Unit 2

Oscillations and waves



by Muhammed Awol 2013 E.C

Unit 2 Oscillations and waves

2.1 Periodic motion (basic concepts)

2.2 Wave motion

2.3 Sound, loudness and the human ear

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A great many things in the world around us oscillate (vibrate) backwards and forwards, up and down, side to side, in and out, etc. Atoms within molecules vibrate and the size of these vibrations is proportional to temperature. Oscillations of charges produce electromagnetic waves: e.g. a current oscillating up and down an aerial produces radio waves. Vibrations of our vocal chords produce sound waves, as do vibrations of strings and of air inside tubes in musical instruments. Parts of machinery, e.g. in washing machines and in cars, vibrate, sometimes when we don't want them to!

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When engineers build large structures like skyscrapers and bridges, they have to understand how the wind or the ways people walk across them will make them oscillate. It is impossible to stop such structures oscillating altogether, but if engineers don't design their structures to control these vibrations, they might end up shaking themselves to pieces.

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Most of these oscillations are periodic. This means that they keep doing exactly the same thing in the same amount of time again and again. In some cases, usually for large objects or structures, each cycle (backwards and forwards, up and down, side to side, in and out, etc.) of the oscillation could take many seconds or even much longer. These are low-frequency oscillations. In other cases there can be hundreds, thousands or even thousands of billions of complete vibrations every second. These are high-frequency oscillations. This predictable time period can be very useful. For example, the predictable time period of pendulums, of masses on springs or of quartz crystals is used to count the passing of time in clocks and watches.

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The way things oscillate can be quite complex, but many oscillations are very close to a special form of periodic motion called **simple harmonic motion**, and more complicated motion can be shown to be simply a sum of simple harmonic motions at different frequencies. This unit will analyse a few examples of oscillating objects performing simple harmonic motion in some mathematical detail.

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2.1 Periodic motion (basic concepts)

By the end of this section you should be able to:

- Describe the periodic motion of a vibrating object in qualitative terms, and analyse it in quantitative terms (e.g. the motion of a pendulum, a vibrating spring, a tuning fork).
- Define simple harmonic motion (SHM) and describe the relationship between SHM and circular motion.
- Derive and use expressions for the frequency, periodic time, displacement, velocity and acceleration of objects performing SHM.
- Draw and analyse $x-t$, $v-t$ and $a-t$ graphs for SHM.
- Use Newton's second law and Hooke's law to derive $\omega = \sqrt{k/m}$.
- Describe the effects: free oscillations, damping, forced oscillations and resonance.
- Analyse the components of resonance and identify the conditions required for resonance to occur in vibrating objects and in various media, including the effects of damping on resonance.
- Explain the energy changes that occur when a body performs SHM.
- Draw and interpret graphs showing the variation of kinetic energy and potential energy of an object performing SHM.
- Relate the energy of an oscillator to its amplitude.
- Solve problems on SHM involving period of vibration and energy transfer.

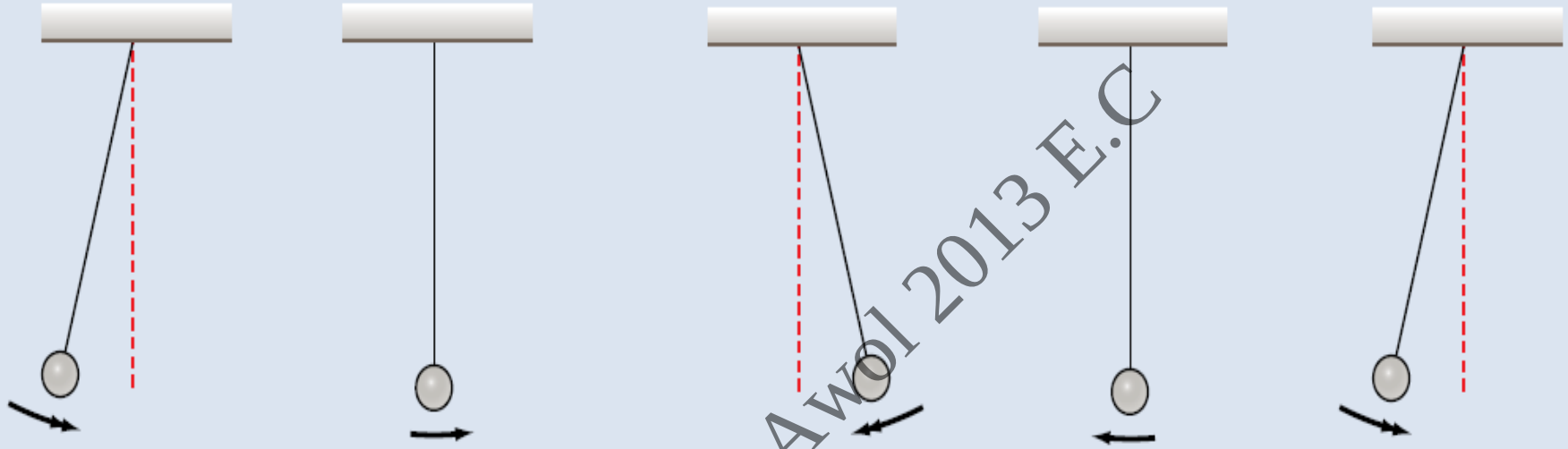
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Periodic oscillations

- Periodic oscillation is defined to be an oscillatory motion that repeats it self at regular interval around a central position such as exhibited by pendulum or by an object on a spring moving up and down.
- This central position is called the equilibrium position and it is the position of the object when it is at rest.
- Whenever an object is displaced from its equilibrium position there is a force that acts towards its original position.
- This force is often referred to as a restoring force, as it tries to restore the system to its equilibrium position.

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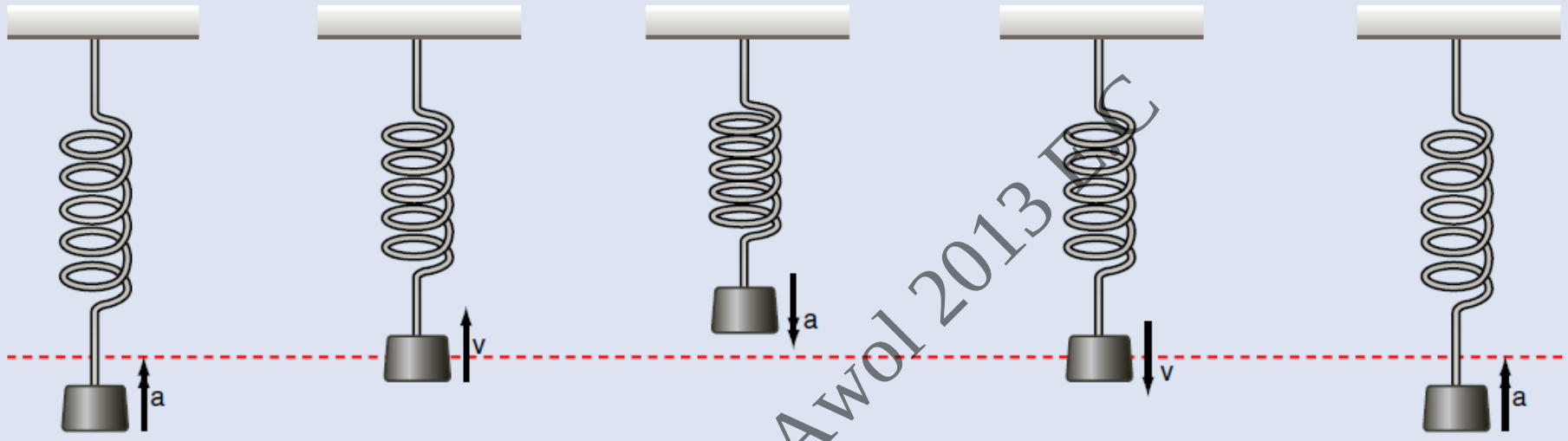
How does a pendulum work?



(a)	(b)	(c)	(d)	(e)
If the pendulum bob is pulled to one side and released, it accelerates back towards its equilibrium position.	When the pendulum bob gets back to the equilibrium position, it is moving relatively fast and, although there is no resultant force now, its inertia keeps it moving.	The pendulum bob keeps moving, slowing down all the time, until it is at the same height as it started and accelerates back towards the equilibrium position.	The pendulum bob passes through the equilibrium position again, going back the other way.	The pendulum bob arrives back at where it started. It has completed one cycle, and will now do the same again, and again . . .

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How does a mass on a spring oscillate?



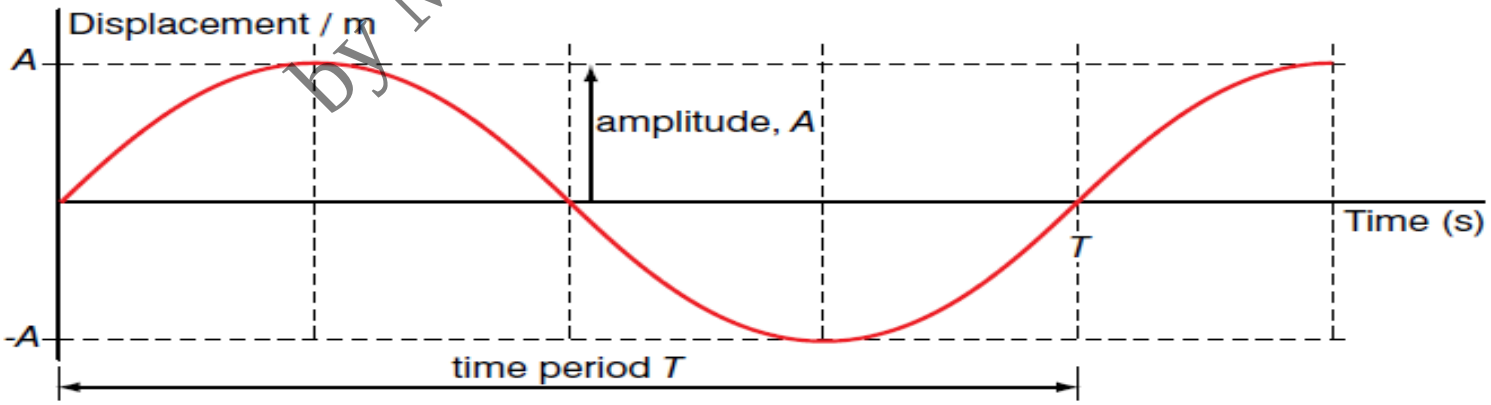
(a) If the mass is pulled down, the tension in the spring is greater the weight of the mass. The resultant force is upwards and the mass accelerates upwards when released.	(b) When the mass gets back to the equilibrium position, it is moving relatively fast and although the weight of the mass balances the tension in the spring and there is no resultant force now, its inertia keeps it moving.	(c) The mass keeps moving, slowing down all the time, until it is as far above the equilibrium point as it started below. The weight of the mass is greater than the tension in the spring, the resultant force is now downwards and the mass accelerates downwards.	(d) The mass passes through the equilibrium position again, going back the other way.	(e) The mass arrives back at where it started. It has completed one cycle, and will now do the same again, and again . . .

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SIMPLE HARMONIC MOTION(SHM)

- **Simple harmonic motion (SHM) is a periodic oscillation of an object about an equilibrium position such that its acceleration is always directly proportional in size but opposite in direction to its displacement. (The acceleration is always towards the equilibrium position.)**
- **It is the oscillations of an object in which the net force can be described by Hooke's law.**
- **If we plot how the displacement of an object performing simple harmonic motion varies with time, we find that the variation is sinusoidal,(*an oscillation that can be described mathematically using sine or cosine functions,* *but throughout the rest of this chapter we will use sine function*)as shown in Figure below.**

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Mathematical expression of Simple Harmonic Motion

The displacement as a function of time *t in any simple harmonic motion* is given by

$$x = A\sin\left(2\pi\frac{t}{T}\right) \text{ or } x = A\cos\left(2\pi\frac{t}{T}\right)$$

where A is amplitude, and the displacement oscillates back and forth with a period T .

Looking carefully at the information above you can see how in one oscillation the displacement starts at 0 rises to a positive amplitude, falls back to zero, falls to a negative amplitude and then rises back to zero.

If $x = A\sin\left(2\pi\frac{t}{T}\right)$, with $\left(\frac{2\pi}{T}t\right)$ expressed in radians:

when $t = 0$	$x = A\sin\left(\frac{2\pi}{T}0\right)$	$= A\sin(0)$	$= 0$
$t = \frac{T}{4}$	$x = A\sin\left(\frac{2\pi}{T}\frac{T}{4}\right)$	$= A\sin\left(\frac{\pi}{2}\right)$	$= A$
$t = \frac{T}{2}$	$x = A\sin\left(\frac{2\pi}{T}\frac{T}{2}\right)$	$= A\sin(\pi)$	$= 0$
$t = \frac{3T}{4}$	$x = A\sin\left(\frac{2\pi}{T}\frac{3T}{4}\right)$	$= A\sin\left(\frac{3\pi}{2}\right)$	$= -A$
$t = T$	$x = A\sin\left(\frac{2\pi}{T}T\right)$	$= A\sin(2\pi)$	$= 0$

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Amplitude

- a simple harmonic oscillator will oscillate with equal displacement on either side of the equilibrium position, as shown in Figure.
- The **maximum displacement** from equilibrium is called the **amplitude** A .
- The units for **amplitude** and **displacement** are the same(is **meter**).

Period and Frequency

- The time to complete one oscillation remains constant and is called the period T . Its units are seconds.
- A concept closely related to period is the frequency of an event.
- Frequency f is defined to be the number of events per unit time.
- For periodic motion, frequency is the number of oscillations per unit time.
- The relationship between frequency and period is: $f = \frac{1}{T}$
- The SI unit for frequency is the *cycle per second, which is defined to be a hertz (Hz):*

$$1\ \text{Hz} = 1\frac{\text{cycle}}{\text{sec}} \text{ or } 1\ \text{Hz} = \frac{1}{\text{s}}$$

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Period and Frequency…

- **and so our mathematical expression for displacement can be written as:**

$$x = A \sin(2\pi ft)$$

Example

A pendulum has a frequency of 4.0 Hz and amplitude of 5.0cm. Determine the displacement after 4.6 seconds.

Solution

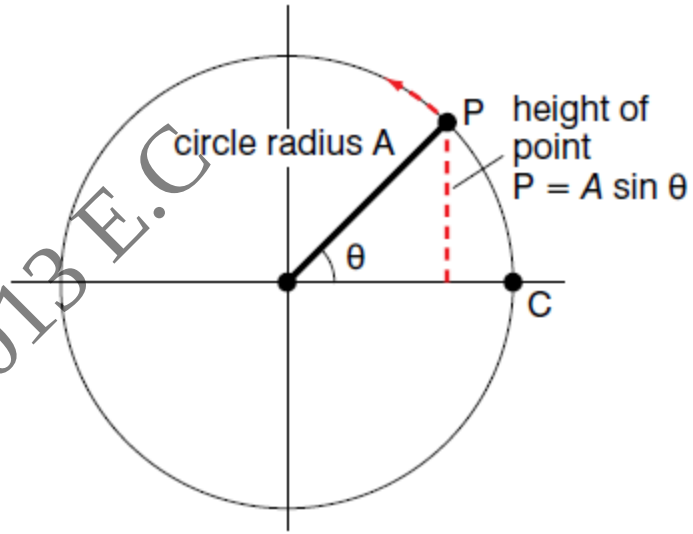
State the equation of SHM	$x = A \sin(2\pi ft)$
Substitute the known values	$x = 0.050 \sin(2\pi \times 4.0 \times 4.6)$
Solve the equation and give the units	$x = 0.029\ m$ *Or* $x = 2.9\ cm$

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Circular motion, SHM and angular frequency

If a point P moves around in circle of radius A, as shown in Figure, starting from point C, then the height of point P, after it has turned through angle θ is given by:

$$h = A\sin\theta$$



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- **simple harmonic motion produced in this manner can give considerable insight into many aspects of oscillations and waves and is very useful mathematically.**
- **we shall indicate some of the major features of this relationship and how they might be useful.**
- **If the point P is rotating at angular speed ω radians per seconds then, after time t seconds, the total angle turned, in radians, is**

$$\theta = \omega t$$

- **and so we can rewrite the equation for the height of point P as**

$$h = A\sin(\omega t)$$

Circular motion, SHM and angular frequency…

- If P goes round in one complete cycle, the angle turned is 2π.
- If P is rotating with a frequency of f cycles per second, the total angle turned per second is $f \times 2\pi$ radians.
- Hence $\omega = 2\pi f$
- In the equation $x = A\sin(2\pi ft)$ we can replace $2\pi f$ by ω and write:

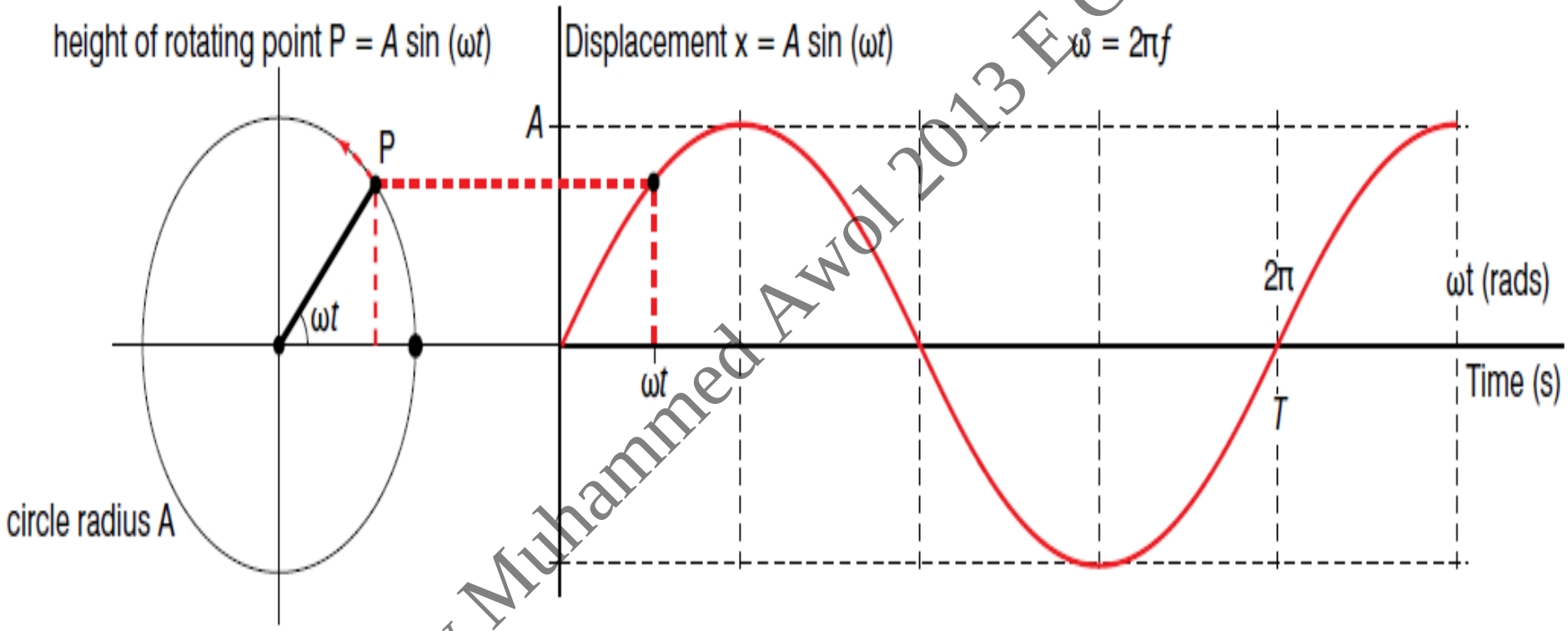
$$x = A\sin(\omega t)$$

- Because of its relationship to frequency, ω is sometimes called angular frequency.

- which is the same as our expression for the height of point P rotating at angular speed ω.
- The height h of point P going round in a circle and the displacement x of an object performing SHM are therefore the same, as shown in Figure on the next slide.

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Circular motion, SHM and angular frequency…

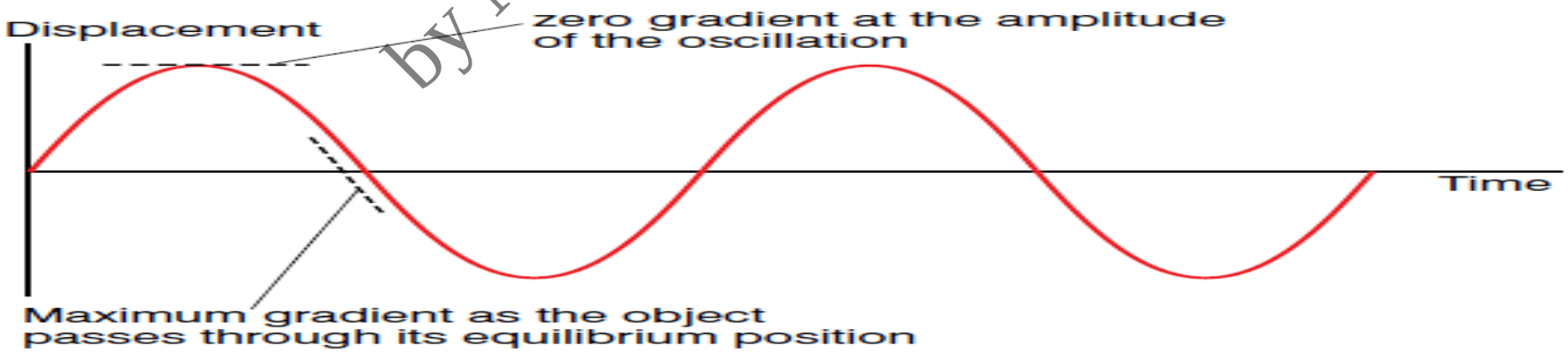


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Displacement, velocity and acceleration in SHM

- The velocity of the oscillating mass is the rate of change of displacement. It becomes zero at the limits of the oscillation. For example, at the top of a pendulum's swing.
- In general, velocity can be found as the gradient of the displacement–time graph.
- At the maximum displacement (the amplitude) the gradient is zero consequently the velocity is zero.
- Therefore, when $x = \pm A$ the velocity is *zero*.
- The maximum velocity occurs when the oscillating object passes through the equilibrium position.

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velocity

The equation for the displacement of the oscillation is $x = A\sin(\omega t)$

The velocity can be found from the time derivative of displacement vector

$$v = \frac{ds}{dt} = \frac{d}{dt}\left(A\sin(\omega t)\right)$$

Therefore the velocity may be written as

$$V = \omega A\cos(\omega t)$$

and the maximum size of velocity (v_o) $v_o = \omega A$

This equation is obtained when $\cos(\omega t) = 1$

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Acceleration

Acceleration is the rate of change of velocity, or the gradient of the velocity time graph.

The equation for acceleration can be found using differential calculus.

$$a = \frac{dv}{dt} = \frac{d}{dt}\,\omega A\cos(\omega t)$$
$$= -\omega^2 A\sin(\omega t)$$

as the expression for x is: $x = A\sin(\omega t)$

The equation can be written as $a = -\omega^2 x$

$a = -\omega^2 x$ ***is the defining equation for SHM.***

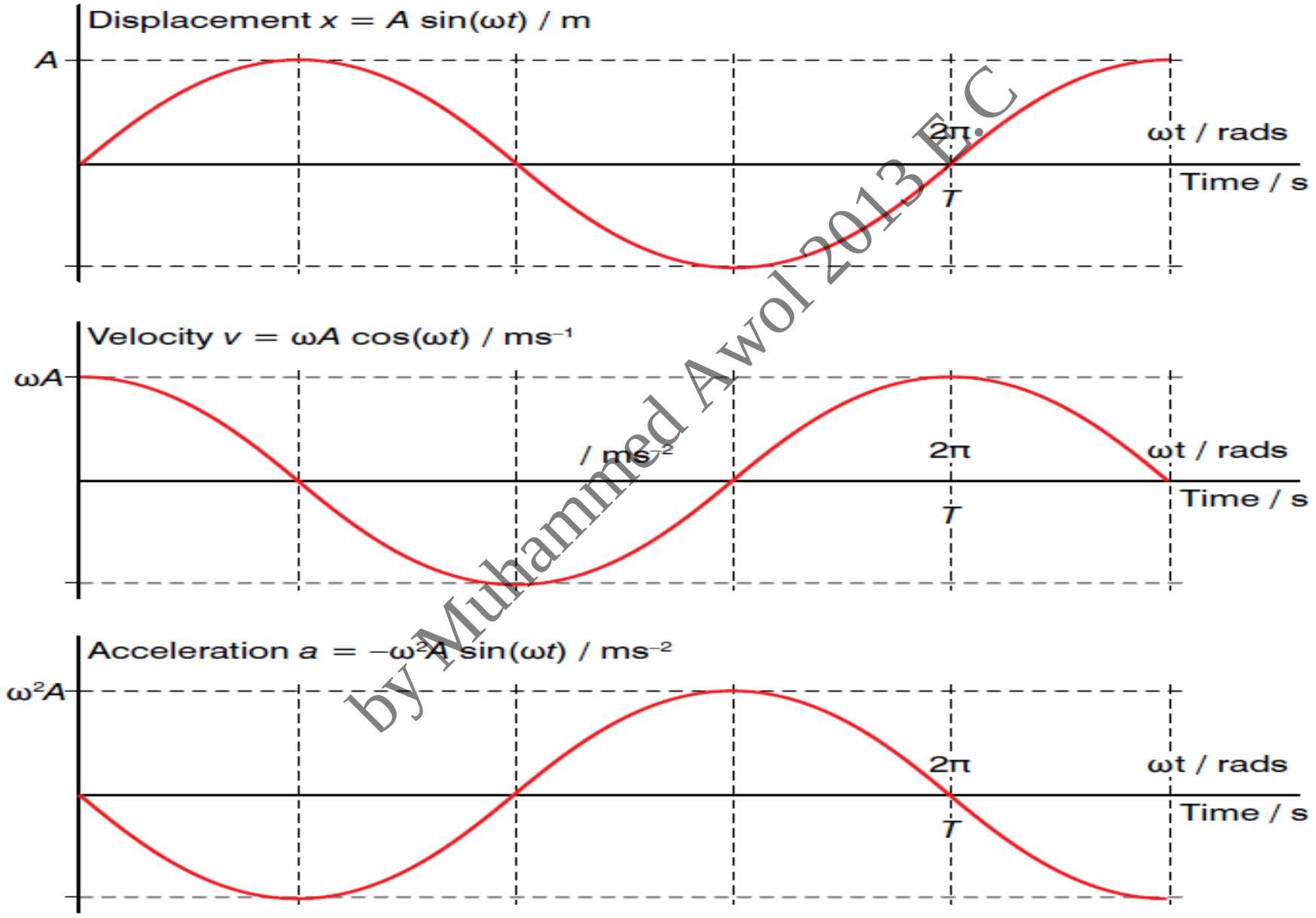
And the maximum size of acceleration (a_o) is: $a_o = \omega^2 A$

This equation is obtained when $\sin(\omega t) = 1$

This happens whenever the mass reaches its maximum displacement.

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Graphs of displacement, velocity and acceleration against time t



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SHM equation summary

- Remember you can use either the sine or cosine function to describe the displacement of a system oscillating with SHM.
- You need to consider when the timing of the oscillation begins.
  - ✓Timing starts with system in its equilibrium position → sine
  - ✓Timing starts with system at its maximum displacement → cosine

The equations are summarized as follows:

	Using sin to describe displacement	Using cos to describe displacement
Displacement	$x = A\sin(\omega t)$	$x = A\cos(\omega t)$
Velocity	$v = \omega A\cos(\omega t)$	$v = -\omega A\sin(\omega t)$
Acceleration	$a = -\omega^2 A\sin(\omega t)$	$a = -\omega^2 A\cos(\omega t)$

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Example

An object moves with simple harmonic motion of amplitude 11 cm and a time period of 2.4 s. Calculate:

a) the frequency

b) the angular frequency

c) the maximum velocity of the object

d) the maximum acceleration of the object

e) the displacement, velocity and acceleration, after 0.5 s, if the object starts from the limit of oscillation at $x = A$.

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Solution

a) Frequency

- $f = \frac{1}{T}$ — *State the relationship to be used*
- $f = \frac{1}{2.4} = 0.42$ Hz — *Substitute in known values and solve, giving the units*

b) Angular frequency

- $\omega = 2\pi f = \frac{2\pi}{T}$ — *State the relationship to be used*
- $\omega = \frac{2\pi}{2.4} = 2.6$ rad/s — *Substitute in known values and solve, giving the units*

c) Maximum velocity

- $v_0 = \omega A$ — *State the relationship to be used*
- $v_0 = \frac{2\pi \times 0.11}{2.4} = 0.29$ m/s — *Substitute in known values and solve, giving the units*

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d) Maximum acceleration

- $a = \omega^2 A$ *State the relationship to be used*
- $a = \left(\frac{2\pi}{2.4}\right)^2 \times 0.11 = 0.75 \text{ m s}^{-2}$ *Substitute in known values and solve, giving the units*

e) Using the cosine equation for displacement, after 0.5 s

- $x = A\cos(\omega t)$ *State the relationship to be used*
- $x = 0.11 \times \cos\left(\frac{2\pi}{2.4} \times 0.5\right)$
  $= 0.11 \times \cos(1.31)$
  $= 0.11 \times 0.259 = 0.028 \text{ m}$

  Substitute in known values and solve, giving the units

The use the appropriate equation to find the velocity, in this case:

- $v = -\omega A\sin(\omega t)$
- $v = -\left(\frac{2\pi}{2.4}\right) \times 0.11 \times \sin(1.31)$
  $= -\left(\frac{2\pi}{2.4}\right) \times 0.11 \times 0.966$
  $= -0.28 \text{ m s}^{-1}$

  Substitute in known values and solve, giving the units

Finally use the defining equation for SHM to find the acceleration.

- $a = -\omega^2 A\cos(\omega t)$ *State the relationship to be used*
- $a = -\left(\frac{2\pi}{2.4}\right)^2 \times 0.11 \times \cos\left(\frac{2\pi}{2.4} \times 0.5\right) = -0.20 \text{ m s}^{-2}$

  Substitute in known values and solve, giving the units

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How does velocity depend on displacement?

We already know that the velocity is zero when $x = \pm A$ at the limits of the oscillation, and that it has its maximum size of $v_o = \omega A$ *when* the mass passes through the equilibrium position.

To get a general expression we need to use a trigonometric identity:

$$\sin^2\theta + \cos^2\theta = 1$$

If $v = -\omega A\cos(\omega t)$ **we have** $v^2 = \omega^2 A^2\cos^2(\omega t)$

If $x = A\sin(\omega t)$ **we have** $\omega^2 x^2 = \omega^2 A^2\sin^2(\omega t)$

$$v^2 + \omega^2 x^2 = \omega^2 A^2\cos^2(\omega t) + \omega^2 A^2\sin^2(\omega t)$$

$$v^2 + \omega^2 x^2 = \omega^2 A^2[\cos^2(\omega t) + \sin^2(\omega t)]$$

$$v^2 + \omega^2 x^2 = \omega^2 A^2 \times 1$$

and so

$$v^2 + \omega^2 x^2 = \omega^2 A^2$$

$$v^2 = \omega^2 A^2 - \omega^2 x^2$$

$$v = \pm\omega\sqrt{A^2 - x^2}$$

This equation shows us that at any given displacement (*x*) *an* oscillating object may have +/– a specific velocity.

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Time periods of real examples of SHM

- In analysing real systems that perform SHM to find their time period we always follow the same procedure.
- We imagine the oscillating mass being displaced from the equilibrium position by **displacement x** and analyse the forces acting on it as a function of its displacement.
- If the system does perform SHM, this resultant force will be a restoring force proportional to *x*:

where k is a constant of proportionality depending on the parameters of the system.

$$F = -kx$$

If we now apply (from Newton's second law), we can replace *F to* write:

$$ma = -kx$$

$$a = -\frac{k}{m}x$$

Comparing this with the defining equation for SHM

$$a = -\omega^2 x$$

we see that

$$\omega^2 = \frac{k}{m}$$

by Muhammad Awal 2018 EC

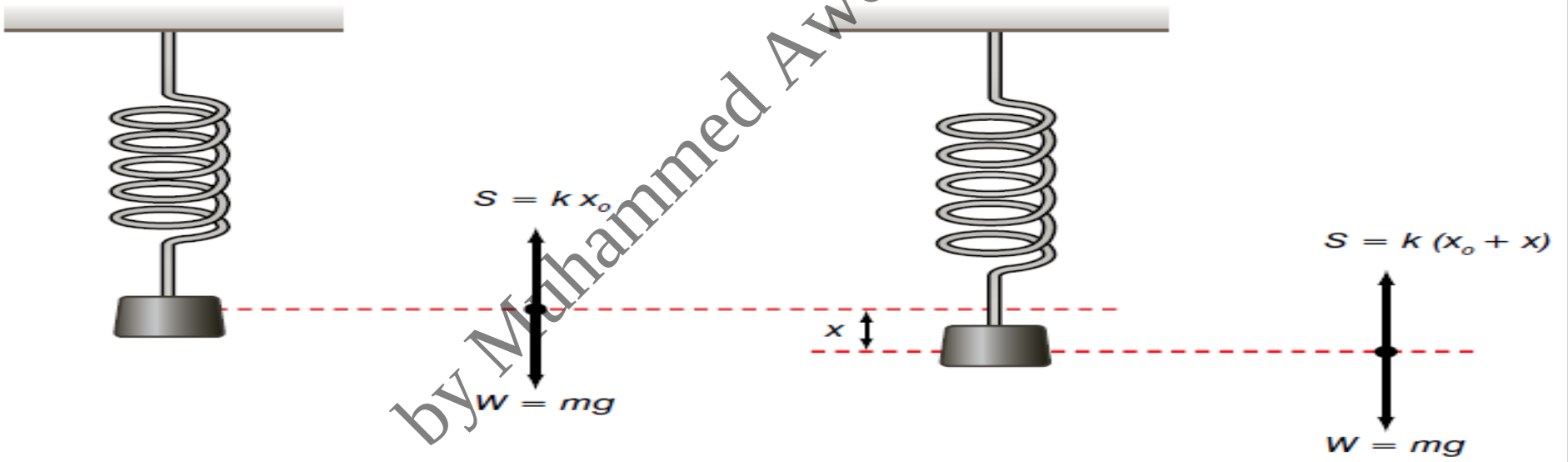
This gives us the angular frequency ω, and from this we can obtain frequency f or time period T by

$$T = \frac{2\pi}{\omega}$$

or

$$f = \frac{\omega}{2\pi}$$

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(a) Spring and mass at equilibrium with spring stretched so that the tension S in the spring is equal and opposite to the weight of the mass.

(b) Mass displaced x downwards from equilibrium position. Tension in spring now greater than the weight of the mass.

The time period of a mass–spring system

- **Consider a *mass m* suspended by a spring of *spring constant k*.**

At equilibrium the tension in the spring is equal and opposite to the weight of the mass

$$S = W$$

$$kx_o = mg$$

and the resultant force downwards on the mass is zero.

When mass is *displaced x* downwards, the tension in the spring, and hence the resultant force downwards, is

$$F = W - S$$

$$F = mg - k(x_o + x)$$

$$F = mg - kx_o - kx$$

But, since $kx_o = mg$, this is

$$F = mg - mg - kx$$

$$F = -kx$$

Newton's second law tells us that, and so

$$ma = -kx$$

and hence

$$a = -\frac{k}{m}x$$

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Comparing this with the general defining equation for SHM

$$a = -\omega^2 x$$

and recalling that

$$\omega = \frac{2\pi}{T}$$

we have

$$\omega^2 = \frac{k}{m}$$

$$\frac{2\pi}{T} = \omega = \sqrt{\frac{k}{m}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

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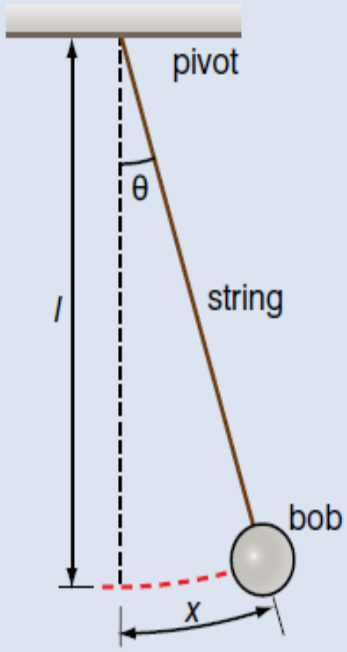
If we make the amplitude of the oscillations too large, however, and we exceed the elastic limit of the spring the above equations are no longer valid and the time period will probably start to become a little longer.

The time period of a simple pendulum

Simple pendulum

A simple pendulum comprises a single mass *m*, *which we treat as* a point mass on a string, length *l*, *(or frictionlessly pivoted rod)* whose mass we ignore.

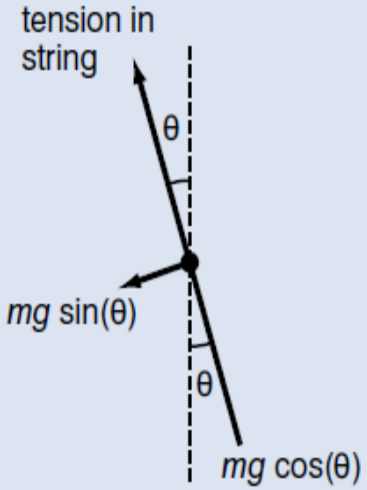
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(a) Pendulum displaced by angle θ. Note: the pendulum bob follows a circular path



(b) Free body force diagram for pendulum bob.



(c) Free body force diagram with resolved into radial and tangential components relative to circular path of bob

- To find the time period of a simple pendulum consider the motion of the bob in a circle radius l about the pivot.
  We analyse the forces acting on the pendulum bob for a displacement x along the circular path that the bob follows, which corresponds to an angular displacement θ, (show on previos slide).
- From the definition of angle measurement, in radians $\theta = \frac{x}{l}$

- There are two forces acting on the pendulum bob:
  - ✓ the tension in the string acting towards the pivot and the weight on the bob acting vertically downwards.
- Since the motion of the bob must be at right angles to the string, we know that forces in the direction of the string, radial with respect to the pivot, cannot contribute to the acceleration of the bob along its circular path, at right angles to the string, tangential with respect to the pivot.

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- **If we resolve the weight, *mg, of the bob parallel and perpendicular* to the string (*shown in Figure*)**
- **The component *mgcos θ* parallel to the string makes no contribution to the tangential motion of the bob.**
- **The resultant tangential restoring force is the component of the bob's weight perpendicular to the string.** $F = -mg\sin\theta$
- **The negative sign tells us that the force is back towards the equilibrium position.**

The acceleration of the bob is $a = \frac{F}{m} = -g\sin\theta$

- **For small angles (less than 10°)** $\theta \approx \sin\theta$ **if we express θ in radians**

And so, for small angles of swing $a = -g\theta = -g\frac{x}{l}$

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And so, comparing this to the defining equation for SHM

$$a = -\omega 2x$$

we obtain

$$\omega^2 = \frac{g}{l}$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{g}{l}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

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- **It is important to remember what approximations we have made to arrive at the above expression for the time period.**

We have assumed that

- ✓ **The mass of the string can be ignored and that the mass can be treated as a point mass.**
- ✓ **The bob is small compared to the length of the string but has a much greater mass.**

For a compound pendulum (such as a swinging metre rule) the time period is better expressed using the relationship:

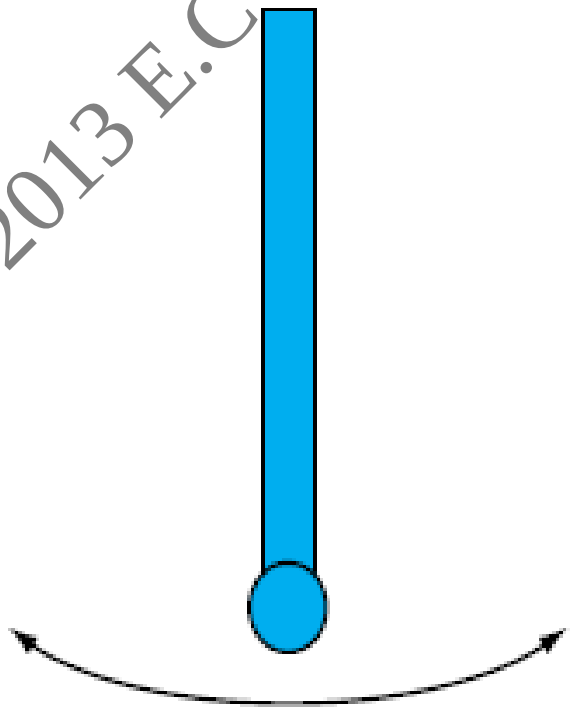
$$T = 2\pi\sqrt{\frac{I}{mgL}}$$

where

I = moment of inertia of pendulum

m = mass of pendulum

L = distance from the pivot to the centre of mass of the pendulum.



A compound pendulum

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Forced oscillations, resonance

And

Damping of oscillations



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Forced oscillations and resonance

free oscillation

- when a body is displaced from its equilibrium position and allowed to oscillate without any external forces acting on it.
- The oscillator then oscillates at its natural frequency.

Forced or driven oscillations

- Forced or driven oscillations occur when a periodic driving force acts on an oscillator.
- This will make the oscillator oscillate at the frequency of the periodic driving force rather than at its natural frequency.

- As long as the frequency of the periodic driving force is not the same as the oscillator's natural frequency, the amplitude of the oscillations is usually relatively small.

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Resonance

- If the frequency of the periodic driving force is the same as the oscillator's natural frequency, energy is transferred easily into the oscillation and the amplitude of the oscillation becomes large, sometimes very large.
- resonance the tendency of a system to oscillate with larger amplitudes when the frequency of the periodic driving force is the same as the natural frequency of the oscillator
- Resonance occurs when:

$$f = f_o$$

where

f = *driving frequency*

f_o = *natural frequency of the system.*

- The natural frequency of the oscillator is often referred to as the resonant frequency.

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Resonance…

- **Resonance occurs in many man-made machines and structures.**

  Forexample- car engines, or unbalanced wheels will create a periodic driving force affecting the whole car, the frequency of this driving force will increase with the car's speed.

  If this frequency becomes the same as the natural frequency of some part of the car that can oscillate, then that oscillation can become very large, and can be the cause of annoying rattles that occur at specific car speeds.

- **If resonance does occur, the large amplitude oscillations can cause damage.**
- **Bridges can collapse or at least oscillate violently, driven by wind or the regular pace of people walking across them.**
- **Troops of marching soldiers often stop marching and walk across bridges out of step to avoid causing this.**

One way to reduce the effects of resonance and its potentially damaging effects is to design machines so that, if they do oscillate, the natural frequency and the frequency of any periodic driving force are never the same.

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Damping of oscillations

- Another way to reduce oscillations is to introduce damping forces.
- damping forces resistive forces that oppose the motion of an oscillator by acting in the opposite direction to its velocity
- The damping force is given by:

$$F_d = -bv$$

where

b = the damping coefficient and is dependent on the medium providing the damping

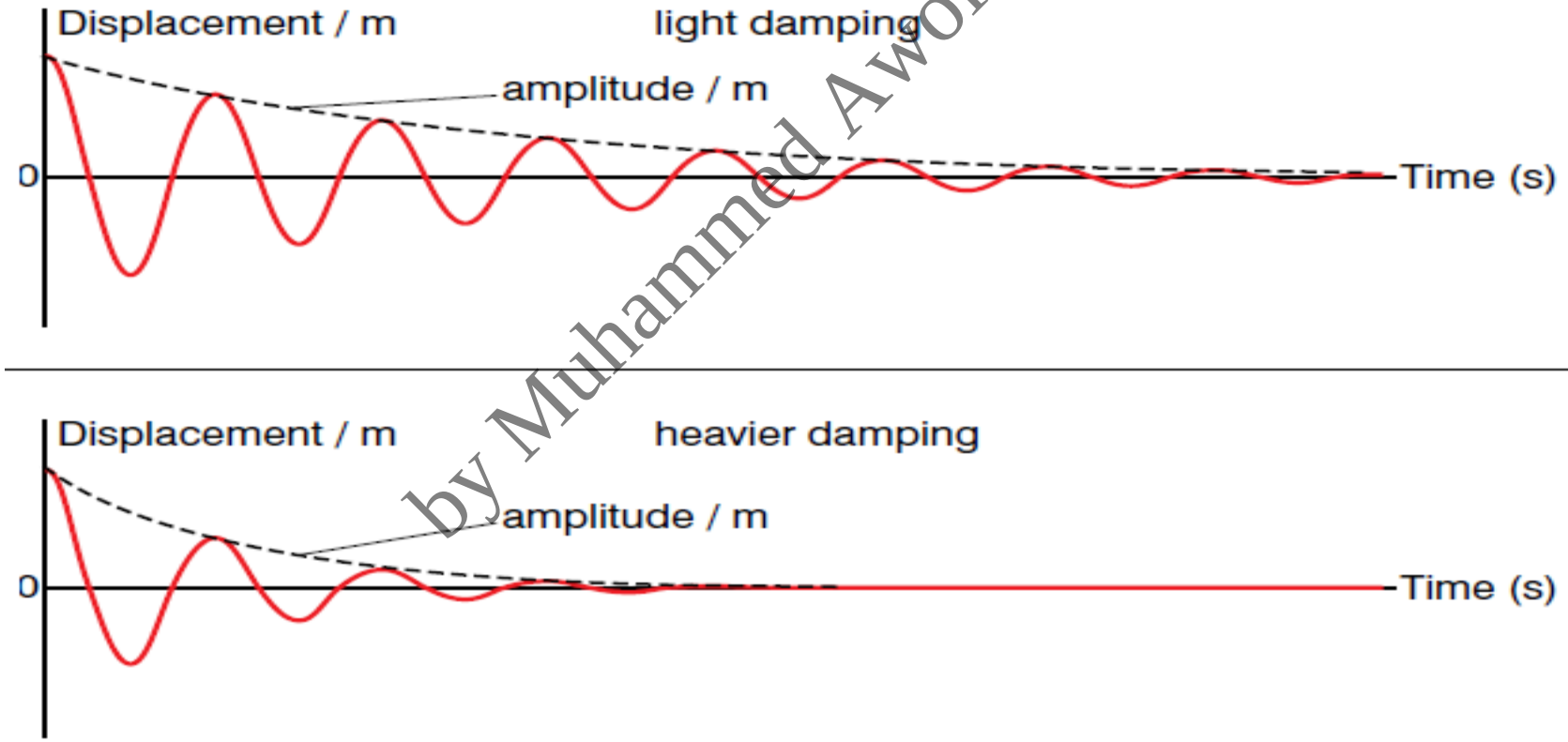
v = the velocity of the object through the medium.

- This equations shows how the resistive force is directly proportional, but opposite, to the velocity.

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Under damping

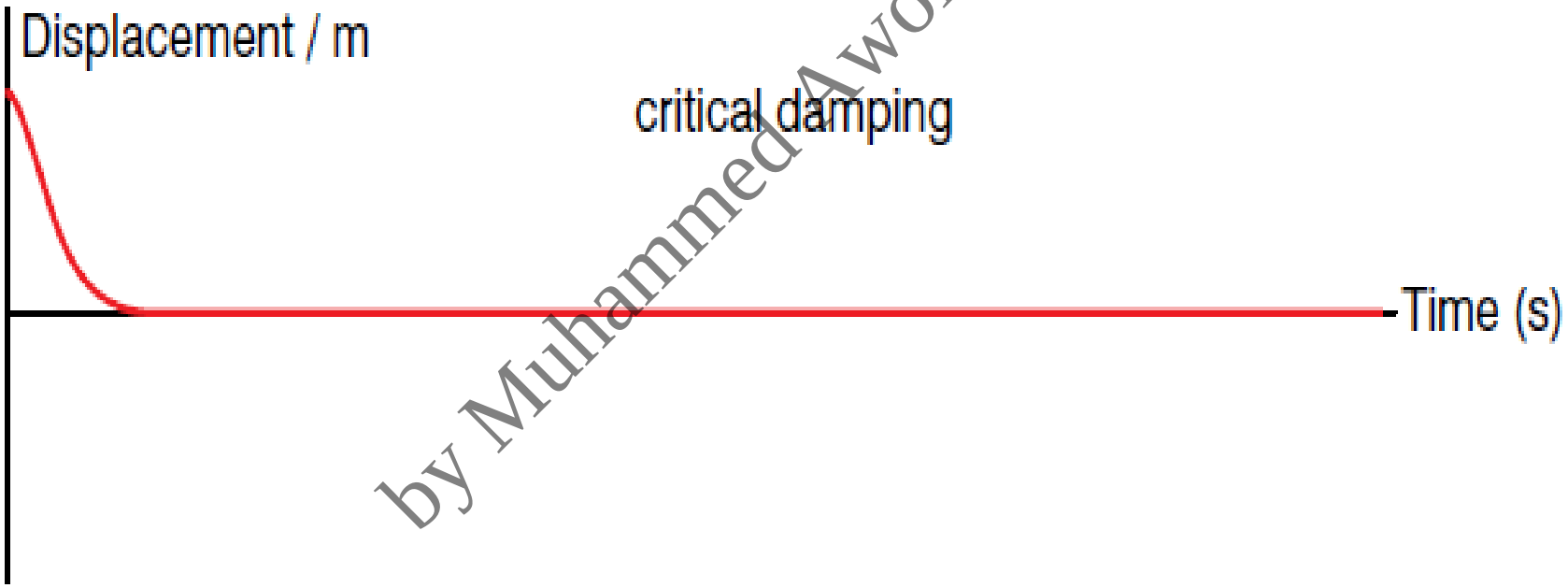
The condition in which damping of an oscillator causes it to return to equilibrium with the amplitude gradually decreasing to zero; system returns to equilibrium faster but overshoots and crosses the equilibrium position one or more times.



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Critical damping

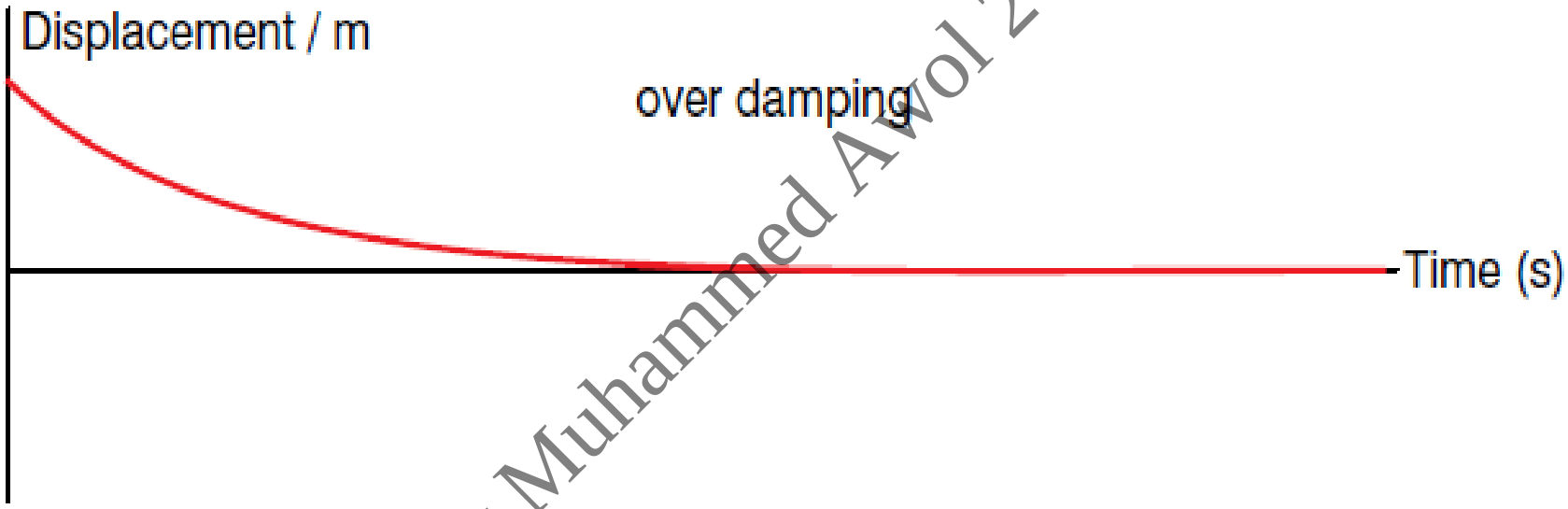
The condition in which the damping of an oscillator causes it to return as quickly as possible to its equilibrium position without oscillating back and forth about this position.



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Over damping

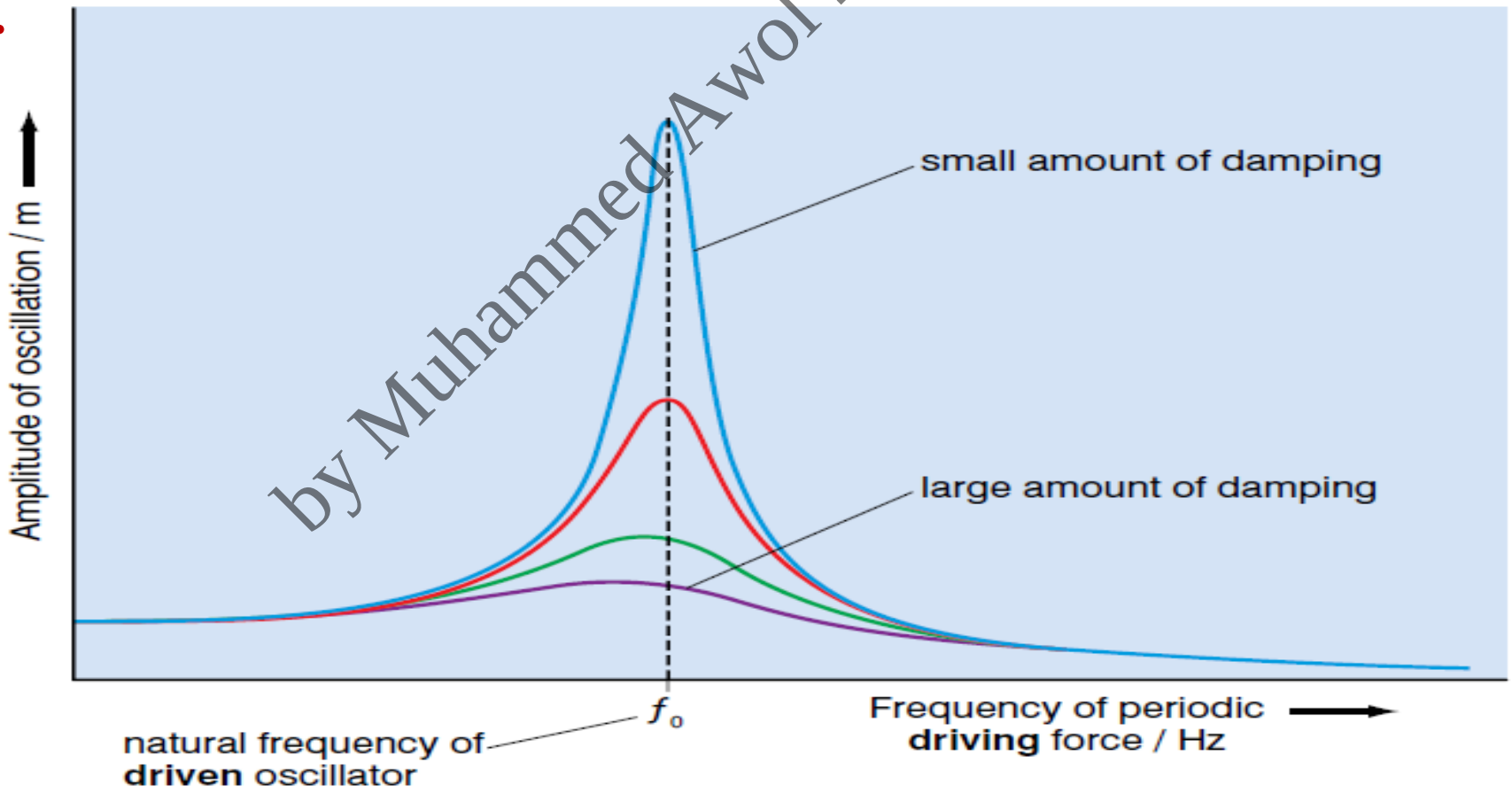
The condition in which damping of an oscillator causes it to return to equilibrium without oscillating; oscillator moves more slowly toward equilibrium than in the critically damped system.



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- **Damping reduces the effects of resonance.**
- **Damping can also be seen that damping also causes a very small reduction in the natural frequency of the oscillator.**

Driven amplitude against driving frequency for forced oscillations of an oscillator with different amounts of damping is shown in the fig. below.



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Energy and the Simple Harmonic Oscillator

- To study the energy of a simple harmonic oscillator, we first consider all the forms of energy it can have We know from Hooke's Law: that the energy stored in the deformation of a simple harmonic oscillator is a form of potential energy given by:

$$PE_{el} = \frac{1}{2}kx^2$$

- Because a simple harmonic oscillator has no dissipative forces, the other important form of energy is kinetic energy KE.
- Conservation of energy for these two forms is:

$$KE + PE_{el} = \text{constant}$$

OR

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}$$

- This statement of conservation of energy is valid for all simple harmonic oscillators.

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- for a simple pendulum we replace the
  velocity with $v = L\omega$
  spring constant with $k = mg / L$
  and the displacement term with $x = L\theta$

Thus

$$\frac{1}{2}mL^2\omega^2 + \frac{1}{2}mgL\theta^2 = \text{constant}$$

- In the case of undamped simple harmonic motion, the energy oscillates back and forth between kinetic and potential, going completely from one to the other as the system oscillates.

- This total energy is equal to the *KE* when $x = 0$ and $v = v_0$, or
- This total energy is equal to the *potential energy* when $x = A$.

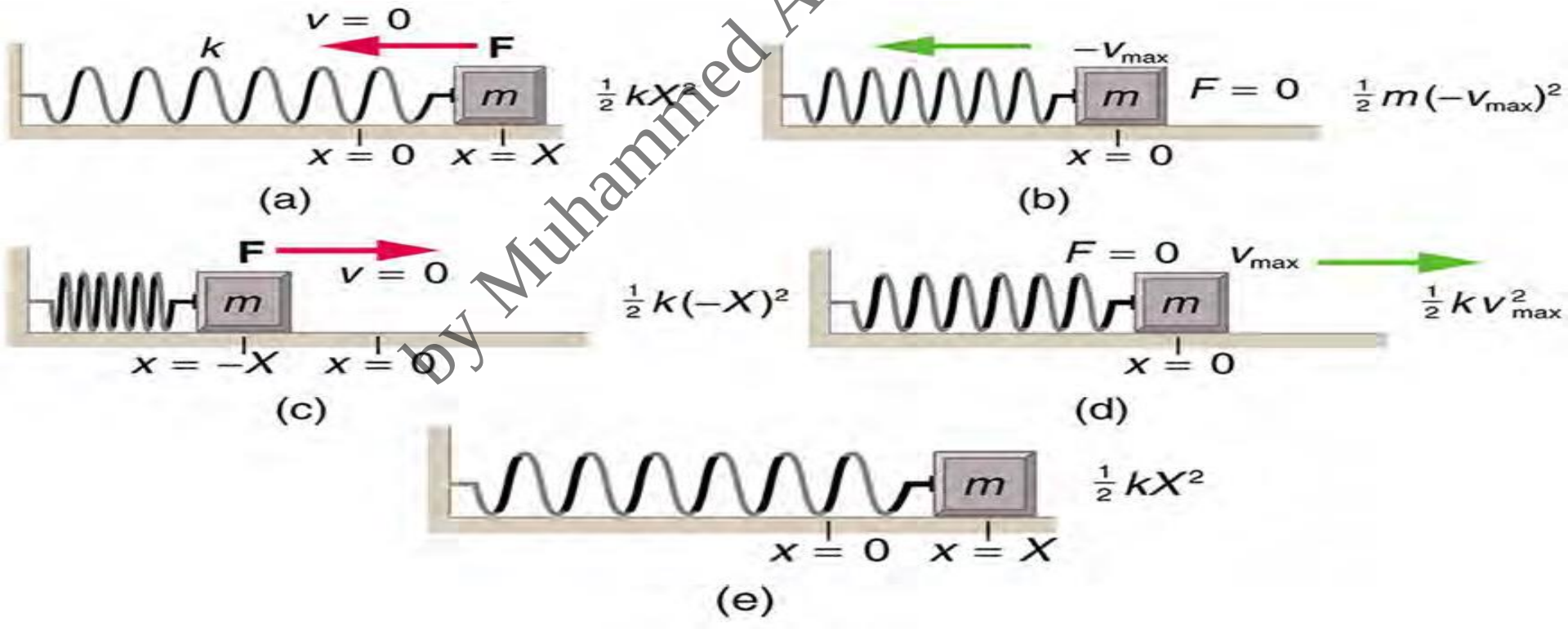
$$\text{Total energy} = \frac{1}{2}mv_0^2 = \frac{1}{2}kA^2$$

$$\text{Total energy} = \frac{1}{2}m\omega^2A^2$$

Note that the total energy of an oscillator is proportional to the amplitude squared.

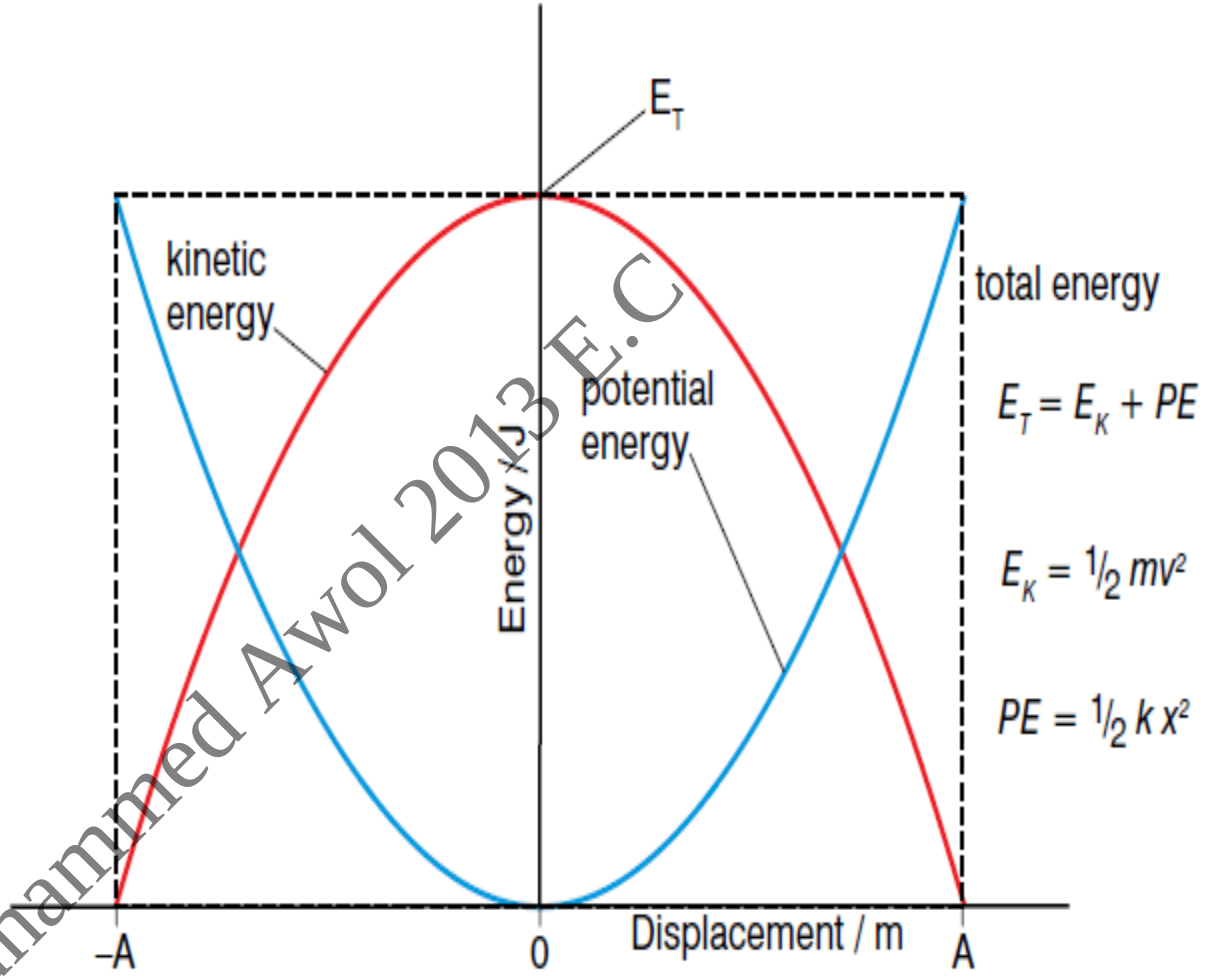
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- So for the simple example of an object on a frictionless surface attached to a spring, as shown again in Figure below, the motion starts with all of the energy stored in the spring.
- As the object starts to move, the elastic potential energy is converted to kinetic energy, becoming entirely kinetic energy at the equilibrium position. It is then converted back into elastic potential energy by the spring, the velocity becomes zero when the kinetic energy is completely converted, and so on.



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The variation of Ek and PE with displacement is shown in Figure.



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- **We can show that these expressions for energy are consistent with our earlier expression for the value of velocity v for any given displacement x.**

Starting with our equation for velocity $v = \omega\sqrt{A^2 - x^2}$

Making v^2 the subject $v^2 = \omega^2(A^2 - x^2)$

Multiplying out the brackets $v^2 = \omega^2 A^2 - \omega^2 x^2$

As this is a mass–spring system we can substitute in *k/m for ω^2*, giving $v^2 = \frac{k}{m}A^2 - \frac{k}{m}x^2$

Substituting v^2 into our Ek equations gives $\frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

Therefore $E_k = total\ energy - PE$

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Worked example 2.3

A block of mass 2.2 kg is attached to a spring with a spring constant of 40 N/m. It is pulled down a distance of 30 cm. Find the blocks kinetic energy as it passes through the equilibrium and determine its velocity at this point.

- $\frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

Using the relationship above, but is this case as the block is passing through its equilibrium position x = 0 so the relationship simplifies to:

- $\frac{1}{2}mv^2 = \frac{1}{2}kA^2$

Substitute in known values and solve, giving:

- $E_k = ½kA^2 = ½ \times 40 \times 0.3^2 = 1.8$ J

The velocity can then easily be determined using the equation for kinetic energy:

- $E_k = ½mv^2$
- $v = \sqrt{2E_K/m}$ *Rearrange to make v the subject*
- $v = \sqrt{((2 \times 1.8) / 2.2)}$ *Substitute known values*
- $v = 1.3$ m/s *Solve and give the unit*

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Superposition and Interference



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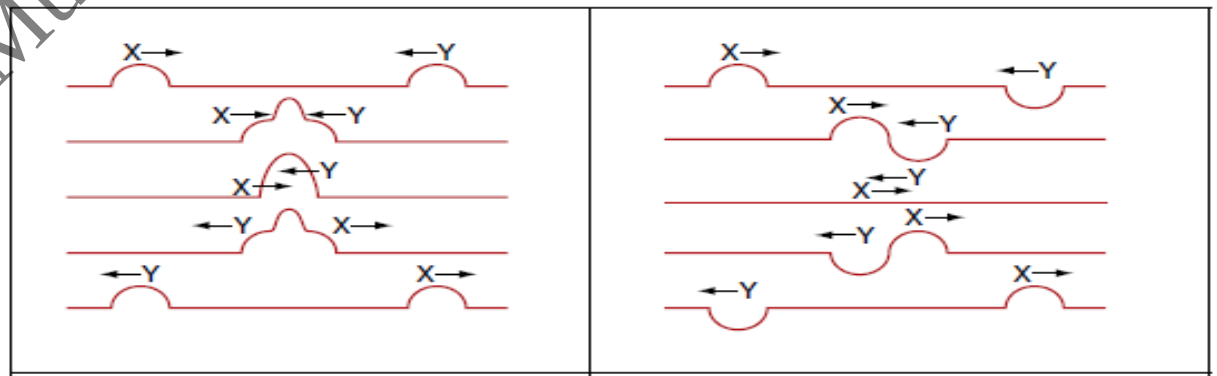
SUPER POSITION

- When two or more waves arrive at the same point, they superimpose themselves on one another.
- More specifically, the disturbances of waves are superimposed when they come together—a phenomenon called superposition.

principle of super position

- when a particle of medium comes under the influence of two or more waves simultaneously, its net displacement is the vector sum of displacement that could occur under the influence of the individual waves.
- The waves pass on through the point and each other and continue on unaffected.

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(a) Pulses of the same sign crossing.

(b) Pulses of opposite sign crossing

CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE

- **Interference is another word for superposition.**
- **When two waves arrive at the same point they interfere with each other.**
- **The instantaneous displacement at that point is the sum of the displacements that would be created separately by each wave.**
- **If we consider just two waves, the result is an oscillation whose amplitude depends on the relative phase of the two waves.**

PHASE is a measurement of the position of a point on a wave after a particular time.

PHASE DIFFERENCE is the angular difference in timing between 2 waves

IN PHASE –Two waves are said to be in phase when the crusts of the two waves are precisely aligned, as are the trough. or The phase difference between the two wave is zero degree.

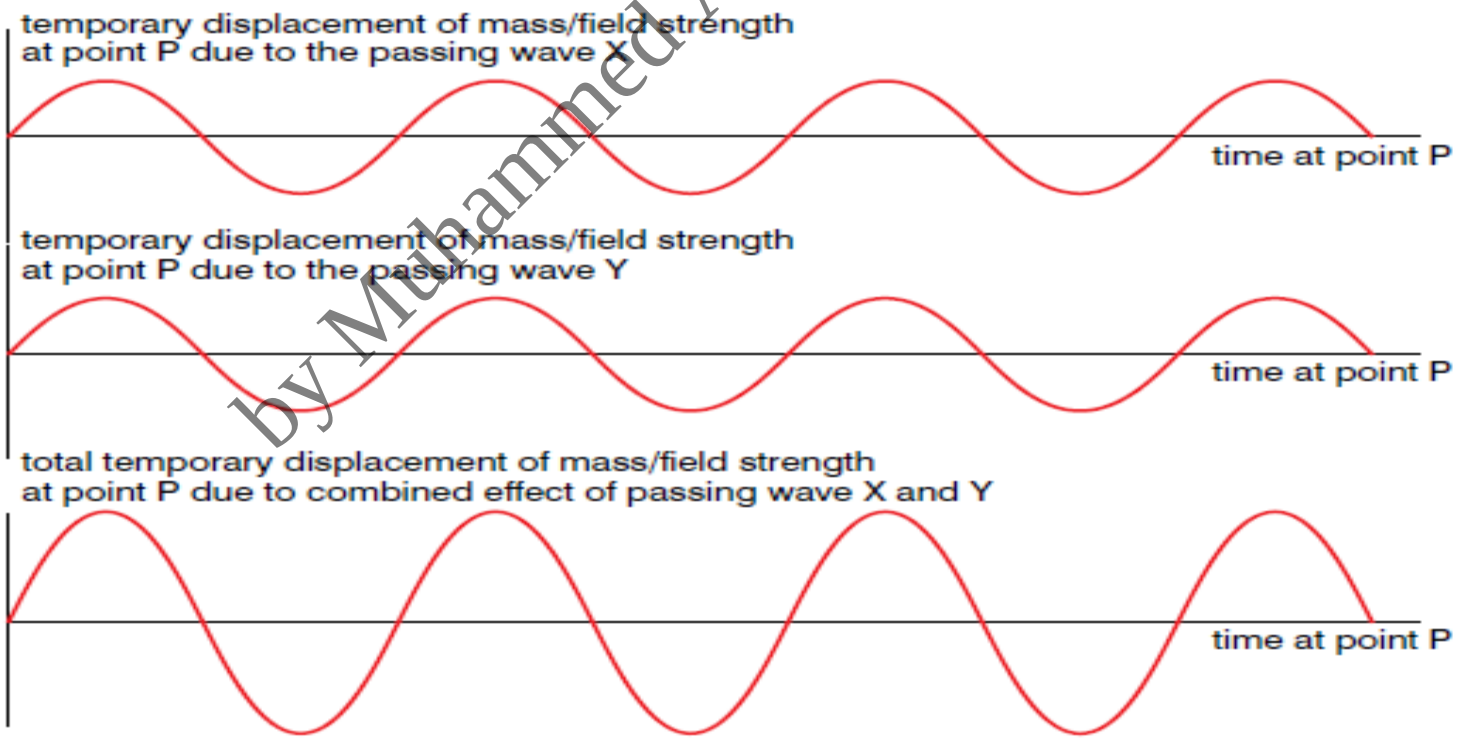
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CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE…

ANTI-PHASE –Two waves are said to be in anti phase when the two waves are precisely aligned crust to trough. or The phase difference between the two wave is 180 degree.

CONSTRUCTIVE INTERFERENCE

- **The production of large oscillations by the superposition of two waves that are in phase with each other.**
- **Giving large amplitude oscillation.**

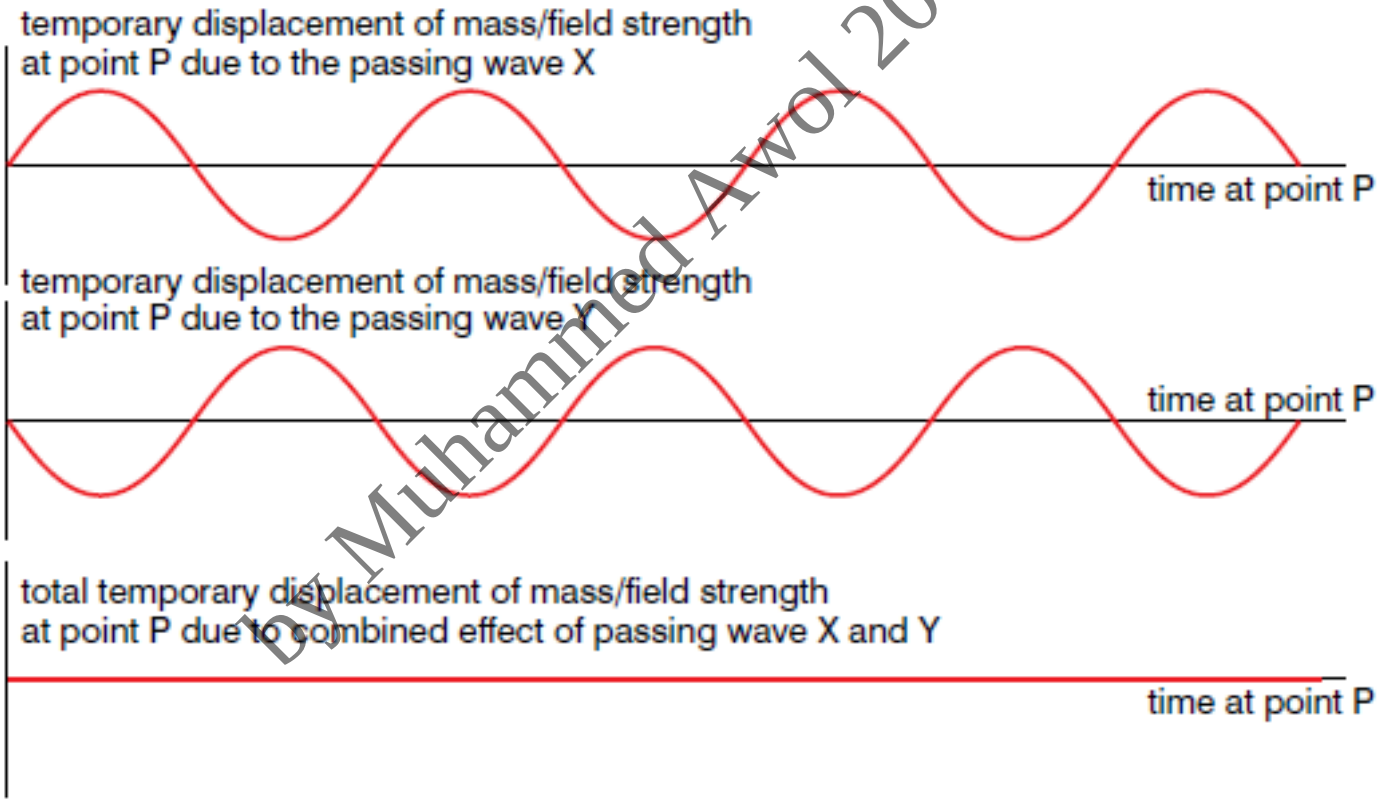


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CONSTRUCTIVE AND DESTRUCTIVE INTERFERENCE…

DESTRUCTIVE INTERFERENCE

- **The cancelling out of oscillations caused by the superposition of two waves that are in Anti-phase.**
- **Giving small or zero resultant oscillations.**

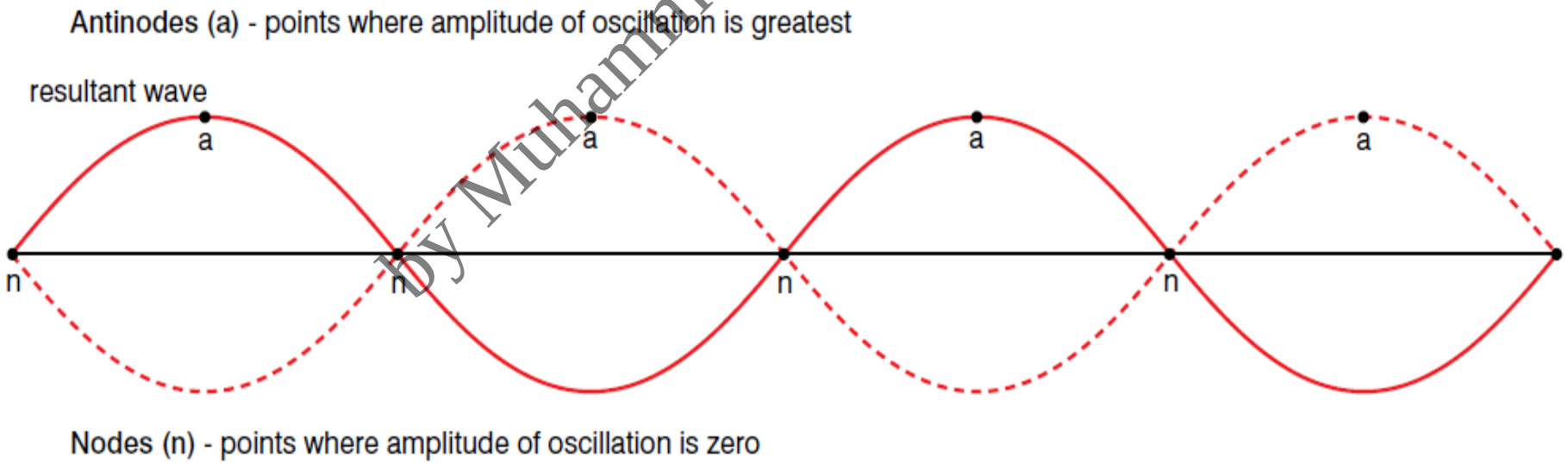


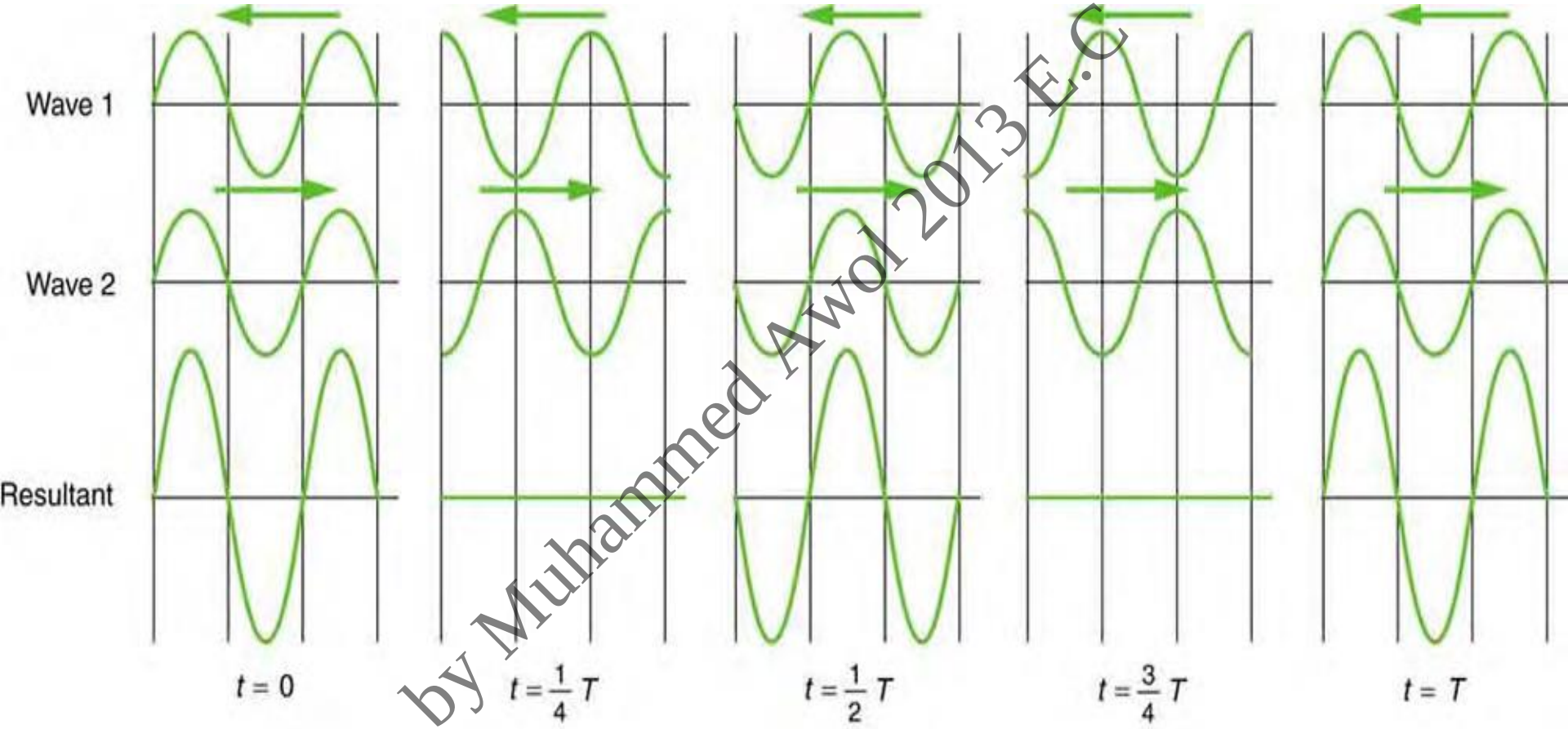
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Standing waves on strings

- **When two waves having same amplitude and frequency and travelling in mutually opposite directions are superimposed the resultant wave formed loss the property of propagation, such a wave is called standing or stationary waves.**
- **Standing waves are also found on the string (musical instrument) and are due to reflections of waves from the end of the string as shown in figure below.**

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Nodes and antinodes

Nodes

- The points where the string does not move.
- More generally, nodes are where the wave disturbance is zero in a standing wave.

Antinodes

- Used to denote The location of maximum amplitude in standing wave

- The location of nodes and antinodes are very important.

- ✓Nodes occur when the distance along the string is equal to $n\lambda / 2$

- ✓Antinodes occur when the distance along the string is equal to $(n+1/2)\ \lambda / 2$.

Where n = an integer number 0, 1, 2, 3, etc.

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The mathematics of standing waves

Two travelling waves moving in opposite directions can be represented by the equations below:

$$y_1 = y_0 \sin(kx - \omega t)$$ **And** $$y_2 = y_0 \sin(kx + \omega t)$$

where

y_0 **is the amplitude of the wave,**
ω is the angular frequency measured in radians per second.
k is equal to $2\pi / \lambda$
x and t are variables for position and time, respectively.

So the resultant wave y equation will be the sum of y_1 and y_2:

$$y = y_0 \sin(kx - \omega t) + y_2 = y_0 \sin(kx + \omega t)$$

We can use a trigonometric identity to simplify this to:

$$y = 2y_0 \cos(\omega t) \sin(kx)$$

This equation shows not only that the wave oscillates in time, but has also these oscillations vary in the *x direction*.

Several maxima occur at x = λ/4, 3λ/4, 5λ/4, these are the antinodes.
Where as at x = 0, λ/2, λ, 3λ/2, the function is zero and so the amplitude is always zero – these are the nodes.

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Wavelength and the length of string

We get stationary waves if the length of the string is a whole number of half wavelengths, i.e. we get stationary waves if

$$n = \frac{2L}{\lambda}$$

where L is the length of the string and n is an integer, or when

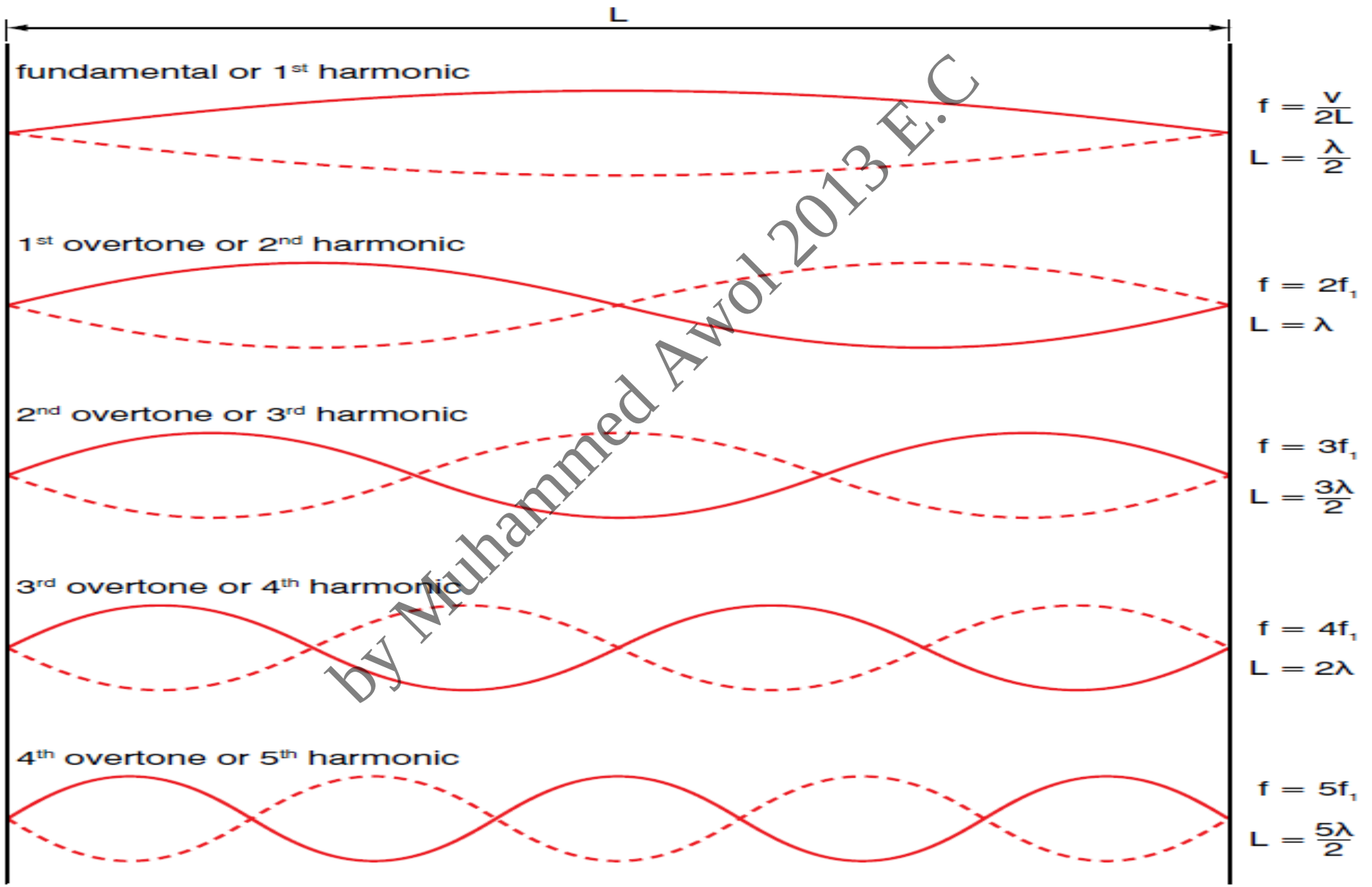
$$\lambda = \frac{2L}{n}$$

Using velocity $v = f\lambda$ we can show that we get standing waves on the string when

$$\frac{v}{f} = \frac{2L}{n} \qquad \textbf{Or} \qquad f = n\frac{v}{2L}$$

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The standing waves that can be formed on a string fixed at both ends for values of n from 1 to 5 shows on figure below.



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- **The lowest frequency of oscillation, when $\lambda = 2L$ and there is just one antinode is known as the fundamental frequency, f_1.**
- **All the other possible oscillation frequencies are integer multiples of this fundamental frequency and are known as harmonics or overtones.(the 2nd harmonics is the first overtone)**

harmonics - are standing waves for which frequencies are integer multiples of the fundamental frequency.

- **The first harmonic is the fundamental frequency.**
- **The harmonic number is the same as the number of antinodes.**
- **A string does not necessarily oscillate at only one of these frequencies; it can oscillate at several different harmonic frequencies at the same time.**
- **The fundamental frequency of a string is clearly determined by its length, but it also depends on the velocity at which travelling waves travel along the string.**
- **As we have already shown this velocity is given by**

$$v = \sqrt{\frac{T}{\mu}}$$

where *T is the tension in the string and μ is the mass per unit* length.

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The fundamental frequency of a string is given by

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

So we can make a string produce a higher note if we make it

- ✓ **Shorter**
- ✓ **Increase the tension or**
- ✓ **replace it with a lighter one.**

These are the parameters that affect the fundamental frequency that the string in a musical instrument produces.

- **These frequencies of oscillation can be thought of as resonant frequencies.**
- **A key difference from oscillators like pendulums and mass-spring systems is that while those oscillators have just one resonant frequency.**
- **A string has multiple resonant frequencies: its fundamental and harmonic frequencies.**

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EXAMPLE

What frequencies can a string vibrate at if it is 30 cm long and the velocity of travelling waves along the string is 120 m/s?

ANSWER

The fundamental frequency is given by:

$$f_1 = n\,\frac{v}{2L}$$

$$f_1 = \frac{120}{2 \times 0.3}$$

$$f_1 = 200 \text{ Hz}$$

Therefore the string can vibrate at 200 Hz, 400 Hz, 600 Hz, 800 Hz, 1000 Hz, 1200 Hz, etc.

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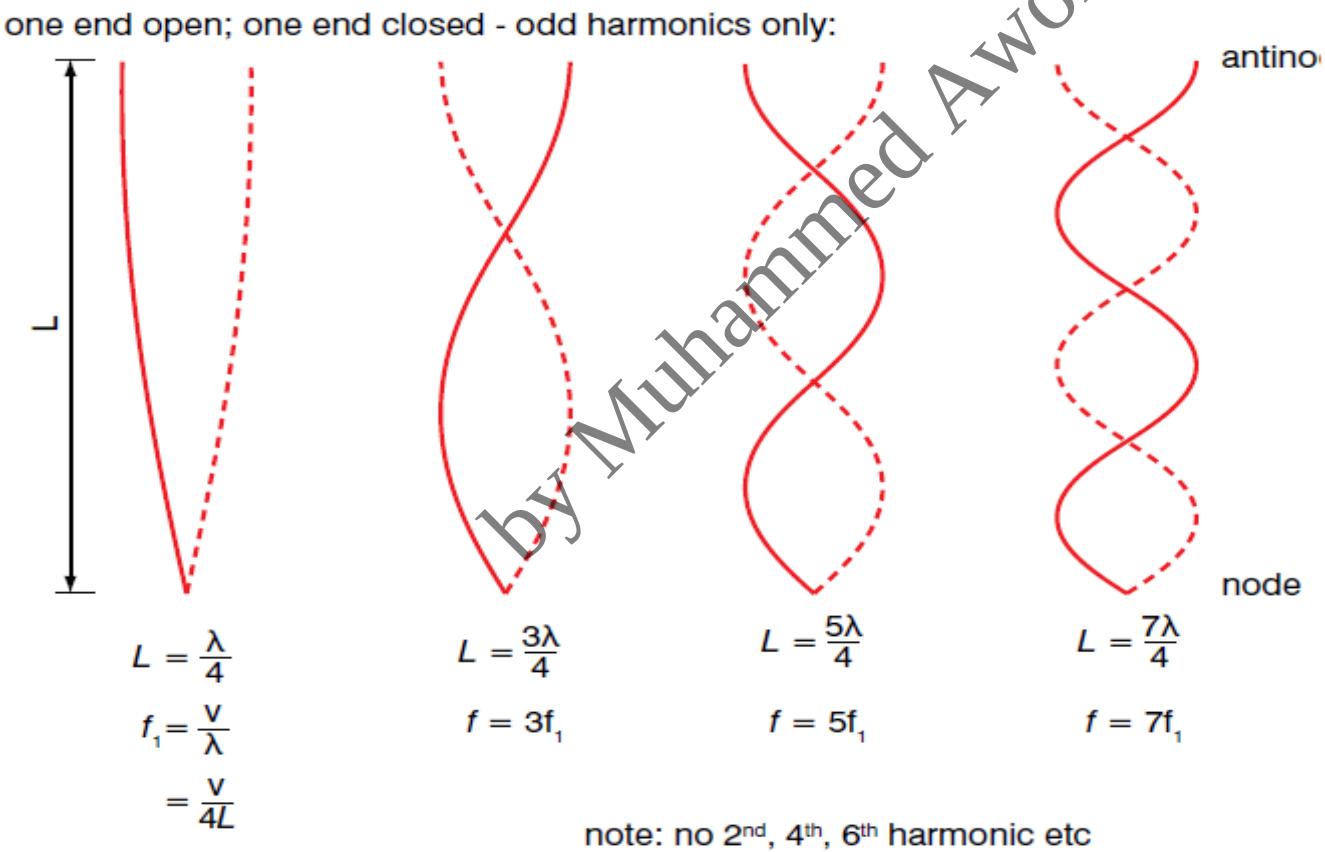
Standing waves in organ pipes

- Wind instruments generate standing waves in the column of air inside them.
- A full analysis of the vibration of the air inside a wind instrument is much more complicated than for a string.
- The vibrating body of air in wind instruments can be varied and complex in shape, but we can arrive at some very good approximations if we stick to straight narrow pipes.
- A simple pipe will need to be open at one end, but could be open or closed at the other.
- Where the end of the tube is closed there is no net oscillation and the sound wave is reflected with a phase shift of π radians.
- There will always be a node at a closed end.
- Where the end of the tube is open there is a large net oscillation and the sound wave is reflected with no phase shift.
- There will always be an antinode at the open end.
- Pipes with one end closed behave differently from those with both ends open.

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one end of the pipe is closed, and the other open

- If one end of the pipe is closed, and the other open, we will get a node at one end and an antinode at the other.
- This creates two differences from standing waves on a string.
  - ✓ The fundamental occurs when the length of the pipe is just a quarter of a wavelength, and
  - ✓ only odd harmonics occur, as shown in Figure below:



For the nth harmonic:

$$f_n = nv/4L$$

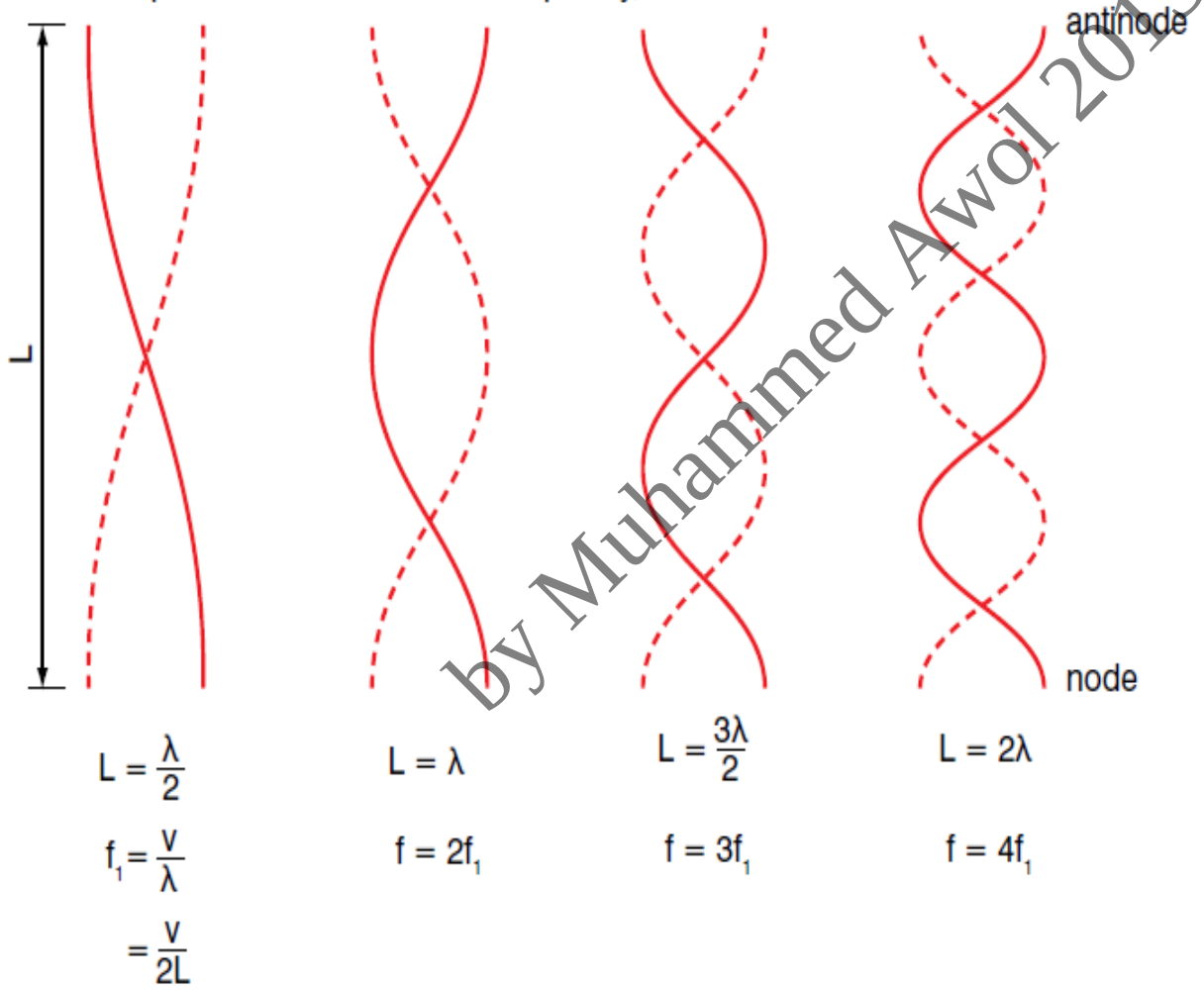
where $n = 1, 3, 5, \ldots$

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both ends of the pipe are open

- If both ends are open, then we get antinodes at both ends.
- Apart from having antinodes at the ends rather than nodes, this produces the same rules for frequencies and harmonics as a string.

both ends open - lower fundamental frequency, but harmonics:



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For the nth harmonic:

$$f_n = nv/2L$$

where $n = 1, 3, 5, \ldots$

EXAMPLE

A narrow pipe is 20 cm long and is open at the top and closed at the bottom. Given that the speed of sound is 340 m/s, what frequency sounds might it be possible to produce by blowing across the top?

ANSWER

The wavelength at the fundamental frequency is four times the length of the pipe and therefore

$$\lambda = 0.2\text{m} \times 4 = 0.8\text{ m}$$

This corresponds to frequency of

$$f = \frac{v}{\lambda} = \frac{340}{0.8} = 425\text{ Hz}$$

And therefore it might be possible to produce sounds at odd multiples of 425 Hz,
i.e. at 425 Hz, 1275 Hz,2125 Hz, etc.

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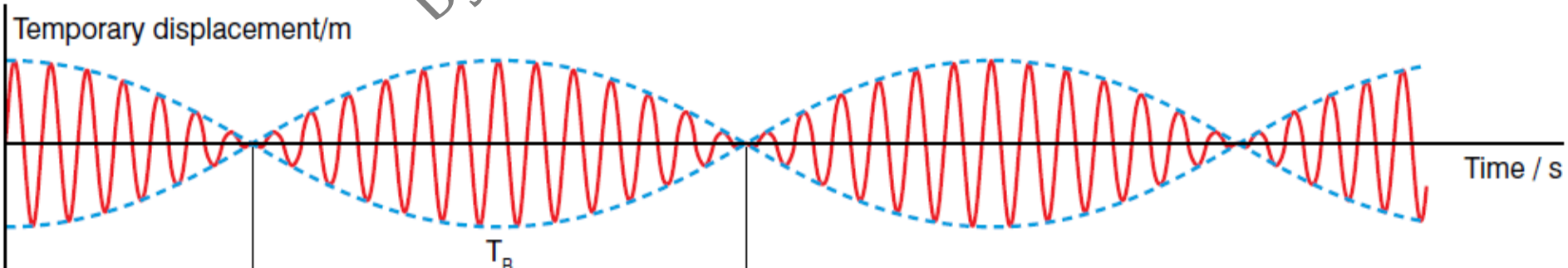
Beats

- If two notes, differing in frequency by a few hertz are played at similar amplitudes, the phenomenon of beats can be heard.
- The resulting sound will be at the average of the two frequencies, but it will get louder and quieter at a frequency equal to the difference between the two frequencies.
- This can be predicted mathematically. If we add two sine waves at frequencies $f + \Delta f$ and $f - \Delta f$, we obtain:

$$\sin(2\pi(f - \Delta f)t) + \sin(2\pi(f + \Delta f)t) = 2\cos(2\pi\Delta f\, t)\sin(2\pi f t)$$

- which describes a sine wave, $\sin(2\pi f t)$, whose slowly varying amplitude is given by $2\cos(2\pi\Delta f\, t)$, where Δf is half of the frequency difference between the two sine waves.
- The result is shown in Figure

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Beats…

- The time between nulls, instants of no sound, is half of the period of $\cos(2\pi\Delta f\, t)$, and is hence the time period corresponding to the frequency difference.
- In other words, if we play two notes at frequencies f_1 and f_2 the resulting sound has a beat frequency of:

$$f_B = |f_2 - f_1| = \frac{1}{T_B}$$

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HOMEWORK

Review questions on page 96 and 97 on your text book question number 1,2,5,7 and 8

HOMEWORK QUASTIONS

1. If the speed of sound in air is 340 m/s, what is the wavelength of a sound wave at 512 Hz?

2. What is the frequency of red light with a wavelength (in free space) of 630 nm?

5. A violin string is 32 cm long and has a mass per unit length of 2 g/m. What tension is required for the string to produce an A, i.e. for its fundamental frequency to be 440 Hz?

7. A pipe, 68 cm long, is open at one end and closed at the other. When air is blown across the open end sound is produced at 110 Hz.

   a) What is the velocity of sound along the pipe?

   b) Blowing harder will produce a higher note. What is the next frequency that the pipe can produce?

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8. In an experiment to measure the speed of sound in air, a speaker directs sound towards a wall, along a path perpendicular to the wall. The wall reflects the sound wave and this produces a standing wave. A microphone and electronic measuring device is used to measure the amplitude of the sound at different distances from the wall. Minimum values of amplitude are detected at 28 cm when the frequency used is 600 Hz.

a) What is the measured speed of sound?

b) Explain why the minimum values of sound are not zero and why they are more difficult to detect further from the wall.

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2.3 Sound, loudness and the human ear

By the end of this section you should be able to:

- Define the intensity of sound and state the relationship between intensity and distance from the source.
- Describe the dependence of the speed of sound on the bulk modulus and density of the medium. Use $v = \sqrt{B/\rho}$
- Give intensity of sound in decibels, and define the terms threshold of pain and threshold of hearing.
- Describe the intensity level versus frequency graph to know which the human ear is most sensitive to.
- Analyse resonance conditions in air columns in quantitative terms.
- Explain the Doppler effect, and predict in qualitative terms the frequency change that will occur in a variety of conditions.
- Describe some practical applications of the Doppler Effect.

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Sound loudness and intensity

- Sound can be used as a familiar illustration of waves. Because hearing is one of our most important senses, it is interesting to see how the physical properties of sound correspond to our perceptions of it.
- Hearing is the perception of sound, just as vision is the perception of visible light. But sound has important applications beyond hearing.
- ✓Ultrasound, for example, is not heard but can be employed to form medical images and is also used in treatment.
- The physical phenomenon of sound is defined to be a disturbance of matter that is transmitted from its source outward. Sound is a wave.
- On the atomic scale, it is a disturbance of atoms that is far more ordered than their thermal motions.
- In many instances, sound is a periodic wave, and the atoms undergo simple harmonic motion. In this text, we shall explore such periodic sound waves.

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loudness the audible strength of a sound, which depends on the amplitude of the sound wave.

Intensity the energy received by each square metre of a surface per second. or power received by a square metre of a surface.

- We know that the energy in an oscillation is equal to the kinetic energy at the equilibrium position

$$= \frac{1}{2} m v_0^2 = \frac{1}{2} m \omega^2 A^2$$

- The mass of air with a cross section of $1m^2$ disturbed by a plane sound wave each second is the density of air ρ times the volume disturbed, and
- The volume disturbed is $1\ m^2$ time the distance travelled by the wave each second (*its velocity*).
- Hence, the intensity of a sound wave is given by $I = \frac{1}{2} \rho v \omega^2 A^2$

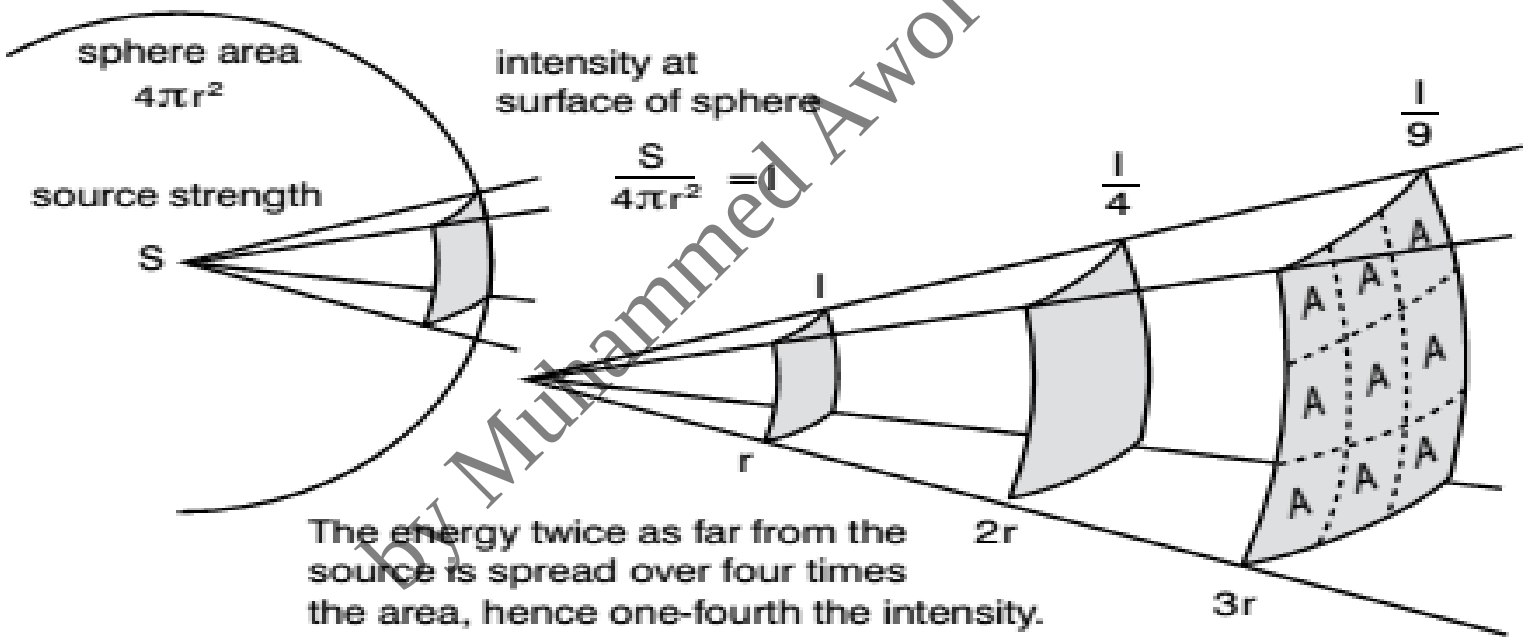
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- ✓ where ρ is the density of the air, v is the velocity of sound, $\omega = 2\pi f$ where f is the frequency and A is the amplitude of oscillation.
- ✓ Hence we see that intensity, measured in W/m^2, is proportional to amplitude squared and to frequency squared.

- **If we assume the sound travels out equally in all directions then the area covered is equal to the surface area of a sphere.**
- **So the equation becomes:**

$$I = \frac{P}{A} = \frac{P}{4\pi r^2}$$

where r is the distance from the source in m.



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The figure shows the intensity from the source varies as an inverse square relationship.

Example

A large explosion is detected 200 m away. The intensity at this distance is measured to be 400 MW/m^2.
A few seconds later the explosion is recorded 3.2 km away.
Find the power of the original explosion and the intensity recorded at the larger distance.

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Answer

$I = \frac{P}{A} = \frac{P}{4\pi r^2}$	*State the relationship to be used*
$P = IA = I \times 4\pi r^2$	*Rearrange to make P the subject*
$P = 400 \times 10^6 \times 4 \times \pi \times 200^2$	*Substitute known values*
$P = 2.0 \times 10^{14}$ W	*Solve for P and give the units*

We can now use the intensity equation to determine the intensity at 3.2km.

$I = \frac{P}{A} = \frac{P}{4\pi r^2}$	*State the relationship to be used*
$I = 2.0 \times 10^{14} / (4 \times \pi \times 3200^2)$	*Substitute known values*
$I = 1.6 \times 10^6$ W/m^2	*Solve for* I *and give the units*

by Muhammed Awol 2012 E.C