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QUESTION 1

Drag and drop the correct answer into the space(s) provided.

A town with a population of 1000 people has an outbreak of the flu.

The growth model of the people who catch this illness is modeled by

$$f(t) = \frac{c}{1 + a\,e^{-bt}}$$

, where

$$\frac{c}{1 + a}$$

is the initial value (when

$t = 0$

), c is the carrying capacity or limiting value, b is a constant determined by the growth of the virus and t is the time in days.

For this particular flu,

$b = 0.406$

, and there is one case at time

$t = 0$

.

Using this model, estimate the number of people who will have the flu after two weeks, and the rate at which that number is changing.

Knowing the limiting value of

$c = 1000$

, since there are only 1000 people in the town, find the value of a. Use

$t = 0$

,

$f(0) = 1$

.

$$f(0) = \frac{1000}{1+a} = 1$$

Solve for a.

$a =$

999 ✓

$$f(t) = \frac{1000}{1 + 999\, e^{-0.406\, t}}$$

At day 14:

$f(14) =$

$$\frac{1000}{1 + 999\, e^{-0.406\,(14)}} \checkmark$$

$f(14) =$

227 ✓ people

The rate of growth is the derivative of the function.

$$f'(t) = \frac{d}{dt}\left(\frac{1000}{1 + 999\, e^{-0.406\, t}}\right)$$

$f'(t) =$

$$\frac{d}{dt}\left[1000\left(1 + 999\, e^{-0.406\, t}\right)^{-1}\right] \checkmark$$

$f'(t) =$

$$\frac{-1000}{\left(1 + 999\, e^{-0.406\, t}\right)^2} \frac{d}{dt}\left(999\, e^{-0.406\, t}\right) \checkmark$$

$f'(t) =$

$$\frac{405594\ e^{-0.406\,t}}{\left(1+999\ e^{-0.406\,t}\right)^2}$$ ✓

At day 14:

$$f'(14)=\frac{405594\ e^{-0.406\,(14)}}{\left(1+999\ e^{-0.406\,(14)}\right)^2}$$

$$f'(14)=$$

71 ✓ people per day.

$\frac{d}{dt}\left[1000\left(1+999\ e^{-0.406\,t}\right)^2\right]$

$\frac{406\ e^{-0.406\,t}}{\left(1+999\ e^{-0.406\,t}\right)^2}$

439

$\frac{1000}{\left(1+999\ e^{-0.406\,t}\right)^2}\ \frac{d}{dt}\left(999\ e^{-0.406\,t}\right)$

$\frac{1000}{1+2a}$

$\frac{1000}{999\ e^{-0.406\,(14)}}$

85

QUESTION 2

Drag and drop the correct answer into the space(s) provided.

The concentration of a chemical substance at time t in a reaction can be found using the concentration formula,

$$c(t) = \frac{15}{9e^{-10t} + 3}$$

.

Using the formula, show that the derivative is always greater than 0 and that the concentration never exceeds 5.

Find the derivative of the concentration formula.

$c'(t) =$

$$\frac{d}{dt}\left[15(9e^{-10t} + 3)^{-1}\right]$$

Rewrite in exponential form.

$c'(t) =$

$$-15(9e^{-10t} + 3)^{-2}\frac{d}{dt}(9e^{-10t} + 3)$$ ✓

$c'(t) =$

$$-15(9e^{-10t} + 3)^{-2}(9e^{-10t})(-10)$$ ✓

$c'(t) =$

$$\frac{1350\ e^{-10t}}{(9e^{-10t} + 3)^2}$$ ✓

Find at $t = 0$

.

$c'(0) =$

$$\frac{1350\ e^{-10(0)}}{(9e^{-10(0)} + 3)^2}$$ ✓

$c'(0) =$

9.375

for

$t > 0$

To find the limiting value of the formula, take the limit of the concentration formula:

$$\lim_{t \to \infty} c(t) = \lim_{t \to \infty} \frac{15}{9e^{-10t} + 3}$$

Substitute in

$t = \infty$

.

$\lim_{t \to \infty} c(t) =$

$\lim_{t \to \infty} \frac{15}{9e^{-10(\infty)} + 3}$ ✓

$\lim_{t \to \infty} c(t) =$

$\frac{15}{0 + 3}$ ✓

$\lim_{t \to \infty} c(t) =$

5 ✓

The concentration limit never exceeds 5.

$-15(9e^{-10t} + 3)^{-2}(9e^{-10t})(10)$

$\lim_{t \to \infty} \frac{15}{9e^{-10(\infty)} + 5}$

3.75

$\frac{15}{1 + 3}$

$15(9e^{-10t} + 3)^{-2} \frac{d}{dt}(15e^{-10t} + 3)$

$$\frac{1350\,e^{1}}{(9\,e^{1}+3)^{2}}$$

$$\frac{1350\,e^{t}}{(9\,e^{-10\,t}+3)^{2}}$$

QUESTION 3

Fill in the blank(s) with the correct answer.

The boiling temperature T (in celsius) of water at different pressures p (in atms) is modeled by the function

$$T(p) = 82.9 + 1.64 \ln p + 17.91\sqrt{p}$$

.

Find the rate of change of the temperature when the pressure is 2 atm, rounded to two decimal places.

The rate of change of the temperature is 7.15 degrees Celsius per atm.

QUESTION 4

Choose the correct answer from the drop-down menu.

Find the derivative of

$$f(x) = (\sin x)^{x}$$

.

$y' =$

$(\sin x)^x(\ln(\sin x) + x\cot x)$ ✓

QUESTION 5

Choose the correct answer from the drop-down menu.

Find the derivative of

$f(x) = (4x)^{\sqrt{x}}$

.

$f'(x) =$

$2x\ln(4x) + 4x$ ✓

QUESTION 6

Choose the correct answer from the drop-down menu.

Find the derivative of

$y = (\sin x)^{\cos x}$

.

$y' =$

$(\sin x)^{\cos x}\ (-\sin x\ln(\sin x) + \cos x\cot x)$ ✓

QUESTION 7

Choose the correct answer from the drop-down menu.

A population of growing bacteria is growing modeled by the function

$N = N_0 e^{kt}$

, where

t

is in hours, the initial amount is

$N_0 = 1200$

, and the growth constant is

$k = 0.45$

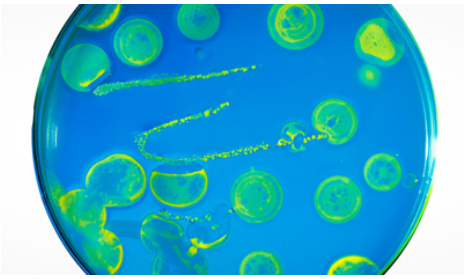
. What is the rate of growth at

$t = 3$

hours.

After 3 hours, the bacteria is growing at a rate of

2083 ✓ bacteria per hour.



QUESTION 8

Choose the correct answer from the drop-down menu.

A deposit of AED 2500 is made into an account that pays interest annually. What is the relative rate of change?

The function modeling the amount of money is

$A(t) = A_0\ 2^{0.5t}$

where

A_0

is the initial amount,

$A_0 = 2500$

.

The relative rate of change is 35% ✓ .

QUESTION 9

Select the correct option.

What is the derivative of

$y = (3x^4 + 5x)^{6x^8 + 9x}$

?

- ✓ $\frac{dy}{dx} = (3x^4 + 5x)^{6x^8 + 9x} \left[(48x^7 + 9)\ln(3x^4 + 5x) + \left(\frac{72x^{10}}{}\right.\right.$
- $\frac{dy}{dx} = (3x^4 + 5x)^{6x^8 + 9x}\ (48x^7 + 9)\ln(3x^4 + 5x) + \left(\frac{72x^{10} +}{}\right.$
- $\frac{dy}{dx} = (3x^4 + 5x)^{6x^8 + 9x}\ (48x^7 + 9)\ln(3x^4 + 5x) + (70x^{11} +$

$\frac{dx}{}$

$\frac{dy}{dx} = (3x^4 + 5x)^{6x^8 + 9x} (48x^7 + 9)\ln(x) + \left(\frac{72x^{10} + 30x^7}{3x^3}\right.$

QUESTION 10

Select the correct option.

What are the derivatives of

$y = (3x)^x$

?

$\frac{dy}{dx} = (3x)^x(\ln(3x) + 3x)$

$\frac{dy}{dx} = (\ln(3x) + 1)$

$\frac{dy}{dx} = (3x)^x(\ln(3x) + 1)$ ✓

$\frac{dy}{dx} = (3x)(\ln(3x) + 1)$

QUESTION 11

Select the correct option.

What are the derivatives of

$y = x^{\cos x}$

?

- $\frac{dy}{dx} = x^{\cos x}\left(\frac{\cos x}{x} - \ln x \cos x\right)$
- $\frac{dy}{dx} = x^{\cos x}\left(\frac{\cos x}{x} + \ln x \sin x\right)$
- $\frac{dy}{dx} = x^{\cos x}\left(\frac{\sin x}{x} - \ln x \sin x\right)$
- $\frac{dy}{dx} = x^{\cos x}\left(\frac{\cos x}{x} - \ln x \sin x\right)$ ✓

QUESTION 12

Select the correct option.

What is the derivative of

$y = (5x)^{x^2+3x}$

?

- [x] $\frac{dy}{dx} = (5x)^{x^2+3x}(\ln(5x)(2x+3)+x+3)$
- [ ] $\frac{dy}{dx} = (5x)^{x^2+3x}\ (\ln(5x)(2x+3)+3x)$
- [ ] $\frac{dy}{dx} = (5x)^{x^2+3x}\ (\ln x(2x+3)+x+3)$
- [ ] $\frac{dy}{dx} = (5x)^{x}(\ln(5x)(2x+3)+x+3)$

QUESTION 13

Select the correct option.

What is the value

y'

for the function

$y = x^{e^x}$

?

- [x] $y' = x^{e^x} e^x \left(\ln x + \frac{1}{x}\right)$ ✓
- [ ] $y' = x^{e^x} e^x (\ln x + x)$
- [ ] $y' = x^{e^x} e^x \left(\ln x + \frac{e^x}{x}\right)$
- [ ] $y' = x^{2e^x} \left(\ln x + \frac{1}{x}\right)$

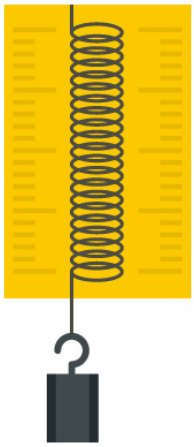
QUESTION 14

The position of spring in motion can be modeled by the function

$s(t) = 2\,e^{-t} \cos t$

.

At what time is the velocity of the spring equal to zero? Select all that apply.



- [x] $t = \frac{3\pi}{4}$
- [ ] $t = \frac{4\pi}{3}$
- [ ] $t = \frac{3\pi}{2}$
- [x] $t = \frac{7\pi}{4}$
- [x] $t = \frac{11\pi}{4}$
- [ ] $t = \frac{4\pi}{11}$

- [ ] $t = \frac{2\pi}{3}$

- [ ] $t = \frac{4\pi}{7}$

QUESTION 15

Find the derivative of

$y = (7x)^{\ln x}$

. Select all that apply.

- [x] $\frac{dy}{dx} = (7x)^{\ln x}\left(\frac{\ln(7x)}{x} + \frac{\ln x}{x}\right)$

- [ ] $\frac{dy}{dx} = (7x)^{\ln x}\left(\frac{7\ln x + \ln x}{x}\right)$

- [ ] $\frac{dy}{dx} = 7^{\ln x} x^{\ln(x-1)} (\ln(7x) + \ln x)$

- [ ] $\frac{dy}{dx} = (x)^{\ln x}\left(\frac{\ln(7x)}{x} + \frac{\ln x}{x}\right)$

- [x] $\frac{dy}{dx} = (7x)^{\ln x}\left(\frac{\ln(7x)+\ln x}{x}\right)$

- [ ] $\frac{dy}{dx} = 7^{\ln x}x^{\ln x}\left(\ln(7x)+\ln x\right)$