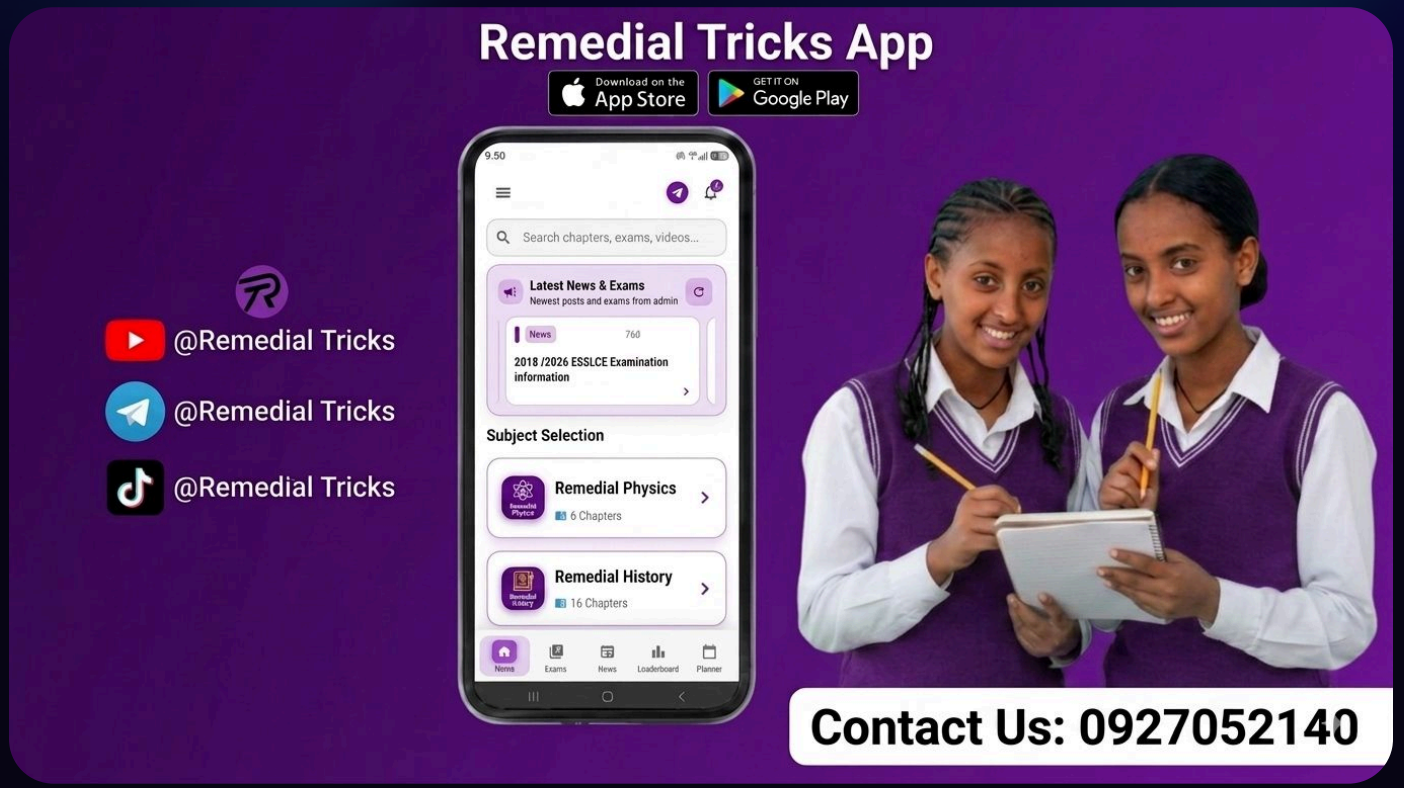
2019 Pre University Remedial Mathematics course content



Unit 1

Further on Sets and the Number System

1.1 Further on Sets

key Concepts

Definition of a Set

A **well-defined collection of distinct objects** called elements. Example: A = {1, 2, 3}

Notation Forms

Roster form: A = {1, 2, 3}
Set-builder form: A = {x | x ∈ ℕ, x < 4}

Set Operations

Union: A ∪ B = {x | x ∈ A or x ∈ B}
Intersection: A ∩ B = {x | x ∈ A and x ∈ B}
Complement: A' = {x | x ∉ A, x ∈ U}
Difference: A \ B = {x | x ∈ A and x ∉ B}

Venn Diagrams

Visual tool to represent set relationships and solve survey/application problems efficiently

Solved Examples

Example 1 — Set Operations:
Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {2, 4, 6, 8}, B = {4, 6, 9}

- A ∪ B = {2, 4, 6, 8, 9}
- A ∩ B = {4, 6}
- A' = {1, 3, 5, 7, 9, 10}
- A \ B = {2, 8}

Example 2 — Survey Problem (Venn Diagram):
In a class of **40 students**, 25 like Math, 20 like Science, and 10 like both.

Using the inclusion-exclusion principle:
$|M \cup S| = 25 + 20 - 10 = 35$
Students who like **neither** = 40 − 35 = **5**

Exam Tip: Always draw a Venn diagram for survey problems. Place the intersection value first, then fill outward.

1.2 The Number System

Number Categories

Natural Numbers ($\mathbb{N}$)

$\{1, 2, 3, \ldots\}$ — counting numbers; closed under **addition** and **multiplication**

Integers ($\mathbb{Z}$)

$\{\ldots, -2, -1, 0, 1, 2, \ldots\}$ — includes negatives; closed under **subtraction**

Rational Numbers ($\mathbb{Q}$)

Numbers of the form p/q where $p, q \in \mathbb{Z}$, $q \neq 0$; includes terminating and repeating decimals

Irrational Numbers

Cannot be expressed as p/q; non-terminating, non-repeating decimals — e.g., $\sqrt{2}$, π, e

Real Numbers ($\mathbb{R}$)

$\mathbb{R} = \mathbb{Q} \cup$ (Irrationals); every point on the number line is a real number

Key Properties: Closure, Commutativity, Associativity, Distributivity, Identity, Inverse

Hierarchy: $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

Number System Comparison

Set	Symbol	Example	Decimal Form
Natural	$\mathbb{N}$	5	5.000…
Integer	$\mathbb{Z}$	-3	$-3.000\ldots$
Rational	$\mathbb{Q}$	$3/4$	0.750…
Irrational	—	$\sqrt{2}$	1.41421…
Real	$\mathbb{R}$	π	3.14159…

Exam Tip: Every rational number is real, but not every real number is rational. On the exam, identify whether a number is rational or irrational by checking if its decimal terminates or repeats.

Unit 2

Solving Equations and Inequalities

2.1 Solving Linear Equations

Key Concepts

Linear Equation (1 variable)

$ax + b = 0 \rightarrow x = -b/a$; e.g., $3x - 9 = 0 \rightarrow x = 3$

Systems of Linear Equations (2 variables)

Solve by substitution, elimination, or graphing

Non-linear Equations

Quadratic $ax^2 + bx + c = 0$; solved by factoring, completing the square, or quadratic formula: $x = (-b \pm \sqrt{(b^2-4ac)}) / 2a$

Discriminant

$\Delta = b^2-4ac \rightarrow \Delta>0$: two real roots; $\Delta=0$: one repeated root; $\Delta<0$: no real roots

Applications

Distance-rate-time, mixture, and age problems

Solved Examples

Example 1 — Elimination Method:

Solve: $2x + y = 7$ and $x - y = 2$

- Step 1: Add equations $\rightarrow 3x = 9 \rightarrow x = 3$
- Step 2: Substitute $\rightarrow 3 - y = 2 \rightarrow y = 1$

Solution: (3, 1)

Example 2 — Quadratic Formula:

Solve: $x^2 - 5x + 6 = 0$

- $a=1, b=-5, c=6$; $\Delta = 25 - 24 = 1$
- $x = (5 \pm 1)/2 \rightarrow x = 3$ or $x = 2$

$x = 2$ or $x = 3$

Exam Tip: Always check your discriminant first. If $\Delta < 0$, there are no real solutions — don't waste time trying to factor.

2.2 Solving Linear Inequalities

Key Concepts

Linear Inequality (1 variable)

$ax + b < 0$; solution is an interval on the number line

Key Rule

Multiplying/dividing by a negative number **REVERSES** the inequality sign

Absolute Value Inequalities

$|x| < a \rightarrow -a < x < a$; $|x| > a \rightarrow x < -a$ or $x > a$

Quadratic Inequalities

$ax^2 + bx + c > 0$ — find roots, test intervals on number line

Systems of Inequalities (2 variables)

Graph each inequality; solution is the overlapping shaded region

Applications

Budget constraints, profit/loss thresholds, speed limits

Solved Examples

Example 1 — Linear Inequality:

Solve: $2x - 3 < 7$

- Step 1: $2x < 10$
- Step 2: $x < 5$

Solution: $(-\infty, 5)$

Example 2 — Absolute Value:

Solve: $|3x - 1| \leq 8$

- $-8 \leq 3x - 1 \leq 8$
- $-7 \leq 3x \leq 9$

$-7/3 \leq x \leq 3$

Example 3 — Quadratic Inequality:

Solve: $x^2 - x - 6 > 0$

- Roots: $x = 3$ and $x = -2$
- Test intervals → solution: $x < -2$ or $x > 3$

$(-\infty, -2) \cup (3, \infty)$

Exam Tip: For quadratic inequalities, always find the roots first, then test a value in each interval.

Unit 3 Relations and Functions

3.1 Relations & 3.2 Functions

key Concepts

Relation

A set of ordered pairs (x, y); Domain = set of all x-values; Range = set of all y-values

Function

A relation where each input (x) has exactly one output (y); vertical line test on graph

Function Operations

$(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, $(f/g)(x)$, $(f\circ g)(x) = f(g(x))$

Classifications

One-to-one (injective): distinct inputs → distinct outputs; Onto (surjective): every y has a preimage; Bijective: both

Inverse Function

f^{-1} exists iff f is one-to-one; swap x and y, then solve for y

Graphs

f and f^{-1} are reflections across the line $y = x$

Solved Examples

Example 1 — Finding Inverse:

$f(x) = 2x + 3$

- Step 1: Write $y = 2x + 3$
- Step 2: Swap x and y → $x = 2y + 3$
- Step 3: Solve for y → $y = (x - 3)/2$

$f^{-1}(x) = (x - 3)/2$

Example 2 — Composition:

$f(x) = x^2$ and $g(x) = x + 1$

- $(f\circ g)(x) = f(g(x)) = f(x+1) = (x+1)^2$

$(f\circ g)(x) = x^2 + 2x + 1$

Exam Tip: To find f^{-1}, always swap x and y first, then solve. Verify by checking $f(f^{-1}(x)) = x$.

3.3 Polynomial & Rational Functions

Key Concepts

Polynomial Function

$P(x) = a_n x^n + \ldots + a_1 x + a_0$; degree n determines shape and end behavior

Remainder Theorem

$P(a)$ = remainder when $P(x)$ is divided by $(x - a)$

Factor Theorem

$(x - a)$ is a factor of $P(x)$ iff $P(a) = 0$

Zeros

Values of x where $P(x) = 0$; found by factoring or synthetic division

Rational Function

$R(x) = P(x)/Q(x)$; undefined where $Q(x) = 0$ (vertical asymptotes)

Horizontal Asymptote

Compare degrees of P and Q: if $\deg(P) < \deg(Q) \rightarrow y = 0$; if equal $\rightarrow$ y = leading coefficient ratio

Solved Examples

Example 1 — Factor Theorem:

$P(x) = x^3 - 6x^2 + 11x - 6$

- Test $P(1)$: $1 - 6 + 11 - 6 = 0$ ✓ $\rightarrow (x-1)$ is a factor
- Divide: $P(x) = (x-1)(x^2-5x+6) = (x-1)(x-2)(x-3)$

Zeros: $x = 1, 2, 3$

Example 2 — Rational Function Asymptotes:

$R(x) = (2x^2 + 1)/(x^2 - 4)$

- Vertical asymptotes: $x = \pm 2$ (where denominator = 0)
- Horizontal asymptote: degrees equal $\rightarrow y = 2/1 = 2$

VA: $x = \pm 2$; HA: $y = 2$

Exam Tip: Use synthetic division to quickly test whether a value is a zero of a polynomial.

3.3 Exponential & Logarithmic Functions

Key Concepts

Exponential Function

$f(x) = a^x$, $a > 0$, $a \neq 1$; domain $\mathbb{R}$, range $(0, \infty)$; passes through $(0,1)$

Logarithmic Function

$f(x) = \log_a(x)$; inverse of a^x; domain $(0,\infty)$, range $\mathbb{R}$

Key Log Laws

$\log_a(mn) = \log_a m + \log_a n$;
$\log_a(m/n) = \log_a m - \log_a n$;
$\log_a(m^n) = n \cdot \log_a m$

Change of Base

$\log_a b = \log b / \log a$

Applications

Population growth $P(t) = P_0 \cdot e^{rt}$; compound interest $A = P(1 + r/n)^{nt}$; radioactive decay

Solved Examples

Example 1 — Solving Exponential Equation:
Solve: $2^x = 32$

- $2^x = 2^5 \rightarrow x = 5$

  $x = 5$

Example 2 — Solving Logarithmic Equation:
Solve: $\log_2(x) = 3$

- $x = 2^3 = 8$

  $x = 8$

Example 3 — Using Log Laws:
Simplify: $\log_3(27) + \log_3(3)$

- $= \log_3(27 \times 3) = \log_3(81) = \log_3(3^4) = 4$

  Answer = 4

Exam Tip: Remember $\log_a(a^n) = n$ and $a^{\log_a x} = x$ — these shortcuts save time on the exam.

3.3 Trigonometric & Special Functions

Key Concepts

Trig Ratios

sin θ = opp/hyp; cos θ = adj/hyp; tan θ = opp/adj; reciprocals: csc, sec, cot

Radian Measure

$180^\circ = \pi$ radians; to convert: multiply degrees × π/180

Pythagorean Identity

$\sin^2\theta + \cos^2\theta = 1$; also: $1 + \tan^2\theta = \sec^2\theta$; $1 + \cot^2\theta = \csc^2\theta$

Power Function

$f(x) = x^n$; even n → symmetric about y-axis; odd n → rotational symmetry about origin

Absolute Value Function

$f(x) = |x|$; V-shaped graph; $|x| \geq 0$ for all x

Signum Function

$\text{sgn}(x) = -1$ if $x<0$; 0 if $x=0$; 1 if $x>0$

Greatest Integer (Floor) Function

$\lfloor x \rfloor$ = largest integer ≤ x; e.g., $\lfloor 3.7 \rfloor = 3$, $\lfloor -1.2 \rfloor = -2$; step graph

Solved Examples

Example 1 — Trig Ratios:
In a right triangle: opposite = 3, adjacent = 4, hypotenuse = 5

- sin θ = 3/5 = 0.6; cos θ = 4/5 = 0.8; tan θ = 3/4 = 0.75

$\sin^2\theta + \cos^2\theta = 0.36 + 0.64 = 1$ ✓

Example 2 — Degree to Radian:
Convert 60° to radians: 60 × π/180 = π/3

$60^\circ = \pi/3$ radians

Example 3 — Floor Function:
$\lfloor 4.9 \rfloor = 4$; $\lfloor -2.3 \rfloor = -3$; $\lfloor 7 \rfloor = 7$

Floor always rounds DOWN toward −∞

Exam Tip: Memorize sin, cos, tan for 30°, 45°, 60°. These standard angles appear in almost every exam.

Unit 4 Geometry
4.1 Congruency & Similarity

Key Concepts

Congruent Triangles

Same shape and size; criteria: SSS, SAS, ASA, AAS, RHS

Similar Figures

Same shape, proportional sides; symbol ~; AA, SAS, SSS similarity

Scale Factor (k)

Ratio of corresponding sides of two similar figures

Ratio of Perimeters

If scale factor = k, then ratio of perimeters = k

Ratio of Areas

If scale factor = k, then ratio of areas = k^2

Proportionality Theorem

A line parallel to one side of a triangle divides the other two sides proportionally

Applications

Map scales, architectural models, shadow problems

Solved Examples

Example 1 — Similar Triangles:
Two similar triangles with corresponding sides 3 cm and 6 cm

- Scale factor k = 6/3 = 2
- Ratio of perimeters = 2:1
- Ratio of areas = 4:1
- If small triangle area = 9 cm^2 → large triangle area = 36 cm^2

Area ratio = $k^2 = 4$; Large area = 36 cm^2

Example 2 — Shadow Problem:
A 2 m pole casts a 3 m shadow. A tree casts a 12 m shadow.

- Using similar triangles: tree height/2 = 12/3

Tree height = 8 m

Exam Tip: Always state the similarity criterion (AA, SAS, SSS) when proving triangles similar. Marks are awarded for justification.

4.2 Regular Polygons & 4.3 Circles

Key Concepts

Sum of Interior Angles

$(n-2) \times 180°$; for hexagon: $(6-2) \times 180° = 720°$

Each Interior Angle (regular)

$(n-2) \times 180° / n$

Each Exterior Angle

$360° / n$; sum of exterior angles always $= 360°$

Circle — Arc Length

$L = r\theta$ (θ in radians); $L = (\theta/360°) \times 2\pi r$

Sector Area

$A = \frac{1}{2}r^2\theta = (\theta/360°) \times \pi r^2$

Angle in Semicircle

Always 90° (Thales' theorem)

Inscribed Angle Theorem

Inscribed angle = ½ × central angle subtending same arc

Tangent-Chord Angle

Equals the inscribed angle on the opposite side

Regular Polygon Properties

Pentagon	5	108°	72°
Hexagon	6	120°	60°
Octagon	8	135°	45°
Decagon	10	144°	36°

Solved Examples

Example 1 — Arc Length:
Circle with r = 6 cm, central angle $\theta = 60°$

- Convert: $60° = \pi/3$ radians
- $L = r\theta = 6 \times \pi/3 = 2\pi \approx 6.28$ cm

Arc Length = 2π cm

Example 2 — Sector Area:
Same circle: r = 6 cm, $\theta = 60°$

- $A = (60/360) \times \pi(6^2) = (1/6) \times 36\pi = 6\pi \approx 18.85$ cm^2

Sector Area = 6π cm^2

Example 3 — Inscribed Angle:
Central angle = 80° → Inscribed angle = 40°

Inscribed angle = ½ × central angle

Exam Tip: For circle problems, always convert degrees to radians when using $L = r\theta$ or $A = \frac{1}{2}r^2\theta$.

4.4 Solid Figures & 4.5 Coordinate Geometry

Key Concepts

Cylinder

$V = \pi r^2 h$; $SA = 2\pi r^2 + 2\pi rh$

Cone

$V = \frac{1}{3}\pi r^2 h$; $SA = \pi r^2 + \pi rl$ (l = slant height = $\sqrt{(r^2+h^2)}$)

Sphere

$V = (4/3)\pi r^3$; $SA = 4\pi r^2$

Frustum (Cone)

$V = (\pi h/3)(R^2 + Rr + r^2)$

Coordinate Geometry

Distance Formula

$d = \sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$

Midpoint Formula

$M = ((x_1+x_2)/2, (y_1+y_2)/2)$

Slope

$m = (y_2-y_1)/(x_2-x_1)$

Equation of Line

$y = mx + b$ (slope-intercept); $ax + by + c = 0$ (general form)

Parallel Lines

$m_1 = m_2$

Perpendicular Lines

$m_1 \times m_2 = -1$

Solved Examples

Example 1 — Distance Formula:

Find distance between A(1, 2) and B(4, 6)

- $d = \sqrt{[(4-1)^2 + (6-2)^2]} = \sqrt{[9 + 16]} = \sqrt{25} = 5$

$d = 5$ units

Example 2 — Volume of Cone:

$r = 3$ cm, $h = 4$ cm, slant height $l = \sqrt{(9+16)} = 5$ cm

- $V = \frac{1}{3}\pi(3^2)(4) = 12\pi \approx 37.7\ cm^3$
- $SA = \pi(3^2) + \pi(3)(5) = 9\pi + 15\pi = 24\pi \approx 75.4\ cm^2$

$V = 12\pi\ cm^3$; $SA = 24\pi\ cm^2$

Example 3 — Perpendicular Lines:

Line 1 has slope $m_1 = 2$. Find slope of perpendicular line.

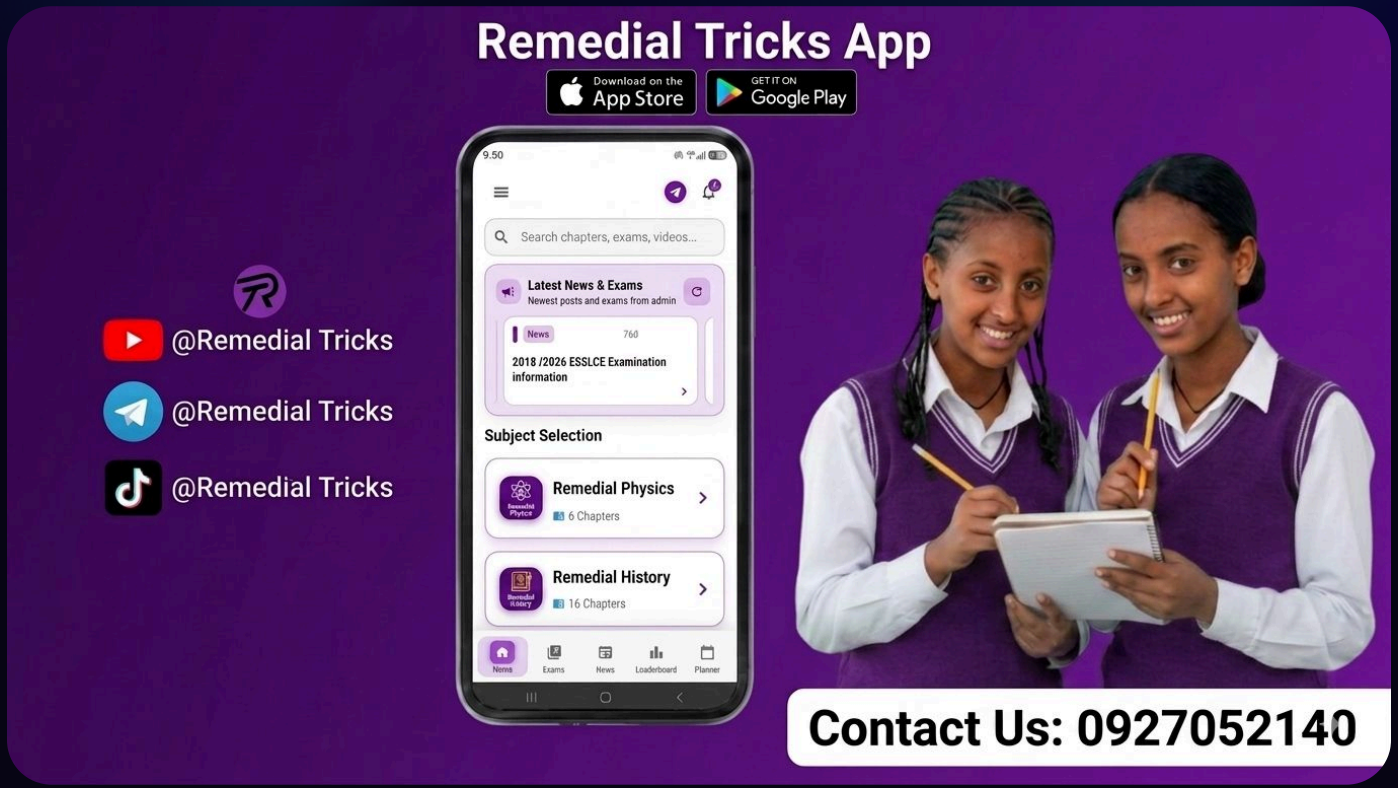
- $m_2 = -1/m_1 = -1/2$

$m_2 = -\frac{1}{2}$

Exam Tip: For coordinate geometry, always label your points clearly and show the substitution step — partial marks are given for correct setup.

🏁 FIRST ROUND MODEL

Chapter 1–5



Unit 5
Vectors and Transformations of the Plane
5.1 Vectors in Two Dimensions

Key Concepts

Scalar vs. Vector

Scalar has magnitude only (e.g., speed); Vector has magnitude and direction (e.g., velocity)

Representation

Vector $\overrightarrow{AB}$ or bold v; component form: $v = (v_x, v_y)$

Vector Operations

Addition: $u+v = (u_1+v_1, u_2+v_2)$; Scalar mult: $kv = (kv_1, kv_2)$

Magnitude

$|v| = \sqrt{(v_x^2 + v_y^2)}$

Dot (Scalar) Product

$u \cdot v = u_1v_1 + u_2v_2 = |u||v|\cos\theta$; if $u \cdot v = 0 \rightarrow$ perpendicular

Cross Product (2D)

$|u \times v| = |u_1v_2 - u_2v_1|$; gives area of parallelogram formed by u and v

Unit Vector

$\hat{u} = v/|v|$; has magnitude 1

Solved Examples

Example 1 — Vector Operations:
$u = (3, 4)$, $v = (1, 2)$

- $u + v = (4, 6)$
- $u \cdot v = 3(1) + 4(2) = 3 + 8 = 11$
- $|u| = \sqrt{(9 + 16)} = \sqrt{25} = 5$
- $|v| = \sqrt{(1 + 4)} = \sqrt{5}$

$u \cdot v = 11$; $|u| = 5$

Example 2 — Angle Between Vectors:
$u = (1, 0)$, $v = (0, 1)$

- $u \cdot v = 0 \rightarrow$ vectors are perpendicular

$\theta = 90^\circ$ (perpendicular)

Example 3 — Area of Parallelogram:
$u = (3, 1)$, $v = (1, 4)$

- $|u \times v| = |3(4) - 1(1)| = |12 - 1| = 11$

Area = 11 sq. units

Exam Tip: If the dot product of two vectors is zero, they are perpendicular. This is a common exam question.

5.2 Transformations of the Plane

Key Concepts

Translation

Shifts every point by vector (a,b); $T(x,y) = (x+a, y+b)$; preserves shape and size

Reflection

Flip across a line; across x-axis: $(x,y) \to (x,-y)$; across y-axis: $(x,y) \to (-x,y)$; across $y=x$: $(x,y) \to (y,x)$

Rotation

Rotate by angle θ about origin: $(x,y) \to (x\cos\theta - y\sin\theta,\ x\sin\theta + y\cos\theta)$

Rotation 90° CCW

$(x,y) \to (-y, x)$; 180°: $(x,y) \to (-x,-y)$; 270° CCW: $(x,y) \to (y, -x)$

Isometries

Translation, reflection, and rotation all preserve distances (congruence)

Dilation

Enlarges/shrinks by scale factor k from a center point; NOT an isometry (changes size)

Solved Examples

Example 1 — Reflection:
Reflect point $P(3, 5)$ across the y-axis

- Rule: $(x, y) \to (-x, y)$

$P' = (-3, 5)$

Example 2 — Rotation 90° CCW:
Rotate $Q(1, 0)$ by 90° counterclockwise about origin

- Rule: $(x, y) \to (-y, x)$

$Q' = (0, 1)$

Example 3 — Translation:
Translate $R(2, 3)$ by vector $(4, -1)$

- $T(2, 3) = (2+4, 3+(-1)) = (6, 2)$

$R' = (6, 2)$

Exam Tip: Memorize the four rotation rules (90°, 180°, 270°, 360°) and the three reflection rules (x-axis, y-axis, y=x). These are tested directly.

Unit 6

Matrices and Determinants

6.1 Matrices & 6.2 Determinants and Cramer's Rule

Key Concepts

Matrix

Rectangular array of numbers; m×n matrix has m rows and n columns

Operations

Addition/Subtraction (same order); Scalar multiplication; Matrix multiplication (cols of A = rows of B)

Special Types

Zero matrix, Identity matrix (I), Diagonal, Symmetric, Transpose (A^T)

Inverse of A (2×2)

$A^{-1} = (1/\det A) \times [d, -b; -c, a]$ where $A = [a,b;c,d]$; exists only if $\det A \neq 0$

Solving Systems

$AX = B \rightarrow X = A^{-1}B$

det(2×2)

$|A| = ad - bc$ for $A = [a,b;c,d]$

det(3×3)

Expand along first row using cofactors: $|A| = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$

Cramer's Rule

For AX=B: $x = \det(A_x)/\det(A)$; $y = \det(A_y)/\det(A)$

Solved Examples

Example 1 — Matrix Inverse & System:

$A = [2, 1; 5, 3]$; $\det A = 2(3) - 1(5) = 1$

- $A^{-1} = (1/1)[3, -1; -5, 2] = [3, -1; -5, 2]$
- Solve $2x+y=4$, $5x+3y=7$:
- $X = A^{-1}[4; 7] = [3(4)+(-1)(7); -5(4)+2(7)] = [5; -6]$

$x = 5, y = -6$

Example 2 — Cramer's Rule:

Solve: $2x + y = 5$, $x - y = 1$

- $\det A = |2,1;1,-1| = -2-1 = -3$
- $\det A_x = |5,1;1,-1| = -5-1 = -6 \rightarrow x = -6/-3 = 2$
- $\det A_y = |2,5;1,1| = 2-5 = -3 \rightarrow y = -3/-3 = 1$

$x = 2, y = 1$

Exam Tip: Always compute det(A) first. If det(A) = 0, the system has no unique solution — state this clearly.

Unit 7 : Statistics and Probability
7.1 & 7.2 Statistics

Key Concepts

Types of Data

Qualitative (categorical) vs. Quantitative (discrete/continuous)

Grouped Data

Organize into class intervals; find class width, midpoint, frequency

Mean

$\bar{x} = \Sigma fx/\Sigma f$ (grouped); $\Sigma x/n$ (ungrouped)

Median

$L + [(n/2 - F)/f] \times h$ (grouped); Middle value (ungrouped)

Mode

Most frequent value; modal class for grouped data

Variance & Standard Deviation

$\sigma^2 = \Sigma f(x-\bar{x})^2/n$; $\sigma = \sqrt{(\text{Variance})}$

Coefficient of Variation

$CV = (\sigma/\bar{x}) \times 100\%$; compares variability across datasets

Graphical Representations

Histogram, frequency polygon, ogive (cumulative frequency curve)

Measures of Central Tendency

Measure	Ungrouped Data	Grouped Data
Mean	$\Sigma x/n$	$\Sigma fx/\Sigma f$
Median	Middle value	$L + [(n/2 - F)/f] \times h$
Mode	Most frequent	Modal class midpoint

Solved Example — Grouped Data

Data: Scores of 20 students in groups:

- [40–50]: f=3, midpoint=45
- [50–60]: f=7, midpoint=55
- [60–70]: f=6, midpoint=65
- [70–80]: f=4, midpoint=75

Step 1: $\Sigma f = 20$

Step 2: $\Sigma fx = 3(45)+7(55)+6(65)+4(75) = 135+385+390+300 = 1210$

Step 3: Mean = 1210/20 = 60.5

Mean = 60.5

Step 4: Median class: $n/2 = 10 \rightarrow$ falls in [50–60]

L=50, F=3, f=7, h=10

Median = $50 + [(10-3)/7] \times 10 = 50 + 10 = 60$

Median ≈ 60

Exam Tip: For grouped data, always identify the median class first by finding the class that contains the n/2-th value.

7.3 Probability

Key Concepts

Fundamental Counting Principle

If event A has m ways and B has n ways → m × n total outcomes

Permutations

$P(n,r) = n!/(n-r)!$ — ordered arrangements; e.g., $P(5,2) = 20$

Combinations

$C(n,r) = n!/[r!(n-r)!]$ — unordered selections; e.g., $C(5,2) = 10$

Binomial Theorem

$(a+b)^n = \Sigma\, C(n,k)\, a^{n-k}\, b^k$; general term: $T(k+1) = C(n,k)a^{n-k}b^k$

Probability of Event E

$P(E) = n(E)/n(S)$; $0 \le P(E) \le 1$

Addition Rule

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$; for mutually exclusive: $P(A \cup B) = P(A)+P(B)$

Multiplication Rule

$P(A \cap B) = P(A)\cdot P(B|A)$; if independent: $P(A \cap B) = P(A)\cdot P(B)$

Complementary Events

$P(A') = 1 - P(A)$

Solved Examples

Example 1 — Basic Probability:
A bag has 3 red and 5 blue balls. One ball drawn at random.

- $P(\text{red}) = 3/8$; $P(\text{blue}) = 5/8$

$P(\text{red}) + P(\text{blue}) = 1$ ✓

Example 2 — Combinations:
How many ways to choose 3 students from a class of 10?

- $C(10,3) = 10!/(3!\times 7!) = (10\times 9\times 8)/(3\times 2\times 1) = 120$

120 ways

Example 3 — Binomial Theorem:
Find the 3rd term of $(x + 2)^5$

- $T(3) = C(5,2)\cdot x^3\cdot 2^2 = 10\cdot x^3\cdot 4 = 40x^3$

3rd term = $40x^3$

Exam Tip: Use combinations (C) when order doesn't matter (selecting a team), permutations (P) when order matters (arranging in a line).

Unit 8: Sequences and Series

Key Concepts

Sequence

An ordered list of numbers following a pattern; e.g., 2, 5, 8, 11, ... (arithmetic)

Arithmetic Sequence

$a_n = a_1 + (n-1)d$; common difference d; e.g., $d = 3, a_1 = 2 \rightarrow a_5 = 14$

Geometric Sequence

$a_n = a_1 \cdot r^{n-1}$; common ratio r; e.g., $r = 2, a_1 = 3 \rightarrow a_4 = 24$

Sigma Notation

$\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$

Arithmetic Series (Partial Sum)

$S_n = n/2 \cdot (a_1 + a_n) = n/2 \cdot [2a_1 + (n-1)d]$

Geometric Series (Partial Sum)

$S_n = a_1(1 - r^n)/(1 - r), r \neq 1$

Infinite Geometric Series

$S_\infty = a_1/(1 - r)$ if $|r| < 1$; diverges if $|r| \geq 1$

Applications

Loan repayments, population growth, savings plans, bouncing ball problems

Solved Examples

Example 1 – Arithmetic Sequence:

Sequence: 3, 7, 11, 15, ... $(a_1 = 3, d = 4)$

Find the 10th term: $a_{10} = 3 + (10 - 1)(4) = 3 + 36 = 39$

Sum of first 10 terms:

$S_{10} = 10/2 \times (3 + 39) = 5 \times 42 = 210$

$a_{10} = 39$; $S_{10} = 210$

Example 2 – Geometric Sequence:

Sequence: 2, 6, 18, 54, ... $(a_1 = 2, r = 3)$

Find the 5th term: $a_5 = 2 \times 3^4 = 2 \times 81 = 162$

$a_5 = 162$

Example 3 – Infinite Geometric Series:

S_∞ for $a_1 = 4, r = 1/2 (|r| < 1)$

$S_\infty = 4/(1 - 1/2) = 4/(1/2) = 8$

$S_\infty = 8$

Exam Tip: Always check $|r| < 1$ before applying the infinite geometric series formula. If $|r| \geq 1$, the series diverges.

Unit 9 : Introduction to Calculus
9.1 Limits & Continuity

Key Concepts

Limit

$\lim_{x \to a} f(x) = L$ means $f(x)$ approaches L as x approaches a (but $x \neq a$)

Limit Laws

Sum, difference, product, quotient, and power rules for limits

Indeterminate Forms

$0/0$ or ∞/∞ — use factoring, rationalization, or L'Hôpital's rule

One-Sided Limits

$\lim_{x \to a^+}$ and $\lim_{x \to a^-}$; limit exists iff both one-sided limits are equal

Continuity

f is continuous at $x = a$ if: (1) $f(a)$ is defined; (2) $\lim f(x)$ exists; (3) $\lim f(x) = f(a)$

Types of Discontinuity

Removable (hole), Jump (one-sided limits differ), Infinite (vertical asymptote)

Solved Examples

Example 1 — Evaluating a Limit (Factoring):

$\lim_{x \to 2}(x^2 - 4)/(x - 2)$

Factor: $(x^2 - 4) = (x + 2)(x - 2)$

Cancel: $\lim_{x \to 2}(x + 2) = 4$

lim = 4

Example 2 — Checking Continuity:

$f(x) = (x^2 - 1)/(x - 1)$ at $x = 1$

$f(1)$ is undefined $\rightarrow$ discontinuity at $x = 1$

But $\lim_{x \to 1}(x + 1) = 2 \rightarrow$ removable discontinuity

Removable discontinuity at x=1 (hole)

Example 3 — Limit at Infinity:

$\lim_{x \to \infty}(3x^2 + 1)/(x^2 - 2) = 3/1 = 3$

lim = 3 (ratio of leading coefficients)

Exam Tip: For $0/0$ forms, always try factoring first. If that fails, try rationalization or L'Hôpital's rule.

9.2 Derivatives

Key Concepts

Definition

$f'(x) = \lim_{h\to 0}[f(x+h) - f(x)]/h$

; geometric meaning: slope of tangent line at x

Power Rule

$d/dx[x^n] = nx^{n-1}$;
$d/dx[c] = 0$; $d/dx[cf] = cf'$

Sum/Difference Rule

$(f \pm g)' = f' \pm g'$

Product Rule

$(fg)' = f'g + fg'$

Quotient Rule

$(f/g)' = (f'g - fg')/g^2$

Chain Rule

$d/dx[f(g(x))] = f'(g(x)) \cdot g'(x)$

Special Derivatives

$d/dx[e^x] = e^x$;
$d/dx[\ln x] = 1/x$;
$d/dx[\sin x] = \cos x$;
$d/dx[\cos x] = -\sin x$

Extreme Values

Set $f'(x) = 0$ to find critical points; use $f''(x)$ to classify: $f'' > 0 \to$ local min; $f'' < 0 \to$ local max

Solved Examples

Example 1 — Power Rule:

$f(x) = 3x^4 - 5x^2 + 2x - 7$

$f'(x) = 12x^3 - 10x + 2$

f'(x) = 12x³ − 10x + 2

Example 2 — Chain Rule:

$f(x) = (2x+1)^5$

Let $u = 2x+1$; $f = u^5$

$f'(x) = 5u^4 \cdot 2 = 10(2x+1)^4$

f'(x) = 10(2x+1)⁴

Example 3 — Critical Points & Max/Min:

$f(x) = x^3 - 3x$

$f'(x) = 3x^2 - 3 = 0 \to x = \pm 1$

$f''(x) = 6x$

$f''(1) = 6 > 0 \to$ local minimum at $x = 1$

$f''(-1) = -6 < 0 \to$ local maximum at $x = -1$

Local max at x=−1; local min at x=1

Exam Tip: Always find $f'(x) = 0$ first for critical points, then use the second derivative test to classify them as max or min.

9.3 Integration

Key Concepts

Antiderivative

$\int f(x)dx = F(x) + C$ where $F'(x) = f(x)$; C is the constant of integration

Power Rule

$\int x^n dx = x^{n+1}/(n+1) + C$ ($n \neq -1$); $\int dx = x + C$; $\int e^x dx = e^x + C$

Special Integrals

$\int (1/x)dx = \ln|x| + C$;
$\int \sin x dx = -\cos x + C$;
$\int \cos x dx = \sin x + C$

Substitution

Let $u = g(x)$;
$\int f(g(x))g'(x)dx = \int f(u)du$

Integration by Parts

$\int udv = uv - \int vdu$

Definite Integral

$\int_a^b f(x)dx = F(b) - F(a)$ (Fundamental Theorem of Calculus)

Area Under Curve

$A = \int_a^b f(x)dx$; Area between curves:
$A = \int_a^b [f(x) - g(x)]dx$

Solved Examples

Example 1 — Power Rule Integration:
$\int (3x^2 + 2x - 5)dx$
$= x^3 + x^2 - 5x + C$

$x^3 + x^2 - 5x + C$

Example 2 — Definite Integral (FTC):
$\int_0^2 x^2 dx = [x^3/3]_0^2 = 8/3 - 0 = 8/3$

$\int_0^2 x^2\,dx = 8/3 \approx 2.67$ sq. units

Example 3 — Substitution:
$\int 2x(x^2+1)^4 dx$; let $u = x^2 + 1$, $du = 2xdx$
$= \int u^4 du = u^5/5 + C = (x^2+1)^5/5 + C$

$(x^2+1)^5/5 + C$

Exam Tip: For definite integrals, always evaluate $F(b) - F(a)$ carefully. A sign error in the final step is the most common mistake.

Unit 10: Introduction to Linear Programming

Key Concepts

Definition

Optimize (maximize or minimize) a linear objective function subject to linear constraints (inequalities)

Feasible Region

The set of all points satisfying all constraints; found by graphing the system of inequalities

Corner Point Theorem

The optimal value of the objective function occurs at a vertex (corner) of the feasible region

Steps to Solve

(1) Define variables; (2) Write objective function; (3) Write constraints; (4) Graph feasible region; (5) Evaluate objective at each vertex; (6) State the optimal solution

Bounded vs. Unbounded

Bounded region has a finite area; unbounded may have no maximum (or no minimum)

Applications

Production planning, resource allocation, diet optimization, transportation problems

Solved Example — Maximization

Maximize $P = 3x + 2y$

Subject to:

$x + y \leq 4$

$x \geq 0$

$y \geq 0$

Step 1: Graph constraints $\rightarrow$ feasible region is triangle with vertices (0,0), (4,0), (0,4)

Step 2: Evaluate P at each vertex:

- $P(0, 0) = 0$

- $P(4, 0) = 12$

- $P(0, 4) = 8$

Maximum P = 12 at (4, 0)

Second Example — Minimization:

Minimize $C = 2x + 3y$ subject to $x + y \geq 3$, $x \geq 1$, $y \geq 1$

Vertices: (1,2), (2,1)

Evaluate C at each vertex:

- $C(1, 2) = 2(1) + 3(2) = 2 + 6 = 8$

- $C(2, 1) = 2(2) + 3(1) = 4 + 3 = 7$

Minimum C = 7 at (2, 1)

Exam Tip: Always list ALL corner points and evaluate the objective function at each one. Never assume the maximum is at the largest x or y value.

Unit 11 : Mathematical Applications in Business
Business Mathematics

Key Concepts

Simple Interest

$I = PRT$; $A = P(1 + RT)$ where P=principal, R=annual rate, T=time in years

Compound Interest

$A = P(1 + r/n)^{nt}$; more frequent compounding → higher return; continuous: $A = Pe^{rt}$

Time Value of Money

A birr today is worth more than a birr in the future due to earning potential

Present Value

$PV = FV/(1 + r)^t$ – how much to invest today to reach a future amount

Annuity (Future Value)

$FV = PMT \times [(1 + r)^n - 1]/r$; regular equal payments over time

Taxation

Income tax brackets; VAT = selling price × tax rate; Net income = Gross – Tax

Applications

Personal budgeting, business profit/loss, loan repayment planning

Solved Examples

Example 1 — Simple Interest:

P = 5,000 \text{ ETB, R = 10%, T = 3 years}

$I = 5,000 \times 0.10 \times 3 = 1,500 \text{ ETB}$

$A = 5,000 + 1,500 = 6,500 \text{ ETB}$

Total Amount = 6,500 ETB

Example 2 — Compound Interest:

Invest $10,000 \text{ ETB}$ at 8% compounded annually for 3 years

$A = 10,000(1 + 0.08)^3 = 10,000 \times 1.2597 = 12,597.12 \text{ ETB}$

A = 12,597.12 ETB

Example 3 — Present Value:

How much to invest today at 10% to have $15,000 \text{ ETB}$ in 2 years?

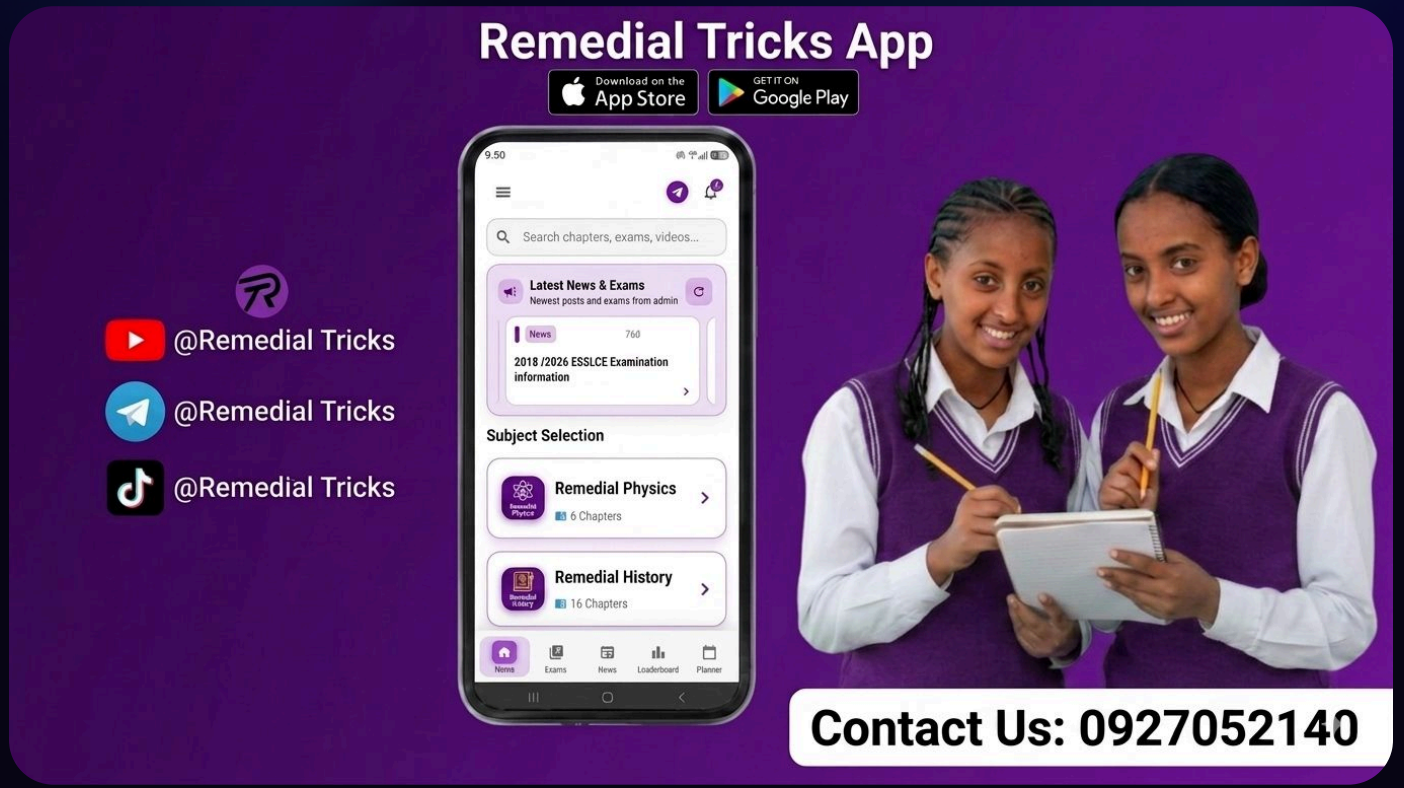
$PV = 15,000/(1.10)^2 = 15,000/1.21 = 12,396.69 \text{ ETB}$

PV ≈ 12,396.69 ETB

Exam Tip: Always identify whether the problem asks for Simple or Compound interest. Compound interest grows faster — the difference becomes significant over many years.

Second Round Model

CHAPTER(6-11)



📋 Revision Course

U1 Sets & Numbers

$A \cup B$, $A \cap B$, A'; $|A \cup B| = |A| + |B| - |A \cap B|$; $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$; rational vs. irrational

U2 Equations

$x = (-b \pm \sqrt{(b^2 - 4ac)})/2a$; $\Delta = b^2 - 4ac$; flip inequality sign when ÷ by negative

U3 Functions

Vertical line test; f^{-1} swap x,y; $\log_a(mn) = \log_a m + \log_a n$; $\sin^2\theta + \cos^2\theta = 1$; $\lfloor x \rfloor$ floor function

U4 Geometry

Interior angle $= (n-2) \times 180°/n$; $L = r\theta$; $d = \sqrt{[(\Delta x)^2 + (\Delta y)^2]}$; $V(\text{cone}) = \frac{1}{3}\pi r^2 h$; area ratio $= k^2$

U5 Vectors

$|v| = \sqrt{(v_x^2 + v_y^2)}$; $u \cdot v = |u||v|\cos\theta$; $u \cdot v = 0 \rightarrow$ perpendicular; rotation/reflection rules

U6 Matrices

$\det(2\times2) = ad - bc$; $A^{-1} = (1/\det)\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$; Cramer's Rule; Gaussian elimination

U7 Stats & Prob

Mean $= \Sigma fx / \Sigma f$; $\sigma = \sqrt{\text{Variance}}$; CV $= (\sigma / \bar{x}) \times 100\%$; $P(E) = n(E)/n(S)$; $C(n,r) = n!/[r!(n-r)!]$

U8 Sequences

$a_n = a_1 + (n-1)d$ (arith.); $a_n = a_1 r^{n-1}$ (geom.); S_n formulas; $S_\infty = a_1/(1-r)$ for $|r| < 1$

U9 Calculus

Limits (factoring for 0/0); $f'(x^n) = nx^{n-1}$; chain rule; $\int x^n dx = x^{n+1}/(n+1) + C$; FTC

U10 Linear Prog.

Feasible region; corner point theorem; maximize/minimize at vertices

U11 Business

$I = PRT$; $A = P(1 + r/n)^{nt}$; $PV = FV/(1+r)^t$; VAT; annuity formula

🏆 THIRD ROUND MODEL

CHAPTER(1-11)

