-6}	Acetone	1.5×10^{-4}
Glass (ordinary)	9×10^{-6}	Glycerin	4.85×10^{-4}
Glass (Pyrex)	3.2×10^{-6}	Mercury	1.82×10^{-4}
Lead	29×10^{-6}	Gasoline	9.6×10^{-4}
Steel	11×10^{-6}	Air	3.67×10^{-3}
Invar	0.9×10^{-6}	Helium	3.665×10^{-3}
Concrete	12×10^{-6}		

Thermal expansion must be included in buildings, concrete highways, and bridges to compensate for changes in dimensions with variations in temperature.

Example

1. A bar is 100cm long at 10^0C and expand by 0.0018cm when heated to 100^0C. What is the coefficient of expansion? **Ans**. $\alpha = 2\times10^{-7}$ /^{0}C
2. A brass bar is 10cm long when it is at a temperature of 15K. What will its length be when it is heated to 150K? $\alpha = 1.9\times10^{-5}K^{-1}$ **Ans**. $\ell = 10.0256$cm
3. A copper bar is 80cm at 15^0C. What is the increase in length when it is heated to 35^0C? $\alpha_{cop} = 1.7\times10^{-70}C^{-1}$ **Ans**. $\Delta\ell = 2.72$mm
4. At 20^0C, the length of a certain steel rod is 15m. What is the length of the steel rod at 0^0C? $\alpha_{steel} = 1.2\times10^{-5}K^{-1}$ **Ans**. $\ell = 14.996$m
5. An aluminum rod is 56cm long at 20^0C. At what temperature will it be 1mm longer? $\alpha_{al} = 2.4\times10^{-5}K^{-1}$. **Ans**. T = 94.4^{0}C
6. A steel wire is 2m long at 50^0C. At what temperature will the length of the wire be 2.0024m? **Ans**. T = 150^0C

Exercise

1. A metal rod is 1m long at 20^0C and 1.005m long at 40^0C. What is the length of the rod at 60^0C? **Ans**. $\ell = 1.01$m
2. The ratio of the thermal expansion coefficient of two different metals is 1:3. What is the corresponding ratio in their change in lengths when they are both heated through the same temperature? (Consider both have equal length)

A) 3:1 B) 1:3 C) 2:3 D) 1:4

3. A metal plate of original area A_0 is heated from T_1 to T_2. If the coefficient of linear expansion of the metal is α, what is the new area of the plate?

   A) $2\alpha A_0 (T_2 - T_1)$ C) $A_0 [1+2\alpha (T_2 - T_1)]$

   B) $A_0 [1+\alpha (T_2 - T_1)]$ D) $A_0 [1+3\alpha (T_2 - T_1)]$

4. A steel rod with cross-sectional area $2cm^2$ undergoes a stretching force of 500N. Find the change in temperature that would elongate the rod by the same amount as the 500N force does.
   Ans. $\Delta T = 1.04^0C$

5. Determine the change in volume of a block of cast iron 5cm×10cm×6cm, when the temperature changes from 15^0C to 47^0C. $\alpha = 10^{-5}K^{-1}$ **Ans**. $\Delta V = 0.288cm^3$

6. The brass bar and the aluminum bar in the following figure are each attached to an immovable wall. At 20°C the air gap between the rods is 1mm. At what temperature will the gap close?

   $\alpha_{brass} = 1.9x10^{-5}/^0C$
   $\alpha_{alum} = 2.5 \times 10^{-5}/^0C$
   Ans. $T = 36.13\ ^0C$



7. A copper wire ($17 \times 10^{-6}/^0C$) and a lead wire ($29 \times 10^{-6}/^0C$) are joined together, end to end. The compound wire has an effective coefficient of linear expansion of $20\times 10^{-6}/°C$. What fraction of the length of the compound wire is copper? **Ans**. $\frac{l_{cop}}{l_{comp}} = 0.75$

Questions from national exam

1. A steel wire is 2m long at a temperature of 50^0C. If $\alpha_{steel} = 1.2\times10^{-5}/^0C$, what would be the length of the wire at a temperature of 150^0C?

   A) 2.24m B) 2.012m C) 2.024m D**)** 2.0024m

2. A flat metal disc has a small hole in it. When heated,

   A) the disc will expand, but the hole will shrink.

   B) the disc will expand and so will the hole.

   C) the disc will expand, but the hole will remain unchanged.

   D) the disc will remain unchanged, but the hole will enlarge.

3. The length of an aluminum rod at a temperature of 20^0C is 3m. What will be the ratio of change in the length of the rod to its original length if its temperature is raised to $120\ ^0C$? $\alpha_{al}=2.31\times10^{-5}/K$

   A) 23% B) 3.2% C) 3% D**)** 0.23%

4. A glass flask whose volume is exactly $1000cm^3$ at 0^0C is totally filled with $1000cm^3$ of mercury at this temperature. When flask and mercury are heated to 100^0C, $15.2cm^3$ of mercury overflow. If $\gamma_{merc} = 1.82\times10^{-4}K^{-1}$, what should be the coefficient of volume expansion of the glass?

   A) $1.52\times10^{-4}/^0C$ B) $1.82\times10^{-4}/^0C$ C**)** $3.00\times10^{-5}/^0C$ D) $1.00\times10^{-5}/^0C$

5. A metal ball has a diameter that is slightly greater than the diameter of a hole that has been cut into a metal plate. The coefficient of linear expansion for the metal from which the ball is made is greater than for the metal of the plate. Which one of the following procedures can be used to make the ball pass through the hole?

   A) Raise the temperatures of the ball and the plate by the same amount.

   B) Heat the ball and cool the plate.

   C) Lower the temperatures of the ball and the plate by the same amount.

   D) There is no way of passing the ball through the hole.

Heat, temperature and thermal energy

Heat:- is a form of energy that flows from high temperature to low temperature region.

Temperature:- is the measure of the degree of hotness or coldness of a body. Temperature can also be defined as the measure of the average molecular kinetic energy.

Specific heat capacity (c):- is the amount of heat required to raise the temperature of *one kilogram* of substance by 1^0C or 1K.*Note that change in temperature is the same in both kelvin and Celsius.*

The amount of heat (Q) given to a substance of mass (m) to change its temperature by ΔT is

$$Q = mc\Delta T$$

What is the SI unit of specific heat capacity?

Different substances have different values of specific heat capacity. Water has the highest value of specific heat capacity which is nearly 4200 J/kg.K

The heat capacity *(C):*- is defined as the amount of energy needed to raise the temperature of a substance by 1°C or 1K. From this definition, we see that if energy Q produces a change ΔT in the temperature of a sample, then

$$Q = C\Delta T$$

Example

1. How much heat is required to change the temperature of 250g of water from 20^0C to 35^0C?
   Ans. Q = 15750J
2. How much heat is required to raise the temperature of 0.2kg water from 20^0C 70^0C?
   Ans. Q= 4.2×10^4J
3. The heat required to raise the temperature of an object from 25^0C to 50^0C is 125kcal. How much heat is needed to raise the temperature of the same object from 40^0C to 60^0C? **Ans**. Q=100kcal
4. The heat required to raise the temperature of an object from 20^0C to 70^0C is 150 kcal. How much heat is needed to raise the temperature of the same object from 50^0C to 80^0C? **Ans**. Q = 90kcal

5. The number of joules of heat energy required to raise the temperature of a body by 1K is:

A) Heat capacity. C) Specific heat capacity.

B) Expansion coefficient. D) Caloric value.

The law of heat exchange

When two substances at different temperature are mixed or in thermal contact, the hotter loses and the colder gains heat. The amount of heat lost by hot substance is equal to the heat gained by the cold one.

Heat lost = Heat gained

$$Q_{lost} = Q_{gained}$$

$$m_{hot}c_{hot}\Delta T_{hot} = m_{cold}c_{cold}\Delta T_{cold}$$

Example

1. A 5kg water at a temperature of 5^0C is mixed with a 1kg of water at a temperature of 95^0C. What is the final temperature of the mixture? **Ans**. $T_{final} = 20^0$C
2. A 400g milk at 80^0C is mixed with a 100g coffee at 40^0C. What is the temperature of the mixture? (Take $c_{coffee} = c_{milk}$ = 4200J/kg.K) **Ans**. $T_{mixture} = 72^0$C
3. A cup, made up of 0.1kg copper and initially at 20^0C, is filled with a 0.2kg coffee initially at

70^0C. What is the final temperature after the coffee and the cup attain equilibrium? c_{copper} = 490J/kg.K, c_{coffee} = 4200J/kg. K **Ans**. $T_f = 67.2^0$C

Exercise

1. A 100g metal at 100^0C is added to a 150g water (c_{water} = 1cal/g.^{0}C) at 40^0C. After thermal equilibrium is reached, the temperature of the water and metal is 50^0C. What is the specific heat capacity of the metal? **Ans**. c_{metal} = 0.3cal/g.^{0}C
2. A 2kg water at 0^0C is mixed with a 6kg water of at 24^0C. What is the final temperature of the mixture? **Ans**. $T_{final} = 18^0$C
3. A piece of iron (c_{iron} = 450J/kg.K) is dropped from a height of 900m above the ground. Assuming all the energy is converted to heat the iron, what will be the increase in temperature of theiron? **Ans**. $\Delta T = 20^0$C
4. A 250 g aluminum (specific heat = 0.215cal/ g.^{0}C) at 115^0C is added to 430g of water (specific heat = 1cal/g.^{0}C) at 25^0C.If heat is not lost to the environment, what is the temperature of the aluminum and the water after thermal equilibrium is reached?

   A) 20^0C B**)** 35^0C C) 30^0C D) 40^0C
5. A cup of coffee at a temperature of 80 ^{0}C is placed on a table in a 20 ^{0}C room. What happens to the thermal energy and molecular motion of the coffee when it is left on the table?

   A) The thermal energy decreases and the molecules move more slowly.

   B) The thermal energy increases and the molecules move more slowly.

   C) The thermal energy decreases and the molecules move faster.

   D)The thermal energy increases and the molecules move faster.

Change of state

A substance usually undergoes a change in temperature when energy is transferred between the substance and its environment. In some cases, however, the transfer of energy doesn't result in a change in temperature. This can occur when the physical characteristics of the substance change from one form to another, commonly referred to as a **phase change**. Two common phase changes are solid to liquid (melting) and liquid to gas (boiling). Any such phase change involves a change in the internal energy, but *no change in the temperature*.

Latent heat of fusion (L_f):- is the amount of heat required to melt **one kilogram** of solid *at the same temperature*. It is also equal to the amount of heat given off by **one kilogram** of molten substance as it solidifies at the same temperature.

For ice $L_f = 3.33\times10^5$J/kg

The quantity of heat required to melt a mass *m* is

$$Q = m\,L_f$$

Latent heat of vaporization (L_v):- is the amount of heat required to vaporize **one kilogram** of liquid at the same temperature. Note that in the revers process, namely condensation, the same amount of heat is given off at the same temperature.

For water $L_v = 2.26\times10^6$J/kg

The quantity of heat required to vaporize a mass *m* is

$$Q = mL_v$$

Example

1. If a1g of ice at -30^0Cis converted to 1g of steam at 120^0C:

Part A: $Q = m_{ice}C_{ice}\,\Delta T$

$= 10^{-3}$ kg (2090J/kg ^{0}C) $[0-(-30)]^0$C = 62.7J

Part B: $Q = m_{ice}L_f$

$= 10^{-3}$kg $(3.33\times10^5$J/kg) = 333J

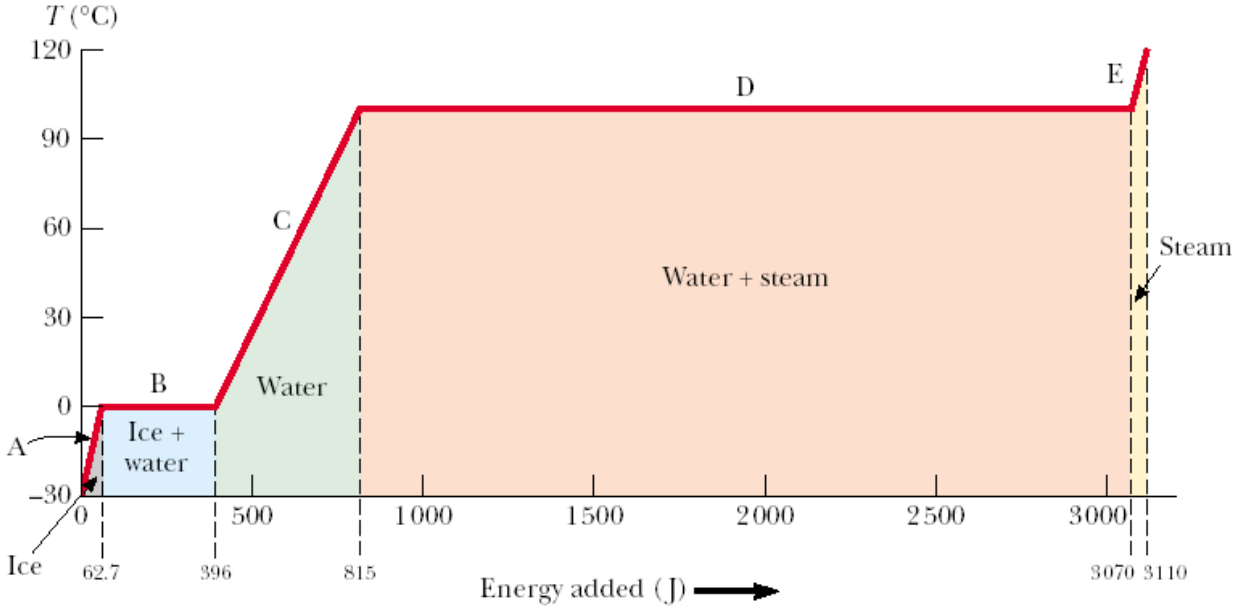
Part C: $Q = m_{water}C_w\Delta T$

$= 10^{-3}$kg (4190J/kg ^{0}C)(100^0C – 0^0C) = 419J

Part D: $Q = m_wL_v = 10^{-3}$kg$(2.26\times10^6$J/kg) = 2260J

Part E: $Q = m_{steam}\,C_{steam}\Delta T$

$=10^{-3}$Kg (2010J/kg^{0}C) (120^0C–100^0C) = 40.2J



2. Calculate the heat required to change 10g of

a) ice ($L_f = 3.33\times10^5$J/kg) at 0^0C to water at 0^0C. **Ans**. Q = 3330J

b) water at 0^0C to water at 100^0C. **Ans**. Q = 4200J

c) water at 100^0C to steam at 100^0C. **Ans**. $Q = 2.26\times10^4$J

3. How much heat is needed to change 2kg of ice at 0^0C to water at 50^0C? **Ans**. $Q = 1.086\times10^5$J

4. How much heat must be removed to cool a 0.8kg water at 20^0C to ice at 0^0C?

Ans. $Q = 3.336\times10^5$J

5. How much heat is required to convert 1 g of ice at –10 ^{0}C to steam at 100^0C? **Ans**. Q= 3033.9J

6. How much water at a temperature of 60^0C must be added to raise the temperature of 1.5kg of water from 20^0C to 45^0C? **Ans**. m = 2.5kg

7. Consider a steam and water, ($m_{water} = m_{steam}$), at 100^0C. Explain why the steam has more burning effect than water even though both of them are at 100^0C?

8. Why is an ice cube at 0^0C more effective than the same mass of water at 0^0C in cooling a drink?

A) The ice has more average kinetic energy than the water.

B) The water particles on the average move faster than the ice.

C) The ice absorbs more heat from the drink in order to melt to water.

D) The water transfers heat to the drink in order to freeze to ice.

Questions from national exam

1. A 5 g ice at -20^0C is dropped into 10g water at 60^0C. What is the final temperature of the system? ($c_{ice} = 2.1\times10^3$J/kg. ^{0}C, $L_f = 3.33\times10^5$J/kg)

   A) -10^0C B) 0^0C C) 20.23^0C D) 10.23^0C

2. Heat travels from one object to another when these objects differ in

   A) specific heat B) temperature C) heat capacity D) state

The Unusual Behavior of Water

Liquids generally increase in volume with increasing temperature and *have volume expansion coefficients about ten times greater than those of solids*. Over a small temperature range, water is an exception to this rule. As the temperature increases from 0^0C to 4^0C, water contracts, so its density increases. Above 4^0C, water exhibits the expected expansion with increasing temperature. The density of water reaches its maximum value of 1000 kg/m^3 at 4^0C.

We can use this unusual thermal expansion behavior of water to explain why a pond freezes slowly from the top down. When the atmospheric temperature drops from 7^0C to 6^0C, say, the water at the surface of the pond also cools and consequently decreases in volume. This means the surface water is denser than the water below it, which has not yet cooled nor decreased in volume. As a result, the surface water sinks and warmer water from below is forced to the surface to be cooled. When the atmospheric temperature is between4^0C and 0^0C, however, the surface water expands as it cools, becoming less dense than the water below it. The sinking process stops, and eventually the surface water freezes. As the water freezes, the ice remains on the surface because ice is less dense than water. The ice continues to build up on the surface, and water near the bottom of the pool remains at 4^0C. Further, the ice forms an insulating layer that slows heat loss from the underlying water, offering thermal protection for marine life. The same peculiar thermal expansion properties of water sometimes cause pipes to burst.

Calorimetry

Calorie:- is the cgs unit of heat. 1cal = 4.1868J

Calorimetry:- is a science to study the measurement of quantity of heat. It involves the use of calorimeter.

Calorimeter:-is an apparatus for measuring amount of heat, specific heat capacity, caloric values, etc.

Caloric value:- is the quantity of heat produced by the complete combustion of a given mass of fuel, usually expressed in joules per kilogram.

Internal energy:- is the total energy of a system that is a function of its temperature and is the sum of kinetic and potential energies of its atoms (molecules). It increases when the temperature of the system is increased.

Heat transfer

There are three ways by which heat can be transferred from one point to another.

i) Conduction:-is the transfer of heat from places of higher temperature in a substance by the interaction of atoms and electrons possessing greater kinetic energy with those possessing small kinetic energy. It takes place by means of successive collisions between neighboring particles without any movement of a particle from its mean position.

Conduction occurs only if there is a difference in temperature between two parts of the conducting medium.

Metals are good thermal conductors because they contain large numbers of electrons that are relatively free to move through the metal and so can transport energy over large distances. Thus, in a good conductor, conduction takes place by means of both the vibration of atoms and the motion of free electrons.

Materials such as asbestos, cork, paper, and, fiberglass are poor conductors.

Gases are also poor conductors because the separation between the particles is so great.

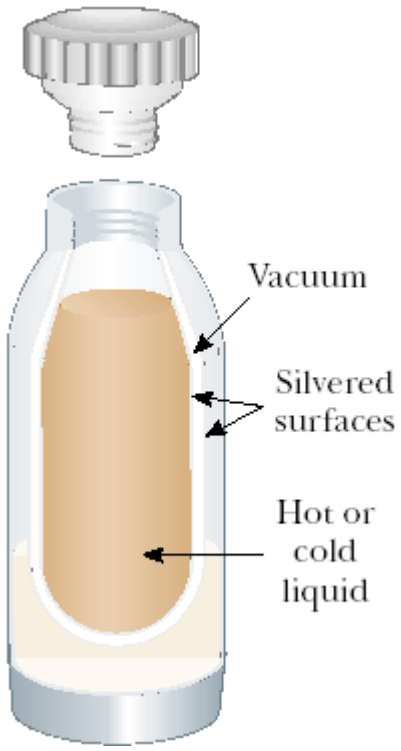
ii) Convection:- is a process by which heat is transferred from one part of a fluid to another by movement of the fluid itself. Hotter fluid becomes less dense and rise up. Cold fluid from the above moves down and get heated. This is a kind of self-stirring mechanism.

iii) Radiation:- is the mode of transfer of heat energy through empty space.

Vacuum flask

The vacuum flask is designed to minimize energy losses by conduction, convection, and radiation. Such a container is used to store either cold or hot liquids for, long periods of time. The standard construction, shown in figure below, consists of a double-walled Pyrex glass vessel with silvered walls.

The space between the walls is evacuated to minimize energy transfer by conduction and convection.
The silvered surfaces minimize energy transfer by radiation because silver is a very good reflector and has very low emissivity.



A cross-sectional view of a vacuum flask, which is used to store hot or cold substances.

Rate of heat flow

The rate of heat flow per unit time is:

i)directly proportional to the area (A):-the more area available, the more heat can flow per unit time

ii) directly proportional to the change in temperature (ΔT):- the larger the temperature difference, the more heat can flow per unit time.

iii) inversely proportional to thickness (Δ*x*):- the thicker the slab, the less heat can flow per unit time.

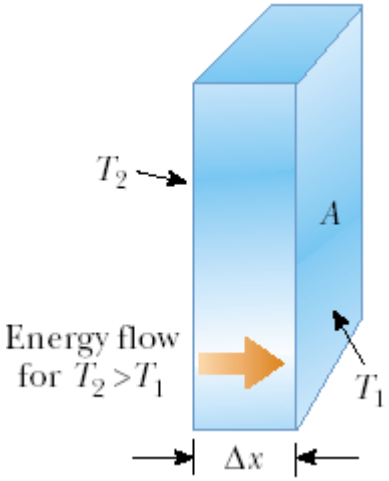
$$\frac{\Delta Q}{\Delta t} = H = kA\frac{\Delta T}{\Delta x}$$

Where k is thermal conductivity of the material

$\frac{\Delta T}{\Delta x}$ is temperature gradient

Note: Materials with

- **large value of k** are **good conductors of heat**.
- **small value of k** are **poor conductors of heat**.



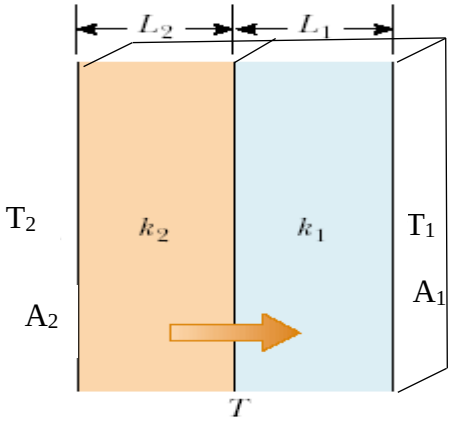
Example

1. What is the rate of heat loss through a $1m^2$ of a glass window (k_{glass}= 0.84W/mK) which is 3mm thick? Outside temperature = –28°C, inside temperature = 22°C. **Ans**. H = 14000W
2. Calculate the rate of heat flow through a copper bar (k_{copp} = 390W/K. m) with A = $4cm^2$ and Δ*x* = 0.5m if there is a temperature difference of 30^0C is maintained between its ends.
   Ans. H = 9.36W
3. Two slabs of thicknessesL_1 and L_2 and thermal conductivities k_1 and k_2 are in thermal contact with each other, as shown in the figure below. The temperatures of their outer surfaces are T_1 and respectively. If $T_2 > T_1$ and both slabs have equal areas, $A_1=A_2=A$, show that
   a) the temperature at the interface is given by:

$$T = \frac{k_1 L_2 T_1 + k_2 L_1 T_2}{k_1 L_2 + k_2 L_1}$$

   b) the rate of energy transfer by conduction through the slabs is given by:

$$H = \frac{A(T_2 - T_1)k_1 k_2}{L_1 k_2 + L_2 k_1}$$



4. For question 3 above if $T_1 = 30^0$C, $T_2 = 100^0$C, k_1 = 400 W/K.m, k_2 = 50W/K.m, L_1 = 2m and L_2 = 1.2m, calculate
   a) the temperature T at the interface. **Ans**. T = 88^0C
   b) the rate of heat flow through the slabs. **Ans**. H = 0.24W

5. The size of a window is 2m x 1m. (k_{glass}= 0.84W/mK)

a) Calculate the heat loss through the windowpane 4mm thick, if the outside temperature is 5^0C and the inside temperature is 25^0C. **Ans**. H = 8400Watts

b) In a very cold countries, builders use a technique called *double glazing* in order to reduce the heat loss though the windows. Now, instead of one pane of glass in the window above, we have two placed right next to each other, so that it is like one pane of glass 8mm thick.

How does this affect the heat flow through the window? Do a calculation to support our answer.

Ans. H=4200W

We can see that by doubling the thickness of the glass, we have halved the heat loss through the window, which is a good thing, but perhaps not enough.

c) What is the effect of placing a space of air, thickness 4*mm*, in between the two panes of glass?(k_{air} = 0.024W/K.m) **Ans**. H = 1793.4Watts.

This is less than one quarter of the original rate of heat loss when we had one pane of glass.

Compare your answers in a), b) and c)

Exercise

1) A plaster ceiling is 3mx 5m x 20mm. If the inside and outside temperatures are 17°C and 7°C, respectively.

a) Calculate the net rate of heat flow through the plaster ceiling if k_{pl}=0.6W/Km. **Ans**. H = 4500Watts

b) A layer of insulating fiberglass (k_{fiberg} = 0.040W/K.m), 40*mm* thick, is placed on top of the ceiling. How this would affect the rate of heat loss through the ceiling? **Ans**. H = 90Watts

Placing a layer of fiberglass over the plaster ceiling will cause the rate of heat loss to be reduced, for two reasons. The *first is that the fiberglass has a lower conductivity* than the plaster and the *second is that the heat has to pass through a thicker solid*.

2) When you stand bare feet on a concrete floor, your feet feel colder than if you were standing on a carpet that lies on the same floor. Since both the carpet and the floor have been in contact for a long time, they are at the same temperature. Why does it feel as if the floor is colder than the carpet?

It appears that the floor is colder than the carpet, but in fact this is not really the case. When we stand bare feet on a cold concrete floor, heat energy is transferred away From our feet very quickly. On the other hand, when we stand on a carpet or a Blanket that is placed on the floor, the heat energy is conducted away much more slowly than when we are on the concrete floor. The ***concrete is a better conductor of heat than the carpet****, which is* ***made from a good insulating material****.*

This makes the floor appear to feel colder than the carpet.

3.Why does a piece of metal feel colder than a piece of wood when they are at the same temperature?

4. When we feel cold, we wear warm clothes. *Clothes do not supply heat to our bodies*, so how do we feel warm?

Questions from national exam

1. A steel rod 20*cm* long has a cross-sectional area of $1.2\times10^{-2}m^2$. What is the rate of conduction through the rod if the opposite ends are maintained at 80^0C and 120^0C? (k = 50W/^{0}C.m)
   A) 80W B**)** 120W C) 140W D) 160W
2. Which of the following combinations of properties would be most desirable for a cooking pot?
   A) High specific heat and low conductivity. C) High specific heat and high conductivity.
   B) Low specific heat and high conductivity. D) Low specific heat and low conductivity.
3. Heat conduction will take place between two objects that are in contact, when there is a different in the:
   A) temperatures. C) thermal conductivities.
   B) heat capacities. D) heat content of the objects.
4. The body surface area is $1.8m^2$; the clothing is 1.0cm thick; skin surface temperature is 33^0C, whereas the outer surface of the clothing is 1.0^0C; the thermal conductivity of the clothing is 0.04W/K.m. What is the rate at which body heat flow out through the clothing of a man?
   A) 22.04W B) 23.04W C) 2.30W D) 230.4W
5. Which of the following combinations of properties would be most desirable for a thermal insulator?
   A) High specific heat and low conductivity. C) High specific heat and high conductivity.
   B) Low specific heat and high conductivity. D) Low specific heat and low conductivity.

Conduction and convection

When a solid and fluid are kept at two distinct temperatures, the rate of heat flow is proportional to the area of contact between the solid and the fluid, and the temperature difference.

$$H = h\,A\Delta T$$

Where *h* is convective surface coefficient which depends on the type of the material of which the substance is made.

Example

1. An element of a kettle that has a total surface area of $25cm^2$ is put into cold water that has a temperature of 22^0C. The temperature of the element of the kettle is 182^0C and *h* =105 W/m^2K. What is the rate of heat flow from the element? **Ans**. H =42W
2. A heater at a temperature of 220^0C is immersed in water at 40^0C. If the surface of the heater is $80cm^2$, what is the rate of heat lose? (*h* = 400W/m^2. K) **Ans**. H = 576W

Thermal Radiation

Stefan-Boltzmann law states "The power radiated over all wavelengths from a surface is proportional to the fourth power of the absolute temperature."

$$H = Ae\,\sigma T^4$$

Where A is surface area of the radiator
$\sigma = 5.67\times10^{-8}W/m^2.K^2$
e is emissivity of the surface

For dull rough surface, e is nearly 1 and for shiny surface, e is near 0.

Example

1. An aluminum sphere has a surface area of $12.5cm^2$ and is at a temperature of 427°C. How much energy does the sphere lose per second, if it only loses energy through radiation? (Take e = 0.6)**Ans**. H = 10.2W
2. How fast is the sphere in question 1 losing energy when the temperature is only 600K? **Ans**. H = 5.51W

~~THE END~~